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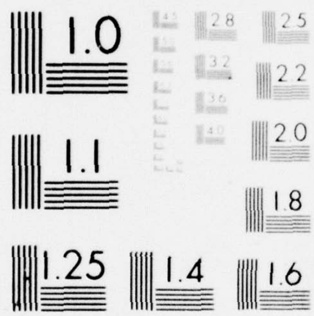

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UNIVERSITY OF SOUTHERN CALIFORNIA

DECISION MAKING WITH THE AID OF A SUBJECTIVE PRIOR DISTRIBUTION

by

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Decision Making with the Aid of a Subjective Prior Distribution

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Abstract

This paper discusses a methodology to establish a subjective prior distribution via an interactive computer program on a CRT. The program asks specific, carefully worded questions. The answers to these questions are checked for logical consistency and are translated into fractiles of a subjective probability distribution. The fractiles as well as other summary measures (like the mean and variance) are printed out. In order to find a conjugate prior distribution for a Bayesian treatment, the mathematical form of a possible conjugate prior needs to be ascertained. The paper illustrates how the parameters of a lognormal distribution might be obtained from the fractiles of the subjectively derived prior.

1. INTRODUCTION

Every firm experiences, once in a while, such a heavy demand for some of its products or services that leads to a complete sellout. When sellout occurs, the firm forgoes potential profit and suffers possible loss of good will. Examples of this can be seen in supermarkets, in TV-repair shops, and at airline counters when customers have to be turned away. Usually one only hears "Sorry, we cannot accommodate your wishes."

Almost invariably, no one keeps track of unsatisfied customers demand. No one, however, denies the importance of knowing the unsatisfied demand. If for no other reason, maybe the manager in charge can do a better job in scheduling available capacity for the future. Without the data on unsatisfied demand, the manager must estimate future demand from incomplete data--sales record only. In other words, the distribution of future demand has to be established from truncated frequency distribution of demand.

Modern statistical decision analysis can help the manager establish a prior distribution from incomplete prior data or no prior data at all. If a sample is taken, then with the aid of Bayes' Theorem the prior distribution can be combined with the likelihood function which reflects the sample information to obtain the posterior distribution. The demand of a certain product or service can be estimated from either the posterior distribution or the prior distribution depending upon whether or not the sample data are available.

One difficulty associated with the assessment of a prior distribution is the assessor's inconsistencies which often occur in responding to a questionnaire. The question of how to discover and remove inconsistencies is of general interest to decision analysts. Another question of interest is how to fit a probability distribution through the assessed fractiles in order to make the subsequent analysis more tractable. Both of these questions are addressed in this paper. The paper offers two computer programs which allow a person to interact with the computer via a graphical device (CRT) during his course of assessment and fit a theoretical probability distribution through the assessed points.

2. PREVIOUS WORK

In the past the researchers were particularly concerned with the methodology of Bayesian analysis, the question was how Bayes' theorem can be used to start with a prior distribution and using experimental data to derive the posterior distribution (see, for example, [5], [14] and [16]). Usually the researchers started with an assumed distribution, say the Bernoulli, Poisson, or normal and then using the Bayesian methodology to derive a particular posterior distribution [2]. During recent years, however, considerable attention has been given to methods for assessment of a prior distribution. Subjective probability has been studied by researchers in various disciplines such as psychology, mathematics, statistics, engineering, and business administration (see reference at the end of the paper). While some of these studies are mainly theoretical or philosophical, others are experimental.

In their text [14], Pratt, Raiffa, and Schlaifer present the method of equally likely subintervals. Subsequently, Raiffa [15] illustrates this method in detail by providing a dialogue between a decision analyst and his client. Schlaifer [19] advocates this method and offers a computer program for fitting a cumulative function through assessed fractiles.

The experimental study conducted by Winkler [23] considers four assessment techniques: (a) Cumulative Distribution Function--assessment of fractiles by means of equally likely subintervals or direct questions regarding fractiles, (b) Hypothetical Future Samples, (c) Equivalent Prior Sample Information, and (d) Probability Density Function. A questionnaire using these techniques was developed and used to elicit prior distributions from 38 selected subjects involved in Winkler's study.

The use of penalty functions, or scoring methods, has been discussed by several researchers as means of encouraging honest assessments. Specifically, de Finetti [3] presents the quadratic scoring rule. Savage [18] derives the general class of strictly proper scoring rules by considering probabilities as special cases of rates of substitutions. Winkler discusses the use of scoring rules and other payoff schemes in [24] and reports his experimental results in [25].

Staël von Holstein and his associates ([20] and [21]) focus on the subject of eliciting the opinions of experts in practical situations rather than laboratory experiments. They discuss probability encoding in the context of decision analysis and propose the use of a probability wheel to facilitate the encoding process.

At the Reliability Conference in 1970, Lin and Schick [10] presented the use of an on-line computer system to assist a person in assessing a prior distribution, which is illustrated by a problem in the reliability field. Again in 1974, Lin and Schick [11] showed how a subjective probability distribution can be derived in the field of maintainability and showed how it can be used by the maintainability engineer in everyday decision making.

The present paper results from the authors' continued effort in making the probability assessment more practical by using modern electronic computers. This paper first offers a newly designed computer program which has incorporated the experience gained from the use of the previous program. To simplify the assessment procedure, the new program: (1) reduces the number of questions significantly, (2) is highly conversational and interactive, (3) checks for consistency as the user answers question by question, (4) uses graphical display rather than the typewriter terminal to help the user observe the assessment process as well as to greatly increase the speed of drawing the assessed probability curves, and (5) plots not only the cumulative function but also the density function. The paper also considers the question of how to fit a theoretical distribution to a subjectively derived distribution. Specifically, it uses a lognormal distribution to illustrate how the fit can be performed, and presents some preliminary findings of a research project which investigates the fitting procedure.

3. METHOD OF PROBABILITY ASSESSMENT

A number of methods have been suggested for the assessment of prior distributions (see, for example, [6], [14], and [23]). Our computer program makes use of the method of equally likely subintervals, which perhaps is the most commonly used approach. The basic idea of this method is to ask the decision maker, at any stage, to divide a given interval into two judgmentally equally likely subintervals.

To begin with, the interval covering all possible values of an uncertain quantity (usually called a random variable) is split into two subintervals and the decision maker is asked to choose which subinterval to bet on. The dividing point is then changed until he feels indifferent between betting on one or the other subinterval. When the indifference point is reached, the decision maker feels that it is equally likely that the actual value of the uncertain quantity will fall above (to the right of) or below (to the left of) this point. The indifference point, which divides the entire interval into two subintervals with equal probabilities, is the median. Next, the decision maker is asked to specify a point which will further divide the subinterval to the left of the median into two equally likely parts. This new point is the first quartile. Similarly, the subinterval to the right of the median may be further divided into two equally likely parts. The decision maker may proceed in this manner to divide any given interval (generated previously) into two equally likely subintervals.

Suppose we let x_k designate the k^{th} fractile of the uncertain quantity $\tilde{x}$, i.e.,

$$P(\tilde{x} \leq x_k) = k, \qquad 0 \leq k \leq 1 .$$

Then, using the method of equally likely subintervals, the decision maker is asked to respond to a series of questions which will lead to a determination of x_k values for such k as .5, .25, .75, etc.

4. COMPUTER PROGRAM AND OUTPUT FOR PROBABILITY ASSESSMENT

The program stores a set of questions for the method of equally likely subintervals. These questions are displayed successively on a CRT; the user responds to the questions by typing his anwers on a teletype. The response to each of the questions is processed immediately and checked for logical consistency.

Assuming you are the user of the program, the first question calls for the lower limit of the probability distribution by asking you to:

> "Specify the largest value such that you feel virtually certain that the actual value of the uncertain quantity will fall above this value."

The second question, on the other hand, calls for the upper limit of the distribution by asking you to:

> "Specify the smallest value such that you feel virtually certain that the actual value of the uncertain quantity will fall below this value."

In terms of the fractile notation described earlier, the first question asks for x_0 and the second question asks for x_1. The program will check to see if x_0 is less than x_1 and if you feel virtually certain that the actual value of the uncertain quantity will lie in between x_0 and x_1.

The third question asks you to divide the interval defined by the limits x_0 and x_1 into two equally likely subintervals. The question says:

> "Specify the value such that you feel it is equally likely that the actual value of the uncertain quantity will fall above or below this value."

The answer to this question yields $x_{.5}$, which should lie in between x_0 and x_1.

The fourth question, which calls for $x_{.25}$, is:

> "Suppose you were told that actual value is less than $x_{.5}$. Specify the value such that it is equally likely that the actual value of the uncertain quantity is either above or below this value."

The program will check to see if this answer lies in between x_0 and $x_{.5}$.

The fifth question, which calls for $x_{.75}$, is:

> "Suppose you were told the actual value is greater than $x_{.5}$. Specify the value such that it is equally likely that the actual value of the uncertain quantity is either above or below this value."

This answer is checked to see if it lies in between $x_{.5}$ and x_1.

At this point, the program further checks for consistency. Specifically, it asks:

> "Now, do you feel it is equally likely that the actual value of the uncertain quantity will lie within the interval between $x_{.25}$ and $x_{.75}$ or outside of this interval?"

If the check is not met, the program will direct you to review and revise each of your previous answers. Otherwise, the program will proceed to ask you to specify the most likely value (the mode).

The assessments thus obtained are summarized on the CRT. The program then fits a smooth cumulative distribution function through the assessed fractiles. At your request, it will plot the cumulative curve and the corresponding density curve. If these graphs do not seem to reflect your judgements about the uncertain quantity, you will be guided by the program to revise your previous responses. Whenever you are satisfied with the assessed distribution, the mean and the standard deviation are computed. In addition, you may ask for .005, .015, .025,..., .995 fractiles of the distribution.

To illustrate the computerized method of probability assessment discussed above, the computer output of an example is presented. In this example, the expert (a manager in charge of scheduling a service) is asked to quantify his judgments concerning the number of people requesting a particular service.* As can be seen from this output, the expert violates some of the axioms of probability and is asked to revise his responses several times.

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*The probability distribution of the number of people is actually a discrete function. However, a continuous distribution is used to approximate the discrete distribution in order to simplify the assessment procedure and the analysis of the service problem.

THIS PROGRAM IS DESIGNED TO ASSIST YOU IN (A) QUANTIFYING YOUR PROBABILITY JUDGMENTS CONCERNING AN UNCERTAIN QUANTITY, (B) CALCULATING THE MEAN AND VARIANCE OF THE PROBABILITY DISTRIBUTION OBTAINED FROM THIS QUANTIFICATION, AND (C) FITTING THE ASSESSED DISTRIBUTION TO A THEORETICAL DISTRIBUTION.

WHAT IS THE UNCERTAIN QUANTITY OF YOUR CONCERN NOW?

number of customers requesting a particular service*

PLEASE RESPOND TO THE FOLLOWING QUESTIONS WITH YOUR CAREFUL JUDGMENTS:

(1) SPECIFY THE LARGEST VALUE SUCH THAT YOU FEEL VIRTUALLY CERTAIN THAT THE ACTUAL VALUE OF THE UNCERTAIN QUANTITY WILL FALL ABOVE THIS VALUE.

340

(2) SPECIFY THE SMALLEST VALUE SUCH THAT YOU FEEL VIRTUALLY CERTAIN THAT THE ACTUAL VALUE OF THE UNCERTAIN QUANTITY WILL FALL BELOW THIS VALUE.

260

SORRY, YOU HAVE MISINTERPRETED THESE TWO QUESTIONS. QUESTION (1) ASKS FOR THE LOWER LIMIT OF THE UNCERTAIN QUANTITY. PLEASE GIVE YOUR NEW ANSWER TO (1).

260

QUESTION (2) ASKS FOR THE UPPER LIMIT OF THE UNCERTAIN QUANTITY. PLEASE GIVE YOUR NEW ANSWER TO (2).

340

YOUR ANSWERS TO THESE TWO QUESTIONS IMPLY THAT YOU FEEL VIRTUALLY CERTAIN THAT THE ACTUAL VALUE OF THE UNCERTAIN QUANTITY WILL LIE IN BETWEEN 260 AND 340. DO YOU AGREE?

no

THEN YOU MUST REVISE YOUR ANSWER(S).

PLEASE TYPE YOUR NEW ANSWER TO (1).

240

PLEASE TYPE YOUR NEW ANSWER TO (2).

340

(3) SPECIFY THE VALUE SUCH THAT YOU FEEL IT IS EQUALLY LIKELY THAT THE ACTUAL VALUE OF THE UNCERTAIN QUANTITY WILL FALL ABOVE OR BELOW THIS VALUE.

280

*user's responses are in small letters

(4) SUPPOSE YOU WERE TOLD THAT THE ACTUAL VALUE IS LESS THAN 280. SPECIFY THE VALUE SUCH THAT IT IS EQUALLY LIKELY THAT THE ACTUAL VALUE OF THE UNCERTAIN QUANTITY IS EITHER ABOVE OR BELOW THIS VALUE.

310

I BELIEVE YOU DIDN'T ANSWER THIS QUESTION CAREFULLY. PLEASE GIVE IT SOME THOUGHT.

DO YOU NOW REALIZE WHAT YOU DID WRONG?

no

YOUR ANSWER TO (4) SHOULD HAVE LIED IN BETWEEN 240 AND 280.
PLEASE TYPE YOUR NEW ANSWER.

270

(5) SUPPOSE YOU WERE TOLD THAT THE ACTUAL VALUE IS GREATER THAN 280. SPECIFY THE VALUE SUCH THAT IT IS EQUALLY LIKELY THAT THE ACTUAL VALUE OF THE UNCERTAIN QUANTITY IS EITHER ABOVE OR BELOW THIS VALUE.

300

(6) NOW, DO YOU FEEL IT IS EQUALLY LIKELY THAT THE ACTUAL VALUE OF THE UNCERTAIN QUANTITY WILL LIE WITHIN THE INTERVAL BETWEEN 270 AND 300 OR OUTSIDE OF THIS INTERVAL?

no

SORRY, YOUR NO ANSWER INDICATES THAT YOUR ANSWERS TO QUESTIONS (3) THROUGH (5) MAY NOT REFLECT YOUR PROBABILITY JUDGMENTS. SO, LET'S REVIEW EACH OF THESE ANSWERS.

(3A) ACCORDING TO YOUR ANSWER TO QUESTION (3), YOU FEEL IT IS EQUALLY LIKELY THAT THE ACTUAL VALUE OF THE UNCERTAIN QUANTITY WLL FALL ABOVE OR BELOW 280. DO YOU AGREE?

yes

(4A) WE NOW LOOK AT YOUR ANSWER TO QUESTION (4). IF THE ACTUAL VALUE OF THE UNCERTAIN QUANTITY WERE LESS THAN 280, WOULD YOU FEEL EQUALLY LIKELY THAT THE ACTUAL VALUE IS EITHER ABOVE OR BELOW 270?

no

PLEASE TYPE YOUR NEW VALUE FOR WHICH YOU WOULD FEEL EQUALLY LIKELY.

265

(5A) FINALLY, YOUR ANSWER TO QUESTION (5) IMPLIED THAT IF THE ACTUAL VALUE OF THE UNCERTAIN QUANTITY WERE GREATER THAN 280 YOU WOULD FEEL EQUALLY LIKELY THAT THE ACTUAL VALUE IS EITHER ABOVE OR BELOW 300. DO YOU STILL AGREE?

yes

(6A) NOW, LET'S CHECK THE CONSISTENCY OF YOUR REVISED ANSWERS. THE LOGICAL CONSISTENCY REQUIRES YOU TO FEEL EQUALLY LIKELY THAT THE ACTUAL VALUE OF THE UNCERTAIN QUANTITY WILL LIE WITHIN THE INTERVAL BETWEEN 265 AND 300 OR OUTSIDE OF THIS INTERVAL. DO YOU FEEL THAT WAY?

yes

(7) SPECIFY THE MOST LIKELY VALUE (THE MODE).

275

(8) GREAT. YOU NOW HAVE DONE YOUR ASSESSMENTS AS SUMMARIZED:

CUM. PROB.	VALUE	CORRESP. QUESTION
0.	240	1
.25	265	4
.50	280	3
.75	300	5
1.00	340	2
MODE	275	7

FROM THESE ASSESSED POINTS, A SMOOTHED CUMULATIVE DISTRIBUTION WILL BE OBTAINED. WOULD YOU LIKE TO SEE THE GRAPHS FOR THE DISTRIBUTION FUNCTION AND THE DENSITY FUNCTION?

yes

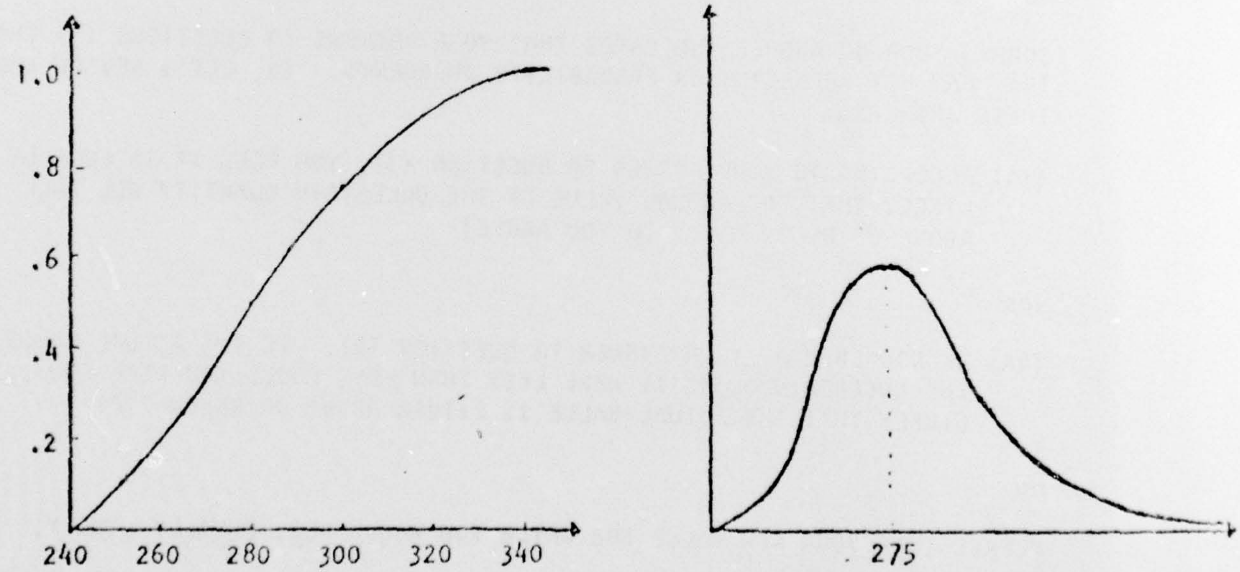


Figure 1 Distribution and Density Functions for Customers Requesting a Particular Service

(9) WOULD YOU ACCEPT THESE GRAPHS AS REASONABLY ACCURATE EXPRESSIONS OF YOUR JUDGMENTS CONCERNING THE UNCERTAIN QUANTITY?

yes

(10) SUMMARY MEASURES OF THIS DISTRIBUTION ARE:

MEAN	283.0439
STANDARD DEVIATION	24.0676

(11) DO YOU WANT TO SEE VARIOUS FRACTILES OF THE DISTRIBUTION

yes

0.005	241.848	**	0.255	265.030	**	0.505	280.340	**	0.755	300.521
0.015	243.032	**	0.265	265.594	**	0.515	281.023	**	0.765	301.574
0.025	244.200	**	0.275	266.160	**	0.525	281.710	**	0.775	302.640
0.035	245.352	**	0.285	266.729	**	0.535	282.399	**	0.785	303.718
0.045	246.488	**	0.295	267.302	**	0.545	283.092	**	0.795	304.809
0.055	247.608	**	0.305	267.878	**	0.555	283.788	**	0.805	305.913
0.065	248.712	**	0.315	268.458	**	0.565	284.488	**	0.815	307.030
0.075	249.800	**	0.325	269.040	**	0.575	285.200	**	0.825	308.160
0.085	250.872	**	0.335	269.625	**	0.585	285.928	**	0.835	309.342
0.095	251.928	**	0.345	270.214	**	0.595	286.672	**	0.845	310.618
0.105	252.968	**	0.355	270.806	**	0.605	287.432	**	0.855	311.986
0.115	253.992	**	0.365	271.401	**	0.615	288.208	**	0.865	313.446
0.125	255.000	**	0.375	272.000	**	0.625	289.000	**	0.875	315.000
0.135	255.982	**	0.385	272.603	**	0.635	289.803	**	0.885	316.616
0.145	256.927	**	0.395	273.213	**	0.645	290.613	**	0.895	318.264
0.155	257.836	**	0.405	273.829	**	0.655	291.429	**	0.905	319.944
0.165	258.709	**	0.415	274.451	**	0.665	292.251	**	0.915	321.656
0.175	259.546	**	0.425	275.080	**	0.675	293.080	**	0.925	323.400
0.185	260.347	**	0.435	275.715	**	0.685	293.915	**	0.935	325.176
0.195	261.111	**	0.445	276.357	**	0.695	294.764	**	0.945	326.984
0.205	261.843	**	0.455	277.005	**	0.705	295.644	**	0.955	328.824
0.215	262.561	**	0.465	277.659	**	0.715	296.556	**	0.965	330.696
0.225	263.270	**	0.475	278.320	**	0.725	297.500	**	0.975	332.600
0.235	263.969	**	0.485	278.987	**	0.735	298.476	**	0.985	334.536
0.245	264.659	**	0.495	279.661	**	0.745	299.484	**	0.995	336.668

(12) DO YOU WANT TO FIT THE ASSESSED DISTRIBUTION TO A THEORETICAL DISTRIBUTION?

no

DO YOU WISH TO QUANTIFY YOUR JUDGMENTS CONCERNING ANY OTHER UNCERTAIN QUANTITY?

no

THANK YOU FOR YOUR COOPERATION. GOOD-BYE.

To quantify his judgement about customer demand at peak periods, the expert may want to use the above assessment technique several times until he feels comfortable with the limits of the distribution. It should be noted here that nothing is assumed, about the nature of the probability distribution that is being assessed. For instance a rather skewed distribution might underlie customer demand and our purpose simply was to ask the expert to give us his best estimates of some of these fractiles. The program then gives us a printout of various fractiles of this distribution. Next, let us consider what one might want to do with these fractiles or summary measures of the assessed distribution.

5. FIT OF A LOGNORMAL DISTRIBUTION TO A SUBJECTIVE DISTRIBUTION

The density function of the lognormal distribution is given by:

$$f(x) = \frac{1}{\beta\sqrt{2\pi}}\, x^{-1} \exp\left[-\frac{1}{2}\left(\frac{\ln x - \alpha}{\beta}\right)^2\right] \qquad x > 0, \quad \beta > 0 \qquad (1)$$

It is well known that the mean $E(\tilde{x})$ and the variance $V(\tilde{x})$ are given by:

$$E(\tilde{x}) = \mu = \exp(\alpha + (\tfrac{1}{2})\beta^2)$$

$$V(\tilde{x}) = \sigma^2 = \mu^2[\exp \beta^2 - 1]$$

The mode of this distribution is at

$$\text{MODE} = \exp[\alpha - \beta^2]$$

While the median or 50th percentile, P_{50}, is at

$$P_{50} = e^{\alpha}.$$

By letting $y = \frac{\ln x - \alpha}{\beta}$ in (1) the 90th percentile was found to be

$$P_{90} = \exp(1.282 + \alpha).$$

Other fractile points can be found in similar fashion.

From the assessment procedure given earlier, several fractile points or the mean and the variance are available in the summary output of the first computer program. Any two fractile points, or a fractile point and the mean, or a fractile point and the variance etc., can be used to determine the parameters of the lognormal distribution. A second program was developed that allows some 20 different input combination pairs in the procedure for determining the parameters of the lognormal distribution. The program also integrates over prespecified ranges and plots the function with the newly found parameters. A typical output is displayed on the following two pages.

A plot of the distribution function is also available. Now this distribution function can be visually compared with the subjectively derived prior distribution using the questionnaire involving the demand for service. If "reasonable" agreement has been achieved the mathematical form of the density has been found. This form is important in order to establish with the incoming data and the likelihood function via Bayes' theorem the posterior distribution. On the other hand, if "reasonable" agreement between the two distribution functions has not been achieved the assessment procedure starts anew. Ultimately, agreement will be found unless the lognormal distribution is not a valid model describing customer demand. But considerable research points to the fact that customer demand for services at peak periods is indeed lognormally distributed.

To fit a lognormal distribution to a subjectively derived distribution will make subsequent analysis of the problem of satisfying customers demand more tractable mathematically. However, there is no standard procedure

LØG NØRMAL DISTRIBUTIØN

```
WHAT IS THE INPUT CØMBINATIØN NUMBER ?15
MEAN= ?283.0439
STD. DEV.= ?24.0676
```

ALPHA	BETA	MEDIAN	MEAN	STD DEV	MØDE	90TH PCTLE TIME
5.6424	0.0800	282.1396	283.0439	24.0676	280.3397	312.6118

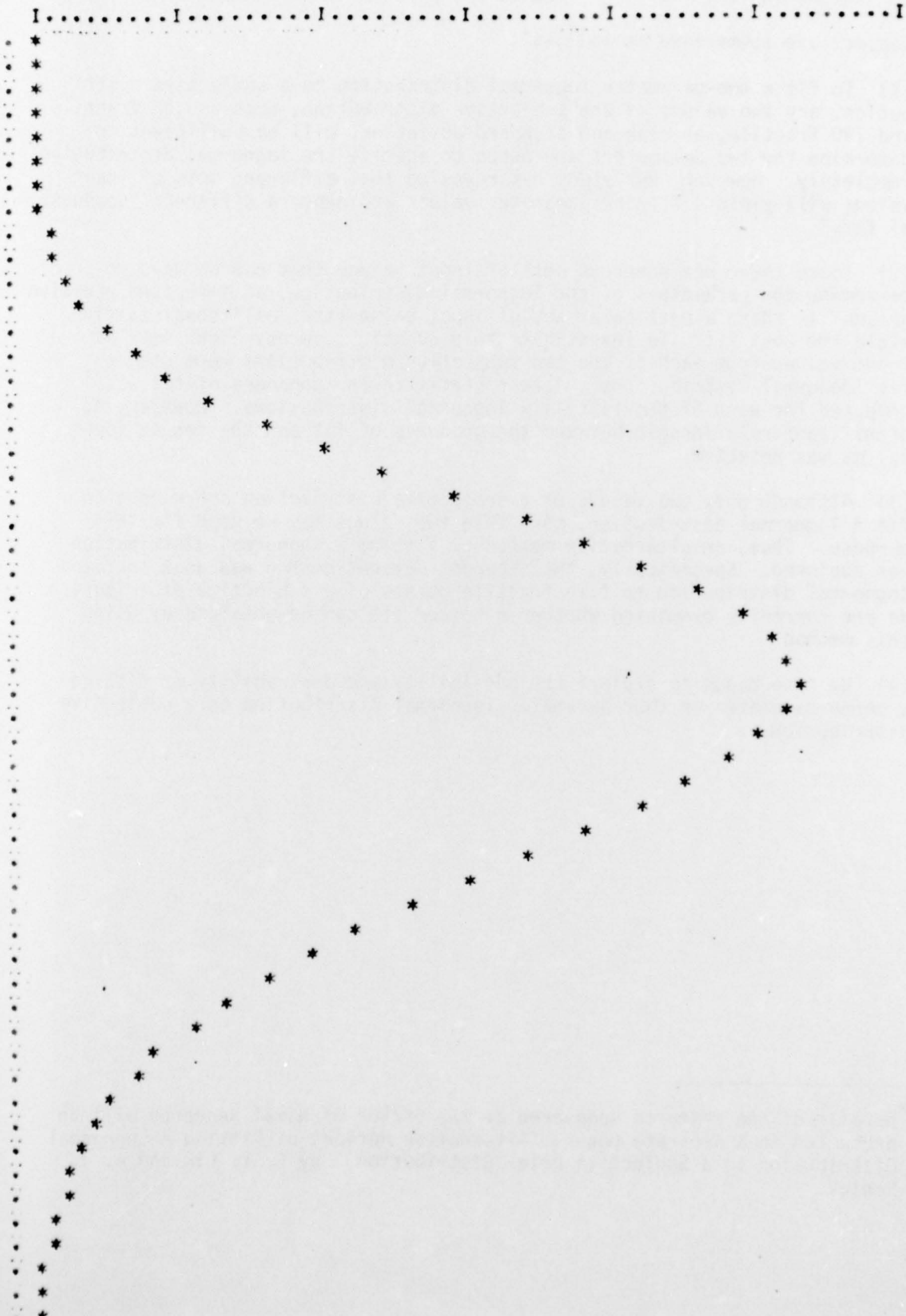
```
DØ YØU WISH TØ INTEGRATE-NØ=0,YES=1,RETURN=2 ?0

DØ YØU WISH TØ PRINT X AND Y-NØ=0,YES=1,RETURN=2 ?1
WHAT IS XMIN,XMAX,DELX
?50,650,25
```

X-VALUES	Y-VALUES
50	0
75	1.83421E-61
100	1.56688E-38
125	1.30210E-24
150	9.54053E-16
175	5.18728E-10
200	2.39752E-06
225	4.05544E-04
250	6.36217E-03
275	1.72265E-02
300	1.23842E-02
325	3.21614E-03
350	3.78106E-04
375	2.38277E-05
400	9.15507E-07
425	2.36932E-08
450	4.46584E-10
475	6.52256E-12
500	7.75782E-14
525	7.82180E-16
550	6.90762E-18
575	5.48863E-20
600	4.01178E-22
625	2.74749E-24
650	1.79027E-26

```
WHAT ARE YMIN,YMAX,XMIN,XMAX,DELX ?0,.02,200,450,3

FØR X:    TØP = 200   BØTTØM = 450   INCREMENT = 3
FØR Y:  LEFT = 0   RIGHT = 2.00000E-02   INCREMENT = 3.33333E-04
```



for performing the fit. The results and progress of our research on this subject are summarized as follows*:

(1) To fit a two-parameter lognormal distribution to a subjective distribution, any two values of the subjective distribution, such as .50 fractile and .90 fractile, or mean and standard deviation, will be sufficient to determine the two parameters and hence to specify the lognormal distribution completely. However, our study has revealed that different sets of input values will yield different parameter values and hence a different "goodness of fit."

(2) Since there are numerous sets of input values that can be used to determine the parameters of the lognormal distribution, an important question arises: Is there a particular set of input values that will consistantly yield the best fit? To investigate this question, twenty-eight sets of input values from each of the two subjective distributions were used to fit lognormal distributions. A test statistic for goodness of fit was computed for each of the fifty-six lognormal distributions. However, no significant relationship between the goodness of fit and the set of input values was detected.

(3) Although only two values of a subjective distribution are needed to fit a lognormal distribution, more than two values may be used for this purpose. Thus, an alternative method of fitting a lognormal distribution was explored. Specifically, the steepest descent method was used to fit a lognormal distribution to five fractile points of a subjective distribution. We are currently examining whether a better fit can be obtained by using this method.

(4) We have begun to explore the possibility and desirability of fitting a three-parameter or four-parameter lognormal distribution to a subjective distribution.

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*Details of the research sponsored by the Office of Naval Research will be presented in a separate paper, "Alternative Methods of Fitting a Lognormal Distribution to a Subjective Prior Distribution," by C. Y. Lin and G. J. Schick.

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This paper discusses a methodology to establish a subjective prior distribution via an interactive computer program on a CRT. The program asks specific carefully worded questions. The answers to these questions are checked for logical consistency and are translated into fractiles of a subjective probability distribution. The fractiles as well as other summary measures (like the mean and variance) are printed out. In order to find a conjugate prior distribution for a Bayesian treatment, the mathematical form of a possible

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conjugate prior needs to be ascertained. The paper illustrates how the parameters of a lognormal distribution might be obtained from the fractiles of the subjectively derived prior.