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IMPROVING THE MANRY-AGGARWAL METHOD FOR DESIGNING MULTI-DIMENSI--ETC(U)
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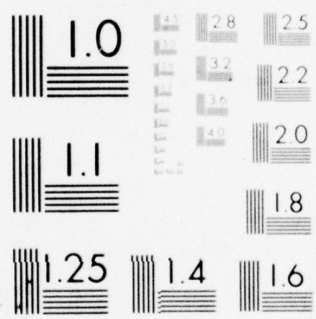

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IMPROVING THE MANRY-AGGARWAL METHOD FOR
DESIGNING MULTI-DIMENSIONAL FIR
DIGITAL FILTERS

by

James W. Daniel

September 1976 CNA-114

IMPROVING THE MANRY-AGGARWAL METHOD FOR DESIGNING

MULTI-DIMENSIONAL FIR DIGITAL FILTERS

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Abstract

M.T. Manry and J.K. Aggarwal recently described an algorithm for use in the design of multi-dimensional FIR digital filters by phase correction. As they observe, their method can be viewed as the steepest descent method for minimizing a certain function $f(x)$: given an approximate solution x_n, a new approximation is $x_{n+1} = x_n + t_n p_n$ where $p_n = -\nabla f(x_n)$ and t_n is chosen by a simple rule. We derive here an improved rule for determining t_n and an improved direction p_n (essentially the Fletcher-Reeves conjugate-gradient direction). The resulting method appears to be two to three times as fast as the Manry-Aggarwal method; the additional cost is primarily in storage, which roughly doubles.

Key words: FIR digital filters; filter design; phase correction; Manry-Aggarwal method.

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1. Introduction

In recent papers [Manry (1976), Manry-Aggarwal (1976)], M.T. Manry and J.K. Aggarwal have proposed a new technique for the design of multi-dimensional FIR digital filters. The reader is referred to those papers for references, applications, and comparisons with other methods; in the interest of brevity, we confine ourselves here to presenting dramatic improvements in the Manry-Aggarwal method. For simplicity and clarity, we follow the lead of [Manry (1976), Manry-Aggarwal (1976)] by presenting our discussion in terms of two-dimensional filters; generalization is obvious and straightforward.

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A two-dimensional FIR digital filter is described by an array $\underline{\underline{h}}$ of filter spatial coefficients h_{mn} for $o \leq m \leq M-1$ and $o \leq n \leq N-1$ for integers M,N. The filter produces output spatial data g_{mn} from input spatial data d_{mn} according to

$$g_{mn} = \sum_{k=o}^{M-1} \sum_{\ell=o}^{N-1} h_{k\ell}\, d_{m-k,n-\ell} \quad .$$

Corresponding to the spatial coefficient array $\underline{\underline{h}}$ is the frequency-domain array $\underline{\underline{H}}$ of coefficients H_{mn} for $o \leq m \leq M-1$ and $o \leq n \leq N-1$ defined by the discrete Fourier transform:

$$(1.1) \qquad H_{k\ell} = \sum_{m=o}^{M-1} \sum_{n=o}^{N-1} h_{mn}\, U^{km}\, V^{\ell n} \; ,$$

$$(1.2) \qquad h_{mn} = (1/MN) \sum_{k=o}^{M-1} \sum_{\ell=o}^{N-1} H_{k\ell}\, U^{-km}\, V^{-\ell n} \; ,$$

where

$$(1.3) \qquad U = \exp(-2\pi j/M),\; V = \exp(-2\pi j/N),\; j = \sqrt{-1} \; .$$

The problem addressed by Manry and Aggarwal is the following. It is desired to design a filter whose spatial coefficients h_{mn} will be zero except for a small number of specified values of m,n and whose corresponding frequency-domain-coefficient amplitudes $|H_{k\ell}|$ will assume (or approximate) prescribed values $A_{k\ell}$ for $o \leq k \leq M-1$ and $o \leq \ell \leq N-1$. For simplicity of presentation it is assumed that the spatial array $\underline{\underline{h}}$ is truncated in the sense that the (possibly) non-zero spatial coefficients h_{mn} are for $o \leq m \leq M_1 - 1$, $o \leq n \leq N_1 - 1$, where $M_1 \leq M$, $N_1 \leq N$; generalization is obvious and straightforward. In this case, computation of the frequency-domain array $\underline{\underline{H}}$ by Equation 1.1 simplifies to

$$(1.4) \qquad H_{k\ell} = \sum_{m=o}^{M_1-1} \sum_{n=o}^{N_1-1} h_{mn} U^{km} V^{\ell n} \quad .$$

2. The Manry-Aggarwal method

Manry and Aggarwal first present a basic iterative step for improving a spatial array $\underline{\underline{h}}^{(i)}$ to an array $\underline{\underline{h}}^{(i+1)}$ which comes closer to having the desired frequency-domain-coefficient amplitudes $A_{k\ell}$. Assuming that $h_{mn}^{(i)} = o$ except for $o \leq m \leq M_1-1$ and $o \leq n \leq N_1-1$, we compute the corresponding frequency-domain array $\underline{\underline{H}}^{(i)}$ by Equations 1.1 or 1.4. We then write these frequency-domain coefficients $H_{k\ell}^{(i)}$ as

$$(2.1) \qquad H_{k\ell}^{(i)} = |H_{k\ell}^{(i)}| \exp(j\theta_{k\ell}^{(i)}) \quad \text{for } o \leq k \leq M-1, \ o \leq \ell \leq N-1.$$

Recall that we desire to have $|H_{k\ell}^{(i)}| = A_{k\ell}$ for $o \leq k \leq M-1$, $o \leq \ell \leq N-1$, for given $A_{k\ell}$. We therefore define a new frequency domain array $\underline{\underline{H}}^{(i+\frac{1}{2})}$ with the same phase as for $\underline{\underline{H}}^{(i)}$ but with the correct amplitude:

$$(2.2) \qquad H_{k\ell}^{(i+\frac{1}{2})} = A_{k\ell} \exp(j\theta_{k\ell}^{(i)}).$$

From $\underline{\underline{H}}^{(i+\frac{1}{2})}$ we compute the corresponding spatial array $\underline{\underline{h}}^{(i+\frac{1}{2})}$ via Equation 1.2. Since we cannot in general expect $\underline{\underline{h}}^{(i+\frac{1}{2})}$ to be truncated, that is to satisfy the requirement that $h_{mn}^{(i+\frac{1}{2})} = o$ except for $o \leq m \leq M_1-1$ and $o \leq n \leq N_1-1$, we must truncate $\underline{\underline{h}}^{(i+\frac{1}{2})}$ to obtain $\underline{\underline{h}}^{(i+1)}$:

$$(2.3) \qquad \hat{h}_{mn}^{(i+\frac{1}{2})} = \begin{cases} h_{mn}^{(i+\frac{1}{2})} & \text{for } o \leq m \leq M_1-1, \ o \leq n \leq N_1-1 \\ o & \text{otherwise} \end{cases} ,$$

The basic iterative step then takes $\underline{\underline{h}}^{(i+1)} = \hat{\underline{\underline{h}}}^{(i+\frac{1}{2})}$.

It is a simple matter to see that $\hat{\underline{\underline{h}}}^{(i+\frac{1}{2})}$ comes closer, in a sense, than $\underline{\underline{h}}^{(i)}$ to achieving the desired frequency-domain-coefficient amplitudes $A_{k\ell}$. More precisely, given any truncated spatial array $\underline{\underline{h}}$, so that $h_{mn} = o$ except possibly for $o \leq m \leq M_1 - 1$, $o \leq n \leq N_1 - 1$, we define

$$(2.4) \qquad f(\underline{\underline{h}}) = \sum_{k=o}^{M-1} \sum_{\ell=o}^{N-1} (|H_{k\ell}| - A_{k\ell})^2$$

where $\underline{\underline{H}}$ is computed from $\underline{\underline{h}}$ via Equations 1.1 or 1.4. Then [Manry (1976)] we can measure the improvement in $\hat{\underline{\underline{h}}}^{(i+\frac{1}{2})}$ over $\underline{\underline{h}}^{(i)}$ by

$$(2.5) \qquad f(\hat{\underline{\underline{h}}}^{(i+\frac{1}{2})}) - f(\underline{\underline{h}}^{(i)}) \leq -MN \sum_{m=o}^{M_1-1} \sum_{n=o}^{N_1-1} |h_{mn}^{(i+\frac{1}{2})} - h_{mn}^{(i)}|^2 = -MN \, \|\hat{\underline{\underline{h}}}^{(i+\frac{1}{2})} - \underline{\underline{h}}^{(i)}\|^2.$$

If we always use the basic iterative step $\underline{\underline{h}}^{(i+1)} = \hat{\underline{\underline{h}}}^{(i+\frac{1}{2})}$, then since f is bounded below and f decreases at each step, we know that $f(\underline{\underline{h}}^{(i)}) - f(\underline{\underline{h}}^{(i+1)})$ converges to zero; by Equation 2.5 this implies that $\|\underline{\underline{h}}^{(i+1)} - \underline{\underline{h}}^{(i)}\|$ converges to zero, and hence [Daniel (1971)] the sequence $\{\underline{\underline{h}}^{(i)}\}$ either converges to a unique point $\underline{\underline{h}}^*$ or it has a continuum of limit points (i.e., an infinite connected set of limit points), an unlikely occurence.

In [Manry (1976)], another important interpretation is given for the step from $\underline{\underline{h}}^{(i)}$ to $\hat{\underline{\underline{h}}}^{(i+\frac{1}{2})}$. The function f in Equation 2.4 should be considered a function of the M_1N_1 variables h_{mn} for $o \leq m \leq M_1 - 1$ and $o \leq n \leq N_1 - 1$. It can then be shown that the gradient ∇f of f is given by (the M_1N_1 low-order components of)

$$(2.6) \qquad -\nabla f(\underline{\underline{h}}^{(i)}) = 2MN(\hat{\underline{\underline{h}}}^{(i+\frac{1}{2})} - \underline{\underline{h}}^{(i)})$$

Therefore we can interpret the basic iterative step from $\underline{\underline{h}}^{(i)}$ to $\hat{\underline{\underline{h}}}^{(i+\frac{1}{2})}$ as a step of length $\frac{1}{2MN}$ in the <u>steepest descent direction</u> $-\nabla f(\underline{\underline{h}}^{(i)})$ for f:

$$(2.7) \qquad \hat{\underline{\underline{h}}}^{(i+\frac{1}{2})} = \underline{\underline{h}}^{(i)} + \frac{1}{2MN}\left[-\nabla f(\underline{\underline{h}}^{(i)})\right]$$

Since we deduced above that if we let $\underline{\underline{h}}^{(i+1)} = \hat{\underline{\underline{h}}}^{(i+\frac{1}{2})}$ then $\|\underline{\underline{h}}^{(i+1)} - \underline{\underline{h}}^{(i)}\|$ converges to zero, it follows from Equation 2.6 that $\nabla f(\underline{\underline{h}}^{(i)})$ converges to zero. Thus we can analyze the convergence of the basic iterative method.

<u>Proposition</u>.

Let the basic iterative method be used, so that $\underline{\underline{h}}^{(i+1)} = \hat{\underline{\underline{h}}}^{(i+\frac{1}{2})}$. Then $\{\underline{\underline{h}}^{(i)}\}$ either converges to a unique point or has a continuum of limit points. In either case, all limits $\underline{\underline{h}}$ satisfy $\nabla f(\underline{\underline{h}}) = \underline{\underline{o}}$.

Manry and Aggarwal observed that in practice the choice $\underline{\underline{h}}^{(i+1)} = \hat{\underline{\underline{h}}}^{(i+\frac{1}{2})}$ gave rather slow convergence; this simply says that $\frac{1}{2MN}$ is not a particularly good step in the steepest descent direction. They therefore propose letting

$$(2.8) \qquad \underline{\underline{h}}^{(i+1)} = \underline{\underline{h}}^{(i)} + t_i(\hat{\underline{\underline{h}}}^{(i+\frac{1}{2})} - \underline{\underline{h}}^{(i)})$$

where t_i is not necessarily chosen as $\frac{1}{2MN}$. Their choice of t_i attempts to accelerate convergence. Specifically, they let c_i minimize $\|\underline{\underline{h}}^{(i)} - [\underline{\underline{h}}^{(i-1)} + c(\underline{\underline{h}}^{(i-1)} - \underline{\underline{h}}^{(i-2)})]\|^2$ with respect to c, giving a simple formula for c_i; they then let $t_i = 1 + c_i$ if this decreases f, but $t_i = \frac{1}{2MN}$ otherwise or if $i < 2$. Numerical experiments show that this choice gives a dramatic improvement, reducing f to a given value in half the work as for the basic, unaccelerated, method. This accelerated method, therefore, is the method proposed in [Manry (1976), Manry-Aggarwal (1976)]; we call it the Manry-Aggarwal method.

3. <u>Improving the step size t_i</u>

Although the Manry-Aggarwal choice of the step size t_i dramatically improves convergence as compared with the unaccelerated basic method, further improvement seems possible at little cost. Suppose that we wish to move in a direction $\underline{\underline{p}}^{(i)}$ from $\underline{\underline{h}}^{(i)}$ to $\underline{\underline{h}}^{(i+1)}$ by $\underline{\underline{h}}^{(i+1)} = \underline{\underline{h}}^{(i)} + t_i \underline{\underline{p}}^{(i)}$; at present we have $\underline{\underline{p}}^{(i)} = \hat{\underline{\underline{h}}}^{(i+\frac{1}{2})} - \underline{\underline{h}}^{(i)}$, but in the next Section we will use different directions. In the frequency domain, we will then have

$$\underline{\underline{H}}^{(i+1)} = \underline{\underline{H}}^{(i)} + t_i \underline{\underline{P}}^{(i)}$$

where $\underline{\underline{P}}^{(i)}$ is computed from $\underline{\underline{p}}^{(i)}$ as $\underline{\underline{H}}^{(i)}$ is from $\underline{\underline{h}}^{(i)}$, namely by Equations 1.1 or 1.4. We will try to choose t_i so as approximately to minimize f, that is,

$$g(t) \equiv \sum_{k=o}^{M-1} \sum_{\ell=o}^{N-1} (A_{k\ell} - |H_{k\ell}^{(i)} + tP_{k\ell}^{(i)}|)^2 , \tag{3.1}$$

as a function of t. This function is difficult to handle because of the term $|H_{k\ell}^{(i)} + tP_{k\ell}^{(i)}|$; writing this for the momemt as $|H+tP|$, we derive a simple approximation.

We have $|H + tP| = \sqrt{(\bar{H} + t\bar{P})(H + tP)}$, where $^-$ denotes complex conjugation. Therefore $|H + tP| = [\bar{H}H + t(\bar{H}P + H\bar{P}) + t^2\bar{P}P]^{\frac{1}{2}} = [|H|^2 + t(\bar{H}P + H\bar{P}) + t^2|P|^2]^{\frac{1}{2}} = |H|[1 + t(\bar{H}P + H\bar{P})/|H|^2 + t^2|P|^2/|H|^2]^{\frac{1}{2}}$. Since we typically expect the steps $t\underline{\underline{p}}^{(i)}$ and $t\underline{\underline{P}}^{(i)}$ to be small, we can approximate the square root $(1 + x)^{\frac{1}{2}}$ above by the terms in t through the second power in the power series $(1 + x)^{\frac{1}{2}} = 1 + 1/2\,x - 1/8\,x^2 + \ldots$. After some rearrangement, this gives

$$|H + tP| \simeq |H| + t(\bar{P}H + P\bar{H})/(2|H|) + t^2\left[\frac{|P|^2}{2|H|} - \frac{(\bar{P}H + P\bar{H})^2}{8|H|^3}\right]. \tag{3.2}$$

For each term in Equation 3.1, by substituting Equation 3.2 and rearranging we therefore have

$$(A - |H + tP|)^2 = A^2 - 2A|H + tP| + |H + tP|^2$$

$$= A^2 - 2A|H + tP| + |H|^2 + t(\bar{P}H + \bar{H}P) + t^2|P|^2$$

$$\approx (A - |H|)^2 + t(\bar{P}H + P\bar{H})(1 - \frac{A}{|H|}) + t^2 \left[(1 - \frac{A}{|H|})|P|^2 + A \frac{(P\bar{H} + \bar{P}H)^2}{4|H|^3} \right].$$

Substituting this approximation in Equation 3.1 gives us a quadratic polynomial at t, whose minimum is easily found to occur at

$$t_i = - \sum_{k=o}^{M-1} \sum_{\ell=o}^{N-1} (P_{k\ell}^{(i)}\bar{H}_{k\ell}^{(i)} + \bar{P}_{k\ell}^{(i)}H_{k\ell}^{(i)})(1 - \frac{A_{k\ell}}{|H_{k\ell}^{(i)}|}) \Bigg/ \Bigg\{ 2|P_{k\ell}^{(i)}|^2 \times \left(1 - \frac{A_{k\ell}}{|H_{k\ell}^{(i)}|}\right) + A_{k\ell} \frac{\left(P_{k\ell}^{(i)}\bar{H}_{k\ell}^{(i)} + \bar{P}_{k\ell}^{(i)}H_{k\ell}^{(i)}\right)^2}{2|H_{k\ell}^{(i)}|^3} \Bigg\} \tag{3.3}$$

This expression is indeed formidable, but it really is easy to compute; the work is small compared to the cost of the two discrete Fourier transforms required at each iteration. If this choice of t_i does not decrease f, then we instead use $t_i = \frac{1}{2MN}$. Using this step-size, we require about 75% of the iterations necessary with the Manry-Aggarwal method to reduce the error f to a given amount. We show no numerical examples here, however, because still better improvements will be made in the next section.

4. Improving the search direction

As we remarked in Section 2, the direction used in the Manry-Aggarwal method is simply the steepest descent direction for the function f of Equation 2.4. Steepest descent, however, is no longer widely used by people in numerical opitmization; various conjugate gradient or conjugate direction methods are much more successful. The so-called variable-metric versions of these methods are unattractive because they require storage in proportion to the square of the number (M_1N_1 in our case) of variables. We therefore opt for the version of conjugate gradients described in [Fletcher-Reeves (1964), Poljak (1969), Polak-Ribiere (1969)].

In this method, the initial direction $\underline{\underline{p}}^{(i)}$ is the steepest descent direction $-\nabla f(\underline{\underline{h}}^{(i)})$. Thereafter we take

$$\text{(4.1)}\qquad \begin{aligned} \underline{\underline{p}}^{(i)} &= - f(\underline{\underline{h}}^{(i)}) + b_{i-1}\underline{\underline{p}}^{(i-1)} \\ b_{i-1} &= \left\langle \nabla f(\underline{\underline{h}}^{(i)}) - \nabla f(\underline{\underline{h}}^{(i-1)}),\ \nabla f(\underline{\underline{h}}^{(i)}) \right\rangle / \|\nabla f(\underline{\underline{h}}^{(i-1)})\|^2 , \end{aligned}$$

where $\langle \cdot,\cdot \rangle$ denotes the usual inner product

$$\langle \underline{\underline{x}}, \underline{\underline{y}} \rangle = \sum_{k=o}^{M_1-1} \sum_{\ell=o}^{N_1-1} x_{k\ell}\, y_{k\ell} .$$

Again, this changed direction costs little to compute in comparison with the two discrete Fourier transforms per iteration.

Thus our final algorithm improving the Manry-Aggarwal method consists of using the search direction given by Equation 4.1 in conjunction with the step size given by Equation 3.11; whenever $f(\underline{\underline{h}}^{(i)} + t_i\underline{\underline{p}}^{(i)})$ is not less than $f(\underline{\underline{h}}^{(i)})$, we revert to the basic iterative step and let $\underline{\underline{h}}^{(i+1)} = \underline{\underline{h}}^{(i+\frac{1}{2})}$, the same as taking a step $t_i = \frac{1}{2MN}$ in the steepest descent direction. This method dramatically improves the Manry-Aggarwal method, as we show in the next section. Before considering numerical experiments, we must compare the storage required by the two methods now under consideration. The Manry-Aggarwal requires on the order of $2MN + 3M_1N_1$ locations (plus terms of lower order). Because of needs to store previous gradients, et cetera, our new method requires on the order of $4MN + 6M_1N_1$ locations. Thus the new method requires about twice the storage.

5. Some simple numerical experiments

We show here a comparison of the Manry-Aggarwal method against our method on two problems from [Manry (1976)]. In both cases we take $M = N = 32$ and $M_1 = N_1 = 8$. Thus there are 64 unknown coefficients h_{mn} to determine in an attempt to match 1024 amplitudes $A_{k\ell}$. The Manry-Aggarwal method requires about 4500 locations of storage compared with about 9000 for our method.

In all cases we start the algorithm by letting $H_{k\ell}^{(-\frac{1}{2})} = A_{k\ell}$ for $o \leq k \leq M-1$, $o < \ell < N-1$, we compute $\underline{\underline{h}}^{(-\frac{1}{2})}$ from $\underline{\underline{H}}^{(-\frac{1}{2})}$ via Equation 1.2, and finally we truncate to let $\underline{\underline{h}}^{(o)} = \hat{\underline{\underline{h}}}^{(-\frac{1}{2})}$. We show the results of 100 iterations on the old method and as many iterations of the new method as needed until the change in f in one iteration is less than 10^{-14} and $\|\underline{\underline{h}}^{(i)} - \underline{\underline{h}}^{(i-1)}\| \leq 10^{-7}$.

Example 1

The desired amplitude response is a ring. More precisely,

$$A_{k\ell} = \begin{cases} 1 \text{ if } 1 < \theta_k^2 + \theta_\ell^2 \leq 4 & \text{for } o \leq k \leq 31, \quad o \leq \ell \leq 31 \\ o.2 \text{ otherwise} \end{cases}$$

where

$$\theta_k = \begin{cases} \dfrac{k\pi}{16} & \text{for } o \leq k \leq 15 \\ \dfrac{(k-32)\,\pi}{16} & \text{for } 16 \leq k \leq 31 \end{cases}$$

With $\underline{\underline{h}}^{(o)}$ determined as above, initially we have

$$f(\underline{\underline{h}}^{(o)}) = .09575649, \quad \|\nabla f(\underline{\underline{h}}^{(o)})\| = 233.$$

After 100 iterations (200 Fourier transforms) the Manry-Aggarwal algorithm has reduced f to a value of .024544519 and $\|\nabla f\|$ to approximately .38. The new algorithm accomplishes this for f in 52 transforms, at which point f is reduced to .024541657 and $\|\nabla f\|$ to .48; after 78 transforms we have $f = .02454094819$ and

$\|\nabla f\| = .0095$, while 110 transforms yield $f = .02454094777$ and $\|\nabla f\| = .000079$. More data are displayed in Tables 5.1 and 5.2 below. Both methods reduce f to a reasonable value in a few iterations, but the new method improves the f-values much better thereafter.

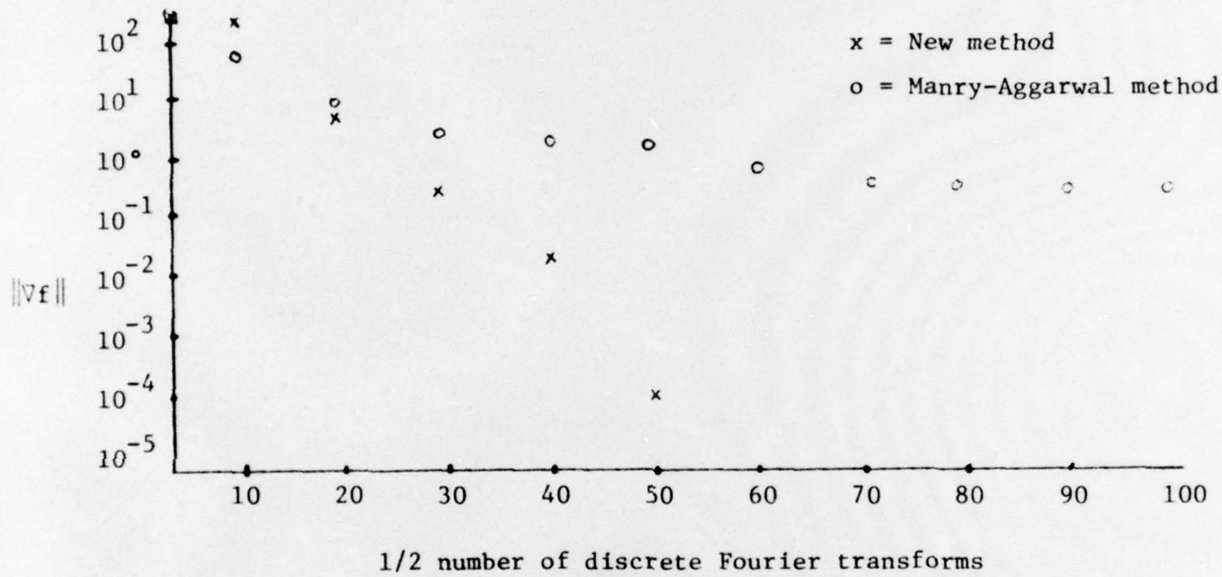

Table 5.1. $\|\nabla f\|$ for Example 1.

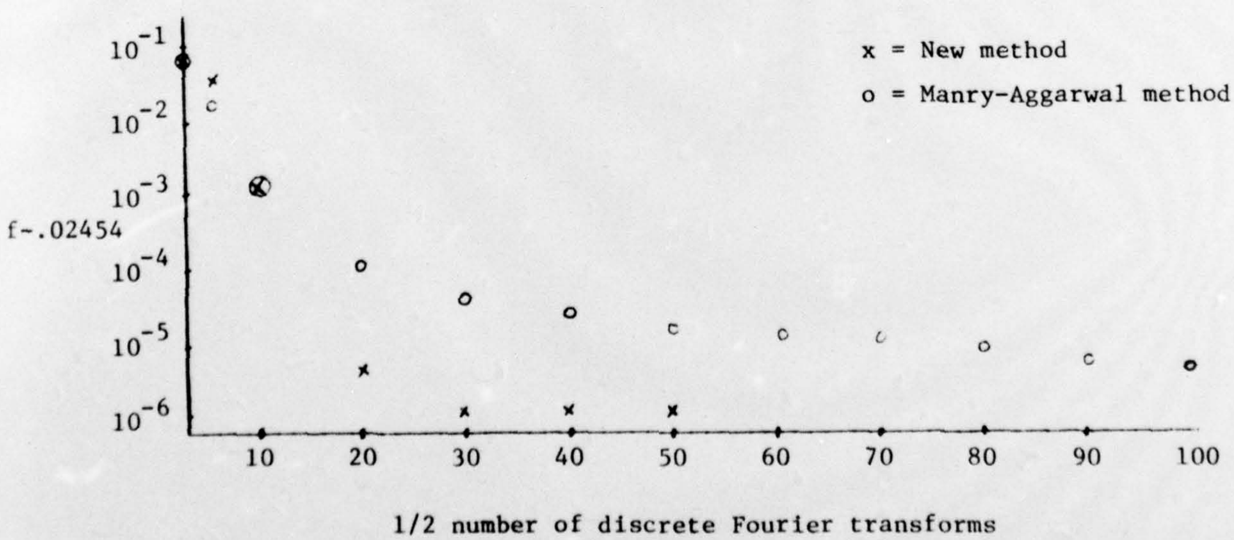

Table 5.2 f for Example 2.

Example 2.

The desired amplitude response is a diamond. More precisely,

$$A_{k\ell} = \begin{cases} 1 \text{ if } |\theta_k| + |\theta_\ell| \leq 2 \\ o \text{ otherwise} \end{cases}$$

where θ_k is as in Example 1. With $\underline{\underline{h}}^{(o)}$ determined as above, initially we have

$$f(\underline{\underline{h}}^{(o)}) = .08179448, \ \|\nabla f(\underline{\underline{h}}^{(o)})\| = 257.$$

After 100 iterations (200 Fourier transforms) the Manry-Aggarwal algorithm has reduced f to a value of .0186557567 and $\|\nabla f\|$ to approximately 3.9. The new algorithm accomplishes this for f in 52 transforms at which point f is reduced to .018574098 and $\|\nabla f\|$ to 14; after 120 transforms we have f = .01800420801 and $\|\nabla f\|$ = .01, while 156 transforms yield f = .01800420785 and $\|\nabla f\|$ = .00007. More data are displayed in Tables 5.3 and 5.4 below.

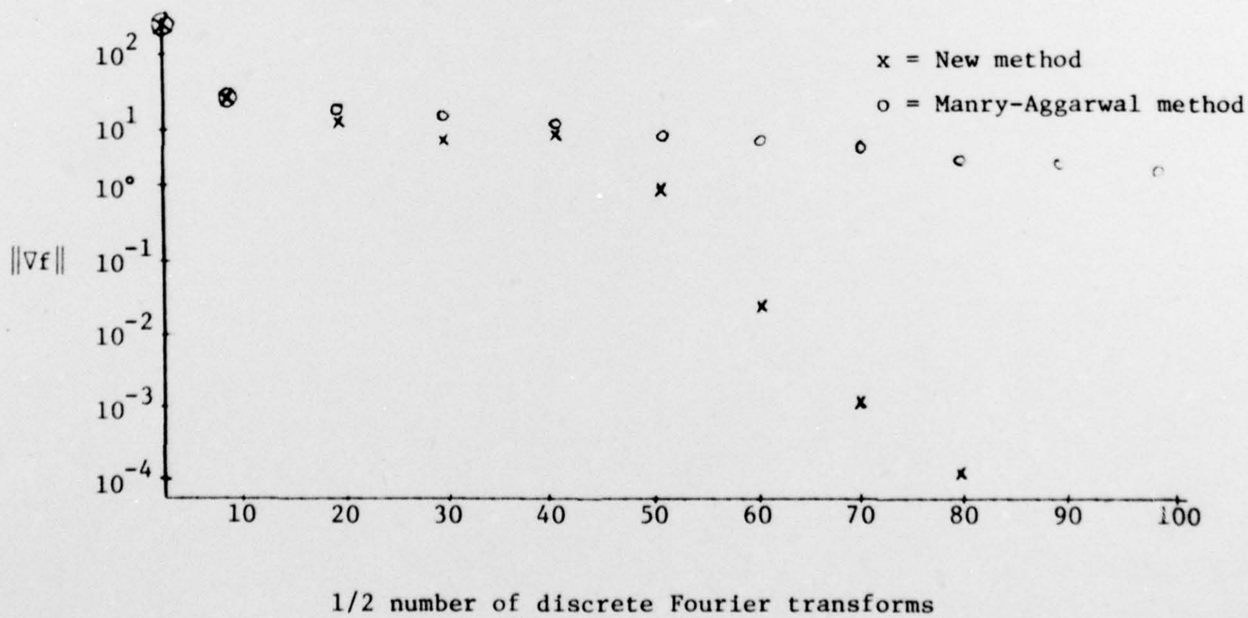

Table 5.3 $\|\nabla f\|$ for Example 2

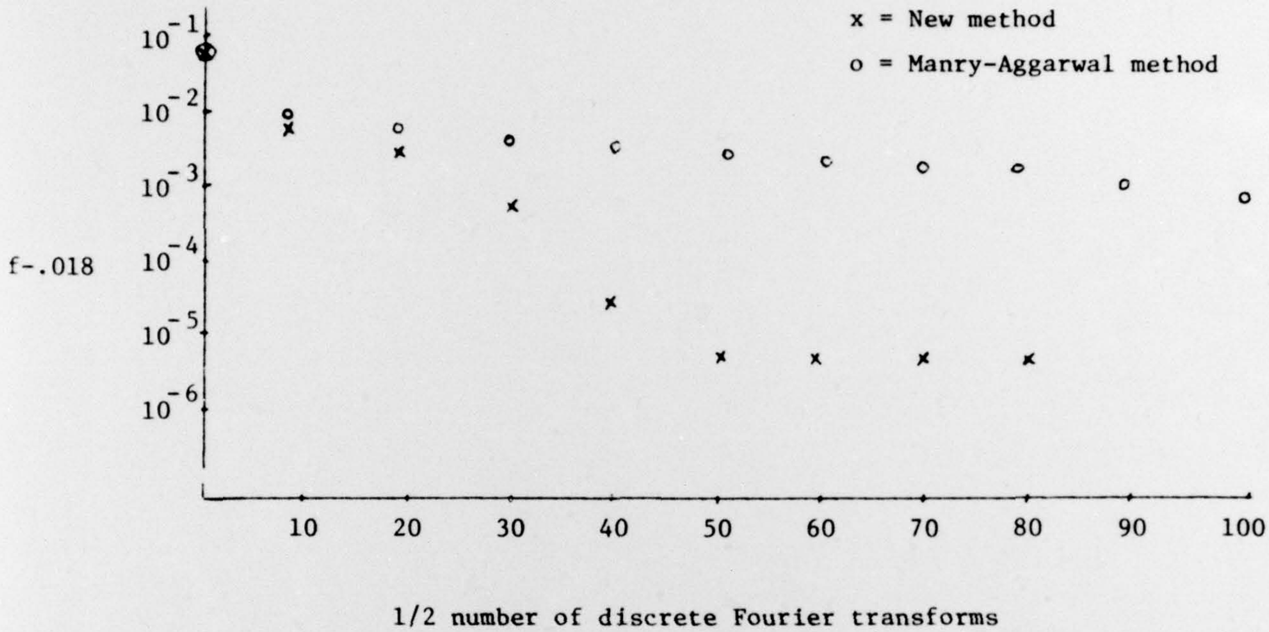

Table 5.4 f for Example 2

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13. ABSTRACT

(GRAD)

M.T. Manry and J.K. Aggarwal recently described an algorithm for use in the design of multi-dimensional FIR digital filters by phase correction. As they observe, their method can be viewed as the steepest descent method for minimizing a certain function $f(x)$: given an approximate solution $x_{(n)}$, a new approximation is $x_{(n+1)} = x_{(n)} + t_{(n)}p_{(n)}$ where $p_{(n)} = -\nabla f(x_{(n)})$ and $t_{(n)}$ is chosen by a simple rule. We derive here an improved rule for determining $t_{(n)}$ and an improved direction $p_{(n)}$ (essentially the Fletcher-Reeves conjugate-gradient direction). The resulting method appears to be two to three times as fast as the Manry-Aggarwal method; the additional cost is primarily in storage, which roughly doubles.

Key Words: FIR digital filters; filter design; phase correction; Manry-Aggarwal method.

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