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RESOLUTION ENHANCEMENT USING ALIAS FILTERING

THESIS

Presented to the Faculty of the School of Engineering of the Air Force Institute of Technology Air University in Partial Fulfillment of the Requirements for the Degree of Master of Science

by

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Approved for public release; distribution unlimited

Preface

The work accomplished in support of this thesis has been a rewarding experience in addition to an educational one. Throughout the course of the research, the support and encouragement of my thesis advisor, Lieutenant Stanley R. Robinson, has been most appreciated. In addition, the suggestions of Dr. John J. Knab and Captain Gregg L. Vaughn have been of great value. I also wish to acknowledge the comments provided by Captain Peter E. Miller during the final weeks of this project.

i



Contents

5

																				Page
Prefa	ce	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	ii
List	of	Fig	ure	es	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	v
Abstr	act		•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	vii
1. 1	ntr	odu	cti	on	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	1
	В	ack	Pro	unc	ł															1
			San	nli	ne	The	eory	,												2
			Ali	as																4
			Gil	b's	P	hend	omer	ion												15
			Int	err	01	atio	on													17
	S	יחוו	ort	ing	R	ese	arch	'n												18
		app	Rol	late	he	Enh	anci	- me	nt	Res	ar	ch								18
			Imr	Lac.	eiv		nag		nec	tra	-									20
			Eng		ano		amo	ree	sin	n at	hd	Exn	ans	ion						22
			FL	equi	ono		oto	10	avi	no		anp	une			-				22
	D	-	rre lor	equi	tat		nt	and	Sc	ODE	•									23
	r	rot	Sel		Lat	So		tio	200	ope	•	•								23
			Sp			Se	Tec	10		•	•	•	•	•						23
			r1.	lle		ion	511 of	Fi	1+4	·	•	•	•	•						24
			Ap		cal	101	01	LT	ILC	13	•	•	•	•	•	•	•			24
			L11	nita	all	ons	•	•	•	•	•	•	•	•	•	•				24
	A	ssu	mp		ns	•	-14	•	•	•	•	•	•	•	•	•	•			24
			Nyo	qui	SL	Sam	pri	ug	•	•	•	•	•	•	•	•	•	•	•	24
			NO	ise	Le	ver	•	•	·		+ 1.				•	•	•	•	•	25
			Qu	ant	ıza	L10	n a	na	ITU	inca	LIC	on E	110	LS	•	•	•	•	•	25
			rr	equ	enc	y D	aca	CO	nice	enc	•	•	•	•	•	•	•	•	•	25
	t	qui	Lpm	ent	•	.:	•	•	•	•	•	•	•	•	•	•	•	•	•	25
			DI	git	ıza	t10	nE	qui	pine	ent	•	•	•	•	•	•	•	•	•	26
			DI	git	al	Com	put	er	•	•	•	•	•	•	•	•	•	•	•	26
	3	um	nar	У	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	20
11.	Pro	ce	lur	e	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	28
	1	nt	rod	uct	ion															28
	,	Te	ati	on	of	Sou	rce	Da	ta											28
			Se	lec	tio	n														28
			Di	oit	172	tio	n													28
	I	hat	F	orm	at	Cha	nge													29
		au	Da	ta	Unr	ack	ing													29
			Da	ta	Pac	kin	0													31
		ine	otr	21	Plo	te	0													31
		pe	De	ta	Blo	ck	Siz													32
			Sa	mnl	inc	CA	512	-	•											33
	,		Ja	mp r	rig		•	•	•											33
		er r	F-	De	one	W C	nec	ifi		tion										33
			F4	1to	r L	in o	tio	n	ivn	thes	is									34
		Ima	r I	Fil	tor	ine			.,	-										35
		• 111 et .	1.1												-	-	-	-	-	

Contents

														Page
	Image Blocking													35
	Filtering and	Sca	lin	g										35
	Reassembly .													36
	Interpolation Enha	ince	men	t										36
	Summary													37
III.	Results	•	•	•	•	•	•	•	•	•	•	•	•	38
	Introduction													38
	Frequency Spectra													38
	Block Size .													38
	Sample Spacing	5 .												40
	Filter Design .													40
	Frequency Spec	ifi	cat	ion										45
	Filter Functio	ons												45
	Filtered Images .													45
	Initial Filter	ing												45
	Alias Removal													47
	Refined Filter	ing												47
	Interpolation Imag	es												51
	Blocked Images													51
	Blocking Disto	rti	on											51
	Interpolation	by	Lin	es										55
	Summary	-,												55
					•									
IV.	Conclusion and Recomm	nend	ati	ons	•	•	•	•	•	•	•	•	•	65
	Introduction .													65
	Alias Filtering Pr	nce	dim	•										65
	Present Design			Ξ.			•							65
	Improved Filte	T D	esi	on										66
	Interpolation Proc	edu	re											67
	Present Method	l	20											67
	Modified Metho	d	•		•	•	•							68
	Summary	<i>.</i>	•											70
	Summary	•	•	•	•	•	•	•	•	•	•	•	•	
Bib	liography	•	•	•	•	•	•	•	•	•	•	•	•	71
App	endix	•	•		•	•	•	•	•	•	•		•	72
	Broomen BBOCEC													72
	Program UNDACY	•	•	•	•	•	•	•	•	•	•	•	•	75
	Program DACKA	•	•	•	•	•	•	•	•	•	•	•	•	79
	Program PACK4 .	•	•	•	•	•	•	•	•	•	•	•	•	20
	Program SPCANAL .	•	•	•	•	•	•	•	•	•	•	•	•	01
	Program DIGFILT .	•	•	•	•	•	•	•	•	•	•	•	•	04
	Program FILLER .	•	•	•	•	•	•	•	•	•	•	•	•	07
	Program SPELL	-		-		-	-							7)

A Day a se

Anderson and

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List of Figures

Figu	ire J	age
1	Continuous and Discrete Signals	3
2	Aliased Frequency Spectrum	5
3	Frequency Spectrum of a Signal Sampled at a Rate Faster than Nyquist	6
4	Frequency Spectrum of an Undersampled Signal	8
5	Aliased Frequency Spectra	10
6	Image Scanning Process	11
7	Scanned Image Signal	12
8	Aliased Frequency Envelope After Filtering	14
9	Frequency Spectra	16
10	Signal Interpolation	19
11	Impulsive Image Spectrum	21
12	Test Image	30
13	Average Frequency Spectrum of One Line of Image	39
14	Average Frequency Spectrum of a Scanned 32-by-32 Block .	41
15	Average Frequency Spectrum of Fourth-Point Sampled Image	42
16	Average Frequency Spectrum of Fifth-Point Sampled Image .	43
17	Average Frequency Spectrum of Seventh-Point Sampled Image	44
18	Average Frequency Spectrum after Processing with Filters	46
19	Test Image after First Alias Filtering and Translation $\ .$	48
20	Test Image after Alias Removal Filtering	49
21	Test Image after Refined Alias Filtering and Translation	50
22	Seventh-Point Sampled Test Image Interpolated Using 32-by-14 Blocks	52
23	Exponential Function	53

La territoria de la

v

List of Figures

Figu	re	Page
24	Interpolated Exponential Function	54
25	Fourth-Point Sampled Test Image Interpolated Line-by-Line	56
26	Fourth-Point Sampled Test Image Printed by Repeating Points Four Times	57
27	Eighth-Point Sampled Test Image Interpolated Line-by-Line	58
28	Eighth-Point Sampled Test Image Printed by Repeating Points Eight Times	59
29	Sixteenth-Point Sampled Test Image Interpolated Line-by-Line	60
30	Sixteenth-Point Sampled Test Image Printed by Repeating Points Sixteen Times	61
31	Sixth-Point Sampled Test Image Interpolated Using 2048-by-3 Blocks	62
32	Twelfth-Point Sampled Test Image Interpolated Using 2048-by-6 Blocks	63

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Abstract

The purpose of the research accomplished was to determine if the bandpass filtering of digitized image data could improve the resolution of an image beyond that of an unfiltered image. The bandpass filters removed the aliased frequency components from the image spectrum and translated them to positions which they would occupy if the image had originally been sampled by the digitizer at a higher rate.

The research was accomplished in three phases. The first phase involved the digitization of a test image and the analysis of frequency spectra obtained by horizontal scanning of the data. The second phase consisted of designing software digital bandpass filters to remove the identifiable aliased frequencies from the spectrum and relocate them. The third phase accomplished the actual filtering of the test image which had been segmented and scanned.

The results of the filtering showed that the removal and relocation of aliased frequency components with the filters designed during this research produced more image distortion than was created by the aliased frequency components. It was discovered, however, that significant increases in resolution could be obtained by using an interpolation procedure.

vii

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RESOLUTION ENHANCEMENT USING ALIAS FILTERING

I. Introduction

The research which resulted from this thesis was based on extensive background work accomplished by other authors which provided a sound theoretical basis for the proposed algorithm. The problem and the scope of its solution were limited to the testing of an alias removal and translation algorithm on a single test image. This problem was attacked in three distinct phases. The spectrum of the image was first analyzed and the effects of sampling were noted. A filter for the removal of aliased frequencies and another for translation of those frequencies were then constructed. Finally, the filtering operation was accomplished on the test image using a block-by-block procedure.

Background

The experiments which are described in this thesis were designed to determine the feasibility of increasing the resolution of a digitized image by using known aliased frequency components to reconstruct the frequency spectrum of a higher-resolution version of the image. For the purpose of this research, resolution enhancement or increasing image resolution will refer to increasing the number of digitized samples used to represent a horizontally scanned line of the image. To accomplish this task using an alias filtering

algorithm, the exact location of the aliased frequencies must be determined. In order to discuss the method used to find these frequencies, the sampling theorem must first be considered.

<u>Sampling Theory</u>. Lathi (Ref 9:89) expresses the sampling theorem as "A bandlimited signal which has no spectral components above a frequency f_m Hz is uniquely determined by its values at uniform intervals less than $1/(2f_m)$ seconds apart." This relationship between time and frequency is given by

$$T_{N} \leq \frac{1}{2f_{m}}$$
(1)

where T_N is the spacing between samples and f_m is the frequency of the highest frequency component. When samples are spaced at intervals of T_N , the rate of sampling is referred to as the Nyquist rate. Stated in other words, if the spacing between samples is less than or equal to T_N , the signal from which the samples were taken can be reconstructed exactly by lowpass filtering.

The frequency spectrum of a signal sampled at the Nyquist rate is related to the frequency spectrum of the unsampled signal. The spectrum of the sampled signal consists of a series of identically shaped frequency envelopes, all of which appear identical to the frequency envelope of the original, unsampled signal. These multiple envelopes lie such that each exactly touches the next. If one envelope is lowpass filtered from this spectrum, a spectrum identical to that of the original signal is obtained. An example of a bandlimited signal, its frequency spectrum, a Nyquist rate sampling of the signal, and the spectrum of the sampled signal are shown in Fig. 1.



Fig. 1. Continuous and Discrete Signals: (A) Continuous bandlimited signal, (B) Frequency spectrum of (A), (C) Nyquist sampling of (A), (D) Frequency spectrum of (C)

If the sampling rate is changed so the new spacing between samples is greater than T_N , the signal can no longer be exactly reconstructed. Once the sampling rate becomes less than the Nyquist rate, the frequency envelopes overlap as shown in Fig. 2. After this overlap has occurred, there is no longer an exact replica of the original frequency spectrum. Thus, the lowpass filtering of the sampled signal spectrum will no longer yield the original spectrum. The overlap which results from sub-Nyquist rate sampling is referred to as alias.

If the signal under consideration is a digitized image, it is possible to think of this signal as a sampled version of the original, continuous image. If the samples of the image are taken along horizontal lines, each line of samples yields a discrete, quantized signal which possesses the properties of a sampled, continuous signal. If the Nyquist sampling rate has been exceeded, the individual frequency envelopes will be separated and no alias error will be present. If, as in the continuous case, the sample rate is less than the Nyquist rate, aliasing will occur, resulting in an inaccurate reconstruction of the original digitized image.

<u>Alias</u>. In order to locate the starting point of the overlap of frequency envelopes, consider the sampled continuous signal previously discussed. Further, consider only one of the multiple frequency envelopes which is symmetric about an arbitrary origin on the frequency axis. If the sample spacing is less than T_N , the two neighboring frequency envelopes are separate from the central envelope as illustrated in Fig. 3. If r=0, samples are infinitesimally close to one another; and, if r=1, sample spacing is exactly T_N . As the distance between samples becomes greater



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than T_N , r becomes greater than one. Thus at r = 2, the distance between samples is $2T_N$. As r increases beyond one, the adjacent envelopes overlap the central envelope as shown in Fig. 4. From Fig. 4, it can be observed that

$$\frac{\partial \omega_{\rm m}}{\rm r} \stackrel{\Delta}{=} \omega_{\rm m} \frac{2-\rm r}{\rm r} \tag{2}$$

and therefore

$$0 \stackrel{4}{=} 2 - r \tag{3}$$

where $_{\Omega}$ represents the fractional coefficient which determines the location of the start of overlap of the aliased spectrum with respect to the frequency $\frac{(0)}{m}$. If the Nyquist sample spacing, T_N , is known, then rT_N is the sample spacing for any other rate. Once r is established, $_{\Omega}$ is determined from equation (3), and the starting point of the frequency overlap is determined based on a spectral length of $\frac{(0)}{m}$.

If an image is initially digitized at a spacing which is greater than T_N , it is still possible to determine the starting point of "significant" overlap. Since most images contain areas of sharp contrast, which represent very high frequency components, it is often impossible to sample an image at a density which will result in an unaliased spectrum. When this circumstance is encountered, an assumption can be made that the spacing of samples produces a negligible overlapping of frequency envelopes. This sample spacing then may be used as an approximation of T_N , the Nyquist sample spacing. Using this rate as a starting point, the overlap due to any increase in the spacing between samples may be found.

In order to eliminate the error due to aliasing, the frequency



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components from adjacent envelopes which are present within the central envelope must be identified. In the case of a continuous signal, it would be impossible to distinguish the central envelope components from those components resulting from alias. In the case of discrete signals, however, it is sometimes possible to distinguish between these two types of components. Examples of the continuous and discrete cases are shown in Fig. 5.

When a digitized image is horizontally scanned, a change in spectral characteristics occurs. The process of scanning converts a two-dimensional image into a one-dimensional signal, with each successive image line appended to the end of the previous line. Such a process is illustrated in Fig. 6. Due to the fact that each line of the image changes very little from the previous line, the onedimensional signal created by scanning appears to be almost periodic with a "period" corresponding to the number of samples in each line. If this signal is fast Fourier transformed, the resulting frequency spectrum is impulsive due to the "periodicity" of the signal. A sufficient number of lines must be used, however, to avoid distortion of the frequency spectrum due to the window created by the fast Fourier transform array. A sample one-dimensional signal and its impulsive frequency spectrum are shown in Fig. 7. The frequency resulting from the "periodicity" of the scanned image, $\mathbf{T}_{_{\rm L}}$, and its harmonics possess the majority of the image spectral power. These frequencies will henceforth be referred to as primary frequency components. The remaining frequencies, occurring at all other positions within the spectrum will henceforth be referred to as secondary frequency components. These components occur at the







Fig. 6. Image Scanning Process: (A) Two-dimensional image, (B) Onedimensional scanned image



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frequency based on the total length of the scanned signal, $\mathrm{T}_{\mathrm{F}}^{},$ and its harmonics.

If such an impulsive signal spectrum was overlapped, as shown in Fig. 5, then it would be possible to distinguish the primary frequencies of the overlapped envelopes from the primary frequencies of the central envelope, provided the value of $_{0}$ was adjusted such that the primary components of the two envelopes did not overlay one another. If the primary frequencies of the overlapped envelope are removed by filters, the resulting spectrum will approximate the envelope of the unaliased spectrum. This procedure is illustrated in Fig. 8.

In order to prevent frequencies from overlaying one another, the proper value of ρ must be obtained. To accomplish this, the value of r, determined by the sample spacing, must be adjusted to lie between one and two. If the spacing between secondary frequencies is invarient, then changing r from one to two will result in the coincidence of the primary frequencies of the central envelope and the overlapped envelope. If r is chosen to be some intermediate value, the overlapped primary frequencies will appear between the central primary frequencies, making a filtering operation possible.

If only the frequencies between zero and $\frac{\omega_m}{r}$ are known, the overlapped primary frequencies may be used to simulate the missing central primary frequencies. In order to satisfy the condition that the spacing between secondary frequency components remain invariant as sample spacing is changed, the number of points comprising the one-dimensional signal must remain constant. This will result in the highest spectral frequency being $\frac{\omega_m}{r}$. In order to obtain the



Fig. 8. Allased Frequency Envelope After Filtering

primary components of the original envelope which lie between $\frac{m_{\rm in}}{r}$ and $m_{\rm m}$, it is necessary to rely on the fact that the fast Fourier transform of a real-valued signal will produce a conjugate symmetric array. Thus the values determining each envelope are conjugate symmetric about the midpoint of the envelope. It can be observed in Fig. 9 that, by using the conjugate of the values which determine the overlapped primary frequencies, the values used to find the missing central envelope primary frequencies are obtained. Therefore, if the primary frequencies of the overlapped power spectrum are filtered from the central envelope between $\frac{2-r}{r}m_{\rm in}$ and $\frac{m_{\rm in}}{r}$ and translated to the positions of the missing primary frequencies from $\frac{m_{\rm in}}{r}$ to $m_{\rm in}$, a restoration of the primary frequency components of the original, unaliased spectrum will be obtained.

If only the primary components are filtered and translated however, discontinuities will exist in the frequency spectrum between these frequencies. Such discontinuities, which result from the missing secondary frequency components, will cause distortion to appear in the reconstructed one-dimensional signal. The distortion caused by missing frequency components is referred to as Gibb's phenomenon.

<u>Gibb's Phenomenon</u>. In order to understand Gibb's phenomenon, the nature of the discrete Fourier series must be considered. Oppenheim and Schafer (Ref 10:88) express the equation for the discrete Fourier series as

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} x(k) e^{j\left(\frac{2\pi}{N}\right)nk}$$
(4)

where x(n) is a periodic, discrete series with one period containing



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exactly N samples and X(k) is a periodic sequence of N Fourier coefficients. If all N coefficients, X(k), are non-zero valued and, for some reason, one or more is changes to a zero value, the series, x(n), will be altered. If the artificially zeroed coefficients occur at the highest values of the index of summation, k, then the alteration will be confined to the high-frequency transitions in the series, x(n). The changes to the original signal will appear as lower frequency oscillations occurring in the vicinity of the rapid changes of the signal which the series represents.

As a result of the discontinuities which occur in the reconstructed image spectrum, some Gibb's phenomenon will be present in the signal obtained from this spectrum. The significance of this presence depends on the magnitude of the discontinuity in the spectrum, the composition of the natural frequencies within the image, and the data distortion caused by Gibb's phenomenon in the impulsive response of the filter functions. In order to perform the linear convolution required for filtering the aliased signal, zeros must be appended to both the signal and the filter function to prevent a circular convolution. Besides causing Gibb's phenomenon, the appending of these zeros results in the interpolation of points between values in the frequency spectra.

Interpolation. As stated in Gold and Raider (Ref 7:199), a function which has been sampled according to the Nyquist criterion may be precisely interpolated by fast Fourier transforming the series of samples, appending any number of zeros to the frequency spectrum above the highest non-zero frequency, and inverse transforming the extended array. This process will produce an interpolated version of the sampled function.

If the function to be interpolated has not been sampled at the Nyquist rate, interpolation will not be exact as a result of the aliasing phenomenon previously discussed. If a continuous, non-zero function is extended by adding zero values to it, the spectrum of the function will be continuous and exhibit Gibb's phenomenon. If a sampling of the same function is performed and zero values appended to that sequence, the fast Fourier transform of that extended sequence will be a periodic series whose values are a sampling of the continuous spectrum with the Gibb's phenomenon present. An example of this is shown in Fig. 10. Because both the filter functions and the source data are non-zero at the point of extension, interpolation will not be exact.

The previous review of interpolation, Gibb's phenomenon, aliasing and sampling theory provide a background for the alias filtering procedure used in this thesis. These principles, along with the work of other authors, contributed to the formulation of this algorithm.

Supporting Research

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Although much work has been accomplished in the areas of image enhancement and image restoration, a search of image processing literature reveals that little has been published in the area of alias removal in images. Many authors have, however, published work which contributes directly to the concept of alias filtering.

Related Enhancement Research. Authors such as Harry C. Andrews (Ref 1,2,3,4, and 11), Richard P. Kruger (Ref 2 and 11), and A.G. Tescher (Ref 1 and 2) have investigated many procedures and developed algorithms designed to improve the quality of existing images and



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restore data which has been degraded in a known manner. None of these procedures, however, address the possibility of using an a priori knowledge of image spectral characteristics to allow restoration of a digitized image degraded by aliased frequency components.

Work has been accomplished on the impulsive nature of the horizontally scanned video signal. Although this work dealt with continuous television image signals, it is, in general, applicable to the discrete domain of digitized images.

Impulsive Image Spectra. L. E. Franks (Ref 6), in "A Model for the Random Video Process," has derived simple expressions for the autocorrelation function and power spectral density of a generalized video signal. All assumptions which he made were based on the statistical properties of image data and, therefore, are independent of data variations due to image content. His results show that the power spectral density of a horizontally scanned image is highly impulsive. The power in the spectrum is concentrated at multiples of the line scan and frame scan rates. An example of this type of spectrum is shown in Fig. 11.

The application of these results to a digital image shows that the same type of spectrum will occur if the image is horizontally scanned. The line scan rate of Franks' video signal corresponds to the "period" of the one-dimensional signal discussed earlier (page 9). This period produces the primary components of the frequency spectrum. The frame scan rate corresponds to the period of the complete array. This period produces the smaller secondary components of the spectrum. This initial formulation of the characteristics of image spectra led to further efforts to use these properties.



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<u>Frequency Compression and Expansion</u>. The research accomplished by Alan R. Billings and Armando B. Scolardo (Ref 5) on a bandwidth compression and expansion system for television signals has shown that the positions of the primary frequency components can be varied by adjusting the line scan rate of the system. The lowering of the scan rate results in the compression of the primary frequency components. Based on an a priori knowledge of this compression factor, the authors demonstrated that the frequencies can then be expanded back to their original locations. The end result was a spectrum containing the primary frequency components, but also containing gaps where secondary frequencies had been lost due to the compression.

<u>Frequency Interleaving</u>. A paper by Leonard S. Golding and Ronald K. Garlow (Ref 8), entitled "Frequency Interleaved Sampling of a Color Television Signal," provided a foundation for the technique of filtering desired primary frequency components from a spectrum. By interleaving the primary frequency components of two different signals, luminance and chrominance, the total bandwidth required to transmit a color television signal was reduced to that used for a black-and-white signal. The two separate color signals were then filtered from one another and used to reconstruct the color television image.

The works described in the preceding paragraphs provide the necessary evidence that the procedure of increasing image resolution using alias filtering is possible. By using the spectral properties of a horizontally scanned image and applying techniques to remove aliased frequencies and adjust the positions of the primary frequency

components, the reconstruction of a higher resolution image spectrum can be accomplished.

Problem Statement and Scope

The purpose of this research was to determine the feasibility of increasing the resolution of a digitized image by selectively filtering the aliased primary frequency components and translating them to positions which would be occupied in an un-aliased, high resolution spectrum. This problem was divided into three distinct areas.

<u>Spectrum Selection</u>. The selection of a spectrum with the correct properties for alias filtering involved two factors. The image spectrum needed was one in which the primary frequency components were readily identifiable. This was to permit the visual verification of the positions of aliased components calculated from the start of frequency overlap, occurring at $\frac{\Omega v_{\rm m}}{r}$. The spacing between primary frequencies also had to be large enough to provide a "buffer" region between aliased primary frequencies and non-aliased primary frequencies. This was required to permit space for filter cut-off frequency bands.

<u>Filter Design</u>. The design of multiple passband filters to remove and translate primary frequency components involved the determination of the passband locations and the selection of functions to provide adequate filtering. The problem of locating the passbands of the filters was tied directly to the selection of the spectrum to be filtered. The problem of selecting functions which provided adequate cut-off bands and minimum distortion of the filtered components was one which proved to be most critical to the results of

the filtering procedure.

<u>Application of Filters</u>. The problem of applying the filters involved the design of a procedure which would accept the input image, produce the selected image spectrum, apply the filters, and then output the processed image.

Limitations. The problem was limited to the restoration of a single test image from a lower-resolution version. The input and output sample spacing to be used for the filtering process was limited to a single set. This was done since the filter design is specific for a given set of input and output sampling rates. The output sample spacing was limited to one compatible with the digitizing equipment, in order to reduce the post-filter manipulation of image data.

Assumptions

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The validity of the algorithm which was used in the reconstruction of an enhanced resolution spectrum was based on certain assumptions.

<u>Nyquist Sampling</u>. As discussed previously, the fact that a digitized image may not have been sampled at the Nyquist spacing requires the designation of an "arbitrary Nyquist rate" for the purpose of determining the location of aliased frequencies. Based on a chosen sample spacing, the assumption made was that any existing alias was negligible. By making this assumption, the frequency overlap due to further undersampling could be calculated and the locations of aliased primary frequencies identified.

Noise Level. Once digitized, the test image was assumed to contain no noise. This assumption allowed the treatment of all frequency components as image data. Thus, the reconstructed image

spectrum was based on the exact chosen target image spectrum, and no attempt to eliminate noise already present in the target image was undertaken.

<u>Quantization and Truncation Errors</u>. The errors introduced into the output image due to quantizations and truncations performed by the filtering software were assumed to be due to the filtering process. This simplified the overall judgment as to the success or failure of the procedure.

<u>Frequency Data Content</u>. The procedure of filtering and translating primary frequencies was based on the assumption that these frequencies carried the majority of image information. The research of Billings and Scolardo (Ref 5) relied on the fact that the line scan period and its harmonics contain the essential information necessary for signal reconstruction. Based on this result, the alias removal and translation procedure processed only the primary frequency components.

Equipment

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The equipment used in support of this research consisted primarily of image digitization equipment and digital computers. The digitization equipment was provided by the Air Force Avionics Laboratory, Wright-Patterson AFB, Ohio. The digital computer used was a Control Data Corporation 6600 System operated by the Aerospace Systems Division, Wright-Patterson AFB, Ohio.

Digitization Equipment. The image digitizer used was a Dicomed Model D57. This device, in conjunction with a Digital Equipment Corporation PDP 11/45 and its peripheral devices, produced the test image used during the research project.
Both the Dicomed Model 15 and PDP 11/45 tape units were employed to record the digitized imageon nine-track magnetic tape. The Dicomed unit was used for the output of the image to the image recorder.

The Dicomed Model D47 image recorder was used to produce the output image from the data stored on nine-track magnetic tape. The single scan speed of this unit is matched to the drive speed of the Dicomed tape unit.

The PDP 11/45 system was employed as an intermediate data processor during the production of the test image. This system was connected through interfaces with the three Dicomed devices, which were capable of functioning as computer peripherals.

<u>Digital Computer</u>. The CDC 6600 System was used to accomplish all image manipulations. These included data formatting, spectral analysis, filter design and realization, and the image filtering procedure. The peripheral devices used included the Digigraphic console, Calcomp plotter, seven-track and nine-track tape drives. The Digigraphic console was used for designing filters, windows, and performing interpolation tests on various spectra. The Calcomp plotter was employed to provide average spectral plots, final filter responses, and tests of the filters. The tape drives were used for input and output of image data as well as the intermediate storage of data in CDC 6600 format.

Summary

The background theory and the explanation of the problem to be resolved by this research provide a foundation for the procedures

used to accomplish image resolution enhancement using alias filtering. The following chapter will discuss the details of these procedures.

II. Procedure

Introduction

The procedure follows the three parts of the problem outlined in the previous chapter. The creation of source data, data reformatting, and spectral plotting procedures support the first problem segment. The filter design and realization procedures support the second problem segment. The actual image filtering supports the solution of the final problem segment. An additional procedure, that of interpolation, was developed in support of the overall goal of increased image resolution.

Creation of Source Data

The initial step in the research was the selection and digitization of the image source data. The procedure used for image selection was primarily qualitative judgment, while the digitization procedures were dictated by equipment parameters and limitations.

<u>Selection</u>. The selection of a test image was made from a collection of photographic positives which were made available by the Air Force Avionics Laboratory. Based on the available images, the test image was chosen for reasonably high contrast to provide as much high-frequency content as possible.

<u>Digitization</u>. The slow scan rate of the image digitizer produced a data stream which was not compatible with the Dicomed tape unit. To circumvent this problem, the data was sent from the digitizer to the PDP 11/45, where it was stored and then transferred onto nine-track magnetic tape. To accomplish this transfer, a Fortran program was developed. A copy of the program source listing is included in the Appendix. The digitized test image is shown in Fig. 12.

Data Format Change

Prior to performing any image manipulations, the data required a change in format. The test image had originally been recorded in an eight-bit-per-picture-element format on nine-track magnetic tape. If the tape was read directly into the memory of the CDC 6600 computer, each 60-bit memory word would contain seven-and-one-half image points, or picture elements. To resolve this problem, Fortran programs were used to convert the source tape into three tapes which had picture elements in a CDC 6600 format.

Data Unpacking. Besides the changing of data format, the unpacking program also changed tape format. The program initially read 2048 picture elements from the nine-track source tape. These elements were placed in a buffer array and then bits were transferred in segments to an output array. The bits were arranged so that the eight bits comprising one picture element were stored in the low-order bits of a 60-bit word. The output array, 2048 words long, was then written onto a seven-track magnetic type. This procedure was repeated until approximately one-third of the source image had been processed. The use of seven-track tapes as an output medium was based on the fact that the CDC 6600 computer used seven-track tape units as its primary system. The division of the test image into thirds was necessary to allow for the standard 2400-foot tape length. The source listing for the unpacking program is



included in the Appendix. Once the data from the test image had been converted to a 60-bit per picture element format, the next procedure was to insure that the reverse formatting could be accomplished.

<u>Data Packing</u>. A data packing program was used to take the seven-track, 60-bit data format and write it in a nine-track, 8-bit format. The details of this operation are exactly the reverse of those of the unpacking program. The source listing for one version of the packing program, designated Pack 4, is included in the Appendix. The "4" designates this version as one which packs an image which has been sampled at every fourth point during horizontal scanning. The program repeats each point four times to attain a 2048 picture element resolution.

The unpacking and packing programs were tested for correctness by unpacking the test image, repacking it, and printing the result. This procedure was accomplished to insure that, if necessary, a complete test image could be reassembled from its unpacked segments. The next procedural step was the plotting of image spectra from the first of the three reformatted tapes.

Spectral Plots

The purpose of the plotting of average frequency spectra of the test image was to select the spectrum with the most easily identifiable alias. In order to provide a "normalized scale" along the frequency axis of the spectral plots, the discrete Fourier series definition of radian frequency was used. The "periodicity" inherent in the fast Fourier transform provides a radian frequency of 2π radians for each "period" of the sequence. Due to the real-valued

data, the fast Fourier transform is conjugate symmetric about the radian frequency π . Therefore, the horizontal frequency axis of the spectral plots was scaled from 0 to π . The vertical axis, labeled "Magnitude in dB," reflected the relative power of each frequency compared to a unity power signal. In order to arrive at this number, the value of

20 log | M

was computed for each complex value, M, in the fast Fourier transform of the image data. These scales were used along the axes of the resulting spectral plots. Two parameters of interest were varied to observe their effect on frequency spectra. The first was the amount of data.

Data Block Size. In order to determine the spectral variation occurring due to the horizontal scanning process, the amount of data which was scanned was varied. Since the amount of data which could be scanned and fast Fourier transformed at one time was limited by computer memory space, it was not possible to use the full 720-by-2048 picture elements present on a single seven-track tape. Therefore, image spectra were produced by scanning segments of an image, henceforth referred to as blocks. The blocks were horizontally scanned and fast Fourier transformed one at a time and then averaged to produce the spectrum of an average block of data. Both the horizontal and vertical dimensions of the block were varied in order to obtain various spacings and amplitude relationships between primary and secondary frequency components. Once this procedure had resulted in the choice of a specific block size, the next parameter to be varied was the image sample spacing.

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<u>Sampling</u>. The sample spacing used during the horizontal scanning process was varied to determine its effects on aliasing and thereby allow the selection of the most favorable average frequency spectrum for filtering. The average image spectra produced from various samplings were then evaluated visually for the presence of aliased primary frequency components. A sample source listing of a spectrum analysis program is listed in the Appendix. The selection of a suitable spectrum for filtering, henceforth referred to as the candidate spectrum, defined the specifications for the alias removal filter. Likewise, the selection of a spectrum to which the candidate spectrum could be restored, henceforth referred to as the target spectrum, set the requirements for a translation filter.

Filter Design

The design of the digital filters to remove the aliased primary frequencies of the candidate image spectrum and translate them to the new positions of the target spectrum was realized using software. The filter designs were first specified in the frequency domain and then synthesized from analytic functions.

<u>Frequency Specification</u>. The frequency specification of the passbands and stopbands for the alias removal and translation filters was accomplished from the visual and mathematic location of the alias components within the candidate spectrum. The removal filter was designed to suppress the aliased primary frequencies of the candidate spectrum, while the translation filter suppressed the un-aliased components. Copies of the candidate spectrum were multiplied by the impulse response of each filter, and the resulting spectrum from the translation filter was then flipped end for end and

appended to the spectrum produced by the removal filter. The resulting combined spectrum was then extended, inverse fast Fourier transformed, and sampled to achieve the correct primary frequency location and number of samples as those of the target spectrum. Once this procedure had been accomplished on the average spectrum of the candidate image, the synthesis of analytic functions which would provide a close approximation of these filters was started.

<u>Filter Function Synthesis</u>. The basic function used to obtain the required passbands for the filter responses was the sinc function. This function, defined as

sinc (x)
$$\stackrel{\Delta}{=} \frac{\sin(\Pi x)}{\Pi x}$$
 (5)

when multiplied by cosine functions of various frequencies, produces multiple passbands in the frequency spectrum of the function. Thus, a function of the form

$$f(n) = \sum_{k=0}^{N-1} A_k \operatorname{sinc} (B_k [n-p]) \cos (C_k [n-p])$$
(6)

where p is the point of symmetry of the filter function, will produce a frequency response containing N passbands. These passbands have an amplitude which is a function of A_k , a passband width which is a function of B_k , and occur at positions within the spectrum which are determined by C_k . The adjustment of these constants was accomplished using the CDC 6600 Digigraphic console. A source listing for the filter design program is included in the Appendix. The filters which had been designed were then used by an application program to filter an image.

Image Filtering

The image filtering program performed the image blocking necessary for the production of the candidate spectrum, applied the two filters to identical copies of the spectrum of each block, combining and scaling the filtered spectra, and reassembled the image from the filtered blocks.

<u>Image Blocking</u>. The test image was blocked in the manner previously described in the section on spectral plots (page 32). The filtering program, however, processed image blocks one at a time instead of averaging them. A series of image lines were first read into a holding array, then each block was taken from the holding array in a left-to-right, horizontal scanning fashion, and processed.

<u>Filtering and Scaling</u>. The application of the filters to each block of image data was accomplished by a multiplication in the frequency domain. The spectrum of each block of image data was interpolated to provide the correct number of points for a linear convolution with the filter responses. The filter responses were likewise interpolated. The scaling procedure used to adjust the length of the filtered spectrum, henceforth referred to as the restored spectrum, was varied during the course of the research.

The initial procedure used was to append zeros to the restored spectrum in order to obtain a multiple of the number of points required in the restored block. The inverse transformation of this extended spectrum was then sampled to achieve the same number of points as the target image block. This procedure employed filter arrays of identical length.

The second procedure used was one in which unequal-length filters were applied to the copies of the candidate spectrum. The filters

were scaled so that, after the combination and truncation procedure, the resulting restored spectrum contained a multiple of the number of points required for the target spectrum. The restored spectrum was then sampled and inverse transformed. After filtering the resulting restored image blocks were then ready for reassembly.

Reassembly. Once filtered, the individual blocks were reassembled into a restored image. Due to the block-by-block filtering, a holding array was also used for storing partial rows of output image data. As blocks were processed, they were reassembled in the same horizontal fashion as they had been removed. Once a complete set of blocks, equal in length to the lines of the target image, was assembled, the output array was read onto seven-track magnetic tape and the next set of lines was read from magnetic tape into the input holding array. In this manner, the entire image was processed. A sample source listing of the filtering program is included in the Appendix. The completely processed image was then packed from seven-track to nine-track tape. The method of appending zeros to a frequency spectrum, which was used during the filtering procedure, generated interest in the use of the same procedure for the direct increase of image resolution.

Interpolation Enhancement

The filtering procedure described in the preceding section evolved into one using interpolation as a direct algorithm for increasing the resolution of a digital image. The interpolation procedure was accomplished by appending zeros to the frequency spectra of blocks or lines of image data, and inverse transforming the extended arrays. A sample interpolation program source listing is

shown in the Appendix. Although not related to the original procedure of resolution enhancement using alias filtering, the interpolation algorithm did support the more general goal of increasing the resolution of a digitized image.

Summary

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The procedures outlined in this chapter followed the basic divisions of the problem. Some were developed from the intermediate results presented in the next chapter. One such procedure was the interpolation algorithm.

III. Results

Introduction

The intermediate results obtained from the frequency spectra, filter designs, and filtered images determined the direction of subsequent work. In addition, the results of the first filtering experiment led to experiments employing sections of the filter program which, in turn, led to the use of interpolation as a resolution enhancement algorithm.

Frequency Spectra

The average frequency spectra produced by variations of the image data block size and sample spacing showed that these parameters had some effect on spectral characteristics. The degree of change within the spectrum due to each parameter varied widely. Initial experiments involving block size variation proved to have the most effect on the frequency spectrum.

<u>Block Size</u>. The variations in the image block size produced changes in the amplitude and spacing relationships of the primary and secondary frequency components. Initial "blocks" of data consisted of one complete line of image data. This spectrum, shown in Fig. 13, revealed a concentration of spectral power at the low frequencies, but no impulsive characteristics. Further experiments, employing the blocking and scanning of multiple line segments of data, did provide impulsive spectral characteristics.

Due to the length of image lines, blocks were altered to use line segments rather than entire lines. The spectrum of the first



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720 lines of the test image, blocked in 32-by-32 segments, revealed the desired impulsive characteristics. As shown in Fig. 14, the 0 to π frequency range contained 16 primary frequencies, corresponding to half of the 32 samples comprising the line segment scanning "period." Based on this spectrum, a 32-by-32 picture element block was used during the initial sample spacing variations.

<u>Sample Spacing</u>. Changes in sample spacing produced variations in the location and spacing of primary frequency components within the spectrum. The first spectrum, produced from 32-by-8 blocks sampled at every fourth point, had four primary frequency components. As sampling was changed to every fifth point, the primary components spread and an aliased component appeared at the high frequency end of the spectrum. The spectra produced from fourth, fifth, and seventh point samplings are shown in Fig. 15, 16, and 17.

Using these results, the candidate and target spectra were chosen. The spectrum produced from the seventh-point sampling was chosen to be the candidate spectrum. The target spectrum was chosen to be the fourth-point sampled version, since this was the next highest sampling density with no observable alias. The results of these frequency spectra experiments provided the specifications for the alias removal and translation filters.

Filter Design

The design of the two digital filters for alias removal and translation resulted in the software realization of filters which met the specifications imposed by the choice of candidate and target spectra.



Fig. 14. Average Frequency Spectrum of a Scanned 32-by-32 Block

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Fig. 15. Average Frequency Spectrum of Fourth-Point Sampled Image



Fig. 16. Average Frequency Spectrum of Fifth-Point Sampled Image

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Fig. 17. Average Frequency Spectrum of Seventh-Point Sampled Image

<u>Frequency Specification</u>. The results of the frequency specification of the two filters provided a basis for the adjustment of the filter functions. The location of the passbands and width of the transition bands within the removal filter resulted from calculations based on the candidate spectrum. The specifications of the translation filter were fixed by the choice of the target spectrum.

The results of a test filtering of the average seventh-point spectrum indicated that the design would accomplish the needed alias removal and translation. The test showed that the aliased components had been removed from the primary frequency components and translated to positions corresponding to those occupied in the fourth-point sampled target spectrum. Based on this result, the design of the filter function was accomplished.

<u>Filter Functions</u>. The result of the design of the filter functions showed that the frequency specifications could be met. Once these design results had been achieved, the filters were tested in the same manner as the frequency specification filters. The result is shown in Fig. 18. The filters were then ready for application to the candidate spectrum produced from the test image.

Filtered Images

The result of the filtering of the test image revealed that the procedure used induced more image distortion than it was designed to eliminate. Although changes introduced to the filters did improve the resulting image, the final image was still highly distorted.

Initial Filtering. The initial filtering of the test image was performed using filters without a window. The image was produced



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using equal-length filters of 256 points. This image, shown in Fig. 19, was almost totally corrupted with noise. Although the low frequency data was still faintly distinguishable, the image had a regularly spaced, horizontal and vertical distortion. In an effort to determine which portion of the filtering software was inducing this distortion, the test image was filtered using only the alias removal filter. The results of this experiment varied from those of the first.

<u>Alias Removal</u>. The result of the alias removal filtering with no translation of the aliased components showed a reduction in the magnitude of distortion. Instead of the previous horizontal and vertical distortion, distortion was now predominantly horizontal, as shown in Fig. 20. This image revealed that an "averaging" of light levels throughout each image block was occurring. This averaging produced the bands of vertical distortion observed. The fact that severe distortion remained resulted in an experiment using refined filters to again attempt both removal and translation of the aliased frequency components.

Refined Filtering. The result of image filtering using windowed filters was better than that of the initial filtering. The image, shown in Fig. 21, revealed a lesser degree of distortion than either previous filtering. Although the overall distortio. level had been lowered, more pronounced vertical bands of distortion were introduced than in the removal filtered image. The fact that distortion levels throughout the filtering remained quite high resulted in an experiment which used interpolation only to achieve an increase in image resolution.







Interpolation Images

The interpolation of points to increase the resolution of a digitized image produced better results than did the filtering experiments. Although good results were obtained at low interpolation ranges, distortion did occur at the higher levels attempted.

<u>Blocked Images</u>. During the search for the source of distortion in the filtering program, a program was designed which interpolated a seventh-point sampled block of image to a fourth-point reconstruction. The results of this experiment, shown in Fig. 22, revealed lower distortion than occurred within any of the filtering programs. The only significant distortion occurred in vertical bands which corresponded to the boundaries of the image blocks. The distortion occurring in blocks in the lower right quadrant of the image was due to exceeding the dynamic range of the recording equipment. This error was later corrected by adjusting interpolation software. In order to explore the nature of the significant boundary distortion, an experiment was performed on the CDC 6600 Digigraphic console.

<u>Blocking Distortion</u>. The blocking distortion encountered in the interpolated image was simulated by the interpolation of an exponential function. The procedures used to extend the spectrum of the test image were applied to an exponential function of the form

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The original function and the result of the inverse transformation of the extended spectrum are shown in Fig. 23 and 24. The distortion in the interpolated version of the function was most pronounced at the initial and terminal regions of the function. Based on this result and those of the interpolations performed on image blocks,





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the interpolation procedures were modified to use entire lines of image data.

Interpolation by Lines. The modification of the interpolation procedure to use entire lines of image data, rather than blocks, resulted in some excellent restorations of the high resolution version of the test image. In order to determine any variation in the degree of interpolation distortion levels, experiments were conducted using three pairs of test images sampled at every fourth point, every eighth point, and every sixteenth point. Each pair of images sampled at a given rate was divided into an interpolated and repeated version. The fourth-point interpolated image and fourth-point repeated image are shown in Fig. 25 and 26. The eighth-point interpolated and repeated images are shown in Fig. 27 and 28. The sixteenth-point interpolated images are shown in Fig. 29 and 30. These images revealed that the level of distortion present in an interpolated image increased as the sample spacing increased.

To provide some intermediate levels of interpolation for comparison to the fourth-, eighth-, and sixteenth-point sampled images previously interpolated, experiments on sixth- and twelfthpoint sampled images were performed. The results of these interpolations are shown in Fig. 31 and 32. The distortion levels present in these images lie between those of the neighboring interpolations previously accomplished.

Summary

The results of the experiments performed show that the procedure of alias removal and translation produces more distortion than it
















was designed to eliminate. Although results were very poor, the search for the cause of the distortion initiated experiments which successfully increased the resolution of an image with no visible distortion. The interpolation of images was found to be a usable technique for image resolution enhancement.

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IV. Conclusion and Recommendations

Introduction

The results of the previous chapter led to the conclusion that the enhancement of image resolution using the alias removal and translation filters designed may not be feasible. These results do, however, support the conclusion that the accurate interpolation of an image signal can increase image resolution. Improved methods, in addition to that of appending zeros to the image spectrum, may provide better interpolation than was accomplished in this study.

Alias Filtering Procedure

<u>Present Design</u>. During the sequence of filtering experiments which were performed, the quality of the filtered image improved. The result of the first experiment using the filters, however, was a virtually unrecognizable image. The elimination of filtering showed that the interpolation of a blocked image does induce some distortion, but no more than 5 to 10 percent of that observed as a result of the first filtering experiment. Therefore, it appeared that the filtering and spectral combination using blocks of data was causing the majority of the image distortion. The second experiment, which re-inserted only the alias removal filter, led to the conclusion that the removal filter did cause much of the distortion. Based on this result, it appeared that the removal of any frequency components from their normal power range would cause image distortion. The removal of significant power levels at higher frequencies from the test image may have accounted for the black horizontal distortion bands which appeared within the second filtered image. The number and spacing of these bands was coincident with the image blocking performed. In addition, the "averaging" of light intensity throughout each block of image may have been a function of the removal of high-frequency components.

The third filtering experiment, which re-inserted the translation filter into the program and used a window to shape both filter responses, showed the least distortion of all. The distortion which did occur again followed the blocking boundaries. Instead of being all black, however, the horizontal distortion was composed of both black and white light levels. In addition, less of the "averaging" phenomenon was observable in other areas of the image. From this result, it was concluded that the re-insertion of the frequency power, which had been removed in the second experiment, lessened the degree of signal distortion in each line segment. The changes in light level within the distortion bands may have resulted from the artificial high frequencies created within the restored spectrum.

Improved Filter Design. The modification of the frequency removal and translation filters to manipulate less spectral power may reduce the amount of distortion observed in the filtered test

image. If the alias removal filter were changed to suppress the aliased primary frequency components to the level of surrounding secondary components, rather than to levels below the surrounding frequencies, the spectral discontinuities would be minimized. If the aliased components were then translated and rescaled to lower power levels, frequency spectrum discontinuities would again be minimized. Filters employing this concept should significantly reduce the horizontal band distortion which dominated the filtering experiment results. Regardless of the outcome of future alias filtering tests, the success encountered using interpolation procedures as a direct technique for increasing resolution warrants further investigation.

Interpolation Procedure

The interpolation procedure used in this research project provided excellent results up to a density of approximately six interpolated points to every original data point. With refinements, it may be possible to extend the interpolation much further.

The conclusion drawn from the various interpolation experiment results was that the end-to-end arrangement of line segments, when interpolated, produced Gibb's phenomenon in the region of the discontinuities occurring at the start and end of each segment. When entire image lines are used, the distortion caused by the artificial discontinuities introduced at the end of each line appears only at normal image boundaries and is not apparent to the eye.

The results of fourth-point, sixth-point, eighth-point, twelfth-point, and sixteenth-point interpolations indicate that Gibb's phenomenon increases within the image as the larger sample

spacings are interpolated back to original resolutions. The larger sample spacings may be thought of as increasingly undersampled signals. Their frequency spectra, once zeros are appended, resemble high-resolution spectra which have been low-pass filtered. It is, therefore, apparent that Gibb's phenomenon will increase as sampling is decreased.

Modified Method. By modifying the image frequency spectrum, the Gibb's phenomenon present due to interpolation may be reduced. If the frequency spectrum of an image were extended and smoothed so that no abrupt changes in frequency power occurred, there could be less Gibb's phenomenon resulting from the interpolation. Two possible methods to accomplish this goal involve the use of a non-zero extension of the frequency spectrum and a smoothed, bandlimited version of the spectrum. In addition to these, a twodimensional interpolation may also produce better interpolation levels.

The extension of an image frequency spectrum by appending a non-zero function could produce a better interpolation than that which used zeros. Preliminary tests of this procedure, carried out on the CDC 6600 Digigraphic terminal, showed that the distortion of an interpolated exponential can be reduced significantly by appending a non-zero function which eliminates the discontinuity caused by using zeros.

Experiments which apply this technique to image spectra should be undertaken. If a scaled version of the low-frequency spectrum could be appended to that spectrum, a better interpolation may result. Another possibility would be to use an analytic function to provide

a smoother reconstruction of the spectrum than is done by appending zeros. The results of such an interpolation, however, will depend on the natural discontinuities within the data, and the level of distortion which is perceptible to the human eye.

The smoothing of the non-bandlimited spectrum of an image may also enable better interpolation. If the image spectrum were filtered in such a way as to retain the low-frequency spectrum at its original power level, but scale the higher frequencies smoothly to zero, then the spectrum produced would be bandlimited. The appending of zeros to this power spectrum would result in an interpolation without error. If the filter could be designed to accomplish this operation with no perceptible loss of the high-frequency data content of the image, then the image data could be interpolated as a bandlimited signal. The level of interpolation attainable without perceptible distortion, again, will rely on any natural discontinuities within the image data and the level of distortion induced by the smoothing.

Experiments should be conducted to determine the feasibility of such a filter design. If the desired effect is achieved, the filtered image would provide the basis for interpolations which could exceed the levels achieved in this research.

In addition to these one-dimensional spectrum experiments, two-dimensional interpolations should also be undertaken. The onedimensional spectra used for interpolation in this thesis resulted from the expansion of the alias removal and translation technique, which relied on the nature of the one-dimensional spectrum produced by scanning an image. A two-dimensional interpolation may permit higher levels of interpolation without perceptible distortion.

Summary

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The conclusions which were made from the results of the experiments were unfavorable to the alias filtering and translation procedure. Based on the filters used, the technique produced very poor quality images. The possibility exists that the improvement of filter designs could reduce the level of distortion in the filtered image. Based on the results of experiments conducted using interpolation to increase image resolution, it was concluded that excellent results could be achieved to certain levels. Further research into the techniques of spectral extension and smoothing could produce even better results. Extension of all interpolation techniques to two dimensions may result in further increases in successful interpolation levels.

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Appendix

Image Processing Programs

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1.

Program	PROCESS:	Image Production Program
Program	UNPACK:	Image Reformatting Program
Program	PACK4:	Image Reformatting Program
Program	SPCANAL:	Spectral Plotting Program
Program	DIGFILT:	Filter Design Program
Program	FILTER:	Filter Application Program
Program	SPECT:	Interpolation Program

DATA IMID/20, 255, 262, 254, 0, 0, 265, 0, 0, 255, 255, 7, 257, 255, 7, DIGITIZED IMASE DATA ONTO NIVE TRACK MAGNETIC TAPE THIS PROGRAM USES PDP 11/45 SUBROUTINES TO LOAD INTEGER BUF= (2048,4), IMID(21), STATUS CALL TWRITE(RUFF(1, J), 1024,0) CALL TWAIT(0) IF (STATUS. VE. 0) GO TO 1100 CALL IID(IMID,0,STATUS) IF (STATUS.ED.1) 60 TO 500 CALL TINIT(STATUS) IF (STATUS.EQ.6) 60 TO 700 IF (STATUS. E0.6) GO TO 900 IN EIGHT PIT WORDS. CALL ID(BJF=(1, J), STATUS) \$271, 385, 393, 396, 432, 447/ CALL PK(BUFF(1, J), 2048) CALL IDR(0, STATUS) 00 300 I=1,512 00 100 J=1,4 4.1=f 002 00 CONTINUE STATUS=0

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WRITE (6,500) Format (" initialization parameter error") Go to 400 FORMAT (" DIGITIZER HARDWARE ERROR") GO TO 400 WRITE (6,1000) FCRMAT (" DIGITIZER TRANSFER ERROR") GO TO 400 WRITE (5,1200) FORMAT (" TAPE TRANSFER ERRO?") CALL IDR(1, STATUS) CALL TWEOF(0) WRITE (6, 300) CALL TRLSE CONTINUE ONE 1100 1000 006 002 001 009 300 001

************************ THIS PROGRAM JUPACKS IMAGE DATA FROM A NÍNE TRACK TAPE IN EIGHT BIT WORDS AND OUTPUTS IT TO A SEVEN TRACK KBLK IS THE NUMBER DF FOUR LINE DATA BLOCKS TO BE INPUT. INPUT DATA IS IN A 2048 WORD PER RECORD FORMAT. THIS LOOP WJRKS WITH 4 LINE BLOCKS DF DATA. DIMENSION IBUFT(274), IP(4,2055) TAPE IN SIXTY 3IT WORDS. KL IS THE BLOCK COUNTER. SET COUNTERS AND STOPS. INITIALIZE ARRAYS 00 20 JI=1,2055 00 10 K=1,274 00 30 L=1,4 I P (L, JI) = 0 I = 0 = 0 = 0LINTOT=720 CONTINUE KBLK = 180CONTINUE CONTINUE LINE=0 (= JX * * * 2000 10 000000000000 C O 0

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IF (KL .6T. KBLK) GO TO 210
THIS LOOP PLACES 4 LINES IN IP(1,1 THRU 2055) TO IP(4,1 THRU 2055).
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     THIS LOOP PLACES 7 SEOUPS OF 8 BITS INTO IP(L,1 THRU 7) 400ULO 15.
JM IS THE STARTING BIT.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 BUFFER IN UNTIL EOR ENCOUNTERED. THEN STOP. THIS IS ONE LINE.
L IS THE LINE NUMBER. JI IS THE ELEMENT OF THAT LINE.
THIS LOOP PLACES IBJF(1 THRU 274) INTO IP(L,1 THRU 2055).
                                                                                                                                                                                                                                                                                                            FORMAT(* EOF ENCOUNTERED DURING INPUT AT LINE *,14)
Go to 90
                                                                                                                                                                                                                                                                                                                                                                                               FORMAT (* JAZITY ERROR DURING INPUT AT LINE *, I4)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               CALL STRING(-8, IBUFT(LOC1), JM, IP(L, JI), 53)
                                                                                                                                                                                               IF (LINE .GT. LINTOT) G0 T0 210
BUFFER IN (1,1) (IBUFT(1),IBUFT(274))
                                                                                                                                                                                                                                                     IF (UNIT(1)) 90,60,70
WRITE(6,65) LINE
                                                                                                                                                                                                                                                                                                                                                                     WRITE (6, 80) LINE
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  L0C1=2*(I-1) + 1
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  00 130 I=1,137
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            JM = (J - 1) + 8 + 1
                                                                                                                                                                  LINE=LINE + 1
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              7,1=L 011 00
                                                                                                         00 150 L=1,4
                                                                                                                                                                                                                                                                                                                                                                                                                          GO TO 250
                          KL=KL + 1
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       1 + 1L = 1L
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           L0C2=2*I
                                                                                                                                                                                                                                                                                                                                                                                                                                                      CONTINUE
CONTINUE
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                CONTINUE
                                                                                                                                           0=1C
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 110
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CALL STRING(-4,IBUFT(LOC1),57,IP(L,JI),53)
CALL STRING(-4,IBUFT(LOC2),1,IP(L,JI),57)
THIS LOOP PLACES 7 5200PS OF 8 BITS INTO IP(L,9 THRU 15) MODULO 15.
2 HALF PIXELS INTO IP(L, 8) 400ULO 15.
                                                                                                                                                                                                                     4 LINES HAVE NOW BEEN UNPACKED INTO IP.
This Loop writes the 4 lines onto a 7 track output tape.
                                                                                                                                                                                                                                                                                                                                                                                                                                               ( +I +
                                                                                                                                                                                                                                                                                                                                                                      -
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                WRITE(6,240) LINE
Format(* 2421TY Error Juring Output at Line *,14)
                                                                                                                                                                                                                                                                                                                                                                      ENCOUNTERED ON 7 TRACK OUTPUT TAPE
                                                                                                                                                                                                                                                                                                                                                                                                                                              FORMAT (* NORMAL TERMINATION OF UNPACK AT LINE
                                                                                                                                               CALL STRING( -8, IBUFT (LOC2), JM, IP (L, JI), 53)
THESE THREE STATEMENTS PLACE
                                                                                                                                                                                                                                                                            WRITE(5) (IP(L, JI), JI=1,2048)
                                                                                                                                                                                                                                                                                                                 F (UNIT(5)) 40,180,230
                                                                                                                                                                                                                                                                                                                                                                                                                             WRITE (6, 220) LINE
                                                                                                           JM=(J-1)*8 + 5
                                                                                                                                                                                                                                                                                                                                                                      FORMAT (* EOF
                                                                                                                                                                                                                                                          00 170 L=1,4
                                                                                          00 120 J=1,7
                                                                                                                                                                                                                                                                                                                                                     WRITE (6, 190)
                                                                                                                                                                                                                                                                                                                                                                                       60 TO 215
ENDFILE 5
                                                                                                                                                                                                                                                                                                                                                                                                                                                                GO TO 250
                  1 + 1l = 1l
                                                                                                                             1 + 1l = 1l
                                                                                                                                                                                                                                                                                                                                   ENDFILE 5
                                                                                                                                                                                                                                                                                                CONTINUE
                                                                                                                                                                                   CONTINUE
                                                                                                                                                                 CONTINUE
                                                                                                                                                                                                   CONTINUE
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     STOP
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       CNE
                                                                                                                                                                 130
                                                                                                                                                                                                                                                                                                                                                                                                        210
                                                                                                                                                                                                                                                                                                                                                                                                                          215
                                                                                                                                                                                                                                                                                                                                   130
                                                                                                                                                                                                                                                                                                                                                                       130
                                                                                                                                                                                                                                                                                                                                                                                                                                             220
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  240
                                                                                                                                                                                                                                                                                                170
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************************** TAPE IN SIXTY BIT WORDS AND DUTPUTS IT TO A NINE TRACK TAPE IN EIGHT BIT WORDS. INPUT DATA IS IN 512 WORD PER RECORD FORMAT. Each output word is repeated four times to attain an output LINTOT IS THE STOP SET AT THE TOTAL NUMBER OF INPUT LINES. THIS PROGRAM PACKS IMAGE DATA FROM A SEVEN TRACK PROGRAM PACK4 (INPUT, JUTPUT, TAPE7, TAPE9, TAPE6=0UTPUT) DIMENSION IPEL (2055), IRUFT (274) INITIALIZE ARRAYS. RECORD LENGTH OF 2048 W020S. DIMENSION INP(512) LINE IS THE LINE COUNTER. SET COUNTERS AND STOPS. 00 10 JI=1,2055 IPEL(JI)=0 00 20 K=1,274 -LINE=LINE + I B U F T (K) = 0LINT07=714 CONTINUE CONTINUE CONTINUE LINE=0 10 200 UM 0 0000000000000000 U

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56 BITS HAVE NOW BEEN PACKED INTO IBUFT. NOW PACK THE LAST 4 BITS. NEXT, PUT THE 1ST FOUR BITS OF THE SAME WORD INTO NEW IBUFT. NOW THERE ARE 2048 PICTURE ELEMENTS IN THE LINE. This Loop places 15 eight bit pixels into two ibuft words. Ji=0 IF(IFL .NE. 0) 60 TO 30 Complete read in of dne line of length 512. This loop expands a 512 line into a 2048 line. CALL STRING(-8, IPEL(JI), 53, IBUFT(LOC1), JM) CONTINUE CALL STRING(-4, IPEL(JI), 53, IBUFT(LOC1), 57) THE LAST 4 BITS ARE NOW IN IBUFT. CALL STRING(-4, IPEL(JI), 57, IBUFT(L022), 1) IF (LINE .GT. LINTOT) GO TO 100 READ IN ONE LINE. READ(7) (IN⁹(II),II=1,512) 00 40 ID=1,4 KI=ID + 4*(JM-1) L0C1=2*(I-1) + 1I PEL (KI) = I NP (JM) 00 50 JM=1,512 JM=(J-1)+8 + 1 00 80 I=1,137 00 60 J=1,7 IFL=EOF(7)1 + 1l = 1l1 + 1f = 1fL 0C2=2*I CONTINUE CONTINUE 3000 00

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111 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	NOW PACK 7 40805 INTO TAUFT(LOC2). DO 70 J=1.7 JH=JJ-1 + 1 Call STRING(-8, FPEL(JJ), 53, FJUFT(LOC2), JM) Call STRING(-8, FPEL(JJ), 53, FJUFT(LOC2), JM) CALL STRING CONTINUE MRITE OUT 63, 13 (FBUFT(L), FSUFT(LOC2), JM) CONTINUE MRITE OUT 63, 13 (FBUFT(L), FSUFT(273)) FCUNIT(9) 30, 100, 95 GO 10 10 30 MRITE (6, 95) LINE FORMAT(* 928, LINE CONTINUE CONTINUE CONTINUE CONTINUE CONTINUE CONTINUE CONTINUE CONTINUE CONTINUE CONTINUE CONTINUE CONTINUE CONTINUE	

***************** **************************** THE LUPUT DATA IS A SEVEN TRACK TAPE IN SIXTY BIT PER WORD FORMAT AND THE OUTPUT IS A SPECTRAL PLOT. : THIS PROGRAM PERFORMS SPECTRAL AVERAGING OF IMAGE DATA BY BLOCKING THE INPUT DATA AND FAST FOURIER DIMENSION PICT (224), IDUMMY (7, 2048), WORK (2), DX (66), X (66) TRANSFORMING IT ON & BLOCK BY BLOCK BASIS. DIMENSION SPICT(64), ID(17), IBUFF(1024), PICT(130) ID(7) /"SAMP. EVERY 7TH PT."/ /.. BC NI ECNITURE "/ (11)CI / SPATIAL FREQJENCY ... EQUIVALENCE (FPICT(1), PPICT(1)) 1. DATA ID(1) /"AVG. SPECT 0- ' DATA ID(3) /"FILE 14 USINS"/ PLOTS(I3UFF, 1024, 44PLOT) LCOUNT IS THE LINE SOUNTER. 10(5) / 32X7 BLOCKS"/ CALL FLOT(0., -4., -3) CALL FLOT(0., 0.03, -3) COMPLEX FPIST(65) ID(13) /" (6)01 DATA OATA DATA DATA DATA CALL

INITIALIZE ARRAYS TO TERO.

LCOUNT=0

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PROGRAM SPCANAL (INPUT, DUTPUT, TAPE5, TAPE5=OUTPUT, PLOT)

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EXTEND THE SAMPLED ARRAY TO PERMIT OBSERVATION OF ALL FREQUENCY i BLOCK LINES INTO 32X7 SECTIONS AND SCAN THE DATA DO 150 I=1,102 THIS LOOP READS IN 7 LINES INTO IDUMMY(7,2048) DO 10 KL=1,7 CONTINUE SAMPLE THE SCANVED JATA AF EVERY 7TH PJINT READ (5) (IJUMMY(KL, KK), KK=1, 2048) CONTINUE PICT (NL) = I DJMMY (JL, NZ) LE = (32*(JZ-1))+32RPICT(I)=PICT(KG) LS = (32 + (J7 - 1)) + 1LCOUNT=LCOUNT+1 00 100 JZ=1,64 00 15 NZ=LS,LE 00 40 I=33,128 JLA=32*(JL-1) 00 30 I=1,32 KG=I*7 00 20 JL=1,7 SPICT(I)=0.0 00 5 I=1,54 COMPONENTS NL =NN+ JLA CONTINUE A Party of Long CONTINUE CONTINUE 1+NN=NN 0=NN 80 500 10 1 10 0 0 82

SPICT(I) = (SPICT(I)) + (((REAL(FPICT(I))) + *2) + ((AIMAG(FPICT(I))) + *2)) CONTINUE TAKE THE TOTAL SPECTRUM FROM ALL BLOCKS, DIVIDE BY THE NUMBER OF BLOCKS, AND PLOT THE AVERAGE SPECTRUM CALL FOURT (PICT, 128, 1, -1, 1, 40%, 2) PLACE THE SPECTRUM OF THE BLOCK IN A HOLDING ARRAY WRITE(6,210) LCOUNT FORMAT(" TERMINATION OF SPECT AT LINE ",110) FAST FOURIES TRANSFORM THE EXTENDED ARRAY WRITE OUT THE VALUE OF THE LINE COUNTER 0X(I)=10*(AL0610(SPIST(I)/6528)) CALL HGRAPH(X, DX, 64, ID, 1, 0, 74) CALL PLOTE(4) X(I)=I*(3,1415926535893764) 00 200 I=1,54 00 50 I=1,64 QPICT(I)=0.0 CONTINUE CONTINUE CONTINUE CONTINUE CALL 210 001100 000 50 0

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: EQUIVALENSE (FILTER(1), QFILTER(1)), (GILTER(1), RFILTER(1)) DATA ID(1) /"SPECT. OF "/ DIMENSION ID(17), IBUFF(1024), X(27), DX(27), Y(50), DY(50) COMPLEX FILTER (26), GILTER(43) QFIL TER(I) =SIN(, 02*3, 14159*4) * (, 2*(SINC((A-25.) *.2)) DIMENSION OFILTER(52), RFI_TER(98), NORK(100) : : "SPATIAL FREQUENCY" (N+N/I+Id)NIS HIM ALIAS REMOVAL FILTER FUNCTION FLOTS (IBUFF, 1024, 4HPL 0T) /"COMB FILTER"/ ID(11) / "MAGNITUDE**2 FLOT(0.,0.03,-31 PLOT(0.,-4.,-3) MOGNIM ... ID(13) /" 00 10 I=1,50 DATA ID(3) (2)01 (S)0I (6)0I DATA DATA DATAC CALL CALL CALL DATA DATA I=V

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PROGRAM DIGFILT(INPUT, 0JTPUT, PLOT)

THIS PROGRAM PLOTS THE FILTER FUNCTION FREQUENCY

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WORD FORMAT AND THE OUTPUT IS A FREQUENCY PLOT.

RESPONSE FOR EVALUATION OF FILTER DESIGN SPECIFICATIONS. THE INPUT DATA IS A SEVEN TRACK TAPE IN SIXTY BIT PER

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RFIL TER(I) = (.1*((SIN2((A-48.)*.05))*COS((A-48.)*.245*3.14159))
                     $+* 2500*((SINC((A-25.0)*.250)) *COS((A-25.0)*1.00*3.14159)))
%+. 3340*((SIVC((A-25.0)*.157))*COS((A-25.0)*.457*3.14159))
                                                                                                                                                           {+* 1400*((SIVC((a-48*)**07))*COS((a-48.)*.69*3.14159)))
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        DY(I)=(REAL(GILTER(I))**2)+(AIMAG(GILTER(I))**2)
Y(I)=(A*3.1415926535993)/48.
                                                                                                                                                                                                                                                                                                                                                                      DX(I) = (REAL(FILTER(I)) **2) + (AIMAG(FILTER(I)) **2)
                                                                                                                                                                                                                                 FAST FOURIER TRANSFORMATION OF FILTER FUNCTIONS
CALL FOURT (DFILTER, 50, 1, -1, 1, WORK, 100)
                                                                                                                                                                                                                                                                            CALL FOURT (REILTER, 95, 1, -1, 1, WORK, 100)
                                                                                                                                                                                                                                                                                                  GENERATION OF ARRAYS FOR PLOTTING
                                                                   ALIAS TRANSLATION FILTER FUNCTION
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   HGRAPH(X, DX, 25, ID, 1, 0, 74)
HGRAPH(Y, DY, 48, ID, 1, 0, 74)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         CALL OF PLOTTING SUBROUTIVES
                                                                                                                                                                                                                                                                                                                                                                                             X(I)=(A*3,1415926535893)/25.
CONTINUE
                                                                                                                                                                                      (*SIN(.01*3.14159*A)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             PLOTE (4)
                                                                                            00 20 I=1,95
                                                                                                                                                                                                                                                                                                                           00 30 I=1,25
                                                                                                                                                                                                                                                                                                                                                                                                                                              00 40 I=1,43
                                                                                                                                                                                                           CONTINUE
                                               CONTINUE
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        CONTINUE
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     CALL
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                CALL
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          CALL
                                                                                                                   I=V
                                                                                                                                                                                                                                                                                                                                                   A=T
                                                                                                                                                                                                                                                                                                                                                                                                                                                                  4=1
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      CNE
                                               20
                                                                                                                                                                                                           00
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i FUNCTION SINC(X) THIS SUBROUTINE DEFINES THE SINC FUNCTION PI=3.1415925535998 IF (ABS(X).LT.1E-7) 50 TO 100 SINC=(SIN(PI*X))/(PI*X) RETURN SINC=COS(PI*X) RETURN SINC=COS(PI*X) RETURN END Statistics of the second 1. Contractor -----100 C 86

************** ****************************** INPUT IMAGE, REMOVING AND TRANSLATING ALLASED FREQUENCY COMPONENTS. THE RESULTING IMAGE IS THEN OUTPUT ON SEVEN TRACK TAPE IN SIXTY BIT PER WORD FORMAT. HICOUNT IS THE COUNTER OF OUTPUT DATA VALUES GREATER THAN 255. LOCOUNT IS THE COUNTER OF OUTPUT DATA VALUES LESS THAN ZERO. THIS PROGRAM FILTERS & LOW RESOLUTION VERSION OF THE COMPLEX CFILTER(137), XFILTER(57), YFILTER(81), ZFILTER(81) COMPLEX SFILTER(81), PFILTER(37), FPICT(57), GPICT(81) PROGRAM FILTER(INPJT, DUTPJT, TAPE5, TAPE7, TAPE6=DUTPUT) DIMENSION PICT(448), WORK(100), RPICT(114), OFILTER(274) DIMENSION GFILTER(114), RFILTER(162), ZPIST(162) INTEGER IJUMMY(14,2048), 0JUMMY(14,512), HICOUNT EQUIVALENCE (PFILTER(1), QFILTER(1)) EQUIVALENCE (SFILTER(1), RFILTER(1)) EQUIVALENCE (CFILTER(1),0"ILTER(1)) EQUIVALENCE (GPICT(1), ZPICT(1)) EQUIVALENCE (FPICT(1), RPICT(1)) INITIALIZE ARRAYS AND COUNTERS LCOUNT IS THE LINE COUNTER 00 90 I=1,228 HICOUNT=0 LOCOUNT=0 COUNT=0

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RFILTER(I) = (, 12*((SIV3((4-32°)*.05))*COS((4-32°)*.245*3.14159))
                                                                                                                                  &+. 3674*((SIV2((A-32.0)*.157))*COS((A-32.0)*.457*3.14159))
&+. 2750*((SIV2((A-32.0)*.250))*COS((A-32.0)*1.00*3.14159)))
                                                                                                                                                                                                                                                                                                                                                                     EXTEND FILTER ARRAYS TO REQUIRED LENGTH FOR CONVOLUTION
                                                                                                                                                                                                                                                                                               $+.1680*((SINC((4-32.)*.07))*20S((4-32.)*.69*3.14159)))
                                                                                                                QFIL TER(I) = SIN(.02+3.14153+4) + (.22+(SING((A-32.)+.2))
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  FAST FOURIER TRANSFORM OF FILTER FUNCTIONS
Call Fourt(2Filter,112,1,-1,1, MORK,100)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              CALL FOURT ( 2FILTER, 150, 1, -1, 1, WORK, 100)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         READ (5) (IDUMMY (KL, KK), K<=1, 2048)
                                                                                                                                                                                                       ALIAS TRANSLATION FILTER FUNCTION
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            READ IN 14 LINES OF INPUT DATA
                                             ALIAS REMOVAL FILTER FUNCTION
D0 100 1=1,49
                                                                                                                                                                                                                                                                                                                      $*SIN (.01*3.14159*4)
                                                                                                                                                                                                                                                                                                                                                                                            00 110 I=50,112
                                                                                                                                                                                                                                                                                                                                                                                                                                                                 00 115 I=97,160
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   00 120 KL=1.14
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     00 260 II=1,51
OFILTER(I) =0.0
                                                                                                                                                                                                                                                                                                                                                                                                                   0FIL TER(I) = 0.0
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      RFILTER(I) =0.0
                                                                                                                                                                                                                                 00 105 I=1,35
                                                                                                                                                                                                                                                                                                                                                                                                                                          CONTINUE
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             CONTINUE
                      CONTINUE
                                                                                                                                                                                   CONTINUE
                                                                                                                                                                                                                                                                                                                                                CONTINUE
                                                                                           A=T
                                                                                                                                                                                                                                                        I=V
                                                                                                                                                                                      100
                                                                                                                                                                                                                                                                                                                                                105
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              115
                                                                                                                                                                                                                                                                                                                                                                                                                                          110
                        60
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  0
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             0
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EXTEND THE DATA ARRAY TO THE REQUIRED LENGTH FOR CONVOLUTION DO 160 I=55,112 BLOCK DATA LINES INTO 32X14 SECTIONS AND SCAN THE DATA CONTINUE SAMPLE THE SCANNED DATA AT EVERY 7TH POINT COPY EXTENDED DATA INTO A DUPLICATE ARRAY PICT (NL) = IDUMMY (JL, NZ) CONTINUE LE = (32 + (JZ - 1)) + 32RPICT(I) =>ICT(KG) (I) 1 CId2=(I) 10IdZ 00 167 I=113,150 ZPICT(I)=0.0 LS = (32 + (JZ - 1)) + 100 130 NZ=LS, LE LCOUNT=LCOUNT+1 00 165 I=1,112 00 240 JZ=1,54 00 140 JL=1,14 00 150 I=1,54 KG=I+7 JLA=32*(JL-1) PPICT(I)=0.0 NL=NN + JLA CONTINUE CONTINUE CONTINUE CONTINUE T+NN=NN 0=NN

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CALL FOURT(OFILTER,224,1,1,-1,WORK,100) Sampling of reconstructed data to achieve output resolution do 200 I=1,112 COMBINATION OF ALIAS REMOVED AND ALIAS REANSLATED SPECTRA FREQUENCY DOMAIN MULTIPLICATION OF DATA WITH FILTERS INVERSE TRANSFORMATION OF RECONSTRUCTED SPECTRUM FAST FOURIER TRANSFORM OF EXTENDED DATA ARRAYS FILP AND JONJUGATE THE TRANSFER FILTERED DATA FOURT (RPICT, 112, 1, -1, 1, 40RK, 100) CALL FOURT (ZPICT, 160, 1, -1, 1, WORK, 100) XFILTER(I) = PFILTER(I) * FPIST(I) YFILTER(I) =SFILTER(I) *GPICT(I) ZFILTER(J) =CONJG(YFILTER(I)) CFILTER(J) = ZFILTER(I) OFILTER(I) = OFILTER(J) CFILTER(I) = XFILTER(I) 00 170 I=1,57 00 180 I=1,81 00 190 I=1,56 00 175 I=1,81 00 195 I=1,81 CONTINUE CONTINUE CONTINUE CONT INUE CONTINUE CONT INUE J=82-I I+95=1 5*1=L CALL 157 170 175 180 190 135

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WRITE OUT 14 LINES 35 FILTERED IMAGE DATA AT A RESOLUTION Equivalent to a 4th point sampled image Do 250 kl=1,14 ODUMMY(I,J)=OFILTER(LXY)/224 TEST FOR JUT OF RANGE CONDITIONS ON FILTERED OUTPUT DATA IF (ODUMMY(I,J).GT.255) GO TJ 280 IF (ODUMMY(I,J).LT.J) GJ TD 290 PLACEMENT OF FILTERED BLOCK IN HOLDING ARRAY FORMAT (* TERMINATION OF SPECT AT LINE *, 18) WRITE (7) (0DUMMY(KL,KK),KK=1,512) Continue WRITE OUT COUNTER INFORMATION • WRITE(6,270) LCOUNT HICOUNT=HICOUNT+1 00UMMY (1, J) =255 L P = (8 * (JZ - 1) + 1)LQ= (8+ (JZ-1)+8) 00 220 J=LP,L0 U0 230 I=1,14 LTP=8*(I-1) LXY=LTP+JJ 50 TO 300 60 10 220 CONTINUE CONTINUE CONTINUE CONTINUE CONTINUE 1+01=00 0=00 200 00540 250 270 082

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00UMMY(I,J)=0 LOCOUNT=LOCOUNT+1 G0 T0 220 CONTINUE WRITE (5,310) HICOUNT WRITE (5,310) HICOUNT=",110) HRITE (5,320) LOCOUNT FORMAT (" LOCOUNT=",110) END 062 300 310

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********************************** HICOUNT IS THE COUNTER OF OUTPUT VALUES GREATER THAN 255 INPUT FROM A SEVEN TRACK INPUT TAPE AND DUTPUTS A THIS PROGRAM INTERPOLATES A LOW RESOLUTION IMAGE HIGH RESOLUTION VERSION OF THE IMAGE TO A SECOND LOCOUNT IS THE COUNTER OF OUTPUT VALUES LESS THAN ZERO PROGRAM SPECT (INPUT, OUTPUT, TAPES, TAPE7, TAPE6=OUTPUT) SEVEN TRACK TAPE USING SIXIY BITS PER WORD. INTEGER IDUMMY(3,2049),00JMYY(3,2048),4120UNT DIMENSION PICT (6144), WORK (100), RPICT (6146) READ (5) (100MMY(0,KK), KK=1,2048) INITIALIZE ARRAYS AND COUNTERS LCOUNT IS THE LINE COUNTER DO 260 KZ=1,240 READ IN THREE INPUT LINES SCAN THE INPUT DATA LCOUNT=LCOUNT+1 JLA=2048*(JL-1) 00 140 JL=1,3 00 100 J=1,3 HICOUNT=0 LOCOUNT=0 CONTINUE COUNT=0 100 000000000000 00 0 0 0

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CALL FOURT (RPICT, 1024, 1, -1, 1, WORK, 100) Append Zeros to Attain the Original Length of the Unsampled Scanned data TEST FOR DUT OF RANGE CONDITIONS IN THE DUTPUT DATA INVERSE FOURIER TRANSFORM THE EXTENDED SPECTRUM CALL FOURT (2PICT, 6144,1,1,-1, WORK, 100) Reconstruct three output lines of IMAGE DATA CONTINUE FAST FOURIES TRANSFORM THE SAMPLED DATA CONTINUE SAMPLE THE JATA AT EVERY STH POINT JOUMMY (I, J) = 2 PICT (LXY) / 1024 PICT (NL) = I DUMMY (JL, NZ) 00 160 K=1027,6146 (5) 1CIc= (I) 10Ida 00 130 NZ=1,2048 00 150 I=1,1024 KG=I*6 00 220 J=1,2048 LTP=2049*(I-1) 00 230 I=1,3 RPICT(K)=0.0 NL=NN + JLA CC+411=XX1 CONTINUE CONTINUE 1+01=00 I+NN=NN 0=00 0=NN 140 150 150

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WRITE(6,270) LCOUNT FORMAT(* TERMINATION OF SPECT AT LINE *,18) WRITE (7) (33UM4Y(KL, KK), KK=1, 2048) IF (ODUMMY(I,J).6T.255) 60 T0 280 IF (ODUMMY(I,J).LT.0) 60 T0 290 OUTPUT THREE LINES OF IMASE DO 250 KL=1,3 WRITE (5,310) LOCOUNT FCRMAT (" LOCOUNT=",110) WRITE (5,320) HICOUNT FORMAT (" HICOUNT=",110) HICOUNT=HICOUNT+1 LOCOUNT=LOCOUNT+1 00UMMY (I, J) =255 0= (f ' 1) XMMN00 50 TO 220 GO TO 300 60 T0 220 CONTINUE CONTINUE CONTINUE CONTINUE CONT INUE END 0220 250 012 082 062 002 310 320

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Frederick Graham Barney was born on March 28, 1947, in Chicago, Illinois. He graduated from Arlington Heights High School in 1965 and attended Colorado State University, where he received a Bachelor of Science in Biological Science in 1969. He then received a commission in the United States Air Force and entered active duty in July, 1969. After graduating from Aircraft Maintenance Officer School in December, 1970, he was stationed at Davis-Monthan AFB, Arizona. He entered the post-graduate Electrical Engineering program at the Air Force Institute of Technology in May, 1974.

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VITA

(14) REPORT DOCUMENTATION PAGE	READ INSTRUCTIONS BEFORE COMPLETING FOR
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RESOLUTION ENHANCEMENT USING ALIAS FILTERING.	MS Thesis
	5. PERFORMING ORG. REPORT NUMB
7. AUTHOR(s)	8. CONTRACT OR GRANT NUMBER()
Capt USAF	2 master's these
9. PERFORMING ORGANIZATION NAME AND ADDRESS	10. PROGRAM ELEMENT, PROJECT, T AREA & WORK UNIT NUMBERS
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by horizontal scanning of the data. The second phase consisted of designing software digital bandpass filters to remove the identifiable aliased frequencies from the spectrum and relocate them. The third phase accomplished the actual filtering of the test image which had been segmented and scanned. The results of the filtering showed that the removal and relocation of aliased frequency components with the filters designed during this research produced more image distortion than was created by the aliased frequency components. It was discovered, however, that significant increases in resolution could be obtained by using an interpolation procedure.

