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VLF Flexural Disk Transducers Using Disks 1 Meter in Diameter

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3 December 1976

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PREFACE

This study was conducted under NUSC Project No. A-720-01, "Transduction Techniques for Navy Sonar Transducers," Principal Investigator, C. L. LeBlanc (Code 316), and Navy Subproject and Task No. SF 11 121 603, Program Manager, C. C. Walker (Code SEA 06H1-2).

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A series of underwater projector designs in which the driving elements are piezoelectric flexural-mode disks 1 m in diameter are studied. The disks are employed either as direct radiators or as drivers for Helmholtz resonators. All the proposed designs have a substantial depth capability (200-500 m), which is obtained by use of liquid-filled interiors containing compliant tubes. The 1 m disk size was chosen because it is well suited to very low frequency (VLF) applications; however, no piezoelectric disks this large have yet been built. Designs using smaller disks at higher frequencies can be obtained readily

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20. (Cont'd)

from the present results by linear scaling.

Because of the low radiation damping of the VLF transducers, the resonance peaks in the response tend to be too high. They must be flattened by use of acousto-electrical feedback, electrical equalization, or both. Helmholtz resonator transducers are suitable either for broadband or narrowband applications, and a number of examples are presented illustrating both types of operation. Internal cavitation is a possible problem in Helmholtz resonators; hence, these transducers have a minimum allowable depth, which may be as high as 100 m for some of the narrowband designs. The direct radiator transducers produce much less source level than the Helmholtz resonator types, but they are free of the minimum depth restriction.

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VLF FLEXURAL DISK TRANSDUCERS USING DISKS 1 METER IN DIAMETER

INTRODUCTION

To illustrate the potential of flexural disk transducers for very low frequencies (<400 Hz), performance calculations have been carried out for underwater transducers using trilaminar disks 1 m in diameter. The disks have various thicknesses, and they are used in different configurations. The trilaminar disks contain two layers of PZT-4 ceramic, arranged so that flexural vibrations are excited piezoelectrically.[1]

The disks are employed either as direct radiators or as drivers for Helmholtz resonators.[2,3] Some of the transducers of each type use a single disk; others use two disks.

The 1 m disk size was chosen because it is large enough to be useful at low frequencies, yet does not appear impractical to build. None this large actually have been built, but there seems to be no reason why the segmented ceramic-construction techniques used successfully in disks up to 0.4 m diameter would not continue to work in larger sizes.

The applicability of the 1 m design data is not as limited as might appear, because designs for other sizes or frequencies can be obtained from the present results by scaling.[4] In this way, the capabilities of flexural disk transducers at any chosen frequency can be determined quickly.

The proposed designs are intended for applications with substantial depth requirements (e.g., 200 to 500 m). The depth capability is achieved by use of liquid-filled interiors containing compliant metal tubes.

BASIS OF CALCULATIONS

Performance calculations are made for the sinusoidal steady state. A lumped-parameter approach, using electroacoustical circuit analysis, is employed. The theory and equations for the Helmholtz resonator transducer will be presented in subsequent publications.

A maximum electric driving field on the ceramic of 4 kV/cm rms is used in the calculations. A maximum ceramic stress of 2000 psi (14 MPa) is allowed. This is a conservative value, and it is predicated on the supposition that pre-stressing of the ceramic will not be feasible.

Although ceramic heating is low at VLF frequencies and cooling of the disks should be good, a restriction on duty cycle still might be required in some cases to avoid overheating.

The source level curves that will be presented are for sine wave drive. Source level curves for random or pseudorandom signals are, of course, lower. It is not sufficient simply to keep the broadband power of such signals no greater than the rated sine wave power. In addition, the signal peaks must not exceed the peak of the rated sine wave. The latter, more severe, restriction reflects the fact that the output of a ceramic transducer is peak limited, not just average-power limited.

TRILAMINAR DISK PROPERTIES

Disks will be considered with thickness-to-radius ratios (h/a) ranging from 0.05 to 0.30. The two limiting disks are described below:

thickness/radius, h/a	0.05	0.30
thickness h	1.0 in. (2.5 cm)	5.9 in. (15 cm)
ceramic thickness	0.33 in. (0.85 cm)	0.75 in. (1.9 cm)
core thickness	0.33 in. (0.85 cm)	4.41 in. (11.2 cm)
max. press. differential	4.1 psi (0.028 MPa)	146 psi (1.0 MPa)
max. head of water	9.1 ft (2.8 m)	327 ft (100 m)
resonance in air	71 Hz	423 Hz

Choice of the minimum thickness (h/a = 0.05) was based on fragility and possible fabrication difficulties. If the thickness were further reduced, gravity stresses would begin to become significant. It might be difficult to keep the disk from warping during construction.

At the maximum thickness considered (h/a = 0.30), thin disk theory begins to become unreliable, and the electromechanical coupling begins to decrease as more of the strain energy is stored in shear deformation. At this thickness the ceramic layers make up only one-quarter of the total thickness; this relative decrease in ceramic would also cause significant loss of electromechanical coupling if carried any further. To minimize this effect, the inner layer, or core, is made of a low-stiffness material like aluminum. A further reason for not considering thicker disks is that their strength at h/a = 0.30 is about as high as can profitably be used. In the Helmholtz resonator the cavity pressure that can be generated is limited by the strength of the disk, but for h/a = 0.30 this pressure is so high that the transducer must be operated at least 100 m deep to avoid internal cavitation.

ACHIEVING BROAD BANDWIDTH

Unless the internal damping is very high the resonances of VLF transducers usually have high Q_M, due to the poor radiation loading associated with the small transducer size. For broadband applications, a sharp resonance peak in the response is not useful. Furthermore, it produces high disk stresses which can lead to ceramic fracture, assuming the ceramic is driven at full voltage in order to maximize the response on the skirts of the resonance. Steps must, therefore, be taken to flatten the resonance, but this does not mean that all benefits of resonance will be lost. Even when well damped, it still contributes very significantly to the source level.

There are three techniques for flattening the response:

1. Mechanoacoustical Dissipation. The mechanoacoustical efficiency η_{ma} is deliberately made low by allowing high losses in the compliant-tube pack and in the potting compounds used for waterproofing the disks. A disadvantage of this method is that the damping varies with temperature. The loss of efficiency may not change the amplifier requirements, since the main burden on the amplifier may be supplying reactive power.

2. Acoustoelectrical Feedback. A signal from a probe hydrophone or a vibration sensor on the disk is fed back to the amplifier input. The effective Q_M is lowered without lowering the efficiency. The disadvantage is expense: an extra cable and a sensor are required.

3. Electrical Equalization to Reduce the Peak. A tuning inductor at the amplifier output will provide some equalization. Sophisticated low-level equalizers can be used at the amplifier input. The difficulty is that the equalizers have no ability to track the transducer resonance if it should shift. Thus, in an array, the transducer resonance could vary with steering angle and get out of synchronism with the equalizer.

A combination of technique 3 with 1 or 2 should produce the best results. Simple damping (1 or 2) produces a response that does not completely fill out the maximum source level envelope. Electrical equalization (3) can produce more desirable response shaping, but it should be used with reasonably well-damped transducers so that small shifts in resonance will not be critical.

TRANSDUCER CONFIGURATIONS

Figure 1A shows a direct-radiator disk transducer. Unless the transducer is to be mounted on a baffle it is clearly advantageous to use two disks in such a transducer, as illustrated. The interior of the transducer contains a liquid (usually oil) which is maintained at the

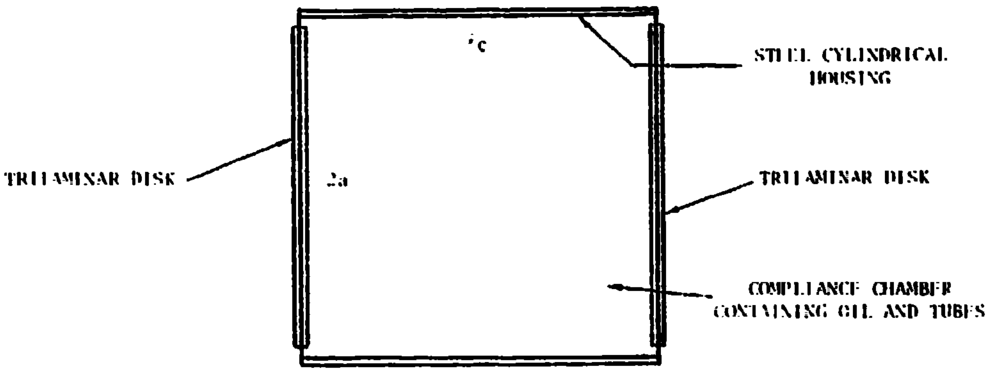

Figure 1A. Direct Radiator, Double-Disk

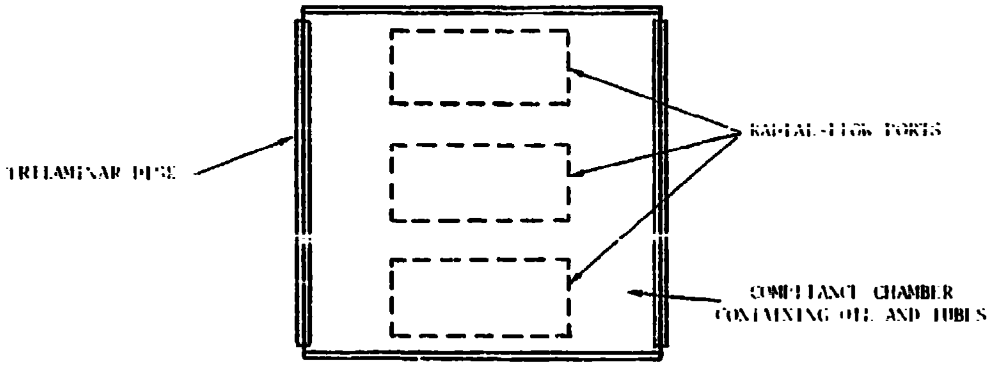

Figure 1B. Helmholtz Resonator, Double-Disk

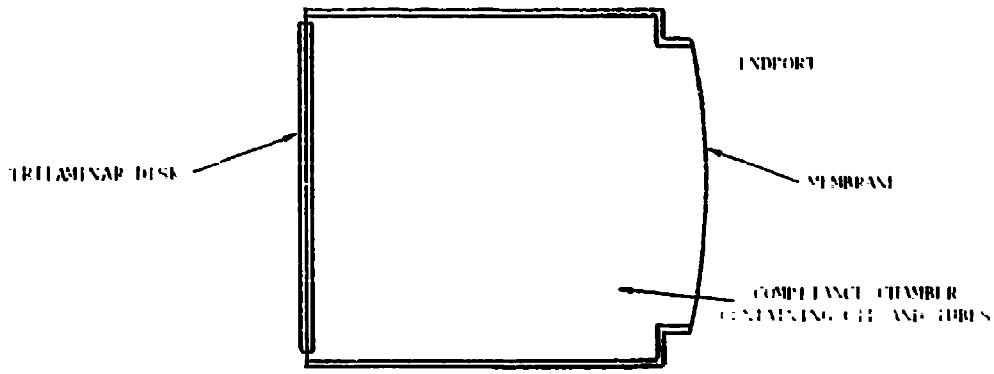

Figure 1C. Helmholtz Resonator, Single-Disk

Figure 1. Flexural Disk Transducer Configurations

same hydrostatic pressure as the ambient sea, so that the disks experience no static pressure differential. To reduce the stiffness of the liquid (i.e., provide the disks with a degree of pressure release), a pack of compliant metal tubes is immersed in the interior liquid.

Figure 1B shows a Helmholtz resonator transducer with two driving disks. It is similar to the transducer of figure 1A except for the radial-flow ports or orifices that have been introduced in the wall of the compliance chamber. The multiplicity of uniformly-spaced ports is equivalent acoustically to one large port. The ports provide the inertance which resonates with the compliance of the liquid-filled cavity containing the compliant tubes. The ports are normally covered with a thin elastomeric membrane to prevent mixing of sea water with the interior liquid, though in some cases use of sea water as the interior fluid may be acceptable.

Figure 1C shows a Helmholtz resonator transducer with a single driving disk. This configuration is more readily recognized as a Helmholtz resonator than the one in figure 1B, but both configurations have the same form of electroacoustical circuit. Use of an endport in this model, while convenient, is not essential. Radial-flow ports could be used as in figure 1B. The endport would then be replaced by a rigid metal cap. In either case, the ports would normally be covered by membranes to prevent sea water entry.

GENERAL PROPERTIES OF 1 METER MODELS

All the transducers under consideration have omnidirectional radiation; they are effectively monopoles. Wall motion of the compliance chamber must be avoided as it would contribute volume velocity 180 degrees out of phase with the principal volume velocity. Hence, the walls of the compliance chamber are made thick enough so that their compliance is small compared with the interior cavity compliance.

The proportion of the cavity to be filled with compliant tubes depends on how much viscous loss can be tolerated. The maximum packing factor used here will be 50 percent. Tubes designed for a 1500 ft (460 m) depth have a compliance about 100 times that of water. They have an effective specific gravity of about 4. These properties are independent (to first order) of tube size.

At the lowest frequencies maximum performance is obtained by making the compliance chamber very long (providing large compliance). However, the length must be limited to meet the size restrictions that are normally imposed. In these studies, 1 m will be taken to be the maximum allowable length (making the transducer a "square cylinder"). At the higher same hydrostatic pressure as the ambient sea, so that the disks experience no static pressure differential. To reduce the stiffness of the liquid (i.e., provide the disks with a degree of pressure release), a pack of compliant metal tubes is immersed in the interior liquid.

Figure 1B shows a Helmholtz resonator transducer with two driving disks. It is similar to the transducer of figure 1A except for the radial-flow ports or orifices that have been introduced in the wall of the compliance chamber. The multiplicity of uniformly-spaced ports is equivalent acoustically to one large port. The ports provide the inertance which resonates with the compliance of the liquid-filled cavity containing the compliant tubes. The ports are normally covered with a thin elastomeric membrane to prevent mixing of sea water with the interior liquid, though in some cases use of sea water as the interior fluid may be acceptable.

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The lowest frequency models tend to be the heaviest. For a weight estimate of such a transducer, consider a double-disk unit with $h/a = 0.1$, having a cavity of length $\ell_c = 1$ m, packed 50 percent with compliant tubes rated for 1500 ft (460 m). The wall thickness of the chamber will be 0.5 in. (1.3 cm), which makes the wall stiffness 50 times greater than the stiffness of the interior cavity. The total weight of this transducer in air will be 2860 kg, or 6290 lb. The liquid and tubes filling the interior account for 69 percent of the weight. Some of the transducers to be described later (with shorter chambers) will have less than half the weight of this example.

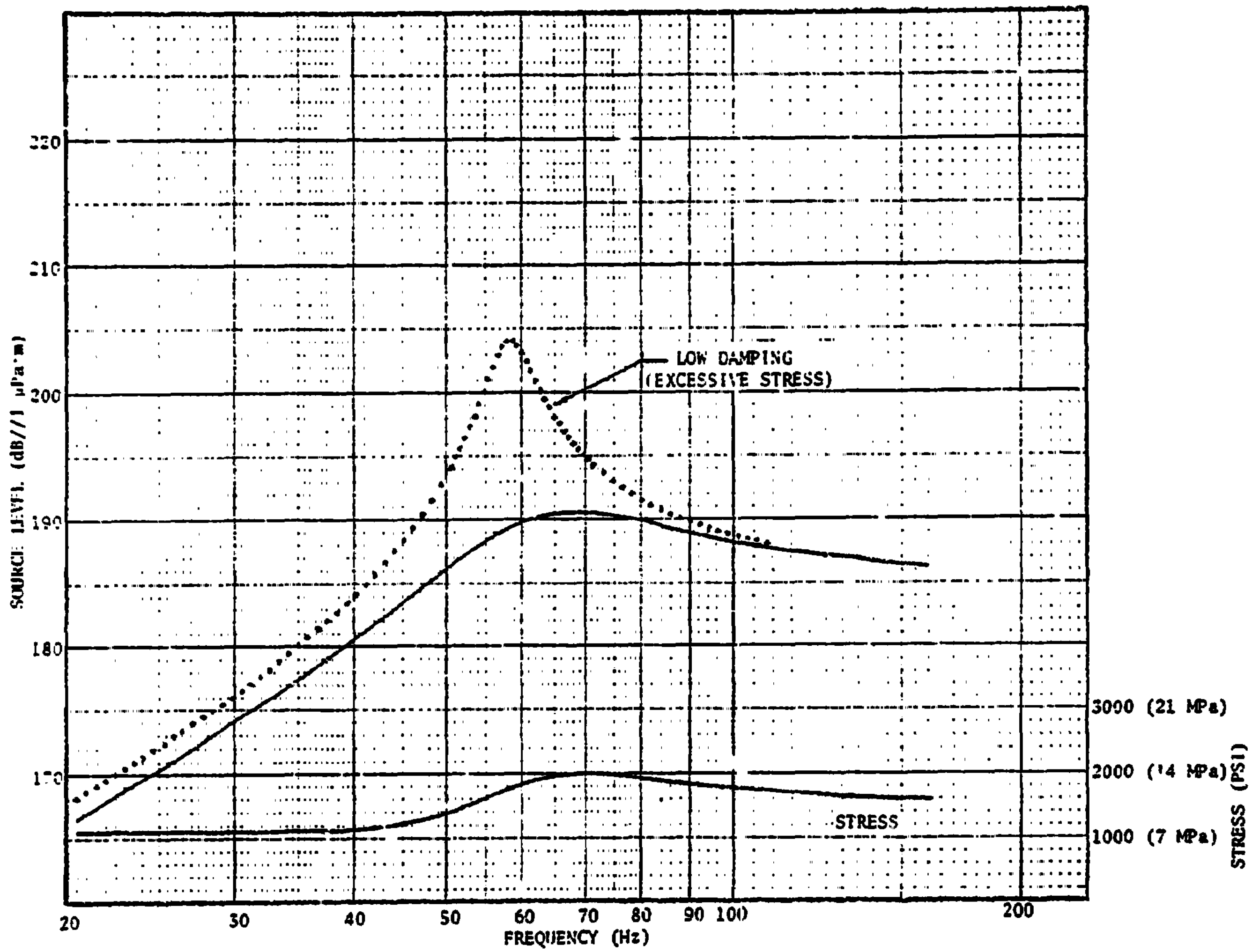
Compliant tubes 1.5 in. (3.8 cm) wide could be chosen for this example, since they are readily available. The total length of tubing required would then be 2170 m (7120 ft), or 1.3 miles. The cost of the tubing would probably be comparable with the cost of the driving disks.

The ratio of the disk compliance to the total compliance (disk plus cavity) is a parameter of interest in considering different designs. This ratio will be designated α. The stiffer the cavity relative to the disk, the higher will be the value of α, with the limit $\alpha \leq 1$. For perfect pressure release, $\alpha = 0$. Another parameter, Q_o, which is the mechanical Q for radiation damping only, provides insight into the design problem. The more miniaturized the transducer (with respect to the wavelength) the higher will be Q_o. The higher the Q_o, the lower will be the achievable efficiency. Thus for narrowband designs, where efficiency is important, a high value of Q_o would be an unfavorable omen.

DIRECT-RADIATOR TRANSDUCERS

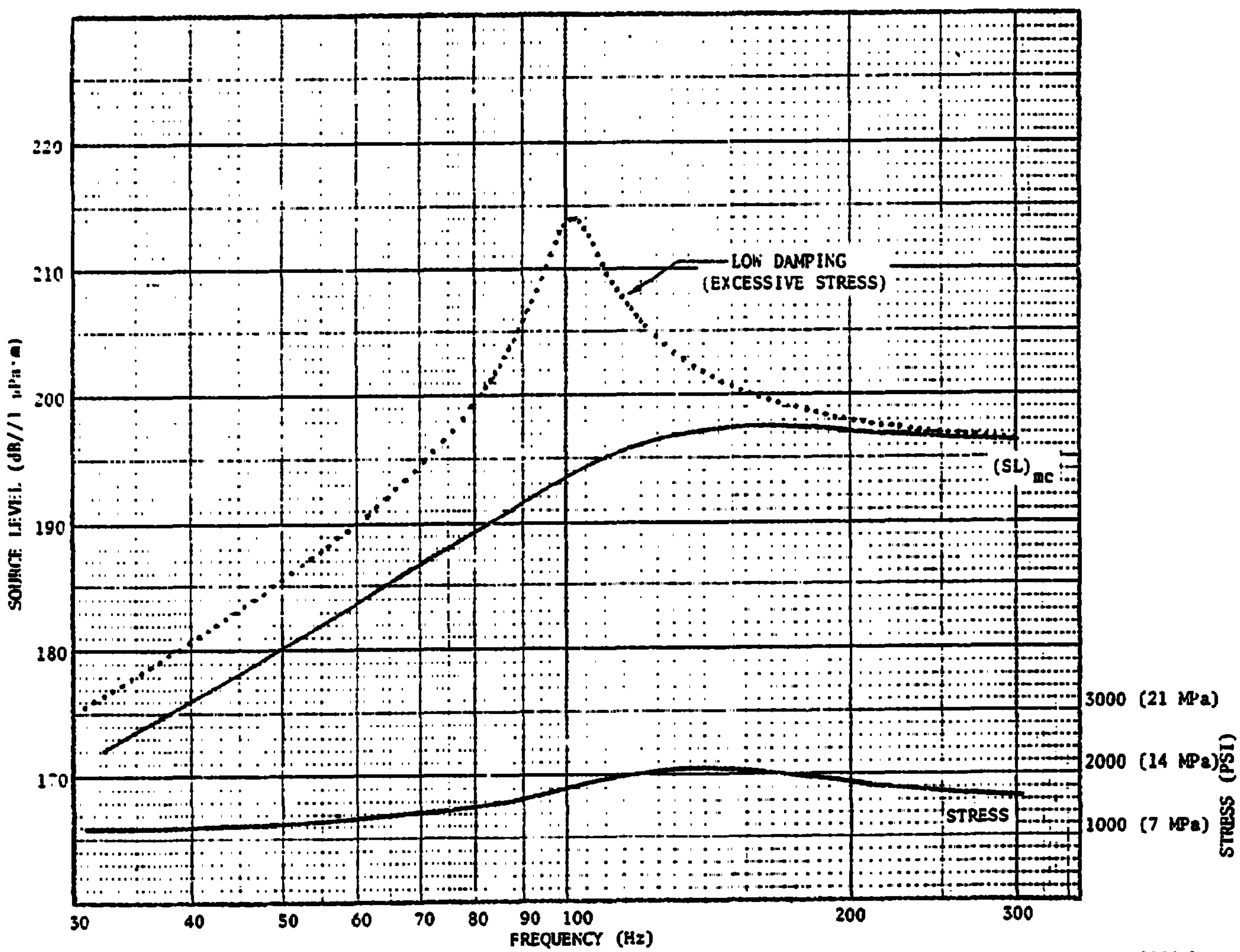
Direct-radiator disks operating at full voltage (4 kV/cm) can accommodate very little response rise at resonance without violating the stress limit. Thus for broadband applications, the flattening of the resonance by damping or other means, as discussed on page 3, must be rigorously carried out.

The first question to be answered about these transducers is how low in frequency can they operate? The thinnest possible disk should be chosen. On page 2 such a disk was described as having $h/a = 0.05$ and a resonance frequency in air of 71 Hz. When the disk functions in a transducer, the radiation mass will push the resonance frequency down and the cavity stiffness will push it up. A depth requirement of 1500 ft (460 m) will be assumed, so the compliant tubes will have 100 times the compressibility of water. To minimize the cavity stiffness, full 50 percent packing factor will be used and the chamber will have its maximum 1 m length.



$h/a = 0.05$ $\alpha = 0.62$ $Q_o = 72$ $\ell_c = 1$ m $2a = 1$ m $Q_M = 1.7$ max. depth = 460 m weight = 2500 kg

Figure 2. 58 Hz Double-Disk Direct Radiator



$h/a = 0.102$ $\alpha = 0.156$ $Q_o = 53$ $\ell_c = 1$ m $2a = 1$ m $Q_m = 0.83$ max. depth = 460 m weight = 2900 kg

Figure 5. 100 Hz Double-Disk Direct Radiator

Figure 4 provides a means for estimating the output of double-disk direct-radiator transducers at other frequencies in their applicable range. The mass-controlled source level is plotted versus the thickness-to-radius ratio h/a. This source level applies above resonance where the response becomes constant. Since only a few dB rise, at most, can be tolerated in the response at resonance without exceeding the stress limit, this mass-controlled source level enables one also to estimate roughly the response throughout resonance. The mass-controlled source level is independent of α. The resonance frequency, on the other hand, does depend on α, or correspondingly on the chamber length ℓ_c. Plots of resonance frequency, with ℓ_c as a parameter, are given in figure 4 for a chamber 50 percent packed with compliant tubes designed for 1500 ft (460 m). For lesser depth capability a shorter chamber could be used to obtain the same resonance frequency.

As an example of the use of figure 4, suppose a broadband transducer with a resonance frequency of 140 Hz is desired. If a chamber length of 0.5 m is used, figure 4 shows that h/a will be 0.173 and the mass-controlled source level will be 199.5 dB. With proper equalization this source level should be maintainable down to about 120 Hz. If the chamber length is 1 m, figure 4 shows that h/a = 0.138 and the source level is 200.0 dB. If the chamber length is 0.2 m, figure 4 shows that h/a = 0.116 and the source level is 197.5 dB.

HELMHOLTZ RESONATOR TRANSDUCERS

The Helmholtz resonator transducer employs a second resonance in addition to the disk resonance. This additional resonance, the Helmholtz resonance, depends on the fluid components of the transducer, namely the cavity compliance and the orifice inertance. By using these components to advantage, the Helmholtz resonator transducer can resonate at lower frequencies than the direct-radiator transducer. Also, the Helmholtz resonance is instrumental in transforming the radiation resistance upward to values that match the mechanical impedance of the disk drivers better. Hence, the source level at the lower end of the passband (where this effect occurs) can undergo a considerable boost before the stress limit of the disks is reached, and it is much higher than that of the direct-radiator transducer.

For narrowband applications the transducer may be designed so that this response peaking at the Helmholtz resonance is very substantial. However, the rise in output at resonance is accompanied by a rise in the ac pressure in the cavity, and this will lead to internal cavitation unless the static pressure is great enough to suppress it. Hence, narrowband Helmholtz resonator transducers have a minimum depth of operation which is required to avoid cavitation.

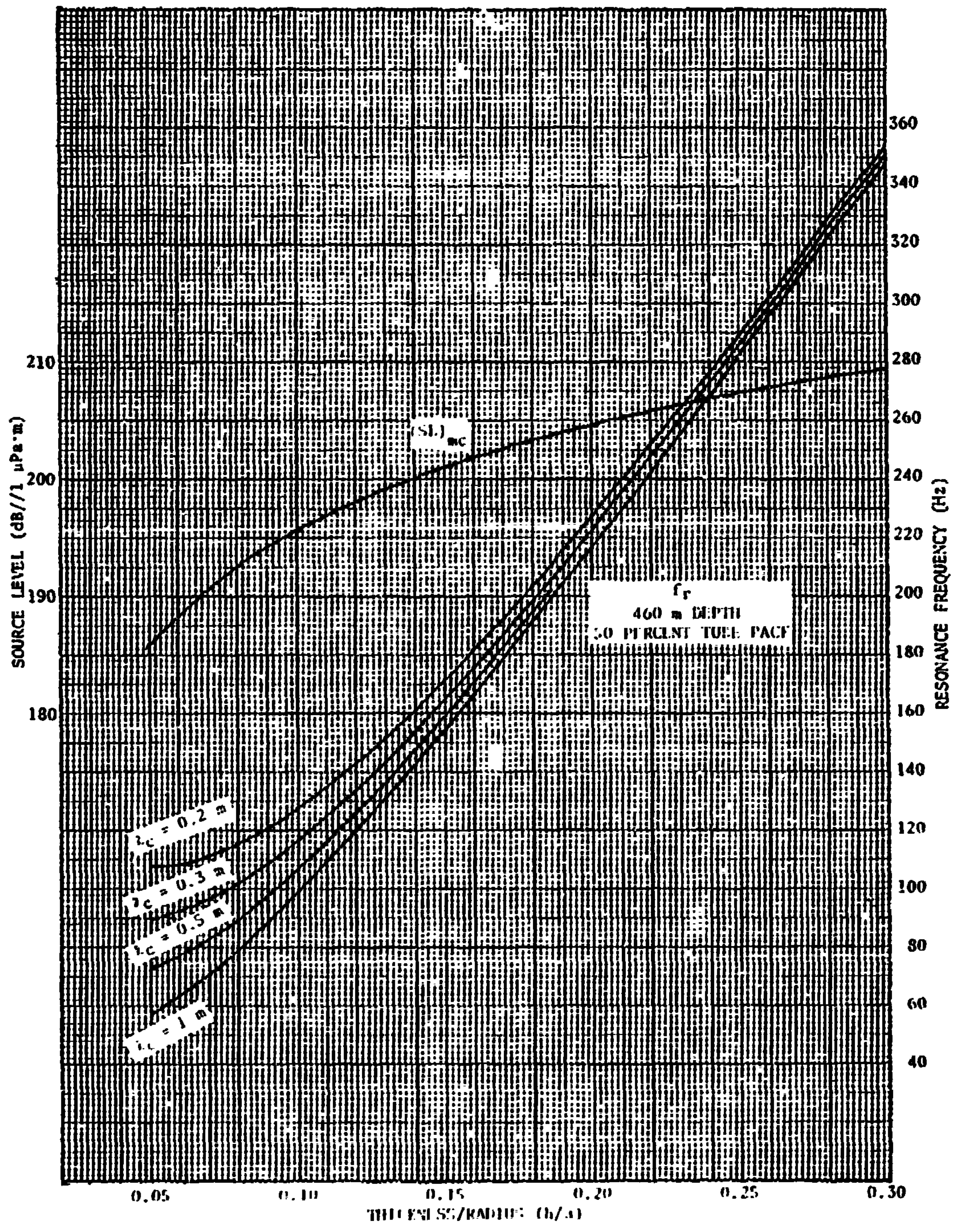


Figure 1. Design Curves for Double Disk Direct Radiators

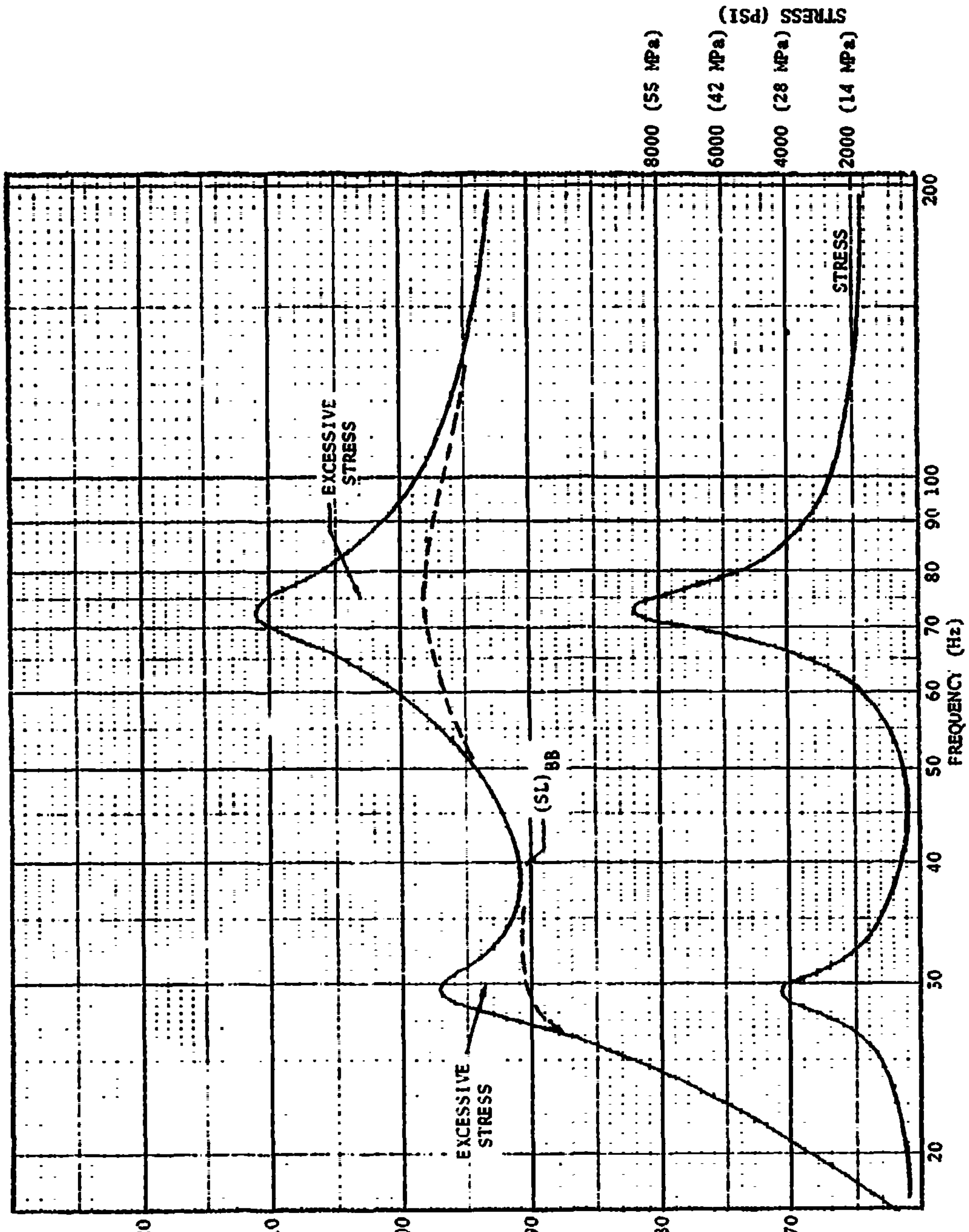
It has been found that maximum source level is obtained by using minimum neck inertance. Hence, the designs presented here will all be minimum inertance designs. There are limits on how low the inertance can be made, however. At low frequencies low inertance requires large cavity compliance for resonance, so the restriction imposed earlier on chamber length ($\ell_c \leq 1$ m) becomes the constraining factor. At higher frequencies the chamber length is no longer limiting, but problems with neck geometry come into play. To minimize inertance one makes the neck length zero, then makes the orifice as large as possible. In figure 1C the limit on orifice size is reached when the orifice diameter becomes the same as that of the chamber (1 m). At this limit the inertance is approximately 1000 kg/m^4, and this value will be used in calculating designs based on figure 1C. For the configuration shown in figure 1B, greater orifice area will be available if $\ell_c > a/2$, so an inertance as low as 500 kg/m^4 will be used for some of the designs based on figure 1B. A secondary benefit of using minimum inertance designs is that the nonlinear orifice effects will be small.

The range of design possibilities is much greater for the Helmholtz resonator transducer than for the direct-radiator transducer, because of the additional design parameters (cavity resonator parameters as well as disk resonator parameters). Design examples will be chosen with the objective of sampling this wide range of possibilities.

As a first example, assume that a broadband transducer with output down to 30 Hz is needed and that the depth requirement is only 800 ft (240 m). The maximum acceptable chamber length of 1 m will be used, and the tube packing factor will be 50 percent. For broadband transducers two disks are always better than one. For a fixed cavity compliance and a fixed resonance frequency, the gain due to the second disk is 4 dB. The maximum voltage response of a moderately-damped double-disk transducer designed to meet the specifications cited above is shown in figure 5. The inertance of the radial ports used in this design is 650 kg/m^4. In figure 5 the lower resonance is the Helmholtz resonance, and the upper resonance is the disk resonance (the ports are essentially blocked at the latter frequency).

It is seen that the stress limit of 2000 psi (14 MPa) is exceeded at both resonances. If equalizers are used to reduce the voltage at both resonances, the attainable response is approximately as shown by the dashed lines in figure 5. The resulting broadband system is capable of a source level of 190 dB//1 μPa·m down to about 28 Hz. The transducer weighs about 2300 kg (5100 lb). If the transducer were redesigned for 1500 ft (460 m), the source level would be 2 dB lower and the weight about 22 percent greater.

If the passband does not have to extend as low as 28 Hz, one would tune the Helmholtz resonance to a higher frequency. The broadband



h/a = 0.073 α = 0.20 Q_o = 98 ℓ_c = 1 m 2a = 1 m max. depth = 240 m min. depth = 6 m

Figure 5. Broadband Double-Disk Helmholtz Resonator

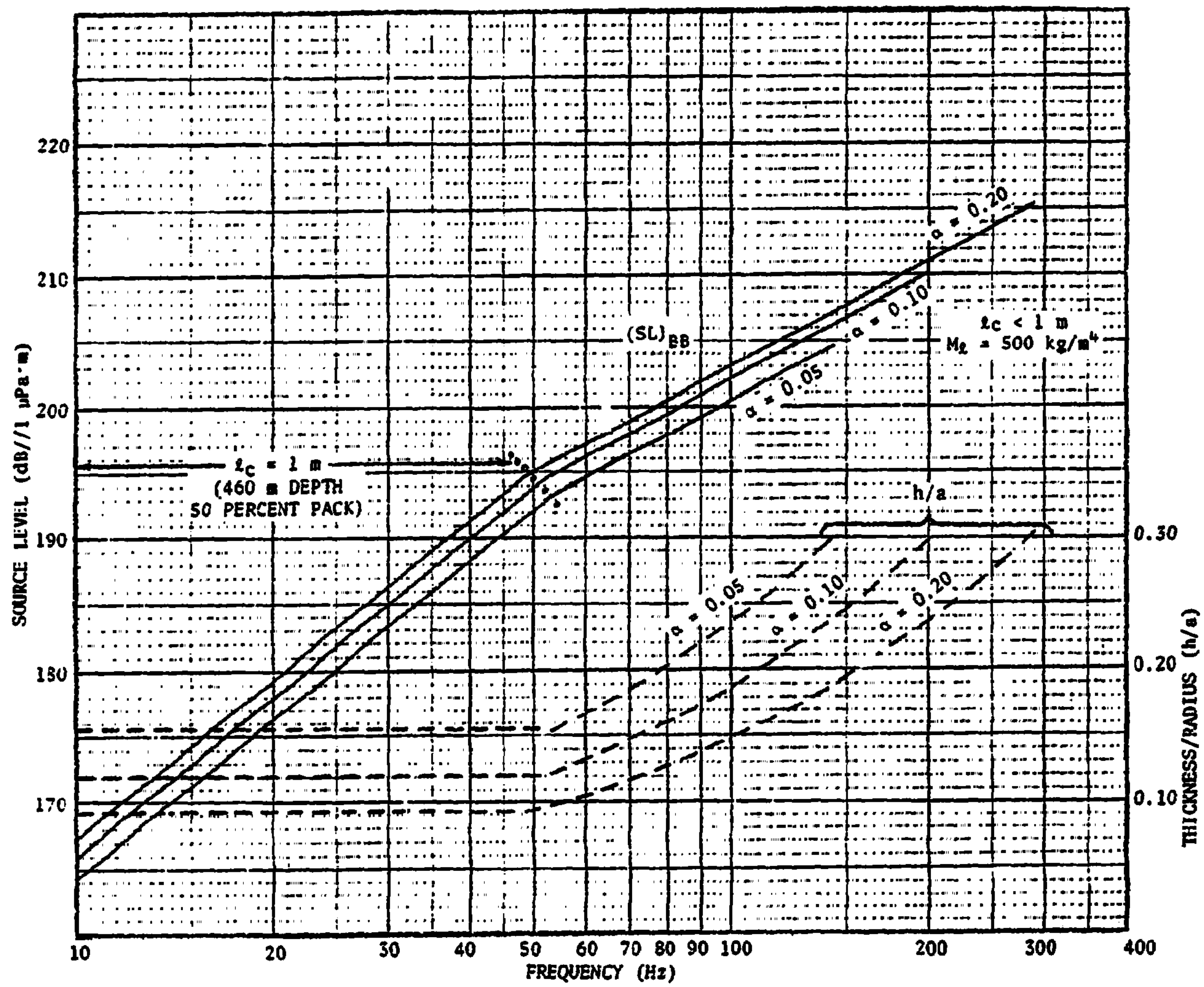
source level would increase. From figure 5 it is seen that the broadband source level $(SL)_{BB}$ is determined by the level of the saddle region between the two resonances. An approximate calculation of the source level at the saddle can be made by computing the Helmholtz resonator response at a frequency of 1.4 f_r and ignoring the effects of the higher resonance. The approximations will make the results err on the conservative side. Curves of the source level calculated in this way are plotted as a function of the Helmholtz resonance frequency in figure 6. The values of h/a required to achieve resonance are also plotted.

Breaks are seen to occur in the curves of figure 6 at about 50 Hz. These are caused by the imposed restriction on the chamber length, $\ell_c \leq 1$ m. At frequencies below the breaks, the chamber length is maintained at 1 m and the resonance is changed by varying the neck inertance. At frequencies above the breaks, the orifice inertance is kept constant at 500 kg/m^4, and the cavity length ℓ_c and disk thickness h are varied to change resonance (ℓ_c remains below 1 m in this region).

A depth requirement of 1500 ft (460 m) and 50 percent tube packing is assumed for figure 6, but this is only relevant below the curve breaks. Above the breaks, one can vary ℓ_c to compensate for a change in tube stiffness or packing factor, thereby keeping the cavity compliance invariant and not affecting the curves. If the depth requirement were reduced to 800 ft (240 m), the break points would move down in frequency and the source level curves below the breaks would be raised 2 dB.

As an example of the use of figure 6, suppose that broadband response is desired down to 60 Hz. The value of α is selected on the basis of the desired rise in response at resonance. The lower the value of α the higher the response rise can be without excessive stress being produced. For broadband very little response rise is required; so values of α in the 0.1 to 0.2 range should be suitable. The curves show that for a resonance frequency of 60 Hz a broadband source level of 196 dB//1 μPa·m is achievable.

For narrowband applications the transducer will operate near the peak of its Helmholtz resonance, and the design will be aimed at allowing a large rise in the response at resonance before the stress limit of the disks is reached. This requires low values of α (in the range of 0.05 to 0.10). A rough estimate of the achievable narrowband source level is obtained by adding 10 dB to the α = 0.05 curve in figure 6 at the chosen resonance frequency. In the high-frequency region of this curve the designs call for thick disks, and a high cavity pressure is required to produce the desired resonance rise. The minimum operating depth to avoid cavitation therefore becomes large.



Broadband Source Level vs. Resonance Frequency

Figure 6. Design Curves for Double-Disk Helmholtz Resonators

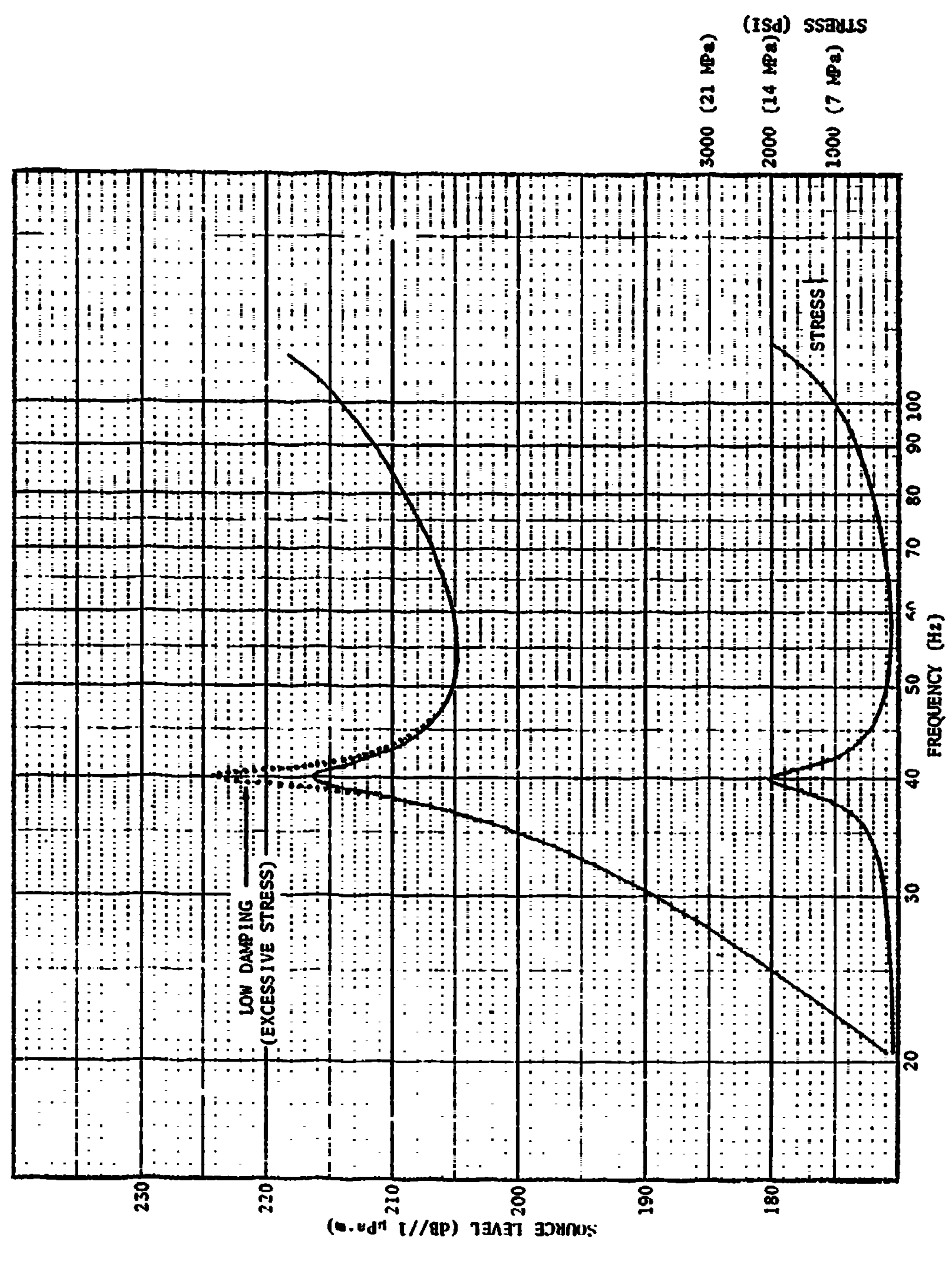
To go beyond the rough estimate mentioned above, a series of narrowband designs will be presented, spaced over the feasible frequency range. The practical range of resonance frequencies for the 1 m disk transducer covers at least a 10:1 span. Naturally, the characteristics vary considerably over this range. At the low end, the resonance rise is potentially very high and is held down to practical levels by internal damping (or feedback). At the high end, radiation damping can become large enough to prevent the resonance rise from reaching desired levels. Another aspect of this radiation phenomenon is that the efficiency is low at the low-frequency end but becomes respectable at the high-frequency end. In low-frequency designs the Helmholtz resonance and the disk resonance tend to be widely spaced, whereas in high-frequency designs they tend to be crowded together.

NARROWBAND EXAMPLES

The narrowband examples will be single-disk Helmholtz resonator designs. In making a rough estimate of source level from figure 6 for these cases, one should subtract 4 to 6 dB to account for the change to a single-disk driver. All designs will use compliant tubes designed for a 1500 ft (460 m) depth.

Figure 7 shows the response of a Helmholtz resonator transducer designed to resonate at 40 Hz. The predicted source level is 196 dB//1 μPa·m (275 W output). The maximum operating depth is 1500 ft (460 m), and the required depth to suppress cavitation is 70 ft (21 m). The maximum allowable chamber length of 1 m has been used, but the tube packing factor has been held down to 25 percent to keep the internal viscous losses moderately low. With the damping adjusted as shown in the figure to keep the stress from exceeding its allowed value of 2000 psi (14 MPa), the efficiency would be about 10 percent. It is expected that 25 percent packing will allow lower damping to be achieved than shown in figure 7. In that case the efficiency will be increased, but it will be necessary to reduce the driving voltage at resonance by use of acoustoelectrical feedback or equalizers. The dotted curve shows the shape of the response for reduced damping. With equalizers one could flatten the top of the resonance curve to fill out the region between the dotted lines at a level of 196 dB, and thus obtain about 5 percent bandwidth.

The primary constraint in this design is the chamber length, ℓ_C = 1 m. Consequently, it has not been possible to use the minimum orifice inertance of 1000 kg/m^4; rather, the inertance is 1690 kg/m^4. The weight of the transducer is 6100 lb (2800 kg). If two driving disks were used, but no changes were made in the compliance cavity or the neck, the source level would go up 4 dB. The disk thickness would increase by the factor 1.26. The main penalty, other than increased cost, would be increased cavitation-suppression depth, which would go



$h/a = 0.138$ $\alpha = 0.07$ $Q_0 = 143$ $\ell_c = 1$ m $2a = 1$ m max. depth = 460 m min. depth = 21 m

Figure 7. 40 Hz Single-Disk Helmholtz Resonator

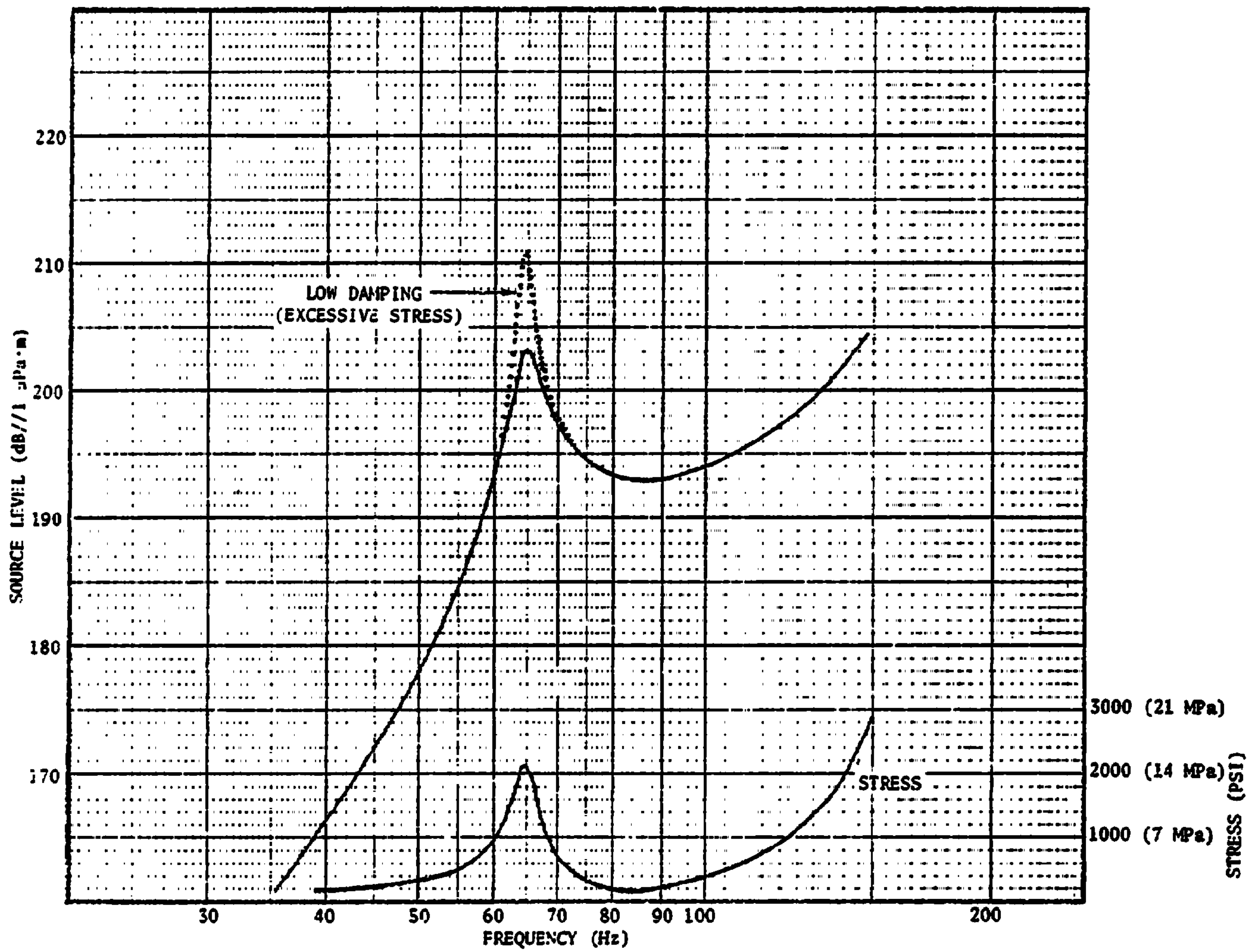
up by the factor 1.59.

Figure 8 shows the response that results when the resonance frequency of the design is raised to 65 Hz. The predicted source level is 203 dB//1 μPa·m (1380 W). The 25 percent packing factor has been retained, but less cavity length is needed at the higher frequency. Advantage is taken of this fact to drop the neck inertance to 1000 kg/m^4. Even with this change the cavity length is only 0.64 m. The disk thickness has gone up, and the minimum operating depth has increased to 93 ft (28 m). The weight of the transducer is 4200 lb (1900 kg). Q_o has dropped from 143 to 52, indicating that the efficiency will be better (probably 30 percent or more). The remarks made for the previous design regarding equalization and the use of two driving disks apply also to this design.

Figure 9 shows results for a resonance frequency of 100 Hz. The predicted source level is 210 dB//1 μPa·m (6900 W). The required chamber length has now been reduced dramatically; for 25 percent packing ℓ_c = 0.28 m. The disk thickness continues to increase, as does also the cavitation-avoidance depth (63 m). The weight of the transducer is 2900 lb (1300 kg). The efficacy of the radiation damping at this frequency is shown by the fact that if all internal losses were removed the resonance peak would rise only another 4 dB. The efficiency should now be in the 50 percent area. If a second driving disk were added, as described for the design of figure 7, the disk thickness would increase to bring h/a to its limit of 0.30. Although the source level would go up 4 dB, the minimum operating depth would increase to 327 ft (100 m).

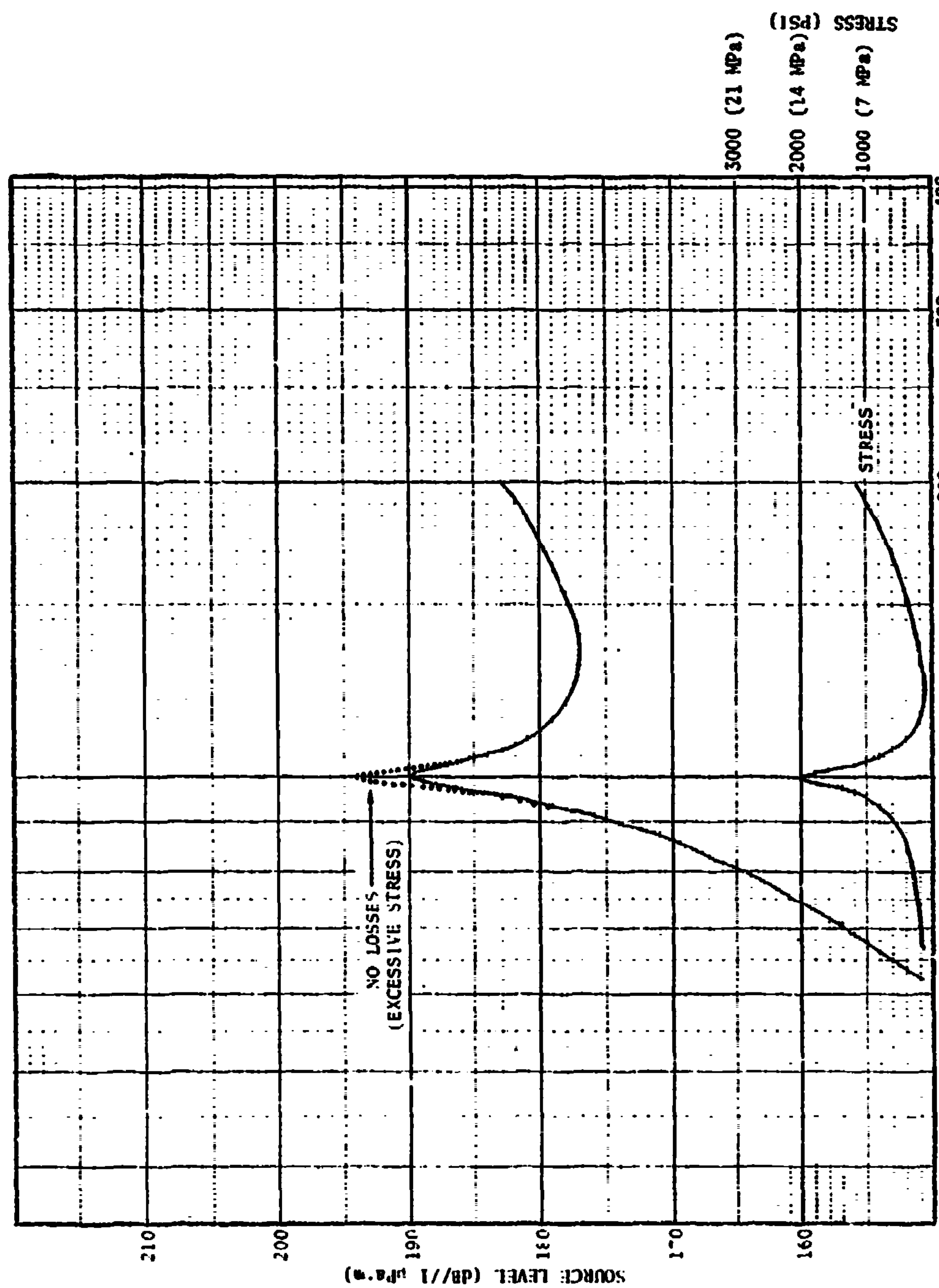
Figure 10 shows results for a resonance frequency of 140 Hz. The predicted source level is 212 dB//1 μPa·m (11 000 W). The chamber length would become impractically small if the packing factor were kept at 25 percent. Reducing the packing factor to 12 percent will make the cavity length 0.3 m and will result in high efficiency. A problem remains, however; even with this increased chamber length the transducer mass will probably be insufficient to keep the transducer from shaking substantially. The radiation from this shaking motion may interfere with the principal radiation. The solution is to use radial-flow ports, symmetrically placed. The ports are self-balancing with respect to vibration and do not require case motion to achieve conservation of momentum. The end of the cavity opposite the driving disk will be closed with a rib-reinforced metal cap. The cap adds considerably to the weight, so the final tally will be about 4400 lb (2000 kg). The option of adding a second driving disk while keeping the rest of the transducer unchanged does not exist for this transducer. The h/a ratio for the double-disk version would far exceed the accepted limit of 0.30.

Figure 11 shows results for a resonance frequency of 200 Hz. The predicted source level is 214 dB//1 μPa·m (17 000 W). In this design



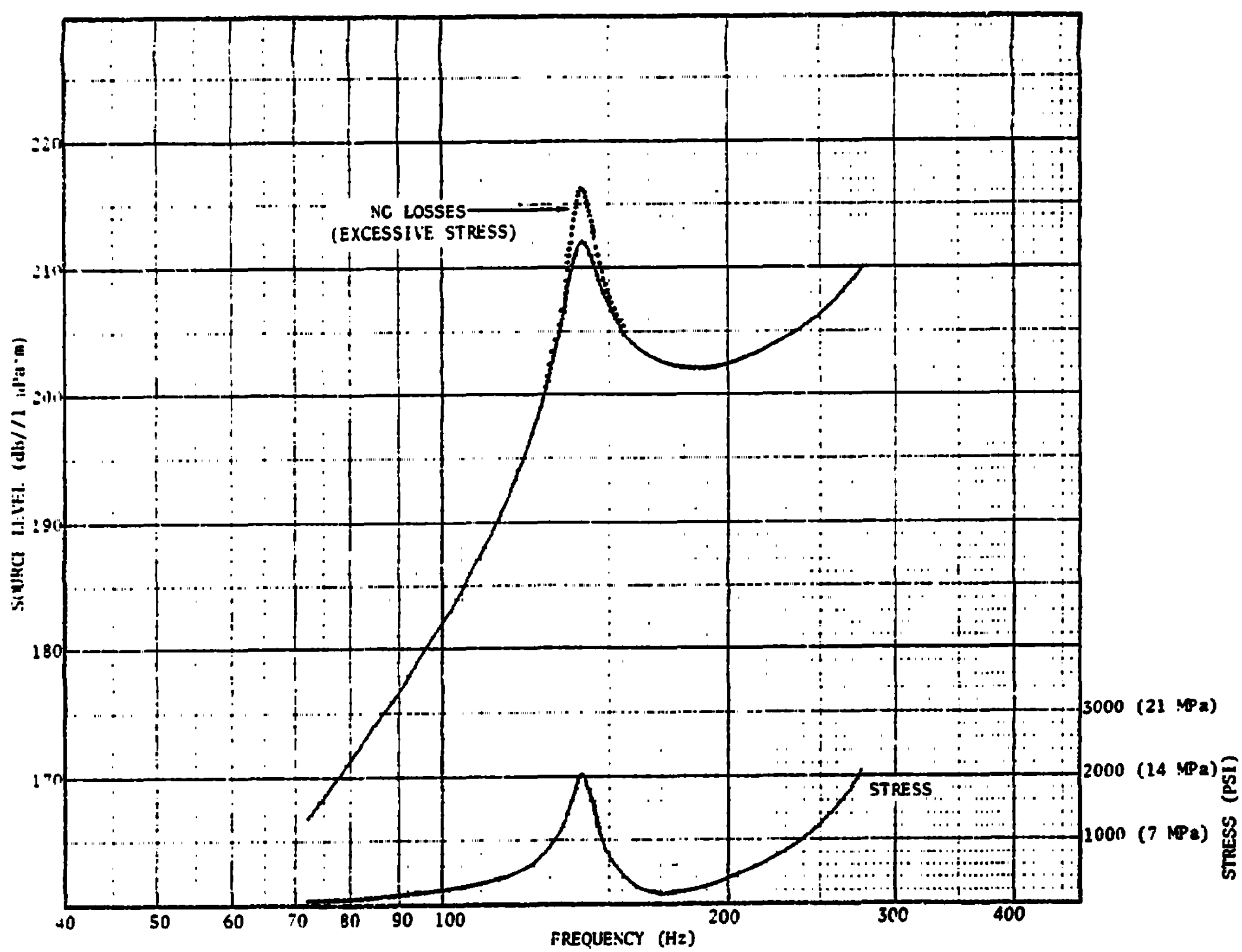
$h/a = 0.16$ $\alpha = 0.07$ $Q_o = 52$ $\ell_c = 0.64$ m $2a = 1$ m max. depth = 460 m min. depth = 28 m

Figure 8. 65 Hz Single-Disk Helmholtz Resonator



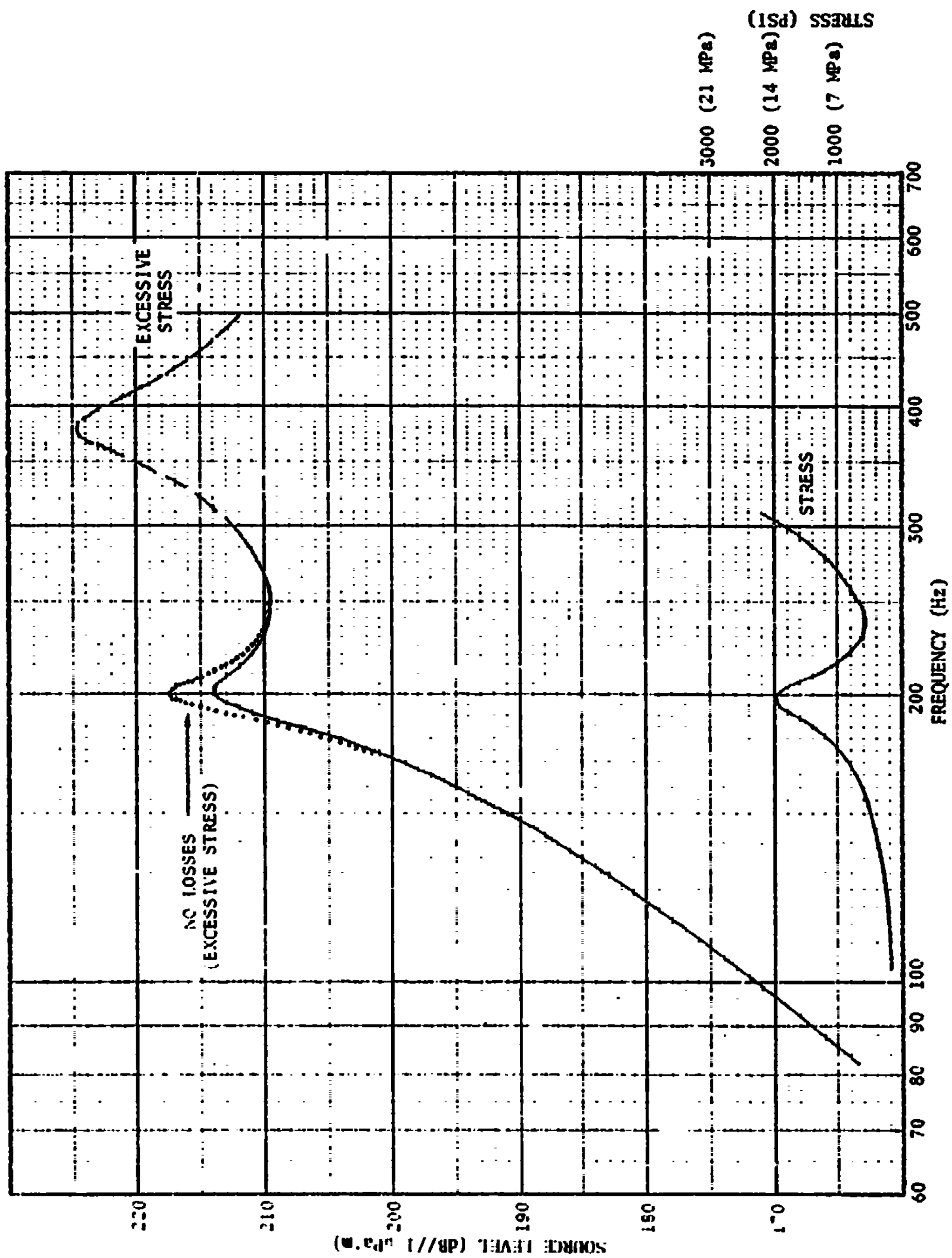
$h/a = 0.239$ $\alpha = 0.05$ $Q_0 = 32$ $t_c = 0.28$ m $2a = 1$ m max. depth = 460 m min. depth = 63 m

Figure 9. 100 Hz Single-Disk Helmholtz Resonator



h/a = 0.28 ε = 0.061 Q_o = 24 ℓ_c = 0.30 m 2a = 1 m max. depth = 460 m min. depth = 87 m

Figure 10. 140 Hz Single-Disk Helmholtz Resonator



h/a = 0.33 α = 0.0719 Q_o = 1. ℓ_c = 0.43 m 2a = 1 m max. depth = 460 m min. depth = 100 m

Figure 11. 200 Hz Single-Disk Helmholtz Resonator

there was difficulty in getting α small enough to permit a substantial rise in the response at resonance before encountering the stress limit. It was necessary to reduce the orifice inertance from the 1000 kg/m^4 value used in the previous three designs to 700 kg/m^4. This change requires radial orifices and a chamber length of at least 0.43 m to accommodate the orifice area. The shaking problem mentioned for the previous design thus will not arise. The weight of the transducer is about 4900 lb (2200 kg). In this transducer the maximum h/a of 0.30 is used, and the cavitation-avoidance depth is 327 ft (100 m). The tube packing factor is only 6 percent.

Designs with resonance frequencies greater than 200 Hz in this 1 m disk line do not appear feasible. In the last example, the disk thickness had reached its maximum value, and the orifice inertance was close to its minimum value, yet the allowable resonance rise was very limited. The disk diameter is about one-eighth wavelength for this example, so this number may be taken as the useful upper size limit for disk-driven Helmholtz resonators.

DISCUSSION OF DESIGNS

In the narrowband series of Helmholtz resonators (figures 7 to 11) the power-to-weight ratio varies almost two orders of magnitude over the range of designs. Normalizing the W/kg values with respect to frequency to obtain a figure of merit,[4] one obtains 2.4×10^{-3} W/kg per Hz at 40 Hz and 3.7×10^{-2} W/kg per Hz at 200 Hz. Thus, this figure of merit improves more than an order of magnitude in going from a resonance frequency of 40 Hz to a resonance frequency of 200 Hz. The upper value is comparable to values achieved in conventional medium-frequency transducers.

A review of figure 6 following the study of the specific examples may be instructive. Above the break in the curves the designs are based on use of minimum inertance and may be considered high-performance designs. Below the break in the curves the performance deteriorates because of the restrictions on chamber length. The break occurs at about 50 Hz, where the disk diameter is about 1/30 of a wavelength. This is the dividing point between restricted and high-performance Helmholtz resonators when the depth capability is 1500 ft (460 m). For shallower depths the dividing point moves to lower frequencies. The restricted performance results from miniaturization of the transducer in wavelength measure ($2a = \ell_c \cdot \lambda/30$).

As an example of a highly-miniaturized transducer, suppose there is a need for a narrowband source at 10 Hz with 1500 ft (460 m) depth capability. A double disk Helmholtz resonator should be able to produce about 175 dB source level (2 W). The required inertance would be 14 000 kg/m^4, so this would definitely be a narrow-neck transducer, in contrast to the wide orifice types used in the designs of figures 8 to 11. The nonlinear effects of the narrow neck would be substantial even at the low power levels anticipated. Prospective users would be unhappy to have to employ an expensive 6000 lb (2727 kg) transducer just to produce 2 W of acoustic power, but at 10 Hz and 1500 ft (457 m) depth there is little choice.

The Helmholtz resonator transducer is basically an underwater version of the bass-reflex loudspeaker. The prescriptions for designing bass-reflex loudspeakers call for making the two uncoupled resonances of the system coincide approximately. When the device is assembled the modal coupling will push the two resonances apart, and the final result is supposed to be a broad flat-topped response boost. Achieving the flat top requires a judicious balancing of acoustic coupling (determined by α) and damping, which is rarely found in hi-fi products. To bring the two uncoupled resonances close together in the underwater transducer calls for low inertance, large h/a, and large α. The response curves show the coupled resonances, and figure 11 gives a fairly representative example of how close these can be brought together under practical conditions. The ratio of resonance frequencies here is 1.9; with further juggling it might be made as low as 1.6. Normal damping comes nowhere close to providing a flat top to the response, unfortunately. As described earlier, sophisticated equalization procedures are necessary to achieve this.

While figure 11 shows a passband near the top of the feasible frequency range, the problems in obtaining a flat-topped response are not much different for the design shown in figure 5, which is for a passband near the bottom of the range. The ratio of resonance frequencies for the latter case is 2.5, only moderately greater than for figure 11. It is concluded that a deliberate effort to follow the loudspeaker design procedure would not lead to any new results not already included in the given examples. However, it should be noted that at very low frequencies ($<$ 30 Hz) the situation will be quite different from the loudspeaker case, since the ratio of resonance frequencies becomes large in this region where no constraints have been placed on the inertance.

The frequency span of the direct-radiator transducer occupies a higher region of the frequency scale than that of the Helmholtz resonator transducer. It extends only to about 60 Hz on the low end but upward to about 350 Hz. A wide region of overlap exists in which the two types of transducers can be compared.

For example, assume that a resonance frequency of 100 Hz is specified. For narrowband applications figure 3 may be compared with figure 9, remembering in figure 3 that stress considerations limit the resonance peak to a source level of 198 dB//1 μPa·m. The single-disk Helmholtz resonator is seen to produce about 12 dB more source level than the double-disk direct radiator. Also it weighs less, since the chamber length is shorter. However, the Helmholtz resonator is restricted to operation at depths greater than 207 ft (63 m), whereas the direct radiator is free of such restrictions. For a look at broadband applications, a double-disk Helmholtz resonator has been compared with the double-disk direct radiator in figure 12. The Helmholtz resonator transducer produces about 7 dB more source level in the passband above 100 Hz. Again, it weighs less than the direct-radiator transducer, but has a minimum depth restriction.

STATE OF THE ART

The basic principles of the underwater Helmholtz resonator transducer have been verified by model experiments in the 140 to 240 Hz range using 6-inch driving disks.[5] However, not all of the features of the designs described above have been investigated experimentally. A major innovation not yet tried is the use of radial-flow ports, as in figure 1B. Even if the estimated inertance of such ports is found in later experiments to be inaccurate, it is expected that the predicted performance will not suffer greatly. Presumably it will be possible to redesign the ports to give the desired inertance.

As mentioned earlier, construction of 1 m disks has not yet been accomplished, but should present no insurmountable difficulties. Cost estimates cannot be made with much accuracy at this time.

The fatigue properties of compliant tubes have not been sufficiently investigated yet. They could be used with confidence in broadband transducers, but in narrowband (highly resonant) Helmholtz resonators their reliability has not been well verified. The worst case would be for the narrowband transducer using maximally-thick disks (h/a = 0.30). The ac cavity pressure could reach 150 psi (1 MPa) peak (about 20 percent of the static pressure at 1500 ft (460 m)).

In addition to the Helmholtz resonator types covered in the design examples, which were chosen on the basis of their estimated usefulness, other variations could have occasional utility. One is the liquid-filled Helmholtz resonator that does not contain compliant tubes. Its merit lies in the fact that it has no depth limitation. The compliance chamber must be large and the walls very thick, so this is a heavy transducer. However, if a VLF transducer is needed to work at the bottom of the ocean, there may be no better approach to the problem. A model of this type of transducer was tested and found to work as expected.

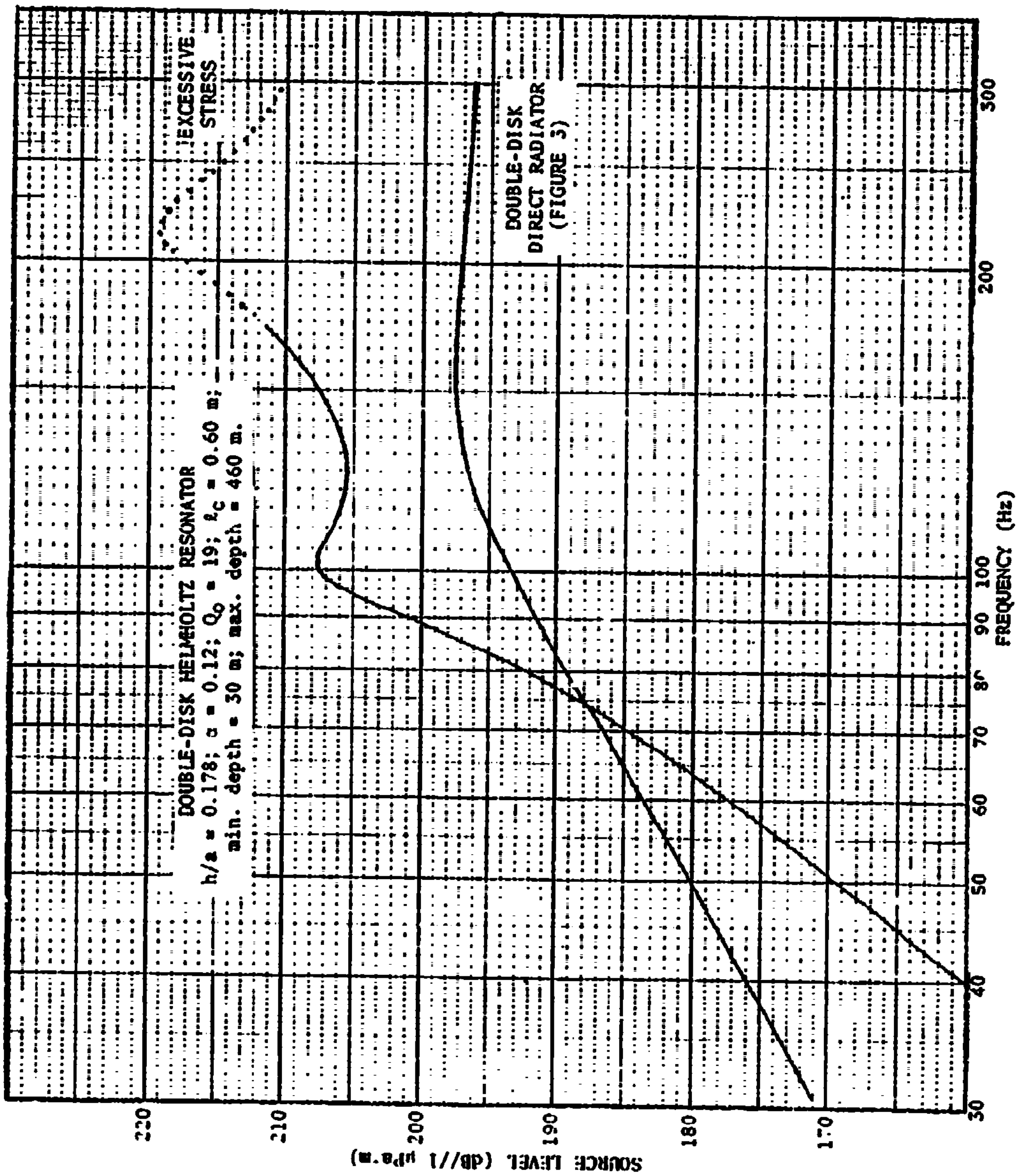


Figure 12. Comparison of Helmholtz Resonator With the Direct Radiator

Another variation, which has not yet been tried, is to use a gas for filling the compliance chamber of the Helmholtz resonator. This approach is believed feasible only for transducers that operate at fixed depth. Air under the pressure of a 500 m head of water is about nine times more compressible than a 50 percent pack of compliant tubes in oil. Hence, if gas is used the chamber size can be reduced dramatically. The payoff would come in performance rather than size for the Helmholtz resonators that are chamber-size limited (those designs that lie below the break in the curves in figure 6). With air substituted for the compliant tube pack, the break in the curves would move down to about 17 Hz, and the source levels at this frequency would increase about 6 dB. This improvement would also apply to the narrowband 10 Hz transducer mentioned on page 23 if it were filled with air.

The partially-explored areas mentioned above are expected to be investigated in further model experiments. Mathematical studies by NUC using finite element methods are also contributing to the advancement of the state of the art.[5]

SCALING OF DESIGNS TO OTHER FREQUENCIES

The design examples given earlier may be changed to other frequencies by linear scaling. When the resonance frequency (f_r) of a given design is changed, all dimensions remain invariant if expressed in terms of sound wavelengths; this defines linear scaling. The transducer shape remains unchanged. The response curves are translated in frequency and shifted in level, but their shape remains unchanged.

Some of the invariants under linear scaling are: h/a, $\frac{\ell_c}{2a}$, $\frac{2a}{\lambda}$, α, Q_0, max. depth, min. depth, W/kg per Hz. Other properties vary as follows:

dia (2a)	$\sim f_r^{-1}$	power	$\sim f_r^{-2}$
weight	$\sim f_r^{-3}$	watts/kg	$\sim f_r$
inertance	$\sim f_r$		

The source level change is -20 log (scale factor), where the scale factor is defined as the ratio of the new frequency to the old. The efficiency would be invariant if the dissipation factors (tangents of the loss angles) were independent of frequency. Assuming such behavior for the dissipation seems like a reasonable first approximation, even though it lacks theoretical support.

Scaling is a convenient method for obtaining a quick view of the range of possible performance of projector designs at some new chosen frequency. Guided by this view, it is easy to steer the design process towards the desired goals, using the conventional design formulas and mathematical models. Of course, as transducers are scaled down in size a limit is finally reached beyond which it is impractical to keep all the relative dimensions the same. Machining tolerances and the end of the size ranges for such items as O-rings, cable connectors, and compliant tubes bring a halt to strict scaling, though scaling may still contribute to the designer's understanding.

Two examples of scaling will be given. First, suppose that the goal is to produce a broadband projector to cover the 350 to 700 Hz octave band. There were four broadband designs included in the original examples, so these will be the prototypes for the new designs. The results of scaling these four to the new frequency band are shown in the first section of table 1. The lower end of the passband (350 Hz) was taken to be 0.85 f_r for the direct radiator and 0.95 f_r for the Helmholtz resonator. This results in the resonance frequencies for the new designs being 412 Hz for the direct radiator and 368 Hz for the Helmholtz resonator. The original design of figure 5 was for 800 ft (240 m) depth. To bring it into conformance with the 1500 ft (460 m) specification requires subtraction of 2 dB from its response.

The scaled designs in table 1 offer a wide range of source levels. There is a considerable gap between the figure 5 and the figure 12 entries in the table. To generate a new design falling between these two entries, use can be made of figure 6. Choosing f_r = 65 Hz and α = 0.15 for the prototype, one obtains h/a = 0.12 and a source level of 197.5 dB//1 μPa·m. Scaling reduces the source level to 182 dB//1 μPa·m for the new design. The disk diameter is 0.18 m, and the minimum operating depth is 57 ft (17 m). Further use of figure 6 will enable additional interpolations to be made in the table. The conclusions drawn from the scaled designs are the same as those previously reached. The Helmholtz resonator gives more source level in relation to size than the direct radiator, but has the onus of a minimum allowed operating depth.

Next, suppose a 200 Hz narrowband projector is desired, but that the source level required is less than that provided by the design of figure 11. If all the narrowband examples given previously are scaled to 200 Hz, a considerable choice of designs will result. The second section of table 1 shows the family of designs that is produced by this approach. Also included are designs that were converted to double-disk models before scaling. The trends are familiar. The larger the transducer, the higher is the source level and the greater is the minimum operating depth. The Q_0 column indicates that the efficiency will improve greatly with size.

Table 1. Designs Derived From 1 Meter Prototypes by Scaling

Prototype Design (Figure No.)	Invariants					Scaled Designs				
	h/a	$\ell_c/2a$	Cav. Depth (m)	α	Q_o	Freq. Hz	Scaling Factor	Dia. (m)	Wt. (kg)	Source Level (dB//1 μPa·m)
First Section										
2	0.05	1	-	0.62	72	350-700	7.10	0.14	7	170
3	0.102	1	-	0.156	53	↓	4.12	0.24	41	185
5 (460 m)	0.093	1	9	0.20	197	↓	12.5	0.08	1	167
12	0.178	0.6	50	0.12	19	↓	3.68	0.27	48	194
Second Section										
7	0.138	1	21	0.07	143	f_r = 200	5.0	0.20	22	182
7 (d.d.)	0.177	1	34	0.07	143	↓	5.0	0.20	24	186
8	0.160	0.64	28	0.07	52	↓	3.08	0.33	66	193
8 (d.d.)	0.205	0.64	47	0.07	52	↓	3.08	0.33	90	197
9	0.239	0.28	63	0.05	32	↓	2.0	0.50	168	204
9 (d.d.)	0.306	0.28	104	0.05	32	↓	2.0	0.50	300	208
10	0.28	0.30	87	0.061	24	↓	1.43	0.70	690	209
11	0.30	0.43	100	0.072	12	↓	1.0	1.0	2410	214

Notes: 1. Depth capability = 460 m.
2. d.d. = double disk

FINAL REMARKS

This report was devoted to presenting predicted performance results for VLF flexural disk transducers. The theory employed and experimental verification data were conspicuously lacking. Some of the missing material is available in the references, but documentation of the Helmholtz resonator work is far from complete at this time. A forthcoming report will cover the theory and design procedures of Helmholtz resonator transducers, and other publications will document the experimental results as they are obtained.

By devoting this report entirely to descriptions of expected performance, the subject could be treated at sufficient length to give a reasonable picture of Helmholtz resonator capabilities. It is hoped that this report will meet the needs of prospective users in determining whether these transducers are applicable to their requirements.

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Subj: Errata in NUSC Technical Report 5509, VLF Flexural Disk Transducers Using Disks 1 Meter in Diameter

1. Addressees are requested to make the following changes in their copies of the report:

Figure 7 — Subtract 20 dB from the source level scale on the ordinate.

Figure 9 — Add 20 dB to the source level scale on the ordinate.

C. T. KINDILIEN
Head, Technical Information Department
By direction of the Commanding Officer