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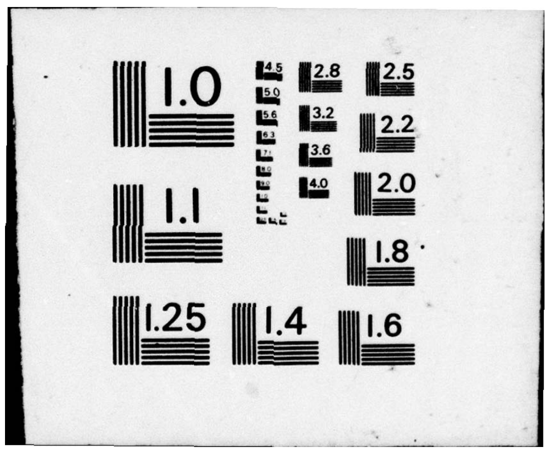
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REPORT SAMSO TR-76-104

(12)

Relation Between Bounce-Averaged Collisional Transport Coefficients for Geomagnetically Trapped Electrons

Space Sciences Laboratory
The Ivan A. Getting Laboratories
The Aerospace Corporation
El Segundo, Calif. 90245

8 December 1976

Interim Report

Prepared for
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This report has been reviewed by the Information Office (OI) and is releasable to the National Technical Information Service (NTIS). At NTIS, it will be available to the general public, including foreign nations.

This technical report has been reviewed and is approved for publication. Publication of this report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

FOR THE COMMANDER

Ronald C. Lawson

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The unweighted bounce average $\langle N_j \rangle$ of the atmospheric density (N_j) along a field line and the weighted bounce average $\langle (B_0/B)N_j \cos^2\alpha \rangle$ are related (for each atmospheric constituent(j) by an integral or derivative with respect to the sine (y) of the equatorial pitch angle (α_0), where $B/B_0 (= y^{-2}\sin^2\alpha)$ is the ratio of local to equatorial magnetic-field intensity and α is the local pitch angle. The unweighted bounce average $\langle N_j \rangle$ is relevant to energy deposition by radiation-belt particles of all species, while the weighted bounce average → next page

alpha

1/y squared sin squared alpha

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19. KEY WORDS (Continued)

cos squared alpha

20. ABSTRACT (Continued)

$\langle (B_{(0)}/B) N_{(j)}^{-1} \cos^2 \alpha \rangle$ is relevant to the description of pitch-angle diffusion of radiation-belt electrons.

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PREFACE

The author is pleased to thank Dr. G. T. Davidson for a preprint introducing (7) as a new approximation for $T(y)$, and Dr. H. H. Hilton for some empirical data on $\langle N_j \rangle$.

CONTENTS

INTRODUCTION

The interaction of geomagnetically trapped electrons with the neutral atmosphere entails both a diffusion in local pitch angle (α) and a loss in electron energy (E). The corresponding transport coefficients $D_{\alpha\alpha}$ and $(dE/dt)_{\nu}$ can be constructed by summing the contributions $D_{\alpha\alpha}^{(j)}$ and $(dE/dt)_j$ from the various atmospheric constituents (j). The subscript ν on dE/dt serves only to emphasize that the corresponding process is essentially frictional, as distinguished from stochastic. The partial transport coefficients $D_{\alpha\alpha}^{(j)}$ and $(dE/dt)_j$ are proportional to the local number density N_j of atmospheric constituent j but are independent [e.g., Walt, 1966] of the local pitch angle (α). Both decrease in magnitude as the energy (E) of the incident electron increases.

Radiation-belt evolution is conventionally described [e.g., Haerendel, 1968] by means of a bounce-averaged Fokker-Planck equation. Thus, it proves convenient to introduce the bounce-averaged partial transport coefficients $D_{yy}^{(j)} = \langle (B_0/B)\, D_{\alpha\alpha}^{(j)} \cos^2\alpha \rangle$ and $\langle (dE/dt)_j \rangle$, where

$$y = (B_0/B)^{1/2} \sin \alpha \tag{1}$$

is the sine of the equatorial pitch angle (α_0) and B_0 is the minimum value of B (the magnetic-field intensity) along the field line (L). The bounce average of any quantity Q is given by

$$\langle Q \rangle = (\Omega_2/2\pi) \oint (Q/v) \sec \alpha \, ds, \tag{2}$$

where v is the speed of the electron, s is the coordinate that measures arc length along $\underset{\sim}{B}$, and $2\pi/\Omega_2$ is the full bounce period defined by setting $Q = \langle Q \rangle = 1$ in (2). The purpose of the present note is to derive an analytical relationship between the contributions of a given atmospheric constituent (j) to D_{yy} and $\langle (dE/dt)_v \rangle$, i.e., to derive an analytical relationship between $\langle (B_0/B) N_j \cos^2\alpha \rangle$ and $\langle N_j \rangle$.

DERIVATION

The essential step in the derivation is to observe that (1) implies $\cos \alpha = [1 - y^2(B/B_0)]^{1/2}$. Thus, it follows from (2) that

$$(2\pi/\Omega_2) \langle (B_0/B) N_j \cos^2\alpha \rangle = \oint \frac{(N_j/v)(B_0/B) \, ds}{[1 - y^2(B/B_0)]^{-1/2}} \tag{3}$$

while

$$y(2\pi/\Omega_2) \langle N_j \rangle = \oint \frac{y \, (N_j/v) \, ds}{[1 - y^2(B/B_0)]^{1/2}} \, . \tag{4}$$

Moreover, the right member of (4) is equal to minus the derivative of the right member of (3) with respect to y. Thus, it follows that

$$\langle (B_0/B) N_j \cos^2\alpha \rangle = \Omega_2(y) \int_y^1 [u/\Omega_2(u)] \langle N_j \rangle \, du, \tag{5}$$

since $\cos^2\alpha \equiv 0$ for $y = 1$. This is the desired result. The practical significance of (5) is that, given either $\langle N_j \rangle$ or $\langle (B_0/B)N_j \cos^2\alpha \rangle$ as a function of y, one can immediately obtain the other by integration or differentiation (respectively).

It can be shown by Jacobian methods [Haerendel, 1968] that the bounce-averaged phase-space density $\bar{f}$ satisfies the Fokker-Planck equation [cf. Walt, 1966]

$$\frac{\partial \bar{f}}{\partial t} = \frac{\Omega_2}{y} \frac{\partial}{\partial y} \left[\frac{y D_{yy}}{\Omega_2} \frac{\partial \bar{f}}{\partial y} \right] - \frac{1}{\gamma^2 v} \frac{\partial}{\partial E} \left[\gamma^2 v \left\langle \left(\frac{dE}{dt} \right)_\nu \right\rangle \bar{f} \right] \tag{6}$$

in the absence of range straggling ($D_{EE} = 0$) and radial diffusion ($D_{LL} = 0$), where γ is the ratio of relativistic mass (m) to rest mass (m_0). The present work has shown that the contributions of any atmospheric constituent (j) to D_{yy} and $\langle (dE/dt)_\nu \rangle$ in (6) are related by virtue of (5).

APPLICATIONS

The bounce period $(2\pi/\Omega_2)$ in a dipolar magnetic field ($\underset{\sim}{B}$) is proportional, at fixed energy (E) and shell parameter (L), to a function called T(y) which is well approximated [Davidson, 1976] by the formula

$$\begin{aligned} T(y) &\approx T(0) - [T(0) - T(1)]y^{3/4} \\ &\approx 1.380173 - 0.639693\, y^{3/4}. \end{aligned} \tag{7}$$

Earlier approximations of $T(y)$ had been given by Hamlin et al. [1961] and Lenchek et al. [1961]. Although $T(y)$ can be generated to arbitrary accuracy by a numerical integration of (2) for $Q = \langle Q \rangle = 1$, the use of (7) in (5) should make the required integral for $\langle (B_0/B) N_j \cos^2\alpha \rangle$ much easier to perform in most instances.

A particularly simple case utilizing (7) is that of a spatially uniform atmospheric density N_j. This occurs in most laboratory devices, for example. (A laboratory device that simulates the geometry of the magnetosphere is commonly called a "terrella.") In situations such as this, for which $\langle N_j \rangle = N_j$ (a constant), it follows from (5) that

$$\langle (B_0/B) N_j \cos^2\alpha \rangle = [Z(y)/T(y)] N_j, \tag{8}$$

where

$$\begin{aligned} Z(y) &= \int_y^1 u\, T(u)\, du \\ &\approx (1/2)(1 - y^2)\, T(0) \\ &\quad - (4/11)(1 - y^{11/4})\, [T(0) - T(1)] \end{aligned} \tag{9}$$

if $T(y)$ is approximated by (7). The present approximation yields $Z(0) \approx 0.457471$, which agrees well with the exact result [Schulz, 1974] that $Z(0) = 16/35$ (≈ 0.457143). An earlier approximation of $Z(y)$, which yielded $Z(0) \approx 0.455533$, had been derived by Schulz [1974] in a different context. It follows from (6) that the Fokker-Planck equation for scattering by a uniform atmosphere is of the form

$$\frac{\partial \bar{f}}{\partial t} = \frac{D_{\alpha\alpha}}{y\,T(y)} \frac{\partial}{\partial y}\left[y\,Z(y)\,\frac{\partial \bar{f}}{\partial y}\right] - \frac{1}{\gamma^2 v}\frac{\partial}{\partial E}\left[\gamma^2 v \left\langle\left(\frac{dE}{dt}\right)_\nu\right\rangle \bar{f}\right], \quad (10)$$

where $D_{\alpha\alpha}$ and $\langle(dE/dt)_\nu\rangle$ are independent of y and of L.

In the real magnetosphere, of course, one does not encounter anything like a uniform atmosphere. However, various investigators have numerically implemented the bounce averages indicated in (3) and (4). The dashed and solid curves [Walt, 1966] shown in Figure 1 represent (respectively) the quantities $(2\,\gamma^2 v^3/c^3 x^2)\,D_{yy}$ and $(-v/m_0c^3)\,\langle(dE/dt)_\nu\rangle$, where x is the cosine of the equatorial pitch angle (hence $x^2 + y^2 = 1$) and c is the speed of light. These particular forms serve to factor out the energy dependence of the transport coefficients, except for *some slowly varying* logarithms which are evaluated at $E \sim 1$ MeV.

Except for the fact that the neutral atmosphere has several constituents (j), one should expect to find

$$(v/m_0c^3)\,\langle(dE/dt)_\nu\rangle$$

$$= \frac{A}{y\,T(y)}\,\frac{d}{dy}\left\{(1 - y^2)\,T(y)\left[\frac{2\gamma^2 v^3}{x^2 c^3}\,D_{yy}\right]\right\}, \quad (11)$$

where A is a dimensionless constant that depends in detail upon the ratio between the inelastic and elastic cross sections of atmospheric molecules or atoms for incident electrons. The "data" points (filled

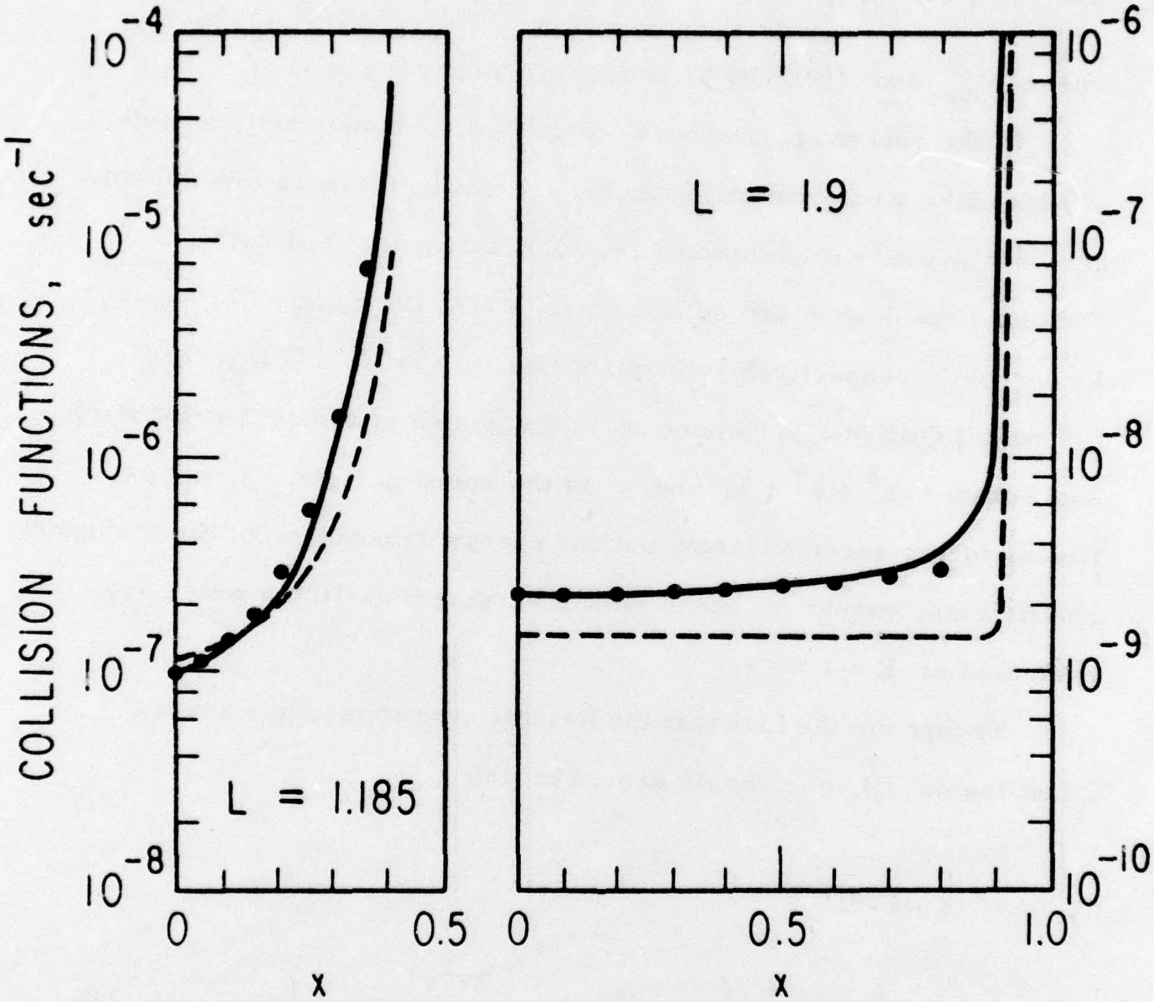

Figure 1. **Magnitudes of $(-v/m_0c^3)\langle(dE/dt)_\nu\rangle$ (solid curve) and $(2\gamma^2v^3/c^3x^2)\,D_{yy}$ (dashed curve) for inner-zone electrons subjected solely to atmospheric collisions [Walt, 1966], together with "data" points derived from dashed curve (and nominally "predicting" the solid curve) by means of (11).**

circles) in Figure 1 serve as an illustration of this idea. The square-bracketed factor in (11) has been differentiated numerically at $L = 1.185$ and treated as a constant (for $x < 0.86$) at $L = 1.9$ for this purpose.

The resulting "data" points (normalized by evaluating A to force agreement at $x = 0$) follow the solid curves rather well. One would not have expected perfect agreement, since the densities (N_j) of the various atmospheric constituents (j) have different altitude profiles and contribute to D_{yy} and $\langle (dE/dt)_\nu \rangle$ with different weights (essentially as Z_j^2 to D_{yy} and as Z_j to $\langle (dE/dt)_\nu \rangle$, where Z_j is the nuclear charge). Thus, the transition from hydrogen dominance to oxygen dominance along field lines on the $L = 1.9$ drift shell may help to account for the departure of the "data" points from the solid curve at large x.

Bounce-averaged atmospheric densities $\langle N_j \rangle$ have been compiled by several investigators [e.g., Cornwall et al., 1965] in the course of studies on radiation-belt protons (for which atmospheric pitch-angle diffusion is an unimportant process). The present result, as summarized by (5), enables one to extract the weighted bounce average relevant to electron pitch-angle diffusion directly from such compilations of $\langle N_j \rangle$. For example, if one has found that $\langle N_j \rangle = \sum_n N_n^{(j)} y^{-n}$ for some range of L values, then it follows from (5) and (7) that

$$\langle (B_0/B)\, N_j \cos^2\alpha \rangle$$

$$= \sum_n \frac{N_n^{(j)}}{T(y)} \left[\frac{1 - y^{2-n}}{(2 - n)} - \frac{1 - y^{2.75-n}}{0.53939\,(11 - 4n)} \right] \qquad (12)$$

Terms corresponding to n = 2 and n = 11/4, if present in (12), reduce to logarithmic form upon expansion. Applications such as this may enable the present discovery (which seems just mildly curious at first sight) to be useful in practice as well as in theory.

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