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A MODEL FOR ESTIMATING JOINT PROBABILITIES OF WEATHER EVENTS,(U)
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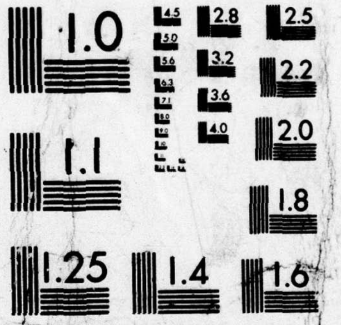

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Reprinted from Preprint Volume: Seventh Conference on Aerospace and Aeronautical Meteorology and Symposium on Remote Sensing from Satellites. Nov. 16-19, 1976; Melbourne, Fla. Published by American Meteorological Society, Boston, Mass.

A MODEL FOR ESTIMATING JOINT PROBABILITIES OF WEATHER EVENTS*

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1. INTRODUCTION

Meteorological satellites have made it possible to obtain information on areal coverage and joint occurrences of meteorological events not heretofore possible. However, a long record of observations is required to derive a climatology of joint occurrences. Routine hourly surface weather observations may be sufficiently concurrent to estimate joint occurrences among sites where such observations are taken. Satellite observations are especially useful in areas where surface observations are unavailable or unsuitable.

Unconditional probabilities of weather events, such as precipitation, freezing temperatures, overcast skies, etc. can be estimated with considerable accuracy from available records. However, the probabilities of such events occurring jointly at two or more locations is often difficult to estimate if the events are not statistically independent. Joint probabilities of a weather event can be estimated either from relative frequencies obtained directly from the data or from a model. Large samples of data must be processed to obtain relative frequencies that are good estimates of true probabilities. Models eliminate the need for data processing to estimate joint probabilities and, in some cases, they can actually provide better estimates of true probabilities than those obtainable from data samples, except in those cases where the data samples are very large.

2. THE MODEL

2.1 Two-site Joint Probabilities

Let us denote the probability that a weather event, W, will occur at site A by $P(W_A)$, at site B by $P(W_B)$, and jointly at the two sites by $P(W_AW_B)$. We know that

$$P(W_AW_B)=P(W_A)\ P(W_B) \qquad (1)$$

if the events at the two sites are statistically independent.

We also know that

$$P(W_AW_B)=P(W_A)=P(W_B) \qquad (2)$$

if the events at the two sites are perfectly correlated and mutually dependent.

If we denote the absence of the event at site A as $P(\overline{W}_A)$, and at site B as $P(\overline{W}_B)$, then $P(W_A)=1-P(\overline{W}_A)$ and $P(W_B)=1-P(\overline{W}_B)$, and we can write Eq. 1 as:

$$\begin{aligned}P(W_AW_B) &= \left[1-P(\overline{W}_A)\right]\left[1-P(\overline{W}_B)\right]\\ &= 1-P(\overline{W}_A)-P(\overline{W}_B)+P(\overline{W}_A)P(\overline{W}_B) \qquad (3)\end{aligned}$$

Eq. 3 applies only in the case when the events are statistically independent. For the case when the events are not statistically independent, the joint probability can be estimated with the expression

$$\hat{P}(W_AW_B)=1-P(\overline{W}_A)-P(\overline{W}_B)+P(\overline{W}_A)P(\overline{W}_B)^{K_1} \qquad (4)$$

$$P(\overline{W}_A)\geqslant P(\overline{W}_B)$$

When there is perfect correlation, $K_1=0$ and Eq. (4) reduces to Eq. (2). When the events are statistically independent, $K_1=1$ and Eq. (4) reduces to Eq. (3). Nearly perfect correlation might be expected when the sites are very close to one another. Statistical independence is approached as the distance between the sites becomes large. Obviously K_1 is a function of the distance between sites A and B.

2.2 Three-site Joint Probabilities

When weather events are statistically independent, three site joint probabilities can be expressed as follows:

$$P(W_AW_BW_C)=\left[1-P(\overline{W}_A)\right]\left[1-P(\overline{W}_B)\right]\left[1-P(\overline{W}_C)\right] \qquad (5)$$

When the events are not independent they can be estimated with the following expression:

$$\begin{aligned}\hat{P}(W_AW_BW_C)=&1-P(\overline{W}_A)-P(\overline{W}_B)-P(\overline{W}_C)+P(\overline{W}_A)P(\overline{W}_B)^{K_1}\\ &+P(\overline{W}_A)P(\overline{W}_C)^{K_1}+P(\overline{W}_B)P(\overline{W}_C)^{K_1}\\ &-P(\overline{W}_A)P(\overline{W}_B)^{K_1}P(\overline{W}_C)^{K_2} \qquad (6)\end{aligned}$$

$$P(\overline{W}_A)\geqslant P(\overline{W}_B)\geqslant P(\overline{W}_C)$$

The K_1 values are obtained from the same function as those used in Eq. (4). The distance

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*This paper has been submitted for publication in the Journal of Applied Meteorology.

between sites A and B, $\overline{AB}$, is the only distance required for finding K_1 in Eq. (4). Distances $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$ are required for determining the necessary K_1 values in Eq. (6) and K_2 is a function of the distance $\overline{AC}$ or $\overline{BC}$, whichever is shorter.

2.3 N-site Joint Probabilities

In the general case, say n sites, joint probabilities of independent events can be expressed as follows:

$$P(W_A W_B \ldots W_n) = \left[1-P(\bar{W}_A\right] \left[1-P(\bar{W}_B\right] \ldots \left[1-P(\bar{W}_n)\right] \qquad (7)$$

When the events are not independent, they can be estimated with the following expression:

$$\hat{P}(W_A W_B \ldots W_n) = 1-P(\bar{W}_A)-P(\bar{W}_B)-\ldots-P(\bar{W}_n)+ P(\bar{W}_A)P(\bar{W}_B)^{K_1}+P(\bar{W}_A)P(\bar{W}_C)^{K_1}+\ldots+P(\bar{W}_{n-1})P(\bar{W}_n)^{K_1}- P(\bar{W}_A)P(\bar{W}_B)^{K_1}P(\bar{W}_C)^{K_2}-P(\bar{W}_A)P(\bar{W}_B)^{K_1}P(\bar{W}_D)^{K_2}-\ldots -P(\bar{W}_{n-2})P(\bar{W}_{n-1})^{K_1}P(\bar{W}_n)^{K_2}+\ldots+P(\bar{W}_A)P(\bar{W}_B)^{K_1}P(\bar{W}_C)^{K_2}\ldots P(\bar{W}_{n-1})^{K_{m-1}}P(\bar{W}_n)^{K_m} \qquad (8)$$

$$P(\bar{W}_A) \geqslant P(\bar{W}_B) \geqslant \ldots \geqslant P(\bar{W}_{n-1}) \geqslant P(\bar{W}_n)$$

K_1 is found for the distance between the two sites with the highest probability of the weather event not occurring, in each term of Eq. (8) in which K_1 appears; K_2 is found for the distance between the site with the third highest probability and the closer of the two sites with higher probabilities of the weather event not occurring, and, so on. The last K in Eq. (8), K_m, is found for the distance between site n, the site with the lowest probability, and its nearest neighbor.

3. AN EXAMPLE

Records of hourly precipitation occurrences, observed in winter and summer, during the 13 year period 1951 through 1963, at the following nine observing sites, shown on Fig. 1, were studied:

LGA-LaGuardia Airport, New York, NY
JFK-Kennedy International Airport, New York, NY
EWR-Newark Airport, NJ
PHL-Philadelphia International Airport, PA
BAL-Baltimore-Washington International Airport, MD
DCA-National Airport, Washington, DC
ADW-Andrews AFB, MD
RIC-Byrd Field, Richmond, VA
RDU-Raleigh-Durham Airport, NC

Each hour, approximately on the hour, a weather observer at each of the above sites went outdoors to make his regular hourly observation. One of the weather elements that he recorded was precipitation. The Federal Meteorological Handbook (1975) describes how the observations are taken.

For this study, only active precipitation occurring at the time of observation was considered as a precipitation event. All types of precipitation (rain, snow, sleet, hail, etc.) and all intensities (very light, light, moderate and heavy) were included.

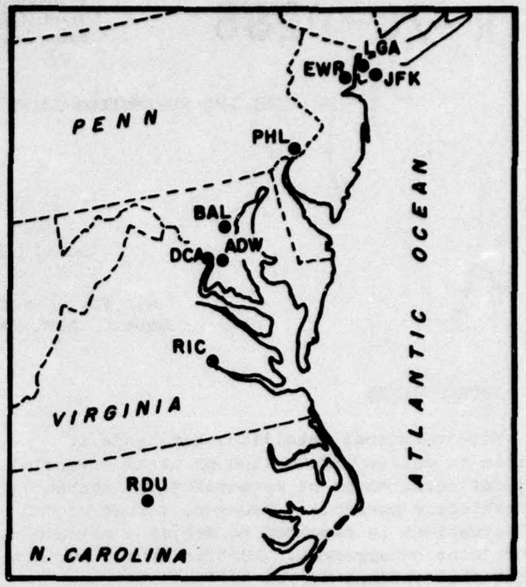

Figure 1. Location of the nine stations whose winter and summer hourly observations of precipitation were studied.

In winter, the observers recorded 0.0641, 0.638 and 12.638 percent of the observations as heavy, heavy or moderate, and all intensities of precipitation, respectively. In summer, the percentages were 0.202, 0.459 and 6.257. Clearly most of the precipitation events were of the light or very light variety.

This study addresses neither the subject of *precipitation intensity nor the amount of precipitation.* The joint occurrences studied are occurrences of active precipitation at two, or more, locations.

3.1 Unconditional Probabilities

The relative frequencies of precipitation as reported on the hourly observations were used as estimates of the unconditional probabilities. Because these relative frequencies were based on 28,080 hourly observations taken at each station in winter and 28,704 hourly observations in summer, they are believed to be good estimates of the true probabilities. The relative frequencies are shown in Table 1.

Table 1. Relative frequencies of precipitation, RF(W), at each of the nine sites under study.

SITE	WINTER	SUMMER
LGA	.1470	.07643
JFK	.1405	.07639
EWR	.1494	.07624
PHL	.1379	.07238
BAL	.1353	.06881
DCA	.1289	.06730
ADW	.1259	.06487
RIC	.1297	.06951
RDU	.1165	.06970

3.2 Two-site Joint Relative Frequencies

As previously stated the probability of precipitation occurring concurrently at two observing sites is a function of the distance between sites. The distances between sites are shown in Table 2.

Table 2. Distances between sites, in nautical miles.

	JFK	EWR	PHL	BAL	DCA	ADW	RIC	RDU
LGA	9	16	95	185	210	214	292	431
JFK		21	94	184	208	213	289	427
EWR			80	169	195	198	279	417
PHL				90	115	119	199	337
BAL					29	30	121	256
DCA						9	93	228
ADW							95	227
RIC								138

The relative frequencies of joint occurrences of precipitation at the 36 pairs of observing sites are shown in Table 3. Because the data samples are large the relative frequencies should be good estimates of true probabilities.

Table 3. Relative frequencies of joint occurrences of precipitation, RF (WW), at each of the thirty-six pairs of observation sites, in winter. Estimated joint probabilities, $\hat{P}$(WW), obtained from the model are shown in parenthesis.

	JFK	EWR	PHL	BAL	ADW	DCA	RIC	RDU
LGA	.125 (.128)	.126 (.124)	.0942 (.0905)	.0738 (.0710)	.0643 (.0659)	.0647 (.0655)	.0551 (.0542)	.0360 (.0365)
JFK		.119 (.121)	.0925 (.0869)	.0718 (.0681)	.0625 (.0634)	.0627 (.0629)	.0536 (.0523)	.0346 (.0353)
EWR			.0991 (.0962)	.0771 (.0751)	.0670 (.0695)	.0676 (.0692)	.0579 (.0568)	.0376 (.0385)
PHL				.0921 (.0862)	.0808 (.0795)	.0808 (.0788)	.0669 (.0640)	.0425 (.0442)
BAL					.104 (.105)	.106 (.105)	.0793 (.0771)	.0516 (.0534)
ADW						.108 (.112)	.0821 (.0800)	.0528 (.0535)
DCA							.0828 (.0796)	.0539 (.0548)
RIC								.0696 (.0697)

3.3 Two-site Estimated Joint Probabilities

Eq. (9), shown below, was found by solving Eq. (4) for K_1 and substituting relative frequencies for probabilities.

$$K_1 = \left\{ \log \left[RF(W_A W_B) + RF(\overline{W}_A) + RF(\overline{W}_B) - 1 \right] - \log RF(\overline{W}_A) \right\} \left\{ \frac{1}{\log RF(\overline{W}_B)} \right\} \qquad (9)$$

$$RF(\overline{W}_A) \geqslant RF(\overline{W}_B)$$

The relative frequencies in Table 3 and the complements of the values found in Table 1, $RF(\overline{W}) = 1 - RF(W)$, were substituted into Eq. (9) and thirty-six solutions were found for K_1, one for each pair of sites.

The K_1 values are given in Table 4 and plotted in Fig. 2. A curve of the form

$$K_\beta = \frac{a_\beta D^\alpha}{b + c_\beta D^\alpha} \qquad (10)$$

was fitted to the data points.

Table 4. Solutions for K_1 obtained from Eq. (9) in winter.

	JFK	EWR	PHL	BAL	ADW	DCA	RIC	RDU
LGA	.167	.168	.398	.556	.625	.622	.702	.845
JFK		.226	.378	.547	.617	.618	.695	.843
EWR			.371	.539	.612	.609	.687	.836
PHL				.367	.455	.457	.574	.770
BAL					.250	.235	.458	.685
ADW						.171	.404	.642
DCA							.399	.643
RIC								.507

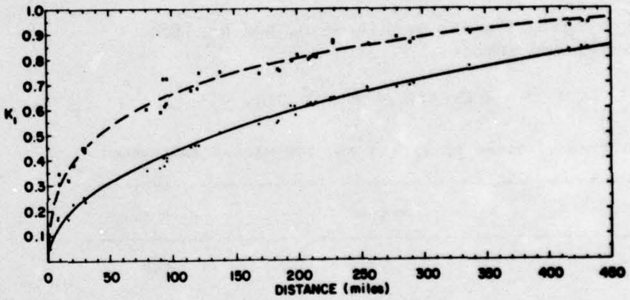

Figure 2. The parameter K_1 plotted as a function of distance. The dots and X's are solutions to Eq. (9) for winter and summer, respectively. The curves are of Eq. (14) with $a_1 = 1.2$ in winter (solid) and $a_1 = 2.7$ in summer (dashed).

Eq. (10) was chosen because of its characteristic shape and because it met the two necessary conditions, that: (1) K_β equals zero when the distance, D, equals zero; and (2) by setting

$$b = D_s^\alpha \qquad (11)$$

where D_s is the distance at which the events are statistically independent, K_β equals one when D equals D_s. At distance D_s when K_β equals one Eq. (10) can be written

$$c_\beta = a_\beta - \frac{b}{D_s^\alpha} \qquad (12)$$

and from Eqs. (11) and (12) it can be seen that $c_\beta = a_\beta - 1$. Thus, if the distance D_s is known, a_β and α are the only parameters that require evaluation.

The parameter α was set equal to one-half and the distance D_s was estimated to be 650 and 550 nautical miles in winter and summer, respectively, based on extrapolation of the points in Fig. 2. Eq. (10) was solved for a_β using the above information. The resulting equation is

$$a_\beta = \frac{K_\beta \,(b-\sqrt{D})}{\sqrt{D}\,(1-K_\beta)} \tag{13}$$

where for winter $b = \sqrt{D_s} = \sqrt{650} = 25.495$, and for summer $b = \sqrt{D_s} = \sqrt{550} = 23.492$.

The 36 K_1 values given in Table 4 together with the distances between the sites were used to find estimates of a_1 for winter. The average value of the 36 a_1's, used as the first guess, was 1.19. Because the points are unevenly distributed along the curve a "best" fit, to the authors' eye, was found by slightly varying the a_1 value from its average. The value chosen was 1.20. The summer average was 2.69 and the value chosen was 2.70.

Substituting Eqs. (11) and (12) into (10) yields:

$$K_\beta = \frac{a_\beta \sqrt{D}}{\sqrt{D_s} + (a_\beta - 1)\sqrt{D}} \tag{14}$$

when $0 \leq D \leq D_s$ and $K_\beta = 1$ when $D > D_s$.

Table 5 gives values of D_s and a_β for winter and summer.

Curves of K_1 are shown in Fig. 2.

Table 5. Values of D_s and a_β for winter and summer.

	Winter	Summer
D_s	650	550
a_1	*1.20*	*2.70*
a_2	1.00	2.30
a_3	0.90	2.15
a_4	0.85	2.10
a_5	0.822	2.08
a_6	0.803	2.068
a_7	0.790	2.062
a_8	0.780	2.057

With a_1 equal to 1.20, Eq. (14) was substituted into Eq. (4) and estimated joint probabilities were found for winter. They are shown in Table 3. The agreement between the relative frequencies and the probabilities estimated by the model is excellent.

3.4 Three-site Joint Relative Frequencies

The nine sites provided 84 combinations of three-site concurrent precipitation occurrences. Sample relative frequencies of these three-site joint occurrences are shown in Table 6.

Table 6. Relative frequencies of joint occurrences of precipitation, $RF(\overline{W}\overline{W}\overline{W})$, at LGA and each of the other twenty-eight pairs of observation sites in winter. Estimated joint probabilities, $P(\overline{W}\overline{W}\overline{W})$, obtained from the model are shown in parenthesis.

	EWR	PHL	BAL	ADW	DCA	RIC	RDU
JFK	.113 (.117)	.0863 (.0869)	.0671 (.0670)	.0585 (.0620)	.0585 (.0616)	.0500 (.0501)	.0322 (.0322)
EWR		.0892 (.0915	.0691 (.0700)	.0602 (.0645)	.0602 (.0642)	.0517 (.0515)	.0331 (.0331)
PHL			.0657 (.0625)	.0573 (.0575)	.0569 (.0569)	.0482 (.0447)	.0298 (.0266)
BAL				.0581 (.0586)	.0590 (.0583)	.0453 (.0442)	.0271 (.0257)
ADW					.0558 (.0582)	.0436 (.0444)	.0256 (.0255)
DCA						.0436 (.0443)	.0258 (.0256)
RIC							.0273 (.0269)

3.5 Three-site Estimated Joint Probabilities

Eq. (15), shown below, was found by solving Eq. (6) for K_2 and substituting relative frequencies for probabilities.

$$K_2 = \Big\{\log\Big[1 - RF(\overline{W}_A\overline{W}_B\overline{W}_C) - RF(\overline{W}_A) - RF(\overline{W}_B) - RF(\overline{W}_C) + RF(\overline{W}_A)RF(\overline{W}_B)^{K_1} + RF(\overline{W}_A)RF(\overline{W}_C)^{K_1} + RF(\overline{W}_B)RF(\overline{W}_C)^{K_1}\Big] - \log RF(\overline{W}_A)RF(\overline{W}_B)^{K_1}\Big\}$$

$$\left\{\frac{1}{\log RF\ (\overline{W}_C)}\right\}$$

$$RF(\overline{W}_A) \geqslant RF(\overline{W}_B) \geqslant RF(\overline{W}_C) \tag{15}$$

The relative frequencies in Table 6 (and the others, not shown, involving sites other than LGA), the complements of the values found in Table 1, and K_1 values using a_1 equal to 1.2 and 2.7 were substituted in Eq. (15) and eighty-four solutions were found for K_2, in winter and summer, one for each set of three sites.

The solutions are plotted in Fig. 3 and sample solutions are shown in Table 7. A good fit to the data points was obtained when a_2 was set equal to 1.00 and 2.30 in winter and summer, respectively.

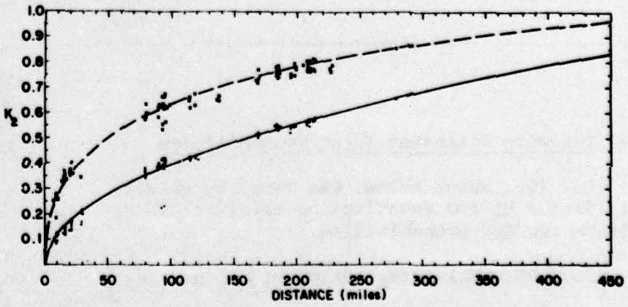

Figure 3. The parameter K_2 plotted as a function of distance. The dots and X's are solutions to Eq. (15) for winter and summer, respectively. The curves are of Eq. (14) with a_2=1.0 in winter (solid) and a_2=2.3 in summer (dashed).

Table 7. Sample solutions for K_2 obtained from Eq. (15) based on relative frequencies of joint occurrences of precipitation at LGA and each of the other twenty-eight pairs of observation sites in winter.

	EWR	PHL	BAL	ADW	DCA	RIC	RDU
JFK	.122	.113	.118	.089	.093	.117	.116
EWR		.138	.149	.122	.125	.159	.157
PHL			.409	.380	.382	.412	.410
BAL				.530	.542	.542	.545
ADW					.546	.562	.569
DCA						.568	.576
RIC							.673

K_1 and K_2 values were substituted into Eq. (6) and estimated 3-site joint probabilities were found for winter. A sample of them is shown in Table 6. There is good agreement between the observed and estimated values.

3.6 Four-site to Nine-site Joint Relative Frequencies

The nine sites provided 126, 126, 84, 36, 9 and 1 combinations of 4, 5, 6, 7, 8 and 9-site concurrent precipitation occurrences, respectively. Sample relative frequencies of these four-site to nine-site joint occurrences are shown in Table 8. In order to conserve space only a few of the possible 382 joint occurrences are shown. They are believed to be typical and include the highest and lowest observed relative frequencies.

Table 8. Sample relative frequencies of joint occurrences of precipitation at four to six sites and corresponding estimated probabilities obtained from the model in winter. In order to conserve space only a few sets of sites are included. They include a range of observed relative frequencies from the lowest to the highest.

Number of Sites	Sites						Winter	
							Observed	Calculated
4			ADW	RDU	PHL	LGA	.0234	.0216
4			RDU	LGA	EWR	JFK	.0305	.0305
4			ADW	RIC	LGA	JFK	.0399	.0422
4			ADW	PHL	RIC	LGA	.0401	.0401
4			ADW	PHL	LGA	EWR	.0546	.0590
4			ADW	LGA	EWR	JFK	.0561	.0620
5		ADW	RDU	RIC	LGA	JFK	.0200	.0221
5		ADW	RDU	PHL	RIC	LGA	.0206	.0206
5		ADW	PHL	RIC	DCA	LGA	.0370	.0388
5		ADW	PHL	DCA	LGA	EWR	.0481	.0557
5		ADW	PHL	LGA	EWR	BAL	.0510	.0561
5		PHL	LGA	EWR	BAL	JFK	.0591	.0631
6	RDU	PHL	RIC	DCA	LGA	JFK	.0188	.0199
6	ADW	RDU	PHL	RIC	LGA	EWR	.0198	.0216
6	ADW	RDU	PHL	LGA	EWR	JFK	.0212	.0217
6	ADW	PHL	RIC	DCA	LGA	JFK	.0344	.0381
6	ADW	PHL	DCA	LGA	BAL	JFK	.0452	.0512
6	PHL	DCA	LGA	EWR	BAL	JFK	.0484	.0549

3.7 Four-site to Nine-site Estimated Joint Probabilities

Relative frequencies were substituted for the probabilities in Eq. (8) and solutions were found for all 126 combinations of 4-site joint precipitation occurrences. The solutions are plotted in Fig. 4. A good fit to the data points was obtained when a_3 was set equal to 0.90 and 2.15 in winter and summer, respectively.

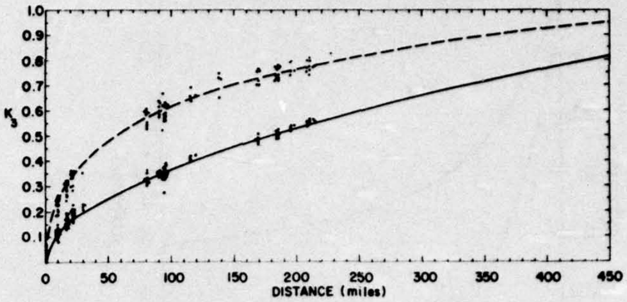

Figure 4. The parameter K_3 plotted as a function of distance. The lower points are based on winter data and the upper points on summer data. The curves are of Eq. (14).

Relative frequencies were substituted for the probabilities in Eq. (8) and solutions were found for all 126 combinations of 5-site joint precipitation occurrences. The solutions are plotted in Fig. 5. A good fit to the data points was obtained when a_4 was set equal to 0.85 and 2.10 in winter and summer, respectively.

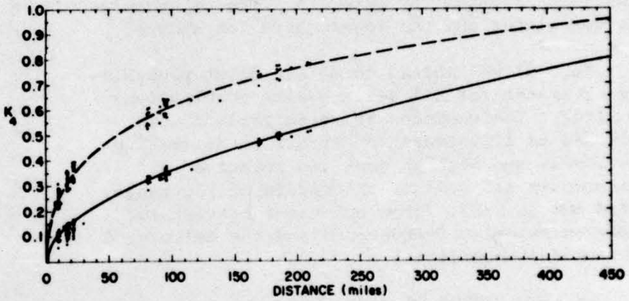

Figure 5. The parameter K_4 plotted as a function of distance. The lower points are based on winter data and the upper points on summer data. The curves are of Eq. (14).

The curves for K_1, K_2, K_3 and K_4 for both winter and summer are shown in Fig. 6. The separation between the curves decreases with increasing subscript. In other words, the value of "a", the only parameter permitted to vary with the number of observation sites, n, decreases less with the addition of each new site. The a_1 through a_4 values are plotted in Fig. 7.

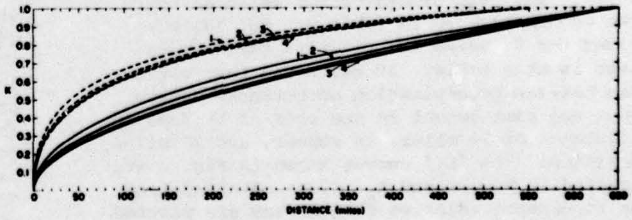

Figure 6. Curves of K_1, K_2, K_3 and K_4 for winter (lower set) and summer (upper set).

Curves were subjectively drawn through the points in Fig. 7 and extrapolated to a total of nine sites. Values for a_5 through a_8 were estimated from these curves to obtain values for use in solving for K_5, K_6, K_7 and K_8. The "a_ϕ" values are shown in Table 5.

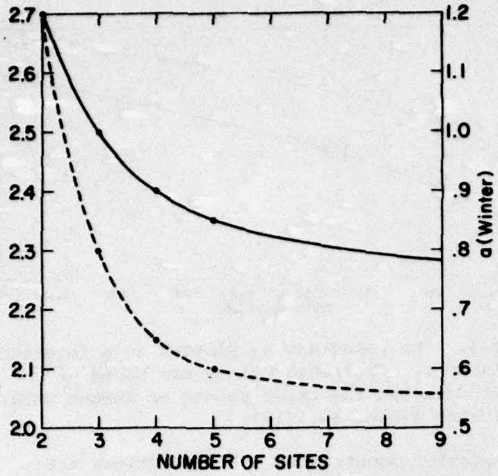

Figure 7. The parameter a plotted as a function of the number of observation sites included in the joint probability estimate. The upper curve is for winter and the lower curve for summer.

Eq. (8) was solved to obtain joint probability estimates for all 4- to 9-site combinations of sites. The examples shown in Table 8 were selected to illustrate the errors in estimating the lowest and highest observed relative frequencies and some in the middle of the range. There was generally good agreement between the observed relative frequencies and the calculated estimated probabilities.

4. AN APPLICATION TO RECURRENCE

At the conclusion of the application of the model to spatial joint occurrences a decision was made to test the model on hourly recurrence of precipitation. Only sets of two events, at time t and t + x hours, were investigated with the following results.

4.1 Recurrence Relative Frequencies

Recurrence relative frequencies of precipitation for the nine East-Coast stations are shown in Figures published by Lund and Grantham (1976). They are tabulated for selected hours from one through 30 in Table 9. Eq. (9) was solved for K_1 using the relative frequencies given in this table. It was found that correlation between precipitation occurrences decays about the same amount in one hour as it does in a distance of 22 miles, in summer, and 27 miles, in winter. The "K_1" curves shown in Fig. 2 are repeated in Fig. 8 and K_1 values obtained from the recurrence relative frequencies are plotted in Fig. 8. The scale is adjusted so that 22 miles equals one hour in summer, and, 27 miles equals one hour in winter. The agreement between spatial K_1 values and temporal K_1 values is excellent.

4.2 Estimated Recurrence Probabilities

Eq. (4) was solved for probabilities every 22 miles in summer and 27 miles in winter. The probability estimates are given in Table 9 corresponding to selected hours from one through 30. Again there is good agreement between the observed relative frequencies and the calculated recurrence probabilities.

Table 9. Observed (OBS) relative frequencies of precipitation recurrences and calculated (CALC) recurrence probabilities using the model derived for joint probabilities in winter. One hour was assumed to equal 27 miles.

LAG (HOURS)	WINTER		
	Miles	OBS	CALC
1	27	.108	.106
2	54	.0967	.0945
3	81	.0879	.0864
4	108	.0803	.0797
5	135	.0737	.0740
10	270	.0493	.0529
15	405	.0342	.0380
20	540	.0256	.0262
25	675	.0210	.0181
30	810	.0184	.0181

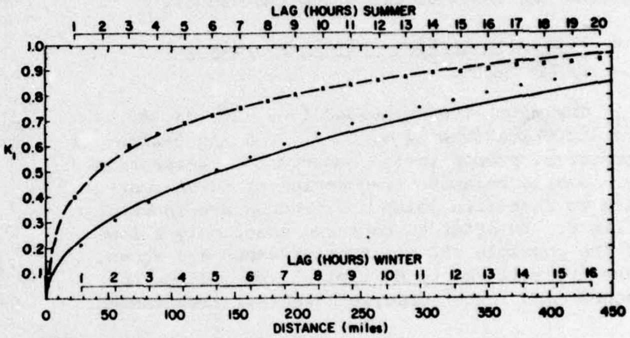

Figure 8. Curves of K_1 obtained from Fig. 2. The dots and X's are solutions to Eq. (9) for winter and summer, respectively, when recurrence relative frequencies are substituted for joint occurrence relative frequencies. A distance of 27 miles, in winter, and 22 miles, in summer, was assumed to be equivalent to one hour of time lag.

5. CONCLUSIONS

The model developed for estimating joint probabilities of weather events is intended for general usage. However, the parameters of the model must be obtained either from data or theory. They will vary with the unconditional probability of the event and the spatial decay of correlation. The test of the model on winter and summer joint-occurrences of precipitation indicates that the parameters are well behaved. Because this is a dependent sample test, the general applicability of the model parameters, even for precipitation occurrences, is not known.

The model is being tested on other weather events.

6. ACKNOWLEDGMENTS

The authors gratefully acknowledge assistance from Professor Donald E. Martin of St. Louis University and Dr. Paul Tsipouras of the Air Force Geophysics Laboratory in the selection of equations for study. Expert computational support was provided by Messrs. James F. Atkinson, Leonard J. Natoli, Kenneth C. Zwirble, of Analysis and Computer Systems, Inc. and Miss Melinda A. Zouvelos, Student Aid from Lowell University. Hourly precipitation data were provided by the USAF Environmental Technical Applications Center and the National Weather Service.

7. REFERENCES

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U.S. Department of Commerce (1975). Federal Meteorological Handbook No. 1, Surface Observations. U.S. Government Printing Office, Washington, D.C., 309 pp.

Security Classification

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)*
Air Force Geophysics Laboratory(LKI)
Hanscom AFB
Massachusetts 01731

2a. REPORT SECURITY CLASSIFICATION
Unclassified

2b. GROUP

(6) 3. REPORT TITLE
A MODEL FOR ESTIMATING JOINT PROBABILITIES OF WEATHER EVENTS

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*
Scientific. Interim.

(10) 5. AUTHOR(S) *(First name, middle initial, last name)*
Iver A. Lund
Donald D. Grantham

(11) 7 Dec 76

(12) 8p.

6. REPORT DATE 7 December 1976	7a. TOTAL NO. OF PAGES 7	7b. NO. OF REFS 2

8a. CONTRACT OR GRANT NO.

(16)

b. PROJECT, TASK, WORK UNIT NOS. 8624 0207

c. DOD ELEMENT 62101F

d. DOD SUBELEMENT (17) 02

(14) 9a. ORIGINATOR'S REPORT NUMBER(S)
AFGL-TR-76-0288

9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*

10. DISTRIBUTION STATEMENT

Approved for public release; distribution unlimited.

11. SUPPLEMENTARY NOTES Reprinted from Preprint Vol, 7th Conf on Aerospace & Aeronautical Meteorology & Symp on Remote Sensing From Satellites, Nov 16-19, 1976, Melbourne, Florida

12. SPONSORING MILITARY ACTIVITY
Air Force Geophysics Laboratory(LKI)
Hanscom AFB
Massachusetts 01731

13. ABSTRACT Meteorological satellites have made it possible to obtain information on areal coverage and joint occurrences of meteorological events not heretofore possible. However, a long record of observations is required to derive a climatology of joint occurrences. Routine, hourly, surface weather observations may be sufficiently concurrent to estimate joint occurrences among sites where such observations are taken.

This paper ~~will~~ describes a model for estimating joint probabilities of weather events. The model was applied to 13 years of hourly winter and summer precipitation observations taken at nine sites along the East Coast of the United States between LaGuardia Airport, N.Y., and Raleigh-Durham Airport, N.C. It requires a knowledge of the unconditional probability of the event, distances between the sites, and a measure of the spatial correlation. Probabilities estimated by the model agree very well with relative frequencies of observed joint occurrences of precipitation.

Satellite observations are needed to derive the parameters of the model and to test it in areas where surface observations are unavailable or unsuitable.

KEYWORDS: Precipitation, Joint occurrences, Spatial correlation

DD FORM 1 NOV 65 1473

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Unclassified 409578