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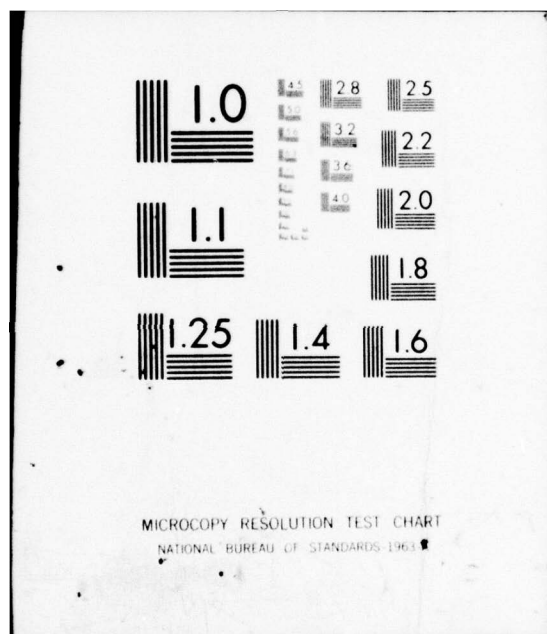
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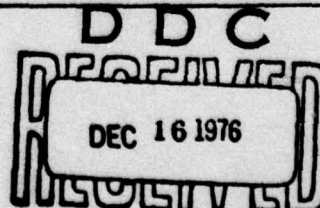
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RESEARCH ON NONLINEAR DYNAMICAL SYSTEMS

Final Technical Report

by

E. F. Infante, Acting Director
Lefschetz Center for Dynamical Systems

19 October 1976

U. S. Army Research Office

Grant DAH CO 4/75/G/0077

1 December 1974 - 31 November 1975

Principal Investigator: Professor J. P. LaSalle

Grant DAA G 29/76/G/0052

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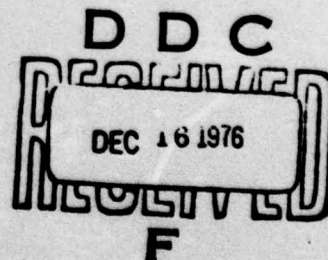
Principal Investigator: Professor J. P. LaSalle

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ABSTRACT

This Final Technical Report summarizes research findings supported by Grants DAH CO 4/75/G/0077 and DAAG 29/76/G/0052 during the period 1 December 1974 - 31 August 1976. The principal areas of research reported are:

- 1) Functional differential equations and their control;
- 2) Linear and nonlinear dynamical systems and their stability;
- 3) Bifurcation theory and applications to plates and shells;
- 4) Hyperbolic systems of conservation laws;
- 5) Stability of nuclear reactor models; and
- 6) Stability of elastic structures.

This report briefly summarizes results and refers to specific publications and reports previously reported to the U. S. Army Research Office.

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I. Foreword

This is the Final Technical Report on the research project entitled Research on Nonlinear Dynamical Systems supported through Grants DAH C04/75/G/0077 during the period 1 December 1974 - 31 November 1975 and DAA G29/76/G/0052 during the period 1 December 1975 - 31 August 1976.

The focus of research in this project was in six inter-related areas: (1) functional differential equations and their control, (2) linear and nonlinear dynamical systems and their stability, (3) bifurcation theory and applications to plates and shells, (4) hyperbolic systems of conservation laws, (5) stability of nuclear reactor models and (6) stability of elastic structures.

A brief summary of major research accomplishments and applications is reported in Section II. Each subsection, reporting on a research area, refers to reports or publications previously reported to the U. S. Army Research Office. These publications and reports are listed in Section III.

The final section reports the personnel supported from these grants.

II. Summary of Research Accomplishments and Applications

This brief outline of research accomplishments and applications attempts to summarize and categorize results obtained by six investigators over a period of time, including 1 December 1974 to 31 August 1976. It must be pointed out that although the results are listed under different categories, they are highly interrelated and that considerable benefit has been derived from the interrelation of the research efforts of the seven principal investigators.

A copy of each of the papers or reports referred to in the text is included with the original of this report.

1. Functional Differential Equations and their Control

References*: [1-10]

In references [1,2,3], Professor H. T. Banks and associates continue to develop mathematical models, based on ordinary and functional differential equations, of biological phenomena of significance. In [1,3] models of enzyme cascades are modeled; in [2] a computational method is developed, arising from a mathematical model for the development of optimal treatment schedules in radiation therapy. In references [4,5], Professors Banks and Burns study the problem of developing convergent computational methods for the determination of solutions and optimal controls of functional differential equations. The importance of this work is in its emphasis on numerical and computational procedures which can be assured to converge.

*References refer to publications listed in Section III.

Professor Hale and students continue in [6-9] to study the qualitative properties of functional differential equations, with emphasis on geometric aspects, such as the existence of periodic solutions and the stability properties of certain types of equations. Reference [10] refers to a just completed full monograph by Professor Hale on the theory of functional differential equations. This research monograph brings up to date the earlier monograph by Professor Hale on this subject.

The emphasis placed on the study of functional equations is that these equations provide models for systems that depend on their past history, and not only on their present state; they naturally arise in numerous applications, such as material science, econometrics and biometrics.

2. Stability of Linear and Nonlinear Dynamical Systems

References: [11-18]

LaSalle, in references [11-15] continues his work on the stability of general dynamical systems. In [11], he presents results in abstract form, applicable to general dynamical systems, and in [12] he exploits vector Liapunov functions, rather than scalar ones, to obtain general stability theorems. In [13], he develops a theory specifically applicable to difference equations, and in [14,15], to nonautonomous differential equations. In [16], Infante and Artstein apply results of the type presented in [14,15] to a specific type of nonautonomous system: oscillators in which the damping coefficient grows unboundedly, and they provide characterization of those rates of growth that lead to asymptotic

stability. This study was motivated by specific applications from metal and plastic hardening phenomena.

Dafermos, in [17], exploits the asymptotic stability of contraction semigroups with specific applications to problems that arise in continuum mechanics. This paper represents an attempt to bring to bear functional analytic approaches to the study of problems arising in mechanics. In [18], Dafermos studies abstract evolution equations and the existence of almost periodic solutions to such equations.

3. Bifurcation Theory and Applications

References: [19-26]

Hale and Mallet Paret, together with their collaborators and students, have continued their studies in bifurcation theory and applications.

Hale in [19], uses bifurcation theory to study the existence of periodic solutions of autonomous functional differential equations. In [20-21], Hale, Mallet-Paret and Chow develop a novel viewpoint in bifurcation theory, and display its use in a large number of applications in problems from continuum mechanics.

Mallet-Paret and Chow develop, in [22], the relationship between Hopf bifurcation and integral averaging, exploiting the advantages of the two viewpoints. In [23-24], Hale presents expositions of a rather novel viewpoint in generic bifurcation and illustrates through examples. The bifurcation of periodic solutions of Duffing's equation with multiple parameters is studied

in [25]. Mallet Paret and Chow, in [26] use the concept of Fuller's Index to characterize certain bifurcation phenomena.

4. Hyperbolic Systems of Conservation Laws

Reference [27]

In [27], Dafermos continues his studies of the partial differential equations that arise in problems governed by systems of conservation laws; in this work he used the behavior of entropy as a criterion for the selection of admissible solutions, among the many available, in problems arising from thermoelasticity.

5. Stability of Nuclear Reactor Models

References [28,29]

In [28,29], Infante and Walker study nonlinear operator equations that arise in the modeling of nuclear reactors. By viewing these equations as generating nonlinear semigroups in appropriate Hilbert spaces, simple, applicable stability criteria of different types are developed.

6. Stability of Elastic Structures

References: [30-35]

In [30,31], Infante and Walker have completed a study on the stability of undamped structures of nonconservative nature. This is a theoretical study that justifies, under appropriate circumstances, the separation of variables approach usually used in the engineering literature. The results are also directly applicable,

in that they are amenable to simple computations.

Plaut, in [32-35], continues to pursue his study of the dynamic behavior of structures subjected to nonconservative or gyroscopic forces. In [32,34], the effect of gyroscopic forces is studied; in [33], the convexity properties of the domains of stability in parameter space are exploited; in [35], the post buckling dynamic behavior of a standard model for an elastic column with a nonconservative load is studied.

III. Publications supported from ARO Grants DAH/CO4/75/G/007 and DAAG/29/76/G/0052. 1 December, 1974 to 31 August, 1976.

These publications are numbered as references to Section II.

- [1] Banks, H.T., Miech, R.P. & Zinberg, D.J., "Nonlinear systems in models for enzyme cascades", Proceedings - U.S. Italy Seminar on Variable Structure Systems with Emphasis on Biology and Economics, Portland, Oregon 5/28-6/1/74. To appear.
- [2] Banks, H.T. (with K. J. Almquist), "A Theoretical and computational method for determining optimal treatment schedules in fractionated radiation therapy", Math. Bio-sciences 29, pp. 159-179 (1976).
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- [4] Banks, H.T. (with J.A. Burns), "Projection methods for hereditary systems", Dynamical Systems, An International Symposium, Vol. I, Academic Press, Inc., 1976, pp. 287-295.
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- [6] Hale, J.K., "Functional differential equations of neutral type", Dynamical Systems, An International Symposium, Vol. I, Academic Press, Inc., 1976, pp. 179-194.
- [7] Hale, J.K., "Periodic solutions of functional differential equations".
- [8] Hale, J.K., "Some remarks and problems in functional differential equations", Lectures given at Un. of Sao Paulo, Sao Carlos, Brasil, July 1975.
- [9] Hale, J.K. (with Pedro Martinez-Amores), "Stability in neutral equations", J. Nonlinear Analysis: Theory, Methods and Applications, to appear.
- [10] Hale, J.K., Theory of Functional Differential Equations, Springer-Verlag, June 1976.

- [11] LaSalle, J.P., "Stability theory and invariance principles", Dynamical Systems, An International Symposium, Vol.1, pp. 211-222, Academic Press, Inc. 1976.
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- [15] LaSalle, J.P., "New stability results for nonautonomous systems", Proceedings - Dynamical Systems Conference, Univ. of Florida, Gainesville, Florida, 3/24-26/76, to appear.
- [16] Infante, E. F. (with Zvi Artstein), "On the asymptotic stability of oscillators with unbounded damping", Quarterly of Applied Mathematics, Vol. 34, No. 2, 7(1976), pp. 195-199.
- [17] Dafermos, C.M., "Contraction semigroups and trend to equilibrium in continuum mechanics", Springer-Verlag, Lecture Notes in Math.
- [18] Dafermos, C.M., "Almost periodic processes and almost periodic solutions of evolution equations".
- [19] Hale, J.K. (with Shui-Nee Chow), "Periodic solutions of autonomous equations".
- [20] Hale, J.K. (with Shui-Nee Chow and J. Mallet-Paret), "Applications of generic bifurcation, I", Archives Rational Mechanics and Analysis, Vol. 59, No. 2, 1975, pp. 159-188, Springer-Verlag, 1975.
- [21] Hale, J.K. (with Shui-Nee Chow, and J. Mallet-Paret), "Applications of generic bifurcation, II", Archives Rational Mechanics and Analysis, Vol. 62 (1976) , pp. 209-235.
- [22] Chow, Shui-Nee (with J. Mallet-Paret), "Integral averaging and bifurcation" (formerly entitled - The Hopf bifurcation and integral averaging), Journal of Diff. Equations, to appear.
- [23] Hale, J.K., "Bifurcation with several parameters", Invited lecture - VII Int. Conf. on nonlinear oscillations, Humboldt Univ., West Germany, 9/8-13/75.

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- [25] Hale, J.K. (with Hildebrando M. Rodrigues), "Bifurcation in the Duffing equation with independent parameters", Proc. Royal Society Edinburgh - Series A, to appear.
- [26] Chow, Shui-Nee and Mallet-Paret, J., "Fuller's index and global Hopf's bifurcation".
- [27] Dafermos, C., "The entropy rate admissibility criterion in thermoelasticity", Accademia Nazionale Dei Lincei Estratto dai Rendiconti della Classe di Scienze fisiche, matematiche e naturali, Serie VIII, Vol. LVII, fasc. 1-2 (Luglio-Agosto)-Ferie 1974.
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- [30] Infante, E.F. (with J. A. Walker), "On the behavior of linear undamped elastic systems perturbed by follower forces", Proceedings - Dynamical Systems Conference, Univ. of Florida, Gainesville, Florida, 3/24-26/76, to appear.
- [31] Infante, E.F. and J. A. Walker, "A perturbation approach to the stability of undamped linear elastic systems subjected to follower forces", J. Math. Analysis and Applications, to appear.
- [32] Plaut, R.H. (with K. Huseyin), "Transverse vibrations and stability of gyroscopic systems", Journal of Structural Mechanics, to appear.
- [33] Plaut, R. H., "Convexity properties and bounds for a class of linear autonomous mechanical systems", Dynamical Systems, An International Symposium, Vol. II, Academic Press, Inc., 1976, pp. 279-283.
- [34] Plaut, R. H., "Determination of eigenvalues in gyroscopic systems".
- [35] Plaut, R.H., "Post-buckling behavior of a double pendulum with partial follower load".

IV. Personnel

1. Faculty Personnel

The following faculty personnel were partially supported from these grants during the period 1 December, 1974 - 31 August, 1976:

H. T. Banks, Associate Professor
Constantine Dafermos, Associate Professor
J. K. Hale, Professor
E. F. Infante, Professor
J. P. LaSalle, Professor and Principal Investigator
J. Mallet-Paret, Assistant Professor
R. Plaut, Research Associate Professor

2. Graduate Student Personnel

The following graduate students were supported partially from these grants during this period:

R. M. Bates, Research Assistant
J. M. Mahaffy, Research Assistant
J. W. Palmer, Research Assistant
D. C. Reber, Research Assistant
F. S. Tsen, Research Assistant
L. Turyn, Research Assistant

3. Staff Personnel

The following staff personnel received partial support during the reported period:

E. M. Addison, Contracts Supervisor

F. J. Beagan, Reproductions

D. P. Libutti, Secretary

D. A. Perry, Reproductions

S. Spinacci, Technical Secretary