SIMILARITY REQUIREMENTS FOR FLUTTER AND OTHER AEROELASTIC MODELS IN A CRYOGENIC WIND TUNNEL

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SUMMARY

A consideration of the requirements for aeroelastic similarity shows the low working temperature of a cryogenic tunnel and an ability to vary temperature both have potential advantages in regard to the choice of suitable stiffness and density scales for an aeroelastic model.

The advantages are incidental to the main purpose of a cryogenic tunnel, which is to achieve high Reynolds numbers.
GENERAL FEATURES OF CRYOGENIC TUNNELS

A cryogenic wind tunnel is a test facility which uses air or another gas at low temperature. Current developments at NASA Langley\(^1,2\) relate to the use of nitrogen at temperatures as low as 100 K. The primary reason for going to a low temperature is the advantage that it offers in the attainment of high values of Reynolds number. This becomes obvious on consideration of the expression for Reynolds number:

\[
\text{Re} = \frac{\rho V \ell}{\mu} = \left(\frac{\rho a}{\mu}\right) M^2,
\]

where
\[\rho = \text{gas density}\]
\[V = \text{velocity}\]
\[\ell = \text{typical linear dimension}\]
\[\mu = \text{viscosity}\]
\[a = \text{speed of sound in gas}\]
\[M = \text{Mach number}.
\]

That is, for a given Mach number and model size,

\[
\text{Re} \propto \left(\frac{\rho a}{\mu}\right).
\]

Relating the properties of the test medium to the static pressure \(p\) and static temperature \(T\) of the flow we have for an ideal gas,

\[
\rho \propto \frac{p}{T}
\]

\[
a \propto T^{\frac{1}{2}}
\]

\[
\mu \propto \frac{T^{\frac{3}{2}}}{T + 117} \quad \text{(Sutherland's law)}
\]

so that,

\[
\text{Re} \propto \frac{p(T + 117)}{T^2}.
\]
Thus for a given Mach number, model size and tunnel operating pressure, Reynolds number increases with decreasing temperature. It can also be shown that the tunnel driving power is, to a first approximation, proportional to $T^4$ and that the quantity $\rho V^2$ is independent of temperature.

The situation is summarised by Kilgore, et al.²: "As the temperature is decreased, the density $\rho$ increases and the viscosity $\mu$ decreases, .... both these changes result in increased Reynolds number. With decreasing temperature the speed of sound $a$ decreases. For a given Mach number, this reduction in the speed of sound results in a reduced velocity $V$ which, while offsetting to some extent the Reynolds number increase due to changes in $\rho$ and $\mu$, provides advantages with respect to dynamic pressure, drive power and energy consumption."

In addition to these benefits, the ability to vary the operating temperature as well as pressure offers certain general advantages concerning aeroelastic effects. For example:

(a) by varying temperature only, the Reynolds number of one model can be varied without changing the quantity $\rho V^2$ which is the primary quantity affecting the static aeroelastic distortion;

(b) the quantity $\rho V^2$, which is important for the simulation of aeroelastic effects, can be varied independently of Reynolds number. For $\rho V^2$ can be increased by increasing operating pressure; if at the same time the operating temperature is raised, the Reynolds number can be kept constant.

It follows that Reynolds number effects and steady aeroelastic effects can be separated more easily than in the situation where only the tunnel pressure can be varied. In addition to these general advantages, the cryogenic tunnel offers features beneficial to unsteady tests with aeroelastic models that are only revealed by a consideration of the requirements for dynamic similarity.

A brief discussion of the possibility of making wing buffeting tests on ordinary wind tunnel models in cryogenic wind tunnels has been given by Mabey³. The following sections relate particularly to the testing of models that are specially designed to reproduce the aeroelastic behaviour of an aircraft.

2 FLUTTER TESTING

Although undoubtedly there will be problems in the construction and instrumentation of flutter models for cryogenic operation, a low temperature,
and the ability to vary temperature can, in principle, make it easier to satisfy the requirements for dynamical similarity in addition to the potential advantages of attaining high Reynolds number.

For flutter testing the usual requirement is that the model and full-scale values must be identical for each of the following three non-dimensional parameters,

- Mach number, \( M \)
- Stiffness parameter, \( \frac{S}{\rho V^2} \)
- Density parameter, \( \frac{\sigma}{\rho} \)

where \( S = \) scaled structural stiffness (e.g. moment per unit angular deflection/\( \ell^3 \))
\( \sigma = \) scaled structural mass (mass of structure/\( \ell^3 \)).

The relative values of other non-dimensional groups (e.g. Reynolds number, Froude number and one relating to structural damping) cannot be completely ignored, but model to full-scale equality for these is not usually necessary for flutter testing.

It follows that the aeroelastic behaviour of an aircraft in level flight at a given speed and altitude will be simulated by a model in a wind tunnel provided the following three equations hold:

\[
\begin{align*}
M_m &= M_a \quad (1) \\
\frac{S_m}{\rho_m V_m^2} &= \frac{S_a}{\rho_a V_a^2} \quad (2) \\
\frac{\sigma_m}{\rho_m} &= \frac{\sigma_a}{\rho_a} \quad (3)
\end{align*}
\]

where suffices \( m \) and \( a \) refer respectively to model and aircraft conditions.
For an ideal gas we have static pressure and speed of sound given by,

\[ \frac{p}{\rho} = RT \]

\[ a^2 = \gamma RT \]

where \( R \) = constant appropriate to the particular gas
\( \gamma \) = ratio of specific heats.

Making use of these relations and equation (1), we can rewrite equations (2) and (3) to express the requirements which must be satisfied so that a model tested in nitrogen will represent an aircraft at an altitude at which the static pressure and temperature are \( p_a \) and \( T_a \). In addition to an equality of Mach number, the requirements are:

\[
\frac{S_m}{\gamma_N p_m} = \frac{S_a}{\gamma_A p_a} \]

\[
\frac{\sigma_{m N}}{p_m} = \frac{\sigma_{m A}}{p_a} \]

\[ \sigma_{m N} = \sigma_{m A} \]

where the suffices \( N \) and \( A \) refer to nitrogen and air respectively. For the present discussion it is permissible to ignore the difference between \( \gamma_N \) and \( \gamma_A \) and between \( R_N \) and \( R_A \), so that we write:

\[ \frac{S_m}{p_m} = \frac{S_a}{p_a} \quad (4) \]

\[ \frac{\sigma_{m N}}{p_m} = \frac{\sigma_{m A}}{p_a} \quad (5) \]

Once the aircraft stiffness, mass and flight altitude have been specified and a convenient tunnel pressure and temperature have been chosen, the required model stiffness and mass are given by:

\[ S_m = \frac{p_m}{p_a} S_a \quad (6) \]

\[ \sigma_m = \frac{p_m T_a}{p_a T_m} \sigma_a \quad (7) \]
We note that the required model stiffness is dependent on the ratio of tunnel to flight pressures and independent of the temperatures; the required model mass is dependent on both the pressures and temperatures. Lowering $T_m$ alone means the required model will be heavier. The ratio of stiffness to mass, often termed the structural efficiency, is for the model,

$$\left(\frac{S}{\sigma}\right)_m = \frac{T_m}{T_a} \left(\frac{S}{\sigma}\right)_a .$$  

(8)

A potential advantage of testing in a cryogenic tunnel becomes apparent on consideration of equations (6), (7) and (8). For tunnel temperatures lower than the temperature of flight, the problems of model design and construction are eased because the required structural efficiency of the model is reduced. Of course the realisation of this advantage depends on overcoming the practical difficulties of constructing a model suitable for low temperatures.

A convenient procedure for model flutter testing entails simulating a series of flight conditions and for each simulated condition measuring some dynamic property, the variation of which indicates the approach to flutter. The measurements are then extrapolated to give a flutter point. Tests made at each constant Mach number should preferably simulate a series of flight altitudes, so that the extrapolated flutter point itself also represents a definite altitude. This procedure is not generally possible when using a conventional wind tunnel in which the static temperature remains almost constant for each Mach number. In this case a given model can correctly represent only one altitude for each Mach number. That is, for each Mach number there is only one tunnel pressure that matches a flight condition in the sense that the correct values of both the stiffness and density parameters are achieved. A change of tunnel pressure away from this value, on its own, means that the combination of effective full-scale pressure and temperature are not compatible with the standard atmosphere. This subject has recently been discussed by Baldock 5.

A cryogenic tunnel, which inherently includes an ability to vary temperature, overcomes the problem. Thus we suppose that a model has been constructed (with $S_m$ and $\sigma_m$ given by equations (6) and (7)) so that at convenient values of tunnel pressure and temperature it represents the aeroplane at a specified altitude.
The simulation of other altitudes requires new values for the tunnel pressure and temperature which are given by rewriting equations (6) and (8),

\[
\begin{align*}
P_m &= p_a \left( \frac{S_m}{S_a} \right) \\
T_m &= T_a \left( \frac{S_m}{S_a} \right)^{\frac{m}{\gamma}}
\end{align*}
\]

where the quantities within brackets are already fixed by the model construction, and \( p_a \) and \( T_a \) refer to the series of altitudes.

Examples showing the application of the similarity rules to particular cases are given in the following table. Each of these examples relate to a model designed initially to simulate the aerelastic behaviour of an aeroplane flying at an altitude of 10km at which, on the basis of a standard atmosphere, the static pressure and temperature are: \( p_a = 0.262 \) bar, \( T_a = 223.2 \) K. In two of the examples A1 and A2 the models are intended for cryogenic operation and are designed to simulate the 10km altitude with a tunnel static temperature of 100 K. In example A1, because it is assumed that the attainment of a high Reynolds number is not important in this case, the comparatively low value of 0.25 bar has been chosen for the tunnel static pressure to match the 10km condition. In example A2 the model is designed for a static pressure which has been increased to 1 bar in order to attain a higher Reynolds number. The other examples, B1 and B2 relate to models designed for use at normal temperatures. It is assumed that they will be tested in a tunnel with a stagnation temperature fixed at 290 K, which corresponds to a reasonably conventional situation. To start from a common design point for the conventional and cryogenic models, static pressures of 0.25 bar (for B1) and 1.0 bar (for B2) are again chosen to match the 10km altitude, but now, because the stagnation temperature is fixed, the design of the models must be related not only to altitude but also to a particular Mach number. The design point of the conventional models of examples B1 and B2 relate to Mach number \( M = 1.0 \) for which, with stagnation temperature fixed at 290 K, the static temperature, on the basis of isentropic flow, will be 242 K.
Examples of cryogenic models compared with conventional models

<table>
<thead>
<tr>
<th>Model</th>
<th>( \frac{S_m}{S_a} )</th>
<th>( \frac{\sigma_m}{\sigma_a} )</th>
<th>( \frac{\left( \frac{S}{\sigma} \right)_m}{\left( \frac{S}{\sigma} \right)_a} )</th>
<th>Altitude simulated</th>
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<td>T_m</td>
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<tr>
<td>CRYOGENIC</td>
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<tr>
<td>A1</td>
<td>0.95</td>
<td>2.1</td>
<td>0.45</td>
<td>0.25</td>
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<td>0.95</td>
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<td>125</td>
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<tr>
<td>A2</td>
<td>3.8</td>
<td>8.4</td>
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<td>1.0</td>
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<td>125</td>
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<tr>
<td>CONVENTIONAL</td>
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</tr>
<tr>
<td>B1</td>
<td>0.95</td>
<td>0.88</td>
<td>1.08</td>
<td>0.25</td>
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<td>242</td>
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<td></td>
<td></td>
<td>(M = 1.0)</td>
</tr>
<tr>
<td>B2</td>
<td>3.8</td>
<td>3.5</td>
<td>1.08</td>
<td>1.0</td>
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<td>242</td>
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* The static pressure 3.8 bar corresponds to a stagnation pressure of 7.2 bar for \( M = 1.0 \) which is compatible with the maximum stagnation pressure of 8.8 bar of the cryogenic tunnel under consideration by NASA Langley.

We note that, whereas the conventional models require a structural efficiency almost the same as that of the aeroplane, the cryogenic models need a structural efficiency less than half that of the aeroplane. In principle this should ease the problem of model design. It is the direct result of the low temperature.

The additional advantages that accrue from the ability to vary temperature become apparent only when the tests move away from the chosen design point which, for our conventional models was chosen to be \( (M = 1.0, 10\text{km}) \). By varying tunnel static pressure and temperature, the models in the cryogenic tunnel can accurately simulate conditions for all variations in altitude and Mach number. The required pressures and temperatures for simulating sea-level are included in the table. In contrast, the models in the conventional tunnel cannot achieve strict similarity for arbitrary combinations of Mach number and altitude. The situation is probably best illustrated by considering the error in the model density parameter \( \left( \frac{\sigma}{\rho} \right)_m \), if the stiffness parameter \( \left( \frac{S}{\rho V^2} \right)_m \) is correctly
reproduced at conditions other than the model design point. For instance for testing at \( M = 0.5 \) and with the stiffness parameter correct for the design altitude, \( h = 10 \text{km} \), both conventional models will be too heavy by the ratio,

\[
\frac{\sigma_m \text{ (actual model)}}{\sigma_m \text{ (correct)}} = \frac{(T_m) M = 0.5}{(T_m) M = 1.0} = \frac{1.14}{1}.
\]

If the test condition is sea-level with \( M = 1.0 \), then the conventional models will be too light by the factor,

\[
\frac{\sigma_m \text{ (actual model)}}{\sigma_m \text{ (correct)}} = \frac{(T_a) h = 10 \text{km}}{(T_a) h = 0} = \frac{0.77}{1}.
\]

These inaccuracies in simulation are overcome by the ability to vary tunnel stagnation temperature inherent in the concept of a cryogenic tunnel. However, it will be realised that the benefits obtained from varying stagnation temperature are not confined to low temperature operation, but would be obtained with a normal temperature tunnel if it were possible to vary stagnation temperature.

It was mentioned earlier that the cryogenic tunnel eased the problem of separating the effects of Reynolds number changes from certain aeroelastic effects. This is true for the effects of distortion under steady aerodynamic load because test pressure can be varied whilst maintaining constant Reynolds number by simultaneous adjustment of temperature. Although in flutter testing it would be preferable to maintain a constant Reynolds number throughout the tests, this will not be possible, in general, because the combinations of pressure and temperature required for altitude simulation will not be the same as those required for constant Reynolds number. For instance in the tabulated examples of cryogenic models the Reynolds number when simulating the aeroelastic conditions of flight at sea-level will be 2.7 times the value when simulating flight at 10km altitude. However, a constant value of Reynolds number could be maintained in combination with the correct values of \( \frac{S}{\rho V^2} \) appropriate to different altitudes, provided errors in the density parameter \( \frac{\rho}{\rho} \) are acceptable.
3 OTHER AEROELASTIC TESTS

It has already been mentioned that the values of other non-dimensional parameters cannot be completely ignored when designing a flutter model. One of these parameters relates the structural damping of the model to the aerodynamic forces and is conveniently written:

\[
\frac{D}{\rho V^4},
\]

where \( D \) is a damping coefficient typical of the model and defined as the moment per unit angular velocity of the model distortion. For tests involving classical coupled flutter, a value for the model not too different from that of the aeroplane is usually considered acceptable. However for other types of aeroelastic model testing it may be necessary to achieve the full-scale value with closer precision. For instance, aeroelastic models used for the direct prediction of buffet response or for investigations of non-linear flutter would not have dynamic similarity unless this parameter were correctly reproduced.

If \( r \) is the relative structural damping expressed as a proportion of the critical damping coefficient it can be shown that,

\[
\frac{D}{\rho V^4} \propto r \left[ \frac{g}{\rho} \left( \frac{S}{\rho V^2} \right) \right]^{\frac{1}{2}}.
\]

That is, provided the density and stiffness parameters are correctly reproduced and the relative damping of the structure is the same as that for the aeroplane, the correct value of the damping parameter will be achieved. Thus, in principle cryogenic models lead to no new difficulties in this respect.

4 CHANGES OF MODEL CHARACTERISTICS WITH TEMPERATURE

It has tacitly been assumed in the previous discussion, that the model properties do not change with temperature. There is no reason why the mass of the model should change by a significant amount, but some change in model stiffness will occur between normal temperature and the low temperatures envisaged in a cryogenic tunnel. For a model in which an aluminium alloy is used for the main stiffness we can expect both the stiffness and the strength to be some 8% greater at the test temperatures as compared to the values at normal temperature\(^6,7,8\). However the increase in strength may be accompanied by an increase in brittleness.
No information on the effect of a low temperature on the damping of constructional materials has so far been located, but in view of the trends in the other elastic properties of aluminium alloys, it would seem unlikely that the change in material damping would be serious. As far as the structural damping of a model is concerned, the contribution from the joints in the construction and the changes that may occur in this contribution when the temperature is lowered are likely to be more relevant than changes in the damping of the material itself.

5 CONCLUSIONS

Flutter testing in a cryogenic tunnel has potential advantages in comparison with testing in a conventional tunnel. These are:

1. A test at low temperature requires the model to be heavier than for normal temperatures. This makes for easier construction.
2. The ability to vary test temperature allows a range of flight altitudes and Mach numbers to be represented with a single model. That is, the correct values of both stiffness and density parameters can be achieved.
3. For a given test pressure and model size, the Reynolds number will be larger.

Realisation of these potential advantages will of course depend on achieving methods of model construction and instrumentation suitable for low temperatures.
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<td>M.J. Goodyer</td>
<td>High-Reynolds number cryogenic wind tunnel.</td>
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<td>2</td>
<td>R.A. Kilgore</td>
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<td>3</td>
<td>D.G. Mabey</td>
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</tr>
<tr>
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   The advantages are incidental to the main purpose of a cryogenic tunnel, which is to achieve high Reynolds numbers.