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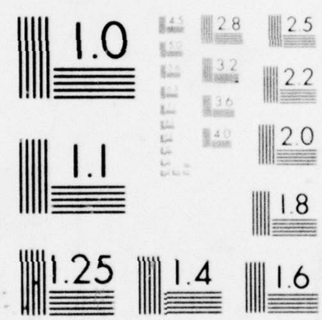

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POSITIVE EIGENFUNCTIONS FOR A CLASS OF SECOND-ORDER ELLIPTIC EQUATIONS WITH STRONG NONLINEARITY

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POSITIVE EIGENFUNCTIONS FOR A CLASS OF SECOND-ORDER ELLIPTIC EQUATIONS WITH STRONG NONLINEARITY †

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ABSTRACT

Let Ω be a bounded domain of $\mathbb{R}^N$. We consider the equation $\mathcal{A}u(x) + f(x, u(x)) = \lambda u(x)$, $\int_\Omega |u|^2(x)dx = R^2 > 0$, where $\mathcal{A}$ is a second-order quasilinear elliptic operator whose coefficients have polynomial growth and f essentially satisfies a sign condition. The existence of positive and negative solutions is proved.

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POSITIVE EIGENFUNCTIONS FOR A CLASS OF SECOND-ORDER ELLIPTIC EQUATIONS WITH STRONG NONLINEARITY†

Philippe Clément

The aim of this note is to prove the existence of positive and negative solutions of the eigenvalue problem:

(1) $Au = \lambda u$ with the L^2 norm of u, $\|u\|$ being a prescribed constant $R > 0$.

Here A is a second-order elliptic operator defined on a bounded domain Ω of $\mathbb{R}^n$, with a strong nonlinearity in its lowest order term. We assume that A has a variational structure. The corresponding problem

(2) $Au = f$ has been considered by several people [1], [2], [3], [4]. In these papers, only a divergence structure condition for A is assumed, however for the eigenvalue problem, such a hypothesis is too weak in general. Moreover [1], [2], [4] also deal with equations of higher order. In such situations, one can expect the existence of infinitely many distinct pairs of solutions with a prescribed norm, provided that A is odd, but not necessarily the existence of positive or negative solutions. Therefore it is convenient to consider the second-order case in itself.

1. Statement of the results

Let Ω be a bounded domain of $\mathbb{R}^N$.

$a: \Omega \times \mathbb{R} \times \mathbb{R}^N \to \mathbb{R}$ measurable for all $(t, \xi) \in \mathbb{R} \times \mathbb{R}^N$ and $C^1(\mathbb{R} \times \mathbb{R}^N; \mathbb{R})$ for almost all $x \in \Omega$, satisfying the following conditions:

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a1) $a(x, -t, -\xi) = a(x, t, \xi)$ for a.a. x in Ω, all (t, ξ) in $\mathbb{R} \times \mathbb{R}^N$.

a2) $a(x, 0, 0) = 0$ for a.a. x in Ω.

If we denote by $a_i(x, t, \xi)$, $i = 1, \ldots, N$, the partial derivative of a with respect to ξ_i and $a_0(x, t, \xi)$ the partial derivative of a with respect to t, we assume:

a3) there exists $C > 0$ and $2 \leq p < \infty$ such that:

$$|a_\alpha(x, t, \xi)| \leq C(k(x) + |t|^{p-1} + |\xi|^{p-1}), \text{ where } |\xi| = |\sum_{i=1}^N \xi_i^2|^{\frac{1}{2}} \text{ for } \alpha = 0, 1, \ldots, N, \text{ and } k \in L^{p/p-1}(\Omega).$$

a4) Leray-Lions conditions:

i) $\sum_{i=1}^N [a_i(x, t, \xi) - a_i(x, t, \xi')] (\xi_i - \xi_i') > 0$ for $\xi \neq \xi'$

ii) $\lim_{|\xi| \to +\infty} (\sum_{i=1}^N a_i(x, t, \xi)\xi_i)/(|\xi| + |\xi|^{p-1}) \to \infty$

for a.a. x in Ω and $|t|$ bounded.

Let $f: \Omega \times \mathbb{R} \to \mathbb{R}$ be measurable for all $t \in \mathbb{R}$ and continuous for almost all x in Ω, satisfying:

f1) $\operatorname{ess\,sup}_{x \in \Omega} \sup_{|s| \leq t} |f(x, s)| \leq K(t)$, for all $t \in \mathbb{R}$

f2) $f(x, t)t \geq 0$ for almost all x in Ω.

Remark. If f doesn't depend on x, f1) is trivially satisfied and f2) is purely a sign condition. Observe that by our assumptions on a, $\varphi(u) := \int_\Omega a(x, u, Du)dx$ is $C^1(W_0^{1,p}, \mathbb{R})$.

We consider the following equation:

$$(3) \qquad \int_\Omega \sum_{i=1}^N a_i(x,u,Du)\, D^i v\, dx + \int_\Omega f(x,u) v\, dx = \lambda \int_\Omega u v\, dx$$

for all $v \in W_0^{1,p}(\Omega) \cap L^\infty(\Omega)$ and $\int_\Omega |u|^2 dx = R^2 > 0$.

By a positive (resp. negative) solution of (3), we mean a pair $(\lambda, u) \in \mathbb{R} \times W_0^{1,p}(\Omega)$ such that u is ≥ 0 a.e. (resp ≤ 0 a.e.), $f(u)$ and $f(u)u$ are in $L^1(\Omega)$, and satisfy (3) and (4):

$$(4) \qquad \int_\Omega \sum_{i=1}^N a_i(x,u,Du)\, D^i u\, dx + \int_\Omega f(x,u) u\, dx = \lambda R^2 .$$

Our main result is:

<u>Theorem 1.</u> If a and f satisfy a1 - a4 and f1 - f2, and if $\lim_{\|u\|_{W^{1,p}} \to \infty} \varphi(u) = +\infty$, then for all $R > 0$, (3) possesses at least one positive and one negative solution.

2. <u>Proof</u>.

a) First observe that if $u \in W_0^{1,p}(\Omega)$, then $u^+ = \sup(u, 0)$ belongs to $W_0^{1,p}(\Omega)$ as well as $|u| = \sup(u, -u)$. Let $\Omega^+ \subset \Omega$ be the support (as a distribution) of u^+. We have $\varphi(|u|) = \varphi(u)$. Indeed $\varphi(|u|) = \int_\Omega a(x, |u|, D|u|)dx = \int_{\Omega^+} a(x,u,Du)dx + \int_{\Omega-\Omega^+} a(x,-u,-Du)dx = \int_{\Omega^+} a(x,u,Du)dx + \int_{\Omega-\Omega^+} a(x,u,Du)dx = \varphi(u)$, by a1).

b) Since we are looking for positive solutions, without loss of generality we can assume that f is odd. Indeed we can replace f by $\tilde{f}$ defined by $\tilde{f}(x,t) = f(x,t)$ for $t \geq 0$ and $\tilde{f}(x,t) = -f(x,-t)$ for $t < 0$. The negative case is similar.

By a1), if f is odd, and (λ, u) is a positive solution, then $(\lambda, -u)$ is a negative one, so we can restrict ourselves to the case of positive solutions, with f odd.

c) We shall first prove the theorem under the additional assumption that f is bounded, let $\psi(u) := \int_\Omega dx \int_0^{u(x)} f(x,t)dt$. It is known that $\psi \in C^1(W_0^{1,p}, \mathbb{R})$. Clearly $\psi \geq 0$. By the compact imbedding of $W_0^{1,p}(\Omega)$ in $L^2(\Omega)$ $(p \geq 2)$, if $u_n \rightharpoonup u$ in $W^{1,p}$, then $u_n \to u$ in L^2 and therefore $\psi(u_n) \to \psi(u)$. We know that $\varphi \in C^1(W_0^{1,p}, \mathbb{R})$. Moreover $\varphi'(u)$, the Frechet derivative of φ at u is bounded by a3) and satisfies [see 5, p. 183]: $u_n \rightharpoonup u$ in $W_0^{1,p}$, $\varphi'(u_n) \rightharpoonup v$ in $(W_0^{1,p})'$ and $\overline{\lim}\ \langle \varphi'(u_n),\ u_n - u \rangle \leq 0$ imply $v = \varphi'(u)$ and $\lim \langle \varphi'(u_n), u_n \rangle = \langle \nu, u \rangle$ †. It easily follows that $\varphi' : W_0^{1,p} \to (W_0^{1,p})'$ is of type (P)‡, [6] and therefore $\varphi: W_0^{1,p} \to \mathbb{R}$ is sequentially weakly lower semicontinuous. Hence $\varphi + \psi$ is also s. w. l. s. For $R > 0$, let $\alpha := \inf_{\nu \in S^R} \varphi(\nu) + \psi(\nu)$, where $S^R := \{u \in W_0^{1,p} \mid \int_\Omega |u|^2 dx = R^2\}$.

Clearly $S^R \neq \emptyset$, so $\alpha < \infty$. Let $u_n \in S^R$ such that $\varphi(u_n) + \psi(u_n) \downarrow \alpha$. Since $\psi \geq 0$ and by the assumption of the theorem, $\|u_n\|_{W^{1,p}} \leq C$, for some $c > 0$.

We have $\varphi(|u_n|) + \psi(|u_n|) = \varphi(u_n) + \psi(u_n)$ and $\| |u_n| \|_{W^{1,p}} = \|u_n\|_{W^{1,p}} \leq C$. By the compact imbedding of $W^{1,p}$ into L^2, and the reflexivity of $W^{1,p}$ there exists $u \in W_0^{1,p}$ with $\int_\Omega |u|^2 dx = R^2$ and $u \geq 0$ such that $|u_n| \rightharpoonup u$ in $W^{1,p}$ and $|u_n| \to u$ in L^2. Therefore, by our previous remark, $\varphi(u) + \psi(u) \leq \underline{\lim}\, \varphi(|u_n|) + \psi(|u_n|) = \alpha$. But $u \in S^R$, so $\varphi(u) + \psi(u) = \alpha$. Hence the minimum of

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† $\langle\, ,\, \rangle$ shall denote the duality between $W_0^{1,p}$ and $(W_0^{1,p})'$.

‡ $u_n \rightharpoonup u$ in $W_0^{1,p}$ implies $\overline{\lim}\ \langle \varphi'(u_n), u_n - u \rangle \geq 0$.

$\varphi + \psi$ on S^R, is achieved at u. Moreover $\varphi + \psi \in C^1(W_0^{1,p}, \mathbb{R})$, $u \to \frac{1}{2} \int_\Omega |u|^2 dx$ is $C^1(W_0^{1,p}, \mathbb{R})$ and the Frechet derivative of the latter function is $\neq 0$ since $u \neq 0$. Thus, by the well-known "Lyusternik principle", there exists $\lambda \in \mathbb{R}$, such that (λ, u) is a positive solution of (3).

d) For $n \in \mathbb{N}$, define $f_n(x,t) = f(x,t)$ if $|f(x,t)| \leq n$, $f_n(x,t) = n$ if $f(x,t) > n$ and $f_n(x,t) = -n$ if $f(x,t) < -n$. Let $\psi_n(u) := \int_\Omega dx \int_0^{u(x)} f_n(x,t)dt$. Since f_n are bounded, we know that for each $n \in \mathbb{N}$, there exists $(\lambda_n, u_n) \in \mathbb{R} \times W_0^{1,p}$, a positive solution of (3), where f is replaced by f_n. We claim that $\|u_n\|_{W_0^{1,p}} \leq C$ for some $C > 0$. Indeed as in c), we have: $\varphi(u_n) + \psi_n(u_n) = \inf_{v \in S^R} \varphi(v) + \psi_n(v)$. Let $u_0 \in S^R \cap L^\infty(\Omega)$. Then $\varphi(u_n) + \psi_n(u_n) \leq \varphi(u_0) + \psi_n(u_0) \leq \varphi(u_0) + \int_\Omega dx \int_0^{u_0(x)} f(x,t)dt < \infty$. By the coercivity of φ and the positivity of ψ_n we get $\|u_n\|_{W^{1,p}} \leq C$. By the reflexivity of $W_0^{1,p}$ and the compact imbedding of $W_0^{1,p}$ into L^2 we can extract a subsequence, still denoted by u_n, such that $u_n \rightharpoonup u$ in $W^{1,p}$ and $u_n \to u$ in L^2. So $u \in S^R$ and u is positive a.e.

We will prove that λ_n is bounded. Indeed, assume that there is a subsequence $\lambda_n \downarrow -\infty$, then we get:

$$\frac{1}{\lambda_n} \langle \varphi'(u_n), u_n \rangle + \frac{1}{\lambda_n} \int_\Omega f_n(x, u_n) u_n \, dx = R^2 . \tag{5}$$

But $\varphi'(u_n)$ is bounded, since u_n is, so $\frac{1}{\lambda_n} \langle \varphi'(u_n), u_n \rangle \to 0$. But then $\frac{1}{\lambda_n} \int_\Omega f_n(x, u_n) u_n \, dx \leq 0$ and $R^2 > 0$ imply that for n big enough we get a contradiction. Next assume now that $\lambda_n \uparrow \infty$, for a subsequence. We have, for all $v \in W_0^{1,p}$:

(6) $$\frac{1}{\lambda_n}\langle \varphi'(u_n), v\rangle + \frac{1}{\lambda_n}\int_\Omega f_n(x,u_n)v\,dx = \int_\Omega u_n v\,dx\ .$$

In particular for $v = u_n$ we get:

(7) $$\int_\Omega \frac{1}{\lambda_n} f_n(x,u_n)u_n\,dx \le C \quad \text{for some } C > 0\ .$$

By assumption f1) and f2), for each $\delta > 0$, there exists $K_\delta > 0$ such that $|f(x,t)| \le K_\delta + \delta f(x,t)t$. [2]. Hence $|f_n(x,t)| \le K_\delta + \delta f(x,t)t$. By using (7), this shows that the sequence $\frac{1}{\lambda_n} f_n(x,u_n)$ is equiintegrable and since $\frac{1}{\lambda_n} f_n(x,u_n) \to 0$ a.e. in Ω, by Vitali's theorem, $\frac{1}{\lambda_n} f_n(x,u_n) \to 0$ in L^1 and by (6), for $v \in W_0^{1,p}(\Omega) \cap L^\infty(\Omega)$, we get: $\int_\Omega uv\,dx = 0$. Hence by density, $\int_\Omega |u|^2\,dx = 0$, a contradiction. Thus λ_n is bounded and we can extract a subsequence $\lambda_n \to \lambda \in \mathbb{R}$.

Since u_n is bounded in $W_0^{1,p}$ and $\varphi': W_0^{1,p} \to (W_0^{1,p})'$ is bounded, $\varphi'(u_n)$ is bounded and we can extract a subsequence converging weakly to $w \in (W_0^{1,p})'$: $\varphi'(u_n) \rightharpoonup w$ in $(W_0^{1,p})'$. We have $\int_\Omega f_n(x,u_n)u_n\,dx = \lambda_n R^2 - \langle \varphi'(u_n), u_n\rangle \le C$. By using the same argument as before, there exists a subsequence $f_n(x,u_n) \to f(x,u)$ in L^1 . Moreover, since $f_n(x,u_n)u_n \ge 0$, by Fatou's lemma, $f(x,u)u \in L^1$. Therefore for all $v \in W_0^{1,p} \cap L^\infty$, we have:

(8) $$\langle w, v\rangle + \int_\Omega f(x,u)\ dx = \lambda \int_\Omega uv\,dx\ .$$

Now, we define $v_n := \inf(u,n)$. It is known that $v_n \in W_0^{1,p} \cap L^\infty$, $v_n \to u$ in $W_0^{1,p}$ and by Lebesgue's dominated convergence theorem and the fact that $f(x,u)u \in L^1$, $f(x,u)v_n \to f(x,u)u$ in L^1 . By putting $v = v_n$ and passing to the limit we have:

(9) $$\langle w, u\rangle + \int_\Omega f(x, u)u dx = \lambda R^2 .$$

Now, we are done provided that we prove that $w = \varphi'(u)$. But $u_n \to u$ in $W_0^{1,p}$, $\varphi'(u_n) \to w$ in $(W_0^{1,p})'$, so it is sufficient to prove that $\overline{\lim} \langle \varphi'(u_n), u_n - u\rangle \leq 0$. We have $\overline{\lim} \langle \varphi'(u_n), u_n - u\rangle = \overline{\lim} \langle \varphi'(u_n), u_n\rangle - \langle w, u\rangle = \overline{\lim} [\lambda_n R^2 - \int_\Omega f_n(x, u_n)u_n dx] - [\lambda R^2 - \int_\Omega f(x, u)u dx] = \int_\Omega f(x,u)u dx - \underline{\lim} \int_\Omega f(x, u)u dx - \underline{\lim} \int_\Omega f_n(x, u_n)u_n dx \leq 0$, by Fatou's lemma. This completes the proof of the theorem. □

<u>Remark 1.</u> With the same arguments, we can allow a more general right hand side and consider the equation:

(10) $$\int_\Omega \sum_{i=1}^N a_i(x, u, Du)D^i v dx + \int_\Omega f(x, u)v dx = \lambda \int_\Omega g(x, u) v dx$$

for all $v \in W_0^{1,p} \cap L^\infty$ with the condition $\int_\Omega |u|^2 dx = R^2 > 0$ replaced by the condition $\int_\Omega dx \int_0^{u(x)} g(x, t)dt = R > 0$. In this case if $g: \Omega \times \mathbb{R} \to \mathbb{R}$ satisfies the Caratheodory condition, and

g1) $g(x, t)t > 0$ for a. a. X in Ω

g2) there exists $1 < q < \infty$ and $C > 0$ such that $|g(x, t)| \leq C(1 + |t|^{q-1})$ and if we assume that p in the hypothesis a3) satisfies $1 < p < \infty$, we can state the following generalization of Theorem 1:

<u>Theorem 1'</u>. If a, f, g satisfy $a_1 - a_4$, $f_1 - f_2$, $g_1 - g_2$ and if we have

i) the injection of $W^{1,p}$ into L^q is compact

ii) $$\lim_{\|u\|_{W^{1,p}}} \int_\Omega a(x, u, Du)dx = +\infty$$

iii) $\lim\limits_{\|u\|_{L^q}} \int_\Omega dx \int_0^{u(x)} g(x,t)dt = +\infty$.

Then the equation (10) possess at least one positive and one negative solution $(\lambda, u) \in W_0^{1,p}(\Omega)$ with $f(x,u)$ and $f(x,u)u \in L^1$, for all $R > 0$.

Remark 2. In the simple case $-\Delta u + f(u) = \lambda u$, $u \in W_0^{1,2}(\Omega)$, our condition on f reduces to: i) $f(0) = 0$ ii) $\underline{\lim}_{t\to+\infty} \frac{f(t)}{t} \geq -c$ for some $c > 0$. If we assume further that $f(u) = \tilde{f}(u)u$, $\tilde{f}(t) \geq 0$, $\tilde{f} \in C^1(\mathbb{R})$, it follows from [7], that there exists an unbounded connected set C^+ of positive solutions in $\mathbb{R} \times C^{1,\alpha}(\Omega)$ $(0 < \alpha < 1)$, containing $(\lambda_0, 0)$ in its closure, where λ_0 is the first eigenvalue of $-\Delta h + \tilde{f}(0)h = \lambda h$, $h|_{\partial\Omega} = 0$. If we suppose that $\lim_{t\to\infty} \tilde{f}(t) = +\infty$, it follows easily that the projection of C^+ on $\mathbb{R}$ contains the interval $[\lambda_0, \infty)$. Therefore in this case the above equation possesses positive solutions with arbitrary norm in $L^2(\Omega)$ and for all $\lambda \geq \lambda_0$. It can happen that all positive solutions are in C^+; it is true, for example, if $\tilde{f}'(t) \geq 0$ for all $t \in \mathbb{R}$. For the case $\tilde{f}'(t) > 0$, see [8]; the case $\tilde{f}'(t) \geq 0$ can be handled by using a simiiar argument to [9]. In this particular situation C^+ is a C^1 curve of solutions in $C^{2,\alpha}(\Omega)$, parametrized by λ and unbounded in $L^2(\Omega)$. Concerning the regularity of the solutions, let us mention that if $f \in C^0(\mathbb{R})$, $f(0) = 0$ and f is monotone, then $u \in C^{1,\alpha}(\Omega)$. For a different approach of this case, see [10].

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Let Ω be a bounded domain of $\mathbb{R}^N$. We consider the equation $\mathcal{A}u(x) + f(x, u(x)) = \lambda u(x)$, $\int_\Omega |u|^2(x)dx = R^2 > 0$, where $\mathcal{A}$ is a second-order quasi-linear elliptic operator whose coefficients have polynomial growth and f essentially satisfies a sign condition. The existence of positive and negative solutions is proved.

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