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Simultaneous direct measurements taken in elevation, azimuth and range from independent instrumentation sources, when aggregated and reduced can often yield sensible estimates. However, strong aberrations and inconsistencies in the estimates derived from data reduction have been known to occur. Two particular factors are investigated in order to partially explain theoretically these elusive deviations and to establish some cause and effect relationship. The first set of factors relates to the relative geometric siting of the instruments as well as the type of measurements recorded · Typical measurements which would be conducive to such aberrations are discussed. The second set of factors relates to the selection of a frame of reference for azimuthal measurement. It is shown that coordinate selection can play for certain types of measurements a significant role in multiplying the GDOP by severalfold from a rationally established minimum basis.

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EFFECT OF INSTRUMENT SITING AND COORDINATE SELECTION ON GDOP IN TARGET TRACKING

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ABSTRACT

Simultaneous direct measurements taken in elevation, azimuth and range from independent instrumentation sources, when aggregated and reduced can often yield sensible estimates. However, strong aberrations and inconsistencies in the estimates derived from data reduction have been known to occur. Two particular factors are investigated in order to partially explain theoretically these elusive deviations and to establish some cause and effect relationship. The first set of factors relates to the relative geometric siting of the instruments as well as the type of measurements recorded. Typical measurements which would be conducive to such aberrations are discussed. The second set of factors relates to the selection of a frame of reference for azimuthal measurement. It is shown that coordinate selection can play for certain types of measurements a significant role in multiplying the GDOP by severalfold from a rationally established minimum basis.

I. INTRODUCTION

In a previous publication [1], a methodology was developed to determine and measure the error contribution on estimated Cartesian coordinates of a target tracked by a complex of instrumentation systems. It is known that, in general, such factors as target position, site location of instruments, type of instruments, type of measurable observations from the instruments and accuracy of instruments affect the error quantity. If $(\tilde{x}, \tilde{y}, \tilde{z})$ refer to the position of the estimated target, the error measured as GDOP is given by

$$\text{GDOP} = \sqrt{\text{var}[\tilde{x}] + \text{var}[\tilde{y}] + \text{var}[\tilde{z}]}$$

This GDOP criterion plays an extremely important role in ascertaining the validity of direct measurements in data reduction techniques to derive statistical estimates of the target state. Simultaneous direct measurements taken in elevation, azimuth and range from independent instrumentation sources at a particular instant of time, when aggregated and reduced using well known statistical techniques, often yield sensible estimates. That is, the GDOP associated with these estimates is significantly consistent with the empirical GDOP values obtained from past computations. However strong aberrations and inconsistencies in the estimates derived from data reduction have been known to occur. In many instances, the cause of these deviations cannot be correlated with such factors as the performance of the tracking instrument, or the reliability of the recorded field information or the propagation of error in the statistical techniques used in analysis and development of estimate measures.

In this paper, two particular factors are investigated in order to explain the cause of some aberrations. Moreover, a theoretical basis is established which 1) identifies and explains such causes, 2) can be used to search for possible means to eliminate such aberrations, and 3) can be exploited to determine rationally the minimum achievable GDOP to be used as a standard basis for comparative purposes.

The first set of factors relates to the relative geometric siting of the instruments as well as the type of measurements recorded. It is shown for example, that the utilization of a radar system in target tracking can yield a data basis which cannot be meaningfully aggregated to form a statistical input basis for target estimation. In other words large GDOP values can result from the possible minimal achievable value. The second set of factors relates to the selection of a frame of reference for azimuthal measurement. Although, in practice an established frame of reference is often used, it is shown that coordinate selection can play for certain types of measurements, particularly those involving photo-optical devices and radars, a significant role in multiplying the GDOP by severalfold from the rationally established minimum basis.

II. THE NORMAL EQUATIONS FOR EQUI-ELEVATION SITING OF INSTRUMENTS RELATIVE TO AN AUXILIARY TARGET.

Consider a situation where n instruments (observation sites), $n \geq 3$, of the same type are located on a circle of radius d in the x-y plane. The system of coordinate axis is so selected that the position of an auxiliary target P_o (an approximate target position whose coordinates are known) is located on the z-axis passing from the center of the circle and normal to its plane. The altitude of P_o measured from the x-y plane is h; thus the coordinates of P_o are $(x_o, y_o, z_o) \equiv$

(0, 0, h). All elevation angle measurements of the auxiliary target P_o from all sites are equal to E. Let

(x, y, z) = coordinates of the true target position P,

(A_i, E_i, R_i) = azimuth angle, elevation angle and range from site i to point P ($i = 1, 2, \ldots, n$),

(A_i^o, E_i^o, R_i^o) = azimuth angle, elevation angle and range from site i to point P_o,

(A_i^m, E_i^m, R_i^m) = actual observed or measured azimuth angle, elevation angle and range.

Each observation site is assumed to operate independently of any observation sites. For convenience, we introduce the following quantities:

$$\Delta x = x - x_o \qquad \Delta A_i = A_i - A_i^o$$
$$\Delta y = y - y_o \quad (1) \qquad \Delta E_i = E_i - E_i^o \quad (2)$$
$$\Delta z = z - z_o \qquad \Delta R_i = R_i - R_i^o$$

where $i = 1, 2, \ldots, n$. Each of the above Δ quantities is unknown since they involve non-measurable variables; however, the quantities

$$\Delta' A_i = A_i^m - A_i^o$$
$$\Delta' E_i = E_i^m - E_i^o \quad (3)$$
$$\Delta' R_i = R_i^m - R_i^o$$

are known exactly. We assume that 1) $\{\Delta' A_i\}$, $\{\Delta' E_i\}$ and $\{\Delta' R_i\}$ are sample observations taken respectively from the normal populations $N(\Delta A_i, \sigma_A^2)$, $N(\Delta E_i, \sigma_E^2)$ and $N(\Delta R_i, \sigma_R^2)$, 2) $\{\Delta' A_i\}$, $\{\Delta' E_i\}$, and $\{\Delta' R_i\}$ form a sequence of independently distributed random variables and 3) these random variables do not depend on the coordinates of any of the sites from which the measurements *are made. The quantities* σ_A, σ_E, *and* σ_R are known characteristics of the instruments.

As shown in [1], a likelihood function can be formulated, and its maximization will yield the following normal equations for $\tilde{\Delta x}$, $\tilde{\Delta y}$, $\tilde{\Delta z}$, the estimates of Δx, Δy, Δz:

$$\frac{\tilde{\Delta x}}{d^2}\left[\frac{1}{\sigma_A^2}\sum_{i=1}^{i=n}\sin^2 A_i^0 + \frac{\sin^2 E\cos^2 E}{\sigma_E^2}\sum_{i=1}^{i=n}\cos^2 A_i^0 + \frac{d^2\cos^2 E}{\sigma_R^2}\sum_{i=1}^{i=n}\cos^2 A_i^0\right]$$
$$+\frac{\tilde{\Delta y}}{d^2}\left(-\frac{1}{\sigma_A^2} + \frac{\sin^2 E\cos^2 E}{\sigma_E^2} + \frac{d^2\cos^2 E}{\sigma_R^2}\right)\sum_{i=1}^{i=n}\sin A_i^0\cos A_i^0$$
$$+\frac{\tilde{\Delta z}}{d^2}\left(-\frac{\sin E\cos^3 E}{\sigma_E^2} + \frac{d^2\sin E\cos E}{\sigma_R^2}\right)\sum_{i=1}^{i=n}\cos A_i^0$$
$$= \frac{1}{d}\left[-\frac{1}{\sigma_A^2}\sum_{i=1}^{i=n}\sin A_i^0\,\Delta' A_i - \frac{\sin E\cos E}{\sigma_E^2}\sum_{i=1}^{i=n}\cos A_i^0\,\Delta' E_i + \frac{d\cos E}{\sigma_R^2}\sum_{i=1}^{i=n}\cos A_i^0\,\Delta' R_i\right] \quad (4)$$

$$\frac{\tilde{\Delta x}}{d^2}\left(-\frac{1}{\sigma_A^2} + \frac{\sin^2 E\cos^2 E}{\sigma_E^2} + \frac{d^2\cos^2 E}{\sigma_R^2}\right)\sum_{i=1}^{i=n}\sin A_i^0\cos A_i^0$$
$$+\frac{\tilde{\Delta y}}{d^2}\left[\frac{1}{\sigma_A^2}\sum_{i=1}^{i=n}\cos^2 A_i^0 + \frac{\sin^2 E\cos^2 E}{\sigma_E^2}\sum_{i=1}^{i=n}\sin^2 A_i^0 + \frac{d^2\cos^2 E}{\sigma_R^2}\sum_{i=1}^{i=n}\sin^2 A_i^0\right]$$
$$+\frac{\tilde{\Delta z}}{d^2}\left(-\frac{\sin E\cos^3 E}{\sigma_E^2} + \frac{d^2\sin E\cos E}{\sigma_R^2}\right)\sum_{i=1}^{i=n}\sin A_i^0$$
$$= \frac{1}{d}\left[\frac{1}{\sigma_A^2}\sum_{i=1}^{i=n}\cos A_i^0\,\Delta' A_i - \frac{\sin E\cos E}{\sigma_E^2}\sum_{i=1}^{i=n}\sin A_i^0\,\Delta' E_i + \frac{d\cos E}{\sigma_R^2}\sum_{i=1}^{i=n}\sin A_i^0\,\Delta' R_i\right] \quad (5)$$

$$\frac{\tilde{\Delta x}}{d^2}\left(-\frac{\sin E\cos^3 E}{\sigma_E^2} + \frac{d^2\sin E\cos E}{\sigma_R^2}\right)\sum_{i=1}^{i=n}\cos A_i^0$$
$$+\frac{\tilde{\Delta y}}{d^2}\left(-\frac{\sin E\cos^3 E}{\sigma_E^2} + \frac{d^2\sin E\cos E}{\sigma_R^2}\right)\sum_{i=1}^{i=n}\sin A_i^0$$
$$+\frac{\tilde{\Delta z}}{d^2}\left(\frac{\cos^4 E}{\sigma_E^2} + \frac{d^2\sin^2 E}{\sigma_R^2}\right)\sum_{i=1}^{i=n}1$$
$$= \frac{1}{d}\left[\frac{\cos^2 E}{\sigma_E^2}\sum_{i=1}^{i=n}\Delta' E_i + \frac{d\sin E}{\sigma_R^2}\sum_{i=1}^{i=n}\Delta' R_i\right] \quad (6)$$

Note that

$$d = h\cot E \quad (7)$$

The simultaneous solution of equations (4), (5) and (6) will yield the values of the estimates $\tilde{\Delta x}$, $\tilde{\Delta y}$, and $\tilde{\Delta z}$. These estimators are unbiased, that is, their corresponding mean is respectively Δx, Δy, and Δz. Further, it is well known that their joint distribution is normal. Formally, the variance-covariance matrix can be computed once expressions for $\tilde{\Delta x}$, $\tilde{\Delta y}$ and $\tilde{\Delta z}$ are obtained. In particular, explicit formulas for the variance can be derived. For notational purpose we let $\sigma_x^2 = \mathrm{var}[\tilde{\Delta x}]$, $\sigma_y^2 = \mathrm{var}[\tilde{\Delta y}]$ and $\sigma_z^2 = \mathrm{var}[\tilde{\Delta z}]$. Then

$$\overline{\mathrm{GDOP}}^2 = \sigma_x^2 + \sigma_y^2 + \sigma_z^2 \quad (8)$$

In what follows, we shall analyze and investigate the characteristics of GDOP for two special classes of problems:

1. In the first case, we assume that n, $n \geq 3$, identical instruments having angle and range measuring capabilities (e.g. radar), are sited equidistantly from each other and are recording simultaneously the position of the target.

2. In the second case, we assume that n = 4 identical instruments having only angle measuring capabilities (e.g. cinetheodolites, telescopes) are located at the vertices of a rectangle and are again recording simultaneously the azimuth and elevation of the target.

For each of these problems we shall study in particular the impact of the elevation angle E upon GDOP. In addition, for the second problem the relation between the selection of the frame of reference upon GDOP will be investigated.

III. EFFECT ON GDOP WHEN TRACKING IN AZIMUTH, ELEVATION AND RANGE WITH n ≥ 3 IDENTICAL INSTRUMENTS

The configuration of the instruments siting is as shown in Figure 1

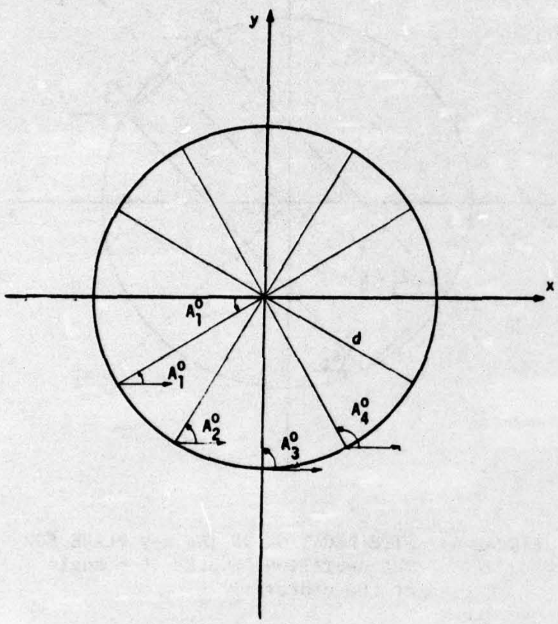

FIGURE 1: SITE LOCATIONS ON THE x-y PLANE, AND AZIMUTH MEASUREMENT UNDER SYMMETRICAL SITING OF INSTRUMENTS

We assume here that all sites are located equidistantly from each other; then, in general

$$A_i^o = 2\pi i/n, \quad i = 1, 2, \ldots, n \qquad (9)$$

The azimuth angles are thus selected as integral multiples of a least value generating a symmetrical configuration of the site geometry. As pointed out in [1], for such a geometry, the various interplaying terms in equation (4), (5) and (6) affected by the azimuth angles, remain invariant under a rotation of the x-y coordinates. More explicitly, if α is the amount of counterclockwise rotation imparted to the x-y coordinates, then

$$\sum_{i=1}^{i=n} \sin\left(\frac{2\pi i}{n} - \alpha\right) = 0 = \sum_{i=1}^{i=n} \cos\left(\frac{2\pi i}{n} - \alpha\right)$$

$$\sum_{i=1}^{i=n} \sin^2\left(\frac{2\pi i}{n} - \alpha\right) = \frac{n}{2} = \sum_{i=1}^{i=n} \cos^2\left(\frac{2\pi i}{n} - \alpha\right) \qquad (10)$$

$$\sum_{i=1}^{i=n} \sin\left(\frac{2\pi i}{n} - \alpha\right) \cos\left(\frac{2\pi i}{n} - \alpha\right) = 0$$

for all n ≥ 3 and all α ≥ 0.

Due to symmetry, $\sigma_x^2 = \sigma_y^2$. The expression for $(GDOP)^2$ is [1]:

$$(GDOP)^2 = \frac{h^2}{n}\left[\frac{4\cos^2 E}{\frac{1}{\sigma_A^2}\sin^2 E + \frac{1}{\sigma_E^2}\sin^4 E\cos^2 E + \frac{h^2}{\sigma_R^2}\cos^4 E} + \frac{1}{\frac{1}{\sigma_E^2}\sin^2 E\cos^2 E + \frac{h^2}{\sigma_R^2}\sin^2 E}\right] \qquad (11)$$

where E, $0 < E < \frac{\pi}{2}$, is the common elevation angle of the instruments. It is evident that E is a decision variable; the instruments could be sited in such a way that a preassigned value of E can be selected, subject to any constraints dictated by physical or other considerations.

It was shown in [1] that when the instruments have 1) angle measuring capability only ($\sigma_R = \infty$) or 2) range measuring capability only ($\sigma_A = \infty$, $\sigma_E = \infty$), optimum elevation angles can be obtained for each case which minimizes GDOP. In the first case, E(opt) = 54°51', while for the second case E(opt) = 35°16'. Neither of these values depend on the target altitude h or the instrument accuracies σ_A, σ_E, σ_R.

On the other hand, it is easily seen from (11) that when the instruments have simultaneous angle and range measuring capabilities (σ_A, σ_E, $\sigma_R < \infty$) the optimum value of E, if it exists, will depend on the target altitude and the instruments accuracies.

We consider now a common situation when $\sigma_A = \sigma_E$ and define the quantity $a = \sigma_A h/\sigma_R$. Expression (11) can be written as:

$$(GDOP)^2 = \frac{\sigma_R^2}{n}\left[\frac{4\cos^2 E}{\frac{\sin^2 E}{a^2} + \frac{\sin^4 E\cos^2 E}{a^2} + \cos^4 E} + \frac{1}{\frac{\sin^2 E\cos^2 E}{a^2} + \sin^2 E}\right] \qquad (12)$$

For $a = \sqrt{10}$, Figure 2 shows the plot of $(GDOP)^2$ as a function of E. The behavior of σ_z^2 is regular: as E varies from 0 to $\frac{\pi}{2}$, σ_z^2 decreases from infinity to 1. The behavior of σ_x^2 and σ_y^2 are quite unexpected: both reach a maximum value for an elevation angle of about 55°.

The plot of the $(GDOP)^2$ function for various values of a^2 is exhibited in Figure 3. Patterns of instability as reflected by high GDOP values can be distinctly noticed on the graph. Also, practical considerations often limit the instrument measuring capability to elevation angles of less than 80°. Thus, as a general rule of thumb, elevation angles E should be selected from the range 30°-50° in order that the estimates of the Cartesian coordinates of the target yield GDOP values which are close to the minimum achievable level.

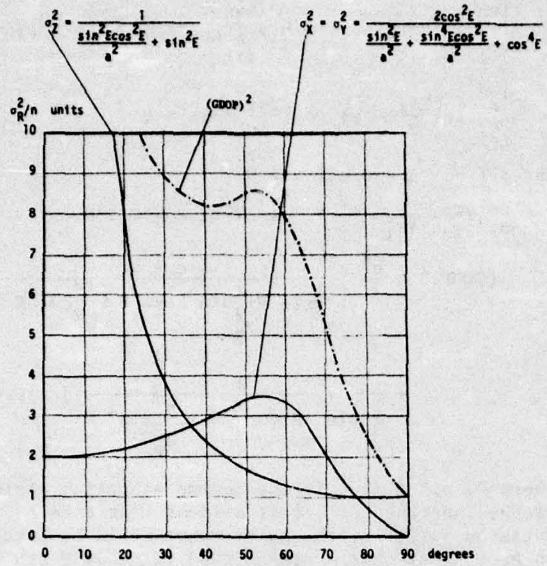

FIGURE 2: $(GDOP)^2$ AS A FUNCTION OF THE ELEVATION ANGLE FOR $a = \sigma_A h/\sigma_R = \sqrt{10}$. MEASUREMENTS TAKEN IN AZIMUTH, ELEVATION AND RANGE.

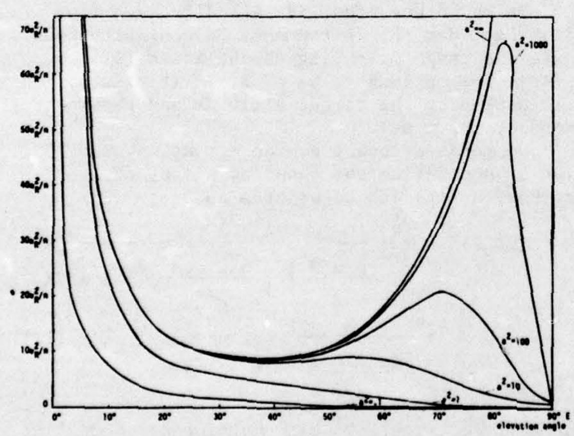

FIGURE 3: $(GDOP)^2$ AS A FUNCTION OF THE ELEVATION ANGLE E FOR DIFFERENT VALUES OF a. MEASUREMENTS TAKEN IN AZIMUTH, ELEVATION AND RANGE

IV. EFFECT ON GDOP WHEN TRACKING IN AZIMUTH AND ELEVATION WITH n = 4 IDENTICAL INSTRUMENTS.

The configuration of the instruments siting is as shown in Figure 4

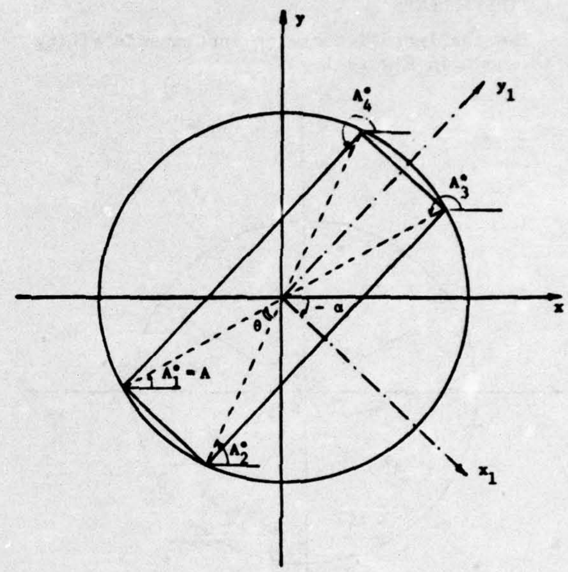

FIGURE 4: SITE LOCATION ON THE x-y PLANE FOR THE 4-STATION PROBLEM (θ = angle at the center).

In general the orientation of the x-y coordinates will affect the estimates of Cartesian coordinates of the target. Thus, in (4), (5) and (6) the terms $\sum_{i=1}^{i=n} \sin A_i^0$, $\sum_{i=1}^{i=n} \cos A_i^0$, $\sum_{i=1}^{i=n} \sin^2 A_i^0$, $\sum_{i=1}^{i=n} \cos^2 A_i^0$ and $\sum_{i=1}^{i=n} \sin A_i^0 \cos A_i^0$ will depend not only on the relative positions of each site i with respect to each other, but also, on their relative position with respect to the x-y coordinates. In turn, the selection of the x-y coordinate system affects GDOP. It has been verified [2], that if

1) $(\sum_{i=1}^{i=n} \cos 2 A_i^0)^2 - (\sum_{i=1}^{i=n} \sin 2 A_i^0)^2 = \text{constant}$

$$\text{and} \quad \sum_{i=1}^{i=n} \sin A_i^0 = 0 = \sum_{i=1}^{i=n} \cos A_i^0 \qquad (13)$$

2) or if readings are taken in elevation and range only, i.e. azimuth measurements are absent (σ_E, $\sigma_R < \infty$, $\sigma_A = \infty$) then the GDOP will be independent of the positioning of the x-y coordinates. Neither of these conditions are satisfied in our present example. We are interested in determining the amount of rotation to be imparted to the x-y axes for minimum and maximum GDOP values. If θ is the angle at the center, then, the azimuth angles A_1^0, A_2^0, A_3^0, and A_4^0 are given by

$$A_1^0 = A$$

$$A_2^0 = A + \theta$$

$$A_3^0 = A + \pi$$

$$A_4^0 = A + \theta + \pi$$

It can be shown that the amount of rotation α to be imparted to the x-y axes for least GDOP is given by ($\theta \neq \pi/2$)

$$\tan 2\alpha = \frac{\sum_{i=1}^{i=4} \sin 2 A_i^0}{\sum_{i=1}^{i=4} \cos 2 A_i^0}$$

$$= \frac{4 \sin(2A + \theta) \cos \theta}{4 \cos(2A + \theta) \cos \theta} = \tan(2A + \theta) \qquad (14)$$

The complete details of the analytic arguments may be found in [2] and will not be repeated here. The solution of (14) yields

$$\alpha = A + \frac{\theta}{2} + \frac{k\pi}{2}, \text{ where } k = 0, \pm 1, \pm 2, \ldots$$

It is easy to verify that this corresponds to selecting the coordinate axes parallel to the sides of the rectangle. This rotation is valid so long as $0 \leq \theta < \pi/2$ and $\pi/2 < \theta \leq \pi$. When $\theta = \pi/2$, the expression for solving α is indeterminate. Actually, this corresponds to the special case when the rectangle shapes into a square, and as previously noted from expression (13), the GDOP for that case is independent of the orientation of the x-y coordinate system. If such an optimum orientation is selected, then

$$A = -\frac{\theta}{2} + \frac{k\pi}{2}, \quad k = 0, \pm 1, \pm 2, \ldots$$

In a similiar fashion, one may show that the worst instrument siting which maximizes GDOP occurs when

$$A = \frac{\pi}{4} - \frac{\theta}{2} + \frac{k\pi}{2}, \quad k = 0, \pm 1, \pm 2, \ldots$$

Let

$$M = \frac{1}{\sigma_A^2} + \frac{\sin^2 E \cos^2 E}{\sigma_E^2} + \frac{d^2 \cos^2 E}{\sigma_R^2}$$

$$N = -\frac{1}{\sigma_A^2} + \frac{\sin^2 E \cos^2 E}{\sigma_E^2} + \frac{d^2 \cos^2 E}{\sigma_R^2} \qquad (15)$$

$$K = \frac{\cos^4 E}{\sigma_E^2} + \frac{d^2 \sin^2 E}{\sigma_R^2}$$

Then, it can be shown that [2]:

$$\frac{(GDOP)_{max}}{(GDOP)_{min}} = \sqrt{\frac{\frac{M(M^2 - 2MN\cos^2\theta + N^2\cos^2\theta)}{(M^2 - N^2\cos^2\theta)^2} + \frac{1}{4K}}{\frac{M}{M^2 - N^2\cos^2\theta} + \frac{1}{4K}}} \qquad (16)$$

The plot of the above function as the elevation angle varies from 0 to $\pi/2$, for various values of θ is shown in Figure 5

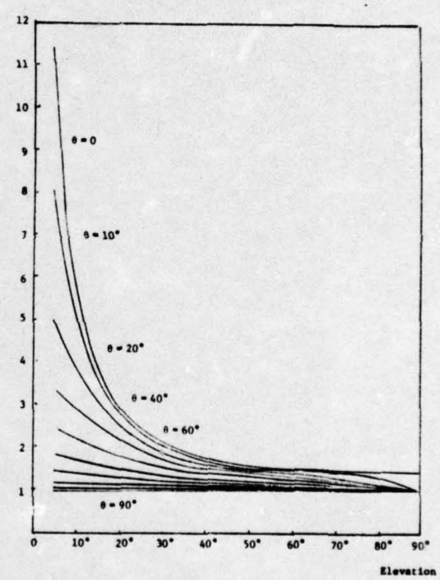

FIGURE 5: $(GDOP)_{max}/(GDOP)_{min}$ AS A FUNCTION OF THE ELEVATION ANGLE E FOR THE 4-STATION PROBLEM. MEASUREMENTS TAKEN IN AZIMUTH AND ELEVATION (θ = angle at the center).

It is interesting to note that for low values of θ and low elevation angles E, the GDOP value is quite sensitive to the selection of the x-y coordinate axes. This sensitivity disappears for all practical purposes when the value of the angle at the center θ exceeds 40°.

V. CONCLUSIONS

The effect on estimation of the Cartesian coordinate position of a target by a system where data measurements are gathered simultaneously in azimuth, elevation and range (e.g. radars) is studied. It is assumed that the number of measurements exceeds the minimum number required thus leading to an overdetermined system of equations. The method of maximum likelihood estimation is used to determine analytically the net contribution to GDOP of the various interplaying variables. It is shown that low GDOP values result when the measuring instruments are positioned so that their approximate elevation angle to the target is in the range $30^{o} - 50^{o}$.

In another example, it is shown that for measurements involving azimuthal data and for certain types of instrument siting, coordinate selection affect the GDOP values and that a minimum GDOP can be achieved by properly selecting the orientation of the coordinate frame of references.

ACKNOWLEDGEMENT

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REFERENCES

1. Sivazlian, B. D., and R. E. Green, "Optimal Instrument Siting for Stationary Target Tracking", NAECON '75 Record, pp. 302-308.

2. Sivazlian, B. D., and R. E. Green, "Asymmetrical Multistation Effects on GDOP in Target Tracking", Technical Report STEWS - ID-74, Systems Management Division, ID, White Sands Missile Range, N.M. September 1974.