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TIME DEPENDENT HOLOGRAPHIC INTERFEROMETRY AND FINITE-ELEMENT ANALYSIS OF HEAT TRANSFER WITHIN A RECTANGULAR ENCLOSURE

Gerald Paul Braun

NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

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AND FINITE-ELEMENT ANALYSIS
OF HEAT TRANSFER WITHIN A
RECTANGULAR ENCLOSURE

by

Gerald Paul Braun

September 1976

Thesis Advisor: D. J. Collins

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Analyzing heat flows experimentally was also explored utilizing holographic interferometry. Specific problems

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Time Dependent Holographic Interferometry and Finite-Element Analysis of Heat Transfer within a Rectangular Enclosure

by

Gerald Paul Braun
Lieutenant, United States Navy
B.S.E.E., The University of Toledo, 1968

Submitted in partial fulfillment of the
requirements for the degree of

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NOMENCLATURE

C_p - specific heat of fluid

D - width of the enclosure

$\triangle$ - area of element triangle

g - gravitational acceleration

Gr_L - Grashof number in L direction = $\frac{gBL^3(T_H-T_C)}{\nu^2}$

L - height of the enclosure

L_i - natural coordinates

N_i - interpolation functions

P - pressure (either wall or fluid)

Pr - Prandtl number = $\frac{\nu}{\alpha}$

Ra_D - Rayleigh number in D direction = $\frac{gBD^3(T_H-T_m)}{\nu}$

Ra_L - Rayleigh number in L direction = $\frac{gBL^3(T_H-T_C)}{\nu}$

T - temperature (either wall or fluid)

t - time relative to beginning of solution

T_C - temperature at cold wall

T_H - temperature at hot wall

T_m - mean temperature of fluid = $\frac{(T_H+T_C)}{2}$

u - velocity in x-direction

v - velocity in y-direction

x - independent coordinate in horizontal direction

y - independent coordinate in vertical direction

α - thermal diffusivity of fluid = $\frac{\chi}{\rho C_p}$

β - coefficient of thermal expansion of fluid

χ - thermal conductivity of fluid

μ - dynamic viscosity of fluid

ν - kinematic viscosity of fluid = $\frac{\mu}{\rho}$

ρ - fluid density

$\Omega^{(e)}$ - domain of integration for element (e)

I. INTRODUCTION

A. HOLOGRAPHIC INTERFEROMETRY

The phenomenon of interference has had a considerable influence on the development of physics. Thomas Young's observation and explanation of the interference of the beams through two holes provided the basis for Fresnel's wave theory of light and the same experiment has been used as the foundation of modern coherence theory.

Derived from interference is the technique of interferometry, now one of the important methods of experimental physics. The father of visible-light interferometry was A. A. Michelson, who was awarded in 1907 the Nobel prize in physics for "his optical instruments of precision and the spectroscopic and metrological investigations he has executed with them." Applications to other spectral regions were more recent: the first use of interferometry in radio astronomy was reported in 1947, and infra-red interference spectroscopy was successfully employed some thirteen years later.

Ever since the wave nature of light was generally accepted, interferometry has been the primary method for making measurements with great accuracy. The very small

wavelength of light, on the order of 5×10^{-5}cm, and the fact that interferometric means are available for detecting changes of only a small fraction of this length, indicates the degree of accuracy which can be achieved. The widespread applications of the method attest to its general usefulness. Interferometry is used for testing optical components, optical gauging of machine tools, studying air flow in wind tunnels, and standardizing the fundamental units of length. Therefore it is understandable that any fundamental improvement or innovation in this interferometric technique would find many applications over a wide field.

Holographic interferometry is just such an innovation. Holography may be described as a photographic technique in which the amplitude and phase characteristics emanating from a coherent light source are recorded and later reproduced. This reproduction assumes the form of a three-dimensional image of the original subject. Holography has widened the scope of interferometry to such a degree that holographic interferometry is now considered a standard tool in engineering laboratories all over the world.

Conventional interferometry can be utilized to make measurements on highly polished surfaces of relatively simple shape. Holographic interferometry extends this

range by allowing measurements to be made on three-dimensional surfaces or arbitrary shape and condition. A roughly processed machine part can now be measured to optical tolerance. Furthermore, with the holographic technique a complex object can be examined interferometrically from many different perspectives, because of the three-dimensional nature of the hologram. A single interferometric hologram is equivalent to many observations with a conventional interferometer. This property is especially useful for observations of such things as fluid flow in a wind tunnel. A third departure of holographic interferometry from conventional interferometry is that an object can be interferometrically examined at two different times; one can detect with wavelength accuracy any changes undergone by an object over a period of time. The present object can thus be compared with itself as it was at an earlier time. This is a great advantage in many fields. For example, a large lens can be tested before and after mounting. Similarly, with the aid of pulsed lasers, a machine part can be interferometrically compared with itself statically as well as dynamically.

Methods of holographic interferometry include single- and double-exposure as well as pulsed laser interferometry.

In this thesis, only single-exposure holographic interferometry was considered since it corresponds to real-time interferometry, that is, a method which allows one to observe changes in a subject as they actually occur.

B. CONCEPT AND HISTORY OF THE FINITE-ELEMENT METHOD

One must often resort to numerical procedures in order to obtain quantitative approximate solutions to linear and nonlinear problems in continuum mechanics. However, regardless of the initial assumptions and the methods used to formulate a problem, if numerical methods are employed in evaluating the results, the continuum is, in effect, approximated by a discrete model in the solution process. This observation suggests a logical alternative to the classical approach, namely, represent the continuum by a discrete model at the onset. One such approach, based on the idea of piecewise approximating continuous fields, is referred to as the finite-element method. Its simplicity and generality make it an attractive candidate for applications to a wide range of engineering problems.

Classically, the analysis of continuous systems often began with investigations of the properties of small differential elements of the continuum under investigation. Relationships were established among mean values of various

quantities associated with the infinitesimal elements, and partial differential equations or integral equations governing the behavior of the entire domain were obtained by allowing the dimensions of the elements to approach zero as the number of elements became infinitely large.

In contrast to this classical approach, the finite-element method begins with investigations of the properties of elements of finite dimensions. The equations describing the continuum may be employed in order to arrive at the properties of these elements, but the dimensions of the elements remain finite in the analysis, integrations are replaced by finite summations, and the partial differential equations of the continuous media are replaced, for example, by systems of algebraic or ordinary differential equations. The continuum with infinitely many degrees of freedom is thus represented by a discrete model possessing a finite number of degrees of freedom. Moreover, if certain completeness conditions are satisfied, then, as the number of finite elements is increased and their dimensions are decreased, the behavior of the discrete system converges to that of the continuous system. A significant feature of this procedure is that, in principle, it is applicable to the analysis of finite deformations of materially

nonlinear, nonhomogeneous bodies of any geometrical shape with arbitrary boundary conditions.

The practice of representing a structural system by a collection of discrete elements dates back to the early days of aircraft structural analysis, when wings and fuselages, for example, were treated as assemblages of stringers, skins, and shear panels. By representing a plane elastic solid as a collection of discrete elements composed of bars and beams, Hennikoff [1941] introduced his "framework method," a forerunner to the development of general discrete methods of structural mechanics. Topological properties of certain types of discrete systems were examined by Kron [1939], who developed systematic procedures for analyzing complex electrical networks and structural systems. Courant [1943] presented an approximate solution to the St. Venant torsion problem in which he approximated the warping function linearly in each of an assemblage of triangular elements and proceeded to formulate the problem using the principle of minimum potential energy. Courant's piecewise application of the Ritz method involves all the basic concepts of the procedure now known as the finite-element method. In 1954, Argyris and his collaborators began a series of papers in which they developed certain

generalizations of the linear theory of structures and presented procedures for analyzing complicated discrete structural configurations in forms easily adapted to the digital computer.

The formal presentation of the finite-element method together with the direct stiffness method for assembling elements was attributed to Turner, Clough, Martin, and Topp [1956], who employed the equations of classical elasticity to obtain properties of a triangular element for use in the analysis of plane stress problems. It was Clough [1960], who first used the term "finite elements" in a later paper devoted to plane elasticity problems.

Concepts of the method became more understandable after 1963 when Besseling [1969], Melosh [1970], Fraeys de Veubeke [1971], and Jones [1972] recognized that the finite-element method was a form of the Ritz technique and demonstrated its generality for handling elastic continuum problems. In 1965, the finite-element method received an even broader interpretation when Zienkiewicz and Cheung [1973] reported that it was applicable to all field problems which could be cast into variational form. During the late 1960's and early 1970's, while mathematicians were working on establishing errors, bounds, and convergence

criteria for finite-element approximations, engineers and other appliers of this same method were also studying similar concepts for various problems in the area of solid mechanics.

Although a major portion of the literature written to date on the finite-element method deals with static and dynamic structural analysis, there has been a continuing steady increase in the number of applications in other fields. The goal of this thesis was to develop a computer program, utilizing the finite-element method, which could accurately analyze laminar natural convection within a vertical rectangular enclosure. The program should be able to properly analyze axisymmetric as well as two-dimensional flows.

II. FUNDAMENTAL THEORY OF FINITE-ELEMENT ANALYSIS

In this section the fundamental theory on which the thesis was based is presented. Highlighted topics include the variational principle, some basic concepts of finite-element analysis and the Ritz technique, and finally the method of weighted residuals featuring the Galerkin criterion. The variational principle and the Galerkin method are looked at in detail in regards to the derivation of finite-element equations.

The finite-element method envisions a solution region as built up of many small, interconnected subregions or elements. Such a model of a problem gives a piecewise approximation to the governing equations. The basic premise of the finite-element method is that a solution region can be analytically modeled or approximated by replacing it with an assemblage of discrete elements. These finite-element discretization procedures reduce the problem to one of a finite number of unknowns by dividing the solution region into elements and by expressing the unknown field variable in terms of assumed approximating or interpolation functions within each element. The interpolation functions

are defined in terms of the value of the field variables at specified points called nodes or nodal points. Nodes usually lie on the element boundaries where adjacent elements are considered to be connected. In addition to boundary nodes, an element may also have a few interior nodes (although this was not the case in the choice of linear and quadratic trinagular elements utilized in this thesis). The nodal values of the field variable and the interpolation functions for the elements completely define the behavior of the field variable within the elements. For the finite-element representation of a particular problem, the nodal values of the field variable become the new unknowns. Once these unknowns are found, the chosen interpolation functions define the field variable throughout the assemblage of elements.

Clearly, the nature of the solution and the degree of approximation depend not only on the size and number of the elements used, but also on the interpolation functions selected. As one would expect, functions cannot be arbitrarily chosen since certain compatibility conditions must be satisfied. Often such functions are selected so that the field variable and/or its derivatives are continuous across adjoining element boundaries. Another important

feature of the finite-element method which sets it apart from other approximate numerical methods is its ability to formulate solutions for individual elements before putting them together to represent the entire problem.

The finite-element method has gained much popularity and has been utilized extensively in recent years because it has, in general, several outstanding advantages. These are the following:

1. Non-homogeneous configurations may be treated with relative simplicity.

2. The elements can be graded in size and shape to follow boundaries of arbitrary shape.

3. Once a computer program has been developed, problems of the same variety can be solved simply by supplying the computer with appropriate data.

There are at least three distinct approaches one may employ in order to obtain finite-element equations of a particular system. In order of <u>increasing</u> versatility they are: (1) the direct approach, (2) the variational principle, and (3) the weighted residuals approach.

The direct approach can be used only for relatively simple problems in which discrete elements may be easily identified. Once these elements have been selected, direct

physical reasoning is introduced to establish the element equations in terms of pertinent variables. The final step is then to combine the element equations to form the governing equations of the complete system.

A detailed explanation of the remaining two approaches will be given in Subsections A, B and C to follow.

Whichever one of these three particular approaches is utilized, the finite-element method follows a systematic step-by-step process when applied to continuum problems. They are:

1. Discretize the continuum.

The entire flow region under study is divided into a series of subregions or elements assumed to be interconnected at a finite number of nodal points; thus a program originally exhibiting an infinite number of degrees of freedom is made finite. The elements used can be triangular, rectangular, or almost any shape. Also, information must be fed into a computer giving global coordinates of the nodes and topology of the system.

Finally, selection of which field variables are to be used to satisfactorily describe solution domain must be indicated at this point in the process.

2. Select interpolation functions, $N_i^{(e)}$

From the nodal values one represents the value of the field variable over the element by means of interpolation functions. Often, although not always, polynomials are selected as these functions because they are easy to integrate and differentiate. The number of nodes and the order of the interpolation polynomials are interrelated. The field variable itself may be a scalar, a vector, or a higher-order tensor.

3. Find the element properties.

Essentially, the problem is solved at the element level. The matrix equations expressing the properties of the individual elements are determined. This can be accomplished by any one of the three approaches previously mentioned: the direct method, the variational principle, or the weighted residual method. The approach used depends entirely on the nature of the particular problem.

4. Assemble the element properties to obtain the system equations.

In this step, one combines the matrix equations expressing the behavior of the elements to form the matrix equations expressing the behavior of the entire solution region or system. The matrix equations for the system

exhibit the same form as the equations for an individual element except that they contain many more terms because they include all the nodes. The basis for this assembly procedure stems from the fact that, at a node where elements are interconnected, the value of the field variable is the same for each element sharing that node.

5. Solve the system equations.

From the previous step, a set of simultaneous equations are derived which can now be solved to obtain the unknown nodal values of the field variable. If these equations are linear, a number of standard solution techniques may be employed; if the equations are nonlinear, their solution is more difficult to obtain, but several alternative approaches do exist that lead to satisfactory results.

6. Make additional computations if desired.

It may be desired to use the solution of the system equations to calculate other important parameters, i.e., from the nodal values of the pressure, one might wish to calculate velocity distributions.

It is worth making mention of the fact that several of the steps in the above process are essentially the same regardless of the type of problem (this thesis was devoted

to the fluid mechanics problem). Thus, only steps three (3) and six (6) might differ for any given situation, in that the equations describing the elements could vary. The other steps would be the same. This generality of the finite-element method is, without doubt, one of its greatest strengths.

A. VARIATIONAL PRINCIPLE

Often, continuum problems have different but yet equivalent formulations, such as a differential formulation and a variational formulation. In the differential case, the problem is to integrate a differential equation or a system of differential equations subject to given boundary conditions. In the classical variational formulation, the problem is to find the unknown function or functions which extremize (maximize, minimize) or make stationary a functional or system of functionals subject to the same specified boundary conditions. The two problem formulations are equivalent because the functions that satisfy the differential equations and their boundary conditions also extremize or make stationary the functionals. This equivalence is apparent from the calculus of variations, which shows that the functionals are extremized or made stationary only when

one or more Euler equations and their boundary conditions are satisfied. Consequently, these equations are precisely the governing differential equations of the problem. To illustrate this duality concept, Appendix B provides a brief review and introduction to some basic ideas of the calculus of variations.

B. FINITE-ELEMENT METHOD AND THE RITZ TECHNIQUE

The Ritz technique is basically a procedure for transforming a continuous medium into an approximated lumped parameter system. A more qualitative definition would be that the Ritz method consists of assuming the form of the unknown adjustable parameters. From this family of trial or coordinate functions, that particular function which renders the functional stationary is then selected. The procedure is to substitute the trial functions into the functional and thereby express the functional in terms of the adjustable parameters. This functional is then differentiated with respect to each parameter, and the resulting equation is set equal to zero. If there are n unknown parameters, there will be n simultaneous equations to be solved for these parameters. By this means, the approximate solution is chosen from the family of assumed solutions.

This procedure does nothing more than give one the "best" solution from the family of assumed solutions. Clearly, then, the accuracy of the approximate solution depends on the choice of trial functions. These trial functions are required to be defined over the whole solution domain and must satisfy at least some and usually all of the boundary conditions. Sometimes, if the general nature of the desired solution is known, the approximation can be improved by choosing the trial functions to reflect this nature. If, by chance, the exact solution is contained in the family of trial solutions, then the Ritz technique gives the exact solution as expected. Generally, the approximation improves as the size of the family of trial functions and the number of adjustable parameters increase. If the trial functions are part of an infinite set of functions that are capable of representing the unknown function to any degree of accuracy, the process of including more and more trial functions leads to a series of approximate solutions which converge to the true solution. Often a family of trial functions is constructed from polynomials of successively increasing degree.

To illustrate the Ritz technique, consider the following simple example. Suppose it is desired to find the general

function $\phi(x)$ satisfying

$$\frac{d^2\phi}{dx^2} = -f(x)$$

with boundary conditions of $\phi(a)=A$ and $\phi(b)=B$ specified. It is assumed that $f(x)$ is a continuous function in the closed interval $[a,b]$. This problem is equivalent to finding the function $\phi(x)$ that minimizes the functional

$$I(\phi) = \int_b^a \left[\tfrac{1}{2}\left(\frac{d\phi}{dx}\right)^2 - f(x)\phi(x) \right] dx$$

which is of the form $I(\phi) = \int_{x_1}^{x_2} F(x,\phi, \phi_x, f(x))dx$

Ignoring the fact that this problem possesses an exact solution, we will attempt to find an approximate solution. According to the Ritz method, the desired solution can be assumed to be approximately represented in $[a,b]$ by a combination of selected trial functions of the form

$$\phi(x) = C_1 \psi_1(x)+C_2 \psi_2(x)+....+C_n \psi_n(x) \quad , \quad a \leq x \leq b$$

where the n constants C_i are the adjustable parameters to be determined. The trial functions should be selected so that the expression for $\phi(x)$ satisfies the boundary conditions regardless of the choice of the constants C_i. Using

polynomials is a simple and convenient way of constructing the trial functions. Therefore

$$\phi(x) \approx (x-a)(x-b)(C_1 + C_2 x + C_3 x^2 + \ldots + C_n x^{n-1})$$

is a possible series of trial functions. When this approximate expression for $\phi(x)$ is substituted into the functional to be minimized, and after the integration has been carried out, the functional is of the form

$$I = I(C_1, C_2, ----, C_n).$$

Since the C_i are required to be chosen such that they minimize I, employing differential calculus, the following partial differential equations are formulated

$$\frac{\partial I}{\partial C_1} = 0, \frac{\partial I}{\partial C_2} = 0, \ldots., \frac{\partial I}{\partial C_n} = 0$$

These n equations are then solved for the n parameters C_i, and the accuracy of the approximate solution depends on the number of C's used in the trial function. Generally, as n increases the accuracy improves. To assess the improvement in accuracy as more C's are utilized, the problem is solved repeatedly by taking successively more terms in the approximation, that is

$$\phi_1(x) \approx (x-a)(x-b)C_1$$

$$\phi_2(x) \approx (x-a)(x-b)(C_1+C_2x)$$

$$\phi_3(x) \approx (x-a)(x-b)(C_1+C_2x+C_3x^2)$$

and so on. By comparing the results at the end of each calculation, the effect on accuracy of adding more terms can be estimated.

The finite-element method and the Ritz technique are essentially equivalent. Each method uses a set of trial functions as the starting point for obtaining an approximate solution; both methods take linear combinations of these trial functions; and both methods seek the combination of trial functions that renders a given functional stationary. The major difference between these approximating methods stems from the fact that the assumed trial functions in the finite-element method are not defined over the entire solution domain, and they must satisfy not just any boundary conditions, but only certain continuity conditions and then only sometimes. Since the Ritz technique uses functions construed over the whole domain, it can be employed only for domains of relatively simple geometric shape. Also, these trial functions associated with the Ritz method are required to satisfy at least some and usually all of the

boundary conditions. In the finite-element method the same geometric limitations exist, but only for the elements. Due to the fact that elements with simple shapes can be assembled to represent exceedingly complex geometries, the finite-element method is a far more versatile tool than the Ritz technique. From a strict mathematical standpoint, the finite-element method is a special case of the Ritz technique only when the piecewise trial functions obey certain continuity and completeness conditions that are stipulated over just the element alone.

C. METHOD OF WEIGHTED RESIDUALS (GALERKIN'S METHOD)

The third and final approach to the finite-element method involves a procedure that is more generalized and straightforward than either of its two predecessors.

The relationship between the well-known Ritz technique and the finite-element method enables one to view the finite-element discretization procedure as simply another means for finding approximate solutions to variational problems. In fact, these finite-element equations were shown to be derived by requiring that a given functional be stationary. This broad variational interpretation is the one most widely used to derive element equations, and it is the

most convenient approach whenever a classical variational statement exists for a given problem.

However, applied scientists and engineers encounter practical problems for which classical variational principles are unknown. In these cases finite-element techniques are still applicable, but more generalized procedures characteristic of the method of weighted residuals must be employed to derive the element equations. Through certain generalizations, finite-element equations may be derived directly from the governing differential equations of the problem without reliance on any classical, quasi-variational, or restricted variational "principles." This procedure allows one to apply the finite-element method to almost all practical problems of mathematical physics.

The method of weighted residuals is a technique for obtaining approximate solutions to linear and nonlinear partial differential equations. It offers still another means with which to formulate the finite-element equations. Applying the method of weighted residuals involves basically two steps. The first step is to assume the general functional behavior of the dependent field variable in some way so as to approximately satisfy the given differential equation along with its associated boundary conditions.

Substitution of this approximation into the original differential equation and boundary conditions then results in some error called a residual. This residual is required to vanish in some average sense over the entire solution domain. The second step entails solving the equation(s) resulting from step one and thereby specializing the general functional form to a particular function, which in turn becomes the approximate solution sought.

To be more specific, the following typical problem is offered. Suppose it is desired to find an approximate functional representation for a general field variable ϕ governed by the differential equation

$$\mathcal{L}(\phi) - f = 0 \tag{2.1}$$

in the domain D bounded by the surface $\sum$. $\mathcal{L}$ is a linear or nonlinear differential operator and the function f is a known function of the independent variables. Also, proper boundary conditions are assumed to be prescribed on $\sum$. The method of weighted residuals is now applied in two steps. First, the unknown exact solution ϕ is approximated by $\hat{\phi}$, where either the functional behavior of $\hat{\phi}$ is completely specified in terms of unknown parameters, or the functional dependence on all but one of the independent variables is given while the functional dependence on the

remaining independent variable is left unspecified. Thus the dependent variable is approximated by

$$\phi \approx \hat{\phi} = \sum_{i=1}^{m} N_i C_i \qquad (2.2)$$

where the N_i are the assumed functions and the C_i are either the unknown parameters or unknown functions of one of the independent variables. The m functions N_i are usually chosen to satisfy the global boundary conditions of the system in question. When $\hat{\phi}$ is substituted into equation 2.1, it is unlikely that this equation will not be satisfied, that is,

$$\mathcal{L}(\hat{\phi}) - f \neq 0$$

but in fact,

$$\mathcal{L}(\hat{\phi}) - f = e$$

where e is the residual or error that results from approximating ϕ by $\hat{\phi}$. The method of weighted residuals seeks to determine the m unknowns C_i in such a way that the error e over the entire solution domain is small. This is accomplished by forming a weighted average of the error and specifying that this weighted average vanish over the solution domain. In other words, m linearly independent weighting functions, W_i, are chosen such that

$$\int_D \left[\mathcal{L}(\hat{\phi}) - f\right] W_i \, dD = \int_D e W_i \, dD = 0, \quad i = 1, 2, \ldots, m \qquad (2.3)$$

The form of the error distribution principle expressed in equation 2.3 depends on the choice of weighting functions. Once these are specified, equation 2.3 represents of a set of m equations, which may be either algebraic or ordinary differential. The second step is to solve for the C_i's and hence obtain an approximate representation of the unknown general field variable ϕ via equation 2.2. There are many linear problems and even some nonlinear problems for which it can be shown that, as $m \to \infty$, $\hat{\phi} \to \phi$, but, in general, studies of convergence and error bounds are scarce.

Due to the broad choice of weighting functions or error distribution principles than can be used, a variety of weighted residual techniques are likewise available. The error distribution principle most often utilized to derive finite-element equations in the field of aeronautics is known as the Galerkin criterion, or Galerkin's method. Here, the weighting functions are chosen to be the same as the approximating functions employed to represent ϕ, that is, $W_i = N_i$ for $i=1,2,\ldots,m$. Therefore Galerkin's method requires that

$$\int_D \left[\mathcal{L}(\hat{\phi}) - f\right] N_i \, dD = 0 \qquad (2.4)$$

In the preceding section pertaining to the Ritz technique,

it was assumed that the entire solution domain was being dealt with. However, because equation 2.1 holds for any point in this region, it also holds for any collection of points defining an arbitrary subdomain or element of the whole domain. Consequently, attention may be focused directly on an individual element by means of a local approximation analogous to equation 2.2, but being defined as valid for only one element at a time. Now the finite-element representations of a general field variable become available. The functions N_i become what are known as the interpolation functions $N_i^{(e)}$ defined over the element, and the C_i are the undetermined parameters, which may be the nodal values of the field variable or its derivatives. Then, from Galerkin's method, the equations governing the behavior of an element of the solution domain may be written as

$$\int_D \left[\mathcal{L}(\phi^{(e)}) - f^{(e)} \right] N_i^{(e)} dD^{(e)} = 0, \quad i=1,2,\ldots,r \qquad (2.5)$$

where, as before, the superscript (e) restricts the range to one element, and

$$\phi^{(e)} = \lfloor N^{(e)} \rfloor \{\phi\}^{(e)}$$

$f^{(e)}$ = forcing function defined over element (e)

r = number of unknown parameters assigned to the element.

There exists a set of equations similar to equation 2.5 for each element of the whole assemblage. Prior to assembling the system equations from the individual element equations, it is required that the choice of approximating functions N_i guarantee the interelement continuity along the boundary necessary for the assembly process. If the field variable is continuous at element interfaces, then C^o continuity exists; if, in addition, first derivatives of the variable are continuous, C^1 continuity is said to occur; if second derivatives are also continuous, a region of C^2 continuity exists; and so on. This is the standard definition and notation utilized for expressing the degree of continuity of a field variable at element junctions. The higher the order of continuity required in the solution, the narrower one's choice of interpolation functions becomes.

With the above definition of continuity in mind, the compatibility and completeness requirements for such interpolation functions may be stated. If the functions appearing under the integrals in the element equations contain derivatives up to the (n+1)th order, then the following stipulations must be satisfied for assurance of convergence as the element size decreases.

Compatibility requirement: At element interfaces, C^n continuity must exist.

Completeness requirement: Within an element, C^{n+1} continuity must exist. These requirements hold regardless of whether the element equations (integral expressions) were derived using the variational technique or the Galerkin methed. For this thesis, n was taken to have a value of zero.

Integration by parts is a convenient way to introduce the natural boundary conditions that must be satisfied on some portion of the system exterior or boundary. Although the boundary terms containing these imposed conditions appear in the equations for each element, during the assembly of the element equations only the boundary elements give nonvanishing contributions. After the assembly process has been completed, the fixed boundary conditions (i.e., specified velocity, pressure or temperature) are conveniently introduced to help simplify the final matrix form of the finite element equation.

III. ANALYSIS OF CONVECTIVE HEAT TRANSFER BETWEEN PARALLEL PLATES

The transfer of heat energy across a fluid layer is accomplished, in general, through the mechanisms of conduction, convection and radiation. This last phenomenon is usually a function of the fluid enclosed between the surfaces and the nature, temperature and configuration of the enclosing boundaries. Radiation takes place independently of the conduction and convection as long as there is no absorption by the fluid, and therefore under these conditions it can be considered separately. The phenomena of conduction and convection are closely interdependent and are usually analyzed together. Buoyancy forces result from differences in density within the fluid and are caused by heat transfer to or from this fluid. Natural convection may then be thought of as fluid motion of the system due to the activation of these buoyance forces. In a two-dimensional plane, such heat transfer across a vertical, enclosed fluid layer is a function of the Grashof number, the Prandtl number and the fluid layer height-to-width ratio (L/D).

Natural convection plays a very important role in materials processing at high temperatures where agitation by other means is impracticable, or where the existence of temperature gradients is an inherent characteristic of the system.

The steady convective motion of a lubricating fluid contained within a long, rectangular enclosure was investigated. Holographic interferometry and numerical approximation were the experimental and theoretical analysis tools, respectively.

The two vertical walls of the enclosure were held at different temperatures, and the top and bottom were deemed perfect insulators (Figure 1). It was considered that the length of the enclosure (7 inches) was sufficiently long in the direction normal to the plane of Figure 1 for the motion to be assumed two-dimensional. Another assumption made was that the fluid motion was laminar. Experimental evidence indicates that such an assumption is valid provided the Rayleigh number based on cavity height is less than about 10^8 (Ra_L in this study was calculated to be 1.018×10^7). Using this value and a value of the Prandtl number of 1.0755×10^4, determined from the ratio of kinematic viscosity to thermal diffusivity of the fluid, a system

Grashof number of 946.4 was calculated. The temperatures of the vertical walls x=0 and x=D were defined to be T_H and T_C respectively. If (T_H-T_C) in degrees Fahrenheit is sufficiently small with respect to T_C, the Boussinesq approximation may be introduced which neglects density variations in inertia terms of the equations of motion, but retains it in the buoyancy term. One final assumption was made that all other relevant thermodynamic and transport properties were independent of temperature and that compressibility and viscous dissipation effects were negligible.

The problem now was to find the time and spatial dependence of the velocities and the temperatures within the system.

The governing differential equations expressing conservation of mass, momentum (both in x- and y-directions) and energy were

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3.1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \frac{1}{\rho}\frac{\partial P}{\partial x}$$

$$+gB(T-T_m) \tag{3.2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - \frac{1}{\rho}\frac{\partial P}{\partial y} \tag{3.3}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \qquad (3.4)$$

The solution to the foregoing set of dynamic equations must satisfy the following boundary conditions on the walls,

$u_o = v_o = 0$, no velocity on any of the four walls

$T = T_H$ or T_C given on the two vertical walls

$P = P_{ATMOS}$, also given on the two vertical walls.

IV. THEORETICAL RESULTS

A. FINITE-ELEMENT ANALYSIS OF THE CONTINUUM

1. Discretization of the Continuum

Since the fundamental premise of the finite-element method is that a continuum or solution domain of arbitrary shape can be accurately modeled by an assemblage of simple shapes, most finite elements are geometrically simple also. This statement especially pertains to the choice of the triangular-shaped element which would represent the unknown system parameters in this study, that is, the velocity, temperature and pressure. The main reason behind this choice was the fact that the three-node flat triangular element is the simplest two-dimensional element available, and hence an assemblage of triangles could always depict a two-dimensional domain with any number of straight sides. The solution domain in this problem was the vertical rectangular enclosure, a relatively simple-shaped continuum which posed no problem for the triangular elements. Twelve (12) elements were utilized to represent the 8.5 inch by 1.875 inch area. They were interconnected to each other and the boundary at a total of thirty-five (35) nodal points, of which twelve (12) were corner nodes (Figure 2).

Arriving at this figure of thirty-five nodes was not an arbitrary process. Once pressure was chosen to be linearly approximated, system velocities and temperature were required to assume polynomial approximation of one degree higher, or quadratic, if the highest solution accuracy was to be achieved. For triangular elements, a complete n th-order polynomial requires $\frac{1}{2}(n+1)(n+2)$ nodes for its specification. Therefore, a 1st order, or linearly approximated, polynomial is associated with a three-node triangle; and a quadratic polynomial relates to a six-node triangle.

The three-node elements, with their nodes on the corners, may be thought of as being superimposed onto the six-node elements. Such elements contain, in addition to the corner nodes, nodes located at the midpoint of each side of the triangle. Twelve triangular elements of the six-node variety may be interconnected to form the solution domain shown in Figure 2; this domain possessing exactly thirty-five nodes.

Each element (6-node and 3-node) specifies uniquely a complete polynomial of the order necessary to give C^o continuity, and hence satisfy the completeness and compatibility requirements for elemental assemblage.

Next, the distinction between local and global node-numbering had to be made. Since each element in the triangular mesh had six nodes, the local nodes were identified as such by starting in the upper left hand corner of each element and numbering counterclockwise around the element. The global node system is a method for uniting these independent elements along with their nodes into one distinct entity. Figure 3 summarizes the relation between local and global numbering for four (4) such elements. This figure defines the system topology or the connectivity of the system.

2. Selection of the Interpolation Functions

In the preceding subsection it was mentioned that linear approximation was used for values representing nodal pressures, while both velocity and temperature varied in a quadratic fashion within the elements. Such a relationship was based on the governing equations of the system, in which the highest order of partial differential equations involving pressure was one, while partial derivatives of u, v and T existed up to second order. Therefore, choosing linear pressures required the remaining three nodal parameters or field variables to take on quadratic approximation.

The functions employed to represent the behavior of these field variables within an element are known as interpolation or approximating functions. Their order within an element depends on the number of degrees of freedom assigned to that element. In this study, two different polynomial series were selected as the first and second order interpolation functions. Associated with these series were coefficients made up of generalized coordinates, that is, independent parameters which specified the magnitude of the prescribed distribution for each field variable (u, v, P, T). These polynomials were represented as follows

$$P(x,y)^{(e)}=C_1{}^{(e)}+C_2{}^{(e)}x+C_3{}^{(e)}y \tag{4.1}$$

for the linear pressure terms, and

$$\phi(x,y)^{(e)}=C_1{}^{(e)}+C_2{}^{(e)}x+C_3{}^{(e)}y+C_4{}^{(e)}x^2+$$
$$C_5{}^{(e)}xy+C_6{}^{(e)}y^2 \tag{4.2}$$

with ϕ being a generalized quadratic field variable (either u, v, or T in this case, and the superscript 'e' standing for element.

The next step in the process was to solve for the generalized coordinate $C_i^{(e)}$ in terms of the as yet unknown field variables. This gave the desired interpolation, but the form of the resulting equations was not convenient. As a final step then, the equations were rearranged until they appeared as

$$P(x,y)^{(e)}=N_1^P(x,y)P_1+N_2^P(x,y)P_2+$$

$$N_3^P(x,y)P_3=\lfloor N^P \rfloor \{ P \} \qquad (4.3)$$

and

$$\phi(x,y)^{(e)}=N_1^\phi(x,y)\phi_1+N_2^\phi(x,y)\phi_2+N_3^\phi(x,y)\phi_3+$$

$$N_4^\phi(x,y)\phi_4+N_5^\phi(x,y)\phi_5+N_6^\phi(x,y)\phi_6=\lfloor N^\phi \rfloor \{ \phi \} \qquad (4.4)$$

where N_i^u, N_i^v, and N_i^T were the specific interpolation functions in equation (4.4) for this study and were all equal in form, i.e., $N_i^u=N_i^v=N_i^T=N_i$.

3. Determination of the Elemental Properties

In this thesis, the Galerkin method was utilized to determine the element properties. This procedure applied at a general node i of an isolated element becomes, in view of equations 3.1-3.4,

$$\int_{\Omega^{(e)}} H_i \left(\frac{\partial u^{(e)}}{\partial x} + \frac{\partial v^{(e)}}{\partial y}\right) dxdy = 0 \qquad (4.5)$$

$$\int_{\Omega^{(e)}} W_i \left[\frac{\mu}{\rho}\left(\frac{\partial^2 u^{(e)}}{\partial x^2} + \frac{\partial^2 u^{(e)}}{\partial y^2}\right) - \frac{1}{\rho}\frac{\partial P^{(e)}}{\partial x} + gB(T^{(e)} - T_m)\right.$$
$$\left. - u^{(e)}\frac{\partial u^{(e)}}{\partial x} - v^{(e)}\frac{\partial u^{(e)}}{\partial y} - \frac{\partial u^{(e)}}{\partial t}\right] dxdy = 0 \qquad (4.6)$$

$$\int_{\Omega^{(e)}} W_i \left[\frac{\mu}{\rho}\left(\frac{\partial^2 v^{(e)}}{\partial x^2} + \frac{\partial^2 v^{(e)}}{\partial y^2}\right) - \frac{1}{\rho}\frac{\partial P^{(e)}}{\partial y} - u^{(e)}\frac{\partial v^{(e)}}{\partial x}\right.$$
$$\left. - v^{(e)}\frac{\partial v^{(e)}}{\partial y} - \frac{\partial v^{(e)}}{\partial t}\right] dxdy = 0 \qquad (4.7)$$

$$\int_{\Omega^{(e)}} W_i \left[\alpha\left(\frac{\partial^2 T^{(e)}}{\partial x^2} + \frac{\partial^2 T^{(e)}}{\partial y^2}\right) - u^{(e)}\frac{\partial T^{(e)}}{\partial x}\right.$$
$$\left. - v^{(e)}\frac{\partial T^{(e)}}{\partial y} - \frac{\partial T^{(e)}}{\partial t}\right] dxdy = 0 \qquad (4.8)$$

where $W_i(x,y)$ and $H_i(x,y)$ are the weighting or interpolation functions, which were taken as

$$W_i = N_i \text{ and } H_i = N_i^P = L_i \text{ (natural coordinates).}$$

The inertia terms in equations 4.6, 4.7 and 4.8 considerably increased the degree of difficulty of this fluid flow problem when compared to an incompressible viscous flow without inertia. This is because the above mentioned equations are nonlinear, thereby forcing an iterate procedure to be introduced and repeated until the $u_{n+1}^{(e)}$, $v_{n+1}^{(e)}$, and $T_{n+1}^{(e)}$

values converged to the previous $u_n^{(e)}$, $v_n^{(e)}$, and $T_n^{(e)}$ solutions. The subscript n runs from zero to some positive number at which the field variable passes a convergence test.

Integrating each term of equations 4.5-4.8 by parts, and making use of the approximations of equations 4.3 and 4.4, the following results on an elemental level were obtained

$$\int_{\Omega^{(e)}} \left(N_i \frac{\partial \lfloor N \rfloor}{\partial x}\right) dxdy \left\{ u \right\} + \int_{\Omega^{(e)}} \left(N_i \frac{\partial \lfloor N \rfloor}{\partial y}\right) dxdy \left\{ v \right\} = 0 \qquad (4.9)$$

$$\int_{\Omega^{(e)}} \frac{\mu}{\rho}\left(\frac{\partial N_i}{\partial x}\frac{\partial \lfloor N \rfloor}{\partial x} + \frac{\partial N_i}{\partial y}\frac{\partial \lfloor N \rfloor}{\partial y}\right) dxdy \left\{ u \right\} - \frac{1}{\rho}\int_{\Omega^{(e)}} \frac{\partial N_i}{\partial x} \lfloor N^P \rfloor \, dxdy \left\{ P \right\} - gB \lfloor N_i \rfloor \left\{ T \right\} - \int_{\Omega^{(e)}} \left(N_i \lfloor N \rfloor \, dxdy\right)\left\{\frac{\partial u}{\partial t}\right\} = \left\{ gBT_m \right\} + \int_C N_i X^* ds \qquad (4.10)$$

$$\int_{\Omega^{(e)}} \frac{\mu}{\rho}\left(\frac{\partial N_i}{\partial x}\frac{\partial \lfloor N \rfloor}{\partial x} + \frac{\partial N_i}{\partial y}\frac{\partial \lfloor N \rfloor}{\partial y}\right) dxdy \left\{ v \right\} - \frac{1}{\rho}\int_{\Omega^{(e)}} \frac{\partial N_i}{\partial y} \lfloor N^P \rfloor \, dxdy \left\{ P \right\} - \int_{\Omega^{(e)}} \left(N_i \lfloor N \rfloor \, dxdy\right)\left\{\frac{\partial v}{\partial t}\right\} = \int_C N_i Y^* ds \qquad (4.11)$$

$$\int_{\Omega^{(e)}} \alpha \left(\frac{\partial N_i}{\partial x}\frac{\partial \lfloor N \rfloor}{\partial x} + \frac{\partial N_i}{\partial y}\frac{\partial \lfloor N \rfloor}{\partial y}\right) dxdy \left\{ T \right\}$$

$$- \int_{\Omega^{(e)}} (N_i \lfloor N \rfloor \, dxdy) \left\{ \frac{\partial T}{\partial t} \right\} = \int_C N_i Z^* ds \qquad (4.12)$$

where $N_i X^* ds$, $N_i Y^* ds$ and $N_i Z^* ds$ are simply lumped-sum contour integrals that introduce the natural boundary conditions for u, v and T respectively. These integral values were labeled QX, QY, and QZ in the computer program. The last term on the left hand side of equations 4.10-4.12 represents the transient nature of the system.

Finally, the element matrix equations were written by inspection from equations 4.9-4.12 and were of the general form

$$\left[K\right]^{(e)} \left\{ \phi \right\}^{(e)} - \left[K_t\right]^{(e)} \left\{ \dot{\phi} \right\}^{(e)} = \left\{ R \right\}^{(e)} \qquad (4.13)$$

where the square matrices $\left[K\right]^{(e)}$ AND $\left[K_t\right]^{(e)}$ are known as stiffness matrices, the column vectors $\left\{ \phi \right\}^{(e)}$ and $\left\{ \dot{\phi} \right\}^{(e)}$ are the nodal field variable and time derivative vectors, respectively. The column vector $\left\{ R \right\}^{(e)}$ signifies the resultant nodal force vector for the element. In the actual computer program, the following identities were used

$$\left[K\right] = \left[TM\right], \quad \left[K_t\right] = \left[CD\right], \quad \left\{\phi\right\} = \left\{X\right\}, \quad \left\{\dot{\phi}\right\} = \left\{\dot{X}\right\},$$

$$\text{and} \quad \left\{R\right\} = \left\{RHS\right\}$$

and the element matrix equations were

$$
\underset{(3r+s)\times(3r+s)}{\begin{bmatrix} \nu[K_1] & [0] & -\frac{1}{\rho}[K_2]^T & g\beta[I] \\ [0] & \nu[K_1] & -\frac{1}{\rho}[K_3]^T & [0] \\ -[K_2] & -[K_3] & [0] & [0] \\ [0] & [0] & [0] & \alpha[K_1] \end{bmatrix}}
\underset{(3r+s)\times 1}{\begin{bmatrix} \{U\}^e \\ \{V\}^e \\ \{P\}^e \\ \{T\}^e \end{bmatrix}}
-
\underset{(3r+s)\times(3r+s)}{\begin{bmatrix} [CD] & [0] & [0] & [0] \\ [0] & [CD] & [0] & [0] \\ [0] & [0] & [0] & [0] \\ [0] & [0] & [0] & [CD] \end{bmatrix}}
\underset{(3r+s)\times 1}{\begin{bmatrix} \{\dot{U}\}^e \\ \{\dot{V}\}^e \\ \{\dot{P}\}^e \\ \{\dot{T}\}^e \end{bmatrix}}
=
\underset{(3r+s)\times 1}{\begin{bmatrix} \{AX\}^e \\ \{QY\}^e \\ \{QZC\}^e \\ \{QZ\}^e \end{bmatrix}}
$$

where, $\{AX\}^e = \{g\beta T_M + QX\}$

In the above assemblage, the individual matrix notation utilized was

$$\left[K_1\right] = K_1(i,j) = \int_{\Omega^e} \left(\frac{\partial N_i}{\partial x}\frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y}\frac{\partial N_j}{\partial y}\right) dxdy$$

$$\left[K_2\right]^T = K_2^T(i,j) = \int_{\Omega^e} \left(\frac{\partial N_i}{\partial x} N_j^P\right) dxdy$$

$$\left[K_3\right]^T = K_3^T(i,j) = \int_{\Omega^e} \left(\frac{\partial N_i}{\partial y} N_j^P\right) dxdy$$

$$\left[K_2\right] = K_2(i,j) = \int_{\Omega^e} \left(\frac{\partial N_j}{\partial x} N_i^P\right) dxdy$$

$$\left[K_3\right] = K_3(i,j) = \int_{\Omega^e} \left(\frac{\partial N_j}{\partial y} N_i^P\right) dxdy$$

$$\left[CD\right] = CD(i,j) = \int_{\Omega^e} N_i N_j \, dxdy \,.$$

Also, gBT_m is the u velocity forcing function, and r and s are the number of nodes where velocity (or temperature) and pressure are interpolated at, respectively. In this study, r=6 and s=3, therefore the element matrices were 21x21 and the element column vectors 21x1.

Once the matrix equations were compiled or assembled on the element level, assembling these properties to obtain the system equations in matrix form also was a relatively simple operation for the digital computer. In essence, the large square matrices were derived by systematically adding

together the contributions of each individual element matrix, and inserting prescribed nodal variables or boundary conditions where applicable. As was brought out in the theoretical section of this thesis, the final assembly now became a system of ordinary differential equations resembling the same format as equation 4.13, i.e.

$$\left[K \right]\left\{ \phi \right\} - \left[K_t \right]\left\{ \dot{\phi} \right\} = \left\{ R \right\} \qquad (4.14)$$

The problem solution was completed when these equations were solved for the nodal parameters $\left\{ \phi \right\}$, subject to the discretized initial conditions.

B. DERIVATION OF ELEMENT MATRICES

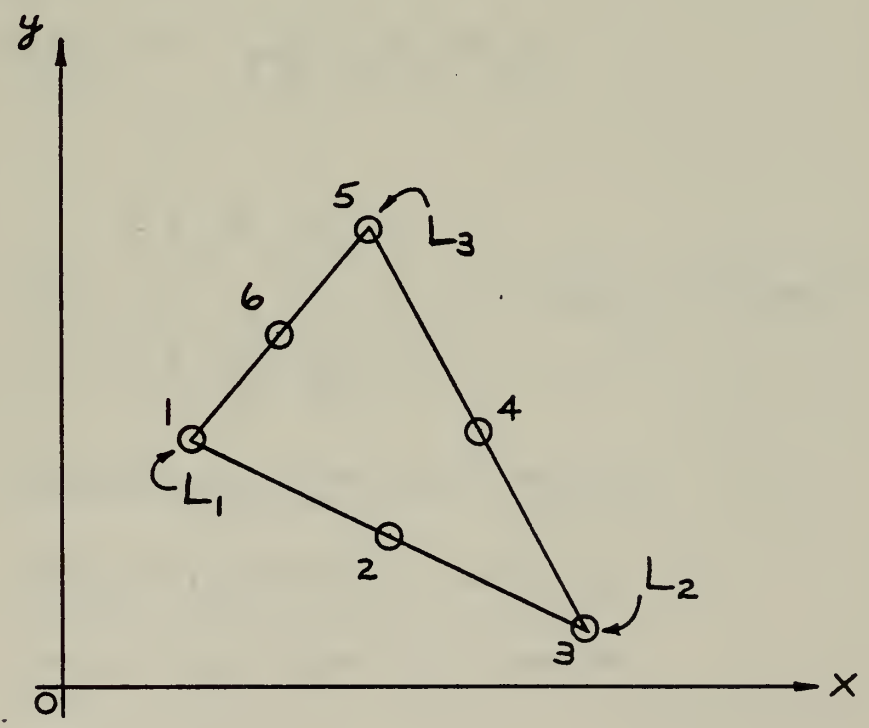
Derivation of various element matrices, referred to previously as simply area integrals over the solution domain, will be discussed in this section. The evaluation of each matrix will be in terms of natural coordinates, that is, weighting functions relating the coordinates of the end nodes to the coordinate of any interior point belonging to the element. The weighting functions are not independent of one another, since their sum must equal unity, i.e.

$$\sum_{i=1}^{n} L_i = 1 \qquad (4.15)$$

where n is the number of external nodes of the element. This expression can be interpreted to mean that one and only one coordinate is associated with node i, having a unit value there and a zero value at every other node. As was previously mentioned in other sections, a general triangular shaped element, such as sketched below, was employed. Then by equation 4.15

$$L_1+L_2+L_3 = 1$$

A cartesian coordinate system is used since the fluid flow is assumed to be two-dimensional. Similar results could be derived using cylindrical coordinates for an axisymmetrically shaped element. The original Cartesian coordinates of a



point in the element can now be linearly related to the new natural coordinates by the equations

$$x = L_1x_1+L_2x_2+L_3x_3 \tag{4.16}$$

and

$$y = L_1y_1+L_2y_2+L_3y_3 \tag{4.17}$$

Solving for the natural coordinates in terms of the Cartesian coordinates gives

$$L_1(x,y) = \frac{1}{2\Delta}(a_1+b_1x+c_1y) \tag{4.18a}$$

$$L_2(x,y) = \frac{1}{2\Delta}(a_2+b_2x+c_2y) \tag{4.18b}$$

and finally

$$L_3(x,y) = \frac{1}{2\Delta}(a_3+b_3x+c_3y) \tag{4.18c}$$

where

$$2\Delta = \begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix} = 2 \text{ (area of triangle 1-2-3)}$$

$a_1=x_2y_3-x_3y_2$, $b_1=y_2-y_3$, $c_1=x_3-x_2$

$a_2=x_3y_1-x_1y_3$, $b_2=y_3-y_1$, $c_2=x_1-x_3$

$a_3=x_1y_2-x_2y_1$, $b_3=y_1-y_2$, $c_3=x_2-x_1$

The interpolation functions N_i for the linear pressures in terms of natural coordinates are merely

$$N_1^P = L_1,\ N_2^P = L_2,\ N_3^P = L_3$$

but those interpolation functions that relate to the velocities and temperatures stemming from quadratic approximation possess the form

$$\begin{aligned} N_1 &= 2L_1^2 - L_1 \\ N_2 &= 4L_1L_2 \\ N_3 &= 2L_2^2 - L_2 \\ N_4 &= 4L_2L_3 \\ N_5 &= 2L_3^2 - L_3 \\ N_6 &= 4L_1L_3 \end{aligned} \qquad (4.19)$$

Another way of envisioning $L_i(x,y)$ for the triangular element is to consider it a ratio of areas. Figure 4 shows how the natural coordinates, often called area coordinates, are related to areas. In this figure, when the point (x_p,y_p) is located on the boundary of the element, one of the area segments vanishes and hence the appropriate area coordinate along that particular boundary is identically zero. For example, if (x_p,y_p) is on line 1-2, then

$$L_3 = \frac{A_3}{\Delta} = 0 \quad \text{since } A_3 = 0$$

There is also a convenient analytical method for integrating area coordinates over the area of a triangular element and involves the formula

$$\int_{A^{(e)}} L_1^{\alpha} L_2^{\beta} L_3^{\gamma} \, dA^{(e)} = \frac{\alpha!\,\beta!\,\gamma!}{(\alpha+\beta+\gamma+2)!} \, 2\Delta$$

A summation of values derived using this formula is presented in Table 4.1 for $(\alpha+\beta+\gamma) \leq 4$.

$$\frac{1}{\Delta} \int_{A^{(e)}} L_1^{\alpha} L_2^{\beta} L_3^{\gamma} \, dA^{(e)} = \frac{A}{B}$$

$\alpha+\beta+\gamma$	α	β	γ	A	B
0	0	0	0	1	1
1	1	0	0	1	3
2	2	0	0	2	12
2	1	1	0	1	12
3	3	0	0	6	60
3	2	1	0	2	60
3	1	1	1	1	60
4	4	0	0	12	180
4	3	1	0	3	180
4	2	2	0	2	180
4	2	1	1	1	180

Table 4.1

With this preliminary work finished, the actual derivation of the element matrices may now begin. In all, five matrices will be completely evaluated while one matrix, $[K_1]$, will have only two of its terms derived, due to the extensive amount of time and paper needed to evaluate $[K_1]$ in total. Beginning with this above-mentioned matrix as it appeared in Subsection A,

$$K_1(i,j) = \nu \int_{\Omega^{(e)}} \left(\frac{\partial Ni}{\partial x}\frac{\partial Ni}{\partial x} + \frac{\partial Ni}{\partial y}\frac{\partial Nj}{\partial y}\right) dxdy \qquad (4.20)$$

where $\Omega^{(e)}$ is the elemental area representing the solution domain, and i and j both vary from one to six. Since $[K_1]$ is an array multiplying the nodal variables of velocity and temperature, it must be correlated with the quadratic interpolation functions of equation 4.19. For the point (1,1), equation 4.20 becomes

$$K_1(1,1) = \nu \int_{\Omega^{(e)}} \left(\frac{\partial N1}{\partial x}\frac{\partial N1}{\partial x} + \frac{\partial N1}{\partial y}\frac{\partial N1}{\partial y}\right) dxdy \qquad (4.21)$$

where

$$\frac{\partial N1}{\partial x} = \frac{\partial L1}{\partial x}(4L_1-1) = \frac{b_1}{2\Delta}(4L_1-1)$$

and

$$\frac{\partial N1}{\partial y} = \frac{\partial L1}{\partial y}(4L_1-1) = \frac{C_1}{2\Delta}(4L_1-1)$$

substituting these values into equation 4.21

$$K_1(1,1) = \nu \int_{\Omega^{(e)}} \left[\frac{b_1^2}{4\Delta^2}(4L_1-1)^2 + \frac{c_1^2}{4\Delta^2}(4L_1-1)^2 \right] \, dxdy$$

$$= \frac{\nu(b_1^2+c_1^2)}{4\Delta^2} \int_{\Omega^{(e)}} (16L_1^2 - 8L_1 + 1) \, dxdy$$

employing Table 4.1 for these three cases above in which $(\alpha+\beta+\gamma) = 2$, 1, and 0 respectively; plus the relationship that $\int_{\Omega^{(e)}} dxdy = \Delta$

$$K_1(1,1) = \frac{\nu(b_1^2+c_1^2)}{4\Delta^2}\left(16 \cdot \frac{2}{12} - 8 \cdot \frac{1}{3} + 1\right)\Delta$$

or finally

$$\boxed{K_1(1,1) = \frac{\nu(b_1^2+c_1^2)}{4\Delta}}$$

Next, consider the point (2,4) where

$$K_1(2,4) = \nu \int_{\Omega^{(e)}} \left(\frac{\partial N_2}{\partial x}\frac{\partial N_4}{\partial x} + \frac{\partial N_2}{\partial y}\frac{\partial N_4}{\partial y} \right) dxdy \qquad (4.22)$$

where

$$\frac{\partial N_2}{\partial x} = 4\left[L_1\left(\frac{b_2}{2\Delta}\right) + L_2\left(\frac{b_1}{2\Delta}\right) \right],$$

$$\frac{\partial N_2}{\partial y} = 4\left[L_1\left(\frac{c_2}{2\Delta}\right) + L_2\left(\frac{c_1}{2\Delta}\right) \right],$$

$$\frac{\partial N_4}{\partial x} = 4\left[L_2\left(\frac{b_3}{2\Delta}\right) + L_3\left(\frac{b_2}{2\Delta}\right) \right],$$

$$\frac{\partial N_4}{\partial y} = 4\left[L_2\left(\frac{c_3}{2\Delta}\right) + L_3\left(\frac{c_2}{2\Delta}\right) \right]$$

substituting these four values into equation 4.22

$$K_1(2,4)=\nu\int_{\Omega^{(e)}}\Big\{16\Big[L_1L_2\Big(\frac{b_2}{2\Delta}\Big)\Big(\frac{b_3}{2\Delta}\Big)+L_2{}^2\Big(\frac{b_1}{2\Delta}\Big)\Big(\frac{b_3}{2\Delta}\Big)$$
$$+L_1L_3\Big(\frac{b_2}{2\Delta}\Big)^2+L_2L_3\Big(\frac{b_1}{2\Delta}\Big)\Big(\frac{b_2}{2\Delta}\Big)+L_1L_2\Big(\frac{c_2}{2\Delta}\Big)\Big(\frac{c_3}{2\Delta}\Big)$$
$$+L_2{}^2\Big(\frac{c_1}{2\Delta}\Big)\Big(\frac{c_3}{2\Delta}\Big)+L_1L_3\Big(\frac{c_2}{2\Delta}\Big)^2+L_2L_3\Big(\frac{c_1}{2\Delta}\Big)\Big(\frac{c_2}{2\Delta}\Big)\Big]\Big\}\,dxdy$$

simplifying again, through the use of Table 4.1

$$K_1(2,4)=\frac{\nu}{3\Delta}(b_2b_3+c_2c_3+2b_1b_3+2c_1c_3+b_1b_2+c_1c_2+b_2{}^2+c_2{}^2)$$

As can be seen, terms in the K_1 matrix can be quite lengthy and require considerable time to derive. On a more positive note though, this square matrix is symmetric and thus only half the terms need be calculated manually.

Next, attention will be focused on the K_2 and K_3 matrices. These two arrays may be considered together since the only difference between the two of them is that b values are associated with $\left[K_2\right]$ and c values with $\left[K_3\right]$. Otherwise, they are identical. These two matrices were given as

$$K_2(i,j) = \int_{\Omega^{(e)}}\Big(\frac{\partial N_j}{\partial x}N_i{}^P\Big)dxdy \qquad (4.23)$$

and

$$K_3(i,j) = \int_{\Omega^{(e)}}\Big(\frac{\partial N_j}{\partial y}N_i{}^P\Big)dxdy \qquad (4.24)$$

Taking the (3,6), equation 4.23 becomes

$$K_2(3,6) = \int_{\Omega^{(e)}} \left(\frac{\partial N_6}{\partial x} N_3^P\right) dxdy$$

where

$$\frac{\partial N_6}{\partial x} = 4\left[L_1\frac{\partial L_3}{\partial x} + L_3\frac{\partial L_1}{\partial x}\right] \text{ and } N_3^P = L_3, \text{ then}$$

substituting above

$$K_2(3,6) = 4\int_{\Omega^{(e)}} \left[L_1\left(\frac{b_3}{2\Delta}\right)+L_3\left(\frac{b_1}{2\Delta}\right)\right] L_3 dxdy$$

$$= \frac{2}{\Delta}\int_{\Omega^{(e)}} (L_1L_3b_3+L_3^2b_1)\,dxdy$$

once again, using Table 4.1

$$\boxed{K_2(3,6) = \frac{1}{6}(2b_1+b_3)}$$

and consequently

$$\boxed{K_3(3,6) = \frac{1}{6}(2c_1+c_3)}$$

Following the same procedure throughout each of these 3x6 matrices, complete $\left[K_2\right]$ and $\left[K_3\right]$ are

$$\left[K_2\right] = \frac{1}{6}\begin{bmatrix} b_1 & b_1+2b_2 & 0 & b_2+b_3 & 0 & b_1+2b_3 \\ 0 & 2b_1+b_2 & b_2 & b_2+2b_3 & 0 & b_3+b_1 \\ 0 & b_1+b_2 & 0 & 2b_2+b_3 & b_3 & 2b_1+b_3 \end{bmatrix}$$

and also

$$\left[K_3\right] = \frac{1}{6}\begin{bmatrix} c_1 & c_1+2c_2 & 0 & c_2+c_3 & 0 & c_1+2c_3 \\ 0 & 2c_1+c_2 & c_2 & c_2+2c_3 & 0 & c_3+c_1 \\ 0 & c_1+c_2 & 0 & 2c_2+c_3 & c_3 & 2c_1+c_3 \end{bmatrix}$$

The next two matrices to be derived, that is $\left[K_2\right]^T$ and $\left[K_3\right]^T$, can simply be written down by inspection of the two above arrays. Thus

$$\left[K_2\right]^T = \frac{1}{6}\begin{bmatrix} b_1 & 0 & 0 \\ b_1+2b_2 & 2b_1+b_2 & b_1+b_2 \\ 0 & b_2 & 0 \\ b_2+b_3 & b_2+2b_3 & 2b_2+b_3 \\ 0 & 0 & b_3 \\ b_1+2b_3 & b_3+b_1 & 2b_1+b_3 \end{bmatrix}$$

while its counterpart is then

$$\left[K_3\right]^T = \frac{1}{6}\begin{bmatrix} c_1 & 0 & 0 \\ c_1+2c_2 & 2c_1+c_2 & c_1+c_2 \\ 0 & c_2 & 0 \\ c_2+c_3 & c_2+2c_3 & 2c_2+c_3 \\ 0 & 0 & c_3 \\ c_1+2c_3 & c_3+c_1 & 2c_1+c_3 \end{bmatrix}$$

Finally, the last elemental matrix to be analyzed is the one associated with the time-dependent nodal parameters. In subsection A this matrix was given as $\left[CD\right]$. For convenience here, let $\left[CD\right] = \left[K_t\right]$, then

$$K_t(i,j) = \int_{\Omega^{(e)}} N_i N_j \, dxdy \qquad (4.25)$$

with both i and j running from one to six. Consider, for example, point (1,5)

$$K_t(1,5) = \int_{\Omega^{(e)}} N_1 N_5 \, dxdy$$

substituting from equation 4.19

$$K_t(1,5) = \int_{\Omega^{(e)}} (2L_1^2 - L_1)(2L_3^2 - L_3)\, dxdy$$

$$= \int_{\Omega^{(e)}} (4L_1^2 L_3^2 - 2L_1 L_3^2 - 2L_1^2 L_3 + L_1 L_3)\, dxdy$$

then from Table 4.1, using $(\alpha+\beta+\gamma)$ four separate times, this term reduces rather easily to

$$\boxed{K_t(1,5) = -\frac{\Delta}{180}}$$

Factoring out a constant of $\frac{\Delta}{180}$, the total K_t matrix takes on the form

$$[K_t] = [CD] = \frac{\Delta}{180}\begin{bmatrix} 6 & 0 & -1 & -4 & -1 & 0 \\ 0 & 32 & 0 & 16 & -4 & 16 \\ -1 & 0 & 6 & 0 & -1 & -4 \\ -4 & 16 & 0 & 32 & 0 & 16 \\ -1 & -4 & -1 & 0 & 6 & 0 \\ 0 & 16 & -4 & 16 & 0 & 32 \end{bmatrix}$$

Which is also a symmetric matrix, thereby allowing faster derivation of the individual terms with less chance of numerical error.

C. STRUCTURE OF COMPUTER PROGRAMS FOR FLOW ANALYSIS

A total of three computer programs analyzing two distinct test cases of fluid flow problems were employed in this thesis. The first was a steady state analysis of Couette flow. This involved determining the solution of the velocity profiles (linear and nonlinear) in a shear- and pressure-induced flow between flat parallel plates.

The upper plate slides in the positive x-direction with a constant velocity u, while the lower plate remains stationary. There is no velocity component normal to the plates, that is, v=0 in the y-direction. The governing equations for this particular fluid flow are

Continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{4.26}$$

Momentum:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\nabla^2 u \tag{4.27}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial y} + \nu\nabla^2 v \tag{4.28}$$

For determination of the velocity profile involving only linear terms, the left hand side of equations 4.27 and 4.28 are set equal to zero (no inertia terms).

A physical representation of the Couette flow analyzed is shown in Figure 5. Node and element numbering is the same as in Figure 2. A pressure gradient of -3 units is directed along the x-axis, i.e., $\frac{dP}{dx} = -3$.

The two remaining programs formed the majority of the theoretical portion of this study. They were devised to carry out the calculations for the analysis of two-dimensional or axisymmetric natural convection heat transfer problems.

The first is the lesser-complex steady state approach, whereby all transient conditions characteristic of the system are assumed to have died out, leaving only those steady state parameters remaining to be solved for. Once the finite element matrix equations describing the system are correctly assembled, a library subroutine (LEQT2F) functioning as a linear equation solver is called and the desired nodal parameters can be calculated. The second program takes into account the previously-neglected time dependence of the system by introducing a type of finite difference integration scheme to solve the transient portion of the governing equations. This integration technique must be an iterative procedure in order to circumvent the problem of nonlinearity similar to that resulting from the addition of inertia terms. Furthermore, the integral is solved at successive time steps, with time being increased until the value of the field variable converges, within tolerance, to that of its steady state counterpart. This segment of the total equation is then algebraically combined with the remaining steady state solution to yield values of nodal field variables with improved accuracy. Gravity acting in the longitudinal direction was taken into account for both the two-dimensional and axisymmetric flows.

The flow region to be studied is first defined, followed by the setting up of coordinate axes (Figure 1). The location of the origin of these axes is in most cases arbitrary, except that for a problem involving axial symmetry the x-axis must coincide with the system axis of symmetry. The flow region is then divided into a mesh of triangular elements, and the nodal points are numbered in the sequence previously described. Once the setting-up of the solution domain is completed, computer analysis of the system with its included boundary conditions can be initiated.

Structure of the steady state fluid mechanics problem will be discussed in detail, primarily because it comprises one entire program and with the addition of the transient stiffness matrix elements, accounts for the main program in the time-dependent study.

The program was coded in FORTRAN IV language and begins with a series of DIMENSION statements, which set up the arrays needed in the calculations. As indicated in statements 0260-0310, storage has been allocated for problems with up to 117 nodes; however larger problems can be considered by simply increasing the dimensions of these matrices. The limit of problem size is dictated by the

core storage of the available machine. Next, the program calls for a declaration of the type of program to be solved (either two-dimensional or axisymmetric); then the appropriate problem label is printed (statements 0360-0410). Before proceeding to input the data describing the finite element mesh, all the matrix arrays must be initialized by setting all terms in these arrays equal to zero.

Statements 0930-1150 read into the program the node numbers and the coordinates of the nodes for the complete finite element mesh. Also, the system topology, the element numbers, and the numbers of the six nodes associated with each element are read. Beginning with statement 1300, the velocity, pressure and temperature conditions within the solution domain are inserted. Correspondingly, conditions are specified for the QX, QY, QZC and QZ indices where the nodal field variables are unknown. Since the solution obtained by the program depends intimately on the body of data, the program is queried to print out all data that have been input. This enables the programmer to check for input data errors. Statement 2970 marks the completion of these steps; the program is now ready to commence work on a particular fluid mechanics problem.

The loop to begin calculating the various element matrices starts with statement 3010. Once the element matrices [TM$] are computed for one element, they are assembled into the master or system stiffness matrix [TM] by the code followed in statements 7740-7790. The non-linear terms appearing in the velocity and temperature expressions of the governing equations are formulated in statements 4110-5900. An iterative process compares each of these terms with a corresponding quantity in the linear symmetric [TM$] matrix until they converge in value. It is this comparison value that is then assembled with all similar values of the other elements to form the system [TM] matrix, which now exhibits or reflects the non-linearity.

Since each element in the triangular mesh has six (6) nodes, the local node numbers are I$ = 1, 2, ...,6. The global node numbers for the element are recovered from the parameter NODE (K,I$), which was read as input data for the element; that is, for element K, the node numbers N(1)=NODE(K,1), N(2)=NODE(K,2), etc. were introduced. Then the code in statements 7740-7790 loads the terms of the elemental matrices into their proper locations in the system matrices. Each time that a term of an element

matrix is placed in a location in the system matrix where another term has already been inserted, this new term is added to whatever value is there. A similar loading process takes place for the right-hand-side column vector in statements 7810-7940.

After all the elements have been processed in this fashion, the assembled system equations are ready to be modified to account for the boundary conditions or phenomena. This is done by statements 7980-8060. Thus, at the conclusion of statement 8060, the system equations possess the form

$$\left[\text{TM}\right]\left\{\text{X}\right\} = \left\{\text{RHS}\right\}$$

where $\left\{\text{RHS}\right\} = \left\{\text{Kq}\right\} = \begin{Bmatrix} \text{QX+gBTm} \\ \text{QY} \\ \text{QZC} \\ \text{QZ} \end{Bmatrix}$, and $\left\{\text{x}\right\} = \begin{Bmatrix} \text{u} \\ \text{v} \\ \text{P} \\ \text{T} \end{Bmatrix}$

Not all of the components of the column vector $\left\{\text{RHS}\right\}$ are known because the Q values at nodes where velocity, pressure or temperature is specified are unknown; that is, at each node i, either u_i, v_i, or T_i is known on the one side, or Kq_i is known on the other. A similar relationship exists at the corner nodes for pressure and QZC values. The only Q's that can be specifically labeled as

heat fluxes are the QZ's, since they directly relate to temperature parameters within the system.

The only thing that remains to be done now is to call a compatible linear equation solver to produce the nodal variables sought. In this case, LEQT2F was chosen because of its speed and accuracy.

The following is a list of the symbols and descriptions utilized in coding the above program:

Symbol	Description
NCASE	interger which specifies the type of problem to be solved: NCASE=1, 2-D plane problem NCASE=2, axisymmetric problem
NN	number of nodes in solution domain
NNCN	number of corner nodes
NE	number of elements
XC(I),YC(I)	global coordinates of node I
NODE(J,I)	J=1,2,....NE; I=1,2,....6 node numbers associated with element J
NVS(I)	node number where velocity or temperature is specified
NPS(I)	node number where pressure is specified
VELU	specified nodal u velocity
VELV	specified nodal v velocity
PNP	specified nodal pressure

Symbol	Description
TNT	specified nodal temperature
NQS(I)	node number where a Q value is specified; QX and QY are specified only at internal nodes, while QZC and QZ may be specified at either external or internal nodes
QXNS	specified nodal value of QX
QYNS	specified nodal value of QY
QZCNS	specified nodal value of QZC
QZNS	specified nodal value of heat flux QZ
XC$(I),YC$(I)	local coordinates of node I
TM$	element stiffness matrix
TM	system stiffness matrix
DEL	area of a triangular element

The program output begins with a statement declaring the type of problem to be solved - either nonlinear two-dimensional or axisymmetrical. Next, all input data are printed and labeled for easy identification. To ensure the validity of the solution, the printed input data should be carefully checked against the intended input. A statement following these printed data identifies which nodal parameters are associated with which system nodes (remembering specifications of the finite element analysis called for a value of both velocities along with a temperature

at each node, while a pressure value could be defined only at the corner nodes). The complete continuum solution follows in the form of a numbered list, in which the integer appearing at the far left of this list designates the node number, or multiple of it in cases above I=35, and the figure on the right representing the value of the nodal variable in double precision.

D. NUMERICAL RESULTS

Complete numerical listings of the field variables for both the Couette flow problem and the steady state heat transfer problem are shown in the two computer program outputs.

The velocity profile for the linear Couette flow, i.e. node numbers 1-5, 6-10, etc., revealed that the finite-element method of analysis agreed with the exact solution of this shear-type flow out to the sixth decimal place. This is evident by the fact that all five of the FEM points lie exactly on the smooth curve depicting the exact solution in Figure 15.

The approximate steady state isotherms of Figure 16 are directly related to the nonlinear temperatures (node numbers 83-117 of the second set of nodal variables) in

the heat transfer problem. These isotherms, or constant nondimensional temperature lines, vary in value from +1.0 on the hot temperature wall, to -1.0 on the cold temperature wall. The equation used for deriving these values at all thirty-five nodal points within the solution domain was

$$\theta = \frac{T-T_M}{T_H-T_M} \tag{4.29}$$

where T is the nodal temperature and T_M is the mean temperature of the fluid defined at $T_M = \frac{(T_H+T_C)}{2}$.

The general shape and relative location of the various isotherms within the rectangular enclosure are somewhat similar to those of comparable heat transfer flows involving different Prandtl numbers, Grashof numbers and L/D ratios. However, due to the relatively low Grashof number of the present system (946.4), there is a total lack of a plateau in the center region of Figure 16 and the closely packed boundary layer flow near the walls is also missing. This boundary layer type flow is characteristic of much higher Grashof numbers such as those found in the comparative examples in [3], [7] and [15] where the Gr_L ranged from 5000 up to 18000.

Based solely on the thirty-five nodal point temperatures available from the solution, the isotherms were sketched as

shown in Figure 16. Lacking additional information, the shape of the contour lines between such nodes were linearly approximated, to a large extent, without speculating as to their exact curvature.

The actual height and width of the enclosure was normalized to y* and x*, respectively, for easier interpretation of the figure.

V. EXPERIMENTAL PROCEDURE

A. ARRANGEMENT OF TEST APPARATUS

The experimental apparatus was arranged so as to allow the study of an essentially two-dimensional fluid flow.

The major components of the apparatus consisted of the test platform, which housed the rectangular enclosure (Figure 6), a control system made up of two water circulators that maintained the vertical copper walls of the test platform at desired temperatures (Figure 7), and a large (250 mm DIA) plano-convex glass lens for reducing the object (8.5 x 1.875 inch vertical rectangular cavity) down to a smaller image size that could be completely captured on the 4x5 inch holographic plate (Figure 11).

The rectangular enclosure holding the fluid under investigation was 8.5 inches high, 7 inches long, and 1.875 inches wide. This test cavity was sandwiched between two plexiglas water reservoirs providing constant circulation by means of manifold connections on their tops and bottoms. Hot and cold water drains were located on top of the left and right reservoirs, respectively. Similarly, on the bottom were the hot and cold water inputs. The

inner walls of these two reservoirs were formed by quarter inch thick oxygen-free copper plates. Also, these same copper plates comprised the principle walls of the rectangular enclosure, with the "side walls" being made of plate glass, in order to allow visual observations. Six thermocouples were attached to each copper wall and then connected to a multichannel recorder for temperature monitoring purposes. The time needed for each copper plate to reach its respective equilibrium temperature once the water circulators had been turned on was approximately 39.8 seconds. For comparison purposes, it took the system just under one hour (58.1 minutes) for the 50-HB-3520 lubricating fluid to attain a steady equilibrium temperature under the same experimental conditions.

An important constraint imposed by interferometry is that the total distance traveled by the object beam must be nearly identical with the total path length of the reference beam, if the index of refraction throughout is uniform. Since the fluid in the rectangular enclosure possessed a refraction index of 1.461 and laser light along the object beam had to travel through 7 inches of this fluid, then the corrected path length through the test cavity was 10.23 inches, or a net increase of over 3 inches.

With this in mind, the equipment was arranged in a semi-elliptical pattern on a heavy table supported at six critical areas by inflated inner tubes. These were to act as stabilizing devices. Equipment could not be arranged on an exact ellipse due to the fact that a distance of four feet, five inches alone was needed from the object to the plano-convex lens out of a total table length of eight feet. Even so, the turning mirrors were located on the apexes of the "shortened" minor axis, the beam splitter at one end of the major axis, and the aqueous hologram holder at the opposite end (Figure 8). Spatial filters were employed to clean up each beam and expand it. A diverging lens was inserted just after the object beam spatial filter in order to expand this beam to proper size before it reached the rectangular test slit. Also, a collimating lens was placed between the spatial filter and hologram holder along the reference beam. Finally, a large diffusing screen made from a piece of developed film mounted on plexi-glas and secured in a rigid metal frame, plus the test platform itself, were placed between the object mirror and the hologram holder (Figures 9 and 10).

Choosing the correct hologram holder is very important in live fringe holography. The reconstructed virtual image

must exactly match the original object. Unfortunately, after the processing of a hologram, the emulsion on its surface tends to dry, thus causing an unwanted displacement of the virtual image. One procedure that may be used to circumvent this problem is to choose a holder that maintains the hologram in aqueous surroundings, such as was utilized in this experiment. Also, in order to keep the hologram perfectly rigid during and after processing, the exposed glass plate was secured in a removable metal frame complete with handle. In this way, exact replacement in the hologram holder after processing posed no problem. Two micrometers built into the holder's top and left side were then used for fine adjustment of the hologram.

The test cavity or rectangular enclosure was filled with a very highly viscous fluid (actually a lubricant) produced by Union Carbide and known as "UCON" 50-HB-3520. This fluid was required to possess physical properties such that Rayleigh numbers, based on cavity width, of the order of 10^4-10^5 could be obtained in the apparatus, at accurately measurable temperature differences. Since (T_H-T_C) was held constant throughout the experiment, only one Rayleigh number was calculated. Its value of 1.0755×10^4 was well within the above tolerance zone. Worth mentioning

is the generally accepted prediction that above $Ra \simeq 2.0 \times 10^4$, the phenomena known as "secondary flows" begin to occur. Obviously, such was not the case in this experiment.

A scribed grid pattern was attached to the back side of the test cavity to assist in alignment of the fringes. Two water circulators were connected by tubes to manifold nipples on the reservoir ends of the test platform. One circulator was set to deliver distilled water at 20°C (cold temp.) and the other at 25°C (hot temp). By using slide valves, the amount of water expended from the circulators could be regulated and controlled.

The experimental procedure was initiated only after the entire system had been carefully aligned. The rectangular enclosure was allowed to sit undisturbed for a period of at least several hours to ensure an equilibrium temperature state throughout the fluid. Then, a shutter was placed directly in front of the beam of a 3 milliwatt, helium-neon continuous wave laser serving as the coherent light source. After an Agfa-Gevaert Inc. 10E75 holographic recording plate was placed in the holder, the shutter was opened and the plate was exposed for one and one-half seconds (Figure 13). This plate was then removed, developed, and replaced in its exact position. Both circulators were

then turned on, and a continuous flow of water at 20°C and 25°C was allowed to cycle through the reservoirs on the test platform. Once fringe lines appear, their visibility can be strengthened by following the procedure outlined in the next subsection.

If one word could be used to describe the single most important factor determining the success or failure of this experiment, it would have to be rigidity. All relatively light-weight gear, such as; the laser, turning mirrors, spatial filters, and the plano-convex lens had to be weighted down to make them immovable. The test platform, in which the rectangular cavity was located, was sufficiently heavy on its own to preclude it from having to be additionally weighted down. The hologram holder already came with a very heavy base attached. All connecting devices, including the tubes transporting heated water to the plexiglas reservoirs and plastic sleeves housing the thermocouple leads were securely taped together to prevent vibration or motion. Any such random vibration would cause the fringe patterns to become lost.

The viewing of these fringes and the subsequent collecting of data can be accomplished by positioning the appropriate camera in a direct line with the rectangular

enclosure, plano-convex lens, and hologram holder (Figure 11). A television monitor (Figure 12) was employed for convenient viewing of the fringe patterns in an area adjacent to the experimental set-up.

B. HOLOGRAPHIC INTERFEROMETRY APPLICATIONS

Holographic interferometry is an excellent technique for developing interference fringe patterns, which may in turn be evaluated to quantitatively provide an accurate temperature field throughout the domain of the system.

Heat convection in a rectangular cavity would be difficult to analyze empirically. However, by replacing the sensors that would ordinarily be used to record temperature changes and flow rates with holographic technology, one can analyze directly the variation of the density fields within the rectangular enclosure. This technique also eliminates the inherent change in temperature and flow pattern caused by the physical insertion of the sensors into the test fluid.

Real-time holographic interferometry allows a continuous flow of information to be recorded at the precise time any changes in the observed fluid occur. Single exposure holograms are utilized with real-time interferometry. A time

sequence can be derived for each different viewing position, with the use of a single developed hologram.

Such an exposure technique consists of recording phase and amplitude information from an object, in this case the rectangular fluid enclosure, onto a holographic plate. The recording is accomplished through the use of a reference and an object (scene) beam, originating from a single source (Figure 13). After processing, the hologram is accurately repositioned in its holder. Illuminating the plate with the original reference beam results in the primary (virtual) image being projected onto the same area as was the object (Figure 14). By focusing the object and virtual image beams onto a film or focal plane, and then adjusting the system so that the two interfering wavefronts (object wave and reconstructed wave) coincide, fringes can be produced.

The hologram now can be finely adjusted to orient the fringes in either a vertical or horizontal reference frame. Likewise at this time, the fringe patterns can be made to appear more visible by varying the beamsplitter to increase the intensity of the reference beam while decreasing that of the scene beam. If the original object is changed or altered in any fashion by the effect of temperature, motion, or pressure, an exact superposition will create a reinforcement

or cancellation of the intensities of the two waves with the result being the establishment of a fringe pattern. A dark fringe is produced whenever the difference between the object and reconstructed wavefront involves an odd factor of $\pi/2$. Bright fringes occur when this difference equals an integer value of 2π.

By inserting a camera in-line with the scene (object) beam, but on the back side of the holographic plate, one can observe and record live fringe data. This technique provides a real time analysis of an unsteady system without the need for expensive and time consuming sensors and calibration.

After processing has been completed, problems arising from live fringe single exposure holography include; displacement of the virtual image due to drying emulsions on the holographic plate, and non-exact replacement of the hologram in its holder. If any relative motion whatsoever has transpired between pieces of the experimental equipment during or after replacement of the hologram, the fringe patterns may be destroyed.

It was this last problem that caused a particularly detrimental effect on the experimental results of this thesis. Somewhere within the system (apparatus arrangement)

there existed a source of motion or a piece of gear slightly off the horizontal reference plane that completely eliminated these fringe formations almost immediately after they evolved. The exact source(s) was never totally isolated, but the possible choices were reduced down to two, the water circulators and/or the support stand of the hologram holder.

VI. CONCLUSIONS

The finite-element method was incorporated into steady state and time dependent computer programs for analyzing laminar convective heat transfer between parallel plates. Two sample cases were tested utilizing the general steady state program. In each case, values of derived field variables compared favorably to either an exact solution, in the case of the Couette flow problem (Figure 15); or to similar theoretical results, in the case of the heat transfer problem (Figure 16). The exact solution of the velocity profile for Couette flow was obtained by programming the analytical expression given by equation 5.5 in [11].

After successful steady state results were achieved, a second computer program was then designed to take into account the previously-neglected transient behavior of the system. A major portion of the steady state fluid mechanics problem was interfaced with a series of subroutines, wherein the time-dependent terms were to be calculated, to yield a total solution to the governing system of equations and associated boundary conditions. Time itself became a limiting factor in the completion of this second program.

A problem associated with the convergence of the field variables remains to be resolved.

In the experimental phase of this thesis, five attempts were made to produce live fringe formations through the use of holographic interferometry. In only one of these attempts was there observed a momentary interference pattern, corresponding to the temperature gradient, across the test section. This observation lasted approximately three (3) seconds after the water heaters/circulators were activated.

The main factor(s) influencing this inability to acquire such live fringes, on film, was the necessary exclusion of the hologram holder from the recording plane because of limited table length and/or the vibrations generated by the water circulators used in the experiment. Either of these detrimental conditions could have eliminated completely the formation of interference fringe patterns.

Due to considerable time delay in the acquisition of some of the experimental apparatus, no further documentation of real time holographic interferometry study could be made beyond the previously-mentioned five attempts.

APPENDIX A

FIGURES

Figure 1
Rectangular Enclosure

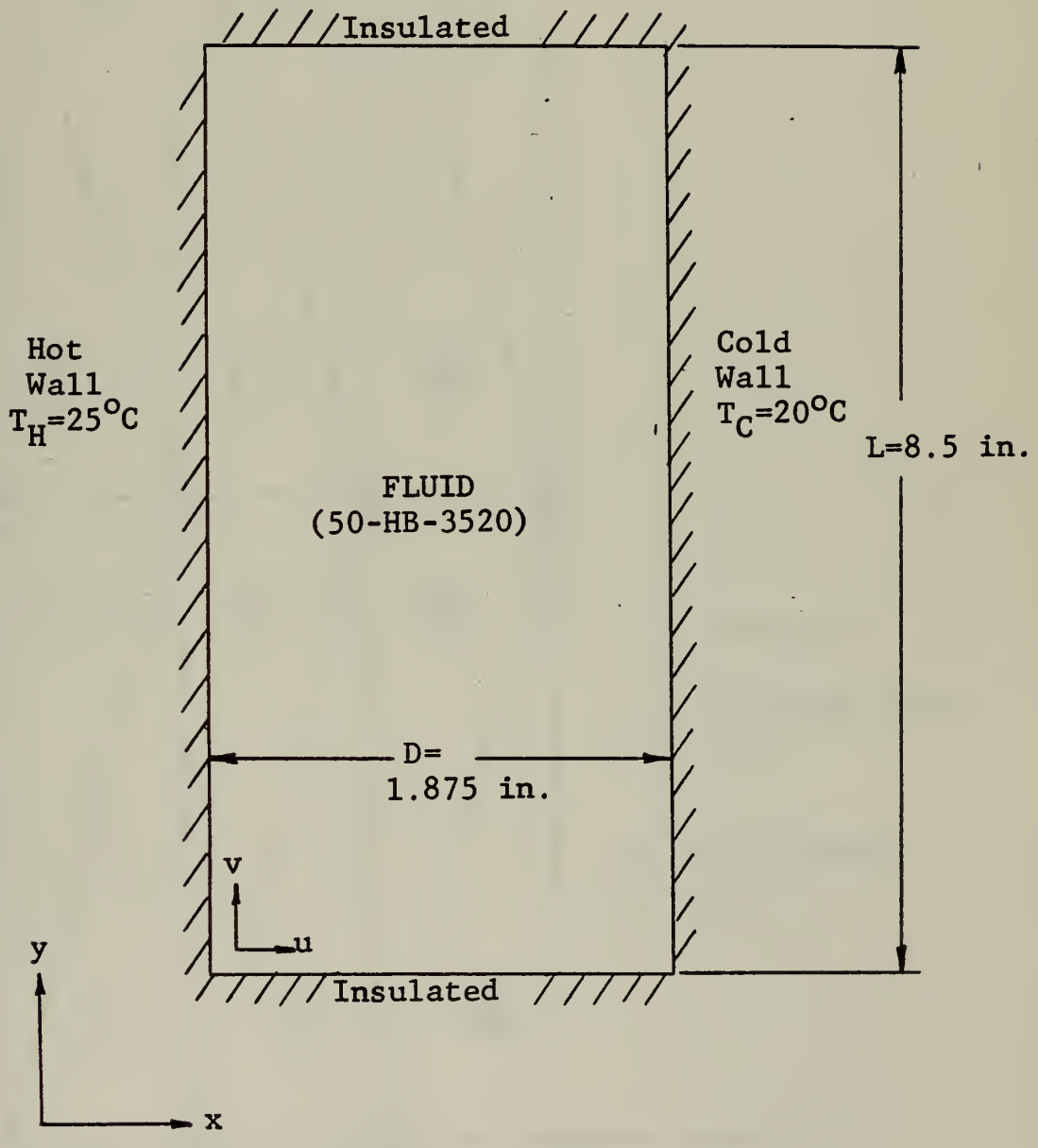


Figure 2
Discretization of Solution Domain

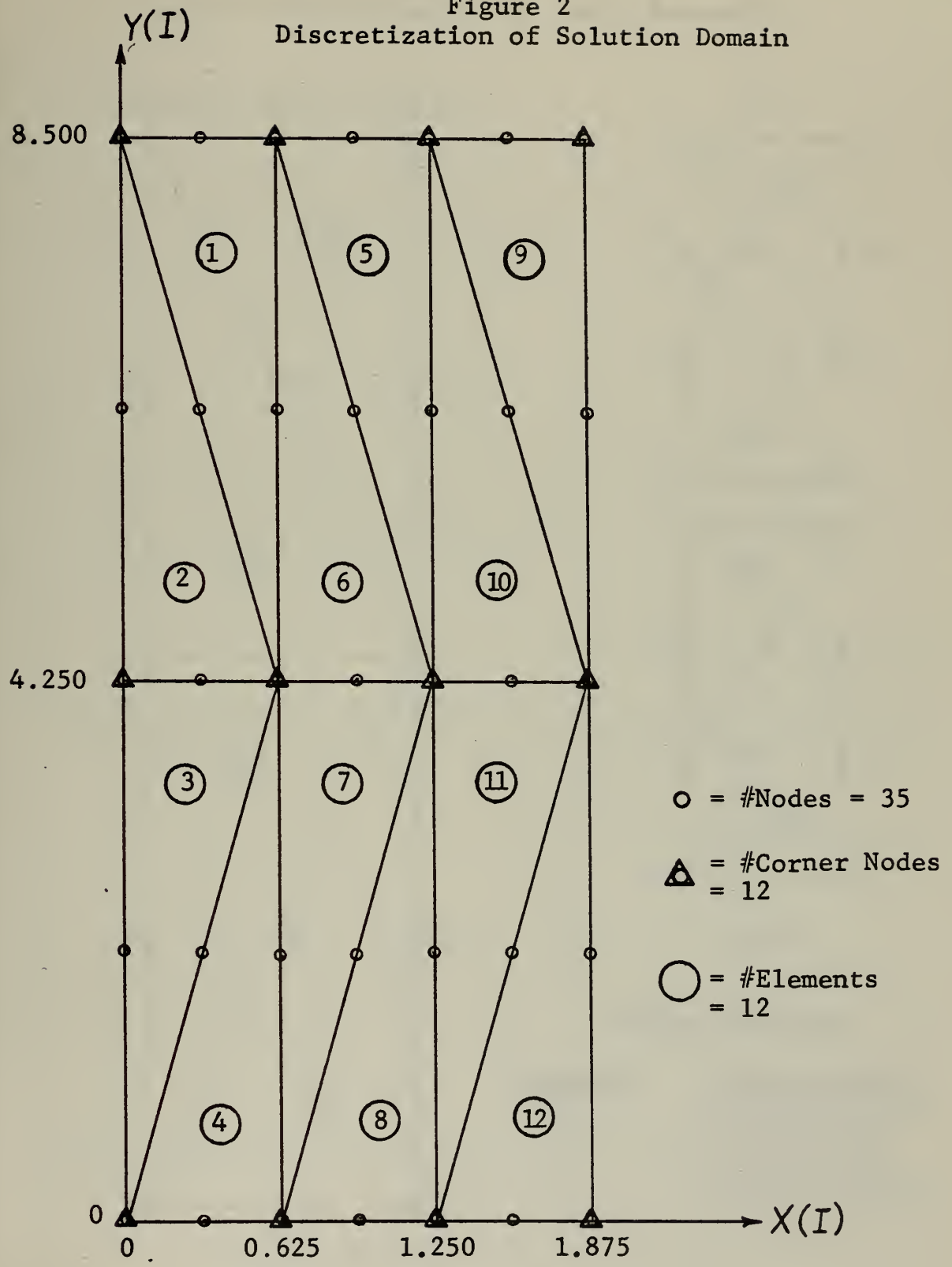
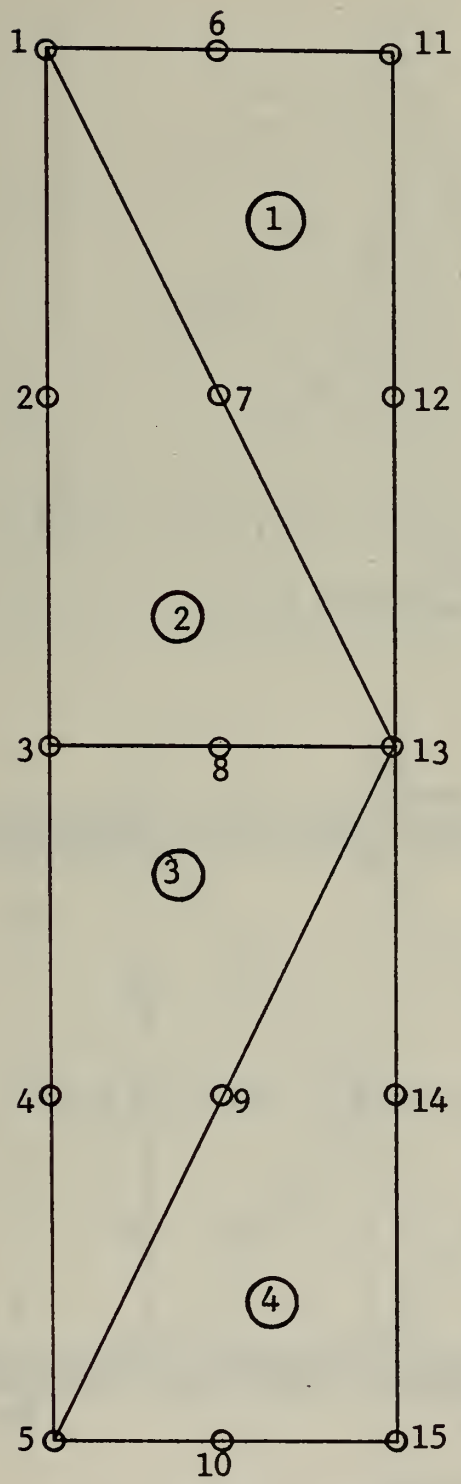
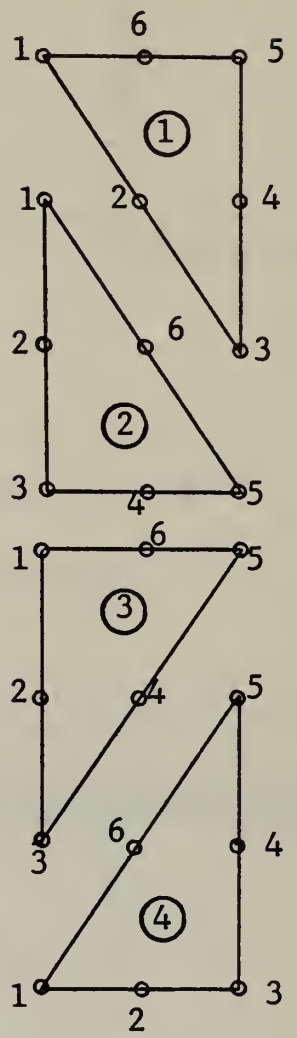


Figure 3
Global Node-Numbering vs. Local Numbering



Global



Local

System Topology

Element#	Global Node#'s
1	1,7,13,12,11,6
2	1,2,3,8,13,7
3	3,4,5,9,13,8
4	5,10,15,14,13,9

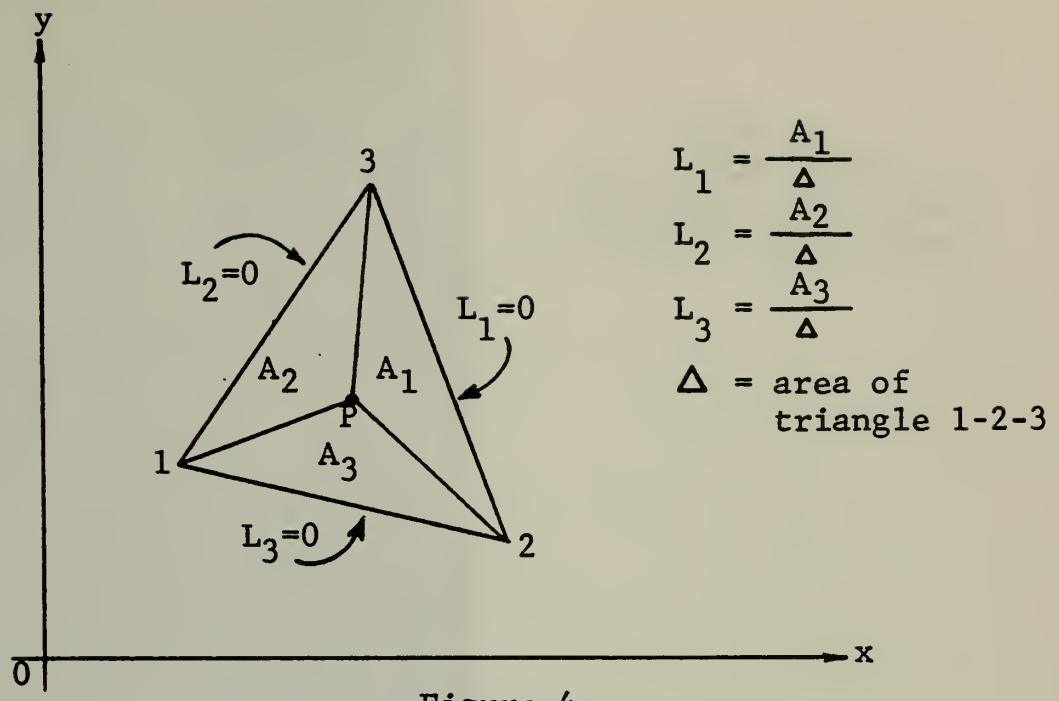


Figure 4
Area Coordinates for a Triangle

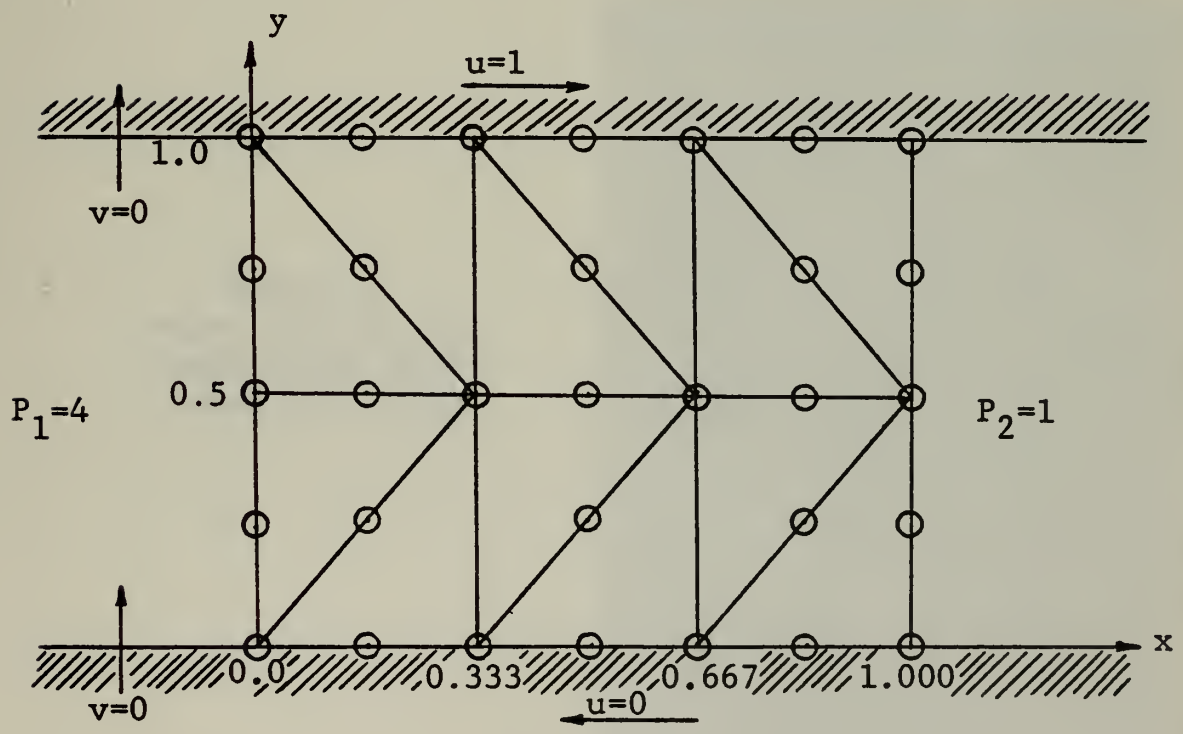


Figure 5
F.E.M. Analysis of Couette Flow

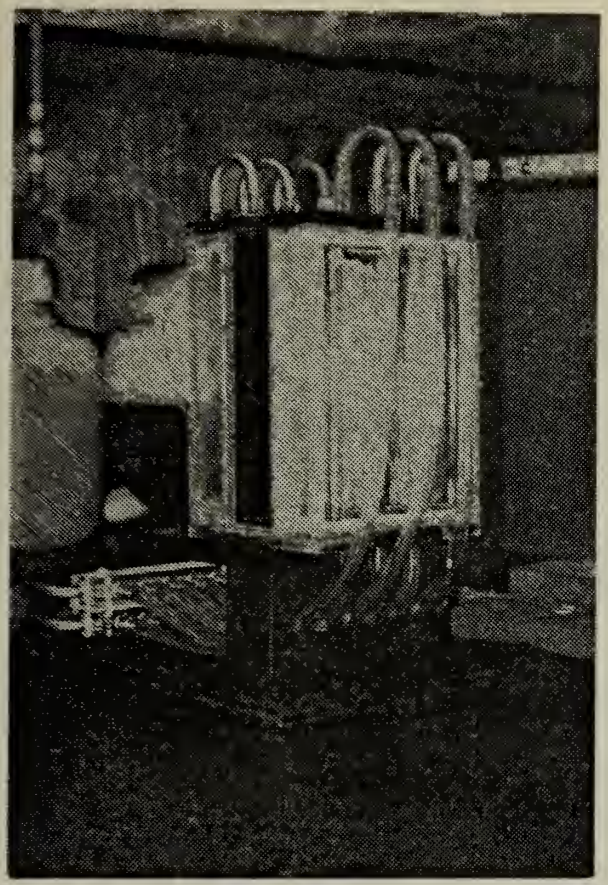


Figure 6
Test Platform with
Rectangular Enclosure
and Water Reservoirs

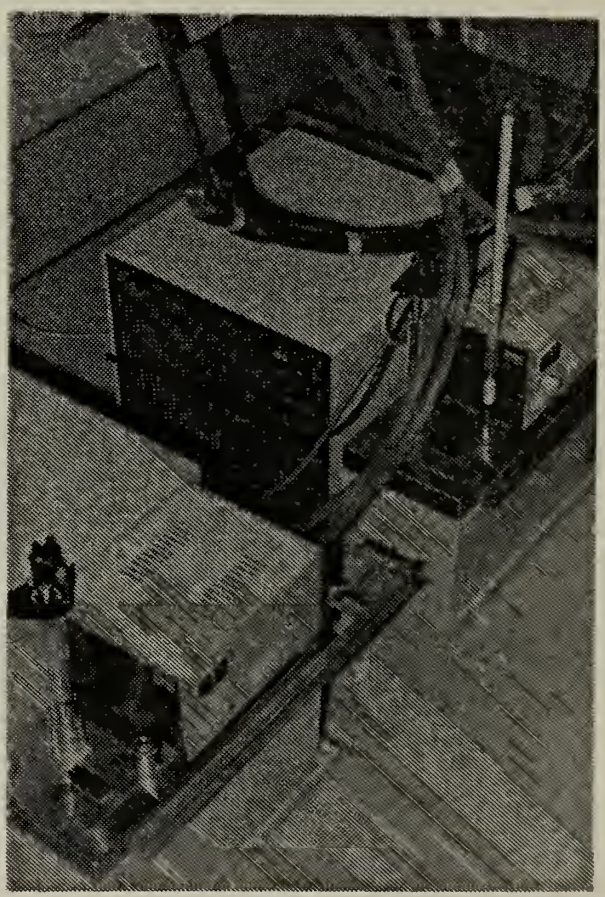
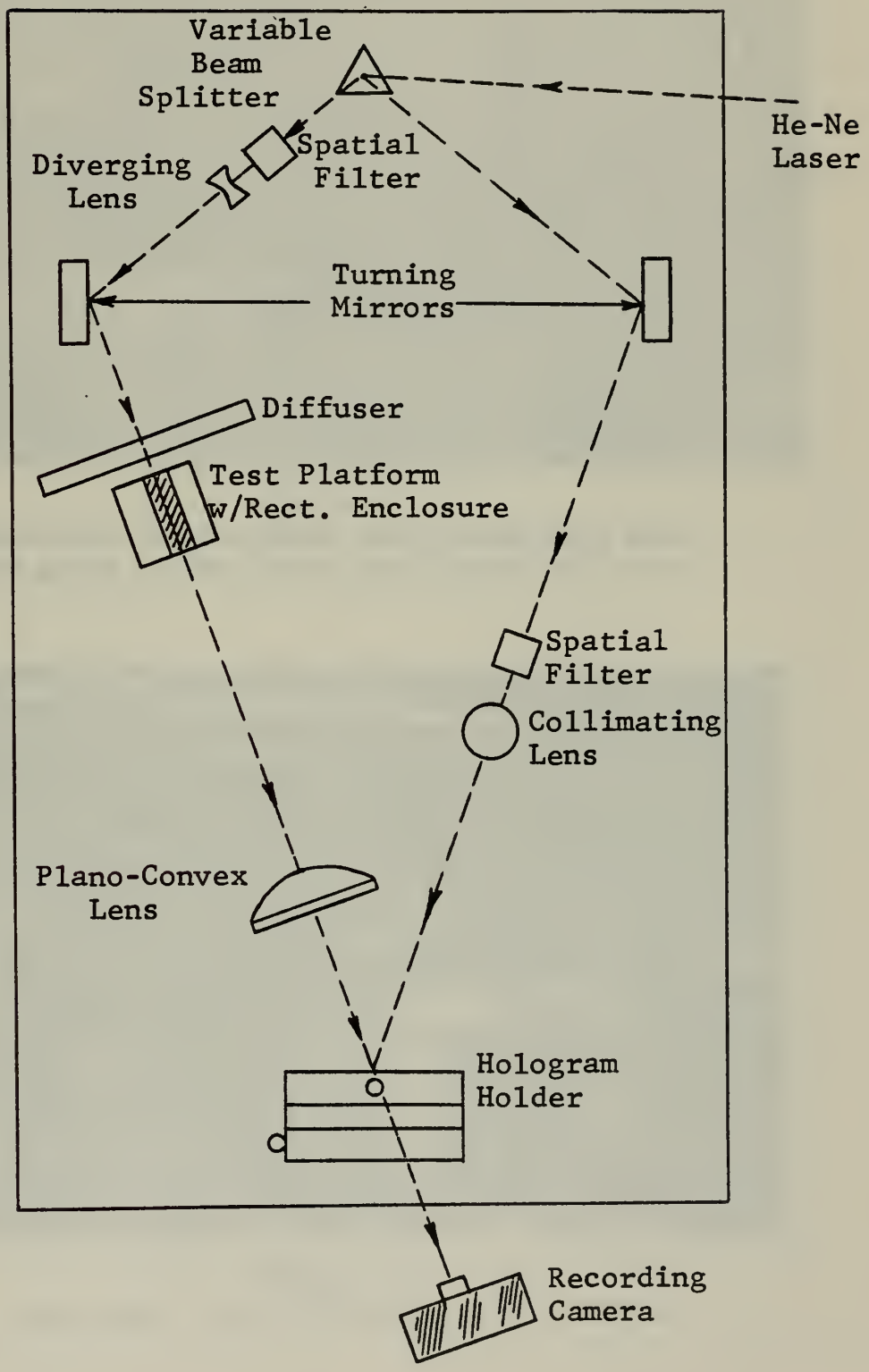


Figure 7
Water Heaters and
Circulators with
Connecting Tubes

Figure 8
Table Arrangement (top view)



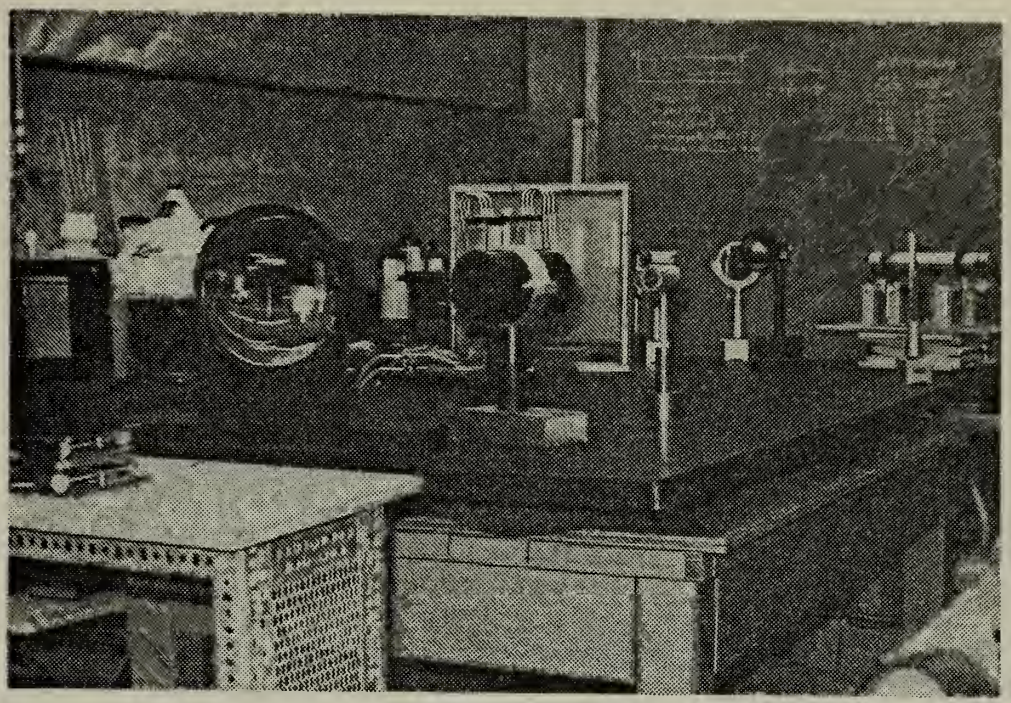


Figure 9
Apparatus Arrangement with Reference Beam
Oriented on the Right-hand-side of Table

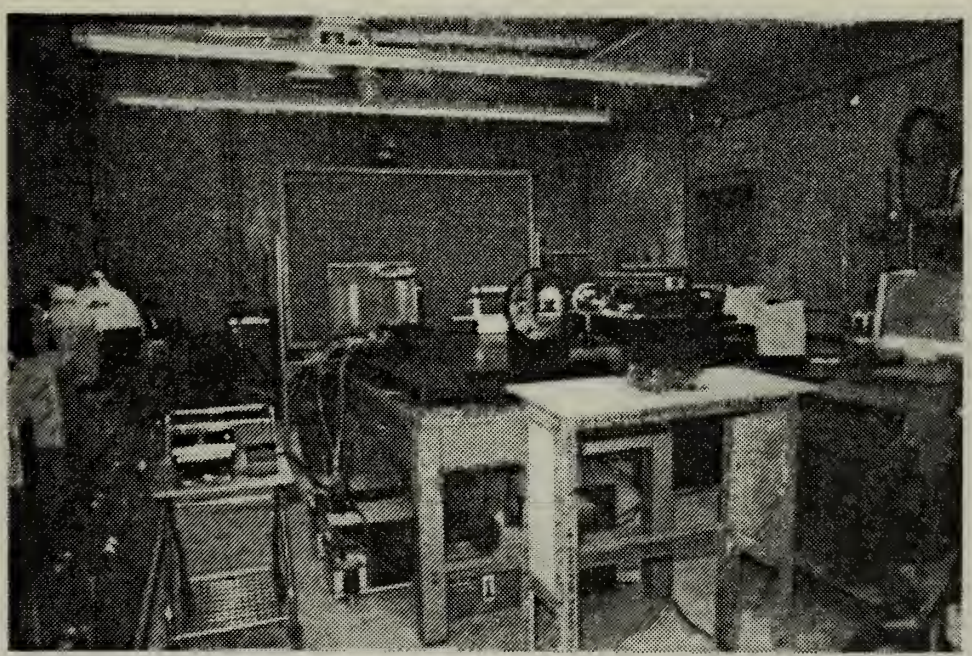


Figure 10
Panoramic View of Experimental Layout

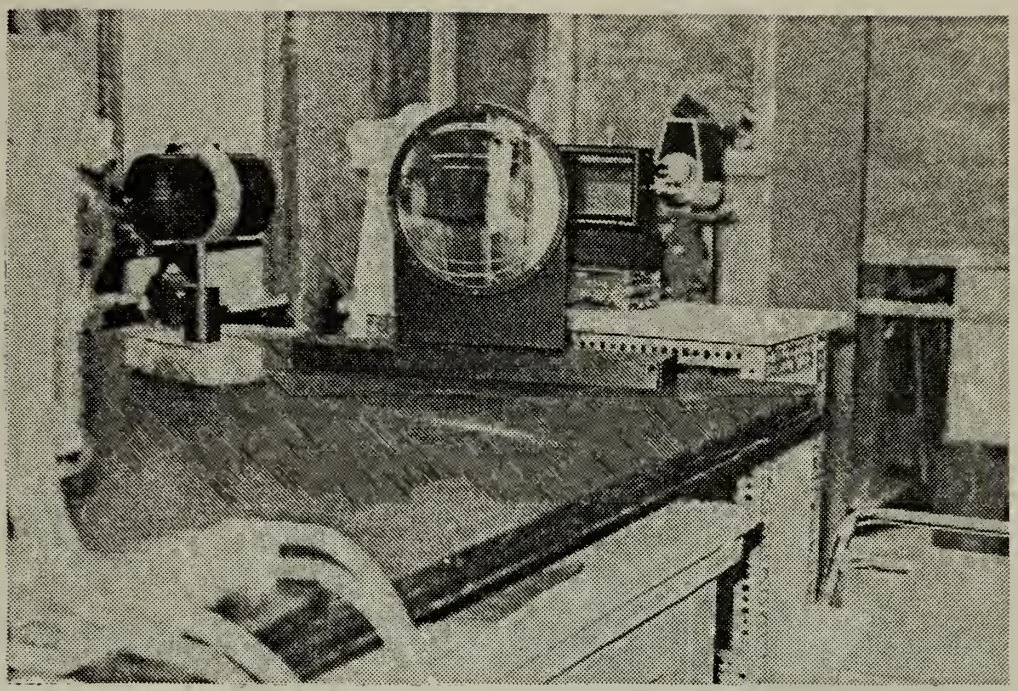


Figure 11
Direct Line-up on Object Beam from
Test Platform to Recording Camera

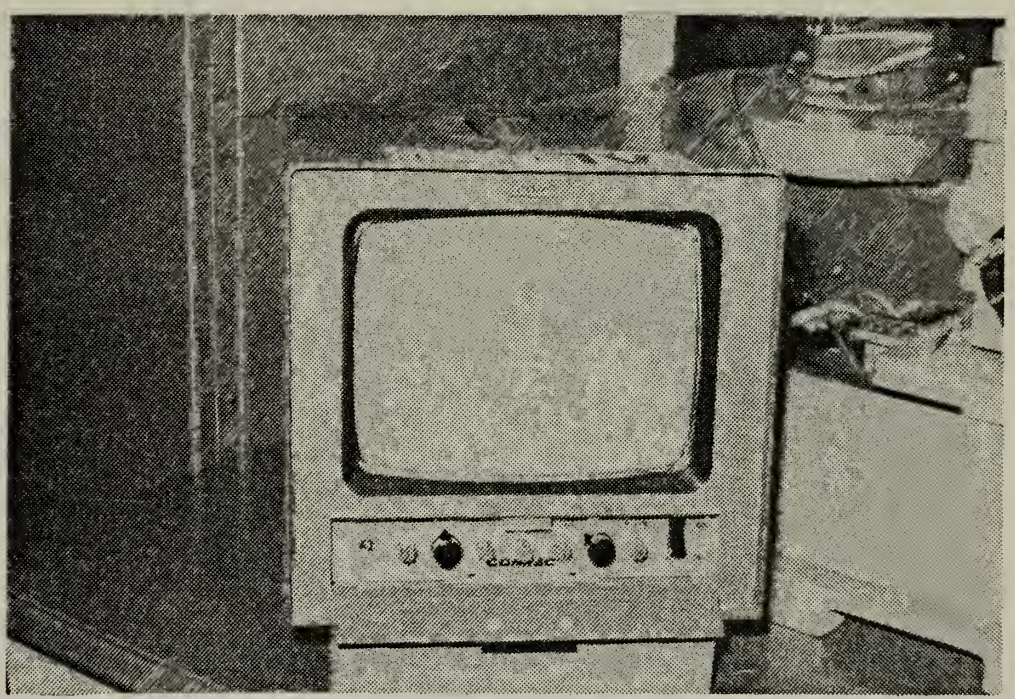


Figure 12
Television Monitor Used for Convenient Viewing of Fringes

Figure 13
Holographic Recording (top view)

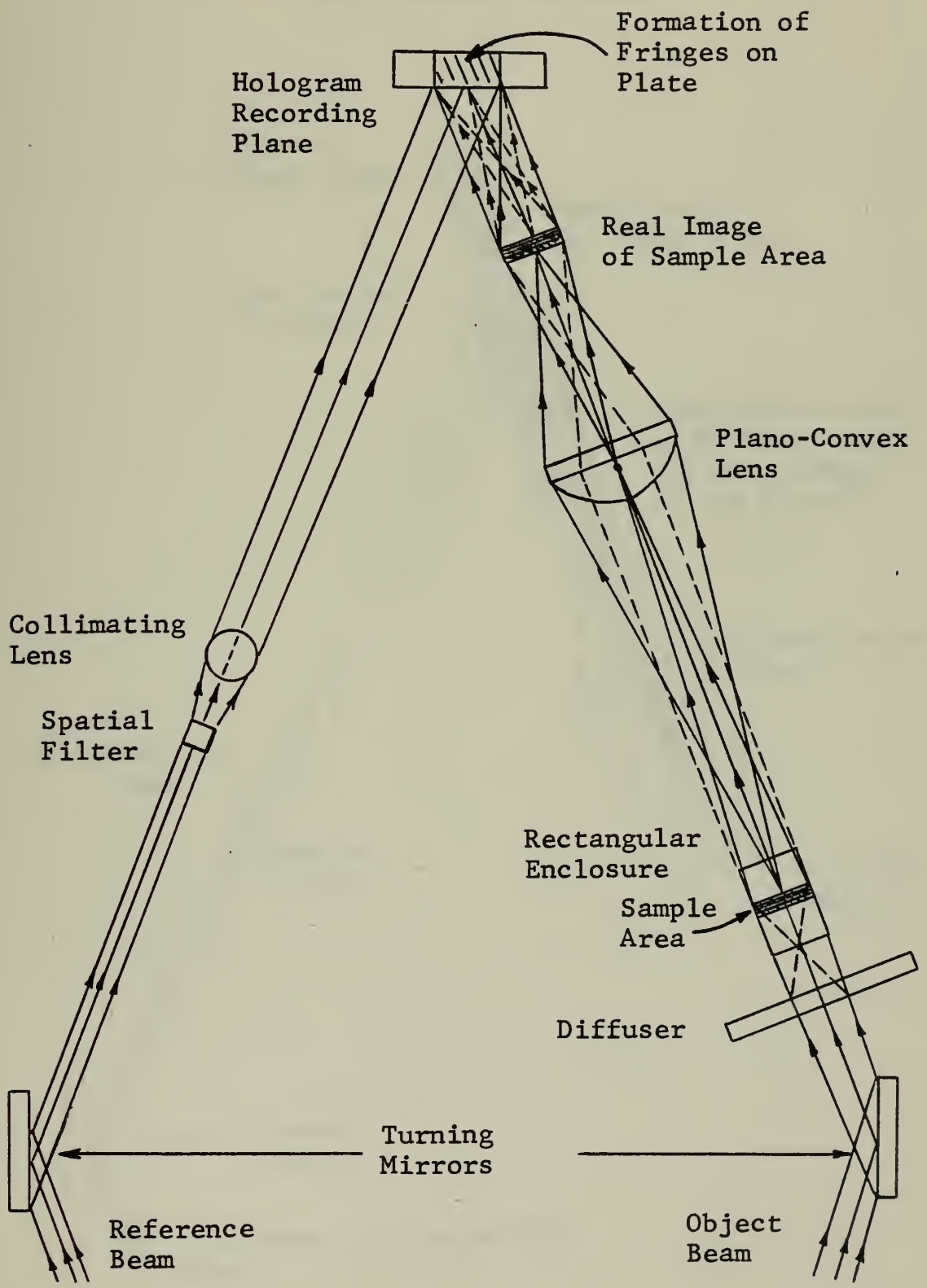
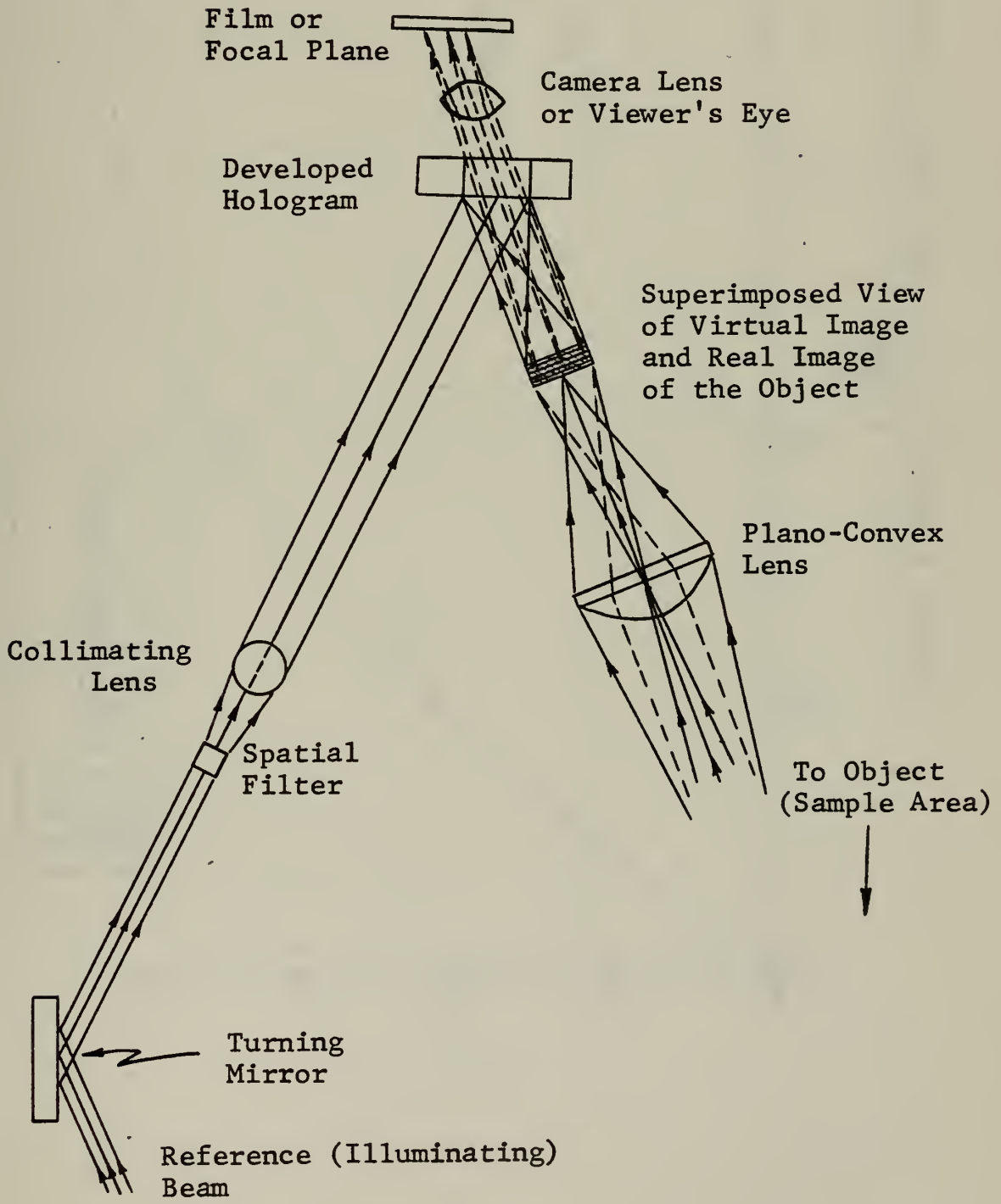


Figure 14
Reconstruction and Recording of
Interference Patterns



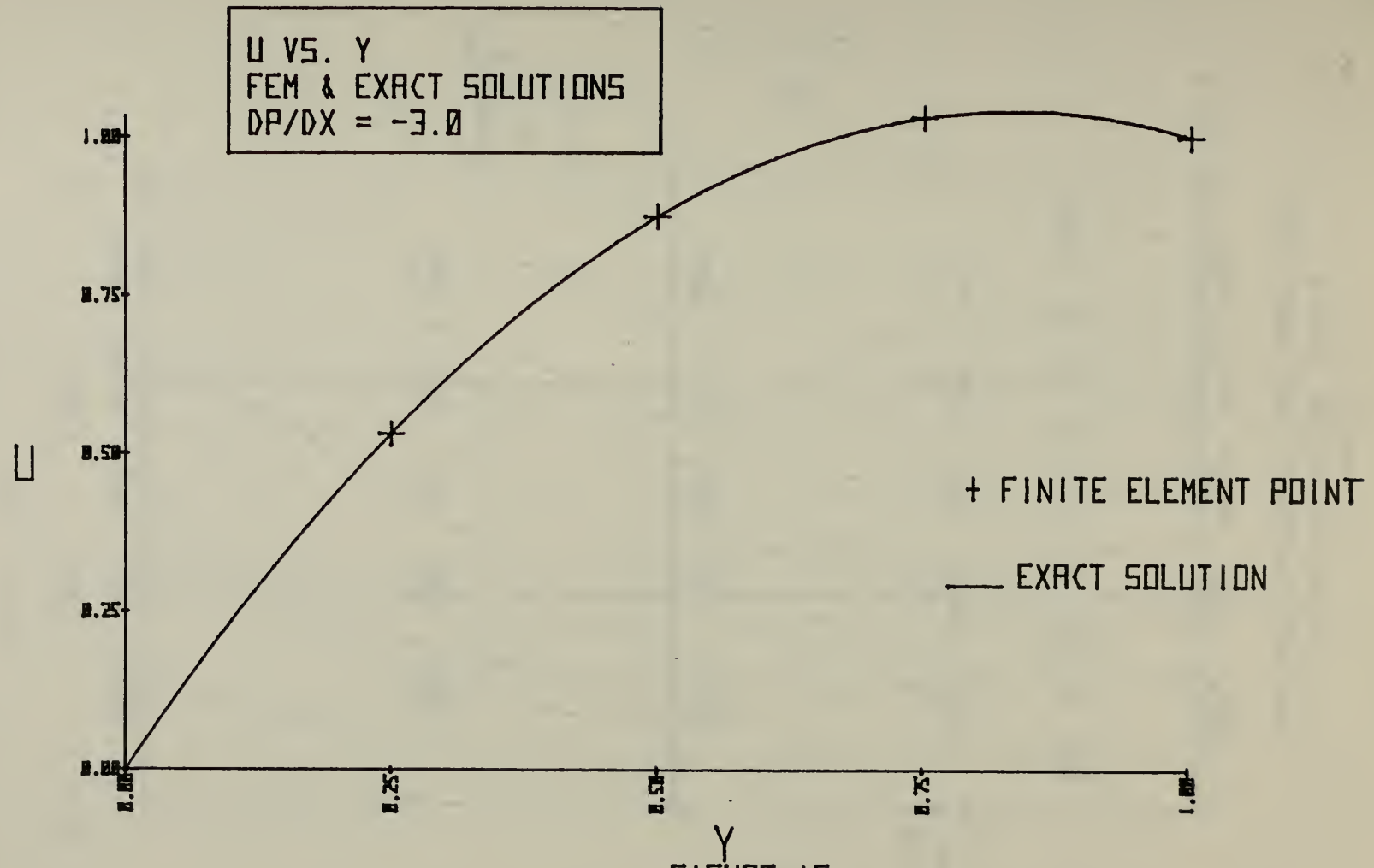
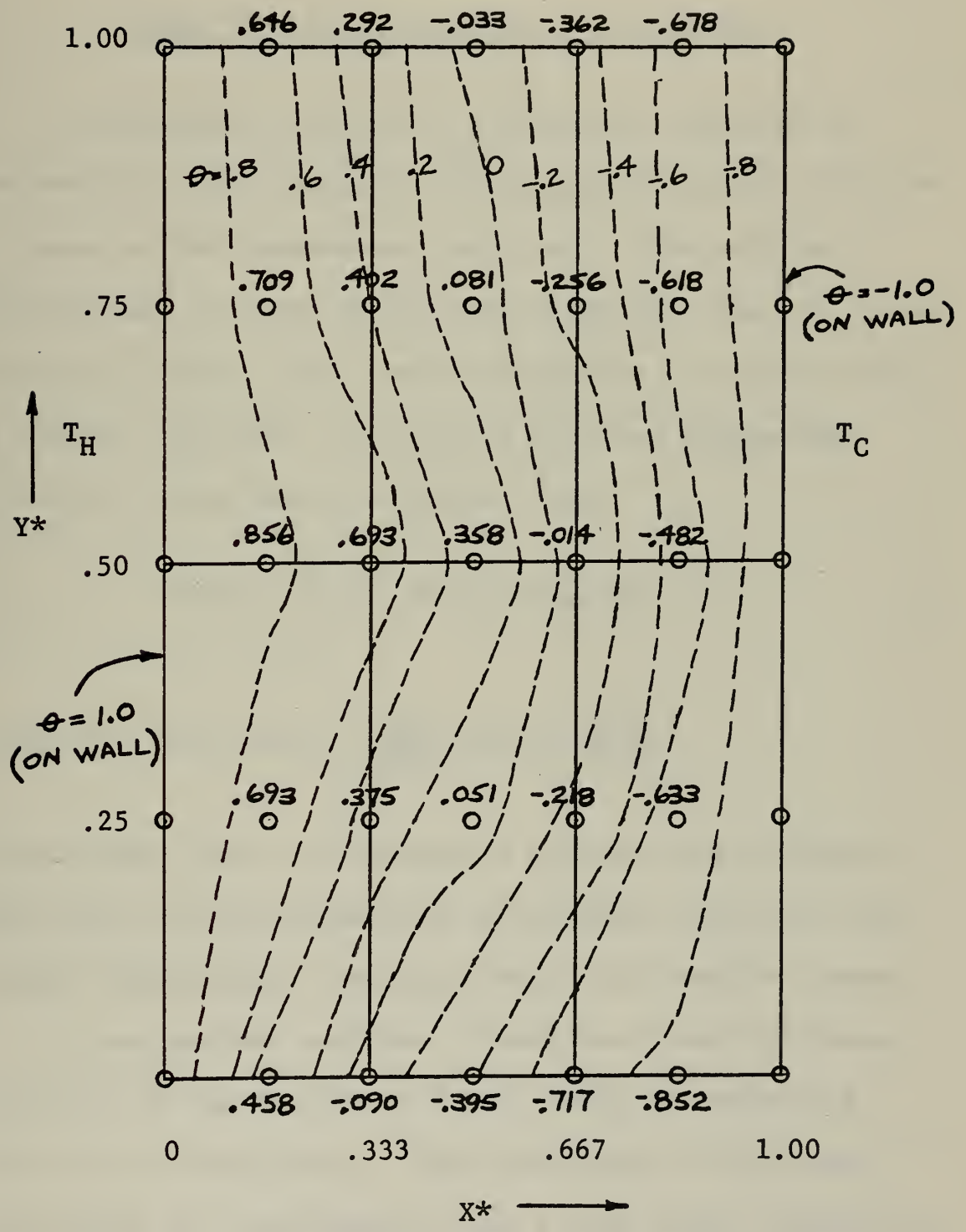


FIGURE 15
VELOCITY PROFILE FOR LINEAR COUETTE FLOW

Figure 16
Steady State Isotherms
(Nonlinear Heat Transfer Problem)



$Gr_L=9.464x10^2$, $Pr=1.0755x10^4$, $L/D=4.53$

APPENDIX B

BRIEF REVIEW ON CALCULUS OF VARIATIONS

A fundamental problem in differential calculus is extremizing (maximizing or minimizing) a function f(x) for a range of the independent variable x. The problem in variational calculus is also extremization; however, it is concerned chiefly with the extremization of a functional. A simple functional, in terms of only one independent variable, would have the typical form

$$I(\phi) = \int_{x_1}^{x_2} F(x, \phi, \phi_x, \phi_{xx})\,dx$$

where $\phi = \phi(x)$ and $\phi_x = \frac{\partial \phi}{\partial x}$, $\phi_{xx} = \frac{\partial^2 \phi}{\partial x^2}$.

Summarizing, the two branches of calculus are related in that both are concerned with an extremum; one deals with number spaces while the other deals with function spaces.

In variational problems a functional which is characteristic of the problem is first formed in terms of a function (or functions). Then variations of this same functional are investigated with a view toward extremizing the functional. In some cases this approach results in a

closed form, exact solution. But more often, the problem must be solved by an approximate method. One such method is the Rayleigh-Ritz technique. This approach is preferable to the direct application of finite difference methodology to solve the differential equation with its associated boundary conditions, because the functional can often be used to assure convergence of the approximate solution.

A simple example of variational calculus is the problem of finding the plane curve joining two points (x_1, y_1) and (x_2, y_2) which has the shortest length. The solution sought here is the function y(x) describing the curve of shortest length; the corresponding functional is the length of the curve given by

$$I(y) = \int_{x_1}^{x_2} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

Using the method of variation of calculus implies that of all the curves

$$Y(x) = y(x) + \varepsilon\gamma(x)$$

which pass through the given end points, the shortest one y(x) must be selected. The problem thus reduces to finding the function y(x) that makes the integral I(y) a minimum.

Generally, in order to minimize the integral

$$I(y) = \int_{x_1}^{x_2} F(x, y, y')dx$$

where $y' = \frac{dy}{dx}$, the function $y(x)$ must satisfy the boundary conditions and the Euler-Lagrange differential equation

$$\frac{\partial F}{\partial y} - \frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) = 0$$

The previous result could be extended to several dependent and independent variables. For example, in order to minimize the integral

$$I(\phi) = \iint_A F(x, y, \phi, \phi_x, \phi_y)dxdy$$

in which ϕ_x and ϕ_y are the partial derivatives of ϕ with respect to x and y, respectively, the general function ϕ must satisfy the Euler-Lagrange differential equation

$$\frac{\partial F}{\partial \phi} - \frac{\partial}{\partial x}\left(\frac{\partial F}{\partial \phi_x}\right) - \frac{\partial}{\partial y}\left(\frac{\partial F}{\partial \phi_y}\right) = 0$$

in addition to the specified boundary conditions.

In the past two decades, since the advent of high speed digital computers, the variational formulation has been quite extensively employed in the fields of structural

and continuum mechanics. Important variational principles such as least work, minimum strain energy, minimum potential energy, and Reissner's variational theorem of elasticity have been well developed and are documented in standard textbooks. However, similar variational principles applicable to fluid mechanic problems have not been as comprehensively developed. Calculus of variations has, until only recently, been utilized sparingly in the field of fluid mechanics.

THIS IS A 2-D NONLINEAR COUETTE FLOW PROBLEM

IBAND= 26

NEQ = 82

NO. OF NODES= 35

NO. OF ELEMENTS= 12

NO. OF CORNER NODES= 12

NNVELS= 14

NNCXY= 21

NNPS= 12

SUMMARY OF NODAL COORDINATES

I	X(I)	Y(I)
1	0.0	1.000
3	0.0	0.500
5	0.0	0.0
11	0.333	1.000
13	0.333	0.500
15	0.333	0.0
21	0.667	1.000
23	0.667	0.500
25	0.667	0.0
31	1.000	1.000
33	1.000	0.500
35	1.000	0.0

LISTING OF SYSTEM TOPOLOGY

ELEMENT NUMBER	NODE NUMBERS					
1	1	7	13	12	11	6
2	1	2	3	8	13	7
3	3	4	5	9	13	8
4	5	10	15	14	13	9
5	11	17	23	22	21	16
6	11	12	13	18	23	17
7	13	14	15	19	23	18
8	15	20	25	24	23	19
9	21	27	33	32	31	26
10	21	22	23	28	33	27
11	23	24	25	29	33	28
12	25	30	35	34	33	29

NODES WHERE VELOCITIES ARE SPECIFIED

I	NODE	U VELOCITY	V VELOCITY
1	1	1.000	0.0
2	5	0.0	0.0
3	6	1.000	0.0
4	10	0.0	0.0
5	11	1.000	0.0
6	15	0.0	0.0
7	16	1.000	0.0
8	20	0.0	0.0
9	21	1.000	0.0
10	25	0.0	0.0
11	26	1.000	0.0
12	30	0.0	0.0
13	31	1.000	0.0
14	35	0.0	0.0

NODES WHERE QX AND QY ARE SPECIFIED

I	NODE	QX	QY
1	7	0.0	0.0
2	8	0.0	0.0
3	9	0.0	0.0
4	12	0.0	0.0
5	13	0.0	0.0
6	14	0.0	0.0
7	17	0.0	0.0
8	18	0.0	0.0
9	19	0.0	0.0
10	22	0.0	0.0
11	23	0.0	0.0
12	24	0.0	0.0
13	27	0.0	0.0
14	28	0.0	0.0
15	29	0.0	0.0
16	2	1.333	0.0
17	3	0.667	0.0
18	4	1.333	0.0
19	32	-0.333	0.0
20	33	-0.167	0.0
21	34	-0.333	0.0

NODES WHERE PRESSURE IS SPECIFIED

I	NODE	PRESSURE
1	1	4.000
2	3	4.000
3	5	4.000
4	11	3.000
5	13	3.000
6	15	3.000
7	21	2.000
8	23	2.000
9	25	2.000
10	31	1.000
11	33	1.000
12	35	1.000

NODAL VARIABLE IS THE U-VELOCITY AT NODES 1 - 35;
THE V-VELOCITY AT NODES 36 - 70;
AND THE PRESSURES AT NODES 71 - 82.

THE FIRST SEQUENCE OF 82 NODAL VARIABLES
REPRESENTS A LINEAR, STEADY STATE SYSTEM;
WHILE THE SECOND SET OF THE 82 VALUES CORRESPONDS
TO A NONLINEAR ANALYSIS OF THE SOLUTION DOMAIN.

A PRESSURE GRADIENT IN THE HORIZONTAL SHEAR
DIRECTION OF -3 UNITS IN MAGNITUDE HAS BEEN ADDED
TO PRODUCE A CURVED VELOCITY PROFILE.

NODE NO.	NODE VARIABLES
1	0.10000000000D 01
2	0.10312499996D 01
3	0.87500000670 00
4	0.53124999610 00
5	0.0
6	0.10000000000D 01
7	0.10312499990 01
8	0.87500000220 00
9	0.53124999910 00
10	0.0
11	0.10000000000D 01
12	0.10312499990 01
13	0.87499999980 00
14	0.53124999950 00
15	0.0
16	0.10000000000D 01
17	0.10312500000D 01
18	0.87499999980 00
19	0.53124999990 00
20	0.0
21	0.10000000000D 01
22	0.10312500020 01
23	0.87500000140 00
24	0.53125000200 00
25	0.0
26	0.10000000000D 01
27	0.10312500060 01
28	0.87499999860 00
29	0.53125000560 00
30	0.0
31	0.10000000000D 01
32	0.10312500140 01
33	0.87499998870 00
34	0.53125001390 00
35	0.0
36	0.0
37	0.60994269640-19
38	-0.71251878180-42
39	-0.60994269640-19
40	0.0
41	0.0
42	0.88102833380-19

43	-0.81333946C69D-42
44	-0.88102833380-19
45	0.0
46	0.0
47	0.19352502520-18
48	-0.11149020410-41
49	-0.19352502520-18
50	0.0
51	0.0
52	0.47096946920-18
53	-0.15718322820-41
54	-0.47096946920-18
55	0.0
56	0.0
57	0.11670534580-17
58	-0.21868403170-41
59	-0.11670534580-17
60	0.0
61	0.0
62	0.29005182730-17
63	-0.26715796970-41
64	-0.29005182730-17
65	0.0
66	0.0
67	0.20080511240-17
68	-0.33565885460-41
69	-0.20080511240-17
70	0.0
71	0.40000000C0D 01
72	0.40000000000D 01
73	0.40000000000D 01
74	0.30000000000D 01
75	0.30000000000D 01
76	0.30000000000D 01
77	0.20000000000D 01
78	0.20000000000D 01
79	0.20000000000D 01
80	0.10000000000D 01
81	0.10000000000D 01
82	0.10000000000D 01

NODE NO.	NODE VARIABLES
1	0.10000000000D 01

```
 2      0.10286874930 01
 3      0.86895844590 00
 4      0.52886443940 00
 5      0.0
 6      0.10000000000 01
 7      0.10291188120 01
 8      0.87082505230 00
 9      0.52910358090 00
10      0.0
11      0.10000000000 01
12      0.10293004470 01
13      0.87078935530 00
14      0.52928248490 00
15      0.0
16      0.10000000000 01
17      0.10295625700 01
18      0.87181955720 00
19      0.52954825460 00
20      0.0
21      0.10000000000 01
22      0.10297264840 01
23      0.87148436690 00
24      0.52970966770 00
25      0.0
26      0.10000000000 01
27      0.10300368900 01
28      0.87285576610 00
29      0.53002333900 00
30      0.0
31      0.10000000000 01
32      0.10301734920 01
33      0.87341990360 00
34      0.53016098300 00
35      0.0
36      0.0
37      0.94093325430-19
38      0.24063440910-19
39     -0.56904218950-19
40      0.0
41      0.0
42      0.12824933130-18
43      0.26110722880-19
44     -0.86538783260-19
45      0.0
```

```
46      0.0
47      0.25383549350-18
48      0.32006190050-19
49     -0.19986923340-18
50      0.0
51      0.0
52      0.56134699840-18
53      0.36351135030-19
54     -0.49119243710-18
55      0.0
56      0.0
57      0.12819175680-17
58      0.37806257670-19
59     -0.12007464690-17
60      0.0
61      0.0
62      0.29622390920-17
63      0.32117633760-19
64     -0.29108457190-17
65      0.0
66      0.0
67      0.20082897640-17
68      0.15102713170-19
69     -0.19908894340-17
70      0.0
71      0.40000000000 01
72      0.40000000000 01
73      0.40000000000 01
74      0.30000000000 01
75      0.30000000000 01
76      0.30000000000 01
77      0.20000000000 01
78      0.20000000000 01
79      0.20000000000 01
80      0.10000000000 01
81      0.10000000000 01
82      0.10000000000 01
```

STEADY STATE FLUID MECHANICS PROBLEM

(HEAT TRANSFER)

THIS IS A 2-D NONLINEAR PROBLEM

IBAND= 26

NEC=117

NO. OF NODES= 35

NO. OF ELEMENTS= 12

NO. OF CORNER NODES= 12

NNVELS= 20

NNCXY= 15

NNPS= 6

NNTS= 10

NNCZC= 6

NNCZ= 25

SUMMARY OF NODAL COORDINATES

I	X(I)	Y(I)
1	0.0	8.500
3	0.0	4.250
5	0.0	0.0
11	0.625	8.500
13	0.625	4.250
15	0.625	0.0
21	1.250	8.500
23	1.250	4.250
25	1.250	0.0
31	1.875	8.500
33	1.875	4.250
35	1.875	0.0

LISTING OF SYSTEM TOPOLOGY

ELEMENT NUMBER	NODE NUMBERS					
1	1	7	13	12	11	6
2	1	2	3	8	13	7
3	3	4	5	9	13	8
4	5	10	15	14	13	9
5	11	17	23	22	21	16
6	11	12	13	18	23	17
7	13	14	15	19	23	18
8	15	20	25	24	23	19
9	21	27	33	32	31	26
10	21	22	23	28	33	27
11	23	24	25	29	33	28
12	25	30	35	34	33	29

NODES WHERE VELOCITIES ARE SPECIFIED

I	NODE	U VELOCITY	V VELOCITY

1	1	0.0	0.0
2	2	0.0	0.0
3	3	0.0	0.0
4	4	0.0	0.0
5	5	0.0	0.0
6	6	0.0	0.0
7	10	0.0	0.0
8	11	0.0	0.0
9	15	0.0	0.0
10	16	0.0	0.0
11	20	0.0	0.0
12	21	0.0	0.0
13	25	0.0	0.0
14	26	0.0	0.0
15	30	0.0	0.0
16	31	0.0	0.0
17	32	0.0	0.0
18	33	0.0	0.0
19	34	0.0	0.0
20	35	0.0	0.0

NODES WHERE QX AND QY ARE SPECIFIED

I	NODE	QX	QY
1	7	0.0	0.0
2	8	0.0	0.0
3	9	0.0	0.0
4	12	0.0	0.0
5	13	0.0	0.0
6	14	0.0	0.0
7	17	0.0	0.0
8	18	0.0	0.0
9	19	0.0	0.0
10	22	0.0	0.0
11	23	0.0	0.0
12	24	0.0	0.0
13	27	0.0	0.0
14	28	0.0	0.0
15	29	0.0	0.0

NODES WHERE PRESSURE IS SPECIFIED

I	NODE	PRESSURE
1	1	1014000.000
2	3	1014000.000
3	5	1014000.000
4	31	1014000.000
5	33	1014000.000
6	35	1014000.000

NODES WHERE TEMPERATURE IS SPECIFIED

I	NODE	TEMPERATURE
1	1	25.000
2	2	25.000
3	3	25.000
4	4	25.000
5	5	25.000
6	31	20.000
7	32	20.000
8	33	20.000
9	34	20.000
10	35	20.000

NODES WHERE QZC IS SPECIFIED

I	NODE	QZC
1	4	0.0
2	5	0.0
3	6	0.0
4	7	0.0
5	8	0.0
6	9	0.0

NODES WHERE HEAT FLUX QZ IS SPECIFIED

I	NODE	HEAT FLUX
1	6	0.0
2	7	0.0
3	8	0.0
4	9	0.0
5	10	0.0
6	11	0.0
7	12	0.0
8	13	0.0
9	14	0.0
10	15	0.0
11	16	0.0
12	17	0.0
13	18	0.0
14	19	0.0
15	20	0.0
16	21	0.0
17	22	0.0
18	23	0.0
19	24	0.0
20	25	0.0
21	26	0.0
22	27	0.0
23	28	0.0
24	29	0.0
25	30	0.0

NODAL VARIABLE IS THE U-VELOCITY AT NODES 1 - 35;
THE V-VELOCITY AT NODES 36 - 70;
THE PRESSURE AT NODES 71 - 82;
AND THE TEMPERATURE AT NODES 83 - 117.

THE SPECIFIED WALL PRESSURES
ARE NORMALIZED TO ONE (1) ATMOSPHERE,
THAT IS, 1014000 DYNES/SQ.CM
(ALL PARAMETER VALUES ARE IN CGS UNITS).

THE FIRST SEQUENCE OF 117 NODAL VARIABLES
REPRESENTS A LINEAR,STEADY STATE SYSTEM;
WHILE THE SECOND SET OF THE 117 VALUES CORRESPONDS
TO A NONLINEAR ANALYSIS OF THE SOLUTION DOMAIN.

NODE NO.	NODE VARIABLE
1	0.0
2	0.0
3	0.0
4	0.0

```
 5      0.0
 6      0.0
 7     -0.15945801720 01
 8     -0.78826727870 01
 9     -0.15945801720 01
10      0.0
11      0.0
12     -0.36495074880 00
13     -0.12038963780 02
14     -0.36495074880 00
15      0.0
16      0.0
17     -0.12166891530 00
18     -0.92233087650 01
19     -0.12166891530 00
20      0.0
21      0.0
22     -0.16051865150 01
23     -0.10709975830 02
24     -0.16051865150 01
25      0.0
26      0.0
27     -0.16866135620 01
28     -0.64584102400 01
29     -0.16866135620 01
30      0.0
31      0.0
32      0.0
33      0.0
34      0.0
35      0.0
36      0.0
37      0.0
38      0.0
39      0.0
40      0.0
41      0.0
42      0.32878138200 00
43     -0.86715052030-23
44     -0.32878138200 00
45      0.0
46      0.0
47      0.48418167430 00
48     -0.12128888260-22
```

```
49      -0.4841818187430 00
50       0.0
51       0.0
52       0.28532060500 00
53       0.42109196680-23
54      -0.28532060500 00
55       0.0
56       0.0
57       0.15892437070-01
58       0.92415574840-23
59      -0.15892437070-01
60       0.0
61       0.0
62      -0.43460776970-01
63       0.11208359430-22
64       0.43460776970-01
65       0.0
66       0.0
67       0.0
68       0.0
69       0.0
70       0.0
71       0.10140000000 07
72       0.10140000000 07
73       0.10140000000 07
74       0.10140370640 07
75       0.10142256570 07
76       0.10140370640 07
77       0.10139803130 07
78       0.10138750670 07
79       0.10139803130 07
80       0.10140000000 07
81       0.10140000000 07
82       0.10140000000 07
83       0.25000000000 02
84       0.25000000000 02
85       0.25000000000 02
86       0.25000000000 02
87       0.25000000000 02
88       0.24166666670 02
89       0.24166666670 02
90       0.24166666670 02
91       0.24166666670 02
92       0.24166666670 02
```

93	0.23333333330 02
94	0.23333333330 02
95	0.23333333330 02
96	0.23333333330 02
97	0.23333333330 02
98	0.22500000000 02
99	0.22500000000 02
100	0.22500000000 02
101	0.22500000000 02
102	0.22500000000 02
103	0.21666666670 02
104	0.21666666670 02
105	0.21666666670 02
106	0.21666666670 02
107	0.21666666670 02
108	0.20833333330 02
109	0.20833333330 02
110	0.20833333330 02
111	0.20833333330 02
112	0.20833333330 02
113	0.20000000000 02
114	0.20000000000 02
115	0.20000000000 02
116	0.20000000000 02
117	0.20000000000 02

NODE NO.	NODE VARIABLE
1	0.0
2	0.0
3	0.0
4	0.0
5	0.0
6	0.0
7	-0.16719673270 01
8	-0.81463882870 01
9	-0.16677090240 01
10	0.0
11	0.0
12	-0.35290662010 00
13	-0.12670865430 02
14	-0.34647969680 00
15	0.0
16	0.0

```
17     -0.9534759753D-01
18     -0.96623861C4D 01
19     -0.8874654693D-01
20      0.0
21      0.0
22     -0.16437945500 01
23     -0.11188367610 02
24     -0.16392742270 01
25      0.0
26      0.0
27     -0.16663660230 01
28     -0.68680556020 01
29     -0.16597517820 01
30      0.0
31      0.0
32      0.0
33      0.0
34      0.0
35      0.0
36      0.0
37      0.0
38      0.0
39      0.0
40      0.0
41      0.0
42      0.39280003440 00
43      0.28717151270-03
44     -0.39167605910 00
45      0.0
46      0.0
47      0.59303609440 00
48      0.61262434900-03
49     -0.59071993840 00
50      0.0
51      0.0
52      0.37670795430 00
53      0.99603754060-03
54     -0.37314213780 00
55      0.0
56      0.0
57      0.63827156120-01
58      0.13760499030-02
59     -0.59875228560-01
60      0.0
```

61	0.0
62	-0.2429545074D-01
63	0.1210812751D-02
64	0.2677263620D-01
65	0.0
66	0.0
67	0.0
68	0.0
69	0.0
70	0.0
71	0.1014000000D 07
72	0.1014000000D 07
73	0.1014000000D 07
74	0.1014038420D 07
75	0.1014261630D 07
76	0.1014038452D 07
77	0.1013999776D 07
78	0.1013884020D 07
79	0.1014060754D 07
80	0.1014000000D 07
81	0.1014000000D 07
82	0.1014000000D 07
83	0.2500000000D 02
84	0.2500000000D 02
85	0.2500000000D 02
86	0.2500000000D 02
87	0.2500000000D 02
88	0.2411490471D 02
89	0.2427187487D 02
90	0.2463978352D 02
91	0.2423258019D 02
92	0.2364452284D 02
93	0.2323056484D 02
94	0.2350586481D 02
95	0.2423194982D 02
96	0.2343829368D 02
97	0.2227636650D 02
98	0.2241562105D 02
99	0.2270150631D 02
100	0.2339548232D 02
101	0.2262750919D 02
102	0.2151348362D 02
103	0.2159431099D 02
104	0.2185937901D 02

```
105       0.2248590917D 02
106       0.2179546544D 02
107       0.2070833022D 02
108       0.2080608577D 02
109       0.2095508338D 02
110       0.2129443533D 02
111       0.2091829352D 02
112       0.2037035832D 02
113       0.2000000000D 02
114       0.2000000000D 02
115       0.2000000000D 02
116       0.2000000000D 02
117       0.2000000000D 02
```

THE KINEMATIC VISCOSITY OF FLUID 50-HB-3520 AT 20 DEGREES C. =
10.90 SQ.CM/SEC

THE DENSITY OF 50-HB-3520 AT 20 DEGREES C. = 1.0596 GM/CC

THE COEFF. OF THERMAL EXPANSION OF 50-HB-3520 AT 20 DEGREES C. =
0.002278/DEGREE C.

THE THERMAL DIFFUSIVITY OF 50-HB-3520 AT 20 DEGREES C. =
0.00103 SQ.CM/SEC

THE GRASHOF NUMBER (GR(L)) =(G*B*L**3*(TH-TC))/V**2 = 946.4

THE U VELOCITY FORCING FUNCTION,G*B*T(INITIAL), = 65.962 CM/SQ.S

```
      IMPLICIT REAL*8(A-H,O-Z,$)                                        COUT0001
C  THE U VELOCITY IS IN THE FIRST NN POSITIONS OF T(I)  (NN=NO. OF NODESCOUT0002
CC  THE V VELOCITY IS IN THE SECOND NN POSITIONS OF T I.E. T(I+NN)      COUT0003
CC  THE P PRESSURE IS IN THE NN+NN+I POSITIONS OF T  I.E. T(I+NN+NN)    COUT0004
CC  THERE ARE NNCN PRESSURE NODES                                       COUT0005
CC                                                                      COUT0006
C                                                                       COUT0007
      DATA NREAD/5/                                                     COUT0008
      DATA NWRITE/6/                                                    COUT0009
      DATA STOP/'STOP'/                                                 COUT0010
                                                                        COUT0011
      DIMENSION TM(82,82)                                               COUT0012
      DIMENSION XC(82), YC(82), NODE(82,6), NVS(82), NCN(82)            COUT0013
      DIMENSION T(82), NVIS(82), NCP(82), NPS(82), Q(82)                COUT0014
      DIMENSION NQS(82), T1(82)                                         COUT0015
      DIMENSION TM$(15,15), N(15), NQIS(82)                             COUT0016
      DIMENSION RHS(82)                                                 COUT0017
      DIMENSION RP$(6), ZP$(6)                                          COUT0018
      DIMENSION XC$(3), YC$(3), WKAREA(12000)                           COUT0019
C                                                                       COUT0020
CC  THE FIRST PART OF THE PROGRAM CAN BE CONSIDERED AS AN INPUT ROUTINE COUT0021
CC  LINE 014 TO  0151 ARE INPUT AND VERIFICATION OF ALL DATA            COUT0022
CC                                                                      COUT0023
CC  THIS WOULD BE PART OF ANY FINITE ELEMENT PROGRAM                    COUT0024
CC                                                                      COUT0025
C                                                                       COUT0026
C  READ IN NUMBER OF NODES AND ELEMENTS AND NO. OF CORNER NODES         COUT0027
      READ (NREAD,3800) NN,NE,NNCN                                      COUT0028
      WRITE(NWRITE,1094)                                                COUT0029
C                                                                       COUT0030
CC    INITIALIZE ALL PARAMETERS                                         COUT0031
C                                                                       COUT0032
      MM = 2*NN+NNCN                                                    COUT0033
C                                                                       COUT0034
      DO 200 I=1,MM                                                     COUT0035
      XC(I) = 0.D0                                                      COUT0036
```

```
      YC(I) = 0.D0                                                      COUT0037
      NVS(I) = 0                                                        COUT0038
      NCN(I) = 0                                                        COUT0039
      NCP(I) = 0                                                        COUT0040
      NPS(I) = 0                                                        COUT0041
      NQS(I) = 0                                                        COUT0042
C                                                                       COUT0043
      DO 200 J=1,6                                                      COUT0044
      NODE(I,J) = 0                                                     COUT0045
  200 CONTINUE                                                          COUT0046
C                                                                       COUT0047
CC                                                                      COUT0048
      DO 300 I=1,MM                                                     COUT0049
      T1(I) = 0.D0                                                      COUT0050
      T(I) = 0.D0                                                       COUT0051
      NVIS(I) = 0                                                       COUT0052
      Q(I) = 0.D0                                                       COUT0053
      NQIS(I) = 0                                                       COUT0054
      RHS(I) = 0.D0                                                     COUT0055
C                                                                       COUT0056
      DO 300 J=1,MM                                                     COUT0057
      TM(I,J) = 0.D0                                                    COUT0058
  300 CONTINUE                                                          COUT0059
C                                                                       COUT0060
CC                                                                      COUT0061
      DO 400 I=1,15                                                     COUT0062
      N(I) = 0                                                          COUT0063
C                                                                       COUT0064
      DO 400 J=1,15                                                     COUT0065
      TM$(I,J) = 0.D0                                                   COUT0066
  400 CONTINUE                                                          COUT0067
C                                                                       COUT0068
CC                                                                      COUT0069
      DO 500 I=1,6                                                      COUT0070
      RP$(I) = 0.D0                                                     COUT0071
      ZP$(I) = 0.D0                                                     COUT0072
  500 CONTINUE                                                          COUT0073
C                                                                       COUT0074
CC                                                                      COUT0075
      DO 600 I=1,3                                                      COUT0076
      XC$(I) = 0.D0                                                     COUT0077
      YC$(I) = 0.D0                                                     COUT0078
  600 CONTINUE                                                          COUT0079
C                                                                       COUT0080
CCC   READ NODE NUMBERS AND COORDINATES                                 COUT0081
CCC                                                                     COUT0082
C                                                                       COUT0083
      DO 700 J=1,NN                                                     COUT0084
```

```
      READ (NREAD,3900) WORD,I,XC(I),YC(I)                              COUT0085
      IF (WORD.EQ.STOP) GO TO 800                                       COUT0086
      NCN(J) = I                                                        COUT0087
  700 CONTINUE                                                          COUT0088
C                                                                       COUT0089
  800 NNCN = J-1                                                        COUT0090
C                                                                       COUT0091
CCCC  THE ARRAY  NCP(J) GENERATES THE GLOBAL PRESSURE INDICES (P1,P2.)  COUT0092
CCCC  THUS PRESSURE NODES ARE LABELED AS CORNER NODES ARE INPUTTED      COUT0093
CCCC  WHEN ONE INPUTS A GLOBAL CORNER NODE FOR J                        COUT0094
CCC                                                                     COUT0095
C                                                                       COUT0096
      DO 900 J=1,NNCN                                                   COUT0097
      NCP(NCN(J)) = J+NN+NN                                             COUT0098
  900 CONTINUE                                                          COUT0099
C                                                                       COUT0100
CCCCC                                                                   COUT0101
CCCCC   SYSTEM TOPOLOGY( ELEMENT NO. AND NODE NUMBERS IN                COUT0102
CCCCCC COUNTER-CLOCKWISE FASHION STARTING AT ANY CORNER NODE            COUT0103
CCCCCC ALWAYS COUNT FROM UPPER LEFT HAND CORNER                         COUT0104
CCC                                                                     COUT0105
C                                                                       COUT0106
      DO 1000 I=1,NE                                                    COUT0107
      READ (NREAD,4000) J,NODE(J,1),NODE(J,2),NODE(J,3),NODE(J,4),NODE(JCOUT0108
     1,5),NODE(J,6)                                                     COUT0109
 1000 CONTINUE                                                          COUT0110
C                                                                       COUT0111
      MAXDIF = 0                                                        COUT0112
C                                                                       COUT0113
      DO 1100 I=1,NE                                                    COUT0114
C                                                                       COUT0115
      DO 1100 J=1,6                                                     COUT0116
C                                                                       COUT0117
      DO 1100 K=1,6                                                     COUT0118
      LL = IABS(NODE(I,J)-NODE(I,K))                                    COUT0119
      IF (LL.GT.MAXDIF) MAXDIF=LL                                       COUT0120
      IBAND = 2*(MAXDIF+1)                                              COUT0121
      NEQ = 2*NN+NNCN                                                   COUT0122
 1100 CONTINUE                                                          COUT0123
C                                                                       COUT0124
      WRITE (NWRITE,3700) IBAND,NEQ                                     COUT0125
C                                                                       COUT0126
CCC   READ NODES WHERE BOTH U AND V VELOCITY IS SPECIFIED               COUT0127
CCC                                                                     COUT0128
C                                                                       COUT0129
      DO 1200 I=1,MM                                                    COUT0130
      READ (NREAD,3900) WORD,NVELS,VELU,VELV                            COUT0131
      IF (WORD.EQ.STOP) GO TO 1300                                      COUT0132
```

```
      NVS(I) = NVELS                                                    COUT0133
      T(NVS(I)) = VELU                                                  COUT0134
      T(NVS(I)+NN) = VELV                                               COUT0135
 1200 CONTINUE                                                          COUT0136
C                                                                       COUT0137
CCC                                                                     COUT0138
CCC  COUNT NODES HAVING SPECIFIED  VELOCITIES                           COUT0139
C                                                                       COUT0140
 1300 NNVELS = I-1                                                      COUT0141
C                                                                       COUT0142
CCCC                                                                    COUT0143
CCCC READ QX AND QY VALUES AT INTERNAL NODES                            COUT0144
C                                                                       COUT0145
      DO 1400 I=1,NN                                                    COUT0146
      READ (NREAD,3900) WORD,NQXY,QXNS,QYNS                             COUT0147
      IF (WORD.EQ.STOP) GO TO 1500                                      COUT0148
      NQS(I) = NQXY                                                     COUT0149
      Q(NQS(I)) = QXNS                                                  COUT0150
      Q(NQS(I)+NN) = QYNS                                               COUT0151
 1400 CONTINUE                                                          COUT0152
C                                                                       COUT0153
CCCC                                                                    COUT0154
CCCC  COUNT NODES HAVING SPECIFIED QX AND QY                            COUT0155
C                                                                       COUT0156
 1500 NNQXY = I-1                                                       COUT0157
C                                                                       COUT0158
CCCC  READ NODE NUMBER AND PRESSURE WHERE SPECIFIED                     COUT0159
CCCC                                                                    COUT0160
C                                                                       COUT0161
      DO 1600 I=1,NN                                                    COUT0162
      READ (NREAD,4100) WORD,NP,PNP                                     COUT0163
      IF (WORD.EQ.STOP) GO TO 1700                                      COUT0164
      NPS(I) = NP                                                       COUT0165
      T(NCP(NPS(I))) = PNP                                              COUT0166
 1600 CONTINUE                                                          COUT0167
C                                                                       COUT0168
CCCC                                                                    COUT0169
CCCC COUNT  BOUNDARY NODES WHERE PRESSURE SPECIFIED                     COUT0170
C                                                                       COUT0171
 1700 NNPS = I-1                                                        COUT0172
C                                                                       COUT0173
CC NQIS IS A LIST OF THE INDICES OF KNOWN QX,QY                         COUT0174
C                                                                       COUT0175
      DO 2000 I=1,NNQXY                                                 COUT0176
      NQIS(I) = NQS(I)                                                  COUT0177
      NQIS(I+NNQXY) = NQS(I)+NN                                         COUT0178
 2000 CONTINUE                                                          COUT0179
C                                                                       COUT0180
```

```
C                                                                       COUT0181
CC                                                                      COUT0182
CC  NVIS IS A LIST OF KNOWN VELOCITY AND PRESSURE INDICES               COUT0183
C                                                                       COUT0184
      DO 2200 I=1,NNVELS                                                COUT0185
      NVIS(I) = NVS(I)                                                  COUT0186
      NVIS(I+NNVELS) = NVS(I)+NN                                        COUT0187
 2200 CONTINUE                                                          COUT0188
C                                                                       COUT0189
C                                                                       COUT0190
      DO 2300 J=1,NNPS                                                  COUT0191
      NVIS(2*NNVELS+J) = NCP(NPS(J))                                    COUT0192
 2300 CONTINUE                                                          COUT0193
C                                                                       COUT0194
C NNHC=NUMBER OF NODES WHERE HEAT TRANSFER COEFFICIENT IS SPECIFIED     COUT0195
      NNHC = 0                                                          COUT0196
C                                                                       COUT0197
CC  NTOTQ=TOTAL NUMBER OF KNOWN QX,QY                                   COUT0198
C                                                                       COUT0199
      NTOTQ = 2*NNQXY                                                   COUT0200
C                                                                       COUT0201
C NTOTVP=TOTAL NUMBER OF KNOWN VELOCITIES, AND PRESSURES                COUT0202
C                                                                       COUT0203
      NTOTVP = 2*NNVELS+NNPS                                            COUT0204
C                                                                       COUT0205
CC  PRINT ALL INPUT DATA                                                COUT0206
C                                                                       COUT0207
      WRITE (NWRITE,4300) NN,NE,NNCN                                    COUT0208
      WRITE (NWRITE,4400) NNVELS                                        COUT0209
      WRITE (NWRITE,4500) NNQXY                                         COUT0210
      WRITE (NWRITE,4600) NNPS                                          COUT0211
      WRITE (NWRITE,4700)                                               COUT0212
C                                                                       COUT0213
      DO 2400 I=1,NNCN                                                  COUT0214
      WRITE (NWRITE,4800) NCN(I),XC(NCN(I)),YC(NCN(I))                  COUT0215
 2400 CONTINUE                                                          COUT0216
C                                                                       COUT0217
      WRITE (NWRITE,4900)                                               COUT0218
C                                                                       COUT0219
      DO 2500 I=1,NE                                                    COUT0220
      WRITE (NWRITE,5000) I,NODE(I,1),NODE(I,2),NODE(I,3),NODE(I,4),NODECOUT0221
     1(I,5),NODE(I,6)                                                   COUT0222
 2500 CONTINUE                                                          COUT0223
C                                                                       COUT0224
      WRITE (NWRITE,5100)                                               COUT0225
C                                                                       COUT0226
      DO 2600 I=1,NNVELS                                                COUT0227
      WRITE (NWRITE,5200) I,NVS(I),T(NVS(I)),T(NVS(I)+NN)               COUT0228
```

```
 2600 CONTINUE                                                          COUT0229
C                                                                       COUT0230
      WRITE (NWRITE,5300)                                               COUT0231
C                                                                       COUT0232
      DO 2700 I=1,NNQXY                                                 COUT0233
      WRITE (NWRITE,5200) I,NQS(I),Q(NQS(I)),Q(NQS(I)+NN)               COUT0234
 2700 CONTINUE                                                          COUT0235
C                                                                       COUT0236
      WRITE (NWRITE,5400)                                               COUT0237
C                                                                       COUT0238
      DO 2800 I=1,NNPS                                                  COUT0239
      WRITE (NWRITE,5600) I,NPS(I),T(NCP(NPS(I)))                       COUT0240
 2800 CONTINUE                                                          COUT0241
      WRITE(NWRITE,1090)                                                COUT0242
      WRITE(NWRITE,1092)                                                COUT0243
      WRITE(NWRITE,1093)                                                COUT0244
 2840 CONTINUE                                                          COUT0245
      DO 2850 I=1,MM                                                    COUT0246
      RHS(I) = 0.D0                                                     COUT0247
      DO 2850 J=1,MM                                                    COUT0248
      TM(I,J) = 0.D0                                                    COUT0249
 2850 CONTINUE                                                          COUT0250
      DO 2860 I=1,15                                                    COUT0251
      DO 2860 J=1,15                                                    COUT0252
      TM$(I,J) = 0.D0                                                   COUT0253
 2860 CONTINUE                                                          COUT0254
CCCCC                                                                   COUT0255
CCCCC                                                                   COUT0256
CCCCC                                                                   COUT0257
CCCCC    END OF  INPUT AND VERIFICATION ROUTINE                         COUT0258
CCCCC                                                                   COUT0259
C                                                                       COUT0260
      DO 3200 K=1,NE                                                    COUT0261
      N1=NODE(K,1)                                                      COUT0262
      N2=NODE(K,2)                                                      COUT0263
      N3=NODE(K,3)                                                      COUT0264
      N4=NODE(K,4)                                                      COUT0265
      N5=NODE(K,5)                                                      COUT0266
      N6=NODE(K,6)                                                      COUT0267
      N7=NODE(K,1)+NN                                                   COUT0268
      N8=NODE(K,2)+NN                                                   COUT0269
      N9=NODE(K,3)+NN                                                   COUT0270
      N10=NODE(K,4)+NN                                                  COUT0271
      N11=NODE(K,5)+NN                                                  COUT0272
      N12=NODE(K,6)+NN                                                  COUT0273
      N13=NCP(NODE(K,1))                                                COUT0274
      N14=NCP(NODE(K,3))                                                COUT0275
      N15=NCP(NODE(K,5))                                                COUT0276
```

```
      XC$(1) = XC(NODE(K,1))                                                    COUT0277
      XC$(2) = XC(NODE(K,3))                                                    COUT0278
      XC$(3) = XC(NODE(K,5))                                                    COUT0279
      YC$(1) = YC(NODE(K,1))                                                    COUT0280
      YC$(2) = YC(NODE(K,3))                                                    COUT0281
      YC$(3) = YC(NODE(K,5))                                                    COUT0282
      A = 1.D0                                                                  COUT0283
      AA = 1.D0                                                                 COUT0284
      A1 = XC$(2)*YC$(3)-XC$(3)*YC$(2)                                          COUT0285
      A2 = XC$(3)*YC$(1)-XC$(1)*YC$(3)                                          COUT0286
      A3 = XC$(1)*YC$(2)-XC$(2)*YC$(1)                                          COUT0287
      B1 = YC$(2)-YC$(3)                                                        COUT0288
      B2 = YC$(3)-YC$(1)                                                        COUT0289
      B3 = YC$(1)-YC$(2)                                                        COUT0290
      C1 = XC$(3)-XC$(2)                                                        COUT0291
      C2 = XC$(1)-XC$(3)                                                        COUT0292
      C3 = XC$(2)-XC$(1)                                                        COUT0293
      DEL = DABS(0.5D0*(XC$(1)*(YC$(2)-YC$(3))+XC$(2)*(YC$(3)-YC$(1))+XCCOUT0294
     1$(3)*(YC$(1)-YC$(2))))                                                    COUT0295
      CONST = (1.D0*A/(3.D0*DEL))*AA                                            COUT0296
      D1=-B1/6.D0                                                               COUT0297
      D2=-B2/6.D0                                                               COUT0298
      D3=-B3/6.D0                                                               COUT0299
      E1=-C1/6.D0                                                               COUT0300
      E2=-C2/6.D0                                                               COUT0301
      E3=-C3/6.D0                                                               COUT0302
      F1=B1/2520.D0                                                             COUT0303
      F2=B2/2520.D0                                                             COUT0304
      F3=B3/2520.D0                                                             COUT0305
      G1=C1/2520.D0                                                             COUT0306
      G2=C2/2520.D0                                                             COUT0307
      G3=C3/2520.D0                                                             COUT0308
      U1=T1(N1)                                                                 COUT0309
      U2=T1(N2)                                                                 COUT0310
      U3=T1(N3)                                                                 COUT0311
      U4=T1(N4)                                                                 COUT0312
      U5=T1(N5)                                                                 COUT0313
      U6=T1(N6)                                                                 COUT0314
      V1=T1(N7)                                                                 COUT0315
      V2=T1(N8)                                                                 COUT0316
      V3=T1(N9)                                                                 COUT0317
      V4=T1(N10)                                                                COUT0318
      V5=T1(N11)                                                                COUT0319
      V6=T1(N12)                                                                COUT0320
      TM$(1,1) = 0.75D0*(B1*B1+C1*C1)*CONST                                     COUT0321
      TM$(1,2) = (B1*B2+C1*C2)*CONST                                            COUT0322
      TM$(1,3) = -TM$(1,2)*0.25D0                                               COUT0323
      TM$(1,4) = 0.                                                             COUT0324
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      TM$(1,6) = (B1*B3+C1*C3)*CONST                                                    COUT0325
      TM$(1,5) = -TM$(1,6)*.25D0                                                        COUT0326
      TM$(3,3) = .75D0*(B2*B2+C2*C2)*CONST                                              COUT0327
      TM$(3,4) = (B2*B3+C2*C3)*CONST                                                    COUT0328
      TM$(3,5) = -TM$(3,4)*0.25D0                                                       COUT0329
      TM$(2,1) = TM$(1,2)                                                               COUT0330
      TM$(2,3) = TM$(1,2)                                                               COUT0331
      TM$(2,2) = 8.D0/3.D0*(TM$(1,1)+TM$(3,3))+2.D0*TM$(1,2)                            COUT0332
      TM$(2,4) = 2.D0*TM$(1,6)+TM$(3,4)+TM$(1,2)+4.D0/3.D0*TM$(3,3)                     COUT0333
      TM$(2,5) = 0.                                                                     COUT0334
      TM$(2,6) = TM$(1,6)+2.D0*TM$(3,4)+TM$(1,2)+4.D0/3.D0*TM$(1,1)                     COUT0335
      TM$(3,1) = TM$(1,3)                                                               COUT0336
      TM$(3,2) = TM$(2,3)                                                               COUT0337
      TM$(3,6) = 0.                                                                     COUT0338
      TM$(6,3) = TM$(3,6)                                                               COUT0339
      TM$(5,5) = .75D0*(B3*B3+C3*C3)*CONST                                              COUT0340
      TM$(4,1) = TM$(1,4)                                                               COUT0341
      TM$(4,2) = TM$(2,4)                                                               COUT0342
      TM$(4,3) = TM$(3,4)                                                               COUT0343
      TM$(4,4) = 8.D0/3.D0*(TM$(3,3)+TM$(5,5))+2.D0*TM$(3,4)                            COUT0344
      TM$(4,5) = TM$(3,4)                                                               COUT0345
      TM$(4,6) = TM$(1,6)+TM$(3,4)+2.D0*TM$(1,2)+4.D0/3.D0*TM$(5,5)                     COUT0346
      TM$(5,1) = TM$(1,5)                                                               COUT0347
      TM$(5,2) = TM$(2,5)                                                               COUT0348
      TM$(5,3) = TM$(3,5)                                                               COUT0349
      TM$(5,4) = TM$(4,5)                                                               COUT0350
      TM$(5,6) = TM$(1,6)                                                               COUT0351
      TM$(6,1) = TM$(1,6)                                                               COUT0352
      TM$(6,2) = TM$(2,6)                                                               COUT0353
      TM$(6,4) = TM$(4,6)                                                               COUT0354
      TM$(6,5) = TM$(5,6)                                                               COUT0355
      TM$(6,6) = 8.D0/3.D0*(TM$(5,5)+TM$(1,1))+2.D0*TM$(1,6)                            COUT0356
                                                                                        COUT0357
C     BEGIN INPUT OF NONLINEAR TERMS                                                    COUT0358
                                                                                        COUT0359
      TM$(1,1)=TM$(1,1)                                                                 COUT0360
     1-(78.D0*U1+48.D0*U2-9.D0*U3+12.D0*U4-9.D0*U5+48.D0*U6)*F1                         COUT0361
     2              -(78.D0*V1+48.D0*V2-9.D0*V3+12.D0*V4-9.D0*V5+48.D0*V6)COUT0362
     3*G1                                                                               COUT0363
      TM$(2,1)=TM$(2,1)                                                                 COUT0364
     1-(48.D0*U1+160.D0*U2-32.D0*U3+16.D0*U4-20.D0*U5+80.D0*U6)*F1                      COUT0365
     2              -(48.D0*V1+160.D0*V2-32.D0*V3+16.D0*V4-20.D0*V5+80.D0*COUT0366
     3V6)*G1                                                                            COUT0367
      TM$(3,1)=TM$(3,1)                                                                 COUT0368
     1-(-9.D0*U1-32.D0*U2-18.D0*U3-16.D0*U4+11.D0*U5-20.D0*U6)*F1                      COUT0369
     2              -(-9.D0*V1-32.D0*V2-18.D0*V3-16.D0*V4+11.D0*V5-20.D0*VCOUT0370
     36)*G1                                                                             COUT0371
      TM$(4,1)=TM$(4,1)                                                                 COUT0372
```

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1-(12.D0*U1+16.D0*U2-16.D0*U3-96.D0*U4-16.D0*U5+16.D0*U6)*F1          COUT0373
2              -(12.D0*V1+16.D0*V2-16.D0*V3-96.D0*V4-16.D0*V5+16.D0*VCOUT0374
36)*G1                                                               COUT0375
 TM$(5,1)=TM$(5,1)                                                   COUT0376
1-(-9.D0*U1-20.D0*U2+11.D0*U3-16.D0*U4-18.D0*U5-32.D0*U6)*F1         COUT0377
2              -(-9.D0*V1-20.D0*V2+11.D0*V3-16.D0*V4-18.D0*V5-32.D0*VCOUT0378
36)*G1                                                               COUT0379
 TM$(6,1)=TM$(6,1)                                                   COUT0380
1-(48.D0*U1+80.D0*U2-20.D0*U3+16.D0*U4-32.D0*U5+160.D0*U6)*F1        COUT0381
2              -(48.D0*V1+80.D0*V2-20.D0*V3+16.D0*V4-32.D0*V5+160.D0*COUT0382
3V6)*G1                                                              COUT0383
 TM$(1,2)=TM$(1,2)                                                   COUT0384
1-(24.D0*U1-32.D0*U2-16.D0*U3-48.D0*U4+4.D0*U5-16.D0*U6)*F1          COUT0385
2              -(24.D0*V1-32.D0*V2-16.D0*V3-48.D0*V4+4.D0*V5-16.D0*V6COUT0386
3)*G1                          -(120.D0*U1+48.D0*U2-16.D0*U3-16.D0*U4-16.COUT0387
4D0*U5+48.D0*U6)*F2                            -(120.D0*V1+48.D0*V2-16.D0*V3-COUT0388
516.D0*V4-16.D0*V5+48.D0*V6)*G2                                      COUT0389
 TM$(2,2)=TM$(2,2)                                                   COUT0390
1-(-32.D0*U1+384.D0*U2+48.D0*U3+192.D0*U4-48.D0*U5+128.D0*U6)*F1     COUT0391
2              -(-32.D0*V1+384.D0*V2+48.D0*V3+192.D0*V4-48.D0*V5+128.COUT0392
3D0*V6)*G1                     -(48.D0*U1+384.D0*U2-32.D0*U3+128.D0*U4-48COUT0393
4.D0*U5+192.D0*U6)*F2                          -(48.D0*V1+384.D0*V2-32.D0*V3+COUT0394
5128.D0*V4-48.D0*V5+192.D0*V6)*G2                                    COUT0395
 TM$(3,2)=TM$(3,2)                                                   COUT0396
1-(-16.D0*U1+48.D0*U2+120.D0*U3+48.D0*U4-16.D0*U5-16.D0*U6)*F1       COUT0397
2              -(-16.D0*V1+48.D0*V2+120.D0*V3+48.D0*V4-16.D0*V5-16.D0COUT0398
3*V6)*G1                       -(-16.D0*U1-32.D0*U2+24.D0*U3-16.D0*U4+4.DCOUT0399
40*U5-48.D0*U6)*F2                             -(-16.D0*V1-32.D0*V2+24.D0*V3-COUT0400
516.D0*V4+4.D0*V5-48.D0*V6)*G2                                       COUT0401
 TM$(4,2)=TM$(4,2)                                                   COUT0402
1-(-48.D0*U1+192.D0*U2+48.D0*U3+384.D0*U4-32.D0*U5+128.D0*U6)*F1     COUT0403
2              -(-48.D0*V1+192.D0*V2+48.D0*V3+384.D0*V4-32.D0*V5+128.COUT0404
3D0*V6)*G1                     -(-16.D0*U1+128.D0*U2-16.D0*U3+128.D0*U4-1COUT0405
46.D0*U5+128.D0*U6)*F2                         -(-16.D0*V1+128.D0*V2-16.D0*V3COUT0406
5+128.D0*V4-16.D0*V5+128.D0*V6)*G2                                   COUT0407
 TM$(5,2)=TM$(5,2)                                                   COUT0408
1-(4.D0*U1-48.D0*U2-16.D0*U3-32.D0*U4+24.D0*U5-16.D0*U6)*F1          COUT0409
2              -(4.D0*V1-48.D0*V2-16.D0*V3-32.D0*V4+24.D0*V5-16.D0*V6COUT0410
3)*G1                          -(-16.D0*U1-48.D0*U2+4.D0*U3-16.D0*U4+24.DCOUT0411
40*U5-32.D0*U6)*F2                             -(-16.D0*V1-48.D0*V2+4.D0*V3-1COUT0412
56.D0*V4+24.D0*V5-32.D0*V6)*G2                                       COUT0413
 TM$(6,2)=TM$(6,2)                                                   COUT0414
1-(-16.D0*U1+128.D0*U2-16.D0*U3+128.D0*U4-16.D0*U5+128.D0*U6)*F1     COUT0415
2              -(-16.D0*V1+128.D0*V2-16.D0*V3+128.D0*V4-16.D0*V5+128.COUT0416
3D0*V6)*G1                     -(48.D0*U1+192.D0*U2-48.D0*U3+128.D0*U4-32COUT0417
4.D0*U5+384.D0*U6)*F2                          -(48.D0*V1+192.D0*V2-48.D0*V3+COUT0418
5128.D0*V4-32.D0*V5+384.D0*V6)*G2                                    COUT0419
 TM$(1,3)=TM$(1,3)                                                   COUT0420
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1-(-18.D0*U1-32.D0*U2-9.D0*U3-20.D0*U4+11.D0*U5-16.D0*U6)*F2          COUT0421
2              -(-18.D0*V1-32.D0*V2-9.D0*V3-20.D0*V4+11.D0*V5-16.D0*VCOUT0422
36)*G2                                                               COUT0423
 TM$(2,3)=TM$(2,3)                                                   COUT0424
1-(-32.D0*U1+160.D0*U2+48.D0*U3+80.D0*U4-20.D0*U5+16.D0*U6)*F2       COUT0425
2              -(-32.D0*V1+160.D0*V2+48.D0*V3+80.D0*V4-20.D0*V5+16.D0COUT0426
3*V6)*G2                                                             COUT0427
 TM$(3,3)=TM$(3,3)                                                   COUT0428
1-(-9.D0*U1+48.D0*U2+78.D0*U3+48.D0*U4-9.D0*U5+12.D0*U6)*F2          COUT0429
2              -(-9.D0*V1+48.D0*V2+78.D0*V3+48.D0*V4-9.D0*V5+12.D0*V6COUT0430
3)*G2                                                                COUT0431
 TM$(4,3)=TM$(4,3)                                                   COUT0432
1-(-20.D0*U1+80.D0*U2+48.D0*U3+160.D0*U4-32.D0*U5+16.D0*U6)*F2       COUT0433
2              -(-20.D0*V1+80.D0*V2+48.D0*V3+160.D0*V4-32.D0*V5+16.D0COUT0434
3*V6)*G2                                                             COUT0435
 TM$(5,3)=TM$(5,3)                                                   COUT0436
1-(11.D0*U1-20.D0*U2-9.D0*U3-32.D0*U4-18.D0*U5-16.D0*U6)*F2          COUT0437
2              -(11.D0*V1-20.D0*V2-9.D0*V3-32.D0*V4-18.D0*V5-16.D0*V6COUT0438
3)*G2                                                                COUT0439
 TM$(6,3)=TM$(6,3)                                                   COUT0440
1-(-16.D0*U1+16.D0*U2+12.D0*U3+16.D0*U4-16.D0*U5-96.D0*U6)*F2        COUT0441
2              -(-16.D0*V1+16.D0*V2+12.D0*V3+16.D0*V4-16.D0*V5-96.D0*COUT0442
3V6)*G2                                                              COUT0443
 TM$(1,4)=TM$(1,4)                                                   COUT0444
1-(24.D0*U1-16.D0*U2+4.D0*U3-48.D0*U4-16.D0*U5-32.D0*U6)*F2          COUT0445
2              -(24.D0*V1-16.D0*V2+4.D0*V3-48.D0*V4-16.D0*V5-32.D0*V6COUT0446
3)*G2                         -(24.D0*U1-32.D0*U2-16.D0*U3-48.D0*U4+4.DOCOUT0447
4*U5-16.D0*U6)*F3                             -(24.D0*V1-32.D0*V2-16.D0*V3-4COUT0448
58.D0*V4+4.D0*V5-16.D0*V6)*G3                                        COUT0449
 TM$(2,4)=TM$(2,4)                                                   COUT0450
1-(-16.D0*U1+128.D0*U2-16.D0*U3+128.D0*U4-16.D0*U5+128.D0*U6)*F2     COUT0451
2              -(-16.D0*V1+128.D0*V2-16.D0*V3+128.D0*V4-16.D0*V5+128.COUT0452
3D0*V6)*G2                    -(-32.D0*U1+384.D0*U2+48.D0*U3+192.D0*U4-4COUT0453
48.D0*U5+128.D0*U6)*F3                        -(-32.D0*V1+384.D0*V2+48.D0*V3COUT0454
5+192.D0*V4-48.D0*V5+128.D0*V6)*G3                                   COUT0455
 TM$(3,4)=TM$(3,4)                                                   COUT0456
1-(4.D0*U1-16.D0*U2+24.D0*U3-32.D0*U4+16.D0*U5-48.D0*U6)*F2          COUT0457
2              -(4.D0*V1-16.D0*V2+24.D0*V3-32.D0*V4+16.D0*V5-48.D0*V6COUT0458
3)*G2                         -(-16.D0*U1+48.D0*U2+120.D0*U3+48.D0*U4-16COUT0459
4.D0*U5-16.D0*U6)*F3                          -(-16.D0*V1+48.D0*V2+120.D0*V3COUT0460
5+48.D0*V4-16.D0*V5-16.D0*V6)*G3                                     COUT0461
 TM$(4,4)=TM$(4,4)                                                   COUT0462
1-(-48.D0*U1+128.D0*U2-32.D0*U3+384.D0*U4+48.D0*U5+192.D0*U6)*F2     COUT0463
2              -(-48.D0*V1+128.D0*V2-32.D0*V3+384.D0*V4+48.D0*V5+192.COUT0464
3D0*V6)*G2                    -(-48.D0*U1+192.D0*U2+48.D0*U3+384.D0*U4-3COUT0465
42.D0*U5+128.D0*U6)*F3                        -(-48.D0*V1+192.D0*V2+48.D0*V3COUT0466
5+384.D0*V4-32.D0*V5+128.D0*V6)*G3                                   COUT0467
 TM$(5,4)=TM$(5,4)                                                   COUT0468
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1-(-16.D0*U1-16.D0*U2-16.D0*U3+48.D0*U4+120.D0*U5+48.D0*U6)*F2      COUT0469
2              -(-16.D0*V1-16.D0*V2-16.D0*V3+48.D0*V4+120.D0*V5+48.DOCOUT0470
3*V6)*G2                       -(4.D0*U1-48.D0*U2-16.D0*U3-32.D0*U4+24.DOCOUT0471
4*U5-16.D0*U6)*F3                                -(4.D0*V1-48.D0*V2-16.D0*V3-32COUT0472
5.D0*V4+24.D0*V5-16.D0*V6)*G3                                              COUT0473
 TM$(6,4)=TM$(6,4)                                                         COUT0474
1-(-32.D0*U1+128.D0*U2-48.D0*U3+192.D0*U4+48.D0*U5+384.D0*U6)*F2    COUT0475
2              -(-32.D0*V1+128.D0*V2-48.D0*V3+192.D0*V4+48.D0*V5+384.COUT0476
3D0*V6)*G2                     -(-16.D0*U1+128.D0*U2-16.D0*U3+128.D0*U4-1COUT0477
46.D0*U5+128.D0*U6)*F3                           -(-16.D0*V1+128.D0*V2-16.D0*V3COUT0478
5+128.D0*V4-16.D0*V5+128.D0*V6)*G3                                         COUT0479
 TM$(1,5)=TM$(1,5)                                                         COUT0480
1-(-18.D0*U1-16.D0*U2+11.D0*U3-20.D0*U4-9.D0*U5-32.D0*U6)*F3        COUT0481
2              -(-18.D0*V1-16.D0*V2+11.D0*V3-20.D0*V4-9.D0*V5-32.D0*VCOUT0482
36)*G3                                                                     COUT0483
 TM$(2,5)=TM$(2,5)                                                         COUT0484
1-(-16.D0*U1-96.D0*U2-16.D0*U3+16.D0*U4+12.D0*U5+16.D0*U6)*F3       COUT0485
2              -(-16.D0*V1-96.D0*V2-16.D0*V3+16.D0*V4+12.D0*V5+16.D0*COUT0486
3V6)*G3                                                                    COUT0487
 TM$(3,5)=TM$(3,5)                                                         COUT0488
1-(11.D0*U1-16.D0*U2-18.D0*U3-32.D0*U4-9.D0*U5-20.D0*U6)*F3         COUT0489
2              -(11.D0*V1-16.D0*V2-18.D0*V3-32.D0*V4-9.D0*V5-20.D0*V6COUT0490
3)*G3                                                                      COUT0491
 TM$(4,5)=TM$(4,5)                                                         COUT0492
1-(-20.D0*U1+16.D0*U2-32.D0*U3+160.D0*U4+48.D0*U5+80.D0*U6)*F3      COUT0493
2              -(-20.D0*V1+16.D0*V2-32.D0*V3+160.D0*V4+48.D0*V5+80.DOCOUT0494
3*V6)*G3                                                                   COUT0495
 TM$(5,5)=TM$(5,5)                                                         COUT0496
1-(-9.D0*U1+12.D0*U2-9.D0*U3+48.D0*U4+78.D0*U5+48.D0*U6)*F3         COUT0497
2              -(-9.D0*V1+12.D0*V2-9.D0*V3+48.D0*V4+78.D0*V5+48.D0*V6COUT0498
3)*G3                                                                      COUT0499
 TM$(6,5)=TM$(6,5)                                                         COUT0500
1-(-32.D0*U1+16.D0*U2-20.D0*U3+80.D0*U4+48.D0*U5+160.D0*U6)*F3      COUT0501
2              -(-32.D0*V1+16.D0*V2-20.D0*V3+80.D0*V4+48.D0*V5+160.DOCOUT0502
3*V6)*G3                                                                   COUT0503
 TM$(1,6)=TM$(1,6)                                                         COUT0504
1-(24.D0*U1-16.D0*U2+4.D0*U3-48.D0*U4-16.D0*U5-32.D0*U6)*F1         COUT0505
2              -(24.D0*V1-16.D0*V2+4.D0*V3-48.D0*V4-16.D0*V5-32.D0*V6COUT0506
3)*G1                          -(120.D0*U1+48.D0*U2-16.D0*U3-16.D0*U4-16.COUT0507
4D0*U5+48.D0*U6)*F3                              -(120.D0*V1+48.D0*V2-16.D0*V3-COUT0508
516.D0*V4-16.D0*V5+48.D0*V6)*G3                                            COUT0509
 TM$(2,6)=TM$(2,6)                                                         COUT0510
1-(-16.D0*U1+128.D0*U2-16.D0*U3+128.D0*U4-16.D0*U5+128.D0*U6)*F1    COUT0511
2              -(-16.D0*V1+128.D0*V2-16.D0*V3+128.D0*V4-16.D0*V5+128.COUT0512
3D0*V6)*G1                     -(48.D0*U1+384.D0*U2-32.D0*U3+128.D0*U4-48COUT0513
4.D0*U5+192.D0*U6)*F3                            -(48.D0*V1+384.D0*V2-32.D0*V3+COUT0514
5128.D0*V4-48.D0*V5+192.D0*V6)*G3                                          COUT0515
 TM$(3,6)=TM$(3,6)                                                         COUT0516
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```
1-(4.D0*U1-16.D0*U2+24.D0*U3-32.D0*U4-16.D0*U5-48.D0*U6)*F1          COUT0517
2              -(4.D0*V1-16.D0*V2+24.D0*V3-32.D0*V4-16.D0*V5-48.D0*V6COUT0518
3)*G1                          -(-16.D0*U1-32.D0*U2+24.D0*U3-16.D0*U4+4.DCOUT0519
40*U5-48.D0*U6)*F3                                -(-16.D0*V1-32.D0*V2+24.D0*V3-COUT0520
516.D0*V4+4.D0*V5-48.D0*V6)*G3                                          COUT0521
 TM$(4,6)=TM$(4,6)                                                      COUT0522
1-(-48.D0*U1+128.D0*U2-32.D0*U3+384.D0*U4+48.D0*U5+192.D0*U6)*F1       COUT0523
2              -(-48.D0*V1+128.D0*V2-32.D0*V3+384.D0*V4+48.D0*V5+192.COUT0524
3D0*V6)*G1                     -(-16.D0*U1+128.D0*U2-16.D0*U3+128.D0*U4-1COUT0525
46.D0*U5+128.D0*U6)*F3                            -(-16.D0*V1+128.D0*V2-16.D0*V3COUT0526
5+128.D0*V4-16.D0*V5+128.D0*V6)*G3                                      COUT0527
 TM$(5,6)=TM$(5,6)                                                      COUT0528
1-(-16.D0*U1-16.D0*U2-16.D0*U3+48.D0*U4+120.D0*U5+48.D0*U6)*F1         COUT0529
2              -(-16.D0*V1-16.D0*V2-16.D0*V3+48.D0*V4+120.D0*V5+48.DOCOUT0530
3*V6)*G1                       -(-16.D0*U1-48.D0*U2+4.D0*U3-16.D0*U4+24.DCOUT0531
40*U5-32.D0*U6)*F3                                -(-16.D0*V1-48.D0*V2+4.D0*V3-1COUT0532
56.D0*V4+24.D0*V5-32.D0*V6)*G3                                          COUT0533
 TM$(6,6)=TM$(6,6)                                                      COUT0534
1-(-32.D0*U1+128.D0*U2-48.D0*U3+192.D0*U4+48.D0*U5+384.D0*U6)*F1       COUT0535
2              -(-32.D0*V1+128.D0*V2-48.D0*V3+192.D0*V4+48.D0*V5+384.COUT0536
3D0*V6)*G1                     -(48.D0*U1+192.D0*U2-48.D0*U3+128.D0*U4-32COUT0537
4.D0*U5+384.D0*U6)*F3                             -(48.D0*V1+192.D0*V2-48.D0*V3+COUT0538
5128.D0*V4-32.D0*V5+384.D0*V6)*G3                                       COUT0539
                                                                        COUT0540
C     THIS ENDS ADDITION OF NONLINEAR TERMS TO THE LOCAL ARRAY          COUT0541
                                                                        COUT0542
      TM$(7,7) = TM$(1,1)                                               COUT0543
      TM$(7,8) = TM$(1,2)                                               COUT0544
      TM$(7,9) = TM$(1,3)                                               COUT0545
      TM$(7,10) = TM$(1,4)                                              COUT0546
      TM$(7,11) = TM$(1,5)                                              COUT0547
      TM$(7,12) = TM$(1,6)                                              COUT0548
      TM$(8,7) = TM$(2,1)                                               COUT0549
      TM$(8,8) = TM$(2,2)                                               COUT0550
      TM$(8,9) = TM$(2,3)                                               COUT0551
      TM$(8,10) = TM$(2,4)                                              COUT0552
      TM$(8,11) = TM$(2,5)                                              COUT0553
      TM$(8,12) = TM$(2,6)                                              COUT0554
      TM$(9,7) = TM$(3,1)                                               COUT0555
      TM$(9,8) = TM$(3,2)                                               COUT0556
      TM$(9,9) = TM$(3,3)                                               COUT0557
      TM$(9,10) = TM$(3,4)                                              COUT0558
      TM$(9,11) = TM$(3,5)                                              COUT0559
      TM$(9,12) = TM$(3,6)                                              COUT0560
      TM$(10,7) = TM$(4,1)                                              COUT0561
      TM$(10,8) = TM$(4,2)                                              COUT0562
      TM$(10,9) = TM$(4,3)                                              COUT0563
      TM$(10,10) = TM$(4,4)                                             COUT0564
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```
TM$(10,11) = TM$(4,5)                                    COUT0565
TM$(10,12) = TM$(4,6)                                    COUT0566
TM$(11,7) = TM$(5,1)                                     COUT0567
TM$(11,8) = TM$(5,2)                                     COUT0568
TM$(11,9) = TM$(5,3)                                     COUT0569
TM$(11,10) = TM$(5,4)                                    COUT0570
TM$(11,11) = TM$(5,5)                                    COUT0571
TM$(11,12) = TM$(5,6)                                    COUT0572
TM$(12,7) = TM$(6,1)                                     COUT0573
TM$(12,8) = TM$(6,2)                                     COUT0574
TM$(12,9) = TM$(6,3)                                     COUT0575
TM$(12,10) = TM$(6,4)                                    COUT0576
TM$(12,11) = TM$(6,5)                                    COUT0577
TM$(12,12) = TM$(6,6)                                    COUT0578
TM$(13,1) = D1                                           COUT0579
TM$(1,13) = TM$(13,1)                                    COUT0580
TM$(14,1) = 0.D0                                         COUT0581
TM$(1,14) = TM$(14,1)                                    COUT0582
TM$(15,1) = 0.D0                                         COUT0583
TM$(1,15) = TM$(15,1)                                    COUT0584
TM$(13,2) = D1+2.D0*D2                                   COUT0585
TM$(2,13) = TM$(13,2)                                    COUT0586
TM$(14,2) = 2.D0*D1+D2                                   COUT0587
TM$(2,14) = TM$(14,2)                                    COUT0588
TM$(15,2) = D1+D2                                        COUT0589
TM$(2,15) = TM$(15,2)                                    COUT0590
TM$(13,3) = 0.D0                                         COUT0591
TM$(3,13) = TM$(13,3)                                    COUT0592
TM$(14,3) = D2                                           COUT0593
TM$(3,14) = TM$(14,3)                                    COUT0594
TM$(15,3) = 0.D0                                         COUT0595
TM$(3,15) = TM$(15,3)                                    COUT0596
TM$(13,4) = D2+D3                                        COUT0597
TM$(4,13) = TM$(13,4)                                    COUT0598
TM$(14,4) = D2+2.D0*D3                                   COUT0599
TM$(4,14) = TM$(14,4)                                    COUT0600
TM$(15,4) = 2.D0*D2+D3                                   COUT0601
TM$(4,15) = TM$(15,4)                                    COUT0602
TM$(13,5) = 0.D0                                         COUT0603
TM$(5,13) = TM$(13,5)                                    COUT0604
TM$(14,5) = 0.D0                                         COUT0605
TM$(5,14) = TM$(14,5)                                    COUT0606
TM$(15,5) = D3                                           COUT0607
TM$(5,15) = TM$(15,5)                                    COUT0608
TM$(13,6) = D1+2.D0*D3                                   COUT0609
TM$(6,13) = TM$(13,6)                                    COUT0610
TM$(14,6) = D1+D3                                        COUT0611
TM$(6,14) = TM$(14,6)                                    COUT0612
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      TM$(15,6) = 2.D0*D1+D3                                            COUT0613
      TM$(6,15) = TM$(15,6)                                             COUT0614
      TM$(13,7) = E1                                                    COUT0615
      TM$(7,13) = TM$(13,7)                                             COUT0616
      TM$(14,7) = 0.D0                                                  COUT0617
      TM$(7,14) = TM$(14,7)                                             COUT0618
      TM$(15,7) = 0.D0                                                  COUT0619
      TM$(7,15) = TM$(15,7)                                             COUT0620
      TM$(13,8) = E1+2.D0*E2                                            COUT0621
      TM$(8,13) = TM$(13,8)                                             COUT0622
      TM$(14,8) = 2.D0*E1+E2                                            COUT0623
      TM$(8,14) = TM$(14,8)                                             COUT0624
      TM$(15,8) = E1+E2                                                 COUT0625
      TM$(8,15) = TM$(15,8)                                             COUT0626
      TM$(13,9) = 0.D0                                                  COUT0627
      TM$(9,13) = TM$(13,9)                                             COUT0628
      TM$(14,9) = E2                                                    COUT0629
      TM$(9,14) = TM$(14,9)                                             COUT0630
      TM$(15,9) = 0.D0                                                  COUT0631
      TM$(9,15) = TM$(15,9)                                             COUT0632
      TM$(13,10) = E2+E3                                                COUT0633
      TM$(10,13) = TM$(13,10)                                           COUT0634
      TM$(14,10) = E2+2.D0*E3                                           COUT0635
      TM$(10,14) = TM$(14,10)                                           COUT0636
      TM$(15,10) = E2+E2+E3                                             COUT0637
      TM$(10,15) = TM$(15,10)                                           COUT0638
      TM$(13,11) = 0.D0                                                 COUT0639
      TM$(11,13) = TM$(13,11)                                           COUT0640
      TM$(14,11) = 0.D0                                                 COUT0641
      TM$(11,14) = TM$(14,11)                                           COUT0642
      TM$(15,11) = E3                                                   COUT0643
      TM$(11,15) = TM$(15,11)                                           COUT0644
      TM$(13,12) = E1+E3+E3                                             COUT0645
      TM$(12,13) = TM$(13,12)                                           COUT0646
      TM$(14,12) = E1+E3                                                COUT0647
      TM$(12,14) = TM$(14,12)                                           COUT0648
      TM$(15,12) = 2.D0*E1+E3                                           COUT0649
      TM$(12,15) = TM$(15,12)                                           COUT0650
  185 CONTINUE                                                          COUT0651
      N(1)=N1                                                           COUT0652
      N(2)=N2                                                           COUT0653
      N(3)=N3                                                           COUT0654
      N(4)=N4                                                           COUT0655
      N(5)=N5                                                           COUT0656
      N(6)=N6                                                           COUT0657
      N(7)=N7                                                           COUT0658
      N(8)=N8                                                           COUT0659
      N(9)=N9                                                           COUT0660
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      N(10)=N10                                                          COUT0661
      N(11)=N11                                                          COUT0662
      N(12)=N12                                                          COUT0663
      N(13)=N13                                                          COUT0664
      N(14)=N14                                                          COUT0665
      N(15)=N15                                                          COUT0666
C                                                                        COUT0667
      DO 3100 I$=1,15                                                    COUT0668
      I = N(I$)                                                          COUT0669
C                                                                        COUT0670
      DO 3100 J$=1,15                                                    COUT0671
      J = N(J$)                                                          COUT0672
      TM(I,J) = TM(I,J)+TM$(I$,J$)                                       COUT0673
 3100 CONTINUE                                                           COUT0674
C                                                                        COUT0675
 3200 CONTINUE                                                           COUT0676
C                                                                        COUT0677
C                                                                        COUT0678
      DO 3300 I=1,NNQXY                                                  COUT0679
      RHS(NQS(I)) = RHS(NQS(I))+Q(NQS(I))                                COUT0680
      RHS(NQS(I)+NN) = RHS(NQS(I)+NN)+Q(NQS(I)+NN)                       COUT0681
 3300 CONTINUE                                                           COUT0682
C                                                                        COUT0683
 3400 CONTINUE                                                           COUT0684
C                                                                        COUT0685
CC   INSERT SYSTEM BOUNDARY CONDITIONS                                   COUT0686
C                                                                        COUT0687
      DO 3500 I=1,MM                                                     COUT0688
C                                                                        COUT0689
      DO 3500 J=1,NTOTVP                                                 COUT0690
      JX = NVIS(J)                                                       COUT0691
      RHS(I) = RHS(I)-TM(I,JX)*T(JX)                                     COUT0692
      TM(I,JX) = 0.D0                                                    COUT0693
      TM(JX,I) = 0.D0                                                    COUT0694
      TM(JX,JX) = 1.D0                                                   COUT0695
      RHS(JX) = T(JX)                                                    COUT0696
 3500 CONTINUE                                                           COUT0697
C                                                                        COUT0698
      M = 1                                                              COUT0699
      ND = 82                                                            COUT0700
      IA = 82                                                            COUT0701
      IDGT = 0                                                           COUT0702
      CALL LEQT2F (TM,M,ND,IA,RHS,IDGT,WKAREA,IER)                       COUT0703
      WRITE (NWRITE,5700)                                                COUT0704
      DO 322 J=1,MM                                                      COUT0705
      TDIFF=DABS(T1(J)-T(J))                                             COUT0706
      EPSLN=1.D-06                                                       COUT0707
      IF(TDIFF-EPSLN) 322,324,324                                        COUT0708
```

```
  322 CONTINUE                                                          COUT0709
  324 CONTINUE                                                          COUT0710
C                                                                       COUT0711
      DO 3600 I=1,MM                                                    COUT0712
      T(I) = RHS(I)                                                     COUT0713
      T1(I)=T(I)                                                        COUT0714
      WRITE (NWRITE,5800) I,T(I)                                        COUT0715
 3600 CONTINUE                                                          COUT0716
      IF(J.NE.MM) GO TO 2840                                            COUT0717
      WRITE(NWRITE,1091)                                                COUT0718
C                                                                       COUT0719
 1090 FORMAT(////,5X,'NODAL VARIABLE IS THE U-VELOCITY AT NODES 1 - 35;'COUT0720
     1,//,5X,'THE V-VELOCITY AT NODES 36 - 70;',                        COUT0721
     2//,5X,'AND THE PRESSURES AT NODES 71 - 82.',////)                 COUT0722
 1091 FORMAT(1H1)                                                       COUT0723
 1092 FORMAT(////,5X,'THE FIRST SEQUENCE OF 82 NODAL VARIABLES',        COUT0724
     1//,5X,'REPRESENTS A LINEAR, STEADY STATE SYSTEM;',                COUT0725
     2//,5X,'WHILE THE SECOND SET OF THE 82 VALUES CORRESPONDS',        COUT0726
     3//,5X,'TO A NONLINEAR ANALYSIS OF THE SOLUTION DOMAIN.',////)     COUT0727
 1093 FORMAT(////,5X,'A PRESSURE GRADIENT IN THE HORIZONTAL SHEAR'      COUT0728
     1,//,5X,'DIRECTION OF -3 UNITS IN MAGNITUDE HAS BEEN ADDED'        COUT0729
     2,//,5X,'TO PRODUCE A CURVED VELOCITY PROFILE.',////)              COUT0730
 1094 FORMAT(1H1,15X,'THIS IS A 2-D NONLINEAR COUETTE FLOW PROBLEM',//) COUT0731
 3700 FORMAT (5X,'IBAND=',I3,//,5X,'NEQ =',I3,//)                       COUT0732
 3800 FORMAT (3I10)                                                     COUT0733
 3900 FORMAT (6X,A4,I10,2F10.0)                                         COUT0734
 4000 FORMAT (7I10)                                                     COUT0735
 4100 FORMAT (6X,A4,I10,F10.0)                                          COUT0736
 4300 FORMAT (5X,'NO. OF NODES=',I3,//,5X,'NO. OF ELEMENTS=',I3,//,     COUT0737
     1 5X,'NO. OF CORNER NODES=',I3,///)                                COUT0738
 4400 FORMAT (5X,'NNVELS=',I3,//)                                       COUT0739
 4500 FORMAT (5X,'NNQXY=',I3,//)                                        COUT0740
 4600 FORMAT (5X,'NNPS=',I3,//)                                         COUT0741
 4700 FORMAT (5X,'SUMMARY OF NODAL COORDINATES',//,                     COUT0742
     1 7X,'I',12X'X(I)',13X,'Y(I)',//)                                  COUT0743
 4800 FORMAT (5X,I3,2(7X,F10.3))                                        COUT0744
 4900 FORMAT(//,5X,'LISTING OF SYSTEM TOPOLOGY',//,5X,                  COUT0745
     1 'ELEMENT NUMBER',20X,'NODE NUMBERS',//)                          COUT0746
 5000 FORMAT (5X,I3,10X,6(5X,I3))                                       COUT0747
 5100 FORMAT (//,7X,'NODES WHERE VELOCITIES ARE SPECIFIED',             COUT0748
     1 //,8X,'I',5X,'NODE',5X,'U VELOCITY',5X,'V VELOCITY',//)          COUT0749
 5200 FORMAT (2X,2(4X,I3),3X,F12.3,3X,F12.3)                            COUT0750
 5300 FORMAT (//,5X,'NODES WHERE QX AND QY ARE SPECIFIED',              COUT0751
     1        //,2X,'      I    NODE',1X,'        QX',10X,'   QY',//)  COUT0752
 5400 FORMAT (//,5X,'NODES WHERE PRESSURE IS SPECIFIED',                COUT0753
     1              //,5X,'I',5X,'NODE',15X,'PRESSURE',//)              COUT0754
 5600 FORMAT (7X,I3,3X,I3,10X,F12.3)                                    COUT0755
 5700 FORMAT (///,5X,'NODE NO.',6X,'NODE VARIABLES',//)                 COUT0756
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5800 FORMAT (9X,I3,5X,D17.10,/)                                    COUT0757
     STOP                                                          COUT0758
     END                                                           COUT0759
```

```
      IMPLICIT REAL*8(A-H,O-Z,$)                                        FLSS0110
      DATA NREAD/5/                                                     FLSS0120
                                                                        FLSS0130
THE U VELOCITY IS IN THE FIRST NN POSITIONS OF X(I)                     FLSS0140
THE V VELOCITY IS IN THE SECOND NN POSITIONS OF X I.E. X(I+NN)          FLSS0150
THE P PRESSURE IS IN THE NN+NN+I POSITIONS OF X I.E. X(I+NN+NN)         FLSS0160
THE T TEMPERATURE IS IN THE NN+NN+NNCN+I PPOSITIONS OF X I.E.           FLSS0170
X(I+NN+NN+NNCN)                                                         FLSS0180
THERE ARE NNCN PRESSURE NODES (NNCN=NUMBER OF CORNER NODES)             FLSS0190
                                                                        FLSS0200
TM MUST BE DIMENSIONED 3*NN+NNCN X 3*NN+NNCN                            FLSS0210
                                                                        FLSS0220
                                                                        FLSS0230
      DATA NWRITE/6/                                                    FLSS0240
      DATA STOP/'STOP'/                                                 FLSS0250
      DIMENSION XC(125),YC(125),NODE(125,6),NVS(125),NCN(125)           FLSS0260
      DIMENSION X(117),NVIS(117),NCP(125),NPS(125),Q(117)               FLSS0270
      DIMENSION NQS(125),T1(117)                                        FLSS0280
      DIMENSION TM$(21,21),N(21),NQIS(117)                              FLSS0290
      DIMENSION RP$(6),ZP$(6),WKAREA(15000)                             FLSS0300
      DIMENSION XC$(3),YC$(3),RHS(117),TM(117,117)                      FLSS0310
                                                                        FLSS0320
SPECIFY WHETHER TWO DIMENSIONAL, INCLUDING NON-LINEAR TERMS,            FLSS0330
(NCASE=1) OR AXISYMMETRIC (NCASE=2)                                     FLSS0340
                                                                        FLSS0350
      READ(NREAD,500)NCASE                                              FLSS0360
      WRITE(NWRITE,600)
      IF(NCASE.EQ.1)GO TO 5                                             FLSS0370
      WRITE(NWRITE,2015)                                                FLSS0380
      GO TO 6                                                           FLSS0390
    5 WRITE(NWRITE,2020)                                                FLSS0400
    6 CONTINUE                                                          FLSS0410
                                                                        FLSS0420
THE FIRST PART OF THE PROGRAM CAN BE CONSIDERED AS AN INPUT ROUTINE     FLSS0430
IN WHICH LINES 540 TO 2970 ARE INPUT VERIFICATION OF ALL DATA.          FLSS0440
SUCH A SECTION WOULD BE PART OF ANY FINITE ELEMENT PROGRAM.             FLSS0460
```

```
C                                                                       FLSS0470
C                                                                       FLSS0480
C  READ IN NUMBER OF NODES AND ELEMENTS AND NO. OF CORNER NODES         FLSS0490
C                                                                       FLSS0500
      READ(NREAD,1005)NN,NE,NNCN                                        FLSS0510
C                                                                       FLSS0520
C  INITIALIZE ALL PARAMETERS                                            FLSS0530
C                                                                       FLSS0540
      MM=2*NN+NNCN                                                      FLSS0550
      MMM=3*NN+NNCN                                                     FLSS0560
      DO 50 I=1,MMM                                                     FLSS0570
      XC(I)=0.D0                                                        FLSS0580
      YC(I)=0.D0                                                        FLSS0590
      NVS(I)=0                                                          FLSS0600
      NCN(I)=0                                                          FLSS0610
      NCP(I)=0                                                          FLSS0620
      NPS(I)=0                                                          FLSS0630
      NCS(I)=0                                                          FLSS0640
      DO 50 J=1,6                                                       FLSS0650
      NODE(I,J)=0                                                       FLSS0660
   50 CONTINUE                                                          FLSS0670
      DO 51 I=1,MMM                                                     FLSS0680
      T1(I)=0.D0                                                        FLSS0690
      X(I)=0.D0                                                         FLSS0700
      NVIS(I)=0                                                         FLSS0710
      Q(I)=0.D0                                                         FLSS0720
      NCIS(I)=0                                                         FLSS0730
      RHS(I)=0.D0                                                       FLSS0740
      DO 51 J=1,MMM                                                     FLSS0750
      TM(I,J)=0.D0                                                      FLSS0760
   51 CONTINUE                                                          FLSS0770
      DO 52 I=1,21                                                      FLSS0780
      N(I)=0                                                            FLSS0790
      DO 52 J=1,21                                                      FLSS0800
      TM$(I,J)=0.D0                                                     FLSS0810
   52 CONTINUE                                                          FLSS0820
      DO 53 I=1,6                                                       FLSS0830
      RP$(I)=0.D0                                                       FLSS0840
      ZP$(I)=0.D0                                                       FLSS0850
   53 CONTINUE                                                          FLSS0860
      DO 54 I=1,3                                                       FLSS0870
      XC$(I)=0.D0                                                       FLSS0880
      YC$(I)=0.D0                                                       FLSS0890
   54 CONTINUE                                                          FLSS0900
C                                                                       FLSS0910
C  READ NODE NUMBER AND COORDINATES                                     FLSS0920
C                                                                       FLSS0930
      DO 100 J=1,NN
```

```
      READ(NREAD,1006)WORD,I,XC(I),YC(I)                                FLSS0940
      IF(WORD.EQ.STOP) GO TO 101                                        FLSS0950
      NCN(J)=I                                                          FLSS0960
  100 CONTINUE                                                          FLSS0970
  101 NNCN=J-1                                                          FLSS0980
                                                                        FLSS0990
THE ARRAY NCP(J) GENERATES THE GLOBAL PRESSURE INDICES (P1,P2.ETC.)     FLSS1000
THUS PRESSURE NODES ARE LABELED AS CORNER NODES AND ARE INPUTED         FLSS1010
WHEN ONE INPUTS A GLOBAL CORNER NODE FOR J                              FLSS1020
                                                                        FLSS1030
      DO 107 J=1,NNCN                                                   FLSS1040
      NCP(NCN(J))=J+NN+NN                                               FLSS1050
  107 CONTINUE                                                          FLSS1060
                                                                        FLSS1070
READ SYSTEM TOPOLOGY (ELEMENT NO. AND NODE NUMBERS IN                   FLSS1080
COUNTERCLOCKWISE FASHION STARTING AT THE UPPER LEFT                     FLSS1090
HAND CORNER NODE).                                                      FLSS1100
                                                                        FLSS1110
      DO 105 I=1,NE                                                     FLSS1120
      READ(NREAD,1010)J,NODE(J,1),NODE(J,2),NODE(J,3),                  FLSS1130
     1NODE(J,4),NODE(J,5),NODE(J,6)                                     FLSS1140
  105 CONTINUE                                                          FLSS1150
      MAXDIF=0                                                          FLSS1160
      DO 108 I=1,NE                                                     FLSS1170
      DO 108 J=1,6                                                      FLSS1180
      DO 108 K=1,6                                                      FLSS1190
      LL=IABS(NODE(I,J)-NODE(I,K))                                      FLSS1200
      IF(LL.GT.MAXDIF) MAXDIF=LL                                        FLSS1210
      IBAND=2*(MAXDIF+1)                                                FLSS1220
      NEQ=3*NN+NNCN                                                     FLSS1230
  108 CONTINUE                                                          FLSS1240
      WRITE(NWRITE,1017)IBAND,NEQ                                       FLSS1250
 1017 FORMAT(5X,'IBAND=',I3,//,5X,'NEQ=',I3,//)                         FLSS1270
                                                                        FLSS1280
READ NODES WHERE BOTH U AND V VELOCITY IS SPECIFIED                     FLSS1290
                                                                        FLSS1300
      DO 110 I=1,MM                                                     FLSS1310
      READ(NREAD,1006)WORD,NVELS,VELU,VELV                              FLSS1320
      IF(WORD.EQ.STOP) GO TO 111                                        FLSS1330
      NVS(I)=NVELS                                                      FLSS1340
      X(NVS(I))=VELU                                                    FLSS1350
      X(NVS(I)+NN)=VELV                                                 FLSS1360
  110 CONTINUE                                                          FLSS1370
                                                                        FLSS1380
COUNT NODES HAVING SPECIFIED VELOCITIES                                 FLSS1390
                                                                        FLSS1400
  111 NNVELS=I-1                                                        FLSS1410
```

```
READ QX AND QY VALUES AT INTERNAL NODES                                 FLSS1420
                                                                        FLSS1430
      DO 125 I=1,NN                                                     FLSS1440
      READ(NREAD,1006)WORD,NQXY,QXNS,QYNS                               FLSS1450
      IF(WORD.EQ.STOP) GO TO 126                                        FLSS1460
      NQS(I)=NQXY                                                       FLSS1470
      Q(NQS(I))=QXNS                                                    FLSS1480
      Q(NQS(I)+NN)=QYNS                                                 FLSS1490
  125 CONTINUE                                                          FLSS1500
                                                                        FLSS1510
COUNT NODES HAVING SPECIFIED QX AND QY                                  FLSS1520
                                                                        FLSS1530
  126 NNQXY=I-1                                                         FLSS1540
                                                                        FLSS1550
READ NODE NUMBER AND PRESSURE WHERE SPECIFIED                           FLSS1560
                                                                        FLSS1570
      DO 130 I=1,NN                                                     FLSS1580
      READ(NREAD,1025)WORD,NP,PNP                                       FLSS1590
      IF(WORD.EQ.STOP)GO TO 135                                         FLSS1600
      NPS(I)=NP                                                         FLSS1610
      X(NCP(NPS(I)))=PNP                                                FLSS1620
  130 CONTINUE                                                          FLSS1630
                                                                        FLSS1640
COUNT BOUNDARY NODES WHERE PRESSURE IS SPECIFIED                        FLSS1650
                                                                        FLSS1660
  135 NNPS=I-1                                                          FLSS1670
                                                                        FLSS1680
READ NODE NUMBER AND TEMPERATURE WHERE SPECIFIED                        FLSS1690
                                                                        FLSS1700
      DO 140 I=1,MM                                                     FLSS1710
      READ(NREAD,1025) WORD,NTEMP,TNT                                   FLSS1720
      IF(WORD.EQ.STOP) GO TO  145                                       FLSS1730
      NVS(I+ NNVELS)=NTEMP                                              FLSS1740
      X(NVS(I+NNVELS)+MM)=TNT                                           FLSS1750
  140 CONTINUE                                                          FLSS1760
                                                                        FLSS1770
COUNT NODES HAVING SPECIFIED TEMPERATURES                               FLSS1780
                                                                        FLSS1790
  145 NNTS=I-1                                                          FLSS1800
                                                                        FLSS1810
READ NODE NUMBERS AND QZC WHERE SPECIFIED                               FLSS1820
                                                                        FLSS1830
      DO 141 I=1,MM                                                     FLSS1840
      READ(NREAD,1025) WORD,NQZC,QZCNS                                  FLSS1850
      IF(WORD.EQ.STOP) GO TO 146                                        FLSS1860
      NQS(NNQXY+I)=NQZC                                                 FLSS1870
      Q(NQS(NNQXY+I)+2*NN)=QZCNS                                        FLSS1880
  141 CCNTINUE                                                          FLSS1890
```

```
C COUNT NODES WHERE QZC IS SPECIFIED                                    FLSS1900
C                                                                       FLSS1910
C                                                                       FLSS1920
  146 NNQZC=I-1                                                         FLSS1930
C                                                                       FLSS1940
C READ NODE NUMBERS AND HEAT FLUX QZ WHERE SPECIFIED                    FLSS1950
C                                                                       FLSS1960
      DO 142 I=1,MM                                                     FLSS1970
      READ(NREAD,1025) WORD,NQZ,QZNS                                    FLSS1980
      IF(WORD.EQ.STOP) GO TO  147                                       FLSS1990
      NQS(NNQXY+NNQZC+I)=NQZ                                            FLSS2000
      Q(NQS(NNQXY+NNQZC+I)+MM)=QZNS                                     FLSS2010
  142 CONTINUE                                                          FLSS2020
C                                                                       FLSS2030
C COUNT NODES WHERE HEAT FLUX QZ IS SPECIFIED                           FLSS2040
C                                                                       FLSS2050
  147 NNQZ=I-1                                                          FLSS2060
C                                                                       FLSS2070
C NQIS IS A LIST OF INDICES OF KNOWN QX,QY,QZC AND QZ                   FLSS2080
C                                                                       FLSS2090
      DO 1140 I=1,NNQXY                                                 FLSS2100
      NQIS(I)=NQS(I)                                                    FLSS2110
      NQIS(I+NNQXY)=NQS(I)+NN                                           FLSS2120
 1140 CONTINUE                                                          FLSS2130
      DO 1141 I=1,NNQZC                                                 FLSS2140
      NQIS(2*NNQXY+I)=NQS(NNQXY+I)+2*NN                                 FLSS2150
 1141 CONTINUE                                                          FLSS2160
      DO 1145 I=1,NNQZ                                                  FLSS2170
      NQIS(2*NNQXY+NNQZC+I)=NQS(NNQXY+NNQZC+I)+MM                       FLSS2180
 1145 CONTINUE                                                          FLSS2190
C                                                                       FLSS2200
C NVIS IS A LIST OF KNOWN VELOCITY, PRESSURE, AND TEMPERATURE INDICES   FLSS2210
C                                                                       FLSS2220
      DO 1150 I=1,NNVELS                                                FLSS2230
      NVIS(I)=NVS(I)                                                    FLSS2240
      NVIS(I+NNVELS)=NVS(I)+NN                                          FLSS2250
 1150 CONTINUE                                                          FLSS2260
      DO 1155 J=1,NNPS                                                  FLSS2270
      NVIS(2*NNVELS+J)=NCP(NPS(J))                                      FLSS2280
 1155 CONTINUE                                                          FLSS2290
      DO 1160 K=1,NNTS                                                  FLSS2300
      NVIS(2*NNVELS+NNPS+K)=NVS(K+NNVELS)+MM                            FLSS2310
 1160 CONTINUE                                                          FLSS2320
C                                                                       FLSS2330
C NNHC=NUMBER OF NODES WHERE HEAT TRANSFER COEFFICIENT IS SPECIFIED     FLSS2340
C                                                                       FLSS2350
      NNHC=0                                                            FLSS2360
                                                                        FLSS2370
```

```
NTOTQ=TOTAL NUMBER OF KNOWN QX,QY,QZC, AND QZ                          FLSS2380
                                                                       FLSS2390
      NTOTQ=2*NNQXY+NNQZC+NNQZ                                         FLSS2400
                                                                       FLSS2410
NTOTVP=TOTAL NUMBER OF KNOWN VELOCITIES, PRESSURES, AND TEMPERATURES   FLSS2420
                                                                       FLSS2430
      NTOTVP=2*NNVELS+NNPS+NNTS                                        FLSS2440
                                                                       FLSS2450
PRINT ALL INPUT DATA                                                   FLSS2460
                                                                       FLSS2470
      WRITE(NWRITE,1035)NN,NE,NNCN                                     FLSS2480
      WRITE(NWRITE,1036)NNVELS                                         FLSS2490
      WRITE(NWRITE,1037)NNQXY                                          FLSS2500
      WRITE(NWRITE,1038)NNPS                                           FLSS2510
      WRITE(NWRITE,1039)NNTS                                           FLSS2520
      WRITE(NWRITE,1034)NNQZC                                          FLSS2530
      WRITE(NWRITE,1040)NNQZ                                           FLSS2540
      WRITE(NWRITE,1041)                                               FLSS2550
      DO 150 I=1,NNCN                                                  FLSS2560
      WRITE(NWRITE,1045)NCN(I),XC(NCN(I)),YC(NCN(I))                   FLSS2570
  150 CONTINUE                                                         FLSS2580
      WRITE(NWRITE,1050)                                               FLSS2590
      DO 155 I=1,NE                                                    FLSS2600
      WRITE(NWRITE,1055)I,NODE(I,1),NODE(I,2),NODE(I,3),               FLSS2610
     1NODE(I,4),NODE(I,5),NODE(I,6)                                    FLSS2620
  155 CONTINUE                                                         FLSS2630
      WRITE(NWRITE,1060)                                               FLSS2640
      DO 160 I=1,NNVELS                                                FLSS2650
      WRITE(NWRITE,1065)I,NVS(I),X(NVS(I)),X(NVS(I)+NN)                FLSS2660
  160 CONTINUE                                                         FLSS2670
      WRITE(NWRITE,1070)                                               FLSS2680
      DO 165 I=1,NNQXY                                                 FLSS2690
      WRITE(NWRITE,1065)I,NQS(I),Q(NQS(I)),Q(NQS(I)+NN)                FLSS2700
  165 CONTINUE                                                         FLSS2710
      WRITE(NWRITE,1080)                                               FLSS2720
      DO 170 I=1,NNPS                                                  FLSS2730
      WRITE(NWRITE,1085)I,NPS(I),X(NCP(NPS(I)))                        FLSS2740
  170 CONTINUE                                                         FLSS2750
      WRITE(NWRITE,1081)                                               FLSS2760
      DO 171 I=1,NNTS                                                  FLSS2770
      WRITE(NWRITE,1085)I,NVS(I+NNVELS),X(NVS(I+NNVELS)+MM)            FLSS2780
  171 CONTINUE                                                         FLSS2790
      WRITE(NWRITE,1083)                                               FLSS2800
      DO 173 I=1,NNQZC                                                 FLSS2810
      WRITE(NWRITE,1085)I,NQS(I+NNQXY),Q(NQS(I+NNQXY)+2*NN)            FLSS2820
  173 CONTINUE                                                         FLSS2830
      WRITE(NWRITE,1082)                                               FLSS2840
      DO 172 I=1,NNQZ                                                  FLSS2850
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      WRITE(NWRITE,1085)I,NQS(I+NNQXY+NNQZC),Q(NQS(I+NNQXY+NNQZC)+MM)    FLSS2860
  172 CONTINUE                                                          FLSS2870
      WRITE(NWRITE,1090)
      WRITE(NWRITE,2055)                                                FLSS8320
      WRITE(NWRITE,1091)
  177 CONTINUE                                                          FLSS2880
      DO 178 I=1,MMM                                                    FLSS2890
      RHS(I)=0.D0                                                       FLSS2900
      DO 178 J=1,MMM                                                    FLSS2910
      TM(I,J)=0.D0                                                      FLSS2920
  178 CONTINUE                                                          FLSS2930
      DO 181 I=1,21                                                     FLSS2940
      DO 181 J=1,21                                                     FLSS2950
      TM$(I,J)=0.D0                                                     FLSS2960
  181 CONTINUE                                                          FLSS2970
                                                                        FLSS2980
END OF INPUT AND VERIFICATION ROUTINE                                   FLSS2990
                                                                        FLSS3000
      DO 300 K=1,NE                                                     FLSS3010
      N1=NODE(K,1)                                                      FLSS3020
      N2=NODE(K,2)                                                      FLSS3030
      N3=NODE(K,3)                                                      FLSS3040
      N4=NODE(K,4)                                                      FLSS3050
      N5=NODE(K,5)                                                      FLSS3060
      N6=NODE(K,6)                                                      FLSS3070
      N7=NODE(K,1)+NN                                                   FLSS3080
      N8=NODE(K,2)+NN                                                   FLSS3090
      N9=NODE(K,3)+NN                                                   FLSS3100
      N10=NODE(K,4)+NN                                                  FLSS3110
      N11=NODE(K,5)+NN                                                  FLSS3120
      N12=NODE(K,6)+NN                                                  FLSS3130
      N13=NCP(NODE(K,1))                                                FLSS3140
      N14=NCP(NODE(K,3))                                                FLSS3150
      N15=NCP(NODE(K,5))                                                FLSS3160
      N16=NODE(K,1)+MM                                                  FLSS3170
      N17=NODE(K,2)+MM                                                  FLSS3180
      N18=NODE(K,3)+MM                                                  FLSS3190
      N19=NODE(K,4)+MM                                                  FLSS3200
      N20=NODE(K,5)+MM                                                  FLSS3210
      N21=NODE(K,6)+MM                                                  FLSS3220
      XC$(1)=XC(NODE(K,1))                                              FLSS3230
      XC$(2)=XC(NODE(K,3))                                              FLSS3240
      XC$(3)=XC(NODE(K,5))                                              FLSS3250
      YC$(1)=YC(NODE(K,1))                                              FLSS3260
      YC$(2)=YC(NODE(K,3))                                              FLSS3270
      YC$(3)=YC(NODE(K,5))                                              FLSS3280
      A=1.D0                                                            FLSS3290
      ZBAR=(YC$(1)+YC$(2)+YC$(3))/3.D0                                  FLSS3300
```

```
      RBAR=(XC$(1)+XC$(2)+XC$(3))/3.D0                                          FLSS3310
      IF(NCASE.EQ.2)A=RBAR                                                      FLSS3320
      AA=1.D0                                                                   FLSS3330
      IF(NCASE.EQ.2)AA=2.D0*3.14159D0*(RBAR)                                    FLSS3340
      A1=XC$(2)*YC$(3)-XC$(3)*YC$(2)                                            FLSS3350
      A2=XC$(3)*YC$(1)-XC$(1)*YC$(3)                                            FLSS3360
      A3=XC$(1)*YC$(2)-XC$(2)*YC$(1)                                            FLSS3370
      B1=YC$(2)-YC$(3)                                                          FLSS3380
      B2=YC$(3)-YC$(1)                                                          FLSS3390
      B3=YC$(1)-YC$(2)                                                          FLSS3400
      C1=XC$(3)-XC$(2)                                                          FLSS3410
      C2=XC$(1)-XC$(3)                                                          FLSS3420
      C3=XC$(2)-XC$(1)                                                          FLSS3430
      DEL=DABS(0.5D0*(XC$(1)*(YC$(2)-YC$(3))+XC$(2)*(YC$(3)-YC$(1))             FLSS3440
     1+XC$(3)*(YC$(1)-YC$(2))))                                                 FLSS3450
      CONST=10.90D0*(1.D0*A/(3.D0*DEL))*AA                                      FLSS3460
      D1=-B1/6.D0                                                               FLSS3470
      D2=-B2/6.D0                                                               FLSS3480
      D3=-B3/6.D0                                                               FLSS3490
      E1=-C1/6.D0                                                               FLSS3500
      E2=-C2/6.D0                                                               FLSS3510
      E3=-C3/6.D0                                                               FLSS3520
      F1=B1/2520.D0                                                             FLSS3530
      F2=B2/2520.D0                                                             FLSS3540
      F3=B3/2520.D0                                                             FLSS3550
      G1=C1/2520.D0                                                             FLSS3560
      G2=C2/2520.D0                                                             FLSS3570
      G3=C3/2520.D0                                                             FLSS3580
      U1=T1(N1)                                                                 FLSS3590
      U2=T1(N2)                                                                 FLSS3600
      U3=T1(N3)                                                                 FLSS3610
      U4=T1(N4)                                                                 FLSS3620
      U5=T1(N5)                                                                 FLSS3630
      U6=T1(N6)                                                                 FLSS3640
      V1=T1(N7)                                                                 FLSS3650
      V2=T1(N8)                                                                 FLSS3660
      V3=T1(N9)                                                                 FLSS3670
      V4=T1(N10)                                                                FLSS3680
      V5=T1(N11)                                                                FLSS3690
      V6=T1(N12)                                                                FLSS3700
      TM$(1,1)=0.75D0*(B1*B1+C1*C1)*CONST                                       FLSS3710
      TM$(1,2)=(B1*B2+C1*C2)*CONST                                              FLSS3720
      TM$(1,3)=-TM$(1,2)*0.25D0                                                 FLSS3730
      TM$(1,4)=0.                                                               FLSS3740
      TM$(1,6)=(B1*B3+C1*C3)*CONST                                              FLSS3750
      TM$(1,5)=-TM$(1,6)*.25D0                                                  FLSS3760
      TM$(3,3)=.75D0*(B2*B2+C2*C2)*CONST                                        FLSS3770
      TM$(3,4)=(B2*B3+C2*C3)*CONST                                              FLSS3780
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      TM$(3,5)=-TM$(3,4)*0.25D0                                          FLSS3790
      TM$(2,1)=TM$(1,2)                                                  FLSS3800
      TM$(2,3)=TM$(1,2)                                                  FLSS3810
      TM$(2,2)=8.D0/3.D0*(TM$(1,1)+TM$(3,3))+2.D0*TM$(1,2)               FLSS3820
      TM$(2,4)=2.D0*TM$(1,6)+TM$(3,4)+TM$(1,2)+4.D0/3.D0*TM$(3,3)        FLSS3830
      TM$(2,5)=0.                                                        FLSS3840
      TM$(2,6)=TM$(1,6)+2.D0*TM$(3,4)+TM$(1,2)+4.D0/3.D0*TM$(1,1)        FLSS3850
      TM$(3,1)=TM$(1,3)                                                  FLSS3860
      TM$(3,2)=TM$(2,3)                                                  FLSS3870
      TM$(3,6)=0.                                                        FLSS3880
      TM$(5,5)=.75D0*(B3*B3+C3*C3)*CONST                                 FLSS3890
      TM$(4,1)=TM$(1,4)                                                  FLSS3900
      TM$(4,2)=TM$(2,4)                                                  FLSS3910
      TM$(4,3)=TM$(3,4)                                                  FLSS3920
      TM$(4,4)=8.D0/3.D0*(TM$(3,3)+TM$(5,5))+2.D0*TM$(3,4)               FLSS3930
      TM$(4,5)=TM$(3,4)                                                  FLSS3940
      TM$(4,6)=TM$(1,6)+TM$(3,4)+2.D0*TM$(1,2)+4.D0/3.D0*TM$(5,5)        FLSS3950
      TM$(6,3)=TM$(3,6)                                                  FLSS3960
      TM$(5,1)=TM$(1,5)                                                  FLSS3970
      TM$(5,2)=TM$(2,5)                                                  FLSS3980
      TM$(5,3)=TM$(3,5)                                                  FLSS3990
      TM$(5,4)=TM$(4,5)                                                  FLSS4000
      TM$(5,6)=TM$(1,6)                                                  FLSS4010
      TM$(6,1)=TM$(1,6)                                                  FLSS4020
      TM$(6,2)=TM$(2,6)                                                  FLSS4030
      TM$(6,4)=TM$(4,6)                                                  FLSS4040
      TM$(6,5)=TM$(5,6)                                                  FLSS4050
      TM$(6,6)=8.D0/3.D0*(TM$(5,5)+TM$(1,1))+2.D0*TM$(1,6)               FLSS4060
      IF(NCASE.NE.1) GO TO 3000                                          FLSS4070
                                                                         FLSS4080
BEGIN INPUT OF NON-LINEAR TERMS                                          FLSS4090
                                                                         FLSS4100
      TM$(1,1)=TM$(1,1)                                                  FLSS4110
     1-(78.D0*U1+48.D0*U2-9.D0*U3+12.D0*U4-9.D0*U5+48.D0*U6)*F1          FLSS4120
     2             -(78.D0*V1+48.D0*V2-9.D0*V3+12.D0*V4-9.D0*V5+48.D0*V6)FLSS4130
     3*G1                                                                FLSS4140
      TM$(2,1)=TM$(2,1)                                                  FLSS4150
     1-(48.D0*U1+160.D0*U2-32.D0*U3+16.D0*U4-20.D0*U5+80.D0*U6)*F1       FLSS4160
     2             -(48.D0*V1+160.D0*V2-32.D0*V3+16.D0*V4-20.D0*V5+80.D0*FLSS4170
     3V6)*G1                                                             FLSS4180
      TM$(3,1)=TM$(3,1)                                                  FLSS4190
     1-(-9.D0*U1-32.D0*U2-18.D0*U3-16.D0*U4+11.D0*U5-20.D0*U6)*F1        FLSS4200
     2             -(-9.D0*V1-32.D0*V2-18.D0*V3-16.D0*V4+11.D0*V5-20.D0*VFLSS4210
     36)*G1                                                              FLSS4220
      TM$(4,1)=TM$(4,1)                                                  FLSS4230
     1-(12.D0*U1+16.D0*U2-16.D0*U3-96.D0*U4-16.D0*U5+16.D0*U6)*F1        FLSS4240
     2             -(12.D0*V1+16.D0*V2-16.D0*V3-96.D0*V4-16.D0*V5+16.D0*VFLSS4250
     36)*G1                                                              FLSS4260
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 TM$(5,1)=TM$(5,1)                                                      FLSS4270
1-(-9.D0*U1-20.D0*U2+11.D0*U3-16.D0*U4-18.D0*U5-32.D0*U6)*F1            FLSS4280
2              -(-9.D0*V1-20.D0*V2+11.D0*V3-16.D0*V4-18.D0*V5-32.D0*VFLSS4290
36)*G1                                                                 FLSS4300
 TM$(6,1)=TM$(6,1)                                                     FLSS4310
1-(48.D0*U1+80.D0*U2-20.D0*U3+16.D0*U4-32.D0*U5+160.D0*U6)*F1          FLSS4320
2              -(48.D0*V1+80.D0*V2-20.D0*V3+16.D0*V4-32.D0*V5+160.D0*FLSS4330
3V6)*G1                                                                FLSS4340
 TM$(1,2)=TM$(1,2)                                                     FLSS4350
1-(24.D0*U1-32.D0*U2-16.D0*U3-48.D0*U4+4.D0*U5-16.D0*U6)*F1            FLSS4360
2              -(24.D0*V1-32.D0*V2-16.D0*V3-48.D0*V4+4.D0*V5-16.D0*V6FLSS4370
3)*G1                          -(120.D0*U1+48.D0*U2-16.D0*U3-16.D0*U4-16.FLSS4380
4D0*U5+48.D0*U6)*F2                           -(120.D0*V1+48.D0*V2-16.D0*V3-FLSS4390
516.D0*V4-16.D0*V5+48.D0*V6)*G2                                        FLSS4400
 TM$(2,2)=TM$(2,2)                                                     FLSS4410
1-(-32.D0*U1+384.D0*U2+48.D0*U3+192.D0*U4-48.D0*U5+128.D0*U6)*F1       FLSS4420
2              -(-32.D0*V1+384.D0*V2+48.D0*V3+192.D0*V4-48.D0*V5+128.FLSS4430
3D0*V6)*G1                     -(48.D0*U1+384.D0*U2-32.D0*U3+128.D0*U4-48FLSS4440
4.D0*U5+192.D0*U6)*F2                         -(48.D0*V1+384.D0*V2-32.D0*V3+FLSS4450
5128.D0*V4-48.D0*V5+192.D0*V6)*G2                                      FLSS4460
 TM$(3,2)=TM$(3,2)                                                     FLSS4470
1-(-16.D0*U1+48.D0*U2+120.D0*U3+48.D0*U4-16.D0*U5-16.D0*U6)*F1         FLSS4480
2              -(-16.D0*V1+48.D0*V2+120.D0*V3+48.D0*V4-16.D0*V5-16.D0FLSS4490
3*V6)*G1                       -(-16.D0*U1-32.D0*U2+24.D0*U3-16.D0*U4+4.DFLSS4500
40*U5-48.D0*U6)*F2                            -(-16.D0*V1-32.D0*V2+24.D0*V3-FLSS4510
516.D0*V4+4.D0*V5-48.D0*V6)*G2                                         FLSS4520
 TM$(4,2)=TM$(4,2)                                                     FLSS4530
1-(-48.D0*U1+192.D0*U2+48.D0*U3+384.D0*U4-32.D0*U5+128.D0*U6)*F1       FLSS4540
2              -(-48.D0*V1+192.D0*V2+48.D0*V3+384.D0*V4-32.D0*V5+128.FLSS4550
3D0*V6)*G1                     -(-16.D0*U1+128.D0*U2-16.D0*U3+128.D0*U4-1FLSS4560
46.D0*U5+128.D0*U6)*F2                        -(-16.D0*V1+128.D0*V2-16.D0*V3FLSS4570
5+128.D0*V4-16.D0*V5+128.D0*V6)*G2                                     FLSS4580
 TM$(5,2)=TM$(5,2)                                                     FLSS4590
1-(4.D0*U1-48.D0*U2-16.D0*U3-32.D0*U4+24.D0*U5-16.D0*U6)*F1            FLSS4600
2              -(4.D0*V1-48.D0*V2-16.D0*V3-32.D0*V4+24.D0*V5-16.D0*V6FLSS4610
3)*G1                          -(-16.D0*U1-48.D0*U2+4.D0*U3-16.D0*U4+24.DFLSS4620
40*U5-32.D0*U6)*F2                            -(-16.D0*V1-48.D0*V2+4.D0*V3-1FLSS4630
56.D0*V4+24.D0*V5-32.D0*V6)*G2                                         FLSS4640
 TM$(6,2)=TM$(6,2)                                                     FLSS4650
1-(-16.D0*U1+128.D0*U2-16.D0*U3+128.D0*U4-16.D0*U5+128.D0*U6)*F1       FLSS4660
2              -(-16.D0*V1+128.D0*V2-16.D0*V3+128.D0*V4-16.D0*V5+128.FLSS4670
3D0*V6)*G1                     -(48.D0*U1+192.D0*U2-48.D0*U3+128.D0*U4-32FLSS4680
4.D0*U5+384.D0*U6)*F2                         -(48.D0*V1+192.D0*V2-48.D0*V3+FLSS4690
5128.D0*V4-32.D0*V5+384.D0*V6)*G2                                      FLSS4700
 TM$(1,3)=TM$(1,3)                                                     FLSS4710
1-(-18.D0*U1-32.D0*U2-9.D0*U3-20.D0*U4+11.D0*U5-16.D0*U6)*F2           FLSS4720
2              -(-18.D0*V1-32.D0*V2-9.D0*V3-20.D0*V4+11.D0*V5-16.D0*VFLSS4730
36)*G2                                                                 FLSS4740
```

```
 TM$(2,3)=TM$(2,3)                                                      FLSS4750
1-(-32.D0*U1+160.D0*U2+48.D0*U3+80.D0*U4-20.D0*U5+16.D0*U6)*F2          FLSS4760
2              -(-32.D0*V1+160.D0*V2+48.D0*V3+80.D0*V4-20.D0*V5+16.D0FLSS4770
3*V6)*G2                                                               FLSS4780
 TM$(3,3)=TM$(3,3)                                                     FLSS4790
1-(-9.D0*U1+48.D0*U2+78.D0*U3+48.D0*U4-9.D0*U5+12.D0*U6)*F2            FLSS4800
2              -(-9.D0*V1+48.D0*V2+78.D0*V3+48.D0*V4-9.D0*V5+12.D0*V6FLSS4810
3)*G2                                                                  FLSS4820
 TM$(4,3)=TM$(4,3)                                                     FLSS4830
1-(-20.D0*U1+80.D0*U2+48.D0*U3+160.D0*U4-32.D0*U5+16.D0*U6)*F2         FLSS4840
2              -(-20.D0*V1+80.D0*V2+48.D0*V3+160.D0*V4-32.D0*V5+16.D0FLSS4850
3*V6)*G2                                                               FLSS4860
 TM$(5,3)=TM$(5,3)                                                     FLSS4870
1-(11.D0*U1-20.D0*U2-9.D0*U3-32.D0*U4-18.D0*U5-16.D0*U6)*F2            FLSS4880
2              -(11.D0*V1-20.D0*V2-9.D0*V3-32.D0*V4-18.D0*V5-16.D0*V6FLSS4890
3)*G2                                                                  FLSS4900
 TM$(6,3)=TM$(6,3)                                                     FLSS4910
1-(-16.D0*U1+16.D0*U2+12.D0*U3+16.D0*U4-16.D0*U5-96.D0*U6)*F2          FLSS4920
2              -(-16.D0*V1+16.D0*V2+12.D0*V3+16.D0*V4-16.D0*V5-96.D0*FLSS4930
3V6)*G2                                                                FLSS4940
 TM$(1,4)=TM$(1,4)                                                     FLSS4950
1-(24.D0*U1-16.D0*U2+4.D0*U3-48.D0*U4-16.D0*U5-32.D0*U6)*F2            FLSS4960
2              -(24.D0*V1-16.D0*V2+4.D0*V3-48.D0*V4-16.D0*V5-32.D0*V6FLSS4970
3)*G2                         -(24.D0*U1-32.D0*U2-16.D0*U3-48.D0*U4+4.D0FLSS4980
4*U5-16.D0*U6)*F3                              -(24.D0*V1-32.D0*V2-16.D0*V3-4FLSS4990
58.D0*V4+4.D0*V5-16.D0*V6)*G3                                          FLSS5000
 TM$(2,4)=TM$(2,4)                                                     FLSS5010
1-(-16.D0*U1+128.D0*U2-16.D0*U3+128.D0*U4-16.D0*U5+128.D0*U6)*F2       FLSS5020
2              -(-16.D0*V1+128.D0*V2-16.D0*V3+128.D0*V4-16.D0*V5+128.FLSS5030
3D0*V6)*G2                    -(-32.D0*U1+384.D0*U2+48.D0*U3+192.D0*U4-4FLSS5040
48.D0*U5+128.D0*U6)*F3                         -(-32.D0*V1+384.D0*V2+48.D0*V3FLSS5050
5+192.D0*V4-48.D0*V5+128.D0*V6)*G3                                     FLSS5060
 TM$(3,4)=TM$(3,4)                                                     FLSS5070
1-(4.D0*U1-16.D0*U2+24.D0*U3-32.D0*U4+16.D0*U5-48.D0*U6)*F2            FLSS5080
2              -(4.D0*V1-16.D0*V2+24.D0*V3-32.D0*V4+16.D0*V5-48.D0*V6FLSS5090
3)*G2                         -(-16.D0*U1+48.D0*U2+120.D0*U3+48.D0*U4-16FLSS5100
4.D0*U5-16.D0*U6)*F3                           -(-16.D0*V1+48.D0*V2+120.D0*V3FLSS5110
5+48.D0*V4-16.D0*V5-16.D0*V6)*G3                                       FLSS5120
 TM$(4,4)=TM$(4,4)                                                     FLSS5130
1-(-48.D0*U1+128.D0*U2-32.D0*U3+384.D0*U4+48.D0*U5+192.D0*U6)*F2       FLSS5140
2              -(-48.D0*V1+128.D0*V2-32.D0*V3+384.D0*V4+48.D0*V5+192.FLSS5150
3D0*V6)*G2                    -(-48.D0*U1+192.D0*U2+48.D0*U3+384.D0*U4-3FLSS5160
42.D0*U5+128.D0*U6)*F3                         -(-48.D0*V1+192.D0*V2+48.D0*V3FLSS5170
5+384.D0*V4-32.D0*V5+128.D0*V6)*G3                                     FLSS5180
 TM$(5,4)=TM$(5,4)                                                     FLSS5190
1-(-16.D0*U1-16.D0*U2-16.D0*U3+48.D0*U4+120.D0*U5+48.D0*U6)*F2         FLSS5200
2              -(-16.D0*V1-16.D0*V2-16.D0*V3+48.D0*V4+120.D0*V5+48.D0FLSS5210
3*V6)*G2                      -(4.D0*U1-48.D0*U2-16.D0*U3-32.D0*U4+24.D0FLSS5220
```

```
4*U5-16.D0*U6)*F3                                   -(4.D0*V1-48.D0*V2-16.D0*V3-32FLSS5230
5.D0*V4+24.D0*V5-16.D0*V6)*G3                                                      FLSS5240
 TM$(6,4)=TM$(6,4)                                                                 FLSS5250
1-(-32.D0*U1+128.D0*U2-48.D0*U3+192.D0*U4+48.D0*U5+384.D0*U6)*F2                   FLSS5260
2              -(-32.D0*V1+128.D0*V2-48.D0*V3+192.D0*V4+48.D0*V5+384.FLSS5270
3D0*V6)*G2                        -(-16.D0*U1+128.D0*U2-16.D0*U3+128.D0*U4-1FLSS5280
46.D0*U5+128.D0*U6)*F3                          -(-16.D0*V1+128.D0*V2-16.D0*V3FLSS5290
5+128.D0*V4-16.D0*V5+128.D0*V6)*G3                                                 FLSS5300
 TM$(1,5)=TM$(1,5)                                                                 FLSS5310
1-(-18.D0*U1-16.D0*U2+11.D0*U3-20.D0*U4-9.D0*U5-32.D0*U6)*F3                       FLSS5320
2              -(-18.D0*V1-16.D0*V2+11.D0*V3-20.D0*V4-9.D0*V5-32.D0*VFLSS5330
36)*G3                                                                             FLSS5340
 TM$(2,5)=TM$(2,5)                                                                 FLSS5350
1-(-16.D0*U1-96.D0*U2-16.D0*U3+16.D0*U4+12.D0*U5+16.D0*U6)*F3                      FLSS5360
2              -(-16.D0*V1-96.D0*V2-16.D0*V3+16.D0*V4+12.D0*V5+16.D0*FLSS5370
3V6)*G3                                                                            FLSS5380
 TM$(3,5)=TM$(3,5)                                                                 FLSS5390
1-(11.D0*U1-16.D0*U2-18.D0*U3-32.D0*U4-9.D0*U5-20.D0*U6)*F3                        FLSS5400
2              -(11.D0*V1-16.D0*V2-18.D0*V3-32.D0*V4-9.D0*V5-20.D0*V6FLSS5410
3)*G3                                                                              FLSS5420
 TM$(4,5)=TM$(4,5)                                                                 FLSS5430
1-(-20.D0*U1+16.D0*U2-32.D0*U3+160.D0*U4+48.D0*U5+80.D0*U6)*F3                     FLSS5440
2              -(-20.D0*V1+16.D0*V2-32.D0*V3+160.D0*V4+48.D0*V5+80.D0FLSS5450
3*V6)*G3                                                                           FLSS5460
 TM$(5,5)=TM$(5,5)                                                                 FLSS5470
1-(-9.D0*U1+12.D0*U2-9.D0*U3+48.D0*U4+78.D0*U5+48.D0*U6)*F3                        FLSS5480
2              -(-9.D0*V1+12.D0*V2-9.D0*V3+48.D0*V4+78.D0*V5+48.D0*V6FLSS5490
3)*G3                                                                              FLSS5500
 TM$(6,5)=TM$(6,5)                                                                 FLSS5510
1-(-32.D0*U1+16.D0*U2-20.D0*U3+80.D0*U4+48.D0*U5+160.D0*U6)*F3                     FLSS5520
2              -(-32.D0*V1+16.D0*V2-20.D0*V3+80.D0*V4+48.D0*V5+160.D0FLSS5530
3*V6)*G3                                                                           FLSS5540
 TM$(1,6)=TM$(1,6)                                                                 FLSS5550
1-(24.D0*U1-16.D0*U2+4.D0*U3-48.D0*U4-16.D0*U5-32.D0*U6)*F1                        FLSS5560
2              -(24.D0*V1-16.D0*V2+4.D0*V3-48.D0*V4-16.D0*V5-32.D0*V6FLSS5570
3)*G1                             -(120.D0*U1+48.D0*U2-16.D0*U3-16.D0*U4-16.FLSS5580
4D0*U5+48.D0*U6)*F3                             -(120.D0*V1+48.D0*V2-16.D0*V3-FLSS5590
516.D0*V4-16.D0*V5+48.D0*V6)*G3                                                    FLSS5600
 TM$(2,6)=TM$(2,6)                                                                 FLSS5610
1-(-16.D0*U1+128.D0*U2-16.D0*U3+128.D0*U4-16.D0*U5+128.D0*U6)*F1                   FLSS5620
2              -(-16.D0*V1+128.D0*V2-16.D0*V3+128.D0*V4-16.D0*V5+128.FLSS5630
3D0*V6)*G1                        -(48.D0*U1+384.D0*U2-32.D0*U3+128.D0*U4-48FLSS5640
4.D0*U5+192.D0*U6)*F3                           -(48.D0*V1+384.D0*V2-32.D0*V3+FLSS5650
5128.D0*V4-48.D0*V5+192.D0*V6)*G3                                                  FLSS5660
 TM$(3,6)=TM$(3,6)                                                                 FLSS5670
1-(4.D0*U1-16.D0*U2+24.D0*U3-32.D0*U4-16.D0*U5-48.D0*U6)*F1                        FLSS5680
2              -(4.D0*V1-16.D0*V2+24.D0*V3-32.D0*V4-16.D0*V5-48.D0*V6FLSS5690
3)*G1                             -(-16.D0*U1-32.D0*U2+24.D0*U3-16.D0*U4+4.DFLSS5700
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     40*U5-48.D0*U6)*F3                                  -(-16.D0*V1-32.D0*V2+24.D0*V3-FLSS5710
     516.D0*V4+4.D0*V5-48.D0*V6)*G3                                                    FLSS5720
      TM$(4,6)=TM$(4,6)                                                                FLSS5730
     1-(-48.D0*U1+128.D0*U2-32.D0*U3+384.D0*U4+48.D0*U5+192.D0*U6)*F1                  FLSS5740
     2              -(-48.D0*V1+128.D0*V2-32.D0*V3+384.D0*V4+48.D0*V5+192.FLSS5750
     3D0*V6)*G1                          -(-16.D0*U1+128.D0*U2-16.D0*U3+128.D0*U4-1FLSS5760
     46.D0*U5+128.D0*U6)*F3                          -(-16.D0*V1+128.D0*V2-16.D0*V3FLSS5770
     5+128.D0*V4-16.D0*V5+128.D0*V6)*G3                                            FLSS5780
      TM$(5,6)=TM$(5,6)                                                            FLSS5790
     1-(-16.D0*U1-16.D0*U2-16.D0*U3+48.D0*U4+120.D0*U5+48.D0*U6)*F1                FLSS5800
     2              -(-16.D0*V1-16.D0*V2-16.D0*V3+48.D0*V4+120.D0*V5+48.D0FLSS5810
     3*V6)*G1                            -(-16.D0*U1-48.D0*U2+4.D0*U3-16.D0*U4+24.DFLSS5820
     40*U5-32.D0*U6)*F3                              -(-16.D0*V1-48.D0*V2+4.D0*V3-1FLSS5830
     56.D0*V4+24.D0*V5-32.D0*V6)*G3                                                FLSS5840
      TM$(6,6)=TM$(6,6)                                                            FLSS5850
     1-(-32.D0*U1+128.D0*U2-48.D0*U3+192.D0*U4+48.D0*U5+384.D0*U6)*F1              FLSS5860
     2              -(-32.D0*V1+128.D0*V2-48.D0*V3+192.D0*V4+48.D0*V5+384.FLSS5870
     3D0*V6)*G1                          -(48.D0*U1+192.D0*U2-48.D0*U3+128.D0*U4-32FLSS5880
     4.D0*U5+384.D0*U6)*F3                           -(48.D0*V1+192.D0*V2-48.D0*V3+FLSS5890
     5128.D0*V4-32.D0*V5+384.D0*V6)*G3                                             FLSS5900
                                                                                   FLSS5910
THIS ENDS ADDITION OF NON-LINEAR TERMS TO THE LOCAL ARRAY                          FLSS5920
                                                                                   FLSS5930
 3000 CONTINUE                                                                     FLSS5940
      TM$(7,7)=TM$(1,1)                                                            FLSS5950
      TM$(7,8)=TM$(1,2)                                                            FLSS5960
      TM$(7,9)=TM$(1,3)                                                            FLSS5970
      TM$(7,10)=TM$(1,4)                                                           FLSS5980
      TM$(7,11)=TM$(1,5)                                                           FLSS5990
      TM$(7,12)=TM$(1,6)                                                           FLSS6000
      TM$(8,7)=TM$(2,1)                                                            FLSS6010
      TM$(8,8)=TM$(2,2)                                                            FLSS6020
      TM$(8,9)=TM$(2,3)                                                            FLSS6030
      TM$(8,10)=TM$(2,4)                                                           FLSS6040
      TM$(8,11)=TM$(2,5)                                                           FLSS6050
      TM$(8,12)=TM$(2,6)                                                           FLSS6060
      TM$(9,7)=TM$(3,1)                                                            FLSS6070
      TM$(9,8)=TM$(3,2)                                                            FLSS6080
      TM$(9,9)=TM$(3,3)                                                            FLSS6090
      TM$(9,10)=TM$(3,4)                                                           FLSS6100
      TM$(9,11)=TM$(3,5)                                                           FLSS6110
      TM$(9,12)=TM$(3,6)                                                           FLSS6120
      TM$(10,7)=TM$(4,1)                                                           FLSS6130
      TM$(10,8)=TM$(4,2)                                                           FLSS6140
      TM$(10,9)=TM$(4,3)                                                           FLSS6150
      TM$(10,10)=TM$(4,4)                                                          FLSS6160
      TM$(10,11)=TM$(4,5)                                                          FLSS6170
      TM$(10,12)=TM$(4,6)                                                          FLSS6180
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TM$(11,7)=TM$(5,1)                                                    FLSS6190
TM$(11,8)=TM$(5,2)                                                    FLSS6200
TM$(11,9)=TM$(5,3)                                                    FLSS6210
TM$(11,10)=TM$(5,4)                                                   FLSS6220
TM$(11,11)=TM$(5,5)                                                   FLSS6230
TM$(11,12)=TM$(5,6)                                                   FLSS6240
TM$(12,7)=TM$(6,1)                                                    FLSS6250
TM$(12,8)=TM$(6,2)                                                    FLSS6260
TM$(12,9)=TM$(6,3)                                                    FLSS6270
TM$(12,10)=TM$(6,4)                                                   FLSS6280
TM$(12,11)=TM$(6,5)                                                   FLSS6290
TM$(12,12)=TM$(6,6)                                                   FLSS6300
TM$(13,1)=D1                                                          FLSS6310
TM$(14,1)=0.D0                                                        FLSS6320
TM$(15,1)=0.D0                                                        FLSS6330
TM$(13,2)=D1+2.D0*D2                                                  FLSS6340
TM$(14,2)=2.D0*D1+D2                                                  FLSS6350
TM$(15,2)=D1+D2                                                       FLSS6360
TM$(13,3)=0.D0                                                        FLSS6370
TM$(14,3)=D2                                                          FLSS6380
TM$(15,3)=0.D0                                                        FLSS6390
TM$(13,4)=D2+D3                                                       FLSS6400
TM$(14,4)=D2+2.D0*D3                                                  FLSS6410
TM$(15,4)=2.D0*D2+D3                                                  FLSS6420
TM$(13,5)=0.D0                                                        FLSS6430
TM$(14,5)=0.D0                                                        FLSS6440
TM$(15,5)=D3                                                          FLSS6450
TM$(13,6)=D1+2.D0*D3                                                  FLSS6460
TM$(14,6)=D1+D3                                                       FLSS6470
TM$(15,6)=2.D0*D1+D3                                                  FLSS6480
TM$(13,7)=E1                                                          FLSS6490
TM$(14,7)=0.D0                                                        FLSS6500
TM$(15,7)=0.D0                                                        FLSS6510
TM$(13,8)=E1+2.D0*E2                                                  FLSS6520
TM$(14,8)=2.D0*E1+E2                                                  FLSS6530
TM$(15,8)=E1+E2                                                       FLSS6540
TM$(13,9)=0.D0                                                        FLSS6550
TM$(14,9)=E2                                                          FLSS6560
TM$(15,9)=0.0                                                         FLSS6570
TM$(13,10)=E2+E3                                                      FLSS6580
TM$(14,10)=E2+2.D0*E3                                                 FLSS6590
TM$(15,10)=E2+E2+E3                                                   FLSS6600
TM$(13,11)=0.D0                                                       FLSS6610
TM$(14,11)=0.D0                                                       FLSS6620
TM$(15,11)=E3                                                         FLSS6630
TM$(13,12)=E1+E3+E3                                                   FLSS6640
TM$(14,12)=E1+E3                                                      FLSS6650
TM$(15,12)=2.D0*E1+E3                                                 FLSS6660
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      CONST4=0.94375D0                                                  FLSS6670
      TM$(1,13)=D1*CONST4                                               FLSS6680
      TM$(1,14)=0.D0                                                    FLSS6690
      TM$(1,15)=0.D0                                                    FLSS6700
      TM$(2,13)=(D1+2.D0*D2)*CONST4                                     FLSS6710
      TM$(2,14)=(2.D0*D1+D2)*CONST4                                     FLSS6720
      TM$(2,15)=(D1+D2)*CONST4                                          FLSS6730
      TM$(3,13)=0.D0                                                    FLSS6740
      TM$(3,14)=D2*CONST4                                               FLSS6750
      TM$(3,15)=0.D0                                                    FLSS6760
      TM$(4,13)=(D2+D3)*CONST4                                          FLSS6770
      TM$(4,14)=(D2+2.D0*D3)*CONST4                                     FLSS6780
      TM$(4,15)=(2.D0*D2+D3)*CONST4                                     FLSS6790
      TM$(5,13)=0.D0                                                    FLSS6800
      TM$(5,14)=0.D0                                                    FLSS6810
      TM$(5,15)=D3*CONST4                                               FLSS6820
      TM$(6,13)=(D1+2.D0*D3)*CONST4                                     FLSS6830
      TM$(6,14)=(D1+D3)*CONST4                                          FLSS6840
      TM$(6,15)=(2.D0*D1+D3)*CONST4                                     FLSS6850
      TM$(7,13)=E1*CONST4                                               FLSS6860
      TM$(7,14)=0.D0                                                    FLSS6870
      TM$(7,15)=0.D0                                                    FLSS6880
      TM$(8,13)=(E1+2.D0*E2)*CONST4                                     FLSS6890
      TM$(8,14)=(2.D0*E1+E2)*CONST4                                     FLSS6900
      TM$(8,15)=(E1+E2)*CONST4                                          FLSS6910
      TM$(9,13)=0.D0                                                    FLSS6920
      TM$(9,14)=E2*CONST4                                               FLSS6930
      TM$(9,15)=0.D0                                                    FLSS6940
      TM$(10,13)=(E2+E3)*CONST4                                         FLSS6950
      TM$(10,14)=(E2+2.D0*E3)*CONST4                                    FLSS6960
      TM$(10,15)=(E2+E2+E3)*CONST4                                      FLSS6970
      TM$(11,13)=0.D0                                                   FLSS6980
      TM$(11,14)=0.D0                                                   FLSS6990
      TM$(11,15)=E3*CONST4                                              FLSS7000
      TM$(12,13)=(E1+2.D0*E3)*CONST4                                    FLSS7010
      TM$(12,14)=(E1+E3)*CONST4                                         FLSS7020
      TM$(12,15)=(2.D0*E1+E3)*CONST4                                    FLSS7030
      CONST2=3.2981D0                                                   FLSS7040
      TM$(1,16)=1.D0*CONST2                                             FLSS7050
      TM$(2,17)=1.D0*CONST2                                             FLSS7060
      TM$(3,18)=1.D0*CONST2                                             FLSS7070
      TM$(4,19)=1.D0*CONST2                                             FLSS7080
      TM$(5,20)=1.D0*CONST2                                             FLSS7090
      TM$(6,21)=1.D0*CONST2                                             FLSS7100
                                                                        FLSS7110
ALPHA1=ALPHA/VISCOSITY                                                  FLSS7120
                                                                        FLSS7130
      ALPHA1=9.4454D-05                                                 FLSS7140
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      CONST1=ALPHA1                                                     FLSS7150
      TM$(16,16)=TM$(1,1)*CONST1                                        FLSS7160
      TM$(16,17)=TM$(1,2)*CONST1                                        FLSS7170
      TM$(16,18)=TM$(1,3)*CONST1                                        FLSS7180
      TM$(16,19)=TM$(1,4)*CONST1                                        FLSS7190
      TM$(16,20)=TM$(1,5)*CONST1                                        FLSS7200
      TM$(16,21)=TM$(1,6)*CONST1                                        FLSS7210
      TM$(17,16)=TM$(2,1)*CONST1                                        FLSS7220
      TM$(17,17)=TM$(2,2)*CONST1                                        FLSS7230
      TM$(17,18)=TM$(2,3)*CONST1                                        FLSS7240
      TM$(17,19)=TM$(2,4)*CONST1                                        FLSS7250
      TM$(17,20)=TM$(2,5)*CONST1                                        FLSS7260
      TM$(17,21)=TM$(2,6)*CONST1                                        FLSS7270
      TM$(18,16)=TM$(3,1)*CONST1                                        FLSS7280
      TM$(18,17)=TM$(3,2)*CONST1                                        FLSS7290
      TM$(18,18)=TM$(3,3)*CONST1                                        FLSS7300
      TM$(18,19)=TM$(3,4)*CONST1                                        FLSS7310
      TM$(18,20)=TM$(3,5)*CONST1                                        FLSS7320
      TM$(18,21)=TM$(3,6)*CONST1                                        FLSS7330
      TM$(19,16)=TM$(4,1)*CONST1                                        FLSS7340
      TM$(19,17)=TM$(4,2)*CONST1                                        FLSS7350
      TM$(19,18)=TM$(4,3)*CONST1                                        FLSS7360
      TM$(19,19)=TM$(4,4)*CONST1                                        FLSS7370
      TM$(19,20)=TM$(4,5)*CONST1                                        FLSS7380
      TM$(19,21)=TM$(4,6)*CONST1                                        FLSS7390
      TM$(20,16)=TM$(5,1)*CONST1                                        FLSS7400
      TM$(20,17)=TM$(5,2)*CONST1                                        FLSS7410
      TM$(20,18)=TM$(5,3)*CONST1                                        FLSS7420
      TM$(20,19)=TM$(5,4)*CONST1                                        FLSS7430
      TM$(20,20)=TM$(5,5)*CONST1                                        FLSS7440
      TM$(20,21)=TM$(5,6)*CONST1                                        FLSS7450
      TM$(21,16)=TM$(6,1)*CONST1                                        FLSS7460
      TM$(21,17)=TM$(6,2)*CONST1                                        FLSS7470
      TM$(21,18)=TM$(6,3)*CONST1                                        FLSS7480
      TM$(21,19)=TM$(6,4)*CONST1                                        FLSS7490
      TM$(21,20)=TM$(6,5)*CONST1                                        FLSS7500
      TM$(21,21)=TM$(6,6)*CONST1                                        FLSS7510
  185 CONTINUE                                                          FLSS7520
      N(1)=N1                                                           FLSS7530
      N(2)=N2                                                           FLSS7540
      N(3)=N3                                                           FLSS7550
      N(4)=N4                                                           FLSS7560
      N(5)=N5                                                           FLSS7570
      N(6)=N6                                                           FLSS7580
      N(7)=N7                                                           FLSS7590
      N(8)=N8                                                           FLSS7600
      N(9)=N9                                                           FLSS7610
      N(10)=N10                                                         FLSS7620
```

```
      N(11)=N11                                                                 FLSS7630
      N(12)=N12                                                                 FLSS7640
      N(13)=N13                                                                 FLSS7650
      N(14)=N14                                                                 FLSS7660
      N(15)=N15                                                                 FLSS7670
      N(16)=N16                                                                 FLSS7680
      N(17)=N17                                                                 FLSS7690
      N(18)=N18                                                                 FLSS7700
      N(19)=N19                                                                 FLSS7710
      N(20)=N20                                                                 FLSS7720
      N(21)=N21                                                                 FLSS7730
      DO 200 I$=1,21                                                            FLSS7740
      I=N(I$)                                                                   FLSS7750
      DO 200 J$=1,21                                                            FLSS7760
      J=N(J$)                                                                   FLSS7770
      TM(I,J)=TM(I,J)+TM$(I$,J$)                                                FLSS7780
  200 CONTINUE                                                                  FLSS7790
  300 CONTINUE                                                                  FLSS7800
      IF(NNQXY.EQ.0) GO TO 310                                                  FLSS7810
      DO 310 I=1,NNQXY                                                          FLSS7820
      RHS(NQS(I))=RHS(NQS(I))+Q(NQS(I))+65.962D0                                FLSS7830
      RHS(NQS(I)+NN)=RHS(NQS(I)+NN)+Q(NQS(I)+NN)                                FLSS7840
  310 CONTINUE                                                                  FLSS7850
      IF(NNQZC.EQ.0) GO TO 312                                                  FLSS7860
      DO 312 I=1,NNQZC                                                          FLSS7870
      RHS(NQS(NNQXY+I)+2*NN)=RHS(NQS(NNQXY+I)+2*NN)+Q(NQS(NNQXY+I)+2*NN)FLSS7880
  312 CONTINUE                                                                  FLSS7890
      IF(NNQZ.EQ.0) GO TO 311                                                   FLSS7900
      DO 311 I=1,NNQZ                                                           FLSS7910
      RHS(NQS(NNQXY+NNQZC+I)+MM)=RHS(NQS(NNQXY+NNQZC+I)+MM)+                    FLSS7920
     1Q(NQS(NNQXY+NNQZC+I)+MM)                                                  FLSS7930
  311 CONTINUE                                                                  FLSS7940
                                                                                FLSS7950
C MODIFICATION OF RHS FOR TM BOUNDARY CONDITIONS                                FLSS7960
C                                                                               FLSS7970
      DO 315 I=1,MMM                                                            FLSS7980
      DO 315 J=1,NTOTVP                                                         FLSS7990
      JX=NVIS(J)                                                                FLSS8000
      RHS(I)=RHS(I)-TM(I,JX)*X(JX)                                              FLSS8010
      TM(I,JX)=0.D0                                                             FLSS8020
      TM(JX,I)=0.D0                                                             FLSS8030
      TM(JX,JX)=1.D0                                                            FLSS8040
      RHS(JX)=X(JX)                                                             FLSS8050
  315 CONTINUE                                                                  FLSS8060
      M=1                                                                       FLSS8070
      ND=117                                                                    FLSS8080
      IA=117                                                                    FLSS8090
      IDGT=0                                                                    FLSS8100
```

```
      CALL LEQT2F (TM,M,ND,IA,RHS,IDGT,WKAREA,IER)                    FLSS8110
      WRITE(NWRITE,2000)                                              FLSS8120
      DO 322 J=1,MMM                                                  FLSS8130
      TDIFF=DABS(T1(J)-X(J))                                          FLSS8140
      EPSLN=1.D-06                                                    FLSS8150
      IF(TDIFF-EPSLN) 322,324,324                                     FLSS8160
  322 CONTINUE                                                        FLSS8170
  324 CONTINUE                                                        FLSS8180
      DO 360 I=1,MMM                                                  FLSS8190
      X(I)=RHS(I)                                                     FLSS8200
      T1(I)=X(I)                                                      FLSS8210
      WRITE (NWRITE,2005) I,X(I)                                      FLSS8220
  360 CONTINUE                                                        FLSS8230
      IF(J.NE.MMM) GO TO 177                                          FLSS8240
      WRITE(NWRITE,2021)                                              FLSS8250
      WRITE(NWRITE,2025)                                              FLSS8260
      WRITE(NWRITE,2030)                                              FLSS8270
      WRITE(NWRITE,2035)                                              FLSS8280
      WRITE(NWRITE,2040)                                              FLSS8290
      WRITE(NWRITE,2045)                                              FLSS8300
      WRITE(NWRITE,2050)                                              FLSS8310
  500 FORMAT(I10)                                                     FLSS8330
  600 FORMAT(1H1,18X,'STEADY STATE FLUID MECHANICS PROBLEM',////)
 1005 FORMAT(3I10)                                                    FLSS8340
 1006 FORMAT(6X,A4,I10,2F10.0)                                        FLSS8350
 1010 FORMAT(7I10)                                                    FLSS8360
 1015 FORMAT(6X,A4,I10,F10.0)                                         FLSS8370
 1016 FORMAT(6X,A4,I10)                                               FLSS8380
 1020 FORMAT(I10,2F10.0)                                              FLSS8390
 1025 FORMAT(6X,A4,I10,F10.0)                                         FLSS8400
 1030 FORMAT(6X,A4,2I10,2F10.0)                                       FLSS8410
 1034 FORMAT(5X,'NNQZC=',I3,//)                                       FLSS8420
 1035 FORMAT(5X,'NO. OF NODES=',I3,//,5X,'NO. OF ELEMENTS=',I3,//,    FLSS8430
     15X,'NO. OF CORNER NODES=',I3,///)                               FLSS8440
 1036 FORMAT(5X,'NNVELS=',I3,//)                                      FLSS8450
 1037 FORMAT(5X,'NNQXY=',I3,//)                                       FLSS8460
 1038 FORMAT(5X,'NNPS=',I3,//)                                        FLSS8470
 1039 FORMAT(5X,'NNTS=',I3,//)                                        FLSS8480
 1040 FORMAT(5X,'NNQZ=',I3,///)                                       FLSS8490
 1041 FORMAT(5X,'SUMMARY OF NODAL COORDINATES',//,                    FLSS8500
     17X,'I',12X'X(I)',13X,'Y(I)',//)                                 FLSS8510
 1045 FORMAT(5X,I3,2(7X,F10.3))                                       FLSS8520
 1050 FORMAT(//,5X,'LISTING OF SYSTEM TOPOLOGY',//,5X,
     1'ELEMENT NUMBER',20X,'NODE NUMBERS',//)                         FLSS8540
 1055 FORMAT(5X,I3,10X,6(5X,I3))                                      FLSS8550
 1060 FORMAT(//,7X,'NODES WHERE VELOCITIES ARE SPECIFIED',            FLSS8560
     1//,8X,'I',5X,'NODE',5X,'U VELOCITY',5X,'V VELOCITY',//)         FLSS8570
 1065 FORMAT(2X,2(4X,I3),3X,F12.3,3X,F12.3)                           FLSS8580
```

```
1070 FORMAT(//,5X,'NODES WHERE QX AND QY ARE SPECIFIED',                    FLSS8590
     1      //,2X,'      I    NODE',1X,'        QX',10X,'   QY',//)       FLSS8600
1075 FORMAT(5X,I3,2(10X,F12.3))                                            FLSS8610
1080 FORMAT(//,5X,'NODES WHERE PRESSURE IS SPECIFIED',                     FLSS8620
     1           //,9X,'I',4X,'NODE',13X,'PRESSURE',//)                   FLSS8630
1081 FORMAT(//,5X,'NODES WHERE TEMPERATURE IS SPECIFIED',                  FLSS8640
     1           //,9X,'I',4X,'NODE',12X,'TEMPERATURE',//)                FLSS8650
1082 FORMAT(//,5X,'NODES WHERE HEAT FLUX QZ IS SPECIFIED',                 FLSS8660
     1           //,9X,'I',4X,'NODE',12X,'HEAT FLUX',//)                  FLSS8670
1083 FORMAT(//,5X,'NODES WHERE QZC IS SPECIFIED',                          FLSS8680
     1           //,9X,'I',4X,'NODE',15X,'QZC',//)                        FLSS8690
1085 FORMAT(7X,I3,3X,I3,10X,F12.3)                                         FLSS8700
1090 FORMAT(////,5X,'NODAL VARIABLE IS THE U-VELOCITY AT NODES 1 - 35;'
     1,//,5X,'THE V-VELOCITY AT NODES 36 - 70;',
     2//,5X,'THE PRESSURE AT NODES 71 - 82;',
     3//,5X,'AND THE TEMPERATURE AT NODES 83 - 117.',////)
1091 FORMAT(////,5X,'THE FIRST SEQUENCE OF 117 NODAL VARIABLES',
     1//,5X,'REPRESENTS A LINEAR,STEADY STATE SYSTEM;',
     2//,5X,'WHILE THE SECOND SET OF THE 117 VALUES CORRESPONDS',
     3//,5X,'TO A NONLINEAR ANALYSIS OF THE SOLUTION DOMAIN.',/////)
1095 FORMAT(5X,3(3X,I3),2(5X,F12.3))                                       FLSS8710
2000 FORMAT(///,5X,'NODE NO.',6X,'NODE VARIABLE',//)                       FLSS8720
2005 FORMAT(9X,I3,5X,D17.10,/)                                             FLSS8730
2015 FORMAT(1H1,21X,'THIS IS A 3-D AXISYMMETRIC PROBLEM',//)               FLSS8740
2020 FORMAT(21X,'THIS IS A 2-D NONLINEAR PROBLEM',//)                      FLSS8750
2021 FORMAT(/////////)                                                     FLSS8760
2025 FORMAT(///,10X,'THE KINEMATIC VISCOSITY OF FLUID 50-HB-3520 AT 20 FLSS8770
     1DEGREES C. =',//,
     221X,'10.90 SQ.CM/SEC',//)
2030 FORMAT(///,10X,'THE DENSITY OF 50-HB-3520 AT 20 DEGREES C. = 1.059FLSS8790
     16 GM/CC',//)                                                         FLSS8800
2035 FORMAT(///,10X,'THE COEFF. OF THERMAL EXPANSION OF 50-HB-3520 AT 2FLSS8810
     10 DEGREES C. =',//,
     221X,'0.002278/DEGREE C.',//)
2040 FORMAT(///,10X,'THE THERMAL DIFFUSIVITY OF 50-HB-3520 AT 20 DEGREEFLSS8830
     1S C. =',//,
     221X,'0.00103 SQ.CM/SEC',//)
2045 FORMAT(///,10X,'THE GRASHOF NUMBER (GR(L)) =(G*B*L**3*(TH-TC))/V**FLSS8850
     12 = 946.4',//)                                                       FLSS8860
2050 FORMAT(///,10X,'THE U VELOCITY FORCING FUNCTION,G*B*T(INITIAL), = FLSS8870
     165.962 CM/SQ.SEC',//,1H1)                                            FLSS8880
2055 FORMAT(////,5X,'THE SPECIFIED WALL PRESSURES',
     1//,5X,'ARE NORMALIZED TO ONE (1) ATMOSPHERE,',
     2//,5X,'THAT IS, 1014000 DYNES/SQ.CM',
     3//,5X,'(ALL PARAMETER VALUES ARE IN CGS UNITS).',////)
      STOP                                                                 FLSS8910
      END                                                                  FLSS8920
```

```
      IMPLICIT REAL*8(A-H,O-Z,$)                                        TIME0001
      DATA NREAD/5/                                                     TIME0002
                                                                        TIME0003
THE U VELOCITY IS IN THE FIRST NN POSITIONS OF X(I)                     TIME0004
THE V VELOCITY IS IN THE SECOND NN POSITIONS OF X I.E. X(I+NN)          TIME0005
THE P PRESSURE IS IN THE NN+NN+I POSITIONS OF X I.E. X(I+NN+NN)         TIME0006
THE T TEMPERATURE IS IN THE NN+NN+NNCN+I PPOSITIONS OF X I.E.           TIME0007
X(I+NN+NN+NNCN)                                                         TIME0008
THERE ARE NNCN PRESSURE NODES (NNCN=NUMBER OF CORNER NODES)             TIME0009
TM MUST BE DIMENSIONED 3*NN+NNCN X 3*NN+NNCN                            TIME0010
                                                                        TIME0011
                                                                        TIME0012
                                                                        TIME0013
      DATA NWRITE/6/                                                    TIME0014
      DATA STOP/'STOP'/                                                 TIME0015
      DIMENSION XC(125),YC(125),NODE(125,6),NVS(125),NCN(125)           TIME0016
      DIMENSION X(117),NVIS(117),NCP(125),NPS(125),Q(117)               TIME0017
      DIMENSION NQS(125),T1(117)                                        TIME0018
      DIMENSION TM$(21,21),N(21),NQIS(117)                              TIME0019
      DIMENSION RP$(6),ZP$(6)                                           TIME0020
      DIMENSION XC$(3),YC$(3)                                           TIME0021
      DIMENSION CD$(21,21)                                              TIME0022
      DIMENSION Y(7,117),W(117,132)                                     TIME0023
      COMMON CD(117,117),TM(117,117),RHS(117)                           TIME0024
                                                                        TIME0025
SPECIFY WHETHER TWO DIMENSIONAL, INCLUDING NON-LINEAR TERMS,            TIME0026
(NCASE=1) OR AXISYMMETRIC (NCASE=2)                                     TIME0027
                                                                        TIME0028
      READ(NREAD,500)NCASE                                              TIME0029
      IF(NCASE.EQ.1)GO TO 5                                             TIME0030
      WRITE(NWRITE,2015)                                                TIME0031
      GO TO 6                                                           TIME0032
    5 WRITE(NWRITE,2020)                                                TIME0033
    6 CONTINUE                                                          TIME0034
                                                                        TIME0035
THE FIRST PART OF THE PROGRAM CAN BE CONSIDERED AS AN INPUT ROUTINE     TIME0036
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IN WHICH LINES 049 TO 0303 ARE INPUT VERIFICATION OF ALL DATA.          TIME0038
SUCH A SECTION WOULD BE PART OF ANY FINITE ELEMENT PROGRAM.             TIME0040
                                                                        TIME0041
                                                                        TIME0042
READ IN NUMBER OF NODES AND ELEMENTS AND NO. OF CORNER NODES            TIME0043
                                                                        TIME0044
      READ(NREAD,1005)NN,NE,NNCN                                        TIME0045
                                                                        TIME0046
INITIALIZE ALL PARAMETERS                                               TIME0047
                                                                        TIME0048
      MM=2*NN+NNCN                                                      TIME0049
      MMM=3*NN+NNCN                                                     TIME0050
      DO 50 I=1,MMM                                                     TIME0051
      XC(I)=0.DO                                                        TIME0052
      YC(I)=0.DO                                                        TIME0053
      NVS(I)=0                                                          TIME0054
      NCN(I)=0                                                          TIME0055
      NCP(I)=0                                                          TIME0056
      NPS(I)=0                                                          TIME0057
      NQS(I)=0                                                          TIME0058
      DO 50 J=1,6                                                       TIME0059
      NODE(I,J)=0                                                       TIME0060
   50 CONTINUE                                                          TIME0061
      DO 51 I=1,MMM                                                     TIME0062
      T1(I)=0.DO                                                        TIME0063
      Y(1,I)=0.DO                                                       TIME0064
      X(I)=0.DO                                                         TIME0065
      NVIS(I)=0                                                         TIME0066
      Q(I)=0.DO                                                         TIME0067
      NQIS(I)=0                                                         TIME0068
      RHS(I)=0.DO                                                       TIME0069
      DO 51 J=1,MMM                                                     TIME0070
      TM(I,J)=0.DO                                                      TIME0071
      CD(I,J)=0.DO                                                      TIME0072
   51 CONTINUE                                                          TIME0073
      DO 52 I=1,21                                                      TIME0074
      N(I)=0                                                            TIME0075
      DO 52 J=1,21                                                      TIME0076
      TM$(I,J)=0.DO                                                     TIME0077
      CD$(I,J)=0.DO                                                     TIME0078
   52 CCNTINUE                                                          TIME0079
      DO 53 I=1,6                                                       TIME0080
      RP$(I)=0.DO                                                       TIME0081
      ZP$(I)=0.DO                                                       TIME0082
   53 CONTINUE                                                          TIME0083
      DO 54 I=1,3                                                       TIME0084
      XC$(I)=0.DO                                                       TIME0085
      YC$(I)=0.DO                                                       TIME0086
```

```
   54 CONTINUE                                                          TIME0087
                                                                        TIME0088
READ NODE NUMBER AND COORDINATES                                        TIME0089
                                                                        TIME0090
      DO 100 J=1,NN                                                     TIME0091
      READ(NREAD,1006)WORD,I,XC(I),YC(I)                                TIME0092
      IF(WORD.EQ.STOP) GO TO 101                                        TIME0093
      NCN(J)=I                                                          TIME0094
  100 CONTINUE                                                          TIME0095
  101 NNCN=J-1                                                          TIME0096
                                                                        TIME0097
THE ARRAY NCP(J) GENERATES THE GLOBAL PRESSURE INDICES (P1,P2.ETC.)     TIME0098
THUS PRESSURE NODES ARE LABELED AS CORNER NODES AND ARE INPUTED         TIME0099
WHEN CNE INPUTS A GLOBAL CORNER NODE FOR J                              TIME0100
                                                                        TIME0101
      DO 107 J=1,NNCN                                                   TIME0102
      NCP(NCN(J))=J+NN+NN                                               TIME0103
  107 CONTINUE                                                          TIME0104
                                                                        TIME0105
READ SYSTEM TOPOLOGY (ELEMENT NO. AND NODE NUMBERS IN                   TIME0106
COUNTERCLOCKWISE FASHION STARTING AT THE UPPER LEFT                     TIME0107
HAND CORNER NODE).                                                      TIME0108
                                                                        TIME0109
      DO 105 I=1,NE                                                     TIME0110
      READ(NREAD,1010)J,NODE(J,1),NODE(J,2),NODE(J,3),                  TIME0111
     1NODE(J,4),NODE(J,5),NODE(J,6)                                     TIME0112
  105 CONTINUE                                                          TIME0113
      MAXDIF=0                                                          TIME0114
      DO 108 I=1,NE                                                     TIME0115
      DO 108 J=1,6                                                      TIME0116
      DO 108 K=1,6                                                      TIME0117
      LL=IABS(NODE(I,J)-NODE(I,K))                                      TIME0118
      IF(LL.GT.MAXDIF) MAXDIF=LL                                        TIME0119
      IBAND=2*(MAXDIF+1)                                                TIME0120
      NEQ=3*NN+NNCN                                                     TIME0121
  108 CONTINUE                                                          TIME0122
      WRITE(NWRITE,1017)IBAND,NEQ                                       TIME0123
 1017 FORMAT(5X,'IBAND=',I3,//,5X,'NEQ =',I3,//)                        TIME0124
                                                                        TIME0125
READ NODES WHERE BOTH U AND V VELOCITY IS SPECIFIED                     TIME0126
                                                                        TIME0127
      DO 110 I=1,MM                                                     TIME0128
      READ(NREAD,1006)WORD,NVELS,VELU,VELV                              TIME0129
      IF(WORD.EQ.STOP) GO TO 111                                        TIME0130
      NVS(I)=NVELS                                                      TIME0131
      X(NVS(I))=VELU                                                    TIME0132
      X(NVS(I)+NN=VELV                                                  TIME0133
  110 CONTINUE                                                          TIME0134
```

```
C COUNT NODES HAVING SPECIFIED VELOCITIES                              TIME0135
C                                                                      TIME0136
C                                                                      TIME0137
  111 NNVELS=I-1                                                       TIME0138
C                                                                      TIME0139
C READ QX AND QY VALUES AT INTERNAL NODES                              TIME0140
C                                                                      TIME0141
      DO 125 I=1,NN                                                    TIME0142
      READ(NREAD,1006)WORD,NQXY,QXNS,QYNS                              TIME0143
      IF(WORD.EQ.STOP) GO TO 126                                       TIME0144
      NQS(I)=NQXY                                                      TIME0145
      Q(NQS(I))=QXNS                                                   TIME0146
      Q(NQS(I)+NN)=QYNS                                                TIME0147
  125 CONTINUE                                                         TIME0148
C                                                                      TIME0149
C COUNT NODES HAVING SPECIFIED QX AND QY                               TIME0150
C                                                                      TIME0151
  126 NNQXY=I-1                                                        TIME0152
C                                                                      TIME0153
C READ NODE NUMBER AND PRESSURE WHERE SPECIFIED                        TIME0154
C                                                                      TIME0155
      DO 130 I=1,NN                                                    TIME0156
      READ(NREAD,1025)WORD,NP,PNP                                      TIME0157
      IF(WORD.EQ.STOP)GO TO 135                                        TIME0158
      NPS(I)=NP                                                        TIME0159
      X(NCP(NPS(I)))=PNP                                               TIME0160
  130 CONTINUE                                                         TIME0161
C                                                                      TIME0162
C COUNT BOUNDARY NODES WHERE PRESSURE IS SPECIFIED                     TIME0163
C                                                                      TIME0164
  135 NNPS=I-1                                                         TIME0165
C                                                                      TIME0166
C READ NODE NUMBER AND TEMPERATURE WHERE SPECIFIED                     TIME0167
C                                                                      TIME0168
      DO 140 I=1,MM                                                    TIME0169
      READ(NREAD,1025) WORD,NTEMP,TNT                                  TIME0170
      IF(WORD.EQ.STOP) GO TO  145                                      TIME0171
      NVS(I+ NNVELS)=NTEMP                                             TIME0172
      X(NVS(I+NNVELS)+MM)=TNT                                          TIME0173
  140 CONTINUE                                                         TIME0174
C                                                                      TIME0175
C COUNT NODES HAVING SPECIFIED TEMPERATURES                            TIME0176
C                                                                      TIME0177
  145 NNTS=I-1                                                         TIME0178
C                                                                      TIME0179
C READ NODE NUMBERS AND QZC WHERE SPECIFIED                            TIME0180
C                                                                      TIME0181
      DO 141 I=1,MM                                                    TIME0182
```

```
      READ(NREAD,1025) WORD,NQZC,QZCNS                                  TIME0183
      IF(WORD.EQ.STOP) GO TO 146                                        TIME0184
      NQS(NNQXY+I)=NQZC                                                 TIME0185
      Q(NQS(NNQXY+I)+2*NN)=QZCNS                                        TIME0186
  141 CONTINUE                                                          TIME0187
                                                                        TIME0188
C COUNT NODES WHERE QZC IS SPECIFIED                                    TIME0189
                                                                        TIME0190
  146 NNQZC=I-1                                                         TIME0191
                                                                        TIME0192
C READ NODE NUMBERS AND HEAT FLUX QZ WHERE SPECIFIED                    TIME0193
                                                                        TIME0194
      DO 142 I=1,MM                                                     TIME0195
      READ(NREAD,1025) WORD,NQZ,QZNS                                    TIME0196
      IF(WORD.EQ.STOP) GO TO  147                                       TIME0197
      NQS(NNQXY+NNQZC+I)=NQZ                                            TIME0198
      Q(NQS(NNQXY+NNQZC+I)+MM)=QZNS                                     TIME0199
  142 CONTINUE                                                          TIME0200
                                                                        TIME0201
C COUNT NODES WHERE HEAT FLUX QZ IS SPECIFIED                           TIME0202
                                                                        TIME0203
  147 NNQZ=I-1                                                          TIME0204
                                                                        TIME0205
C NQIS IS A LIST OF INDICES OF KNOWN QX,QY,QZC AND QZ                   TIME0206
                                                                        TIME0207
      DO 1140 I=1,NNQXY                                                 TIME0208
      NQIS(I)=NQS(I)                                                    TIME0209
      NQIS(I+NNQXY)=NQS(I)+NN                                           TIME0210
 1140 CONTINUE                                                          TIME0211
      DO 1141 I=1,NNQZC                                                 TIME0212
      NQIS(2*NNQXY+I)=NQS(NNQXY+I)+2*NN                                 TIME0213
 1141 CONTINUE                                                          TIME0214
      DO 1145 I=1,NNQZ                                                  TIME0215
      NQIS(2*NNQXY+NNQZC+I)=NQS(NNQXY+NNQZC+I)+MM                       TIME0216
 1145 CONTINUE                                                          TIME0217
                                                                        TIME0218
C NVIS IS A LIST OF KNOWN VELOCITY, PRESSURE, AND TEMPERATURE INDICES   TIME0219
                                                                        TIME0220
      DO 1150 I=1,NNVELS                                                TIME0221
      NVIS(I)=NVS(I)                                                    TIME0222
      NVIS(I+NNVELS)=NVS(I)+NN                                          TIME0223
 1150 CONTINUE                                                          TIME0224
      DO 1155 J=1,NNPS                                                  TIME0225
      NVIS(2*NNVELS+J)=NCP(NPS(J))                                      TIME0226
 1155 CONTINUE                                                          TIME0227
      DO 1160 K=1,NNTS                                                  TIME0228
      NVIS(2*NNVELS+NNPS+K)=NVS(K+NNVELS)+MM                            TIME0229
 1160 CONTINUE                                                          TIME0230
```

```
NNHC=NUMBER OF NODES WHERE HEAT TRANSFER COEFFICIENT IS SPECIFIED
      NNHC=0
NTOTQ=TOTAL NUMBER OF KNOWN QX,QY,QZC, AND QZ
      NTOTQ=2*NNQXY+NNQZC+NNQZ
NTOTVP=TOTAL NUMBER OF KNOWN VELOCITIES, PRESSURES, AND TEMPERATURES
      NTOTVP=2*NNVELS+NNPS+NNTS
PRINT ALL INPUT DATA
      WRITE(NWRITE,1035)NN,NE,NNCN
      WRITE(NWRITE,1036)NNVELS
      WRITE(NWRITE,1037)NNQXY
      WRITE(NWRITE,1038)NNPS
      WRITE(NWRITE,1039)NNTS
      WRITE(NWRITE,1034)NNQZC
      WRITE(NWRITE,1040)NNQZ
      WRITE(NWRITE,1041)
      DO 150 I=1,NNCN
      WRITE(NWRITE,1045)NCN(I),XC(NCN(I)),YC(NCN(I))
  150 CONTINUE
      WRITE(NWRITE,1050)
      DO 155 I=1,NE
      WRITE(NWRITE,1055)I,NODE(I,1),NODE(I,2),NODE(I,3),
     1NODE(I,4),NODE(I,5),NODE(I,6)
  155 CONTINUE
      WRITE(NWRITE,1060)
      DO 160 I=1,NNVELS
      WRITE(NWRITE,1065)I,NVS(I),X(NVS(I)),X(NVS(I)+NN)
  160 CONTINUE
      WRITE(NWRITE,1070)
      DO 165 I=1,NNQXY
      WRITE(NWRITE,1065)I,NQS(I),Q(NQS(I)),Q(NQS(I)+NN)
  165 CONTINUE
      WRITE(NWRITE,1080)
      DO 170 I=1,NNPS
      WRITE(NWRITE,1085)I,NPS(I),X(NCP(NPS(I)))
  170 CONTINUE
      WRITE(NWRITE,1081)
      DO 171 I=1,NNTS
      WRITE(NWRITE,1085)I,NVS(I+NNVELS),X(NVS(I+NNVELS)+MM)
  171 CONTINUE
      WRITE(NWRITE,1083)
```

```
      DO 173 I=1,NNQZC                                                          TIME0279
      WRITE(NWRITE,1085)I,NQS(I+NNQXY),Q(NQS(I+NNQXY)+2*NN)                     TIME0280
  173 CONTINUE                                                                  TIME0281
      WRITE(NWRITE,1082)                                                        TIME0282
      DO 172 I=1,NNQZ                                                           TIME0283
      WRITE(NWRITE,1085)I,NQS(I+NNQXY+NNQZC),Q(NQS(I+NNQXY+NNQZC)+MM)          TIME0284
  172 CONTINUE                                                                  TIME0285
      T=0.D0                                                                    TIME0286
  177 CONTINUE                                                                  TIME0287
      DO 178 I=1,MMM                                                            TIME0288
      RHS(I)=0.D0                                                               TIME0289
      DO 178 J=1,MMM                                                            TIME0290
      TM(I,J)=0.D0                                                              TIME0291
  178 CONTINUE                                                                  TIME0292
      IF(T.GT.0.D0) GO TO 180                                                   TIME0293
      DO 179 I=1,MMM                                                            TIME0294
      DO 179 J=1,MMM                                                            TIME0295
      CD(I,J)=0.D0                                                              TIME0296
  179 CONTINUE                                                                  TIME0297
  180 CONTINUE                                                                  TIME0298
      DO 181 I=1,21                                                             TIME0299
      DO 181 J=1,21                                                             TIME0300
      TM$(I,J)=0.D0                                                             TIME0301
      CD$(I,J)=0.D0                                                             TIME0302
  181 CONTINUE                                                                  TIME0303
                                                                                TIME0304
END OF INPUT AND VERIFICATION ROUTINE                                           TIME0305
                                                                                TIME0306
      DO 300 K=1,NE                                                             TIME0307
      N1=NODE(K,1)                                                              TIME0308
      N2=NODE(K,2)                                                              TIME0309
      N3=NODE(K,3)                                                              TIME0310
      N4=NODE(K,4)                                                              TIME0311
      N5=NODE(K,5)                                                              TIME0312
      N6=NODE(K,6)                                                              TIME0313
      N7=NODE(K,1)+NN                                                           TIME0314
      N8=NODE(K,2)+NN                                                           TIME0315
      N9=NODE(K,3)+NN                                                           TIME0316
      N10=NODE(K,4)+NN                                                          TIME0317
      N11=NODE(K,5)+NN                                                          TIME0318
      N12=NODE(K,6)+NN                                                          TIME0319
      N13=NCP(NODE(K,1))                                                        TIME0320
      N14=NCP(NODE(K,3))                                                        TIME0321
      N15=NCP(NODE(K,5))                                                        TIME0322
      N16=NODE(K,1)+MM                                                          TIME0323
      N17=NODE(K,2)+MM                                                          TIME0324
      N18=NODE(K,3)+MM                                                          TIME0325
      N19=NODE(K,4)+MM                                                          TIME0326
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```
      N20=NODE(K,5)+MM                                                          TIME0327
      N21=NODE(K,6)+MM                                                          TIME0328
      XC$(1)=XC(NODE(K,1))                                                      TIME0329
      XC$(2)=XC(NODE(K,3))                                                      TIME0330
      XC$(3)=XC(NODE(K,5))                                                      TIME0331
      YC$(1)=YC(NODE(K,1))                                                      TIME0332
      YC$(2)=YC(NODE(K,3))                                                      TIME0333
      YC$(3)=YC(NODE(K,5))                                                      TIME0334
      A=1.D0                                                                    TIME0335
      ZBAR=(YC$(1)+YC$(2)+YC$(3))/3.D0                                          TIME0336
      RBAR=(XC$(1)+XC$(2)+XC$(3))/3.D0                                          TIME0337
      IF(NCASE.EQ.2)A=RBAR                                                      TIME0338
      AA=1.D0                                                                   TIME0339
      IF(NCASE.EQ.2)AA=2.D0*3.14159D0*(RBAR)                                    TIME0340
      A1=XC$(2)*YC$(3)-XC$(3)*YC$(2)                                            TIME0341
      A2=XC$(3)*YC$(1)-XC$(1)*YC$(3)                                            TIME0342
      A3=XC$(1)*YC$(2)-XC$(2)*YC$(1)                                            TIME0343
      B1=YC$(2)-YC$(3)                                                          TIME0344
      B2=YC$(3)-YC$(1)                                                          TIME0345
      B3=YC$(1)-YC$(2)                                                          TIME0346
      C1=XC$(3)-XC$(2)                                                          TIME0347
      C2=XC$(1)-XC$(3)                                                          TIME0348
      C3=XC$(2)-XC$(1)                                                          TIME0349
      DEL=DABS(0.5D0*(XC$(1)*(YC$(2)-YC$(3))+XC$(2)*(YC$(3)-YC$(1))             TIME0350
     1+XC$(3)*(YC$(1)-YC$(2))))                                                 TIME0351
      CONST=10.90D0*(1.D0*A/(3.D0*DEL))*AA                                      TIME0352
      D1=-B1/6.D0                                                               TIME0353
      D2=-B2/6.D0                                                               TIME0354
      D3=-B3/6.D0                                                               TIME0355
      E1=-C1/6.D0                                                               TIME0356
      E2=-C2/6.D0                                                               TIME0357
      E3=-C3/6.D0                                                               TIME0358
      F1=B1/2520.D0                                                             TIME0359
      F2=B2/2520.D0                                                             TIME0360
      F3=B3/2520.D0                                                             TIME0361
      G1=C1/2520.D0                                                             TIME0362
      G2=C2/2520.D0                                                             TIME0363
      G3=C3/2520.D0                                                             TIME0364
      U1=T1(N1)                                                                 TIME0365
      U2=T1(N2)                                                                 TIME0366
      U3=T1(N3)                                                                 TIME0367
      U4=T1(N4)                                                                 TIME0368
      U5=T1(N5)                                                                 TIME0369
      U6=T1(N6)                                                                 TIME0370
      V1=T1(N7)                                                                 TIME0371
      V2=T1(N8)                                                                 TIME0372
      V3=T1(N9)                                                                 TIME0373
      V4=T1(N10)                                                                TIME0374
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      V5=T1(N11)                                                        TIME0375
      V6=T1(N12)                                                        TIME0376
      TM$(1,1)=0.75D0*(B1*B1+C1*C1)*CONST                               TIME0377
      TM$(1,2)=(B1*B2+C1*C2)*CONST                                      TIME0378
      TM$(1,3)=-TM$(1,2)*0.25D0                                         TIME0379
      TM$(1,4)=0.                                                       TIME0380
      TM$(1,6)=(B1*B3+C1*C3)*CONST                                      TIME0381
      TM$(1,5)=-TM$(1,6)*.25D0                                          TIME0382
      TM$(3,3)=.75D0*(B2*B2+C2*C2)*CONST                                TIME0383
      TM$(3,4)=(B2*B3+C2*C3)*CONST                                      TIME0384
      TM$(3,5)=-TM$(3,4)*0.25D0                                         TIME0385
      TM$(2,1)=TM$(1,2)                                                 TIME0386
      TM$(2,3)=TM$(1,2)                                                 TIME0387
      TM$(2,2)=8.D0/3.D0*(TM$(1,1)+TM$(3,3))+2.D0*TM$(1,2)              TIME0388
      TM$(2,4)=2.D0*TM$(1,6)+TM$(3,4)+TM$(1,2)+4.D0/3.D0*TM$(3,3)       TIME0389
      TM$(2,5)=0.                                                       TIME0390
      TM$(2,6)=TM$(1,6)+2.D0*TM$(3,4)+TM$(1,2)+4.D0/3.D0*TM$(1,1)       TIME0391
      TM$(3,1)=TM$(1,3)                                                 TIME0392
      TM$(3,2)=TM$(2,3)                                                 TIME0393
      TM$(3,6)=0.                                                       TIME0394
      TM$(5,5)=.75D0*(B3*B3+C3*C3)*CONST                                TIME0395
      TM$(4,1)=TM$(1,4)                                                 TIME0396
      TM$(4,2)=TM$(2,4)                                                 TIME0397
      TM$(4,3)=TM$(3,4)                                                 TIME0398
      TM$(4,4)=8.D0/3.D0*(TM$(3,3)+TM$(5,5))+2.D0*TM$(3,4)              TIME0399
      TM$(4,5)=TM$(3,4)                                                 TIME0400
      TM$(4,6)=TM$(1,6)+TM$(3,4)+2.D0*TM$(1,2)+4.D0/3.D0*TM$(5,5)       TIME0401
      TM$(6,3)=TM$(3,6)                                                 TIME0402
      TM$(5,1)=TM$(1,5)                                                 TIME0403
      TM$(5,2)=TM$(2,5)                                                 TIME0404
      TM$(5,3)=TM$(3,5)                                                 TIME0405
      TM$(5,4)=TM$(4,5)                                                 TIME0406
      TM$(5,6)=TM$(1,6)                                                 TIME0407
      TM$(6,1)=TM$(1,6)                                                 TIME0408
      TM$(6,2)=TM$(2,6)                                                 TIME0409
      TM$(6,4)=TM$(4,6)                                                 TIME0410
      TM$(6,5)=TM$(5,6)                                                 TIME0411
      TM$(6,6)=8.D0/3.D0*(TM$(5,5)+TM$(1,1))+2.D0*TM$(1,6)              TIME0412
      IF(NCASE.NE.1) GO TO 3000                                         TIME0413
                                                                        TIME0414
BEGIN INPUT OF NON-LINEAR TERMS                                         TIME0415
                                                                        TIME0416
      TM$(1,1)=TM$(1,1)                                                 TIME0417
     1-(78.D0*U1+48.D0*U2-9.D0*U3+12.D0*U4-9.D0*U5+48.D0*U6)*F1         TIME0418
     2             -(78.D0*V1+48.D0*V2-9.D0*V3+12.D0*V4-9.D0*V5+48.D0*V6)TIME0419
     3*G1                                                               TIME0420
      TM$(2,1)=TM$(2,1)                                                 TIME0421
     1-(48.D0*U1+160.D0*U2-32.D0*U3+16.D0*U4-20.D0*U5+80.D0*U6)*F1      TIME0422
```

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2              -(48.D0*V1+160.D0*V2-32.D0*V3+16.D0*V4-20.D0*V5+80.D0*TIME0423
3V6)*G1                                                              TIME0424
 TM$(3,1)=TM$(3,1)                                                   TIME0425
1-(-9.D0*U1-32.D0*U2-18.D0*U3-16.D0*U4+11.D0*U5-20.D0*U6)*F1         TIME0426
2              -(-9.D0*V1-32.D0*V2-18.D0*V3-16.D0*V4+11.D0*V5-20.D0*VTIME0427
36)*G1                                                               TIME0428
 TM$(4,1)=TM$(4,1)                                                   TIME0429
1-(12.D0*U1+16.D0*U2-16.D0*U3-96.D0*U4-16.D0*U5+16.D0*U6)*F1         TIME0430
2              -(12.D0*V1+16.D0*V2-16.D0*V3-96.D0*V4-16.D0*V5+16.D0*VTIME0431
36)*G1                                                               TIME0432
 TM$(5,1)=TM$(5,1)                                                   TIME0433
1-(-9.D0*U1-20.D0*U2+11.D0*U3-16.D0*U4-18.D0*U5-32.D0*U6)*F1         TIME0434
2              -(-9.D0*V1-20.D0*V2+11.D0*V3-16.D0*V4-18.D0*V5-32.D0*VTIME0435
36)*G1                                                               TIME0436
 TM$(6,1)=TM$(6,1)                                                   TIME0437
1-(48.D0*U1+80.D0*U2-20.D0*U3+16.D0*U4-32.D0*U5+160.D0*U6)*F1        TIME0438
2              -(48.D0*V1+80.D0*V2-20.D0*V3+16.D0*V4-32.D0*V5+160.D0*TIME0439
3V6)*G1                                                              TIME0440
 TM$(1,2)=TM$(1,2)                                                   TIME0441
1-(24.D0*U1-32.D0*U2-16.D0*U3-48.D0*U4+4.D0*U5-16.D0*U6)*F1          TIME0442
2              -(24.D0*V1-32.D0*V2-16.D0*V3-48.D0*V4+4.D0*V5-16.D0*V6TIME0443
3)*G1                        -(120.D0*U1+48.D0*U2-16.D0*U3-16.D0*U4-16.TIME0444
4D0*U5+48.D0*U6)*F2                    -(120.D0*V1+48.D0*V2-16.D0*V3-TIME0445
516.D0*V4-16.D0*V5+48.D0*V6)*G2                                      TIME0446
 TM$(2,2)=TM$(2,2)                                                   TIME0447
1-(-32.D0*U1+384.D0*U2+48.D0*U3+192.D0*U4-48.D0*U5+128.D0*U6)*F1     TIME0448
2              -(-32.D0*V1+384.D0*V2+48.D0*V3+192.D0*V4-48.D0*V5+128.TIME0449
3D0*V6)*G1                   -(48.D0*U1+384.D0*U2-32.D0*U3+128.D0*U4-48TIME0450
4.D0*U5+192.D0*U6)*F2                  -(48.D0*V1+384.D0*V2-32.D0*V3+TIME0451
5128.D0*V4-48.D0*V5+192.D0*V6)*G2                                    TIME0452
 TM$(3,2)=TM$(3,2)                                                   TIME0453
1-(-16.D0*U1+48.D0*U2+120.D0*U3+48.D0*U4-16.D0*U5-16.D0*U6)*F1       TIME0454
2              -(-16.D0*V1+48.D0*V2+120.D0*V3+48.D0*V4-16.D0*V5-16.D0TIME0455
3*V6)*G1                     -(-16.D0*U1-32.D0*U2+24.D0*U3-16.D0*U4+4.DTIME0456
40*U5-48.D0*U6)*F2                     -(-16.D0*V1-32.D0*V2+24.D0*V3-TIME0457
516.D0*V4+4.D0*V5-48.D0*V6)*G2                                       TIME0458
 TM$(4,2)=TM$(4,2)                                                   TIME0459
1-(-48.D0*U1+192.D0*U2+48.D0*U3+384.D0*U4-32.D0*U5+128.D0*U6)*F1     TIME0460
2              -(-48.D0*V1+192.D0*V2+48.D0*V3+384.D0*V4-32.D0*V5+128.TIME0461
3D0*V6)*G1                   -(-16.D0*U1+128.D0*U2-16.D0*U3+128.D0*U4-1TIME0462
46.D0*U5+128.D0*U6)*F2                 -(-16.D0*V1+128.D0*V2-16.D0*V3TIME0463
5+128.D0*V4-16.D0*V5+128.D0*V6)*G2                                   TIME0464
 TM$(5,2)=TM$(5,2)                                                   TIME0465
1-(4.D0*U1-48.D0*U2-16.D0*U3-32.D0*U4+24.D0*U5-16.D0*U6)*F1          TIME0466
2              -(4.D0*V1-48.D0*V2-16.D0*V3-32.D0*V4+24.D0*V5-16.D0*V6TIME0467
3)*G1                        -(-16.D0*U1-48.D0*U2+4.D0*U3-16.D0*U4+24.DTIME0468
40*U5-32.D0*U6)*F2                     -(-16.D0*V1-48.D0*V2+4.D0*V3-1TIME0469
56.D0*V4+24.D0*V5-32.D0*V6)*G2                                       TIME0470
```

```
 TM$(6,2)=TM$(6,2)                                                      TIME0471
1-(-16.D0*U1+128.D0*U2-16.D0*U3+128.D0*U4-16.D0*U5+128.D0*U6)*F1        TIME0472
2                -(-16.D0*V1+128.D0*V2-16.D0*V3+128.D0*V4-16.D0*V5+128.TIME0473
3D0*V6)*G1                      -(48.D0*U1+192.D0*U2-48.D0*U3+128.D0*U4-32TIME0474
4.D0*U5+384.D0*U6)*F2                           -(48.D0*V1+192.D0*V2-48.D0*V3+TIME0475
5128.D0*V4-32.D0*V5+384.D0*V6)*G2                                       TIME0476
                                                                        TIME0477
 TM$(1,3)=TM$(1,3)                                                      TIME0478
1-(-18.D0*U1-32.D0*U2-9.D0*U3-20.D0*U4+11.D0*U5-16.D0*U6)*F2            TIME0479
2                -(-18.D0*V1-32.D0*V2-9.D0*V3-20.D0*V4+11.D0*V5-16.D0*VTIME0480
36)*G2                                                                  TIME0481
 TM$(2,3)=TM$(2,3)                                                      TIME0482
1-(-32.D0*U1+160.D0*U2+48.D0*U3+80.D0*U4-20.D0*U5+16.D0*U6)*F2          TIME0483
2                -(-32.D0*V1+160.D0*V2+48.D0*V3+80.D0*V4-20.D0*V5+16.DOTIME0484
3*V6)*G2                                                                TIME0485
 TM$(3,3)=TM$(3,3)                                                      TIME0486
1-(-9.D0*U1+48.D0*U2+78.D0*U3+48.D0*U4-9.D0*U5+12.D0*U6)*F2             TIME0487
2                -(-9.D0*V1+48.D0*V2+78.D0*V3+48.D0*V4-9.D0*V5+12.D0*V6TIME0488
3)*G2                                                                   TIME0489
 TM$(4,3)=TM$(4,3)                                                      TIME0490
1-(-20.D0*U1+80.D0*U2+48.D0*U3+160.D0*U4-32.D0*U5+16.D0*U6)*F2          TIME0491
2                -(-20.D0*V1+80.D0*V2+48.D0*V3+160.D0*V4-32.D0*V5+16.DOTIME0492
3*V6)*G2                                                                TIME0493
 TM$(5,3)=TM$(5,3)                                                      TIME0494
1-(11.D0*U1-20.D0*U2-9.D0*U3-32.D0*U4-18.D0*U5-16.D0*U6)*F2             TIME0495
2                -(11.D0*V1-20.D0*V2-9.D0*V3-32.D0*V4-18.D0*V5-16.D0*V6TIME0496
3)*G2                                                                   TIME0497
 TM$(6,3)=TM$(6,3)                                                      TIME0498
1-(-16.D0*U1+16.D0*U2+12.D0*U3+16.D0*U4-16.D0*U5-96.D0*U6)*F2           TIME0499
2                -(-16.D0*V1+16.D0*V2+12.D0*V3+16.D0*V4-16.D0*V5-96.D0*TIME0500
3V6)*G2                                                                 TIME0501
 TM$(1,4)=TM$(1,4)                                                      TIME0502
1-(24.D0*U1-16.D0*U2+4.D0*U3-48.D0*U4-16.D0*U5-32.D0*U6)*F2             TIME0503
2                -(24.D0*V1-16.D0*V2+4.D0*V3-48.D0*V4-16.D0*V5-32.D0*V6TIME0504
3)*G2                           -(24.D0*U1-32.D0*U2-16.D0*U3-48.D0*U4+4.DOTIME0505
4*U5-16.D0*U6)*F3                               -(24.D0*V1-32.D0*V2-16.D0*V3-4TIME0506
58.D0*V4+4.D0*V5-16.D0*V6)*G3                                           TIME0507
 TM$(2,4)=TM$(2,4)                                                      TIME0508
1-(-16.D0*U1+128.D0*U2-16.D0*U3+128.D0*U4-16.D0*U5+128.D0*U6)*F2        TIME0509
2                -(-16.D0*V1+128.D0*V2-16.D0*V3+128.D0*V4-16.D0*V5+128.TIME0510
3D0*V6)*G2                      -(-32.D0*U1+384.D0*U2+48.D0*U3+192.D0*U4-4TIME0511
48.D0*U5+128.D0*U6)*F3                          -(-32.D0*V1+384.D0*V2+48.D0*V3TIME0512
5+192.D0*V4-48.D0*V5+128.D0*V6)*G3                                      TIME0513
 TM$(3,4)=TM$(3,4)                                                      TIME0514
1-(4.D0*U1-16.D0*U2+24.D0*U3-32.D0*U4+16.D0*U5-48.D0*U6)*F2             TIME0515
2                -(4.D0*V1-16.D0*V2+24.D0*V3-32.D0*V4+16.D0*V5-48.D0*V6TIME0516
3)*G2                           -(-16.D0*U1+48.D0*U2+120.D0*U3+48.D0*U4-16TIME0517
4.D0*U5-16.D0*U6)*F3                            -(-16.D0*V1+48.D0*V2+120.D0*V3TIME0518
5+48.D0*V4-16.D0*V5-16.D0*V6)*G3                                        TIME0519
```

```
 TM$(4,4)=TM$(4,4)                                                      TIME0519
1-(-48.D0*U1+128.D0*U2-32.D0*U3+384.D0*U4+48.D0*U5+192.D0*U6)*F2        TIME0520
2              -(-48.D0*V1+128.D0*V2-32.D0*V3+384.D0*V4+48.D0*V5+192.TIME0521
3D0*V6)*G2                    -(-48.D0*U1+192.D0*U2+48.D0*U3+384.D0*U4-3TIME0522
42.D0*U5+128.D0*U6)*F3                        -(-48.D0*V1+192.D0*V2+48.D0*V3TIME0523
5+384.D0*V4-32.D0*V5+128.D0*V6)*G3                                      TIME0524
 TM$(5,4)=TM$(5,4)                                                      TIME0525
1-(-16.D0*U1-16.D0*U2-16.D0*U3+48.D0*U4+120.D0*U5+48.D0*U6)*F2         TIME0526
2              -(-16.D0*V1-16.D0*V2-16.D0*V3+48.D0*V4+120.D0*V5+48.D0TIME0527
3*V6)*G2                      -(4.D0*U1-48.D0*U2-16.D0*U3-32.D0*U4+24.D0TIME0528
4*U5-16.D0*U6)*F3                             -(4.D0*V1-48.D0*V2-16.D0*V3-32TIME0529
5.D0*V4+24.D0*V5-16.D0*V6)*G3                                           TIME0530
 TM$(6,4)=TM$(6,4)                                                      TIME0531
1-(-32.D0*U1+128.D0*U2-48.D0*U3+192.D0*U4+48.D0*U5+384.D0*U6)*F2        TIME0532
2              -(-32.D0*V1+128.D0*V2-48.D0*V3+192.D0*V4+48.D0*V5+384.TIME0533
3D0*V6)*G2                    -(-16.D0*U1+128.D0*U2-16.D0*U3+128.D0*U4-1TIME0534
46.D0*U5+128.D0*U6)*F3                        -(-16.D0*V1+128.D0*V2-16.D0*V3TIME0535
5+128.D0*V4-16.D0*V5+128.D0*V6)*G3                                      TIME0536
 TM$(1,5)=TM$(1,5)                                                      TIME0537
1-(-18.D0*U1-16.D0*U2+11.D0*U3-20.D0*U4-9.D0*U5-32.D0*U6)*F3           TIME0538
2              -(-18.D0*V1-16.D0*V2+11.D0*V3-20.D0*V4-9.D0*V5-32.D0*VTIME0539
36)*G3                                                                  TIME0540
 TM$(2,5)=TM$(2,5)                                                      TIME0541
1-(-16.D0*U1-96.D0*U2-16.D0*U3+16.D0*U4+12.D0*U5+16.D0*U6)*F3          TIME0542
2              -(-16.D0*V1-96.D0*V2-16.D0*V3+16.D0*V4+12.D0*V5+16.D0*TIME0543
3V6)*G3                                                                 TIME0544
 TM$(3,5)=TM$(3,5)                                                      TIME0545
1-(11.D0*U1-16.D0*U2-18.D0*U3-32.D0*U4-9.D0*U5-20.D0*U6)*F3            TIME0546
2              -(11.D0*V1-16.D0*V2-18.D0*V3-32.D0*V4-9.D0*V5-20.D0*V6TIME0547
3)*G3                                                                   TIME0548
 TM$(4,5)=TM$(4,5)                                                      TIME0549
1-(-20.D0*U1+16.D0*U2-32.D0*U3+160.D0*U4+48.D0*U5+80.D0*U6)*F3         TIME0550
2              -(-20.D0*V1+16.D0*V2-32.D0*V3+160.D0*V4+48.D0*V5+80.D0TIME0551
3*V6)*G3                                                                TIME0552
 TM$(5,5)=TM$(5,5)                                                      TIME0553
1-(-9.D0*U1+12.D0*U2-9.D0*U3+48.D0*U4+78.D0*U5+48.D0*U6)*F3            TIME0554
2              -(-9.D0*V1+12.D0*V2-9.D0*V3+48.D0*V4+78.D0*V5+48.D0*V6TIME0555
3)*G3                                                                   TIME0556
 TM$(6,5)=TM$(6,5)                                                      TIME0557
1-(-32.D0*U1+16.D0*U2-20.D0*U3+80.D0*U4+48.D0*U5+160.D0*U6)*F3         TIME0558
2              -(-32.D0*V1+16.D0*V2-20.D0*V3+80.D0*V4+48.D0*V5+160.D0TIME0559
3*V6)*G3                                                                TIME0560
 TM$(1,6)=TM$(1,6)                                                      TIME0561
1-(24.D0*U1-16.D0*U2+4.D0*U3-48.D0*U4-16.D0*U5-32.D0*U6)*F1            TIME0562
2              -(24.D0*V1-16.D0*V2+4.D0*V3-48.D0*V4-16.D0*V5-32.D0*V6TIME0563
3)*G1                         -(120.D0*U1+48.D0*U2-16.D0*U3-16.D0*U4-16.TIME0564
4D0*U5+48.D0*U6)*F3                           -(120.D0*V1+48.D0*V2-16.D0*V3-TIME0565
516.D0*V4-16.D0*V5+48.D0*V6)*G3                                         TIME0566
```

```
      TM$(2,6)=TM$(2,6)                                                 TIME0567
     1-(-16.D0*U1+128.D0*U2-16.D0*U3+128.D0*U4-16.D0*U5+128.D0*U6)*F1   TIME0568
     2              -(-16.D0*V1+128.D0*V2-16.D0*V3+128.D0*V4-16.D0*V5+128.TIME0569
     3D0*V6)*G1                    -(48.D0*U1+384.D0*U2-32.D0*U3+128.D0*U4-48TIME0570
     4.D0*U5+192.D0*U6)*F3                         -(48.D0*V1+384.D0*V2-32.D0*V3+TIME0571
     5128.D0*V4-48.D0*V5+192.D0*V6)*G3                                  TIME0572
                                                                        TIME0573
      TM$(3,6)=TM$(3,6)                                                 TIME0574
     1-(4.D0*U1-16.D0*U2+24.D0*U3-32.D0*U4-16.D0*U5-48.D0*U6)*F1        TIME0575
     2              -(4.D0*V1-16.D0*V2+24.D0*V3-32.D0*V4-16.D0*V5-48.D0*V6TIME0576
     3)*G1                         -(-16.D0*U1-32.D0*U2+24.D0*U3-16.D0*U4+4.DTIME0577
     40*U5-48.D0*U6)*F3                            -(-16.D0*V1-32.D0*V2+24.D0*V3-TIME0578
     516.D0*V4+4.D0*V5-48.D0*V6)*G3                                     TIME0579
      TM$(4,6)=TM$(4,6)                                                 TIME0580
     1-(-48.D0*U1+128.D0*U2-32.D0*U3+384.D0*U4+48.D0*U5+192.D0*U6)*F1   TIME0581
     2              -(-48.D0*V1+128.D0*V2-32.D0*V3+384.D0*V4+48.D0*V5+192.TIME0582
     3D0*V6)*G1                    -(-16.D0*U1+128.D0*U2-16.D0*U3+128.D0*U4-1TIME0583
     46.D0*U5+128.D0*U6)*F3                        -(-16.D0*V1+128.D0*V2-16.D0*V3TIME0584
     5+128.D0*V4-16.D0*V5+128.D0*V6)*G3                                 TIME0585
      TM$(5,6)=TM$(5,6)                                                 TIME0586
     1-(-16.D0*U1-16.D0*U2-16.D0*U3+48.D0*U4+120.D0*U5+48.D0*U6)*F1     TIME0587
     2              -(-16.D0*V1-16.D0*V2-16.D0*V3+48.D0*V4+120.D0*V5+48.D0TIME0588
     3*V6)*G1                      -(-16.D0*U1-48.D0*U2+4.D0*U3-16.D0*U4+24.DTIME0589
     40*U5-32.D0*U6)*F3                            -(-16.D0*V1-48.D0*V2+4.D0*V3-1TIME0590
     56.D0*V4+24.D0*V5-32.D0*V6)*G3                                     TIME0591
      TM$(6,6)=TM$(6,6)                                                 TIME0592
     1-(-32.D0*U1+128.D0*U2-48.D0*U3+192.D0*U4+48.D0*U5+384.D0*U6)*F1   TIME0593
     2              -(-32.D0*V1+128.D0*V2-48.D0*V3+192.D0*V4+48.D0*V5+384.TIME0594
     3D0*V6)*G1                    -(48.D0*U1+192.D0*U2-48.D0*U3+128.D0*U4-32TIME0595
     4.D0*U5+384.D0*U6)*F3                         -(48.D0*V1+192.D0*V2-48.D0*V3+TIME0596
     5128.D0*V4-32.D0*V5+384.D0*V6)*G3                                  TIME0597
THIS ENDS ADDITION OF NON-LINEAR TERMS TO THE LOCAL ARRAY               TIME0598
                                                                        TIME0599
 3000 CONTINUE                                                          TIME0600
      TM$(7,7)=TM$(1,1)                                                 TIME0601
      TM$(7,8)=TM$(1,2)                                                 TIME0602
      TM$(7,9)=TM$(1,3)                                                 TIME0603
      TM$(7,10)=TM$(1,4)                                                TIME0604
      TM$(7,11)=TM$(1,5)                                                TIME0605
      TM$(7,12)=TM$(1,6)                                                TIME0606
      TM$(8,7)=TM$(2,1)                                                 TIME0607
      TM$(8,8)=TM$(2,2)                                                 TIME0608
      TM$(8,9)=TM$(2,3)                                                 TIME0609
      TM$(8,10)=TM$(2,4)                                                TIME0610
      TM$(8,11)=TM$(2,5)                                                TIME0611
      TM$(8,12)=TM$(2,6)                                                TIME0612
      TM$(9,7)=TM$(3,1)                                                 TIME0613
      TM$(9,8)=TM$(3,2)                                                 TIME0614
```

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TIME00615
TIME00616
TIME00617
TIME00618
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TIME00659
TIME00660
TIME00661
TIME00662
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```
TM$(9,9)=TM$(3,3)
TM$(9,10)=TM$(3,4)
TM$(9,11)=TM$(3,5)
TM$(9,12)=TM$(3,6)
TM$(10,7)=TM$(4,1)
TM$(10,8)=TM$(4,2)
TM$(10,9)=TM$(4,3)
TM$(10,10)=TM$(4,4)
TM$(10,11)=TM$(4,5)
TM$(10,12)=TM$(4,6)
TM$(11,7)=TM$(5,1)
TM$(11,8)=TM$(5,2)
TM$(11,9)=TM$(5,3)
TM$(11,10)=TM$(5,4)
TM$(11,11)=TM$(5,5)
TM$(11,12)=TM$(5,6)
TM$(12,7)=TM$(6,1)
TM$(12,8)=TM$(6,2)
TM$(12,9)=TM$(6,3)
TM$(12,10)=TM$(6,4)
TM$(12,11)=TM$(6,5)
TM$(12,12)=TM$(6,6)
TM$(13,11)=D1
TM$(14,11)=D0
TM$(15,11)=D0
TM$(13,12)=D1+D0
TM$(14,12)=D0*D2
TM$(15,12)=D0
TM$(13,13)=2
TM$(14,13)=D1+D2
TM$(15,13)=D0
TM$(13,14)=D2
TM$(14,14)=D0
TM$(15,14)=D0
TM$(13,15)=D2+D3
TM$(14,15)=D0*D3
TM$(15,15)=D0
TM$(13,16)=2
TM$(14,16)=D2+D3
TM$(15,16)=D0
TM$(13,17)=D3
TM$(14,17)=D0
TM$(15,17)=D0
TM$(13,18)=D1+D3
TM$(14,18)=D0*D3
TM$(15,18)=D0
TM$(13,19)=2
TM$(14,19)=D1+D3
TM$(15,19)=D0
TM$(13,20)=E1
TM$(14,20)=D0
TM$(15,20)=D0
TM$(13,21)=E1+E2
TM$(14,21)=D0*E2
TM$(15,21)=D0
TM$(13,22)=2
TM$(14,22)=E1+E2
TM$(15,22)=D0
TM$(13,23)=E0
TM$(14,23)=D0
TM$(13,24)=E2
TM$(14,24)=D0
TM$(13,25)=E2
TM$(14,25)=D0
TM$(13,26)=E2
TM$(14,26)=D0
TM$(13,27)=E2
TM$(14,27)=D0
TM$(13,28)=E2
TM$(14,28)=D0
TM$(13,29)=E2
TM$(14,29)=D0
TM$(13,30)=E2
TM$(14,30)=D0
TM$(13,31)=E2
TM$(14,31)=D0
TM$(13,32)=E2
TM$(14,32)=D0
TM$(13,33)=E2
TM$(14,33)=D0
TM$(13,34)=E2
TM$(14,34)=D0
TM$(13,35)=E2
TM$(14,35)=D0
TM$(13,36)=E2
TM$(14,36)=D0
TM$(13,37)=E2
TM$(14,37)=D0
TM$(13,38)=E2
TM$(14,38)=D0
TM$(13,39)=E2
TM$(14,39)=D0
TM$(13,40)=E2
TM$(14,40)=D0
TM$(13,41)=E2
TM$(14,41)=D0
TM$(13,42)=E2
TM$(14,42)=D0
TM$(13,43)=E2
```

```
TM$(15,9)=0.0                                                    TIME0663
TM$(13,10)=E2+E3                                                 TIME0664
TM$(14,10)=E2+2.D0*E3                                            TIME0665
TM$(15,10)=E2+E2+E3                                              TIME0666
TM$(13,11)=0.D0                                                  TIME0667
TM$(14,11)=0.D0                                                  TIME0668
TM$(15,11)=E3                                                    TIME0669
TM$(13,12)=E1+E3+E3                                              TIME0670
TM$(14,12)=E1+E3                                                 TIME0671
TM$(15,12)=2.D0*E1+E3                                            TIME0672
CONST4=0.94375D0                                                 TIME0673
TM$(1,13)=D1*CONST4                                              TIME0674
TM$(1,14)=0.D0                                                   TIME0675
TM$(1,15)=0.D0                                                   TIME0676
TM$(2,13)=(D1+2.D0*D2)*CONST4                                    TIME0677
TM$(2,14)=(2.D0*D1+D2)*CONST4                                    TIME0678
TM$(2,15)=(D1+D2)*CONST4                                         TIME0679
TM$(3,13)=0.D0                                                   TIME0680
TM$(3,14)=D2*CONST4                                              TIME0681
TM$(3,15)=0.D0                                                   TIME0682
TM$(4,13)=(D2+D3)*CONST4                                         TIME0683
TM$(4,14)=(D2+2.D0*D3)*CONST4                                    TIME0684
TM$(4,15)=(2.D0*D2+D3)*CONST4                                    TIME0685
TM$(5,13)=0.D0                                                   TIME0686
TM$(5,14)=0.D0                                                   TIME0687
TM$(5,15)=D3*CONST4                                              TIME0688
TM$(6,13)=(D1+2.D0*D3)*CONST4                                    TIME0689
TM$(6,14)=(D1+D3)*CONST4                                         TIME0690
TM$(6,15)=(2.D0*D1+D3)*CONST4                                    TIME0691
TM$(7,13)=E1*CONST4                                              TIME0692
TM$(7,14)=0.D0                                                   TIME0693
TM$(7,15)=0.D0                                                   TIME0694
TM$(8,13)=(E1+2.D0*E2)*CONST4                                    TIME0695
TM$(8,14)=(2.D0*E1+E2)*CONST4                                    TIME0696
TM$(8,15)=(E1+E2)*CONST4                                         TIME0697
TM$(9,13)=0.D0                                                   TIME0698
TM$(9,14)=E2*CONST4                                              TIME0699
TM$(9,15)=0.D0                                                   TIME0700
TM$(10,13)=(E2+E3)*CONST4                                        TIME0701
TM$(10,14)=(E2+2.D0*E3)*CONST4                                   TIME0702
TM$(10,15)=(E2+E2+E3)*CONST4                                     TIME0703
TM$(11,13)=0.D0                                                  TIME0704
TM$(11,14)=0.D0                                                  TIME0705
TM$(11,15)=E3*CONST4                                             TIME0706
TM$(12,13)=(E1+2.D0*E3)*CONST4                                   TIME0707
TM$(12,14)=(E1+E3)*CONST4                                        TIME0708
TM$(12,15)=(2.D0*E1+E3)*CONST4                                   TIME0709
CONST2=3.2981D0                                                  TIME0710
```

```
      TM$(1,16)=1.D0*CONST2                                          TIME0711
      TM$(2,17)=1.D0*CONST2                                          TIME0712
      TM$(3,18)=1.D0*CONST2                                          TIME0713
      TM$(4,19)=1.D0*CONST2                                          TIME0714
      TM$(5,20)=1.D0*CONST2                                          TIME0715
      TM$(6,21)=1.D0*CONST2                                          TIME0716
                                                                     TIME0717
ALPHA1=ALPHA/VISCOSITY                                               TIME0718
                                                                     TIME0719
      ALPHA1=9.4454D-05                                              TIME0720
      CONST1=ALPHA1                                                  TIME0721
      TM$(16,16)=TM$(1,1)*CONST1                                     TIME0722
      TM$(16,17)=TM$(1,2)*CONST1                                     TIME0723
      TM$(16,18)=TM$(1,3)*CONST1                                     TIME0724
      TM$(16,19)=TM$(1,4)*CONST1                                     TIME0725
      TM$(16,20)=TM$(1,5)*CONST1                                     TIME0726
      TM$(16,21)=TM$(1,6)*CONST1                                     TIME0727
      TM$(17,16)=TM$(2,1)*CONST1                                     TIME0728
      TM$(17,17)=TM$(2,2)*CONST1                                     TIME0729
      TM$(17,18)=TM$(2,3)*CONST1                                     TIME0730
      TM$(17,19)=TM$(2,4)*CONST1                                     TIME0731
      TM$(17,20)=TM$(2,5)*CONST1                                     TIME0732
      TM$(17,21)=TM$(2,6)*CONST1                                     TIME0733
      TM$(18,16)=TM$(3,1)*CONST1                                     TIME0734
      TM$(18,17)=TM$(3,2)*CONST1                                     TIME0735
      TM$(18,18)=TM$(3,3)*CONST1                                     TIME0736
      TM$(18,19)=TM$(3,4)*CONST1                                     TIME0737
      TM$(18,20)=TM$(3,5)*CONST1                                     TIME0738
      TM$(18,21)=TM$(3,6)*CONST1                                     TIME0739
      TM$(19,16)=TM$(4,1)*CONST1                                     TIME0740
      TM$(19,17)=TM$(4,2)*CONST1                                     TIME0741
      TM$(19,18)=TM$(4,3)*CONST1                                     TIME0742
      TM$(19,19)=TM$(4,4)*CONST1                                     TIME0743
      TM$(19,20)=TM$(4,5)*CONST1                                     TIME0744
      TM$(19,21)=TM$(4,6)*CONST1                                     TIME0745
      TM$(20,16)=TM$(5,1)*CONST1                                     TIME0746
      TM$(20,17)=TM$(5,2)*CONST1                                     TIME0747
      TM$(20,18)=TM$(5,3)*CONST1                                     TIME0748
      TM$(20,19)=TM$(5,4)*CONST1                                     TIME0749
      TM$(20,20)=TM$(5,5)*CONST1                                     TIME0750
      TM$(20,21)=TM$(5,6)*CONST1                                     TIME0751
      TM$(21,16)=TM$(6,1)*CONST1                                     TIME0752
      TM$(21,17)=TM$(6,2)*CONST1                                     TIME0753
      TM$(21,18)=TM$(6,3)*CONST1                                     TIME0754
      TM$(21,19)=TM$(6,4)*CONST1                                     TIME0755
      TM$(21,20)=TM$(6,5)*CONST1                                     TIME0756
      TM$(21,21)=TM$(6,6)*CONST1                                     TIME0757
      IF(T.GT.0.D0) GO TO 185                                        TIME0758
```

```
      CONST3=-DEL/180.D0                                                TIME00759
      CD$(1,1)=6.D0*CONST3                                              TIME00760
      CD$(1,2)=0.D0                                                     TIME00761
      CD$(1,3)=-CONST3                                                  TIME00762
      CD$(1,4)=-4.D0*CONST3                                             TIME00763
      CD$(1,5)=-CONST3                                                  TIME00764
      CD$(1,6)=0.D0                                                     TIME00765
      CD$(2,1)=0.D0                                                     TIME00766
      CD$(2,2)=32.D0*CONST3                                             TIME00767
      CD$(2,3)=0.D0                                                     TIME00768
      CD$(2,4)=16.D0*CONST3                                             TIME00769
      CD$(2,5)=-4.D0*CONST3                                             TIME00770
      CD$(2,6)=16.D0*CONST3                                             TIME00771
      CD$(3,1)=-CONST3                                                  TIME00772
      CD$(3,2)=0.D0                                                     TIME00773
      CD$(3,3)=6.D0*CONST3                                              TIME00774
      CD$(3,4)=0.D0                                                     TIME00775
      CD$(3,5)=-CONST3                                                  TIME00776
      CD$(3,6)=-4.D0*CONST3                                             TIME00777
      CD$(4,1)=-4.D0*CONST3                                             TIME00778
      CD$(4,2)=16.D0*CONST3                                             TIME00779
      CD$(4,3)=0.D0                                                     TIME00780
      CD$(4,4)=32.D0*CONST3                                             TIME00781
      CD$(4,5)=0.D0                                                     TIME00782
      CD$(4,6)=16.D0*CONST3                                             TIME00783
      CD$(5,1)=-CONST3                                                  TIME00784
      CD$(5,2)=-4.D0*CONST3                                             TIME00785
      CD$(5,3)=-CONST3                                                  TIME00786
      CD$(5,4)=0.D0                                                     TIME00787
      CD$(5,5)=6.D0*CONST3                                              TIME00788
      CD$(5,6)=0.D0                                                     TIME00789
      CD$(6,1)=0.D0                                                     TIME00790
      CD$(6,2)=16.D0*CONST3                                             TIME00791
      CD$(6,3)=-4.D0*CONST3                                             TIME00792
      CD$(6,4)=16.D0*CONST3                                             TIME00793
      CD$(6,5)=0.D0                                                     TIME00794
      CD$(6,6)=32.D0*CONST3                                             TIME00795
      CD$(7,1)=CD$(1,1)                                                 TIME00796
      CD$(7,2)=CD$(1,2)                                                 TIME00797
      CD$(7,3)=CD$(1,3)                                                 TIME00798
      CD$(7,4)=CD$(1,4)                                                 TIME00799
      CD$(7,5)=CD$(1,5)                                                 TIME00800
      CD$(7,6)=CD$(1,6)                                                 TIME00801
      CD$(8,1)=CD$(2,1)                                                 TIME00802
      CD$(8,2)=CD$(2,2)                                                 TIME00803
      CD$(8,3)=CD$(2,3)                                                 TIME00804
      CD$(8,4)=CD$(2,4)                                                 TIME00805
      CD$(8,5)=CD$(2,5)                                                 TIME00806
```

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      CD$(19,21)=CD$(4,6)                                              TIME0855
      CD$(20,16)=CD$(5,1)                                              TIME0856
      CD$(20,17)=CD$(5,2)                                              TIME0857
      CD$(20,18)=CD$(5,3)                                              TIME0858
      CD$(20,19)=CD$(5,4)                                              TIME0859
      CD$(20,20)=CD$(5,5)                                              TIME0860
      CD$(20,21)=CD$(5,6)                                              TIME0861
      CD$(21,16)=CD$(6,1)                                              TIME0862
      CD$(21,17)=CD$(6,2)                                              TIME0863
      CD$(21,18)=CD$(6,3)                                              TIME0864
      CD$(21,19)=CD$(6,4)                                              TIME0865
      CD$(21,20)=CD$(6,5)                                              TIME0866
      CD$(21,21)=CD$(6,6)                                              TIME0867
  185 CONTINUE                                                         TIME0868
      N(1)=N1                                                          TIME0869
      N(2)=N2                                                          TIME0870
      N(3)=N3                                                          TIME0871
      N(4)=N4                                                          TIME0872
      N(5)=N5                                                          TIME0873
      N(6)=N6                                                          TIME0874
      N(7)=N7                                                          TIME0875
      N(8)=N8                                                          TIME0876
      N(9)=N9                                                          TIME0877
      N(10)=N10                                                        TIME0878
      N(11)=N11                                                        TIME0879
      N(12)=N12                                                        TIME0880
      N(13)=N13                                                        TIME0881
      N(14)=N14                                                        TIME0882
      N(15)=N15                                                        TIME0883
      N(16)=N16                                                        TIME0884
      N(17)=N17                                                        TIME0885
      N(18)=N18                                                        TIME0886
      N(19)=N19                                                        TIME0887
      N(20)=N20                                                        TIME0888
      N(21)=N21                                                        TIME0889
      DO 200 I$=1,21                                                   TIME0890
      I=N(I$)                                                          TIME0891
      DO 200 J$=1,21                                                   TIME0892
      J=N(J$)                                                          TIME0893
      TM(I,J)=TM(I,J)+TM$(I$,J$)                                       TIME0894
  200 CONTINUE                                                         TIME0895
      IF(T.GT.0.DO) GO TO 300                                          TIME0896
      DO 210 I$=1,21                                                   TIME0897
      I=N(I$)                                                          TIME0898
      DO 210 J$=1,21                                                   TIME0899
      J=N(J$)                                                          TIME0900
      CD(I,J)=CD(I,J)+CD$(I$,J$)                                       TIME0901
  210 CONTINUE                                                         TIME0902
```

```
  300 CONTINUE                                                          TIME0903
      IF(NNQXY.EQ.0) GO TO 310                                          TIME0904
      DO 310 I=1,NNQXY                                                  TIME0905
      RHS(NQS(I))=RHS(NQS(I))+Q(NQS(I))+65.962D0                        TIME0906
      RHS(NQS(I)+NN)=RHS(NQS(I)+NN)+Q(NQS(I)+NN)                        TIME0907
  310 CONTINUE                                                          TIME0908
      IF(NNQZC.EQ.0) GO TO 312                                          TIME0909
      DO 312 I=1,NNQZC                                                  TIME0910
      RHS(NQS(NNQXY+I)+2*NN)=RHS(NQS(NNQXY+I)+2*NN)+Q(NQS(NNQXY+I)+2*NN)TIME0911
  312 CONTINUE                                                          TIME0912
      IF(NNQZ.EQ.0) GO TO 311                                           TIME0913
      DO 311 I=1,NNQZ                                                   TIME0914
      RHS(NQS(NNQXY+NNQZC+I)+MM)=RHS(NQS(NNQXY+NNQZC+I)+MM)+            TIME0915
     1Q(NQS(NNQXY+NNQZC+I)+MM)                                          TIME0916
  311 CONTINUE                                                          TIME0917
                                                                        TIME0918
C MODIFICATION OF RHS FOR TM AND CD BOUNDARY CONDITIONS                 TIME0919
                                                                        TIME0920
      DO 315 I=1,MMM                                                    TIME0921
      DO 315 J=1,NTOTVP                                                 TIME0922
      JX=NVIS(J)                                                        TIME0923
      RHS(I)=RHS(I)-TM(I,JX)*X(JX)                                      TIME0924
      TM(I,JX)=0.D0                                                     TIME0925
      TM(JX,I)=0.D0                                                     TIME0926
  315 CONTINUE                                                          TIME0927
      IF(T.GT.0.D0) GO TO 321                                           TIME0928
      DO 320 I=1,NTOTVP                                                 TIME0929
      K=NVIS(I)                                                         TIME0930
      Y(1,K)=X(K)                                                       TIME0931
      DO 316 J=1,MMM                                                    TIME0932
      CD(J,K)=0.D0                                                      TIME0933
  316 CD(K,J)=0.D0                                                      TIME0934
      CD(K,K)=1.D0                                                      TIME0935
      RHS(K)=0.D0                                                       TIME0936
  320 CONTINUE                                                          TIME0937
      NY=117                                                            TIME0938
      NL=0                                                              TIME0939
      M=70                                                              TIME0940
      JSKF=0                                                            TIME0941
      MAXDER=6                                                          TIME0942
      IPRT=1                                                            TIME0943
      H=1.D-08                                                          TIME0944
      HMIN=1.D-12                                                       TIME0945
      HMAX=5.D-02                                                       TIME0946
      RMSEPS=1.D-03                                                     TIME0947
      IF(T-0.D0) 323,323,321                                            TIME0948
  323 TEND=1.D-08                                                       TIME0949
      GO TO 325                                                         TIME0950
```

```
  321 TEND=T                                                              TIME0951
  325 CONTINUE                                                            TIME0952
      CALL SDESOL(Y,YL,T,TEND,NY,NL,M,JSKF,MAXDER,IPRT,H,HMIN,HMAX,       TIME0953
     1RMSEPS,W)                                                           TIME0954
      IF(T.GT.5.D-02) GO TO 324                                           TIME0955
      DO 322 J=1,MMM                                                      TIME0956
      TDIFF=DABS(T1(J)-Y(1,J))                                            TIME0957
      T1(J)=Y(1,J)                                                        TIME0958
      EPSLN=1.D-06                                                        TIME0959
      IF(TDIFF-EPSLN) 322,177,177                                         TIME0960
  322 CONTINUE                                                            TIME0961
  324 CONTINUE                                                            TIME0962
      WRITE(NWRITE,2000)                                                  TIME0963
      DO 360 I=1,MMM                                                      TIME0964
      WRITE(NWRITE,2005) I,Y(1,I)                                         TIME0965
  360 CONTINUE                                                            TIME0966
      WRITE(NWRITE,2021)                                                  TIME0967
      WRITE(NWRITE,2025)                                                  TIME0968
      WRITE(NWRITE,2030)                                                  TIME0969
      WRITE(NWRITE,2035)                                                  TIME0970
      WRITE(NWRITE,2040)                                                  TIME0971
      WRITE(NWRITE,2045)                                                  TIME0972
      WRITE(NWRITE,2050)                                                  TIME0973
      WRITE(NWRITE,2055)                                                  TIME0974
  500 FORMAT(I10)                                                         TIME0975
  600 FORMAT(1H1,18X,'TIME-DEPENDENT FLUID MECHANICS PROBLEM',////)
 1005 FORMAT(3I10)                                                        TIME0976
 1006 FORMAT(6X,A4,I10,2F10.0)                                            TIME0977
 1010 FORMAT(7I10)                                                        TIME0978
 1015 FORMAT(6X,A4,I10,F10.0)                                             TIME0979
 1016 FORMAT(6X,A4,I10)                                                   TIME0980
 1020 FORMAT(I10,2F10.0)                                                  TIME0981
 1025 FORMAT(6X,A4,I10,F10.0)                                             TIME0982
 1030 FORMAT(6X,A4,2I10,2F10.0)                                           TIME0983
 1034 FORMAT(5X,'NNQZC=',I3,//)                                           TIME0984
 1035 FORMAT(5X,'NO. OF NODES=',I3,//,5X,'NO. OF ELEMENTS=',I3,//,        TIME0985
     15X,'NO. OF CORNER NODES=',I3,///)                                   TIME0986
 1036 FORMAT(5X,'NNVELS=',I3,//)                                          TIME0987
 1037 FORMAT(5X,'NNQXY=',I3,//)                                           TIME0988
 1038 FORMAT(5X,'NNPS=',I3,//)                                            TIME0989
 1039 FORMAT(5X,'NNTS=',I3,//)                                            TIME0990
 1040 FORMAT(5X,'NNQZ=',I3,///)                                           TIME0991
 1041 FORMAT(5X,'SUMMARY OF NODAL COORDINATES',//,                        TIME0992
     17X,'I',12X'X(I)',13X,'Y(I)',//)                                     TIME0993
 1045 FORMAT(5X,I3,2(7X,F10.3))                                           TIME0994
 1050 FORMAT(5X,'LISTING OF SYSTEM TOPOLOGY'//,5X                         TIME0995
     1'ELEMENT NUMBER',20X,'NODE NUMBERS',//)                             TIME0996
 1055 FORMAT(5X,I3,10X,6(5X,I3))                                          TIME0997
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 1060 FORMAT(//,7X,'NODES WHERE VELOCITIES ARE SPECIFIED',                TIME0998
     1//,8X,'I',5X,'NODE',5X,'U VELOCITY',5X,'V VELOCITY',//)             TIME0999
 1065 FORMAT(2X,2(4X,I3),3X,F12.3,3X,F12.3)                              TIME1000
 1070 FORMAT(//,5X,'NODES WHERE QX AND QY ARE SPECIFIED',                 TIME1001
     1       //,2X,'      I    NODE',1X,'       QX',10X,'   QY',//)     TIME1002
 1075 FORMAT(5X,I3,2(10X,F12.3))                                         TIME1003
 1080 FORMAT(//,5X,'NODES WHERE PRESSURE IS SPECIFIED',                   TIME1004
     1       //,5X,'I',5X,'NODE',15X,'PRESSURE',//)                     TIME1005
 1081 FORMAT(//,5X,'NODES WHERE TEMPERATURE IS SPECIFIED',                TIME1006
     1       //,5X,'I',5X,'NODE',15X,'TEMPERATURE',//)                  TIME1007
 1082 FORMAT(//,5X,'NODES WHERE HEAT FLUX QZ IS SPECIFIED',               TIME1008
     1       //,5X,'I',5X,'NODE',15X,'HEAT FLUX',//)                    TIME1009
 1083 FORMAT(//,5X,'NODES WHERE QZC IS SPECIFIED',                        TIME1010
     1       //,5X,'I',5X,'NODE',23X,'QZC',//)                          TIME1011
 1085 FORMAT(7X,I3,3X,I3,10X,F12.3)                                      TIME1012
 1095 FORMAT(5X,3(3X,I3),2(5X,F12.3))                                    TIME1013
 2000 FORMAT(///,5X,'NODE NO.',6X,'NODE VARIABLE',//)                    TIME1014
 2005 FORMAT(9X,I3,5X,D17.10,/)                                          TIME1015
 2015 FORMAT(1H1,5X,'THIS IS A 3-D AXISYMMETRIC PROBLEM',//)             TIME1016
 2020 FORMAT(1H1,5X,'THIS IS A 2-D NON-LINEAR PROGRAM',//)               TIME1017
 2021 FORMAT(/////////)                                                  TIME1018
 2025 FORMAT(///,10X,'THE KINEMATIC VISCOSITY OF FLUID 50-HB-3520 AT 20 TIME1019
     1DEGREES C. = 10.90 SQ.CM/SEC',//)                                  TIME1020
 2030 FORMAT(///,10X,'THE DENSITY OF 50-HB-3520 AT 20 DEGREES C. = 1.059TIME1021
     16 GM/CC',//)                                                       TIME1022
 2035 FORMAT(///,10X,'THE COEFF. OF THERMAL EXPANSION OF 50-HB-3520 AT 2TIME1023
     10 DEGREES C. = 0.002278/DEGREE C.',//)                             TIME1024
 2040 FORMAT(///,10X,'THE THERMAL DIFFUSIVITY OF 50-HB-3520 AT 20 DEGREETIME1025
     1S C. = 0.00103 SQ.CM/SEC',//)                                      TIME1026
 2045 FORMAT(///,10X,'THE GRASHOF NUMBER (GR(L)) =(G*B*L**3*(TH-TC))/V**TIME1027
     12 = 946.4',//)                                                     TIME1028
 2050 FORMAT(///,10X,'THE U VELOCITY FORCING FUNCTION,G*B*T(INITIAL), = TIME1029
     165.962 CM/SQ.SEC',//)                                              TIME1030
 2055 FORMAT(///,10X,'THE SPECIFIED WALL PRESSURES ARE NORMALIZED TO ONETIME1031
     1 (1) ATMOSPHERE',//)                                               TIME1032
      STOP                                                               TIME1033
      END                                                                TIME1034



      SUBROUTINE SDESOL (Y,YL,T,TEND,NY,NL,M,JSKF,MAXDER,IPRT,H,HMIN,    SDE    10
     1HMAX,RMSEPS,W)                                                     SDE    20
C                                                                        SDE    30
CC    ------------------------------------------------------------------SDE    40
C                                                                        SDE    50
C     SUBROUTINE SDESOL IS A DRIVER ROUTINE FOR SUBROUTINE LDASUB.       SDE    60
C     ITS PURPOSE IS TO SET UP THE NECESSARY REFERENCES TO A LARGE       SDE    70
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C     BLOCK OF AUXILLARY STORAGE, AND  OBTAIN INITIAL VALUES OF          SDE   80
C     DERIVATIVES.                                                       SDE   90
C     THE CALLING SEQUENCE FOR SDESOL IS                                 SDE  100
C                                                                        SDE  110
C CALL SDESOL(Y,YL,T,TEND,NY,NL,M,JSKF,MAXDER,IPRT,H,HMIN,HMAX,RMSEPS,W)SDE  120
C                                                                        SDE  130
C     WHERE THE PARAMETERS ARE DEFINED AS FOLLOWS.                       SDE  140
C                                                                        SDE  150
C        Y        - ARRAY DIMENSIONED (7,NY).  THIS ARRAY CONTAINS THE   SDE  160
C                   DEPENDENT VARIABLES AND THEIR SCALED DERIVATIVES.    SDE  170
C                   Y(J+1,I) CONTAINS THE J-TH DERIVATIVE OF THE I-TH VARSDE  180
C                   IABLE TIMES H**J/J-FACTORIAL, WHERE H IS THE CURRENT SDE  190
C                   STEPSIZE.  ON FIRST ENTRY THE CALLER SUPPLIES THE    SDE  200
C                   INITIAL VALUES OF EACH VARIABLE IN Y(1,I).  ON SUB-  SDE  210
C                   SEQUENT ENTRIES IT IS ASSUMED THE ARRAY HAS NOT      SDE  220
C                   BEEN CHANGED.  TO INTERPOLATE TO NON-MESH POINTS,    SDE  230
C                   THESE VALUES CAN BE USED AS FOLLOWS.  IF H IS THE    SDE  240
C                   CURRENT STEPSIZE AND VALUES AT TIME  T+E  ARE        SDE  250
C                   NEEDED, LET  S = E/H  AND THEN                       SDE  260
C                                                                        SDE  270
C                                                  JS                    SDE  280
C                        I-TH VARIABLE AT  T+E  IS SUM Y(J+1,I)*S**J     SDE  290
C                                                  J=0                   SDE  300
C                                                                        SDE  310
C                   THE VALUE OF JS IS OBTAINED IN THE CALLING PROGRAM   SDE  320
C                   BY  JS = IABS(JSKF/10)                               SDE  330
C        YL       - ARRAY OF NL VARIABLES WHICH APPEAR LINEARLY.         SDE  340
C        T        - CURRENT VALUE OF THE INDEPENDENT VARIABLE (TIME)     SDE  350
C        TEND     - END TIME                                             SDE  360
C        NY       - NUMBER OF DIFFERENTIAL EQUATIONS AND NONLINEAR       SDE  370
C                   VARIABLES.                                           SDE  380
C        NL       - NUMBER OF LINEAR VARIABLES                           SDE  390
C        M        - NUMBER OF VARIABLES INCLUDED IN THE ERROR TEST       SDE  400
C        JSKF     - AN INDICATOR USED BOTH ON INPUT AND OUTPUT           SDE  410
C                   ON INPUT, JSKF = -1 INDICATES A RESTART CALL TO      SDE  420
C                   SDESOL.  JSKF = 0 INDICATES AN INITIAL CALL TO       SDE  430
C                   SDESOL.  JSKF > 0 INDICATES A CONTINUATION OF THE    SDE  440
C                   PREVIOUS CALL TO SDESOL.  JSKF < -1 MAY HAVE RESULTEDSDE  450
C                   FROM THE USER NEGLECTING TO TEST FOR ERROR RETURNS   SDE  460
C                   FROM SDESOL.  BECAUSE OF THIS POSSIBILITY, JSKF < -1 SDE  470
C                   RESULTS IN TERMINATION OF THE RUN WITH THE           SDE  480
C                   APPROPRIATE COMMENT.                                 SDE  490
C                   ON OUTPUT, JSKF CONSISTS OF TWO DIGITS AND SIGN,     SDE  500
C                   + OR - QP.  Q IS THE ORDER OF THE FORMULA CURRENTLY   SDE  510
C                   BEING USED.  P INDICATES THE TYPE OF RETURN, AS      SDE  520
C                   FOLLOWS.                                             SDE  530
C                   JSKF > 0, P = 1 IS THE NORMAL RETURN                 SDE  540
C                   JSKF < 0 IS AN ERROR RETURN, WITH THE FOLLOWING      SDE  550
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C                MEANINGS.                                              SDE   560
C                     P = 1  ERROR TEST FAILURE FOR H > HMIN            SDE   570
C                     P = 3  CORRECTOR FAILED TO CONVERGE FOR H > HMINSDE   580
C                     P = 4  CORRECTOR FAILED TO CONVERGE FOR FIRST     SDE   590
C                            ORDER METHOD                               SDE   600
C                     P = 5  ERROR RETURN FROM SUBROUTINE NUITSL        SDE   610
C                     P = 6  ERROR RETURN FROM SUBROUTINE DERVAL        SDE   620
C     MAXDER   - MAXIMUM ORDER DERIVATIVE THAT SHOULD BE USED IN        SDE   630
C                METHOD.  IT MUST BE NO GREATER THAN SIX.               SDE   640
C     IPRT     - INTERNAL PRINT CONTROL INDICATOR FOR LCASUB.           SDE   650
C                     IPRT = 0  NO PRINT                                SDE   660
C                     IPRT > 0  PRINT COUNTERS, STEPSIZE, CURRENT TIMESDE   670
C                               AND VALUES OF DEPENDENT VARIABLES AT    SDE   680
C                               EACH STEP.                              SDE   690
C     H        - CURRENT STEPSIZE.  AN INITIAL VALUE MUST BE SUPPLIED   SDE   700
C                BUT NEED NOT BE THE ONE WHICH MUST BE USED, SINCE THESDE   710
C                SUBROUTINE WILL CHOSE A SMALLER ONE IF NECCESSARY TO  SDE   720
C                KEEP THE ERROR PER STEP SMALLER THAN THE SPECIFIED    SDE   730
C                VALUE.  IT IS BETTER TO UNDERESTIMATE THE INITIAL     SDE   740
C                STEPSIZE THAN TO OVERESTIMATE IT.  THE STEPSIZE IS    SDE   750
C                NORMALLY NOT CHANGED BY THE USER.                     SDE   760
C     HMIN     - MINIMUM STEPSIZE ALLOWED                               SDE   770
C     HMAX     - MAXIMUM STEPSIZE ALLOWED                               SDE   780
C     RMSEPS   - THE ERROR TEST CONSTANT.  THE ROOT-MEAN-SQUARE OF      SDE   790
C                THE SINGLE STEP ERROR ESTIMATES, ER(I), DIVIDED BY    SDE   800
C                YMAX(I) = (MAXIMUM TO CURRENT TIME OF Y(I) ) MUST BE  SDE   810
C                LESS THAN EPS.  THE STEPSIZE AND/OR THE ORDER         SDE   820
C                ARE VARIED TO ACHIEVE THIS.                           SDE   830
C     W        - SCRATCH STORAGE ARRAY.  MUST BE AT LEAST 13*NY + 5*NLSDE   840
C                LOCATIONS, PLUS THOSE REQUIRED FOR STORAGE OF THE     SDE   850
C                MATRIX  PW (SEE DESCRIPTION OF SUBROUTINE JACMAT).    SDE   860
C                THE STORAGE OF PW WILL NORMALLY REQUIRE NO MORE THAN  SDE   870
C                N**2 + 2*N LOCATIONS, AND IF COMPACT STORAGE TECH-    SDE   880
C                NIQUES ARE USED, CAN BE MUCH FEWER.                   SDE   890
C                                                                      SDE   900
C     ---------------------------------------------------------------SDE   910
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION Y(7,1), YL(1), W(1)
      IF (JSKF.GT.0) GO TO 120
      IF (JSKF.LT.-1) GO TO 140
      N = NY+NL
      IF (JSKF.LT.0) GO TO 110
C                                                                      SDE   970
CC    IF THIS IS THE FIRST ENTRY, OBTAIN VALUES OF THE DERIVATIVES.    SDE   980
      CALL DERVAL (Y,YL,T,N,NY,W,KRETR)
      IF (KRETR.NE.0) GO TO 130
C                                                                      SDE  1010
CC    NOW SET UP STORAGE BLOCKS IN THE W ARRAY.  THIS NEEDS TO BE DONE SDE  1020
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C      ONLY INITIALLY AND ON RESTARTS.                                          SDE  1030
C                                                                               SDE  1040
C         THE ARRAY   SAVE   STARTS AT LOCATION  1      IN THE W ARRAY         SDE  1050
C         THE ARRAY   YLSV   STARTS AT LOCATION  NSVL   IN THE W ARRAY         SDE  1060
C         THE ARRAY   YMAX   STARTS AT LOCATION  NYMAX  IN THE W ARRAY         SDE  1070
C         THE ARRAY   ER     STARTS AT LOCATION  NER    IN THE W ARRAY         SDE  1080
C         THE ARRAY   ESV    STARTS AT LOCATION  NESV   IN THE W ARRAY         SDE  1090
C         THE ARRAY   F1     STARTS AT LOCATION  NF1    IN THE W ARRAY         SDE  1100
C         THE ARRAY   DY     STARTS AT LOCATION  NDY    IN THE W ARRAY         SDE  1110
C         THE MATRIX  PW     STARTS AT LOCATION  NPW    IN THE W ARRAY         SDE  1120
C                                                                               SDE  1130
  110 NSVL = 7*NY+1
      NYMAX = NSVL+NL
      NER = NYMAX+NY
      NESV = NER+NY
      NF1 = NESV+NY
      NDY = NF1+N
      NPW = NDY+N
  120 JS = JSKF
      CALL LDASUB (Y,YL,T,TEND,N,NY,M,JS,KF,MAXDER,IPRT,H,HMIN,HMAX,
     1RMSEPS,W,W(NSVL),W(NYMAX),W(NER),W(NESV),W(NF1),W(NDY),W(NPW))
C                                                                               SDE  1240
C     CODE JSKF ON RETURN FROM LDASUB                                           SDE  1250
C                                                                               SDE  1260
      JSKF = ISIGN(JS*10+IABS(KF),KF)
      RETURN
  130 JSKF = -6
      RETURN
  140 PRINT 1, JSKF
      STOP
C                                                                               SDE  1330
C                                                                               SDE  1340
    1 FORMAT ('0IT IS AN ERROR TO ENTER SDESOL WITH JSKF = ',I10//
     1 '   RUN HAS BEEN TERMINATED.')
      END
      SUBROUTINE LDASUB (Y,YL,T,TEND,N,NY,M,JSTART,KFLAG,MAXOR,IPRT,H,
     1HMIN,HMAX,RMSEPS,SAVE,YLSV,YMAX,ER,ESV,F1,DY,PW)
C                                                                               LDA    30
C     SUBROUTINE LDASUB IS A MODIFICATION OF SUBROUTINE DFASUB                  LDA    40
C     WHICH IS DUE TO R. L. BROWN AND C. W. GEAR.  DFASUB IS DOCUMENTED         LDA    50
C     IN THE REPORT                                                             LDA    60
C        DOCUMENTATION FOR DFASUB--                                             LDA    70
C        BY R. L. BROWN AND C. W. GEAR                                          LDA    80
C        REPORT UIUCDCS-R-73-575, JULY 1973                                     LDA    90
C        UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN                             LDA   100
C        URBANA, ILLINOIS  61801                                                LDA   110
C     THIS REPORT IS AVAILABLE FROM THE NATIONAL TECHNICAL INFORMATION          LDA   120
C     SERVICE OF THE U. S. DEPARTMENT OF COMMERCE UNDER ACCESSION NUMBER        LDA   130
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C     CCO-1469-225.                                                          LDA  140
C                                                                            LDA  150
C     THE MODIFICATION HERE IS DOCUMENTED IN THE REPORT                      LDA  160
C     A PROGRAM FOR THE NUMERICAL SOLUTION OF LARGE SPARSE SYSTEMS OF        LDA  170
C     ALGEBRAIC AND IMPLICITLY DEFINED STIFF DIFFERENTIAL EQUATIONS          LDA  180
C        BY RICHARD FRANKE                                                   LDA  190
C        REPORT NPS53FE76051, MAY 1976                                       LDA  200
C        NAVAL POSTGRADUATE SCHOOL                                           LDA  210
C        MONTEREY, CALIFORNIA  93940                                         LDA  220
C                                                                            LDA  230
C     ------------------------------------------------------------------     LDA  240
C                                                                            LDA  250
C     THE CALLING SEQUENCE FOR LDASUB IS                                     LDA  260
C                                                                            LDA  270
C     CALL LDASUB(Y,YL,T,TEND,N,NY,M,JSTART,KFLAG,MAXOR,IPRT,H,HMIN,         LDA  280
C     HMAX,RMSEPS,SAVE,YLSV,YMAX,ER,ESV,F1,DY,PW)                            LDA  290
C                                                                            LDA  300
C     WHERE THE PARAMETERS ARE DEFINED AS FOLLOWS.                           LDA  310
C        Y       - ARRAY DIMENSIONED (7,NY).  THIS ARRAY CONTAINS THE        LDA  320
C                  DEPENDENT VARIABLES AND THEIR SCALED DERIVATIVES.         LDA  330
C                  Y(J+1,I) CONTAINS THE J-TH DERIVATIVE OF THE I-TH VAR     LDA  340
C                  IABLE TIMES H**J/J-FACTORIAL, WHERE H IS THE CURRENT      LDA  350
C                  STEPSIZE.  ON FIRST ENTRY THE CALLER SUPPLIES THE         LDA  360
C                  INITIAL VALUES OF EACH VARIABLE IN Y(1,I) AND AN          LDA  370
C                  ESTIMATE OF THE INITIAL VALUES OF THE DERIVATIVES         LDA  380
C                  IN Y(2,I).  ON SUBSEQUENT ENTRIES IT IS ASSUMED THAT      LDA  390
C                  THE ARRAY HAS NOT BEEN CHANGED.  TO INTERPOLATE TO        LDA  400
C                  NON-MESH POINTS, THESE VALUES CAN BE USED AS FOLLOWS.     LDA  410
C                  IF H IS THE CURRENT STEPSIZE AND VALUES AT TIME T+E       LDA  420
C                  NEEDED, LET S = E/H  AND THEN                             LDA  430
C                                                                            LDA  440
C                                                    NQ                      LDA  450
C                       I-TH VARIABLE AT  T+E IS     SUM Y(J+1,I)*S**J       LDA  460
C                                                    J=0                     LDA  470
C                                                                            LDA  480
C                  THE VALUE OF NQ IS OBTAINED IN THE CALLING PROGRAM        LDA  490
C                  BY  NQ = JSTART.                                          LDA  500
C                                                                            LDA  510
C        YL      - ARRAY OF NL = N - NY VARIABLES WHICH APPEAR LINEARLY.     LDA  520
C                  THE USER SUPPLIES INITIAL VALUES FOR THESE VARIABLES.     LDA  530
C        T       - CURRENT VALUE OF THE INDEPENDENT VARIABLE (TIME)          LDA  540
C        TEND    - END TIME                                                  LDA  550
C        N       - TOTAL NUMBER OF VARIABLES                                 LDA  560
C        NY      - NUMBER OF DIFFERENTIAL EQUATIONS AND NONLINEAR            LDA  570
C                  VARIABLES.                                                LDA  580
C        M       - NUMBER OF VARIABLES INCLUDED IN THE ERROR TEST.           LDA  590
C                  THIS NUMBER CAN BE NO GREATER THAN NY.  IF IT IS          LDA  600
C                  GREATER THAN NY, NY VARIABLES ARE USED IN THE ERROR       LDA  610
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C          TEST.                                                          LDA  620
C     JSTART - INPUT AND OUTPUT INDICATOR.                                LDA  630
C          ON INPUT JSTART HAS THE FOLLOWING MEANINGS.                    LDA  640
C               <0  THIS INDICATES A RE-START FROM A PREVIOUS             LDA  650
C                   POINT FOLLOWING TERMINATION OF THE RUN OR             LDA  660
C                   SOLUTION OF ANOTHER PROBLEM DURING THE SAME           LDA  670
C                   RUN.  PARAMETERS IN THE CALLING SEQUENCE              LDA  680
C                   MUST HAVE BEEN PRESERVED FROM THE PREVIOUS            LDA  690
C                   USE, PARTICULARLY THE ARRAYS                          LDA  700
C                       SAVE, YLSV, ESV, AND PW.                          LDA  710
C                   THESE ARRAYS MUST BE SAVED AFTER A CALL               LDA  720
C                   TO SUBROUTINE LDASAV, WHICH ALSO SAVES                LDA  730
C                   NECESSARY PARAMETERS INTERNAL TO LDASUB.              LDA  740
C               =0  INDICATES AN INITIAL CALL TO LDASUB.  THE             LDA  750
C                   ROUTINE INITIALIZES ITSELF, SCALES THE                LDA  760
C                   DERIVATIVES IN Y(2,I) AND THEN PERFORMS THE           LDA  770
C                   INTEGRATION UNTIL T > TEND.                           LDA  780
C               >0  INDICATES THE SOLUTION IS TO BE CONTINUED.            LDA  790
C                   AFTER THE INITIAL ENTRY IT IS NEITHER                 LDA  800
C                   DESIRABLE NOR NECESSARY TO RE-ENTER WITH              LDA  810
C                   JSTART = 0, SINCE THIS RE-INITIALIZES                 LDA  820
C                   THE CODE, BEGINNING WITH A FIRST ORDER                LDA  830
C                   METHOD AGAIN.                                         LDA  840
C          ON OUTPUT, JSTART IS SET TO THE VALUE OF NQ, THE               LDA  850
C          ORDER OF THE FORMULA CURRENTLY BEI-& 0 & &                    LDA  860
C     KFLAG  - THE COMPLETION CODE INDICATOR, WITH THE FOLLOWING         LDA  870
C          MEANINGS                                                       LDA  880
C               +1  THE INTEGRATION WAS SUCCESSFUL                        LDA  890
C               -1  ERROR TEST FAILURE FOR H > HMIN                       LDA  900
C               -3  CORRECTOR FAILED TO CONVERGE FOR H > HMIN             LDA  910
C               -4  CORRECTOR FAILED TO CONVERGE FOR FIRST                LDA  920
C                   ORDER METHOD                                          LDA  930
C               -5  ERROR RETURN FROM SUBROUTINE NUITSL                   LDA  940
C     MAXOR  - MAXIMUM ORDER DERIVATIVE THAT SHOULD BE USED IN THE       LDA  950
C          METHOD.  IT MUST BE NO GREATER THAN SIX.  IF IT IS            LDA  960
C          GREATER THAN SIX, THE MAXIMUM ORDER USED WILL BE SIX.         LDA  970
C     IPRT   - INTERNAL PRINT CONTROL INDICATOR                          LDA  980
C               = 0  NO PRINT                                             LDA  990
C               > 0  PRINT COUNTERS, STEPSIZE, CURRENT TIME               LDA 1000
C                    AND VALUES OF DEPENDENT VARIABLES AT                 LDA 1010
C                    EACH STEP.                                           LDA 1020
C     H      - CURRENT STEPSIZE.  AN INITIAL VALUE MUST BE SUPPLIED      LDA 1030
C          BUT NEED NOT BE THE ONE WHICH WILL BE USED, SINCE THE         LDA 1040
C          SUBROUTINE WILL CHOOSE A SMALLER ONE IF NECCESSARY TO         LDA 1050
C          KEEP THE ERROR PER STEP SMALLER THAN THE SPECIFIED            LDA 1060
C          VALUE.  IT IS BETTER TO UNDERESTIMATE THE INITIAL             LDA 1070
C          STEPSIZE THAN TO OVERESTIMATE IT.  THE STEPSIZE IS            LDA 1080
C          NORMALLY NOT CHANGED BY THE USER.                             LDA 1090
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C        HMIN     - MINIMUM STEPSIZE ALLOWED                                    LDA 1100
CCC      HMAX     - MAXIMUM STEPSIZE ALLOWED                                    LDA 1110
CCC      RMSEPS   - THE ERROR TEST CONSTANT.  THE ROOT-MEAN-SQUARE OF           LDA 1120
CCC                 THE SINGLE STEP ERROR ESTIMATES,  ER(I), DIVIDED BY        LDA 1130
CCC                 YMAX(I) = (MAXIMUM TO CURRENT TIME OF Y(I)) MUST BE        LDA 1140
CCC                 LESS THAN RMSEPS.  THE STEPSIZE AND/OR ORDER ARE           LDA 1150
CCC                 VARIED TO ACHIEVE THIS.                                    LDA 1160
CCC      SAVE     - AN ARRAY OF LENGTH AT LEAST 7*NY                            LDA 1170
CCC      YLSV     - AN ARRAY OF LENGTH AT LEAST NL                              LDA 1180
CCC      YMAX     - A VECTOR OF LENGTH NY WHICH CONTAINS THE MAXIMUM            LDA 1190
CCC                 OF EACH Y SEEN SO FAR.  ON THE FIRST CALL, THESE WILL      LDA 1200
CCC                 BE INITIALIZED AS YMAX(I) = MAX(1,|Y(1,I)|)                LDA 1210
CCC      ER       - A VECTOR OF LENGTH NY                                       LDA 1220
CCC      ESV      - A VECTOR OF LENGTH NY                                       LDA 1230
CCC      F1       - A VECTOR OF LENGTH N = NY + NL                              LDA 1240
CCC      DY       - A VECTOR OF LENGTH N = NY + NL                              LDA 1250
CCC      PW       - AN ARRAY IN WHICH THE  J  MATRIX COMPUTED                   LDA 1260
CCC                 IN SUBROUTINE JACMAT WILL BE STORED.  SIZE WHICH           LDA 1270
CCC                 MUST BE ALLOWED IS DETERMINED BY THE STORAGE TECH-         LDA 1280
CCC                 NIQUE USED FOR IT, BUT NORMALLY WON'T BE MORE THAN         LDA 1290
CCC                 N**2 + 2*N LOCATIONS, THE LATTER 2*N BEING REQUIRED        LDA 1300
CCC                 BY THE LINEAR EQUATION SOLVER.                             LDA 1310
CCC                                                                            LDA 1320
C       ------------------------------------------------------------------     LDA 1330
        IMPLICIT REAL*8 (A-H,O-Z)
        DIMENSION Y(7,1), YL(1), SAVE(7,1), YMAX(1), ER(1), YLSV(1), F1(1)
       1, PERT(6,3), COF(21), ESV(1), DY(1), PW(1), SAV(1), A(29)
        EQUIVALENCE (A(8),BND), (A(9),BR), (A(10),E), (A(11),EDWN),
       1(A(12),ENQ1), (A(13),ENQ2), (A(14),ENQ3), (A(15),EPS), (A(16),EUP)
       2,(A(17),HNEW), (A(18),PEPSH), (A(19),IDOUB), (A(20),IWEVAL),
       3(A(21),K), (A(22),LCOPYL), (A(23),LCOPYY), (A(24),MAXDER),
       4(A(25),M1), (A(26),NL), (A(27),NQ), (A(28),NS), (A(29),NW)
C                                                                              LDA 1410
CCC     ------------------------------------------------------------------     LDA 1420
CCC                                                                            LDA 1430
CCC     THE COEFFICIENTS IN THE PERT ARRAY ARE USED FOR ERROR TESTING AND      LDA 1440
CCC     CHANGING STEPSIZE AND NEED TO BE ACCURATE TO ONLY A FEW DIGITS.        LDA 1450
CCC                                                                            LDA 1460
C       ------------------------------------------------------------------     LDA 1470
        DATA PERT/4.D0,9.D0,16.D0,25.D0,36.D0,49.D0,9.D0,16.D0,25.D0,36.D0
       1,49.D0,64.D0,1.D0,1.D0,.25D0, 2.7889D-2,1.70569D-3,6.83929D-5/
C       ------------------------------------------------------------------     LDA 1500
CCC                                                                            LDA 1510
CCC     THE ENTRIES IN THE COF ARRAY ARE THE COEFFICIENTS FOR THE STIFFLY      LDA 1520
CCC     STABLE METHODS USED IN THIS PROGRAM AND ARE TO BE THE MACHINE          LDA 1530
CCC     PRECISION EQUIVALENTS OF THE FOLLOWING CONSTANTS.                      LDA 1540
CCC                                                                            LDA 1550
C       -1                                                                     LDA 1560
```

```
C     -3/2 , -1/2                                                          LDA 1570
CC    -11/6 , -1 , -1/6                                                    LDA 1580
CC    -25/12 , -35/24 , -5/12 , -1/24                                      LDA 1590
CC    -137/60 , -15/8 , -17/24 , -1/8 , -1/120                             LDA 1600
CCC   -147/60, -203/90 , -49/48 , -35/144 , -7/240 , -1/720                LDA 1610
                                                                           LDA 1620
C     ------------------------------------------------------------------LDA 1630
      DATA COF/-1.D0,-1.5D0,-.5D0,-1.8333333333333D0,-1.D0,
     1-1.6666666666667D-1,-2.0833333333333D0,-1.4583333333333D0,
     2-4.1666666666667D-1,-4.1666666666667D-2,-2.8333333333333D0,
     3 -1.875D0,-7.0833333333333D-1,-1.25D-1,-8.3333333333333D-3,
     4-2.45D0,-2.2555555555556D0,-1.0208333333333D0,
     5-2.4305555555556D-1,-2.9166666666667D-2,-1.3888888888889D-3/
      IF (JSTART) 100,110,150
C     ------------------------------------------------------------------LDA 1680
CC    IF THIS IS A RESTART ENTRY, RESTORE Y AND YL FROM THE SAVE AND     LDA 1690
CC    YLSV ARRAYS, WHERE THEY WERE SAVED BY A PREVIOUS CALL TO LDASAV.   LDA 1700
C     ------------------------------------------------------------------LDA 1710
  100 CALL COPYZ (Y,SAVE,LCOPYY)
      CALL COPYZ (YL,YLSV,LCOPYL)
      GO TO 150
C     ------------------------------------------------------------------LDA 1750
CC    IF THIS IS THE FIRST CALL, INITIALIZE YMAX, SCALE DERIVATIVES, ANDLDA 1760
CC    INITIALIZE INDICATORS AND SET ORDER TO ONE.                        LDA 1770
CC    FOR DOUBLE PRECISION, SET LCOPYL = 14*NY AND LCOPYL = 2*NL IF      LDA 1780
CC    SUBROUTINE COPYZ IS IN SINGLE PRECISION.                           LDA 1790
C     ------------------------------------------------------------------LDA 1800
  110 NL = N-NY
      LCOPYY = 7*NY
      LCOPYL = NL
      M1 = MIN0(M,NY)
      EPS =DSQRT(DFLOAT(M1))*RMSEPS
      MAXDER = MIN0(MAXDR,6)
      IF (IPRT.LE.0) GO TO 120
      PRINT 3, N,NL,RMSEPS,TEND,H
      PRINT 4
  120 NS = 0
      NW = 0
C                                                                        LDA 1920
      DO 130 J=1,NY
      YMAX(J) =DMAX1(1.D0,DABS(Y(1,J)))
  130 Y(2,J) = Y(2,J)*H
C                                                                        LDA 1960
      NQ = 1
      BR = 1.D0
      ASSIGN 190 TO IRET
C     ------------------------------------------------------------------LDA 2000
C     SET COEFFICIENTS FOR THE ORDER CURRENTLY BEING USED.               LDA 2010
```

```
C     E IS A TEST FOR ERRORS OF THE CURRENT ORDER NQ                    LDA 2020
CC    EUP IS TO TEST FOR INCREASING THE ORDER, EDWN FOR DECREASING THE  LDA 2030
CC    ORDER.                                                            LDA 2040
C     ------------------------------------------------------------------LDA 2050
  140 K = NQ*(NQ-1)/2
      CALL COPYZ (A(2),COF(K+1),NQ)
      K = NQ+1
      IDOUB = NQ
      ENQ1 = .5D0/NQ
      ENQ2 = .5D0/K
      ENQ3 = .5D0/(NQ+2)
      PEPSH = EPS**2
      E = PERT(NQ,1)*PEPSH
      EUP = PERT(NQ,2)*PEPSH
      EDWN = PERT(NQ,3)*PEPSH
      BND = (EPS*ENQ3)**2
      IWEVAL = 1
      GO TO IRET, (190,200,490,570)
  150 IF (H.EQ.HNEW) GO TO 190
C     ------------------------------------------------------------------LDA 2210
CC    IF CALLER HAS CHANGED H, RESCALE DERIVATIVES TO REFLECT THAT HNEW LDA 2220
CC    WAS USED ON THE LAST CALL.                                        LDA 2230
C     ------------------------------------------------------------------LDA 2240
      R = H/HNEW
      ASSIGN 190 TO IRET
      GO TO 610
C     ------------------------------------------------------------------LDA 2280
CC    SET JSTART TO NQ, THE CURRENT ORDER OF THE METHOD, BEFORE EXIT,   LDA 2290
CC    AND SAVE THE CURRENT STEPSIZE IN HNEW.                            LDA 2300
C     ------------------------------------------------------------------ LDA 2310
  160 JSTART = NQ
      HNEW = H
      RETURN
  170 NS = NS+1
      IF (IPRT.LE.0) GO TO 180
C     ------------------------------------------------------------------LDA 2370
CC    PRINT DATA IF DESIRED BY USER                                     LDA 2380
C     ------------------------------------------------------------------LDA 2390
      PRINT 1, NS,NW,NQ,H,T,(Y(1,I),I=1,NY)
      IF (NL.GT.0) PRINT 2, (YL(I),I=1,NL)
  180 CONTINUE
      IF (KFLAG.LT.0) GO TO 160
      IF (T.GE.TEND) GO TO 160
C     ------------------------------------------------------------------LDA 2450
CC    TAKE ANOTHER STEP IF T < TEND                                     LDA 2460
C     ------------------------------------------------------------------LDA 2470
      JSTART = 1
C     ------------------------------------------------------------------LDA 2490
```

```
C     SAVE DATA FOR TRIAL WITH A SMALLER TIMESTEP IF THIS STEP FAILS    LDA 2500
C     ------------------------------------------------------------------LDA 2510
  190 CALL COPYZ (SAVE,Y,LCOPYY)
      CALL COPYZ (YLSV,YL,LCOPYL)
      RACUM = 1.D0
      KFLAG = 1
      HOLD = H
      NQOLD = NQ
      TOLD = T
  200 T = T+H
      HINV = 1.D0/H
C     ------------------------------------------------------------------LDA 2610
CC    COMPUTE PREDICTED VALUES BY EFFECTIVELY MULTIPLYING DERIVATIVE    LDA 2620
CC    VECTOR BY PASCAL TRIANGLE MATRIX                                  LDA 2630
CC    ------------------------------------------------------------------LDA 2640
C                                                                       LDA 2650
      DO 210 J=2,K
      J3 = K+J-1
C                                                                       LDA 2680
      DO 210 J1=J,K
      J2 = J3-J1
C                                                                       LDA 2710
      DO 210 I=1,NY
  210 Y(J2,I) = Y(J2,I)+Y(J2+1,I)
C                                                                       LDA 2740
C                                                                       LDA 2750
      DO 220 I=1,NY
  220 ER(I) = 0.D0
C                                                                       LDA 2780
C     ------------------------------------------------------------------LDA 2790
CC    DO UP TO THREE CORRECTOR ITERATIONS.  CONVERGENCE IS OBTAINED WHENLDA 2800
CC    CHANGES ARE LESS THAN BND WHICH IS DEPENDENT ON THE ERROR TEST    LDA 2810
CC    CONSTANT.  THE SUM OF CORRECTIONS IS ACCUMULATED IN ER(I).  IT IS LDA 2820
CC    EQUAL TO THE K-TH DERIVATIVE OF Y TIMES H**K/(K-FACTORIAL*A(K)),  LDA 2830
CC    AND THUS IS PROPORTIONAL TO THE ACTUAL ERRORS TO THE LOWEST POWER LDA 2840
CC    OF H PRESENT, WHICH IS H**K.                                      LDA 2850
CC    ------------------------------------------------------------------LDA 2860
C                                                                       LDA 2870
      DO 270 L=1,3
      CALL DIFFUN (Y,YL,T,HINV,DY)
      IF (IWEVAL.LT.1) GO TO 230
C     ------------------------------------------------------------------LDA 2910
CC    IF THERE HAS BEEN A CHANGE OF ORDER OR THERE HAS BEEN TROUBLE     LDA 2920
CC    WITH CONVERGENCE, PW IS RE-EVALUATED PRIOR TO STARTING THE        LDA 2930
CC    CORRECTOR ITERATION.  IWEVAL IS THEN SET TO -1 AS AN INDICATOR    LDA 2940
CC    THAT IT HAS BEEN DONE.  NEWPW IS SET NONZERO TO INDICATE TO       LDA 2950
CC    SUBROUTINE NUITSL THAT A NEW PW HAS BEEN PROVIDED.                LDA 2960
C     ------------------------------------------------------------------LDA 2970
```

```
      CALL JACMAT (Y,YL,T,HINV,A(2),N,NY,EPS,DY,F1,PW)
      KFLAG = 1
      IWEVAL = -1
      NW = NW+1
      NEWPW = 1
  230 CALL NUITSL (PW,DY,F1,N,NY,EPS,YMAX,NEWPW,KRRET)
      IF (KRRET.NE.0) GO TO 600
      IF (NL.LE.0) GO TO 250
C                                                                     LDA 3060
      DO 240 I=1,NL
  240 YL(I) = YL(I)-F1(I+NY)
C                                                                     LDA 3090
  250 CONTINUE
      DEL = 0.D0
C                                                                     LDA 3120
      DO 260 I=1,NY
      Y(1,I) = Y(1,I)-F1(I)
      Y(2,I) = Y(2,I)+A(2)*F1(I)
      ER(I) = ER(I)+F1(I)
      DEL = DEL+(F1(I)/DMAX1(YMAX(I),DABS(Y(1,I))))**2
  260 CONTINUE
C                                                                     LDA 3190
      IF (L.GE.2) BR =DMAX1(.9D0*BR,DEL/DEL1)
      DEL1 = DEL
      IF (DMIN1(DEL,BR*DEL*2.D0).LE.BND) GO TO 330
  270 CONTINUE
C                                                                     LDA 3240
C     ---------------------------------------------------------------LDA 3250
C     THE CORRECTIOR ITERATION FAILED TO CONVERGE IN 3 TRIES.  VARIOUS LDA 3260
C     POSSIBILITIES ARE CHECKED FOR.  IF H IS ALREADY HMIN AND PW HAS LDA 3270
C     ALREADY BEEN RE-EVALUATED, A NO CONVERGENCE EXIT IS TAKEN.      LDA 3280
C     OTHERWISE THE MATRIX PW IS RE-EVALUATED AND/OR (IN THAT ORDER) THELDA 3290
C     STEP IS REDUCED TO TRY AND GET CONVERGENCE.                     LDA 3300
C     ---------------------------------------------------------------LDA 3310
      T = TOLD
      IF (IWEVAL) 280,300,290
  280 IF (H.LE.HMIN*1.00001D0) GO TO 310
  290 RACUM = RACUM*.25D0
  300 CONTINUE
      GO TO 560
  310 KFLAG = -3
C     ---------------------------------------------------------------LDA 3390
C     RESTORE Y AND YL AFTER CONVERGENCE FAILURE                      LDA 3400
C     ---------------------------------------------------------------LDA 3410
  320 CALL COPYZ (Y,SAVE,LCOPYY)
      CALL COPYZ (YL,YLSV,LCOPYL)
      H = HOLD
      NC = NQOLD
```

```
      GO TO 170
C     ------------------------------------------------------------------LDA 3470
CC    THE CORRECTOR CONVERGED, SO NOW THE ERROR TEST IS MADE.           LDA 3480
C     ------------------------------------------------------------------LDA 3490
  330 D = 0.D0
C                                                                       LDA 3510
      DO 340 I=1,M1
      YM =DMAX1(DABS(Y(1,I)),YMAX(I))
  340 D = D+(ER(I)/YM)**2
C                                                                       LDA 3550
      IWEVAL = 0
      IF (D.GT.E) GO TO 380
C     ------------------------------------------------------------------LDA 3580
CC    THE ERROR TEST IS OKAY, SO THE STEP IS ACCEPTED.  IF IDOUB        LDA 3590
CC    NOW BECOMES NEGATIVE, A TEST IS MADE TO SEE IF THE STEP SIZE      LDA 3600
CC    CAN BE INCREASED AT THIS ORDER OR ONE HIGHER OR ONE LOWER.        LDA 3610
CC    THE CHANGE IS MADE ONLY IF THE STEP CAN BE INCREASED BY AT        LDA 3620
CC    LEAST 10%.  IDOUB IS SET TO NQ TO PREVENT FURTHER TESTING         LDA 3630
CC    FOR A WHILE.  IF NO CHANGE IS MADE, IDOUB IS SET TO 9.            LDA 3640
C     ------------------------------------------------------------------LDA 3650
      IF (K.LT.3) GO TO 360
C                                                                       LDA 3670
      DO 350 J=3,K
C                                                                       LDA 3690
      DO 350 I=1,NY
  350 Y(J,I) = Y(J,I)+A(J)*ER(I)
C                                                                       LDA 3720
  360 KFLAG = 1
      IDOUB = IDOUB-1
      IF (IDOUB) 410,370,510
  370 CALL COPYZ (ESV,ER,M1)
      GO TO 510
C     ------------------------------------------------------------------LDA 3780
CC    THE ERROR TEST FAILED.  IF JSTART = 0, THE DERIVATIVES IN THE     LDA 3790
CC    SAVE ARRAY ARE UPDATED.  TESTS ARE THEN MADE TO FIX THE STEPSIZE  LDA 3800
CC    AND PERHAPS REDUCE THE ORDER.  AFTER RESTORING AND SCALING THE    LDA 3810
CC    Y VARIABLES, THE STEP IS RETRIED.                                 LDA 3820
C     ------------------------------------------------------------------LDA 3830
  380 IF (JSTART.GT.0) GO TO 400
C                                                                       LDA 3850
      DO 390 I=1,NY
  390 SAVE(2,I) = Y(2,I)
C                                                                       LDA 3880
  400 KFLAG = KFLAG-2
      IF (H.LE.HMIN) GO TO 550
      T = TOLD
      IF (KFLAG.LE.-5) GO TO 530
  410 PR2 = (D/E)**ENQ2*1.2D0
```

```
      L = 0
      IF (NQ.LE.1) GO TO 430
      D = 0.D0
C                                                                      LDA 3970
      DO 420 J=1,M1
      YM =DMAX1(DABS(Y(1,J)),YMAX(J))
  420 D = D+(Y(K,J)/YM)**2
C                                                                      LDA 4010
      PR1 = (D/EDWN)**ENQ1*1.3D0
      IF (PR1.GE.PR2) GO TO 430
      PR2 = PR1
      L = -1
  430 IF (KFLAG.LT.0.OR.NQ.GE.MAXDER) GO TO 450
      D = 0
C                                                                      LDA 4080
      DO 440 J=1,M1
      YM =DMAX1(DABS(Y(1,J)),YMAX(J))
  440 D = D+((ER(J)-ESV(J))/YM)**2
C                                                                      LDA 4120
      PR1 = (D/EUP)**ENQ3*1.4D0
      IF (PR1.GE.PR2) GO TO 450
      PR2 = PR1
      L = 1
  450 R = 1.D0/DMAX1(PR2,1.D-5)
      IF (KFLAG.LT.0.OR.R.GE.1.1D0) GO TO 460
      IDOUB = 9
      GO TO 510
  460 NEWQ = NQ+L
      K = NEWQ+1
      IF (NEWQ.LE.NQ) GO TO 480
      R1 = A(NEWQ)/DFLOAT(NEWQ)
C                                                                      LDA 4250
      DO 470 J=1,NY
  470 Y(K,J) = ER(J)*R1
C                                                                      LDA 4280
  480 CONTINUE
C     ---------------------------------------------------------------LDA 4300
CC    IF THE STEP WAS OKAY, SCALE THE Y VARIABLES IN ACCORDANCE      LDA 4310
CC    WITH THE NEW VALUE OF H.  IF KFLAG < 0, HOWEVER, USE THE       LDA 4320
CC    SAVED VALUES (IN SAVE AND YLSV).  IN EITHER CASE, IF THE ORDER LDA 4330
CC    HAS CHANGED IT IS NECESSARY TO FIX CERTAIN PARAMETERS BY CALLING LDA 4340
CC    THE PROGRAM SEGMENT AT STATEMENT NUMBER 140.                   LDA 4350
C     ---------------------------------------------------------------LDA 4360
      IDOUB = NQ
      IF (NEWQ.EQ.NQ) GO TO 490
      NQ = NEWQ
      ASSIGN 490 TO IRET
      GO TO 140
```

```
  490 IF (KFLAG.GT.0) GO TO 500
      RACUM = RACUM*R
      GO TO 560
  500 R =DMAX1(DMIN1(HMAX/H,R),HMIN/H)
      H = H*R
      IWEVAL = 1
      ASSIGN 510 TO IRET
      GO TO 610
C                                                                     LDA 4500
  510 DO 520 I=1,M1
  520 YMAX(I) =DMAX1(DABS(Y(1,I)),YMAX(I))
C                                                                     LDA 4530
      GO TO 170
C     ------------------------------------------------------------------LDA 4550
C     THE ERROR TEST HAS NOW FAILED THREE TIMES, SO THE DERIVATIVES ARE LDA 4560
C     IN BAD SHAPE.  RETURN TO FIRST ORDER METHOD AND TRY AGAIN.  OF    LDA 4570
C     COURSE, IF NQ = 1 ALREADY, THEN THERE IS NO HOPE AND WE EXIT WITH LDA 4580
C     KFLAG = -4.                                                     LDA 4590
C     ------------------------------------------------------------------LDA 4600
  530 IF (NQ.EQ.1) GO TO 540
      NQ = 1
      ICOUB = 1
      ASSIGN 570 TO IRET
      GO TO 140
  540 NQOLD = 1
      KFLAG = -4
      GO TO 320
  550 KFLAG = -1
      GO TO 170
C     ------------------------------------------------------------------LDA 4710
C     THIS SECTION RESTORES THE SAVED VALUES OF Y AND YL, SCALING THE   LDA 4720
C     Y DERIVATIVES AS NECESSARY, AND THEN RETURNS TO THE PREDICTOR LOOPLDA 4730
C     ------------------------------------------------------------------LDA 4740
  560 H = HOLD*RACUM
      H =DMAX1(HMIN,DMIN1(H,HMAX))
  570 RACUM = H/HOLD
      R1 = 1.D0
C                                                                     LDA 4790
      DO 580 J=2,K
      R1 = R1*RACUM
C                                                                     LDA 4820
      DO 580 I=1,NY
  580 Y(J,I) = SAVE(J,I)*R1
C                                                                     LDA 4850
C                                                                     LDA 4860
      DO 590 I=1,NY
  590 Y(1,I) = SAVE(1,I)
C                                                                     LDA 4890
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```
      CALL COPYZ (YL,YLSV,LCOPYL)
      IWEVAL = 1
      GO TO 200
  600 KFLAG = -5
      GO TO 160
C     ------------------------------------------------------------LDA 4950
CC    THIS SECTION SCALES THE Y DERIVATIVES BY R**J                LDA 4960
C     ------------------------------------------------------------ LDA 4970
  610 R1 = 1.D0
C                                                                  LDA 4990
      DO 620 J=2,K
      R1 = R1*R
C                                                                  LDA 5020
      DO 620 I=1,NY
  620 Y(J,I) = Y(J,I)*R1
      GO TO IRET, (190,510)
C                                                                  LDA 5050
CC    ------------------------------------------------------------LDA 5070
CCC   THIS SECTION ALLOWS FOR RESTARTS AFTER SOLVING ANOTHER PROBLEM, ORLDA 5080
CCC   HAVING TERMINATED THE CURRENT COMPUTER RUN.  SUBROUTINE LDASAV   LDA 5090
CCC   SAVES THE NECESSARY VALUES WHICH ARE INTERNAL TO LDASUB.  FOR    LDA 5100
CCC   DOUBLE PRECISION, WITH COPYZ IN SINGLE PRECISION, THE NUMBER OF  LDA 5110
CCC   LOCATIONS TO BE SAVED AND RESTORED, LCOPYS AND LCOPYR, MUST BE   LDA 5120
CCC   SET TO 58.                                                       LDA 5130
CCC   IT IS ASSUMED THAT IN ADDITION TO THE VARIABLES IN THE ARRAY A   LDA 5140
CCC   SAVED BY CALLING LDASAV, THE USER ALSO SAVES THE ARRAYS SAVE,    LDA 5150
CCC   YLSV, YMAX, ESV, AND PW.                                         LDA 5160
CCC                                                                    LDA 5170
CCC   TO RESTART THE USER FIRST CALLS LDARST TO RESTORE THE VALUES SAVEDLDA 5180
CCC   BY LDASAV, THEN RE-ENTERS LDASUB WITH JSTART < 0, AND WITH THE   LDA 5190
CCC   OTHER PARAMETERS THE SAME AS RETURNED FROM THE LAST ENTRY TO     LDA 5200
CCC   LDASUB, PARTICULARLY THOSE ARRAYS MENTIONED ABOVE.               LDA 5210
C     ------------------------------------------------------------LDA 5220
      ENTRY LDASAV(SAV)
      LCOPYS = 29
      CALL COPYZ (SAV,A,LCOPYS)
      CALL COPYZ (SAVE,Y,LCOPYY)
      CALL COPYZ (YLSV,YL,LCOPYL)
      RETURN
C                                                                  LDA 5290
      ENTRY LDARST(SAV)
      LCOPYR = 29
      CALL COPYZ (A,SAV,LCOPYR)
      RETURN
C                                                                  LDA 5340
CC    ------------------------------------------------------------LDA 5350
CCC                                                                LDA 5360
C                                                                  LDA 5370
```

```
    1 FORMAT (2I5,I2,1P2E10.2,7E14.6/(32X,7E14.6))
    2 FORMAT (32X,1P7E14.6)
    3 FORMAT ('1  N =',I3,'   NL =',I3,'   RMSEPS =',1PE9.2,'   TEND ='
     1  ,E9.2,'   H =',E9.2//)
    4 FORMAT ('   NS   NW Q    H',8X,'T   ',8X,'Y(1,*) AND YL(*)'//)
      END
      SUBROUTINE COPYZ(S,Y,L)
C     ------------------------------------------------------------------COP    30
CC                                                                      COP    40
CC    THIS SUBROUTINE COPIES THE ARRAY Y, OF LENGTH L, INTO THE ARRAY S COP    50
CC                                                                      COP    60
C     ------------------------------------------------------------------COP    70
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION S(1),Y(1)
      IF(L.LE.0)RETURN
      DO 100 J=1,L
  100 S(J) = Y(J)
      RETURN
      END
      SUBROUTINE DERVAL (Y,YL,T,N,NY,W,KERET)
C     THIS SUBROUTINE SUPPLIES THE INITIAL VALUES OF THE DERIVATIVES
C     OF THE NODAL PARAMETERS TAKEN FROM STEADY STATE SYSTEM ANALYSIS.
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION Y(7,1), YL(1), W(1)
      Y(2,7)=-9.428681D01
      Y(2,8)=-9.079442D01
      Y(2,9)=-9.428681D01
      Y(2,12)=-1.012731D02
      Y(2,13)=-2.894024D02
      Y(2,14)=-1.012731D02
      Y(2,17)=-1.057558D02
      Y(2,18)=-3.616863D01
      Y(2,19)=-1.057558D02
      Y(2,22)=-7.0622577D01
      Y(2,23)=-3.162976D02
      Y(2,24)=-7.062277D01
      Y(2,27)=-1.322337D02
      Y(2,28)=-3.825550D01
      Y(2,29)=-1.322337D02
      Y(2,42)=0.D0
      Y(2,43)=0.D0
      Y(2,44)=0.D0
      Y(2,47)=0.D0
      Y(2,48)=0.D0
      Y(2,49)=0.D0
      Y(2,52)=0.D0
      Y(2,53)=0.D0
      Y(2,54)=0.D0
```

```
      Y(2,57)=0.D0
      Y(2,58)=0.D0
      Y(2,59)=0.D0
      Y(2,62)=0.D0
      Y(2,63)=0.D0
      Y(2,64)=0.D0
      Y(2,88)=-4.157413D-01
      Y(2,89)=-3.571020D-01
      Y(2,90)=-3.156324D-01
      Y(2,91)=-3.571020D-01
      Y(2,92)=-4.157413D-01
      Y(2,93)=2.162937D-01
      Y(2,94)=2.371216D-01
      Y(2,95)=1.476146D-01
      Y(2,96)=2.371216D-01
      Y(2,97)=2.162937D-01
      Y(2,98)=-3.861831D-02
      Y(2,99)=-6.708371D-02
      Y(2,100)=-5.798223D-02
      Y(2,101)=-6.708371D-02
      Y(2,102)=-3.681831D-02
      Y(2,103)=1.207203D-01
      Y(2,104)=1.948975D-01
      Y(2,105)=2.023086D-01
      Y(2,106)=1.948975D-01
      Y(2,107)=1.207203D-01
      Y(2,108)=-2.532375D-01
      Y(2,109)=-2.842183D-01
      Y(2,110)=-3.355962D-01
      Y(2,111)=-2.842183D-01
      Y(2,112)=-2.532375D-01
      KERET=0
      RETURN
      END
      SUBROUTINE NUITSL (PW,DY,F1,N,NY,EPS,YMAX,NEWPW,KRET)
C     ------------------------------------------------------------NUI    20
CCC   THE PURPOSE OF THIS SUBROUTINE IS TO SOLVE A                NUI    30
CCC   LINEAR SYSTEM OF EQUATIONS FOR THE NEWTON ITERATES WHEN THE NUI    40
CCC   CORRECTOR EQUATION IS BEING SOLVED.  UPON ENTRY TO THIS SUBROUTINENUI    50
CCC   THE SYSTEM OF EQUATIONS TO BE SOLVED IS   J W = -F  , WHERE  NUI    60
CCC      J  IS STORED IN PW UPON ENTRY                            NUI    70
CCC      W  IS RETURNED IN F1                                     NUI    80
CCC      -F IS STORED IN DY UPON ENTRY                            NUI    90
CCC                                                               NUI   100
CCC   THIS SUBROUTINE IS GENERALLY SUPPLIED BY THE USER, ALTHOUGH THERE NUI  110
CCC   ARE SOME STANDARD FORMS AVAILABLE.  FOR EXAMPLE, THIS VERSION NUI   120
CCC   ASSUMES THAT PW IS STORED IN FULL STORAGE MODE IN AN NXN MATRIX. NUI  130
C     IF NEWPW = 1, AN LU DECOMPOSITION IS DONE, NEWPW IS SET TO ZERO  NUI   140
```

```
C     AND FORWARD AND BACKWARD SUBSTITUTION FOR THE SOLUTION IS DONE.    NUI  150
CCC   IF NEWPW = 0, ONLY FORWARD AND BACKWARD SUBSTITUTION FOR THE       NUI  160
CCC   SOLUTION IS NECESSARY.                                            NUI  170
CCC                                                                     NUI  180
CCC   NOTE THAT THE PARAMETERS EPS AND YMAX ARE USEFUL IF AN ITERATIVE  NUI  230
CCC   METHOD IS USED TO SOLVE THE SYSTEM OF EQUATIONS.                  NUI  240
CCC                                                                     NUI  250
CCC   THE CALLING SEQUENCE FOR THIS SUBROUTINE IS                       NUI  260
CCC                                                                     NUI  270
CCC   CALL NUITSL(PW,DY,F1,N,NY,EPS,YMAX,NEWPW,KRET)                    NUI  280
CCC                                                                     NUI  290
CCC   WHERE THE PARAMETERS ARE DEFINED AS FOLLOWS.                      NUI  300
CCC                                                                     NUI  310
CCC      PW       - THE J MATRIX CALCULATED IN SUBROUTINE JACMAT        NUI  320
CCC      DY       - THE RIGHT HAND SIDE OF THE LINEAR SYSTEM TO BE SOLVEDNUI  330
CCC      F1       - THE SOLUTION IS RETURNED IN THE ARRAY F1            NUI  340
CCC      N        - SAME AS IN LDASUB, TOTAL NUMBER OF VARIABLES        NUI  350
CCC      NY       - SAME AS IN LDASUB, NUMBER OF DIFFERENTIAL EQUATIONS NUI  360
CCC                 AND NONLINEAR VARIABLES                            NUI  370
CCC      EPS      - L2 ERROR CONSTANT USED IN LDASUB                    NUI  380
CCC      YMAX     - MAXIMUM VALUES OF Y(1,I) SEEN UP TO THE CURRENT TIME NUI  390
CCC      NEWPW    - INDICATES WHETHER A NEW J MATRIX HAS BEEN COMPUTED  NUI  400
CCC                    =1  INDICATES A NEW J MATRIX HAS BEEN COMPUTED   NUI  410
CCC                        SINCE THE LAST ENTRY TO NUITSL.  NEWPW       NUI  420
CCC                        SHOULD BE SET TO ZERO IF SOME PREPROCESSING  NUI  430
CCC                        SUCH AS LU DECOMPOSITION MUST BE DONE ON A   NUI  440
CCC                        NEW  J  MATRIX.                              NUI  450
CCC                    =0  INDICATES THE J MATRIX IS THE SAME AS WHEN   NUI  460
CCC                        NUITSL WAS LAST ENTERED                      NUI  470
CCC      KRET     - RETURN INDICATOR                                    NUI  480
CCC                    =0  NORMAL RETURN                                NUI  490
CCC                    =1  ERROR RETURN.  SOLUTION OF EQUATIONS COULD   NUI  500
CCC                        NOT BE OBTAINED.                             NUI  510
CCC                                                                     NUI  520
C     ------------------------------------------------------------------NUI  530
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION PW(1), DY(1), F1(1), YMAX(1)
      NL = N-NY
      IF (NEWPW.EQ.0) GO TO 100
      NEWPW = 0
      NN=N**2+1
      NNN=NN+N
      CALL LUDATF (PW,PW,N,N,0,D1,D2,PW(NN),PW(NNN),F1,IER)
      IF (IER.EQ.0) GO TO 100
      KRET = 1
      RETURN
  100 CALL LUELMF (PW,DY,PW(NN),N,N,F1)
      KRET = 0
```

```
      RETURN
      END
      SUBROUTINE DIFFUN(Y,YL,T,HINV,DY)
C     THIS SUBROUTINE EVALUATES THE SYSTEM'S GOVERNING SET OF
CC    EQUATIONS AT A GIVEN TIME AND GIVEN VALUES OF THE NODAL
C     PARAMETER AND ITS DERIVATIVE.
      IMPLICIT REAL*8(A-H,O-Z,$)
      DIMENSION Y(7,1),YL(1),DY(1)
      COMMON CD(117,117),TM(117,117),C(117)
      DO 200 I=1,117
      DY(I)=-C(I)
      DO 100 J=1,117
  100 DY(I)=DY(I)+CD(I,J)*Y(2,J)*HINV+TM(I,J)*Y(1,J)
  200 CONTINUE
      RETURN
      END
      SUBROUTINE JACMAT(Y,YL,T,HINV,A2,N,NY,EPS,DY,F1,PW)
C     THIS SUBROUTINE EVALUATES THE JACOBIAN MATRIX AT THE GIVEN TIME
CC    AND AT CURRENT VALUES OF THE DEPENDENT VARIABLES, ORDER,
C     AND STEPSIZE.
      IMPLICIT REAL*8(A-H,O-Z,$)
      DIMENSION Y(7,1),PW(117,1)
      COMMON CD(117,117),TM(117,117),C(117)
      AH=-A2*HINV
      DO 200 I=1,117
      DO 200 J=1,117
  200 PW(I,J)=TM(I,J)+AH*CD(I,J)
      RETURN
      END
```

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