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PRELIMINARY RESULTS FOR SINGLE AIRFOIL RESPONSE TO LARGE NONPOTENTIAL FLOW DISTURBANCES

P.V.K.Perumal
Graduate Research Assistant

F. Sisto
Professor and Head

Department of Mechanical Engineering
Stevens Institute of Technology
Hoboken, New Jersey 07030, USA

SUMMARY

An attempt is made to evaluate the unsteady response of a flat plate airfoil to large nonpotential flow disturbances in the form of a translating rectangular grid of eddy-array. A suitable streamfunction to represent the translating nonpotential vortex-array is chosen. The method of analysis adopted is the singularity distribution principle in combination with the time-marching technique. The problem is solved in two stages, namely, (1) auxiliary solution and (2) time-marching solution. By auxiliary solution is meant the solution of the problem which completely neglects the presence of the wake vortex sheet and treats time as a parameter; this results in a steady flow type of analysis. The time-marching part of the analysis increments time by equal steps starting from zero time, makes use of the auxiliary solution, keeps track of the shedding and growth of the wake vortex sheet, evaluates the unsteady response, and continues along the time-axis up to any specified maximum time limit. Preliminary numerical results from a computer program are presented.

SYMBOLS

b	Semichord length
C_l	Lift coefficient
C_m	Moment coefficient
g	Eddy intensity parameter
g_l	Strength of lth youngest wake vortex
G_S	Kernel function defined in text
h,k	Sides of rectangular eddy cell
J_k	Bessel function of kth order
$K(\tilde{S},x)$	Kernel function defined in text
L	Lift force (+ve up)
M	Moment about leading edge (+ve clockwise)
n	Time step number; summation index
N	Maximum number of wake vortices handled by program
p	Pressure
r,θ	Polar coordinates
S	Arc length along wake
$\tilde{S}$	Location vector associated with S
t	Time
T	Period
u,v	x- and y-components of velocity
U,V	Mean translational velocity components of freestream
$\tilde{V}$	Total velocity vector
w	Complex velocity potential
W	Mean freestream velocity; wake vortex sheet
x,y	Space fixed coordinates
x_0,y_0	Initial location coordinates for (ξ,η)-coordinates
z	Complex variable x+iy
Z1,..,Z6	Variables defined in text
β	Direction of mean freestream velocity
γ	Bound vortex strength
γ_w	Wake vortex distribution
Γ	Bound circulation strength
Γ'	Strength of a general vortex

ε	A small value ($<<1$)
ζ	Complex variable $\xi+i\eta$
θ	Inclination of eddy-array with foil
ξ,η	Translating natural coordinates; transformed plane coordinates
ρ	Fluid density
ϕ	Variable defined in text
ψ	Stream function

SUBSCRIPTS:

A	Auxiliary
E	Due to eddies
l	Due to lth youngest wake vortex
M	Mixed
n	nth time step
r	Radial component
S	Due to steady translation
U	Unsteady
θ	Tangential component

SUPERSCRIPTS:

'	Nondimensional quantity; dummy variable
l	Due to lth youngest wake vortex
-	Complex conjugate

BACKGROUND

The unsteady aerodynamic response of airfoils due to a time-variant incoming flow field leads to many undesirable results. Some of these deleterious effects are: (1) loss of aerodynamic performance (2) structural failure due to aerodynamic overloading (3) vibration which leads to other problems like fatigue and wear (4) noise due to the acoustic dipoles which always accompany such fluctuating forces (5) laminar-turbulent boundary layer transition (6) cavitation and the attendant pitting and erosion of working surfaces. A second class of unwanted phenomena originates when the unsteady response and the resulting direct effects such as physical movement of the foils start interacting strongly with the incoming flow thus establishing a closed feedback system. These phenomena could be collectively termed as loss in stability. Classical flutter, stall flutter, surge and rotating stall are some of the well known members of this group.

The type of time-variant inlet flow dealt with will depend on the engineering application for the theory. In an axial turbomachine the flow disturbances exciting the blades of a particular row are either due to gross inlet distortion, to wakes from the blades of the preceding row and structural members like struts, or to turbulence. All these three types of flow disturbances could be assumed to consist of eddies having definite wavelengths. The wavelengths associated with these eddies in terms of the blade chord length may be taken to be very large for gross inlet distortion, of unit order for wakes from preceding blade rows, and very small for freestream turbulence. Since the effects of viscosity and turbulence in fluid flows is to make the flow rotational these eddies in all likelihood are nonpotential. The flow velocities associated with them could range from very small to the order of the mean flow velocity. Since one is mainly interested in short durations of these flows such as the time to travel a few chord lengths distance these eddies may be taken to be 'frozen', stable, or having no mutual interference. In the present work an attempt is made to formulate a reasonable model for such time-variant, nonpotential, incoming flows and the attendant unsteady response of a single thin airfoil is evaluated.

PROBLEM STATEMENT

Consider (Fig. 1) the two-dimensional, unsteady, rotational flow field of a 'frozen' rectangular grid of eddies with alternating signs of equal circulation strength translating at a constant velocity U,V with respect to a system of space-fixed coordinates x,y. The fluid involved is assumed to be incompressible and inviscid. The field of velocity components associated with the eddies alone, neglecting the constant translational motion, is represented by the streamfunction

$$\psi(\xi,\eta) = g \cos\frac{\pi\xi}{k} \cos\frac{\pi\eta}{h}$$

where ξ,η is a system of natural coordinates translating with the grid at the constant velocity U,V; g is an intensity parameter for the strength of the nonpotential eddies; K and h are parameters representing the unequal lengths of the sides of the rectangle which defines one eddy cell.

Let a flat plate airfoil of chord length 2b be placed along the x-axis with the midchord point at the origin of the x,y-coordinate system. If θ is the angle of rotation of the ξ,η-coordinates with respect to the x,y-coordinates and x_0, y_0 is the location of the origin of the ξ,η-coordinate system at zero time, then the unsteady rotational incoming flow to which the airfoil is subject to is represented by the streamfunction

$$\psi(x,y,t) = Uy - Vx + g\cos\frac{\pi}{k}\left[(x-x_0-Ut)\cos\theta + (y-y_0-Vt)\sin\theta\right]\cos\frac{\pi}{h}\left[-(x-x_0-Ut)\sin\theta + (y-y_0-Vt)\cos\theta\right]$$

The problem under consideration is to evaluate the unsteady response of the flat plate airfoil subject to such an incoming flow. Further assumptions made in solving the problem are: (i) the flow about the foil remains attached (ii) the introduction of the foil into the incoming flow does not change the total circulation of the whole system and (iii) the foil remains stationary.

STUDY OF STREAMFUNCTION

In order to obtain an idea of the streamline pattern represented by the assumed streamfunction one can assign, without losing any essential features, zero values for the orientation parameters x_0, y_0 and θ, and choose to plot the instantaneous streamline pattern at zero time. On normalizing the resulting streamfunction with respect to the semichord length b, and the mean velocity components with respect to the eddy-intensity parameter and the semichord one gets

$$\begin{aligned}\psi' &\equiv \psi/g \\ &= \frac{Ub}{g}\,\frac{y}{b} - \frac{Vb}{g}\,\frac{x}{b} + \cos\left[\pi\,\frac{b}{k}\,\frac{x}{b}\right]\cos\left[\pi\,\frac{b}{h}\,\frac{y}{b}\right]\end{aligned}$$

Using a computer program the stencils for the streamline patterns of ψ' have been plotted in Figs. 2 and 3 for two sets of parametric values. Fig. 2 represents the streamline pattern for nontranslating eddy-array flow. For this case the streamlines are also lines of constant vorticity.

It can also be shown that the assumed streamfunction: (i) satisfies the continuity equation for the flow of an incompressible fluid (ii) represents a flow field which is in general rotational (iii) involves rectangular eddy cells each of total circulation strength equal to $4g(k/h + h/k)$ (iv) represents streamlines with 2k and 2h as the wavelengths (the stencil sides) of the distance in the x- and y-directions respectively and (v) the upwash created by the inlet flow streamfunction is periodic only when the direction of the mean inlet velocity satisfies the constraint

$$\operatorname{Tan}\beta = \frac{(m+n)\frac{k}{b}\sin\theta + (m-n)\frac{h}{b}\cos\theta}{(m+n)\frac{k}{b}\cos\theta - (m-n)\frac{h}{b}\sin\theta}$$

and when β satisfies the above constraint the nondimensional period of the upwash is given by

$$T' \equiv \frac{T}{\left[\frac{b}{U}\right]} = \begin{cases} (m+n)\frac{k}{b}\cos\theta - (m-n)\frac{h}{b}\sin\theta & m \neq n \\ 2\,\frac{k}{b}\cos\theta & m = n \end{cases}$$

PROPOSED METHOD

The method of solution adopted to solve the present problem is the singularity distribution principle in combination with a time-marching technique. The problem is solved in two stages, namely, (1) auxiliary solution and (2) time-marching solution. By 'auxiliary' solution is meant the solution of the problem which completely neglects the presence of the wake vortex sheet (which is always present in such an unsteady flow situation) and treats time t as a parameter; hence in solving for the auxiliary solution the unsteady terms Ut and Vt in the incoming flow streamfunction can be lumped with the orientation parameters x_0 and y_0 respectively. The time-marching part of the solution increments time by equal steps starting from zero time, makes use of the auxiliary solution at every time step, keeps track of the shedding and growth of the wake vortex sheet, evaluates the unsteady aerodynamic responses (lift and moment)at every time step, and continues along the time-axis up to a specified maximum time limit.

The question arises as to whether one is justified in solving the problem of an airfoil located in a rotational flow field using the method of singularities. Since it is assumed in the present theory that the fluid is inviscid it is not possible for the airfoil to exert any nonconservative force on the fluid particles and thus change the rotationality of the incoming flow. Also, since it is assumed that the flow about the foil remains always attached, the only way other than pure convective effects, by which the rotationality of the fluid particles in the flow field can change is due to the shedding of wake vortex sheet which arises solely due to the unsteadiness of the total bound circulation around the airfoil. Since the time-marching technique employed takes complete care of the unsteady wake vortex sheet starting from zero time the use of conventional method of singularities (satisfying the nonpenetration and Kutta conditions) is justified in the present work.

AUXILIARY SOLUTION

According to the earlier stated definition of 'auxiliary problem', assuming no wake vortex sheet to be present and considering time t only as a parameter, the upwash along the airfoil is to be calculated from the streamfunction

$$\psi_A(x,y) = Uy - Vx + g\cos\frac{\pi}{k}\left[(x-x_0)\cos\theta + (y-y_0)\sin\theta\right]\cos\frac{\pi}{h}\left[-(x-x_0)\sin\theta + (y-y_0)\cos\theta\right]$$

where suffix A denotes 'auxiliary'; x_0 and y_0 incorporate in them the temporal terms Ut and Vt also. For the sake of simplicity it is chosen not to indicate by symbols in the foregoing and following expressions the fact that time t is a parameter.

The upwash (normal velocity component) along the foil becomes

$$\begin{aligned} v_A(x,0) &= -\left.\frac{\partial\psi}{\partial x}\right|_{y=0} \\ &= V + g\frac{\pi}{k}\cos\theta\,\sin\frac{\pi}{k}\left[(x-x_0)\cos\theta - y_0\sin\theta\right]\cos\frac{\pi}{h}\left[-(x-x_0)\sin\theta - y_0\cos\theta\right] \\ &\quad - g\frac{\pi}{h}\sin\theta\,\cos\frac{\pi}{k}\left[(x-x_0)\cos\theta - y_0\sin\theta\right]\sin\frac{\pi}{h}\left[-(x-x_0)\sin\theta - y_0\cos\theta\right] \end{aligned}$$

Decomposing the upwash into one due to steady translation (denoted by suffix S) and one due to the presence of eddies (denoted by suffix E) as $v_A \equiv v_S + v_E$ one gets after considerable simplification

$$v_S(x,0) = V$$

$$v_E(x,0) = \frac{g}{2b}\left[Z_2\cos Z_5\sin(Z_2\frac{x}{b}) - Z_2\sin Z_5\cos(Z_2\frac{x}{b}) + Z_1\cos Z_6\sin(Z_1\frac{x}{b}) - Z_1\sin Z_6\cos(Z_1\frac{x}{b})\right]$$

The new constants involved in the above equation have the following expressions

$$Z_1 = \pi(\frac{b}{k}\cos\theta + \frac{b}{h}\sin\theta) \qquad Z_2 = \pi(\frac{b}{k}\cos\theta - \frac{b}{h}\sin\theta)$$

$$Z_3 = \pi(\frac{b}{h}\cos\theta + \frac{b}{k}\sin\theta) \qquad Z_4 = \pi(\frac{b}{h}\cos\theta - \frac{b}{k}\sin\theta)$$

$$Z_5 = \left[\frac{x_0}{b}\right]Z_2 + \left[\frac{y_0}{b}\right]Z_3 \qquad Z_6 = \left[\frac{x_0}{b}\right]Z_1 - \left[\frac{y_0}{b}\right]Z_4$$

Using the Söhngen inversion formula (Ref. 1) to solve the singular integral equation

$$\frac{1}{2\pi}\int_{-b}^{b}\frac{\gamma(x')}{x - x'}\,dx' = v(x,0)$$

the expressions for the bound vortex distributions become

$$\begin{aligned} \gamma_E(\phi) = 2\frac{g}{b}\Big[&-\tfrac{1}{2}\{Z_2J_0(Z_2)\sin Z_5 + Z_1J_0(Z_1)\sin Z_6\}\frac{\sin\phi}{1+\cos\phi} \\ &+ \{Z_2J_1(Z_2)\cos Z_5 + Z_1J_1(Z_1)\cos Z_6\}\sin\phi \\ &+ \sum_{k=1}^{\infty}(-1)^k\{Z_2J_{2k+1}(Z_2)\cos Z_5 + Z_1J_{2k+1}(Z_1)\cos Z_6\}\sin\overline{2k+1}\phi \\ &- \sum_{k=1}^{\infty}(-1)^k\{Z_2J_{2k}(Z_2)\sin Z_5 + Z_1J_{2k}(Z_1)\sin Z_6\}\sin 2k\phi\Big] \end{aligned}$$

$$\gamma_S(\phi) = 2V\sqrt{\frac{1 - \cos\phi}{1 + \cos\phi}}$$

where the variable ϕ has been defined by the expression $x \equiv b \cos\phi$. It may be noted that even though the expression for the auxiliary bound vortex distribution involves infinite series its zeroth and higher moments about the midchord have expressions which involve only finite number of terms. Only these moments are needed in the time-marching solution.

TIME-MARCHING SOLUTION

In the present analysis the unsteadiness of the resultant flow field has three origins: (i) unsteady nature of the incoming flow; this is taken care of by the streamfunction assumed (ii) unsteadiness of the bound vortex distribution and (iii) unsteady nature of the shedding and growth of the wake vortex sheet. In order to take care of the unsteadiness of the bound vortex distribution and the wake vortex sheet a time-marching scheme can be devised as follows.

Decomposing (Ref. 2, 3) the bound vortex distribution and the total circulation into two components, namely, one due to 'auxiliary' solution (solution of flow entirely neglecting the presence of wake) denoted by suffix A and another due to 'unsteady' solution (solution of flow entirely due to the presence of wake) denoted by suffix U

$$\Gamma(t) = \Gamma_A(t) + \Gamma_U(t) \qquad \text{.......................................(1)}$$

where

$$\Gamma_A(t) = \int_{-b}^{b} \gamma_A(x,t)\, dx$$

$$\Gamma_U(t) = \int_{-b}^{b} \gamma_U(x,t)\, dx$$

Making use of the condition that the presence of the foil (and hence the presence of the wake) do not change the total circulation of the whole system one gets

$$\Gamma(t) + \int_{W(t)} \gamma_W(S)\, dS = 0 \qquad \text{.......................................(2)}$$

where $W(t)$ denotes the entire instantaneous line element representing the wake vortex sheet; S is the arc length along the line element $W(t)$ measured from the trailing edge of the foil; and $\gamma_W(S)$ is the wake vortex distribution (Fig. 4).

Rearranging Eq. (1) and using Eq. (2)

$$-\Gamma_A(t) = -\Gamma(t) + \Gamma_U(t)$$

$$= \int_{W(t)} \gamma_W(S)\, dS + \int_{-b}^{b} \gamma_U(x,t)\, dx$$

$$= \int_{W(t)} \gamma_W(S)\, dS + \int_{-b}^{b} \left[\int_{W(t)} K(\vec{S},x)\, \gamma_W(S)\, dS \right] dx$$

where $K(\vec{S},x)$ denotes the bound vortex distribution along the foil due to the presence of a unit vortex at vector distance $\vec{S}$ from the trailing edge; $\vec{S}$ is the vector from trailing edge associated with a point on the vortex sheet whose arc length is S (Fig. 4). Changing the order of integration and grouping terms

$$-\Gamma_A(t) = \int_{W(t)} \left[1+G_S\right] \gamma_W(S)\, dS$$

after defining a new kernel to be $G_S \equiv \int_{-b}^{b} K(\vec{S},x)\, dx$ where G_S means the total circulation around the foil due to a unit vortex located at vector distance $\vec{S}$. The general expressions for $K(\vec{S},x)$ and G_S are derived in Appendix A.

The above integral equation for wake vortex strength will be solved in a time-marching fashion making use of (i) the concept (Ref. 3) that the wake which is a continuous distribution of point vortices may be approximated by an equivalent discrete distribution of point vortices; the discretization scheme adopted in this work is to represent the elemental wake vortex sheet shed during any one time step by a single point vortex of strength equal to the total circulation around the element and positioned at a meanpoint of the element; (ii) the condition that the wake vortices are free in the sense that they move with the fluid element to which they are 'attached'. The details of the solution scheme follows.

At time t+Δt denoting the far end (Fig. 4) of the wake vortex sheet (i.e., location at time t+Δt of the fluid particle which marked the birth of the wake vortex sheet at time t = 0) by $S_0(t+\Delta t)$ one gets from the wake vortex integral equation

$$\Gamma_A(t+\Delta t) = \int_{S_0(t+\Delta t)}^{b} [1 + G_S]\, \gamma_W(S)\, dS$$

Measuring time in steps of equal length Δt and dividing the wake vortex sheet into segments formed during each time step (Fig. 4)

$$\Gamma_A(\overline{n+1}\,\Delta t) = \sum_{l=1}^{n} \int_{S_{l-1}(\overline{n+1}\Delta t)}^{S_l(\overline{n+1}\Delta t)} [1 + G_S]\gamma_W(S)\, dS + \int_{S_n(\overline{n+1}\Delta t)}^{b} [1 + G_S]\gamma_W(S)\, dS$$

where $S_l(\overline{n+1}\,\Delta t)$ denotes the location at time $\overline{n+1}\,\Delta t$ of the fluid particle which marked the birth of the wake vortex sheet at time $l\Delta t$.

Discretizing the wake segments, that is, replacing each segment of continuous vortices by a single vortex

$$-\Gamma_A(\overline{n+1}\,\Delta t) = \sum_{l=1}^{n} \left[1 + G_S^{\ell}\right] g_l + \left[1 + G_S^{n+1}\right] g_{n+1}$$

where g_l denotes the constant circulation strength (equal to the total vorticity of the fluid particles constituting the elemental wake segment formed during the time interval $\overline{l-1}\Delta t<t<l\Delta t$) of the lth youngest discrete wake vortex which is located at a mean point of the wake segment it represents. Thus

$$g_{n+1} = \frac{-\Gamma_A(\overline{n+1}\,\Delta t) - \sum_{l=1}^{n} \left[1 + G_S^{l}\right] g_l}{1 + G_S^{n+1}}$$

where G_S^l represents G_S when the arc length involved in the evaluation of G_S is the one with respect to the location of the lth youngest wake vortex.

The location vector $\hat{S}_l(\overline{n+1}\,\Delta t)$ for the lth youngest wake vortex at time $\overline{n+1}\,\Delta t$ may be found by the following predictor-corrector scheme adopted from Ref. 3

$$\hat{P}_l = \hat{S}_l(n\Delta t) + \Delta t\, \hat{V}_l(\hat{S}_l(n\Delta t))$$

$$\hat{S}_l(\overline{n+1}\Delta t) = \hat{S}_l(n\Delta t) + \Delta t \left[\frac{\hat{V}_l(\hat{S}_l(n\Delta t)) + \hat{V}_l(\hat{P}_l)}{2}\right]$$

where $\hat{V}_l$ is the convective velocity for the lth wake vortex. Obviously, such a predictor-corrector scheme is necessary due to the fact that the value of the convective velocity keeps changing with the convection. The convective velocity $\hat{V}_l$ for the lth wake vortex is composed of the following components

$$\hat{V}_l = \hat{V}_{\text{freestream}} + \hat{V}_{\substack{\text{auxiliary bound}\\\text{vortices}}} + \sum_k \hat{V}_k + \hat{V}_{\substack{\text{unsteady bound}\\\text{vortices}}}$$

where the summation index k stands for all wake vortices except the lth. The term 'freestream' includes the steady translation and eddy-array components. Assuming that the elemental portion of the wake vortex sheet being shed does not contribute to the convection of the wake all the information needed to calculate $\hat{V}_l$ is known. In the limit of small time steps this assumption is valid.

Due to the limitation of the computer memory it is not possible to represent the wake vortex by more than a reasonably large number of discrete free vortices. This calls for a scheme by which one will keep track of only a finite number of wake vortices and allow for a rational "overflow" of the remaining wake vortices. Such a scheme follows. It may be seen from the results of Appendix A that for large values of the location vector $\hat{S}$ the kernel G_S approaches zero. Hence if it is agreed to handle a maximum of, say, only N number of wake vortices and it is expected that, in general, the older wake vortices would have been swept away farther from the foil the time-marching equation for the strength of (n+1)th nascent wake vortex becomes

$$g_{n+1}\Big|_{n \geq N} = \frac{-\Gamma_A(\overline{n+1}\ t) - \sum\limits_{k=1}^{n-N+1} g_k - \sum\limits_{l=n-N+2}^{n} [1 + G_S^l] g_l}{1 + G_S^{n+1}}$$

UNSTEADY RESPONSE EXPRESSIONS

Starting from Euler's equation of motion it can be shown that for the flow of an incompressible, inviscid fluid about a flat plate airfoil the loading, lift (positive upward), and moment about the leading edge (positive clockwise) are given by

$$\Delta p(x,t) = \rho \left[u(x,t)\gamma(x,t) + \int_{-b}^{x} \frac{\partial\gamma}{\partial t}\, dx \right]$$

$$L(t) = \rho \left[\int_{-b}^{b} u(x,t)\gamma(x,t)\, dx + b\frac{\partial}{\partial t} \int_{-b}^{b} \gamma(x,t)\, dx - \frac{\partial}{\partial t} \int_{-b}^{b} x\gamma(x,t)\, dx \right]$$

$$M(t) = -\rho \left[\int_{-b}^{b} (x+b)u(x,t)\gamma(x,t)\, dx + \frac{3}{2} b^2 \frac{\partial}{\partial t} \int_{-b}^{b} \gamma(x,t)\, dx - \frac{\partial}{\partial t} \int_{-b}^{b} (\frac{x^2}{2} + bx)\gamma(x,t)\, dx \right]$$

where $u(x,t)$ is the resultant tangential velocity along the foil position and $\gamma(x,t)$ is the bound vortex distribution.

Decomposing the tangential velocity and bound vortex distribution into steady translation (suffix S), eddy-presence (suffix E) and unsteady (suffix U) parts and grouping the response expressions into auxiliary (suffix A), unsteady (suffix U) and mixed (suffix M) contributions one gets after omitting to indicate the dependency on independent variables

$$\frac{1}{\rho} L_A = U \int_{-b}^{b} \gamma_S\, dx + \int_{-b}^{b} u_E\gamma_E\, dx + b\frac{\partial}{\partial t} \int_{-b}^{b} \gamma_E\, dx - \frac{\partial}{\partial t} \int_{-b}^{b} x\gamma_E\, dx + U\int_{-b}^{b} \gamma_E\, dx + \int_{-b}^{b} u_E\gamma_S\, dx$$

$$\frac{1}{\rho} L_U = \int_{-b}^{b} u_U\gamma_U\, dx + b\frac{\partial}{\partial t} \int_{-b}^{b} \gamma_U\, dx - \frac{\partial}{\partial t} \int_{-b}^{b} x\gamma_U\, dx$$

$$\frac{1}{\rho} L_M = \int_{-b}^{b} u_U\gamma_S\, dx + \int_{-b}^{b} u_U\gamma_E\, dx + U \int_{-b}^{b} \gamma_U\, dx + \int_{-b}^{b} u_E\gamma_U\, dx$$

$$\frac{1}{\rho} M_A = -U \int_{-b}^{b} (x+b)\gamma_S\, dx - \int_{-b}^{b} (x+b)u_E\gamma_E\, dx - \frac{3}{2} b^2 \frac{\partial}{\partial t} \int_{-b}^{b} \gamma_E\, dx + \frac{\partial}{\partial t} \int_{-b}^{b} (\frac{x^2}{2} + bx)\gamma_E\, dx$$
$$- U \int_{-b}^{b} (x+b)\gamma_E\, dx - \int_{-b}^{b} (x+b)u_E\gamma_S\, dx$$

$$\frac{1}{\rho} M_U = - \int_{-b}^{b} (x+b)u_U\gamma_U\, dx - \frac{3}{2} b^2 \frac{\partial}{\partial t} \int_{-b}^{b} \gamma_U\, dx + \frac{\partial}{\partial t} \int_{-b}^{b} (\frac{x^2}{2} + bx)\gamma_U\, dx$$

$$\frac{1}{\rho} M_M = - \int_{-b}^{b} (x+b)u_U\gamma_S\, dx - \int_{-b}^{b} (x+b)u_U\gamma_E\, dx - U \int_{-b}^{b} (x+b)\gamma_U\, dx - \int_{-b}^{b} (x+b)u_E\gamma_U\, dx$$

Defining the lift and moment coefficients to be

$$C_l \equiv \frac{L}{\frac{1}{2}\rho W^2\ 2b} \qquad C_m \equiv \frac{M}{\frac{1}{2}\rho W^2\ (2b)^2}$$

one gets the different contributions to be

$$C_l = C_{lA} + C_{lU} + C_{lM}$$

$$C_m = C_{mA} + C_{mU} + C_{mM}$$

where

$$C_{lA} = C_{lA1} + C_{lA2} + C_{lA3} + C_{lA4} + C_{lA5} + C_{lA6}$$

$$C_{lU} = C_{lU1} + C_{lU2} + C_{lU3}$$

$$C_{lM} = C_{lM1} + C_{lM2} + C_{lM3} + C_{lM4}$$

$$C_{mA} = C_{mA1} + C_{mA2} + C_{mA3} + C_{mA4} + C_{mA5} + C_{mA6}$$

$$C_{mU} = C_{mU1} + C_{mU2} + C_{mU3}$$

$$C_{mM} = C_{mM1} + C_{mM2} + C_{mM3} + C_{mM4}$$

Nondimensionalizing the time t, distance x, bound vortex strength γ, and the tangential velocity u according to the following definitions

$$t' \equiv \frac{tU}{b} \; ; \quad x' \equiv \frac{x}{b} \; ; \quad \gamma' \equiv \frac{\gamma b}{g} \; ; \quad u' \equiv \frac{ub}{g}$$

one gets after arranging each of the expressions involved to be the product of nondimensionalized terms

$$C_{lA1} = \frac{Ub}{g}\left[\frac{g}{bW}\right]^2 \int_{-1}^{1} \gamma'_S \, dx'$$

$$C_{lA2} = \left[\frac{g}{bW}\right]^2 \int_{-1}^{1} u'_E \gamma'_E \, dx'$$

$$C_{lA3} = \frac{Ub}{g}\left[\frac{g}{bW}\right]^2 \frac{\partial}{\partial t'} \int_{-1}^{1} \gamma'_E \, dx'$$

$$C_{lA4} = -\frac{Ub}{g}\left[\frac{g}{bW}\right]^2 \frac{\partial}{\partial t'} \int_{-1}^{1} x'\gamma'_E \, dx'$$

$$C_{lA5} = \frac{Ub}{g}\left[\frac{g}{bW}\right]^2 \int_{-1}^{1} \gamma'_E \, dx'$$

$$C_{lA6} = \left[\frac{g}{bW}\right]^2 \int_{-1}^{1} u'_E \, \gamma'_S \, dx'$$

$$C_{lU1} = \left[\frac{g}{bW}\right]^2 \int_{-1}^{1} u'_U \, \gamma'_U \, dx'$$

$$C_{lU2} = \frac{Ub}{g}\left[\frac{g}{bW}\right]^2 \frac{\partial}{\partial t'} \int_{-1}^{1} \gamma'_U \, dx'$$

$$C_{lU3} = -\frac{Ub}{g}\left[\frac{g}{bW}\right]^2 \frac{\partial}{\partial t'} \int_{-1}^{1} x'\gamma'_U \, dx'$$

$$C_{lM1} = \left[\frac{g}{bW}\right]^2 \int_{-1}^{1} u'_U \gamma'_S \, dx'$$

$$C_{lM2} = \left[\frac{g}{bW}\right]^2 \int_{-1}^{1} u'_U \gamma'_E \, dx'$$

$$C_{lM3} = \frac{Ub}{g}\left[\frac{g}{bW}\right]^2 \int_{-1}^{1} \gamma'_U \, dx'$$

$$C_{lM4} = \left[\frac{g}{bW}\right]^2 \int_{-1}^{1} u'_E \gamma'_U \, dx'$$

$$C_{mA1} = -\frac{1}{2}\frac{Ub}{g}\left[\frac{g}{bW}\right]^2 \left[\int_{-1}^{1} x'\gamma'_S \, dx' + \int_{-1}^{1} \gamma'_S \, dx'\right]$$

$$C_{mA2} = -\frac{1}{2}\left[\frac{g}{bW}\right]^2 \left[\int_{-1}^{1} x' u'_E \gamma'_E \, dx' + \int_{-1}^{1} u'_E \gamma'_E \, dx'\right]$$

$$C_{mA3} = -\frac{3}{4}\frac{Ub}{g}\left[\frac{g}{bW}\right]^2 \left[\frac{\partial}{\partial t'} \int_{-1}^{1} \gamma'_E \, dx'\right]$$

$$C_{mA4} = \frac{1}{2}\frac{Ub}{g}\left[\frac{g}{bW}\right]^2 \left[\frac{1}{2}\frac{\partial}{\partial t'} \int_{-1}^{1} x'^2 \gamma'_E \, dx' + \frac{\partial}{\partial t'} \int_{-1}^{1} x'\gamma'_E \, dx'\right]$$

$$C_{mA5} = -\frac{1}{2}\frac{Ub}{g}\left[\frac{g}{bW}\right]^2 \left[\int_{-1}^{1} x'\gamma'_E \, dx' + \int_{-1}^{1} \gamma'_E \, dx'\right]$$

$$C_{mA6} = -\frac{1}{2}\left[\frac{g}{bW}\right]^2 \left[\int_{-1}^{1} x' u'_E \gamma'_S \, dx' + \int_{-1}^{1} u'_E \gamma'_S \, dx'\right]$$

$$C_{mU1} = -\frac{1}{2}\left[\frac{g}{bW}\right]^2 \left[\int_{-1}^{1} x' u'_U \gamma'_U \, dx' + \int_{-1}^{1} u'_U \gamma'_U \, dx'\right]$$

$$C_{mU2} = -\frac{3}{4}\frac{Ub}{g}\left[\frac{g}{bW}\right]^2 \left[\frac{\partial}{\partial t'} \int_{-1}^{1} \gamma'_U \, dx'\right]$$

$$C_{mU3} = \frac{1}{2}\frac{Ub}{g}\left[\frac{g}{bW}\right]^2 \left[\frac{1}{2}\frac{\partial}{\partial t'} \int_{-1}^{1} x'^2 \gamma'_U \, dx' + \frac{\partial}{\partial t'} \int_{-1}^{1} x'\gamma'_U \, dx'\right]$$

$$C_{mM1} = -\frac{1}{2}\left[\frac{g}{bW}\right]^2 \left[\int_{-1}^{1} x' u_U' \gamma_S' \, dx' + \int_{-1}^{1} u_U' \gamma_S' \, dx'\right]$$

$$C_{mM2} = -\frac{1}{2}\left[\frac{g}{bW}\right]^2 \left[\int_{-1}^{1} x' u_U' \gamma_E' \, dx' + \int_{-1}^{1} u_U' \gamma_E' \, dx'\right]$$

$$C_{mM3} = -\frac{1}{2}\,\frac{Ub}{g}\left[\frac{g}{bW}\right]^2 \left[\int_{-1}^{1} x' \gamma_U' \, dx' + \int_{-1}^{1} \gamma_U' \, dx'\right]$$

$$C_{mM4} = -\frac{1}{2}\left[\frac{g}{bW}\right]^2 \left[\int_{-1}^{1} x' u_E' \gamma_U' \, dx' + \int_{-1}^{1} u_E' \gamma_U' \, dx'\right]$$

Using the results of the auxiliary solution for the bound vortex distributions and the following expressions for the tangential velocity components

$$u_S' = \frac{Ub}{g}$$

$$u_E' = \tfrac{1}{2}\,[Z_4 \cos Z_6 \sin(Z_1 x') - Z_4 \sin Z_6 \cos(Z_1 x') - Z_3 \cos Z_5 \sin(Z_2 x') + Z_3 \sin Z_5 \cos(Z_2 x')]$$

one gets after carrying out the integrations involved

$$\int_{-1}^{1} \gamma_S' \, dx' = 2\pi \frac{Vb}{g}$$

$$\int_{-1}^{1} x' \gamma_S' \, dx' = -\pi \frac{Vb}{g}$$

$$\int_{-1}^{1} \gamma_E' \, dx' = \pi[Z_2 J_1(Z_2)\cos Z_5 + Z_1 J_1(Z_1)\cos Z_6 - Z_2 J_0(Z_2)\sin Z_5 - Z_1 J_0(Z_1)\sin Z_6]$$

$$\int_{-1}^{1} x' \gamma_E' \, dx' = \pi[J_1(Z_2)\sin Z_5 + J_1(Z_1)\sin Z_6]$$

$$\begin{aligned}\int_{-1}^{1} x'^2 \gamma_E' \, dx' = \frac{\pi}{2}\Big[&\{(Z_2 - \tfrac{4}{Z_2})J_1(Z_2) + 2J_0(Z_2)\}\cos Z_5 + \{(Z_1 - \tfrac{4}{Z_1})J_1(Z_1) + 2J_0(Z_1)\}\cos Z_6 \\ &- Z_2 J_0(Z_2)\sin Z_5 - Z_1 J_0(Z_1)\sin Z_6\Big]\end{aligned}$$

$$\begin{aligned}\int_{-1}^{1} u_E' \gamma_E' \, dx' = \frac{\pi}{4}\Big[&\{Z_2 J_0(Z_2)\sin Z_5 + Z_1 J_0(Z_1)\sin Z_6\}\{Z_4 \cos Z_6 \sin Z_1 - Z_3 \cos Z_5 \sin Z_2\} \\ &+ 3\{Z_2 J_1(Z_2)\cos Z_5 + Z_1 J_1(Z_1)\cos Z_6\}\{J_0(Z_2) Z_3 \sin Z_5 - J_0(Z_1) Z_4 \sin Z_6\} \\ &+ \{Z_2 J_1(Z_2)\cos Z_5 + Z_1 J_1(Z_1)\cos Z_6\}\{Z_4 \sin Z_6 \cos Z_1 - Z_3 \sin Z_5 \cos Z_2\} \\ &+ 2\{Z_2 J_0(Z_2)\sin Z_5 + Z_1 J_0(Z_1)\sin Z_6\}\{J_0(Z_1) Z_4 \sin Z_6 - J_0(Z_2) Z_3 \sin Z_5\}\Big]\end{aligned}$$

$$\begin{aligned}\int_{-1}^{1} x' u_E' \gamma_E' \, dx' = \frac{\pi}{8}\Big[&2\{Z_2 J_0(Z_2)\sin Z_5 + Z_1 J_0(Z_1)\sin Z_6\}\{Z_3 \cos Z_5 \sin Z_2 - Z_4 \cos Z_6 \sin Z_1\} \\ &+ \{Z_2 J_0(Z_2)\sin Z_5 + Z_1 J_0(Z_1)\sin Z_6\}\{J_0(Z_2) Z_3 \sin Z_5 - J_0(Z_1) Z_4 \sin Z_6\} \\ &+ \{Z_2 J_0(Z_2)\sin Z_5 + Z_1 J_0(Z_1)\sin Z_6\}\{Z_3 \sin Z_5 \cos Z_2 - Z_4 \sin Z_6 \cos Z_1\}\Big]\end{aligned}$$

$$\int_{-1}^{1} u_E' \gamma_S' \, dx' = \frac{\pi}{2}\frac{Vb}{g}\,[-Z_4 \cos Z_6 \sin Z_1 - 2J_0(Z_1) Z_4 \sin Z_6 + Z_3 \cos Z_5 \sin Z_2 + 2J_0(Z_2) Z_3 \sin Z_5]$$

$$\begin{aligned}\int_{-1}^{1} x' u_E' \gamma_S' \, dx' = \frac{\pi}{4}\frac{Vb}{g}\Big[&2Z_4 \cos Z_6 \sin Z_1 + Z_4 \sin Z_6 \{J_0(Z_1) + \cos Z_1\} - 2Z_3 \cos Z_5 \sin Z_2 \\ &- Z_3 \sin Z_5 \{J_0(Z_2) + \cos Z_2\}\Big]\end{aligned}$$

Other integrals involved in the expressions for the various components of the response coefficients cannot be evaluated in closed form; they have to be obtained by quadrature. This calls for a special scheme to handle in closed form the integration over a small region around the leading edge which is a singular point. Adopting such a scheme based on the fact that close to the leading edge ($x' = -1$) one can approximate γ_U' by $C/\sqrt{x'+1}$ where C is a constant and making use of the following expression for the unsteady component of the tangential velocity along the foil position

$$u_U' = \sum_{\substack{n = \text{Wake} \\ \text{vortices}}} - \frac{g_n'}{2\pi}\left[\frac{y_n'}{(x_n' - x')^2 + y_n'^2}\right]$$

one gets

$$\int_{-1}^{1} \gamma_U' \, dx' \approx 2\varepsilon\, \gamma_U'(-1+\varepsilon) + \int_{-1+\varepsilon}^{1} \gamma_U' \, dx'$$

$$\int_{-1}^{1} x'\gamma_U' \, dx' \approx -\frac{2}{3}\,\varepsilon(3-\varepsilon)\, \gamma_U'(-1+\varepsilon) + \int_{-1+\varepsilon}^{1} x'\gamma_U' \, dx'$$

$$\int_{-1}^{1} x'^2\gamma_U' \, dx' \approx \frac{2}{15}\,\varepsilon(15-10\varepsilon+3\varepsilon^2)\, \gamma_U'(-1+\varepsilon) + \int_{-1+\varepsilon}^{1} x'^2\gamma_U' \, dx'$$

$$\int_{-1}^{1} u_U'\gamma_U' \, dx' \approx -\frac{1}{2\pi}\sum_n g_n'y_n'\left[\frac{2\varepsilon\, \gamma_U'(-1+\varepsilon)}{(x_n'+1)^2 + y_n'^2} + \int_{-1+\varepsilon}^{1} \frac{\gamma_U'}{(x_n'-x')^2 + y_n'^2}\, dx'\right]$$

$$\int_{-1}^{1} x'u_U'\gamma_U' \, dx' \approx -\frac{1}{2\pi}\sum_n g_n'y_n'\left[\frac{-(2/3)\varepsilon(3-\varepsilon)\, \gamma_U'(-1+\varepsilon)}{(x_n'+1)^2 + y_n'^2} + \int_{-1+\varepsilon}^{1} \frac{x'\gamma_U'}{(x_n'-x')^2 + y_n'^2}\, dx'\right]$$

$$\int_{-1}^{1} u_U'\gamma_S' \, dx' \approx -\frac{Vb}{g}\sum_n \frac{g_n'y_n'}{x_n'^2 + y_n'^2}$$

$$\int_{-1}^{1} x'u_U'\gamma_S' \, dx' \approx \frac{1}{2}\frac{Vb}{g}\sum_n \frac{g_n'y_n'}{x_n'^2 + y_n'^2}$$

$$\int_{-1}^{1} u_U'\gamma_E' \, dx' \approx -\frac{1}{2\pi}\left[\sum_n \frac{g_n'y_n'}{x_n'^2 + y_n'^2}\right]\int_{-1}^{1} \gamma_E' \, dx'$$

$$\int_{-1}^{1} x'u_U'\gamma_E' \, dx' \approx -\frac{1}{2\pi}\left[\sum_n \frac{g_n'y_n'}{x_n'^2 + y_n'^2}\right]\int_{-1}^{1} x'\gamma_E' \, dx'$$

$$\int_{-1}^{1} u_E'\gamma_U' \, dx' \approx \varepsilon\gamma_U'(-1+\varepsilon)\left[\{-Z_4\sin(Z_1+Z_6) + Z_3\sin(Z_2+Z_5)\}\right] + \int_{-1+\varepsilon}^{1} u_E'\gamma_U' \, dx'$$

$$\int_{-1}^{1} x'u_E'\gamma_U' \, dx' \approx -\frac{1}{3}\,\varepsilon(3-\varepsilon)\, \gamma_U'(-1+\varepsilon)\left[\{-Z_4\sin(Z_1+Z_6) + Z_3\sin(Z_2+Z_5)\}\right] + \int_{-1+\varepsilon}^{1} x'u_E'\gamma_U' \, dx'$$

where ε is a small number very much smaller than unity.

LIMITING CASES

In the present theory if the eddy-intensity parameter is made zero it is possible to get the classical result for the case of a flat plate airfoil in a uniform parallel flow. By assuming the length of one side of the rectangular eddy to be very large the present work can simulate stratified flows over thin foils. It can be seen from the assumed inlet flow streamfunction that the freestream velocity components along the foil position for the case of $\theta=0$, $x_0=0$, $y_0=0$ and $V=0$ becomes

$$v(x,0,t) = g\frac{\pi}{k}\sin\frac{\pi}{k}(x-Ut)$$

$$u(x,0,t) = U$$

Thus for this special set of parametric values the upwash is that of a sinusoidal gust and if the quantity $g\pi/k$ is relatively small in comparision with U then the formulation is exactly the same as that of the classical transverse sinusoidal gust problem. However it should be noted that the present theory does not (i) limit itself to a perturbation type of analysis (ii) assume the spatial location and temporal variation of the wake vortex sheet (iii) assume the temporal variation of the bound vortex distribution; these are some of the modeling assumptions in the existing theories for sinusoidal gust problems. Another important difference to be noted is that in the sinusoidal gust theories nothing is said explicitly about the velocity distribution of the freestream flow field except the specification of the upwash whereas in the equivalent limiting case of the present work the flow field continues to be that of a translating rectangular grid of nonpotential eddies. Hence it is not expected that the results of this limiting case will be identical to those of the sinusoidal gust theories even after allowing for the modeling simplifications noted.

RESULTS AND RECOMMENDATION

A computer program has been developed to obtain numerical results for the response coefficients making use of the expressions derived in this theory. The results of this program is compared with the linear theory of Sears (Ref. 6) in Table 1. k is the reduced frequency and v_0 is the gust amplitude. In Table 2 the effect of large amplitudes of sinusoidal gust on the response is given. It has been checked that the program gives satisfactory values for uniform parallel flow response when the eddy strength is made very small. Figure 6 shows the shape of trailing vortex sheets at the instant when the foil has traveled through four sinusoidal gust wavelengths. It is intended to develop a similar analysis for the case of a cascade.

APPENDIX A

Consider a flat plate airfoil of chord length 2b to be located at the origin (Fig. 5) of Z-plane along the real axis in the flow field of a vortex of strength Γ' located at a general point Z_0. The expressions for the bound vortex distribution and the total circulation, subject to nonpenetration and Kutta conditions, may be derived as follows. The conformal mapping function

$$2z = \zeta + \frac{b^2}{\zeta}$$

maps the flat plate into a circle of radius b as shown in Fig. 5. Let ζ_0 be the mapping of Z_0. Consider a vortex of strength $-\Gamma'$ to be placed at the 'image' point $b^2/\bar{\zeta}_0$. The complex potential for the system in the transformed plane is

$$w(\zeta) = i\frac{\Gamma'}{2\pi}\log(\zeta-\zeta_0) - i\frac{\Gamma'}{2\pi}\log(\zeta-\frac{b^2}{\bar{\zeta}_0})$$

Making use of the following expression for the complex conjugate velocity

$$u - iv = \frac{dw}{d\zeta}$$

and the fact that on the circle under consideration $\zeta = b(\cos\theta + i\sin\theta)$ the expressions for the Cartesian components of velocity can be shown to be

$$u\Big|_{|\zeta|=b} = -\frac{\Gamma'}{2\pi}\frac{1}{b}\frac{(\xi_0^2+\eta_0^2-b^2)\sin\theta}{(\xi_0^2+\eta_0^2+b^2-2\xi_0 b\cos\theta-2\eta_0 b\sin\theta)}$$

$$v\Big|_{|\zeta|=b} = \frac{\Gamma'}{2\pi}\frac{1}{b}\frac{(\xi_0^2+\eta_0^2-b^2)\cos\theta}{(\xi_0^2+\eta_0^2+b^2-2\xi_0 b\cos\theta-2\eta_0 b\sin\theta)}$$

Measuring θ anticlockwise from the ξ-axis the polar components of velocity are given by

$$V_r = u\cos\theta + v\sin\theta$$

$$V_\theta = -u\sin\theta + v\cos\theta$$

and hence

$$V_r = 0$$

$$V_\theta = \frac{\Gamma'}{2\pi b}\frac{\xi_0^2+\eta_0^2-b^2}{(\xi_0^2+\eta_0^2+b^2-2\xi_0 b\cos\theta-2\eta_0 b\sin\theta)}$$

This proves that the circle $|\zeta|=b$ is a streamline and hence the satisfaction of the nonpenetration condition.

At the point $\theta=0$ corresponding to the 'trailing edge' Z=b the tangential velocity is

$$V_\theta\Big|_{\theta=0} = \frac{\Gamma'}{2\pi b}\frac{\xi_0^2+\eta_0^2-b^2}{\xi_0^2+\eta_0^2+b^2-2\xi_0 b}$$

Adding a circulation to make the tangential velocity vanish at the trailing edge (Kutta condition) and resubstituting b cosθ=ξ; b sinθ=η

$$V_\theta = \frac{\Gamma'}{2\pi b}\left[\frac{\xi_0^2 + \eta_0^2 - b^2}{\xi_0^2 + \eta_0^2 + b^2 - 2\xi_0\xi - 2\eta_0\eta} - \frac{\xi_0^2 + \eta_0^2 - b^2}{\xi_0^2 + \eta_0^2 + b^2 - 2\xi_0 b}\right]$$

From page 46 of Ref. 5 one gets the relation between the bound vortex distribution and the tangential velocity component, after noting the difference in the transformation functions used, to be

$$\gamma(\xi;\eta) = -\frac{2V_\theta}{\sin\theta} = -\frac{2bV_\theta}{\eta}$$

Hence, using the relation $\eta = \sqrt{b^2-\xi^2}$ for the upper half of the circle, one gets for b=1

$$\gamma(\xi) = \frac{\Gamma'}{\pi}\,\frac{\xi_0^2 + \eta_0^2 - 1}{\sqrt{1 - \xi^2}}\left[\frac{1}{\xi_0^2 + \eta_0^2 + 1 - 2\xi_0} - \frac{1}{\xi_0^2 + \eta_0^2 + 1 - 2\xi_0\xi - 2\eta_0\sqrt{1 - \xi^2}}\right]$$

The kernel function $K(\hat{S},x)$ needed in the time-marching solution takes the form

$$K(\hat{S},x) = \frac{1}{\pi}\,\frac{\xi_0^2 + \eta_0^2 - 1}{\sqrt{1 - \xi^2}}\left[\frac{1}{\xi_0^2 + \eta_0^2 + 1 - 2\xi_0} - \frac{1}{\xi_0^2 + \eta_0^2 + 1 - 2\xi_0\xi - 2\eta_0\sqrt{1 - \xi^2}}\right].$$

where $\hat{S}$ *corresponds* to $x_0 + iy_0$ and by inverting the mapping function one gets

$$\xi = x \qquad R = \sqrt{(x_0^2 - y_0^2 - 1)^2 + 4x_0^2y_0^2}$$

$$\xi_0 = x_0 + \sqrt{R}\cos\frac{\phi}{2} \qquad \phi = \mathrm{Tan}^{-1}\,\frac{2x_0y_0}{x_0^2 - y_0^2 - 1}$$

$$\eta_0 = y_0 + \sqrt{R}\sin\frac{\phi}{2}$$

where Tan^{-1} stands for principal value.

For the particular case of η_0=0 the result for the bound vortex distribution reduces to

$$\gamma(\xi) = \frac{\Gamma'}{\pi}\sqrt{\frac{1 - \xi}{1 + \xi}}\,\frac{2\xi_0}{1 + \xi_0^2 - 2\xi_0\xi}\,\frac{\xi_0 + 1}{\xi_0 - 1}$$

$$= \frac{\Gamma'}{\pi}\sqrt{\frac{1 - \xi}{1 + \xi}}\,\frac{1}{x_0 - \xi}\sqrt{\frac{x_0 + 1}{x_0 - 1}}$$

This agrees with the result reported in Ref. 2.

The total circulation around the flat plate foil for b=1 is given by

$$\Gamma(\xi_0,\eta_0) = \int_{-1}^{1}\gamma(\xi)\,d\xi$$

$$= \Gamma'\left[\frac{\xi_0^2 + \eta_0^2 - 1}{\xi_0^2 + \eta_0^2 + 1 - 2\xi_0} - 1 - \frac{2}{\pi}\,\mathrm{Tan}^{-1}\,\frac{2\eta_0}{\xi_0^2 + \eta_0^2 - 1}\right]$$

Hence the kernel function G_S needed in the time-marching solution becomes

$$G_S = \frac{\xi_0^2 + \eta_0^2 - 1}{\xi_0^2 + \eta_0^2 + 1 - 2\xi_0} - 1 - \frac{2}{\pi}\,\mathrm{Tan}^{-1}\,\frac{2\eta_0}{\xi_0^2 + \eta_0^2 - 1}$$

For the particular case of η_0=0 one gets from above result for total circulation

$$\Gamma(\xi_0) = \Gamma'\left[\frac{\xi_0 + 1}{\xi_0 - 1} - 1\right]$$

which also entirely agrees with the result of Ref. 2.

REFERENCES

1. Söhngen, H. "Die Lösungen der Integralgleichung und deren Anwendung in der Tragflügeltheorie" Mathematische Zeitschrift, Band 45, 1939, pp. 245-264.
2. Von Karman, T. and Sears, W.R. "Airfoil Theory for Non-Uniform Motion" Journal of Aeronautical Sciences, Vol. 5, No. 10, 1938, pp. 379-390.
3. Giesing, J.P. "Unsteady Two-Dimensional Potential Flow with Lift" Douglas Aircraft Company, Inc., Report No. LB(32144), 1965.
4. Gradshteyn, I.S. and Ryzhik, I.M. Table of Integrals, Series, and Products Seventh Printing, New York, Academic Press, 1973.
5. Durand, W.F. (Editor) Aerodynamic Theory Vol. II Berlin, Julius Springer, 1935.
6. Sears, W.R. "Some Aspects of Non-Stationary Airfoil Theory and its Practical Application" J. Aero. Scs., Vol. 8, No. 3, 1941, pp. 104-108.

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Table I
Sears Function S and Fundamental of Present Result P for $v_0/U = .01$

k	$\lvert S(k) \rvert$	$\angle S(k)$	$\lvert P(k) \rvert$	$\angle P(k)$
0.02	0.96583	-4.439	0.96498	-1.906
0.05	0.91431	-8.060	0.91867	-4.289
0.10	0.83735	-11.258	0.85440	-7.368
0.50	0.52647	-4.797	0.56922	-19.385
1.00	0.38957	18.861	0.29153	-32.947
5.00	0.17820	-117.097	0.69366	2.331

Table II
Effect of Gust Amplitude on Response

$$\frac{C_1}{(v_0/U)} = A_0 + A_1\cos\omega t + A_2\cos 2\omega t + A_3\cos 3\omega t + A_4\cos 4\omega t + A_5\cos 5\omega t + A_6\cos 6\omega t + B_1\sin\omega t + B_2\sin 2\omega t + B_3\sin 3\omega t + B_4\sin 4\omega t + B_5\sin 5\omega t$$

v_0/U	0.01	0.05	0.10	0.50	1.00
A_0	-0.00093	0.00318	0.00679	0.00731	0.00383
A_1	0.68843	0.68861	0.68932	0.70759	0.73617
A_2	-0.00090	-0.00400	-0.00783	-0.03637	-0.06334
A_3	-0.00013	-0.00041	-0.00116	-0.02216	-0.07056
A_4	-0.00012	-0.00017	-0.00019	0.00083	0.00613
A_5	-0.00011	-0.00019	-0.00030	-0.00734	-0.05103
A_6	-0.00011	-0.00020	-0.00036	-0.01065	-0.07169
B_1	-5.32406	-5.32486	-5.32668	-5.35415	-5.38706
B_2	-0.00063	-0.00254	-0.00488	-0.02338	-0.04263
B_3	-0.00008	0.00016	0.00107	0.03087	0.12100
B_4	-0.00005	-0.00013	-0.00029	-0.00441	-0.01860
B_5	-0.00001	-0.00009	-0.00026	-0.00534	-0.02160

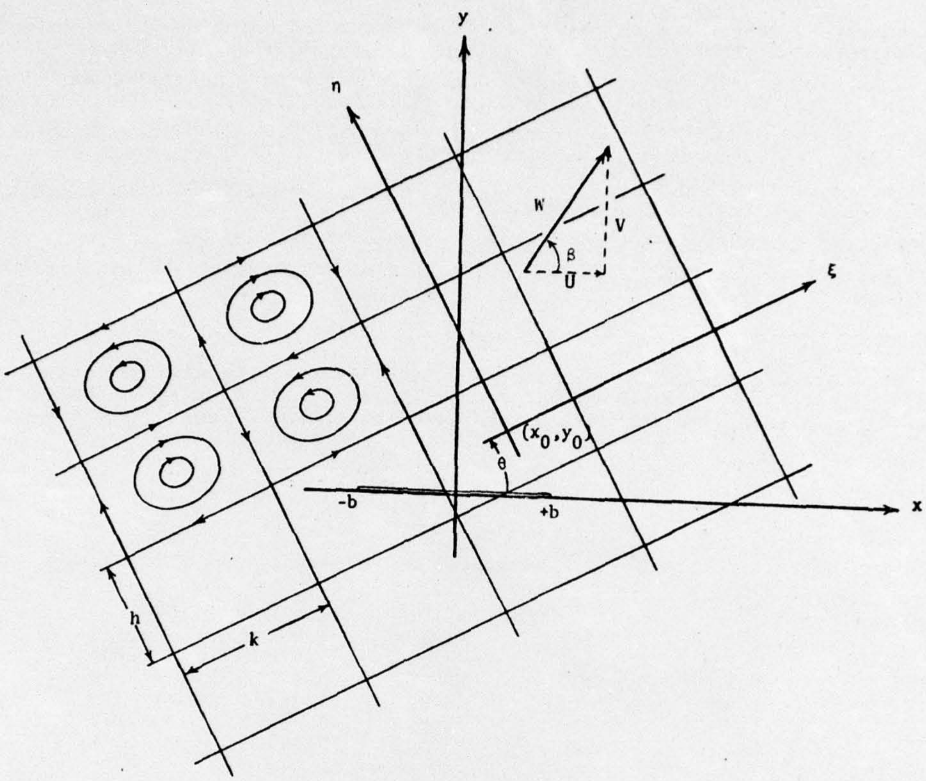


Fig.1

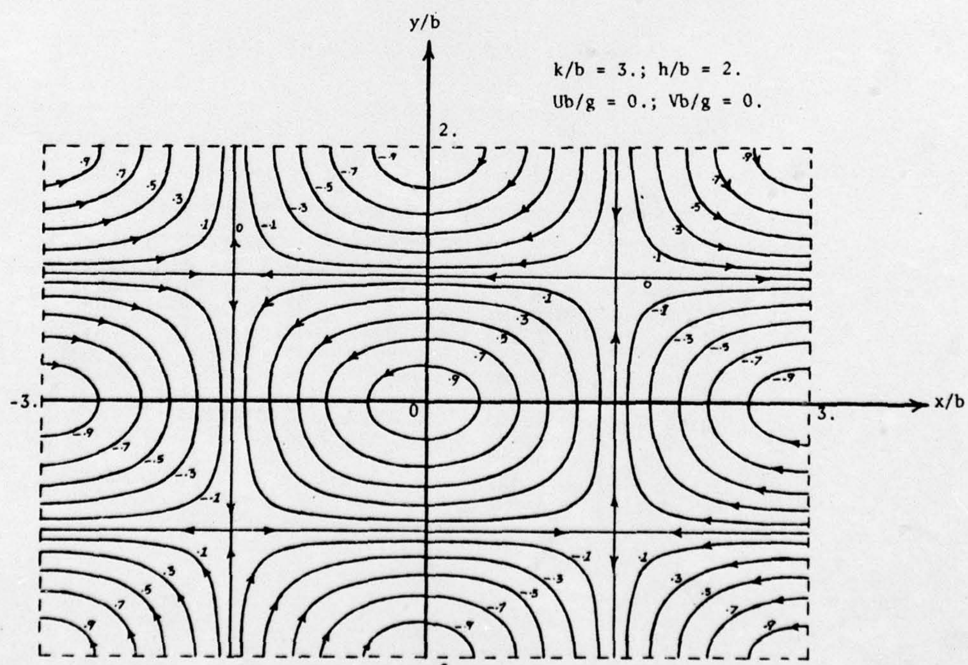
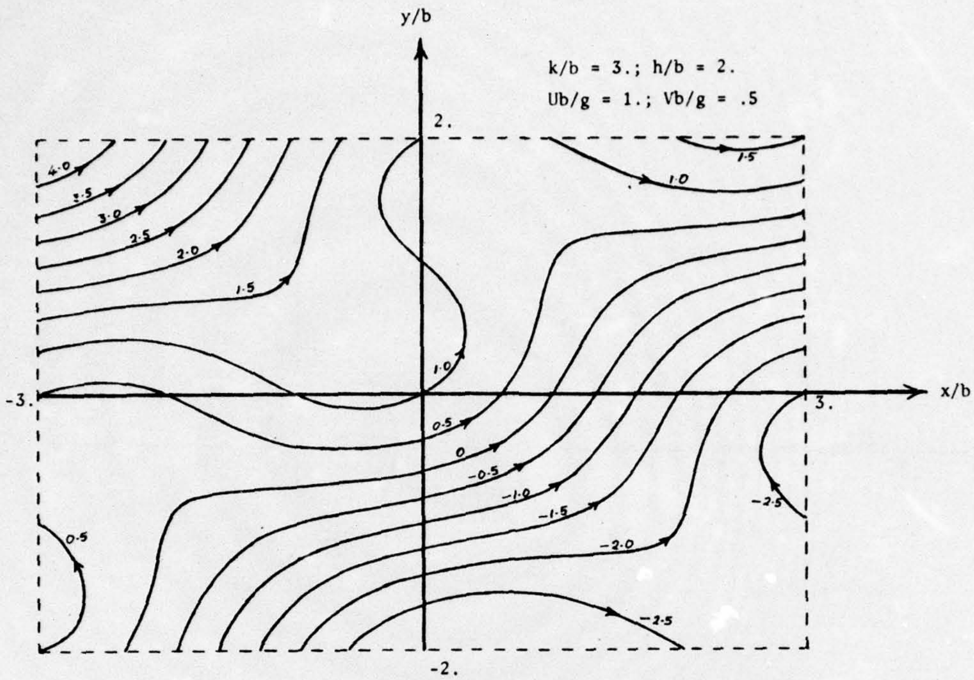


Fig.2


y/b
k/b = 3.; h/b = 2.
Ub/g = 1.; Vb/g = .5
2.
-3.
3.
x/b
-2.
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0
-0.5
-1.0
-1.5
-2.0
-2.5
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1.0
0.5
-2.5
-2.5

Fig.3

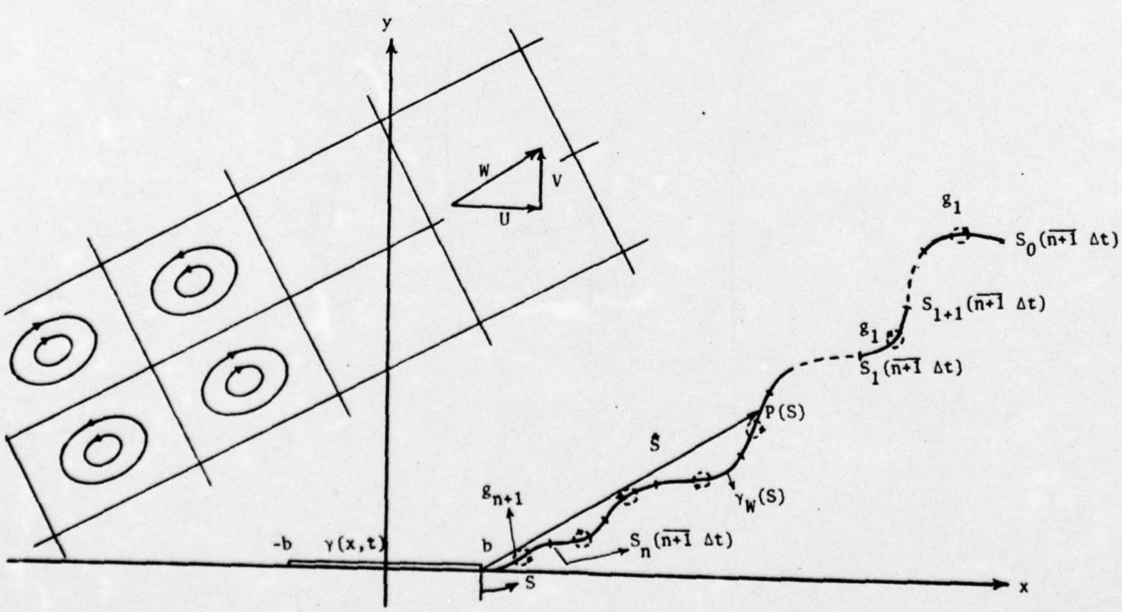
y
W
V
U
x
-b
γ(x,t)
b
S
g_{n+1}
$S_n(\overline{n+1}\ \Delta t)$
$\hat{S}$
$\gamma_W(S)$
P(S)
g_1
$S_1(\overline{n+1}\ \Delta t)$
$S_{1+1}(\overline{n+1}\ \Delta t)$
g_1
$S_0(\overline{n+1}\ \Delta t)$

Fig.4

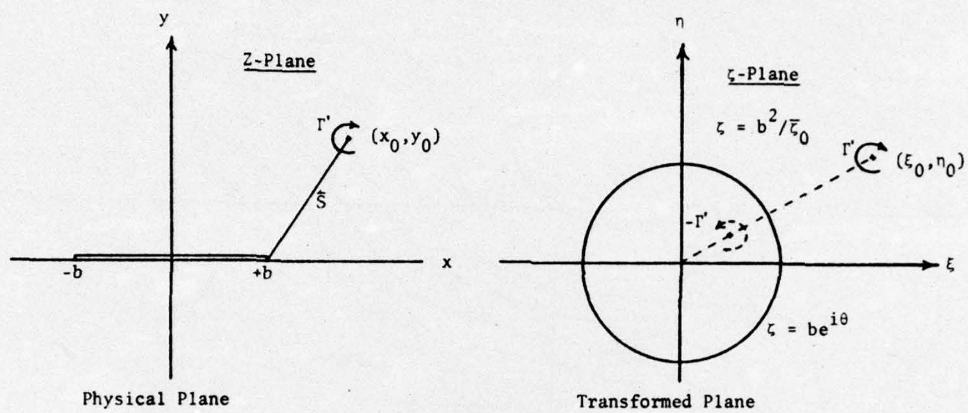
y
Z-Plane
Γ'
(x_0, y_0)
$\bar{\bar{S}}$
-b
+b
x
Physical Plane
η
ζ-Plane
$\zeta = b^2/\bar{\zeta}_0$
Γ'
(ξ_0, η_0)
-Γ'
ξ
$\zeta = be^{i\theta}$
Transformed Plane

Fig.5

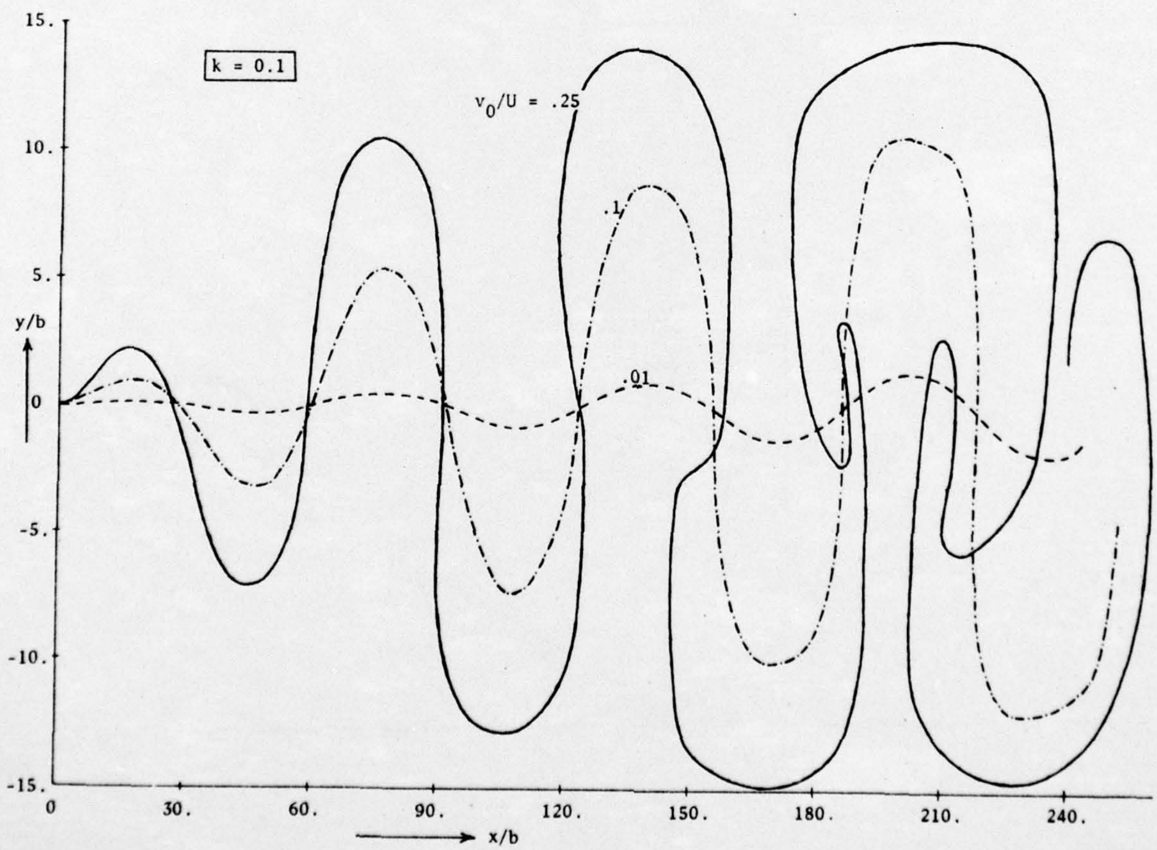
k = 0.1
v_0/U = .25
.1
.01
y/b
x/b
15.
10.
5.
0
-5.
-10.
-15.
0
30.
60.
90.
120.
150.
180.
210.
240.

Fig.6

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The unsteady response of a flat plate airfoil to large non-potential flow disturbances is evaluated in the form of a translating rectangular grid of eddy-array. The singularity distribution principle in combination with the time-marching technique was used as the method of analysis.