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SPECULAR SCATTERING OF ACOUSTIC WAVES FROM A ROUGH SURFACE IN THE FRAUNHOFER AND FRESNEL APPROXIMATIONS

APPROVED BY SUPERVISORY COMMITTEE:

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Daniel Rayburn Melton

Austin, Texas October, 1968

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ABSTRACT

This dissertation presents the results obtained in an experimental and theoretical study of the scattering of underwater acoustic waves from a rough surface. Four rough surfaces 32 in. by 32 in. in size were made of a pressure-release material. The reliefs of these surfaces were approximately Gaussian and were characterized by an exponential covariance function. The amplitude and relative phase of a pulsed wave scattered in the specular direction were measured at a frequency of 95.8 kHz for grazing angles from 6 deg to 80 deg. Both the directional source with a beamwidth of 9 deg measured at the half-power points and the omnidirectional receiver were placed at a alant-line distance of 60 in. from the scattering surface for all measurements. The measurements were used to calculate the amplitude fluctuations, the phase fluctuations, and the scattering coefficient. Theoretical formulas were developed for the fluctuations and the scattering coefficient in the Fraunhofer and the Fresnel approximations for the opecular direction.

The theoretical approach of Gulin was used successfully in the Fresnel approximation to predict the observed amplitude fluctuations for values of the Rayleigh parameter less than 0.6. The experimental amplitude fluctuations reached a maximum at a Rayleigh parameter of 2.5 and for values between 3.5 and 7.4 the fluctuations disclosed a spread of 33 - 65 percent (relative to the average amplitude).

Gulin's theory in both the Fraunhofer and Fresnel approximations agreed with the experimental phase fluctuations to within 6 percent (relative to a radian) for values of the Rayleigh parameter less than 0.6. The experimental phase fluctuations varied from 12 to 250 percent (relative to a radian) over a range of values of Rayleigh parameters from 0.2 to 7.4.

The theoretical development in the Fraunhofer approximation had to be extended to the Fresnel approximation in order to obtain values for the scattering co-fficient that were in good agreement with the measured

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values in the specular direction. As the value of the Rayleigh parameter increased the difference between the theoretical values and the measured values of the scattering coefficient increased in both the Fraunhofer and Fresnel approximations. For values of the Rayleigh parameter near 1.8 the Fresnel theory predicted values within 2 dB of the experimental values of the scattering coefficient. When the Rayleigh parameter for the surface was 3.6 (7.2), the theoretical value for the scattering coefficient was 5 dB (10 dB) smaller than the experimental value. The need for the Fresnel theory was evident in that for values of the Rayleigh parameter near 7.2 the Fraunhofer theory predicted values that were 22 dB smaller than the experimental scattering coefficient.

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CHAPTER I INTRODUCTION

The problem of the scattering of waves from a rough surface has interested physicists ever since Rayleigh (1896) first predicted the scattered field from a sinusoidal surface. His solution was limited to normal incidence and was applicable only when the wavelengths of the surface and the incident radiation were much larger than the surface amplitude. It was 67 years later before an exact solution for the problem of the reflection of a plane sound wave from a pressure-release corrugated surface was obtained by Uretsky (1963). He made no simplifying assumptions concerning the frequency of the incident radiation or the geometrical parameters of the sinusoidal surface. Uretsky (1965) has published an expanded paper in which he gives more details regarding his solution of this problem. Barnard, Horton, Miller, and Spitznogle (1966) have given experimental results which show that Uretsky's theory provides a satisfactory prediction of scattered sound field from a pressure-release sinusoidal surface when the amplitude of the sinusoid is comparable to the wavelength of the incident radiation.

The number of papers published in the area of underwater acoustics in the last 15 years has shown the growing interest in understanding the scattering of underwater sound by the sea bottom and the sea surface. The topography of these surfaces is neither periodic nor deterministic; in practice they can only be described by their statistical properties. The first general theory which predicted the field scattered by a perfectly reflecting statistically rough surface, the irregularities being large compared with the wavelength, was presented by Isakovich (1952). The problem was solved in the Kirchhoff approximation; that is, it was assumed that the field at each point of the surface could be represented as the sum of the incident wave and a wave reflected from the plane tangent to the surface at the given point. In addition, the Fraunhofer (farfield) approximation was used throughout the paper. The rough surface that

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was studied by Isakovich was isotropic and characterized by a Gaussian covariance function.

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Eckart (1953) presented a theory which predicted the field scattered by a pressure release or rigid surface that was randomly rough and isotropic. His theory for a directional source leads to useful expressions for two limiting cases:

(1) when the wavelength of the incident radiation is much greater than the largest values of the surface relief, and

(2) when the wavelength is much less than the surface relief. The Kirchhoff and the Fraunhofer approximations were employed throughout the paper. Eckart did not consider any specific covariance functions.

Apparently, the first theory in the Fresnel approximation (as opposed to the Fraunhofer approximation) was presented by Feinstein (1954). The method of physical optics (Kirchhoff approximation) was used to predict the field scattered from a randomly rough surface characterized by a Gaussian covariance function. Also, he employed the exponential covariance function to describe the field scattered from a one-dimensional rough surface. All formulas were restricted to the specular direction.

Clay (1960) extended the theory of Eckart (1953) to include an omnidirectional source. A Gaussian covariance function for the rough surface was assumed for the calculations. Considering only the long wavelength case, Clay compared his theoretical results for the amplitude fluctuations to the fluctuations of the sound reflected from the sea surface measured by Brown and Ricard (1960). The numerical calculations of scattered sound had the same dependence on the source-receiver separation as the experimental data.

Formulas for the amplitude and phase fluctuations of a sound wave reflected from a statistically uneven surface were derived by Gulin (1952). His theoretical approach was similar to that of Eckart (1953) except that all calculations were made in the Fresnel approximation and were confined to the specular direction. Only the long wavelength case in the Kirchhoff approximation was considered. The details were given for the derivation of the fluctuations of a spherical wave scattered from a rough surface characterized by a onedimension ... quasi-harmonic Gaussian covariance function. The details in the development using a two-dimensional Gaussian covariance function were omitted. The theoretical results for the amplitude fluctuations obtained with the one-dimensional quasi-hermonic Gaussian covariance function were compared with the results of an experimental investigation made by Gulin and Malyshev (1962) of the amplitude fluctuations of sound signals reflected from the undulating surface of the sea. The experimental amplitude fluctuations increased linearly with the Rayleigh parameter up to values of the latter equal to approximately 0.7 and corresponded to the theoretical law over this range of Rayleigh parameters. For Rayleigh parameters between 0.7 and 3.5 there was a well defined alternation of experimental amplitude coefficient of variation between maximum and minimum values. This saturation effect at large values of the Rayleigh parameter could not possibly be explained by means of the theoretical analysis, as the region of applicability of the analytical formulas was limited by the condition that the Rayleigh parameter must be less than one. It was stated by these authors that the conclusion of the maxima and minima in the saturation region of the coefficient of variation should be considered as a tentative result and in need of further verification. No experimental data were given for the phase fluctuations.

The problem of the scattering of waves from a rough surface has been the subject of at least one monograph written by Beckmann and Spizzichino (1963). This book provides an excellent survey of the literature. Beckmann's approach to the problem is more general than that of Isakovich (1952) and that of Eckart (1953) in that it is not restricted to the special cases of a short wavelength or a long wavelength assumptior. Beckmann employed both the Kirchhoff and the Fraunhofer approximation throughout the theoretical analysis. A rough surface characterized by a Gaussian ervariance function was investigated. Later, Beckmann (1965) discussed the scattering of a wave from a rough surface characterized by an exponential covariance function. Medwin (1966) modified Beckmann's theory to account for the error inherent in the common assumption of uniform insonification over a fixed area.

Horton and Muir (1967) have employed the theory of Eckart (1953) to study the scattering from isotropically rough surfaces whose covariance functions are exponential, Gaussian, and sinusoidal. The effect of the radiation pattern of the source upon the scattered intensity was discussed by inserting an analytical expression for the transducer pattern. In a companion paper Horton, Mitchell, and Barnard (1967) produced surprising experimental evidence of the validity of a modified form of Eckart's theory in the short wavelength case. These experimental fudies were performed on a model surface whose covariance . unction was exponential. The theory was developed in the Kirchhoff and the Fraunhol'er approximations.

Of the references quoted here only two, Feinstein (1954) and Gulin (1962), have studied the problem of scattering from a rough surface in the F esnel approximation. Neither have compared their results with the correspording solutions in the Fraunhofer approximation. One might question whether or not the extension of the mathematical analysis of the problem from the Fraunhofer to the Freshel approximation would give better agreement between the theoreti-Cal and experimental realts. No one to the author's knowledge has presented a solution in the Fresnel approximation to the problem of scattering from a rendemly rough surface whose statistical properties are given by an exponential covariance function. One is further motivated to investigate the scattering of acoustic waves from this type of surface since there are available model surfaces that are characterized by an exponential covariance function. These model surfaces whose statistical properties were analyzed by Horton, Horinan, and Hemkins (1952) give the excellent means of experimental verification of the theoretical results.

The plan of study will be the following. In Chapter II -general expressions are derived for the amplitude and phase fluctuations of an acoustic wave scattered in the spacular direction from a randomly rough surface. The theoretical approach is that of Gulin (1962). In the same chapter general expressions are obtained for the scattering coefficient derived in a manner similar to that presented by Beckmann (1963). Throughout Chapter II there is a parallel presentation of the integral expressions obtained in the Fraunhofer and Fresnel approximation. The expressions for the amplitude and phase fluctuations will be valid only for Rayleigh parameters less than one whereas the formulas for the scattering coefficient should be correct for any value of the Rayleigh parameter. The calculations in this initial study shall be confined to the specular direction where the mathematical analysis is less cumbersome.

In Chapter III the problem is specialized to the case where the statistical properties of the rough surface are described by an exponential covariance function. The integral expressions occurring in Chapter II are solved giving formulas for the amplitude fluctuations, phase fluctuations, and scattering coefficient that can be used to describe the experimental results.

The statistical properties of the model surfaces used in the experimental work are discussed in Chapter IV. This section presents a list of the experimental equipment and gives an explanation of the experimental procedures used to measure the fluctuations and the scattered intensity.

The experimental data are compared with the theoretical formulas in Chapter V, a general discussion is presented in Chapter VI, and the conclusions of this study are stated in Chapter VII.

The theory presented in Chapters II and III and in Appendixes A and C can best be followed by referring to the block diagrams in Figs. 1 and 2.



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FIGURE 1 PRIMARY CALCULATIONS



CHAPTER II THEOREFICAL APPROACH

General expressions are derived for the amplitude and phase fluctuations, and for the scattering coefficient in the specular direction for an acoustic wave scattered from a randomly rough surface. The theoretical approach that is used is the method presented by E. P. Gulin (1962). Gulin's notation will be maintained in this paper.

A. The Pressure Field

The coordinates of the source $(0,0,z_1)$ and the receiver $(L,0,z_2)$ are illustrated in Fig. 5. ψ_1 is the grazing angle. ψ_2 is the receiving angle measured with respect to the x-axis. The axis of the directional source is directed toward the point on the surface labeled 0'. The x-y plane is oriented so that the average height of the interface above this plane is zero. R_{10} is the distance from the source to the point 0' which is located on the x-y plane. R_1 is the distance from the source to any arbitrary point on the randomly rough surface. R_1' is the distance from the source to a point on the x-y plane directly above or below the arbitrary point on the scattering surface. The distances R_{20} , R_2 , and R_2' to the receiver are defined in the same way.

The total pressure field p of a sound wave reflected from an uneven surface can be written in terms of the Helmholtz integral, Baker and Copson (1959),

$$p = \frac{1}{L_{R}} \iint \left[-p \frac{\partial}{\partial n} \left(\frac{i k R_{2}}{R_{2}} \right) + \frac{e R_{2}}{R_{2}} \frac{\partial p}{\partial n} \right] ds , \qquad (1)$$

(2)

where <u>n</u> is the outward normal to the surface (in a direction away from source and receiver). The wave incident on the surface is assured to be a sperical wave given by

$$P'_0 = \frac{\frac{1kR_1}{R_1}}{R_1}$$

8



The field at the surface S is specified after the example of Eckart (1953) for the special case of a "pressure release" surface. The boundary conditions at the surface S are

$$p'_{0} + p = 0$$
, and (3).

10

(4)

(6)

$$\frac{Qu}{Qb_1} = \frac{Qu}{Qb_2} \quad .$$

The application of these boundary conditions corresponds to the Kirchhoff approximation in the problem of sound scattering by rough surfaces. The applicability of this approximation has been formulated by Brekhovskikh (1952) and Luakovich (1952), and has been discussed by Beckmann (1953). Substituting Eqs. (2), (3), and (4) into Eq. (1) one obtains

$$\mathbf{p} = \frac{\mathbf{i}\mathbf{k}}{4\pi} \int \int \left[\left(\frac{\partial R_1}{\partial \mathbf{n}} + \frac{\partial R_2}{\partial \mathbf{n}} \right) + \left(\frac{\mathbf{i}\lambda}{2\pi R_1} \frac{\partial R_1}{\partial \mathbf{n}} + \frac{\mathbf{i}\lambda}{2\pi R_2} \frac{\partial R_2}{\partial \mathbf{n}} \right] \frac{\mathbf{e}^{\mathbf{i}\mathbf{k}(R_1 + R_2)}}{\mathbf{R}_1 \mathbf{R}_2} \mathbf{as} , \quad (\mathcal{I}) = \frac{\mathbf{e}^{\mathbf{i}\mathbf{k}(R_1 + R_2)}}{\mathbf{R}_1 \mathbf{R}_2} \mathbf{as} ,$$

where A is the wavelength of the incident radiation. If it is assumed

 $R_1 \gg \lambda$, and $R_2 \gg \lambda$

the second group of terms can be neglected, which reduces Eq. (5) to

$$p = \frac{1k}{4\pi} \iint \frac{1}{R_1 R_2} \left(\frac{\partial R_1}{\partial n} + \frac{\partial R_2}{\partial n} \right) e^{\frac{1k(R_1 + R_2)}{4G}} dG .$$
(7)

In the case of irregularities of sufficiently shall slope differentiation along the normal can be approximately replaced by differentiation along the negative z-axis, and integration over the rough surface by integration over the x-y plane. Thus, Eq. (7) becomes

$$p = -\frac{ik}{4\pi} \int \int \frac{1}{R_1 R_2} \left(\frac{\partial R_1}{\partial z} + \frac{\partial R_2}{\partial z} \right) e^{ik(R_1 + R_2)} dxdy \quad . \tag{8}$$

Introducing the new terms z'_1 , z'_2 , and F which are depicted in Fig. 4, one can show that

$$\frac{\partial R_1}{\partial z} = -\frac{z_1'}{R_1} , \quad \frac{\partial R_2}{\partial z} = -\frac{z_2'}{R_2}$$

Eq. (8) becomes

$$p = \frac{ik}{4\pi} \int \int \frac{1}{R_1 R_2} \left(\frac{z_1'}{R_1} + \frac{z_2'}{R_2} \right) e^{ik(R_1 + R_2)} dxdy \quad . \tag{9}$$

<u>F</u> is the deviation of the surface relief from the average height. If it is assumed that |F| is much less than the minimum of z_1 and z_2 , then

$$R_{1} = \left[x^{2} + y^{2} + (z_{1} - F)^{2}\right]^{\frac{1}{2}}$$

can be written as

$$R_{1} = \left[x^{2} + y^{2} + z_{1}^{2}\right]^{1/2} \left(1 - \frac{R_{1}}{2} + y^{2} + z_{1}^{2}\right)$$

"Seckman (1965) inserts the "local" scattering geometry of the normal derivative at the surface in his theoretical approach. This method introduces the term

$$\frac{1 + \sin \psi_1 \sin \psi_2 - \cos \psi_2 \cos \psi_3}{\sin \psi_1 (\sin \psi_1 + \sin \psi_3)}$$

which reduces to one in the specular direction $(v_1 = v_p)$.

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Defining (see Fig. 3)

$$R'_{1} = (x^{2} + y^{2} + z_{1}^{2})^{1/2}$$
,

13

(10)

(11)

(13)

one can write R1 as

$$R_{1} \approx R_{1}' - \frac{z_{1}'}{R_{1}'}$$

Similarly

$$R_2 = \left[(L - x)^2 + y^2 + (z_2 - F)^2 \right]^{1/2}$$

and R2 can be written as

$$R_{2} = \left[(L-x)^{2} + y^{2} + z_{2}^{2} \right]^{1/2} \left[1 - \frac{Fz_{2}}{(L-x)^{2} + y^{2} + z_{2}^{2}} \right]$$

Defining

$$R_2' = \left[(L - x)^2 + y^2 + z_2^2 \right]^{1/2}$$
, (12)

one can write Rg 48

$$R_2 = R_2' - \frac{R_2'}{R_2}$$

The approximations given by Eqs. (11) and (13) reduce Eq. (9) to

$$\mathbf{F} = \frac{\mathbf{i}\mathbf{k}}{\mathbf{k}} \int \int \frac{\mathbf{e} \left(\mathbf{R}_{1}^{*} + \mathbf{R}_{2}^{*}\right)}{\mathbf{R}_{1}^{*} \mathbf{R}_{2}^{*}} \left[\left(\mathbf{R}_{1}^{*} + \mathbf{R}_{2}^{*}\right) - \mathbf{F} \left(\mathbf{R}_{1}^{*} + \mathbf{R}_{2}^{*}\right) - \mathbf{F} \left(\mathbf{R}_{1}^{*} + \mathbf{R}_{2}^{*}\right) \right]$$
$$-\mathbf{i}\mathbf{k}\mathbf{F} \left(\mathbf{R}_{1}^{*} + \mathbf{R}_{2}^{*}\right)$$

Since $F \ll z_1$ and $F \ll z_2$, the second term in the square brackets can be neglected. The pressure becomes

$$p = \frac{ik}{4\pi} \int \int \frac{e^{ik(R_1'+R_2')}}{R_1'R_2'} \left(\frac{z_1}{R_1'} + \frac{z_2}{R_2'}\right) e^{-ikF\left(\frac{z_1}{R_1'} + \frac{z_2}{R_2'}\right)} e^{ikF\left(\frac{z_1}{R_1'} + \frac{z_2}{R_2'}\right)} dxdy \quad . \quad (14)$$

14

(15)

If the acoustic wavelength, λ , is greater than the surface relief, F(x,y), or if the grazing angle is small, so that the following condition holds

$$\left| kF\left(\frac{z_1}{\overline{R_1^{T}}} + \frac{z_2}{\overline{R_2^{T}}}\right) \right| < 1$$

then

$$e^{-ikF\left(\frac{z_1}{R_2^{i}}+\frac{z_2}{R_2^{i}}\right)} \approx 1 - ikF\left(\frac{z_1}{R_1^{i}}+\frac{z_2}{R_1^{i}}\right) \qquad (16)$$

This approximation separates Eq. (14) into a nonfluctuating component

$$P_{0} = \frac{1k}{4\pi} \iint \frac{1}{R_{1}^{2}R_{2}^{2}} \left(\frac{2}{R_{1}^{2}} + \frac{2}{R_{2}^{2}} \right) e^{\frac{1}{2}k(R_{1}^{2} + R_{2}^{2})} e^{\frac{1}{2}k(R_{$$

and a scattered, fluctuating component

$$P_{1} = \frac{k^{2}}{L_{x}} \iint \frac{1}{K_{1}^{2}R_{2}^{2}} \left(\frac{z_{1}}{R_{1}^{2}} + \frac{z_{2}}{R_{2}^{2}}\right)^{2} e^{ik(R_{1}^{\prime} + R_{2}^{\prime})} F(x,y) dxdy \quad . \quad (18)$$

The pressure p₀ reflected from a plane "pressure release" surface is evaluated for the specular direction in Appendix C.

B. Amplitude and Phase Fluctuations in the Fraunhofer Approximation

15

(19)

The transition from the expressions for the sound field to expressions for the amplitude and phase fluctuations can be carried out by using a method utilized by Chernov (1960). The total pressure given by Eqs. (17) and (18)

$$p = p_0 + p_1$$

can be written as

$$Ae^{i\Phi} = A_e^{i\Phi} + A_e^{i\Phi}$$
, or

$$\frac{A}{A_{o}} e^{i(\Phi - \Phi_{o})} = 1 + \frac{A_{l}}{A_{o}} e^{i(\Phi_{l} - \Phi_{o})}$$

<< A_

A₇

Assuming that

one can set

$$A = A_0 + \delta A$$
, and

These approximations, with the use of the logarithm and $\ln (1 + \epsilon) \approx \epsilon$ when $|\epsilon| \ll 1$, reduce Eq. (19) to

$$\frac{\delta A}{A_{o}} + i \delta \Phi \approx \frac{A_{1}}{A_{o}} e^{i(\Phi_{1} - \Phi_{o})}$$

Since the right side of this equality is Eq. (18) divided by $A_0 e^{-\circ}$, one has for the variation in the amplitude

$$\frac{\delta A}{A_{o}} = \frac{k^{2}}{4\pi A_{o}} \int \int \frac{1}{R_{1}^{\prime}R_{2}^{\prime}} \left(\frac{\frac{1}{R_{1}^{\prime}} + \frac{z}{R_{2}^{\prime}}}{R_{1}^{\prime}}\right)^{2} \cos\left[k(R_{1}^{\prime} + R_{2}^{\prime}) - \Phi_{0}\right] F(x,y) dxdy , \quad (20)$$

and for the variation in phase

$$\delta \Phi = \frac{k^2}{4\pi A_0} \int \int \frac{1}{R_1' R_2'} \left(\frac{z_1}{R_1'} + \frac{z_2}{R_2'} \right)^2 \sin \left[k(R_1' + R_2') - \Phi_0 \right] F(x,y) dxdy \quad . (21)$$

In the Fraunhofer approximation (Appendix A, Eq. (A-20) for the specular direction

$$R_{1}' + R_{2}' \approx R_{10} + R_{20}$$

 s_{\odot} that the variation in amplitude is

$$\frac{\delta A}{A_{o}} = \frac{k^{2} \sin^{2} \psi \cos \left[k(R_{10} + R_{20}) - \Phi_{o}\right]}{\pi A_{o}^{R} 10^{R} 20} \int \int F(x_{1}, y) dx_{1} dy \quad . \quad (22)$$

where (Appendix A, Eq. (A-1)

$$\mathbf{x}_{1} = \mathbf{x} - \frac{\mathbf{z}_{1}}{\mathbf{z}_{1} + \mathbf{z}_{2}}$$

The variation in phase is

$$\mathcal{E} = \frac{k^{2} \sin^{2} \psi \sin\left[k(R_{10} + R_{20}) - \Phi_{0}\right]}{\pi A_{0}R_{10}R_{20}} \int \int F(x_{1}, y) dx_{1} dy \quad . \tag{23}$$

A_o and Φ_o are given by Eqs. (C-3) and (C-4) in Appendix C. These values reduce Eqs. (22) and (23) to

$$\frac{\delta A}{\delta} = 0$$
, and (24)

$$\delta \phi = \frac{k \sin \psi}{l(\Delta_1 + \Delta_2)} \int_{-\Delta_1} \int_{-L} F(x_1, y) dx_1 dy ,$$

16

(25)

where Δ_1 , Δ_2 , and l are the dimensions of the active scattering area defined in Appendix B. $\delta\Phi$ is zero only if the integration extends over a significant area of the surface. If the active scattering region is small compared to the correlation distance, $\delta\Phi$ is likely to differ from zero for each single measurement.

Since $F(x_1,y)$ is a random variable, the quantities of interest are the ensemble averages. It is assumed that the surface relief is distributed so that

$$\langle \delta \Phi \rangle = 0$$
 .

Since the variation in amplitude is identically zero, one can write

$$\left\langle \frac{\delta A}{A_{o}} \right\rangle = 0 \quad , \qquad (27)$$

without reference to the distribution of the surface relief.

The variance of a random variable v is defined as the average square of the deviation from the mean value

$$D(\mathbf{v}) = \left\langle \left(\mathbf{v} - \langle \mathbf{v} \rangle\right)^2 \right\rangle = \left\langle \mathbf{v}^2 \right\rangle - \left\langle \mathbf{v}^2 \right\rangle^2 \qquad (28)$$

The square root of the variance,

$$\sigma = \sqrt{D(v)} , \qquad (29)$$

is called the stendard deviation. Thus, utilizing Eq. (27), one can write for the relative standard deviation of the amplitude the expression

$$A = \frac{1}{A_0} \sqrt{\langle (\delta A)^2 \rangle} , \qquad (50)$$

which hereafter will be called the amplitude fluctuations. In the same way

$$\sigma_{\phi} = \sqrt{\langle (\delta \phi)^2 \rangle}, \qquad (3)$$

will represent the phase fluctuations.

-17

(26)

Eq. (24) indicates that the amplitude fluctuations in the specular direction are zero in the Fraunhofer field zone. To determine the phase fluctuations in the specular direction in the Fraunhofer approximation Eq. (25) is squared and averaged to give

$$\left\langle (\Xi^{1})^{2} \right\rangle = \frac{k^{2} \sin^{2} \psi}{\ell^{2} (\Delta_{1} + \Delta_{2})^{2}}$$

$$\times \int_{-\Delta_{1}}^{\Delta_{2}} \int_{-\ell}^{\ell} \int_{-\ell}^{\Delta_{2}} \int_{-\ell}^{\ell} \left\langle F(x_{1}, y)F(x_{1}^{*}, y^{*}) \right\rangle dx_{1} dy dx_{1}^{*} dy^{*} ,$$

where $\langle F(x_1,y)F(x_1',y') \rangle$ is defined as the spatial covariance function of the surface displacements. A complete solution of the problem will be delayed until Chapter III where by Eq. (56) an exponential convariance function is specified.

C. Amplitude and Phase Fluctuations in the Fresnel Approximation

The Freenel approximation for the expansion of $R'_1 + R'_2$ in the specular direction is given by Eq. (A-19) in Appendix A,

$$R'_1 + R'_2 = R_{10} + R_{20} + \frac{1}{R} (x_1^2 \sin^2 \psi + y^2)$$
, (55)

where

$$R = \frac{2R_{10}R_{20}}{R_{10} + R_{20}}$$
(34)

Substitution of Eq. (33) into Eq. (20) gives for the variation in amplitude

$$\frac{4A}{A_{o}} = \frac{k^{2} \sin^{2} \psi}{\pi R_{10} R_{20} A_{o}} \int \int \cos \left[\frac{k x_{1}^{2} \sin^{2} \psi}{R} + \frac{k y^{2}}{R} - \phi_{o}^{\prime} \right] F(x_{1}, y) dx_{1} dy \quad , \quad (35)$$

where

$$\phi'_0 = \phi_0 - k(R_{10} + R_{20})$$
 (36)

18

(32)

A_o and Φ_o are given by Eqs. (C-10) and (C-11) in Appendix C. Eqs. (33) and (21) give for the variation in phase

$$\delta\Phi = \frac{k^2 \sin^2 \psi}{\pi R_{10} R_{20} A_0} \int \int \sin \left[\frac{k x_1^2 \sin^2 \psi}{R} + \frac{k y^2}{R} - \Phi_0' \right] F(x_1, y) dx_1 dy \quad . \quad (37)$$

Assuming that the surface relief is distributed such that $\langle F(x_1,y) \rangle = 0$, the ensemble average of Eqs. (35) and (37) gives

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$$\left\langle \frac{\delta A}{A_0} \right\rangle = 0$$
, and (38)

(40)

It is of interest to point out that in the Fraunhofer approximation $\delta A/A_{o}$ is identically zero, whereas in the Fresnel case it is only the average value $\langle \delta A/A_{o} \rangle$ that vanishes.

Squaring and averaging Eq. (55) and utilizing Eq. (58) one obtains for the square of the amplitude fluctuations

$$\frac{\langle (5A)^2 \rangle}{A_0^2} = \frac{k^4 \sin^4 \psi}{\pi^2 R_{10}^2 R_{20}^2 A_0^2} \int \int \int \cos \left[\frac{k x_1^2 \sin^2 \psi}{R} + \frac{k y^2}{R} - e_{ij}^2 \right]$$

$$\times \cos \left[\frac{k x_1'^2 \sin^2 \psi}{R} + \frac{k y'^2}{R} - \frac{1}{2} \right]$$

$$\times \left\langle F(x_1, y) F(x_1', y') \right\rangle dx_1 dy dx_1' dy'$$

Eqs. (37) and (39) give for the square of the phase fluctuations

$$\left< (\delta \Phi)^2 \right> = \frac{k^4 \sin^4 \psi}{\pi^2 R_{10}^2 R_{20}^2 A_0^2} \int \int \int \int \sin \left[\frac{k x_1^2 \sin^2 \psi}{R} + \frac{k y^2}{R} - \phi_0^2 \right]$$

$$\times \sin \left[\frac{k x_1'^2 \sin^2 \psi}{R} + \frac{k y'^2}{R} - \phi'_0 \right]$$

$$\times \left\langle F(x_1, y) F(x_1', y') \right\rangle dx_1 dy dx_1' dy' .$$

20

(41)

(42)

The development of Eqs. (40) and (41) will be continued in Eqs. (67) and (73) of Chapter III, respectively.

D. Scattering Coefficient in the Fraunhofer Approximation

The Fraunhofer approximation in the specular direction is the substitution from Eq. (A-20) in Appendix A of

 $R_1' + R_2' = R_{10} + R_{20}$

into the exponential of Eq. (14). With $z_1 = R_{10} \sin \psi$ and $z_2 = R_{20} \sin \psi$ Eq. (14) becomes

$$\int \frac{ik(ein y)e^{ik(R_{10}+R_{20})}}{2\pi R_{10}R_{20}} \int \int e^{-2ikFeint}dx_{1}dy$$
.

The intensity of the scattered wave is defined as

$$I = \frac{(pp^*)}{2pv}$$

where

p is the density,

is the velocity of the medium through which the wave is propagating,

is the complex conjugate of the pressure p.

The product ρv is called the specific acoustic impedance of the medium. Forming the product pp^* from Eq. (42) and taking the ensemble average, one obtains

$$\left< pp \right> = \frac{k^2 \sin^2 \psi}{4\pi^2 R_{10}^2 R_{20}^2}$$

$$\times \iiint \left\langle e^{-2ik(\sin\psi)[F(x_{1},y)-F(x_{1}',y')]} \right\rangle dx_{1} dy dx_{1}' dy'$$

It is assumed that the surface relief of the rough surface is characterized by a bivariate Gaussian distribution function, Eckart (1953) and Horton and Muir (1967), defined by

$$W_{2}(x_{1}, y, F, x_{1}', y', F') = \frac{1}{2\pi \sqrt{\langle F_{0}^{2} \rangle^{2} - c^{2}}}$$
$$\frac{\langle F_{0}^{2} \rangle F^{2} - 2FF'C + \langle F_{0}^{2} \rangle F'^{2}}{2(\langle F_{0}^{2} \rangle^{2} - c^{2})}$$
$$\times e^{2(\langle F_{0}^{2} \rangle^{2} - c^{2})}$$

vhere

$$C = \left\langle F(x_1, y)F(x_1', y') \right\rangle , \qquad (45)$$

is the spatial covariance function of the surface displacements, and $\langle F_0^2 \rangle$ is the mean-square amplitude of the surface irregularities. It can be shown that with this function one obtains

$$(e^{-2ik(sink)(F-F')}) = e^{-4k^2 \langle F_0^2 \rangle (sin^2 v) \left(1 - \frac{c}{\langle F_0^2 \rangle}\right)}$$
(46)

21

(43)

(44)

24

Eq. (46) reduces Eq. (43) to

$$\left< pp^{*} \right> = \frac{k^{2} \sin^{2} \psi}{4\pi^{2} R_{10}^{2} R_{20}^{2}} e^{-4k^{2}} \left< \frac{F_{0}^{2}}{F_{0}^{2}} \right> \sin^{2} \psi}$$

$$\times \int \int \int \int e^{4k^2 C \sin^2 \psi} dx_1 dy dx_1' dy'$$

The scattering coefficient σ_s is defined as the ratio of the intensity scattered from a rough surface to the intensity reflected from a plane surface,

$$\sigma_{\rm g} = \frac{\langle \mathbf{I} \rangle}{\mathbf{I}_{\rm Q}} = \frac{\langle \mathbf{p} \mathbf{p}^* \rangle}{\langle \mathbf{p}_{\rm Q} \mathbf{p}_{\rm Q} \rangle} \qquad (48)$$

The term $p_0 p_0^*$, which is proportional to the intensity reflected from a plane surface, is given in Appendix C by Eq. (C-5),

$$p_0 p_0^* = \frac{k^2 l^2 (\Delta_1 + \Delta_2)^2 \sin^2 \psi}{x^2 R_{10}^2 R_{20}^2}$$

where Δ_1 , Δ_2 , and I are the dimensions of the active scattering region discussed in Appendix B. Thus, the scattering coefficient in the Fraunhofer approximation in the specular direction is given by



 $\times \int_{-\Delta_1}^{\Delta_2} \int_{-\Delta_1}^{1} \int_{-1}^{1} \int_{0}^{4\mu^2 \text{Cais}^2 \psi} dx_1 dy dx_1^2 dy^2$

(49)

22

(47)

The investigation of σ_s will be continued with Eq. (75) in Chapter III. E. Scattering Coefficient in the Fresnel Approximation

The Fresnel approximation in the specular direction is the substitution of Eq. (A-19) of Appendix A

 $R_1' + R_2' = R_{10} + R_{20} + \frac{1}{R} (x_1^2 \sin^2 \psi + y^2)$

into the exponential of Eq. (14). Putting $z_1 = R_{10} \sin \psi$ and $z_2 = R_{20} \sin \psi$, one obtains

$$p = \frac{ik (\sin \psi)e^{ik(R_{10}+R_{20})}}{2\pi R_{10}R_{20}} \int \int e^{\frac{ik}{R} (x_1^2 \sin^2 \psi + y^2) - 2ikF \sin \psi} dx_1 dy \quad . (50)$$

Forming the product pp", taking the ensemble average, one obtains

$$\left| pp^{*} \right\rangle = \frac{k^{2} \operatorname{ain}^{2} *}{4\pi^{2} R_{10}^{2} R_{20}^{2}}$$

$$\times \int \int \int \int e^{i} \left[\frac{k}{R} \left(x_{1}^{2} - x_{1}^{*2} \right) \operatorname{ain}^{2} \psi + \frac{k}{R} \left(y^{2} - y^{*2} \right) \right]$$

$$\times \left\langle e^{-2ik(\operatorname{sin}\psi) \left[F(x_{1},y) - F(x_{1}^{*},y^{*}) \right] } \right\rangle \operatorname{dx}_{1} \operatorname{dydx}_{1}^{*} \operatorname{dy}^{*} .$$

$$\left\langle e^{-2ik(\operatorname{sin}\psi) \left[F(x_{1},y) - F(x_{1}^{*},y^{*}) \right] } \right\rangle$$

with the aid of Eq. (46), Eq. (51) can be written as

$$\langle pp^{*} \rangle = \frac{k^{2} \sin^{2} \psi}{4\pi^{2} R_{10}^{2} R_{20}^{2}} e^{-4k^{2}} \langle F_{0}^{2} \rangle \sin^{2} \psi$$

$$\times \int \int \int \int e^{i} \left[\frac{k}{R} \left(x_{1}^{2} - x_{1}^{*2} \right) \sin^{2} \psi + \frac{k}{R} \left(y^{2} - y^{*2} \right) \right]$$

$$\times e^{4k^{2} C \sin^{2} \psi} dx, dy dx' dy'$$

$$(52)$$

The intensity reflected from a plane surface in the Fresnel approximation is proportional to the expression given by Eq. (C-12) in Appendix C,

$$p_0 p_0^* = \frac{K}{(R_{10} + R_{20})^2}$$
, (53)

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where

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$$\kappa = \left[c_{\ell} \left[c_{\Delta_{1}} + c_{\Delta_{2}} \right] - s_{\ell} \left[s_{\Delta_{1}} + s_{\Delta_{2}} \right] \right]^{2} + \left[s_{\ell} \left[c_{\Delta_{1}} + c_{\Delta_{2}} \right] + c_{\ell} \left[s_{\Delta_{1}} + s_{\Delta_{2}} \right] \right]^{2}$$
(54)

The Fresnel integrals, S and C, are defined by Eq. (C-9) in Appendix C. The scattering coefficient is the ratio of Eqs. (52) and (55),


CHAPTER III

THEORY USING AN EXPONENTIAL COVARIANCE FUNCTION

The statistical properties of the model surfaces, which shall be discussed in Chapter IV, suggest that an exponential function best describes the spatial covariance of the rough surfaces employed in the experimental study. The exponential spatial covariance function of the surface displacements is given by

$$c = \left\langle F(x_{1}, y)F(x_{1}', y') \right\rangle = \left\langle F_{0}^{2} \right\rangle e^{-\frac{1}{a} \left[(x_{1} - x_{1}')^{2} + (y - y')^{2} \right]^{1/2}}, \quad (56)$$

where <u>a</u> is the spatial correlation interval of the surface displacements and $\langle F_0^2 \rangle$ is the mean-square amplitude of the surface irregularities. The distance <u>a</u>, for which C will drop to the value e⁻¹, has the physical significance that it is a measurement of the relative distance between the hills and valleys on the randomly rough surface. The square root of $\langle F_c^2 \rangle$ has the physical meaning that it represents the average heights of the hills and the average depth of the valleys.

A. Amplitude and Phase Fluctuations in the Fraunhofer Approximation

The amplitude fluctuations in the Fraunhofer approximation are zero irrespective of the form of the spatial covariance function as shown by Eq. (24) in Chapter II.

The phase fluctuations in the Fraunhoser approximation are determined by the substitution of Eq. (56) into Eq. (32) to give

$$\left< (80)^2 \right> = \frac{k^2 \left< F^2 \right> \sin^2 \psi}{k^2 \left(\Delta_1 + \Delta_2\right)^2}$$

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 $\times \int_{-\Delta_{1}}^{\Delta_{2}} \int_{-\Delta_{1}}^{L} \int_{-\lambda_{1}}^{\Delta_{2}} \int_{-\lambda_{1}}^{L} e^{\frac{1}{a} \left[\left[\left[x_{1} - x_{1}^{\prime} \right]^{2} + \left(y - y^{\prime} \right]^{2} \right]^{\frac{1}{2}} \right]^{\frac{1}{2}}} dx_{1} dy dx_{1}^{\prime} dy$

CHAPTER III

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where <u>a</u> is the spatial correlation interval of the surface displacements and $\langle F_o^2 \rangle$ is the mean-square amplitude of the surface irregularities. The distance <u>a</u>, for which C will drop to the value e⁻¹, has the physical significance that it is a measurement of the relative distance between the hills and valleys on the randomly rough surface. The square r it of $\langle F_o^2 \rangle$ has the physical meaning that it represents the average heights of the hills and the average depth of the valleys.

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The phase fluctuations in the Fraunhorer approximation are determined by the substitution of Eq. (56) into Eq. (32) to give

$$\left< (80)^2 \right> = \frac{k^2 \left< F^2 \right> \sin^2 y}{t^2 (\Delta_1 + \Delta_2)^2}$$

$$\times \int \int \int \int \int \int e^{\frac{1}{2} \left[\left(x_1 - x_1^* \right)^2 + \left(y - y^* \right)^2 \right]^{1/2}} dx_1 dy dx_1^* dy^*$$

(57)

Since Δ_1 and Δ_2 are not equal, it is convenient to make the preliminary change of variables (see Fig. 5)

$$u = x_1 - \frac{\Delta_2 - \Delta_1}{2}$$
$$u' = x_1' - \frac{\Delta_2 - \Delta_1}{2}$$

and introduce

$$\Delta = \frac{\Delta_1 + \Delta_2}{2}$$

With these changes Eq. (57) becomes

$$\left< (\delta \Phi)^2 \right> = \frac{k^2 \left< F_0^2 \right> \sin^2 \Psi}{\ell^2 (\Delta_1 + \Delta_2)^2}$$

$$\times \int_{-\Delta}^{\Delta} \int_{-\Delta}^{4} \int_{-\frac{1}{2}}^{4} \left[(u-u')^{2} + (y-y')^{2} \right]^{1/2} du dy du' dy' . (60)$$

27

(58)

(59)

(61)

(62)

The approximate size of the active scattering region is 24 by 24 as defined in Appendix B.

If one introducer relative coordinates as shown in Fig. 5 and

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defined by

and center of gravity coordinates defined by

$$x_0 = \frac{1}{2} (u + u')$$

 $y_0 = \frac{1}{2} (y + y')$



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Eq. (60) recomes 스 1 2(Δ-x₀) 2(1-y₀) 당[[] [

The integrals in Eq. (63) can not be computed in open form. Further discussion shall be restricted to the case when the dimensions of the region essential to integration over ξ and η are considerably less than those of the irradiated part of the surface. That is, it is assumed that

 $-2(\Delta - x_{0}) - 2(\ell - y_{0})$

$$a \ll 2\Delta, 2l$$
 . (64)

dx_dy_d\$dŋ

(63)

In keeping with this approximation one may extend the range of integration over $\boldsymbol{\xi}$ and η to infinity, so that Eq. (63) becomes

$$\delta\Phi)^{2} = \frac{4k^{2} \langle F_{o}^{2} \rangle \sin^{2} \psi}{\ell^{2} (\Delta_{1} + \Delta_{2})^{2}} \times \int_{0}^{\Delta} \int_{0}^{\ell} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} e^{-\frac{1}{a} (\xi^{2} + \eta^{2})^{1/2}} dx_{o} dy_{o} d\xi d\eta \qquad (65)$$

The integration may be performed readily by changing to polar coordinates, and one finds for Eq. (65)

$$\sigma_{\Phi} = \sqrt{\langle (\delta \Phi)^2 \rangle} = \left(2k \sqrt{\langle F_0^2 \rangle} \sin \psi \right) \approx \sqrt{\frac{\pi}{\ell(\Delta_1 + \Delta_2)}} \quad . \tag{66}$$

This equation shows that the phase fluctuations in the Fraunhofer approximation are proportional to the Rayleigh parameter. $2k \sqrt{\langle P_o^2 \rangle} \sin \psi$ (for Rayleigh parameters much less than one), and to the spatial correlation interval, a. The fluctuations are inversely proportional to the square root of the approximate insonified area, $2\ell (\Delta_1 + \Delta_2)$.

B. Amplitude and Phase Fluctuations in the Fresnel Approximation

Substituting Eq. (56) into Eq. (40) of Chapter II, cne finds for the square of the amplitude fluctuations

$$\frac{\left(\left(\delta A\right)^{2}\right)}{A_{0}^{2}} = J_{1} + J_{2} , \qquad (67)$$

(68)

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where

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$$=\frac{k^{4}\left\langle F_{0}^{2}\right\rangle \sin^{4}\psi}{2\pi^{2}R_{10}R_{20}R_{0}^{2}A_{0}^{2}}\int_{-l}^{l}\int_{-l}^{l}\int_{-l}^{l}\int_{-l}^{l}\int_{-l}^{l}\frac{1}{a}\left[\left(x_{1}-x_{1}^{\prime}\right)^{2}+\left(y-y^{\prime}\right)^{2}\right]^{1/2}$$

$$\times \cos \left[\frac{k}{R} \sin^{2} \psi \left(x_{1}^{2} - x_{1}^{\prime 2} \right) + \frac{k}{R} \left(y^{2} - y^{\prime 2} \right) \right] dx_{1} dy dx_{1}^{\prime} dy^{\prime} , \text{ and}$$

$$J_{2} = \frac{k^{4} \left\langle F_{0}^{2} \right\rangle \sin^{4} \psi}{2\pi^{2} R_{10}^{2} R_{20}^{2} A_{0}^{2}} \int_{-L}^{L} \int_{-L}^{-L} \int_{-L}^{L} \int_{-L}^{-\frac{1}{2}} \int_{-L}^{L} \left(x_{1} - x_{1}^{\prime} \right)^{2} + \left(y - y^{\prime} \right)^{2} \right]^{1/2}$$

$$\times \cos \left[\frac{k}{R} \sin^{2} \psi \left(x_{1}^{2} + x_{1}^{\prime 2} \right) + \frac{k}{R} \left(y^{2} + y^{\prime 2} \right) - 2\Phi_{0}^{\prime} \right] dx_{1} dy dx_{1}^{\prime} dy^{\prime} .$$

$$(69)$$

The details in the integration of J_1 are given in Appendix D. The result is

$$J_{1}(a) = \frac{2k^{2} \langle F_{0}^{2} \rangle \sin^{2} \psi}{\pi \left\{ \left[C_{1} \left[C_{2} \left[C_{2} + C_{2} \right] - S_{1} \left[S_{1} + S_{2} \right] \right]^{2} + \left[C_{1} \left[S_{2} + S_{2} \right] + S_{1} \left[C_{1} + C_{2} \right] \right]^{2} \right\}} \right\}$$

$$\times \left\{ \operatorname{arccot} \left[\frac{R}{2ka t \Delta_{1} \sin^{2} \psi} \left[t^{2} + \Delta_{1}^{2} \sin^{4} \psi + \frac{R^{2}}{4k^{2}a^{2}} \right]^{1/2} \right]$$

$$+ \operatorname{arccot} \left[\frac{R}{2ka t \Delta_{2} \sin^{2} \psi} \left[t^{2} + \Delta_{2}^{2} \sin^{4} \psi + \frac{R^{2}}{4k^{2}a^{2}} \right]^{1/2} \right] \right\} ,$$
(70)

where C and S are the Fresnel integrals defined by Eq. (C-9) in Appendix C. A discussion of the J_2 term is given in Appendix E. The investigation shows that

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$$J_2 \ll J_1 \tag{71}$$

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so that the amplitude fluctuations as given by Eq. (67) can be approximated

$$\frac{\sqrt{\langle (\delta A)^2 \rangle}}{A_0} \approx \sqrt{J_1} \qquad (72)$$

Substituting Eq. (56) into Eq. (41) of Chapter II, one obtains for the phase fluctuations

$$\sqrt{\langle (\delta\Phi)^2 \rangle} = \sqrt{J_1 - J_2} , \qquad (73)$$

where J_1 and J_2 are given by Eqs. (68) and (69). Since $J_2 \ll J_1$, the amplitude and phase fluctuations are approximately equal for small Rayleigh parameters so that

$$\frac{\sqrt{\langle (\delta A)^2 \rangle}}{A_0} \approx \sqrt{\langle (\delta \Phi)^2 \rangle} \approx \sqrt{J_1} \quad . \tag{74}$$

Thus, the Fraunhofer approximation shows the phase fluctuations to be significantly larger than the amplitude fluctuations (which are zero) whereas the Fresnel approximation shows the amplitude and phase fluctuations to be approximatley of the same magnitude.

C. Scattering Coefficient in the Fraunhofer Approximation

Substituting Eq. (56) into Eq. (49) of Chapter II, one obtains

$$\sigma_{s} = \frac{e^{-4k^{2} \langle F_{0}^{2} \rangle \sin^{2} \psi}}{4t^{2} (\Delta_{1} + \Delta_{2})^{2}}}$$

$$\times \int_{-\Delta_{1}}^{\Delta_{2}} \int_{-k}^{k} \int_{-k}^{\Delta_{2}} \int_{-k}^{k} e^{4k^{2} \langle F_{0}^{2} \rangle} (\sin^{2}\psi) e^{-\frac{1}{a} \left[(x_{1} - x_{1}^{*})^{2} + (y - y^{*})^{2} \right]^{1/2}} dx_{1} dy dx_{1}^{*} dy^{*}$$

$$= \int_{-k}^{\Delta_{1}} \int_{-k}^{k} e^{-\lambda_{1}^{*} - \lambda_{1}^{*}} dx_{1} dy dx_{1}^{*} dy^{*}$$

 $g = 4k^2 \langle F_o^2 \rangle \sin^2 \psi$

and refer to \underline{g} as the Beckmann parameter (which in the specular direction is the square of the Rayleigh parameter).

32

(76)

The integrand can be written as



Eq. (75) becomes

Set



Since the integrals occurring in Eq. (78) are the same as those occurring in Eq. (57) for the phase fluctuations, one can write immediately from the square of Eq. (66),

$$\sigma_{s} = e^{-4k^{2} \langle F_{o}^{2} \rangle \sin^{2} \psi} \left[1 + \frac{a^{2} \pi}{\ell(\Delta_{1} + \Delta_{2})} \sum_{m=1}^{m=\infty} \frac{1}{m!m^{2}} \left(\frac{4k}{\sqrt{F_{o}^{2}}} \right) \sin^{2} \psi \right] \quad .(79)$$

for the scattering coefficient in the specular direction in the Fraunhofer approximation.

D. Scattering Coefficient in the Fresnel Approximation

Substituting Eq. (56) into Eq. (55) of Chapter II, one obtains

33.

(80)

$$\sigma_{s} = \frac{k^{2}(R_{10} + R_{20})^{2} \sin^{2} \psi}{4\pi^{2}R_{10}^{2}R_{20}^{2}K} e^{-g}$$

$$\times \int_{-\Delta_{1}}^{\Delta_{2}} \int_{-\ell}^{\ell} \int_{-\Delta_{1}}^{\Delta_{2}} \int_{-\ell}^{\ell} e^{i\left[\frac{k}{R}\left(x_{1}^{2} - x_{1}^{2}\right)\sin^{2}\psi + (y^{2} - y^{2})\right]}$$

$$- \frac{1}{2}\left[(x_{1} - x_{1}^{2})^{2} + (y - y^{2})^{2}\right]^{1/2}$$

$$x e^{ge} dx_1 dy dx_1' dy'$$

Using the infinite summation defined by Eq. (77), one obtains

$$\sigma_{s} = \frac{k^{2}(R_{10} + R_{20})^{2} \sin^{2} \psi}{4\pi^{2}R_{10}^{2}R_{20}^{2}K} e^{-\varepsilon} \left\{ \int_{-\Delta_{1}}^{\Delta_{2}} \int_{-\lambda_{1}}^{\lambda_{1}} \int_{-\lambda_{1}}^{\lambda_{2}} e^{i\left[\frac{k}{R}(x_{1}^{2} - x_{1}^{2})\sin^{2}\psi\right]} + \frac{k}{R}(y^{2} - y^{2})\right] dx_{1} dy dx_{1}^{*} dy^{*} + \sum_{m=1}^{m=\omega} \frac{g^{m}}{m!}$$

$$\times \int_{-\Delta_{1}}^{\Delta_{2}} \int_{-\lambda_{1}}^{\lambda_{1}} \int_{-\lambda_{1}}^{\lambda_{2}} e^{i\left[\frac{k}{R}(x_{1}^{2} - x_{1}^{2})\sin^{2}\psi + \frac{k}{R}(y^{2} - y^{2})\right]}$$

$$\times e^{-\frac{m}{4}\left[(x_{1} - x_{1}^{*})^{2} + (y - y^{*})^{2}\right]^{1/2}} dx_{1} dy dx_{1}^{*} dy^{*}$$

$$(81)$$

Integrating the first set of integrals as shown in Appendix C and separating the second set of integrals into their real and imaginary parts, one obtains 34



The integration of the second group of integrals containing the sine function can be accomplished by following the procedure given for the integration of the J_1 term in Appendix D. The parallel development produces the integral (see Eq. (D-3) in Appendix D)

 $I_{s} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} e^{-\frac{1}{2}(\xi^{2}+\eta^{2})^{1/2}} \sin\left[\frac{2k}{R}\sin^{2}\psi\left(x_{o}+\frac{\Delta_{2}}{2}-\frac{\Delta_{1}}{2}\right)\xi + \frac{2k}{R}y_{o}\eta\right]d\xi d\eta$

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which can be reduced to

$$I_{s} = \int_{0}^{2\pi} \int_{0}^{\infty} e^{-\rho/a} \sin\left[\rho \sqrt{M^{2} + N^{2}} \cos \varphi\right] \rho d\rho d\varphi$$

From Gradshteyn and Ryzhik (integral 3.715-13, page 402), one obtains $I_s = 0$. Thus, the expression for the coefficient σ_s becomes

$$f_{S} = e^{-g} \left\{ 1 + \frac{k^{2}(R_{10} + R_{20})^{2} \sin^{2} \psi}{4\pi^{2}R_{10}^{2}R_{20}^{2}K} \sum_{m=1}^{m=\infty} \frac{g^{m}}{m!} + \frac{\lambda^{2}}{2} \int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{\sqrt{2}} \cos\left[\frac{k}{R} (x_{1}^{2} - x_{1}^{2}) \sin^{2} \psi + \frac{k}{R} (y^{2} - y^{2})\right] - \frac{m}{2} \left[(82) - \frac{m}{4} \left[(x_{1} - x_{1}^{2})^{2} + (y - y^{2})^{2} \right]^{1/2} + \frac{1}{2} dx dw^{2} x^{2} dy^{2} \right] \right\}$$

The integrals appearing in Eq. (82) are the same as the integrals appearing in in Eq. (63). Thus, the scattering coefficient calculated in the Fresnel approximation can be written as

$$\sigma_{s} = e^{-\varepsilon} \left[1 + \frac{2}{\varepsilon} \sum_{m=1}^{m + \varepsilon} \frac{\varepsilon}{m} J_{1}(\frac{3}{m}) \right]$$
(85)

where $J_1\left(\frac{3}{m}\right)$ is given by Eq. (70).

CHAPTER IV EXPERIMENTAL APPARATUS AND PROCEDURES

It is seen from the theory developed in Chapter II that one must know the statistical properties of the rough surface in order that the predictions of the theory can be compared with measurements on the rough surface. In particular, if $F(x_1,y)$ is the elevation of the reflecting surface, one must know the autocovariance $\langle F(x_1y)F(x_1',y')\rangle$ as shown by Eqs. (32), (40), (41), (49), and (55), and the bivariate distribution function as illustrated by Eq. (44).

The problem of selecting a topographic surface to be modeled has been discussed by Horton, Mitchell, and Barnard (1967). The model surface was generated from maps of aeromagnetic intensity over a portion of the Canadian Shield. Before these maps were used to construct the model, the two-dimensional autocovariance functions of these maps were studied by Horton, Hempkins, and Hoffman (1964). Details of the mathematical analysis used in the investigation of such maps are given by Horton, Hoffman, and Hempkins (1962).

A summary of the results obtained in these investigations will be given. The Genedian maps were contoured in gammas $(1y = 10^{-5} \text{ Oe})$. The vertical relief of the model surface was obtained from this aeromagnetic dimension using the scale 10y = 1/32 in. The factor 1 mile = 1 in. was used to convert the area of the aeromagnetic maps into model surfaces of size 32 in. by 32 inches. The theory in Chapters II and III is based on the assumption that the reflecting surface is pressure release (see Eq. (3), and the models were constructed from a material of small acoustic impedance compared with the acoustic impedance of the vater in which the surfaces are submerged. The model was made of low-density expanded polystryrene by the Construction Service Company of Fort Worth, Texas. A square grid of sample points drawn 1/2 in. apart on the central one-fourth of the map surface was used to compute the autocovariance function. This square grid gave 1059 values of the elevation.

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The autocovariance function was computed from a formula given by Horton, Hoffman, and Hempkins (1962)

$$C(s,t) = \frac{1}{(N-s)(M-t)} \sum_{i=1}^{N-s} \sum_{j=1}^{M-t} F(i,j)F(i+s, j+t) , \quad (84)$$

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where

N = M = 33 is the number of samples along each 16 in. grid line,

F is the surface relief,

s and t are units of shift along the grid, and

C is the autocovariance function.

Profiles of the autocovariance function along the <u>s</u> and <u>t</u> directions are plotted in Fig. 6. The profiles show that the surface is possibly isotropic for values of <u>s</u> and <u>t</u> less than one inch. The mean-square surface relief, $\langle F_0^2 \rangle$, is 0.132 in.² A curve that will pass through the three points nearest the origin and approximately average the other anisotropic points for values of <u>s</u> and <u>t</u> greater than one inch is the exponential autocovariance function given by

$$C(r) = 0.132 e^{-\frac{r}{2.55}}$$
, (85)

where 2.55 in. is the correlation length defined as the distance over which the covariance function is reduced by 1/e. Eq. (85) corresponds to Eq. (56) in Chapter III, where

$$\langle F_0^2 \rangle = 0.132 \text{ in.}^2$$

a = 2.55 in.
r = $[(x_1 - x_1')^2 + (y - y')^2]^{1/2}$

Although the properties of the model surface do not satisfy the assumption of isotropy on which the theory in Chapters' II and III is based, the agreement is close enough to justify a comparison between theory and experiment. Since no one has measured experimentally the bivariate



probability density for a sample of topography, it is not possible to justify directly the choice given by Eq. (44) at the present time. However, Horton, Mitchell, and Barnard (1967) give convincing reasons why the bivariate Gaussian distribution may be used. The reader is referred to a thesis by Mitchell (1966) for further details concerning the model surface.

Three other model surfaces described statistically by the same exponential covariance function were constructed. The statistical properties of the four model surfaces are given in Table I. The ratio $\sqrt{\langle F_o^2 \rangle} / \lambda$ was calculated for a frequency of 95.800 kHz (wavelength $\lambda = 0.608$ in.).

The experiments were performed with the model surface submerged in a water-filled tank with dimensions 15 ft wide, 60 ft long, and 12 ft deep. The surface was suspended vertically with its center approximately 46 in. below the waterline. An x,y,z mechanical positioning system mounted on the top of the tank was used to position the source and receiver in the experiment. Figure 7 shows the laboratory tank, positioning system, and electronic equipment used in the study. The source and receiver were mounted on two identical columns with the movement in the x,y,z directions read from a counter (π milli-inches per smallest division or 518.510 divisions per inch on the counter). Also, the mechanism allows the projector and receiver to be rotated about the vertical axis in increments of 0.1 deg.

The source was a transducer that has an active face diameter of 12.5 cm and a resonance frequency of 94.0 kHz. The directivity pattern of the piston transducer is shown in Fig. 8. The beamwidth is approximately 9 deg measured between points where the radiated intensity falls to one-half the intensity along the forward axis (-3 dB points).

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The receiver was a small cylinder of outside diameter 0.082 in., inside diameter 0.057 in., and length 0.125 inch. Its directivity pattern is approximately cumidirectional as shown by Fig. 9. The same side of the probe was directed along the slant-line distance to the model surfaces throughout the experiment.

Model	rms Relief	Correlation Length	Relative Slope	Ratio	
	$\left< \left< \frac{F_{o}^{2}}{in} \right> \right.$	in.	$\sqrt{F_0^2}/a$	$\sqrt{F_o^2}/\lambda$	
I	0.091	2.55	0.0357	0.150	
II	0.182	2.55	0.0714	0.300	
III	0.364	2.55	0.1428	0.600	
IV	0.364	5,10	0.0714	0.600	

TABLE I STATISTICAL PROPERTIES OF THE MODEL SURFACES

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Model	rms Relief	Correlation Length a	Relative Slope	Ratio
	$\left< \left< \frac{F_{o}^{2}}{10} \right>$ in.	in.	$\sqrt{F_o^2}/a$	$\sqrt{F_o^2}/\lambda$
I	0.091	2.55	0.0357	0.150
II	0.182	2.55	0.0714	0.300
III	0.364	2.55	0.1428	0.600
IV	0.364	5.10	0.0714	0.600

TABLE I									
STATISTICAL	PROPERTIES	OF	THE	MODEL	SURFACES				

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SOURCE AND RECEIVER SEPARATED BY 10 fr INPUT VOLTAGE: 15 V PRESEWIDTH: 640 JAKEE RESONANCE FREQUENCY: 94.0 kHz

> FIGURE 8 DIRECTIVITY PATTERN TRANSDUCER DRL No. 116(1257) FREQUENCY: 95.800 kHz



Figure 10 is a block diagram of the arrangement of the electronic and acoustic equipment. The continuous oscillator signal is gated into signals of 640 μ sec duration at a rate of 28 times per second. These pulses are 95 cm \approx 37 in. long in the water and contain 61 cycles at 95.800 kHz. The pulsed signals are used as the input of the power amplifier that drives the projecting transducer. The received signal scattered from the pressure release model surface is amplified and is displayed on the oscilloscope. The peak-to-peak voltage of the received signal is recorded. At the same instant the relative phase of the received signal is measured by recording the time delay of the scattered pulse relative to a continuous wave from the oscillator. The counter allows the experimenter to maintain the frequency to within one cycle throughout the data collecting.

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All measurements were made with the projector and receiver in the specular direction. With the projector placed 60.0 in. from the origin on the rough surface and with the receiver placed 60.0 in. from the same origin (the origin lies on an imaginary plane that passes through the average height of the rough surface), the grazing angle was varied from 80 deg to 6 deg in increments of 2 deg. Figure 11 shows samples of the oscilloscope pictures for grazing angles of 50 and 80 deg.

The center of the received pulse is recorded since this represents a scattered pressure due to the total insonified area. The pulse to the left of the largest signal in Fig. 11a for a grazing angle of 80 deg is attributed to the side lobes of the projector.

If the source and the receiver are displaced as a rigid body parallel to the scattering surface, one would obtain different values for the amplitude and relative phase as the dominant scattered energy returns from different parts of the rough surface. Consequently, the sequence of values for the amplitude and relative phase constitute samples of a stochastic variable. Therefore, one is concerned not with specific values but rather with average values. In the process of collecting the data, ten different values of the amplitude and relative phase were obtained by insonifying ten different regions of the model surface. To insonify the different regions, the projector and receiver





were moved as a rigid body in the vertical direction to ten different positions in increments of one inch. Figure 12 shows three of the ten different positions of the area that the projector insonifies (numbering 1, 6, and 10). The ellipses indicate the approximate size of the insonified area (-3 dB) for a grazing angle of 40 deg. The active scattering area for each of the ten data runs is confined to a rectangular region 14.8 in. by 18.4 inches.

The amplitude fluctuations were measured for any one grazing angle with the receiver confined to the specular direction by recording the peak-to-peak voltage of the received signal for the ten different positions of the transmitter and receiver. The average amplitude

1 10

$$A_{av} = \frac{1}{10} \sum_{i=1}^{1} A_i$$
 (86)

was used to calculate the variance

$$D(A) = \frac{1}{10} \sum_{i=1}^{i=10} (A_i - A_{av})^2 , \qquad (87)$$

from which the standard deviation

$$\sigma = \sqrt{D(A)}$$
(83)

was obtained. The relative standard deviation (the amplitude fluctuations) is the ratio of the standard deviation to the average amplitude

$$\sigma_{\rm A} = \frac{1}{A_{\rm av}} \sqrt{D(A)} \qquad (89)$$

The phase fluctuations were measured by recording the time delay of the scattered pulse signal relative to a continuous wave that was displayed on the oscilloscope. The time for a wavelength of the cw signal was measured on the oscilloscope (10.2 µsec for 95.6 kHz); the ratio of these two times multiplied by 360 deg gives the relative phase \uparrow in degrees. The average phase of the ten different measurements



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i=10 $\Phi_{av} = \frac{1}{10} \sum_{i=1}^{2} \Phi_{i}$

was used to calculate the variance

$$D(\Phi) = \frac{1}{10} \sum_{i=1}^{i=10} (\Phi_i - \Phi_{av})^2 , \qquad (91)$$

from which the standard deviation

$$\sigma_{\Phi} = \sqrt{D(\Phi)} \tag{92}$$

was obtained. In this case, the standard deviation represents the phase fluctuations.

The reference used to determine the scattering coefficient was obtained experimentally by measuring the peak-to-peak voltage of the pulsed signal reflected from a plane pressure release surface of the same dimensions, 32 in. by 32 in., as the rough surface. Assuming that the pressure of the underwater sound at the receiver is proportional to the voltage output f the receiving hydrophone, one defines the scattering coefficient for each individual measurement as the ratio of the scattered intensity from the rough surface to the reflected intensity from the plane surface. Thus, the experimental scattering coefficient can be written as

$$\sigma_{Q}(\psi_{i}) = \left(\frac{v_{s}}{v_{p}}\right)^{2}$$

(93)

(90)

vhere

t, is the grazing angle,

V₃ is the peak-to-peak voltage of the pulsed signal scattered from the rough surface, and

V is the peak-to-peak voltage of the same pulsed signal reflected from the plane surface.

The average scattering coefficient, which is the correct coefficient to compare with the theory is

$$\sigma_{e}(j) = \frac{1}{10} \sum_{i=1}^{i=10} \left[\frac{v_{s}(i,j)}{v_{p}(j)} \right]^{2}$$
(94)

where

<u>i</u> is the sample number (i = 1,2,...10), and <u>j</u> is the grazing angle (j = 2,4,...80 deg).

The scattering coefficient expressed in decibels is

$$\sigma_{\rm e}({\rm dB}) = 10 \log_{10} \sigma_{\rm e}$$
 . (95)

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CHAPTER V RESULTS

A. Amplitude Fluctuations

The experimental results for the amplitude fluctuations in the specular direction for a frequency of 95.8 kHz are presented in Figs. 13 and 14. These results were obtained from averages of readings from ten different insonified regions of the rough surface by using Eqs. (86) through (89) of Chapter IV. The theoretical predictions of the amplitude fluctuations in the Fraunhofer approximation, given by Eq. (24), and in the Fresnel approximation, given by Eqs. (70) and (72) are illustrated in Fig. 13. A beamwidth of 9 deg measured at the half-power points (-3 dB) of the main lobe of the incident beam, as shown in Fig. 8 of Chapter IV, was used to calculate the dimensions of the insonified area.

The experimental data were used to construct the dependence of the amplitude fluctuations on the Rayleigh parameter, $2k \sqrt{F_o^2} \sin \psi$. Figure 13 pertains to measurements made on the model surface whose statistical properties were characterized by a root-mean-square relief,

 $\sqrt{F_0^2}$, of 0.091 inch. From the graph it is evident that, within 5% fluctuations, the experimental results correspond to the theoretical law in the Freenel approximation for Rayleigh parameters less than 0.6. The theoretical law in the Fraunhofer approximation of zero amplitude fluctuations gives incorrect predictions for any value of the Rayleigh parameter. The experimental amplitude fluctuations for values of the Rayleigh parameter greater than one could not possibly be explained by means of the theoretical analysis, as the region of applicability of the analytical formulas is limited by the condition given by Eq. (15) that 2k $\sqrt{F_0^2}$ sin $\psi < 1$.

When the projector and receiver were aimed at each other over a distance of 10 ft the amplitude fluctuations were measured to be less than one-half percent. Thus, changes in the properties of the water



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FIGURE 13 AMPLITUDE FLUCTUATIONS IN THE SPECULAR DIRECTION PRESSURE RELEASE RANDOM SURFACE

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through which the transmitted and received pulsed wave propagated, and inaccuracies in positioning the projector and receiver contributed less than 0.5% to the amplitude fluctuations shown in Figs. 13 and 14.

The experimental data of the amplitude fluctuations depicted in Fig. 14 are the outcome of combining the results obtained from three different model surface, I, II, and III. For small values of the Rayleigh parameter up to 1.7 there is an apparent increase of the amplitude fluctuations from 10% to 40%, which is proportional to the value of this parameter. For values of the parameter greater than 2.5 the growth of the amplitude fluctuations ceases and it reaches its saturation point. For larger values of the parameter there is no further increase in the fluctuations, and the amplitude fluctuations disclose a spread of about 33 - 65% in the values. The graph shows a well-defined alternation of the amplitude fluctuations between maximum and minimum values. The fluctuation maxima are situated at 2k $\sqrt{F_0^2}$ sin $\psi \approx 2.5$, 4.2. 5.8, and 6.7; the minima at 2k $\sqrt{F_0^2}$ sin $\psi \approx 3.4$, 4.9, and 6.4.

The measurements illustrated in Fig. 14 verify the tentative results obtained by Gulin and Malyshev (1962) who scattered sound signals from the surface of the sea. Their results, which were stated in Chapter I, show that the amplitude fluctuations reach a saturation point when the Rayleigh parameter has the value 0.7 and for larger Rayleigh parameters the fluctuations experience an alternation between maximum and minimum values of 25 to 50%.

B. Phase Fluctuations

The experimental results for the phase fluctuations in the specular direction for a frequency of (5.8 kHz are presented in Figs. 15) and 16. These results were obtained from averages of readings from ten different insonified regions of the rough surface by using Eqs. (90) through (92) of Chapter IV. The theoretical predictions of the phase fluctuations in the Fraunhofer approximation, given by Eq. (66), and in the Fresuel approximation, given by Eqs. (70) and (74), are depicted in Fig. 15. A 9 deg beamwidth was used in the calculation for Δ_1 , Δ_2 , and is appearing in the formulas.

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B. Phase Fluctuations

The experimental results for the phase fluctuations in the specular direction for a frequency of (5.8 kHz are presented in Figs. 15 and 16. These results were obtained from averages of readings from ten different insonified regions of the rough surface by using Eqs. (90) through (92) of Chapter IV. The theoretical predictions of the phase fluctuations in the Fraunhofer approximation, given by Eq. (66), and in the Fresnel approximation, given by Eqs. (70) and (74), are depicted in Fig. 15. A 9 deg beamwidth was used in the calculation for Δ_1 , Δ_2 , and I appearing in the formulas.



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PHASE FLUCTUATIONS IN THE SPECULAR DIRECTION PRESSURE RELEASE RANDOM SURFACE FIGURE 16

When the projector and receiver were aimed at each other over a distance of 10 ft the phase fluctuations were measured to be less than 5%. This small fluctuation in phase was caused by the inaccuracy in moving the source and receiver as a rigid body relative to the model surface, and also was brought about by local changes in the properties of the water through which the acoustic wave propagated.

The dependence of the phase fluctuations on the Rayleigh parameter for measurements made with the model surface that has a root-meansquare relief of 0.091 in. is shown in Fig. 15. The graph shows that the theoretical law in the Fraunhofer approximation gives predicted values within 4% of the values obtained using the Fresnel approximation for values of the Rayleigh parameter less than one. Over this range of Rayleigh parameters it is evident that, within 15% fluctuations, the experimental results which are expressed in percent relative to a radian, agree with both theoretical approaches. The phase fluctuations resulting from the irregularities on the model surface, for values of the Rayleigh parameter between 0.2 and 0.35 (theory predicts 7 to 14%) are concealed possibly by the fluctuations caused by the wave traveling directly from the source to the receiver and the wave scattered from the edge of the surface interfering with the wave scattered from the topograph of the rough surface (experimental results show about 25% relative to a radian). With this understanding one can assert that both theoretical procedures agree. within 6% fluctuation, with the experimental results for values of the Rayleigh parameter less than 0.6. The apparent good agreement between the experimental results and the theoretical predictions in the Fraunhofer approximation for values of the Rayleigh parameter between 1.225 and 1.67 is surprising but should be treated as purely coincidental since the theory is not valid for Rayleigh parameters greater than one.

The experimental phase fluctuations illustrated in Fig. 16 are the consequence of incorporating the measurements obtained from three model surfaces. I, II, and III. The dependence of the phase fluctuations on the Rayleigh parameter is not as well defined as the amplitude fluctuations on this parameter. Nevertheless, in general the phase fluctuations increase with the Rayleigh parameter from a low value of

12% to a high value of 250%. It is understood that the percentage for the experimental phase fluctuations is percent relative to a radian. The fluctuation maxima are situated at $2k \sqrt{\langle F_0^2 \rangle} \sin \psi \approx 1.5$, 3.3, 4.6, and 6.0; the minima at $2k \sqrt{\langle F_0^2 \rangle} \sin \psi \approx 1.8$, 3.8, 5.3, and 6.8. Since the author is unaware of any other experimental investigation of the phase fluctuations of an underwater acoustic pulsed wave scattered from a randomly rough surface, no comparison can be made and these results should be considered tentative and in need of further verification.

When one attempts to compare the experimental phase fluctuations with the corresponding amplitude fluctuations, one is confronted with a problem. The issue is understood when it is realized that the experimental amplitude fluctuations given by Eq. (89) represent a relative standard deviation, whereas the experimental phase fluctuations given by Eq. (92) represent an absolute standard deviation. The experimental amplitude fluctuations in Fig. 13 are in percent relative to the average amplitude, whereas the phase fluctuations in Fig. 15 are in percent relative to a radian. The Fraunhofer and Fresnel theory for the amplitude and phase fluctuations, using the same convention for the ordinate scales, are presented in Figs. 13 and 15 in percent.

C. Scattering Coefficient

The experimental results for the scattering coefficient in the specular direction for a frequency of 95.8 kHz are presented in Figs. 17, 18, 19, and 20. These results were obtained from averages of readings from ten different inconified regions of the rough surface by employing Eqs. (95) through (95) of Chapter IV. The theoretical predictions of the scattering coefficient in the Fraunhofer approximation, given by Eq. (79), and in the Fresnel approximation, given by Eq. (85), are shown in the four graphs. The discussions of the insonified area, Δ_1 , Δ_2 , and i that appear in the formulas were calculated for a 9 deg beauwidth income at the half-power points of the main lobe of the incident beam.

Figure 17 shows t's dependence of the scattering coefficient on the Rayleigh parameter and the grazing angle, \$, for the model surface that has so ras belief of 0.091 in. and a correlation length of 2.55 inches.


From the graph it is evident that, within 3 dB, the experimental results correspond to the theoretical law in the Fresnel approximation over the complete range of the Rayleigh parameters from values of 0.20 to 1.85. The theoretical law in the Fraunhofer approximation gives good agreement with the experimental results up to a value of 1.3 for the Rayleigh parameter. But for larger values of the parameter the Fraunhofer theory predicts lower values for the scattering coefficient than the experimental results show. At a Rayleigh parameter of 1.85 the Fraunhofer law gives values of the coefficient that are 4 dB smaller than the Fresnel predictions.

The scattering coefficient measured with the model surface that has an rms relief of 0.182 in. and a correlation length of 2.55 in. is illustrated in Fig. 18. Again, it is evident from the graph that the theory in the Fresnel approximation predicts the experimental values better than the Fraunhofer theory. The Fresnel theory predicts the experimental results to within 3 dB for Rayleigh parameters less than 2.35. For larger values of the Rayleigh parameter the Fresnel theory predicts values within 4.5 dB of the measured scattering coefficient. The Fraunhofer theory gives good agreement, within 3 dB, with the measured values of the scattering coefficient for values of the Rayleigh parameter less than 1.25. But for larger values of this parameter the theoretical law in the Fraunhofer approximation deviates notably from the measured coefficient, more than 16 dB for a Rayleigh parameter of 3.7.

Figure 19 describes the measurements and predicted values of the scattering coefficient for the model surface that has an rms relief of 0.364 in. and a correlation length of 2.55 inches. As in the two previous cases the theory derived using the Presnel approximation agrees with the experimental results far better than does the Fraunhofer theory. The Fresnel theory is within 10 dB of the measured values of the scattering coefficient whereas the Fraunhofer theory is within 22 dB for the complete range of Rayleigh parameters shown in Fig. 19.

A comparison of Figs. 17, 18, and 19 shows that the theoretical law in the Fresnel approximation gives progressively worse agreement with the experimental results as the rms relief of the model surface increases. The reason for this disparity for the rougher surfaces is attributed to

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FIGURE 19 SCATTERING COEFFICIENT IN THE SPECULAR DIRECTION PRESSURE RELEASE RANDOM SURFACE

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the breakdown of the approximation, $\frac{\partial}{\partial n} = \frac{-\partial}{\partial z}$, used in deriving Eq. (8) from Eq. (7) in Chapter II. As shown in Table I of Chapter IV the slope of the irregularities on the surfaces increases at the same rate as the rms relief of the first three model surfaces. Apparently the slopes for model surface III, results depicted in Fig. 19, are too great to justify the replacement of differentiation along the normal with differentiation along the negative z-axis.

Figure 20 illustrates the experimental measurements and the theoretical predictions of the scattering coefficient for the model surface that has an rms relief of 0.364 in. and a correlation length of 5.10 inches. The irregularities on this model surface are characterized by the same slopes as the model surface studied in Fig. 18. The theory in the Fresnel approximation predicts values for the coefficient within 7 dB of the measured values whereas the Fraunhofer gives values within 19 dB.



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CHAFTER VI GENERAL DISCUSSION

A. Scattering Coefficient

In the previous chapter the dimensions of the insonified area, Δ_1 , Δ_2 , and l, which appear in the theoretical expressions, are calculated from Eqs. (B-1), (B-2), and (B-3), in Appendix C by using a 9 deg beamwidth for the incident radiation. Also, the mathematical analysis was simplified by approximating the elliptical scattering region with a rectangular scattering area as shown in Fig. 5a of Chapter III. The 9 deg beamwidth was chosen because

- (1) It is the beamwidth measured at the half-power points (-3 dB) of the main lobe of the directivity pattern of the incident radiation illustrated in Fig. 8 of Chapter IV, and
- (?) it gives the best agreement of the theoretical law in the Freshel approximation with the experimental scattering coefficient measured from the "smoothest" model surface, rms relief of 0.091 in., as shown in Fig. C1.

Figure 21 illustrates the influence the while of the beamwidth, and hence the dimensions that the insonified area nation the theoretical predictions. At the Rayleigh parameter of 1.8 the 8 deg beamwidth results in predicted values 1.8 dB smaller than the observed scattering coefficient, the 9 deg beamwidth predicts 0.7 dB larger values, the 10 deg beamwidth predicts 4.6 dB larger values, and the 12 deg beamwidth gives values that are 3.2 dB larger than the measured values of the scattering coefficient.

The effect that a change in the beamwiiith of the incident radiation has on the Fresnel theory for the roughest model surface, rms relief of 0.364 in., is illustrated in Fig. 22. With reference to a value of 7.2 for the Rayleigh parameter it is evident from the graph that a 6 deg beauvidth results in a theoretical value for the scattering coefficient that is 16 dB smaller than the measured coefficient, an 8 deg beauvidth predicts a 13.5 dB smaller value, a 9 deg beauvidth gives a 9 dB smaller value, 10 deg beauvidth gives a 4.5 dB lower value, and a 12 deg beauvidth results in a 5 dB lower value.





Even though the 12 deg been results in predicted values that give best agreement with the observed scattering coefficient for the roughest surface, a 9 deg beamwidth was chosen since this selection agrees best with the experimental results of the smoothest surface shown in Fig. 21. Table I in Chapter IV shows that the slopes of the irregularities of model surface III are four times as great as the slopes on model surface I, the smoothest surface. One would expect the theory, which is subject to the approximation $\partial/\partial n \approx -\partial/\partial z$, to give its best predictions for the experimental results obtained with the model surface that is characterized by the least slopes of the irregularities.

Figure 23 is a plot of Eqs. (53) and (54) of Chapter II,



which is proportional to the intensity in the Freenel approximation of a wave scattered from a plane surface (the denominator of the scattering coefficient). The change in the intensity with change in the beauwidth is the result of the presence of the Fresnel integrals, C and S, in the expression for p.p. This relation has been discussed thoroughly by Leizer (1966). A result of his study was that the term K tends to unity in a fluctuating manner as the number of Fresnel zones enclosed within the rectangular insonified area increases. Figure 23 exhibits this behavior very clearly for as the beamvidth of the incident radiation 1: increased without changing the source distance (R_{10}), receiver distance (R_{20}), and the frequency, the number of Presnel zones snelosed in the inconified area increases. The graphs show that as the beazwidth increases from 6 deg to 12 deg the term p_p_ approaches $1/(R_{10} + R_{20})^2$, which is the straight line with the value 69.5 x 10⁻⁶. Figure b-4 in Appendix B shows that a 9 deg beauwidth produces an inconified area that encloses only one Frescel zone. A 6 day bearvidth vill insozify an area less than one Fresnel zone whereas a 12 deg beauwidth will inzonify

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FIGURE 23 RELATIVE INTENSITY (FRESNEL) REFLECTED IN THE SPECULAR DIRECTION FROM A PLANE PRESSURE RELEASE SURFACE

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approximately the first two Fresnel zones. Thus, the solution for the intensity reflected from a plane surface calculated in the Fresnel approximation for an active scattering region that is rectangular will become steady as the value K = 1 is approached, and in turn the scattering coefficient will become stable.

Leizer (1965) has shown also that the stability of the intensity reflected from a plane surface is strongly governed by the shape of the insonified area used in the calculations in the Fresnel approximation. His study has shown that when an insonified area of elliptical shape is used in the calculation of $p_0 p_0^*$ the term K will fluctuate between constant limits 0 and 4. The indicated minima (K = 0) and maxima (K = 4) are obtained when an even or odd number of Fresnel zones, respectively, exactly fit into the elliptical region. Thus, if an elliptical insonified area had been used in the derivation of the scattering coefficient in this paper, an infinite value for this coefficient would have resulted whenever a beamwidth was chosen that insonified an even number of Fresnel zones. The information of primary importance obtained from Leizer's study was that the solution for the reflected intensity would not stabilize as the number of Fresnel zones encompassed by the elliptical insonified area increased, whereas the colution would stabilize as the number of Fresnel zones enclosed by the rectangular. insonified area increased.

This phenomenon does not occur experimentally because the insonified area is not well-defined as Fig. 8 shows. Consequently, the fluctuating behavior of the solution in the Freenel approximation could be eliginated if better approximations of the transducer beam patterns could be inserted into the theory. Morton and Muir (1967) have used the expression



to discribe the effect of the radiation pattern of the source on the scattered intensity. This relation describes a transducer pattern whose major lobe insomifies an area mAB of elliptical shape on the surface, and that loes not have any minor lobes. Their calculations were made in the Fraunhofer approximation. It is conceivable that the theory presented in the current paper could be improved if the above approximation of the radiation pattern of the source could be inserted into the theoretical development in the Freshel approximation. However, one may encounter insurmountable analytical difficulties.

The scattered intensity calculated by using the statistical parameters of model surface I and model surface III is plotted versus the grazing angle in Figs. 24 and 25, respectively. The term pp^* is the numerator of the scattering coefficient and represents the intensity except for a factor of 1/2Z where Z is the specific acoustic impedance of the medium. The graphs show t there is only a small change in the intensity as the beamwidth varies from 8 deg to 12 deg compared with the fluctuating behavior of the formulas derived using the plane surface as depicted in Fig. 23. Thus, the variability of the scattering coefficient with beamwidth, as illustrated in Figs. 21 and 22, is attributed mainly to the fluctuating behavior of the theoretical expression for the intensity reflected from a plane surface, the denominator of the scattering coefficient.

B. Fraunhofer and Fresnel Approximations for the Intensity

The theoretical scattering coefficient in the Fraunhofer approximation gave progressively worse predictions for the observed scattering coefficient as the rms relief $\sqrt{\langle F_0^2 \rangle}$ and/or as the Rayleigh parameter of the rough surface increased. This deviation between the Fraunhofer theory and the experimental results was evident in Figs. 17, 18, and 19 of Chapter V. Thus, the Fraunhofer approximation predicts incorrect values for the scattering coefficient as defined by Eq. (48) of Chapter II, except possibly for the very "smoothest" surfaces.

This phenomenon is reversed when only the intensity of the wave is considered. From Eq. 48 the intensity is

$$\langle I \rangle = \frac{p_0 p_0}{2\rho v} \sigma_s$$
 (96)



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FIGURE 24 RELATIVE INTENSITY IN THE SPECULAR DIRECTION PRESJURE RELEASE RANDOM SURFACE



In the Fraunhofer approximation $p_0 p_0^*$ is given by Eq. (C-5) in Appendix C, and σ_s is given by Eq. (79) in Chapter III. These relations give for the Fraunhofer intensity

$$\left\langle I \right\rangle_{FR} = \frac{k^2 \ell^2 (\Delta_1 + \Delta_2)^2 \sin^2 \psi}{2\rho v \pi^2 R_{10}^2 R_{20}^2} e^{-g} \left[1 + \frac{a^2 \pi}{\ell (\Delta_1 + \Delta_2)} \sum_{m=1}^{m=\infty} \frac{g^m}{m!m^2} \right] . \quad (97)$$

In the Fresnel approximation p_{OO}^{*} is given by Eq. (C-12) in Appendix C, and σ_{s} by Eq. (83) in Chapter III. These expressions give for the Fresnel intensity

$$\langle I \rangle_{FL} = \frac{K}{2\rho v (R_{10} + R_{20})^2} e^{-g} \left[1 + \frac{2}{g} \sum_{m=1}^{m=\infty} \frac{g^m}{m!} J_1(\frac{a}{m}) \right] .$$
 (98)

The ratio of the Fraunhofer to the Fresnel intensity is

an constructions.

$$\frac{\langle I \rangle_{F^{F}}}{\langle I \rangle_{FL}} = \frac{k^{2} \ell^{2} (R_{10} + R_{20})^{2} (\Delta_{1} + \Delta_{2})^{2} \sin^{2} \psi \left[1 + \frac{a^{2} \pi}{\ell (\Delta_{1} + \Delta_{2})} \sum_{m=1}^{m=\infty} \frac{g^{m}}{m! m^{2}} \right]}{\pi^{2} R_{10}^{2} R_{20}^{2} K} \left[1 + \frac{2}{g} \sum_{m=1}^{m=\infty} \frac{g^{m}}{m!} J_{1}(\frac{a}{m}) \right]}$$
(99)

Figure 26 is a plot of this ratio in lecibels given by the relation

$$dB = 10 \log_{10} \left(\frac{\langle I \rangle_{FR}}{\langle I \rangle_{FL}} \right)$$

for the plane surface and the four model surfaces listed in Table I of Chapter IV. A 9 deg beamwidth was used in the calculations. At a grazing angle of 80 deg the ratio for the plane surface is 12 dB, for surface I the ratio is 8 dB, for surface II the ratio is 0.7 dB, and for surfaces III and IV the ratio is less than 0.2 dB. From the graph it is evident that for grazing angles greater than 30 deg the intensity calculated in the Fraunhofer approximation is within 1 dB of the intensity calculated in the



Fresnel approximation for the roughest model surface. But for the plane surface the two theories differ by as much as 13 dB for grazing angles greater than 30 deg. Also, the effect that the correlation length has on the approximations is revealed. As the correlation length of a rough surface decreases, the intensity derived using the Fraunhofer approximation approaches the values predicted by the Fresnel approximation. Therefore, the theoretical law for the intensity scattered from a rough surface calculated in the Fraunhofer approximation will give poor agreement with the observed intensity for the smoother surfaces and good agreement for the rougher surfaces.

Figure 27 is a plot of the same ratio vs the Rayleigh parameter for model surfaces I, II, and III. For Rayleigh parameters greater than 4 for the numerical example used throughout this paper, the Fraunhofer approximation will give values for the intensity that are within 1 dB of the values predicted by the theory using the Fresnel approximation. Thus, the choice to use either the Fraunhofer or Fresnel approximation to study the <u>intensity</u> scattered from a surface will depend upon the roughness. A study of the <u>scattering coefficient</u> will require the Fresnel approximation in every case except for a very smooth surface where the Rayleigh parameter is less than one.

C. Application of Model Studies to Ocean and Shallow Water Studies

The slant line distance from the source to the scattering surface in the model studies was 5 ft. A common distance from source to the sea bottom for ocean studies is 5000 ft, which requires a factor of 1000 to scale the model studies to an actual situation in the ocean. Shallow water studies are conducted in many instances at a depth of 100 ft. In this case a scale factor of 20 is needed. The type of source, diameter of source, and frequency of operation needed for these studies to be directly applicable are listed in Table II. Also, the rms relief and the correlation length of the model surfaces, listed in Table I of Chapter IV, should be scaled to conform with the shallow water and ocean studies.

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TABLE II

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د د SCALE OF EXPERIMENTAL MODEL STUDIES TO OCEAN AND SHALLOW WATER STUDIES

Experimental Case	Scale Factor	Distance from Source to Surface ft	Diameter of Source ft	Frequency	Type of Source
Shallow Water	ଝ	100	8.20	4.79 kHz	Sonar Transducer Mounted on Ship
Ocean	1000	5000	410	95.8 Hz	Array Mounted on Ocean Bottom
Model	pt	ſ	0.410	95.8 ktz	Transducer

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CHAPTER VII CONCLUSIONS

The theoretical approach of Gulin (1962) has been used successfully to predict the observed amplitude and phase fluctuations of an acoustic wave scattered in the specular direction from a rough surface that is described statistically by an exponential covariance function. The theoretical expressions were limited to values of the Rayleigh parameter less than one. The comparison between experimental and theoretical results showed the necessity of the Fresnel theory to describe the amplitude fluctuations. Fraunhofer and Fresnel approximations gave equally good agreement with the experimental phase fluctuations.

The experimental amplitude and phase fluctuations were presented for values of the Rayleigh parameter less than 7.4. The observed amplitude fluctuations in the specular direction for the model studies showed the same behavior as the measurements made in the sea by Gulin and Malyshev (1962), who limited their study to values of the Rayleigh parameter less than 3.5. The author is unaware of any other measurements of the phase fluctuations of underwater acoustic waves scattered from a rough surface. Since accurate measurements of the phase fluctuations are difficult, further experimental studies are desirable.

The theoretical development had to be extended to the Fresnel approximation to obtain values for the scattering coefficient that were in good agreement with the measured values. Even though the theoretical model produced values, which fluctuated notably whenever only a few Fresnel zones were insonified, the theory in the Fresnel approximation gave significantly better agreement with the experimental scattering coefficient than the Fraunhofer approximation for the four randomly rough model surfaces used in this work.

The theory of rough surface scattering presented in this paper used the following assumptions or simplifying procedures:

- 1. It was assumed that all the radiation that impinged on the rough interface was either reflected or scattered.
- 2. Multiple scattering was neglected.

- 3. Shadowing effects were neglected.
- 4. The radius of curvature of the scattering elements was taken to be much greater than the wavelength of the incident radiation. This restriction along with assumptions 1, 2, and 3 allowed the field at each point of the surface to be represented as the sum of the incident wave and a wave reflected from the plaue tangent to the surface at the given point. These statements constitute the Kirchhoff approximation.
- 5. The distances of the source and the receiver from the scattering surface were assumed to be much greater than a wavelength of the incident radiation.
- 6. The irregularities were assumed to have sufficiently small slopes so that differentiation along the normal to the surface can be approximately replaced by differentiation along the z-axis. Beckmann (1963) avoided making this assumption by inserting into his theory the complete "local" scattering geometry. However, the additional terms obtained by making this insertion reduced to uni'y whenever the problem was restricted to the special case of specular scattering.
- 7. Edge effects caused by the finite size of the model surfaces were neglected in the theory.
- 8. The amplitude of excitation of the incident radiation was accumed to be practically constant over the active scattering region.
- 9. The theory for the amplitude and phase fluctuations was restricted to values of the Rayleigh parameter less than one.
- 10. The probability-density function that governs the simultaneous occurrence of the elevations on the rough surface was assumed to be a bivariate Gaussian distribution.
- 11. The rough surface was assumed to be randomly rough, isotropic, and stationary in the wide sense.
- 12. All calculations were restricted to the specular direction.

- 13. The Fresnel approximation of neglecting cubic and higher ordered terms was employed.
- 14. The active scattering area was assumed to be large enough so that many surface irregularities were enclosed within this region.
- 15. In the application of the theoretical formulas to an actual situation it was assumed that the active scattering region was the insonified area irrespective of the number of Fresnel zones insonified.
- 16. The beam pattern of the energy radiated from the source was approximated by a cone that had a beamwidth measured at the half-power points (-3 dB) of the actual transducer radiation pattern.
- 17. The elliptical insonified area resulting from the intersection of the conical beam of inside... radiation with the scattering surface was approximated by a rectangular active scattering region.

Although this list of assumptions is lengthy, they are utilized by nearly all workers in this field. The substantiated agreement between experiment and theory shows the assumptions are physically acceptable. The major innovation of the present study is assumption 13. Although the Fresnel assumption has been utilized in a few theoretical studies, nowhere has a systematic study been made of the importance of the Fresnel terms.

In future studies it may be possible to withdraw some of these assumptions at the cost of mathematical simplicity. For example, it may be possible to remove assumption 16 by using a better approximation for the radiation pattern of the source such as was discussed by Horton and Nuir (1967). It is very lively that restriction 12 could be generalized somewhat to include the case of any receiving angle independent of the grazing angle in the plane of incidence.



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THE FRAUNHOFER AND FRESNEL APPROXIMATIONS

Figure 3 depicts the coordinate system with the origin at O. The geometry of the scattering of acoustic waves from a randomly rough surface with the origin at O' is illustrated in Fig. A-1. As in Fig. 3 the source Q and the receiver A are confined to the x-z plane. This translation of the origin along the x-axis is expressed as

 $x = x_1 + R_{10} \cos \psi_1$

and for the specular direction can be written as

$$x_1 = x - \frac{Lz_1}{z_1 + z_2}$$
 (A-1)

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With reference to Fig. A-1 the distance R_1 can be expanded in terms of R_{10} . With the angle between r and R_{10} denoted as θ_1 , the law of cosines gives

$$R_{1} = \left[R_{10}^{2} + r^{2} - 2R_{10}r \cos \theta_{1} \right]^{1/2} . \qquad (A-2)$$

This equation can be expressed in terms of the directional angles, α_1 and γ_1 , of the unit vector, \vec{n}_1 , directed from the origin 0' toward the source Q,

$$\vec{n}_1 = \vec{e}_1 \cos \alpha_1 + \vec{e}_2 \cos \gamma_1$$
,

and writing r as

Thus, vith

$$\vec{n}_1 = |\vec{r}| |n_1| \cos \theta_1 = r \cos \theta_1 = x_1 \cos \alpha_1 + 2 \cos \gamma_1$$

ř. ř = r² = x₁² + y² + z²



R₁ becomes

$$R_{1} = \left(R_{10}^{2} - 2R_{10}x_{1}\cos\alpha_{1} - 2R_{10}z\cos\gamma_{1} + x_{1}^{2} + y^{2} + z^{2}\right)^{1/2} \quad . \quad (A-3)$$

 R_1 can be expressed in terms of the grazing angle ψ_1 ,

$$\alpha_{\underline{1}} = 100 \text{ deg} - \psi_{\underline{1}}$$

$$\cos \alpha_{\underline{1}} = -\cos \psi_{\underline{1}}$$

$$\gamma_{\underline{1}} = 90 \text{ deg} - \psi_{\underline{1}}$$

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as

$$R_{1} = R_{10} \left(1 + \frac{2x_{1} \cos \psi_{1}}{R_{10}} - \frac{2z \sin \psi_{1}}{R_{10}} + \frac{x_{1}^{2} + y^{2} + z^{2}}{x_{10}^{2}} \right)^{1/2} \quad . \quad (A-4)$$

Expending Eq. (A-4), one obtains for R

$$R_{1} = (R_{10} + x_{1} \cos \psi_{1} - z \sin \psi_{1}) + \left(\frac{x_{1}^{2} \sin^{2} \psi_{1}}{2R_{10}} + \frac{y^{2}}{2R_{10}} - \frac{x_{1}^{2} \cos^{2} \psi_{1}}{2R_{10}} + \frac{y^{2}}{2R_{10}} + \frac{y$$

$$\frac{1}{2R_{10}} - \frac{1}{R_{10}} + \frac{1}{R_{10}} + \frac{1}{2R_{10}^2} +$$

$$\frac{x_1^2}{2R_{10}^2} \operatorname{sin} \left\{ \frac{x_1^2}{2R_{10}^2} \operatorname{sin}^2 \left\{ \frac{x_1^2}{2R_{10}^2} \operatorname{sin}^2 \left\{ \frac{x_1^2}{2R_{10}^2} \operatorname{sin}^2 \left\{ \frac{x_1^2}{2R_{10}^2} \operatorname{cos}^2 \right\} \right\} + \frac{x_2^2}{2R_{10}^2} \operatorname{cos}^2 \left\{ \frac{x_1^2}{2R_{10}^2} \operatorname{cos}^2 \left\{ \frac{x_1^2}{2R_{10}^2} \right\} + \operatorname{higher order terms} \right\}$$

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(A-5)

In a similar way
$$R_2$$
 can be written in terms of R_{20} to give

$$R_2 = (R_{20} - x_1 \cos \psi_2 - z \sin \psi_2) + \left(\frac{x_1^2 \sin^2 \psi_2}{2R_{20}} + \frac{y^2}{2R_{20}} + \frac{y^2}{2R_{20}} + \frac{x_1^2 \sin \psi_2 \cos \psi_2}{R_{20}}\right) + \left(\frac{x_1^2 \sin^2 \psi_2}{2R_{20}} + \frac{x_1^2 \cos \psi_2}{R_{20}}\right) + \left(\frac{x_1^2 \sin^2 \psi_2 \cos \psi_2}{2R_{20}} + \frac{x_1^2 \cos \psi_2}{R_{20}} + \frac{x_1^2 \cos^2 \psi_2}{2R_{20}} + \frac{x_1^2 \cos^2$$

Adding Eqs. (A-5) and (A-6), one obtains

$$\begin{split} & R_1 + R_2 = \left[R_{10} - R_{20} + x_1(\cos \psi_1 - \cos \psi_2) - z(\sin \psi_1 + \sin \psi_2) \right] \\ & + \left[x_1^2 \left(\frac{\sin^2 \psi_1}{2R_{10}} + \frac{\sin^2 \psi_2}{2R_{20}} \right) + y^2 \left(\frac{1}{2R_{10}} + \frac{1}{2R_{20}} \right) \right] \\ & + z^2 \left(\frac{\cos^2 \psi_1}{2R_{10}} + \frac{\cos^2 \psi_2}{2R_{20}} \right) + x_1 z \left(\frac{\sin \psi_2 \cos \psi_2}{R_{20}} - \frac{\sin \psi_1 \cos \psi_1}{R_{10}} \right) \right] \\ & + \left[x_1^3 \left(\frac{\sin^2 \psi_2 \cos \psi_2}{2R_{20}^2} - \frac{\sin^2 \psi_1 \cos \psi_1}{2R_{10}^2} \right) + z^3 \left(\frac{\sin \psi_1 \cos^2 \psi_1}{2R_{10}^2} - (A-7) \right) \right] \\ & + \frac{\sin \psi_2 \cos^2 \psi_2}{2R_{20}^2} + x_1^2 \left(\frac{\sin^3 \psi_1}{2R_{10}^2} + \frac{\sin^5 \psi_2}{2R_{20}^2} \right) \\ & + x_1 z^4 \left(\frac{\cos^3 \psi_2}{2R_{20}^2} - \frac{\cos^3 \psi_1}{2R_{10}^2} \right) + x_1 y^2 \left(\frac{\cos \psi_2}{2R_{20}^2} - \frac{\cos \psi_1}{2R_{10}^2} \right) \\ & + zy^2 \left(\frac{\sin \psi_1}{2R_{20}^2} + \frac{\sin \psi_2}{2R_{20}^2} \right) \right] \\ & + higher order terms \end{split}$$

In the integrals appearing in Chapter II the z dependency is taken out of $R_1 + R_2$ as illustrated in Eqs. (11) and (13) so that one is interested in $R_1' + R_2'$. Thus, for z = 0, $R_1 = R_1'$ and $R_2 = R_2'$ and Eq. (A-7) reduces to

$$R_{1}' + R_{2}' = \left[R_{10} + R_{20} + x_{1}(\cos \psi_{1} - \cos \psi_{2})\right] + \left[x_{1}^{2}\left(\frac{\sin^{2}\psi_{1}}{2R_{10}} + \frac{\sin^{2}\psi_{2}}{2R_{20}}\right) + y^{2}\left(\frac{1}{2R_{10}} + \frac{1}{2R_{20}}\right)\right] + \left[x_{1}^{2}\left(\frac{\sin^{2}\psi_{2}\cos\psi_{2}}{2R_{20}^{2}} - \frac{\sin^{2}\psi_{1}\cos\psi_{1}}{2R_{10}^{2}}\right)\right]$$

$$(A-8)$$

+
$$x_1 y^2 \left(\frac{\cos \psi_2}{2R_{20}^2} - \frac{\cos \psi_1}{2R_{10}^2} \right) \right]$$
 + quartic and higher order terms

The Fraunhofer approximation is the neglect of the quadratic and higher order terms in Eq. (A-8). The quadratic terms are insignificant whenever

$$\kappa \left[\frac{2}{2} \left(\frac{\sin^2 \psi_1}{2R_{10}} + \frac{\sin^2 \psi_2}{2R_{20}} \right) + \ell^2 \left(\frac{1}{2R_{10}} + \frac{1}{2R_{20}} \right) \right] < 1 \qquad (A-9)$$

vhere

k is the wavenumber,

- Δ_2 is the maximum dimension of the active scattering region in the $+x_1--direction,$ and
- \underline{l} is the maximum dimension of the active scattering region in the +y--direction (Δ_2 and l are discussed in detail in Appendix B).

It is seen from solving Eq. (A-9) for k_{20} that the receiver will be in the Fraunhofer zone (farfield) when

$$R_{20} \gg \frac{k \left(\Lambda_{2}^{2} \sin^{2} \psi_{2} + l^{2} \right)}{2 - \frac{k}{R_{10}} \left(\Lambda_{2}^{2} \sin^{2} \psi_{1} + l^{2} \right)} \qquad (A-10)$$

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The Fresnel approximation is the neglect of the cubic and higher order terms in Eq. (A-8). The receiver will be situated in the Fresnel zone when

$$k \left[\Delta_{2}^{4} \left(\frac{\sin^{2} \psi_{2} \cos \psi_{2}}{2R_{20}^{2}} - \frac{\sin^{2} \psi_{1} \cos \psi_{1}}{2R_{10}^{2}} \right) + \Delta_{2} \ell^{2} \left(\frac{\cos \psi_{2}}{2R_{20}^{2}} - \frac{\cos \psi_{1}}{2R_{10}^{2}} \right) \right] \ll 1 . (A-11)$$

Solving for R_{20} , one obtains

$$R_{20} \gg \left[\frac{\frac{k \cos \psi_2 \left(\Delta_2^{5} \sin^2 \psi_2 + \Delta_2 l^2 \right)}{\frac{k \cos \psi_1}{2} \left(\Delta_2^{5} \sin^2 \psi_1 + \Delta_2 l^2 \right)}}{\frac{k \cos \psi_1}{R_{10}} \left(\Delta_2^{5} \sin^2 \psi_1 + \Delta_2 l^2 \right)} \right]^{1/2} .$$
 (A-12)

Thus, the receiver lies within the Fresnel zone when

$$\frac{k\left(\Delta_{2}^{2}\sin^{2}v_{2}+t^{2}\right)}{2-\frac{k}{R_{10}}\left(\Delta_{2}^{2}\sin^{2}v_{1}+t^{2}\right)} \gg R_{20}$$

$$\approx \left[\frac{k\cos v_{2}\left(\Delta_{2}^{3}\sin^{2}v_{1}+c_{2}t^{2}\right)}{\frac{k\cos v_{1}\left(\Delta_{2}^{3}\sin^{2}v_{1}+c_{2}t^{2}\right)}{R_{10}}\right]^{1/2} \cdot (A-13)$$

The Fraunhofer approximation to Eq. (A-3) can be written as

$$R_1 + R_2 - R_{10} + R_{20} + ex_1$$
 (A-14)

where

$$\mathbf{c} = \mathbf{cos} \, \mathbf{v}_1 - \mathbf{cos} \, \mathbf{v}_2 \, \mathbf{.}$$
 (A-17)

The Fresnel approximation to Eq. (A-8) can be written as

$$R'_{1} + R'_{2} = R_{10} + R_{20} + cx_{1} + \frac{1}{R_{s}}x_{1}^{2} + \frac{1}{R}y^{2}$$
, (A-16)

where

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$$R_{s} = \frac{2R_{10}R_{20}}{R_{10}\sin^{2}\psi_{2} + R_{20}\sin^{2}\psi_{1}}, \text{ and } (A-17)$$

$$R = \frac{2R_{10}R_{20}}{R_{10} + R_{20}} \qquad (A-18)$$

For the specular direction

¥1 = ∛2 = ∛

and Eq. (A-8) reduces to

 $R_{1}' + R_{2}' = R_{10} + R_{20} + \frac{1}{R} \left(x_{1}^{2} \sin^{2} \psi + y^{2} \right)$

+ quartic and higher order terms

Note that the linear and cubic terms Five vanished so that the relation is exact through cubic terms for the Freenel approximation in the specular direction. The Fraunhofer approximation in the specular direction is simply

$$R'_1 + R'_2 = R_{10} + R_{20}$$
 (A-20)

When the Fraunhofer approximation, Eq. (A-20) or the Fresnel approximation. Eq. (A-19), is made in Eqs. (14), (17), (20), or (21) of Chapter II only the approximations

$$R_1^{+}R_2^{+} = R_{10}^{-}R_{20}^{+}$$
,
 $R_1^{+} = R_{10}^{-}$, and (A-21)
 $R_2^{+} = R_{20}^{-}$,

90

(A-19)

The Fresnel approximation to Eq. (A-8) can be written as

$$R'_{1} + R'_{2} = R_{10} + R_{20} + ex_{1} + \frac{1}{R_{s}}x_{1}^{2} + \frac{1}{R}y^{2}$$
, (A-16)

where

$$R_{g} = \frac{2R_{10}R_{20}}{R_{10}\sin^{2}\psi_{2} + R_{20}\sin^{2}\psi_{1}} , \text{ and } (A-17)$$

$$R = \frac{2R_{10}R_{20}}{R_{10} + R_{20}} \qquad (A-18)$$

For the specular direction

$$\Psi_1 = \Psi_2 = \Psi$$

and Eq. (A-8) reduces to

$$R_1' + R_2' = R_{10} + R_{20} + \frac{1}{R} \left(x_1^2 \sin^2 \psi + y^2 \right)$$

+ quartic and higher order terms

Note that the linear and cubic 145%s have vanished so that the relation is exact through cubic terms for the Freenel approximation in the specular direction. The Fraunhofer approximation in the specular direction is simply

$$R_1' + R_2' = K_{10} + R_{20}$$
 (A-20)

When the Fraunhofer approximation, Eq. (A-20) or the Fresnel approximation. Eq. (A-19), is made in Eqs. (14), (17), (20), or (21) of Chapter II only the approximations

$$R_1'R_2' = R_{10}R_{20}$$
,
 $R_1' = R_{10}$, and (A-21)
 $R_2' = R_{20}$,

90

(A-19)

are used in all the terms of the integrand except in the term $k(R_1^{\prime} + R_2^{\prime})$ that represents the phase. What is implied by these approximations is that at distances from the uneven surface. large compared to the dimensions of the active scattering region, the amplitude of the pressure wave produced by any particular surface element differs only slightly from that produced by any other element. On the other hand the relative phase of the pressures produced at the receiver by any two surface elements depends on the differences in the distance of the two elements from the receiver, and for distant points this difference is practically independent of P_1 and R_2^{\prime} . As a consequence, one must use the entire expressions given in Eqs. (A-19) and (A-20) in the term $k(R_1^{\prime} + R_2^{\prime})$ appearing in the integrands of Eqs. (14), (17), (20), and (21).

Consider a numerical calculation of the Fraunhofer and Fresnel distances, R_{20} , as given by Eqs. (A-10) and (A-12). The source is placed 60.0 in. (R_{10}) from the surface. With the source and receiver maintained in the specular direction, the grazing angle of the source is varied. It is desired to know where the receiver must be placed relative to the surface (R_{20}) so that the experimental data can be compared with theory that was developed using either the Fraunhofer or the Frasnel approximation. These positions of the receiver are referred to as the Fraunhofer field zone or the Freenel field zong, whichever the case may be. The calculations are made for a frequency of 95.8 kHz and for a 9 deg beauvidth of the source. The active region of the surface is taken to be the inconified area whose dimensions are calculated from Eqs. (B-1), (B-2), and (B-3) in Appendix 8. The results for this plane surface example are illustrated in Fig. A-2. The calculations show that it is physically impossible to place the receiver in the Fraunhofer field zone for specular reflection from a plang surface. The receiver sust be placed at distances much greater than 44 in. (R_{go}) from the pressure release plane surface at 95.8 kHz in order that it remain in the Freenel field zone at all receiving angles. The abrupt change in the boundary of the Fresnel field some at 22 deg receiving angle is the result of the finite dimensions of the reflecting surface, that is, A, is held constant = 16 in. for grazing angles less than 22 dec.



In Chapter VI it is shown (see Fig. 27) that as the roughness (the Rayleigh parameter which for a plane surface is zero) increases. the Fraunhofer field zone moves in from infinity to some finite distance from the rough surface. Figure 27 of Chapter VI shows for the numerical example cited that for values of the hayleigh parameter greater than 4 the Fraunhofer approximation predicts quantities that are well within 1 dB of the values obtained using the Fresnel approximation. The interpretation of this behavior is that for very rough surfaces the irregularities on the surface act as individual scatterers. Thus, one should not insert into Eqs. (A-10) and (A-12) the dimensions of the active scattering region, Δ_2 and l, but should substitute the dimensions of the surface irregularities. For example, if one took the correlation length <u>a</u> (2.55 in. for three of the model surfaces) for the maximum x and y dimensions, Eq. (A-10) would give for the previously cited example for a grazing angle of 40 deg

$$R_{20} >> 15$$
 in.

for the Fraunhofer field zone.

In summary, for Rayleigh parameters much less than one the dimensions of the active scattering region are used in Eqs. (A-10) and (A-12). When only a few Fresnel zones on the surface are insonified the dimensions of the active scattering region are taken to be the dimensions of the insonified area (see Appendix B). For Rayleigh parameters much greater than one the dimensions of the irregularities on the surface are used. A satisfactory approximation of the size of the irregularities is the correlation length, \underline{a} .

 \mathcal{D}
APPENDIX B

ACTIVE SCATTERING REGION

What regions of the scattering surface are the most important in contributing to the total field at a given receiving point when the surface is illuminated by a source at a given point? Beckmann (1963) has stated that the answer is "the first few Fresnel zones" located on the scattering surface. For an omnidirectional source one would use the dimensions of the first few Fresnel zones as the limits on the integrals appearing in Chapter II.

In the experimental work a directional source was used, which has a directivity pattern illustrated in Fig. B-1. Only a finite portion of the scattering surface is insonified. The insonified area will be defined as the intersection of the half-power points (-3 dB) of the main lobe of the directivity pattern with the scattering surface. It is of interest to determine both the dimensions of the first few Fresnel zones and the dimensions of the irradiated surface.

The length of the irradiced surface is determined by the angular width of the directivity characteristic of the transmitter, the slope of its axis, and the distance from the transmitter to the scattering surface. The dimensions of the insonified area can be derived in the following manner. From Fig. B-1

$$A = 180 \ \deg - \frac{P}{2} - V_1$$

where ψ_1 is the grazing angle and ϕ is the beauvidth measured at the half-power points of the major lobe. The relations

D = 180 deg - A = $\frac{9}{2} + \frac{1}{2}$, and B = 180 deg - $\frac{9}{2} - C = -\frac{9}{2} + \frac{1}{2}$,

give for the linear dimensions Δ_1 and Δ_2 in the x_1 direction

$$\Delta_1 = R_{10} \cos \psi_1 - r_1 \cos(\psi_1 + \frac{1}{2}) , \text{ and}$$
$$\Delta_2 = r_2 \cos(\psi_1 - \frac{1}{2}) - R_{10} \cos \psi_1 .$$



From the law of sines

$$r_{1} = \frac{R_{10} \sin \psi_{1}}{\sin(\psi_{1} + \frac{\varphi}{2})} , \text{ and}$$

$$r_{2} = \frac{R_{10} \sin \psi_{1}}{\sin(\psi_{1} - \frac{\varphi}{2})}$$

give for \triangle_1 and \triangle_2 ,

$$\Delta_{1} = R_{10} \left[\cos \psi_{1} - \cot(\psi_{1} + \frac{\phi}{2}) \sin \psi_{1} \right] , \qquad (B-1)$$

$$\Delta_{2} = R_{10} \left[-\cos \psi_{1} + \cot(\psi_{1} - \frac{\phi}{2}) \sin \psi_{1} \right] . \qquad (B-2)$$

With the projector confined to the x_1 -z plane the linear dimensions of the insonified area in the y direction is calculated easily from the right triangle in Fig. P-2a to give

$$l = R_{10} \tan \frac{\sigma}{2}$$

The insonified area, shown in Fig. B-2b, is an ellipse with a semimajor axis given by

$$a = \frac{1}{2} (\Delta_1 + \Delta_2)$$

and a semiminor axis given by

$$b = \frac{l(\Delta_1 + \Delta_2)}{\sqrt{10\Delta_1\Delta_2 - 3(\Delta_1^2 + \Delta_2^2)}}$$

The area of an ellipse is mab, so that

The stand in the second of the

area =
$$\frac{\pi l (\Delta_1 + \Delta_2)^2}{2\sqrt{10\Delta_1 \Delta_2} - 3(\Delta_1^2 + \Delta_2^2)}$$

Kerr (1951) gives formulas for the calculation of the dimensions of the Fresnel zones. The geometrical arrangement of source and receiver for these calculations is shown in Fig. B-3. The origin lies on the

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(B-3)

(B++

(B-6





reflecting plane directly below the source. The receiver is in the specular direction. The locus of all points in the x - y plane, from which the secondary radiation arrives at the receiver with a constant phase difference δ with respect to the direct radiation along r, is given by the relation

 $R_1' + R_2' - r = \delta$

 $R_{10} + R_{20} - r = \delta_0$

 $\delta_n = \delta_n = n \frac{\lambda}{2} ,$

The path difference δ_{α} via the geometrical point of reflection is

or

$$\delta_n = R_{10} + R_{20} - r + n \frac{\lambda}{2} \qquad (B-7)$$

where $\underline{\lambda}$ is the wavelength of the incident radiation. Expressing the ellipses in terms of L, δ_n, z_1 , and z_2 one obtains for the x coordinate of the center of the ellipse

 $x_{on} = \frac{I_{i}}{2} \left\{ 1 - \frac{\left(\frac{z_{2} - z_{1}}{L}\right)^{2}}{\left[\frac{8}{L} + \sqrt{1 + \left(\frac{z_{2} - z_{1}}{L}\right)^{2}}\right]^{2}} - 1 \right\} = \frac{I_{i}z_{1}}{z_{1} + z_{2}}, \quad (B-8)$

where the origin appleted in Fig. B-3 has been transformed to the point of geometrical reflection by means of Eq. (A-1) in Appendix A. The semiminor axis is

$$\mathbf{b}_{n} = \pm \frac{L}{2} \sqrt{\left(\frac{b}{L}\right)^{2}} + \frac{2b}{L} \sqrt{1 + \left(\frac{2}{L} - \frac{z}{2}\right)^{2}}$$

$$\times \sqrt{1 - \frac{\left(\frac{z_{1} + z}{L}\right)^{2}}{\frac{L}{L}}} - \frac{\left(\frac{z_{1} + z}{L}\right)^{2}}{\frac{z_{2}}{L}} - \frac{1}{2} - \frac{1}{2}$$

100

and the second of the second second

(8-9)

and the semimajor axis is

 $a_{n} = b_{n} \sqrt{1 + \frac{1}{\left[\frac{\delta_{n}}{L} + \sqrt{1 + \left(\frac{z_{2} - z_{1}}{L}\right)^{2}}\right]^{2}} - 1}$ (B-10)

The x intercepts relative to the goometrical point of reflection are

 $\Delta_{ln} = a_{n} - x_{on} , \text{ and } (B-ll)$

$$A_{2n} = a_n + x_{on}$$
 (B-12)

From Eq. (B-5) the y intercepts are

$$u_{n} = \pm \frac{b_{n}}{2a_{n}} \sqrt{10\Delta_{1}\Delta_{2} - 3\Delta_{1}^{2} + \Delta_{2}^{2}}$$
(B-13)

These three equations for the Fresnel zones correspond to Eqs. (B-1), (B-2), and (B-3) for the irradiated area.

Consider a numerical calculation of the dimensions of the insonified area and the dimensions of the first three Fresnel zones. The source is placed 60.0 in. (R_{10}) from the surface and the receiver is maintained in the specular direction at the distance of 60.0 in. (R_{20}) from the surface. The beamwidth of the projector (source) is 9 deg. The results are depicted in Fig. B-4 for a frequency of 95.8 kHz for grazing angles 20 and 60 deg. The calculations show that the dimensions of the insonified area are slightly larger than the dimensions of the first Fresnel zone. The scattering surface boundary shows the relative size of the model surfaces to the insonified area.

If the beauwidth of the source is varied from 6 deg to 13 deg for the situation just described, the insonified area will enclose slightly less than one Freenel zone to almost two Freenel zones.

It is difficult to accertain with analytical precision the part of the desttoring surface that should be taken as the active desttering region. However, since only a few Fresnel zones are insonified in the experimental study in this paper, the insonified area will be taken as the active



scattering region. Thus, the limits of the integrals appearing in Chapter II and III will be taken as the values of Δ_1 , Δ_2 , and l calculated from Eqs. (B-1), (B-2), and (B-3), respectively.

It is noted that the approximate dimensions of the insonified area Δ_1 , Δ_2 , and l are used as the limits rather than the exact dimensions of the insonified areas <u>a</u> and <u>b</u>. This approximation is equivalent to integrating over a rectangular area, which approximates closely the elliptical area.

Table B-I lists the values for Δ_1 , Δ_2 , and ℓ for grazing angles from 6 deg to 80 deg for the previous cited numerical example for a 9 deg beamwidth of source. The dimensions Δ_1 , Δ_2 , and ℓ are calculated from Eqs. (B-1), (B-2), and (B-3), respectively. The irradiated surface area for grazing angles from 6 deg through 20 deg is calculated from the relation

 $2l(\Delta_1 + \Delta_2)$

whereas the area for grazing angles from 22 deg degrees to 80 deg is calculated from Eq. (B-6).

Grazing Angle	4	<u>^2</u>	1	Insonified Area
Degrees	Inches	Inches	Inches	Sq. Inches
6	16.00	16.00	4.72	302.21
8	16.00	16.00	4.72	302.21
10	16.00	16.00	4.72	302.21
12	16.00	16.00	4.72	302.21
14	14.84	16.00	4.72	291.22
16	13.44	16.00	4.72	278,06
18	12,30	16.00	4.72	267.28
20	11.35	16.00	4.72	258.32
55	10.55	15.65	4.72	211,05
24	9.87	14.10	4.72	190,06
26	9.28	12.84	4.72	173.35
58	8.76	11.81	4.72	159.72
30	8,31	10.93	4.72	148.38
32	7.91	10.20	4.72	138,30
54	7.56	9.56	4.72	130.61
36	7.25	9.01	4.72	123.53
34	6,91	8.53	4.72	117,36
40	6,72	8,11	4.72	111.94
42	6.49	7.73	4.72	107.15
ليك	6,29	7.40	4.72	102.80
46	6.10	7.10	4.72	99.10
4 33	2.3*	6,84	4.72	95.71
50 0	<u>نې</u> د 1	6.60	4.72	98.66
Ĵż	ý.67	6.39	4.72	89,62
24	5.92	6.19	4.72	97.49
54	5.41	6.02	4.72	85,22
38	3.31	3.86	4.72	<u>93.21</u>
£.3). ? ?	3.71	4.72	81,39
6 <i>3</i>	5.15	5.38	4.72	79.76
64	5.06	5.40	L.72	78,29
626.	4.09	5.%	4.72	76.96
¥,11 ³⁴	te "Gr	5.26	4.72	75.74
* {)	ù . 1963	5.17	4.72	7 4. 73
4.9	L . 94	5.10	4.72	13.80
*1	¥.%	3.03	N. 72	72.99
**	4.77	4.96	4.72	72.28
-4	4.73	6.91	4.72	71.68
² Қ.	4.75	L.96	4.72	71.17

DIMENSIONS OF INSONIFIED AREA FOR VARIOUS GRAZING ANGLES

1.1

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Grazing Angle	^ <u>1</u>	4 ₂	l	Insonified Area
Degrees	Inches	Inches	Inches	Sq. Inches
6	16.00	16.00	4.72	302.21
8	16.00	16.00	4.72	302.21
10	16.00	16.00	4.72	302.21
12	16.00	16.00	4.72	302.21
14	14.84	16.00	4.72	291.22
16	13.44	16.00	4.72	278.06
18	12.30	16.00	4.72	267.28
20	11.35	16.00	4.72	258.32
22	10.55	15.65	4.72	211.05
24	9.87	14.10	4.72	190,06
26	9.28	12.84	4.72	173.35
28	8.76	11.81	4.72	159.72
30	8.31	10.93	4.72	148.38
32	7.91	10.20	4,72	138.30
54	7.56	9.56	4.72	130.61
30	7.25	9.01	4.72	123.53
34	6.97	8.53	4.72	117.36
40	6,12	8.11	4.72	111.94
42	6.49	7.73	4.72	107.15
ليل	6,29	7.40	4.72	102.80
46	6.10	7.10	4.72	99.1 0
1,35	3.44	6.84	4.72	95.71
50	2.74	ĕ.60	4.72	92.66
ţa.	3.65	6.19	4.78	8 9,92
54	5.92	6.19	4.72	97.43
36	5.41	6.02	L 73	<i>8</i> 5.22
5A	5.31	5.86	4.79	93.21
0	3.12	3.71	4.72	\$1.39
63	5.15	3.39	L.72	79.76
54	5.06	÷ .Lõ	4.72	78, 29
646	L.99	5.%	4.72	76,96
4,3	4.94	5.26	4.72	⇒.*\$
رائه	4. % 3	5.17	4. <i>1</i> 2	74.13
<u>†</u> 3	6. 3 4	5.10	4.72	73.80
F C.	•.**	5.03	L.72	72.99
74;	14 . T .	4.9 6	4.72	12.28
•3,	4.73	5.91	4.72	71,68
Charles -	¥.13	4. 6 8	4. 72	1 74.17

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的。 「我們們們們對於的影響的影響的影響」。 和我們們們們們們們們們的。 我們們的的。 我們們的。 我們們的。 我們們的。 我們們們的。 我們們們的。 我們們們的。 我們們們的。 我們們的。 我們們們的。 我們們的。 我們們的 我們們的 我們們的。 我們們的 我們們的

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TABLE B-1									
DIMENSIONS	OF	INSONINIED	AREA	FOR	VARIOUS	GRAZING	ANGLES		

APPENDIX C

EVALUATION OF THE PRESSURE REFLECTED FROM A PLANE SURFACE

In Chapter II, Eq. (17), we encountered the integral

$$p_{Q} = \frac{ik}{4\pi} \int_{-\Delta_{1}} \int_{-t} \frac{1}{R_{1}^{'}R_{2}^{'}} \left(\frac{z_{1}}{R_{1}^{'}} + \frac{z_{2}}{R_{2}^{'}} \right) e^{ik(R_{1}^{'}+R_{2}^{'})} dx_{1} dy , \quad (C-1)$$

where the limits Δ_1 , Δ_2 , and l are the dimensions of the active scattering region defined in Appendix B. Calculations shall be limited to the specular direction.

The Fraunhofer approximation is the neglect of the quadratic and higher order terms in the expansion of the term $R_1' + R_2'$ appearing in the exponential of Eq. (C-1). For the specular direction and for the Fraunhofer approximation Eq. (A-14) in Appendix A reduces to

 $R_1^* + R_2^* = R_{10} + R_{20}$

Setting $z_1 = R_{10} \sin \psi$, $z_2 = R_{20} \sin \psi$ and $R_1' = R_{10}$ $R_2' = R_{20}$ in the denominator of Eq. (C-1), one obtains

 $P_{0} = \frac{k!(A_{1} + A_{1}) \sin k}{kH_{1} + K_{20}} \left[\frac{k}{2} + k(R_{10} + R_{20}) \right], \text{ or } (C-2)$

 $A_{0} = \frac{kI(\Delta_{1} + \dots)^{2} d\ln v}{\pi R_{10} R_{20}}$ (C-3)

$$\phi_{0} = \frac{\pi}{2} \cdot k(R_{10} \cdot R_{20})$$
 (C-4)

vhere

A, is the applitude, and

• is the plase.

The pressure squared $p_{o}p_{o}^{*}$, which is proportional to the intensity reflected from a plane surface, is

$$p_{0}p_{0}^{*} = \frac{k^{2}l^{2}(\Delta_{1} + \Delta_{2})^{2} \sin^{2} \psi}{\pi^{2}R_{10}^{2}R_{20}^{2}} \qquad (C-5)$$

For the Fresnel approximation the quadratic terms are retained in the expansion of $R'_1 + R'_2$. The desired expansion given by Eq. (A-19) in Appendix A

$$R_1' + R_2' \approx R_{10} + R_{20} + \frac{1}{R} (x_1^2 \sin^2 \psi + y^2)$$

reduces Eq. (C-1) to

.

$$P_{0} = \frac{ik \sin \psi}{2\pi R_{10} R_{20}} e^{ik(R_{10} + R_{20})} \int_{-\Delta_{1}}^{\Delta_{2}} e^{ik\left(\frac{x_{1}^{2} \sin^{2}\psi}{R}\right)} dx_{1} \int_{-k}^{k} e^{i\frac{k}{R}y^{2}} dy \quad . \quad (C-6)$$

The integrals can not be integrated in closed form but yield infinite series. They are called Fresnel integrals and are defined as

$$C(u) = \int_{0}^{u} \cos\left(\frac{\pi}{2} u^{2}\right) du$$

(0-7)

$$S(u) = \int_{0}^{1} \pi in \left(\frac{\pi}{2} u^{2}\right) du$$

Integration of Eq. (C-6) yields

$$p_{0} = \frac{\left\{ \left[C_{\ell} \left[C_{\Delta_{1}} + C_{\Delta_{2}} \right] - S_{\ell} \left[S_{\Delta_{1}} + S_{\Delta_{2}} \right] \right]^{2} + \left[S_{\ell} \left[C_{\Delta_{1}} + C_{\Delta_{2}} \right] + C_{\ell} \left[S_{\Delta_{1}} + S_{\Delta_{2}} \right] \right]^{2} \right\}^{1/2}}{(R_{10} + R_{20})}$$

$$= \frac{i \left\{ \pi + k(R_{10} + R_{20}) - \arctan \left[\frac{C_{\ell} \left[C_{\Delta_{1}} + C_{\Delta_{2}} \right] - S_{\ell} \left[S_{\Delta_{1}} + S_{\Delta_{2}} \right]}{S_{\ell} \left[C_{\Delta_{1}} + C_{\Delta_{2}} \right] + C_{\ell} \left[S_{\Delta_{1}} + S_{\Delta_{2}} \right]} \right\} \right\}$$

$$(C-8)$$

where

$$C_{\Delta_{1}} = C \left(\Delta_{1} \sqrt{\frac{2k}{\pi R}} \sin \psi \right) ,$$

$$L_{\Delta_{2}} = C \left(\Delta_{2} \sqrt{\frac{2k}{\pi R}} \sin \psi \right) ,$$

$$C_{1} = C \left(l \sqrt{\frac{2k}{\pi R}} \right) ,$$

$$S_{\Delta_{1}} = S \left(\Delta_{1} \sqrt{\frac{2k}{\pi R}} \sin \psi \right) ,$$

$$S_{\Delta_{2}} = S \left(\Delta_{2} \sqrt{\frac{2k}{\pi R}} \sin \psi \right) ,$$

$$S_{\Delta_{2}} = S \left(\Delta_{2} \sqrt{\frac{2k}{\pi R}} \sin \psi \right) ,$$

$$S_{\Delta_{2}} = S \left(L_{2} \sqrt{\frac{2k}{\pi R}} \sin \psi \right) ,$$

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(c-9)

The amplitude is

$$A_{o} = \frac{\left\{ \left[C_{\ell} \left[C_{\Delta_{1}} + C_{\Delta_{2}} \right] - S_{\ell} \left[S_{\Delta_{1}} + S_{\Delta_{2}} \right] \right]^{2} + \left[S_{\ell} \left[C_{\Delta_{1}} + C_{\Delta_{2}} \right] + C_{\ell} \left[S_{\Delta_{1}} + S_{\Delta_{2}} \right] \right]^{2} \right\}^{1/2}}{(R_{10} + R_{20})}, (C-10)$$

and the phase is

$$\Phi_{0} = \pi + k(R_{10} + R_{20}) - \arctan \left[\frac{C_{\ell}(C_{\Delta_{1}} + C_{\Delta_{2}}) - S_{\ell}(S_{\Delta_{1}} + S_{\Delta_{2}})}{S_{\ell}(C_{\Delta_{1}} + C_{\Delta_{2}}) + C_{\ell}(S_{\Delta_{1}} + S_{\Delta_{2}})} \right] . \quad (C-11)$$

The pressure squared is

$$p_{0}p_{0}^{*} = \frac{\left[c_{\ell} \left[c_{\Lambda_{1}} + c_{\Lambda_{2}} \right] - s_{\ell} \left[s_{\Lambda_{1}} + s_{\Lambda_{2}} \right] \right]^{2} + \left[s_{\ell} \left[c_{\Lambda_{1}} + c_{\Lambda_{2}} \right] + c_{\ell} \left[s_{\Lambda_{1}} + s_{\Lambda_{2}} \right] \right]^{2}}{(R_{10} + R_{20})^{2}} \cdot (C-12)$$

The amplitude and phase obtained in the Fraunhofer approximation can be obtained from the amplitude and phase derived in the Fresnel approximation if it is assumed that the argument of the Fresnel integrals is very small so that

> $C(u) \approx u$, and $S(u) \approx 0$.

Physically, this approximation implies that both the source and receiver must be moved to infinity with the restriction that the source have a very narrow beam spread so that the dimensions of the insonified area, Δ_1 , Δ_2 , and ! are very small compared to the distance from the surface to the source.

These statements assert that it is theoretically impossible to predict the intensity reflected from a plane surface in the specular direction in the Fraunhofer approximation. A comparison of Eqs. (C-4) and (C-11) will substantiate the invalidity of the Fraunhofer approximation

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in predicting the correct phase. One would expect a phase change at the receiver relative to the source of π radians at the surface and $k(R_{10} + R_{20})$ radians as a result of the travel path. Figure C-1 depicts the phase in the Fraunhofer and Fresnel approximation for the case where

f = 95.8 kHz, k = 10.328 in.⁻¹, $R_{10} = R_{20} = 60.0$ in.

The values for Δ_1 , Δ_2 , and t for a 9 deg beamwidth of source are given in Appendix B. The term $\pi + k(R_{10} + R_{20})$ is not included in the calculations. For this numerical example the Fresnel phase lies between 20 and 30 deg for grazing angles greater than 30 deg. The Fraunhofer phase is -90 deg for all grazing angles.

The amplitude in the Fraunhofer approximation, Eq. (C-3), and the amplitude in the Fresnel approximation, Eq. (C-10), are plotted for this numerical example on a relative scale for various grazing angles in Fig. C-2. Over a wide range of grazing angles (40 to 80 deg) the Fraunhofer amplitude is approximately 4.1 times the Freenel amplitude.

The term $p_0 p_0^*$ that is proportional to the intensity in the Fraunhofer approximation, Eq. (C-5), and the term pp^* in the Fresnel approximation, Eq. (C-12) are plotted for this numerical example in Fig. C-5. The Fraunhofer intensity is over 16 times the intensity in the Fresnel approximation for grazing angles greater than 20 deg.



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FIGURE C-1 RELATIVE PHASE IN THE SPECULAR DIRECTION PLANE PRESSURE RELEASE SURFACE

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80 2 RECEIVER DISTANCE (R 20): 60 in. SOURCE DISTANCE (R 10): 60 in. 60 FREQUENCY: 95.8 kHz BEAMWIDTH: 9 deg 50 FRAUNHOFER GRAZING ANGLE (Ψ) – deg 9 g 20 õ 20 ŝ 1000 2005 2000 RELATIVE INTENSITY 0 °J 61 Q

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FIGURE C-3 RELATIVE INTENSITY REFLECTED IN THE SPECULAR DIRECTION FROM A PLANE PRESSURE RELEASE SURFACE



INTEGRATION OF J1 TERM

In Chapter III, Eq. (68), we encountered the integral

$$J_{1} = \frac{k^{4} \langle F_{0}^{2} \rangle \sin^{4} \cdot \int_{2\pi^{2}R_{10}^{2}R_{20}^{2}A_{0}^{2}} \int_{-\lambda_{1}}^{\lambda_{2}} \int_{-\ell}^{\ell} \int_{-\lambda_{1}}^{\lambda_{2}} \int_{-\ell}^{\ell} e^{-\frac{1}{a} \left[(x_{1} - x_{1}')^{2} + (y - y')^{2} \right]^{1/2}} e^{-\frac{1}{a} \left[(x_{1} - x_{1}')^{2} + (y - y')^{2} \right]^{1/2}} \\ \times \cos \left[\frac{k}{R} \sin^{2} \psi \left(x_{1}^{2} - x_{1}'^{2} \right) + \frac{k}{R} (y^{2} - y'^{2}) \right] dx_{1} dy dx_{1}' dy' \quad .$$

Making the change in variables defined by Eq. (58), one finas for J_1

$$J_{1} = \frac{k^{4} \langle F_{0}^{2} \rangle \sin^{4} \psi}{2\pi^{2} R_{10}^{2} R_{20}^{2} A_{0}^{2}} \int_{-\Delta}^{\Delta} \int_{-\ell}^{\ell} \int_{-\Delta}^{\Delta} \int_{-\ell}^{\ell} e^{-\frac{1}{a} \left[(u-u')^{2} + (y-y')^{2} \right]^{1/2}} \\ \times \cos \left[\frac{k}{R} (u^{2} - u'^{2}) \sin^{2} \psi + \frac{k}{R} (\Delta_{2} - \Delta_{1}) (u - u') \sin^{2} \psi \right] \\ + \frac{k}{R} (y^{2} - y'^{2}) dudu' dydy' .$$

Introducing relative and center of gravity coordinate: defined by Fqs. (61) and (62), one obtains

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where

 $J_1 = J_1(\Delta_1, \Delta_2) + J_1(\Delta_2, \Delta_1)$

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Making the assumption (see Eq. 64) that a << 22,21

one can write for J_{+}

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$$J_{+} = \frac{k^{4} \langle F_{0}^{2} \rangle \sin^{4} \psi}{\pi^{2} R_{20}^{2} R_{20}^{2} A_{0}^{2}} \int_{0}^{0} \int_{0}^{\ell} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} e^{-\frac{1}{a} (\xi^{2} + \eta^{2})^{1/2}} e^{-\frac{1}{a} (\xi^{2} + \eta^{2})^{1/2}} (D-2)$$

$$\times \cos \left[\frac{2k}{R} \sin^{2} \psi \left| x_{0} + \frac{\Delta_{2}}{2} - \frac{\Delta_{1}}{2} \right| \xi + \frac{2k}{R} y_{0} \eta \right] dx_{0} dy_{0} d\xi d\eta \quad .$$

Consider the double integral

$$I = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} e^{-\frac{1}{a} \left(\frac{g^2 + \eta^2}{2} \right)^{\frac{1}{2}}} \cos \left[\frac{2k}{R} \sin^2 \psi \left(x_0 + \frac{\Delta_2}{2} - \frac{\Delta_1}{2} \right) \frac{g}{g} + \frac{2k}{R} y_0 \eta \right] dg d\eta$$
(D-3)

and set

 $\xi = \rho \cos \varphi$ $\rho \sin \varphi$ η

to get

$$I = \int_{0}^{2\pi} \int_{0}^{\infty} e^{-\alpha/a} \cos\left[n(N\cos\varphi + N\sin\varphi)\right] d\varphi d\varphi ;$$

where

$$M = \frac{2k}{R} \left(\sin^2 \psi \right) \left(x_0 + \frac{\Delta_0}{2} - \frac{\Delta_1}{2} \right)$$

$$H = \frac{2k}{R} y_0 \quad .$$
(p-4)

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M = P cos y ll = P sin y

to obtain

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$$I = \int_{0}^{2\pi} \int_{0}^{\infty} e^{-\phi/\hbar} \cos\left[\phi \int N^{2} + N^{2} \cos(\phi - \gamma)\right] d\phi d\phi$$

The term $(\varphi - \gamma)$ ranges from $-\gamma$ to $2\pi - \gamma$ as φ goes through its range of \circ to 2π ; the integrand assumes all the values that it would if the value of γ were zero. Thus,

$$I = \int_{0}^{2\pi} \int_{0}^{\infty} e^{-\rho/a} \cos\left[\rho \sqrt{M^2 + N^2} \cos \varphi\right] \rho d\rho d\varphi$$

From Gradshteyn and Ryzhik (integral 3.715-19, p. 402) one finds for the integral

$$\int_{0}^{2\pi} \cos\left[\rho \sqrt{M^{2} + N^{2}} \cos \rho\right] d\varphi = 2\pi J_{0} \left[\rho \sqrt{M^{2} + N^{2}}\right]$$

where $\boldsymbol{J}_{_{O}}$ is the zeroth ordered Bessel function. The expression for I becomes

$$I = 2\pi \int_{0}^{\infty} e^{-\rho/a} \rho J_{0} \left[\rho \sqrt{M^{2} + N^{2}} \right] d\rho$$

Using an integral solution (6.623-2, p. 712) in Gradshteyn and Ryzhik, one obtains for I

$$I = \frac{2\pi}{6} \left[\frac{1}{\frac{1}{R^2} + \left(\frac{1}{R} \frac{n \ln^2 y}{R}\right)^2 \left(x_0 + \frac{\Delta_1}{2} - \frac{\Delta_1}{2}\right)^2 + \left(\frac{2\pi}{R}\right)^2 y_0^2} \right]^{3/2} ; (D-5)$$

which gives for J.

$$J_{+} = \frac{2k^{4} (F^{2}) \sigma k^{2}}{r \sigma R_{10}^{2} R_{2}^{2} R_{2}^{2} A_{2}^{2}}$$

$$= \frac{A}{2} \int \frac{4x_{1} + x_{2}}{\left[\frac{1}{R^{2}} + \left(\frac{1}{R}\right)^{2} + \left(\frac{1}{R}\right)^{2}\right]^{2} + \left(\frac{2}{R}\right)^{2} + \left(\frac{2}{R}\right)$$

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Integrating over \underline{y}_{0} by employing the table of integrals by Gradshteyn and Ryzhik (integral 2.271-5, p. 86), one gets

$$J_{+} = \frac{kl \left\langle F_{0}^{2} \right\rangle R^{3}}{4 \pi a R_{10}^{2} R_{20}^{2} A_{0}^{2} \sin^{2} \psi} \times \int_{0}^{\Delta} \frac{dx_{0}}{\left[\left(x_{0}^{2} + \frac{\Delta_{2}}{2} - \frac{\Delta_{1}}{2} \right)^{2} + \left(\frac{R}{2ka \sin^{2} \psi} \right)^{2} \right] \sqrt{\left(x_{0}^{2} + \frac{\Delta_{2}}{2} - \frac{\Delta_{1}}{2} \right)^{2} + \left(\frac{R}{2ka \sin^{2} \psi} \right)^{2} + \frac{l^{2}}{\sin^{4} \psi}} \quad (D-7)$$

Set

. .

$$v = x_0 + \frac{\Delta_2}{2} - \frac{\Delta_1}{2}$$

to get for J_+

$$J_{+} = \frac{k\ell \langle F_{0}^{2} \rangle R^{3}}{4\pi a R_{10}^{2} R_{20}^{2} A_{0}^{2} \sin^{2} \psi}$$

$$\times \int_{\frac{\Delta_{0} - \Delta_{1}}{2}}^{\frac{\Delta_{0}}{2}} \left[\frac{\sqrt{2} + \left(\frac{R}{2ka \sin^{2} \psi}\right)^{2}}{\sqrt{\sqrt{2} + \left(\frac{R}{2ka \sin^{2} \psi}\right)^{2} + \frac{\ell^{2}}{\sin^{4} \psi}} \right]$$
(D-8)

Set

$$A = \left(\frac{R}{2ka \sin^2 \psi}\right)^2 , \text{ and}$$

$$B = \left(\frac{R}{2ka \sin^2 \psi}\right)^2 + \frac{\chi^2}{\sin^2 \psi}$$
(D-9)

to get

$$J_{*}(.._{1}, \Delta_{2}) = \frac{\kappa (\langle F_{0}^{2} \rangle R^{3})}{4 \kappa e R_{10}^{2} R_{20}^{2} A_{0}^{2} c \ln^{2} * \Delta_{2}^{2} \Delta_{1}} \int_{2}^{\Delta_{2}} \frac{dv}{(v^{2} + A) \sqrt{v^{2} + B}}$$

Integrating over \underline{y}_{0} by employing the table of integrals by Gradshteyn and Ryzhik (integral 2.271-5, p. 86), one gets

$$J_{+} = \frac{\frac{kl \langle F_{0}^{2} \rangle_{E}^{3}}{4\pi a R_{10}^{2} R_{2C}^{2} A_{0}^{2} \sin^{2} \psi}}{\left[\left[x_{0}^{2} + \frac{\Delta_{2}}{2} - \frac{\Delta_{1}}{2} \right]^{2} + \left(\frac{R}{2ka \sin^{2} \psi} \right)^{2} \right] \sqrt{\left[x_{0}^{2} + \frac{\Delta_{2}}{2} - \frac{\Delta_{1}}{2} \right]^{2} + \left(\frac{R}{2ka \sin^{2} \psi} \right)^{2} + \frac{i^{2}}{\sin^{4} \psi}}$$
(D-7)

Set

$$v = x_0 + \frac{\Delta_2}{2} - \frac{\Delta_1}{2}$$

to get for J_+

$$J_{+} = \frac{k! \langle F_{0}^{2} \rangle R^{5}}{4\pi a R_{10}^{2} R_{20}^{2} A_{0}^{2} \sin^{2} \psi}$$

$$\times \int_{\frac{\Delta_{1} - \Delta_{1}}{2}}^{\Delta_{2}} \left[\sqrt{2} + \left(\frac{R}{2ka \sin^{2} \psi} \right)^{2} \right] \sqrt{v^{2} + \left(\frac{R}{2ka \sin^{2} \psi} \right)^{2} + \frac{t^{2}}{\sin^{4} \psi}}$$
(D-8)

Set

$$A = \left(\frac{R}{2k_{0}} \sin^{2} \psi\right)^{2}, \text{ and}$$

$$B = \left(\frac{R}{2k_{0}} \sin^{2} \psi\right)^{2} + \frac{e^{2}}{\sin^{2} \psi}$$
(D-9)

to get

$$J_{*}(\Delta_{1}, \Delta_{2}) = \frac{\kappa (\langle P_{0}^{2} \rangle R^{3})}{4 \kappa R_{10}^{2} R_{20}^{2} A_{0}^{2} \sin^{2} \pi} \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{dv}{(v^{2} + A)} \sqrt{v^{2} + E}$$

From this equation one can write

$$J_{+}(\Delta_{2},\Delta_{1}) = \frac{k! \langle F_{0}^{2} \rangle R^{3}}{4\pi a R_{10}^{2} R_{20}^{2} A_{0}^{2} \sin^{2} \psi} \int_{\frac{\Delta_{1}-\Delta_{2}}{2}}^{\Delta_{1}} \frac{dv}{(v^{2} + A) \sqrt{v^{2} + B}}$$

and since

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$$J_{1} = J_{+}(\Delta_{1}, \Delta_{2}) + J_{+}(\Delta_{2}, \Delta_{1})$$

one obtains

$$J_{1} = \frac{k! \langle F_{0}^{2} \rangle R^{3}}{4\pi a R_{10}^{2} R_{20}^{2} A_{0}^{2} \sin^{2} \psi} \int_{-\Delta_{1}}^{\Delta_{2}} \frac{dv}{(v^{2} + A) \sqrt{v^{2} + B}}$$

J₁ can be written as

$$J_{1} = \frac{k! \langle P_{0}^{2} \rangle R^{3}}{4\pi e R_{10}^{2} R_{0}^{2} A_{0}^{2} \sin^{2} \psi}$$

$$\times \left[\int_{0}^{\Delta_{1}} \frac{dv}{(v^{2} + A)\sqrt{v^{2} + B}} + \int_{0}^{\Delta_{2}} \frac{dv}{(v^{2} + A)\sqrt{v^{2} + B}} \right]$$

Change the variable of integration to

to get for J_l

$$J_{1} = \frac{k^{2} \langle v_{0}^{2} \rangle R^{3}}{4\pi a R_{10}^{2} R_{20}^{2} A_{0}^{2} \sin^{2} \psi} \left(\frac{-1}{A \sqrt{B}}\right) \left[\int_{\infty}^{-1} \frac{dw}{w^{2} + \left(\frac{1}{A} - \frac{1}{B}\right)}\right]$$

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where

and

$$w_{1} = \sqrt{\frac{1}{B} + \frac{1}{\Delta_{1}^{2}}}$$
$$w_{2} = \sqrt{\frac{1}{B} + \frac{1}{\Delta_{2}^{2}}}$$

With the use of

$$\int \frac{4x}{a + x^2} = \frac{1}{\sqrt{a}} \arctan\left(\frac{x}{\sqrt{a}}\right)$$

J1 becomes

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$$I_{1} = \frac{k! \langle F_{0}^{2} \rangle R^{3}}{4\pi a R_{10}^{2} R_{20}^{2} A_{0}^{2} a \ln^{2} \psi} \left[\frac{1}{A} \left(\frac{A}{B-A} \right)^{1/2} \right] \left\{ \pi - \frac{1/2}{A} - \frac{1/2}{B-A} \right\} = \arctan \left[\frac{(A)^{1/2}}{A} \left(\frac{A}{B-A} \right)^{1/2} \right]$$

The substitution of the expressions for A and B, Eq. (D-9), and the expression for A_0 , Eq. (C-10) of Appendix C, into J_1 gives

$$J_{1} = \frac{2k^{2} \langle F_{0}^{2} \rangle \sin^{2} \psi}{\pi \left(\left[C_{4} + C_{\Delta_{2}} \right]^{2} + S_{4} \left[S_{\Delta_{1}} + S_{\Delta_{2}} \right]^{2} + \left[C_{4} \left[S_{\Delta_{1}} + S_{\Delta_{2}} \right] + S_{4} \left[C_{\Delta_{1}} + C_{\Delta_{2}} \right] \right]^{2} \right)}$$

$$\times \left\{ \frac{4 \operatorname{recot} \left[\frac{R}{2 \operatorname{ka} \Delta_{1} t \sin^{2} \psi} \left(t^{2} + \Delta_{1}^{2} \sin^{4} \psi + \frac{R^{2}}{4 \operatorname{k}^{2} \operatorname{a}^{2}} \right)^{1/2} \right]}{4 \operatorname{k}^{2} \operatorname{k}^{2} \left[\frac{R}{2 \operatorname{ka} \Delta_{1} t \sin^{2} \psi} \left(t^{2} + \sum_{n=1}^{2} \sin^{4} \psi + \frac{R^{2}}{4 \operatorname{k}^{2} \operatorname{a}^{2}} \right)^{1/2} \right]} \right\}$$

$$(D-10)$$

where C and C are the Presson integrals defined by Eq. (C-9) in Appendix C.

For the special case where

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$$\frac{h^{2}\left(r_{1}^{2}\right) \circ lo^{2}}{\pi \left[c_{1}c_{2}\right] - c_{2}c_{1}\int_{-\infty}^{\infty} \left[c_{2}c_{1} + c_{1}c_{2}\right]\right]}$$

$$\times \operatorname{arcond} \left[\frac{h}{4\pi i c_{1}} + c_{1}c_{2}\right] \times \left[c_{1}c_{2}\right] + c_{1}c_{2}\right]$$

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$$= argument of the Frein $\sqrt{\frac{2\pi}{88}} \sin v \gg 1$,$$

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(0-1)

S and C both approach one-half, and Eq. (D-11) reduces to

$$J_{1} = \frac{4k^{2} \langle F_{0}^{2} \rangle \sin^{2} \psi}{\pi}$$
(D-12)
× $\operatorname{arccot} \left[\frac{R}{2kat\Delta_{0}} \sin^{2} \psi \left(t^{2} + \Delta_{0}^{2} \sin^{4} \psi + \frac{R^{2}}{4k^{2}a^{2}} \right)^{1/2} \right] ,$

which can be compared with the J_1 term obtained by Gulin (1962), the result of using a Gaussian covariance function.

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APPENDIX E

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DISCUSSION OF J, TERM

In Chapter III, Eq. (69), we encountered the integral

$$J_{2} = \frac{k^{4} \langle F_{0}^{2} \rangle \sin^{4} \psi}{2\pi^{2} R_{10}^{2} R_{20}^{2} R_{0}^{2}} \int_{-\Delta_{1}}^{\Delta_{2}} \int_{-\ell_{1}}^{\ell_{1}} \int_{-\ell_{1}}^{\Delta_{2}} \int_{-\frac{1}{2}}^{\ell_{1}} e^{-\frac{1}{4} \left[(x_{1} - x_{1}^{\prime})^{2} + (y - y^{\prime})^{2} \right]^{1/2}}$$

$$\times \cos \left[\frac{k}{R} \sin^{2} \psi \left[x_{1}^{2} + x_{1}^{\prime 2} \right] + \frac{k}{3} (y^{2} + y^{\prime 2}) - 2\psi_{0}^{\prime} \right] dx_{1} dy dx_{1}^{\prime} dy^{\prime}$$
(E-1)

Making the change of integration variables as depicted in Fig. 5, the author could not find an analytical expression for J_2 , but approximate expressions showed J_2 to be very small compared to J_1 .

The conclusion that J_1 is much less than J_2 is borne out by examining the work of Gulin (1962) who has evaluated the integrals J_1 and J_2 for the Gaussian form of the covariance function. Numerical evaluations of his expressions are illustrated in Fig. E-1 for the case

> $R_{10} = R_{20} = 60.0 \text{ in.}$ $\sqrt{8^2} = 0.001 \text{ in.}$

> > a = 2.35 in. k = 10.35 in.⁻¹

which correspond to a frequency of 95,800 kHz and a wavelength of 0.508 inch. The values used for Δ_1 , Δ_2 , and l are listed in Table 8-1 in Appendix B. At a Hayleigh parameter of 0.7 (the theory is valid only for Rayleigh parameters less than one) the applitude fluctuations. $(J_1 - J_2)^{1/2}$, exceed the phase fluctuations, $(J_1 - J_2)^{1/2}$, by approximately 4 percent. Since for small Rayleigh parameters the applitude and phase

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fluctuations are approximately of the same magnitude, Gulins' expressions for J_1 and J_2 show, for the example cited, that

 $r^{2} \ll r^{1}$

Figure E-2 shows that the J_1 term for the case of an exponential covariance function does not differ much from the J_1 term for the case of an Gaussian covariance function, especially in the region of Rayleigh parameters less than one where the theory is valid. Thus, one concludes that the Gulin result of $J_2 \ll J_1$ for the Gaussian case is applicable also to the exponential case.


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