MRL-TN-371

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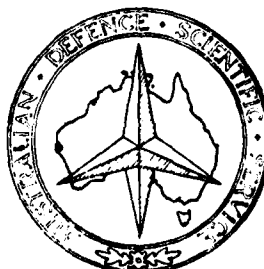
MARIBYRNONG VICTORIA

TECHNICAL NOTE

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FORM-FUNCTIONS FOR MULTI-COMPONENT PROPELLANT CHARGES
INCLUDING INHIBITED GRAINS AND SLIVER BURN

J. Stals

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SEPTEMBER, 1975

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UNCLASSIFIED

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1. Security Grading/Release
   Limitation :

   (a) Document content :
       Unclassified

   (b) This Page :
       Unclassified

2. Document Type/Number :

   MRL-TN-371

3. Document Date :

   SEPTEMBER, 1975

4. Title and Sub-Title :

   FORM-FUNCTIONS FOR MULTI-COMPONENT PROPELLANT CHARGES
   INCLUDING INHIBITED GRAINS AND SLIVER BURN

5. Personal Author   : J. Stals

6. Corporate Author   : Materials Research Laboratories,
                         Maribyrnong, Victoria

7. Abstract (See reverse)

8. Descriptors :

   Propellant Grains
   Grain Shapes
   Area
   Inhibition
   Burning Rate
   Interior Ballistics

9. Cosati Classifications :

   1901

11. Task Reference (RD/P8):

   TASK 71/11

10. Library distribution :

    MRL (Maribyrnong)
    MRL (S.A. Branch)
    MRL (N.S.W. Branch)
    WRE
    Russell Library
    Campbell Park Library

12. Sponsoring Agency
    Reference :

13. Cost Code :

    21804

14. Imprint :   MARIBYRNONG - MRL - 1975

A B S T R A C T

Cubic form-functions in burning surface regression distance are derived and compiled for granulations that are used in current U.S.A., U.K. and Australian ammunition.   Functions for specific inhibition of grain surface, sliver burn for multi-perforated grains, and unequal webs in seven perforated grains, are included.   The functions are of general use in propellant grain design, closed-chamber and interior ballistic computations. Their resolution into igniter and propellant segment contributions as a function of grain surface regression, facilitates studies on multi-granular, multi-propellant charges, mortar charges, deterred grains, programmed or non-isochronous ignition, and the design of specific, or maximum progressivity granulations.

A

C O N T E N T S

C O N T E N T S

(Continued)

NOTATION

		Unit
a_1	)	m^{-1}
a_2	) Geometric grain constants	m^{-2}
a_3	)	m^{-3}
B	Differential of z with respect to r	m^{-1}
C_i	Mass of propellant segment i	kg
Cap	Initial volume available to propellant gases	m^3
D	Outer grain diameter or width	m
d	Perforation diameter or smaller diameter of grain	m
EP	Packing efficiency of propellant charge	-
f	Fraction of grain web unburnt	-
j	Total number of propellant segments	-
$K_{0,1,2,3}$	Form-factors	-
l	Slit length in slotted-hole flake grains	m
L	Length of grain	m
n_i	Number of grains in propellant segment i	-
N	Number of grain perforations	-
P_r	Differential of S with respect to r	m
r	Grain surface regression distance	m
R	Ratio of grain surface area to its initial value, i.e. grain progressivity	-
s	Slit width in slotted hole flake grains	m
S	Surface area that is burning at time t	m^2
S_o	Initial surface area of the grain	m^2
t	Time interval after primer ignition	s
u	2r	m

V_o	Initial volume of propellant grain	m^3
V	Volume of propellant grain at time t	m^3
V_a	Free volume left in charge container	m^3
w	Grain web	m
z	Fraction of charge burnt at time t	$-$
δ_i	Weight fraction of segment i in total charge	$-$
ρ	Grain density	$kg\ m^{-3}$
θ	Form-factor	$-$
θ,θ^1)		
)		
ω,χ)	Subtended angles as shown in grain sliver	rad
)		
ϕ,ϕ_1)	burn diagrams	
)		
β,β_1)		
π	Constant	$-$
ϵ	Spheroid eccentricity factor	$-$

Subscripts

e,E)	
)	Denote grain end surfaces in slivers
EI,EE)	
i	Segment i of total propulsion charge
I	Primer or igniter
$1,L$)	
)	Denote grain lateral surfaces in slivers
LI,LL)	
max	Maximum value of subscripted variable
i,o	Inner and outer webs
s	Segment variable
w	Variable value when $r = web/2$
1 for r	Variable value when small sliver is all burnt
2 for r	Variable value when large sliver is all burnt
3 for r	Variable value when residual slivers are all burnt, $N = 19$

FORM-FUNCTIONS FOR MULTI-COMPONENT PROPELLANT CHARGES

INCLUDING INHIBITED GRAINS AND SLIVER BURN

A. INTRODUCTION

The optimisation of the gasification rate for a propulsion charge and the design of granulations to achieve it, and the desired weapon performance, requires in part, a quantitative formulation of the total propulsion charge form-function. Namely, the fraction of charge burnt (z) at any time (t), as a function of grain geometry and surface regression distance.

In the past, z has been expressed in terms of :

(i) The fraction of web unburnt (f)[1,2,7-11,17]

$$z = (1-f)(1+\theta f) \tag{1}$$

or
$$z = K_0 + K_1(1-f) + K_2(1-f)^2 + K_3(1-f)^3 \tag{2}$$

where the form-factors θ, K_0 to K_3 were approximated from grain geometry or the empirical fitting of experimental pressure-time records. For closed-chamber data, the geometric values of θ are generally 0.2-0.3 too low as a result of grain heterogeneities and the violation of Piobert's law (see Section B). In a gun, however, erosive burning results in greater than predicted grain surface regression rates and gives a compensating approximation for the variation of z with time.

In addition, the variable f loses physical reality *for multiperforated*, and other, grain shapes that result in slivers upon web burnthrough. The form-factors θ and K_0 to K_3 are adjusted empirically to compensate for the latter sliver burn phenomena, or a compensating ballistic web size is invoked.

(ii) The surface area (S) of the grain[1]

$$z = (1-(S/S_o)^2)(1+\theta)^2/4\theta \tag{3}$$

where S_o is the initial surface area of the grain and θ is a form-factor.

The treatment of geometric and chemical heterogeneities of the grain, its specific inhibited surfaces, in terms of surface area, is inconvenient.

(iii) The regression distance (r) of the burning grain surface.[2-6,12-16]

$$z_i = \sum_{i=1}^{j} a_i r^i \quad , \quad i = 1,2, \ldots j \qquad (4)$$

where the grain geometric coefficients a_i are explicitly expressed in terms of grain dimensions.

Equality (4) has physical reality for all grain shapes and it can be readily used to treat radial chemical gradients in grains, such as those due to deterrents and inhibitors. The regression distance may be measured experimentally upon extinguishment of the grain flame, and its time derivative is the useful grain surface regression rate (dr/dt). In addition, sliver burn may be explicitly formulated in terms of r. Further, the use of the explicit geometric constants a_i facilitates parametric studies of the effect of grain geometry on propulsion charge performance in the weapon.

Since a cubic polynomial in r is sufficient to define most grain geometries, we shall use Eq (5) as the basic form-function in this work and relate it to other grain properties in Section B.

$$z = a_1 r + a_2 r^2 + a_3 r^3 \qquad (5)$$

The geometric constants of the grain, viz. a_1, a_2 and a_3, are derived and listed in the Appendix for the following grain shapes. Selective surface inhibition is allowed for in (b) and (d)-type grain shapes.

    (a) Solid right prismatic   (i) rectangular and ribbon

                                    (ii) cubic

                                    (iii) square flake

    (b) Mortar propellants   (i) disc and cord

                                    (ii) slotted-hole flake

    (c) Ellipsoids   (i) sphere

                                    (ii) oblate spheroid

                                    (iii) prolate spheroid

                                    (iv) symmetric rolled-ball

    (d) Cylindrical   (i) cord, of b(i)

                                    (ii) tubular

                                    (iii) multiperforated (N) for :

I   N = 0,1,2 ....., '! prior to sliver formation and web burnthrough,

II   N = 7,19   during sliver burn,

II   N = 7   unequal webs, prior to and during sliver burn.

(e) Slotted tube

The above shapes cover the granulations that are used in Australian ammunition. They also incorporate most USA and UK gun and mortar propellant configurations. Specific rocket motor grain configurations have been treated in Refs. 19 to 25. A useful handbook of equations for mass and area property calculations for various geometric shapes is listed in Ref. 27.

Some of the form functions for the more common granulations have been published previously;[2-6],[12-16] they are included for completeness. The compilation of functions given in the appendix forms a useful reference for R & D work on propulsion charge design, interior ballistic and closed chamber computations, and eliminates the need for time consuming derivation thereof.

B.   FORM-FUNCTION AND CHARGE PARAMETERS

Historically, it has been assumed that a solid propellant grain burns under the following conditions;[1-4]

(a)   Each grain is chemically and physically homogeneous in all directions and there is no cracking or mechanical degradation of the grain during the ignition and combustion processes.

(b)   The segment of the total charge in question consists of chemically and geometrically identical grains and these ignite simultaneously and uniformly within the entire segment of the charge.

(c)   Each grain is ignited instantaneously over all its uninhibited external surface (P. Vieille,[6] 1893), with the speed of igniter flame spread being much greater than the initial propellant burning rate.

(d)   The uninhibited grain surface burns in parallel layers and perpendicular to the surface with the same surface regression rate over the entire burning surface (G. Piobert,[7] 1839).

Each of the above assumptions are violated to varying degrees. For example, (a) and (b) hold under ideal propellant production conditions. These are rarely achieved, and difficult to reproduce. Vieille's assumption and (b) hold accurately for porous propellant beds such as artillery charges; they break down however for compact, high loading density small arms charges in which non-isochronous ignition may occur. Piobert's law is only semi-quantitative, particularly within perforations of multiperforated grains, and is strongly dependent on (a) to (c).

The above violations cannot be easily quantified, nor would they be statistically reproducible between lots of the same propellant. Assumptions (a) to (d) are thus retained and used to provide geometric form functions that are useful, if semi-quantitative, in propellant grain design, the control of propellant mass burning rate, and subsequent computations on interior ballistics and closed chamber performance.

The fraction of charge that is burnt (z) at any time (t), is the sum of the primer (z_I) and propellant component (z_i) contributions

$$z = \delta_I z_I + \sum_{i=1}^{j} \delta_i z_i \quad , \quad i = 1, 2, \ldots j \qquad (6)$$

where the total charge is made up of j segments that may vary in chemical formulation and/or grain geometry and dimensions. The weight proportion of igniter, I, and propellant component, i, are denoted by δ_I and δ_i respectively.

If V_i and V_{oi} denote the respective volumes of a *grain* at time t and initially, we have

$$z_i = 1.0 - V_i/V_{oi} \qquad (7)$$

and for segment i of the propellant charge, Eq (5) becomes

$$z_i = a_{1i} r + a_{2i} r^2 + a_{3i} r^3 \qquad (8)$$

Upon differentiating Eq (8) with respect to r we obtain

$$dz_i/dr = a_{1i} + 2a_{2i} r + 3a_{3i} r^2 \equiv B_i \text{ (say)} \qquad (9)$$

If S_{oi} is the initial surface area of the grain and S_i is the burning surface area at time t, then

$$a_{1i} = S_{oi}/V_{oi} \qquad (10)$$

and

$$B_i = S_i/V_{oi} \qquad (11)$$

The ratio (R_i) of the grain surface area at time t to its initial value is thus

$$R_i = S_i/S_{oi} = B_i/a_{1i} \qquad (12)$$

hence a maximum value of R_i exists if $a_{3i} < o$, o $r = - a_{2i}/3a_{3i}$ r_{max}, and $a_{2i} > o$, thus;

$$R_{i\ max} = 1.0 - a_{2i}^2/3a_{3i}a_{1i} \qquad (13)$$

Let us define P_r as the incremental variation of S as a function of r, that is

$$P_{ri} = dS_i/dr = V_{oi}(2a_{2i} + 6a_{3i}r) \qquad (14)$$

or

$$P_{ri} = S_{oi}(2a_{2i} + 6a_{3i}r)/a_{1i} \qquad (15)$$

where for a given regression distance r the grain is :

(a) Progressive if $P_r > 0.0$. S increases with increase in r to give an increased propellant gasification rate and burning surface area.

(b) Neutral for $P_r = 0.0$. A constant burning surface and propellant gasification rate is maintained.

(c) Degressive if $P_r < 0.0$. The burning surface and gas evolution rate decrease with increase in r and t.

If ρ_i, C_i and n_i denote grain density, mass and number of grains respectively in segment i of the charge then;

$$n_i = C_i/\rho_i V_{oi} \qquad (16)$$

and the *segment charge variables* (sub s) for segment i become

$$S_{si} = n_i S_i \qquad (17)$$

$$P_{sri} = n_i P_{ri} \qquad (18)$$

5

$$V_{si} = n_i V_i \qquad (19)$$

where z_i and R_i are independent of n_i.

Let δ_i denote the weight fraction of segment i in the total charge of mass C, then

$$C_i = \delta_i C \qquad (20)$$

and

$$z_i = \delta_i z \qquad (21)$$

where

$$1.0 = \delta_I + \sum_{i=1}^{j} \delta_i \qquad (22)$$

and

$$C = C_I + \sum_{i=1}^{j} C_i \qquad (23)$$

The *total charge variables* are therefore :

$$S = \sum_{i=1}^{j} S_{si} = \sum_{i=1}^{j} (n_i B_i V_{oi}) \qquad (24)$$

$$P_r = \sum_{i=1}^{j} [n_i V_{oi}(2a_{2i} + 6a_{3i}r)] \qquad (25)$$

$$R = \sum_{i=1}^{j} (\delta_i B_i / \rho_i) / \sum_{i=1}^{j} (\delta_i a_{1i} / \rho i) \qquad (26)$$

$$V = \sum_{i=1}^{j} V_{si} \qquad (27)$$

where z is obtained from Eqs (8) and (6).

6

Let Cap, EP and V_a denote the volume available (e.g. chamber, cartridge, etc.) to the propellant charge prior to shot-start, the packing efficiency and free air-space respectively, then

$$V_a = Cap - V \tag{28}$$

$$EP = V_a/Cap \tag{29}$$

The variables a_1, a_2, a_3, hence B, S, R, P_r and z, are derived in the Appendix for the grain shapes listed in Section A.

C. CONCLUSIONS

The best form of granulation ballistically, is the one that will give the highest muzzle velocity for a given charge weight and chemical composition, within the tube pressure limits for which the weapon was designed. The most progressive grain shape is therefore desirable. The listed form-functions permit the determination of the granulation, or combinations thereof, that will produce a charge of a given, or maximum, progressivity.[26]

In addition, the compiled form-functions and charge parameters facilitate the study of :

(a) Multi-granular and multi-propellant charges, e.g. zoned artillery ammunition, blends of small arms powders.

(b) The importance of the contribution of the primer to the propulsion charge energy, e.g. mortar charges, propellant actuated devices.

(c) The effect of inhibiting specific grain surfaces, e.g. rocket motors, propellant actuated devices, very progressive grains.

(d) The presence of radial chemical gradients within the grain, e.g. deterred small arms grains; nitroglycerine migration; solvent, plasticizer and moisture gradients; sandwich or layered grains.

(e) Sliver burn, e.g. multiperforated grains.

(f) Unequal webs in multiperforated grains.

(g) Non-isochronous charge ignition, e.g. small arms charges, composite and moulded charges, programmed ignition, caseless ammunition.

(h) The modular use of the charge mass burning rate law in charge design, closed chamber and interior ballistic simulations.

7

The effects of non-uniform erosive burning of the grain surface (e.g. within perforations), and the statistical variations in grain shape, dimensions, perforation asymmetry and chemical composition, have been omitted from the compiled form-functions and charge parameters.

D. ACKNOWLEDGEMENTS

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F. APPENDICES

(1) Solid Right Prismatic
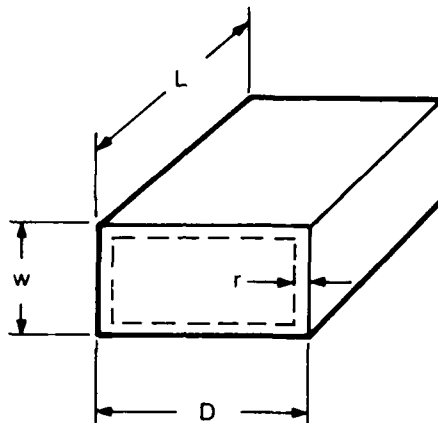
$$S_o = 2(LD + Lw + Dw)$$

$$V_o = LDw$$

Hence

$$S = 2[(L-u)(D-u) + (L-u)(w-u) + (D-u)(w-u)]$$

$$V = (L-u)(D-u)(w-u)$$

where

$$u = 2r$$

and

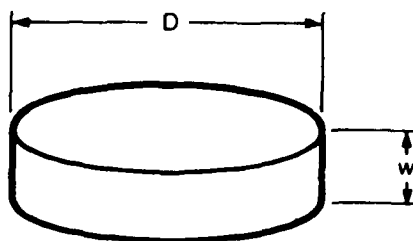$$z = 1 - V/V_o = a_1 r + a_2 r^2 + a_3 r^3$$

when

$$a_1 = 2(LD+Lw+Dw)/LDw$$

$$a_2 = -4(L+D+w)/LDw$$

$$a_3 = 8/LDw$$

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(2) Mortar Grain Shapes

(a) Disc or Cord

(i) Uninhibited

$$S_0 = \pi D(0.5D+w)$$

$$V_0 = 0.25 \pi D^2 w$$

Hence

$$S = \pi(D-u)[0.5(D-u) + (w-u)]$$

$$V = 0.25 \pi (D-u)^2 (w-u)$$

and

$$z = 1-V/V_0 = a_1 r + a_2 r^2 + a_3 r^3$$

for

$$a_1 = 2(D+2w)/Dw$$

$$a_2 = -4(2D+w)/D^2 w$$

$$a_3 = 8/D^2 w$$

(ii) Inhibited Wall V_0 is unchanged

$$S_0 = 0.5 \pi D^2$$

Hence

$$S = 0.5 \pi D^2 = S_0 \qquad \text{i.e. constant burning surface area}$$

$$V = 0.25 \pi D^2 (w-u)$$

$$z = 1-V/V_0 = a_1 r + a_2 r^2 + a_3 r^3$$

when

$$a_2 = a_3 = 0.0$$

$$a_1 = 2/w$$

Inhibited Ends V_O is unchanged

$$S_O = \pi\, Dw$$

Hence

$$S = \pi\, w(D-u)$$

$$V = 0.25\,\pi\, w(D-u)^2$$

and

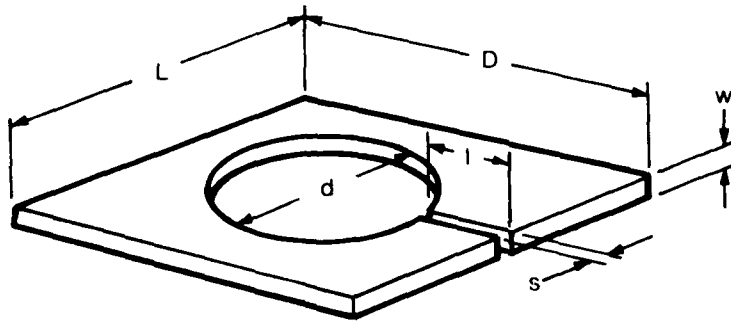$$z = 1 - V/V_O = a_1 r + a_2 r^2 + a_3 r^3$$

when

$$a_3 = 0.0$$

$$a_1 = 4/D$$

$$a_2 = -4/D^2$$

(b)   **Slotted-Hole Flake**

$$L = D = d + 2l$$

Assume $l > w$, i.e. no slivers are formed.

$$S_O = 2(LD + Lw + Dw + lw - sw - ls) + \pi\, d(w - 0.5d)$$

$$V_O = w(LD - ls - 0.25\,\pi\, d^2)$$

Hence $S = 2[(L-u)(D-u) - (l-u)(s+u) + (w-u)(L+D+l-s-4)] + \pi\,(d+u)\cdot$
$\cdot[(w-u) - 0.5(d+u)]$

$$V = (w-u)[(L-u)(D-u) - (l-u)(s+u) - 0.25\,\pi\,(d+u)^2]$$

and $z = 1 - V/V_O = a_1 r + a_2 r^2 + a_3 r^3$

for $a_1 = 2w(L+D+l-s+0.5\,\pi\, d)/V_O + 2/w$

$$a_2 = -4(2w+L+D+l-s-0.25\,\pi\, w+0.5\,\pi\, d)/V_O$$

$$a_3 = 2(8 - \pi)/V_O$$

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(3) **Spheroidal Shapes**

(a) **Sphere**

$$S_o = \pi D^2$$

$$V_o = \pi D^3/6$$
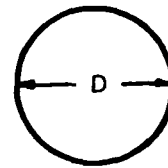
Hence $\quad S = \pi (D-u)^2$

$$V = \pi (D-u)^3/6$$

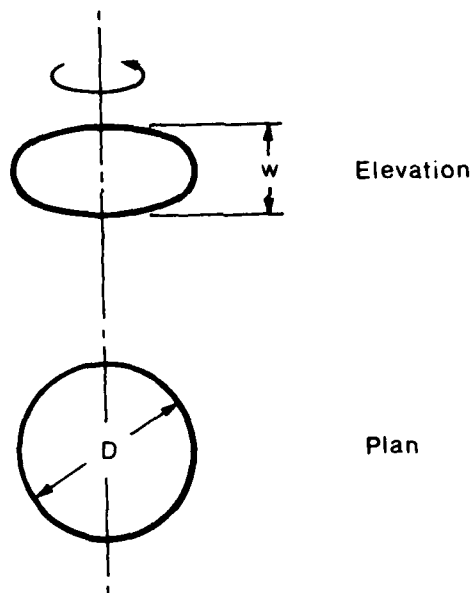and $\quad z = 1-V/V_o = a_1 r + a_2 r^2 + a_3 r^3$

for $\quad a_1 = 6/D$

$$a_2 = -12/D^2$$

$$a_3 = 8/D^3$$

(b) **Oblate Spheroid**

Elevation

Plan

$$\varepsilon = (1-w^2/D^2)^{\frac{1}{2}} = \text{eccentricity}$$

$$S_o = 0.5 \pi D^2 + 0.25 \pi w^2 (\ln[(1+\varepsilon)/(1-\varepsilon)])/\varepsilon$$

$$V_o = \pi D^2 w/6$$

Hence for

$$\varepsilon_u = [1-(w-u)^2/(D-u)^2]^{\frac{1}{2}}$$

$$S = 0.5 \pi (D-u)^2 + 0.25 \pi (w-u)^2 (\ln[(1+\varepsilon u)/(1-\varepsilon u)])/\varepsilon u$$
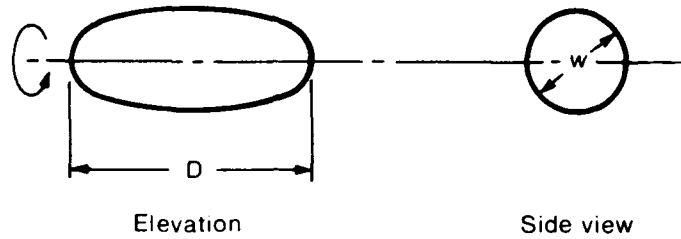
$$V = \pi (D-u)^2 (w-u)/6$$

13

and $\quad z = 1-V/V_o = a_1 r + a_2 r^2 + a_3 r^3$

for $\quad a_1 = (2D^2 + 4Dw)/D^2 w$

$\quad\quad a_2 = -(8D + 4w)/D^2 w$

$\quad\quad a_3 = 8/D^2 w$

(c) Prolate Spheroid

Elevation   Side view

Let $\quad \epsilon = (1-w^2/D^2)^{\frac{1}{2}}$

Then $\quad S_o = 0.5 \pi w(w + D \arcsin(t)/t)$

$\quad\quad V_o = \pi Dw^2/6$

For $\quad \epsilon_u = [1-(w-u)^2/(D-u)^2]^{\frac{1}{2}}$

$\quad\quad S = 0.5 \pi (w-u)[(w-u) + (D-u) \arcsin (\epsilon u)/\epsilon u]$

$\quad\quad V = \pi (D-u)(w-u)^2/6$

and $\quad z = 1-V/V_o = a_1 r + a_2 r^2 + a_3 r^3$

for $\quad a_1 = (4wD+2w^2)/Dw^2$

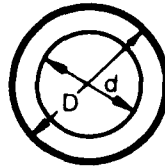$\quad\quad a_2 = -(4D+8w)/Dw^2$

$\quad\quad a_3 = 8/Dw^2$

(d) Rolled-Ball : symmetric sphere of radius D/2 cut by two parallel planes at a distance w apart.

Elevation

$\quad S_o = \pi Dw + 0.5 \pi d^2$

$\quad V_o = 0.25 \pi wd^2 + \pi w^3/6$

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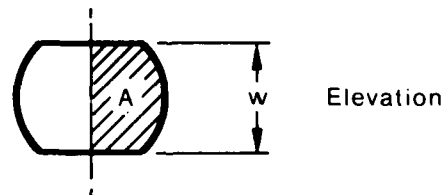
Plan

Hence $\quad S = \pi(D-u)(w-u) + 0.5\,\pi[d-u(w-D+d)/w]^2$

$\quad\quad\quad V = 0.25\,\pi\,(w-u)[d-u(w-D+d)/w]^2 + \pi(w-u)^3/6$

and $\quad\quad z = 1-V/V_o = a_1 r + a_2 r^2 + a_3 r^3$

for

$$a_1 = [w^2-d(D-w-1.5d)]/[w(0.25d^2+w^2/6)]$$

$$a_2 = [2D-3w-4d-(D-3d)(D-d)/w]/[w(0.25d^2+w^2/6)]$$

$$a_3 = [5/3-2(D-d)/w+(D^2-d^2)/w^2]/[w(0.25d^2+w^2/6)]$$

$\quad\quad\quad\quad$ along CB $\quad d_u = d(w-u)/w$

$\quad\quad\quad\quad$ along EB $\quad d_u = d-u(w-D+d)/w$

$\quad\quad\quad\quad$ approximate EFB by line EB
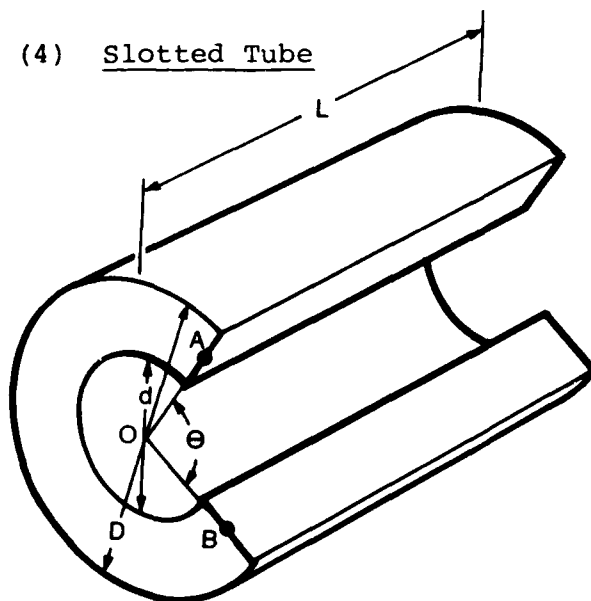
Alternative case: $\quad$ for D-d = w; we get

$$a_1 = (w^2-0.5d^2)/w(0.25d^2+w^2/6)$$

$$a_2 = -12/0.5d^2+w^2/3)$$

$$a_3 = 4/w(0.5d^2+w^2/3)$$

(4) Slotted Tube

$w = D-d$

Take θ with respect to sector AOB of radius (D+d)/4.

Then $\theta^1 = \theta + 4u/(D+d)$ rad

$$S_O = (0.5\pi - 0.25\theta)[D^2 - d^2 + 2L(D+d)] + L(D-d)$$

$$V_O = (0.5\pi - 0.25\theta)L(D^2 - d^2)0.5$$

Hence

$$S = [0.5\pi - 0.25(\theta + 4u/(D+d))][(D-u)^2 - (d+u)^2 + 2(L-u)(D+d)] + (L-u)(D-d-2u)$$

$$V = [0.25 - 0.125(\ +4u/(D+d))](L-u)[(D-u)^2 - (d+u)^2]$$

and $\quad z = 1 - V/V_O = a_1 r + a_2 r^2 + a_3 r^3$

for

$$a_1 = [L(D-d) + (0.5\pi - 0.25\theta)(D+d)(2L+D-d)]/V_O$$

$$a_2 = -2[(\pi - \theta/2)(D+d) + 2L + D - d]/V_O$$

$$a_3 = 8/V_O$$

(5) <u>Cylindrical</u>

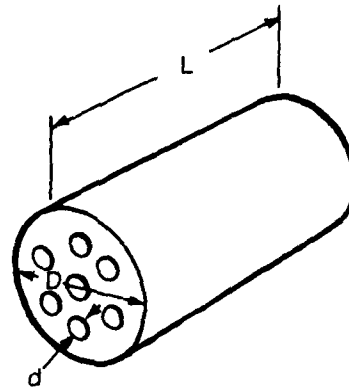   (a)   <u>Cord</u>, N=O, See 2(a)

    (b)   <u>Symmetrical Perforated Grains, Equal Webs</u>

        (i)   <u>Uninhibited, Prior to Web Burnthrough, Any N</u>

$$S_O = \pi L(D+Nd) + 0.5\pi(D^2 - Nd^2)$$

$$V_O = 0.25\pi L(D^2 - Nd^2)$$

Hence $S = \pi(L-u)[D-u+N(d+u)] + 0.5\pi[(D-u)^2 - N(d+u)^2]$

$$V = 0.25\pi(L-u)[(D-u)^2 - N(d+u)^2]$$

and $\quad z = 1 - V/V_O = a_1 r + a_2 r^2 + a_3 r^3$

for

$$a_1 = 2[2L(D+Nd)+D^2-Nd^2]/L(D^2-Nd^2)$$

$$a_2 = -4[2(D+Nd)-L(N-1)]/L(D^2-Nd^2)$$

$$a_3 = -8(N-1)/L(D^2-Nd^2)$$

(ii)   Uninhibited, During Sliver Burn, $N=7$[12]
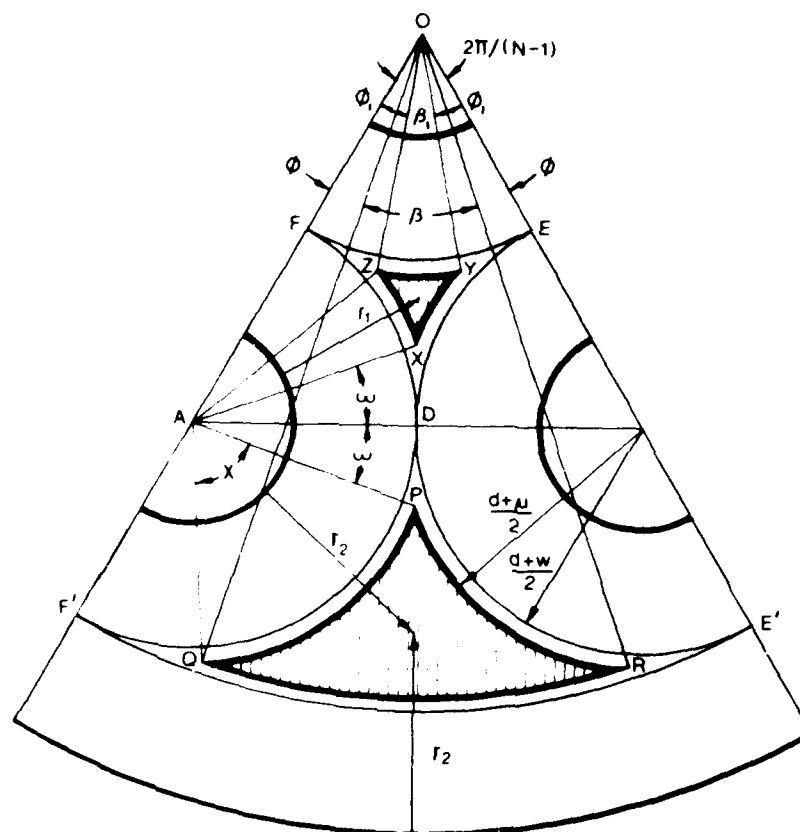
Assume 6 symmetric perforations are arranged on a circle of radius 0.25 (D+d)

Let OA = $0.25(D+d)$

AD = $OA \sin(\pi/(N-1)) = OA \times 0.5$

OZ = $AQ = AZ = 0.5(d+u)$

OQ = $0.5(D-u)$

ω = $\arccos(AD/AQ)$

OAQ = $\arccos[(OA^2+AQ^2-OQ^2)/2OA.AQ]$

χ = $OAQ-\omega-\pi(0.5-1/(N-1))$

ϕ = $\arccos[(OA^2+OQ^2-AQ^2)/2OA.OQ]$

$$\beta \quad = \quad 2(\pi/(N-1)-\phi)$$

$$\phi_1 \quad = \quad \arccos[OA/2.OZ]$$

$$\beta_1 \quad = \quad 2\pi/(N-1) - 2\phi_1$$

then

area of lateral surfaces of N-1 large, outer slivers S_L is, for any regression distance $w > u > u_2$;

$$S_L \quad = \quad (N-1)(L-u)(\beta.OQ+2\chi.AQ)$$

area of lateral surfaces of N-1 small, inner slivers S_1 is, for $w > u > u_1$;

$$S_1 \quad = \quad (N-1)(L-u)(3\ OZ.\beta_1)$$

area of two end surfaces of N-1 large, outer slivers S_E, for $w > u > u_2$;

$$S_E \quad = \quad 2(N-1)\{OQ \sin (0.5\beta)[(2AQ \sin (0.5\chi))^2-(OQ \sin (0.5\beta))^2]^{\frac{1}{2}}$$
$$+ 0.5OQ^2(\beta-\sin(\beta))-AQ^2(\chi-\sin(\chi))\}$$

area of two end surfaces of N-1 small, inner slivers S_e, for $w > u > u_1$;

$$S_e \quad = \quad 2(N-1)OZ^2\{\sin^2(0.5\ \beta_1)\ 1.73206-1.5[\beta_1-\sin(\beta_1)]\}$$

total area of sliver surfaces is therefore

$$S \quad = \quad S_L + S_E + S_1 + S_e$$

Hence $\qquad V \quad = \quad (S_E + S_e)(L-u)0.5$

$\qquad\qquad\qquad z \quad = \quad 1-V/V_o$

i.e. $\qquad\qquad z \quad = \quad 1-0.5(S_E+S_e)(L-u)/V_o \qquad\qquad w < u$

where $\qquad\qquad V_o \quad = \quad 0.25\pi L(D^2-Nd^2)$ as before, similarly S_o.

Thus $\qquad\qquad R \quad = \quad S/S_o$

and $\qquad\qquad P_r \quad = \quad dR/dr$

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The grain dimensions, together with u = w/2 may be used to determine the fraction of charge burnt prior to slivering.* The grain's progressivity is optimised by increasing z at u = w/2, as close to 1.0 as physically possible, since sliver burn is a degressive burn phenomena. For N=7, typical grain dimensions are d=0.1D, L=2.25D and therefore P_r=0.37 and $Z(u=w/2) = 0.85$ with r=1.532 w/2 at z=1.0.

Optional relation (a) of the general cubic type for z can be used if the fraction z_w of charge burnt when u = w, and r_2 and r_1 - the regression distances at which the large and small slivers are all-burnt, are known i.e.[4](a)

$$z = z_S + z_S = a_o + a_1 r + a_2 r^2 + a_3 r^3$$

where $a_3 = 0.0$

$$a_o = z_w - (1-z_w)[w^2 + 2w(r_S - w)] \qquad 0.5w \cdot r \cdot r_1$$

$$a_o = z_w - (1-z_w)[w^2 + 2w(r_S - w)] \qquad r_1 < r < r_2$$

$$a_1 = 2(1-z_w)(r_S - 2w) \qquad 0.5w < r < r_1$$

$$a_1 = 2(1-z_w)(r_S - 2w) \qquad r_1 < r < r_2$$

$$a_2 = 1 - z_w \qquad 0.5w < r < r_2$$

Optional relation (b) During slivering, using a linear approximation,[12]

$$S = S_w[1 - (z - z_w)/(1 - z_w)] \qquad 0.5w < r < r_2$$

generate a table of S versus $z_w < z < 1.0$, and interpolate for the required value of S.

---

* (i) $r = r_o$ when $r = 0.5w$ i.e.

$$r_o = 0.125 \, D - 0.375d$$

web burns through to form six large and six small slivers.

(ii) $r = r_1$ when $\beta_1 = 0$ i.e.

$$r_1 = 0.14433757D - 0.35566243 \, d$$

small slivers complete burning

(iii) $r = r_2$ when $\beta = 0$ i.e.

$$r_2 = 0.16930476D - 0.33069524 \, d$$

large slivers complete burning.

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Similarly, for prior to sliver burn

$$S = S_o - z(S_o - S_w)/z_w \qquad\qquad 0 < r < 0.5w$$

for constant values S_w and z_w, which are calculable from the previous more accurate relations. Option (b) enables significant saving of space and time for the computer simulation, and is a reasonable approximation for L/D > 1.0. It may also be utilised for grains other than 7 perforations, i.e. any N.

(iii) Uninhibited, Sliver Burn, N=19
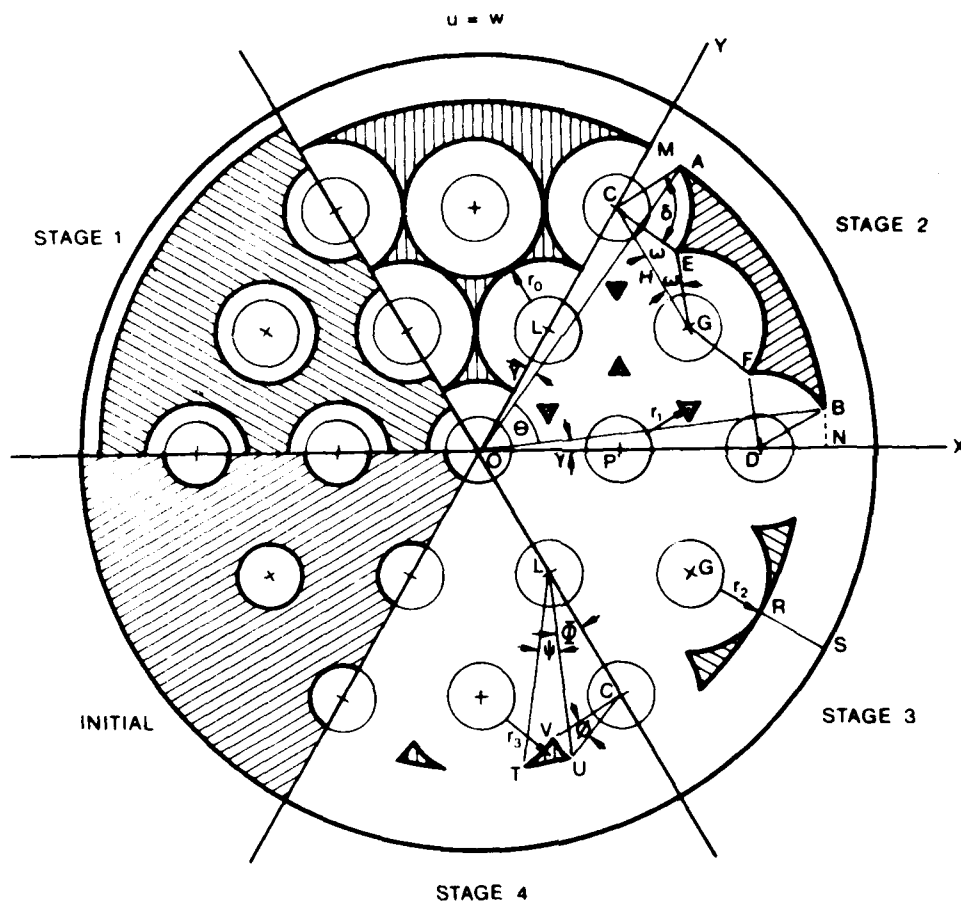
Slivers 1. Inner set, 6 small, as for N=7

2. Second row, 6 small, identical to inner set

3. Third row, 12 small, identical to inner set

4. Outer set, two stage sliver burn

    (a) 6 large slivers

    (b) 12 residual slivers

Stages of burn

1 Prior to slivering, $\qquad u < w = (D-5d)/6$

2 Burning of 24 small and 6 large slivers, $\qquad r_o < r < r_1$

3 Burning of six large slivers, $\qquad r_1 < r < r_2$

4 Burning of 12 residual slivers, $\qquad r_2 < r < r_3$

where $\qquad r_o = 0.0833 \cdot D - 0.4166 \cdot d$

$\qquad r_1 = 0.09622504\, D - 0.40377496\, d$

$\qquad r_2 = 0.10566243\, D - 0.39433757\, d$

and $\qquad r_3 = 0.11286984\, D - 0.38713016\, d$

Stage 1 - Prior to Slivering $\qquad u < w = (D-5d)/6$

Identical to 5(b)(i), N=19

Stage 2 $\qquad\qquad (D-5d)/6 < u < 2r_1$

(a) 24 small slivers

Identical to 5(b)(iii), thus: $S_s = S_1 + S_e$ with (N-1)=24

$$V_s = 0.5\, S_e\, (L-u)$$

(b) 6 large slivers

$$OB = OA = O.S.\ (D-u)$$

$$OC = CD = (D+d)/3$$

$$AC = GE = 0.5\ (d+u)$$

$$CL = OC/2 = CG$$

$$CH = OC/4$$

$$OG = 0.866026\ CD$$

$$\gamma = \text{arcos}[(OC^2 + OA^2 - AC^2)/2OC.OA]$$

$$\theta_1 = \pi/3 - 2\gamma$$

$$\text{arc } AB = OB.\theta_1$$

$$OCA = \text{arcos}[(OC^2 + AC^2 - OA^2)/2OC.AC]$$

$$\omega_1 = \text{arcos}(CH/AC)$$

$$\delta = OCA - \omega_1 - \pi/3$$

$$\text{arc } AE = AC.\delta$$

$$\text{arc } EF = AC.(\pi - 2\omega_1)$$

Lateral area of large slivers is therefore :

$$S_L^1 = 6(L-u)(AB+2AE+EF)$$

$$HE = AC \sin(\omega_1)$$

$$MCA = \pi - OCA$$

Area of ends of large slivers

$$S_E^1 = 12[0.5.OA.AB-0.433013CD^2-CG.HE-0.5GE.EF-AC.AE-CA^2MCA]$$

Hence $$S_S = S_L^1 + S_E^1$$

and $$V_S = 0.5 S_E^1 (L-u)$$

Thus for stage 2, with S_o and V_o identical to section 5(b).

$$S = S_s + S_S$$

$$V = V_s + V_S$$

$$z = 1-V/V_o$$

and $$R = S/S_o$$

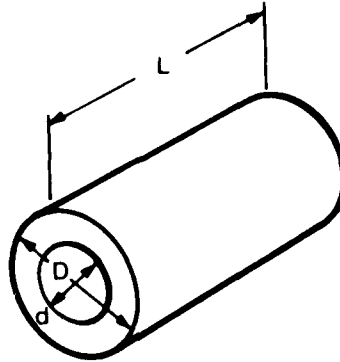## Stage 3 - Continued burn of 6 large slivers

where $$S = S_S$$

$$V = V_S$$

$$z = 1-V/V_o$$

and $$R = S/S_o$$

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Stage 4 - Burning of 12 sub-slivers

These are almost identical to the large slivers of 7 perforated grains, as given in section 5(b)(ii).

(iv) Inhibited External Wall, Prior to Sliver Burn

$$w = D-d$$

$$S_o = 0.5\pi(2dLN+D^2-Nd^2)$$

$$V_o = 0.25\pi L(D^2-Nd^2)$$

Hence

$$S = 0.5\pi[2N(d+u)(L-u)+D^2-N(d+u)^2]$$

$$V = 0.25\pi(L-u)[D^2-N(d+u)^2]$$

and

$$z = 1-V/V_o = a_1 r + a_2 r^2 + a_3 r^3$$

for

$$a_1 = 2(2NdL+D^2-Nd^2)/L(D^2-Nd^2)$$

$$a_2 = 4N(L-2d)/L(D^2-Nd^2)$$

$$a_3 = -8N/L(D^2-Nd^2)$$

(v) Inhibited External Wall and Ends, Prior to Sliver Burn

$$S_o = \pi dLN$$

$$V_o = 0.25\pi L(D^2-Nd^2)$$

Hence

$$S = \pi L(d+u)$$

$$V = 0.25\pi L(D^2-(d+u)^2 N)$$

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and
$$z = 1-V/V_o = a_1 r + a_2 r^2 + a_3 r^3$$

for

$$a_3 = 0.0$$

$$a_1 = 4dN/(D^2-Nd^2)$$

$$a_2 = -N/(D^2-Nd^2) = a_1 d$$

(vi) Inhibited External Wall, During Sliver Burn, N=7

Prior to sliver burn,
general case.
5(b),(i)
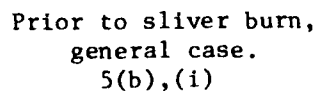
Assume a symmetric perforation pattern; a three-stage sliver burn is
obtained.
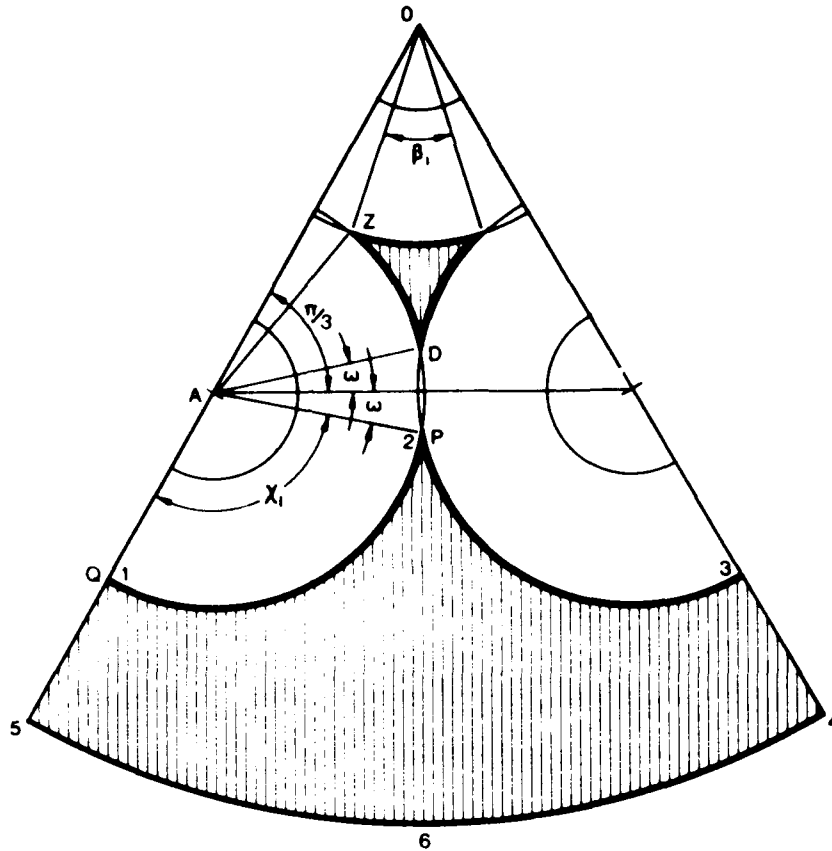
Stage 1 - Geometry of *inner* slivers is identical to case 5(b)(ii) for the limits $0.5w < r < r_1$.

$$S_1 = (N-1)(L-u)(3.OZ.\beta_1)$$

and

$$S_e = 2(N-1)OZ^2[\sin^2(0.5\beta_1) \times 1.73206 - 1.5(\beta_1 - \sin(_1))]$$

For the *huge outer sliver 123465*, we have :

$$\chi_1 = 2\pi/3 - \omega$$

$$\omega = \text{arcos } (AD/AP)$$

$$AD = 0.5 \text{ } OA$$

$$AP = AQ = 0.5(d+u)$$

$$PQ = AP.\chi_1$$

$$OA = 0.25 \text{ } (D+d)$$

Area of lateral surfaces S_{LI} is therefore :

$$S_{LI} = (L-u)\{\pi D + 2(N-1)AP.\chi_1\}$$

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and area of two end surfaces S_{EI} is :

$$S_{EI} = 2\{ \pi D^2/4 - 2(N-1)(OAD)_\Delta - 2(N-1)(ADP)_\Delta - 2(N-1)(APQ)_{sect.}\}$$

where $\quad (AOD)_\Delta = 0.25 \sqrt{3}\ OA^2$

$\qquad\qquad (ADP)_\Delta = 0.5 \times AD \times AP \times \sin(\omega)$

$\qquad\qquad (APQ)_{sect} = 0.5 \times AP^2 \times \chi_1$

total area of sliver surfaces is

$$S_1 = S_1 + S_{LI} + S_e + S_{EI}$$

and since $\quad V_1 = (S_e + S_{EI})(L-u)0.5$

$\qquad\qquad z_1 = 1.0 - (S_e + S_{EI})(L-u)0.5/V_o$

where $\qquad V_o = 0.25\pi L (D^2 - Nd^2)$

Stage 2 - Continued burn of outer sliver under the limits :

$$r_1 < r < 0.25(D+3d) \equiv w$$

where $\qquad S_2 = S_{LI} + S_{EI}$

$\qquad\qquad V_2 = S_{EI}\ 0.5\ (L-u)$

and $\qquad\qquad z_2 = 1.0 - 0.5\ S_{EI}\ (L-u)/V_o$

Stage 3 - Residual burn of six outer sliver remnants under the limits:

$$w < r < u_2 = (5D^2 - 6Dd + 5d^2)^{\frac{1}{2}}/8$$

$$S_3 = S_{LI3} + S_{LE3}$$

where $\qquad S_{LI3} = (N-1)(L-u)(\beta.OQ + 2.\chi.AQ)$

$\qquad\qquad \beta = 2(\pi/(N-1) - \phi)$

$\qquad\qquad \phi = arcos[(\theta A^2 + OQ^2 - AQ^2)/2.OA.OQ)]$

$\qquad\qquad OQ = 0.5\ D$

$\qquad\qquad \chi = OAQ - \omega - \pi/3$

$\qquad\qquad OAQ = arcos[(OA^2 + AQ^2 - OQ^2)/2.OA.AQ)]$

and $\quad S_{EI3} = 2(N-1)\{OQ \sin (0.5\beta) [(2.AQ.\sin(0.5\chi))^2 - (OQ \sin(0.5\beta))^2]^{\frac{1}{2}}$

$$+ 0.5 \ OQ^2 (\beta - \sin(\beta)) - AQ^2 (\chi - \sin(\chi))\}$$

thus $\quad V_3 = 0.5 \ S_{EI3} \ (L-u)$

and $\quad z_3 = 1.0 - 0.5 \ S_{EI3} \ (L-u)/V_o$

(c) Symmetrically Perforated Grains, Unequal Webs, Uninhibited

Prior to sliver burn, that is, until $2r$ is greater than or equal to the shortest web, the form-function is identical to case 5(b)(i).

(i) Sliver Burn, Inner Web Smaller than Outer Web

(a) large outer sliver if $r_1 < 0.5 \ \omega_o$
or (b) six outer + six residual inner slivers if $r_1 > 0.5 \ \omega_o$

$w_i/2 < r \leqslant (f_1, \ \omega_o/2)$ STAGE 2

large outer sliver + 6 inner slivers

STAGE 1

STAGE 3
sequel to 2(a)
6 outer slivers
for $u > \omega_o$

Prior to slivering, $r < 0.5 \ w_i$, as for general case 5(b)(1).

INITIAL
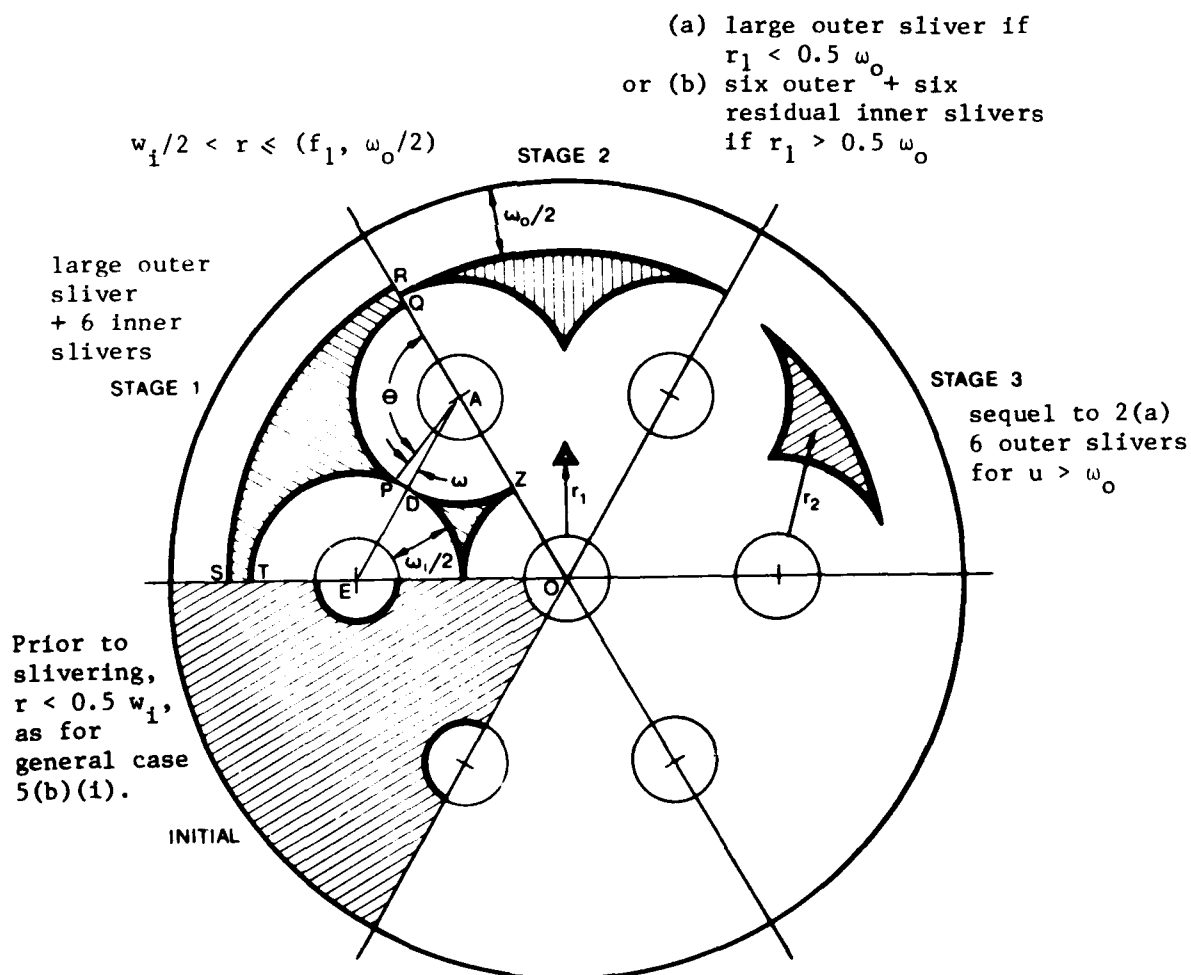
* Assume perforations are symmetrically spaced and are of equal diameter.
  For both cases :

$$r_1 = 0.07735027 \, d + 0.57735027 \, w_i$$

$$D = 3d + 2w_o + 2w_i$$

$$r_2 = \{0.25(D^2-d^2)+(1-\sqrt{3}D/2)(d+w_i)^2\}/(D+(1-\sqrt{3})d-\sqrt{3}w_i)$$

Stage 1

$$r > 0.5 \, w_i \quad \text{and} \quad r < (0.5 \, w_o, \, r_1)$$

cf. equal web case 5(b)(ii).

(a) six inner slivers

$$S_1 = (N-1)(L-u)(3 \, OZ.\beta_1)$$

and

$$S_e = 2(N-1) \, OZ^2\{1.73206 \sin^2(0.5\beta_1) - 1.5 \, x$$

$$x \, (\beta_1-\sin(\beta_1))\}$$

where

$$OZ = (AQ = AZ = 0.5)(d+u)$$

$$\beta_1 = 2(\, /(N-1)-\phi_1)$$

$$\phi_1 = \text{arcos}(OA/2OZ)$$

$$OA = d + w_i$$

(b) large outer sliver

For

$$\theta = 2^\pi/3-\omega$$

$$\omega = \text{arcos} \, (^{AD}/AP)$$

$$AD = 0.5 \, OA$$

$$OA = d + w_i$$

$$AP = 0.5 \, (d+u) \equiv OZ$$

$$S_{OL} = (L-u)[\pi(D-u) + 6.\theta.(d+u)]$$

And if

$$PD = AD.\tan(\omega)$$

$$AE = 2.AD = OA$$

$$OD = OA.0.5\sqrt{3}$$

$$S_{EL} = 0.5\pi(D-u)^2-3 \, OA^2(\sqrt{3}+\tan(\omega))-(2 \, -3w)d+u)$$

Then

$$S = S_1 + S_e + S_{OL} + S_{EL}$$

and

$$z = 1.0-0.5 \, (S_e + S_{EL})(L-u)/V_o$$

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Stage 2

(a)   For $r_1 < r < 0.5\ w_o$

$$S = S_{OL} + S_{EL}$$

$$z = 1.0 - 0.5\ S_{EL}\ (L-u)/V_o$$

(b)   If $r_1 > 0.5\ w_o$ and $r_1 > r > 0.5\ w_o$

$$S = S_1 + S_e + S_L + S_E$$

$$z = 1.0 - 0.5\ (S_e + S_E)(L-u)/V_o$$

where $S_L = (N-1)(L-u)(\beta.OQ + 2\chi.AQ)$

$\qquad AQ = 0.5\ (d+u)$

$\qquad OQ = 0.5\ (D-u)$

$\qquad \chi = OAQ-\omega-\pi/3$

$\qquad \omega = arcos\ (AD/AQ)$

$\qquad AD = 0.5\ OA$

$\qquad OA = d + w_i$

$\qquad OAQ = arcos[\ (OA^2+AQ^2-OG^2)/2OA.AQ]$

$\qquad \beta = 2(\pi/6-\phi)$

$\qquad \phi = arcos[\ (OA^2+OQ^2-AQ^2)/2OA.OQ]$

and $S_E = 2(N-1)\{OQ.sin(0.5\beta)[\ (2AQ.sin(0.5\chi))^2 - (OQsin(0.5\beta))^2]^{\frac{1}{2}}$
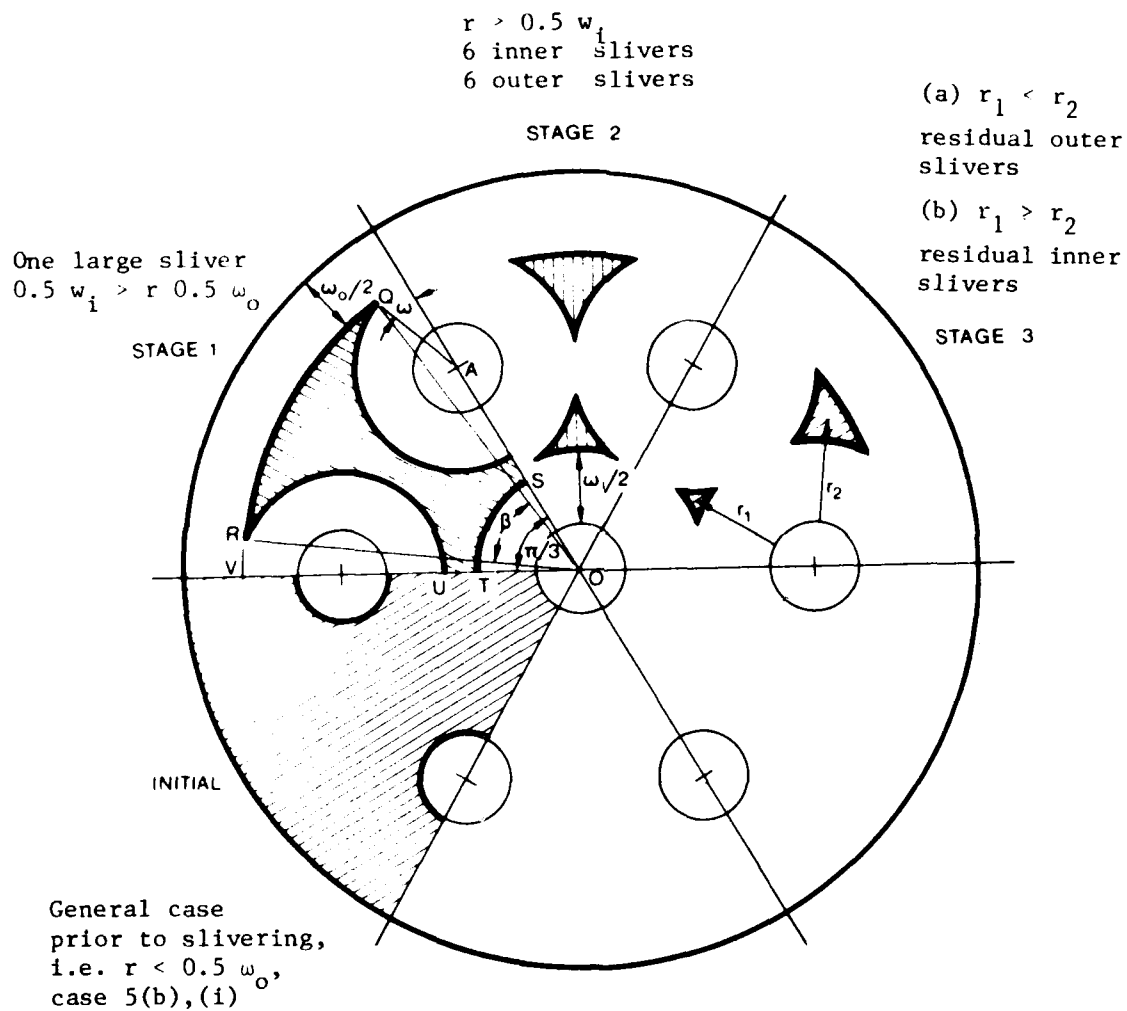
$$+ 0.5\ OQ^2(\beta-sin(\beta)) - AQ^2(\chi-sin(\chi))\}$$

Stage 3

If $r_1 < 0.5\quad w_o$ and $r_2 > r > (0.5\ w_o,r_1)$

$$S = S_L + S_E$$

$$z = 1.0 - 0.5\ S_E\ (L-u)/V_o$$

r > 0.5 w_i
6 inner slivers
6 outer slivers

STAGE 2

(a) $r_1 < r_2$
residual outer
slivers

(b) $r_1 > r_2$
residual inner
slivers

One large sliver
0.5 w_i > r 0.5 ω_o

STAGE 1

STAGE 3

INITIAL

General case
prior to slivering,
i.e. r < 0.5 ω_o,
case 5(b),(i)

Case 2 - Sliver Burn

Stage 1

$$0.5w_o < r < (0.5w_i, r_2)$$

One large sliver for which :

$$S_{LL} = 6(L-u)\{VQ + 2.QR + TS\}$$

$$VQ = 0.5 \ (D-u)\beta$$

$$\beta = 2(\pi/6-\phi)$$

$$\phi = arcos[(OA^2 + OQ^2 - AQ^2)/2OA.OQ]$$

$$OA = d + w_i$$

$$OQ = 0.5 \ (D-u)$$

$$AQ = 0.5 \ (d+u)$$

$$QR = 0.5 \ (d-u)(\pi-\omega)$$

$$(\pi-\omega) = arcos[(AQ^2 + OA^2 - OQ^2)/2AQ.OA]$$

$$TS = \pi(d+u)/6$$

and $$S_{EE} = 0.5\{\pi D^2 - 3u(2D-u) - 7\pi(d+u)^2 + 48A^*\}$$

$$S = S_{LL} + S_{EE}$$

$$z = 1.0 - 0.5 \ S_{EE} \ (L-u)/V_o$$

where $$A^* = \tfrac{1}{4}(d+u)(d+w_i)\sin(\pi-\omega) + {}^1/8(d+u)^2\omega - {}^1/8(D-u)^2\epsilon$$

and $$\epsilon = arcos[(OQ^2+OA^2-AQ^2)/2OQ.OA]$$

Stage 2

$$0.5w_i < r < (r_1, r_2)$$

c.f. case 1, stages 1 and 3(b)

$$S = S_1 + S_e + S_L + S_E$$

$$z = 1.0 - 0.5(S_e + S_E)(L-u)/V_o$$

Stage 3

(a) If $r_1 < r_2$ $\qquad$ $S = S_L + S_E$

$\qquad\qquad\qquad\qquad\qquad$ $z = 1.0 - 0.5\, S_E(L-u)/V_o$

(b) If $r_2 > r_1$ $\qquad$ $S = S_1 + S_e$

$\qquad\qquad\qquad\qquad\qquad$ $z = 1.0 - 0.5\, S_e(L-u)/V_o$

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