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Columbia University
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DEPARTMENT OF CIVIL ENGINEERING
AND ENGINEERING MECHANICS

Strain Energy Expressions of Rings of Rectangular, T- and I- Section,
Suitable for the Dynamic Analysis of Ring-Stiffened Cylindrical Shells

by

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Strain Energy Expressions of Rings of Rectangular, T- and I- Section, Suitable for the Dynamic Analysis of Ring-Stiffened Cylindrical Shells.

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Hans H. Bleich

Office of Naval Research
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ABSTRACT

Strain energy expressions are obtained for rings of rectangular, T- and I- section. The expressions are intended for use in the dynamic analysis of ring stiffened cylindrical shells. The approach is essentially a generalization of the convertional, approximate analysis of straight beams, i.e. the influence of shear stresses, and of direct stresses at right angles to the axis of the beams is neglected.

1. INTRODUCTION

The results obtained in this study will facilitate the determination of the modes of free vibation and/or of the dynamic response of ring-stiffened cylindrical shells. Expressions for the potential energy of the stiffeners in their most general state of displacement will be derived. The strain energies are obtained on the basis of "beam theory," i.e. the expressions are generalizations of those conventionally used for straight beams when shear effects and stresses at right angles to the axis of the beam are ignored. The approach is similar to the treatment of straight beams of thin-walled section in Ref. 1, Chapter 5, where the sections are considered to be built up from a number of flat plates for each of which the strain energy from elementary beam theory is known. Making the assumption that the shape of the cross section does not change, the energy in each plate can be expressed by the global coordinates of the bar. Allowing for the continuity of strains wherever two of the plates are jointed, strain energy expressions and the location of the shear center are obtained.

In the present treatment the elements of which the bars consist are either not flat, or not straight. The sections shown in Figs. 1a, b, and c will be treated. The ring in Fig. 1a is of simple rectangular cross section; it is really a flat plate of annular shape. In Fig. 1b the section is a T. The web is again an annular flat plate, while the flange is a short segment of a cylindrical shell. The third case, Fig. 1c, consists of three parts of similar nature. Expressions for the strain

energies of the elements are derived in the Appendices, using appropriately simplified relations available from plate and shell theories. For deformation of the entire bar in the plane of curvature a generalization of conventional beam theory is used, which assumes that plane sections remain plane, and also at right angles to the deformed center line.

The expressions in the Appendices could also be used to treat nonsymmetric cases like U- or L- stiffeners, Figs. 1d, e. These cases are not included because their use seems rather unlikely.

The strain energy expressions V obtained may be used in various, fairly obvious ways. They may be utilized to find the boundary conditions at the stiffeners for the a priori known partial differential equations for vibrations of the shell by using Hamilton's principle,

$$\int (V - T)\,dt = \text{extremum} \qquad (1)$$

and applying calculus of variation with respect to the shell coordinates x and ϕ, Fig. 2. V and T are the strain energy and the kinetic energy, respectively.

As an alternative, one can use a Raleigh-Ritz approach and introduce appropriate approximation for the shell displacements u, v, w, Fig. 3, into Eq. (1).

As a further alternative, one may introduce into Eq. (1) the expressions

$$u = U_n(x)\ \cos(n\phi + \alpha)$$

$$v = V_n(x)\ \sin(n\phi + \alpha) \qquad (2)$$

$$w = W_n(x)\ \cos(n\phi + \alpha)$$

where α is a phase angle. This substitution reduces the partial differential equations of the shell to three ordinary, simultaneous ones in U, V and W.

For either of the alternatives, a suitable expression for the strain energy of the cylindrical shell may be found in Ref. 2.

II. STIFFENING RINGS OF RECTANGULAR CROSS SECTION, FIG. 1a

Fig. 4a shows a portion of the shell of thickness t and radius a and an interior stiffener of depth d and thickness h. The depth d is a nominal one, measured from the center surface of the shell to the innermost edge of the stiffener, Fig. 4a. This figure also shows the centroid O of the stiffener cross section, and the radius R_O of the centroidal circle. Fig. 4b shows the original and the displaced center lines of shell and stiffener, and the displacements u_O, w_O of the centroid O as well as the rotation β. The out-of-plane displacement is v_O, but cannot be indicated in Fig. 4b.

Using Eqs. (A-13) and (A-22) for the portions of the strain energy of the stiffener in and out of the plane of curvature, respectively,

$$V = \frac{1}{2} \int [\frac{EZ}{R_O^3} (\frac{\partial^2 w_O}{\partial \phi^2} + w_O)^2 + \frac{EA}{R_O} (w_O + \frac{\partial v_O}{\partial \phi})^2 + \\ + \frac{EI_O}{R_O^3} (\frac{\partial u_O}{\partial \phi^2} + R_O \beta)^2 + \frac{GJ_O}{R_O^3} (\frac{\partial u_O}{\partial \phi} - R_O \frac{\partial \beta}{\partial \phi})^2] d\phi \tag{3}$$

where

$$Z = \frac{hd^3}{12}, \quad A = hd, \quad I_O = \frac{h^3 d}{12}, \quad J_O = \frac{h^3 d}{3}, \quad R_O = a - \frac{d}{2} \tag{4}$$

The value of Z is approximate, but suitable if d << a. The displacements u_O, w_O and β can be expressed from geometry by the shell displacements at point A, Figs. 4a, 4b,

$$w_O = w_A, \quad \beta = -\frac{\partial w_A}{\partial x}, \qquad u_O = u_A - \frac{d}{2}\beta \equiv u_A + \frac{d}{2}\frac{\partial w_A}{\partial x} \tag{5}$$

The quantity v_O appears in Eq. (3) only in the form $\frac{\partial v_O}{\partial \phi}$. To express this derivative, the equality of the strains in shell and stiffener at point A, Fig. 4a, is used

$$\frac{1}{a}\frac{\partial v_A}{\partial \phi} + \frac{1}{a} w_A = \frac{w_O}{R_O} + \frac{1}{R_O}\frac{\partial v_O}{\partial \phi} - \frac{d}{2aR_O}\left(\frac{\partial^2 w_A}{\partial \phi^2} + w_A\right) \tag{6}$$

The left hand side of this equation is the membrane strain in the shell at point A, while the right hand side is obtained from Eq. (A-15) for $\eta = d/2$. Noting the relation between a and R_O gives

$$\frac{\partial v_O}{d\phi} = \frac{d}{2a}\frac{\partial^2 w_A}{\partial \phi^2} + \frac{R_O}{a}\frac{\partial v_A}{\partial \phi} \tag{7}$$

Substitution into Eq. (3) gives the strain energy in the ring stiffener in terms of u_A, v_A and w_A,

$$V = \frac{1}{2}\int\left[\frac{EZ}{R_O^3}\left(\frac{\partial^2 w_A}{\partial \phi^2} + w_A\right)^2 + \frac{EA}{R_O}\left(w_A + \frac{d}{2a}\frac{\partial^2 w_A}{\partial \phi^2} + \frac{R_O}{a}\frac{\partial v_A}{\partial \phi}\right)^2 + \right.$$

$$+ \frac{EI_O}{R_O^3}\left(\frac{\partial^2 u_A}{\partial \phi^2} + \frac{d}{2}\frac{\partial^3 w_A}{\partial x \partial \phi^2} - R_O\frac{\partial w_A}{\partial x}\right)^2 \tag{8}$$

$$\left. + \frac{GJ_O}{R_O^3}\left(\frac{\partial u_A}{\partial \phi} + a\frac{\partial^2 w_A}{\partial x \partial \phi}\right)^2\right] d\phi$$

If desired, one could introduce the approximation $R_O \simeq a$, but the resulting simplification is hardly worthwhile.

III. STIFFENING RINGS OF T-SECTION, FIG. 1b

Figure 5a shows the shell and an interior stiffener and all dimensions. The centroid of the stiffener is at a distance e from the middle surface of the shell. Figures 5b and 5c show the web and flange in their original and in their displaced positions, respectively. Also shown are the centroids O_w and O_1 and their displacements.

The strain energy of the entire stiffener due to displacements w_o and v_o is

$$V(v_o,w_o) = \frac{1}{2} \int [\frac{EZ_o}{R_o^3} (\frac{\partial^2 w_o}{\partial \phi^2} + w_o)^2 + \frac{EA_o}{R_o} (w_o + \frac{\partial v_o}{\partial \phi})^2] d\phi \quad (9)$$

where $Z_o = I_o$ and A_o are the moment of inertia and the area of the section, respectively.

The strain energies due to the displacement in the x-direction and rotation for the web, V_w, and for the flange, V_1, are according to Eqs. (A-22) and (A-32), respectively,

$$V_w(u_w,\beta) = \frac{1}{2} \int [\frac{EI_w}{R_w^3} (\frac{\partial^2 u_w}{\partial \phi^2} + R_w\beta)^2 + \frac{GJ_w}{R_w^3} (\frac{\partial u_w}{\partial \phi} - R_w \frac{\partial \beta}{\partial \phi})^2] d\phi \quad (10)$$

$$V_1(u_1,\beta) = \frac{1}{2} \int [\frac{EI_1}{R_1^3} (\frac{\partial^2 u_1}{\partial \phi^2} + R_1\beta)^2 + \frac{GJ_1}{R_1^3} (\frac{\partial u_1}{\partial \phi} - R_1 \frac{\partial \beta}{\partial \phi})^2] d\phi \quad (11)$$

where the section properties are defined by

$$I_w = \frac{dh^3}{12}, \quad J_w = \frac{dh^3}{3}, \quad I_1 = \frac{t_1 b^3}{12}, \quad J_1 = \frac{bt_1^3}{3} \tag{12}$$

The displacements w_O, u_1 and u_w can be expressed by the equivalent quantities at point A. The relations are

$$w_O = w_A, \quad u_w = u_A - \frac{d}{2}\beta, \quad u_1 = u_A - d\beta, \quad \beta = -\frac{\partial w_A}{\partial x} \tag{13}$$

To express $\frac{\partial v_O}{\partial \phi}$, the strain in the shell at point A, and the strain in the web at the same point are equated

$$\frac{1}{a} w_A + \frac{1}{a}\frac{\partial v_A}{\partial \phi} = \frac{1}{R_O}\left(w_O + \frac{\partial v_O}{\partial \phi}\right) - \frac{e}{R_O(R_O+e)}\left(\frac{\partial^2 w_O}{\partial \phi^2} + w_O\right) \tag{14}$$

The right-hand side of this equation is Eq. (A-15) for $\eta = e$. After simplification

$$\frac{\partial v_O}{\partial \phi} = \frac{e}{a}\frac{\partial^2 w_A}{\partial \phi^2} + \frac{R_O}{a}\frac{\partial v_A}{\partial \phi} \tag{15}$$

The total strain energy is the sum of Eqs. (9), (10) and (11). After substitution of Eqs. (13) and (15)

$$V = \frac{1}{2}\int\{\frac{EI_O}{R_O^3}\left(\frac{\partial^2 w_A}{\partial\phi^2} + w_A\right)^2 + \frac{EA_O}{R_O a^2}\left(aw_A + e\frac{\partial^2 w_A}{\partial\phi^2} + R_O\frac{\partial v_A}{\partial\phi}\right)^2 +$$

$$+ \frac{EI_w}{R_1^3}\left[\frac{\partial^2 u_A}{\partial\phi^2} + \frac{d}{2}\frac{\partial^3 w_A}{\partial x\partial\phi^2} - \left(a - \frac{d}{2}\right)\frac{\partial w_A}{\partial\phi}\right]^2 +$$

$$+ \frac{EI_1}{R_1^3}\left[\frac{\partial^2 u_A}{\partial\phi^2} + d\frac{\partial^3 w_A}{\partial x\partial\phi^2} - (a - d)\frac{\partial w_A}{\partial\phi}\right]^2 + \tag{16}$$

$$+ G\left(\frac{J_w}{R_w^3} + \frac{J_1}{R_1^3}\right)\left(\frac{\partial u_A}{\partial\phi} + a\frac{\partial^2 w_A}{\partial x\partial\phi}\right)^2\}d\phi$$

This expression can be somewhat simplified, if desired, by noting that $d \lll a$, and that for thin-walled sections $I_w \lll I_1$. Using also

$$R_O \simeq R_1 \simeq R_w \simeq a - d \simeq a - 2d \simeq a$$

one obtains

$$V = \frac{1}{2a^3}\int[EI_O\left(\frac{\partial^2 w_A}{\partial\phi^2} + w_A\right)^2 + EA_O\left(aw_A + e\frac{\partial^2 w_A}{\partial\phi^2} + a\frac{\partial v_A}{\partial\phi}\right)^2 +$$

$$+ EI_1\left(\frac{\partial^2 u_A}{\partial\phi^2} + d\frac{\partial^3 w_A}{\partial x\partial\phi^2} - a\frac{\partial w_A}{\partial\phi}\right)^2 + G(J_w + J_1)\left(\frac{\partial u_A}{\partial\phi} + a\frac{\partial^2 w_A}{\partial x\partial\phi}\right)^2]d\phi \tag{17}$$

IV. I-BEAM STIFFENING RINGS, FIG. 1c

Figure 6a shows a cross section of the shell and of the stiffening ring inside the shell. The centroid of the web and of the entire stiffener is at O. The properties of the web and its displacements will carry the subscript O. The two flanges are of equal dimensions, b by t_1, and their centroids are O_1,O_2, respectively. The displacements of the two centroids carry the subscripts 1, 2, respectively.

Just as in Sections I and II, the portion of the strain energy $V(w_O,v_O)$ is given by Eq. (A-13),

$$V(w_O,v_O) + \frac{1}{2}\int[\frac{EZ}{R_O^3}(\frac{\partial^2 w_O}{\partial\phi^2} + w_O)^2 + \frac{EA}{R_O}(w_O + \frac{\partial v_O}{\partial\phi})^2]\,d\phi \qquad (18)$$

where A and Z are the area, and the moment of inertia of the entire section.

The portion V_O of the strain energy of the web due to the displacements u_O and β is according to Eq. (A-22)

$$V_O(u_O,\beta) = \frac{1}{2}\int[\frac{EI_O}{R_O^3}(\frac{\partial^2 u_O}{\partial\phi^2} + R_O\beta)^2 + \frac{GJ_O}{R_O^3}(\frac{\partial u_O}{\partial\phi} - R_O\frac{\partial\beta}{\partial\phi})^2]d\phi \qquad (19)$$

where

$$I_O = \frac{dh^3}{12}\,, \qquad J_O = \frac{dh^3}{3}\,, \qquad R_O = a - e - \frac{d}{2} \qquad (20)$$

The portions of the strain energy due to the displacements of the two flanges u_i and β are, respectively, from Eq. (A-32)

$$V_1(u_1,\beta) = \frac{1}{2}\int[\frac{EI}{R_1^3}(\frac{\partial^2 u_1}{\partial\phi^2} + R_1\beta)^2 + \frac{GJ}{R_1^3}(\frac{\partial u_1}{\partial\phi} - R_1\frac{\partial\beta}{\partial\phi})^2]d\phi \quad (21)$$

$$V_2(u_2,\beta) = \frac{1}{2}\int[\frac{EI}{R_2^3}(\frac{\partial^2 u_2}{\partial\phi^2} + R_2\beta)^2 + \frac{GJ}{R_2^3}(\frac{\partial u_2}{\partial\phi} - R_2\frac{\partial\beta}{\partial\phi})^2 d\phi \quad (22)$$

where u_1, u_2 are the displacements of the flanges 1 and 2, respectively, and

$$I = \frac{t_1 b^3}{12}, \quad J = \frac{t_1^3 b}{3}, \quad R_1 = a - e, \quad R_2 = a - e - d \quad (23)$$

Referring to Fig. 6b, the displacements, except v_i, can be expressed by the displacements of the shell at point A,

$$w_O = w_A, \quad \beta = -\frac{\partial w_A}{\partial x}, \qquad u_O = u_A - (\frac{d}{2} + e)\beta = u_A + (\frac{d}{2} + e)\frac{\partial w_A}{\partial x},$$
$$u_1 = u_A - e\beta = u_a + e\frac{\partial w_A}{\partial x} \qquad u_2 = u_A - (d + e)\beta = u_A + (d + e)\frac{\partial w_A}{\partial x} \quad (24)$$

The equality of the hoop strains at A in the shell and in the stiffener, with $\eta = e + d/2$ gives

$$\frac{1}{a}(w_A + \frac{\partial v_A}{\partial\phi}) = \frac{1}{R_O}(w_O + \frac{v_O}{\partial\phi}) - \frac{d + ze}{R_O(2R_O + d + 2e)}(\frac{\partial^2 w_O}{\partial\phi^2} + w_O)$$

and after simplification

$$\frac{\partial v_O}{\partial\phi} = \frac{d + 2e}{2a}\frac{\partial^2 w_A}{\partial\phi^2} + \frac{R_O}{a}\frac{\partial v_A}{\partial\phi} \quad (25)$$

Using Eqs. (24) and (25), the four Eqs. (18),(19),(21) and (22) become

$$V(w_O,v_O) = \frac{1}{2}\int[\frac{EZ}{R_O^3}(\frac{\partial^2 w_A}{\partial\phi^2} + w_A)^2 + \frac{EA}{R_O}(w_A + \frac{d+2e}{2a}\frac{\partial^2 w_A}{\partial\phi^2} + \frac{2a-d-2e}{2a}\frac{\partial v_A}{\partial\phi})^2]d\phi$$

$$V_O(u_O,\beta) = \frac{1}{2}\int[\frac{EI_O}{R_O^3}(\frac{\partial^2 u_A}{\partial\phi^2} + \frac{d+2e}{2}\frac{\partial^3 w_A}{\partial x\partial\phi^2} - R_O\frac{\partial w_A}{\partial x})^2 + \frac{GJ_O}{R_O^3}(\frac{\partial u_A}{\partial\phi} + a\frac{\partial^2 w_A}{\partial x\partial\phi})^2]d\phi$$

$$V_1(u_1,\beta) = \frac{1}{2}\int[\frac{EI}{R_1^3}(\frac{\partial^2 u_A}{\partial\phi^2} + e\frac{\partial^3 w_A}{\partial x\partial\phi^2} - R_1\frac{\partial w_A}{\partial x})^2 + \frac{GJ}{R_1^3}(\frac{\partial u_A}{\partial\phi} + a\frac{\partial^2 w_A}{\partial x\partial\phi})^2]d\phi$$

$$V_2(u_2,\beta) = \frac{1}{2}\int\{\frac{EI}{R_2^3}[\frac{\partial^2 u_A}{\partial\phi^2} + (d+e)\frac{\partial^3 w_A}{\partial x\partial\phi^2} - R_2\frac{\partial v_A}{\partial\phi}]^2 + \frac{GJ}{R_2^3}(\frac{\partial u_A}{\partial\phi} + a\frac{\partial^2 w_A}{\partial x\partial\phi})^2\}d\phi \tag{26}$$

The entire strain energy V is the sum of the four terms, Eqs. (26). However, some approximations seem permissible. The first term in the relation for V_O contains the factor $I_O = dh^3/12$, which is very small in comparison to the similar terms in V_1 and V_2, where $I = t_1 b^3/12$. For thin-walled sections the term multiplied by I_O is negligible. Further $e = (t + t_1)/2$ is small compared to d or R. Using $e \simeq 0$ is thus a reasonable approximation. Assuming further that $d \lll a$, one may use approximately $R_O \simeq R_1 \simeq R_2 \simeq a$. The end result is

$$V = \frac{1}{2a^3} \int [EZ\left(\frac{\partial^2 w_A}{\partial \phi^2} + w_A\right)^2 + Ea^2 A\left(w_A + \frac{d}{2a}\frac{\partial^2 w_A}{\partial \phi^2} + \frac{\partial v_A}{\partial \phi}\right)^2 +$$

$$+ EI\left(\frac{\partial^2 u_A}{\partial \phi^2} - a\frac{\partial w_A}{\partial \phi}\right)^2 + EI\left(\frac{\partial^2 u_A}{\partial \phi^2} + d\frac{\partial^3 v_A}{\partial x \partial \phi^2} - a\frac{\partial w_A}{\partial x}\right)^2 +$$

$$+ G(J_O + 2J)\left(\frac{\partial u_A}{\partial \phi} + a\frac{\partial^2 w_A}{\partial x \partial \phi}\right)^2] d\phi \qquad (27)$$

V. SUMMARY

Strain energy expressions have been obtained for ring stiffeners of the types shown in Figs. 1a, b and c. The results are, respectively, given by Eqs. (8), (17) and (27). The stiffeners shown in Fig. 1 are all on the inside of the shell. However the results obtained are also applicable to stiffeners of the same type on the outside of the shell. In such cases the quantities d and e occurring in Eqs. (8), (17) and (27) must be replaced by $-d, -e$, respectively.

Possible applications for the strain energy expressions are indicated in the Introduction. The suggested uses require an expression for the strain energy of shell panels adjoining the stiffeners. Such an expression is available in Ref. 2. Defining the shell displacements u,v and w as shown in Fig. 3, the strain energy of a panel is

$$U = \frac{E}{2(1-\nu^2)} \frac{h}{a} \iint [a^2 u_x^2 + (v_\phi + w)^2 + 2a\nu u_x (v_\phi + w) + \frac{1-\nu}{2}(u_\phi + a v_x)^2] dx d\phi$$

$$+ \frac{E}{24(1-\nu^2)} \frac{h^3}{a^3} \iint [a^4 w_{xx}^2 + (w_{\phi\phi} + w)^2 + \frac{1-\nu}{2}((a w_{x\phi} + u_\phi)^2 \qquad (28)$$

$$+ \frac{3(1-\nu)}{2} a^2 (v_x - w_{x\phi})^2 + 2\nu a^2 w_{xx}(w_{\phi\phi} - v_\phi) - 2a^3 u_x w_{xx}] dx d\phi$$

where t is the shell thickness, and the subscripts x or ϕ indicate partial derivatives with respect to x or ϕ. The double integrals extend over the area of the shell panel.

APPENDIX 1.

Strain energy of a circular curved bar due to deformation v, w in the plane of curvature, Fig. A-1.

Consider an element of a curved bar, Fig. A-2, the cross section of which is symmetric to the plane of curvature. The element is stressed by a moment M and a direct force N, both in the plane of curvature. The small deflections of a point P in the location ϕ can be described by the radial component w and the tangential component v, Fig. A-1. Figs. A-1 and 2 show I-beams with unequal flanges, which are generalizations of the three specific cases, Figs. 1a, b and c, needed in the body of the report.

The approach used is a generalization of the simple Navier-Euler theory in straight beams, which assumes that only stresses in the axial direction contribute materially to the strain energy. Stresses in the radial direction and shear stresses, both of which must exist, are thus assumed to contribute only negligibly to the strain energy. The assumption that plane cross sections remain plane and at right angles to the deformed center line of the bar, leads to the distribution of bending stresses as obtained in the classical treatment of Winkler-Resal. The following is not concerned with a re-derivation of the equations for the stresses, but with the formulation of strain energy expressions in terms of derivatives of v and w. Such expressions seem not to be available in the literature.

Consider an element of the bar of length ds in its original and in its distorted shape, Fig. A-2. The centroidal axis of original length ds will be lengthened by Δds, and the angle between the two faces will change, as shown in the same figure, by $\frac{\Delta ds}{R} + \Delta d\phi$.

Considering an element dA at a radial distance η from the centroid, one can compute strain and stress in the element as function of Δds and $\Delta d\phi$

$$\sigma = E\left(\frac{1}{R}\frac{\Delta ds}{d\phi} + \frac{\eta}{R+\eta}\frac{\Delta d\phi}{d\phi}\right) \tag{A-1}$$

As usual, the quantity Z is introduced

$$Z = R\int \frac{\eta^2}{R+\eta}\, dA \equiv -R^2 \int \frac{\eta dA}{R+\eta} \tag{A-2}$$

If the depth of the bar is small compared to the radius R, the value of Z is practically identical to the moment of inertia I. In the body of the report, it will be assumed that $Z \simeq I$. Comparing the resultants of the stresses given by Eq. (A-1), and M and N, one can determine Δds and $\Delta d\phi$,

$$\frac{\Delta d\phi}{d\phi} = \frac{R^3 M}{EZ}$$

$$\frac{\Delta ds}{d\phi} = \frac{R}{EA}\left(N + \frac{M}{R}\right) \tag{A-3}$$

The strain energy dV in the element of length $ds = Rd\phi$ being equal to the work done by the forces M and N during the distortion of the element, one finds

$$V = \int dV\ R d\phi = \frac{1}{2} \int [\frac{EZ}{R} (\frac{\Delta d\phi}{d\phi})^2 + \frac{EA}{R} (\frac{\Delta ds}{d\phi})^2] d\phi \qquad (A-4)$$

The quantities $\frac{\Delta d\phi}{d\phi}$ and $\frac{\Delta ds}{d\phi}$ are to be expressed in terms of v and w, which are the components of the displacement of the centroid O of the cross section. Figuare A-3 shows the original and the distorted element superimposed on each other. Using polar coordinates, $\rho(\phi) = R + w(\phi)$, the curvature of the original center line is $1/R$, while the curvature $1/R_1$ of the distorted center line is, with $\rho = R + w$,

$$\frac{1}{R_1} = \frac{\rho^2 + 2(\frac{d\rho}{d\phi})^2 - \rho \frac{d^2\rho}{d\phi^2}}{[\rho^2 + (\frac{d\rho}{d\phi})^2]^{3/2}} \simeq \frac{1}{R+w} - \frac{1}{(R+w)^2} \frac{d^2w}{d\phi^2} \qquad (A-5)$$

The approximate result is obtained by using the fact that $(\frac{dw}{d\phi})^2 << (R+w)^2$. Forming the expression $1/R_1 - 1/R$, and allowing for $w << R$, and $ds \equiv R d\phi$, one finds

$$\frac{1}{R_1} - \frac{1}{R} = -\frac{w}{R(R+w)} - \frac{1}{(R+w)^2} \frac{d^2w}{d\phi^2} \simeq -\frac{1}{R^2} (\frac{d^2w}{d\phi^2} + w) \qquad (A-6)$$

In addition to above relation there is a geometric one between R, R_1, Δds and $\Delta d\phi$, which can be read from Fig. A-3. The total length of the distorted axis, $ds + \Delta ds$, must equal the new radius R_1 multiplied by the angle α enclosed by the two faces of the deformed element. Thus

$$R_1\alpha \equiv R_1\left(\frac{ds}{R} + \Delta d\phi + \frac{\Delta ds}{R}\right) = \Delta s + \Delta ds \qquad \text{(A-7)}$$

Rearranging and dividing by $R_1 R\, d\phi$ gives

$$\frac{1}{R_1} - \frac{1}{R} = \frac{1}{R}\frac{\Delta d\phi}{d\phi} + \frac{\Delta ds}{d\phi}\frac{R_1 - R}{R^2 R_1} \simeq \frac{1}{R}\frac{\Delta d\phi}{d\phi} \qquad \text{(A-8)}$$

The approximation in Eq. (A-8) is permissible because $(R_1-R)/R$ is inherently a small quantity in comparison to unity. Equations (A-6) and (A-8) furnish

$$\frac{\Delta d\phi}{\phi d} = -\frac{1}{R}\left(\frac{d^2 w}{d\phi^2} + w\right) \qquad \text{(A-9)}$$

In conjunction with Eq. (A-3) this relation leads to the well-known differential equation for the radial displacement w, see Ref. (3).

An additional geometric relation can be obtained from Fig. A-4. The end points A and B of the element ds displace to A' and B', respectively. The distances $\overline{CA'}$ and $\overline{CB'}$ follow from Fig. A-4, where quantities which are small of higher order are neglected.

$$\overline{CA'} = dw - vd\phi - dvd\phi \simeq dw - \frac{1}{R}vds$$

$$\overline{CB'} = Rd\phi + wd\phi + dv + dwd\phi \simeq ds + \frac{1}{R}wds + dv \qquad \text{(A-10)}$$

Further

$$\frac{1}{ds}\overline{A'B'} \equiv 1 + \frac{\Delta ds}{ds} = \sqrt{\left(1 + \frac{w}{R} + \frac{dv}{ds}\right)^2 + \left(\frac{dw}{ds} - \frac{v}{R}\right)^2} \qquad \text{(A-11)}$$

Expanding the square root by the binomial law and neglecting higher order terms gives

$$\frac{\Delta ds}{d\phi} = w + \frac{dv}{d\phi} \tag{A-12}$$

Substitution into Eq. (A-4) gives finally the strain energy expression

$$V = \frac{1}{2} \int [\frac{EZ}{R^3} (\frac{d^2w}{d\phi^2} + w)^2 + \frac{EA}{R} (w + \frac{dv}{d\phi})^2] d\phi \tag{A-13}$$

It is noted that the second Eq. (A-3) and Eq. (A-12) give the differential equation

$$\frac{w}{R} + \frac{dv}{ds} = \frac{1}{EA} [N + \frac{M}{R}] \tag{A-14}$$

Conventional texts contain only an approximation of the equation where the term M/R does not appear.

It will also be necessary to have an expression for the strain in a location η, Fig. A-2. Using Eqs. (A-1), (A-9) and (A-12) one finds

$$\varepsilon_\phi = \frac{1}{R} \frac{\Delta ds}{d\phi} + \frac{\eta}{R+\eta} \frac{\Delta ds}{d\phi} = \frac{1}{R}(w + \frac{dv}{d\phi}) - \frac{\eta}{R(R+\eta)} (\frac{d^2w}{d\phi^2} + w) \tag{A-15}$$

APPENDIX 2.

Strain energy of a thin annular plate, displaced at right angles to its middle plane, Fig. 5.

The web of a ring stiffener of a shell, Fig. 1a, b or c, may be considered as an annular plate. An expression for the strain energy of such a plate in polar coordinates can be found in Ref. (4 p. 346, Eq. (o)). This expression is a double integral over three major terms. The sum of the first two terms can be recognized as due to the direct stresses, while the third is due to the shear stresses. Let $V = V_1 + V_2$, and using the symbol $\bar{u}$ for the normal displacements of the plate,

$$V_1 = \frac{D}{2} \iint \left[\left(\frac{\partial^2 \bar{u}}{\partial r^2} + \frac{1}{r} \frac{\partial \bar{u}}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \bar{u}}{\partial \phi^2} \right)^2 - \right.$$

$$\left. -2(1-\nu) \frac{\partial^2 \bar{u}}{\partial r^2} \left(\frac{1}{r} \frac{\partial \bar{u}}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \bar{u}}{\partial \phi^2} \right)^2 \right] r dr d\phi \qquad \text{(A-16a)}$$

$$V_2 = \frac{D}{2} 2(1 - \nu) \iint \left(\frac{1}{r} \frac{\partial^2 \bar{u}}{\partial r \partial \phi} - \frac{1}{r^2} \frac{\partial^2 \bar{u}}{\partial \phi^2} \right)^2 r dr d\phi \qquad \text{(A-16b)}$$

Consistent with the assumption that the only stresses contributing materially to the strain energy V_1 of bending are direct stresses σ_ϕ, the term D in (A-16a) is to be evaluated for $\nu = 0$, while the value $\nu \neq 0$ is retained in Eq. (16-b) for shear effects. Further, as in elementary beam theory, it is assumed that the radial axis of the cross section remains a straight line

$$\bar{u}(r) \equiv \bar{u}(\eta) = u + \eta\beta \tag{A-17}$$

where u is the displacement of the centroid and β the rotation of the cross section. Using $r = R + \eta$, $dr = d\eta$, and Eq. (A-17), one has the relations

$$\frac{\partial^2\bar{u}}{\partial r^2} \equiv \frac{\partial^2 u}{\partial \eta^2} = 0, \qquad \frac{\partial\bar{u}}{\partial r} \equiv \frac{\partial\bar{u}}{\partial\eta} = \beta$$

$$\frac{\partial^2\bar{u}}{\partial\phi^2} = \frac{\partial^2 u}{\partial\phi^2} + \eta\,\frac{\partial^2\beta}{\partial\phi^2}, \quad \frac{\partial\bar{u}}{\partial\phi} = \frac{\partial u}{\partial\phi} + \eta\,\frac{\partial\beta}{\partial\phi}, \quad \frac{\partial^2\bar{u}}{\partial r\partial\phi} = \frac{\partial\beta}{\partial\phi} \tag{A-18}$$

After substitution of these relations into Eqs. (A-16) one can integrate with respect to $\eta = r - R$. Using the fact that d/R is small compared to unity, the various integrals can be approximated

$$\int \frac{dr}{r} = \ell n\,\frac{1+d/2R}{1-d/2R} \simeq \frac{d}{R}, \quad \int \frac{dr}{r^2} = \frac{d}{R^2-d^2/4} \simeq \frac{d}{R^2},$$

$$\int \frac{dr}{r^3} = -\frac{1}{2(R+d/2)^2} + \frac{1}{2(R-d/2)^2} \simeq \frac{d}{R^3} \tag{A-19}$$

One obtains thus

$$V_1 = \frac{1}{2}\,\frac{Edh^3}{12}\int \frac{1}{R^3}\left(\frac{\partial^2 u}{\partial\phi^2} + R\beta\right)^2 d\phi \tag{A-20}$$

and

$$V_2 = \frac{1}{2} \frac{Gdh^3}{3} \int \frac{1}{R^3} \left(\frac{\partial u}{\partial \phi} - R \frac{\partial \beta}{\partial \phi}\right)^2 d\phi \qquad \text{(A-21)}$$

As a check on the adequacy of the simplifications made in Eqs. (19) consider a rigid body rotation of the annular plate with respect to the axis $\phi = + \pi/2, \phi = - \pi/2$, where $u = \Delta \cos \phi$, $\beta = \frac{\Delta}{R} \cos \phi$ and Δ is the displacement at $\phi = 0$. Substitution into Eqs. (A-20, 21) indicates that for this displacement $V_1 = V_2 \equiv 0$, as required for a rigid body displacement.

The total strain energy for out-of-plane displacements is thus

$$V = \frac{1}{2R^3} \int \left[\frac{Edh^3}{12} \left(\frac{\partial^2 u}{\partial \phi^2} + R\beta\right)^2 + \frac{Gdh^3}{3} \left(\frac{\partial u}{\partial \phi} - R \frac{\partial \beta}{\partial \phi}\right)^2\right] d\phi \qquad \text{(A-22)}$$

The two coefficients appearing in the integrand are the values EI and GJ appearing in the equivalent expressions for a straight beam. For $R \to \infty$, Eq. (A-22) thus furnishes the conventional value for V for a straight beam.

APPENDIX 3.

Strain energy stored in the flanges of rings of T- and I-Sections

Let the displacements of an arbitrary point P on the middle surface of a flange, Fig. 7, be designated by the symbols u_P, v_P, w_P while the displacements of the centroid O of the undeformable, rectangular cross section are u,v,w and the rotation is β.

Only sections which are symmetric to the plane of curvature are treated so that the strain energy separates into the sum of two terms

$$V = V_1(w,v) + V_2(u,\beta) \tag{A-23}$$

one term depending only on w and v, the other on u and β. The term V_1 has already been obtained in Appendix 1, Eq. (A-13) and the associated strain in Eq. (A-15).

The second term, V_2, for T- or I-sections will be derived in the body of the report, using an expression for the web alone derived in Appendix 2, in conjunction with a relation to be derived here, treating flanges of rectangular cross section, b x t, as short pieces of cylindrical shells. See Fig. A-7. Assuming that the cross section does not change its shape, the displacements of a point P in the location η, z = 0, Fig. A-7, are

$$w_P = -\eta\beta, \quad u_P = u \qquad v_P = -\frac{\eta}{R}\frac{\partial u}{\partial \phi} \tag{A-24}$$

The strain energy $V_2(u,\beta)$ can be divided in a portion due to hoop strains ε_s, and one due to shear strains $\gamma, V_2 = V_{2\varepsilon} + V_{2\gamma}$. Allowing again for the fact that direct stresses other than σ_s are negligible, the value of Poisson's ratio in $V_{2\varepsilon}$ is assumed to vanish

$$V_{2\varepsilon} = \frac{E}{2} \iiint \varepsilon_s^2 \, d\eta dz(R + z)d\phi \qquad \text{(A-25)}$$

while the usual value of ν is retained in

$$V_{2\gamma} = \frac{G}{2} \iiint \gamma^2 d\eta dz(R + z)d\phi \qquad \text{(A-26)}$$

When evaluating Eq. (A-25) it is assumed that the bending strains do not vary significantly through the thickness, $t << d$, $t <<< R$, so that

$$V_{2\varepsilon} = \frac{Et}{2} \iint \varepsilon_s^2 \, d\eta(R + z)d\phi \simeq \frac{EtR}{2} \iint \varepsilon_s^2 \, d\eta d\phi \qquad \text{(A-27)}$$

where the value ε_s at $z = 0$ is to be used. Reference [5, Eq.(5-b) on p. 209] gives for $z = 0$, after substitution of Eqs. (A-24)

$$\varepsilon_s = \frac{1}{R} \frac{\partial v_p}{\partial \phi} + \frac{w_p}{R} = - \frac{\eta}{R^2} \frac{\partial^2 u}{\partial \phi^2} - \frac{\eta}{R} \beta$$

Equation (A-27) gives thus finally

$$V_{2\varepsilon} = \frac{1}{2} \frac{Etd^3}{12} \int \frac{1}{R^3} \left(\frac{\partial 2_u}{\partial \phi^2} + R\beta\right)^2 d\phi \qquad \text{(A-28)}$$

The above integral vanishes, as required, for the rigid body motion $u = \Delta \cos \phi$, $\beta = \frac{\Delta}{R} \cos \phi$.

To evaluate the integral in Eq. (A-26) use is made of Ref. [5, Eq. (5-c)]. The value of the shear strain at an arbitrary point A, Fig. A-7, is expressed by the values at points P on the center plane

$$\gamma_A = \frac{1}{R+z} \frac{\partial u_p}{\partial \phi} + \frac{R+z}{R} \frac{\partial v_p}{\partial \phi} - \frac{\partial^2 w_p}{\partial x \partial \phi} \left(\frac{z}{R} + \frac{z}{R+z}\right) \qquad \text{(A-29)}$$

Substitution of Eqs. (A-24) gives

$$\gamma_A \equiv \gamma = - z \frac{2R+z}{R^2(R+z)} \frac{\partial u}{\partial \phi} + z \frac{2R+z}{R(R+z)} \frac{\partial \beta}{\partial \phi} \simeq - 2z \left(\frac{1}{R^2} \frac{\partial u}{\partial \phi} - \frac{1}{R} \frac{\partial \beta}{\partial \phi}\right) \qquad \text{(A-30)}$$

The approximation used utilizes the fact that max $z = t/2 <<< a$. The value of the integral in Eg. (A-26) becomes thus

$$V_{2\gamma} = \frac{1}{2} \frac{Gbt^3}{3} \int \frac{1}{R^3} \left(\frac{\partial u}{\partial \phi} - R \frac{\partial \beta}{\partial \phi}\right)^2 d\phi \qquad \text{(A-31)}$$

As necessary, this value vanishes for the rigid body motion tested on Eq. (A-28).

The total value of $V_2(u,\beta)$ is

$$V_2(u,\beta) = \frac{1}{2} \frac{Etd^3}{12} \int \frac{1}{R^3} \left(\frac{\partial^2 u}{\partial \phi^2} + R\beta\right)^2 d\phi + \frac{1}{2} \frac{Gbt^3}{3} \int \frac{1}{R^3} \left(\frac{\partial u}{\partial \phi} - R \frac{\partial \beta}{\partial \phi}\right)^2 d\phi \qquad \text{(A-32)}$$

In the limit, $R \to \infty$, Eq. (A-32) furnishes the usual expression for the strain energy of a straight bar.

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5. W. Flügge, Stresses in Shells, 2nd. ed., J. Springer 1973, pp. 208, 209, Eqs. (5.5 a-c).

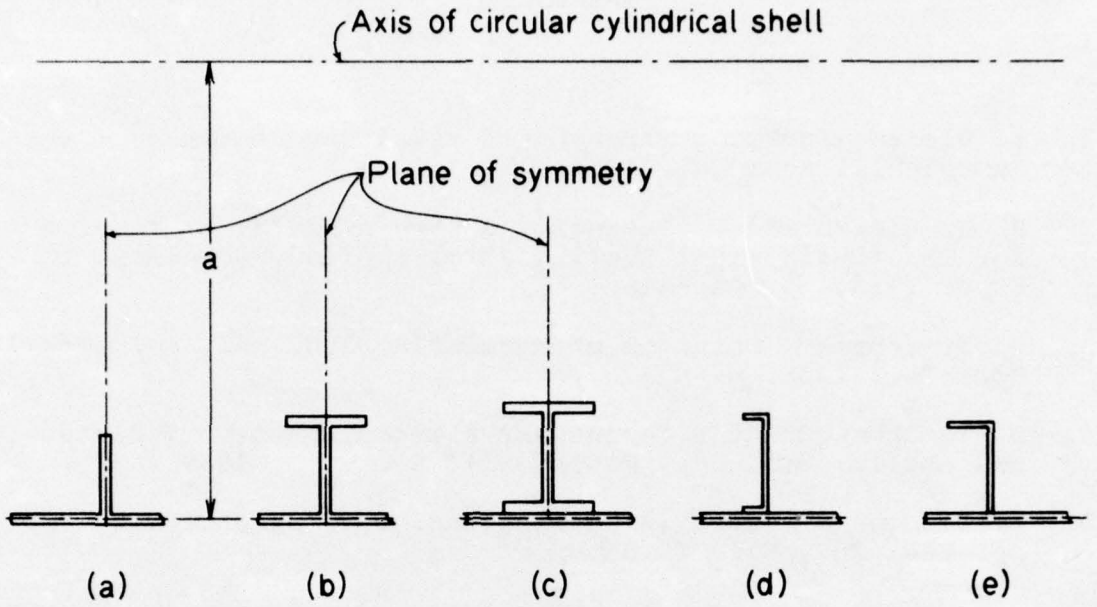

Fig. 1. Cross sections of ring stiffeners

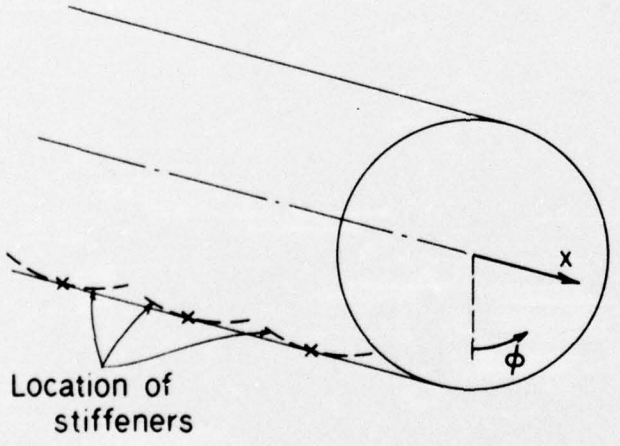

Fig. 2. Shell coordinates x,ϕ

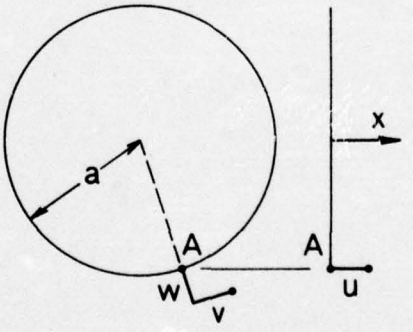

Fig. 3. Components u,v,w of shell displacements

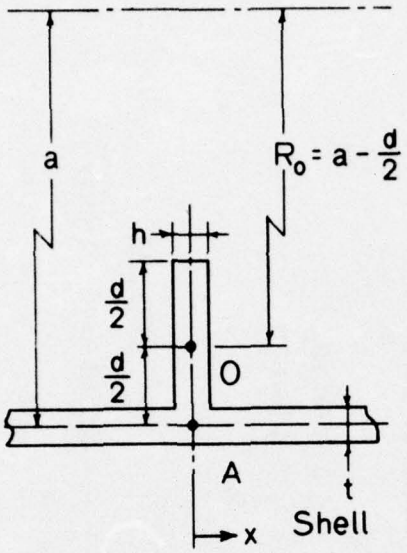
a
$R_o = a - \frac{d}{2}$
h
$\frac{d}{2}$
$\frac{d}{2}$
O
A
t
x
Shell

Fig. 4a

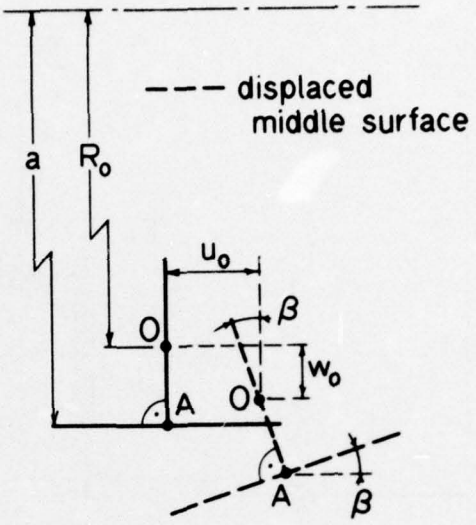
displaced
middle surface
a
R_o
u_o
β
O
w_o
A
O
A
β

Fig. 4b

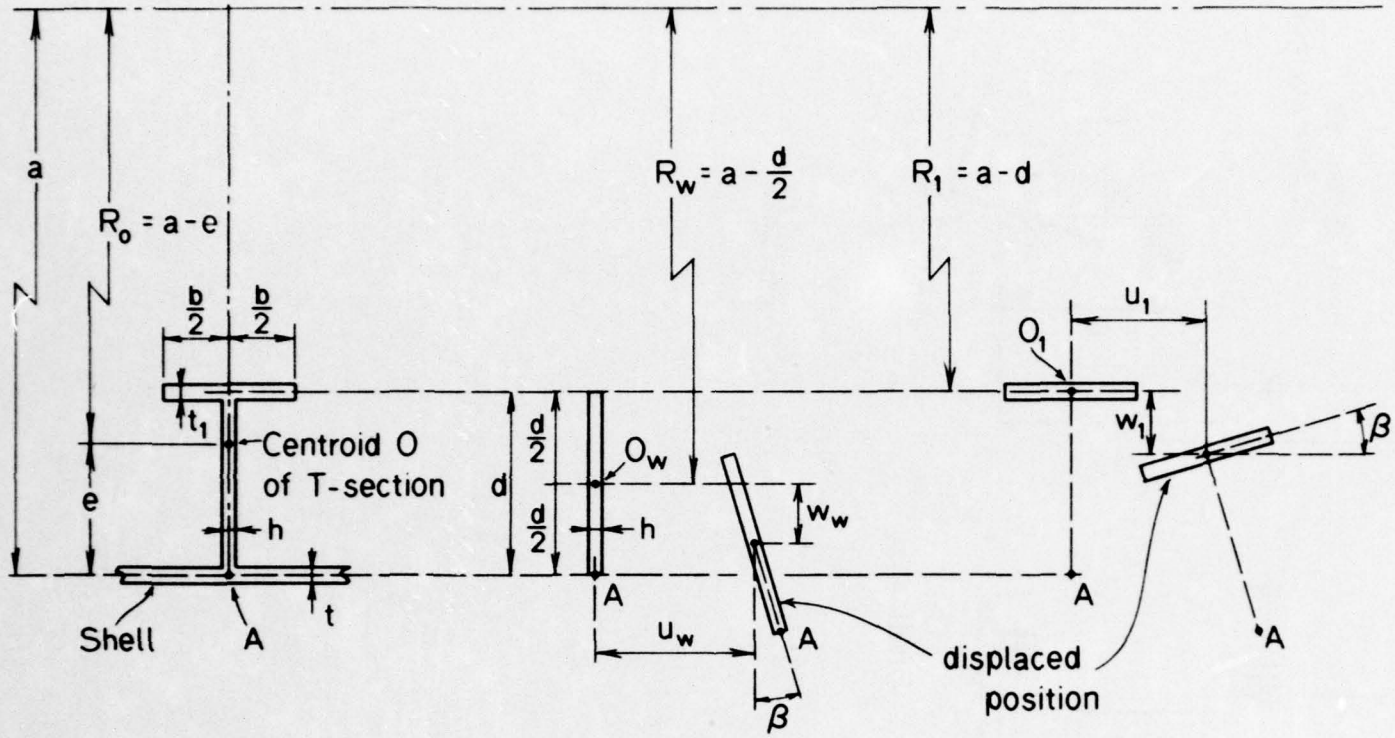

Fig. 5a

Fig. 5b

Displaced web

Fig. 5c

Displaced flange

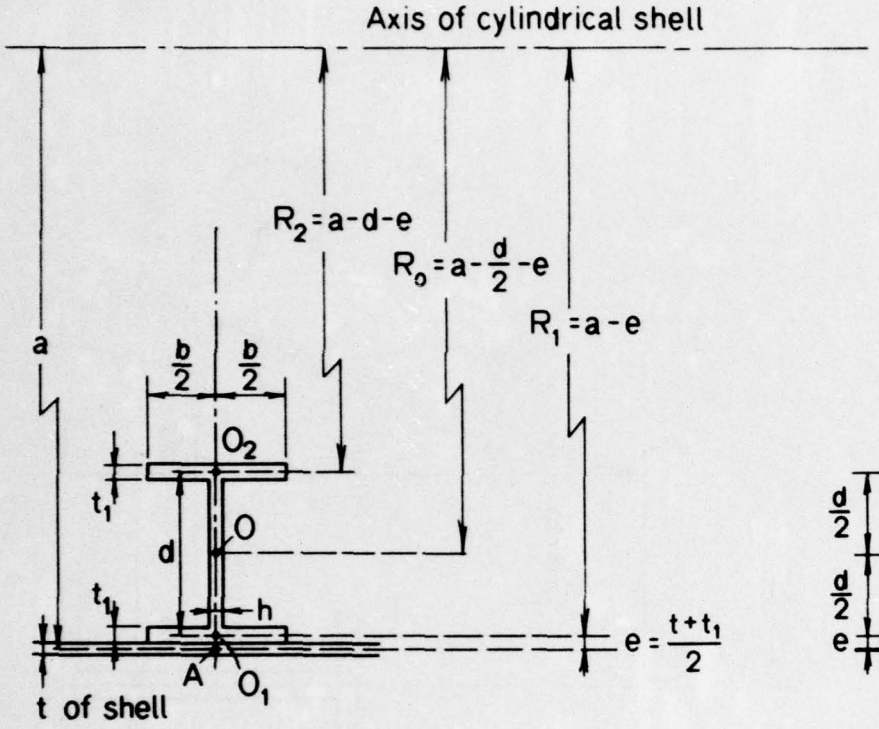

Fig. 6a

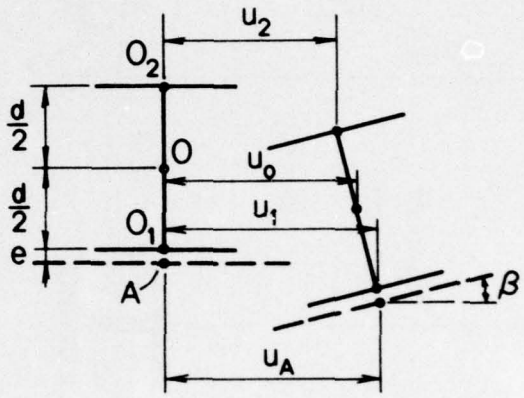

Fig. 6b

Displaced section

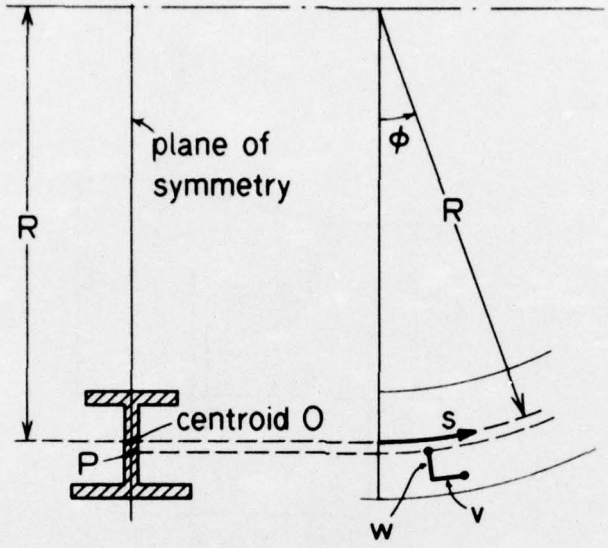
plane of
symmetry
R
φ
R
centroid O
s
P
w
v

Fig. A-1

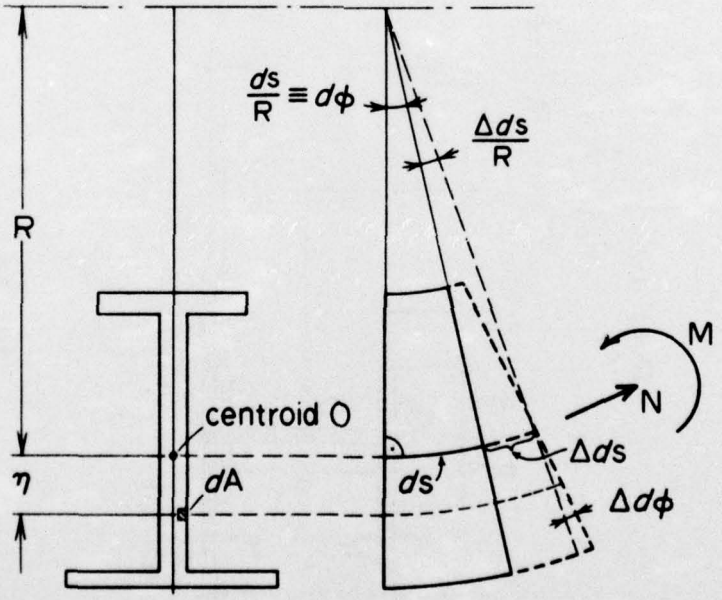
ds/R ≡ dφ
Δds/R
R
M
N
centroid O
η
dA
ds
Δds
Δdφ

Fig. A-2

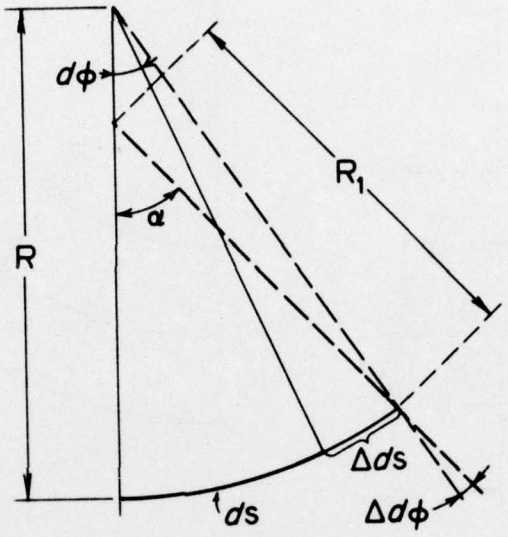

Fig. A-3

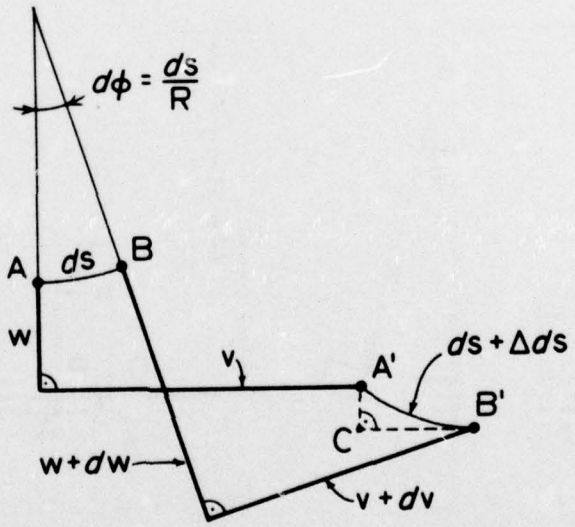

Fig. A-4

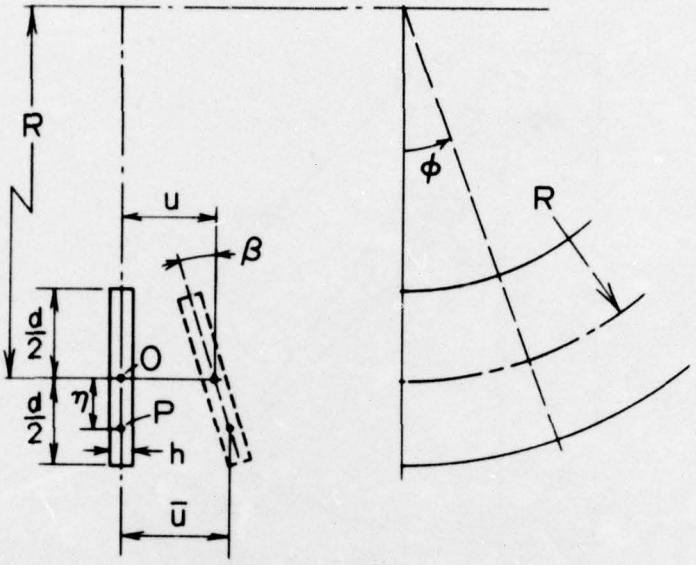

Fig. A-5

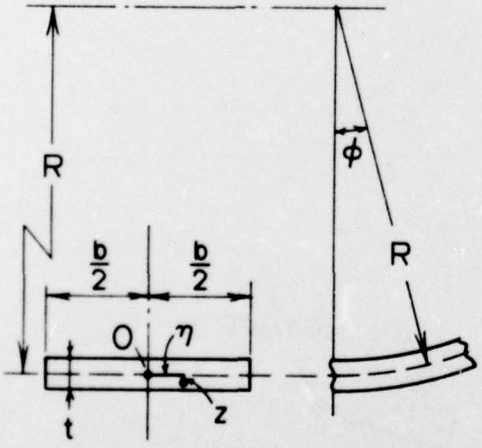

Fig. A-6

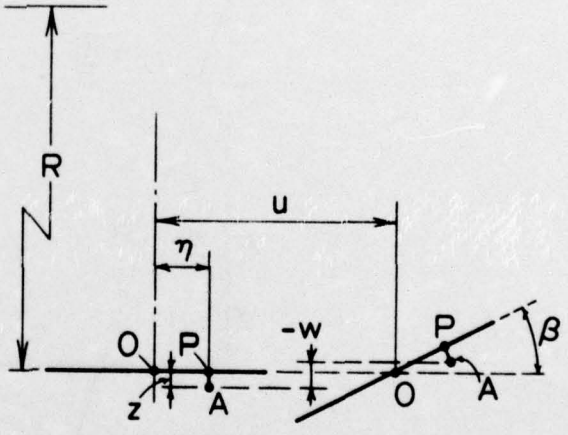

Fig. A-7

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