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PRACTICAL CONSIDERATIONS IN GRANULARITY MEASUREMENTS.(U)
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PRACTICAL CONSIDERATIONS IN GRANULARITY MEASUREMENTS

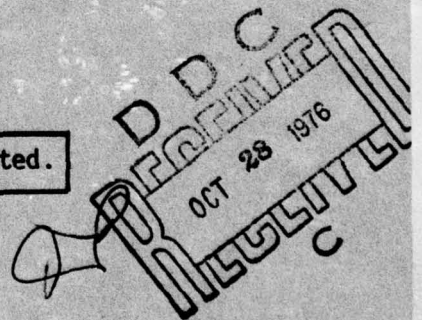
Dynamics and Environmental Evaluation Branch
Reconnaissance and Weapon Delivery Division

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Final Report for Period December 1974 - April 1975

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This report has been reviewed and is approved for publication.

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Abstract (cont'd)

improved the reliability of the RMS granularity calculation. The probability density distribution of the density data for a flashed specimen with pelloids removed is very near a Gaussian density distribution after data imperfections are removed.



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FOREWORD

Edward L. Gliatti and Bruce Kress of the Dynamics and Environmental Evaluation Branch (AFAL/RWF-2), Reconnaissance and Weapon Delivery System and Image Analysis, Avionics Laboratory, Wright-Patterson Air Force Base, Ohio, co-authored this report.

The research was conducted from December 1974 to April 1975.

The report was conducted under work unit number 20040516 entitled "Reconnaissance and Surveillance Subsystem Stem Computer Analysis." This was part of a symposium conducted on May 12, 1975 in Denver, Colorado.

We acknowledge Bill Marshall of Mead Corporation, Dayton, Ohio for consulting us in making microdensitometer measurements. Jack Lewis and Tom Zonars of AF Avionics Laboratory developed the film test specimens and gave their assistance in helping work the section on film preparation. Ralph Pinney patiently performed the microdensitometer scanning, and Al Bowling assisted in developing computer software. Both are from AFAL/RWF-2.

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SUMMARY

Grain noise will affect the quality of images recorded on photographic film. In Modulation Transfer Function (MTF) analysis, noise will degrade the MTF. Using the microdensitometer to measure grain noise, the effects of noise in Modulation Transfer Function Analysis can be estimated.

More research in this area of practical granularity measurement is needed. The next effort should include a good study of the accuracy and repeatability of measurement using the microdensitometer. A study to model the granularity mathematically should be carried out. Then granularity effects can be mathematically minimized in the MTF.

SECTION I
INTRODUCTION

The measurement of film granularity has been studied by many investigators and the quest for this data has followed many different approaches. These include a visual measure of graininess, the use of granularity data in photographic system analysis as a noisy communication channel, the attempt to model its effects as an aid during image enhancement, to name a few. Various measures of granularity exist which include RMS granularity, Selwyn's law granularity, autocorrelation, correlogram, Wiener Spectrum, and other subjective and objective measures.

Granularity, as defined by Kodak, is the objective measurement, with a densitometer having a small aperture, of the local density variations that give rise to the sensation of graininess. This measurement is usually made by scanning across a uniformly exposed area with a microdensitometer and from the trace data calculating the mean and standard deviation. This standard deviation is the usual number given for granularity often called RMS granularity.

This appears to be a relatively simple routine. However, as we have discovered, this analysis is full of potential pitfalls. This paper hopes to expose these pitfalls and to inform the inquirer interested in granularity of how to perform granularity measurements and the possible inaccuracies resulting. Some of this exists in the literature, but here we are attempting to tie it together.

At the Dynamics and Environmental Evaluation Branch of the Air Force Avionics Laboratory, all types of imagery analysis are performed including exposure uniformity, density distribution for exposure control, density profiles as a function of x and y, studies of photographic image motion, target detail detection and contrast analysis, image reconstruction and enhancement, x-y mensuration and gridding, and most importantly, MTF analysis for measuring system performance. In support of these areas, we decided to compile a composite library of carefully measured granularity data for various films used by the Air Force with various apertures. However, in attempting to do this, we discovered that many problems exist that invalidated our initial data.

Obtaining valid granularity data requires extremely good quality control in both production of the specimens and the measurement procedure. Included in this are a physical plant that has a thermally stable clean room for the microdensitometer and highly skilled operators, computers, and software. Additionally, the pitfalls previously mentioned include the preparation of test materials, scanning and focussing procedures, data processing techniques, and careful examination of the end product. These items are discussed together with the results we obtained scanning step wedges on Kodak 3414 film with a 1x80 micron slit. This film was analyzed due to its wide deployment throughout the Air Force. Also, this aperture was selected contrary to those normally used for granularity for measurement sake, but to obtain the data for the aperture size that is most frequently used for edge gradient analysis.

SECTION II
PREPARATION OF PHOTOGRAPHIC FILM TEST SAMPLES

The accuracy of granularity measurements is definitely dependent on the test specimen quality. The usual method of making test specimens involves modulating the exposing light using an Eastman Kodak number 5 photographic step tablet made of colloidal carbon suspended in gelatin, sandwiched in plastic and glass. When very fine grain films are to be analyzed for granularity, the possible impression of granularity from the step tablet modulator must be considered. In this case, the noise of the colloidal carbon step tablet modulators is impressed over the granularity of the fine grain film during the exposure.

Due to the modulation problem, the use of a projection sensitometer, or flashed exposure, is suggested for the exposure of samples for granularity testing. This projection sensitometer precisely controls the uniformity of illumination over the exposure plane, the exposure time, and intensity. This type of projection sensitometer is desirable for efficient and time saving exposure of uniform areas of precise density required for accurate and repeatable granularity testing and evaluation.

It is therefore recommended that granularity measurements of very fine grain films be exposed in controlled cameras, camera sensitometers, specular light sensitometers, etc., where precisely controlled exposure may be made without the use of contact modulators.

The photosensitive test samples to be analyzed for granularity should be processed in the chemistry used for the actual systems application that the granularity data is to support. The developer and fixer chemistry effect to some degree, the granularity and the relief image profile of the final processed product. Precise control of the processing chemistry, temperature, time, and agitation should be employed. Overfixing and overdrying of the exposed film should be avoided because they result in excessive hardening and shrinking of the gelatin emulsion.

Films with gelatin backings (pelloids) should be given special consideration when analyzed for granularity. The microdensitometer operator should use a scanning optical system with sufficiently restricted depth of focus to assure that the pelloid layer is out of focus when the microdensitometer optics are focussed on the grain structure in the emulsion layer. If this is not possible, then the pelloid backing should be carefully removed with a bleach solution, taking care not to scratch the film base or damage the image layer of the exposed and processed film.

Figures 1, 2, and 3 illustrate the microdensitometer traces of density as a function of linear position. The three figures are traces of density wedges exposed using a colloidal carbon process, uniformly flashed projection, and uniformly flashed projection with pelloids removed. It is interesting to note that the carbon processed and flashed exposure have a low frequency modulation compared to the uniformly flashed specimen which has had the pelloids removed. A comparison of Figures 1 and 2 with Figure 3 shows this effect.

Additionally, the exposure level must be considered in manufacturing the test specimens. It is good practice to use a color temperature for the exposing light that simulates the expected environment of the photographic film.

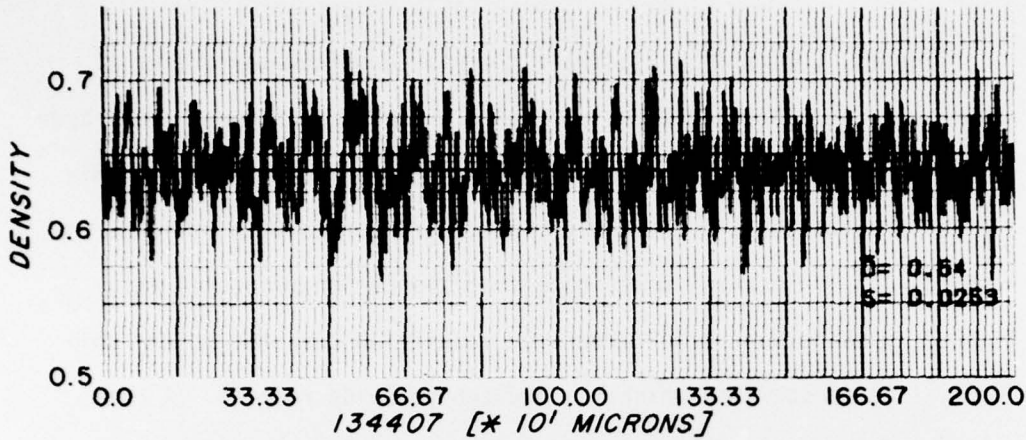


FIGURE 1 COLLOIDAL CARBON TABLET MODULATION ON 3414 FILM

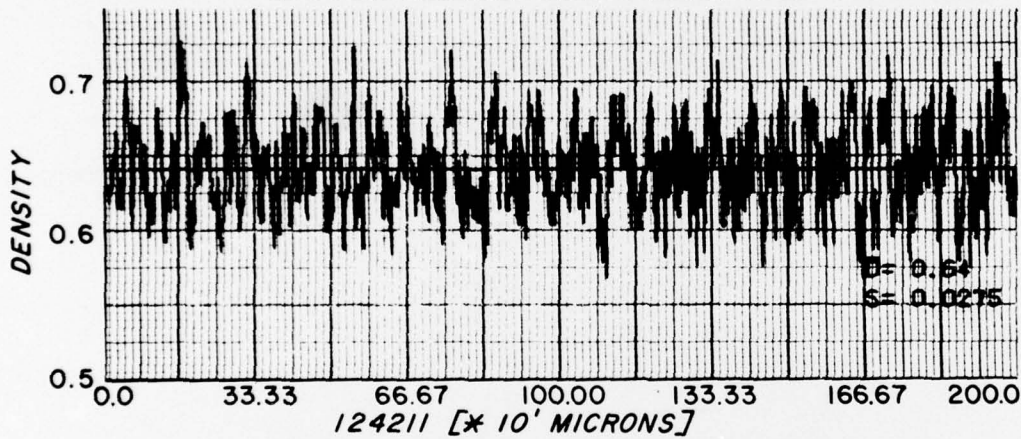


FIGURE 2 UNIFORMLY FLASHED 3414 FILM

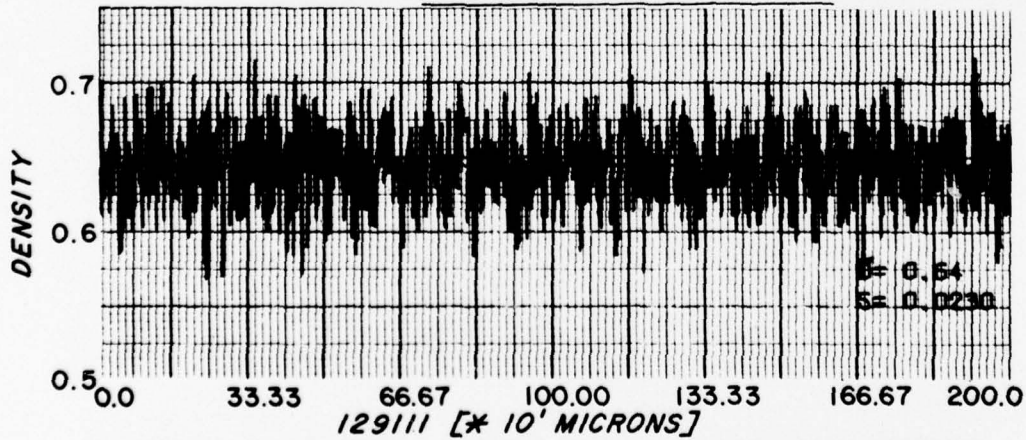


FIGURE 3 UNIFORMLY FLASHED 3414 FILM (PELLOIDS REMOVED)

SECTION III
SCANNING PROCEDURE

The measuring equipment used in these granularity tests was a Mann-Data microdensitometer using white light with a 1x80 micrometer aperture. The travel direction is perpendicular to the long axis of the slit. The data density values were sampled at each micron of travel and recorded digitally on magnetic tape for data reduction by the granularity computer routines.

Assuming the granularity characteristics of the film isotropic, the scan path should not affect the granularity measurement. Each scan is over a straight line of 2100 points (2.1mm) taken from a uniform density area. Assuming that the density wedging and spikes, due to scratches and dirt, can be eliminated by computer processing, no microscopic selection of scan line was made except to avoid obvious defects. The scan speed was selected at the value used commonly for edge gradient analysis on 3414 film. The microdensitometer density calibration is established at the extreme linear limits of the H and D curve for 3414 film.

A very important aspect of good scanning is linked to the procedure for obtaining best focus. The lens and aperture, located above the specimen, converges the light from the source to the plane of the specimen. The light diverging from the specimen is collected with another aperture and lens, and the light is assigned a density

value by the electro-optical system of the microdensitometer. After a visual focus, the lower lens is first adjusted to given optimum focus in the pelloid layer (assuming the test specimen has pelloids), then optimum focus in the film base, and finally in the grain structure of the emulsion. The upper lens is adjusted correspondingly to obtain minimum indicated density. A strip chart recorder is connected to the electrical signal output of the microdensitometer. By a series of scans at incremental focus changes, the focus can be finely tuned by watching the strip chart recorder and looking for the best modulation (largest deviations in amplitude).

With these lens adjustments in the microdensitometer, a change of three microns along the focal axis from the optimum focal point will produce very noticeable effects in the trace density values. Several factors may change the location of the grain with respect to the focal plane namely, film base thickness, foreign particles on the underside of the film, and the amount of exposure. Therefore, a refocus should be made for each scan since the factors influencing grain location and optimum focus position may vary as much as ten micrometers. This can seriously alter the accuracy of granularity measurements.

To illustrate, two areas of different density were chosen from a 3414 flash exposed film sample with pelloids removed. Figure 4, trace A is the microdensitometer trace of a uniform density area

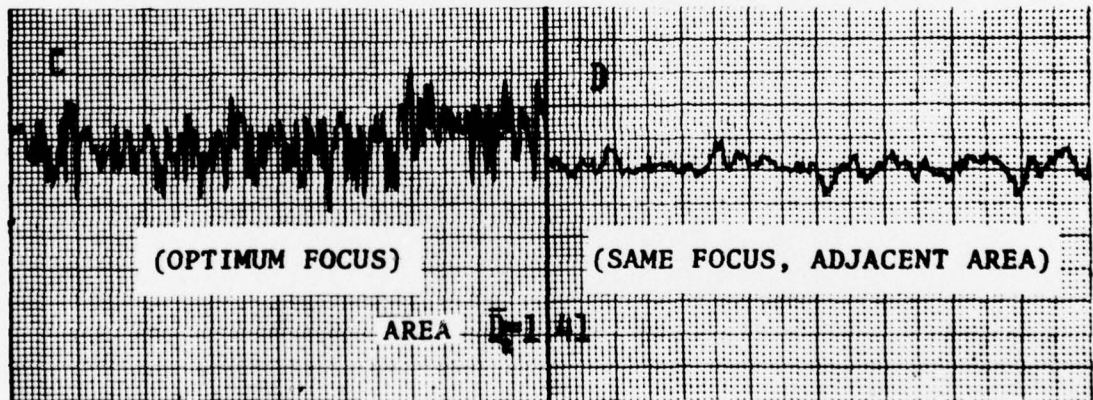
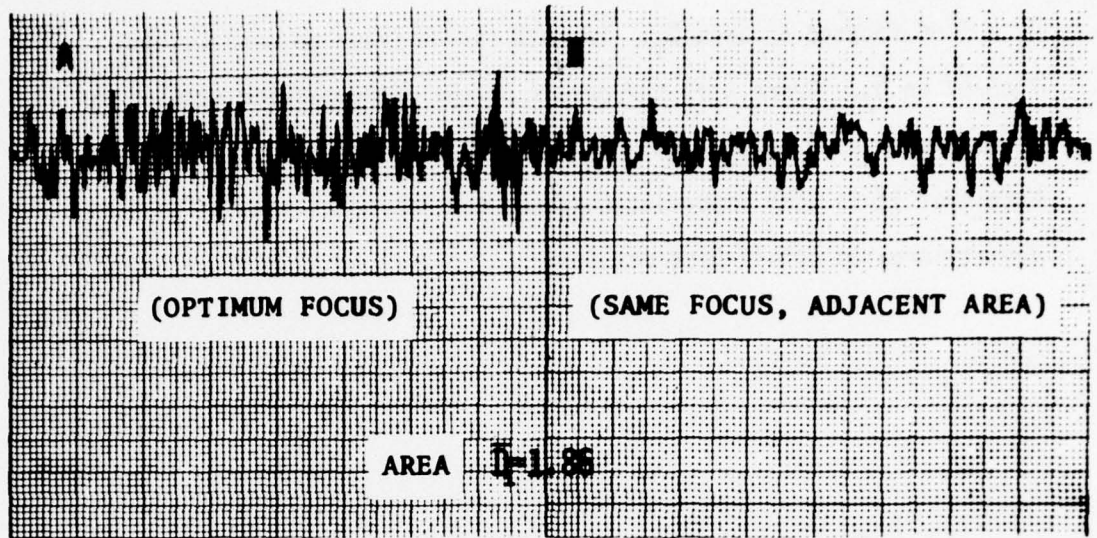


FIGURE 4 EXAMPLES OF CRITICAL FOCUS

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$\bar{D}_1 = 1.86$ focused on area \bar{D}_1 . Trace B is the trace on area \bar{D}_1 , but focused on \bar{D}_2 . Trace C is the result of the microanalyzer scan of \bar{D}_2 focused on area \bar{D}_2 . Finally, trace D is a scan of \bar{D}_2 focused on \bar{D}_1 . Note that it is very important to focus the microdensitometer on each density area to obtain best results.

SECTION IV
DATA ANALYSIS

Inaccurate granularity results can arise from incorrect data analysis. These inaccuracies come about by improper setting of parameters in the digitizing process and by the methods employed to process the data with regard to removing the effects of pinholes, spikes, scratches, foreign particles, and density wedging. A spike is usually caused by a pinhole scratch or a foreign particle. This section discusses these problems.

The density values from the microdensitometer are digitized for every micron of travel. The resulting digitized data were processed to remove pinholes or spikes, due to minute scratches and foreign particles. These defects produce density values with a much greater amplitude deviation from the mean value than produced by granularity alone. Without the removal of this data, the granularity results are incorrect. The procedure for removal of these effects is discussed in the following paragraph.

The sampled data are processed to estimate the mean and the standard deviation by the usual equations. For removing spikes, it is assumed that the data has a probability distribution that is nearly normal. Then,

$$P\{|D(x_i) - \bar{D}| > 3S\} < 0.003$$

where, $P\{\}$ = Probability of the event in brackets
 $D(x_i)$ = Density value at a discrete position
 $x_i = 1, 2, 3, \dots, N$ (in micrometers)
 S The sample standard deviation of $D(x_i)$ calculated
 by

$$S^2 = \frac{1}{(N-1)} \sum_{i=1}^N (D(x_i) - \bar{D})^2$$

\bar{D} The expected value of $D(x_i)$

$$\bar{D} = \frac{1}{N} \sum_{i=1}^N D(x_i).$$

The routine to eliminate spikes tests the data to find all values of $D(x_i)$ for which,

$$|D(x_i) - \bar{D}| > 3S.$$

This cutoff limit is arbitrary. All points which exceed this limit plus three points on each side of these points are eliminated. By scanning an extra hundred microns, the data can be shifted to fill in the gap due to the eliminated points. Thus, the final set of data are the original, with the exception of spikes, concatenated with the extra data shifted so that a total of 2000 points result. For this adjusted

data, a new mean and standard deviation is calculated and the data are again checked for spikes. Figure 5 shows a very bad spike. Figure 6 is the same data with the spikes removed.

One of the most frequently occurring defects in granularity samples is density wedging or trending, whereby, a variation in exposure across the sample causes a trend in density across the sample. Although careful preparation of the sample can minimize the magnitude of this wedging some residual trends invariably occur. Several techniques could be employed to remove the trend; least square polynomial fitting, linear fitting over the whole data set or over equal segments of the data. It was arbitrarily decided to segment the data and use a linear fit.

The data was partitioned into sixteen equal segments containing 125 samples each and a linear regression line was computed for the best fit on each segment. (The sixteen regression lines for each segment are shown in Figure 6.) Then the data for each regression line are rotated to give a zero slope and the mean for the segment is taken as the mean of all sixteen segments. Figure 7 shows density wedging. Figure 8 is the same data as in Figure 7, but spikes and wedging have been removed.

The mean and variance are estimated from the processed data. The resulting standard deviation is the RMS granularity.

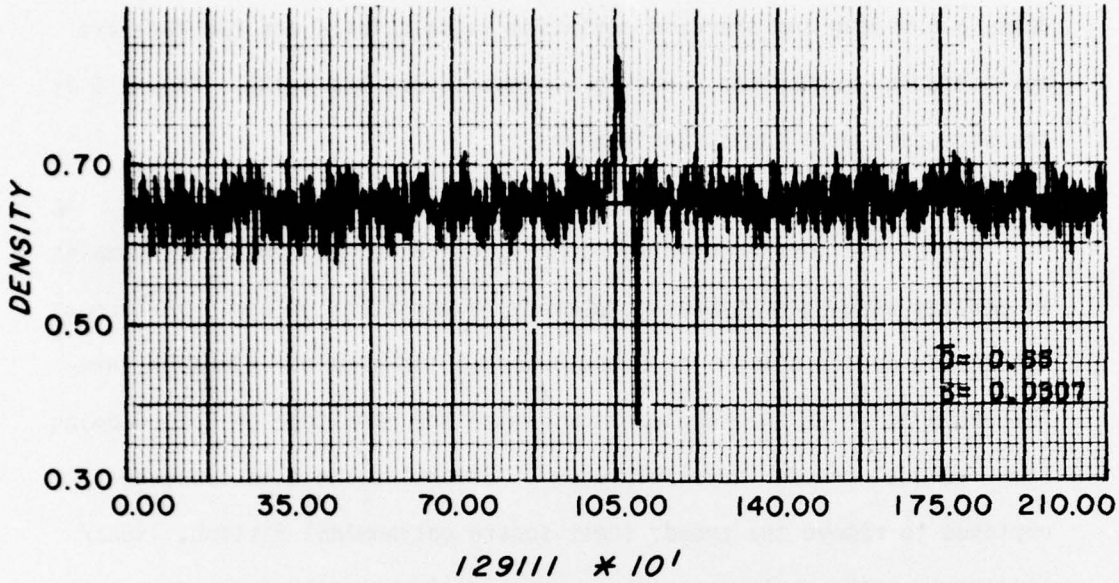


FIGURE 5 UNIFORM FLASHED 3414 FILM (LARGE NOISE SPIKES)

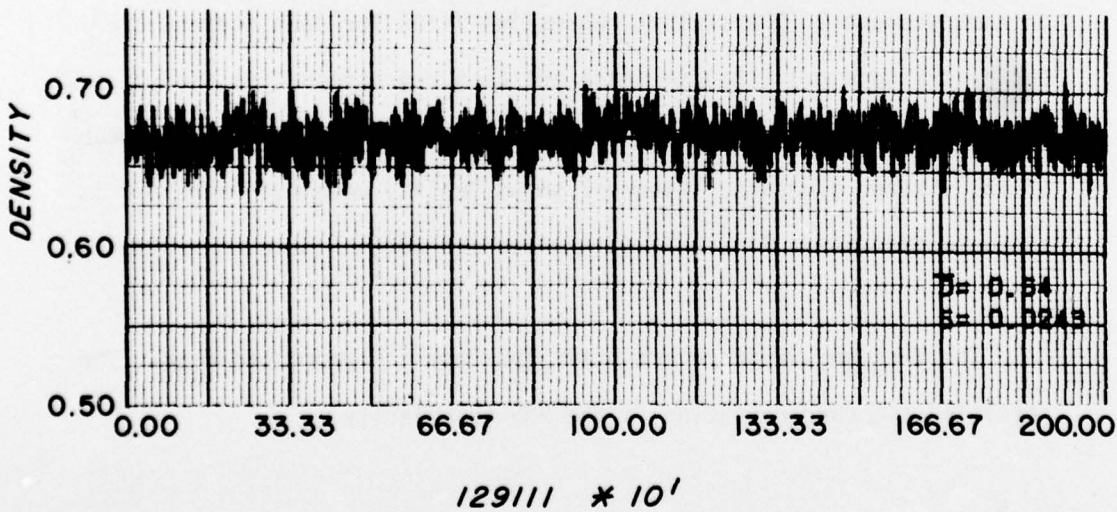


FIGURE 6 UNIFORM FLASHED 3414 FILM (LARGE NOISE SPIKES REMOVED)

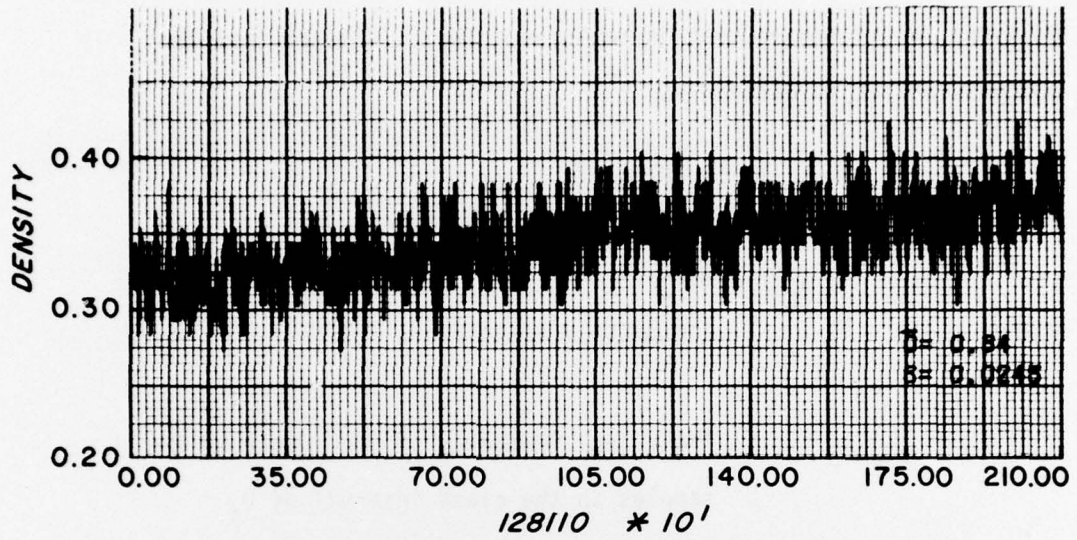


FIGURE 7 UNIFORM FLASHED 3414 EXPOSURE ILLUSTRATING DENSITY WEDGING

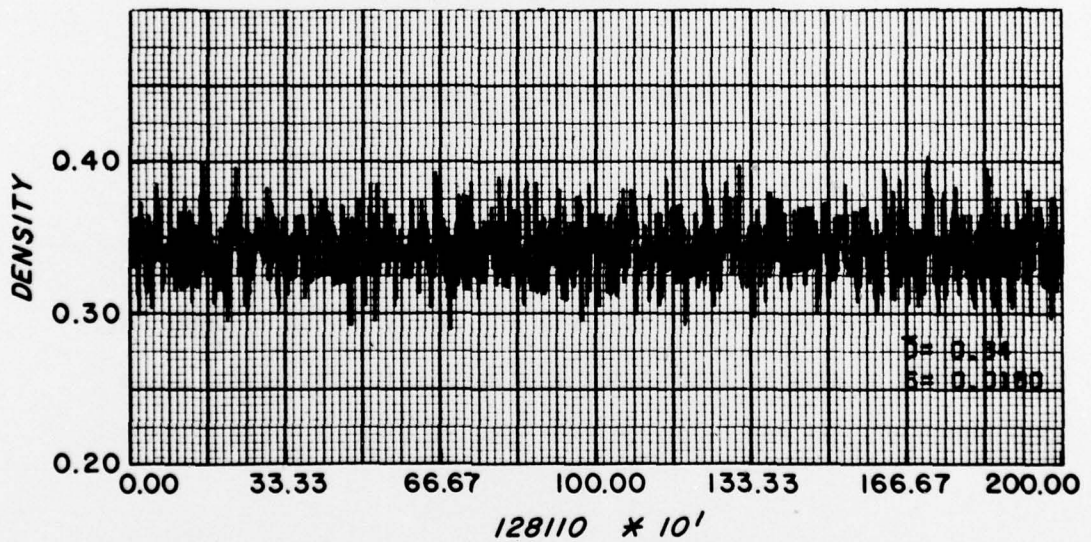


FIGURE 8 UNIFORM FLASHED 3414 EXPOSURE (NOISE SPIKES & DENSITY WEDGING REMOVED)

To test to see if the granularity is distributed normally, or Gaussian, a Chi-Square test is employed. The Chi-Square test for goodness of fit is defined as

$$\chi^2 = \sum_{j=1}^K \frac{[d_j - F(D_j)]^2}{F(D_j)}$$

- where
- D_j The density value at x_j
 - d_j The actual frequency of occurrence of the samples in the class interval of D_j
 - $F(D_j)$ The number of occurrences expected in the class interval
 - K Number of class intervals

For determining the confidence level in this Chi-square test, the number of degrees of freedom is $K-3$ since the mean and variance are already estimated from the actual data. To apply the Chi-Square test, $F(D_j)$ should be greater than five for a good test. If the value was less than five, the class interval was grouped with the next class.

SECTION V
MEASUREMENT RESULTS

Granularity was measured on three samples of fine grain Kodak 3414 film. One specimen was prepared with a colloidal carbon step tablet. The other two specimens were made by using the recommended projected sensitometer method, both with pelloids removed and pelloids remaining. These specimens were scanned with a Mann-Data microdensitometer at a scan speed of .01mm/minute with an aperture of 1x80 microns, with the density data sampled every micron, and an analog filter set at .56 Hz. Five wedges were scanned on each of the three samples corresponding closely to average density levels of .32, .64, 1.1, 1.5, and 2.1.

Representative plots of density versus position are shown in Figures 1, 2, and 3. Note that the specimens with pelloids have a definite low frequency modulation. Also, from Table I, note that the RMS granularity values (S), are lower where the pelloids have been removed as compared to specimens where pelloids remain. The Chi-squared test indicates that the removal of pelloids results in the data being more normally distributed. In fact, five out of five of the flashed specimens with pelloids removed passed the Chi-Square test at the 95% confidence level.

In scanning a given uniform density area, it is important that proper focus be established. Again take note of Figure 4. The microdensitometer should be focussed at each density area. Even though this

TABLE 1
 STATISTICAL SUMMARY OF GRANULARITY
 MEASUREMENT (1x80 MICRON APERTURE, 3414 FILM)

	\bar{D}	S	S _D	DOF	χ^2	ACCEPT
CARBON SPECIMEN	.33	.0306	.0291	7	45.0	NO
	.64	.0335	.0327	8	15.9	NO
	1.02	.0374	.0354	8	22.2	NO
	1.48	.0359	.0315	8	46.6	NO
	2.09	.0439	.0410	7	7.0	YES
FLASHED SPECIMEN	.30	.0275	.0259	8	33.2	NO
	.64	.0300	.0278	8	26.0	NO
	1.12	.0337	.0322	7	68.7	NO
	1.55	.0333	.0313	7	17.9	NO
	2.31	.046	.0422	8	6.5	YES
FLASHED SPECIMEN PELLOIDS REMOVED	.34	.0239	.0180	7	12.4	YES
	.65	.0251	.0230	6	8.3	YES
	1.12	.0242	.0231	8	13.4	YES
	1.57	.0289	.0277	8	5.9	YES
	2.23	.0323	.0315	8	9.7	YES

\bar{D} Average Density
 S Raw Sample Standard Deviation
 S_D Detrended Sample Standard Deviation
 DOF Degrees of Freedom
 χ^2 Chi Squared Computed Values

ACCEPTABLE 95% CONFIDENCE

DOF	χ^2
6	13
7	14
8	15

process is employed, the influence of the pelloids most definitely changes the RMS granularity for that area. (See Table I.) Figure 9 shows the relationship of RMS granularity to density. As expected, the granularity increases with density.

To properly analyze the data, spikes that occur from scratches and foreign particles and the wedging that results from uneven light distribution when the wedge was made, must be eliminated. Routines that compute the mean and variance and eliminate spikes and wedging are very important. (See Figures 5, 6, 7, and 8 for examples of spike removal and elimination of wedging.) From Table I, the detrended RMS granularity S_D is less than S , the RMS granularity before trends were eliminated. Figure 10 shows the amplitude distribution for unprocessed data observed as compared to an expected Gaussian distribution. Figure 11 is the same as 10 except the data used for the observed distribution has had the spikes eliminated and trends removed. It is important to note that removing data imperfections, spikes and trends, by the somewhat arbitrary technique previously described allows the granularity probability distribution to be very comparable to a Gaussian probability distribution.

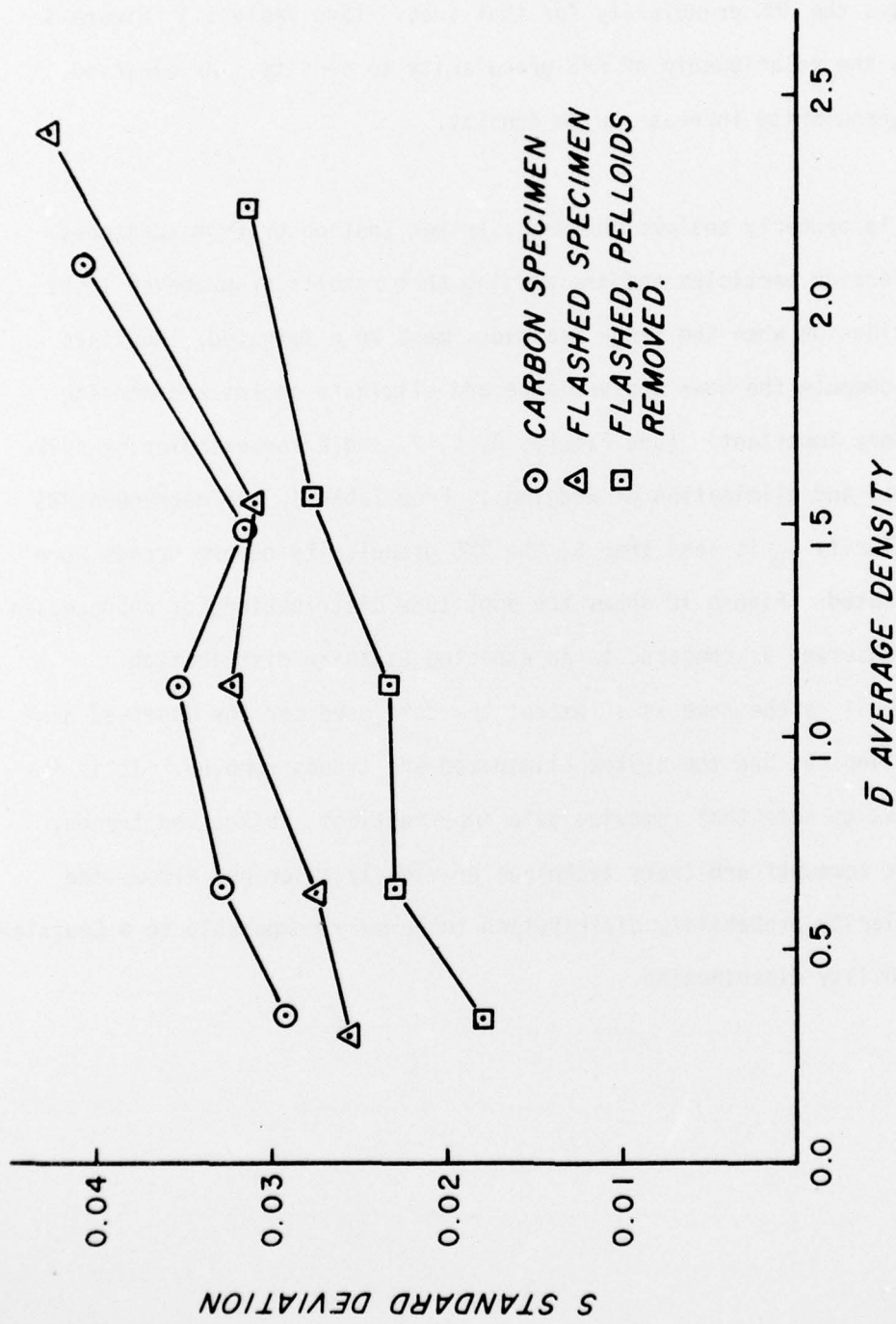


FIGURE 9 STANDARD DEVIATION AS A FUNCTION OF AVERAGE DENSITY

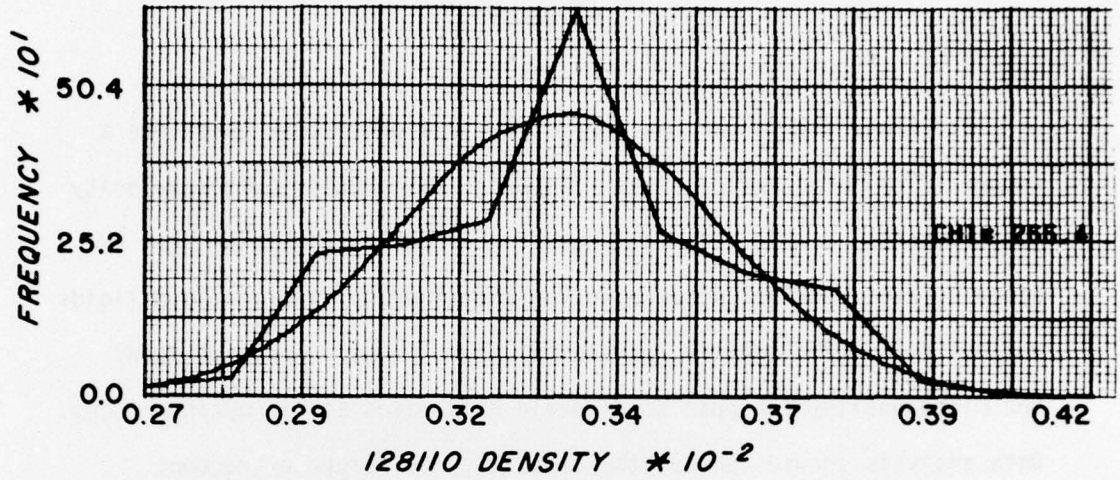


FIGURE 10 DENSITY DISTRIBUTION BEFORE DATA PROCESSING

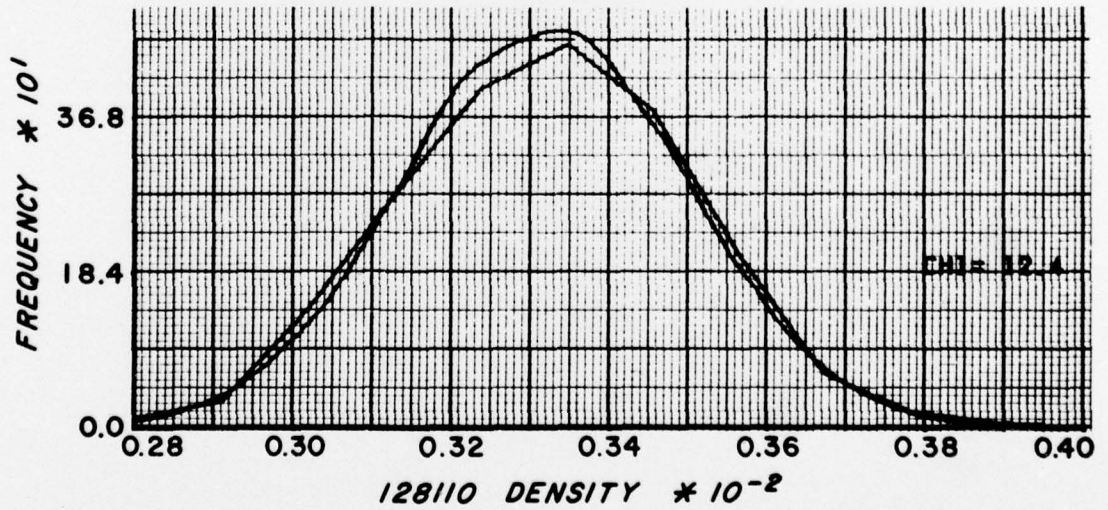


FIGURE 11 DENSITY DISTRIBUTION (NOISE SPIKES AND DENSITY WEDGING REMOVED) GAUSSIAN DISTRIBUTION SHOWN AS A COMPARISON

SECTION VI
CONCLUSIONS

The experience from this project has indicated that there are a number of critical factors which influence the accuracy of granularity measurements. Sample preparation must be free of modulation when exposed to avoid adding patterns to the actual granularity. The pelloids in 3414 film influence the granularity measurement. For each scan, the microdensitometer must be properly calibrated and in optimum focus. Data analysis should include the capability to remove extraneous data and trends before the RMS granularity is calculated.

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