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UNIVERSITY OF COLORADO, BOULDER, COLORADO 80302



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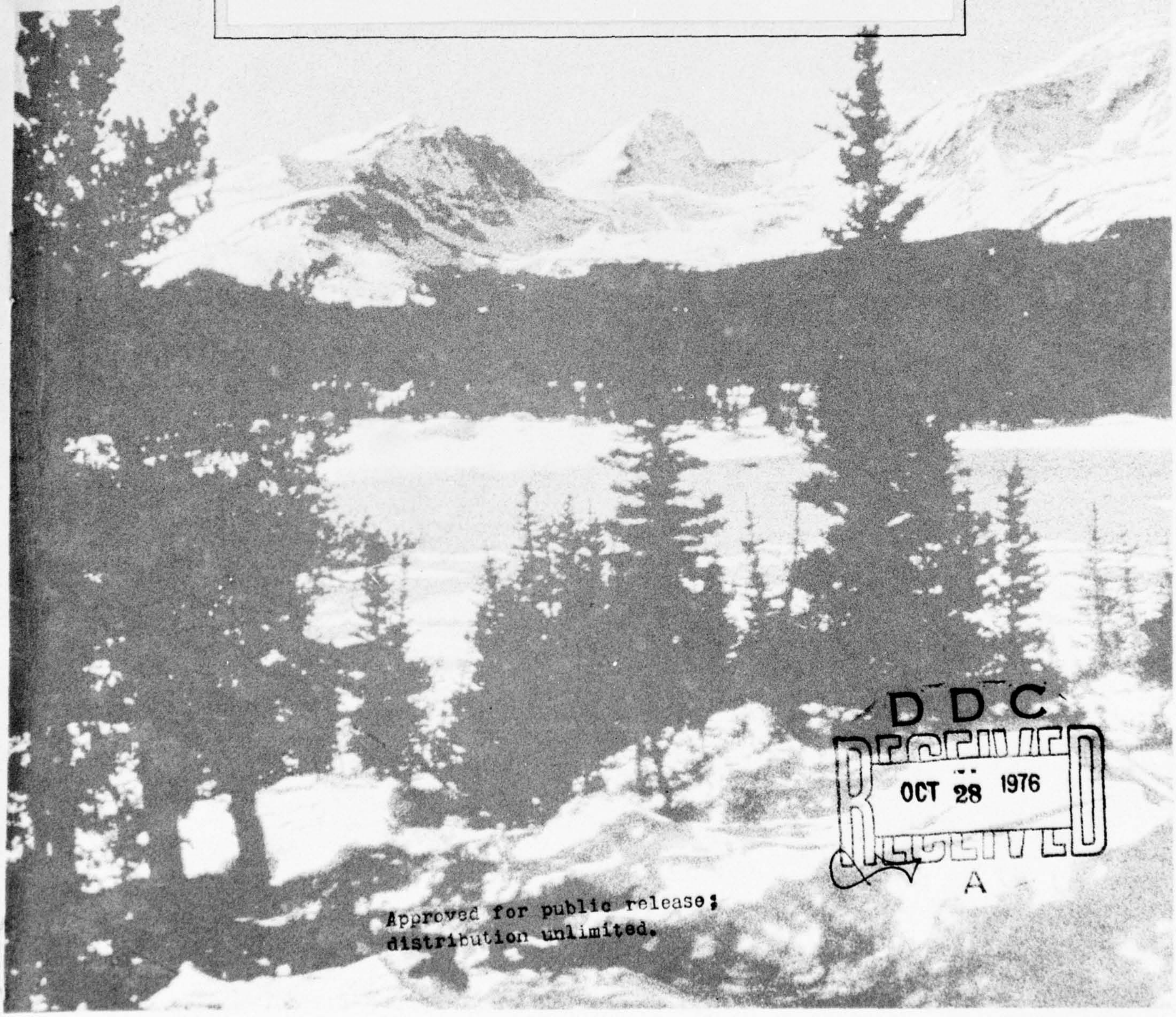



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FINAL REPORT - Attachment A1

#F44620-73-C-0003

LINEAR INHOMOGENEOUS THEORY OF $CO_2$-ENHANCED

LANGMUIR TURBULENCE

Martin V. Goldman

include in 3/c Prep in coord w/

Memorandum #2 to ~~Peacock~~ abstract

CU #1019 - A1

April 1976

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Memorandum (Number Two) to Nicol Peacock and the Culham Laboratories Plasma Physics Group, England, concerning experimental collaboration with Martin V. Goldman, Principal Investigator of this contract. Title describes theory dealt with.

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UNIVERSITY OF COLORADO
BOULDER, COLORADO 80302

Department of
Astro-Geophysics

28 April 1976

MEMORANDUM

TO: N. Peacock and the Culham Group

(Memorandum Number Two)

FROM: M. V. Goldman

SUBJECT: Linear Inhomogeneous Theory of $CO_2$-Enhanced Langmuir Turbulence

I. The scattering cross section for the plasma line is given roughly by:

$$\sigma = \frac{r_e^2}{\alpha^2} N_e \delta\Omega_s I , \tag{1}$$

where,

$$r_e^2 = \left(\frac{e^2}{m_e c^2}\right)^2 = 7.9 \times 10^{-26} \text{ cm}^2 , \tag{2}$$

$$\alpha^2 = \frac{k_D^2}{k_s^2} = k_D^2 \left(\frac{4\omega_i^2}{c^2} \frac{\sin^2\theta_s}{2}\right)^{-1} , \tag{3}$$

$$\delta\Omega_s = \delta\phi \cos(\theta_s - \delta\theta_s) - \delta\phi \cos(\theta_s + \delta\theta_s) , \tag{4}$$

$$I \equiv \lim_{V \to \infty} \frac{\langle |\delta E(k_\gamma)|^2 \rangle}{4\pi\Theta_e V} , \tag{5}$$

$$\left( I = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \lim_{VT \to \infty} \frac{\langle |\delta E(k_\gamma, \omega)|^2 \rangle}{4\pi\Theta_e VT} \right)$$

In this expression, $N_e$ is the total number of scattering electrons, which is equal to

$$N_e = n_o \ell r_o 2\phi_c \delta r \ , \tag{6}$$

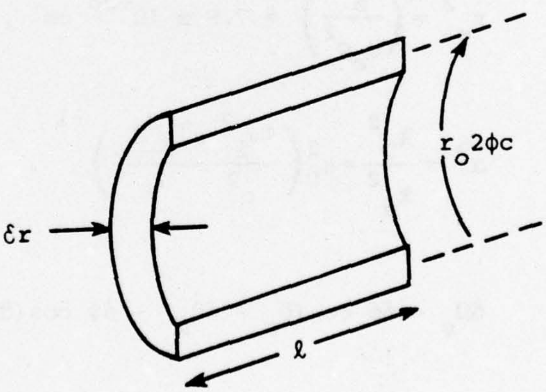
where $n_o \approx 10^{19} cm^{-3}$ is the density of electrons in the neighborhood where $\omega_{CO_2} = \omega_{pe}$, $\ell$ is the axial size of the ruby spot (smaller than the $CO_2$ spot), $r_o$ is the radius at which $n(r_o) \approx n_o$, $\phi_c$ is the maximum azimuthal angle for which the $CO_2$ beam reaches the frequency matched radius:

$$\phi_c \approx \sqrt{2} \left( \frac{3}{2\alpha^2} + 1.7 \sqrt{\frac{m_e}{m_i}} \ \alpha^{-1} \right)^{1/2} \ , \tag{7}$$

and $\delta r$ is the *radial thickness over which* $I$ is close to its maximum.

FIGURE 1

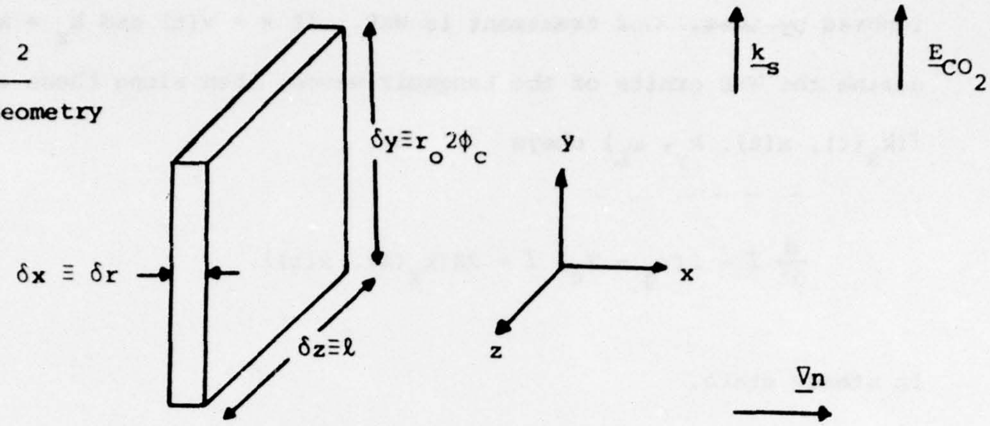
Interaction Volume




II. For simplicity in treating plasma wave convection effects, we shall replace the volume in Fig. 1 by a slab, as illustrated in Figure 2:

FIGURE 2

Slab Geometry




Note that, in this geometry the Langmuir waves probed by the ruby laser (driven by $\underline{E}_{CO_2}$) are propagating in the y-direction. Inhomogeneity is allowed only in the x-direction.

III. The spectrum, $I$, of Langmuir waves excited by the electron-ion decay instability is the solution to a WKB kinetic equation:

$$\frac{\partial I}{\partial t} + \frac{\partial \omega_L}{\partial k_x}\frac{\partial I}{\partial x} + \frac{\partial \omega_L}{\partial x}\frac{\partial I}{\partial k_x} - 2\left(\gamma_g - \gamma_c - \gamma^{NL}(\{I\})\right) I = 2S \,. \quad (8)$$

Here, we have assumed a convective instability, for which the growth rate is $\gamma_g$. $\gamma_c$ is the linear collisional-damping rate. S is an appropriate spontaneous emission term, and $\gamma^{NL}$ is a possible nonlinear saturation term which would be a functional of $I$. In memo 1, we estimated $\delta x$ and $I$ by ignoring x and t-dependence in (8), and using a nonlinearly saturated $I$. In this memo, we will ignore the nonlinear term $\gamma^{NL}$ and find $\delta x$ and $I$ by finding a steady state inhomogeneously saturated $I$. This is in the spirit of Perkins and Flick, but has physical effects ignored by them. Our treatment is WKB. If $x = x(t)$ and $k_x = k_x(t)$ define the WKB orbits of the Langmuir waves, then along these orbits, $I(k_x(t), x(t); k_y, \omega_L)$ obeys

$$\frac{d}{dt} I - 2(\gamma_g - \gamma_c) I = 2S(k_x(t), x(t)). \tag{9}$$

In steady state,

$$\frac{d}{dt} \equiv \frac{\partial k_x}{\partial t}\frac{\partial}{\partial t} + \frac{\partial x}{\partial t}\frac{\partial}{\partial t} . \tag{10}$$

The WKB orbits are the solution to Hamilton's equations

$$\frac{\partial k_x}{\partial t} = -\frac{\partial \omega_L}{\partial x}, \quad \frac{\partial x}{\partial t} = \frac{\partial \omega_L}{\partial k_x} \tag{11}$$

$$\omega_L = (\omega_p^2(x) + 3v_e^2[k_x^2 + k_y^2])^{1/2} . \tag{12}$$

We assume a linear density profile:

$$\frac{\partial \omega_p}{\partial x} = \frac{\omega_p}{2L} . \quad \text{(ignore dependence on x on right-side)} \tag{13}$$

The solutions to (11) and (12) are then

$$k_x = -\frac{\omega_p t}{2L} , \tag{14}$$

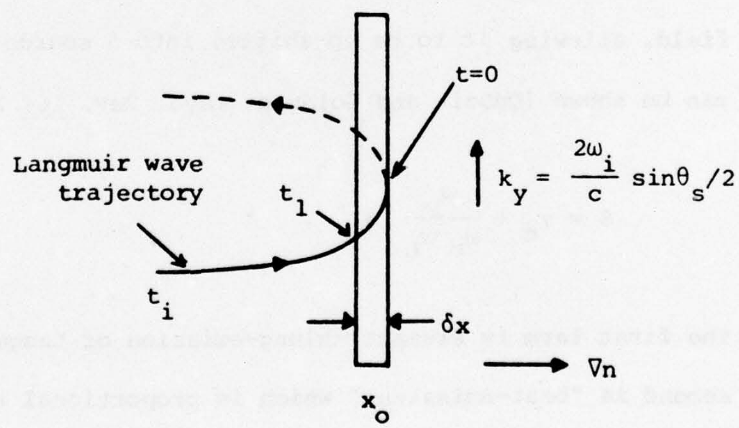
$$x = x_o - \frac{3}{4}\frac{v_e^2 t^2}{L} = x_o - \frac{3Lk_x^2}{k_D^2} , \tag{15}$$

where $x_o$ is the reflection point of the Langmuir wave. In addition to (14) and (15) we have the constants of the motion, $k_y$ and $\omega_L$. Thus, $x_o$ is the solution to

$$\omega_L^2 = (\omega_p^2(x_o) + 3v_e^2 k_y^2) . \tag{16}$$

We must follow a Langmuir wave from a point $x < x_o$ to $x_o$ where it has its $\underline{k}$ in the y-direction, and can be probed by the ruby. The weakness of our theory is that WKB tends to break down at the reflection point. Nevertheless, we have shown (see Appendix) that this same WKB method can predict the Perkins-Flick threshold to within a factor of two, under the relevant conditions ($T_e >> T_i$) for their non-WKB theory. Figure 3 shows the trajectory of a Langmuir wave which can be probed by the ruby:

FIGURE 3




We wish to study orbits for which the spectrum, $I$, is a maximum at $x = x_o$, and is one-half of its maximum at $x = x_o - \delta x$. This will determine $\delta x$. (This is a different criterion from memo 1.)

IV. The kinetic equation, (9) may be integrated to give

$$I(t=0) = e^{2A}\left[1 + \int_{t_i}^{0} dt'\, e^{-2\int_{t_i}^{t'} \Gamma_g(t)dt}\, S(t')\right], \quad (17)$$

$$A = \int_{t_i}^{0} dt\, \Gamma_g(t)dt, \quad (18)$$

$$\Gamma_g(t) \equiv \gamma_g(t) - \gamma_c. \quad (19)$$

Here, we are letting $t = 0$ be the time at which reflection occurs ($x = x_o$, $k_x = 0$), and $t_i < 0$ be some initial time (see Fig. 3). We have assumed $I = 1$ at $t = t_i$, corresponding to an equilibrium spectrum. The dominant contribution to the spontaneous emission comes from the "beating" of low-frequency ion Cerenkov emission with the $CO_2$ electric field, allowing it to be up-shifted into a source for Langmuir waves. It can be shown (DuBois and Goldman, Phys. Rev. 164, 207 (1967)) that:

$$S = \gamma_c + \frac{\omega_p}{\omega_o - \omega_L}\gamma_g. \quad (20)$$

The first term is Bremsstrahlung-emission of Langmuir waves, and the second is "beat-emission," which is proportional to the $CO_2$ pump intensity through $\gamma_g$. If we write $\gamma_g = \Gamma_g + \gamma_c$, the dominant contribution from the right-side of (17) is from the integral over $\Gamma_g\omega_p/(\omega_o-\omega_L)$, which may be performed exactly:

$$I(t=0) = \frac{\omega_p}{\omega_o-\omega_L}\, e^{2A}, \quad (e^{2A} \gg 1). \quad (21)$$

Now it remains to do the integral A given in equation (18). For this we require an expression for the growth rate $\gamma_g$. When $\gamma_g << \omega_a$, the *growth rate is given by*

$$\gamma_g = \frac{(\hat{\underline{k}}\cdot\underline{E}_o)^2\omega_p}{32\pi n\Theta} \operatorname{Im} \left[(k^2/k_D^2)\, \varepsilon\, (\underline{k},\, \omega_L-\omega_o)\right]^{-1}, \tag{22}$$

where $\varepsilon$ is the low frequency dielectric function, and $\underline{E}_o$ is the <u>local</u> value of $CO_2$ electric field. When $T_e >> T_i$, ion-acoustic waves are long-lived, and a resonant approximation for $\varepsilon$ may be used. Then (22) and (18) may be combined to give the Perkins-Flick "threshold" (see appendix). However, when $T_e \approx T_i$, as in this experiment, one cannot use a resonant approximation. In fact, the instability should really be thought of as the decay of a pump photon into a Langmuir wave plus discrete ions. Rather than use the exact $\varepsilon$ (see, for example, DuBois and Goldman, Phys. Rev. Lett. <u>14</u>, 544 (1965)), an adequate approximation at equal temperatures is

$$\operatorname{Im} \left[(k^2/k_D^2)\, \varepsilon\right]^{-1} \approx \frac{\omega_o-\omega_L}{kc_s}\, e^{-(\omega_o-\omega_L)^2/2k^2c_s^2}, \tag{23}$$

$$c_s = 1.7\, v_i = 1.7\, (m_e/m_i)^{1/2}\, v_e\ , \tag{24}$$

This "resonant function" has a maximum value of $e^{-1/2} = 0.61$ when

$$\omega_o - \omega_L(k) = kc_s\ , \tag{25}$$

which therefore constitutes the frequency-matching condition. Both the maximum value and (25) are very close to the exact values.

We shall include another effect missing from the Perkins-Flick analysis: the swelling of the $CO_2$ electric field due to geometric optics. If $I_{VAC}$ is the incident vacuum intensity of the $CO_2$ beam,

$$I_{VAC} \equiv \frac{cE_{VAC}^2}{8\pi} , \tag{26}$$

then the mean intensity (averaged over the Airy oscillations) as a function of distance is given by

$$I_{LOC}(x) = \frac{4I_{VAC}}{2\eta(x)} , \quad \eta(x) \geq \eta_{min} \tag{27}$$

$$\eta_{min} \equiv (c/\omega_o L)^{1/3} . \tag{28}$$

In (27), the factor 4 is due to constructive interference between the incident and reflected $CO_2$ fields (this should really be considered more carefully in the true cylindrical geometry). $\eta$ is the index of refraction,

$$\eta(x) = (1 - \omega_p^2(x)/\omega_o^2)^{1/2} , \tag{29}$$

$\eta_{min}$ is the minimum index of refraction compatible with geometric optics. The maximum possible swelling occurs at the point where $\eta(x) = \eta_{min}$. The factor 1/2 in equation (27) represents an average over the Airy oscillations.

The size of $E_o^2$ in equation (22) is therefore given by,

$$\frac{E_o^2}{32\pi} = \frac{I_{VAC}}{2c}\frac{1}{\eta(x)} \ . \tag{30}$$

We take x = 0 as the reflection point for the $CO_2$. Since $x_o$ is sufficiently close to zero we may use a common scale length L, and write

$$\omega_p^2(x)/\omega_o^2 \equiv 1 + x/L \ , \tag{31}$$

so that,

$$\eta(x) = (-x/L)^{1/2} \ . \tag{32}$$

In this convention x is negative at all points before the reflection point. If we now insert (31) and (32) into (15), we obtain,

$$\eta^2 = -x/L = 3\frac{k^2}{k_D^2} + 2(1 - \omega_L/\omega_o) \ , \tag{33}$$

where $k^2 = k_x^2 + k_y^2$. Combining (22), (23), (30), and (33), the growth rate $\gamma_g$ along the trajectory characterized by $k_y$ and $\omega_L$ is

$$\gamma_g(k_x) = \frac{k_y^2}{k^2}\frac{I_{VAC}\,\omega_p}{2cn_o\theta} \times \frac{1}{\left[(3k^2/k_D^2) + 2(1-\omega_L/\omega_o)\right]^{1/2}}$$

$$\times \frac{(\omega_o-\omega_L)}{kc_s} e^{-(\omega_o-\omega_L)^2/2k^2c_s^2} \ . \tag{34}$$

The factor $k_y^2/k^2$ represents the angular factor, the factor $[\,]^{-1/2}$, geometric swelling, and the last factor is the appropriate frequency-mismatch (or resonant) function at equal temperatures. If we introduce the frequency-matched wavenumber, $k_m$, as the solution to equation (25), then the last two factors in (34) may be written as

$$\frac{1}{\left[(3k^2/k_D^2) + (2\mu k_m/k_D)\right]^{1/2}} \frac{k_m}{k} e^{-k_m^2/2k^2} , \qquad (35)$$

$$\mu \equiv c_s k_D/\omega_o \approx 1.7\ (m_e/m_i)^{1/2} \ . \qquad (36)$$

We must choose $k_m$ (the location of the frequency-matched point along a trajectory) so as to maximize the integrated growth rate. Since $k \geq k_y$, and the exponential must be prevented from getting large, we choose $k_m = k_y$. This allows us to expand

$$e^{-k_m^2/2k^2} \approx 1 - k_y^2/2k^2 \ , \qquad (37)$$

We are also guaranteed that $3k^2/k_D^2 >> 2\mu k_y/k_D$, provided that

$$k_y/k_D >> 2\mu/3 \approx 10^{-2} \ , \qquad (38)$$

which is well satisfied for the experiment, since $\alpha$ values are well below 100. Thus, we can approximate the expression in (35) as

$$\frac{k_D k_y}{\sqrt{3}k^2}\left(1 - \frac{k_y^2}{2k^2}\right) , \tag{39}$$

and the growth rate $\gamma_g$ in (34) becomes,

$$\gamma_g(k_x) = \bar{I}_o \omega_p (k_y^4/k^4)(1 - (k_y^2/2k^2)) , \tag{40}$$

where

$$\bar{I}_o \equiv \frac{I_o}{4cn_o\Theta} , \tag{41}$$

is the dimensionless pump intensity at the point of maximum Langmuir growth (the Langmuir reflection point), and

$$I_o = \frac{2I_{VAC}}{\eta(x_o)} = \frac{2I_{VAC}}{(3k_y^2/k_D^2)^{1/2}} , \tag{42}$$

is the (swollen) pump intensity in physical units at the same point. To recapitulate the meaning of the factors in (40), $k_y^2/k^2$ is the angular factor $(\hat{\underline{k}}\cdot\hat{y})^2$, $k_y/k$ is a pump-diminishing factor (due to geometric optics) as the Langmuir ray passes through points of lower density, and $(k_y/k)(1 - k_y^2/2k^2)$ represents the effects of imperfect frequency matching, which become important away from the point $k = k_y$. The integral for the integrated growth rate A in equation (18) may now be performed. It is best to use $K = k_x/k_y = -\omega_p t/2k_y L$ as a variable, in place of t (see equation (14)). Then,

$$A = \int_{t_i}^{0} dt \left[\gamma_g(k_x(t)) - \gamma_c\right]$$

$$= 2Lk_y \bar{I}_o \int_0^{K_i} dK \frac{1}{(1+K^2)^2}\left[1 - \frac{1}{2(1+K^2)}\right] - 2Lk_y K_i \frac{\gamma_c}{\omega_p}$$

$$= 2Lk_y \bar{I}_o \left[\frac{5}{16}\tan^{-1}K + \frac{5}{16}\frac{K}{1+K^2} - \frac{K}{8(1+K^2)^2}\right]_0^{K_i}$$

$$-2Lk_y K_i \frac{\gamma_c}{\omega_p} \quad , \tag{43}$$

Here, $K_i = k_{xi}/k_y$ should be chosen as the point where $\gamma_g(k_{xi}) = \gamma_c/2$. It is easy to show that the dominant terms in this limit ($K >> 1$) give

$$A = 2Lk_y \bar{I}_o \frac{5}{16}\tan^{-1}(\infty)$$

$$= Lk_y \bar{I}_o \frac{5\pi}{16} \quad . \tag{44}$$

V. We can estimate the spatial width $\delta x$ by taking the integration in equation (43) to some point $K_{min}$ greater than zero, rather than to zero. Define,

$$A(K_{min}) = 2Lk_y \int_{K_{min}}^{K_i} dK \left[\gamma_g(K) - \gamma_c\right]/\omega_p \quad , \tag{45}$$

Then, we wish

$$e^{2A(K_{min})} = \frac{1}{2} e^{2A(0)} \quad , \tag{46}$$

in order that $K_{min}$ define the point at which $I$ is one-half maximum. Thus,

$$A(K_{min}) = A(0) - \frac{\ell n2}{2} . \tag{47}$$

It can be shown by expansion about $K_{min} = 0$ that

$$A(K_{min}) \cong A(0)(1 - 16K_{min}/5\pi) , \tag{48}$$

so

$$K_{min} = \frac{\ell n2}{2Lk_y\bar{I}_o} . \tag{49}$$

By invoking equation (15), we see that

$$\delta x = 3L \frac{k_{x\,min}^2}{k_D^2} = \frac{3(\ell n2)^2 L}{4} \frac{1}{(k_D L\bar{I}_o)^2} , \tag{50}$$

VI. Now we can combine everything to get a final expression for $\sigma$ as a function of $I_o$, $\alpha$, L, and other parameters such as $\Theta$, $r_o$, and $\phi_c$, which, like L and $\alpha$ depend on the exact time prior to peak compression. The data can be fit quite well on the low $I_o$ side of your curve (first four points) by this theory, with L about 0.25 cm. The point where $I_o = 2 \times 10^{10}$ W/cm$^2$ requires the addition of the nonlinear term in the kinetic equation. (We are working on this now.)

Our result is therefore,

$$\frac{\sigma}{r_e^{\,2}} = \frac{1}{\alpha^2} n_o \ell r_o 2\phi_c \delta x \delta\Omega_s \left( \frac{\alpha}{\mu} e^{2A} \right) , \tag{51}$$

where $\mu$ is defined by equation (36). Inserting from (44) and (50), with $\phi_c \approx \sqrt{3}/\alpha$ from (7),

$$\frac{\sigma}{r_e^{\,2}} = \frac{1}{\alpha^2} n_o \ell r_o \frac{2\sqrt{3}}{\alpha} \delta\Omega_s \left[ \frac{3(\ell n 2)^2}{4} \frac{L}{(k_D L\bar{I}_o)^2} \right] \cdot \left[ \frac{\alpha}{\mu} e^{(5\pi/8) L k_y \bar{I}_o} \right] , \tag{52}$$

This may be rewritten as

$$\frac{\sigma}{r_e^{\,2}} = 0.73 \left( \frac{m_i}{m_e} \right)^{1/2} \frac{1}{\alpha^2} \frac{n_o r_o \ell \delta\Omega_s e^{(5\pi/8) L k_y \bar{I}_o}}{k_D^{\,2} L \bar{I}_o^{\,2}} , \tag{53}$$

This is a very strong function of temperature since $\alpha^2 = k_D^{\,2}/k_y^{\,2} \propto \Theta^{-1}$, and $\bar{I}_o \propto I_o/\Theta \propto \Theta^{-3/2}$. Thus,

$$\frac{\sigma}{r_e^{\,2}} \propto \frac{r_o \Theta^5 e^{CL\Theta^{-3/2}}}{L} . \tag{54}$$

As peak compression is approached, $L\Theta^{-3/2}$ decreases, and $\sigma/r_e^{\,2}$ probably decreases, although the non-exponential factors are increasing.

Now we put in parameters which are independent of time prior to peak compression:

$(m_i/m_e)^{1/2} = 60.6$ (deuterium)

$n_o = 9.2 \times 10^{18}$ (at the frequency-matched point)

$\ell = 0.25$ (spot size)

$\delta\Omega_s = d\psi(\cos 5.6^\circ - \cos 10.4^\circ) = 6.53 \times 10^{-3}$ (with mask)

$$\frac{\sigma}{r_e^2} = 6.64 \times 10^{17} \frac{r_o e^{(5\pi/8)Lk_y\bar{I}_o}}{\alpha^2 k_D^2 L\bar{I}_o^2} . \quad (55)$$

Parameters associated with L = 0.25 cm are

$L = 0.25$ cm, $r_o = 0.055$ cm

$\Theta_e = 0.87$ keV $= 1.39 \times 10^{-9}$ ergs $= 1.39 \times 10^{-16}$ J

$k_y = 1.26 \times 10^4$ $(\theta_s = 8^\circ)$

$k_D = 1.4 \times 10^5$

$\alpha = k_D/k_y = 11.1$

$$\bar{I}_o = \frac{2I_{VAC}\alpha}{4cn_o\Theta\sqrt{3}} = I_{VAC} \times 8.35 \times 10^{-14} \quad (56)$$

with $I_{VAC}$ in units of $W/cm^2$. Thus,

$$\frac{\sigma}{r_e^2} = \frac{8.67 \times 10^{30}}{I_{VAC}^2} e^{5.17 \times 10^{-10} I_{VAC}} . \quad (57)$$

| $I_{VAC}$ | $\sigma/r_e^2$ (calculated) | mV |
|---|---|---|
| $0.5 \times 10^{10}$ | $(3.5 \times 10^{11})(13.3) = 4.6 \times 10^{12}$ | 9.4 |
| $1.1 \times 10^{10}$ | $(7.2 \times 10^{10})(311) = 2.2 \times 10^{13}$ | 46 |
| $1.25 \times 10^{10}$ | $(5.5 \times 10^{10})(6.4 \times 10^{2}) = 3.56 \times 10^{13}$ | 73 |
| $2.1 \times 10^{10}$ | $(1.97 \times 10^{10})(5.2 \times 10^{4}) = 10^{15}$ | 2000 |
| $1.4 \times 10^{10}$ | $(4.4 \times 10^{10})(1.4 \times 10^{3}) = 6 \times 10^{13}$ | 126 |

The detector mV readings in the above chart have been calculated by assuming a linear photon counter, and $3 \times 10^{-10}$ J of scattered energy for a 110 mV signal strength and a 5J incident ruby beam.

Thus, the measured

$$\frac{\sigma}{r_e^2} = \frac{\pi d^2}{4r_e^2} \frac{E_{scattered}}{E_{inc}} = \frac{(7.1\text{x}10^{-2})}{(2.81\text{x}10^{-13})^2} \frac{(3\text{x}10^{-10})}{5}$$

$$= 5.37 \times 10^{13} , \tag{58}$$

for d = 0.3 cm. Therefore,

$$\left.\frac{\sigma}{r_e^2}\right|_{measured} = V \frac{5.37 \times 10^{13}}{110} = V \times 4.88 \times 10^{11} , \tag{59}$$

where V is the signal strength in MV. The theoretical curve is shown in Figure 4.

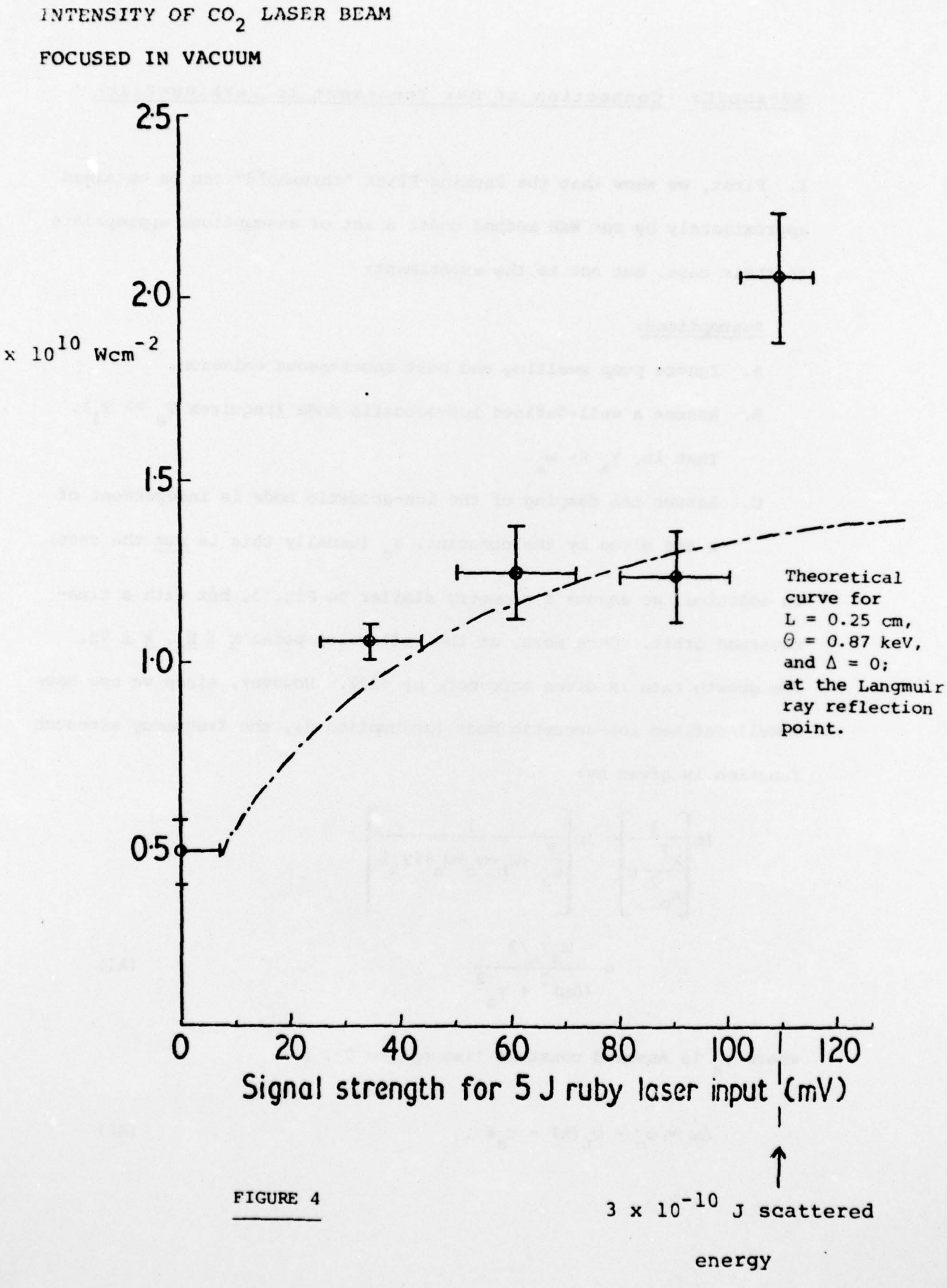

INTENSITY OF $CO_2$ LASER BEAM
FOCUSED IN VACUUM
2·5
2·0
$\times 10^{10}$ Wcm$^{-2}$
1·5
1·0
0·5
0
20
40
60
80
100
120
Signal strength for 5 J ruby laser input (mV)
Theoretical curve for $L = 0.25$ cm, $\Theta = 0.87$ keV, and $\Delta = 0$; at the Langmuir ray reflection point.
$3 \times 10^{-10}$ J scattered energy

FIGURE 4

## APPENDIX: Connection of Our Treatment to Perkins-Flick

I. First, we show that the Perkins-Flick "threshold" can be obtained approximately by our WKB method under a set of assumptions appropriate to their case, but not to the experiment:

Assumptions:

A. Ignore pump swelling and beat spontaneous emission.

B. Assume a well-defined ion-acoustic mode (requires $T_e >> T_i$). That is, $\gamma_a << \omega_a$.

C. Assume the damping of the ion-acoustic mode is independent of k and given by the constant, $\gamma_c$ (usually this is not the case).

In addition, we assume a geometry similar to Fig. 3, but with a time-reversed orbit. Once more, at the reflection point $\underline{k} \parallel \underline{E}_o$, $\underline{k} \perp \underline{\nabla}n$. The growth rate is given once-more by (22). However, since we now have a well-defined ion-acoustic mode (assumption B), the frequency mismatch function is given by:

$$\mathrm{Im}\left[\frac{1}{\frac{k^2}{k_D^2}\varepsilon}\right] = \mathrm{Im}\left[\frac{1}{\frac{2}{\omega_a}(\omega_L - \omega_o + \omega_a + i\gamma_a)}\right]$$

$$= \frac{\omega_a \gamma_a/2}{(\Delta\omega)^2 + \gamma_a^2} \qquad \text{(A1)}$$

where $\gamma_a$ is assumed constant (assumption C), and

$$\Delta\omega = \omega_o - \omega_L(k) - c_s k \ . \qquad \text{(A2)}$$

Here $c_s$ is the sound speed:

$$c_s = (\theta_e + 3\theta_i)^{1/2}/M_i^{1/2} \ . \qquad \text{(A3)}$$

Thus, the integrated growth rate is (see equation (22) above)

$$A = \int_0^{t_f} dt \left[\gamma_g(k_x(t)) - \gamma_c\right]$$

$$= \frac{E_o^2\omega_p}{32\pi n\Theta}\frac{\gamma_a}{2}\int_0^{t_f} dt \ \frac{c_s k(k_y^2/k^2)}{(\Delta\omega)^2 + \gamma_a^2} - \gamma_c t_f \ , \qquad \text{(A4)}$$

where $t_f$ is a final time, and $k_y^2/k^2 = (\hat{k}\cdot\hat{e}_o)^2$. Also, $k = (k_x^2(t) + k_y^2)^{1/2}$. With $K = k_x/k_y$ as a variable, and $k_x = (\omega_p/2L)t$, (A4) becomes

$$A = \Lambda_o^2 \frac{Lk_y}{8}\left(\frac{c_s k_y}{\gamma_a}\right)\int_0^{K^f} dK \ \frac{(1+K^2)^{-1/2}}{1+(\Delta\omega)^2/\gamma_a^2} - \gamma_c t_f \ , \qquad \text{(A5)}$$

$$\Lambda_o^2 \equiv \frac{E_o^2}{4\pi n\Theta} \ , \qquad \text{(A6)}$$

$$K^f = \frac{\omega_p}{2L}\frac{t_f}{k_y} \ . \qquad \text{(A7)}$$

In order to do the integral, we note that,

$$\frac{\Delta\omega}{\gamma_a} = \frac{\omega_o - \omega_L - c_s(k_x^2+k_y^2)^{1/2}}{\gamma_a} = \left(\frac{c_s k_y}{\gamma_a}\right)\times\left[\frac{\omega_o-\omega_L}{c_s k_y} - (1+K^2)^{1/2}\right]. \qquad \text{(A8)}$$

Intrinsic to the approximation (assumption B) is that

$$r \equiv \frac{\gamma_a}{c_s k_y} << 1 \quad . \tag{A9}$$

We also designate the frequency-matched wavenumber by the condition $(\omega_o - \omega_L) = c_s k_m$, so that, if $K_m \equiv k_m/k_y$, then equation (A8) may be rewritten as:

$$\frac{\Delta\omega}{\gamma_a} = \frac{1}{r}\left[K_m - (1 + K^2)^{1/2}\right] \quad , \tag{A10}$$

so the integral in (A5) is of the form

$$I_1 = \int_0^{K_f} dK \, \frac{(1 + K^2)^{-1/2}}{1 + r^{-2}\left[K_m - (1 + K^2)^{1/2}\right]^2} \quad . \tag{A11}$$

In this limit of large $r^{-2}$, the integrand peaks for $K_m \approx 1$ and $K << 1$ so we may approximate the integral is

$$I_1 = \int_0^{K_f} dK \, \frac{1}{1 + r^{-2}\,\frac{K^4}{4}} \approx \int_0^{\infty} dK \, \frac{1}{1 + r^{-2}\,\frac{K^4}{4}}$$

$$= \sqrt{2r} \int_0^{\infty} du \, \frac{1}{1+u^2} = \sqrt{2r}\, \frac{\pi}{4\sin\pi/4} \, . \tag{A12}$$

The main contribution to (A5) is therefore

$$A = \Lambda_o^{\,2}\, \frac{Lk_y}{8}\, \frac{1}{r} \sqrt{2r}\, \frac{\pi}{4\sin\pi/4} \tag{A13}$$

$$A \approx \frac{1}{5.1} \Lambda_o^2 \frac{Lk_y}{r^{1/2}} \quad . \tag{A14}$$

If we define "threshold" as the value where A = 5, as do Perkins and Flick, then the threshold condition we find from (A14) is

$$\left.\frac{E_o^2}{4\pi n\theta}\right|_{WKB} \geq \frac{5.1\ A(\gamma_a/c_s k_y)^{1/2}}{Lk_y} \quad , \quad A = 5 \ . \tag{A15}$$

By comparison, Perkins and Flick find as their threshold (equation 39, p. 2017, Phys. Fluids 14 (1971):

$$\left.\frac{E_o^2}{4\pi n\theta}\right|_{P.F.} = \frac{1.6(1 + 3T_i/T_e)}{k_y L} A \left(\frac{\nu_i}{\omega_a}\right)^{1/2} \ . \tag{A16}$$

Since $T_i >> T_e$, and $\nu_i = 2\gamma_a$ (relation between particle and wave dissipation), our threshold (A15) is higher than theirs by a factor of 2.25, but all dimensional factors are correctly predicted. The discrepancy is due to the use of WKB and, in particular, arises because the mismatch function (denominator in (A11)) is so highly peaked close to the reflection point (K = 0) when r >> 1. For the $T_e = T_i$ case we treated in the main-body of this memo, the mismatch function is much broader, and WKB should be an even better approximation! In our treatment we have the additional physics of (i) pump swelling along the Langmuir ray-path, (ii) pump-dependent (beat) spontaneous emission of Langmuir waves, and (iii) ion discreteness.

It is interesting to note that if we take $\gamma_a = c_s k_y$ in (A15), and $\nu_i = 2\gamma_a$, $T_i = T_e$ in (A16), then (A15) predicts a threshold lower than the Perkins-Flick threshold by a factor of 0.56. We also nete that if we take into account effects (i) and (iii) above, but not beat emission (ii), then (44) gives a "threshold"

$$\frac{E_{LOC}^2}{4\pi n\theta} > \frac{128}{5\pi}\ \frac{A}{Lk_y} = \frac{8.1A}{Lk_y}\ , \tag{A15}$$

which is within 10% of the Perkins-Flick result (A16) at equal temperatures, and with $\nu_i = 2\omega_a$. If spontaneous emission is taken into account, then A in (A15) should be replaced by $\bar{A} - \frac{1}{2}\ln(\omega_o/\omega_o - \omega_L)$ $= \bar{A} - \frac{1}{2}\ln(\omega_o/c_s k_y)$, so the threshold (A15) becomes

$$\frac{E_{LOC}^2}{4\pi n\theta} \geq \frac{8.1\left[\bar{A} - \frac{1}{2}\ln\ (\frac{1}{1.7}\sqrt{\frac{m_i}{m_e}}\ \alpha)\right]}{Lk_y}\ , \tag{A16}$$

where $e^{2\bar{A}} = \left[\omega_o/(\omega_o-\omega_L)\right]e^{2A} = I$ ($\bar{A}$ measures the effective amplitude e-folding as though amplifying from equilibrium). Thus, if $\bar{A} = 5$, the term in brackets above becomes $5 - \frac{1}{2}\ln 10^3 = 1.5$, and it is fair to say that beat spontaneous emission lowers the "threshold" by more than a factor of three. Of course, in terms of $I$ this effect is much stronger, and without beat emission we would have calculated a spectrum and cross section three orders of magnitude smaller!