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QUEUING MODELS FOR MULTIPLE CHAMBER LOCKS

by

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and

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ABSTRACT

Several models for predicting mean waiting times of river traffic at a multiple chamber lock were developed and tested. Mean waiting times predicted by the M/G/1 model differed significantly from observed times. Analysis of possible causes of failure of this model suggested a limited queue length M/G/1 model for one chamber, from which more accurate predictions were derived. For the two chamber system, an M/G/1 model with random batch size was developed. This model yields a lower bound for mean waiting time. These last two models can be used to predict system performance under various operating conditions.

QUEUING MODELS FOR MULTIPLE CHAMBER LOCKS

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0. INTRODUCTION

The analysis of queuing delays experienced by river traffic at locks presents a formidable challenge because of the special features of lock operations. One early attempt [1] used the simple M/M/1 model and obtained results that were reasonably consistent with the limited data available at the time. However, that model does not permit an evaluation of the effects of changes in operating procedures or capital improvements in lock configuration. Furthermore, detailed studies of lock operations suggested that the assumptions underlying the M/M/1 model are not consistent with observed arrival and service times. These inadequacies of simple queuing models prompted the development of simulation models described in [2].

This paper reports on several queuing models developed to assist in the analysis of operations at a particular lock (Number 26) in the Mississippi River. The purposes of this analysis were, first, to predict mean waiting times at higher traffic rates than had been experienced, and, second, to aid in estimating the change in average waiting times that would result from changed lock configurations or operations.

0.1 LOCK OPERATIONS

Lock 26 consists of a pair of chambers each 110 feet wide, but of different lengths (600' and 360'). Adjacent to the lock, and extending across the river, is a dam which maintains the upstream water level several meters higher than the downstream pool level. A typical user of the lock is a tow, consisting of a tug boat and several barges lashed alongside and/or ahead.

A transit of the lock generally requires the following steps:

1. The lock supervisor signals the tow to begin its approach. The gate opens.
2. The tow maneuvers into the chamber.
3. The gate closes, the water level in lock changes to match level of the other pool, the other gate opens.
4. The tow maneuvers out of chamber.

Figure 1 shows a tow just leaving the small chamber and another about to enter the large one.

Several variations of this procedure occur. If several small tows fit into the chamber simultaneously and share one cycle of Step 3, this is known as a pass through lockage. By contrast, very large tows cannot fit into the chamber, and so are broken down into groups of barges each of which requires Steps 2-4. If two (or three) groups are required, the procedure is called a double (or triple) lockage. A double lockage takes more than twice as long as a single because three cycles of Step 3 are required instead of one and furthermore the chamber is blocked while the tow is broken down to chamber sized groups on one side and made up on another. Another type of procedure is known as a knock-out lockage in which a tow rearranges its barges to fit into the chamber and blocks the approach

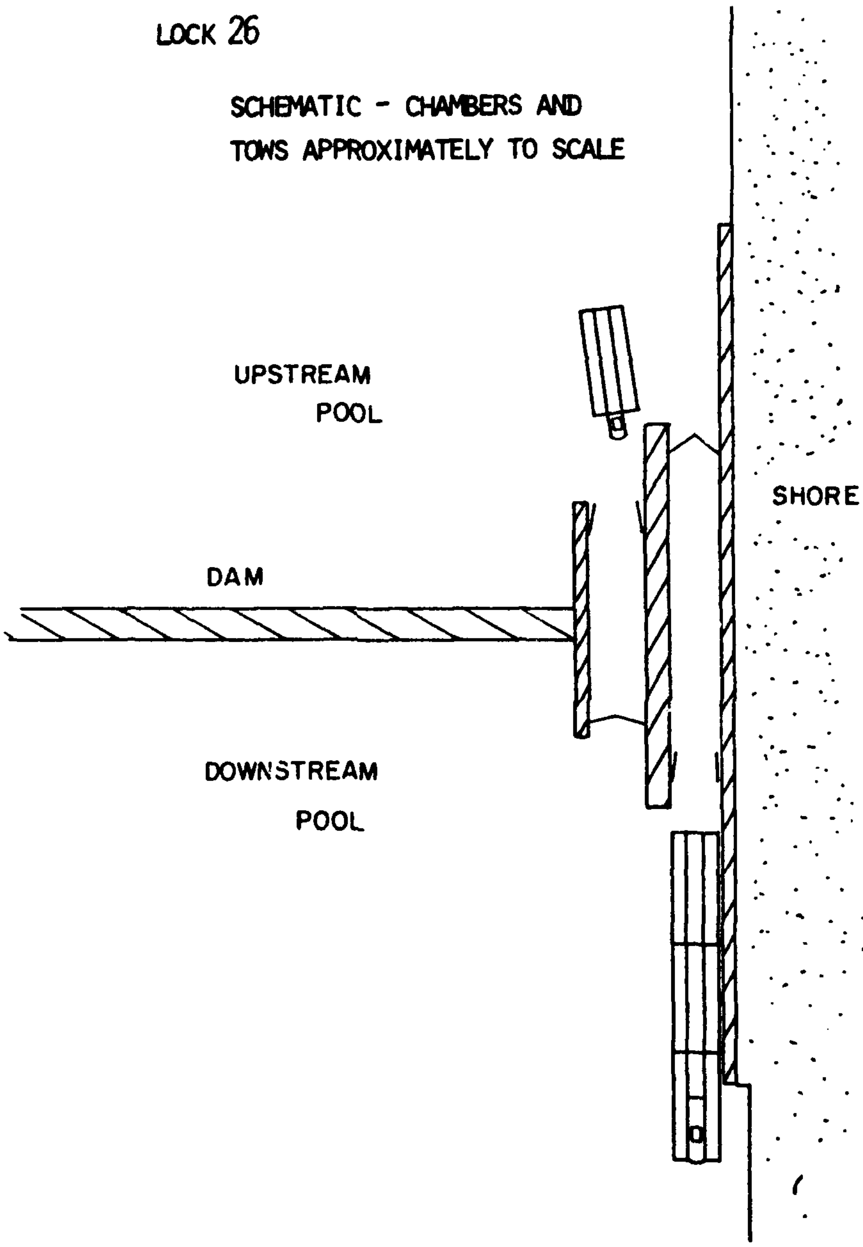
LOCK 26
SCHEMATIC - CHAMBERS AND
TOWS APPROXIMATELY TO SCALE
UPSTREAM
POOL
SHORE
DAM
DOWNSTREAM
POOL

FIG. 1

to the chamber while doing so. A knock-out lockage effectively occupies the lock for about twice as long as a single lockage, on the average.

Other special features of lock operations serve to complicate the queuing analysis. There are, in fact, two queues; one on the upstream side of the lock and the other on the downstream side. The arrival rate of tows varies in time, (see Tables A1, A2 in the Appendix) as does the distribution of tow sizes and hence the relative frequency of single, double, triple and knock-out lockages. The decision process by which a particular tow is selected from the queue for passage through one of the chambers is complex and poorly understood.

The balance of this paper describes how some of these features were found to be important for predicting waiting times and how representations of the more significant ones were incorporated into the models we developed.

1. MODEL I - INDEPENDENT M/G/1 QUEUES

The simplest analysis of this system is based on the assumption that the two chambers are independent; each is characterized by time homogeneous Poisson arrival stream of tows and independent service times. Further, the existence of separate upstream and downstream queues is ignored. The waiting time in queue can be computed for each chamber from the well known M/G/1 model

$$W_Q = \lambda E[S^2]/2(1 - \lambda E[S])$$

where

λ = mean arrival rate.

$E[S]$ = mean service time.

$E[S^2]$ = mean squared service time.

The results of this calculation are shown in Table 1.

TABLE 1

	λ (Tows/Hr.)	E[S] (Hrs.)	$E[S^2]$	W_Q (Hrs.)	W_0(Hrs.)
600' Chamber	.646	1.466	2.753	16.79	6.53
360' Chamber	.4865	.8523	1.320	.549	.814

The last column is the observed mean waiting time. The moments of the arrival and service time distributions were estimated from data extracted from the lock operating log and represent all tows arriving during the 26 week period beginning March 1, 1974, a total of 5171 observations.

A number of data items are recorded in the lock operating log for each tow, including the times that the tow signals its arrival, the lock supervisor signals the tow to begin its approach, and the tow clears the exit gate. These times are entered into a computer data base, from which the moments of the waiting time and lockage time distributions were calculated by the Corps of Engineers. Unfortunately, lockage time as defined is not the same as service time because there is always an additional interval between exit of one tow and the time of the approach signal of the next. If the next tow is heading in the same direction, the lock must be cycled through Step 3; if in the opposite direction, the exiting tow must clear the approach channel. The mean and variance of this extra lost time, which is very small compared with lockage time, were estimated from the log records of 160 consecutive tows and added to the lockage time mean and variance to arrive at estimates of $E[S]$ and $E[S^2]$.

1.1 Analysis of M/G/1 Model Failure

It is clear that the independent M/G/1 queuing model is not a satisfactory predictor of waiting times, and the causes of model failure must be sought. The existence of separate upstream and downstream queues at both chambers does not seem the likely source of difficulty since the simple model overestimates the 600' chamber waiting time and underestimates at the 360' chamber.

Closer analysis of the arrival data revealed that the arrival rate of tows is not, in fact, homogeneous in time. On the average, fewer tows arrive during the night hours and weekends than during the daylight and week days. We shall argue that this nonhomogeneous arrival pattern can partly explain the underestimate at the small chamber, but not the overestimate at the larger chamber.

Consider a system that experiences an arrival rate λ_1 for a fraction p_1 of the time and another arrival rate λ_2 for a fraction p_2. If a long time elapses between change of arrival rates, so that the system spends most of its time in "steady state" at one arrival rate or the other, the time average wait in queue will be approximately $p_1W(\lambda_1) + p_2W(\lambda_2)$ where $W(\lambda)$ is the steady state expected waiting time given the constant mean arrival rate λ. On the other hand, if the time between changes of arrival rate is very short the system behaves as if the mean arrival rate were constant, with $\lambda = p_1\lambda_1 + p_2\lambda_2$. It seems plausible that the mean waiting time for a nonconstant arrival system lies between these bounds, i.e.

$$W(p_1\lambda_1 + p_2\lambda_2) \leq W \leq p_1W(\lambda_1) + p_2W(\lambda_2) .$$

While the outer inequality follows from the convexity of $W(\lambda)$, we have not been able to verify the complete inequality.

Thus our conjecture implies that a nonhomogeneous Poisson arrival process will give rise to a larger mean waiting than the homogeneous process which has the same average arrival rate. Since, however, the observed average waiting time at the large chamber is much smaller than that predicted by the M/G/1 model, the failure of the homogeneous arrival assumption is not the likely cause of model failure. The next candidate for investigation is the assumed independence of chambers. It seems clear that the larger tows would prefer to use the larger chamber, particularly those that are too big to transit the small chamber in a single lockage. On the other hand, when waiting times become too long at the large chamber, the small chamber becomes more attractive. A poorly specified but state dependence queue switching rule apparently operates, and the next two models we describe were developed in an effort to approximate this very complex situation.

2. MODEL II - M/G/1 WITH FINITE QUEUE LENGTH

The simplest state-dependent rule is the finite queue length rule: if the number in the system exceeds N, the arrival does not join the 600' chamber queue, but is diverted to the 360' chamber.

2.1 Derivation of an Approximate Formula for Average Wait in the M/G/1 Queue with Finite Capacity N

Let

λ = Poisson arrival rate of all potential customers

W = average amount of time an entering customer spends in the system

P_n = proportion of time there are n customers in the system

$n = 0,1, \ldots, N$.

Now

$$W = W_1 + W_2$$

where

W_1 = average remaining service time of customer in service (if any) when the new customer arrives

W_2 = average additional time (to W_1) an entering customer spends in the system, i.e. his service time plus the service time of the customers ahead of him in queue.

By conditioning on the state of the system when the customer arrives we obtain[+]

$$W_1 \doteq \frac{1 - P_0 - P_N}{1 - P_N} \frac{E[S^2]}{2E[S]}$$

$$W_2 = \frac{P_0}{1 - P_N} E[S] + \sum_{i=1}^{N-1} \frac{P_i}{1 - P_N} iE[S]$$

where S is a service time random variable. Hence,

[+] For a justification of this approximation, see Nozaki and Ross [5].

$$W = \frac{E[S][L - NP_N] + E[S]P_0 + [1 - P_0 - P_N]E[S^2]/2E[S]}{1 - P_N}$$

where $L = \sum_{i=1}^{N} iP_i$ = average number of customers in the system.

By using the well known queuing identity $L = \lambda_e W$

λ_e = effective arrival rate = $\lambda(1 - P_N)$, we obtain

$$W \doteq \frac{\lambda E[S](1 - P_N)W - NP_N E[S] + E[S]P_0 + (1 - P_0 - P_N)E[S^2]/2E[S]}{1 - P_N}$$

or

$$W = \frac{E[S]P_0 + \frac{E[S^2]}{2E[S]}(1 - P_0 - P_N) - NP_N E[S]}{(1 - P_N)(1 - \lambda E[S])} \quad . \tag{1}$$

By noting that the average arrival rate of customers must equal the average departure rate we obtain

$$\lambda(1 - P_N) = (1 - P_0)/E[S] \quad . \tag{2}$$

Hence the only unknown is P_0 . This, however, may be approximated by using its value obtained in the special case when the service distribution is exponential. In this case we have (see [3], p. 146)

$$P_0 = (1 - \lambda E[S])/[1 - (\lambda E[S])^{N+1}] \tag{3}$$

and W is thus obtained from (1), (2), and (3).

2.2 Computation for Finite Queue Model - 600' Chamber

1) Assume all tows with 2 or more barges would use the 600' chamber if possible.

   Total arrivals 5171 (in 6 months)

   less 0,1 barge tows - 1357

   3814 Total = .873 Tows/hr. = λ

2) Tows observed arriving at 600' chamber (not counting pass through) = 2822 (in 6 months) λ_e = .646 (see Table 1).

3) The difference between these arrival rates is the diversion rate to the smaller chamber. Let P_N be the probability that an arrival sees $(N - 1)$ waiting plus 1 in process. Hence

$$1 - P_N = \lambda_e/\lambda = .740 .$$

4) Computation of busy fraction: effective arrival rate = service rate × busy fraction

$$\lambda_e = \frac{1 - P_0}{E[S]}$$

$$1 - P_0 = \lambda_e E[S] = .947 ,$$

   where $E[S]$ and $E[S^2]$ are from Table 1.

5) Estimate N the maximum number of tows in system (using as an approximation the proportion of time the equivalent M/M/1 queue is empty, i.e. Equation (3)). This yields

$$(\lambda E[S])^{N+1} \doteq 1 + (\lambda E[S] - 1)P_0 = 6.329$$

$$(N + 1) \log \lambda E[S] \doteq \log 6.329$$

$$N \doteq 6.46$$

6) Compute W from Equation (1) and obtain $W = 8.403$, the predicted mean time in system. By comparison, $W_0 + E[S]$ from Table 1 gives the observed average time in system as 7.904 hrs.

The simple M/G/1 model with arrival rate λ_e can be viewed as a queue switching model in which customers arrive at rate λ, but switch to the small chamber at random with constant probability $p = 1 - \lambda_e/\lambda$. In practice, switching is much more likely to occur when the large chamber queue is long, thus reducing the tail of the waiting time distribution. This analysis explains why the M/G/1 model overpredicts the waiting time, and why the finite queue model provides much better agreement with observed waiting times.

It should be noted that we did not choose N so that our analytical expression for W [Equation (1)] would be in agreement with the observed value of W. Had we done so, there would be no independent verification of the model from the data. Rather, N was estimated from the observed effective arrival rate, Equation (3), and the assumption that all tows with 2 or more barges prefer the large chamber. This last assumption is a somewhat subjective interpretation of the data in Table 2.

We have assumed homogeneous Poisson arrivals at the 600' chamber in this analysis. A close inspection of the arrival data showed that the larger tows (3 or more barges) exhibited no significant difference in mean arrival rate among hours of the day or days of the week. The smaller tows showed significantly higher arrival rates during the day than at night, and during the week than on weekends. These results support the idea that the large tows represent long distance traffic and operate around the clock seven days a week. Local traffic, by contrast, is predominantly small tows. Since most of the small tows use the small chamber, the assumption of homogeneous Poisson arrivals at the large chamber is consistent with the data.

TABLE 2*

Tow Size (Barges)	Number Transiting 600' Chamber	360' Chamber	% Transiting 600' Chamber
0	41	737	5
1	19	536	3
2	119	445	21
3	241	264	48
4	352	170	67
5	111	32	78
6	216	50	81
7	73	8	90
8	198	14	93
9	179	9	95
10	98		100
11	109		100
12	401	4	99
13	71		100
14	103		100
15	475	1	100
16 or more	52		100

*The total tows in this table is less than the number arriving during the 26 week period beginning March 1, 1974 because the number of barges was not recorded correctly for all arriving tows.

2.3 Use of Model II to Analyze Changes in Operating Procedure

Suppose that mooring facilities clear of the approach channel were provided so that tows that require reconfiguration before transiting as a single lockage could do so prior to making their approach. The effect of this change would be that knock-out lockages would be converted to single lockages. The resulting shift in the service time distribuiton can be computed from the data below:

Lockage Type	Number	Fraction	E[S]	$E[S^2]$	
600' single	522	.185	.736	.645	
600' double	1550	.550	1.902	4.165	
600' knock-out	744	.264	1.109	1.141	
	2816		1.466	2.753	(weighted average)

If all knock-out lockages become singles, the singles would then be 45% of the total, and doubles would still be 55%. The new weighted average values for $E[S]$ and $E[S^2]$ would then be 1.378 and 2.583.

This change in operating procedure will thus increase the service rate of the 600' chamber and presumably reduce the waiting time for tows using it. To estimate the new mean waiting time some assumption must be made about the behavior of the tow boat captains in electing which chamber to use under the new conditions.

Suppose, for example that this decision is based only on the large chamber queue length, and hence N, as estimated under the old conditions, remains constant (as does λ). Then we obtain $1 - P_0 = .932$ from Equation (3), using the new value of $E[S]$, and then compute $1 - P_N = .774$ from Equation (2). The effective arrival rate, $\lambda_e = .676$, and $W = 7.96$ hrs. from Equation (1).

In contrast, assume that the same fraction of tows elect the small chamber, so that λ_e remains constant. Then the busy fraction, $1 - P_0 = .890$ and $N = 4.66$ (see steps 4 and 5 of the last section). Equation (1) then yields $W = 5.93$ hrs.

It can be argued that these estimates bracket the true value since presumably the queue length at the small chamber is also considered in the decision process. If fewer tows are diverted to the small chamber in consequence of the first assumption, that queue is shorter and hence less unattractive. Consequently, the value of N should be somewhat smaller under the new conditions. However, under the second assumption, the average queue at the large chamber is substantially shorter, thus making it less likely that tows will be diverted, so λ_e should increase somewhat. Further analysis is limited by lack of detailed understanding of the queue switching behavior.

3. MODEL III - M/G/1 BATCH ARRIVALS

Model II is based on one approximation of customer behavior in switching from one queue to another, i.e., that all customers, who would otherwise like to use the 600' chamber, choose the 360' when the number in queue at the large chamber equals N - 1 . This parameter is estimated from an assumption about who prefers the large chamber and from the observed arrival rate. Under vastly different conditions of arrival rate and waiting times for the two chambers, it is not all clear what customer behavior would be.

The difficulties arising from customer queue switching can be avoided by taking a different point of view, i.e., regarding the lock, which consists of two chambers, as a service facility for processing barges (and tow boats). Thus, we treat the barges as customers and suppose they arrive in batches of random size. The individual barges are allowed then to be serviced in either the large or small chamber depending on which one is free. If neither is free, they wait in a common waiting line which feeds into either chamber. Since this model allows a tow with many barges to be served in the small chamber and assumes a constant mean arrival rate of barges, it will underestimate the average waiting time of a barge.

Since the analysis of M/G/2 systems is complicated when service rates are unequal, we seek an "equivalent" M/G/1 system that has approximately the same service characteristics. We are thus led to consider an M/G/1 queue with random size batch arrivals. The batch size distribution can be estimated from the tow size data. The remaining problems are to derive the formula for the mean waiting time of a customer (now a barge, not a tow) and a method of estimating the service time distribution of a single channel equivalent system.

3.1 Derivation of Mean Waiting Time in a M/G/1 Model with Random Batch Size

Define:

W_Q = average amount of time a customer (a barge) spends waiting in queue.

V = average amount of work in the system - average amount of time it would take the server to complete service of all customers (barges) presently in the system.

λ = Poisson arrival rate of batches.

N = random variable representing the number of customers in a batch.

We have the well-known result (see, for example, [6]) relating W_Q and V, namely,

$$V = \lambda E[N]E[S]W_Q + \frac{E[S^2]}{2} \lambda E[N] \; . \tag{4}$$

However, if a batch arrives when the current workload is V, then the average amount of time an average customer from the batch will spend in queue is

$$\begin{aligned} W_Q &= V + E[\text{waiting time due to those in his batch}] \\ &= V + E[W_B] \; . \end{aligned} \tag{5}$$

Now $E[W_B]$ can be computed by conditioning on the number in the batch. If α_n is the proportion of batches of size n, then a customer chosen at random (from among all customers) will come from a batch of size n with probability equal to $n\alpha_n / \sum_j j\alpha_j$, and thus

$$E[W_B] = \sum_n E[S] \frac{(n-1)}{2} n\alpha_n / \sum_j j\alpha_j \tag{6}$$

$$= E[S](E[N^2] - E[N])/2E[N]$$

because if there are n in this batch then the average customer will see $\frac{n-1}{2}$ customers from his batch in front of him. Hence, from (4), (5), and (6), we obtain

$$W_Q = \frac{(E[S](E[N^2] - E[N])/2E[N]) + \lambda E[N]E[S^2]/2}{1 - \lambda E[N]E[S]} \, . \tag{7}$$

3.2 Approximating a Two Server System by a Single Server System

Consider a 2 server system in which S_1 and S_2 represent service random variables for the respective servers. If both servers are continuously busy, then the output stream would be a superposition of 2 renewal processes and thus

$$E\text{ (number departures by time } t) \sim \frac{t}{E[S_1]} + \frac{t}{E[S_2]}$$

$$\text{Var (number departures by time } t) \sim \frac{t \text{ Var } [S_1]}{(E[S_1])^3} + \frac{t \text{ Var } [S_2]}{(E[S_2])^3}$$

(see Reference [3]). Hence, to approximate by a single server system, the service distribution would need to be such that

$$t/E[S] = t/E[S_1] + t/E[S_2]$$

and

$$t \text{ Var } [S]/(E[S])^3 = t \text{ Var } [S_1]/(E[S_1])^3 + t \text{ Var } [S_2]/(E[S_2])^3$$

or, in other words,

(8) $$E[S] = (1/E[S_1] + 1/E[S_2])^{-1}$$

(9) $$\text{Var } [S] = (E[S])^3\left(\text{Var } [S_1]/(E[S_1])^3 + \text{Var } [S_2]/(E[S_2])^3\right) .$$

3.3 Calculations for Two Server Random Batch Model for Barges

The first step is to estimate the mean and variance of equivalent barge service time at each chamber. This can be done based on the assumption that the service time observed for a tow of size j (where j = number of barges + 1) is the sum of j independent random variables, each with mean μ and variance σ^2 .

Let S_{ij} = service time of the i^{th} tow of size j .

Then $ES_{ij} = j\mu$,

$\text{Var } (S_{ij}) = j\sigma^2$,

and $E\left[S_{ij}^2\right] = j\sigma^2 + j^2\mu^2$.

Let n_j = number of tows observed of size j .

Then $E \sum_{i=1}^{n_j} S_{ij} = n_j j\mu$

and $E \sum_{i=1}^{n_j} S_{ij}^2 = n_j j\sigma^2 + n_j j^2\mu^2$.

Summing over all tow sizes, we obtain

$$E \sum_{j=1}^{19} \sum_{i=1}^{n_j} S_{ij} = \mu \sum_{j=1}^{19} jn_j$$

$$E \sum_{j=1}^{19} \sum_{i=1}^{n_j} S_{ij}^2 = \sigma^2 \sum_{j=1}^{19} jn_j + \mu^2 \sum_{j=1}^{19} j^2 n_j ,$$

which give rise to the estimators

$$\hat{\mu} = \sum \sum S_{ij} / \sum jn_j \; ; \; \hat{\sigma}^2 = \left(\sum \sum S_{ij}^2 - \hat{\mu}^2 \sum j^2 n_j \right) / \sum jn_j \; .$$

It can be shown that these estimators are biased, but that the bias is negligibly small for the sample sizes available to us.

These estimators, for the 28,246 barges and tow boats that transited the 600' chamber, are $\mu_{600} = .1476$ and $\sigma_{600}^2 = .02597$. The results for the 360' chamber are $\mu_{360} = .3164$, $\sigma_{360}^2 = .1600$ based on tows containing 5940 barges and boats. From (8) and (9), we calculate $E[S] = .10065$ and $E[S^2] = .02352$.

From the lock log data, we calculate the moments of the tow size distribution: $E[N] = 6.600$ $E[N^2] = 70.447$, and the average arrival rate of units = 7.884. Substituting these values in (7) yields $W_Q = 2.78$ hrs. From the lock log data, we calculate the observed average waiting time as 5.97 hrs. To obtain the average time in system, the average time spent by a barge during a transit of the lock must be added. This number is the average tow service time, weighted by the tow size, and is 1.360 hrs. The model mean time in system is 4.14 hrs. while the observed average time is 7.33 hrs. The model thus provides a lower bound on time in system, as expected.

4. SUMMARY AND CONCLUSIONS

The simple M/G/1 model of independent channels is not consistent with observed mean waiting times. Possible causes of failure of this simple model were examined. The complex behavior of tows in selecting the small chamber when the large chamber queue is long was identified as the most significant aspect of lock operations not captured by this model.

The M/G/1 finite queue size model was developed as an approximation to this queue switching behavior. The time in system predicted by this model was quite close to that observed at the 600' chamber. We expect this model to give good results for the 600' chamber unless operating conditions change substantially.

A lower bounding model was developed for the two chamber system. This model implicitly assumes that barges will join a common queue and be processed at the next available chamber. It will underestimate barge waiting times since, in fact, large tows will wait for the 600' chamber even when the 360' chamber is idle. This model development required derivation of the mean wait for a M/G/1 queue with random size batch arrivals and also the appropriate service time moments for a single channel queue that approximates a system of parallel channels having different service times. We feel this model will provide useful lower bounds on average time in system even under operating conditions very different from the current mode.

Models II and III require essentially the same data that would be required for a simulation model, i.e., the moments of the tow size distribution, and the service time distribution conditional on tow size, and the arrival rate for each tow size. From such data, unconditional service time moments and other parameters required by the model can be caculated, and waiting times estimated. At least for preliminary analysis of policy options, these models represent a very inexpensive alternative to simulation models.

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APPENDIX

The two tables in this appendix show the nature of the time dependence of the mean arrival rate. The data includes all tows arriving at the lock for the 26 weeks beginning at midnight (hour 1) on Friday (day 1), March 1, 1974.

TABLE A1

NUMBER OF TOWS ARRIVING BY DAY AND NUMBER OF BARGES

NUMBER OF BARGES	DAY 1	2	3	4	5	6	7	SUM	SUM OF SQUARES
0	129	112	88	106	135	115	95	780	88,640
1	76	69	46	101	105	90	90	577	50,079
2	85	66	69	92	83	95	82	572	47,444
3	68	75	72	66	81	65	78	505	36,659
4	74	77	93	70	69	80	59	522	39,596
5	22	17	21	32	20	13	18	143	3,131
6	36	35	35	36	44	41	39	266	10,180
7	12	13	9	9	14	11	13	81	961
8	31	24	30	30	39	34	24	212	6,590
9	22	36	25	32	22	22	29	188	5,238
10	19	13	17	8	19	10	12	98	1,488
11	17	19	16	15	16	15	11	109	1,733
12	46	50	64	56	59	61	69	405	23,811
13	11	9	9	13	9	7	13	71	751
14	18	21	12	10	18	11	13	103	1,623
15	65	56	79	74	59	81	62	476	32,964
16	0	4	7	4	7	7	5	34	204
17	3	0	1	1	0	2	4	11	31
18	2	2	2	0	1	1	1	9	15
19 or more	1	1	0	2	4	0	1	9	23
SUM	737	699	695	757	804	761	718	5,171	
SUM OF SQUARES	50,461	43,479	42,887	52,673	60,048	55,417	46,096		

TABLE A2

NUMBER OF TOWS ARRIVING BY HOUR OF DAY AND NUMBER OF BARGES

NUMBER OF BARGES	HOUR 1	2	3	4	5	6	7	8	9	10	11	12	13
0	27	19	26	24	28	27	31	39	48	54	40	28	45
1	11	11	17	13	8	13	15	33	22	46	36	47	45
2	17	13	17	22	17	15	21	52	22	22	19	24	24
3	19	24	18	15	21	11	20	26	23	27	22	32	28
4	23	25	24	15	18	32	20	24	25	20	21	24	19
5	9	10	11	8	7	5	3	7	5	5	5	6	4
6	8	12	15	7	5	15	7	10	13	12	10	4	9
7	6	2	6	1	4	2	4	2	3	2	4	2	1
8	6	7	4	7	10	8	16	9	13	5	6	7	10
9	3	8	10	6	9	9	16	9	7	9	4	5	6
10	6	4	6	5	6	2	1	4	4	3	3	5	3
11	5	5	4	2	6	5	1	4	5	4	4	6	3
12	20	17	14	20	20	20	14	23	15	18	15	17	12
13	1	5	4	8	5	0	2	3	3	1	2	1	2
14	3	1	2	2	3	4	5	5	4	6	5	4	4
15	19	16	21	18	26	20	13	18	21	24	27	30	19
16	1	2	1	0	3	4	1	0	1	2	2	1	1
17	0	0	0	2	0	0	0	0	0	1	1	1	0
18	0	0	0	0	0	1	0	0	1	0	1	0	1
19 or more	0	0	0	0	1	0	0	0	1	1	0	0	1
SUM	184	181	200	175	197	193	190	268	236	262	227	244	237
SUM OF SQUARES	3,088	2,829	3,362	2,703	3,365	3,529	3,410	7,800	5,582	7,892	5,389	6,568	6,551

TABLE A2 (Continued)

NUMBER OF BARGES	14	15	16	17	18	HOUR 19	20	21	22	23	24	SUM	SUM OF SQUARES
0	41	35	44	30	24	42	24	30	28	25	21	780	27,374
1	29	28	31	25	26	24	29	29	16	5	18	577	17,127
2	41	34	30	24	22	23	24	25	23	25	16	572	15,308
3	21	9	26	15	15	24	30	16	20	20	23	505	11,383
4	23	21	17	18	22	23	26	20	24	17	21	522	11,660
5	3	2	4	6	8	5	9	10	4	2	5	143	1,005
6	8	8	14	12	13	13	16	12	20	11	12	266	3,262
7	5	8	3	2	1	5	1	4	5	5	3	81	355
8	9	11	11	13	8	9	7	10	8	8	10	212	2,044
9	11	12	10	8	3	4	6	11	7	7	8	188	1,688
10	6	7	2	5	5	2	3	4	2	4	6	98	462
11	5	4	2	4	4	6	4	7	6	8	5	109	553
12	16	9	15	23	8	18	17	15	17	23	19	405	7,189
13	2	4	4	3	6	1	4	3	3	2	2	71	287
14	6	2	2	7	5	4	6	5	6	7	5	103	507
15	16	17	12	19	15	19	27	21	22	13	23	476	9,946
16	1	2	1	1	1	1	1	2	3	2	0	34	70
17	1	0	1	0	0	1	1	0	2	0	0	11	15
18	1	0	3	0	0	0	0	0	0	0	1	9	15
19 or more	0	1	1	0	0	1	0	0	1	0	1	9	9
SUM	245	214	233	215	186	225	235	224	217	184	199	5,171	
SUM OF SQUARES	6,089	4,544	5,613	4,057	3,144	5,035	5,089	4,272	3,971	3,062	3,315		