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## TRANSIENT TEMPERATURES PRODUCED IN SOLID CYLINDERS BY A NUCLEAR THERMAL PULSE

Ennis F. Quigley

September 1976

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## I. INTRODUCTION

The atmospheric detonation of a nuclear weapon produces a high temperature air plasma which is commonly referred to as a "fireball". This fireball begins to emit thermal radiation, mostly in the visible and infrared regions of the spectrum, into the surrounding atmosphere immediately after it is formed. Although the fireball has a finite diameter, it is considered to be a point source for calculational purposes, and the thermal radiation environment produced is expressed in terms of a time dependent thermal irradiance. The general characteristics of this irradiance at any point outside the fireball are shown by the curve in Figure 1. This curve is generally approximated by

$$H(t) = H_0 \left[ \frac{2.2 \left(\frac{t}{t_0}\right)^2}{1 + 1.69 \left(\frac{t}{t_0}\right)^{3.6}} + \frac{0.206 e^{-3.6 \left(\frac{t}{t_0} - 1.18\right)^2}}{1 + \left(\frac{t}{1.6t_0}\right)^{10}} \right] \quad (1)$$

where  $t_0$  is a function of the weapon yield and  $H_0$  is a function of weapon yield, distance from point of weapon detonation, and transmittance of the atmosphere. Table 1 contains values of  $t_0$ ,  $H_0$  and  $\int_0^\infty H(t) dt$  for several weapon yields and distances. In theory the thermal environment is of infinite duration.

TABLE I. Nuclear Pulse Parameters

Yield (Kt)	Distance (km)	$t_0$ (s)	$H_0$ (MWm <sup>-2</sup> )	$\int_0^\infty H(t) dt$ (MJm <sup>-2</sup> )
1	0.32	0.04	11.32	1.25
10	0.70	0.11	11.26	3.26
100	1.51	0.31	8.96	7.51
1000	3.26	0.87	4.65	14.08

However, for practical applications the time duration of the environment is taken to be  $10t_0$  since  $\int_0^{10t_0} H(t) dt = 0.85 \int_0^\infty H(t) dt$  and  $H(t) < 0.03 H_0$

for  $t > 10t_0$ .

The effects of the thermal environment on exposed materials are due to the absorption of all or part of the radiant energy incident on the exposed surfaces of the materials. The absorption of this radiant energy by the lateral surface of a cylinder will result in a transient temperature rise, a knowledge of which is essential for predicting the response of the cylinder for those effects which are a function of temperature or temperature change. Because of the transient heating, a numerical

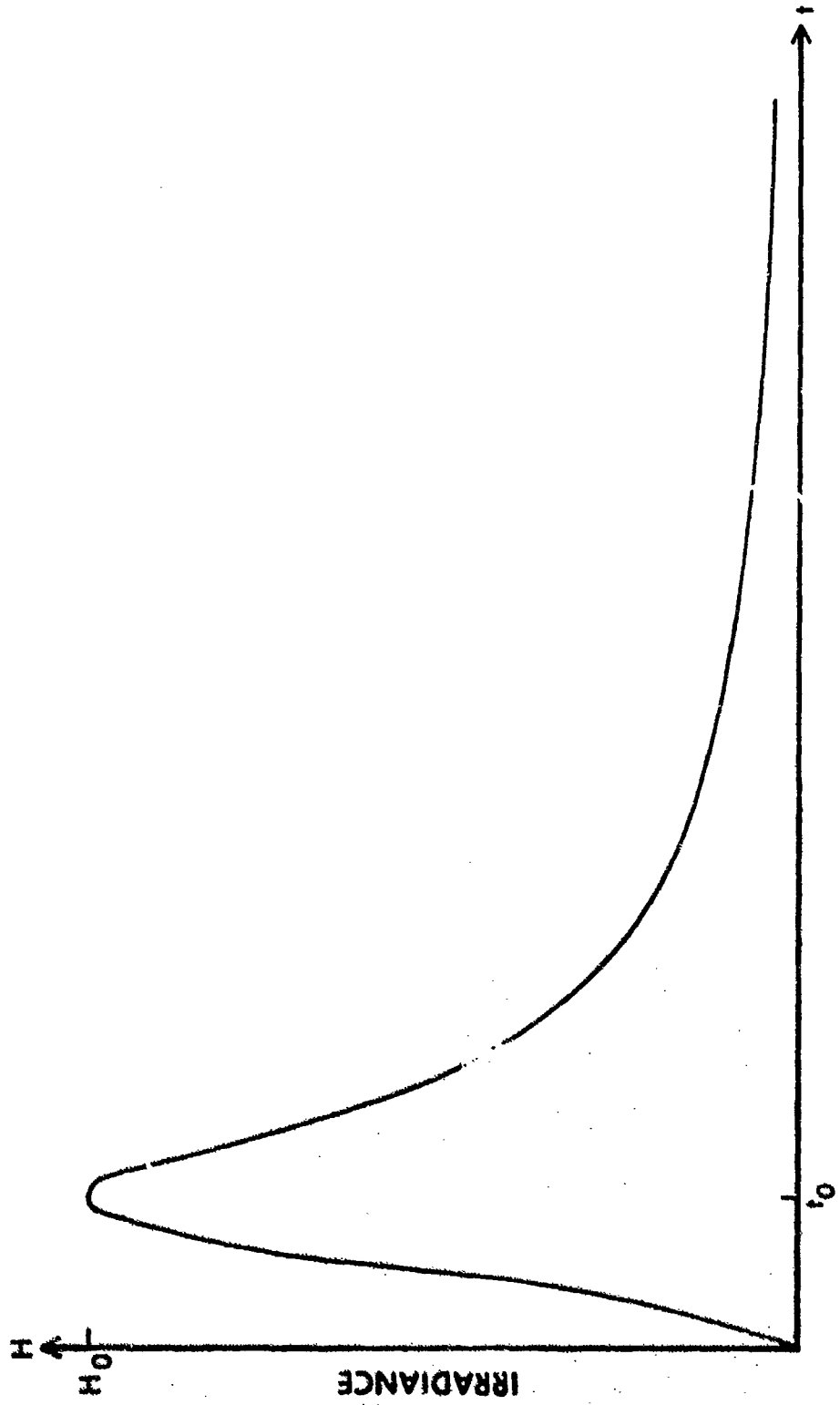


Figure 1. Nuclear Thermal Pulse

calculation of the temperature using finite difference or finite element methods is generally employed. Although these methods are of great value for solving a specific problem, they are not convenient as analytical methods for nondimensional representation or parametric analysis of the temperature field in the cylinder.

This report describes a deviation of the transient temperature field in an isotropic, homogeneous, finite length, solid cylinder of radius  $r_0$  whose lateral surface is subjected to heating by a nuclear thermal radiation environment. Ojalvo's modified separation-of-variables method<sup>1</sup> is used to solve the transient heat conduction equation under the following assumptions:

1. the thermal properties of the cylinder are independent of temperature,
2. the thermal radiation is absorbed at the surface,
3. convection and radiation heat losses by the cylinder can be neglected,
4. the transient temperature field in the cylinder is independent of the axial coordinate, and
5. the initial temperature field in the cylinder is uniform.

## II. TEMPERATURE EQUATIONS

The thermal irradiance at the lateral surface of the cylinder is expressed by

$$H(\theta, t) = \begin{cases} H_0 f(t) \cos \theta & 0 \leq \theta \leq \frac{\pi}{2} \\ 0 & \frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2} \\ H_0 f(t) \cos \theta & \frac{3\pi}{2} \leq \theta \leq 2\pi \end{cases} \quad (2)$$

where  $f(t)$  is the time dependent portion of (1). The transient temperature field in the cylinder is governed by the following equations

$$\nabla^2 T(r, \theta, t) = \frac{c\rho}{k} \frac{\partial T(r, \theta, t)}{\partial t} \quad (3)$$

where

$$\nabla^2 = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \quad (4)$$

<sup>1</sup> J.U. Ojalvo, "Conduction with Time-Dependent Heat Sources and Boundary Conditions," *International Journal of Heat and Mass Transfer*, Vol 5, 1962, pp. 1105-1109.

$$\frac{\partial T}{\partial r} = \begin{cases} \frac{H_0 f \cos \theta}{\kappa} & 0 \leq \theta \leq \frac{\pi}{2} \\ 0 & \frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2} \\ \frac{H_0 f \cos \theta}{\kappa} & \frac{3\pi}{2} \leq \theta \leq 2\pi \end{cases} \quad (5)$$

at  $r = r_0$ ,

$$\frac{1}{r} \frac{\partial T}{\partial \theta} = 0, \quad (6)$$

at  $\theta = 0, \pi$  (because of symmetry), and

$$T(r, \theta, 0) = 0^*. \quad (7)$$

The modified separation-of-variables method assumes that the solution of (3) can be expressed as

$$T = \sum_m \sum_n \psi_{mn}(t) \Phi_{mn}(r, \theta) + T_0(r, \theta) f(t). \quad (8)$$

By substituting (8) into (3) one has

$$\sum \sum \psi_{mn} \nabla^2 \Phi_{mn} + \nabla^2 T_0 f = \frac{c\beta}{\kappa} \left[ \sum \sum \psi_{mn} \Phi_{mn} + T_0 f \right]. \quad (9)$$

The method further assumes that

$$T_0 = \sum a_{mn} \Phi_{mn} \quad (10)$$

and that either

$$\nabla^2 T_0 = 0 \quad (11)$$

or

$$\nabla^2 T_0 = \sum b_{mn} \Phi_{mn} \quad (12)$$

---

\* No generality is lost by assuming  $T = 0$  since the temperature of the cylinder can be obtained by adding the initial constant temperature to  $T$  and only the temperature difference is required for the stress and displacement fields.



The substitution of (10) and (12)\* into (9) yields

$$\sum \sum \psi_{mn} \nabla^2 \Phi_{mn} = \frac{c\rho}{\kappa} \sum \sum \left[ \dot{\psi}_{mn} f + a_{mn} \dot{f} - \frac{\kappa}{c\rho} b_{mn} f \right] \Phi_{mn} \quad (13)$$

If (13) is equated termwise and divided by  $\psi_{mn} \Phi_{mn}$  and if  $-\lambda_{mn}^2$ 's are chosen for the separation constants, one obtains the following differential equations:

$$\nabla^2 \Phi_{mn} + \lambda_{mn}^2 \Phi_{mn} = 0 \quad (14)$$

and

$$\dot{\psi}_{mn} + \lambda_{mn}^2 \frac{\kappa}{c\rho} \psi_{mn} = -a_{mn} \dot{f} + \frac{\kappa}{c\rho} b_{mn} f \quad (15)$$

The solution of (14) and (15) along with  $T_0$  when substituted into (8) provides the transient temperature field in the cylinder.

The boundary and initial conditions for  $\Phi_{mn}$ ,  $T_0$ , and  $\psi_{mn}$  are obtained by substituting (8) into (5), (6), and (7). This substitution yields

$$\sum \sum \psi_{mn} \frac{\partial \Phi_{mn}}{\partial r} = \begin{cases} \left[ \frac{H_0}{\kappa_0} \cos \theta - \frac{\theta T_0}{r} \right] f & 0 \leq \theta \leq \frac{\pi}{2} \\ -\frac{\partial T_0}{\partial r} f & \frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2} \\ \left[ \frac{H_0}{\kappa_0} \cos \theta - \frac{2T_0}{r} \right] f & \frac{3\pi}{2} \leq \theta \leq 2\pi \end{cases} \quad (16)$$

at  $r = r_0$ ,

$$\sum \sum \psi_{mn} \frac{\partial \Phi_{mn}}{r \partial \theta} = -\frac{\partial T_0}{r \partial \theta} f \quad (17)$$

at  $\theta = 0, \pi$ , and

$$\sum \sum \psi_{mn} \Phi_{mn} = -T_0 f \quad (18)$$

\* The use of (12) is more general than the use of (11) since (11) is a special case of (12) in which all the  $b_{mn}$ 's are zero.

at  $t = 0$ . From (16), (17), (18) and Figure 1 the following boundary and initial conditions for  $\Phi_{mn}$ ,  $T_0$ , and  $\Psi_{mn}$  are deduced:

$$\frac{\partial \Phi_{mn}}{\partial r} = 0^* \quad (19)$$

at  $r = r_0$ ,

$$\frac{\partial \Phi_{mn}}{r \partial \theta} = 0^* \quad (20)$$

at  $\theta = 0, \theta, \pi$

$$\frac{\partial T_0}{\partial r} \left\{ \begin{array}{ll} \frac{H_0}{\kappa} \cos \theta & 0 \leq \theta \leq \frac{\pi}{2} \\ 0 & \frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2} \\ \frac{H_0}{\kappa} \cos \theta & \frac{3\pi}{2} \leq \theta \leq 2\pi \end{array} \right. \quad (21)$$

at  $r = r_0$ ,

$$\frac{\partial T_0}{r \partial \theta} = 0 \quad (22)$$

at  $\theta = 0, \pi$ , and

$$\Psi_{mn} = 0 \quad (23)$$

at  $t = 0$ .

Equation (14) is a two dimensional Helmholtz's differential equation, the general solutions of which are of the form

$$\Phi_{0n} = J_0(\lambda_{0n} r) (A_{0n} + B_{0n} \theta) \quad (24a)$$

for  $m = 0$  and

$$\Phi_{mn} = (A_{mn} J_m(\lambda_{mn} r) + B_{mn} Y_m(\lambda_{mn} r)) (C_m \cos m\theta + D_m \sin m\theta) \quad (24b)$$

for  $m > 0$ .

In order for these solutions to be well behaved at  $r = 0$ , the  $B_{mn}$ 's must be equal to zero. Also from (20) it is readily seen that the  $B_{0n}$ 's and the  $D_m$ 's must also equal zero and that  $m = 0, 1, 2, 3, \dots$ . Equation (24) can now be rewritten as

$$\Phi_{mn} = A_{mn} J_m(\lambda_{mn} r) \cos m\theta \quad (25)$$

\* This equation insures a homogeneous boundary condition for (14).

The substitution of (25) into (19) gives

$$J_m'(\lambda_{mn} r_0) = 0 \quad (26a)$$

or

$$(\lambda_{mn} r_0) \frac{J_{m+1}(\lambda_{mn} r_0)}{J_m(\lambda_{mn} r_0)} = m \quad (26b)$$

From (26a) one sees that the  $\lambda_{mn} r_0$ 's are the n-th positive roots of (26a).

Equation (11) cannot be used in determining  $T_0$ . The solution of this two dimensional Laplace's differential equation which is well behaved at  $r = 0$  and satisfies (22) is

$$T_0 = C_0 + \sum_{m=1}^{\infty} C_m r^m \cos n \theta \quad (27)$$

the substitution of (27) into (21) yields

$$\sum_{m=1}^{\infty} m C_m r_0^{m-1} \cos m \theta = \begin{cases} \frac{H}{\kappa} \cos \theta & 0 \leq \theta \leq \frac{\pi}{2} \\ 0 & \frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2} \\ \frac{H}{\kappa} \cos \theta & \frac{3\pi}{2} \leq \theta \leq 2\pi \end{cases} \quad (28)$$

By expressing the right side of (28) in terms of the following Fourier cosine series:

$$\frac{H_0}{\kappa} \left[ \frac{1}{\pi} + \frac{\cos \theta}{2} + \frac{2}{\pi} \sum_{n=2,4,6,\dots}^{\infty} \frac{(-1)^{\frac{n}{2}+1}}{(n^2-1)} \cos n \theta \right], \quad (29)$$

one can rewrite (28) as

$$\sum_{m=1}^{\infty} m C_m r_0^{m-1} \cos m \theta = \frac{H_0}{\kappa} \left[ \frac{1}{\pi} + \frac{\cos \theta}{2} + \frac{2}{\pi} \sum_{\substack{n=2 \\ \text{even}}}^{\infty} \frac{(-1)^{\frac{n}{2}+1}}{(n^2-1)} \cos n \theta \right] \quad (30)$$

It is readily seen from (30) that (27) cannot satisfy the boundary conditions at  $r = r_0$  because of the first term of (27). This term represents the uniform heating of the entire lateral surface of the cylinder, and the form of  $T_0$  which is obtained by solving the uniform

heating case is

$$T_o = \frac{H_o}{2\pi\kappa r_o} r^2 \quad (31)$$

Consequently, it is assumed that

$$T_o = \frac{H}{2\pi\kappa r_o} r^2 + C_o + \sum_{m=1}^{\infty} C_m r^m \cos m \theta \quad (32)$$

where the values of the  $C_m$ 's, except for  $C_o$ , are obtained by equating (30) termwise. From (30) one sees that

$$C_1 = \frac{H_o}{2\kappa} \quad (33)$$

and

$$C_m = -\frac{2H_o \cos\left(\frac{m\pi}{2}\right)}{\pi m(m^2-1)\kappa r_o^{m-1}} \quad (34)$$

for  $m > 1$ .

In order to solve

$$\nabla_{mn}^2 \psi_{mn} + \frac{\kappa}{c\rho} \lambda_{mn}^2 \psi_{mn} = -a_{mn} f + \frac{\kappa}{c\rho} b_{mn} f \quad (15)$$

the coefficients,  $a_{mn}$  and  $b_{mn}$ , must be evaluated. The  $a_{mn}$ 's can be evaluated from (10) and (32) and the  $b_{mn}$ 's from (12) and (32). From (10) and (32)

$$\frac{H_o}{2\pi\kappa r_o} r^2 + C_o + \sum C_m r^m \cos m \theta = a_{oo} + \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} a_{mn} J_m(\lambda_{mn} r) \cos m \theta \quad (35)$$

Equating (35) termwise one obtains

$$\frac{H_o}{2\pi\kappa r_o} r^2 + C_o = a_{oo} + \sum_{n=1}^{\infty} a_{on} J_o(\lambda_{on} r) \quad (36)$$

and

$$C_m r^m = \sum_{n=1}^{\infty} a_{mn} J_m(\lambda_{mn} r). \quad (37)$$

Using the orthogonality properties of the Bessel functions, the  $a_{mn}$ 's can be evaluated from

$$a_{on} = \frac{\frac{H_0}{2\pi\kappa r_0} \int_0^{r_0} r^3 J_0(\lambda_{on} r) dr + C_0 \int_0^{r_0} r J_0(\lambda_{on} r) dr}{\int_0^{r_0} r J_0^2(\lambda_{on} r) dr} \quad (38)$$

and

$$a_{mn} = C_m \frac{\int_0^{r_0} r^{m+1} J_m(\lambda_{mn} r) dr}{\int_0^{r_0} r J_m^2(\lambda_{mn} r) dr} \quad (39)$$

The integration of (38) yields

$$a_{00} = \frac{4H_0 r_0}{\pi\kappa} + C_0 \quad (40a)$$

for  $\lambda_{00} = 0$  and

$$a_{on} = \frac{2H_0 r_0}{\pi\kappa (\lambda_{on} r_0)^2 J_0(\lambda_{on} r_0)} \quad (40b)$$

for  $n=1$ . For the remaining  $a_{mn}$ 's the integration of (39) yields

$$a_{mn} = \frac{2C_m \lambda_{mn} r_0^{m+1} J_{m+1}(\lambda_{mn} r_0)}{[(\lambda_{mn} r_0)^2 - n^2] J_m^2(\lambda_{mn} r_0)} \quad (41)$$

By substituting (26b), (33) and (34) into (41) one obtains

$$a_{m1} = \frac{H_0 (\lambda_{m1} r_0)^2}{\kappa [(\lambda_{m1} r_0)^2 - 1] J_1(\lambda_{m1} r_0)} \quad (42a)$$

for  $m \geq 1$  and

$$a_{mn} = - \frac{2H_0 r_0^2 \cos(m\pi/2)}{\pi\kappa [m^2 - 1] [(\lambda_{mn} r_0)^2 - m^2] J_m(\lambda_{mn} r_0)} \quad (42b)$$

for  $m \geq 1$  and  $n > 1$ . From (12) and (32)

$$\frac{2H_0}{\pi\kappa r_0} = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} b_{mn} J_m(\lambda_{mn} r) \cos m\theta \quad (43)$$

Using the orthogonality properties of the Bessel functions and the cosine functions, the  $b_{mn}$ 's can be evaluated from

$$b_{mn} = \frac{2H_0}{\pi \kappa r_0} \frac{\int_0^{r_0} r J_m(\lambda_{mn} r) dr \int_0^{2\pi} \cos m\theta d\theta}{\int_0^{r_0} r J_m^2(\lambda_{mn} r_0) dr \int_0^{2\pi} \cos^2 m\theta d\theta} \quad (44)$$

The integration of (44) yields

$$b_{00} = \frac{2H_0}{\pi \kappa r_0} \quad (45a)$$

and

$$b_{mn} = 0 \quad (45b)$$

for  $m \geq 0$  and  $n \geq 1$ .

For  $\lambda_{00} = 0$  equation (15) reduces to

$$\dot{\psi}_{00} = -a_{00} \dot{f} + \frac{\kappa}{c\rho} b_{00} f, \quad (46)$$

the integration of which is

$$\psi_{01} = -a_{00} f(t) + \frac{\kappa}{c\rho} b_{00} \int_0^t f(t) dt + B_{00} \quad (47)$$

From the initial condition equation (23), one sees that  $B_{00}$  must equal zero. For the remaining  $\lambda_{mn}$ 's, the general solution of (15) can be obtained by using the integrating factor  $e^{\lambda_{mn} \frac{\kappa}{c\rho} t}$ .

The use of this factor results in

$$\dot{\psi}_{mn} = -a_{mn} e^{-\lambda_{mn} \frac{\kappa}{c\rho} t} \int_0^t \lambda_{mn}^2 \frac{\kappa}{c\rho} e^{\lambda_{mn} \frac{\kappa}{c\rho} t} f(t) dt + B_{mn} e^{-\lambda_{mn} \frac{\kappa}{c\rho} t}. \quad (48)$$

One also sees from (23) that the  $B_{mn}$ 's must equal zero. The integration of the right hand side of (48) by parts yields

$$\psi_{mn} = -a_{mn} f(t) + a_{mn} \lambda_{mn}^2 \frac{\kappa}{c\rho} e^{-\lambda_{mn} \frac{\kappa}{c\rho} t} \int_0^t e^{\lambda_{mn} \frac{\kappa}{c\rho} t} f(t) dt \quad (49)$$

The substitution of (25), (32), (40), (42), (45), (47) and (49) into gives

$$\begin{aligned}
 T = \frac{2H_0}{\pi \rho c r_0} & \left\{ \int_0^t f(t) dt + \sum_{n=1}^{\infty} \frac{e^{-\lambda_{0n}^2 \frac{\kappa}{c\rho} t}}{J_0(\lambda_{0n} r_0)} \left[ \int_0^t e^{\lambda_{0n}^2 \frac{\kappa}{c\rho} t} \right. \right. \\
 & \left. \left. x f(t) dt \right] J_0(\lambda_{0n} r) + \frac{\pi}{2} \sum_{n=1}^{\infty} \frac{[\lambda_{1n} r_0]^2 e^{-\lambda_{1n}^2 \frac{\kappa}{c\rho} t}}{[(\lambda_{1n} r_0)^2 - 1] J_1(\lambda_{1n} r_0)} \right. \\
 & \left. x \left[ \int_0^t e^{\lambda_{1n}^2 \frac{\kappa}{c\rho} t} f(t) dt \right] J_1(\lambda_{1n} r) \cos \theta - 2 \sum_{m=2}^{\infty} \sum_{n=1}^{\infty} \frac{[\lambda_{mn} r_0]^2}{[m^2 - 1]} \right. \\
 & \left. x \frac{\cos(m\pi/2) e^{-\lambda_{mn}^2 \frac{\kappa}{c\rho} t}}{[(\lambda_{mn} r_0)^2 - m^2] J_m(\lambda_{mn} r_0)} \left[ \int_0^t e^{\lambda_{mn}^2 \frac{\kappa}{c\rho} t} f(t) dt \right] \right. \\
 & \left. x J_m(\lambda_{mn} r) \cos m \theta \right\} \quad (50)
 \end{aligned}$$

where  $f(t)$  is the time dependent portion of (1).

### III. NUMERICAL RESULTS

For simplicity and generality, the derived temperature field equation will be numerically evaluated in terms of the following dimensionless quantities

$$\begin{aligned}
 r^* &= \frac{r}{r_0} & t^* &= \frac{t}{t_0} \\
 \beta^* &= \frac{\kappa t_0}{c\rho r_0^2} & \gamma^* &= \frac{T}{\frac{2H_0 t_0}{\pi \rho c r_0}}
 \end{aligned} \quad (51)$$

In addition, (1) is approximated by the following Fourier series:

$$f(t) = H_0 \left[ \frac{C_0}{2} + \sum_{l=1}^{\rho} C_l \cos \frac{\pi l t}{5t_0} + d_l \sin \frac{\pi l t}{5t_0} \right] \quad (52)$$

If (1) were used in evaluating (50), the integrals would have to be computed numerically. Since the use of (52) allowed for the direct integration of these integrals, such a representation is both logical and convenient, regardless of the number of terms required for accuracy. All of the calculations were carried out by using a UNIVAC 1108 digital computer and the number of terms used in evaluating the double series solution were such as to insure at least three digit convergence.

Figures 2, 3, and 4 are plots of the radial temperature distribution for various values of  $\beta^*$ ,  $t^*$  and  $\theta$ . A comparison of the temperatures calculated using (50) and those calculated using CINDA-3G<sup>2</sup>, a finite differencing heat transfer computer program, shows a five percent or less difference in the calculated temperatures. Approximately ninety seconds of machine time is required to calculate the radial temperatures using (50) for paired values of  $\beta^*$  and  $t^*$  and two values of  $\theta$ .

#### IV. CONCLUSION

The analytical expression of the transient temperature field in an isotropic, homogeneous, finite length, solid cylinder whose lateral surface is subjected to the heating by a nuclear thermal radiation environment has been derived. This expression provides a convenient means for the nondimensional representation and parametric analysis of the temperature field in the cylinder. In addition, this temperature equation can be used in the analysis of those effects dependent on temperature or temperature change, e.g., thermal stress in a cylinder.

<sup>2</sup> J.D. Gaski, Chrysler Improved Numerical Differencing Analyzer for 3rd Generation Computers, TN-AP-87-287, October 20, 1967, Chrysler Corporation Space Division, New Orleans, Louisiana.



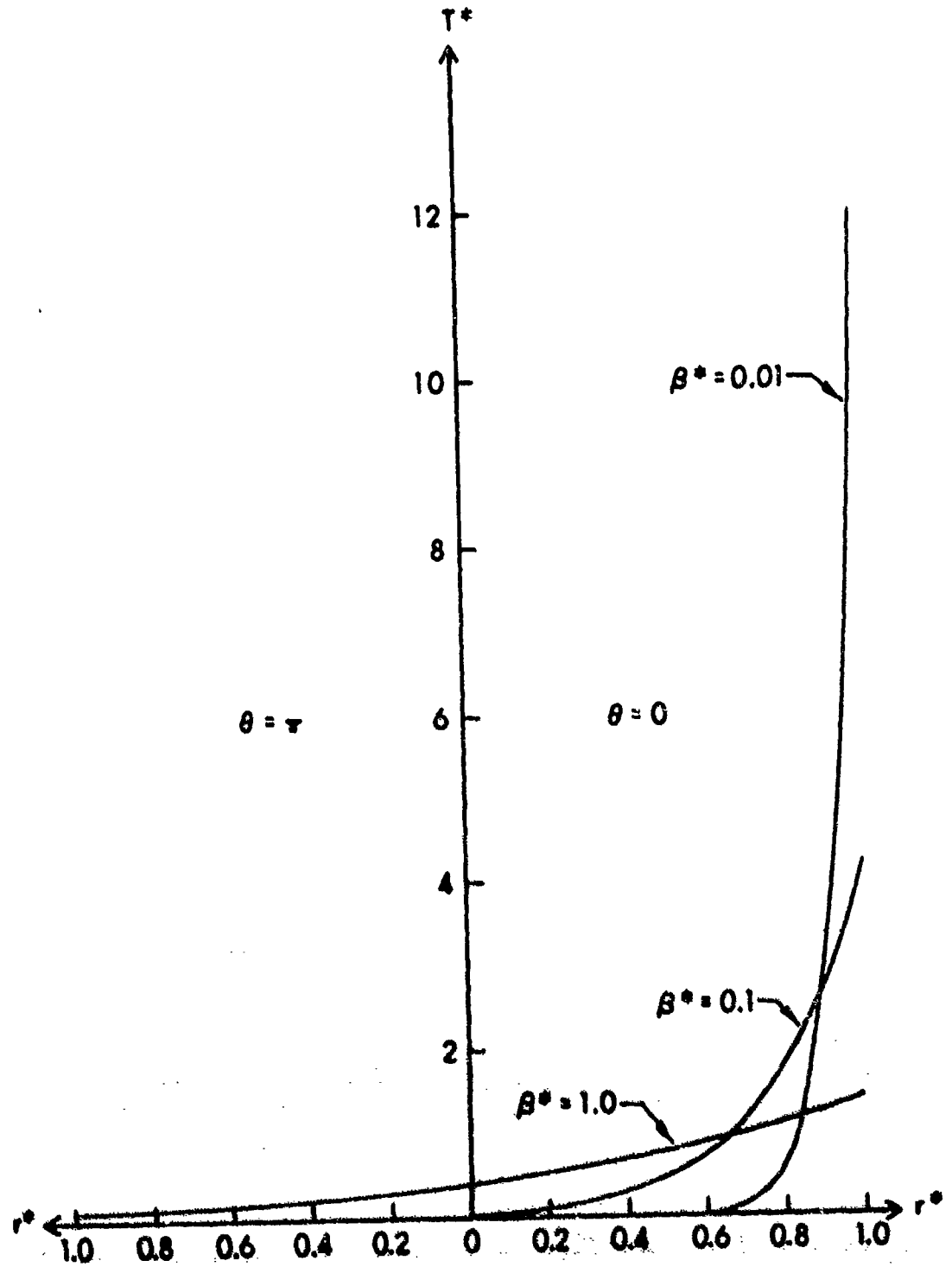


Figure 2. Radial Temperature Distribution on  $\theta = 0$  and  $\pi$  for  $t^* = 1.0$  and  $\beta^* = 0.01, 0.1, \text{ and } 1.0$ .

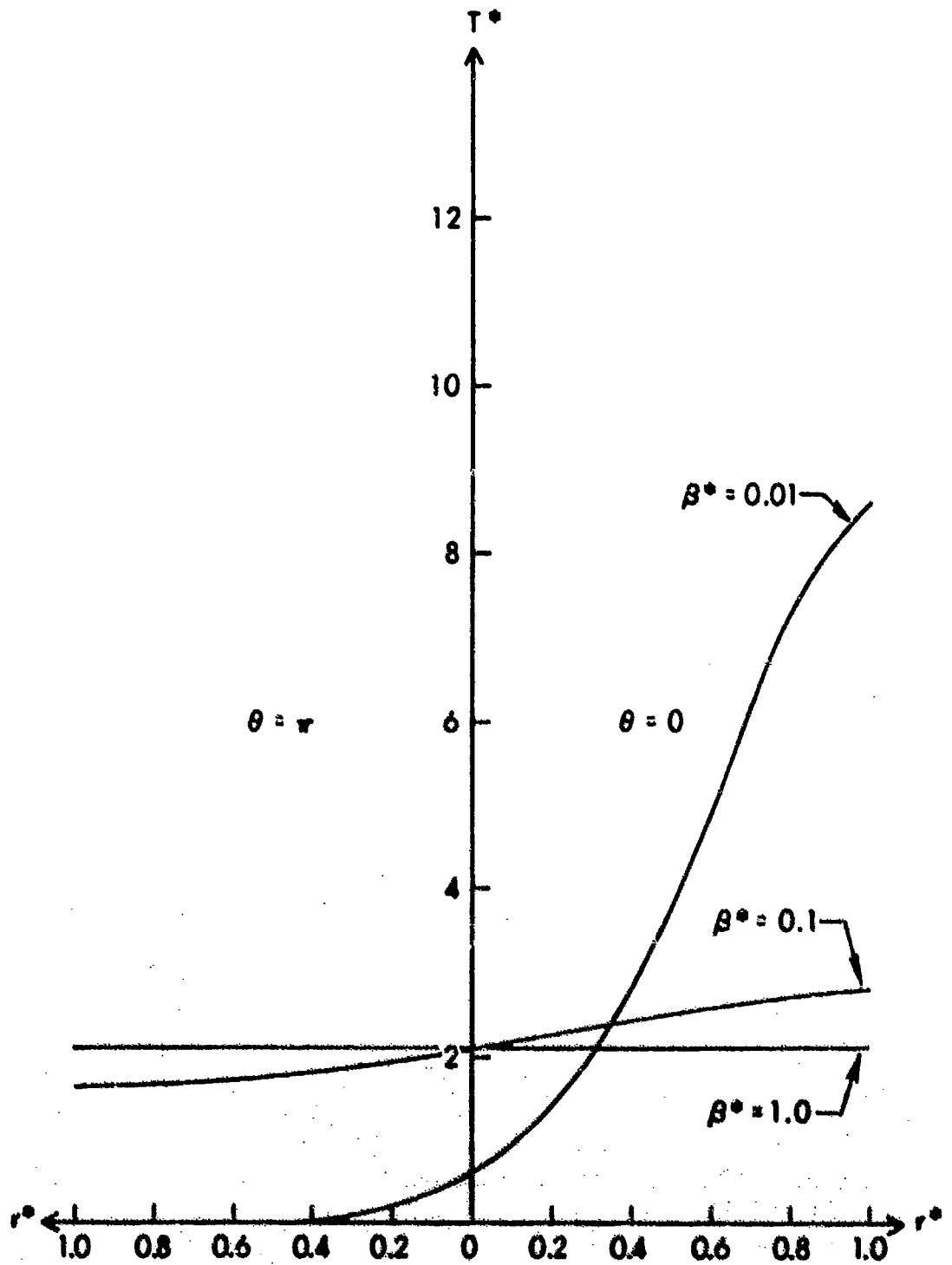


Figure 3. Radial Temperature Distribution on  $\theta = 0$  and  $v$  for  $\tau^* = 10.0$  and  $\beta^* = 0.01, 0.1, \text{ and } 1.0$ .

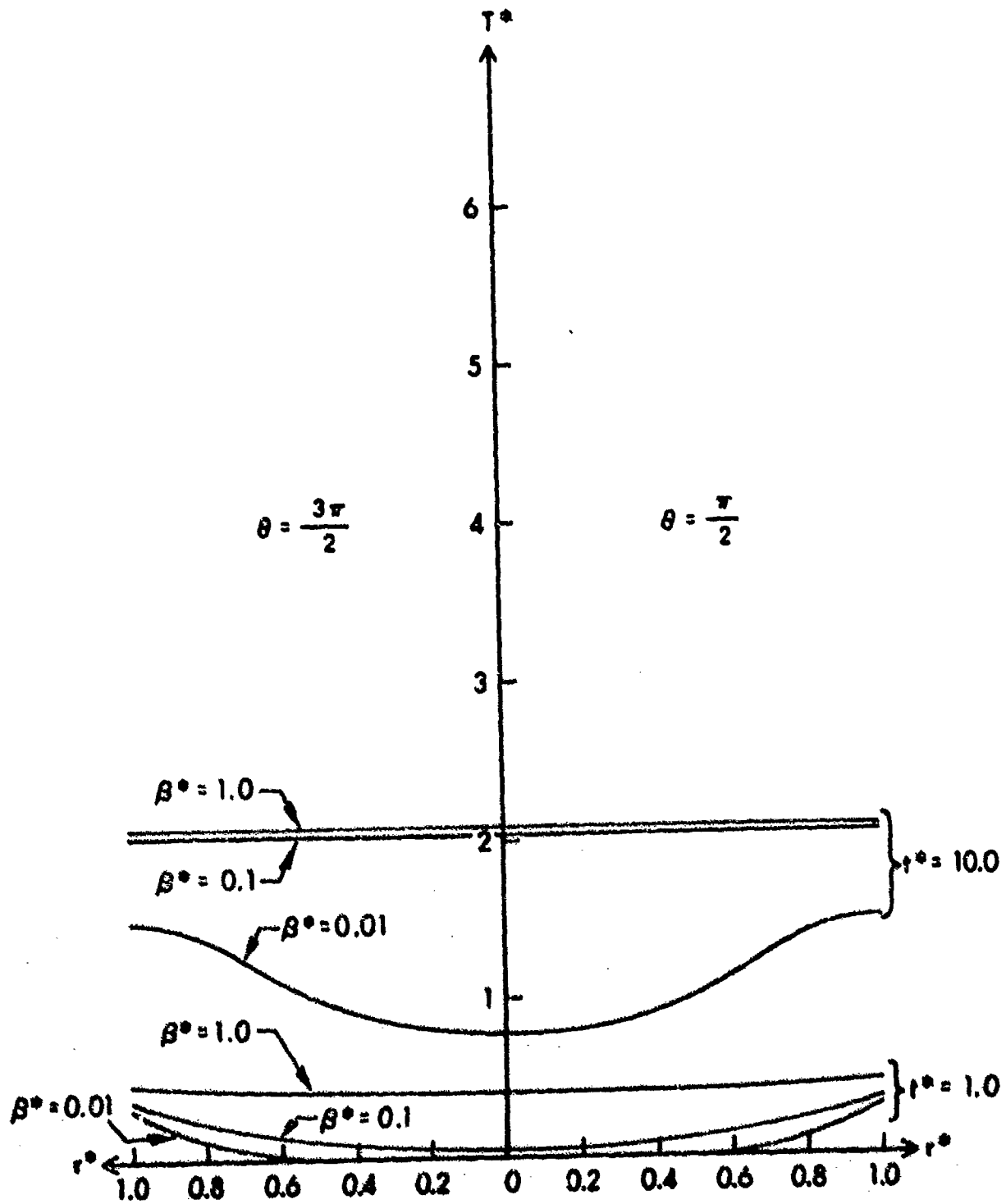


Figure 4. Radial Temperature Distribution on  $\theta = \frac{\pi}{2}$  and  $\frac{3\pi}{2}$  for  $t^* = 1.0$  and  $10.0$  and  $\beta^* = 0.01, 0.1, \text{ and } 1.0$

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## GLOSSARY OF TERMS

$a_{mn}$	= Coefficients defined by (38) and (39)
$b_{mn}$	= Coefficients defined by (44)
$c$	= Specific heat
$f, f(t)$	= Time dependent portion of (1)
$h$	= Half length of cylinder
$r, \theta, z$	= Cylindrical coordinates
$r_0$	= Radius of cylinder
$r^*, t^*, T^*$	= Dimensionless variables defined by (51)
$t$	= Time
$t_0$	= Rise time of nuclear thermal pulse
$A_m, A_{mn}, B_m$	= Unknown coefficients in (24)
$B_{mn}, C_m, D_m$	= Coefficient defined by
$C_l$	= Coefficient defined by
$C_m$	= Coefficient defined by
$H, H(t)$	= Time dependent irradiance of nuclear thermal value
$H_0$	= Maximum irradiance of nuclear thermal pulse
$J_m(x), Y_m(x)$	= Ordinary Bessel function of argument $x$
$J_m^1(x)$	= $\frac{dJ_m(x)}{dx}$
$T, T(r, \theta, t)$	= Temperature in cylinder
$T_0, T_0(r, \theta)$	= Part of solution to
$\beta$	= Dimensionless variable defined by
$\kappa$	= Thermal conductivity
$\lambda_{mn}$	= Separation constant of (13)
$\rho$	= Density
$\phi_{mn}$	= Part of solution to (3)
$\psi_{mn}$	= Part of solution to (3)

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