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Henry C. Pusey, *Director*

Rudolph H. Volin

J. Gordan Showalter

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PREFACE

The increasing importance of computer programs to the shock and vibration community underscores the need to gather together program capability and dissemination information. This book contains critical reviews and summaries of available shock and vibration computer programs. It is hoped that this book will be a valuable tool to readers for use in selecting the best software to solve their problems.

The book is divided into two sections. In the first section, computer programs suitable for particular classes of problems are considered. Both special purpose and general purpose programs are included. In all cases, considerable care has been given to providing details of availability of the programs.

In the second section of the book, the capabilities of readily available general purpose programs are treated. The methods used for solving dynamics problems are outlined and the techniques employed for incorporating various material properties are scrutinized. Finally, we attempt to discern which programs are the most appropriate for particular problems.

Every reasonable effort has been made by the editors to assure completeness of the reviews and summaries. General requests for information on available programs were distributed through the mail, at meetings, and were placed in many technical journals. In addition, specific requests were sent to names supplied by the authors. The information received was then passed on to the appropriate author. Final decisions as to the contents of the individual chapters rested with the authors responsible for that area.

The cooperation of the authors in the preparation of comprehensive chapters is greatly appreciated. Their efforts have been massive, requiring the collection of information on available programs, the preparation of questionnaires for program developers and users, the critical review of the programs' descriptive material and documentation, and the verification of sources of availability. In several instances the authors have even applied most of the computer programs in their area to benchmark example problems.

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October 1975

WALTER AND BARBARA PILKEY
Charlottesville, Virginia

CONTRIBUTORS

Robert E. Ball
Department of Aeronautics
Naval Postgraduate School
Monterey, California 93940

George Belonogoff
Data Processing
Bechtel International Corporation
Box 3965
San Francisco, California 94119

Ted Belytschko
Department of Materials Engineering
University of Illinois at Chicago Circle
Chicago, Illinois 60680

James E. Bernard
Highway Safety Research Institute
University of Michigan
Huron Parkway at Baxter Road
Ann Arbor, Michigan 48105

James C. Bowers
Electrical Engineering Department
University of South Florida
Tampa, Florida 33620

Frank M. Bugg
Vibrations Analysis Branch, ED 23
Marshall Space Flight Center
Huntsville, Alabama 35812

Jimmy M. Cawthorn
Acoustics and Noise Reduction Divison
NASA Langley Research Center
Hampton, Virginia 23665

Carl J. Costantino
Department of Civil Engineering
City College of New York
140th and Convent Streets
New York, New York 10010

Roy R. Craig, Jr.
Texas Institute for Computational Mechanics
University of Texas at Austin
Austin, Texas 78705

Suresh K. Desai
Commonwealth Associates, Inc.
209 E. Washington Avenue
Jackson, Michigan 42901

Vera F. Dunder
Data Processing
Bechtel International Corporation
P. O. Box 3965
San Francisco, California 94119

Loren D. Enochsen
Advanced Development Group
Time/Data Corporation
1050 E. Meadow Circle
Palo Alto, California 94303

Ronnie E. Gillian
Acoustics and Noise Reduction Division
NASA Langley Research Center
Hampton, Virginia 23665

Robert Greif
Department of Mechanical Engineering
Tufts University
Medford, Massachusetts 02155

John K. Haviland
Department of Engineering Science
and Systems
University of Virginia
Charlottesville, Virginia 22901

Walter Herrmann
Solid Dynamics Research Department
Sandia Laboratories
Albuquerque, New Mexico 87115

Garnett C. Horner
Research Laboratories for the Engineering
Sciences
University of Virginia
Charlottesville, Virginia 22901

Andrew Jay
Structures Division
Pratt and Whitney Aircraft
East Hartford, Connecticut 06108

C. P. Johnson
Texas Institute for Computational Mechanics
The University of Texas at Austin
Austin, Texas 78712

Anthony J. Kalinowski
Engineering Mechanics Staff
Navy Underwater Systems Center
New London, Connecticut 06320

Arturs Kalnins
Department of Mechanical Engineering and
Mechanics
Lehigh University
Bethlehem, Pennsylvania 18015

Manohar P. Kamat
Engineering Science and Systems
Virginia Polytechnic Institute and State University
Blacksburg, Virginia 24061

Dean Karnopp
Department of Mechanical Engineering
University of California at Davis
Davis, California 95616

Roger E. Kaufman
Department of Mechanical Engineering
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

Leonard Meirovitch
Department of Engineering Science and Mechanics
Virginia Polytechnic Institute and State University
Blacksburg, Virginia 24061

Charles A. Miller
Department of Civil Engineering
City College of New York
140th and Convent Streets
New York, New York 10010

A. J. Molnar
Mechanics Department
Westinghouse Research Laboratories
Beulah Road
Pittsburgh, Pennsylvania 15235

Nicholas F. Morris
Civil Engineering Department
Polytechnic Institute of New York
333 Jay Street
Brooklyn, New York 11201

Frederick C. Nelson
Department of Mechanical Engineering
Tufts University
Medford, Massachusetts 02155

Vernon H. Neubert
Department of Engineering Science and Mechanics
The Penn State University
University Park, Pennsylvania 16802

Walter D. Pilkey
Department of Engineering Science and Systems
University of Virginia
Charlottesville, Virginia 22901

D. Hurley Robbins
Biomathematics Department
Highway Safety Research Institute
The University of Michigan
Ann Arbor, Michigan 48105

Lynn Seaman
Poulter Laboratory
Stanford Research Institute
Menlo Park, California 94025

Ali Seireg
Department of Mechanical Engineering
University of Wisconsin
Madison, Wisconsin 53706

William Shapiro
Mechanical Engineering Laboratory
The Franklin Institute Research Laboratories
20 N. Parkway
Philadelphia, Pennsylvania 19103

Gary A. Shaw
Electrical Engineering Department
University of South Florida
Tampa, Florida 33620

Paul Smith
Reactor Division
Los Alamos Scientific Laboratory
Los Alamos, New Mexico 87545

James A. Stricklin
Aerospace Engineering Department
Texas A & M University
College Station, Texas 77843

C. Sundararajan
Department of Design Engineering
The Lummus Company Canada, Ltd.
Willowdale, Toronto, Canada

N. Thomas Tsai
Ship Performance Department, Code 1548
Naval Ship Research and Development Center
Bethesda, Maryland 20034

Phillip Underwood
Lockheed Palo Alto Research Laboratory
3251 Hanover Street
Palo Alto, California 94304

C. Visser
Mechanics Department
Westinghouse Research Laboratories
Beulah Road
Pittsburgh, Pennsylvania 15235

Thein Wah
Civil and Mechanical Engineering
Texas A & I University
Kingsville, Texas 78363

Lawrence I. Weingarten
Division 1544
Sandia Laboratories
Albuquerque, New Mexico 87115

Robert A. Wilke
Aerospace Engineering Department
Texas A & M University
College Station, Texas 77843

Yoshiaki Yamada
Institute of Industrial Science
University of Tokyo
7-22-1, Roppongi, Minato-ku
Tokyo 106, Japan

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I. Computer Programs

Multiple Energy Domain Systems

Dean Karnopp

Department of Mechanical Engineering

INTRODUCTION

Almost all shock and vibration computer programs deal exclusively with systems composed of mechanical elements representing compliance, inertial, and dissipative effects in structures. Yet many practical problems involve interactions among mechanical, hydraulic, pneumatic, magnetic, electrical, thermal and other types of elements which can interchange energy. Vibration engineers are often understandably reluctant to extend their analyses very far into unfamiliar energy domains, but there are cases in which the essence of a shock or vibration problem resides in the energy transduction between some energy domain and the mechanical part of the system.

In principle, even the simplest air suspensions or hydraulic or pneumatic shock absorbers when applied to vibration control problems result in multiple energy domain systems. When such devices can be adequately characterized by mechanical compliant or dissipative models, then they may be easily incorporated into a standard vibration analysis. However, at a more detailed level, the internal pressure and flow variables are not easy to predict using vibration computer codes.

For other classes of problems, it is very difficult to use a standard vibration program effectively. Consider, for example, the oscillations in systems containing electric motors, electrodynamic or electrohydraulic shakers, hydraulic power generating equipment, or hydrostatic drives. Equipment failures in such systems may properly be attributed to excessive vibration, but an analysis of the cause of the vibration requires a general type of system study.

Finally, there is the class of active shock and vibration control systems which requires a careful analysis of the vibratory system and the dynamics of the force or motion control effectors before a stable and effective control strategy can be designed. Such systems almost always involve energy transduction from the sensors, through the controller, and to the effectors. With the lowering of cost of the signal processing equipment which seems to be a constant factor in the electronics industry, it is likely that active systems for vibration control will move from aerospace to more mundane applications. For this class of system, also, most vibration computer programs are of limited utility because the servomechanism components involved are not easily incorporated into the analysis.

BASIC TYPES OF PROGRAMS

There are many possible ways to use existing computer programs to aid in the analysis and design of multiple energy domain vibratory systems. Below, several categories of programs are discussed briefly and their merits for the type of systems under discussion are noted.

Equation Integraters and Analyzers

These programs, which are discussed in detail elsewhere in this volume, are extremely versatile since they require only a set of equations and make no presumptions about the type of system which generated the equations. On the other hand, the programs are of no help in modeling the system or in deriving the equations from the system model. Relatively small structural or parametric changes in the system's mathematical model may require a tedious reformulation of the system equations before the computer can be of any aid. This fact tends to limit the usefulness of such programs during the design phase of a vibration control system.

Digital-Analog Simulators

Parts of certain systems are conveniently described by block diagrams, signal flow graphs, or transfer functions. This type of system description is particularly convenient when some components function as isolating amplifiers so that back-effect interactions between components can be neglected. Analog computers are suited to the study of such systems since parametric studies are readily accomplished by adjusting potentiometer knobs. Digital programs which accept the same sort of block diagram system description required for analog computers are available and have the advantage that the careful scaling necessary because of the analog computer's relatively small dynamic range is not necessary.

Generally, digital programs cost more to run than analog computer programs, but the set-up and debugging time for the analog computer can be large. A more basic problem is that for most shock and vibration problems, the generation of the block diagram system model requires a good deal of human skill and time. This is because large parts of most systems contain no isolating amplifiers so that back-effects between components result in many signal flow loops. Again, these computer programs come to the aid of the analyst only after he has put in considerable effort.

Single Domain Formulators, Analyzers and Simulators

Over the years, many computer programs have been developed which accept data describing a physical system involving a single energy domain in a form closely related to the analyst's own idea of how the system should be modeled using a rather small set of ideal elements. The finite element vibration programs described elsewhere are examples of this type of program, as are the electrical circuit design programs. These programs essentially formulate the equations of motion implied by the lumped parameter model and then can perform analyses of the properties of the equations or can perform direct time domain simulations of the system response to given inputs. These programs therefore allow the computer to play a large role in the design of systems, since a designer is spared a good deal of manipulation of parametric expressions if he chooses to modify the system to improve its characteristics.

The only way that a single domain analysis program can be used for a multiple energy domain system is by analogy. For example, an electric circuit design program can be used for an electromechanical system if an equivalent circuit is made for the mechanical part of the system. Similarly, a mechanical vibration program can be used for a hydro-mechanical system only if the hydraulic components can be replaced with equivalent mechanical elements. For simple systems, the use of analogy is reasonable, but for complex systems the process is tedious and needlessly confusing, particularly when parameter values are translated from one energy domain to another.

Multiple Domain Formulators, Analysers and Simulators

It is possible to build a program suited to multiple energy domain systems only if one has a uniform way to describe all physical systems with a small set of elemental components. Very few unified physical system theories have been extensively developed and therefore the number of corresponding programs is small. We shall describe the ENPORT program based on bond graph system descriptions in some detail.

TWO FORMULATORS CONTRASTED

Programs which merely analyze equation sets or human-generated block diagrams are of less interest than those which also formulate equations for multiple energy domain vibration control systems. Since single domain formulators for mechanical systems are described elsewhere in this volume, we will discuss only a leading electrical system formulator program and contrast it with a general purpose formulator program.

A recent good example of an electrical circuit program is SPICE (Simulation Program with Integrated Circuit Emphasis) which has supplanted several earlier programs. A summary of the capabilities of this program is given below:

Simulation Program with Integrated Circuit Emphasis (SPICE) [1]

Date: SPICE version 1Q issued 1 March, 1974.
Capability: This program will handle electrical circuit elements including semi-conductor devices. The analysis portion of the program contains three parts; a) nonlinear dc analysis with provision for stepping an input source to obtain a set of static transfer curves, b) small-signal steady state sinusoidal and noise analysis, c) nonlinear, time-domain analysis.
Limitations and Restrictions: The program is limited to 400 nodes and 200 total elements of which no more than 100 can be semiconductor devices.
Input: This program utilizes a free format input language defined in a User's Guide included in Ref. [1].
Output: A wide variety of tabular data outputs and line printer plots are available depending on the analysis options chosen.
Language: The basic program contains 8000 Fortran IV statements.
Hardware: Approximately 40,000 decimal words on a CDC 6400 computer are required. An overlaid version can be executed in approximately 25,000 decimal words.
Usage: The program has been developed and used over a several year period at Berkeley and now has been exported widely.
Developer: See Ref. [1]. The development was sponsored by the National Science Foundation, Grant GK-17931.
Availability: Program is available from Electronics Research Laboratory, College of Engineering, University of California, Berkeley, California 94720.

This program could be used for vibratory systems if one were willing to use equivalent circuits. The nonlinear capability and the sinusoidal response analysis could be quite useful. On the other hand, it is annoying to have to convert masses, spring constants, and damper parameters into electrical parameters. Also, the nonlinear elements which are user defined or built-

in are set up to model semi-conductor elements, not those often found in vibratory systems.

Finally, some specific problems arise whenever a single-domain program is used for a multiple domain problem. For example, while mutual inductance elements are allowed in this program, mutual capacitance elements are not. Thus, an electrical element representing a mass matrix can be found, but no element is allowed to represent a compliance matrix. Similarly, loops of inductors and branches of capacitors are prohibited, although systems containing analogous elements may occur in mechanical systems.

Consider now a program specifically designed for multiple energy domain systems, ENPORT.

ENPORT [2]

Date: 1972

Capability: This program handles all types of physical systems as long as they are represented by a bond graph model, [3]. In particular, mechanical systems containing coupled masses, spring constant or compliance matrices, component modes, rigid bodies, and dampers, may be coupled to hydraulic, pneumatic, electrical, or thermal elements. Equations of motion are formulated, output equations for requested variables are found, eigenvalues are computed, and time domain simulation can be performed.

Method: The program organizes equations following the type of causality assignment procedure presented in Ref. [3], but with modifications to make it more suitable for computer use, Ref. [4]. Standard programs for eigenvalue analysis and matrix exponential simulation are incorporated.

Limitations and Restrictions:

The program handles linear systems only.

Bonds	Total maximum	= 65
	external	= 50
	internal	= 15
Multiport elements maximum		= 65
Number of ports, C.I.R. maximum		= 5
	0, 1	= 5
Independent energy variables		= 20
Dependent energy variables		= 10
Number of sources		= 10
Number of resistances		= 10

Input: This program utilizes a free format input language defined in Ref. [2] and [3].

Output: A good deal of graph processing information is always provided for the purpose of checking the interpretation of the input data. Tabular data from simulations as well as line printer plots are available by requests.

Language: The program is written in Fortran IV and is contained on about 9000 cards.

Hardware: Present version of the program was developed on a CDC 6500 machine and is in use presently on a Burroughs 6700 and various IBM machines. Core requirements for present version are similar to those for SPICE (above) but less powerful versions have operated on computers as small as an IBM 1100.

Usage: Approximately 30 industrial and academic institutions have obtained ENPORT decks. Academic use has been continuous for several years at several locations.

Developer: Professor R. C. Rosenberg, Michigan State University, East Lansing, Michigan 48823. The development was unsponsored.

Availability: Card deck is available from the developer for less than $50.

The ENPORT program has much to recommend it when multiple energy domain vibratory systems must be analyzed. All energy domains are treated in the same fashion and with compatible units. (The metric system is most convenient with power measured in watts in all energy domains). The input data may be read directly from a system model bond graph and only the primitive physical parameters need be supplied, such as masses, capacitances, damping coefficients, etc. The program can handle algebraic loops and derivative causality automatically. Once the equations are formulated and composite parameters computed by ENPORT, standard analysis programs can be used to extend the analytical power of the program. For modest size problems, the programs listed in Ref. [5] are often used. These programs will print transfer function polynomials and yield plots of frequency response when supplied with vector-matrix equations from ENPORT, for example. For active control systems, the optimal control routines contained in this collection can also prove useful.

The basic difficulty to the use of ENPORT is that one must learn to model dynamic systems using bond graphs. This sort of disadvantage is inescapable if one desires to handle multiple energy domain systems without using a human being to organize the component equations algebraically into the system equations. The alternative of casting all components into an equivalent electrical form, as would be required with SPICE, or into an all mechanical form, as would be required with any all-mechanical element program, is hardly more attractive. In fact, when one is forced to deal with multiple energy domain systems, a uniform approach to modeling the system dynamics is useful whether a computer program such as ENPORT is available or not.

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Transfer Function Analysis

James C. Bowers
Gary A. Shaw

University of South Florida

INTRODUCTION

Laplace Transform theory and the formulation of transfer functions comprise an important technique for the representation and analysis of linear, lumped parameter, multidegree of freedom, vibrational systems. This chapter begins with a brief review of the theory of Laplace transforms and their application to the formulation of transfer functions. Four computer programs, possessing various transfer function analysis capabilities, are then examined in some detail. The program features are demonstrated by means of examples and a method for representing certain piecewise-linear systems is described.

NOMENCLATURE

c = Viscous damping
I = Moment of inertia
j = $\sqrt{-1}$
k = Spring stiffness
L{} = Laplace transform operator
m = Mass
P = Centrifugal force
s = Complex frequency variable
σ = Real part of the complex frequency variable
ω = Imaginary part of the complex frequency variable

LAPLACE TRANSFORMATION

The Laplace transform is a linear transform applicable only to linear time functions, including linear differential equations with constant coefficients. The Laplace transform is recognized as a powerful analytic tool since it transforms a differential equation in the time-domain into an algebraic equation in the complex frequency-domain. The algebraic equation is easily solved for the desired response variable in the complex frequency-domain, and an inverse transform can be performed to obtain the time-domain response [1,2]. A transfer function, which represents the ratio of any two time dependent system variables transformed into the complex frequency-domain, provides much information and insight regarding the frequency response of a system. While Laplace transforms and transfer functions are familiar to many, a brief review will insure the necessary background to understand the examples that follow.

The Laplace transform of a linear differential equation exists, if the transform integral converges. Therefore, in order that the time function f(t) be Laplace transformable, it is sufficient that

$$\int_0^\infty |f(t)| e^{-\sigma_1 t} dt < \infty \tag{1}$$

for some real, positive σ_1 [1]. The Laplace transform of f(t) is defined as

$$L\{f(t)\} \equiv \int_0^\infty f(t) e^{-st} dt \tag{2}$$

where $s = \sigma + j\omega$ represents the complex frequency variable. The transform integral given by Eq. (2) has been used to derive tables of Laplace transforms for commonly encountered functions [3].

An important theorem for applying Laplace transforms directly to differential equations is now stated.

> Theorem: the Laplace transform of the derivative of a time function is given by s times the Laplace transform of the function minus the value of the function at t = 0+.

The proof of this theorem follows from the definition of the Laplace transform.

$$L\left\{\frac{df(t)}{dt}\right\} = \int_0^\infty \frac{df(t)}{dt} e^{-st} dt \tag{3}$$

$$= \int_0^\infty e^{-st} d[f(t)]$$

$$= [e^{-st} f(t)]_0^\infty + s \int_0^\infty f(t)\, e^{-st} dt$$

$$= sF(s) - f(0+)$$

This theorem is generalized in Table 1 along with the integration and linearity properties of Laplace transforms.

Table 1 Laplace Transform Properties

Property	t-domain	s-domain
1st derivative	$f'(t)$	$sF(s) - f(0+)$
2nd derivative	$f''(t)$	$s^2F(s) - sf(0+) - f'(0+)$
nth derivative	$f^n(t)$	$s^nF(s) - s^{n-1}f(0+) - s^{n-2}f'(0+) - \ldots - f^{(n-1)}(0+)$
integration	$f(t)dt$	$\frac{1}{s}[F(s) + f^{-1}(0+)]$
linearity	$af_1(t) + bf_2(t)$	$aF_1(s) + bF_2(s)$

In a linear system composed of a finite number of lumped parameters (discrete masses, springs, dampers), the ratio of any two time dependent system variables can be expressed in the frequency domain as a ratio of two polynominals in s, provided all initial conditions are assumed to be zero. This ratio is referred to as a transfer function; the general form is given by Eq. (4),

$$T(s) = \frac{K(a_m s^m + a_{m-1}s^{m-1} + a_{m-2}s^{m-2} + \ldots + a_1 s + a_o)}{b_n s^n + b_{n-1}s^{n-1} + g_{n-2}s^{n-2} + \ldots + b_1 s + b_o} \qquad (4)$$

where $m \leq n$. The transfer function usually is written to express the ratio of an "output" variable to an "input" variable. Consider, for example, the undamped mass-spring system of Fig. 1.

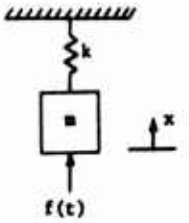


Fig. 1 Simple mass-spring

The applied force, f(t), represents the input to the system and the mass displacement, x(t), is chosen as the output. A transfer function that expresses the ratio of output displacement to input force can be obtained from Eq. (5), which is the differential equation of motion for the system.

$$m\frac{d^2x(t)}{dt^2} + kx(t) = f(t) \qquad (5)$$

Referring to Table 1, the transformation of Eq. (5) to the frequency domain is given by,

$$m[s^2X(s) - sx(0+) - x'(0+)] + k[X(s)] = F(s) \qquad (6)$$

In order to obtain the desired transfer function, the initial displacement, x(0+), and the initial velocity, x'(0+), must be assumed to be zero. Then Eq. (6) may be written,

$$ms^2X(s) + kX(s) = F(s) \qquad (7)$$

or,

$$\frac{X(s)}{F(s)} = \frac{1}{ms^2 + k} \qquad (8)$$

This transfer function expresses the relation between displacement and applied force in the frequency domain. The requirement that all initial conditions be zero is a general requirement that is imposed on all transfer functions. The transfer function of Eq. (8) is rewritten in factored form

as,

$$\frac{X(s)}{F(s)} = \frac{1/m}{(s + j\frac{k}{m})(s - j\frac{k}{m})} \tag{9}$$

The roots of the denominator of Eq. (9) are referred to as the poles of the transfer function. If the forcing function is harmonic, the complex frequency variable, s, will assume the value jω where ω represents the radian frequency of the forcing function. As the value of ω approaches k/m, the denominator approaches zero and the transfer function tends toward infinity. Therefore the poles (denominator roots) of the transfer function represent the resonant or natural frequencies of the system. A similar interpretation can be made regarding the roots of the numerator of a transfer function. The numerator roots are referred to as zeros and they correspond tc the frequencies for which no output is obtained, regardless of the magnitude of the input.

The frequency response of a transfer function is often represented by a plot of the magnitude of the ratio of output to input as a function of frequency. This type of plot is referred to as a Bode[1] plot. The phase shift, which represents the time delay between the input and output, may also be plotted as a function of frequency.

Another important technique for obtaining the transfer function of a system is from experimental data. In many practical cases, the parameters required to formulate a discrete element model of a system are not readily obtainable. In such cases, a Bode plot of the system response can be obtained experimentally and a close approximation to the system transfer function can be formulated from the Bode plot [4,5].

Once a desired transfer function is obtained, either analytically from the differential equation or experimentally from the Bode plot, there are several types of analyses that may be performed.

a. The denominator of the transfer function may be factored to obtain information about the poles (damped and undamped natural frequencies) of the system.

b. The numerator of the transfer function may be factored to obtain information about the zeros (frequencies at which no output occurs) of the system.

c. The frequency response (magnitude and phase) of the system may be evaluated over a specified range of frequencies.

d. The sensitivity of the output to changes in the system parameters may be determined.

e. The time-domain response of the system to an arbitrary input may be determined.

Each of these five types of analysis can be programmed and performed with the aid of a computer. In the case of analyses (a) and (b), programs for determining the roots of nth-degree polynomials are available with virtually all general purpose computers and are not significant in themselves. Melsa and Jones [6] describe in detail a simple FORTRAN program which uses a modified Barstow method for finding the roots of polynomials with real coefficients. Melsa and Jones [6] also provide the complete listing for a FORTRAN program to compute and plot the frequency response of a transfer function. While these programs are easy to implement and extremely fast to execute, the input format is cumbersome and the subroutines are executed independently, resulting in a piecewise analysis. Although the literature includes other examples of such programs, the main emphasis in this chapter is on examining those programs which offer a unified system of

[1] After H. W. Bode whose name is prominently associated with the use of such plots.

analysis, with user-oriented input format and a high degree of flexibility in output format.

PROGRAMS EVALUATED

The four programs evaluated in this chapter are SUPER*SCEPTRE, NET-2, SYNAP and CSMP III. Each of these programs possesses analysis capabilities beyond those required merely for transfer function analysis. SUPER*SCEPTRE is an extension of the SCEPTRE network analysis program and can be used to analyze one-dimensional, multidegree of freedom mechanical systems, digital logic systems and control systems, in addition to transfer functions. NET-2 is another network analysis program which has been expanded to include analysis of digital logic systems and control systems as well as transfer functions. SYNAP is designed to automatically formulate the transfer function for an electrical network and to analyze user-specified transfer functions. CSMP III is a well known continuous system simulation program which includes the capability for representing general transfer functions and system elements. Each of these four programs is examined in detail in the following sections. Four sample problems are presented and analyzed to illustrate the required coding procedures and program features. Only those program features that are directly related to transfer function and vibration analysis are discussed.

SUPER*SCEPTRE

(System for Circuit Evaluation and Prediction of Transient Radiation Effects) [8]

Date: Updated version released January, 1975.
Capability: Nonlinear time-domain response of electrical networks, one-dimensional multidegree of freedom mechanical systems, digital logic, linear transfer functions and control systems.
Method: State-variable formulation of the describing equations for a system. Transient solution obtained by variable step numerical integration with a choice of three explicit and one implicit integration methods.
Limitations and Restrictions: No frequency response curves possible when program used in an interdisciplinary mode. No polynomial root determination. Transfer functions higher than 36th order must be represented by interconnecting appropriate combinations of lower order transfer functions.
Input: Field-free, user-oriented input language. Transfer functions modeled by specifying numerator and denominator roots or coefficients. Electrical and mechanical systems specified by entering element type, values, and interconnections. Digital devices are specified by calling "built-in" logic models.
Output: Any system variables may be requested as output. Solution points printed in tabular format and linear plots.
Language: Primarily FORTRAN with some assembly language I/O routines.
Hardware: Designed for IBM 360/370 with 238K-bytes core or CDC 6000 series with 115.8K-words.
Usage: Preliminary version released January 1974. Final version released January 1975 and is already operative at about 20 installations. Technical manual and loading instructions distributed with tape.
Developers: J. C. Bowers, J. E. O'Reilly, G. A. Shaw
Electrical Engineering Department
University of South Florida
Tampa, Florida 33620
Sponsored by the Army Material Command CAD-E Council under DA project number 1E762708A090-24-07.
Availability: Magnetic tape and installation documentation distributed for a handling charge of $100. Send purchase order, check or letter of intent to purchase to:

Dr. James C. Bowers
Electrical Engineering Department
University of South Florida
Tampa, Florida 33620

Make checks payable to the Electrical Engineering Department.

SUPER*SCEPTRE Program Structure

Transfer function representations are often used in electrical network analysis and synthesis procedures. For this reason, the SCEPTRE network analysis program was recently expanded to include a transfer function modeling and analysis capability. Several other analysis capabilities were also added to SCEPTRE, and the resulting software package, which consists of the SCEPTRE program and a preprocessor, is called SUPER*SCEPTRE. The preprocessor is essentially transparent to the user since it accepts standard SCEPTRE coding and the processed output is automatically passed to the SCEPTRE program. The preprocessor portion of SUPER*SCEPTRE has been in use for less than two years. However, the SCEPTRE program, which performs the actual analyses, has been in use on live jobs for over 6 years and has undergone extensive testing and modification to refine its performance.

The transfer function model provided in SUPER*SCEPTRE is represented diagrammatically as shown in Fig. 2.

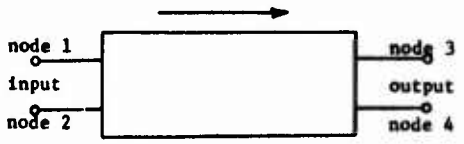


Fig. 2 Transfer function model

Note that the transfer function model possesses four terminals or nodes. The input is applied between nodes 1 and 2 and the output quantity appears between nodes 3 and 4. The arrow above the transfer function block is directed from the input to the output to symbolize the direction of signal flow. Before describing the uses of the transfer function model, the SUPER*SCEPTRE input language is described.

The input language employs a structured, field-free format that is easy to learn and use. The language consists of descriptive headings, subheadings and statements syntactically constructed from user-derived model names, component names, node names, and value specifications. The headings and subheadings required for transfer function analysis are listed in Table 2.

Table 2 SUPER*SCEPTRE Headings and Subheadings

```
1.  TRANSFER FUNCTION DESCRIPTION
     MODEL NAME
2.  MODEL DESCRIPTION
     MODEL NAME (TYPE)
3.  CIRCUIT DESCRIPTION
     ELEMENTS
      FUNCTIONS
      OUTPUTS
     RUN CONTROLS
4.  END
```

The TRANSFER FUNCTION DESCRIPTION heading card is required whenever one or more transfer function models are specified. The first MODEL card follows

the TRANSFER FUNCTION DESCRIPTION card and indicates the beginning of a transfer function specification. The MODEL card must include a user supplied model name of up to 18 alphanumeric characters. The transfer function is entered by specifying the constant multiplier, the numerator coefficients or roots and the denominator coefficients or roots. If the coefficients are specified, they must be in descending order, separated by commas and enclosed by parentheses. If instead the roots are specified, they are also separated by commas, but they may be in any order and must not be enclosed by parentheses. The general format for a transfer function model specification is:

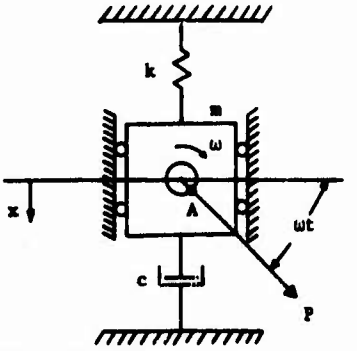
```
TRANSFER FUNCTION DESCRIPTION
MODEL NAME
K = Multiplying constant
NUMERATOR = Coefficients or roots
DENOMINATOR = Coefficients or roots
```

If desired, the symbolic names NUMERATOR and DENOMINATOR may be shortened to N and D respectively.

Once the desired transfer function models are specified, the forcing function must be specified, desired outputs must be requested, and appropriate run controls specified. These specifications are made following the CIRCUIT DESCRIPTION heading card. The following examples will illustrate the exact form for each entry.

Example 1 SUPER*SCEPTRE Analysis of a Second-Order System with Forced Vibrations

Figure 3 shows the model of a spring-suspended motor of mass m, constrained to displace in the verticle direction only [7].



Data
m = 1 lb·s²/ft
k = 100 lb/ft
c = 10 lb·s/ft
P = .1 lb
ω = 62 rad/s

Fig. 3 Spring-suspended motor

Assume that the motor runs at a constant angular speed, ω, and that its rotor is slightly out of balance, as indicated in Fig. 3 by the eccentric mass at point A. The imbalanced rotor creates centrifugal force P, resulting in a verticle component of force, f(t), equal to P sin ωt. The equation of motion for the steady state response of this system is

$$m\ddot{x} + c\dot{x} + kx = f(t) \tag{10}$$

Referring to Table 1, Eq. (10) can be written in the frequency domain as

$$ms^2X(s) + csX(s) + kX(s) = F(s) \tag{11}$$

provided the initial acceleration, velocity and displacement of the motor are zero. Note that the actual transforms of f(t) and x(t) are not required since we are interested only in their ratio. Solving Eq. (11) for the desired transfer function ratio results in Eq. (12).

$$\frac{X(s)}{F(s)} = \frac{1/m}{s^2 + \frac{c}{m}s + \frac{k}{m}} \tag{12}$$

The parameter values indicated in Fig. 3 can be substituted in Eq. (12) to obtain:

$$\frac{X(s)}{F(s)} = \frac{1}{s^2 + 10s + 100} \tag{13}$$

The SUPER*SCEPTRE transfer function model for this example, with the forcing function applied to the input, is shown in Fig. 4.

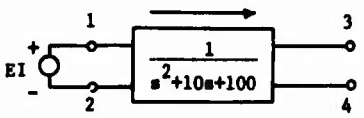


Fig. 4 Transfer function model and input forcing function

In SUPER*SCEPTRE, the input forcing function is represented by a voltage source connected between the input nodes. The voltage source can represent any input quantity such as force or displacement. The only requirement is that the value of the voltage source correspond to the value of the actual forcing function or input to the system. The voltage source, hereafter referred to also as the forcing function, must be assigned a symbolic name, up to five characters in length and beginning with the letter E. In this example, the voltage source is assigned the name EI. The next step is to assign a number to each node of the transfer function model. The node numbers may be chosen in any arbitrary fashion. The complete SUPER*SCEPTRE listing can now be formulated, based on the diagram of Fig. 4.

```
TRANSFER FUNCTION DESCRIPTION
 MODEL UNBALANCED MOTOR
  K = 1
  NUMERATOR = (1)
  DENOMINATOR = (1, 10, 100)
CIRCUIT DESCRIPTION
 ELEMENTS
  EI, 2-1 = X1 (.1*SIN(62*TIME))
  T1, 1-2-3-4 = MODEL UNBALANCED MOTOR
 OUTPUTS
  EI(FORCE), EOT1 (DISP)
 RUN CONTROLS
  STOP TIME = .5
END
```

Fig. 5 SUPER*SCEPTRE program listing for the spring-suspended motor

Although it is not required, indentation is used in the listing of Fig. 5

to distinguish headings, subheadings and the entries under each subheading. Following the TRANSFER FUNCTION DESCRIPTION heading, the user-derived name, UNBALANCED MOTOR, is assigned to the transfer function model. Then the multiplying constant, numerator coefficient, and denominator coefficients of the transfer function are specified. In this example, the specifications for the multiplying constant, K, and the numerator polynomial could have been omitted. If left unspecified, K defaults to 1 and the numerator defaults to a zero-order polynomial with a value of 1.

The node connections between the transfer function model and the forcing function, as well as the value of the forcing function, must be specified under the ELEMENTS subheading of the CIRCUIT DESCRIPTION. The name of the forcing function is entered first, followed by its node connections and value. The value of the forcing function is determined in a mathematical expression, denoted in this example by X1. In order to specify the node connections for a transfer function model, an arbitrary model designator, in this case T1, must first be assigned. The node connections are then specified by entering the model designator, T1, followed by the input nodes, output nodes and model name.

The OUTPUTS subheading is used to indicate the variables that are desired as output from the simulation. In Fig. 5, the values of EI and EOT1 are requested. The output variable of any transfer function model is always referred to by suffixing the user-derived model designator to the symbolic name EO. In this example, T1 was arbitrarily chosen for the model designator, so the output of the transfer function becomes EOT1. The requests shown under the OUTPUTS subheading will generate a printed tabular listing of EI and EOT1 as functions of simulation time. Print-plots of EI and EOT1 will also be generated. The names enclosed by parentheses will cause the printed and plotted values of EI and EOT1 to be labeled FORCE and DISP respectively. The use of this renaming feature is optional. The desired simulation STOP TIME is indicated under the RUN CONTROLS subheading. A number of additional entries are permitted under the RUN CONTROLS subheading. The listing is terminated with an END card.

The listing of Fig. 5 represents the typical input required to obtain the time-domain response of a given transfer function. Note that the forcing function is specified in the form of a mathematical expression in the time-domain. In general, the forcing function can be any arbitrary function of time. If the forcing function is not easily expressed in closed form, it may be represented by entering ordered pairs of data in a tabular format. Each ordered pair represents the value of the function at a specific point in time. Linear interpolation is employed in the program to obtain functional values that occur between the specified points in time.

Example 2 SUPER*SCEPTRE Analysis of a Torsional System with Four Degrees of Freedom

A step torque is applied to the first disk of the torsional system shown in Fig. 6. The problem is to determine the resulting angular velocity and displacement of the fourth disk for a period of 10 seconds immediately following application of the step torque.

The transfer function relating the angular velocity of the fourth disk to the applied torque is given by Eq. (14).

$$\frac{\Omega(s)}{T(s)} = \frac{s}{\frac{I^4}{k^3}s^8 + \frac{7I^3}{k^2}s^6 + \frac{16I^2}{k}s^4 + 13Is^2 + 3} \qquad (14)$$

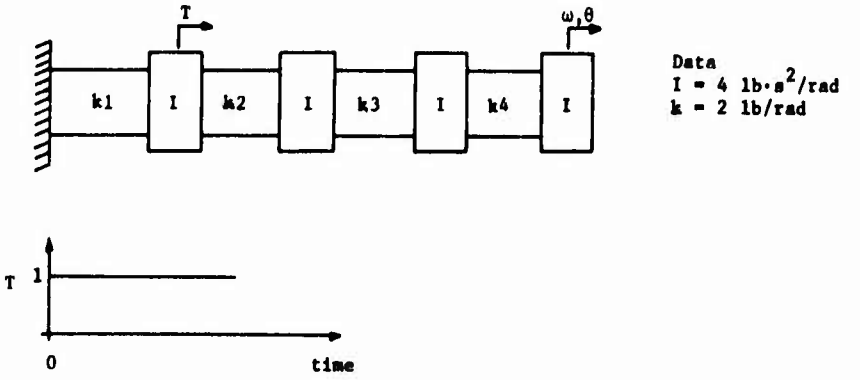


Fig. 6 Torsional system and parameter data

The values indicated in Fig. 6 for the moment of inertia, I, and torsional stiffness, k, can be substituted into Eq. (14) to obtain Eq. (15),

$$\frac{\Omega(s)}{T(s)} = \frac{s}{32s^8 + 112s^6 + 128s^4 + 52s^2 + 3} \qquad (15)$$

Now the angular velocity can be obtained from Eq. (15) and the corresponding angular displacement can be obtained by integration of the angular velocity. Recall from Table 1 that integration in the time-domain is equivalent to division by s in the frequency-domain. A diagram of the SUPER*SCEPTRE representation for this system is shown in Fig. 7, where the angular velocity output of the first transfer function model is integrated by the second transfer function model to obtain angular displacement.

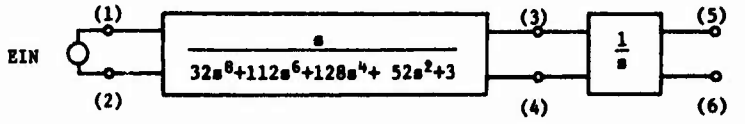


Fig. 7 SUPER*SCEPTRE model for the torsional system

As in the previous example, the input forcing function is represented by a voltage source, EIN. The SUPER*SCEPTRE listing shown in Fig. 8 is also similar to the previous example. However, two transfer function models are now required, and additional node numbers have been added.

In this listing, N and D, the shortened forms for NUMERATOR and DENOMINATOR, have been used to specify the transfer functions. Note also that parentheses are omitted when specifying the roots of the numerator or denominator. The constant multiplier specification is given but is not required.

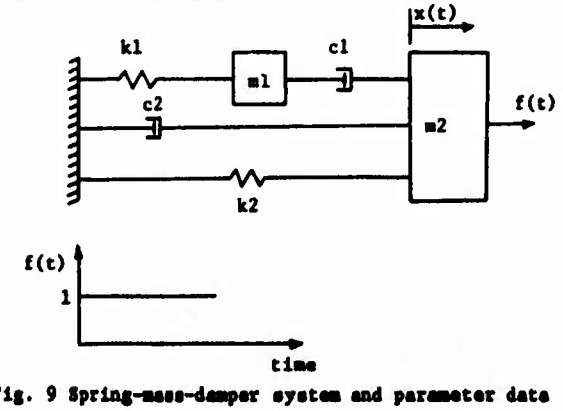
Under the ELEMENTS subheading, EIN is specified as a unit step function and the node connections for the transfer function models are specified. The model designators G1 and G2 are assigned to the transfer function models. Under the OUTPUTS subheading the forcing function, EIN, velocity output of the first transfer function, EOG1, and displacement output of the second transfer function, EOG2, are requested. The specification under the RUN CONTROLS subheading indicates the simulation stop time.

```
TRANSFER FUNCTION DESCRIPTION
 MODEL TORSIONAL SHAFT
  K = 1
  N = 0
  D = (32, 0, 112, 0, 128, 0, 52, 0, 3)
 MODEL INTEGRATOR
  K = 1
  D = 0
CIRCUIT DESCRIPTION
 ELEMENTS
  EIN, 2-1 = 1
  G1, 1-2-3-4 = MODEL TORSIONAL SHAFT
  G2, 3-4-5-6 = MODEL INTEGRATOR
 OUTPUTS
  EIN(TORQUE), EOG1(OMEGA), EOG2(THETA)
 RUN CONTROLS
  STOP TIME = 10
END
```

Fig. 8 SUPER*SCEPTRE program listing for the torsional system

Example 3 SUPER*SCEPTRE Analysis of a Spring-Mass-Damper System with Two Degrees of Freedom

In this example, a step force is applied to mass m2 as indicated in Fig. 9. The displacement of m2 is desired for the first ten seconds after application of the step input.



Data	
m1 =	1 lb.s^2/in.
m2 =	1000 lb.s^2/in.
c1 =	10 lb.s/in.
c2 =	50 lb.s/in.
k1 =	1000 lb/in.
k2 =	100 lb/in.

Fig. 9 Spring-mass-damper system and parameter data

The transfer function relating the displacement, x(t), to the applied force, f(t), can be obtained from the differential equation for the system. Vernon [9] describes an alternate method for obtaining transfer functions which does not require formulation of the time-domain differential equation. The method introduces the concepts of mechanical impedance and mobility to obtain the transfer function directly in the frequency-domain. The desired transfer function, obtained by either of these methods, is

$$\frac{X(s)}{F(s)} = \frac{m_1 s^2 + c_1 s + k_1}{m_1 m_2 s^4 + (m_1 c_1 + m_1 c_2 + m_2 c_1) s^3 + (m_1 k_1 + m_2 k_2 + c_1 c_2) s^2 + (c_1 k_1 + c_1 k_2 + c_2 k_1) s + k_1 k_2} \qquad (16)$$

The parameter values indicated in Fig. 9 can be substituted into Eq. (16) to obtain

$$\frac{X(s)}{F(s)} = \frac{s^2 + 10s + 1000}{1000\ s^4 + 10060\ s^3 + 101500\ s^2 + 61000 + 100000} \qquad (17)$$

The SUPER*SCEPTRE transfer function model and program listing for Eq. (17) could be easily formulated by following the procedures outlined in the first two examples. However, instead, an alternate method for SUPER*SCEPTRE analysis of this system is described. Although the alternate method does not employ transfer functions, it greatly simplifies the analysis of many vibrational systems and is therefore worthy of mention.

An Alternate Approach to Vibration Analysis Using SUPER*SCEPTRE

The MECHANICAL DESCRIPTION heading of SUPER*SCEPTRE can be used to analyze one-dimensional, multidegree of freedom mechanical systems composed of any of the elements listed in Table 3:

Table 3 Mechanical Elements of SUPER*SCEPTRE

ELEMENT TYPE	REQUIRED ELEMENT NAME PREFIX
Mass	M
Moment of Inertia	J
Spring Stiffness	K
Viscous Damping	D
Coulombic Friction	C
Acceleration Source	G
Velocity Source	U
Force Source	R
Torque Source	R

Analysis using the MECHANICAL DESCRIPTION heading eliminates the need for writing any differential or algebraic equations to describe the system. Instead, the user merely specifies the elements comprising the system, their interconnections and values. The program automatically formulates the describing equations for the system, based on the topological description provided by the user. To illustrate the simplicity of this approach, the system of Fig. 9 is analyzed. The first step is to determine the interconnections or nodes of the system. A node is identified as a point of common velocity between two or more elements. Each node is then assigned an arbitrary but unique number. The inertial reference plane constitutes one of the system nodes and all mass accelerations and velocities are computed with respect to this reference plane. The system is redrawn in Fig. 10, with appropriate node numbers added.

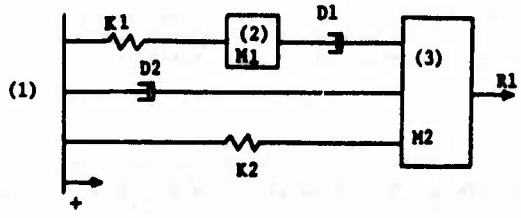


Fig. 10 Spring-mass-damper system prepared for input

In addition to adding the node numbers shown in Fig. 10, some of the element names have been modified so they have the required prefixes indicated in Tabel 3. The SUPER*SCEPTRE listing for the example can now be written. For each element of the system, the element name, node connections and value are specified under the ELEMENTS subheading as shown in Fig. 11.

```
MECHANICAL DESCRIPTION
 ELEMENTS
  R1, 1-3 = 1
  M1, 2-1 = 1
  M2, 3-1 = 1000
  D1, 3-2 = 10
  D2, 3-1 = 50
  K1, 2-1 = 1000
  K2, 3-1 = 100
 OUTPUTS
  AM1, VM1, SM1, FM1, VM2
  FK1, FK2, FD1, FD2, R1
 RUN CONTROLS
  STOP TIME = 10
  INTEGRATION ROUTINE = IMPLICIT
END
```

Fig. 11 SUPER*SCEPTRE Program Listing for the Spring-Mass-Damper System

The types of output that can be obtained from this simulation include accelerations, velocities, displacements and forces. Output requests are made under the OUTPUTS subheading by adding an appropriate status prefix to the element name. For example, the status prefix for acceleration is A, so in the listing, the acceleration of mass M1 is obtained by entering AM1 under OUTPUTS. Similarly, the velocity and displacement of mass M1 are designated by VM1 and SM1 respectively. The algebraic sum of all forces acting on M1 is obtained by requesting FM1. The forces transmitted by spring K1 and dashpot D1 are obtained by the entries FK1 and FD1 respectively. The choice of outputs from a simulation is left entirely to the user. In this example, the IMPLICIT integration routine has been specified. This routine is designed to handle stiff equations for systems with a large variation in eigenvalues. If no integration routine is specified, an exponential integration routine is used.

The MECHANICAL DESCRIPTION heading can also be used to simulate systems containing ideal gears, time-varying nonlinear elements and electromechanical devices. Each of the previous examples can be programmed using the MECHANICAL DESCRIPTION heading.

Example 4 SUPER*SCEPTRE Analysis of a Piecewise-Linear System

This example is included to demonstrate a method for the analysis of piecewise-linear systems. The method relies somewhat on techniques common to the analysis of control systems. A comprehensive treatment of control system theory and its application to linear vibration analysis is given by Vernon [5]. The method to be used here involves representing the linear portion of a system by one or more transfer functions and the nonlinear portion by an appropriate piecewise-linear model. The models are then interconnected to form a feedback control system.

Before examining a nonlinear system, the control system representation for the linear spring-mass system of Fig. 12 is developed.

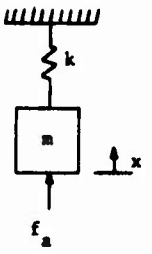


Fig. 12 Linear spring-mass-system

The transfer functions for the mass and spring elements are formulated separately. The total force acting on the mass is equal to the applied force, f_a minus the spring force, f_k.

$$f_a - f_k = m\ddot{x} \tag{18}$$

The frequency-domain representation of Eq. (18) is,

$$F_a(s) - F_k(s) = ms^2X(s) \tag{19}$$

and the transfer function becomes

$$\frac{X(s)}{F_a(s) - F_k(s)} = \frac{1}{ms^2} \tag{20}$$

The equation for the spring force is

$$f_k = kx \tag{21}$$

and the transfer function representation is given by Eq. (22).

$$\frac{F_k(s)}{X(s)} = k \tag{22}$$

The block diagram representations for each of these transfer functions is shown in Figs. 13 (a), (b).

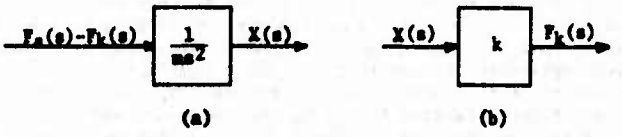


Fig. 13 Transfer function representations (a) mass; (b) spring

The transfer function blocks of Fig. 13 can be interconnected to represent the complete system of Fig. 12. However, the blocks must be interconnected so that the dependency between mass displacement and spring force is properly

preserved. Fig. 14 illustrates the proper connection. The circle is referred to as a summing junction and is used to indicate that the spring force is subtracted from the applied force to obtain the total force acting as the mass.

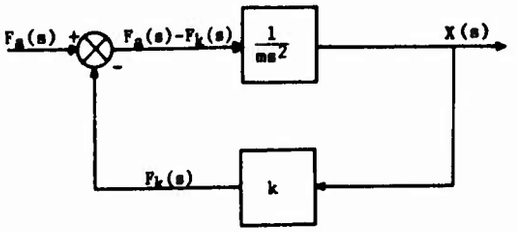


Fig. 14 Feedback control system representation of a linear mass-spring system

Now consider the spring-mass system of Fig. 15(a), which contains a discontinuous, linear elastic spring. If the initial position of the mass is taken midway between the spring gap, the static load-displacement diagram has the form given in Fig. 15 (b).

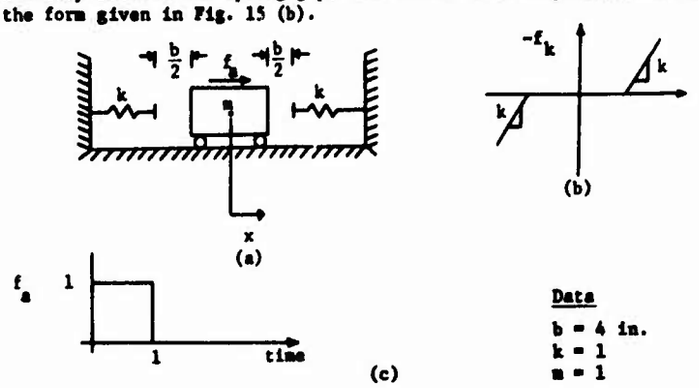


Fig. 15 (a) Spring-mass system; (b) static load curve; (c) input and parameter data

The complete system may be represented by the previously developed control system of Fig. 14, provided the linear spring block is now replaced by a piecewise-linear block as shown in Fig. 16.

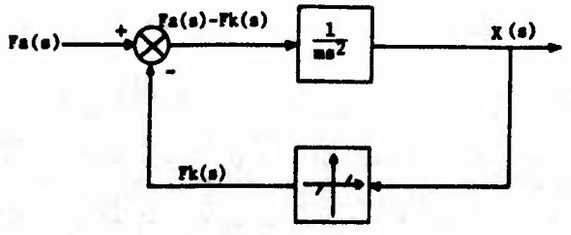


Fig. 16 Feedback control system representation using a piecewise-linear model

The control system of Fig. 16 can be represented in SUPER*SCEPTRE by a transfer function model and a piecewise-linear model. A number of piecewise-linear models are built-in to the SUPER*SCEPTRE program. The built-in model required for this example is identified by the name DEAD BAND. Since the transfer function model and piecewise-linear model have four terminals each, the control system is first redrawn as shown in Fig. 17. Note that the input forcing function is now represented by EFA, and the effect of the summing junction is accounted for by the terminal connections shown. As in the previous examples, each node connection is assigned an arbitrary number.

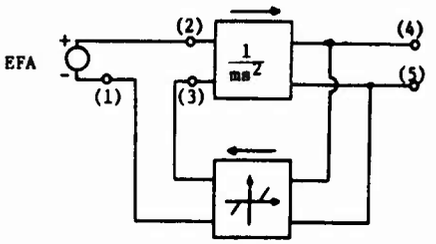


Fig. 17 SUPER*SCEPTRE representation of the feedback control system

Assume that the input consists of a force applied for 1 second as shown in Fig. 15 (c) and the transient response for the first 20 seconds is desired. Then an appropriate SUPER*SCEPTRE listing is presented in Fig. 18.

```
TRANSFER FUNCTION DESCRIPTION
 MODEL MASS
  D = 0, 0
MODEL DESCRIPTION
 MODEL NONLINEAR SPRING (DEAD BAND)
  B = 4
  M = 1
CIRCUIT DESCRIPTION
 ELEMENTS
  EFA, 1-2 = TABLE 1
  T1, 2-3-4-5 = MODEL MASS
  T2, 4-5-3-1 = MODEL NONLINEAR SPRING
 FUNCTIONS
  TABLE 1 = 0,0, 0,1, 1,1, 1,0, 20,0
 OUTPUTS
  EFA, EOT1(DISP), EOT2(FK)
 RUN CONTROLS
  STOP TIME = 20
END
```

Fig. 18 SUPER*SCEPTRE program listing for the piecewise-linear system

Several new entries are introduced in this listing. A MODEL DESCRIPTION heading is inserted to specify the piecewise-linear model. The type of model, DEAD BAND, is specified on the MODEL name card. Seven other types of piecewise-linear models are also available, including a general purpose model which can be modified to represent many different characteristics. The width of the dead band region is specified by the B entry under the MODEL name card. The non-zero slope, which in this example represents spring stiffness, is specified by M.

Since the input function, EFA, is a pulse, for which no simple mathematical expression exists, it is specified by a table of ordered pairs of

data points. The data points of the table are listed under the FUNCTIONS subheading, with time as the first point and the functional value as the second point in each ordered pair. The remaining entries in the listing have been discussed in the previous examples.

The listings for the previous four examples provide a good indication of the transfer function analysis capabilities and coding requirements of SUPER*SCEPTRE. The User's Manual provides a much more detailed description of the program features and includes 14 mechanical analysis examples and 5 control system examples. SUPER*SCEPTRE is intended primarily for time-domain analysis, however, the frequency response and pole-zero locations of individual transfer functions can also be obtained. Since the frequency response capability is limited to individaul transfer functions and is not documented in the User's Manual, it is omitted from discussion in this survey.

These four sample problems are used again in the following sections to illustrate the capabilities and coding requirements of the three remaining programs. The types of analysis performed on the previously described vibrational systems may be modified slightly from one program to another. Nevertheless, the figures and problem descriptions will not be repeated and the transfer functions describing the systems will remain unaltered.

NET-2 NETWORK ANALYSIS PROGRAM [10]

ate: Updated version released September, 1973.

Capability: Nonlinear time-domain response and linearized frequency-domain response of electrical networks, boolean logic, linear transfer functions and control systems.

Method: Transient solution obtained by fixed or variable step numerical integration using a trapezoidal implicit integration method with user selected step size.

Limitations and Restrictions: No polynomial root determination. Transfer functions higher than second order must be represented by interconnecting appropriate combinations of lower order transfer functions or system elements. No automatic integration error control for linear systems and a step size must be specified.

Input: Field-free, user-oriented input language. Transfer functions modeled by specifying numerator and denominator coefficients for any of seven "built-in" transfer function models. Electrical systems and control systems specified by entering element type, value, and interconnections. Boolean logic devices and system elements specified by calling "built-in" models.

Output: Time and frequency response variables available for output including magnitude and phase characteristics of transfer functions. Solution points printed in tabular format or plotted in linear, semilog or log-log format.

Language: 97% FORTRAN IV, 3% Assembler

Hardware: Designed for CDC 6000 series with 113_8 K-words of core. An IBM 360/370 version is also available but has not been updated to reflect recent improvements and extended analysis capabilities. The IBM version requires 400 K-bytes of core.

Usuage: Originally released in 1972 and currently operative at about 20 installations.

Developer: Allan F. Malmberg
Braddock, Dunn and McDonald, Inc.
6500 Convair Road
El Paso, Texas 79925
Sponsored by the Defense Nuclear Agency Under Subtask TC022

Availability: CDC 6000 version, Release 9, or IBM 360/370 version, Release 8, and two copies of the User's Manual may be obtained by sending a 2400 foot, 1/2 inch reel of magnetic tape to:
General Electric Company - TEMPO
ATTN: DASIAC/ESPIG
P. O. Drawer QQ
Santa Barbara, Ca. 93102

The tape must be accompanied by a letter requesting the program. The source program is classified CONFIDENTIAL, and the security office address to which the tape is to be shipped must be included in the requesting letter.

NET-2 Program Structure

NET-2 is a general purpose network analysis program which has undergone extensive testing and revision over a period of several years. Nine separate releases of NET-2 have been made, the most recent in September, 1973. Each new release has included revisions or improvements to enhance the program execution or analysis capabilities. The features of NET-2 which are of interest in transfer function analysis include: built-in transfer function models, system elements, transient and frequency response analysis.

The built-in transfer function models available in NET-2 are listed in Table 4, along with their input specification formats. The specification format includes the transfer function name, input node, output node and the appropriate coefficients. The transfer function names may seem unusual, but they actually contain information about the form of the transfer function they represent. For example, XFCPn is an acronym of sorts for a transfer function with complex poles. The lower case n at the end of the transfer function name represents a user-derived alphanumeric suffix that must begin with a number. If a transfer function does not have the form of one of the built-in models, then it must be represented in some other manner. One possible representation for a higher order transfer function is to factor the transfer function to obtain a product of lower order terms. Each term can then be represented by a built-in model and the models can be connected in series to form the overall transfer function.

Table 4 NET-2 transfer function models

TRANSFER FUNCTION		INPUT FORMAT
$\frac{1}{s+a}$	XFPn	IN OUT a
$\frac{s+a}{s+b}$	XFZPn	IN OUT a,b
$\frac{s}{s+a}$	XFSPn	IN OUT a
$\frac{1}{s^2+as+b}$	XFCPn	IN OUT a,b
$\frac{s+a}{s^2+bs+c}$	XFZCPn	IN OUT a,b,c
$\frac{s^2+as+b}{(s+c)(s+d)}$	XFCZDPn	IN OUT a,b,c,d
$\frac{s^2+as+b}{s^2+cs+d}$	XFCZCPn	IN OUT a,b,c,d

Any one of the built-in transfer function models provided in NET-2 may be represented diagrammatically as shown in Fig. 19. Note that the transfer function model possesses two terminals or nodes. Before presenting applications for the transfer function model, the NET-2 input language is described.

The input language is organized as a series of entries, with each entry composed of one or more lines. The various lines are written at specified indentation levels so the complete input has the appearance of an outline form.

The first line of an entry always starts at the left margin and any subsequent lines of the entry begin at the first level of indentation or higher. Elements are entered by specifying their symbolic names with user-derived suffixes added to insure uniqueness. In addition to specifying the desired transfer functions and forcing function, one or more STATEn entries must be included to specify the type of analysis and outputs desired. The following examples will illustrate the exact form for these entries. Since built-in models are not available for transfer functions of degree greater than two, the systems of Examples 2 and 3, with transfer functions of degree 8 and 4 respectively, are not analyzed.

Fig. 19 NET-2 transfer function model

Example 5 NET-2 Analysis of a Second-Order System with Forced Vibrations

In this example, the transient response and the frequency response of the spring-suspended motor of Example 1 are required. The transfer function for the system is given by Eq. (23)

$$\frac{X(s)}{F(s)} = \frac{1}{s^2 + 10s + 100} \tag{23}$$

The NET-2 transfer function model and forcing function corresponding to Eq. (23) are shown in Fig. 20.

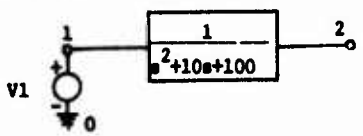


Fig. 20 NET-2 representation for the spring-suspended motor

The input forcing function is represented in NET-2 by a voltage source,V1. The transfer function corresponds to the fourth entry in Table 4 with a = 10 and b = 100. The node numbers are arbitrarily assigned, except for the zero-node designation, which must be assigned as shown. The complete NET-2 listing for this example is presented in Fig. 21.

```
XFCP1 1 2 10,100
X1 = .1*SIN(62*TIME)
V1 1  0  X1
STATE1
 TIME 0 (100) .5
 PRINT N(2)
 PLOT N(2)
 PLOT N(1)
STATE2
 FREQ .16 (50*) 16
 PRINT A(2-0/1-0) A'(2-0/1-0)
 PLOT LINLOG A(2-0/1-0) A'(2-0/1-0)
END
```

Fig. 21 NET-2 program listing for the spring-suspended motor

The first entry in the listing specifies that the built-in transfer function XFCP1 is connected with 1 as the input node, 2 as the output node. The value of coefficients a and b are 10 and 100 respectively. The second entry, X1, is a mathematical expression describing the sinusoidal input. The third entry specifies the node connections for the voltage source, V1, and indicates its value is given by the mathematical expression, X1.

The information following the STATE1 entry is included to obtain the time response of the system. The numbers following the TIME specification indicate a simulation start time of 0, and a stop time of .5, with 100 equally spaced increments or time steps taken by the integration routine to reach the desired stop time. There are several other methods for specifying the simulation stop time and integration step sizes. The simulation can be divided into a number of segments, each with a different step size. Also, specific time points at which the response is to be computed can be entered. Another method, which is illustrated in the next example, is to provide a maximum step size and a termination condition. The crucial consideration in any of these methods is to insure that the step size is not so large that significant integration errors are introduced. Since there is no built-in control of the integration error when simulating linear systems, a poor choice for integration step size may lead to significant integration errors. On the other hand, an arbitrarily small step size may result in excessive computer execution time. An optimum step size can usually be determined from a knowledge of the system eigenvalues, or by making several runs to assess the effect of different step sizes.

The PRINT N(2) entry produces a printed listing of the variable at node 2 as a function of time. The PLOT entries produce linear plots of the indicated node variables as a function of time.

The information following the STATE2 entry is included to obtain the frequency response of the system. The numbers following the FREQ specification indicate an initial frequency of .16 hertz, a final frequency of 16 hertz and 50 intermediate frequency steps. The asterisk indicates that the frequency steps are to be logarithmic. The PRINT entry requests the magnitude and phase of the transfer function as a function of frequency. The symbol A indicates magnitude, while the symbol A' indicates phase. The numbers in parentheses indicate the node pairs for which the magnitude and phase ratios are desired. Thus A(2-0/1-0) represents the magnitude of the variable at node 2 (referenced to node 0), divided by the magnitude of the variable at node 1 (referenced to node 0). The PLOT entry is similarly formulated and includes a request for a semilog plot.

Example 6 NET-2 Analysis of a Piecewise-Linear System

The transient response of the piecewise-linear system described in Example 4 can be obtained using NET-2. The technique of representing the system in terms of a feedback control system is again employed. The control system configuration for this system is shown in Fig. 22.

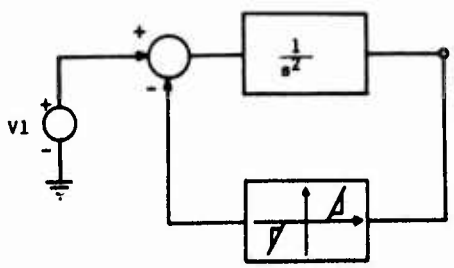


Fig. 22 Feedback control system representation for the piecewise-linear system

Notice, however, that the transfer function for the mass, s^{-2}, does not conform to any of the forms listed in Table 4. Fortunately, since s^{-1} corresponds to integration in the time-domain, the transfer function can be represented by two integrators connected in series. The resulting configuration is shown in Fig. 23.

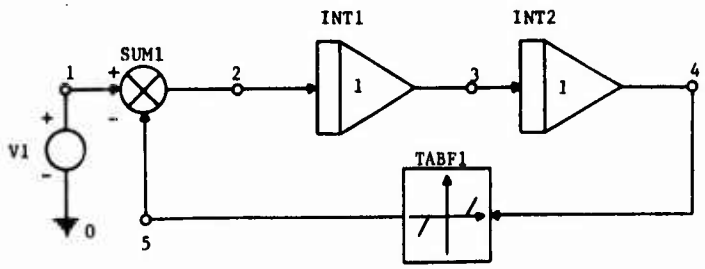


Fig. 23 NET-2 representation of the feedback control system

The format for specifying an integrating element in NET-2 is,

```
        INTn  IN  OUT  K
```

where: INTn = symbolic name
IN = input node
OUT = output node
K = integrating gain factor

The summing junction shown in Fig. 23 is represented in NET-2 by a SUM element. The SUM element may have as many inputs as desired and one output, which is the algebraic sum of all inputs. The specification format is,

```
        SUMn  OUT  IN1  IN2
```

where: SUMn = symbolic name
OUT = output node
IN1, IN2 = input nodes (a minus sign prefix indicates subtraction)

The dead band characteristic is represented using the TABF element. This element delivers a quantity at the output node which is an empirical function of the input node quantity. The empirical function is specified by a table of order pairs of data points. The specification format is,

```
        TABFn  TABLEm  OUT  IN
```

where: TABFn = symbolic name
TABLEm = table name
OUT = output node
IN = input node

Utilizing the three system elements described above, the NET-2 listing for the piecewise-linear system is formulated as shown in Fig. 24. The ordered pairs of data points in TABLE 1 are sufficient to describe the dead band characteristic. Each line of a TABLE entry is one ordered pair of data points. The first point is the independent variable and the second point is the dependent variable. The independent variable in TABLE 1 is automatically identified by NET-2 as the variable at the input node to the TABF element. The dependent variable in TABLE 1 is the variable associated with the output node of the TABF element. Linear interpolation is employed to determine the value of the dependent variable when the value of the independent variable falls between two of the specified data points. It is apparent that the TABF element can be used to represent any number of piece-wise-linear characteristics. The value of the input voltage source, V1, is determined from TABLE 2. TABLE 2 is included to describe the input to the system, which is a pulse and is therefore not easily represented in a mathematical expression. TIME is explicitly declared as the independent variable of TABLE 2. Two-dimensional

tables are also permitted in NET-2, but not required for these examples.

```
INT1 2 3 1
INT2 3 4 1
SUM1 2 1 -5
TABF1 TABLE1 5 4
TABLE1
     -52   -50
     -2     0
      2     0
      52    50
V1 1 0 TABLE2(TIME)
TABLE2
      0     0
      0     1
      1     1
      1     0
      10    0
TERMINATE = TIME-20
MAXSTEP = .2
STATE1
 PLOT N(1)
 PLOT N(2)
END
```

Fig. 24 NET-2 program listing for the piecewise-linear system

The TERMINATE entry provides a method for terminating a simulation if a specified condition is met. The general form of the TERMINATE entry is:

TERMINATE = value,

where value is any mathematical expression. Termination occurs as soon as the value of the mathematical expression becomes greater than zero. Any time a TERMINATE entry is included in a run, a transient response computation is begun and terminated whenever the termination value becomes positive. In this example, the simulation will terminate when the value of TIME exceeds 20. TIME is automatically incremented during the simulation after each integration step. However, a MAXSTEP entry must be included to limit the maximum step size that is taken.

The desired output quantities, in the form of node variables are requested in plotted format under the STATE1 entry. The variable at node 1 represents the input force and the variable at node 2 represents the output displacement of the mass. The completed listing is terminated with an END card.

The listings for Examples 5 and 6 provide a good indication of the transfer function analysis capabilities and coding requirements of NET-2. The User's Manual includes a more comprehensive discussion of the STATEn entry and the many possible formats for generating families of output data curves. Additional system elements are also discussed and one example is presented that uses several of the system elements. However, no specific transfer function or control system examples are included in the manual.

Symbolic Network Analysis Program (SYMAP) [11, 12]

Date: July, 1973.
Capability: Time-domain response, frequency-domain response, sensitivities, and pole-zero locations of linear electrical networks and transfer functions.
Method: Transient response obtained for transfer functions by computing the

inverse Laplace transform and evaluating the resulting function at the specified points in time.

Limitations and Restrictions: Transient response for step, ramp, and pulse inputs only. No plots.

Input: Field-free, user-oriented input language. Transfer functions modeled by specifying numerator and denominator coefficients and the degree of their associated s-term. Linear electrical networks specified by entering element types, values, and interconnections.

Output: Transient response, frequency response, and sensitivities may be printed in tabular form. Poles and zeros of transfer functions may be requested.

Language: FORTRAN

Hardware: Designed for CDC 6000 series with 100$_8$K-words of core. Use of dynamic allocation makes core requirements job dependent.

Usuage: Currently operative at several installations. Technical manual available from NTIS [11].

Developers: B. A. Hass, E. J. Mock, J. R. Pistacchi
TRW Systems Group
Rendondo Beach, California

Sponsored by the Defense Nuclear Agency under Program Element 61102H, Project WDNE1301, Subtask TC022.

Availability: SYNAP is approved for public release and can be obtained by sending a 1/2 inch magnetic tape and letter of request to:
AFWL/ELP
Air Force Systems Command
Kirtland AFB
New Mexico 87117
Attn: SCEPTRE Project Officer

SYNAP Program Structure

SYNAP was developed to provide a mathematical modeling capability to the Air Force Weapons Laboratory system analysis code, and a mathematical interface to currently available circuit analysis codes such as SCEPTRE. The program derives literal or numeric transfer functions for linear electrical networks and also analyzes user-defined transfer functions. The program has not had extensive use but seems to perform reliably.

A transfer function model, or "branch" as it is referred to in SYNAP, may be represented diagrammatically as shown in Fig. 25. Note that the transfer function model or branch possesses only two nodes.

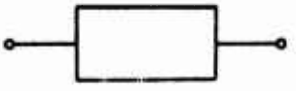


Fig. 25 SYNAP transfer function model

A field-free, user-oriented input language is provided for describing the transfer functions, desired outputs, and analysis modes to the SYNAP program. For topological circuit descriptions, the input language is identical to the SCEPTRE input language. For transfer function descriptions, the language consists of the headings and subheadings shown in Table 5.

Table 5 SYNAP Headings and Subheadings

1. GRAPH DESCRIPTION
   - BRANCHES
   - VALUES
   - OUTPUTS
   - SENSITIVITIES
   - RUN CONTROLS
2. END

The GRAPH DESCRIPTION heading is required whenever one or more transfer functions are specified for analysis. The coefficients and node connections for each transfer function are specified under the BRANCHES subheading. The general format for specifying a transfer function under the BRANCHES subheading is:

INPUT NODE - OUTPUT NODE (NUMERATOR/DENOMINATOR)

The numerator coefficients must be specified first and separated from the denominator coefficients by a slash. Within the numerator or denominator specifications, the coefficientr may be entered in any desired order, but each coefficient must be immediately followed by a number indicating the degree of the associated s-term. Symbolic names may be used to represent some or all of the coefficients. Symbolic names are required in order to compute the sensitivity of a transfer to changes in the numerator or denominator coefficients. The sensitivities are obtained by computing the partial derivative of the transfer function with respect to the indicated symbolic, or literal, coefficients. The coefficient terms can also be expressed as a product of literals and numbers.

The VALUES subheading is used to specify the actual value of any literal coefficients. The value of all literals must be specified in order to obtain a transient or frequency response run. The remaining subheadings are described in the following examples. These examples also provide a clear indication of the format for each entry.

Example 7 SYNAP Analysis of a Second-Order System with Forced Vibrations

Several of the transfer function analysis capabilities of SYNAP are illustrated in the following analysis of the spring-suspended motor of Example 1. The transfer function relating motor displacement to applied force is given by Eq. (24).

$$\frac{X(s)}{F(s)} = \frac{1}{s^2 + 10s + 100} \tag{24}$$

The corresponding SYNAP model for this transfer function is represented diagramatically in Fig. 26. No additional elements or sources are required by SYNAP to simulate the complete system. Arbitrary node numbers are assigned to each of the two model nodes. The actual coefficient of the second term in the denominator of the transfer function is replaced by an arbitrary symbolic name, K1. The substitution of a symbolic name in place of the actual coefficient permits computation of the sensitivity of the transfer function to changes in the coefficient value. Symbolic names, or literals, may be substituted for any of the coefficients in order to compute the corresponding sensitivities.

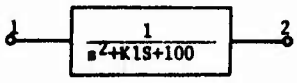


Fig. 26 SYNAP model for the spring-suspended motor

The SYNAP listing for this example is presented in Fig. 27. The input node and output node of the transfer function model are specified under the BRANCHES subheading of the GRAPH DESCRIPTION. Following the node specification, and enclosed by parenthesis, the coefficients of the transfer function are specified. The numerator coefficients are separated from the denominator coefficients by a slash. Each coefficient entry is immediately followed by an entry specifying the degree of the s-term associated with that coefficient.

```
GRAPH DESCRIPTION
 BRANCHES
  1-2(1,0/1,2,K1,1,100,0)
 OUTPUTS
  2/1
 VALUES
  K1=10
 SENSITIVITIES
  K1
 RUN CONTROLS
  TRANSIENT RESPONSE=STEP
  STOP TIME=5
  STEP SIZE=.01
  INITIAL FREQUENCY=.16
  FINAL FREQUENCY=16
  FREQUENCY STEP=.16
END
```

Fig. 27 SYNAP program listing for the spring-suspended motor

The entry under the OUTPUTS subheading requests the output of the transfer function between nodes 1 and 2, where 2 is the output and 1 is the input. In this example, no other entry under OUTPUTS is possible.

The VALUES subheading is used to specify the actual value of the coefficient represented by K1. The actual value is required for computation of the transient and frequency responses. Specification of K1 under the SENSITIVITIES subheading results in computation of the sensitivity of the transfer function to changes in K1, expressed in terms of the real and imaginary parts. The sensitivity is computed at each frequency point included in the frequency response solution.

The RUN CONTROLS subheading includes requests for both a transient and a frequency response computation. In the original statement of the problem, the forcing function for this system was shown to be sinusoidal. However, SYNAP can compute the transient response only for a step, ramp, or pulse forcing function. The step response is requested in this example merely to illustrate the required format. The step input will be applied to node 1, since it is implied as the input node in the specifications under both the BRANCHES subheading and the OUTPUTS subheading. The transient response is obtained by computing the inverse Laplace transform of the transfer function at specified intervals. Since numerical integration is not used to obtain the transient response, the STEP SIZE entry merely determines the values of time for which the response is computed. The step size does not influence the accuracy of the transient response solution. If no step size is specified, the indicated STOP TIME is divided into ten equal size increments to provide ten solution points. The frequency response is requested by entering the desired initial frequency, final frequency, and frequency step size.

The output produced by SYNAP consists of printed data only. The transient response data consists of a tabular listing of the transfer function output as a function of time. The frequency response output consists of a tabular listing of the magnitude and phase of the transfer ratio as a function of frequency. The sensitivity output consists of the real and imaginary parts of the sensitivity function computed at each point of the frequency response.

Example 8 SYNAP Analysis of a Torsional System with Four Degrees of Freedom

The transient response and pole-zero locations of the torsional system described in Example 2 are obtained using SYNAP. A step torque is applied as before with the transient response desired for the first 10 seconds. The transfer function relating angular velocity to applied torque is given by Eq. (25).

$$\frac{\Omega(s)}{T(s)} = \frac{s}{32s^8 + 112s^6 + 128s^4 + 52s^2 + 3} \qquad (25)$$

The angular velocity must be integrated to also obtain angular displacement, and the complete SYNAP model is shown in Fig. 28.

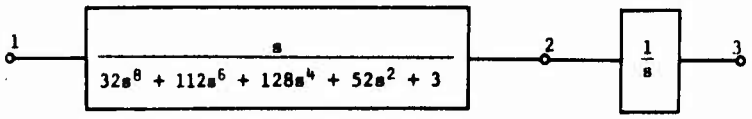


Fig. 28 SYNAP model for the torsional system

The program listing for this example is presented in Fig. 29. Two entries are required under the BRANCHES subheading, one for each transfer function. Only even powers of s occur in the denominator of the first transfer function, and no consideration is required for the missing odd terms when specifying the transfer function coefficients.

```
GRAPH DESCRIPTION
 BRANCHES
  1-2(1,1/32,8,112,6,128,4,52,2,3,0)
  2-3(1,0/1,1)
 OUTPUTS
  2/1, 3/1
 RUN CONTROLS
  TRANSIENT RESPONSE=STEP
  STOP TIME= 10
  STEP SIZE=.1
  PRINT POLES
END
```

Fig. 29 SYNAP program listing for the torsional system

The first entry under the OUTPUTS subheading requests the value of the output variable at node 2, which represents angular velocity, as determined by applying the input step to node 1. Similarly, the second entry requests the value of the output variable at node 3, which represents the angular displacement, as determined by applying the input step to node 1. In addition to computing the output at node 3, SYNAP also computes and prints the overall transfer function between node 1 and node 3. Every output request automatically produces a print-out of the overall transfer function between the two nodes specified in the output request. This feature permits complicated overall transfer functions to be derived for a system represented by several simpler transfer functions. For example, the linear spring-mass system discussed in Example 4 was represented by two simple transfer functions which were then interconnected in a feedback arrangement to represent the complete system. The overall transfer function for the linear system could be obtained with SYNAP.

The first three entries under the RUN CONTROLS subheading were described in Example 7. The entry PRINT POLES will cause the zeros and poles of the transfer functions to be computed and printed.

Example 9 SYNAP Analysis of a Spring-Mass-Damper System with Two Degrees of Freedom

The spring-mass-damper system of Example 3, which was previously analyzed in the time-domain using the MECHANICAL DESCRIPTION of SUPER*SCEPTRE, is now

analyzed using the transfer function approach of SYNAP. The previously derived transfer function is given by Eq. (26).

$$\frac{X(s)}{F(s)} = \frac{s^2 + 10s + 1000}{1000s^4 + 10060s^3 + 101500s^2 + 61000s + 100000} \qquad (26)$$

The SYNAP model for Eq. (26) consists of a single transfer function block or branch as shown in Fig. 30. Arbitrary node numbers are assigned as before.

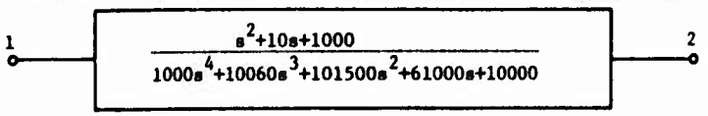


Fig. 30 SYNAP model for the spring-mass-damper system

In addition to computing the transient response, the frequency response and pole-zero locations are also determined. The appropriate program listing is presented in Fig. 31. The entries under each subheading are similar to the entries described for Examples 7 and 8. The frequency response data, entered under the RUN CONTROLS subheading, includes a FREQUENCY MULTIPLIER specification. The FREQUENCY MULTIPLIER specification can be used in place of, or in conjunction with, the FREQUENCY STEP specification. When only the frequency multiplier is specified, each new frequency step is determined by multiplying the previous value of the frequency by the frequency multiplier.

```
GRAPH DESCRIPTION
 BRANCHES
  1-2(1,2, 10,1, 1000,0/1000,4, 10060,3, 101500,2, 61000,1,
      100000,0)
 OUTPUTS
  2/1
 RUN CONTROLS
  TRANSIENT RESPONSE = STEP
  STOP TIME = 10
  STEP SIZE = .1
  INITIAL FREQUENCY = .01
  FINAL FREQUENCY = 2
  FREQUENCY MULTIPLIER = 20
  PRINT POLES
END
```

Fig. 31 SYNAP program listing for the spring-mass-damper system

A solution for the dead-band problem of Example 4 can not be obtained with SYNAP. The SYNAP program is restricted to linear systems and has no provisions for the formulation of piecewise-linear models. However, the listings for the previous three examples provide a good indication of the transfer function analysis capabilities and coding requirements of SYNAP. Four of the examples in the User's Manual deal solely with the analysis of user-specified transfer functions. The manual examples also include sample listings from each run.

CONTINUOUS SYSTEMS MODELING PROGRAM III (CSMP III) [13,14]

Date: Updated version released about 1971.
Capability: Nonlinear transient response of continuous systems represented by algebraic equations, differential equations and various functional blocks.

Method: Numerical integration of the system equations with a choice of five fixed step routines (rectangular, trapezoidal, Simpson's, second-order Adams, Runge-Kutta), two variable step routines(fourth-order Runge-Kutta, fifth-order Milne predictor-corrector) and a routine for stiff equations.

Limitations and Restrictions: No frequency response or polynomial root determination.

Input: Field-free, user-oriented input language. Transfer functions, mathematical functions, and boolean logic represented by functional blocks. Numerator and denominator coefficients entered in appropriately dimensioned arrays.

Output: The inputs or outputs of any functional block may be requested as output. Solution points printed in tabular format and linear or log plots as functions of the independent variable, time.

Language: FORTRAN IV and IBM Assembler.

Hardware: Designed for IBM 360/370 with a minimum partition of 102K-bytes of core.

Usage: CSMP III is an extended version of CSMP S/360 and has been used extensively in many scientific and engineering applications. Technical manual available [13].

Developers: IBM Program Product

Availability: CSMP III is distributed on a rental basis for $91/mo.($346/mo. with interactive graphic feature). Inquiries and arrangements for program rental should be conducted through the nearest IBM sales office.

CSMP III Program Structure

CSMP III is a general purpose program for simulating the dynamic response of continuous systems. CSMP III is an extended version of the CSMP - S/360 program, released by IBM about 1967. Since its release, CSMP - S/360, and later CSMP III, has been used extensively in numerous scientific and engineering applications. Systems are represented in CSMP III through the use of function blocks which perform operations such as integration, differentiation, function generation and logical operations. There are 42 built-in function blocks. New function blocks may be created through various combinations of standard blocks or FORTRAN subroutines. Only a few of the standard function blocks are required in the examples that follow.

The transfer function block provided in CSMP III, as well as many of the other function blocks, is represented diagrammatically as shown in Fig. 32. The input and output are represented by arbitrary symbolic names, in this case F and X respectively. There are never any nodes or node numbers associated with CSMP III function blocks.

Fig. 32 CSMP III transfer function block

CSMP III employs a field-free, user-oriented input language that follows the standard format established for continuous system simulation languages [15]. A CSMP III program may be comprised of three primary segments:

1. INITIAL
2. DYNAMIC
3. TERMINAL

The INITIAL segment is used exclusively for the computation of initial condition values prior to beginning the transient computations. Similarly, the TERMINAL segment includes computations that are performed only after completion of a transient computation. The INITIAL and TERMINAL segments are optional and are not required in any of the examples that follow. The DYNAMIC segment includes the complete description of the system dynamics and

must always be included. Within the DYNAMIC segment, statements describing the required function blocks are entered. In general, a statement consists of a variable equated to a function block or some combination of function blocks. The representation for a function block consists of the function name and one or more arguments, as required by the function. For example, the general form for specifying an integrator is:

```
          Y = INTGRL(IC,X)
where     Y = integral or output
     INTGRL = function name
         IC = initial condition
          X = integrand or input.
```

The function block for a transfer function is specified in a similar manner. However, two additional entries are required; the STORAGE and TABLE entries. Let the general form of a transfer function be defined by Eq. (27).

$$\frac{Y(s)}{X(s)} = \frac{\sum_{i=1}^{m} a_i s^i + a_{m+1}}{\sum_{j=1}^{n} b_j s^j + b_{n+1}} \tag{27}$$

Then the format for specifying a transfer function block is:

```
          Y = TRANSF (N, B, M, A, X)
          STORAGE B(N+1), A(M+1)
          TABLE B(1 - [N+1]) = B(1), B(2), ..., B(N+1)...
                A(1 - [M+1]) = A(1), A(2), ..., A(M+1)
where     Y = output
     TRANSF = function name
          N = highest degree of denominator
          B = an array in which the denominator coefficients are stored
          M = highest degree of numerator
          A = an array in which the numerator coefficients are stored
          X = input
    STORAGE = indicates the array dimensions follow
      TABLE = indicates the array names and corresponding coefficients follow.
```

In addition to specifying the function blocks required in a simulation, statements specifying the desired outputs and run control data are entered. The exact format for these specifications is illustrated in the following examples.

Example 10 CSMP III Analysis of a Second-Order System with Forced Vibrations

The transient response of the spring-suspended motor of Example 1 is desired for a period of 0.5 seconds after application of the sinusoidal forcing function. The transfer function describing the system is restated in Eq. (28).

$$\frac{X(s)}{F(s)} = \frac{1}{s^2 + 10s + 100} \tag{28}$$

The CSMP III function block corresponding to Eq. (28) may be represented as shown in Fig. 33. The input force is represented by the variable name F, and the output displacement is represented by the variable name X.

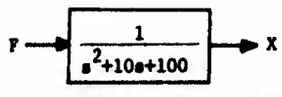


Fig. 33 CSMP III representation for the spring-suspended motor

A CSMP III listing for this example can now be formulated from Fig. 33 and a knowledge of the function block formats, forcing function and run control format. The complete listing is shown in Fig. 34.

The indentations shown in Fig. 34 are included merely for clarity. The listing is composed solely of a DYNAMIC segment. The output displacement, X, is equated to the function block representation for the transfer function. The arguments of the function block are entered as previously described. The STORAGE entry requests 3 storage locations for the denominator coefficients and 1 for the numerator coefficient. The actual coefficients are specified in the TABLE entry. The denominator coefficients have been specified first, starting with the coefficient of s^1 and ending with the coefficient of s^0. The three periods at the end of the entry indicate the entry is contined on the next line. The numerator coefficient is specified on the continued line. In the following entry, the forcing function, F, is computed. The remaining entries specify the run controls and output requests.

```
DYNAMIC
     X = TRANSF (2, B, 0, A, F)
     STORAGE B(3), A(1)
     TABLE   B(1-3) = 10., 1., 100.,...
             A(1)   = 1.
     F = .1*SIN(62*TIME)
TIMER FINTIM = .5, OUTDEL = .01
PRINT F, X
OUTPUT X
LABEL MOTOR DISPLACEMENT
END
STOP
ENDJOB
```

Fig. 34 CSMP III program listing for the spring-suspended motor

The TIMER entry can be used to specify the increments for which printed or plotted outputs are desired, the duration of the simulation, the integration step size, and a minimum permissible step size. In this example, the duration of the simulation is indicated by the FINTIM specification, and the desired output increment is indicated by the OUTDEL specification. Printed solution points of the forcing function, F, and motor displacement, X, are requested. A print-plot of the displacement is produced by the OUTPUT entry. The LABEL entry indicates the plot is to be labeled. MOTOR DISPLACEMENT. The END card signifies the listing for a run is completed. The STOP card is always included and separates the listing from any user-supplied FORTRAN subroutines. The ENDJOB card signifies the end of the input data stream for this job.

The listing of Fig. 34 represents the typical coding required to obtain the time-domain response of a given transfer function. There are several additional run controls and options that can be specified. Several of these will be illustrated in the examples that follow.

Example 11 CSMP III Analysis of a Torsional System with Four Degrees of Freedom

A step torque is applied to the torsional system described in Example 2. The transient response for the first 10 seconds is required. The appropriate transfer function is restated in Eq. (29).

$$\frac{\Omega(s)}{T(s)} = \frac{s}{32s^8 + 112s^6 + 128s^4 + 52s^2 + 3} \tag{29}$$

Since the angular displacement is also desired, an integrator is required in

addition to the transfer function block. The representation for the combined transfer function and integrator blocks is shown in Fig. 35.

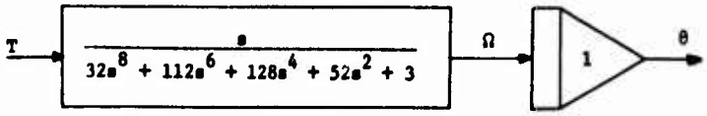


Fig. 35 CSMP III representation for the torsional system

The program listing for this example is presented in Fig. 36. The variables F, Ω, and θ have been assigned the names TORQUE, OMEGA and THETA respectively. The function block representing the transfer function is specified in the manner previously described. The format for specifying the integrator is similar, but requiries only one entry. The arguments of the integrator function, in their respective order, represent the initial condition and the integrand. The input torque is specified as a PARAMETER, since it is constant throughout the simulation.

The TIMER entry includes a specification for step size, DELT, of 0.05. If DELT is not specified, the program assigns a value equal to 1/16 of OUTDEL. Since no integration method has been specified, the variable-step Runge-Kutta integration is used. This integration method will automatically reduce the step size to meet the integration error criteria. When using the fixed-step integration routines, the value of DELT has a direct effect on the integration error.

```
DYNAMIC
     OMEGA = TRANSF (8,B,1,A, TORQUE)
     STORAGE B(9), A(2)
     TABLE B(1-9) = 0.,52.,0.,128.,0.,112.,0.,32.,3.,...
          A(1-2) = 1.,0.,
     THETA = INTGRL (0., OMEGA)
PARAMETER TORQUE = 1.
TIMER FINTIM = 10., OUTDEL = .1, DELT = .05
PRINT TORQUE, OMEGA,THETA
OUTPUT OMEGA
OUTPUT THETA
END
STOP
ENDJOB
```

Fig. 36 CSMP III program listing for the torsional system

The torque, angular velocity, and angular displacement are requested by the PRINT entry. Print-plots of the angular velocity and angular displacement are requested by the two OUTPUT entries.

Example 12 CSMP III Analysis of a Spring-Mass-Damper System with Two Degrees of Freedom

In this example, the translational system described in Example 3 is analyzed for a unit step input. The transient response for a period of 10 seconds is required. The transfer function relating output displacement to input force is restated in Eq. (30).

$$\frac{X(s)}{F(s)} = \frac{s^2 + 10s + 1000}{1000s^4 + 10060s^3 + 101500s^2 + 61000s + 100000} \qquad (30)$$

The CSMP III functional block corresponding to Eq. (30) may be represented as shown in Fig. 37.

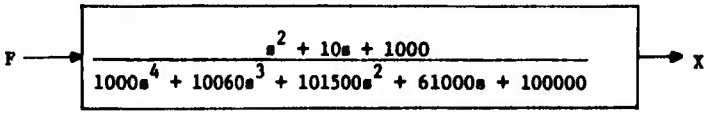


Fig. 37 CSMP III representation for the spring-mass-damper system

The CSMP III listing for this example is essentially the same as the listing for Example 10. However, several new entries have been introduced for the purpose of illustration. The complete listing is presented in Fig. 38.

```
DYNAMIC
     X = TRANSF (4,B,2,A,F)
     STORAGE B(5), A(3)
TABLE B(1-5) = 61000., 101500., 10060., 1000., 100000.,...
      A(1-3) = 10., 1., 1000.
PARAMETER F = 1.
TIMER FINTIM = 10., PRDEL = .1
METHOD STIFF
PRINT X, F
RANGE
END
STOP
ENDJOB
```

Fig. 38 CSMP III program listing for the spring-mass-damper system

The PRDEL entry specifies the TIME increment for which the variables specified in the PRINT entry are to be printed. No output plots are requested in this listing. The integration routine for STIFF equations is requested by the METHOD entry. The minimum and maximum values that the variables assume during the simulation is obtained from the RANGE entry. Output generated by the RANGE entry gives the minimum and maximum value of each variable; the times at which the minimum and maximum occur, and the TIME interval over which the RANGE values were obtained.

Example 13 CSMP III Analysis of a Piecewise-Linear System

The transient response for the piecewise-linear system treated in Examples 4 and 6 is desired. The input to the system is a unit force applied for one second. The representation for the system in Example 4 is also appropriate for this analysis, and is repeated in Fig. 39.

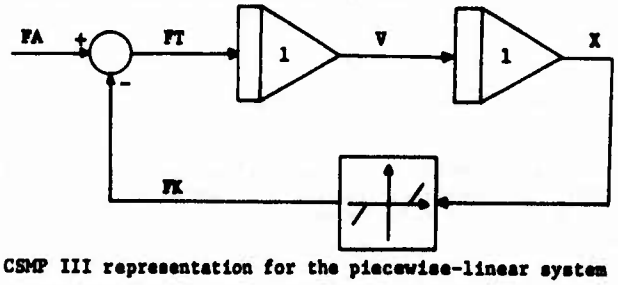


Fig. 39 CSMP III representation for the piecewise-linear system

In the program listing, the integrators and dead band characteristic are rep-

resented by appropriate functional blocks. The summing junction is represented by a simple algebraic equation. The complete listing is given in Fig. 40.

```
DYNAMIC
  FT = FA - FK
   V = INTGRL(0., FT)
   X = INTGRL(0., V)
  FK = DEADSP(-2.,2.,X)
  FA = AFGEN(PULSE1, TIME)
FUNCTION PULSE1 = 0., 1., 1., 1., 1.0001,0.,20.,0.
TIMER FINTIM = 20., OUTDEL = .2, DELT = .2
PRINT FA, FK, FT, V, X
OUTPUT X, V
END
STOP
ENDJOB
```

Fig. 40 CSMP III program listing for the piecewise-linear system

Several new entries are introduced in this example. The functional block AFGEN can be used to specify any arbitrary function in a piecewise-linear fashion. In this example, the input force, FA, is determined for any value of TIME by AFGEN. The first argument of the AFGEN block, PULSE1,merely identifies a set of ordered pairs that describe the curve of the input force. The second argument, TIME, identifies the independent variable. The independent variable is always the first data point in each ordered pair and must be strictly monotonic increasing. The FUNCTION entry is used to specify the ordered pairs that describe the PULSE1 function. Thus, for a given value of TIME, AFGEN performs a linear interpolation on the data points of PULSE1 to determine the value of the input force, FA. The dead band characteristic is referred to as dead space in CSMP III, and is represented by the functional block DEADSP. The arguments of DEADSP, in the order of their occurrance, are: the negative or smaller value of the x-intercept (-2.), the positive or larger value of the x-intercept (2.), and the independent variable (X). The non-zero slope of the DEASP block is fixed at a value of 1. If a slope other than 1 is required, the output of the DEADSP block may be multiplied by the desired slope.

The remaining entries in the listing have been discussed in the previous examples. The program listings for this and the previous examples provide a good indication of the transfer function analysis capabilities and coding requirements of CSMP III. An optional graphics feature is also available with CSMP III. The graphics feature utilizes a CRT display to provide an interactive mode of analysis.

As with the other programs discussed in this chapter, CSMP III possesses many capabilities that are not directly related to transfer function analysis. For instance, this example was formulated without the use of the transfer function block. A full description of the numerous function blocks and the graphic feature is included in the Program Reference Manual. As can be seen from the program listings, CSMP III is intended primarily for time-domain analysis. There are no provisions for frequency analysis or determination of transfer function poles and zeros.

SUMMARY AND PROGRAM COMPARISONS

Transfer function representation of linear vibrational systems provides a systematic method of analysis, and often yields important insight into the frequency behavior of a system. Transfer functions may be formulated from the differential equations of the system. Transfer functions can also be formulated directly in the frequency-domain or derived from experimental data.

Many of the techniques developed for modeling and analyzing control systems are also useful in analyzing vibrational systems.

Obviously, the four problems analyzed in this chapter do not represent even a small sampling of the possible applications for transfer function analysis. However, the examples do illustrate the types of analysis that can be performed on individual and interconnected transfer functions. The examples also provide an indication of the coding requirements and analysis capabilities of the programs included in this evaluation. Table 6 provides a comparison of the more important aspects of each program, and summarizes much of the information presented in the text. Several of the entries in Table 6, ,such as the rerun capability, are not discussed elsewhere, since their meaning is clear.

Table 6 Comparison of Programs

Property	Program			
SYMBOLS: X = YES, NA = NOT APPLICABLE, I = IBM 360/370, C = CDC 6000	SUPER*SCEPTRE	NET-2	SYNAP	CSMP III
INPUT				
Field-free input format	x	x	x	x
Transfer functions	x	x	x	x
Topological mechanical system descriptions	x			
Topological electrical network descriptions	x	x	x	
Topological control system descriptions	x	x	x	
TRANSFER FUNCTION ANALYSIS				
Time-domain response	x	x	x	x
Frequency-domain response		x	x	
Pole and zero determination			x	
Sensitivity			x	
Overall transfer function determination			x	
Maximum order	36	2	none	none
OUTPUT				
User selected outputs	x	x	x	x
Tabular printed output	x	x	x	x
Linear print-plots	x	x		x
Log print-plots		x		x
Polar print-plots		x		
Composite print-plots	x	x		x
Interface with user plot routines	x	x		x
Print-plots of one variable vs. another	x	x		
CRT display				x
RUN CONTROLS				
Conditional termination	x	x		x
Reruns	x	x	x	x
Maximum step size	x	x	x	x
Minimum step size	x		NA	x
Starting step size	x		NA	
Absolute integration error	x		NA	x
Relative integration error	x		NA	x
Arbitrary simulation start time	x	x	x	
Selection of specific points for solution		x		
Swept variables other than time or frequency		x		
INTEGRATION METHODS				
Multiple integration schemes	x		NA	x
Variable step error control	x		NA	x

Property	Program			
SYMBOLS: X = YES NA = NOT APPLICABLE I = IBM 360/370 C = CDC 6000	SUPER•SCEPTRE	NET-2	SYNAP	CSMP III
MODELS				
Expandable model library	x	x	x	x
Library of standard analog elements		x		x
Library of standard piecewise-linear elements	x			x
SOFTWARE INFORMATION				
FORTRAN	x	x	x	x
Assembler	x	x		x
Preprocessor	x			
Interactive				x
Error diagnostics	x	x	x	x
Distributed by developer	x	x	x	
Distributed by software center				x
Cost	$100	none	none	$91/mo
Cost with graphics feature	NA	NA	NA	$346/mo
DOCUMENTATION				
User's Manual	x	x	x	x
Technical Manual	x		x	x
MISCELLANEOUS FEATURES				
Arbitrary forcing functions	x	x		x
Standard math functions	x	x		x
Nonprocedural processing	x	x	x	x
Accepts FORTRAN subroutines	x	x		x
Accepts comment cards	x	x	x	x
Accepts algebraic equations	x	x		x
Solves differential equations	x	x		x
Solves general algebraic loops				x
Machine compatibility	I,C	I,C	C	I

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Dynamics of Spacecraft Structures

Leonard Meirovitch

Virginia Polytechnic Institute and State University

INTRODUCTION

The problems associated with the dynamics of spacecraft structures are much more diverse than those associated with civil structures or even aircraft structures. As with aircraft, there is a weight penalty involved. Indeed, the fact that spacecraft must be lifted from the earth surface and placed in a certain orbit in space requires that the structure be made as light as possible. But the similarities end here, as spacecraft must operate in an entirely different environment than aircraft. As an example, differential gravity forces are generally more important than aerodynamic forces for spacecraft, while the opposite is true for aircraft.

The missions for which spacecraft are designed often require that they retain a certain orientation in space. The most common cases are those in which the spacecraft must retain a relatively fixed orientation with respect to an inertial space, and those in which the spacecraft must point toward the earth at all times. It is also possible that the orientation of certain parts of the spacecraft (such as solar panels) must remain fixed in an inertial space while other parts (such as antennas) must point toward the earth. Because spacecraft are free to rotate in space, rather than being restrained by reactions as in the case of civil structures, certain controls must be exerted on the spacecraft to enable them to fulfill their mission. Controls can be passive or active. Passive controls are of two types: spin stabilization and gravity-gradient stabilization. Spin stabilization is based on the fact that, in the absence of external torques, a rigid body spinning about the axis of maximum or minimum moment of inertia tends to maintain its orientation in space. On the other hand, gravity-gradient stabilization is based on the fact that differential-gravity torques tend to align the axis of minimum moment of inertia or an orbiting spacecraft with the local vertical. Active controls are generally achieved by control jets or by momentum exchange devices, such as control-moment gyros (CMG), reaction wheels, and rotors of dual spin or a multispin spacecraft.

In general, the orbital motion of a spacecraft is given and the problem reduces to that of controlling its attitude. For a rigid spacecraft, the mathematical formulation consists of three simultaneous ordinary differential equations for the rotational motion. The problem becomes appreciably more involved when the spacecraft is flexible, as additional degrees of freedom must be introduced to describe the displacements of the flexible parts relative to a given frame. There are two basic types of mathematical models for flexible parts, namely discrete and distributed. Discrete models, also known as lumped models, are described by ordinary differential equations; and distributed models, also known as continuous models, are described by partial differential equations. For the spacecraft as a whole, discrete models lead to a set of simultaneous ordinary differential equations. On the other hand, spacecraft containing distributed parts are described by a "hybrid" set of equations, in the sense that the rotational motion is described by ordinary differential

equations and the elastic displacements relative to the rotating frame are described by partial differential equations. For practical reasons, a hybrid system must be "discretized". The two most common discretization procedures are the finite element method and the assumed modes method, where the latter is often called the Rayleigh-Ritz method. The first expresses the continuous elastic displacements inside an element in terms of displacements at given points on the boundaries of the element. On the other hand, the second expresses the continuous elastic displacements of a given flexible member as a series of space-dependent admissible functions multiplied by time-dependent generalized coordinates. All of these methods are discussed in Ref. 1.

Another way of simulating a flexible spacecraft is by regarding continuous flexible members as consisting of rigid elements interconnected by springs. This approach has originated in conjunction with the analysis of spacecraft structures [2 and 3] and enjoys a certain degree of popularity with spacecraft dynamicists, as witnessed by the number of computer programs based on the approach. A refinement of this approach consists of regarding the interconnected elements not as rigid but as flexible [4], which brings the approach closer to the finite element method.

The computer programs in the area of dynamics of flexible spacecraft tend to be tailored to specific spacecraft. Nevertheless, in virtually every case attempts have been made to generalize the program so as to accommodate a larger class of configurations. The programs are generally very extensive and they are modular in nature, in the sense that they consist of a group of subprograms integrated into a single general program. Because of this, the programs do not lend themselves to the same clear cut classification and objective evaluation as would a single computational algorithm designed to solve a given problem, such as the eigenvalue problem of a real symmetric matrix.

COMPUTER PROGRAMS

Flexible Spacecraft Dynamics (FSD)

The FSD computer program simulates attitude and flexural motions of generalized spacecraft with long tubular appendages subjected to orbital perturbing forces including gravitational forces, solar pressure, temperature gradients, aerodynamic drag, and magnetic torques. Internal forces due to dampers, thrusters, and momentum wheels are also simulated.

(The FSD Program has the following features:)

1. The flexible appendages can be extended or retracted at a variable rate. All inertial forces due to Coriolis, centrifugal, linear, and angular acceleration are simulated during deployment.

2. The program can be run in sequences of parametric runs to study the effect of variations of one or more parameters or initial conditions. The program can also be stopped and restarted during a computer run to change a parameter in order to simulate a flight operations procedure such as activation of a thruster.

3. Up to 10 flexible members can be simulated. The members can have arbitrary origin and angular position relative to the body axes. Each element may have a different stiffness, coefficient of thermal expansion, projected area, drag coefficients, unit mass, temperature gradient, tip mass, length, and deployment velocity. In addition, each member can have either one of two data sets associated with mass, area, and internal force integrals. This permits, for instance, the simulation of a spin-stabilized spacecraft with wire transverse elements and interlocked tubular elements on the spin axis.

4. For gravity-gradient stabilized spacecraft, a libration damper can be simulated consisting of a long damper boom that can rotate relative to the primary body axes. A magnetic hysteresis or eddy current damper will dissipate energy during the relative motion.

One version of the FSD Program (Dual-Spin) generalized this option into a secondary body with flexible appendages and additional degrees of relative

motion. This Dual-Spin version also simulates nutation dampers situated on the despun portion of a Dual-Spin spacecraft. Nutation dampers simulated can be either a pendulum type with viscous or hysterisis damping or a viscous ring damper.

5. Various body torquing devices are available to simulate spacecraft operation. Thrusting of the spacecraft can be simulated either for pure moments or for a thrust vector whose line of action is arbitrary with respect to the center of mass. The effect of thrusting on orbit parameters is also computed.

6. A simple control law is provided for thrusting which can be activated by sun sighting. Complex control laws can be added if required. Torquing due to electromagnetic or residual magnetic dipoles is simulated. The earth's magnetic field is represented by recent multi-term spherical harmonic model developed by Goddard Space Flight Center. The capability to simulate torques developed by constant or variable speed momentum wheel is also available.

The equations of motion describe the response of the system in terms of the rotations of a reference system with respect to the inertial space and the motions of various parts of the system relative to this reference system. The generalized coordinates are of two types, discrete and distributed. Consequently, the equations of motion constitute a hybrid set of differential equations, which is discretized by the assumed modes method. Typical admissible functions are cantilever modes. Other appropriate admissible functions can be used such as those for cantilever beams with tip masses.

The time history of the dependent variables is obtained by numerical integrations of the equations of motion using the Adams-Moulton method and a fourth order Runge-Kutta starter. The Adams-Moulton integrator uses a predictor-corrector technique to select the appropriate integration time step that will minimize computer time and maintain the desired accuracy. The accuracy bounds for the dependent variable are specified by the user.

Language: The majority of the program language is FORTRAN. A small portion of the input routines is in assembly language.

Hardware: IBM 360

Accuracy and Reliability: The accuracy and validity of the program has been demonstrated by many comparisons with analytical solutions. In addition, the comparison of simulated data and flight data from the RAE-I satellite has been remarkably good. This comparison was made with the RAE-I in various symmetrical and asymmetrical configurations with the boom at various lengths up to 750 feet. For an in-orbit dynamics experiment, the FSD Program accurately predicted the critical spin rate for instability of the spin axis booms of the IMP-I satellite. In addition to the above mentioned flight programs, the computer program was the prime method of simulating the flexible dynamics of the Lunar RAE satellite and the IMP-J (spinning, radial wires) spacecraft.

The program has been operated on a daily basis for more than three years. Any anomalies and discrepancies that have been uncovered have been corrected or are minor. The program is presently considered to be highly reliable.

Contact: The computer program was developed by AVCO Systems Division for NASA Goddard Space Flight Center. In addition to being used in the Goddard ISEE flight program, it is currently being modified, maintained, and operated by Computer Sciences Corporation, System Sciences Division, for NASA Goddard and other agencies.

Availability: The FSD Program was developed under government funding, hence is public property. Further information can be obtained from:

J. V. Fedor
NASA Goddard Space Flight Center
Greenbelt, Maryland

R. M. Davis
Computer Sciences Corporation
Silver Springs, Maryland

Subjective Comments: The FSD Program has several noteworthy advantages over other existing programs. With respect to simulation of spacecraft with long flexible deployable appendages, it is doubtful that other programs have comparable accuracy for large displacements. It is not necessary to determine an equilibrium configuration before solving the equations of motion as many linearized solutions require.
Although the program is large (450 bytes) for the IBM 360 system because of the higher order terms, three flexible modes and many options, it is relatively easy to operate. The options can be invoked by input controls. Because the program is so generalized, there have been occasions when unexpected results have led to new insights into spacecraft dynamic behavior. These results, that were later confirmed by analysis, have had an impact on spacecraft design and flight operations.

LPARL Flexible Spacecraft Attitude Dynamics Programs (LPAKL)

The Lockheed Palo Alto Research Laboratory (LPARL) consists of a series of multipurpose digital computer programs for simulation of the attitude dynamics and control of complex spacecraft structures representable as rotationally interconnected rigid and flexible bodies. The arrangement of bodies in the model is arbitrary, except that "closed loops" of bodies are excluded, i.e. the system of bodies and interconnections must form a "topological tree".

The LPARL multibody programs can also be used to study spacecraft attitude stability. Using the nonlinear programs, a simple numerical method of sequential perturbation of initial conditions gives directly the numerical values of the coefficient matrices of the linearized equations corresponding to linearization about any chosen state. Using standard computer routines for eigenvalues of linear systems, stability analysis is carried out by examining the non-positivity of the real part of the eigenvalues. Thus, in the context of computer integration of the equations of motion, linearized equations (in numerical form) and the associated stability analysis are a direct by-product of the multibody nonlinear equations.

The series of programs, based on formulations described in [3 - 9], represents a natural evolution from the "simple" case of a tree of rigid bodies to the case of a tree of flexible bodies. This last program is currently under development and will be completed this year. Intermediate models consist of topological trees of rigid bodies with terminal flexible bodies, and chains of flexible bodies. In these multibody programs, flexibility of a given body can be suppressed, reducing it to a rigid body. In this sense, the more general programs will logically supersede the less general ones. However, the redundancy in these programs is limited only to the model, because different programs are based on different formulations of multibody dynamics. In view of the vast complexity of such multipurpose programs, the availability of distinct formulations provides a unique method for verification of the validity of output and hence a very high degree of reliability for the correctness of the programs.

The LPARL programs represent a hybrid synthesis of large-angle nonlinear multi-rigid-body dynamics with distributed coordinate representation of linear elastic deformations used in structural mechanics. The determination of deformation functions is obtained from separate structural mechanics programs (e.g. REXBAT, SNAP, etc.).

The programs integrate the equations of motion for the models using the integrator described in "JPL Technology Utilization Document No. CP-2308, VODQ/SVDQ/DVDQ, Variable Order Integrators for the Numerical Solution of Ordinary Differential Equations"; this is a variable self-adjusting step-size integrator. In all the programs, interconnections between bodies in the tree are described by parametrized inter-body rotation matrices, i.e. by a set of gimbals having 0, 1, 2, or 3 degrees of rotational freedom. The gimbal axes can be sequentially locked, and the nominal orientation of the gimbal axes can be input by appropriate gimbal mounting rotation matrices.

The system configuration is described entirely by input data. The only subroutines required from the user are those which give the external force and torque on each body, the gimbal torques (e.g. due to torsional springs, dampers,

motors, etc.), and any auxiliary differential equations, such as for control systems. These subroutines have access to the instantaneous state of the system. Gravitational effects are built into the dynamical equations of motion and need not be user specified.

The program output consists of a tabular listing of the state history, as well as plots of these time histories generated on the SC-4020 plotter.

Programs

i. NBODY I and NBODY(F) I (F: Flexible bodies)

Based on formulation described in [3, 5, and 6]. Separate summation procedures are involved for eliminating moments of the unknown interbody forces and constraint torques arising at gimbals with fewer than three degrees of freedom. NBODY(F) I normally makes use of fixed-base cantilever mode shapes for terminal flexible appendages.

ii. NBODY II and NBODY(F) II

Based on formulation described in [7] and using certain key ideas described in [9]. Base points are shifted from barycenters to interconnection (hinge) points, about which interbody forces have zero moments. Equations of motion are written directly for sub-trees "outboard" of a given interconnection, and formulation is simplified. Computer program runs slightly faster than NBODY I series.

iii. NBODY III and NBODY(F) III

Based on formulation described in [9]. The incidence matrix for each topological tree configuration controls the systematic contribution from the motion of each individual body to the overall dynamic equations. The force equation of the system is maintained in the program, which provides the translational motion of the main body and other bodies. It also simplifies the coefficient matrices of the torque equations about the hinge points. The vibration equations of the terminal flexible bodies are independent. Reduction of the degree-of-freedom at a specific hinge point can be handled easily by eliminating the corresponding gimbal rotational angle and the associated torque component equation. This program is very efficient.

iv. P-NBODY and P-NBODY(F) (P: Perturbation)

Based on [8] and extensive additional revisions by J. Y. L. Ho. The multibody combining procedure employed has no closed-form analytical description of the final equations, but is implemented on the computer by means of a recursive computational scheme which in effect compiles the equation of motion in numerical rather than analytical form. In this program, the assumption is made that the total motion of the flexible multibody spacecraft is representable as the superposition of a nominal motion, obtained by conceptually "rigidizing" each flexible body but preserving localized flexibility at the joints, and a small perturbation corresponding to the linear elastic oscillations of the flexible appendages. The perturbation equations are thus variational equations, i.e. equations linearized about a nominal trajectory of the associated rigidized system. P-NBODY thus provides linear equations in analytical form.

v. Remote Manipulator System (RMS)

This program is based on the model of a chain of flexible interconnected bodies,

and derives its name from its application to the Space Shuttle's Remote Manipulator System. Formulation of the program is given in [4]. It employs three classes of mode shape functions for interconnected flexible bodies, which are not restricted to beams.

Language: The series programs consist of an assemblage of subroutines written in FORTRAN V compatible with the UNIVAC 1108-1110/Exec 8 operating system.

Contact: The program was developed by Lockheed Palo Alto Research Lab.

Availability: It can be obtained on a case by case basis by contacting

G. Margulies
Lockheed Palo Alto Research Laboratory
Palo Alto, California 94087

J. H. L. Ho
Lockheed Palo Alto Research Laboratory
Palo Alto, California 94087

Subjective Comments: The LPARL programs have been used to study various multi-body gravity-gradient configurations (without flexible bodies), and were successfully validated with similar work at Bell Laboratories (in the mid 1960's). They were used for a study of the attitude motions of the spinning SKYLAB (with flexible booms) at NASA, MSFC. Finally, an extensive comparison was made with JPL's MBDY Program.

Martin Marietta Computer Program (MMCP)

The MMCP is a general-purpose computer program system for dynamic simulation and stability analysis of passive and actively controlled spacecraft. The program system will be used in design of attitude control systems and for evaluation of total system performance including both time domain response and frequency domain stability investigations. The program provides capability to examine total system dynamic characteristics including interaction effects between rigid and/or flexible bodies, control systems and a wide range of environmental loadings.

The program system has the capability of redimensioning itself to suit the problem at hand. For example, there may be a trade-off between the total number of bodies and the number of admissible functions (or normal modes) considered to represent each body. The program is redimensioned so as to make efficient use of the available computer memory.

The development of the program system has used the most general form of Lagrange's equations, including auxiliary nonholonomic, rheonomic conditions of constraint. Lagrange multipliers are calculated and used as interaction forces/torques in such a manner that the prescribed constraints are maintained. The nonlinear flexible/rigid dynamic coupling effects are considered in unabridged fashion for each of the bodies of the system and for the system as a whole. Normal vibration modes may be used in representing elastic deformation of a given body member, but an adequate series of admissible functions suffices.

Nonlinear time domain response is calculated for the synthesized dynamic system through use of the Runge-Kutta numerical integration algorithm; state vector concepts are embodied, thus the dynamic system is represented by a series of nonlinear first order differential equations including both plant dynamics and controller dynamics.

For frequency domain studies, the system of nonlinear first order equations is numerically linearized resulting in a set of first order differential equations involving state variable perturbations. The linearized equations correspond to the autonomous system; that is, they are in the form $\dot{y} = Ay$.

The so-called QR algorithm is used to extract closed-loop roots of the characteristic matrix A. Also, a similarity transformation is performed on A, giving a similar dynamic system A* which can be easily manipulated to provide forward-loop, return-loop and loop-gain transfer functions. The transfer functions are automatically displayed in Bode, Nichols or Nyquist form. Also, the program has capability to generate root-locus displays.

Language: The entire program is coded in FORTRAN IV with exception of the job control language which, of course, is installation dependent. Double precision arithmetic is used with all real numbers (not integers) in the IBM and UNIVAC versions. The CDC version does not require double precision due to its 60-bit word length.

Hardware: The program is currently running on the IBM 360/370, and will be implemented on UNIVAC 1108 and CDC 6500 computers by July 1975.

Reliability: As a monitor on accuracy and reliability of the program system, total angular and linear momentum vectors as well as system kinetic and potential energies are always calculated and displayed. The results of all "torque-free" demonstration problems considered to date are positive in that system momentum is conserved accurately and total energy (kinetic plus potential) decreases monotonically.

Contact: The program was developed by MMC for NASA, Goddard Space Flight Center. Contact

Carl Bodley
Martin Marietta Corp.
Denver, Colorado 80201

Availability: Final documentation is scheduled to be delivered to the NASA (Goddard Space Flight Center, Greenbelt, Maryland) by May 1, 1975. It will be available through the Computer Software Management and Information Center (COSMIC), University of Georgia, Athens, Georgia, 30601, by approximately mid 1975.

Subjective Comments: This dynamic simulation program is, by design, a general purpose program for synthesizing and analyzing complex structural-mechanical systems. The program system can, on option, be used to examine, in detail, nonlinear time domain response and/or linear frequency domain response.

MBDY Attitude Dynamics Subroutine Package (MBDYADSP)

The MBDYADSP is a collection of eight programs. All the routines in this package are based on the rigid-body tree equations of [3] as rearranged in [5] to eliminate constraint torques. However, three of the subroutines allow the attachment of flexible appendages to any of the rigid bodies of the tree, and a constant spin of the appendage base may also be accommodated. The only situation (within the tree concept) that cannot be directly handled by one or another of these programs is that of a chain of bodies where intermediate bodies (rather than terminal bodies) in the chain are flexible. However, in this case, one can sometimes redefine the "boundaries" of a flexible appendage so that it is always a terminal appendage.

Since these are subroutines, the user must develop his own "calling" programs, integration subroutines, matrix equation solving (or matrix inversion) routines, etc.

Several of the routines are simply linearized or partially linearized versions of other routines in the package and, as such, are directly interchangeable with them, i.e., virtually no modification is required in the calling program. The presence of these linearized versions allows the user some latitude in trading off speed and accuracy for most efficient problem solving. All of the routines also provide the option of "rigidizing" a joint between rigid bodies or prescribing motion about that joint in a very simple manner. Following is a description of the subroutines.

Capabilities and Methods Used by Subroutines

1. MBDY (see also [10])

MBDY is designed to solve the rotational dynamics equations for a system of rigid bodies, hinge-connected in the form of a topological tree. The solution is given in terms of the inertial angular acceleration components of one body, arbitrarily chosen as the reference body, and in terms of the relative angular

accelerations of all the remaining rotational degrees of freedom.

The mathematical approach is that described in [5] and expanded in some detail in [10]. The resulting matrix equation is of the form $Ax = B$, where x is the desired angular acceleration solution vector (Nx1), A is a real symmetric time-variable matrix (NxN), and B is a vector of forcing functions (Nx1), (N = no. of system degrees of rotational freedom). This equation is then solved by a standard matrix equation solving routine each time the subroutine is entered with new forcing function values (e.g., at every numerical integration step).

ii. MBDYTL (see also [10])

MBDYTL is designed to solve exactly the same problem as MBDY. Again, the mathematical approach is that used for MBDY except that the equations are subjected to a strict linearization with respect to the reference body's angular velocity and acceleration as well as to the hinge rotation angles and their derivatives. The matrix A thus becomes a constant symmetric matrix which may be inverted only once to allow the calculation $x = A^{-1}B$ to be performed each time the subroutine is entered with new forcing function values. A much faster solution for x thus results with reasonably good accuracy for many problems.

iii. MBDYL (see also [10])

This routine is identical to MBDYTL except that further simplification of the system equations, beyond strict linearization, is accomplished. No body-to-body coordinate transformations are done at all in MBDYL. Thus, this routine will produce solutions at a slightly faster rate than MBDYTL, but with less accuracy.

iv. MBDYPL (see also [10])

MBDYPL is designed to solve the same problem as MBDY.

Using the same approach as for MBDY, but assuming that only some of the hinge rotations (and their derivatives) are small, the equations may be partially linearized so that at least a portion of A becomes constant. The constant part of A may then be inverted in the initialization process and x is solved for by matrix partitioning. The result is a somewhat faster solution to the problem than MBDY can provide, if the size of A is relatively large (at least 10 x 10), and if linearization is justified.

v. MBDYM

MBDYM is designed to solve the same problem as MBDY and the method of solution is the same as for MBDY. However, the algorithms used in the routine to form the equation $Ax = b$ have been modified, and in some cases simplified, to speed up the solution. One significant modification changes the manner in which the system is described by substituting the concept of a "connection" rather than a hinge. A connection is defined as a one, two, or three degree-of-freedom joint.

vi. MBDYFR

MBDYFR is designed to solve the rotational dynamics equations for a system of rigid bodies, hinge-connected in the form of a topological tree, where each of the rigid bodies may have a single flexible appendage. Further, any rigid body carrying a flexible appendage is permitted to spin provided that the spin rate is nominally constant (with small variation).

As in MBDY, etc., solutions are desired for inertial angular acceleration components of the rigid reference body and for relative accelerations of the remaining rigid bodies about their hinges. In addition, motion of the flexible bodies must be obtained.

The approach taken is that described in [11]. Briefly, this involves developing finite element models of each elastic appendage, assuming small deformations, and taking into account the appendage base's nominal motion (if spin is present). The homogeneous form of the resulting vibration equation is then transformed to an uncoupled set of coordinates, i.e., modal coordinates, and truncated by deleting unwanted modes of deformation. If spin is present, the resulting modal vibration equations are complex and must be converted to an equivalent real form by taking advantage of certain eigenvector orthogonality relationships.

The appendage equations may then be coupled to each other as well as directly to the rigid-body-tree formulation which remains in almost the identical form shown in [3, 10]. Reference [12] shows the system equation development in great detail. Again, the result is a matrix equation of the form Ax = B, where A is a time variable, symmetric matrix. Its order is determined by the number of rigid body degrees of freedom and the number of modal coordinates retained by the user to describe elastic appendage deformations.

Because the appendage equations have been derived in first order form to allow transformation to an uncoupled coordinate set (for the case of spin), two appendage modal coordinates appear in the solution vector, x, for each eigenvalue retained in its model.

vii. MBDYFN

This subroutine solves the same problem as MBDYFR, except that no spinning appendages are permitted. All appendage base bodies must have a nominally zero spin rate.

The approach is much the same taken for MBDYFR [12]. However, the resulting modal models of each appendage have real eigenvalues and eigenvectors. The system equation, Ax = B, is still solved at every entry with new forcing functions, since A is time variable (and symmetric).

viii. MBDYFL

The subroutine solves the same problem as MBDYFN. However, hinge rotations and their derivatives are restricted to be "small" as are the reference body angular rates.

The method of solution is the same as for MBDYFN but, in addition, a strict linearization process is applied to the equations under the assumption that the hinge motion is small. This results in a constant A matrix and allows its inversion once and for all in the initialization process.

Language: UNIVAC 1108 FORTRAN V

Reliability: The subroutine MBDYM which solves the rotational dynamics equations for the unrestricted rigid-body tree, and on which the other routines are based, has been checked against an older version, MBODY. MBODY in turn was compared to a similar program (see LPARL), and the two agreed to within Univac 1108 roundoff error. The test problem consisted of 7 rigid bodies and 11 degrees of freedom. The LPARL program took approximately .023 seconds for 500 repeated evaluations of the matrix equation (without solution of that equation) while MBDYM was timed on the JPL Univac 1108 at approximately .025 seconds for the same 500 evaluations.

The rigid-body routines in both their unrestricted and linearized versions have been used to simulate T.V. camera slewing dynamics on both Mariner Mars Orbiter 1971 and the Mariner Venus/Mercury Spacecraft (MVM) 1973 as well as vehicle dynamics (including magnetometer boom, high-gain antenna, solar panels, and T.V. platform) during a trajectory correction maneuver of the MVM craft. All of these simulations

agreed very closely with gyro and control system telemetry received from the spacecraft. The two routines which handle the nonspinning case with flexible appendages have also been dynamically checked to within round-off error against older, special-purpose programs written to solve Likins' hybrid-coordinate formulation. MBDYFR, which handles the spinning appendage case, is the only routine which has not, as of this writing, been dynamically checked against some other independent program.

Contact: The program was developed by the Jet Propulsion Laboratory. Contact
Gerald E. Fleischer
Jet Propulsion Laboratory
Pasadena, California 91103

Availability: The complete MBDY Subroutine Package is available from COSMIC (The Computer Software Management and Information Center), Barrow Hall University of Georgia, Athens, Ga. 30601) (404-542-3265). The cost is a very nominal handling charge.

Spinning Structural Analysis Program Group (SSAPG)

The SSAPG consists of three programs: (1) Modified NASTRAN Spinning Structural Analysis Program; (2) Data Generation Program; and (3) Skylab Response Program. The group or programs performs a structural analysis of a rotating structure, yielding its eigenvectors, eigenvalues, and time response. (Although written as two separate programs, the Data Generation Program and Skylab Response Program may be considered conceptually a single entity). This program group was developed to analyze a deliberately spinning Skylab; however, it is sufficiently general to permit analysis of any system of bodies that may be described as a core or central rigid body to which are attached an arbitrary number of flexible appendages and (also) a second rigid body. A controller in the form of Control Moment Gyroscopes (CMG's) is also added as a module for providing closed loop attitude control of a space vehicle. However, the program group is not limited to spacecraft. The modified NASTRAN Spinning Structural Analysis Program can be used to analyze structures spinning at constant angular velocities, such as rotating shafts, blades of spinning turbines, and rotating linkages.

a. The modified NASTRAN Spinning Structural Analysis Program is a finite element program that accepts the structural properties describing the system (written in NASTRAN format). It provides the system eigenvalues and eigenvectors. Given the axis of rotation, the modified NASTRAN program will generate the centrifugal force matrix and the resulting geometric stiffness matrix, the Coriolis acceleration matrix, and the centripetal acceleration matrix. Combining these matrices with the elastic stiffness matrix and mass matrix generated by the original unmodified NASTRAN, a complete set of matrices is now available for the original NASTRAN to use in performing its eigenvalue analysis.

Although the original NASTRAN complex eigenvalue subroutine was inefficient (slow) for the large scale spinning Skylab being analyzed, it is understood that subsequent modifications to NASTRAN have eliminated this deficiency. If the new NASTRAN eigenvalue analysis still is too lengthy, other more efficient eigenvalue routines will have to be substituted, such as the fine EISPACK developed by Argonne Laboratories. The modified NASTRAN Spinning Structural Analysis Program provides capability for performing analyses of rotating structures. This was accomplished by incorporating spin equations into NASTRAN (NASA Structural Analysis Program). The NASTRAN program is unaltered as far as original capabilities are concerned. The spin equations are coded into a dummy subroutine already in NASTRAN which is called by DUMMOD1 entering a DMAP statement referring to this dummy subroutine. The user of this program must be able to construct input data for NASTRAN (including input to the spin equations) and also understand some basic NASTRAN terminology.

b. The Data Generation Program constructs a number of matrices needed for the Skylab Response Program. The required inputs are the eigenvalues, eigenvectors, and grid point coordinates furnished by the modified NASTRAN Spinning Structural Analysis Program. It is limited by storage to a maximum of 30 eigen-

values and 300 degrees of freedom. A more detailed description of the eigenvalue analysis is included in [13] and [14].

Using input from an eigenvalue program, the NASTRAN program, and experimental results, this program computes data that will be used by a SKYLAB response program as constant input. This program evolved into a separate unit because many lengthy computations in the response program may be held constant while other critical terms varied over a series of runs. The output from this program is stored on magnetic tape and used as an input tape to the response program.

c. The Skylab Response Program uses as direct inputs the outputs of the Data Generation Program (since the two can be considered conceptually as a single program). Its outputs are time domain responses of the attitude, attitude rate, and linear and angular displacements of the system components. This can be modified to print out any desired system state as a time response. The program is described in more detail in Enclosure 3, including the assumed CMG closed-loop controller. This program is limited to a maximum characteristic matrix [A] ($Q = AQ$, when Q is the state vector) of 68 x 68. This limits the number of eigenvectors to 30.

This program computes the coefficients of a set of linear differential equations (the set is variable in size). The coefficients are put into matrix form; this matrix is inverted yielding a simultaneous solution for the highest order derivatives. The set of differential equations is then numerically integrated with respect to time via a 4th order Runge-Kutta method. It is the purpose of this program to speed up the calculation time for these mathematical operations.

Language: The programs of this group are all written in FORTRAN IV.
Reliability and Accuracy: The same as the overall NASTRAN.
Developer: The program was developed by Computer Sciences Corporation for NASA, Marshall Space Flight Center (MSFC). Contact
Dr. Sherman M. Seltzer
Marshall Space Flight Center
Huntsville, Alabama 35812
Availability: The programs are available from MSFC at no cost, either as program listings or actual documentation.
Subjective Comments: The SSAPG was developed to perform the above described analysis for large scale rotating structures consisting of connected rigid bodies and attached flexible appendages. The Unified Flexible Spacecraft Simulation Program (UFSSP) performs part of the task retaining nonlinearities. The disadvantage of this more accurate (nonlinear) protrayal of the equations of motion is that the dimension of the problem that can be attacked and implemented successfully by a computer is considerably less than can be done for the linearized SSAPG. In the development of the latter, it was felt that the steady state equilibrium (i.e., a constant spin rate about which perturbations are permitted) was sufficiently accurate since most systems envisioned are either stable or altered to be so; hence, large state displacements and rotations are not expected.

Structural Performance Analysis and Redesign (SPAR)

SPAR is a general purpose, finite element structural analysis computer program. It models complicated aerospace structures as assemblages of elements such as beams, plates, and shells. Current capabilities include static analysis for point loads or motions, inertial loads, temperature distributions, and pressure loads; normal mode analysis including effect of prestress and spectral shift; and bifurcation buckling calculation.

Any problem of the types $Ax = y$ or $(A - \lambda B)x = 0$ can be solved where x and λ are unknown and A and B are any linear combination of the system mass, stiffness, and geometric stiffness matrices.

The outstanding characteristics of SPAR include:

High computation speed - up to 20 times as fast as NASTRAN for large problems.

Low computer core requirement - many significant problems can be worked using demand keyboards.

Simplified minimum input.

The high performance of SPAR is achieved by extensive use of the executive routines and the secondary storage available on third-generation computers, the fact that the program is modularized, an efficient library system, and efficient sparse matrix routines and iteration procedures.

A data base comprised of automatically-maintained direct access libraries (DAL's) is used to provide a means of information exchange between an array of totally independent absolute programs (processors). The DAL's are generated, serviced, and interrogated using a system of routines collectively called IMSYS (information management system). IMSYS is written almost entirely in FORTRAN, and operates nearly identically on UNIVAC, CDC, and IBM systems.

A data set already in a DAL may be replaced by using IMSYS to enter into the DAL a new data set having the same name. Among other services, IMSYS provides the ability to interrogate DAL's to determine their contents, delete data sets, selectively copy data sets from one DAL to another, to copy (and subsequently retrieve) entire DAL's from disk or drum to tape, etc.

Through IMSYS, DAL's can be created and utilized in a wide variety of ways, without concern for DAL internal format, or for the procedure implemented by IMSYS routines in actually executing the data transmissions.

The crucial factor in implementing the DAL approach was to minimize the input-output activity required by IMSYS without using an excessive amount of core storage for directories, and without restricting DAL generality (e.g. the form and number of data sets in a single DAL, the number of separate DAL's simultaneou ly activated, etc.). The method used in IMSYS, which involves dynamic swapping of segments of DAL tables of contents between secondary storage and small in-core directory area, has shown excellent performance. In SPAR applications, most DAL operations have been observed to require a single input or output access.

Other significant features include:

The computer execution costs achieved by the basic static solution routines are very close to the minimum that can possibly be attained using direct solution procedures, generally affording substantial savings when compared with the costs associated with constant or variable-width band matrix, "active column", or partitioning solution methods used in other programs. A number of run-time comparisons with other well-known finite element programs have been made. In no case has any other program been found to execute as fast as SPAR, even for structures ideally suited to band matrix or active column methods; and in large problems very substantial differences in run time (e.g., factors of 10 or more) have often been observed.

SPAR attains its low execution costs through the use of a solution technique based on a direct elimination procedure which (unlike band-matrix, active column, etc.) techniques avoids virtually all unnecessary arithmetic operations by recognizing in essentially complete detail the sparsity characteristics of network stiffness matrices.

The element repertoire includes a very general class of beam elements and 3 and 4-nose plate/shell membrane and bending elements (isotropic, orthotropic, or aeolotropic). Beam elements may include effects of shear center/centroid offsets, transverse shear deflection, and non-uniform torsion. Section properties (moments of inertia, area, shear deflection constants, torsion constants, principal axis orientation, shear center location, etc.) may be input directly, or the program will compute them for many types of sections (wide-flanges, boxes, tube, angles, zees, channels, tees), given only the section dimensions as input.

Plate/shell membrane and bending element formulations based on "hybrid" variational methods are included, providing substantial improvements in the accuracy of displacements and stresses. Three-dimensional liquid and solid elements are planned for the near future.

Loadings include point forces and moments at joints, pressure loads, non-zero specified joint motions (in "oblique" directions, if required), and thermal loading. Oblique joint restraint is allowed.

The data input apparatus is designed to minimize the amount of manual effort required to prepare data decks for large-scale applications. Extensive use is made of "libraries" of beam and shell section properties, material constants, etc., in generating problem definitions. This method usually reduces greatly the amount of manual effort (and probability of error) in preparing data decks for large structures. For example, the section properties of a beam are "defined" by referring to the applicable set of data in one of the libraries. Accordingly, the detailed definition of each unique section appears in the input data only once, regardless of how many elements have that particular section.

Multi-dimensional "network generators" of input data for element definitions, position coordinates, constraint, applied loading, etc. are provided.

Input: An arbitrary number of data sets may be entered into a DAL using IMSYS. When a program requests IMSYS to insert a data set into a DAL, the following information must be given:

A "name" (several words, which may be numeric or alpha-numeric) uniquely identifying the data set.

An error status code for the data set.

Information describing the size, configuration, and position (in core) of the data set.

IMSYS then stores the data set in the DAL, and updates the DAL table of contents enabling the data set to be retrieved from the DAL by direct acces.

Output: To extract a data set from a DAL, the following information is supplied to IMSYS:

The name of the data set.

The position in core where the data set is to be stored, when retrieved from the DAL.

An error action code indicating to IMSYS the action to be taken if various abnormalities arise; e.g. if no such data set exists, or if found to be marked in error, or if there is not sufficient space in core to hold the data set, etc.

Language: FORTRAN IV, UNIVAC 1108, EXEC 8 only, (A CDC 6600 version will be completed simultaneously for NASA Langley Research Center.), Assembler, Map.

Usage: The program has not yet had extensive use but application is starting on several NASA Marshall Space Flight Center (MSFC) projects including the Space Shuttle, the Large Space Telescope, and the Spacelab.

Contact: Larry Kiefling
Marshall Space Flight Center
Huntsville, Alabama 35812

W. D. Whetstone
Lockheed Palo Alto Research Laboratory
Palo Alto, California 94304

James C. Robinson
NASA Langley
Hampton, Virginia 23665

The program was developed by Lockheed Missiles and Space Company (LMSC). The development was funded by MSFC with contributions from NASA Langley and the U.S. Army.

Availability: It has been submitted to COSMIC for release.

Subjective Comments: Some of the advantages of SPAR are: low computer execution cost, large capacity, demand terminal operational capability, single input logic, hybrid polygon elements, modular logic, and flexible output capability.

Unified Flexible Spacecraft Simulation Program (UFSSP)

UFSSP is designed to provide a generalized analytical tool for simulating the

dynamic response of multibodied, controlled, flexible space vehicles subjected to environmental and operational disturbances.

UFSSP accepts as a configurational model any general system of up to eighteen bodies interconnected in a topological tree configuration with terminal flexible members. Up to three rotational degrees of freedom are allowed between adjacent bodies, and the interconnection points can move with respect to the bodies.

The program user basically supplies the number of bodies in his model, the topology and geometry of their interconnections, the mass properties for each body, and a modal description for each flexible body. The digital program then utilizes an inductive algorithm [15] to synthesize and integrate the complete dynamic, kinematic and control equations of the specific system model. Thus, a wide variety of space vehicle configurations can be readily simulated by merely modifying the program's input data; no derivation of dynamic equations is ever required of the user.

The spacecraft model is permitted to undergo a completely arbitrary large-angle motion, and prescribed forces and torques may be applied to all bodies. The flexible characteristics of the terminal members of the model are described by means of three-dimensional orthogonal deformation functions (structural dynamic modes). These modes provide a realistic description of the vehicle's flexibility with as many degrees of freedom as is necessary for a particular problem. The required input structural dynamic modes are derived separately in any of several computer codes (e.g., NASTRAN, SAMIS, TRW's Structural Modal Analysis Program). An automated input routine allows the user to input the modal data either on cards, magnetic tape or as a disk file.

The UFSS program provides a complete simulation system by incorporating basic subroutines for automated calculation of the following system characteristics: rigid and flexible body dynamics, generalized control system actuation, interconnection and internal flexible member dynamic loads, generalized Keplerian orbit parameters, and environmental disturbance effects.

The control subroutine (CONTROL) forms the active forces and torques that act upon the dynamic system. CONTROL accepts automated sensor inputs (e.g., dynamic/orbital variables, unit vectors to the earth, sun, etc.) from the dynamics subroutines, and returns forces and torques to appropriate control points on user-defined bodies.

The structure of CONTROL is such that the user can very quickly and efficiently synthesize his specific control system. This is due to the fact that CONTROL contains a series of packaged subroutines that typify the most common elements normally encountered in control system analysis (e.g., single and double pole networks, hysteresis, dead-zone etc.). The user can also code part or all of his control system in FORTRAN if he so desires.

The environmental disturbance subroutine provides automated calculation of gravity-gradient, geomagnetic, aerodynamic and solar radiation pressure disturbances.

Determination of the dynamic loads within the system model is divided into two basic phases. In the first phase, the forces and moments acting at each interconnection between adjacent bodies are calculated within the program via information available from the dynamics subroutines. In the second phase, the mode-acceleration method is utilized to calculate internal loads within any desired member of the structural model for a given terminal body. These internal loads are calculated by a separate, stand-along program operating upon a special loads history tape generated by the UFSS Program. Thus, if only the interconnection loads are desired, these can be obtained from the UFSS Program time history run itself by merely setting the LOAD option flag to "ON".

Significantly, the present UFSS Program simulates the dynamics of rapidly spinning flexible bodies whose structure can be defined by a space curve (e.g., rotors, booms, antennae) or by a plate model (e.g., a solar array). The method of handling the spinning of flexible bodies is based on addition of a modified displacement function (curvature shortening effect) for each flexible body, thereby including the centrifugal stiffening effect as well as other acceleration effects. This method utilizes the non-spinning real modes as input for each flexible body; all spin corrections are computed internally by the program.

Language: The program is written exclusively in FORTRAN IV.
Hardware: The program is operational on both the CDC 6000 series computers and on the UNIVAC 1108 system. Additional equipment includes a total of four tape or disk units, plus standard printers and readers as well as a Calcomp plotter. The core storage requirement for the program is approximately 146K octal words. A time-share, rigid-body version has also been implemented which requires only 58K octal words of core.
Usage: The UFSS program is presently checked out and operational at the following installations: TRW Systems Group, the NASA Marshall Space Flight Center (MSFC), the NASA Johnson Space Center (JSC), and ERNO Raumfahrttecknik GmbH (Bremen, Germany).
Contact: D. J. Ness
TRW Systems Group
Redondo Beach, California 90278
Availability: The UNIVAC 1108 Versions are available from TRW Systems Group.
Subjective Comments: In addition to routine simulation tasks, the program is ideally suited to provide simulation support in proposal and other short-duration efforts due to its rapid turn-around time, far reaching generality, and operational status. It has been used in this manner at TRW for studies on such vehicles as the Background Measurement Satellite, Pioneer F/G, Space Shuttle, and the ESRO/ESTEC Geosynchronous Observatory Satellite (GEOS).

A Skylab I simulation effort provides a prime example of the versatility and rapid response available with UFSSP. Reacting to a pre-launch request from the NASA Marshall Space Flight Center (MSFC) TRW and NASA personnel used the program to analyze a potentially serious problem in the Apollo Telescope Mount (ATM) deployment system. The problem involved dynamic interaction between the flexible deployment cables, the deployment motor dynamics, and the reaction control system on the S-IVB Orbital Workshop (OWS). Within 48 hours from task initiation, the latest version of the UFSS Program was installed and operational on the MSFC computing system, and simulation results were available for the complete two-body deployment model--including the three-axis reaction control system of the S-IVB, the deployment motor dynamics, the flexible cable dynamics and multiple options for studying cable breakage and motor failures.

The program was also used to simulate the constrained deployment of the damaged OWS solar array system prior to the successful "space repair" work performed by Skylab Commander Pete Conrad and his crew. The deployment equations for the individual solar array wings, including their sliding motions and separation from the face of the OWS, were coded in the control subroutine; simulation results for the system were completed once again within 48 hours.

Westinghouse Electric Computer Program (WECP)

The WECP simulates the dynamics of a spacecraft consisting of an arbitrary tree structure of multiple rigid members, with or without partially constrained joints (i.e., joints with less than 3 degrees of freedom). This includes discretized models of elastic continua.

The following items can be included in the dynamics: gravity gradient and solar pressure effects, passive (damping and elastic reaction) and active (controller) internal moments, orbit eccentricity, and termal (direct earth and direct plus reflected solar) effects. The RAE-1 application computations include specific values for hinge moment coefficients.

The general tree structure of rigid members can include segmented rigid models of elastic continua. Internal elastic reactions are then formed through multiplication of a rigidity constant by a deformation gradient, obtained as a weighted sum of incremental rotation components per unit segment length. Values for weightings are supplied by numerical differencing theory. Damping reactions are computed from relative angular rates, easily obtained from a programmed algorithm. External torques (e.g., due to solar radiation and gravity gradient) conform to known physical relationships.

The computer program is subdivided into five parts. Initial setup (Part 0), General system constants (Part 3) and General numerical integration (Part 4) are applicable to any tree-structure satellite. Parts 1 and 2 contain the necessary modeling and inputs to apply the program to the Radio Astronomy Explorer (RAE-1) satellite.

The program can be adapted to various satellite configurations by changing the satellite parameters, the control parameters, and the initial conditions.

Typical satellite parameters are as follows: Total number of rigid members; number of locked hinge axes and their identification; the inertia matrix and mass of each member; undeformed relative orientations of adjacent members (reference shape); hinge coefficients; incidence matrix and connection matrix as defined in [2]; effective surface area for solar radiation; thermal bending constants (proportional to absorptivity/conductivity ratios). On the other hand, typical control parameters and initial conditions are as follows: Initial angular rate and orientation of each member (e.g., the latter may conform to the previously described undeformed shape); number of orbits to be simulated and the number of computer print intervals desired per orbit; numerical integration error control parameters which trade off economy vs. accuracy.

The entire program is based on [2, 3]. The program is fully documented in [16]. The documentation includes annotated Fortran listings, with specific equations cited from accompanying supporting analysis and from biographical references.

Language: The program is written exclusively in FORTRAN IV.

Usage: The program has been successfully applied to the RAE-1 satellite. The results compared favorably with an independent Lagrangian analysis [17].

Error control in 4th order Runge Kutta integration can be set to 10^{-4} rad and 10^{-6} rad/sec for direction cosines and angular rates, respectively. Model fidelity has been demonstrated adequate for a 4-hour simulated duration.

Stable computation has been observed with orbit eccentricities up to one-tenth.

Contact: J. L. Farrell
Westinghouse Electric Corporation
Baltimore, Maryland

The program was developed by Westinghouse for NASA, Goddard Space Flight Center.

Availability: The program is available in the form of [16] at the Clearinghouse for Federal Scientific and Technical Information, Springfield, Virginia 22151.

Subjective Comments: A positive feature of this program is that it is fully documented [16]. The documentation includes full annotated listings and analysis, as well as straightforward direction for program usage.

Discretization of a continuum is successfully modeled, including specific coefficient formulations and adjustments for thermal deformations.

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Torsional Systems

N. T. Tsai

Naval Ship Research and Development Center

INTRODUCTION

A torsional system is one of the major media for mechanical power transmission and, thus, its dynamic behavior has been the subject of many studies. A typical torsional energy transmission system is the drive train of an automobile. Another commonly used system is the propulsion system of a marine vessel as shown in Fig. 1. It consists of shafts, bearings, propellers, gear trains and turbine or reciprocating engines. The vibration analysis of such a system usually involves longitudinal, transverse, and torsional modes and their coupled motions. Of these motions only the torsional vibration will be examined in this chapter. Other modes of vibration can be found in the chapter reviewing rotating machinery.

The purpose of this chapter is to discuss the functioning and the capabilities of available computer programs on torsional vibration analysis. A short description of the algorithms most widely used in torsional analysis will be presented as background information for the summaries of programs that follow. Because of limited resources, this review may not be complete.

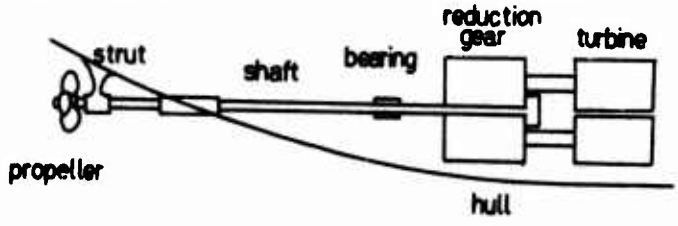

Fig. 1 Ship propulsion system

METHODS OF TORSIONAL ANALYSIS

Depending on the purpose of the torsional analysis, there are different methodologies to meet one's requirements, such as periodic response, transient response, optimization, and strength. For most torsional systems the primary design requirements are the result of a periodic response or harmonic analysis. The typical information needed includes natural frequencies, modal shapes, and critical speeds. This periodic analysis has been successfully handled by the well known Holzer method and the extended-Holzer or Prohl-Myklestad method. On the other hand, recent advances in technology have required the transient analysis of complex torsional systems. Several analysis techniques such as the finite element and bond graph methods have been utilized for this purpose. They will be described briefly here with the Holzer type transfer matrix method which can also handle the transient analysis with a normal mode approach.

Holzer-Myklestad-Prohl Method

In the Holzer method [1], the shaft elements are considered as massless springs with torsional stiffness and the disks are considered as lumped masses. A transfer matrix is developed for each segment to relate the state variables across the element or station. Thus, the Holzer method utilizes transfer matrices to propagate the effect of assumed and known boundary conditions through an N station system model. The independent variable is frequency, from which the mode shape is determined. If the mode shape satisfies the boundary conditions the assumed frequency is a natural frequency. Both trial-and-error and iteration procedures have been used in determining the frequency.

When damping is considered the transfer matrix is not affected, but the mass and stiffness elements are replaced by complex quantities. Equivalent stiffness and inertia are used in the cases of geared torsional systems. Similar approaches are used in modeling the branches in the torsional systems. If the gear ratio is n, then the values of the stiffness and inertia of the geared shaft are multiplied by n^2. Although the Holzer method calculates the response from one end to the other end, a different algorithm has been proposed in which the response of a branched system is obtained by calculating it from the junctions and then simultaneously from all the branches toward their respective ends [2].

When the shaft is considered as a beam structure for the purpose of obtaining the shear stress and bending moment of the shaft under torsional motion, the Prohl-Myklestad method can be used to compute progressively the response of the system from one station to the next in a manner similar to the Holzer method. The shaft is considered as a lumped mass connected by the massless rod elements. This method can also be used to calculate the natural frequencies and mode shapes of the shaft whirling, which is coupled flexure-torsional vibration.

Finite Element Method

In this method, the equations of motion of a discrete model are solved by either modal superposition or direct integration to obtain the transient response of torsional systems. Because the available finite element programs such as NASTRAN [3] are rather large compared with programs based on the Holzer method, this method has been used mostly in the transient analysis of complex torsional systems.

In the modal method only the vibration modes of the structure in a selected frequency range are used as degrees of freedom, thereby reducing the transient equations into an uncoupled form for easy solution. In the direct integration method, the degrees of freedom are the displacements at grid points.

The modal method is usually more efficient in problems where a small fraction of all modes is sufficient to produce the desired accuracy. The direct method is usually more efficient for problems in which a large fraction of the vibration modes are required to produce the desired accuracy.

Method of Electric Circuit Analog

This method of analysis [4,5] is similar to the method of four-pole parameters [6] in which torque is analogous to voltage, velocity to current, inertia to inductance, flexibility to capacitance, damping to resistance, and gear to transformer. Thus, the torsional system is considered as an equivalent electric circuit and treated as a circuit problem. This implies that the system is linear and can be represented mathematically by a set of differential equations with constant coefficients. This approach has the advantage that the entire body of circuit analysis, including computing algorithms, can be used directly where applicable. Steady-state and tran-

sient analyses have been carried out with this method. Branched torsional systems can be modeled easily with the electric circuit analog. In addition, a modular approach is possible with this method.

Parameters and impedance of components of a torsional system can be measured and used as building blocks for the construction of other models. Programs using this method are proprietary in nature. However, since there are many electric circuit programs available, users can establish the circuit block diagrams and conduct the analysis with the circuit analysis program.

Method of Bond Graphs

The method of bond graphs is specifically designed to expedite the modeling of physical dynamic systems in which power interactions are important. Although this method has been in existence for more than a decade, many engineers are not familiar with its advantages. Recent studies [7,8,9] indicate that this method can be successfully applied to the analysis of various geared torsional vibrations.

In this method, the physical systems are treated in a manner similar to that of the method of electric circuit analog. However, the junction structure of the bond graph method has much more variety than the Kirchhoff structure used in the electric circuit theory. In addition, this method can be used to model the torsional system and its interface with other energy media, such as that in a torsional-electric or torsional-hydraulic system. A bond graph is made up of elements indicating where energy is generated, stored or dissipated, connected by lines called bonds. Each bond represents a uni-modal power transmission. The modular approach is used to construct the model. Both steady-state and transient analyses can be conducted with the bond graph method. It is capable of analyzing nonlinear torsional vibrations in lumped or distributed models.

Selecting a Method

In selecting the method of analysis the user should keep in mind what his requirements are. In general, a torsional analysis method should provide the capabilities to model the following aspects of a dynamic system throughout the prescribed range of operations:

1. Speed transmission through gear sets, rollers, belt-pulley, etc.
2. Friction; both friction to ground (bearing) and friction associated with slippage (roller drive, for example). Both linear and nonlinear friction models should be provided.
3. Distributed shaft span and gear span, to include systems for which the wave length of torsional mode energy is of the same order or shorter than one or more shafts in the gear span.
4. Lumped mass and compliance to account for such dynamic effects as gear inertia, gear tooth compliance, and flywheel inertia.
5. Backlash phenomena of meshing gear pairs. This phenomena causes considerable dynamic loads in torsional systems where transient vibration is of a primary concern.

PROGRAM SUMMARIES

There are a large number of computer programs available for torsional analysis, ranging from single shaft Holzer programs for natural frequencies to transient torsional dynamics programs for nonlinear responses of complex geared systems. These computer programs have been classified into two categories: (1) general purpose programs which are general structural dynamics programs and thus capable of handling complex torsional systems, and (2) special purpose programs which are developed for the special needs of tor-

sional analysis. Since the general structural dynamics programs are reviewed in another chapter here and in [10], only a few programs will be discussed here with the understanding that other structural dynamics programs can also be used for torsional analysis. A more detailed summary of the available special purpose programs follows the discussion of general purpose programs. Note that two recent papers on rotor bearing systems [11] and structural members [12] have also included torsional programs in their surveys.

General Purpose Programs

The most familiar general purpose structural analysis program probably is NASTRAN. It has the capabilities to do nonlinear transient analysis. But it is usually used in the linear transient analysis and harmonic analysis of torsional systems. Using the algorithm of the finite element method, NASTRAN is suitable for large complex systems. Because of the large size and rigid structure of NASTRAN, it is not recommended for the occasional user or for simple analyses. Only if the program is available in-house and experienced programmers are available, will NASTRAN be considered for general torsional analysis.

On the other hand, there are other, simpler structural dynamic programs available for linear torsional problems. One of them is the General Bending Response Program [13] and SHAFTRAN [14] for the analysis of marine propulsion systems as well as ship vibrations. The algorithm of the Prohl-Myklestad method is used in the development of these programs, which can handle coupled flexure-torsional problems in addition to torsional vibration. A short summary of these two programs is listed here:

General Bending Response Program (GBRP)

Date: First version was issued in 1962.
Capability: Steady-state and transient vibration analyses of torsional systems with damping
Method: Finite difference method
Limitation: Applicable to linear systems
Input: Physical parameters of torsional systems. Sampled time history of the forces acting on the system in the transient analysis.
Output: Displacement, torque, and rotation in tables and/or SC 4020 plots
Language: FORTRAN IV
Hardware: CDC 6000 Series, UNIVAC 1108, IBM 360 (100 to 150K Core on CDC). The core requirement of the newer version is strictly a function of the application.
Usage: The program has been in application since 1962 on ships, propulsion systems, and various machinery. In addition to torsional analysis, it can handle transverse, longitudinal vibration and shaft whirling problems.
Developer: Elizabeth Cuthill and Francis Henderson
Computation and Mathematics Department
NSRDC
Bethesda, MD 20084
Availability: Available through developer
Other Comment: Program is well documented and tested. An NSRDC Report, "An Updated Guide to the Use of General Bending Response Program (GBRP)," by M.E. Golden and F. Henderson is in preparation for the newer version of the program.

SHAFTRAN

Date: Developed in 1971
Capabilities: Vibration of a general marine propulsion system due to steady-state excitation from the propeller.
Method: Transfer matrix method
Limitation: Applicable to linear systems
Input: Physical parameters of the systems under investigation
Output: Natural frequencies, mode shape, and critical speed as well as forced damped responses of the system such as stress and deflection under steady load.
Language: FORTRAN IV
Hardware: CDC 6600 Computer
Usage: It has been checked for accuracy with a number of exact solutions for simple structures. Results of illustrative examples for application are presented in the documentation.
Developer: Marine Vibration Associates
Belmont, MA 02178
Availability: Available through the developer.
Other Comment: The program can handle a variety of shaft-type problems such as whirling and flexiable rotor motion. The program treats the shaft elements as simple Euler-Bernoulli beams or as generalized Timoshenko beams.

In addition to the general purpose structural dynamic programs, general purpose simulation languages such as the Continuous Systems Simulation Language (CSSL) of CDC and Continuous Systems Modeling Program (CSMP) of IBM can also be used to calculate the responses of geared torsional systems [15]. To use these programs the user should provide the dynamic equations and physical parameters as input while the computing systems will supply the integration and processing capabilities.

Bond Graph Programs

There are two bond graph programs available for torsional analysis. ENPORT-4 [16], which is discussed in the chapter by Karnopp, has been used in many computer systems for various dynamic system simulations. Less distributed and not as well documented is the program DBOND [17], which has been specifically modified for the analysis of complex nonlinear torsional systems [9]. Both programs may be obtained by users from their respective developers. Because these programs have been developed for general dynamic systems, they are very simple to use once the modeling process is understood. Compared with other general purpose programs, bond graph programs provide a simpler and graph-oriented simulation program.

Special Purpose Programs

Summaries of the available special purpose programs are listed below. Some of the summaries are taken from information supplied by developers of the programs.

CADENSE-22

Date: September 1970
Capability: Torsional critical speeds of a geared system
Method: Holzer Method
Input: Physical parameters and operational range of torsional systems
Output: Natural frequencies, mode shapes, and critical speeds
Language: FORTRAN IV

Hardware: CDC 6600 (20000 words)
Usage: Applicable to systems with branches, gears, epicyclic gears, and elastic torsional connections.
Developer: Mechanical Technology, Inc.
968 Albany-Shaker Road
Latham, NY 12110
Availability: Available through developer
Other Comment: Able to handle planetary gears. Steady-state analysis.

CADENSE-23

Date: September 1970
Capability: Damped torsional response of geared systems
Method: Holzer Method
Input: Physical parameters and frequency range of geared systems
Output: Torque and displacement distribution and gear tooth force
Language: FORTRAN IV
Hardware: CDC 6600 (40000 words)
Usage: Applicable to systems with gears, branches, single reduction gear and constraint to ground with stiffness and damping
Developer: Mechanical Technology, Inc.
968 Albany-Shaker Road
Latham, NY 12110
Availability: Available through developer
Other Comment: Can accept excitation in the form of gear manufacturing error. Steady-state analysis.

CADENSE-24

Date: February 1973
Capability: Transient response of geared systems with backlash and branches
Method: Fourth-order Rung-Kutta Method
Input: Physical parameters, torque or velocity history, initial displacement or velocity
Output: Torque, displacement in time domain with plots of torque
Language: FORTRAN IV
Hardware: CDC 6600, UNIVAC 1108 (64000 words)
Usage: Dynamic loads on gears and rolling mills, drive train start-up, etc.
Developer: Mechanical Technology, Inc.
968 Albany-Shaker Road
Latham, NY 12110
Availability: Available through developer
Other Comment: Has been verified and runs effectively on those cases studied [18].

TAGS

Capability: Critical frequencies and forced undamped response of geared systems
Method: Holzer type transfer matrix method
Input: Physical parameters of torsional systems
Output: Torsional critical frequencies and forced dynamic responses
Hardware: CDC 6600 (32000 words)
Usage: Applicable to branched gear train including lumped inertia, torsional spring to ground and external loads.
Developer: S. M. Wang
I.B.M. Research Center
Yorktown Heights, NY 10598

Availability: Professor I.E. Morse, Jr.
University of Cincinnati
Cincinnati, OH 45221
Other Comment: Shear stress may be calculated. See Ref. [19] for detailed information.

TABU

Capability: Steady-state response of branched systems
Method: Stiffness matrix method
Input: Physical parameters of torsional systems
Output: Natural frequencies and mode shapes
Language: FORTRAN IV
Hardware: CDC 6600 (30000 words)
Usage: Linear branched torsional systems
Developer: Southwest Research Institute
8500 Culebra Road
San Antonio, TX 78284
Availability: Available through developer
Other Comment: A forcing function and modal damping may be included to calculate shear stress.

TOFA

Date: August 1971
Capability: Frequency response of shaft-disk systems
Method: Holzer Method
Input: Physical parameters, limited to 50 segments
Output: Natural frequencies and mode shapes
Language: FORTRAN IV
Hardware: CDC 6600 (12000 words)
Usage: Applicable to rotor systems in coolant pumps of nuclear reactors.
Developer: Atomic Energy of Canada, Limited
Chalk River Nuclear Laboratories
Chalk River, Ontario
Availability: Available through developer

Rotating, Twisted Beam

Capability: Steady-state response of torsional structures
Method: Linear lumped parameter model with Holzer-type method
Input: Lumped parameters of torsional structures
Output: Natural frequencies and mode shapes
Hardware: FORTRAN IV
Usage: Applicable to torsional and coupled torsional-flexure motions
Developer: Rochester Applied Science Associates, Inc.
Rochester, NY 14601
Availability: Available through developer or COSMIC Program No. LAR-11461
Other Comment: Can also handle twisted beam stress analysis

COSMIC Program No. MFS-2485

Capability: Steady-state responses of linear systems
Method: Holzer Method
Input: Lumped parameters of torsional systems
Output: Natural frequencies and mode shapes
Language FORTRAN IV
Hardware: IBM 360

Usage: Free-free and free-fixed shaft systems
Developer: Rocketdyne
6633 Canoga Avenue
Canoga Park, CA 91304
Availability: Available through developer or COSMIC Program No. MFS-2485
Other Comment: Another version of this program (COSMIC Program No. MFS-1488) is designed for IBM 7094 computing systems.

TWIST

Date: January 1975
Capability: Static and free dynamic torsional systems, including gears.
Method: Holzer-type transfer matrix method
Input: Lumped parameters of torsional systems
Output: Natural frequencies and mode shapes
Language: FORTRAN IV
Hardware: CDC 6000 Series, IBM 370, UNIVAC 1108
Usage: Applicable to twisting motion of torsional systems under steady loading.
Developer: P. Y. Chang
W. D. Pilkey, University of Virginia, Charlottesville, VA
Availability: The Structural Members User's Group, Ltd.
P. O. Box 3958
Charlottesville, VA 22903
Other Comment: Mass can be modeled as lumped or continuous, or a combination of both.

CONCLUDING COMMENTS

1. Of the many special purpose programs available, most are developed for the steady-state analysis of geared systems. Although some can be extended to transient analysis, only program CADENSE-24 provides the transient analysis model with backlash capability.
2. The general purpose programs can meet the need of most users. These programs are different from each other in methodology and size. Thus, the advice of a specialist is recommended in selecting and applying the appropriate program.
3. There is a need for a user-oriented program capable of doing both steady-state and transient analyses. The program must be well-documented, easy to learn, and accompanied by good input and output formats and processors. The bond graph programs EMPORT-4 and DBOND seem to meet these requirements. But most users are not familiar with this method.
4. The results of the torsional analysis should be compared with the performance of a complex torsional system to assess the effect of the various physical parameters in the programs. At present, only a few developers have attempted to do so (see references [9] and [18], for example). Some comprehensive test data are needed to conduct a meaningful validation of these programs.

ACKNOWLEDGMENT

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Crash Simulation

M. P. Kamat

Virginia Polytechnic Institute and State University

INTRODUCTION

In our present day environment, crashes of light vehicles like automobiles, general aviation aircraft, and helicopters are a frequent occurrence. A light vehicle crash is a dynamic phenomenon involving a complex interaction between structural and inertial behavior. Under crash conditions the structural response is characterized by large deformations, inelastic material behavior, and fracture under highly impulsive loads.

In attempting to make vehicles crashworthy, one of the prime considerations is the passenger trauma, that is to say, the degree of the severity of forces and acceleration vectors experienced by the passenger. USAAVLABS study [1] reveals that for light fixed-wing and rotary-wing aircraft the peak accelerations during a majority of crashes were below human tolerance limits. These test results, together with those of NASA's rather limited tests (under advisement by FAA) on light fixed-wing eight passenger aircraft [2], appear to indicate that with an efficient absorption of the kinetic energy in the structure of a light vehicle the majority of crashes could be made survivable. Needless to say, it is implicitly assumed herein that crashworthy restraint systems and adequate fire protection ensure passenger survivability.

At least until extremely efficient restraint harnesses are developed, the most practical way of ensuring survivability under front end collisions is to improve the vehicle's interior energy absorption behavior [3]. DeHaven [4] aptly points out that "safe transportation of people in any type of vehicle must of necessity apply the practical principles which are used by every packaging engineer to protect goods in transit." One of these basic principles is that the inside of the container must be designed to cushion and distribute impact forces over the maximum area of the contents and have yield qualities to increase deceleration time.

TYPES OF ENERGY ABSORBERS

In principle, an energy absorber is a device which has some form of a mechanism for dissipating the kinetic energy of a body under impact. Most energy absorbing devices fall under one of five categories: (i) Hydraulic or pneumatic; (ii) Material Deformation; (iii) Friction; (iv) Extrusion; and (v) Compaction and/or Compression. This classification is based on the prime mechanism of energy absorption in each case.

Energy absorbers of the first kind are very common on most modern day vehicles in the form of shock absorbers. However, in spite of their reusability, shock absorbers of this kind would be too heavy and inefficient for application in vehicle collision.

Energy absorbers of the second kind rely on buckling, inelastic yielding, crushing, shearing or tearing of the material for energy absorption. The material may be either ductile or brittle, and may even be strain-rate sensitive. Because the mechanism of energy absorption in this case (failure) is an irreversible process, such energy absorbers obviously cannot be reused. Nonetheless they are more efficient than the hydraulic or pneumatic kind because of their high energy dissipation density and stroke efficiency. Frangible or collapsible tubes, honeycomb cores, and balsa wood are energy absorbers of this kind.

Energy can also be dissipated through the mechanism of friction. Vehicle brakes are an example of an energy absorber wherein this mechanism is used to dissipate the kinetic energy by converting it into heat and mechanical energy. The energy dissipated is, however, directly dependent on the coefficient of friction and the magnitude of the normal pressure. Although the latter could be controlled, the former is dependent on the environment and may change with time due to wear of the two surfaces in contact. As a result, such absorbers are not adequately reliable to find applications in vehicle collisions requiring high energy-dissipation.

Extrusion devices utilize the principle of extrusion of materials to dissipate energy partly by compression of the material and partly by viscous effects. They are velocity sensitive, and thus their energy dissipation capacity is automatically controlled to suit impact conditions. Extrusion devices are very efficient energy absorbers because of their reliability and ability to withstand repeated impacts. Inspite of their high initial cost, they may be used in preference to devices belonging to the material deformation category because unlike the latter, the former are not one-shot devices.

Energy absorbers in the last category utilize the principle of compressing low density gases in high strength containers. Besides yielding high specific energies they can be maintained in a collapsed state prior to use and thus save space. A combination of the phenomena of compaction and material deformation is used in vehicle cushions provided by a loose and arbitrary arrangement of steel drums and rubber tires [8].

An extensive bibliography and details on the specifics of a variety of energy absorbers can be found in References [5] through [8].

ENERGY ABSORPTION PREDICTION

There are essentially three ways of predicting absorption of kinetic energy in impact. These are (i) Analytical (ii) Numerical and (iii) Hybrid.

Analytical

This technique can be used only for those energy absorption devices for which the governing differential equations of the mathematical model of the device can be formulated and solved to obtain closed form or numerical solutions. Such cases are in general rare, although under very simplified assumptions such techniques have been successfully used [6, 7] for analysing certain types of energy absorption devices. The drawback of these analyses is that they are essentially quasi-static and do not account for effects like strain-rate sensitivity of the material and randomness of the loadings involved. They can at best be used for qualitative purposes only.

Numerical

Energy absorption, especially through material deformation is a highly nonlinear phenomenon involving a structure of complex geometry and material properties under time varying loads of magnitudes that cause the structure to buckle, warp, undergo large deformations, and respond inelastically. Because of these complexities, the prediction of energy absorption becomes a

formidable task which defies a deterministic theoretical solution. Improved constitutive models have emerged over the past few years [9, 10] which attempt to describe material behavior quite adequately in most cases. And, although a lot more may be desirable in this regard, this to a certain extent removes a part of the arbitrariness and indeterminancy of the phenomenon of material deformation due to impact and makes it more amenable to analysis through approximate numerical techniques like the finite element displacement method.

An excellent review of some of the simulators available for analysis of the response of vehicles in impact can be found in references [9] and [11]. These simulators and a few others will be examined for their capabilities with regard to energy absorption in the next section. It need be mentioned in passing that attention is being restricted here to simulators which can analyze energy absorbers belonging to the material deformation category only. Multi-purpose simulators for analyzing absorbers of other kinds are, to the author's best knowledge non-existent and recourse has to be made to simplified analytical techniques for their analyses.

Hybrid

This technique attempts to exploit experimentally observed features of an energy absorption device to obtain an improved response through a rigorous numerical analysis. This approach is gaining popularity with investigators since it tends to minimize the deterioration of the quality of response resulting from a lack of account of localised effects or from use of an inappropriate constitutive model for numerical analysis. In this technique the constitutive model is most often replaced by experimentally obtained load-response data wherein localised effects are also accounted for in a gross sense. Application of this technique has been successfully accomplished in but a few simplified loading situations, and significant work remains to be done to extend its scope and make it cost effective with numerical technique.

SIMULATORS FOR ENERGY ABSORPTION PREDICTION

Simulators which are rather broad in scope, well documented, non-proprietary and which could be used mainly for prediction of energy absorption through material deformation will be considered next for their capabilities. It must be remarked however, that there exists a host of special purpose simulators designed specifically for a particular type of energy absorber. Although broad details about some of these simulators are available no documentation exists, as they are intended for in-house use only. One such simulator is the one developed by Hirsch [8]. Still another is the one developed by Warner of Brigham Young University [12] for analyzing hydro-cell cushions.

1. MARC-CDC, ANSYS, DYCAST

These simulators, although broad in their scope, can also be used for the prediction of energy absorption through material deformation under controlled conditions of impact. However, several additional features, not presently in these simulators, would certainly enhance their usefulness for prediction of energy absorption. These simulators cannot presently account for strain-rate effects which may well be important under certain circumstances, and, with the exception of MARC-CDC, the other two simulators do not allow for large strains, which appears to be a serious disadvantage as regards energy absorption prediction. Although there is a provision for impulsive and random loadings in MARC-CDC and ANSYS, they, like all other simulators to be described below, do not provide any information regarding probabilities of failure or probable estimates of the important response parameters. MARC-CDC

& ANSYS are very well documented but DYCAST is presently under development by the Grumman Aerospace Corporation under joint NASA/FAA sponsorship.

2. ACTION

This simulator, currently being developed at the Virginia Polytechnic Institute for NASA Langley [13], although not as broad in scope as MARC-CDC, ANSYS, and DYCAST, has nearly the same capabilities for nonlinear transient analysis. As an additional feature, ACTION also admits experimental data in the form of a load-response surface instead of the usual stress-strain curve. This hybrid feature, when fully developed, may extend the scope of this simulator quite significantly. With the exception of [14], the simulation capabilities of this program remain relatively untested.

3. WHAM II & III

This simulator, developed by Belytschko [15], possesses most of the desirable features for energy absorption prediction. It accounts for large strains, strain-rate effects and can model compacting materials. In addition to general transient loading, it admits impulsive but not random loading. It can predict dynamic as well as elastic and plastic buckling. It has been used very successfully for the analysis of automotive sheet metal under impact and crash loadings [16]. This simulator thus appears to possess a tremendous potential for use in the analysis of vehicles for crashworthiness, and for energy absorption prediction.

4. LANDIT

This simulator developed by the Jet Propulsion Laboratory [17] is a very special purpose simulator which predicts the dynamic landing response characteristics of axisymmetric impact attenuating vehicles consisting of a rigid payload and a crushable impact limiter system. Impact is assumed to occur against a rigid barrier whose roughness is accounted for in the analysis. It has a very limited and rather crude description of the crushable material properties and, as such, the energy absorption prediction is only approximate.

5. KRASH

This simulator, developed by Witlin & Gamon of Lockheed California, is a hybrid simulator which uses experimentally based frame element stiffness and masses for nonlinear transient response. Structural inelasticity is accounted for with the help of stiffness reduction factors which in essence are specified by the user based on prior experience. Although limited in terms of element library, correlation of simulation and experiments has been extraordinarily good when applied to helicopter crashes [18]. The program is designer oriented and thus would serve as a good tool for preliminary crashworthiness design studies.

6. HVOSM

This simulator developed by McHenry, et al. of Calspan [19] could be regarded more as a hybrid simulator than otherwise. The simulator has been used mainly for modeling automobiles in crash situations. The vehicle is modeled using lumped masses and springs. The deformable barrier is characterized by specifying its stiffness per unit of area, which is input as a fifth order polynomial in deflection, the coefficients of which are supplied by the user based on previous experience with similar barriers. Although the simulator provides an excellent description of vehicle rigid body kinematics in three dimensions, its suspension and tire forces, and phenomena like skidding,

cornering, etc., the material models of the barrier and the vehicle proper are deficient and rely heavily on the experience and intuition of the designer for their intrinsic values. In short, its usefulness for energy absorption prediction is rather limited at the present time.

7. BARRIER VII

This simulator developed by Powell [20] is a simulator, more of a hybrid type because of its rather crude and user-dependent model of the vehicle. The vehicle is modeled as a rigid body surrounded by springs whose stiffness constants are specified by the user--a task which requires a lot of ingenuity and prior knowledge of the deformation characteristics of vehicles of similar type. It lacks the good vehicle rigid body kinematics description of HVOSM but provides a wide collection of elements like beams, cables, columns, springs, friction and viscous dampers at the disposal of the user for modeling a strictly two-dimensional deformable barrier. A unique feature of this simulator is a rather sophisticated logic for vehicle-barrier interaction and separation.

It appears that the simulator developed by Thompson [21] would also qualify as a hybrid simulator. The simulator however is proprietary, and details about its capabilities other than those in [21] are not available.

Information regarding use and availability of the non-proprietary simulators described can be obtained by contacting their developers or custodians listed in the Appendix.

CONCLUSIONS

A study of the various simulators available to date reveals that much more by way of an analysis capability may be desirable than what is available. But then, assuming cost is not a consideration, is a strict rigorous numerical analysis of a highly complex phenomenon, if at all feasible, capable of answering all the questions with any fidelity? If not, is the hybrid approach which appears to be gaining popularity more cost-effective and reliable and the route which research investigators should adopt in future? Several factors have to be weighed very seriously before we can even attempt to answer such questions. The size and speed of computers to come, our improved understanding and modeling of material behavior under dynamic conditions, ease with which local effects like cross-sectional deformations, joint eccentricities,and joint compliances can be modeled, reliability of designer's intuition, success in extending hybrid approach to general loadings, and above all, cost of simulation will undoubtedly be some of the controlling factors.

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APPENDIX

This section provides a brief description of peculiarities and availability of the simulators referred to in another section.

MARC-CDC [22]

Capability: A static or dynamic nonlinear analysis (large deflections, plasticity and creep) of structures and continua. The program has an extensive finite element library consisting of some 19 finite elements.
Method: The program uses Von-Mises yield criterion with isotropic and kinematic hardening. The solution technique is the modified Newton-Raphson method coupled with explicit time integration of the nonlinear equations of motion.
Language: FORTRAN
Hardware: Program runs on CDC, IBM and UNIVAC machines.
Usage: The program has been used extensively and is found to be reliable in most cases. It is the best known and most widely used program for nonlinear analysis.
Developer: David Hibbit and Associates
MARC Analysis Research Corporation
105 Medway Street
Providence, Rhode Island 02906
Availability: The source or object form of the program along with documentation is available from the developer at a fee which is negotiable.

ANSYS [23]

Capability: A static or dynamic nonlinear structural analysis and heat transfer analysis.
Method: Finite element displacement method. The program has seventeen different finite elements available for nonlinear analysis. It uses the incremental method of solution accounting for plasticity with isotropic and kinematic hardening. It uses the wave-front method coupled with an explicit time integration scheme for the solution of the nonlinear equations of motion.
Language: FORTRAN
Hardware: The program runs on CDC, IBM and UNIVAC machines.
Usage: The program has been used extensively especially by the nuclear industry and some indications of its reliability are available.
Developer: John A. Swanson
Swanson Analysis Systems, Inc.
870 Pine View Drive
Elizabeth, Pa., 15037
Availability: Program and documentation are available from the developer at a fee which is negotiable.

Dynamic Crash Analysis of Structures (DYCAST)

Capability: A static or dynamic nonlinear analysis (large deflections and plasticity) of structural models.

Method: Finite element displacement method. To the author's best knowledge it is the only simulator of its kind which has the most sophisticated capability for modeling plastic deformations using the most up-to-date theories of plasticity. It uses the self-correcting incremental method and an explicit time integration scheme to solve the nonlinear equations of motion.
Language: FORTRAN
Hardware: The program runs on CDC and IBM machines
Usage: Portions of this program are operational at the NASA Langley Research Center.
Developer: Harry Armen, Jr. and Associates
Research Dept , Plant 35
Grumman Aerospace Corporation
Bethpage, N. Y., 11714
Availability: The program, still under development, will be released at the appropriate time through COSMIC, 112 Barrow Hall, University of Georgia, Athens, Ga., 30602

Analyzer of Crash Transients in Inelastic or Nonlinear Range (ACTION) [13]

Capability: A static or dynamic nonlinear analysis (large deflections and plasticity).
Method: Finite element displacement method. It uses Von Mises yield criterion with kinematic hardening. The method used is the minimization of a functional which guarantees the satisfaction of the nonlinear equations of motion. Implicit time integration scheme is used for dynamic analysis.
Language: FORTRAN
Hardware: The program runs on CDC and IBM machines.
Usage: Portions of this program are operational at the NASA Langley Research Center, Hampton, Va., 23365
Developer: Robert J. Melosh and Associates
Department of Engineering Science and Mechanics
Virginia Polytechnic Institute and State University
Blacksburg, Va., 24061
Availability: The program, still under development, will be released at the appropriate time through COSMIC, 112 Barrow Hall, University of Georgia, Athens, Ga., 30602

WHAM II AND III [24]

Capability: A static or dynamic nonlinear analysis (large deflections, plasticity and creep) of structures and continua.
Method: Finite element displacement method. It uses the incremental method with dynamic relaxation and explicit integration to solve the nonlinear equations of motion. It is claimed to be a very fast simulator for transient analysis with speeds varying from 500 to 5000 element time steps per cpu second depending upon the type of element used.
Language: FORTRAN
Hardware: The program runs on CDC, IBM and UNIVAC machines.
Usage: In its limited usage there are indications of a rather high degree of performance and good simulation capability.
Developer: T. Belytschko
Department of Materials Engineering
University of Illinois at Chicago
Box 4348
Chicago, Illinois 60680
Availability: The source program cards are available from the developer at no cost.

Landing Dynamics Program for Axisymmetric Impact Attenuating Vehicles (LANDIT) [17]

Capability: Dynamic landing response of axisymmetric vehicles consisting of a rigid payload and a crushable limiter system.
Method: The nonlinear equations of motion are solved numerically in an incremental fashion.
Language: FORTRAN
Hardware: The program runs on IBM machines.
Usage: The program being very special purpose has had very limited usage and no indication of reliability is available.
Developer: A. C. Knoell
Jet Propulsion Laboratory
California Institute of Technology
Pasadena, California 91109
Availability: The program is available in source form at a cost of $550 from
COSMIC
112 Barrow Hall
University of Georgia
Athens, Ga., 30602

KRASH [25]

Capability: The program predicts the dynamic response of lumped masses interconnected by nonlinear beam elements. Each lumped mass has six degrees of freedom.
Method: The Euler's equations of motion for each mass are linearized incrementally and integrated numerically using a modified predictor corrector method. In each time step the increments of internal forces and moments are determined from the conventional stiffness matrix modified by user specified stiffness reduction factors corresponding to each of the six degrees of freedom.
Language: FORTRAN
Hardware: The program runs on IBM machines.
Usage: The program has had very limited usage for simulating helicopter crashes. Correlation of simulation and experimental results has been shown to be extraordinarily good.
Availability: The program is documented in Reference [25]. Additional details about the program are available from:
Eustis Directorate
U. S. Army Air Mobility Research and Development Laboratory
Fort Eustis, Va., 23604

Highway Vehicle Object Simulation (HVOSM) [26]

Capability: The program predicts the dynamic interaction of a vehicle with a deformable barrier. The vehicle is modeled as an assemblage of lumped sprung and unsprung masses interconnected by rigid links and springs.
Method: The Euler's equations of motion for each mass are linearized incrementally and integrated using Runge-Kutta or Adams-Moulton methods. Deformation characteristics of the vehicle and barrier are user specified.
Language: FORTRAN
Hardware: The program runs on CDC and IBM machines.
Usage: The program has been used fairly extensively but because it is hybrid its reliability is strongly influenced by the user.
Developer: R. R. McHenry
Calspan Corporation
4455 Genesee Street
Buffalo, N. Y., 14221
Availability: The program along with the documentation is available at no cost upon submitting a magnetic tape to

Dave Solomon
HRS-40, NASA Building
Department of Transportation
Federal Highway Administration
Washington, D. C., 20590

BARRIER VII [27]

Capability: The program predicts the dynamic interaction of a vehicle with a deformable barrier. The vehicle and the barrier are modeled using a variety of finite elements. However, the models are strictly two-dimensional.

Method: Finite element displacement method. The program uses an incremental method coupled with an explicit integration scheme to solve the linearized equations of motion.

Language: FORTRAN

Hardware: The program runs on CDC machines.

Usage: The program has had limited use and no indication of the reliability is available since its reliability is greatly influenced by the user.

Developer: Graham Powell
Department of Civil Engineering
University of California
Berkeley, California 94720

Availability: The program along with the documentation is available at no cost upon submitting a magnetic tape to
L. C. Meczkowski
HRS-12, Fairbanks Highway Research Building
Department of Transportation, Federal Highway Administration
McClean, Va., 20590

Highway Vehicle Simulation

James E. Bernard

Highway Safety Research Institute

INTRODUCTION

The problems of vehicle handling appeared in the literature as long ago as 1946, when the pioneering work of Olley [1] was published. Subsequent investigators developed linearized equations whose solution would yield the trajectory of a vehicle subject to time-varying steering or braking. More recently, efforts have been made to analyze various nonlinear aspects of the vehicle system, including, most notably, nonlinear tire properties. Perhaps the best source of an overview of this subject has been given by Ellis [2].

Since the equations of vehicle motion can become quite difficult to handle in the general case, it is not surprising that simulation has been a tool frequently used by vehicle dynamicists. Perhaps the best known early computer simulation was developed in 1961 by Ellis [3], who developed a three degrees of freedom analog-computer model for studying the lateral motion of an articulated vehicle. Since that time, the advent of more and more sophisticated computing equipment has led to the possibility of simulations of increasing complexity. Presently, many research facilities make use of highly nonlinear passenger car simulations with at least fourteen degrees of freedom, including six degrees of freedom for the vehicle body (the so-called sprung mass), a vertical or "wheel hop" degree of freedom for each wheel (or unsprung mass), and a spin degree of freedom for each wheel.

These many degrees of freedom can, of course, prove to be an unnecessary burden. We have therefore listed in Table 1 a set of user guidelines relating user goals to levels of sophistication in the simulations.

Of course, the range of validity of linear analysis and numerical integration of nonlinear equations overlap. Linear analysis is an appropriate tool to solve for key measures of directional response at a constant speed and at lateral accelerations on the order of .3 g or less, and the nonlinear calculations are meant to cover the entire range of vehicle performance. However, it is seldom appropriate to use nonlinear calculations where linear calculations will suffice, since (1) the convenience of the closed-form solutions is lost, and (2) the results are no more accurate than the linearized calculations.

It is important to note that in both the linear and nonlinear analyses there is a wide spectrum of sophistication which may be applied. In the case of linear analysis, a variety of useful information may be obtained from a very simple two degree of freedom model. The only input data required are the cornering stiffness of the tires, the wheelbase, and the weight and fore-aft location of the c.g. To have any hope of matching vehicle test results, however, several additional features are required, namely, (1) a roll degree of freedom, (2) compliance of the steering system, and (3) a more sophisticated model of the tire, including measures of camber and aligning moment.

Table 1 Guidelines for the Simulation of Handling Response

Objective	Tool	Remarks
Understeer/oversteer gradient, response time	Linear analysis	It can be very difficult to match empirical results with calculations. Extremely careful instrumentation is required, and steering compliance may be difficult to quantify.
Trends in performance in the nonlinear range (over .3 g on dry surface)	Nonlinear simulation. Neglect nonlinearities in the suspensions, steering lash, Ackerman effects and steering compliance, and load sensitive tire properties.	As the simulated maneuvers become more severe, the trends become questionable.
Accurate predictions from .3 g to .5 g	Requires accurate steering wheel-road wheel relationship, load sensitive lateral forces and aligning moments, roll steer, camber.	Careful tire modeling accurate to ten degrees slip angle and at least 1.5 times the static load may be necessary.
Accurate predictions over .5 g	Add bump stops, accurate tire data to extreme loads and angles.	Static distance to bump stops is important to ensure realistic fore-aft lateral load transfer distribution.

The same choices may be found in nonlinear simulations. In attaining increased accuracy of the solution, increasing expenses accrue resulting from the acquisition of additional input data, the numerical integration of more complex equations, and in the analysis of additional output.

Various levels of nonlinear calculations are given in the table. It should be emphasized that these levels are concerned with the prediction of vehicle response on a dry surface in the absence of drive torque or braking. If significant longitudinal forces during a steering maneuver are to be simulated, the predictive task becomes much more difficult, requiring a very careful analysis of the combined longitudinal and lateral forces at the tire-road interface, carefully chosen[1] brake dynamometer data and/or a detailed analysis of the engine-transmission-differential system. Prediction of vehicle performance on a wet surface remains a speculative undertaking because of the variability of the shear forces at the tire-road interface with small changes in water depth.

[1]It should perhaps be noted here that calculations of brake torque, given brake parameters such as drum radius, lining descriptors, etc., is not within the state of the art. Thus line pressure vs. brake torque points measured using a brake dynamometer should be procured for computations in which the absolute level of brake torque is important.

Several programs appropriate for use in all but the first task of Table 1 will be considered in this review. Linear analysis will not be considered here, since the code can be generated without extreme difficulty based on any of several well known formulations (see, for example, [4, 5]). This is not, we emphasize, to suggest that linear analysis is somehow trivial or inappropriate—while the code is not difficult to generate, the gathering of appropriate parameters for use in the linear analysis remains a difficult task, as does vehicle testing in the linear range.

The programs to be considered here will be reviewed in three areas, viz.:

(1) the tire model, which dictates the range of validity of the solution to directional response analysis and the usefulness of the simulation for combined braking and turning maneuvers
(2) special features such as algorithms allowing impact with outside objects
(3) documentation.

In addition, a short history of each program will be presented, along with a prognosis for future improvements.

The run costs for these programs, which are highly variable with the maneuver simulated and the computer hardware, will not be discussed in detail. In general, however, one should expect to run passenger car simulations on a digital computer for about $5/simulated second, with rapidly escalating costs for impact calculations and specialized graphics. Commercial vehicle simulations, with their attendant additional degrees of freedom for tandem axles and trailers, may be expected to be significantly more expensive.

Initially, four passenger car directional and ride response simulations will be presented. This will be followed by a review of available commercial vehicle simulations and some general remarks pertaining only to ride analysis.

PASSENGER CAR SIMULATIONS

HVOSM

The Highway-Vehicle-Object Simulation Program, now commonly called HVOSM, has been under development at Calspan Corporation (formerly Cornell Aeronautical Laboratories) since 1966. The development of HVOSM has been funded by the Federal Highway Administration (formerly the Bureau of Public Roads) to serve as a tool in the analysis of pre-crash safety and post-crash performance after impacting certain kinds of fixed objects. This program remains the most comprehensive numerical tool for the analysis of passenger car braking and handling, with exclusive features such as terrain tables allowing virtually arbitrary roadway inputs, and various tire options programmed to compute lateral and fore-aft forces resulting from tire/rough roadway interactions.

The shear forces at the tire-road interface are computed based on a cubic fit to measured tire data for the lateral forces, with rather rudimentary μ-slip curves for braking calculations. All tire shear force calculations are based on load-varying input parameters. A spin degree of freedom is included for each wheel, and the tire properties may vary from wheel to wheel.

In spite of the load-varying input properties, a cubic fit cannot adequately[2] match measured tire data across the entire range of slip angles likely to be encountered. Thus, it may be necessary to "tune" the curve fit to the measured data to emphasize either high or low slip angles.

Coupling between lateral and longitudinal shear forces is accomplished through the use of an updated version of the "friction ellipse" concept which may be expected to lead to reasonable results.

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[2]This is, of course, a matter of judgment. It is our experience that while the cubic fit may sometimes work out well, one is likely to encounter a mismatch on the order of 15% or so in the mid slip angle range.

HVOSM remains the only program with vehicle dynamics orientation to attempt impact calculations[3]. However, the representation of the impact dynamics remains very rudimentary and the barrier analysis is quite limited in scope.

The results calculated using HVOSM may be displayed using perspective views of the automobile-terrain. These graphics are plainly superior to anything else available. Perhaps the only serious unfavorable aspects of the graphics package and indeed the entire HVOSM package relate to documentation.

The documentation for HVOSM is spread out over at least ten separate volumes. Unfortunately, no one volume will suffice, since mathematical details of the various important mathematical models are presented in the order they were developed rather than in order of importance.

A remedy for this problem is currently being provided at Calspan, where comprehensive documentation for the latest version of HVOSM is being produced under sponsorship of FHWA. This documentation is due to be completed in early 1976.

The present version of HVOSM may be obtained from David Solomon, Chief, Environmental Design and Control Division, Federal Highway Administration, HRS-41, 400 7th Street, S.W., Washington, D.C. 20590.

Information on the ongoing improvements may be obtained from David J. Segal, Research Mechanical Engineer, Transportation Safety Department, Calspan Corporation, Buffalo, New York 14221.

The HSRI Passenger Car Simulation

The HSRI passenger car simulation has evolved from commercial vehicle simulations developed under sponsorship of the Motor Vehicle Manufacturers Association (MVMA). The program entails fifteen degrees of freedom including body motions, wheel bounce degrees of freedom, and wheel spin. Impact cannot be simulated, and the range of validity of the roll angles is limited in that changes in the track due to roll, as seen in a plan view, are neglected[4]. While the surface of the simulated roadway need not be smooth, fore-aft and lateral forces due to road undulations cannot be predicted.

The HSRI tire model was recently developed, under sponsorship of the National Highway Traffic Safety Administration (NHTSA), to allow accurate calculations of the shear forces at the tire-road interface across the entire range of slip angles likely to be encountered in limit maneuvers. The resulting algorithm, which entails user input normal pressure distribution at the tire-road interface and load sensitive input parameters, is capable of matching measured tire data within five percent or less, a substantial improvement over the previously available algorithms. This added accuracy is extremely useful if the simulated maneuver covers all ranges of sideslip angles, rather than remaining entirely in a high or low angle range.

The shear forces at the tire-road interface are computed through integration of the shear stresses over the contact patch. The closed-form solution to the integration yields both longitudinal and lateral shear forces, and thus obviates the necessity of any "friction ellipse" type operations. The calculations of the longitudinal-lateral force interactions have been found to closely match measured data.

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[3]The BARRIER programs [6] are useful for vehicle-barrier impact simulation. These programs, however, emphasize the barrier analysis and use a simplified vehicle analysis. Illinois Institute of Technology Research Institute is currently working on a more sophisticated vehicle-barrier interaction algorithm under FHWA sponsorship. R. L. Chiapetta may be contacted at IITRI, 10 W. 35th Street, Chicago, Ill., 60616 for details.

[4]In fact, this means that the numerical solution becomes invalid during the later stages of a rollover maneuver.

It should also be noted that the HSRI simulation is quite economical to run. These economies derive, in the main, from the methodology of the wheel spin calculations as explained in [7].

The documentation, which is complete and up to date, was published as an appendix to the technical report associated with [8]. Further information may be obtained from James Bernard, Highway Safety Research Institute, The University of Michigan, Huron Parkway and Baxter Road, Ann Arbor, Michigan 48105.

The University of Tennessee Passenger Car Simulation

The equations of motion and code for this simulation were developed by Frank Speckhart of the University of Tennessee while serving a temporary appointment with NHTSA. The model contains 19 degrees of freedom, including the usual ten degrees for the sprung and unsprung masses, plus time lags for the shear force build-up at each tire and a steering degree of freedom allowing the calculation of vehicle trajectory with "free" steering.

This simulation is distinguished by a very careful front and rear suspension analysis, in which equations of motion are derived based on the assumption of an inclined roll axis. The tire shear forces are computed based on the Calspan tire model as presented in [9].

While no user-oriented documentation has been presented in the published literature, a summary of the pertinent mathematical models is given in [10]. User information and further technical details may be obtained from F. H. Speckhart, Mechanical and Aerospace Engineering Department, University of Tennessee, Knoxville, Tennessee.

The NHTSA/APL Hybrid Computer Simulation

A hybrid computer simulation for studying vehicle dynamic response to braking and steering inputs is available at the Applied Physics Laboratory of the Johns Hopkins University (APL) [11]. This simulation has evolved from the hybrid simulation [12, 13] developed by the Bendix Research Laboratories for the National Highway Traffic Safety Administration. The tire model is quite similar to the algorithm used in the HVOSM simulation [14]. Current capabilities include simulation of vehicles with the following types of suspensions: (1) independent front and rear, (2) independent front and solid rear axle, (3) solid front and rear axles, and (4) solid front axle and solid rear axle with dual tires. Current enhancement plans include the addition of a tag rear axle with dual tires to permit simulation of buses and large trucks and a trailer model to permit simulation of towed vehicles.

The operation of the simulation is through a CRT and keyboard console. A dedicated printer copies all the output and input from the CRT for permanent hard copy. Special APL written software permits complete user selection of output variables and alteration of input data such as vehicle and tire model parameters. The interactive capability enhances the simulation usefulness and provides increased engineer/simulation productivity. The APL staff has an excellent track record for maintenance and improvements of the hybrid facility and cooperation with outside users.

The hybrid simulation can be used at APL in support of publicly funded contracts. For information on costs and for scheduling contact Paul Bohn, Applied Physics Laboratory, 11100 Johns Hopkins Road, Laurel, Maryland 20810.

COMMERCIAL VEHICLE SIMULATIONS

Introduction

While the dynamic performance of articulated vehicles has been studied by many investigators [15], the development of commercial vehicle simulations began in

the late 1960's. There are now three separate, well-documented simulations devoted to the analysis of commercial vehicles, as will be explained below.

AVDS3

The Articulated Vehicle Dynamic Simulations, or so-called AVDS programs, were developed at the IITRI, 10 W. 35th Street, Chicago, Ill., 60616. Documentation for the latest version, AVDS3, was published in 1973 [16].

AVDS3 simulates the dynamic response of tractor single-, double-, or triple-articulated vehicles. The dynamic characteristics of the vehicle system subject to road, vehicle, and environmental factors can be evaluated through computations of steering and braking demands for the specified vehicle while negotiating a prescribed trajectory. Such a procedure has been validated with generally good results [17].

The inverse procedure of the AVDS3 program—the input is the trajectory and the output is the steering and braking—is a distinct advantage in many analyses. For example, specific trajectories of interest can be studied for the demand on the driver-vehicle system. This is not easily done with programs which calculate trajectory based on driver input, since difficulties are encountered in matching the output trajectory with the trajectory of interest.

The tire model is a cubic fit for the lateral shear forces. However, in this case, the longitudinal forces are calculated based on the required longitudinal deceleration rather than computed based on "μ-slip" type relationships. Thus no wheel spin degree of freedom is necessary (or possible), and the lateral forces are modified in the presence of braking using a friction ellipse.

Since the longitudinal deceleration is input rather than output, AVDS3 is not particularly useful for braking analysis. This is particularly true when antiskid brakes are involved[5]. Further, to facilitate the inverse methodology, many simplifications were made—quasi-static load transfer is used in lieu of roll and pitch degrees of freedom, and tandem axles are not considered.

The HSRI Commercial Vehicle Simulations

The MVMA Truck and Tractor-Trailer Braking and Handling Project was begun at HSRI in mid-1971 with the expressed purpose of establishing a digital computer-based mathematical method for predicting the longitudinal and directional response of trucks and tractor-trailers. Two computer programs have been produced as a result of this project—a straight-line braking program for straight trucks, tractor-semitrailers, and tractor-semitrailer-full trailer combinations [18], and a combined braking and directional response program for trucks and tractor-semitrailers [19]. Some special features of these programs are discussed below.

To allow large payloads without unduly large axle loads, many trucks and articulated vehicles make use of tandem-axle suspensions. These suspensions commonly have a mechanism for "load leveling," that is, an attempt to maintain equal loading on each of the tandem axles in the presence of road irregularities. This mechanism may also cause unequal load distribution during braking, which may, in fact, result in so-called "brake hop." Thus, since the normal force at the tire-road interface has an important effect on the braking process, a careful analysis of tandem suspensions has been performed.

Seven separate tandem suspensions may be chosen by the user for use in the straight-line braking program. These include air spring suspensions, various four spring suspensions, and a walking beam suspension. Two tandem suspensions are user options in the braking and directional response simulation—

[5]Most air-braked vehicles built after March, 1975, will be equipped with air brakes to aid in compliance with Federal Motor Vehicle Safety Standard 121.

the simplest four spring suspension and a walking beam suspension. Validation runs for these latter two suspensions have been performed in both the straight-line braking and the braking and directional response program.

The brake systems commonly employed on commercial vehicles require special attention not usually necessary for hydraulic-braked vehicles. The brake system model may conveniently be divided into three sections. In a tractor-trailer air-brake system, the driver applies the brakes by operating a treadle valve which controls the air pressure at the brakes. In the first section of the model, the relationship between pressure at the treadle valve and the line pressure at the brakes on each axle is computed as a function of time. The time delay and the rise-time characteristics of the air brake system are represented in the simulation.

In the second section of the brake system model the relationship between line pressure and brake torque is modeled. The program user has two options: he may either input a table of brake torque for increasing line pressure, or ask the simulation to calculate a relationship for torque vs. line pressure, based on brake models contained in the computer program.

The third section of the brake model contains the antilock brake system simulation. This system is set up in a quite general form so the user may call for any of a wide variety of antilock control logic.

Unfortunately, the documentation for these programs rests in several separate volumes. This problem is currently being remedied at HSRI where complete and up to date straight-line braking documentation is being produced for publication in May, 1975, and braking and handling documentation for publication in July, 1975. Information on these programs may be obtained from James Bernard, Highway Safety Research Institute, The University of Michigan, Huron Parkway and Baxter Road, Ann Arbor, Michigan 48105.

The Cornell General Vehicle Model

Work on the simulation of articulated vehicles has proceeded at Cornell University since the pioneering analysis and simulation of Mikulcik [20]. Several basic changes and refinements have been made, many of which have appeared in the literature (for example, [21, 22]).

The present Cornell model allows the user to "construct" the vehicle using a building block approach. Thus a straight truck, tractor-semitrailer, and doubles and triples combinations may be modeled with minimal inconvenience. (Note this is the only available doubles or triples handling simulation in which the user specifies driver input commands and the trajectory is computed.) The axles may be modeled as either massless or having mass. In either case, a full six degrees of freedom are included for each sprung mass, and the suspensions are assumed to be made up of springs and dashpots having linear characteristics.

The tire model is a modification of a formulation developed at HSRI [23]. Since the formulation makes use of a closed-form integration of the shear stresses at the tire-road interface, no "friction ellipse" type of calculations are necessary to compute the interactions between the lateral and longitudinal forces.

An explanation of this model will be presented in [24]. There is, however, no published information in the form of a user's manual. The details of the use of the program may be obtained from A. I, Krauter, Shaker Research Corporation, Northway 10 Executive Park, Ballston Lake, New York 12019.

The Systems Technology Model

An analysis of truck and bus handling was performed by Systems Technology under NHTSA sponsorship during the period June 1972-March 1974. In the course of this contract, nonlinear equations of motion for a three-axle straight truck and an intercity bus were derived and implemented as digital computer simulations.

The tire shear forces are computed based on the Calspan tire model as presented in [9]. This simulation is distinguished by the capability to compute the effects of sloshing of liquid cargo as explained in [25].

The liquid cargo is assumed to be inviscid and incompressible, and the fluid flow is assumed to be initially irrotational and remain that way. The equations are for a cylindrical tank with circular cross section, and they are derived for an arbitrary liquid cargo level up to a point, then they are specialized to the half full case. The angular displacement of the plane of the water surface defines the wave motion, and the pitch and roll angles are assumed to be small perturbations. Only the first harmonics of these wave motions are included in the analytical representation, and this gave good correlation with the full scale results.

A source list of the program, and details of the pertinent mathematical models are presented in [25]. For further information, contact G. L. Teper, Systems Technology, Inc., 13766 South Hawthorne Boulevard, Hawthorne, California 90250.

SOME REMARKS CONCERNING THE SIMULATION OF RIDE

Each of the simulations in the preceding review (excepting AVDS3 which is not a useful tool for ride analysis) entail six degrees of freedom for the sprung mass. Thus accelerations at the passenger position can be calculated for the car or truck in question. However, it may well turn out that these simulations are not a particularly accurate tool for ride analysis, because the beam vibration of the sprung mass makes a significant contribution to the total sprung mass acceleration.

Beam vibration can also be handled in the simulation. For example, it may be assumed that the total motion response of the vehicle is given by a linear superposition of the beam bending modes and rigid body modes

$$\bar{S}(x,t) = \sum_{i=1}^{n} a_i(t) f_i(x) + \text{rigid body motion} \qquad (1)$$

where $\bar{S}$ is the vector describing the motion of some point in the vehicle, the f_i are mode shapes, and the time varying coefficients a_i are the solutions to ordinary differential equations of the form

$$\ddot{a}_i + \omega_{Ni}^2 a_i + 2\xi_i \omega_{Ni} \dot{a}_i = \sum_j b_{ij} F_j(t) \qquad (2)$$

where

$\omega_{Ni}\sqrt{1-\xi_i^2}$ is the "resonant frequency" of the ith mode;

ξ_i is the dissipation associated with the ith mode;

b_{ij} are so-called influence coefficients;

F_j are the applied forces from the suspension, engine mounts, etc.

The parameters ω_N, ξ_i, and b_{ij} may be obtained either from an experimental setup wherein the body is suspended by soft springs and subject to sinusoidal forces at various points along the frame, or from a detailed finite element numerical analysis. In either case, significant expense is involved. Further, the sources of some of the F_i, such as engine mounts, may be difficult to analyze. Appropriate references on these topics are [26] and [27].

The important point here is that while the simulations considered in this review may be used in a ride analysis, the results may be flawed due to the rigid body assumptions. However, the expense of a more meaningful analysis, including beam vibration, may be difficult to justify in many applications.

The most cost-efficient procedure in ride simulation is likely to be to neglect all side-to-side dynamics and concentrate efforts on a "bicycle" model entailing front and rear wheel motions and rigid body dynamics. Even when nonlinearities in the suspensions are included, the system will easily fit on most analog or hybrid computer facilities, thus enabling the user to perform economical calculations demonstrating ride trends for various roadways of interest. This methodology obviates the necessity to procure more input data than is justified by the accuracy of the calculations.

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Cable Systems

Nicholas F. Morris

Polytechnic Institute of New York

INTRODUCTION

Structures whose main load-carrying elements are cables are not a recent development in civil engineering. Suspension bridges have often been used to span long distances efficiently and in an aesthetically pleasing manner. Suspended railway electrification lines and guyed towers are other examples of cable-supported structures which have been used for many years. Recently, however, more daring structural applications of cables have been proposed. Roofs supported by cable nets have been built in many areas. These structures differ from more traditional cable-supported structures in that no stiffening elements have been designed into the structural system. It is accepted as a design premise that the cable net can carry a load by its increased nonlinear stiffness under deformation. While suspension bridges require nonlinear equations if they are analyzed by the deflection theory, the linearized deflection theory will result in a fairly accurate description of suspension bridge behavior. Cable nets, however, are usually much more nonlinear than suspension bridges. A simple deflection theory is not adequate for their analysis. This fact has long been recognized and a large amount of research work has been carried out on the static analysis of cable nets [1]. Unfortunately, the dynamic response problem has not received as much consideration. This lack of attention to dynamic analysis does not seem to have resulted in the design of any structures which have behaved poorly under wind load. It must be noted, however, that most previously constructed cable net roofs carry a fairly stiff roofing system along with a relatively large roof dead load. Future designs will not have these chacteristics; roofing will probably have very light membranes with very little dead weight.

Cable systems are used quite extensively in ocean engineering work as mooring lines. While the problems involved in their design seem quite different at first, from the design problems of cable net roofs, there does exist a great deal of similarity. The main dynamic load acting on each element is a random process; for mooring lines the process is due to wave action, while for cable nets the process arises from the wind field. Both sets of loads are a result of fluid-elastic forces arising from an assumed Gaussian velocity field for the fluid. As shall be seen the analytical work done on cable systems for both roof nets and mooring lines, although carried out independently, lead to the same computational procedures.

NOTATION

The following notation has been employed

A	=	cross-sectional areas of a cable element
a_{ij}	=	numerator of displaced element's direction cosine with respect to the global x axis, Eq. (2)

B_{ij} = factor containing nonlinear terms in cable tension, Eq. (4)
C = damping matrix for the net
C_{ir} = elements of the damping matrix
D_{ij} = linear factor in the computation of cable tension, Eq. (5)
D = displacement matrix for the net
$\dot{D}$ = velocity matrix
$\ddot{D}$ = acceleration matrix
E = modulus of elasticity
i,j = designation of end nodes for an arbitrary element
K = stiffness matrix for the net
L_{ij} = original length of the element i j
M = mass matrix of the net
m_{ij} = elements of the mass matrix
m_i = lumped mass at node i
P = load matrix
T_{ij} = tension in the element i j
T_{ijo} = original tension in the element i j
x_i,y_i,z_i = Cartesian coordinates of node i in the net
u_i,v_i,w_i = displacements of node i in the x, y and z directions

BACKGROUND

Cable nets are often highly indeterminate structural systems. Since their analysis usually requires a nonlinear formulation, some work has been carried out on their description by different mathematical models. In general, two basic approaches have been utilized [1]: continuous and discrete methods. In the first method the structure is considered as a continuous element. This representation is obviously true for a single cable but, of course, general nets are made up of discrete cables. An attempt is made, however, to replace the discrete configuration with a continuous anisotopic membrane. The membrane equations can then be solved with the aid of finite differences or the Galerkin method. If finite differences are employed, the continuous approach will be efficient only if the resulting algebraic equations have a lower order than those arising in the discrete method of analysis. This condition can occur if the number of cables and nodes in the structure is quite large. While the continuous method has been widely employed in the static analysis of cable nets, it does not seem to have been carried out for general nonlinear dynamic analysis. Galerkin's method has been used in conjunction with a continuous representation to solve for the dynamic response of double layered planar cable nets [2]. It was found that, while the method was less efficient than the finite element method for nonlinear static and dynamic analyses, it was a reasonable procedure for the computation of linear dynamic response. However, Galerkin's method does have the defect of requiring different programs for each new set of boundary conditions.

In general, it does not appear that the continuous method will be used in dynamic response programs. It is not general enough to analyze various net configurations. There are also problems involved in the description of interaction between layers in double layered cable nets. The familiar problem of programming general boundary conditions also arises in this formulation, although cable nets cannot have very many different support conditions; a cable is either restrained or free. The discrete, or finite element, method has been used to develop dynamic response programs. While it may be less efficient than the continuous method for a particular problem, the discrete method has the advantage of greater generality. It is also easier to program, and fit in with multi-purpose computer programs.

The various formulations of the finite element method differ in the choice of basic element. Some researchers have represented cable nets as a configuration

of straight cable elements while others have attempted a more accurate description with the aid of curved cable elements. The latter formulation is obviously more accurate, albeit, somewhat more difficult to program for nonlinear analysis. Most civil engineering applications, however, appear to have used straight elements. This is valid because cable nets are usually highly tensioned and the geometric surfaces that they span are made up of shallow curves. The largest discrepancy between the two types of discretization methods will, therefore, arise in the analysis of a single, loose cable. A larger number of straight elements will then be required to achieve the same accuracy as a given number of curved elements. This may not be too severe a restriction since the order of the algebraic system remains manageable in either case.

The dynamic equations which arise in the discrete method of analysis are easily derived. If the elements are taken as straight, the inertia equation for the x direction at an arbitrary node i can be written as [2]

$$\sum_j \left(T_{ij} \frac{a_{ij}}{L_{ij}}\right) + X_i - \sum_j m_{ij} \ddot{u}_j + \sum_r c_{ir} \dot{v}_r \tag{1}$$

in which

$$a_{ij} = x_i + u_i - x_j - u_j \tag{2}$$

$$T_{ij} = T_{ijo} + \left(\frac{AE}{L^3}\right)_{ij} (B_{ij} + D_{ij}) \tag{3}$$

$$B_{ij} = 0.50\,[(u_i - u_j)^2 + (v_i - v_j)^2 + (w_i - w_j)^2] \tag{4}$$

$$D_{ij} = (x_i - x_j)(u_i - u_j) + (y_i - y_j)(v_i - v_j) + (z_i - z_j)(w_i - w_j) \tag{5}$$

Similar equations can be written for the y and z directions [3]. It should be noted that the summations over j in Eq. (1) represent the effect of adjacent nodes around the node i. If the lumped mass approach is used, $m_{ij} = m_i$ for $i = j$ and $m_{ij} = 0$ for $i \neq j$. The summation over r includes all vibrating nodes; they all enter into the damping matrix.

Eq. (1), and its counterparts for the y and z directions, can be visualized as the equations of motion for a space truss with initial tensions. They are nonlinear equations for two reasons. First of all, the terms B_{ij} introduce the effect of large displacements on the tension magnitudes. Secondly, the use of displacements in the expression for a_{ij} reveals that the equations are written with respect to the deformed position of the net. Therefore, if B_{ij} is set equal to zero and a_{ij} is taken as $x_i - x_j$, Eq. (1) is the linear equation for a space truss with initial tensions. It must be noted, however, that the initial tensions must exist in the equivalent space truss. Most cable nets have quite arbitrary geometries which would be unstable as trusses unless they were stabilized by the initial cable tensions.

Eq. (1) and the other two equations of motion may be cast into the familiar form of the displacement method of analysis

$$[K(D)]\,\{D\} + \{P\} = [M]\,\{\ddot{D}\} + [C]\,\{\dot{D}\} \tag{6}$$

K(D), however, depends on the value of the displacement matrix which is not known until the equations have been solved. It is, of course, possible to use the formulation of the force method of analysis in the description of the cable system. In fact, in certain problems such as the analysis of single cables, this approach will probably be more efficient computationally. Most work done on cable nets, however, has utilized the displacement method of analysis. The

writer knows of no nonlinear dynamic analysis of cable nets carried out with the aid of the force method.

At least four general solutions to Eq. (6) have been published [2,4,5,6]. Except for the work of Shore and Modjtahedi, who used a perturbation approach, some form of implicit method was chosen to solve the dynamic equations. Leonards and Recker used Holzer's method, Morris and Birnstiel used Newmark's linear acceleration method, while Argyris, Dunne and Angelopoulos used a finite element time interpolation function. Researchers solving the linear form of Eq. (6) have also chosen an implicit method of analysis. It is felt that for cable net analysis an unconditionally stable algorithm is required. This may not necessarily be true; as an example, the method used in [6] is not unconditionally stable. However, the results found therein appear to be quite good. As is customary in structural dynamics, Eq. (6) can be solved either directly or with the aid of the modal response method. Leonards and Recker, and Argyris, Dunne and Angelopoulos used the direct method while Morris and Birnstiel used both methods. Of course, when the modal superposition method is applied to Eq. (6) the resulting system of equations is not uncoupled as it is in the linear case. An equation of the same form as Eq. (6) is obtained, but the order of the matrices involved should be much smaller if the method is to be efficient.

It might be of interest at this point to consider some of the computational difficulties involved in the nonlinear dynamic response analysis of cable nets. These difficulties arose in the writer's work but they appear to be general difficulties in other nonlinear analyses as well [7]. A minor problem is the choice of mass matrix. Comparison of lumped mass solutions with results of experimental tests made by Jensen [8] on both double layered planar nets and hyperbolic nets reveals that the lumped mass eigenvalues and mode shapes agree very closely with empirical values. Therefore the lumped mass method seems reasonable for cable nets. A major problem is the choice of time step. The difficulties here are enormous! In a linear dynamic analysis the choice of too large a time step reveals itself quite rapidly; the displacements become excessive and a cutoff value can be put into the program to stop computations. For the nonlinear form of Eq. (6), this will not happen; a solution will always occur and the displacements will not be excessive. A quick glance at the results will not be sufficient to determine that the results are meaningless; the solution must be followed through to check whether the values are reasonable or not. There are, in fact, two different time step errors that have occurred in the solution of cable nets acted upon by a harmonic forcing function. The first error arises when the time step is completely wrong, even for the linear solution. When this is the situation, the response goes to a certain amplitude and undergoes a very slight vibration about the amplitude. This error can be avoided by basing the time step on the linear solution to the same problem, a procedure which requires a linear analysis prior to the nonlinear analysis. However, if a time step is employed which is the same as, or slightly less than, the stable time increment for the linear solution, a second error might still arise. In this case, the solution behaves correctly for a few cycles of loading until an amplitude trend gradually sets in. The maximum amplitude in one direction increases slightly at each cycle while the maximum amplitude in the other direction becomes slightly smaller; a steady state solution never occurs. This behavior is disconcerting in a cable net because one would expect larger amplitude in one direction (against tension) than in another (with tension). However, the error is revealed quite clearly because the tension values are completely in error. Strangely enough, this behavior does not seem to be dependent on the nonlinearity of the response. Some highly nonlinear response computations have not exhibited this peculiarity while it has been found in only slightly nonlinear response problems. Whether this pattern occurs or not seems to be dependent on the relative magnitude of the in-plane displacements. The more important they are, the more likely it is that this behavior will appear.

When the modal superposition method is applied, the problem of time step appears in a different manner. For use with the linear acceleration procedure, a time step of $T_{min}/6$, where T_{min} is the smallest period of all the modes employed, appears to work quite well for both linear and nonlinear response computations. Making the time step smaller does not change the results obtained.

However, the basic problem still remains in another form: the choice of how many modes must be used to obtain accurate results. Unfortunately, as could be expected, the number of modes required for a nonlinear solution is usually larger than that required for a linear solution. Hence the choice of modes to be used must be decided by trial and error just as the time step is. However, there is some advantage to the modal response method; it seems to arrive at an answer even if this answer is not completely correct. For example, if a sufficient number of modes is not used, the displacements may be approximated quite well although the tensions are incorrect. It should be noted, however, that the errors in tension usually reveal themselves by jumps at a node. That is, for a cable continuous across a node, instead of virtually the same tension in each segment, there is a jump in value with the value on one side of the node too large and that on the other side too small. The correct tension should then be close to their average values. This fact makes it possible to salvage results obtained with the aid of the modal superposition methods, even if the number of modes chosen is not large enough. Probably the greatest advantage of the modal superposition method is that where the loading pattern allows it to be applied, the time step is much larger than that required for the direct method; in some cases, a factor around 10 is involved. The large increase in time step is even more important than the reduction in size of the algebraic system involved in Eq. (6).

Any implicit solution to Eq. (6) requires, in some form, the solution of an equivalent nonlinear static problem at each time step. The time step is so small, however, that no sophisticated methods are required to solve the equations; a simple iteration will converge in two or three trials at most. In some nonlinear systems [9], it has been possible to solve the equations once at each time step and add the errors involved at the next time step. This has been tried for cable nets under harmonic forcing functions but it has not been successful [2]. It must be pointed out that a harmonic forcing function is the most critical load that can be used to check a procedure. A noniterative approach might work for other loading conditions.

The computation of mode shapes and eigenvalues prior to attempting a dynamic solution would seem to be a necessity. Any direct procedure such as the Givens-Householder method should be suitable wherever system size is not a problem. However, this writer has had difficulty in using simple iteration schemes for cable systems. Either convergence was not obtained, or it was obtained very slowly. This result could be expected because many net systems are highly degenerate. Even where different modes do not have the same frequencies, the eigenvalues are close together. For double layer systems the mass terms on the two layers may differ widely. Iteration procedures are quite sensitive to this mass distribution, even though the eigenvalues and mode shapes may not be.

As stated earlier, the main loads that a cable net, used as either a roof or a mooring line, is subjected to will be stochastic in nature, arising from a Gaussian velocity field. (While earthquake loading is an obvious design consideration for roofs, their large in-plane stiffness appears to be such that wind load will be more critical.) In ocean engineering, a large amount of research work has been carried out to arrive at a suitable loading description due to wave action. For roof nets, some work has been done on wind loading but the problem is complex and it will probably be some time before a realistic loading pattern will be decided upon. It is obvious, however, that random vibration theory will be required in any solution. At the present time, a computationally efficient procedure for computing the random vibration of a geometrically nonlinear system with a large number of unknowns does not seem to be available. About the only procedure which could be employed would be a computer simulation of the wind field over the cable net and its resulting loading pattern. The cable net could then be solved using Eq. (6). There are two defects in this procedure. First it requires that several simulated loading patterns be applied to arrive at statistically valid results. For a nonlinear dynamic analysis, the computer time would be prohibitive. Secondly, uncertainty in the loading, at the present time, is so large that a refined nonlinear solution does not seem to be reasonable.

Although random vibration theory cannot be easily applied to nonlinear

response problems, it can be readily employed in linear response computations of root mean square values for displacements and tensions. This knowledge has led many researchers to attempt a linear formulation of the dynamic equations. Fortunately, for both wind and wave forces, a semi-linear analysis is possible. The wind velocity pattern at a given point may be described by a slowly changing mean velocity and a fluctuation, or gusting, about that mean velocity. The mean velocity varies so slowly that its effect on the structure is static, while the dynamic effect is due to the fluctuating forces. It can be assumed, therefore, that the dynamic response is a linear perturbation about the static equilibrium position under the mean velocity. The stiffness of the structure under the dynamic load is computed by substituting the static equilibrium displacements and tensions in the equations for K(D) of Eq. (6). Of course, the nonlinear static equations must be employed in solving for the displaced position of the net under the mean wind load. This procedure has been widely employed in both civil and ocean engineering analysis of cable nets. It will result in accurate answers with an enormous saving in computer time.

It seems evident, therefore, that in order to solve practical problems for cable systems, three subroutines must be available in any program, a nonlinear static subroutine, a linear mode and eigenvalue computation subroutine, and a linear dynamic response subroutine. The latter two subroutines are standard in any general computer package. The nonlinear static subroutine is available in such systems as STRUDL [10] and NASTRAN [11]. However, a search of their literature does not indicate whether initial tensions can be read into their space truss programs. STRUDL does have a mechanism for forming new stiffness matrices but this writer has not tried to utilize it. As pointed out earlier, a cable net is a space truss whose members are in initial tension; therefore, all that is needed to use any general computer system is the ability to incorporate this tension into the stiffness matrix. This was done for the ELAS system by Alpay and Utku [12].

PROGRAMS AVAILABLE

There does not exist at the present time a large commercial market for computer programs devoted exclusively to cable systems. Although only small modifications of existing programs may be required to obtain semi-linear solutions, the manuals for general programs do not state if, or how, this could be done. Most of the available programs presented herein are programs developed by researchers in the field of cable dynamics. In general, such programs suffer from the defects of poor user documentation and lack of generality. However, no judgment can be made on any individual program. The following programs are commercial programs which are available for analysis of cable networks:

1. Aska-Group, ISK Stuttgart
   Pfaffenwalding 27, 7000 Stuttgart 80,
   Germany

   This is a general nonlinear dynamic program for three-dimensional cable nets. It uses a finite element time representation and is not unconditionally stable [6].

2. Atkins Research and Development
   Woodcote Grove
   Ashley Road, Epsom, Surrey, England

   This program uses a semi-linear analysis of three-dimensional cable systems. Dynamic equations are solved by Newmark's β method [13].

3. Lockheed Electronics Company, Inc.
   U.S. Highway 22
   Plainfield, New Jersey 07061

This program utilizes a semi-linear solution to two dimensional cable-bouy systems under various current velocity profiles.

4. Swanson Analysis Systems, Inc.
   870 Pine View Drive
   Elizabeth, Pa. 15037

   A general nonlinear cable system program is available. It uses implicit integration of the basic equations and either a direct or modal response solution.

The following programs have been reported in recent research literature on cable systems; the principal investigator is listed:

5. J. W. Leonard
   Civil Engineering Department
   Illinois Institute of Technology
   Chicago, Illinois 60608

   This program solves the forced response of general nonlinear cable systems [4]. It uses Holzer's method and is commercially available.

6. N. F. Morris
   Civil Engineering Department
   Polytechnic Institute of New York
   Brooklyn, N.Y. 11357

   This is a series of programs for linear and nonlinear response using both the modal response method and the direct method. The work was sponsored by the National Science Foundation and program listings are public [2].

7. S. Shore
   Civil Engineering Department
   University of Pennsylvania
   Philadelphia, Pa. 19104

   This program solves the forced steady state response of cable nets under a harmonic forcing function [5]. The frequency-amplitude curve is computed with a numerical form of perturbation. It requires modal superposition.

8. W. C. Knudsen
   Doctoral thesis, April 1971
   University of California, Berkely 94720

   Knudsen developed semi-linear programs for cable nets under wind loading and earthquake. Both computer simulation of the wind load and random vibration theory are utilized along with both direct and modal response solutions.

9. S. Utku
   Duke University
   School of Engineering
   Durham, North Carolina 27708

   This system is a conversion of the ELAS program to solve initially stressed members. It has a semi-linear procedure for solving random vibration problems [12].

10. NOAA National Data Buoy Center

This program solves the two dimensional problem of mooring cables with the aid of a semi-linear solution [14]. It is documented and has been applied at other centers [15]. Random vibration theory has been used.

11. R. F. Dominquez
Texas A and M University
College Station, Texas 77843

This program utilizes the flexibility method in the solution of cable systems. The analysis is semi-linear. A distinctive feature of this program is its use of complex coordinates in a more accurate description of damping [16].

12. S. A. Crist
Air Force Academy
Colorado 80840

Several computer programs have been developed for the steady state response of trailing wires behind an orbiting sapcecraft [17,18].

13. S. T. Hong
Department of Civil Engineering
Washington University
Seattle, Washington 98105

A semi-linear solution is programmed for long, taut mooring lines. It utilizes a frequency domain analysis [19].

14. K. P. Kerney
Naval Ship Research and Development Center
Bethesda, Maryland 20034

Two dimensional motion of an inextensible cable under harmonic forcing is described by this program. A semi-linear approach is used [20].

15. J. K. Arnston
Doctoral thesis, 1974
University of Minnesota
Minneapolis, Minn. 55455

This program appears to be a semi-linear solution to the dynamic response problem [21]. It has been used in the random vibration analysis of circular roofs.

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Offshore Structures Analysis

V. Dunder
G. Belonogoff
Bechtel International Corporation

INTRODUCTION

This chapter deals predominantly with the dynamic analysis of offshore structures. Dynamic (and in some cases nonlinear) analysis is becoming economically important for the following reasons:

1. For the more costly structures, small design refinements can give rise to large construction cost savings.
2. In complex systems, unsophisticated static analysis methods can result in a higher risk in terms of property and life.
3. Because offshore structures are being built in deeper water and are being exposed to harsher environmental conditions than in the past, these structures can frequently no longer be adequately described by static analysis.
4. The computational capability and speed of the digital computer can reduce production schedules.
5. Both the public and regulatory agencies are concerned about the possible environmental impact of proposed designs.
6. The response of floating rigs is not compatible with static analysis.

In general, the response of a marine structure must account for the action of ocean surface waves, current, wind, gravity loading, and other loading conditions. In complex systems, the fluid-structure-soil interaction must be simulated to obtain a realistic structural model. The response of fixed-base platforms, floating rigs, or large submerged structures, including certain marine components such as risers, pipelines, and moorings, are discussed.

NOMENCLATURE

A_p = Projected area of a unit length of a member
B = Volume of water replaced by a unit length of a member
C = Damping factor
C_D = Drag coefficient
$\bar{C}_D$ = Modified drag coefficient
C_M = Mass (inertial) coefficient
$\bar{C}_M$ = Modified mass (inertial) coefficient
$\hat{C}_M$ = Mass coefficient accounting for a fluid flow disturbance
d_1, d_2, d_3 = Dimensions of the object in the x,y,z directions
E = Modulus of elasticity
F = Internal forces
f = Fluid pressure force
f_D = Fluid drag force
f_H = Horizontal fluid pressure

f_I = Inertial fluid force
f_W = Wind pressure
g = Acceleration due to gravity
H = Mean water depth
h = Wave height
h_n = Coefficient in the wave profile calculation
h_{nl} = Nonlinear coefficient in the wave profile calculation
I = Moment of inertia
K = Stiffness matrix
K_L = Linear strain stiffness matrix
K_{NL} = Nonlinear strain stiffness matrix
k = $2\pi/\ell$
ℓ = Wavelength
M_v = Virtual mass matrix
M_a = Added mass
N = Highest index number in the series
P = Load vector
p_c = Wave pressure at the crest
p_n = Wave pressure normal to the member surface
p_t = Wave pressure at the trough
q = Buoyant or in-air weight of the riser
R = Transfer function
S_h = Response spectrum
t = Time
$u,\dot{u},\ddot{u}$ = Structural displacement, velocity, acceleration
$v,\dot{v},\ddot{v}$ = Fluid displacement, velocity, acceleration
W = Wind velocity
α,β = Coefficients
γ = Specific weight of the fluid
$\Delta\omega$ = Increment of frequency
ϵ = Wave phase angle
ζ = Coordinate of the wave surface
θ = Phase angle
π = Rayleigh distribution
ρ = Fluid density
ρ_A = Air density
σ = Variance of the spectrum
σ_R = Variance of the response spectrum
τ = Tension (axial force)
Φ = Mode shape matrix
ω = Circular frequency of the wave

FLUID-STRUCTURE INTERACTION

The fluid-structure interaction in offshore platform analysis is usually simulated by hydrodynamic forces. These forces may or may not be related to the response of the structure, depending on the degree of the simplification.

In the deterministic approach, the action of regular maximum waves or random waves may be used. The parameters of such waves are taken from oceanographic data of certain locations. Maximum wave characteristics (height, period) from the last 50 to 100 years are considered. Data are available through National Oceanographic Data Center.

The regular waves can induce harmonic response of the structures if the structural period is close to that of the wave. Wind-induced waves are usually of random character. For evaluation of random wave effects, the stochastic evaluation is preferable. The wave is assumed to be composed of several regular (periodic) waves, and an evaluation of the wave energy is used in the computation of structural response. The wave spectral density function and its application is briefly described in another section. Since the computation of hydrodynamic forces in both

stochastic and deterministic analysis is based on the same principles, the differences in approach are mentioned only where necessary.

The calculation of the hydrodynamic forces is primarily based upon:

1. Computation of the pressure based on the fluid acceleration and velocity and on the structural characteristics and response
2. Computation of the water particle kinematics

WAVE FORCES

Evaluation of the pressure forces depends on the relative size of the structure and elements and the wave length. For truss or frame structures, the Morrison [1] equation is usually used. Morrison has derived the pressure forces (force per unit length) for vertical piles and has expressed them as a combination of drag forces f_D and inertial forces f_I. Froude-Krilov pressure is considered negligible. The pressure force according to Morrison is

$$f = f_D + f_I \tag{1}$$

Inertia forces of fluid particles are proportional to the local acceleration of the fluid. Drag forces are proportional to the fluid particle velocity. For flexible structures, the relative velocity and relative acceleration are required. The (modified) Morrison equation can therefore be written in a form

$$f = 1/2\ C_D \rho A_p (\dot{v} - \dot{u})|\dot{v} - \dot{u}| + C_M \rho B \ddot{v} - M_a \ddot{u} \tag{2}$$

where C_D and C_M are empirically derived drag and mass coefficients; $\dot{v}$ and $\ddot{v}$ are fluid velocity and acceleration, respectively; $\dot{u}$ and $\ddot{u}$ are structural velocity and acceleration; ρ is fluid density; A_p and B denote the projecting area and volume of the unit length of a member; and M_a is the added mass. The added mass is usually considered to be

$$M_a = (C_M - 1)\rho B \tag{3}$$

where the difference in tangential and normal directions with regard to the axis of the member were neglected.

Fluid kinematics of an undisturbed wave can be computed using various theories. Model tests on circular cylinders indicate that a value of 2.0 for C_M will give conservative values of inertial forces. In practice, C_M is commonly assigned a value of 1.5. However, the same study shows values of C_D varying from 1.2 to 4.4. Both the drag force and the inertial force depend on the Reynolds number and on the irregularity of the waves. The effect of scattering cannot be included because the computation is based on the assumption that the wave motion is undisturbed by the structure. The Morrison equation was derived for an isolated vertical cylindrical body (tubular beam) whose diameter is small compared with its length.

No coupling in the computation of forces on individual members is considered. (Only the phase of the passing wave is accounted for.)

For large-sized structural members, the Froude-Krilov forces should be included. The effect of the Froude-Krilov forces becomes very important when the size of the structure approaches half the wavelength. The difference between the pressure at the crest p_c and the pressure at the

trough p_t must be considered. Paulling [2] has linearized the expression for the pressure force f. The linearized expression is

$$f = \int_a p_n da + \bar{C}_D(\dot{v} - \dot{u}) + \bar{C}_M(\ddot{v} - \ddot{u}) \tag{4}$$

where p_n is the computed pressure from the undisturbed wave; $\bar{C}_D$ and $\bar{C}_M$ are drag and mass coefficients; a is the surface of the member; $\dot{v}$ and $\ddot{v}$ are the fluid velocity and acceleration, respectively; and $\dot{u}$ and $\ddot{u}$ are the structural velocity and acceleration. This linearization is necessary for a stochastic approach. The first term in Eq. (4) represents the Froude-Krilov force; the second term represents a linearized drag force; and the third term represents the simplified mass inertia effect. More accurate procedures of linearization are required when drag forces are the primary forces. According to some authors, the expression $\bar{C}_M\ddot{v} - (K_M - 1)\ddot{u}$ should be used instead of the last term. A more rigorous evaluation based on the integral representing the Froude-Krilov force uses Bernoulli's equation and is computed from the wave velocity potential. This evaluation can be found in [2].

If the dimensional characteristics of the structure increase to the same order as the wavelength, the wave scattering becomes more important than the viscous effects. Potential flow theory is commonly used in calculating wave pressure and wave forces on large submerged structures such as oil storage tanks and underwater habitats. Reid and Bretschneider [3] have proposed the computation of horizontal forces on large rectangular submerged structures in the following form:

$$f_H = \gamma \tilde{C}_M \left(\frac{h}{2}\right) d_2 d_3 \frac{\cosh k\,(z + H)}{\cosh kH} (\cos \theta_1 - \cos(\theta_1 + kd_1)) \tag{5}$$

where $\tilde{C}_M$ is the mass coefficient due to the flow disturbance; d_1, d_2, and d_3 are dimensions of the object in the x, y, and z directions; $k = 2\pi/\ell$; γ is specific weight of fluid; h and ℓ are wave height and wavelength; H is the water depth; x, y, and z are coordinates; and θ_1 is phase position of the leading edge of the object with respect to the wave.

This equation is based on the assumption that only inertial forces have an important effect on the total pressure. From this equation, some other diffraction theories have emerged. Garrison and Chow [4] have outlined a diffraction theory in which submerged objects have arbitrary shapes. Further references may be found in [5-8].

Morrison's equation is usually preferred in the analysis of tubular offshore structures. No such preferred theory exists for large-sized structures.

For simplicity, no vectorial summation to the wave kinematics is made.

WAVE THEORIES

The kinematics of a wave vary with the parameters of wavelength, height, and water depth. Lambrakos and Brannon [9] have divided the theories of calculating the wave velocity and acceleration into two groups: Stokes theories and non-Stokes theories. In structural computation, the Stokes-type theories are most often used even though they do not best fit the experimental data. The Stokes waves propagate without shape deformation and are periodic in space and in time. Their equations usually include nonlinear terms to describe formation of eddies and similar effects. Stokes fifth-order theory is the extension developed and tabulated by Skjelbreia and Hendrickson [10], and is applicable in medium to deep

water. In medium deep water, Airy theory [8] is used. In shallow water where the ratio of wavelength to depth is relatively large, cnoidal theory [11] should be used. This classification has been presented by Dean [12]. Statistics introduced in the computation of wave forces by Borgman [13] require simpler, preferably linear, wave theories. The linear filter technique introduced by Reid [14] was studied by Hudspeth and others [15]. Hudspeth compares linear filter technique and stream function representation [16] with experimental data. The study does not favor the linear theory; however, fair approximations were obtained.

A regular wave profile can be described in linear theory by

$$\zeta(t) = h \cos (kx - \omega t) \tag{6}$$

where ζ is surface profile of the wave, h is the amplitude, k is the wave number ($k = 2\pi/\ell$), ω is the circular frequency of the wave, x is the coordinate in the direction of the wave, and t is the time. In the Airy linear theory, h is linearly proportional to the wave height h. The Stokes higher order theory gives the surface profile by

$$\zeta(t) = \sum_{n=1}^{N} \bar{h}_n \cos n (kx - \omega t) \tag{7}$$

where the terms are similar in meaning to those in Eq. (6), except that $\bar{h}_n$ does not depend linearly on the wave height. Equations (6) and (7) describe the surface of a regular periodic sea. As pointed out by Froude [17] in 1905, an irregular wave can be built up from simple regular waves. Using a Fourier series,

$$\zeta(t) = \sum_{n=1}^{N} h_n \cos (k_n x - \omega_n t + \varepsilon_n) \tag{8}$$

The terms are again similar in meaning to those in Eqs. (6) and (7), except that ω_n is the circular frequency of the nth wave and ε_n is the phase angle of nth wave. This representation of irregular waves was introduced by Denis and Pierson [18] in their description of the statistical means of predicting the response of a ship when the wave energy spectrum and the response to regular waves are known.

SPECTRAL DENSITY FUNCTION

In spectral analysis, the average energy over the wavelength and per unit area of the sea surface can be computed as a sum of the kinetic energy of orbital motion and potential energy of water level change [19]. The distribution of energy over the wave frequency $S_h(\omega)$ can be computed. This distribution is called the wave spectral density function, the wave energy spectrum, or simply the wave spectrum. Basically, a single harmonic surface wave is characterized by its height h and length ℓ.

The real sea consists of waves having a wide spectrum of wavelengths or frequencies. To help visualize this kind of wave, harmonic waves of amplitude h_n, frequency ω_n (or corresponding length ℓ_n), and phase angle ε_n for $n = 1, 2...N$ can be considered. If the N discrete frequencies are uniformly spaced at intervals $\Delta\omega$ over a range $\omega_a < \omega < \omega_b$, the resulting wave height will be periodic, with a period equal to $2\pi/\Delta\omega$, and will possess a mean square value equal to one half the sum of the squares of

the individual amplitudes $h_n^2/2$, which is the computed component of the spectra.

The wave spectrum is used in stochastic analysis to compute the structural response spectrum. The wave spectrum proposed by Pierson and Moskowitz [20] was recommended by the International Ship Structures Congress and by the International Towing Tank Conference and is frequently used for computing structural response. The Pierson-Moskowitz spectrum is given by

$$S_h(\omega) = \frac{\alpha g^2}{\omega^5} \exp\left(-\beta\left(\frac{g}{\omega w}\right)^4\right) \tag{9}$$

where g is the acceleration due to gravity; ω is the circular wave frequency; w is the wind speed (64 feet above the sea surface); and α and β are constants (for the North Sea, $\alpha = 0.0081$, $\beta = 0.74$ may be representative).

Some more details on the application of spectral density function are in a paragraph "Stochastic Analysis."

Recent results of the research on the growth mechanism of a wind-wave field, combined with actual wave measurements of extreme sea states in the North Sea, show that the Pierson-Moskowitz-type spectrum may not always be adequate for the description of extreme and operational sea states. A more sharply peaked wave spectrum is proposed in [21]. The importance of the spectral shape of a wave field for structural behavior is considered in this paper.

SOIL-STRUCTURE INTERACTION

Soil properties strongly affect the response of fixed-base offshore structures. Modeling of a pile by short-beam elements and modeling of the soil by springs are both cumbersome procedures.

In the choice of a computer program for analysis, preference should be given to programs that allow simpler pile modeling, computed as a substructure or equivalent beam spring. An iterative solution in the application of otherwise linear analysis may be necessary. Special provisions for soil-structure interaction are in McDonnell's STRUDL and in MARCS. Both are programs for the analysis of fixed-base offshore structures. Some useful remarks regarding the soil-pile interaction may be found in [22, 23].

EQUATIONS OF MOTION

Linear Analysis

Finite element analysis is normally used in the solution of the response of complex structures. Linear analysis is in many cases applicable. To reduce transient analysis of offshore structures to linear analysis, the drag forces expression in Eq. (2) must be simplified. When this is done, the equations of motion can be expressed as

$$M_v\ddot{u} + C\dot{u} + Ku = P(\dot{v},\ddot{v}) \tag{10}$$

where M_v is the virtual mass; C is the structural and viscous damping; K is the linear stiffness matrix; P is the load vector; u, $\dot{u}$, and $\ddot{u}$ are the structural displacement, velocity, and acceleration, respectively;

and $\dot{v}$ and $\ddot{v}$ are the fluid velocity and acceleration. Equation (10) conforms to the usual finite element analysis fundamental system. However, the following ditferences should be noted:

1. Virtual mass is composed of the structural lumped mass and added mass (see Eq. (3)). Mass moments of inertia may have to be included.
2. The damping coefficient C is higher than usual because of fluid-structure interaction (viscous damping). A total damping of 2% has been considered by Maddox [24].
3. Because loading vector P varies in time (generally independently from joint to joint), voluminous data, representing hydrodynamic and hydrostatic forces, are generated.
4. The stiffness matrix can be full or reduced depending on whether rotational degrees of freedom are considered in the analysis or whether another reduction of the dynamic system is provided. As a rule, an extensive reduction of the degrees of freedom with a consistent approach in the reduction of all matrices is used.

In the computation of the virtual mass, the difference between the component tangential to the member and the component normal to the member is seldom considered. The proper consideration of the added mass will lead to a nondiagonal mass matrix. In the computation of added mass, the changes in submerged parts and in air structural parts may have to be considered. Mean water level may usually be taken as the limit for the computation of virtual mass. Mass moments of inertia, computed for a reduced system, have been used by Penzien [25]. Equation (10) can be solved by a general finite element analysis program such as SAP, STRUDL, STARDYNE, etc., but handling of the data (both input and output) is impractical unless both a preprocessor and a postprocessor exist. The efficiency of the transient analysis, which solves the structural response due to the high number of forcing functions, is very important. For more details on linear transient analysis and general purpose computer programs, see [26].

Integrated programs for linear time history analysis of offshore structures (WAVE-FORCE and STRUDL, MARCS) were recently introduced into service bureaus. Even though the equations for the computation of virtual mass, drag forces, and buoyant forces seem simple, they require complex structural geometry and transformation matrices generally involved in the assembly of stiffness element matrices. The same input of geometrical data and the transfer of the computed values in the formats applicable in general purpose programs are necessary. The integrated programs have an advantage in that the transfer of the data is transparent to the user. However, integrated programs are probably available only to large engineering companies or offices. Existing programs are reviewed in Appendix I.

Nonlinear Analysis

Nonlinearities in the equations of motion may be caused by three things:

1. Nonlinear fluid-structure interaction
2. Large displacements and small or large strains
3. Nonlinear elastic or elastoplastic properties

Nonlinear fluid structure interaction is peculiar to offshore structures analysis. When viscous forces are predominant, that is, where Morrison's equation (Eq. (2)) in its full nonlinear expression should be considered for the computation of hydrodynamic forces, an incremental formulation and iterative solution are usually used. When nonconservative hydrodynamic forces are predominant, the updated Lagrangian equation of motion is preferred. The nonlinear equation of motion (linearized with respect to the time increment dt) can be expressed in a form

$$ {}^{t}M_{v}\,{}^{t+dt}\ddot{u} + \bar{C}_D\left({}^{t+dt}\dot{v} - {}^{t+dt}\dot{u}\right)\left|{}^{t+dt}\dot{v} - {}^{t+dt}\dot{u}\right| + C\,{}^{t+dt}\dot{u} $$

$$ + {}^{t}_{t}K_{L}\,du + {}^{t}_{t}K_{NL}\,du = {}^{t+dt}P - {}^{t}_{t}F \qquad (11) $$

where ${}^{t}M_v$ is the virtual mass at time t; $\bar{C}_D$ is the drag coefficient; C is the structural and viscous damping; ${}^{t}_{t}K_L$ is the linear strain incremental stiffness matrix based on the strain computed at time t and the configuration at time t; ${}^{t}_{t}K_{NL}$ is the nonlinear strain incremental stiffness matrix based on the strain computed at time t and the configuration at time t; ${}^{t+dt}P$ are the loads (other than forces due to added inertial and drag effect) at time t+dt; ${}^{t}_{t}F$ are internal forces at time t in the configuration at time t; ${}^{t+dt}\ddot{u}$, ${}^{t+dt}\dot{u}$ are acceleration and velocity of the structure, du is the increment of the displacement and ${}^{t+dt}\dot{v}$, ${}^{t+dt}\dot{v}$ are the acceleration and velocity of the fluid at time t+dt. The total displacement at time t+dt is given by the sum of the displacement ${}^{t}u$ at time t and the increment

$$ {}^{t+dt}u = {}^{t}u + du \qquad (12) $$

Equation (11) is the modified form of the equation of motion given by Bathe [27]. In this equation, the effects of hydrodynamic forces are taken into account and the drag coefficient generally depends on time, but this time dependence is seldom considered.

At present, hydrodynamic nonlinearities are considered in the design of structures undergoing large motion and should usually be included in the analysis of semisubmersible platforms and their components, such as risers and flowlines, pipelines, and moorings. The computation of structural displacement can be included in simplified form if the structure is regarded as a rigid body. This motion and the hydrodynamic forces it causes can be described by iterative procedures, e.g., WAVE-LOAD [44]. If rigid body motion is considered, the strain energy change does not contribute in the variational equations and the resulting equation can be written in the form

$$ {}^{t+dt}M_{v}\,{}^{t+dt}\ddot{u} + \bar{C}_D\left({}^{t+dt}\dot{v} - {}^{t+dt}\dot{u}\right)\left|{}^{t+dt}\dot{v} - {}^{t+dt}\dot{u}\right| $$

$$ + C\,{}^{t+dt}\dot{u} = {}^{t+dt}P \qquad (13) $$

The meaning of the variables is the same as in Eq. (11). In practice, the system of six equations for six degrees of freedom is solved in time. Masses, mass moments of inertia, and resulting forces from the drag components are all reduced to the center of gravity of the platform. By iterating on Eq. (13), the forces on individual members can be calculated to any desired accuracy. Finally, the stresses in structural members of the platform can be solved, using general structural analysis programs. The equations in the above form were presented by Paulling in [2].

Large displacement nonlinearities, including strain energy contributions, must be considered in flexible structures. Because of the greatly increased volume of computation, only simple tasks are usually solved. Geometrical nonlinearities should be considered in the analysis of pipelaying, risers, and flowlines, and in computation of the response of moorings and underwater cable structures. Special programs are usually employed for these tasks.

There is, in the public domain, probably no general purpose or integrated program for solution of underwater, flexible composite types of structures. For simpler types of structures, finite difference solutions

rather than the finite element method are being used. Burke [28] has derived the following differential equation of motion of riser and similar basically vertical beam-type components:

$$M_v\ddot{u} + \bar{C}_D(\dot{v} - \dot{u})|\dot{v} - \dot{u}| + EIu^{IV} + \tau u^{II} + qu^{I} = P \qquad (14)$$

where the dot indicates a partial derivative with respect to time and the Roman numerals denote partial derivatives with respect to the depth in the ocean; E and I are modulus of elasticity and moment of inertia, respectively; τ is tensile force in the riser (variable with the depth) and q is the buoyant weight of the submerged riser or the weight of the riser in air. Other variables have the same meaning as before. Particular boundary conditions are needed to complete the formulation. The application of the finite difference method for solution seems to be straightforward and is therefore often used; however the variability of the boundary conditions may be limited, or at least cumbersome.

Material nonlinear analysis is not usual in the practical design of offshore structures, but two contributions should be mentioned. Selna [29], in 1971, in an analysis of a structure, described the reduction of stiffness due to yielding. Steel bilinear hysteric material was considered. Even though the local yielding of joints with regard to shakedown may be of reasonable importance, not much has been done in that field yet. Recently Kaul and Penzien [30] used stochastic analysis to compute the yielding of offshore structures. In this study, linearized equations that incorporated probability relations were employed.

It should be noted that these new developments are generally not publicly accessible, because complex programs are needed and verification of these programs may require considerable effort before a program is applicable in design. Verification of the results of computation is limited by the lack of experimental data or closed-forms solutions (or similar computations).

As a special problem, equations of motion for cable systems may be considered. The structural elements are unstable, unless loaded for a certain configuration. Material nonlinearity and large strains might be important.

SOLUTION TECHNIQUES

Deterministic

Equations of motion as presented in the previous paragraph must be solved in the time domain, and appropriate numerical methods must be used.

It is obvious that relatively slow motion is induced by the waves. The stability of the solution is related to the frequencies of the structure. Unless special provision is made (e.g., filtering out high frequencies by damping, or using modal analysis where applicable), many time-integration steps may be required in the solution of the response. Modal analysis is preferable to time history analysis in solving linear systems of equations because it allows otherwise dependent equations to be decoupled. If a linearized system of equations is considered, the response of the structure can be expressed as

$$u = \Phi y \qquad (15)$$

where Φ are mode shapes derived from homogeneous equations of motion. From the orthogonality conditions, equation

$$M_v^* \ddot{y} + C^* \dot{y} + K^* y = P^* \tag{16}$$

can be obtained, using generalized matrices

$$M_v^* = \phi^T M_v \phi$$

$$K^* = \phi^T K \phi = \omega^2 M^*$$

$$C^* = \phi^T C \phi$$

$$P^* = \phi^T (C_M \ddot{v} + C_D \dot{v}) \tag{17}$$

Modal analysis may be thought of as a purely mathematical exercise for determining orthogonal functions and generalized coordinates that transform systems of coupled ordinary differential equations into independent equations that can be solved by the Duhamel integral.

Note that Eq. (17) includes hydrodynamic forces only. In linear analysis, superposition can be used and the response to various loadings can be superposed. In nonlinear analysis, iterative methods are used to solve equations linearized with regard to the time step. An equilibrium or energy balance check may be an important feature of a program for nonlinear response analysis.

In deterministic analysis, the maximum magnitudes of displacements and stresses that occurred during a given time interval are considered and evaluated. Because the allowable stresses can vary with the slenderness ratio, amount of reversible stress ratio, etc., comparison of maximum values may be a tedious task. It must be remembered that the directions of wave approach, operating conditions, and drilling conditions all must be considered. Postprocessing of the results of the response analysis may therefore be a valuable feature of an integrated computer program.

Changes in the National Codes applicable to stress analysis can pose some problems. The updating of the program may not be fast enough to reflect those code changes.

Linear analysis allows simple superposition. In nonlinear analysis if the nonlinearities are not of major importance, superposition is still used. This may be thought of as a process of linearization, analogous to developing a Fourier series and considering the first term only.

Deterministic analysis is at present a standard approach of evaluating structural design. The following codes are applicable: API [31], AISC [32], ACI [33], AWS [34]. For further details on design, see also [35]. A review of deterministic analysis is given in Fig. 1.

Stochastic

In the analysis of offshore structures, stochastic analysis is generally preferred to deterministic analysis whenever the wave effects are predominant.

The main reason lies in the difficulty of composing a realistic wave train. The random process describing the ocean waves can be assumed stationary over a short period of time. When linearized, the dynamic wave forces have zero-mean and Gaussian characteristics. Known statistical methods can therefore be used to evaluate the distribution of the response function and the probability of occurrence of the maximum peaks.

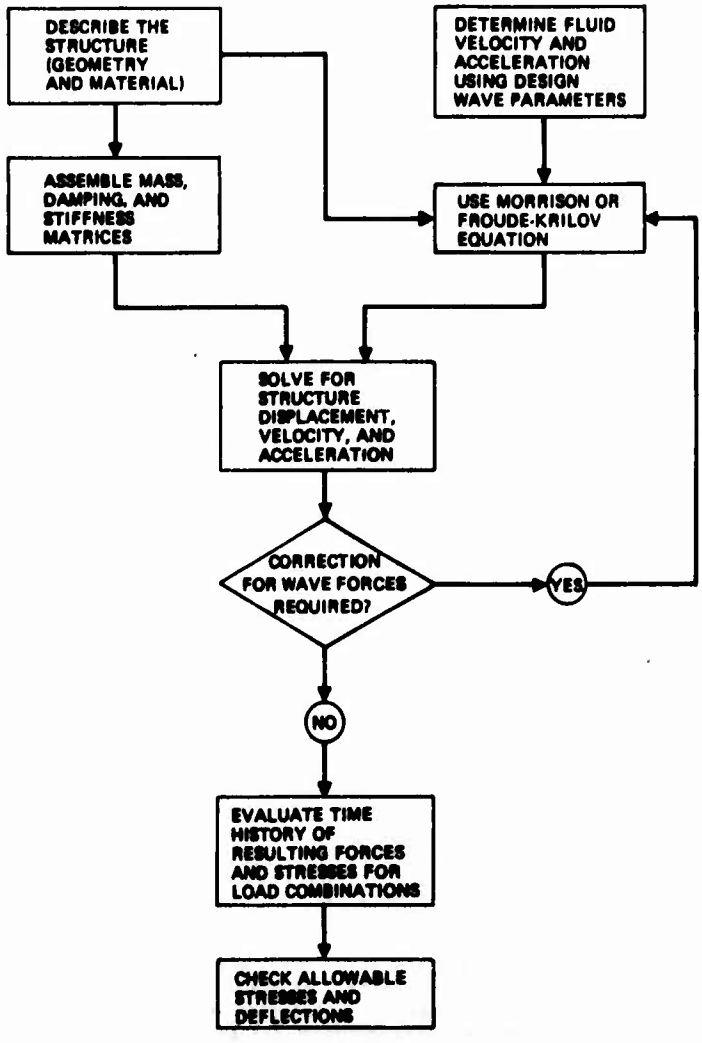

Fig. 1 Deterministic solution

Penzien and others [26, 36, 37] have published several papers using a stochastic analysis of offshore structures. The equation of motion (Eq. (1)) or the linearized equation of motion (Eq. (11)) may be used for predicting the response. The response spectrum S_R can be related to the wave spectrum S_h by

$$S_R(\omega) = S_h(\omega)\,|R(\omega)|^2 \tag{18}$$

where R is the transfer function. By definition, the transfer functions are amplitude quantities of the dynamic deflections or stresses of the given lumped mass model, excited by unit amplitude wave forces.

Wave spectrum computation and its application in stochastic analysis are based on the following:

1. A Gaussian phase relationship
2. A continuous spectrum of waves that may be superimposed linearly
3. A stationary and ergodic process

With the wave spectrum and the response spectrum derived from the wave spectrum, the statistical properties of the response can be obtained. In particular, if the phenomenon is described by the Gaussian distribution, the average value of the response will be given by

$$\bar{R} = \bar{C}\sqrt{\sigma^2} \tag{19}$$

where σ^2 is the variance or area under the spectrum. Values of the coefficient $\bar{C}$ have been given by Cartwright and Longuet-Higgins [38]. It is a property of distributions that processes linearly related to Gaussian processes are themselves Gaussian. The variance σ_R^2 of the response is equal to

$$\sigma_R^2 = \int_0^\infty S_R(\omega)\,d\omega \tag{20}$$

The stochastic evaluation of the hydrodynamic forces can be simply described as follows:

1. Find the wave pressures (loads) of regular waves of unit height for a number of different frequencies. The loads must be determined for different angles between the structure and the direction of the waves. Waves that will result in a given wave density spectrum should be used.
2. Transfer functions are found in principle by solving the response of the structure to the loads specified in 1. The amplitudes of the response (displacement, stress, velocity, etc.) plotted in the frequency domain are the transfer functions. When the linear superposition property is used, the response spectrum is obtained as a product of wave spectrum and a quadratic of the transfer function.

The parameter describing the statistical distribution of the amplitudes of the response is directly related to the area under the response spectrum curve. As the wave spectrum is described by parameters related to observed wave statistics and the response spectrum is directly related to the wave spectrum, statistical long-term predictions of the response can be performed.

When the stresses are computed, the standard deviation of the stresses S_R from the mean level can be calculated. The Rayleigh distribution

$$P(S_R) = 1 - \exp\left(\frac{-S_R^2}{2\sigma_R^2}\right) \tag{21}$$

where $P(S_R)$ is the probability level, can be found using Eq. (21). In particular, the most probable largest stress during a time t can be found from

$$S_{R\ max} = \sqrt{2\sigma_R^2 \ln(t/T)} \qquad (22)$$

where T is the average period of the response. Stochastic methods have an advantage in a complex evaluation of environmental forces, fatigue, corrosion, etc.; however, they are applicable only if they can be combined with long-term experience. The stochastic evaluation is used mainly in Europe, where it is frequently required by law. The process of evaluation is shown in Fig. 2.

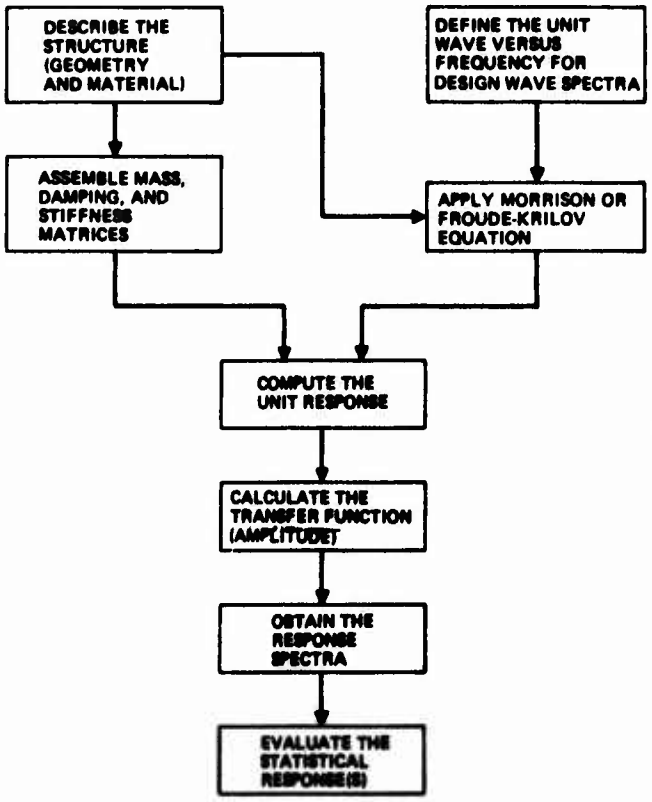

Fig. 2 Stochastic solution

LOADING CONDITIONS

Current

Computation of hydrodynamic forces in the general case includes wave forces as well as the current-induced forces. The current is usually specified by a linearly varying velocity profile. A nonlinear velocity pro-

file can be used when large submerged structures are analyzed, in which case a variable mud line level may be considered with regard to scouring.

It can be concluded from present calculations that large structures in a depth of 300 feet or more should be provided with scour protection whenever the seabed consists of fine sand [39]. Offshore structures located where significant steady currents are encountered can be excited in the fundamental or higher modes. Oscillations in the flow directions can be excited in velocities that are one-quarter the velocity of those currents required for cross-flow excitation [40].

Buoyancy

In the analysis of floating structures where a considerable change of the total moment of buoyant forces can occur because of the rocking motion of the structure, buoyancy forces require special attention.

Wind

Wind pressure force is frequently computed according to API standards

$$fw = 1/2\ C_D \rho W^2 A_p \tag{23}$$

where C_D is the wind drag coefficient, ρ is the air density, W is the wind speed, and A_p is the projected area. For hurricanes, special formulas may have to be considered.

Ice

Ice load is usually specified by the thickness of the ice and by the ice-breaking force. The interaction of the structure with the sea ice causes cracking of the ice sheets. The effect of cracks on the strength properties of sea ice was investigated by Mohaghegh [41], who has developed expressions for the fracture strength of sea ice. Thickness, velocity of ice flow, temperature, shape of the structure, and other variables affect the force of the impact of ice sheets on the structure [42]. Sometimes formations of ice on the structure can considerably increase gravity loads.

Collision

Fenders and dolphins are built to minimize the collision effects between a ship and a structure. For collision design, the equation of motion according to Cummins which employs impulse response functions, is desirable. Data on these functions are presented in [43].

Other Loads

Gravity loads of the structure and equipment must be considered during construction and operational states. Drilling and production live loads must be specified according to purpose and service requirements. In higher seismic zones, the earthquake motions may be required. Detail specifications of loads must precede the structural analysis.

TYPE OF STRUCTURES

The computational procedure and applicable computer program(s) for the solution of structures depend mainly on the assumptions made for modeling the physical situation. As a rule, the relative rigidity of the structural parts allows the overall structure to be broken into smaller units for the purpose of analysis. For example, flexible parts (e.g., risers and moorings) and the rigid portion of the semisubmersible rig are usually analyzed separately. Detail stress concentrations in tubular joints are handled by the discretized shell element model with boundary conditions obtained from analysis of the overall structure.

Fixed-Base Platforms

Fixed-base platforms up to 500 feet in height are usually analyzed by quasistatic methods that neglect inertial forces. The frequency of these platforms is high compared with the frequency of the waves, and the damping effect of the water is sufficient to exclude the possibility of vibration that is induced by regular waves. For these platforms, programs such as MARCS and STARDYNE can be used. For structures over 500 feet, linear or nonlinear dynamic (time history) analysis is preferred, and programs such as WAVE-LOAD + STRUDL [44], MARCS [45], and WAVE + SAP [46] are applicable. The fundamental period of a three-dimensional truss-type offshore structure constructed in 1000 feet of water is predicted to range from 7 to 9 seconds depending on the amount of taper. (The taper is designed to minimize wave action.) The damping force due to the drag term in the wave force equation is not great enough to prevent large amplitudes if waves with periods matching the periods of the structure occur in a regular wave train.

Fixed-base platforms are sometimes subject to long oceanic trips from the assembly site to the construction site. The intermediate position requires careful consideration to account for towing conditions and erection. Programs for simulation of the uprighting process of an assembled platform from a horizontal position to a vertical position have been written. They are FLAP [47] and OPUS [48]. A program readily available through a service bureau is LAUNCH [45], which is actually a part of the MARCS system.

Uprighting of large rigs (400 feet tall, 7000 tons, and $14 million) demands special attention to the uprighting procedure. Obviously the uprighting conditions must be an important part of the offshore structure analysis package.

Semisubmersible Rig

Semisubmersible rigs respond with relatively large motions to the action of the waves. The motion increases with the increased wave height only up to a certain limit. The rapid fall of the transfer functions starts at frequencies that correspond to a wavelength greater than about twice the overall dimensions of the rig in the direction of the waves. A semisubmersible should be able to withstand some minimum amount of flooding without sinking or capsizing.

The description of the behavior of floating platforms is usually based on the assumption that the platform moves as a rigid body described by six degrees of freedom. Hydrodynamic forces must be synthesized, restraint forces due to mooring must be computed, and stability must be evaluated. Programs for evaluating motions of semisubmersible rigs are often based on a stochastic approach. Examples of such programs are WAVE-LOAD + STRUDL and the Esso Production computer programs. These are described in [49]. For reference, see also [50].

Submerged Structures

Since the wave viscous effect for large-sized structures can be neglected, the fluid structure interaction is simplified and conventional linear analysis programs can be used. The analysis of storage tanks that float and those that stand on the seabed, either fully submerged or penetrating the sea surface, are discussed in [6]. No special-purpose programs in the public domain were found for reference.

Riser

In addition to its own weight, the applied tension, and the hydrodynamic forces of wave and current, the riser must also resist stresses resulting from lateral displacements due to rig offset. Nonlinear, large displacement analysis must be applied. Calculations of random responses have been given by Tucker and Murtha [51].

In this paper, the relationship between response spectra and input spectra are developed through a series of intermediate steps each of which is either linear or linearized so that the random riser response variables are also Gaussian. Wave spectra may be presently satisfactory for wave forecasting but not for riser response. It has been demonstrated that different spectra lead to greatly different responses. The riser is very sensitive to the spectral density distribution, and the nonlinearities in computation are of primary importance. Therefore, at present, the time history deterministic analysis seems to be a better approach. The input motion of the semisubmersible rig must be computed for the riser.

This computation for the riser is usually done by specialized programs. Bechtel [52] and Exxon Research Corporation (RISDYN, DYNRUN) have compared the riser computations and have had very good agreement. The above programs are proprietary, and no publicly available programs are known to exist.

During critical environmental conditions, the riser may be disengaged and pulled onto the rig, resulting in a pendulum type of motion. For computation of this mode of the riser, a special program was developed at Bechtel [53].

Pipeline

A special program to analyze the dynamic response of an offshore pipeline was written at Esso Production Research Company [54]. The differential equation of motion was expressed in finite difference form and the Crank-Nicholson time-averaging technique was used for integration. Bilinear springs are included to represent the pipe-stinger interface. Dynamic motions are induced in the pipe and stinger by specifying the barge motions as a function of time. Experimental data for the pitch, roll, and heave of the barge during 6- to 8-foot seas were used. The program results were compared with a simple closed-form analytic solution of a beam response.

In deep water and for large-diameter pipe, tension and stingers are required to control pipelaying stresses and possible buckling failure. Sometimes a beam element stiffness is used, together with an iterative equilibrium guess of the deflected pipe. The stepwise computation should include fluid drag forces which are proportional to the square of the relative velocity.

Engineering Technology Analysts, Inc., offers computational capability using a finite-element program called PIPELAY, which can solve two- or three-dimensional nonlinear problems [55]. Brief references concerning the types of loads to be considered in submarine pipeline can be found in [56].

Cables and Mooring

Submerged moorings and cable structures require special analysis because of their negligible rigidity in bending. Since equilibrium for these structures is possible only for certain deflected shapes, computations starting from an assumed initial shape are generally unstable. Choo and Casarella have described methods for the dynamic analysis of cables [57]. A finite element analysis program for submerged cables was developed at Bechtel [58].

Joints

The entire structure is analyzed as a space frame to obtain the internal joint reactions. These reactions are used to compute the detail stress distribution in joints by means of the thin-shell finite element theory. In the finite element method, an automatic mesh generation scheme is desirable for the intersection of tubular members. Since the finite element that is based on the third-order polynomial (Pascal triangle) may be overstiff, the calculated moment stress concentration is usually higher than the moment stress calculation measured in experiments. Some details on stress analysis are referred to in [59,60]. Visser [61] refers to the SATE computer program, which is used for analysis of tubular joints. Lockheed Structural Analysis Services has illustrated the computation of stress concentrations in [62]. The early analysis of stress concentration in joints was based on the properties of Fourier series, and various programs, based on [63,64], for the analysis of T-joints, Y-joints, and K-joints were developed. Background data for Tubular Joints Design Rules in American Codes were reviewed in terms of static ultimate strength and fatigue by Marshall in [65]. A finite element analysis program for tubular joints was reported in 1969 by Greste [66]. Experimental studies related to joints were described in [67].

SURVEY OF PROGRAMS

Only programs with specific capabilities developed for offshore structures analysis are listed here. Many programs are proprietary, and some programs may have been omitted for lack of information. Information about these programs was obtained from published papers, user's manuals, and the response from the technical public to a questionnaire. This questionnaire has been mailed to many offshore structures contractors engaged in design, and to other consulting firms selected from the Directory [68]. The authors are aware that the list is incomplete and apologize to developers for any failure to include a particular program. A summary of the programs is found in Appendix I.

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APPENDIX I

COMPUTER PROGRAMS

WAVE-LOAD [44]

Capability: Fixed-base offshore structure, semisubmersible platforms
Method: Rigid body motion (6 degrees of freedom), Morrison's equation, Stokes and Airy wave theories, Pierson-Moskowitz spectral analysis, soil-pile interaction (STRUDL), integration and iterative force evaluation within WAVE-LOADS, finite element analysis
Limitation: Basically linear-elastic analysis; transient analysis of structures with more than 300 joints is generally not recommended, but the capability exists
Interacting Program: STRUDL
Postprocessing: Preliminary API code check
Affiliation: McDonnell Douglas Automation Company
Box 516
St. Louis, Missouri 63166
Availability: Engineering/consulting firm

MARCS [45]

Capability: Fixed-base offshore structures
Method: Stokes wave theory; coupled pile-soil interaction (STRAN); time history analysis under future development; deterministic, finite element, linear analysis
Limitation: Limited choice in wave representation
Interacting Program: STRAN
Affiliation: Synercom Technology, Inc.
6300 Hillcroft, Suite 303
Houston, Texas 77036
Availability: Purchase or license

WAVE-SAP [46, 72]

Capability: Fixed-base structures
Method: Linear time history analysis, simulation of soil-pile (special program), solution feasible for large number of degrees of freedom, finite element analysis
Limitation: Linear analysis only
Interacting Program: SAP 1.9
Postprocessing: Stress evaluation
Affiliation: Bechtel International Corporation
350 Mission Street
San Francisco, California 94105
Availability: Proprietary

[24]

Capability: Fixed-base platforms
Method: Nonlinear time history analysis; modified Airy wave theory;

Morrison's equation; stress cycles for fatigue evaluation are counted; differences in added mass in tangential and normal directions are considered; deterministic, finite element analysis
Affiliation: Esso Production Research
P.O. Box 2189
Houston, Texas 77001
Availability: Proprietary

DISMAR/CARGON [71]

Capability: Floating platforms, fixed-base platforms
Method: Time history analysis, substructure analysis, evaluates nonlinear equations of motion, stochastic evaluation of final results can be produced, finite element analysis
Limitations: Basically linear structural analysis
Interacting Program: NASTRAN
Affiliation: Teconomare S.p.A., Italy
Availability: Proprietary

MOSAS [69]

Capability: Semisubmersible and floating structures
Method: Rigid body motion (6 degrees of freedom), Morrison's equation (linearized), Froude-Krilov force, linear wave theory
Limitation: 150 joints, 160 members
Interacting Program: STRUDL
Postprocessing: Statistical evaluation (stress transfer functions for certain members)
Affiliation: Shell, Rotterdam, Netherlands
Availability: Proprietary

[74]

Capability: Floating Platforms
Method: Froude-Krilov forces are used with linearized viscous forces; rigid body motion for 6 degrees of freedom for the semisubmersible rig; Pierson-Moskowitz spectra and stochastic analysis, including flooding stability evaluation, are performed
Limitation: Linerized equations
Postprocessing: Fatigue evaluation
Affiliation: Esso Production Research Co.
P.O. Box 2189
Houston, Texas 77001
Availability: Proprietary

Mobile Offshore Units [70]

Capability: Semisubmersible platform
Method: Rigid body motion (6 degrees of freedom), several wave spectra are available, stress transfer functions, finite element analysis
Limitation: In statistical evaluation, nonlinear drag forces are neglected
Interacting Program: STRUDL
Postprocessing: Stress concentration in joints
Comment: Miner Law is fatigue criterion
Affiliation: Bureau Veritas (Classification Society)
Oslo, Norway
Availability: Proprietary

LAUNCH [73]

Capability: Fixed-base offshore structures
Method: Launching, lifting, and flooding included
Limitation: Interactive static analysis
Affiliation: Synercom Technology, Inc.
6300 Hillcroft, Suite 303
Houston, Texas 77036
Availability: Purchase or license

OPUS [48]

Capability: Offshore platform upending simulation
Interacting Program: IBM System/360 timeshore software
Affiliation: Earl & Wright
657 Howard Street
San Francisco, CA. 94105
Availability: Consulting basis

Riser Dynamics [52, 53]

Capability: Nonlinear dynamic analysis of riser
Method: Based on finite difference method and Morrison's equation; Airy, Stokes, or cnoidal wave theories are used; time history analysis for large horizontal displacement and various boundary conditions is available
Limitation: Two-dimensional analysis
Postprocessing: Comparative studies of response due to various wave parameters and riser pretensioning
Affiliation: Bechtel International Corporation
350 Mission Street
San Francisco, CA. 94105

ETA/PIPLAY [55]

Capability: Finite element nonlinear (geometric) analysis for three-dimensional pipe laying operations
Method: Stokes V wave theory used
Limitation: Maximum of 100 joints
Affiliation: Engineering Technology Analysts, Inc.
4140 Southwest Freeway
Houston, Texas 77027
Availability: Consulting basis

NLIN [58]

Capability: Submerged cable structures
Method: Nonlinear static and dynamic finite element analysis, uses three-dimensional general model, curved cable element, towing and deployment of cable system for time history analysis
Limitation: Hydrodynamic analysis of current-induced forces only
Affiliation: Bechtel International Corporation
350 Mission Street
San Francisco, CA. 94105
Availability: Proprietary

WAVMAS/SPACE III [75]

Capability: Finite element method for beam and shell elements
Method: Wave forces based on Stokes V theory
Limitation: Maximum of 500 joints to represent structure
Interacting Program: SPACE III
Postprocessing: Plotting of structural geometry is available
Affiliation: ODECO
1600 Canal Street
New Orleans, Louisiana 70161
Availability: Proprietary

[75]

Capability: Beam and shell finite element analysis
Method: Dynamic capability for studying barge or ship motion, stochastic approach used
Interacting Program: NASTRAN is the main program; SCORES used for ship motion
Postprocessing: Plotting of structure geometry is available
Affiliation: J. J. Henry Co.
West Park Drive
Moorestown, New Jersey 08057
Availability: Consulting basis or service bureau

Semisubmersible Mooring Motions

Capability: Semisubmersible rig and mooring lines
Method: Nonlinear finite element analysis; Stokes and non-Stokes wave theories used; the dynamic portion is based on harmonic analysis
Limitation: Maximum of 1500 joints
Postprocessing: Plotting of structure and resultant forces and moments
Affiliation: Ocean Oil International Engineering
3019 Mercedes Boulevard
New Orleans, Louisiana 70114
Availability: Consulting basis (contact Hector Pazos of above)

[78]

Capability: Fixed-base structures
Method: Dynamic finite element, a feature for analyzing stress concentration at joints, Stokes V wave theory used
Interacting Program: STRUDL
Postprocessing: Checks for compliance with AISC and API code
Affiliation: De Long Corp, Hersent Offshore, Inc.
24 Broadway
New York, New York 10006
Availability: Proprietary

[77]

Capability: Program uses pipe (curved beam) element to model the structure
Method: Based on finite difference method and Airy wave theory, deterministic or stochastic solution of dynamic problem for both soil structure and wave-structure interaction
Limitation: Maximum of 500 joints to describe the structure
Affiliation: Clemson University,

Civil Engineering Department
Clemson, South Carolina 29631
Availability: University program available to public

Capability: Static analysis of beam and pipe elements
Method: Includes AISC code check, wave theory includes Airy and Stokes V
Limitation: Maximum of 100 joints to describe the structure
Postprocessing: Plotting of structure geometry is available
Affiliation: Shell Oil Company
P.O. Box 2099
Houston, Texas 77001
Availability: Proprietary

Frames

R. R. Craig, Jr.
C. P. Johnson
Texas Institute for Computational Mechanics
The University of Texas at Austin

INTRODUCTION

Although most, if not all, general purpose programs having the capability of solving structural dynamics problems possess the capability of solving frame problems, the present chapter is restricted to a discussion of programs whose primary purpose is the solution of dynamic (and, in some cases, static) problems involving frame structures. Undoubtedly many frame analysis programs exist which were not discovered in the authors' search for programs. However, the programs which are described in this chapter do solve a broad range of frame structural dynamics problems.

The key ingredient in all frame programs is the frame element, a one-dimensional element with six degrees of freedom at each end. Frame elements may be assembled to form structures ranging from continuous beams to three-dimensional frame structures. Some programs described herein permit structures of arbitrary geometry to be described. Several programs are restricted to specialized models of building frames. The types of dynamic behavior represented are: eigenvalue/eigenvector extraction, transient response, harmonic response, and seismic (shock spectrum) response. Some programs are restricted to linearly-elastic response of structures, while others permit material and/or geometric nonlinearity. All programs reviewed are based on the finite element displacement method.

SOLUTION PROCEDURES

Equations of Motion

The equations of motion for the structure may be written in the form [1]

$$[M]\{\Delta\ddot{x}\} + \{\Delta F^{i}\} = \{\Delta F^{e}\} \tag{1}$$

where $\{\Delta F^{e}\}$ is the incremental load vector, $\{\Delta F^{i}\}$ contains incremental internal loads including damping, $\{\Delta x\}$ is the vector of incremental displacements, $[M]$ is the mass matrix, and superscript dots denote time derivatives. For linearly-elastic structures with linear viscous damping Eq. (1) becomes

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F^{e}\} \tag{2}$$

where $[C]$ is the damping matrix and $[K]$ is the stiffness matrix.

Static Analysis

Some programs which treat dynamic analysis of frame structures also make a provision for static analysis. For static analysis of linearly-elastic structures the equilibrium equation

$$[K]\{x\} = \{F^e\} \tag{3}$$

is solved. In most programs an attempt has been made to make the equation solver for Eq. (3) as efficient as possible by taking advantage of the symmetry and bandedness of [K]. Static analysis of structures exhibiting nonlinear response (geometric or material) is performed by employing incremental loading together with the tangent stiffness matrix as indicated by

$$[K_T]\{\Delta x\} = \{\Delta F^e\} \tag{4}$$

An iterative solution is required for Eq. (4) since the $[K_T]$ computed at configuration $\{x\} + \{\Delta x\}$ will generally not be the same as the $[K_T]$ computed at configuration $\{x\}$ and used in Eq. (4). This is illustrated by the one-dimensional load-deflection curve below

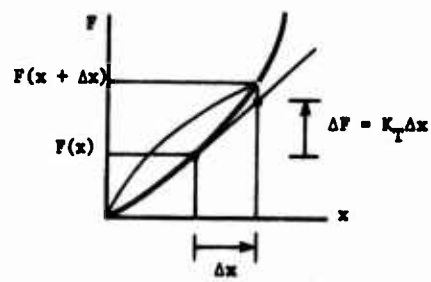

Fig. 1 Nonlinear load-deflection curve

Dynamic Analysis

The types of dynamic analysis available in the frame programs described in this chapter include: eigenvalue/eigenvector extraction (modes and frequencies), transient response, harmonic response, and seismic (shock spectrum) response.

Modes and Frequencies

The eigenvalue problem for linearly-elastic structures may be written

$$[K]\{X\} - \omega^2[M]\{X\} = \{0\} \tag{5}$$

where ω is the circular natural frequency of free vibration and $\{X\}$ is the mode shape vector. If the structure is restrained such that no rigid-body motion is allowed, the values of ω will all be positive. There will be a value of $\omega = 0$ corresponding to each rigid body mode present.

Three types of methods are currently used in structural dynamics software [1]: (1) matrix transformation methods, e.g. Householder's method or Jacobi's method; (2) vector iteration; and (3) determinant search (usually with Sturm sequence checks). The Householder method, which transforms a matrix to tridiagonal form, may be followed by the QR algorithm which computes eigenvalues of the tridiagonal matrix and by inverse iteration to determine the eigenvectors. The Jacobi method transforms a matrix to diagonal form and, at the same time, produces the eigenvectors. Since matrix transformation methods such as the Householder method and Jacobi's method produce all eigenvalues at one time, they are not generally employed for large problems. Vector iteration and determinant search, on the other hand, permit determination of a sub-set of the total number of eigenvalues and eigenvectors of the system. In these latter methods it is also possible to take advantage of the bandedness of the original matrices.

Transient Analysis

Problems classified as transient analysis problems are those governed by Eq. (1), where $\{\Delta F^e\}$ is a vector containing specified functions of time, and where initial conditions are specified. The case where components of $\{\Delta F^e\}$ are harmonic functions of time will be treated separately below as will the case where the excitation is prescribed by shock spectra.

For linear transient response problems, governed by Eq. (2), either of two methods for dealing with the time dependent nature of the problem may be employed: (1) mode superposition, or (2) direct integration. In mode superposition the normal modes are used to uncouple the equations of motion. Damping is usually defined in such a manner that the damping matrix is also diagonal when transformed to modal coordinates. For viscous damping, the resulting equations have the form

$$\ddot{z}_j + 2\zeta_j\omega_j\dot{z}_j + \omega_j^2 z_j = p_j(t) \tag{6}$$

where $z_j(t)$ is the time-dependent amplitude of the jth normal mode, ζ_j is the damping factor, ω_j is the circular natural frequency, and p_j is the forcing function. In the programs reviewed here, the forcing functions are assumed to be piecewise linear. The use of piecewise linear forcing functions, $p_j(t)$, in Eq. (6) permits a closed-form solution of this equation to be used. On the other hand, Eq. (6) can also be solved by step-by-step numerical integration.

The direct integration method involves a step-by-step numerical integration of the original coupled equations of motion, e.g. Eq. (2). If wave propagation effects are important, i.e. if the high-frequency modes of the system contribute significantly to the response, direct integration should be used. Otherwise, mode-superposition is more desirable since modes having frequencies above a stated cut-off frequency can be eliminated. Of the linear analysis programs reviewed in this chapter, only two employ direct integration of the original coupled equations.

Direct integration of the coupled equations of motion has been used in the programs which treat nonlinear transient response. Integration is performed by assuming a form for the acceleration during a time step of length Δt. The method based on linear variation of acceleration with respect to time is known to be unstable in the presence of vibration modes with periods exceeding approximately one third of the time step. Belytschko [1] notes that the use of a conditionally stable method, such as the linear acceleration method, on elastic-plastic problems could lead to an undetected "arrested instability." The constant average acceleration method is stable for all values of Δt and produces results of acceptable accuracy if Δt is short in comparison with the periods of dominant modes present in the response.

The solution of a nonlinear transient response problem is similar in some respects to a nonlinear static analysis in that an incremental solution is employed. For example, Eq. (1) may be written

$$[M]\{\Delta\ddot{x}\} + [C_T]\{\Delta\dot{x}\} + [K_T]\{\Delta x\} = \{\Delta F^e\} \tag{7}$$

where $[K_T]$ is the tangent stiffness matrix and $[C_T]$ is a so-called tangent damping matrix. The tangent stiffness matrix must account for material nonlinearity and geometric nonlinearity. The latter is sometimes referred to as the "P-Δ effect." As in a nonlinear static analysis, it is necessary to evaluate member forces at the end of a time step (load increment) and apply corrective forces if (dynamic) equilibrium is not satisfied due to changes that have taken place in $[K_T]$ and $[C_T]$ over the time step.

Harmonic Analysis

For linear behavior of structures subject to harmonic excitation it is useful to have available the closed form solution of Eq. (6), consisting of the steady-state solution and the starting transient solution. Several programs reviewed here provide this capability.

Seismic Analysis; Shock Spectrum

The analysis of the response of a structure to earthquake motion may either be deterministic or non-deterministic. A deterministic analysis may be carried out using the transient analysis capability of a program if the time history of the input is known. Alternatively, information regarding approximations to the peak response can be determined using the shock spectrum method [3]. If, for a given excitation $p_j(t)$ Eq. (6) is integrated to determine the velocity $\dot{z}_j(t)$, the result will depend on the value of the damping factor ζ_j and the natural frequency ω_j (or the period T_j). The spectral velocity is defined as the maximum value of $\dot{z}_j(t)$, i.e.

$$S_v(\zeta,T) \equiv \max_t \left(\dot{z}(\zeta,T;t)\right) \tag{8}$$

Since the spectral velocity gives the maximum velocity response of a given mode due to a given input function, it is necessary to combine the effects of all participating modes in order to determine the total response. Since the shock spectrum does not contain phase information, the contributions of various modes to the total response can only be combined in an approximate manner. Two such methods are: (1) root-mean-square, and (2) sum of absolute values.

Special Solution Procedures

Although most programs reviewed are restricted to solving particular problems for frame structures, one program [2] is designed primarily for teaching concepts of matrix structural analysis and structural dynamics. It includes commands for executing all of the standard matrix operations, e.g. load, print, add, multiply, invert, partition, etc. In addition, it has commands for generating element matrices, assembling system matrices by the direct stiffness method, solving for element stresses, determining transient response, and other structural mechanics oriented commands.

MODELING

As stated previously, the key ingredient in all frame analysis programs is the frame element. Several programs allow arbitrary configurations of structures to be assembled from frame elements; others treat specialized models of building frames. The latter type will be described first, and then there will be mention of the various elements available.

Special Building Models

Several of the programs reviewed deal with special models for multi-story building frames. In each of these the principal assumption is that each floor level may be assumed to be rigid in its own plane. Thus, in those programs dealing with three-dimensional frames, the rigid-body degrees of freedom of each floor consist of two translational components plus a rotation about the vertical. In some plane frame programs the assumption of floors rigid in their own plane may also be enforced.

Weaver and his colleagues have developed computer programs based on the "tier building" model [4,5,6]. This is described as follows [6]:

> The tier building model is a specialized case of the general problem of rigid bodies elastically connected by structural space framing. The model consists of floor and roof diaphragms that are rigid in their own planes, flexible normal to their planes, and are interconnected by structural framing and shear walls. The framing spans between joints which are arranged in a three-dimensional grid-work pattern oriented parallel to a set of three orthogonal reference axes. Columns and shear walls are vertical and parallel to the z reference axis, whereas beams framing into the columns and shear walls are parallel to the x and y reference axes. Shear wall configurations must conform to the grid-work pattern and may run only between joints. Bracing must either lie in planes parallel to the x,z plane or the y,z plane and must span between adjacent floors and between adjacent grid lines.

An alternative model, not restricted to rectangular geometry, is employed in [7]. This model is based on a substructure concept, wherein beams, columns and shear walls are assembled to form a plane frame. The plane frames are then connected together by rigid floor diaphragms to form the three-dimensional structure.

Element Properties

Stiffness Matrix (Linear) of Frame Element

The straight frame element of uniform cross section is described in [8] and in other standard references on matrix structural analysis or finite element analysis. Shear deformation is allowed in some cases. For three-dimensional frameworks there are six degrees of freedom (d.o.f.) at each node (end). Special cases such as the plane frame element (three d.o.f.), space truss (three d.o.f.), plane truss (two d.o.f.) and beam (two d.o.f.) are available as separate elements in some programs. Tapered elements and curved elements are available in other programs.

Geometric Stiffness Matrix of Frame Element

For large displacements involved in an analysis of post-buckling or of inelastic behavior, the geometric stiffness matrix [8] may be used to account for the effect of the axial force on the lateral displacements, the "P-Δ effect" referred to earlier.

Special Elements

In some structures the nature of the joints makes it desirable to have available procedures for handling finite-dimension joints, partial fixity at joints, or releases at joints. The finite-dimension joint is treated by attaching short rigid links to the ends of the usual frame elements. Partial fixity is treated by use of local springs.

One program [5] provides a twentyfour d.o.f. rectangular prism element for modeling the soil near a building and a rigid block with six degrees of freedom for modeling a massive foundation.

Mass Matrices

Some of the programs reviewed include consistent mass matrices for the frame elements [8], while others employ either a lumped mass model or the model of elastically-connected rigid bodies. The latter is employed in the special building frame programs which model the floor and roof slabs as bodies which are rigid in their own plane. Then the inertia properties of the slab, together with any beam or column mass included, are determined using two components of translation of the mass center in the plane of the slab and a rotation about the normal to the slab.

Damping

Most of the programs reviewed permit viscous damping to be incorporated in some manner, either by assuming a damping matrix of the form

$$[C] = \alpha[M] + \beta[K] \tag{9}$$

or by specifying damping factors ζ_i, to be used in Eq. (6). The former will be referred to as proportional damping, the latter as modal damping. Another form of damping, not widely available in the frame programs reviewed, is called structural damping. In this case, the damping is assumed to be proportional to the elastic force, but 180 degrees out of phase with the velocity.

INPUT/OUTPUT

One feature that potentially distinguishes a frame analysis program from a general purpose computer program is the I/O format. For example, the special purpose programs for buildings have features for input and output of data relative to stories and bays. In most cases the input format is closely related to a more-or-less standard finite element program input, i.e. nodal data, material data, element data, connectivities, boundary conditions, loads, etc.

All of the programs reviewed, with the exception of three, are essentially batch programs with specified input formats. Two programs [2,9] are designed for convenient interactive, or remote terminal, execution. Both have free-field input format with data fields being separated by commas. Both programs

can also be executed in a "remote batch" or batch mode, i.e. if the problem is large the input data can be submitted from a time-share terminal but the job is then executed in the normal batch mode. Free-field input is also employed in the program of [10].

The output of most programs consists of printer output of time histories of displacements of floors or node points, time history of internal forces or maximum values of internal forces, and mode shapes. Several programs permit plotting of mode shapes, time histories of specified displacements, and/or moment and shear diagrams.

PROGRAM SUMMARIES

In the program summaries which follow and in Table 1 the authors have attempted to present an accurate description of the programs reviewed. The information presented was obtained from a questionnaire filled out by most program developers and from program documentation, when supplied. It is the authors' hope that no gross misrepresentations appear in the program summaries which follow.

Structures and Matrix Interpretive System (SMIS74) [2]

Date: July 1974.
Capability: Standard matrix operations (load, add, invert, etc.); special structural mechanics commands to formulate element stiffness and mass matrices, assemble system matrices by direct stiffness method, and solve transient response problem.
Methods: Eigensolver - QR method with inverse iteration for eigenvectors; transient response - linear acceleration step-by-step solution of modal equations.
Limitations and Restrictions: Matrices are stored in a single vector whose size is set at time of execution; auxiliary matrix storage on two external files; eigensolution restricted to 80 d.o.f.
Documentation: Users' Manual and Programmers' Guide contains complete description of all operations (i.e. matrix operations and special structural analysis operations) including input formats; sample static analysis and dynamic analysis.
Input: Free-field, with data fields separated by commas. Operation code word (e.g. LOAD), matrix names, and numerical data (e.g. matrix size) are input. Program can be operated interactively, i.e. one "card" at a time, or as a batch job.
Output: Matrix PRINT can be used to output any matrix, e.g. static displacement, modal matrix, etc. Member forces in a static analysis are output by the FORCE command. Crude line printer plot of vectors available.
Language: FORTRAN IV, special system calls associated with allocation of matrix storage at time of execution.
Hardware: CDC 6600, 36K octal words plus matrix storage
Developers: Eric B. Becker and Roy R. Craig, Jr.,
Texas Institute for Computational Mechanics, and
Dept. of Aerospace Engineering and Engineering Mechanics,
The University of Texas at Austin
Availability: $200, purchase from
Dr. Roy R. Craig, Jr.
Texas Institute for Computational Mechanics
The University of Texas at Austin
Austin, Texas 78712
Comments: Thoroughly documented. Designed especially as an instructional tool for teaching introductory matrix structural analysis and structural dynamics courses. Convenient free-field input format and interactive execution permit step-by-step solution of problems, including interactive

design problems. Batch solution can be employed for large problems if desired. Modularity of program permits relatively easy addition of further operations if desired.

Inelastic Analysis of Tier Buildings (INELASTIER) [4]

Date: May 1972.
Capability: Static and transient response of plane frames or of rectangular buildings using tier building model; inelastic behavior of members.
Methods: Incremental loading, linear acceleration method for transient response.
Limitations and Restrictions: Standard program - Max. 10 stories, 5 bays in both x and y directions, 20 joints per floor.
Documentation: Author's Ph.D. dissertation available as a technical report [2] including theory, input data formats, examples, listing, and sample output.
Input: Fixed format, data input organized according to stories and bays.
Output: Floor displacements, member end actions, joint displacements; maximum values or values at each load step.
Language: FORTRAN IV (H)
Hardware: IBM 360/67, 500K bytes.
Developer: James L. Bockholt,
Department of Civil Engineering
Stanford University
(Prof. William W. Weaver, Jr., supervising professor)
Availability: $240, purchase from
William Weaver, Jr., Prof. Struct. Engr.
Department of Civil Engineering
Stanford University
Stanford, Calif. 94305.
Comments: Documentation appears to be especially good. Complete theory, examples including run times, detailed input format descriptions, and programming notes should make program usage straightforward. A related plane frame program, INELAS2D, is described in the same reference. Current limitations for this program are: 20 stories, 10 bays.

Soil - Foundation - Structure Interaction During Earthquake Excitations (SOILTIER) [5]

Date: June 1971.
Capability: Analysis of modes and frequencies, and of transient response of rectangular buildings using the tier building model and including effects of the soil and structural foundation on transmission of specified bedrock accelerations to the building.
Methods: Finite element representation of frame members and soil; rigid-body representation of foundation motion and in-plane floor motion; modes and frequencies; step-by-step direct integration for transient response.
Limitations and Restrictions: Standard program - Maximum 7 elements in x and y, 4 elements in z, 128 d.o.f. in response calculations, 50 segment forcing functions in x and y directions.
Documentation: Author's Ph.D. dissertation available in a technical report [5] including theory, examples, and listing.
Input: Fixed-format. Portions of the stiffness and mass matrices of the tier building are generated in SOILTIER, Part 2. These matrices are then given to SOILTIER as input data.
Output: Time histories and maximum values of accelerations, velocities, and displacements. Maximum story shears.
Language: FORTRAN IV (H)
Hardware: IBM 360/67

Developer: Gregg E. Brandow
Department of Civil Engineering
Stanford University
(Prof. William W. Weaver, Jr., supervising professor)
Availability: $240, purchase from
William Weaver, Jr., Prof. Structural Engineering,
Department of Civil Engineering
Stanford University
Stanford, Calif. 94305

Dynamics of Tier Building (DYNATIER) [6]

Date: June 1970
Capability: Modes and frequencies, and response of structure to piecewise linear forcing for rectangular space frames
Methods: Eigensolver - Jacobi method; transient response - closed form solution of modal equations for piecewise linear excitation; tier building model.
Limitations and Restrictions: 21 stories, 63 d.o.f. per floor
Documentation: Author's thesis for Engineer degree available as a technical report [6], including brief summary of theory, input data formats, listing, and sample problem. Thesis includes description of companion static analysis program, STATIER.
Input: Fixed format; geometry and member properties related to stories and bays.
Output: Natural frequencies and mode shapes, displacement-time histories, member force-time histories, maximum member forces, static analysis. Plots of mode shapes and displacement-time histories.
Language: FORTRAN IV (H)
Hardware: IBM 360/67, 330K bytes
Developer: Gregg Everett Brandow
Dept. of Civil Engineering
Stanford Univeristy
(Prof. William W. Weaver, Jr., supervising professor)
Availability: $240, purchase from
William Weaver, Jr., Prof. Struct. Engr.
Dept. of Civil Engineering
Stanford University
Stanford, Calif. 94305
Comments: Well documented. Tier building model may result in a substantial reduction of the number of degrees of freedom required in the eigensolution.

Three-Dimensional Analysis of Building Systems (XTABS) [7]

Date: February 1974
Capability: Linear analysis of frame and shear wall buildings subjected to static and earthquake loadings; modes and frequencies; nonsymmetric, nonrectangular buildings that have frames and shear walls located arbitrarily in plan can be considered.
Methods: Structure idealized as frames connected to floor diaphragms rigid in their own planes; special elements for shear panel and finite joint; piecewise linear forcing with closed form integration of modal equations; response spectrum analysis with r.m.s. modal combination or summation of absolute values of modal contributions.
Limitations and Restrictions: Dynamic storage allocation for major arrays in blank common. Documentation has formulas for computing required storage.
Documentation: Technical report describes building model and data input formats; listing; sample input and output.

Input: Fixed format; story and frame data; frame, colur , and shear panel data; loading data.
Output: For complete building - story displacements for various load cases, mode shapes and natural periods; for each frame - lateral frame displacements for each load case, member forces, story shear at each level of frame for various load cases.
Language: FORTRAN IV
Hardware: CDC 6400, IBM 370, 55K octal
Developers: E. L. Wilson and H. H. Dovey
Earthquake Engineering Research Center
University of California at Berkeley
Availability: $100 (surface mail) $125 (air mail)
NISEE/ Computer Applications
Davis Hall
University of California
Berkeley, CA 94729
(make checks payable to: The Regents of the University of California)
Comments: Description of data input appears adequate. Discussion of special building model based on frame substructures is difficult to follow. Authors comment that use of the program is "questionable" unless the frames are arranged in a "reasonably rectangular fashion." Authors provide some suggestions for modeling practical structures.

Dynamic Analysis of General Structures (DAGS, DAGSINE, DAGTRAN, DAGSMIC) [9]

Date: Jan. 1974 (DAGS), Feb. 1975 (DAGS Response Programs).
Capability: Modes and frequencies, transient response, buckling, response to harmonic excitation, and response spectrum of general frame structures.
Methods: Eigensolver - determinant search with Sturm sequence checks; transient response - closed-form solution for harmonic or piecewise linear excitation.
Limitations and Restrictions: 300 d.o.f. and 100 joints, other restrictions specified in documentation
Documentation: Users' Manuals briefly summarize theory, describe elements and program options, provide input data forms, and provide numerous examples including problem set-up, input data, and output.
Input: Free-field; program may be run interactively or as a batch job; input data sheet forms provided.
Output: Mode shapes and frequencies, displacement and moment time histories, displacement and moment response spectra; plots of deformed structure, moment and shear plots by member.
Language: FORTRAN
Hardware: Honeywell 400, CDC 6000, XDS Sigma 9
Developers: Structural Dynamic Research Corporation
Availability: Lease; use on national time share vendors; license (= purchase) for DAGS is $20,000. Other prices available from developer,
Structural Dynamics Research Corporation
5729 Dragon Way
Cincinnati, Ohio 45227
Comments: These appear to be high quality programs for production type usage. Documentation and form of input are especially user oriented. SAGS is a companion static analysis program.

A Computer Program for Plane Frame Analysis (PFVIBAT) [10]

Date: March 1974
Capability: Modes and frequencies and response to harmonic excitation for linearly elastic plane frames.

Methods: Eigensolver - determinant search.
Limitations and Restrictions: 100 joints, 100 members as currently dimensioned.
Documentation: Technical report briefly summarizes theory, describes input formats, and provides examples. Report also describes how the eigensolution can be used for the solution of harmonic loading, transient loading, and random loading problems.
Input: Free-field input consists of operation cards and data cards.
Output: For each natural mode of the forced vibration solution the following are printed: displacements (mode shape), support reactions, member end displacements, and member end forces. Modal masses are given. Displacements and bending moments in forced and free vibration may be plotted.
Language: FORTRAN IV
Hardware: IBM 360/65 202K bytes
Developers: Bengt Akesson and Harold Tagnfors
Division of Solid Mechanics
Chalmers University of Technology
Availability: $200 for punched-card copy of source program from
Division of Solid Mechanics
Chalmers University of Technology
Fack, S-402 20
Göteborg, Sweden

Dynamic Analysis of Linear Frames (DALF1) and Dynamic Analysis of Nonlinear Frames (DANF1) [11]

Date: August 1971
Capability: static response; transient response of linear (DALF1) and geometrically nonlinear (DANF1) plane frames.
Methods: Constant average acceleration; geometric nonlinearity treated by incremental solution using geometric stiffness matrix; force imbalance at end of each time step due to nonlinearity is applied (with minus sign) at beginning of succeeding step.
Limitations and Restrictions: 100 members, 75 nodes, 21 d.o.f. bandwidth
Documentation: Developer's M.S. thesis contains very thorough description of theory underlying the program and presents numerical results for several example problems. Data preparation guide available separately.
Input: Joint coordinates, member connectivities, member properties, supports and constraints and loads are input in fixed format. Generation options available for specifying dynamic loading.
Output: Printer output of displacement time history and moment time history. Plots of time histories of displacement, velocity, and acceleration at a specified degree of freedom may be produced.
Language: FORTRAN IV
Hardware: CDC 6600
Developer: Larry M. Bryant,
Dept. of Civil Engineering
The University of Texas at Austin
(Dr. C. P. Johnson, supervising professor)
Availability: $50 (DALF1) or $50 (DANF1) to purchase program from
Mr. Larry M. Bryant (or Dr. C. P. Johnson)
Dept. of Civil Engineering
The University of Texas at Austin
Austin, Texas 78712
Comments: Theory of solving geometrically nonlinear problems well presented in developer's thesis. The data preparation and problem set-up information is sketchy. Results obtained using program, e.g., post-buckling behavior of a cantilever, show good agreement with other solutions.

Inelastic Dynamic Response of Plane Structures (DRAIN 2D) [12]

Date: April 1973
Capability: Dynamic response of inelastic two-dimensional structures of arbitrary configuration resulting from earthquake type ground motions. Static loads may be applied prior to dynamic loading, but behavior under static loads may not be inelastic.
Methods: Constant average acceleration step-by-step numerical integration. Incremental solution with force imbalance corrections applied at succeeding time steps. Geometric stiffness matrix using static gravity loads approximates "P-Δ effect."
Limitations and Restrictions: Most large matrices are stored in blank COMMON. User's Guide describes calculation of required storage and procedure for allocating it. Data automatically transferred in blocks to scratch storage if core storage is exceeded.
Documentation: Technical report contains thorough description of theory and procedures involved and contains a User's Guide which presents input data formats with an example data set which is provided on the program tape when program is purchased. Several example problems are solved, and input data listings for these problems are provided.
Input: Fixed format data including structure geometry, element data, nodal loadings, and earthquake time history.
Output: Static displacements; force and displacement time histories and envelope (maximum) values of nodal forces including accumulated plastic strains and plastic hinge rotations for each element.
Language: FORTRAN IV
Hardware: CDC 6400, 76K octal words.
Developers: Amin E. Kanaan and Graham H. Powell
Earthquake Engineering Research Center
The University of California at Berkeley
Availability: $150 (surface mail USA), $200 (air mail USA), $250 (surface mail outside USA), $300 (air mail outside USA) from
NISEE
Davis Hall
University of California
Berkeley, CA 94729
(Make check payable to: "The Regents of the University of California")
Comments: Theory is well presented. Section describing procedure for addition of elements to program may be useful. Several instructive examples are presented. User's Guide appears to provide adequate information.

Dynamic Analysis of Inelastic Space Frames (DYNAMIC) [13]

Date: 1970
Capability: Analysis of dynamically loaded space frames with geometric and material nonlinearity.
Methods: Extension of concept of yield hinge to yield zone; incremental solution with acceleration taken as constant initial acceleration to compute incremental displacement, but constant average acceleration to compute incremental velocity.
Limitations and Restrictions: 100 members, 50 joints
Documentation: Ph.D. thesis by Fereydoon Farhoomand (Prof. R. K. Wen, supervising professor) contains theory, numerical results for three example problems, and program listing with definition of program labels (arrays, parameters, etc.); no input data format was provided to authors, and developers do not indicate existence of any.
Input: Not available to authors.
Output: Not available to authors.
Language: FORTRAN, except matrix inversion routine in COMPASS.
Hardware: CDC 6500

Developers: R. K. Wen and F. Farhoomand,
Dept. of Civil Engineering
Michigan State University
Availability: Available from
Prof. R. K. Wen
Dept. of Civil Engineering
Michigan State University
East Lansing, Mich. 48824
Comments: Lack of input data formats and sample problems with input listings leads the authors to conclude that the developers have not previously considered transferring the program. Due to limited number of programs available to treat inelastic behavior of space frames, and due to the developers' expressed willingness to transfer the program, it has been included in this review of available frame dynamics programs.

Frame (FRAME) [14]

Date: October 1973.
Capability: Static analysis, modes and frequencies, and transient displacement by mode superposition and direct integration of linearly elastic frames (beams, trusses, grids, etc.)
Methods: Eigensolver - power method and Jennings' method; transient response-mode superposition and Chan's method
Limitations and Restrictions: Not stated
Documentation: Data preparation manual, mathematical formulations, numerical techniques, and sample problem solutions. (The developers did not supply any of the documentation to the authors, so it is not known how extensive or helpful this documentation would be.)
Input: Not available to authors at time of writing.
Output: Not available to authors at time of writing.
Language: FORTRAN for the most part.
Hardware: Univac 1100 series, 65 K words
Developers: Structural Analysis Group, R & D Division
Nippon Univac Co.
Tokyo, Japan
Availability: Computing service (charges not stated) provided by
Nippon Univac Co.
17-51, Akasaka 2-Chome, Minato-ku
Tokyo 107 JAPAN
Comments: The developers did not provide the authors with a sample of the documentation. Apparently the program is available only on a computing service basis.

STRU-PAK [15] including
Two-Dimensional Truss Modal Analysis Program (2DTMAP)
Two-Dimensional Frame Modal Analysis Program (2DFMAP)
Three-Dimensional Truss Modal Analysis Program (3DTMAP)
Three-Dimensional Frame Modal Analysis Program (3DFMAP)
Grid Modal Analysis Program (GRIDMAP)

Date: May 1970
Capability: Modes and frequencies of the respective categories of frame-type structures.
Methods: Gaussian elimination of massless degrees of freedom followed by Householder-Sturm solution for eigenvalues and inverse iteration for eigenvectors.
Limitations and Restrictions: 75 d.o.f., 100 members, 50 joints, 20 mode shapes (all programs)
Documentation: Users' Guide for each program containing brief review of theory, input data formats, and sample problems including problem set-up, input listing, and output listing.

Input: User creates free-field input file and executes program interactively.
Output: Stiffness matrix (optional), natural frequencies, mode shapes, displacement-mass products, and orthogonality check are printed.
Language: Not stated by developer; presumably FORTRAN IV.
Hardware: CDC
Developers: TRW Systems Group, Redondo Beach, Calif.
Availability: Local CDC Cybernet Center, or Applications Services
CONTROL DATA CORPORATION
P. O. Box O HQW05G
Minneapolis, Minn. 55440
Comments: Very well documented programs. Free-field input and interactive execution are desirable features.

Analysis of Plane Frames Subjected to Forced Vibrations (#253) [16]

Date: Not given.
Capability: Response of linearly-elastic plane frames to harmonic excitation.
Method: Basically a static analysis using $(K - \omega^2 M)X = P$ with ω specified. Gaussian elimination and Cholesky square root methods for equation solvers.
Limitations and Restrictions: 600 members, 400 joints; maximum size of stiffness matrix is 30,000.
Documentation: User's Manual including very brief description of solution method, summary of input cards, computer print-out description, and definitions of variables used in input data.
Input: Joint coordinates, member connectivities, member properties, supports and constraints, and loads are input in fixed format.
Output: Displacements and member forces
Language: FORTRAN
Hardware: Univac 1108
Developer: Dr. S. S. Tezcan, P. E., Electronic Calculus, Inc.
Availability: Electronic Calculus Inc.
468 Park Avenue South
New York, N. Y. 10016
or University Computing Co. (UCC)
Utility Network Center
Comments: ECI has a large number of users of its programs among structural consulting firms.

Dynamic Modal Analysis of Large Space Bar Structures (#658C) [17]

Date: Not given.
Capability: Modes and frequencies, transient response to specified time-dependent loading or shock spectrum. 1940 El Centro (N-S component) spectrum incorporated into program, but other spectra may be specified.
Methods: Eigensolver - Jacobi method determines all eigenvalues and vectors; transient response - Duhamel integral for undamped case is cited, but documentation does not indicate if or how this is used for damped structure.
Limitations and Restrictions: Documentation states, "No strict limits exist for the maximum number of joints and members in a bar structure which may be analysed by this program." On a Univac 1108 the maximum number of joints is 2500, the maximum number of d.o.f. is 4000, and the maximum number of time points is 1000.
Documentation: User's Manual including summary of theory employed, summary of input cards, computer print-out description, and definition of variables used in input data.
Input: Joint coordinates, member properties, member connectivities, lumped masses, damping factors, forcing function ordinates or response spectrum.

Output: Modes and frequencies, spectral displacements for the supplied accelerogram or forcing functions, modal and maximum probable (RMS) inertia forces.
Language: FORTRAN
Hardware: Univac 1108
Developer: Dr. S. S. Tezcan, P. E.
Electronic Calculus, Inc.
Availability: Electronic Calculus Inc.
468 Park Avenue South
New York, New York 10016
or University Computing Co. (UCC) Utility Network Centers
Comments: ECI has a large number of users of its programs among structural consulting firms.

Frame Modal Analysis (FMA) [18]

Date: Nov. 1970
Capability: Modes and frequencies of space frame
Methods: Jacobi
Limitations and Restrictions: 200 d.o.f. (single precision version), 140 d.o.f. (double precision version)
Documentation: Technical report briefly describes modeling procedures, lists input, describes output, and provides two sample problems, each with problem set-up, input data listing, output listing, and summary of results. There is also program listing.
Input: Data input by NAMELIST, usual frame data except that some substructuring is provided for.
Output: Input data, section (substructure) and system K and M matrices, dynamic matrix, orthogonality check, and frequencies and mode shapes. Section K and M may be punched or saved on tape.
Language: FORTRAN IV
Hardware: IBM 360, 370K bytes
Developer: R. J. Guyan
Space Division Rockwell International
12214 Lakewood Blvd.
Downey, CA 90741
Availability: COSMIC
Barrow Hall, University of Georgia
Athens, Ga., 30601
Comments: The developer states that FMA has not been used at the Space Division in several years, but has been replaced by more general programs such as NASTRAN.

DDAM

Date: Developed September 18, 1969.
Capability: DDAM is a program for the dynamic design/analysis of structures composed of beams and/or springs. DDAM can accommodate up to 120 static degrees of freedom (DOF) and up to 48 dynamic DOF. Input consists of joint coordinates, material properties, member cross-sectional properties, and joint connection data. The DDAM program analyzes a 2 or 3 dimensional structure composed of beams and/or springs to determine the structure's dynamic response to mechanical shock loading.
Method: The program utilizes the stiffness matrix method, beam finite elements, discrete mass, and normal mode methods in calculating natural frequencies, mode shapes, inertial loads, deflections, and internal forces and moments for three-dimension frame beam structures.
Input: Coordinates are used to define the spatial location of the structure's joints or nodes. The precise number of coordinates required to define the location of a given joint is dependent upon the number of static degrees of freedom per joint chosen for the problem. For instance, the analysis of a three-dimensional frame would require that translational

DOF in the directions of the X, Y, Z axes be included as well as the rotational DOF about these axes. (The shorter notation TX, TY, TZ, RX, RY, RZ will be used to denote these DOF below.) Similarly, a two-dimensional truss would only require TX and TY. In all, five combinations of the six DOF are possible: TX-TY, TX-TY-TZ, TX-TY-RZ, TZ-RX-RY, TX-TY-TZ-RX-RY-RZ. The joints of the structure may be interconnected by beams or springs. Each beam element may consist of any number of series pieces or sub-sections. Each piece may have a unique orientation in space and may possess geometric and material properties, unlike those of the adjacent pieces. The final piece in a series may be pinned (released) to slide or rotate in any desired direction(s). Springs may be either translational or rotational. The dynamic properties of the structure may be defined in terms of weight or mass and may be associated with any of the six possible DOF at a given joint. The input spectrum acceleration values must be provided for each mode.

Output: Output consists of the overall structural stiffness matrix, dynamic flexibility, natural frequencies, mode shapes, modal weights, joint displacements and member forces, deflections, and stresses.

Language: FORTRAN IV.

Hardware: GE 635

Developer: Electric Boat Division
General Dynamics Corp.
Groton, Conn. 06340

Availability: Contact: Superintendent of Shipbuilding
Conversion and Repair
U.S.N.
3rd. Avenue and 29th Street
Brooklyn, N. Y. 11232
Attn: Larry Gordon (212) 965-5417
Gerald Greenidge

Also: For commercial availability, contact G. E. Information Services, or Computer Sciences Corporation INFONET

Qualified individuals only.

Computer Methods in Advanced Structural Analysis [19]

Date: 1973

Capabilities: This is a textbook which includes discussion of the following structural dynamics programs together with program listings:
Chapter 1: Free Vibrations of Structural Frames with Lumped Masses
Chapter 2: Free Vibrations of Continuous Beams and Rigid Frames with Distributed Masses
Chapter 3: Undamped Forced Motion of Structural Frames with Lumped Masses

Documentation: Textbook [19]

Language: FORTRAN IV

Developer: C. K. Wang
Department of Civil and Environmental Engineering
University of Wisconsin
Madison, Wisconsin 53706

Comments: The above programs are intended primarily for use in conjunction with a course based on the text. The style of the text would make the programs difficult to use as "stand alone" programs.

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14. "FRAME," Nippon Univac Co., Tokyo, Japan, Oct. 1973.

15. "STRU-PAK," (2DTMAP, 3DTMAP, 2DFMAP, GRIDMAP), Control Data Corporation, Minneapolis, Minn., 1970.

16. "PROGRAM #253 Analysis of Plane Frames Subjected to Forced Vibrations," User's Manual of Electronic Calculus Programs Available on UCC Utility Network, Electronic Calculus Inc., New York, pp. 253-1 through 253-14.

17. "PROGRAM #658C, Dynamic Modal Analysis of Large Space Bar Structures (Frames, Grids, Trusses)," User's Manual of Electronic Calculus Programs Available on UCC Utility Network, Electronic Calculus Inc., New York, pp. 658C-1 through 658C-27.

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Table 1. Survey of Evaluated Programs

	SMIS74 [2]	INELASTIER [4]	SOILTIER [5]	DYNATIER [6]	XIABS [7]	SAGS/DAGS [9]	PFVIBAT [10]	DALFI/DANFI [11]	DRAIN 2D [12]	DYNAMIC [13]	FRAME [14]	STRUPAK [15]	ECI#253 [16]	ECI#658C [17]	FHA [18]
1. TYPE OF STRUCTURE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Continuous beam	X	o	o	o	o	X	X	X	o	o	X	X	X	X	o
Plane truss	X	o	o	o	o	X	X	o	o	o	X	X	o	X	o
Plane frame	X	o	o	o	o	X	X	X	X	o	X	X	X	X	o
3D truss	X	o	o	o	o	X	o	o	o	o	X	X	o	X	o
3D frame	X	X	X	X	X	X	o	o	o	X	X	X	o	X	X
Grillage	X	o	o	o	o	X	o	o	o	o.	X	X	o	X	o
Shear wall	o	o	o	o	X	o	o	o	o	o	o	o	o	o	o
2. TYPE OF ANALYSIS															
Static deflection (linear)	X	o	o	o	X	o	X	o	X	o	X	o	o	o	o
Static deflection (geom. nonlinearity)	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o
Static deflection (geom. and material nonlin.)	o	o	o	o	o	o	o	o	X	o	o	o	o	o	o
Static element forces (moment, shear, axial)	X	o	o	o	o	o	X	o	X	o	X	o	o	o	o
Static substructure analysis	o	X	X	X	o	o	X	o	o	o	o	o	o	o	o
Modes and frequencies	X	o	X	X	X	X	X	o	o	o	X	X	o	X	X
Transient displ. (linear, mode superposition)	X	o	o	X	X	X	o	o	o	o	X	o	o	o	o

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Transient displ. (linear, direct integration)	o	o	X	o	o	o	o	X	X	o	X	o	o	o	o
Transient displacement (geom. nonlin.)	o	o	o	o	o	o	o	X	o	o	o	o	o	o	o
Transient displacement (geom. and material nonlin.)	o	X	o	o	o	o	o	o	X	X	o	o	o	o	o
Dynamic substructure analysis	o	X	X	X	o	o	o	o	o	o	o	o	o	o	o
Random vibration	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o
Shock spectrum (seismic)	o	o	o	o	X	X	o	o	o	o	X	o	o	X	o
Response to harmonic excitation	o	o	o	o	o	X	X	o	o	o	o	o	X	o	o
Buckling	X	o	o	o	o	X	o	X	o	o	o	o	o	o	o
Matrix operations	X	o	o	o	o	o	o	o	o	o	?	o	o	o	o
3. ELEMENT TYPES															
Stiffness matrix:															
2D Truss	X	o	o	o	o	X	o	o	X	o	X	X	o	o	o
3D Truss	X	o	o	o	o	X	o	o	o	o	X	X	o	o	o
Beam	X	o	o	o	o	X	X	X	X	o	X	X	o	o	o
2D Frame	X	o	o	o	o	X	X	X	X	o	X	X	X	o	o
3D Frame	X	X	X	X	X	X	o	o	o	X	X	X	o	X	X
Triangular shear panel	X	o	o	o	o	o	o	o	o	o	o	o	o	o	o
Quadrilateral shear panel	o	o	o	X	X	o	o	o	X	o	o	o	o	o	o
Tapered	o	o	o	o	o	X	o	o	o	o	X	o	o	X	o
Curved	o	o	o	o	o	X	o	o	o	o	o	o	o	X	o
3D Solid	o	o	X	o	o	o	o	o	o	o	o	o	o	o	o
Mass matrix:															
Consistent	X	o	o	o	o	X	X	X	o	o	X	o	X	o	X
Lumped	X	X	X	X	X	X	o	X	X	X	o	X	o	X	X

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Rigid body	o	X	X	X	o	o	X	o	o	o	o	o	o	o	o
Geometric stiffness (or equivalent)	X	X	o	o	o	X	X	X	X	o	o	o	o	o	o
Consistent forces	o	o	o	o	o	o	o	X	o	o	o	o	o	o	o
Special joint (linear)	o	o	o	o	o	X	o	o	o	o	o	X	o	X	o
Special joint (nonlinear)	o	o	o	o	o	o	o	o	X	X	o	o	o	o	o
Special boundary condition	o	o	o	o	o	X	X	o	o	o	o	o	X	X	X
Viscous dashpot	o	o	X	o	o	o	o	X	o	o	o	o	o	o	o
4. MATERIAL PROPERTIES															
Isotropic linearly-elastic	X	o	X	X	X	X	X	X	o	o	X	X	X	X	X
Temperature dependent	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o
Nonlinear elastic	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o
Elastic plastic	o	o	o	o	o	o	o	o	o	X	o	o	o	o	o
Elastic-strain hardening	o	o	o	o	o	o	o	o	X	o	o	o	o	o	o
Ramberg-Osgood	o	X	o	o	o	o	o	o	o	o	o	o	o	o	o
5. SYSTEM DAMPING															
Viscous modal	X	o	o	X	X	o	o	o	o	o	X	o	o	X	o
Viscous proportional	o	o	X	o	o	X	o	o	X	X	o	o	o	o	o
Other (structural, input coefficients, etc.)	o	X	o	o	o	o	o	o	X	o	X	o	o	o	o
6. INPUT															
Free-field	X	o	o	o	o	X	X	o	o	o	?	X	o	o	X
Fixed format	o	X	X	X	X	o	o	X	X	X	?	o	X	X	o
Rectangular frames only		X	X	X	X	o	o	o	o	o	o	o	o	o	o
Arbitrary frame geometry	X	o	o	o	o	X	X	X	X	X	X	X	X	X	X

7. PRINTER OUTPUT	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Error diagnostics	X	o	o	o	o	X	X	o	X	o	X	?	?	?	X
Input data	X	X	X	X	X	X	X	X	X	X	?	X	X	X	X
Static analysis:															
Total structure-deflections	X	X	X	X	X	X	X	o	o	o	X	o	o	o	o
Total structure-moment	o	o	o	o	o	o	X	o	o	o	X	o	o	o	o
Total structure-shear	o	o	o	o	o	o	o	o	o	o	X	o	o	o	o
Member deflection	o	o	o	o	o	o	o	o	X	o	X	o	o	o	o
Member forces (axial, moment, shear)	X	o	o	o	X	X	o	o	X	o	X	o	o	o	o
analysis:															
Mode shapes and natural frequencies	X	o	X	X	X	X	X	o	o	o	X	X	o	X	X
Displacement time history	X	X	X	X	X	X	o	X	X	X	X	o	o	o	o
Velocity time history	o	o	o	o	o	o	o	X	o	o	X	o	o	o	o
Acceleration time history	o	o	o	o	o	o	o	o	o	o	X	o	o	o	o
Element force histories	o	X	X	X	o	X	o	X	X	o	o	o	o	o	o
Displacement spectrum or steady-state response	o	o	o	o	o	X	o	o	o	o	o	o	X	X	o
Force (moment, etc.) spectrum or steady-state response	o	o	o	o	o	X	o	o	o	o	o	o	X	X	o
8. PLOTS															
Line printer plot of specified function	X	o	o	o	o	o	o	o	o	o	o	o	o	o	o
Mesh	o	o	o	o	o	X	o	o	o	o	o	o	o	o	o
Deformed structure (static, dynamic, mode shape)	o	X	X	X	o	X	X	o	o	o	X	o	o	o	o
Force (moment, etc.) history plot	o	?	?	?	o	X	X	o	o	o	o	o	o	o	o
Displacement history of specified d.o.f.	o	?	?	?	o	o	o	X	o	o	X	o	o	o	o
Velocity history of specified d.o.f.	o	o	o	o	o	o	o	X	o	o	o	o	o	o	o

	1	2	3	4	5	6	7	8	9	10.	11	12	13	14	15
Acceleration history of specified d.o.f.	o	o	o	o	o	o	o	X	o	o	o	o	o	o	o
Input force history	o	o	o	o	o	o	o	X	o	o	X	o	o	o	o
9. OTHER POST PROCESSING															
Stresses, etc. compared with codes	o	o	o	o	o	o	o	o	o	o	X	o	o	o	o
10. SOFTWARE INFORMATION AND COMPUTER REQUIREMENTS															
Size of deck (x 100)	30	30	30	30	20	110	35	8	47	?	300	?	?	?	20
In-core program?	N	N	N	N	Y	N	Y	Y	Y	?	N	?	?	?	Y
Min. core (K words/K bytes)	36	V	V	V	55	15	224	60	76	?	65	?	?	?	390
No. external files	3	V	V	V	3	12	0	0	3	?	30	?	?	?	4
Operational on	C	I	I	I	C,I	M,C,S	I	C	C	C	U	C	U	U	I
Under active development?	N	N	N	N	N	N	Y	Y	N	N	Y	N	N	N	N
Actively maintained?	Y	N	N	N	N	Y	Y	Y	N	N	Y	Y	Y	Y	N
Interactive execution	X	o	o	o	o	o	o	o	o	o	X	X	o	o	o
Restart capability	o	o	o	o	o	o	o	o	o	o	X	o	o	o	o
11. DOCUMENTATION															
Data preparation manual	X	X	X	X	X	X	X	X	X	o	X	X	X	X	X
Programmer's manual	X	X	X	X	o	o	X	o	o	o	o	o	o	o	o
Mathematical formulation	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
Numerical techniques	o	X	X	X	o	o	X	X	o	X	X	o	o	X	o
Complete examples (input listing, output listing, etc.)	X	E	E	X	X	X	X	E	E	E	X	X	E	E	X
Listing available	X	X	X	X	X	o	X	X	o	X	o	o	o	o	o
Updates	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o

C=CDC, E=partial example, H=Honeywell, I=IBM, N=No, S=XDS, U=Univac, V=Variable, X=Available in program, (o)=not available in program, (?)=insufficient information supplied

Nonlinear Transient Response of Solids

Walter Herrmann

Sandia Laboratories

INTRODUCTION

In this chapter we will consider computer programs capable of solving problems involving the transient mechanical response of solid bodies. A conspicuous feature of such response is the propagation of stress waves. We will be particularly concerned with high stresses at which material response is nonlinear.

Transient problems can be disconcertingly complicated, even for seemingly simple initial and boundary conditions. A localized impact or explosion may induce stress waves which reverberate throughout the body, perhaps to cause failure at a location remote from the excitation. Computations with sufficient resolution to follow stress waves through several reverberations in the body can be done by intelligent use of the programs to be described here, although they are often time consuming and expensive due to the detail which may be required to ensure that the important physical phenomena are retained.

Applications requiring stress-wave analysis arose in the early 1940's in connection with nuclear weapon design. By the 1950's finite difference methods were developed to handle these problems. Their growth in capability kept pace with computer development, and to some extent influenced the development of large scientific computers. With an initial emphasis on very high pressure and temperature phenomena, shear strength was neglected in early calculations, and solid materials were modelled as compressible fluids. While elastic-plastic and other material descriptions were soon introduced, a legacy of this early history is an emphasis on complete thermodynamic descriptions which correctly account for irreversible heating of materials during rapid dynamic deformations.

By the early 1960's, with the introduction of realistic models of material strength, these methods found wide application to nuclear weapon effects, to conventional ordnance problems such as interior and terminal ballistics and fragmentation, and to a rapidly expanding range of non-military applications ranging from explosive forming and processing, percussive rock breakage and explosive excavation to reactor accident analysis. While finite element methods have been subsequently extended to treat transient problems in solid bodies, this chapter will be limited to finite difference methods.

Before discussing particular computer programs, some general material on discontinuities, program structuring, material constitutive equations and failure will be presented to provide the reader with some guidance in the choice of numerical method for his problem. Various finite difference methods will then be outlined, and specific computer programs implementing these methods will be mentioned.

DISCONTINUITIES

A salient feature of the propagation of high amplitude stress waves in solid materials is the presence of shock waves. Even if shock waves are not intro-

duced by discontinuities in the initial/boundary data, they may arise spontaneously in the body by steepening of compressive waves due to nonlinear response of the material. While, of course, shock waves cannot exist as mathematical discontinuities in real materials when viewed on a molecular scale, shock waves may nevertheless be very thin. For example, Barker [1] and Schuler [2] have observed shock waves in metals and polymers with rise times less than the resolution of current instrumentation (a few ns). Corresponding shock widths are less than 10^{-9} m. Even in more dissipative materials, such as porous materials [3], shock wave thicknesses may be several orders of magnitude smaller than body dimensions.

Another feature of stress wave propagation is the presence of acceleration waves; that is discontinuities in stress and strain gradients. While again their description as discontinuities is an idealization, very thin acceleration waves have been observed in experiments [4].

The presence of shock and acceleration waves implies that stress wave solutions possess a substantial very high frequency content. Any numerical method which relies on a discretization of space will act as a low pass filter, removing components with a wavelength smaller than twice to four times the spatial mesh size. It is almost always prohibitive, in engineering calculations, to provide a sufficiently fine numerical grid so that physically realistic shock and acceleration wave thicknesses are resolved. Consequently, special provisions are necessary to accommodate shock and acceleration waves.

Shock Fitting

There are two methods of dealing with discontinuities. The first is called the method of shock fitting. Shock waves are considered to be mathematical discontinuities which are treated as internal moving boundaries. Discretized numerical methods are used in the continuous regions between shocks, and their solutions are joined across shocks by means of the shock jump relations. The positions of shocks are not known a priori, they must be obtained as part of the solution. Acceleration waves are usually ignored, and give rise to some numerical noise, although they may also be treated as further internal moving boundaries.

Shock fitting obviously involves severe complications in logic to keep track of numerous moving boundaries, particularly when shock waves can interact with each other, with material interfaces and with boundaries to give rise to further shocks. While the method has been applied successfully in one dimension [5, 6, 7] and to some extent in two dimensions [8], computer programs are usually limited to specific kinds of problems in which shock waves are known to propagate in certain regions and in certain directions.

A variant of the shock fitting method utilizes a discrete mesh in characteristic directions, rather than along constant space and time directions. Since acceleration waves and higher order discontinuities propagate in characteristic directions, these can also be treated as discontinuities in such a method. The positions of characteristics are not known a priori (except in the linear case) and the computational mesh itself is part of the solution. Nevertheless, the method has been used quite successfully in one dimension [6, 9, 10, 11]. In particular, the SWAP code [9] utilizes the fact that the shock jump relations reduce to the acoustic relations for infinitely weak shocks, and hence can be used on characteristics. Smooth motions are, in effect, represented as a series of weak shocks. The logic of handling wave interactions is thereby reduced to manageable size, and the SWAP code can handle a wide variety of problems. The RICSHAW code [11] also has a logic structure which allows a wide variety of problems to be handled. Techniques of this kind are often extremely efficient in comparison with other methods when narrow stress pulses propagate long distances through otherwise relatively quiescent regions. Two-dimensional methods using a bi-characteristic computational grid have proved to be relatively difficult to implement due to the greatly increased complexity of the logic, and have not been widely used. Due to their specialized nature, shock fitting codes are not included in what follows.

Dissipative Methods

A second method of dealing with shock waves involves the use of dissipation to limit the high frequency content of solutions. In effect, shock waves are broadened so that they may be represented as continuous profiles with a reasonable computational grid. The dissipation may be internal to the numerical method, as for example in the Lax-Wendroff technique [12], or it may be introduced as an artificial viscosity [13], added, in effect, in the material constitutive equations. Artificial viscosity methods generally have some internal dissipation [14], while techniques with high internal dissipation often require the use of an added artificial viscosity for at least some problems [12].

The magnitude of internal dissipation is normally a function of the spatial mesh size. The same effect is achieved in artificial viscosity schemes by making the viscous coefficients a function of mesh size. Both methods have essentially the same effect. Shock waves are spread into steep but continuous waves whose width is a more or less fixed multiple of the spatial mesh size.

If the scheme properly conserves energy, the dissipation converts the energy in the excluded high frequency components into shock heating, and the proper entropy change across the shock wave is obtained. If the shock is not spread over a sufficient number of meshes, however, components near the cut-off frequency are amplified, and spurious numerical oscillations appear behind the shock [15]. It is usually found for most methods that such noise is eliminated if shocks are spread over three to four mesh widths, although it is possible to spread shocks over as little as two mesh widths with the appearance of some limited noise behind the shock. Dissipation also serves to spread acceleration waves, which otherwise also lead to numerical noise. Since artificial viscosity methods allow control of shock spreading through simple adjustment of the viscous coefficients, they are often preferred to methods which rely on internal dissipation, in which the shock spreading is not adjustable.

Except in the linear elastic case, most methods are dissipative to some extent, and may attenuate components with frequencies below the cut-off frequency. Some methods also show dispersion in that components of different frequencies will propagate with different velocities. It is obviously desirable to use a scheme with little or no dispersion and with a very sharp frequency cut-off. Analysis is difficult because, of course, the problem is nonlinear, and simple mode superposition is not applicable. However, use of an artificial viscosity which depends on the second or higher power of the velocity gradient [13] represents an attempt to achieve a sharp frequency cut-off. Most programs to be described also use a linear viscosity [16] to damp low level noise.

Some distortion of the solution by dispersion and attenuation may always be expected. Care is always necessary in the use of any numerical method for nonlinear wave propagation problems to be sure that, on the one hand, adequate high frequency content is retained and shocks are not spread to the point where the solution is meaningless, and on the other hand, spurious numerical noise does not override the solution. Since these effects are mesh size dependent, comparison of computations with different mesh sizes can reveal their presence. It is always necessary to scrutinize extensive plotted output of several calculations with different mesh sizes whenever a new type of problem is initiated, to be sure that the solution is not affected to an unacceptable degree by numerical artifacts.

NUMERICAL METHODS

There are basically three types of descriptions of a motion, depending on the choice of coordinate system. In the Eulerian description, the motion is referred to spatial coordinates, and the computational grid is fixed in space with material flowing through it. In the Lagrangian description, the motion is referred to a material coordinate system. The mapping of the computational grid in space appears to distort with the material. Finally, convected

coordinates may be used to describe the motion. In this system, the computational grid moves through space, either with the material, or independently. In addition, there are a number of hybrid descriptions.

For solid materials, the Lagrangian description is the most straightforward. Since the same material occupies a particular computational mesh throughout the motion, it is a simple matter to keep track of the history which that material has experienced. Consequently, it is relatively easy to incorporate history dependent material descriptions, such as plasticity, viscoelasticity or phase changes and chemical reactions. The Lagrangian description is almost always used in one-dimensional methods, and all of the one-dimensional programs cited below are of this type. In two or three dimensions, the Lagrangian description is also preferable as long as distortions do not become so extreme that the map of the Lagrangian meshes in space becomes disordered. This may happen in many problems, as for example in the ejecta region of a crater due to a very high velocity impact or explosion, or in a shaped charge jet.

Two techniques have been developed for alleviating problems of large distortion in Lagrangian methods. One is a rezone technique, in which portions of the solution which are becoming disordered are periodically remapped onto a new ordered computational grid. The new grid may be defined point by point by hand, or automatic mesh generators may be used. Mesh quantities for the new grid are interpolated from those of the old grid. It should be noted that interpolation of quantities representing the history of the material is somewhat uncertain, and some loss of definition of history parameters is unavoidable when any type of rezoning is used.

The second technique for alleviating problems associated with large distortions in Lagrangian methods in multi-dimensions is the use of sliding interfaces. In interface regions between two materials which can be expected to slide over one another, the computational grid is arranged so that sliding may occur. In most schemes, the interface may have zero or finite friction, and may also separate and reclose. Sliding interfaces are commonly used between explosive gasses and solid materials, for example, or between a ballistic projectile and target material. Sliding interfaces may also be used between bodies which may collide during the calculation. They may be introduced either initially or, in conjunction with rezoning, during the calculation in regions undergoing either tensile fracture or adiabatic shear failure. They may also be used, with sliding and separation suppressed, to merely introduce a change in the computational grid size.

For problems in which very large distortions predominate, the Eulerian description may be necessary. Two methods have been developed. In one, the material transport terms are included in the difference algorithms for the partial derivatives from the outset. In the other, at each time step a Lagrangian calculation is performed, followed by a separate calculation of the convective transport terms. Obviously, severe distortions or flows pose no difficulties for these methods. However, special provisions must be made for locating material surfaces and interfaces, otherwise these will diffuse rapidly through the computational grid. In one technique, care is taken in the transport of material from one mesh to another to identify the material in the acceptor mesh, and to transport only that material until it is exhausted in the donor mesh. Material surfaces and interfaces are therefore defined only to within one mesh width, but they do not progressively diffuse. In another technique, the actual positions of surfaces and interfaces is calculated, either by a method analogous to shock fitting, or by introducing Lagrangian "tracer particles." It may be noted that since the material residing in a given Eulerian mesh at a given time may have resided in other meshes at earlier times, it is difficult to define its history, unless Lagrangian tracers are used to label material particles as they move through the Eulerian grid. This is not usually done, due to the extra computational complexity and expense, so that most Eulerian techniques are usually limited in their ability to handle history-dependent material descriptions.

Convected coordinate methods offer possibilities of combining the best features of Lagrangian and Eulerian techniques, but have so far received

relatively less attention. Since the motion of the computational grid can be specified independently of that of the material, it is possible, in principle, to have the grid follow the material in some portions of the motion, while in effect continuously rezoning other portions which might be undergoing severe distortions. Obviously, the method suffers the same problems in defining material histories as Eulerian methods, unless Lagrangian tracers are used.

Several hybrid schemes have been developed. In the most obvious, the early portion of a problem during which severe distortions are expected are performed with an Eulerian method. After the motion subsides somewhat, the solution is mapped onto a Lagrangian grid, and carried on with a Lagrangian method which is better able to describe complicated material behavior [17, 18]. Numerous problems of explosions and hypervelocity impact have been handled this way.

Another hybrid technique uses a Lagrangian method in one portion of the problem and an Eulerian description in another, coupled across a suitable interface [19]. Such a description is useful for problems involving two materials, one of which is easily deformable such as a fluid, and the other of which is much less deformable such as an elastic-plastic solid. It has been applied successfully to fluid flows through thick deformable pipes and to deformable projectiles moving through fluids.

More complicated hybrid schemes have also been devised. For example, one method uses a Lagrangian description in one coordinate direction and an Eulerian description in another [20]. Computer programs based on hybrid methods tend to be written for a specific application, and consequently are difficult to adapt to other problems. With few exceptions, the user oriented programs described below are based either on the Lagrangian or the Eulerian method.

No mention has been made of the specific numerical algorithms which have been used. Virtually without exception, user-oriented multi-purpose computer programs use explicit conditionally stable algorithms. This greatly facilitates data storage and handling in the computer, which, for large multidimensional problems, becomes a limiting factor in machine efficiency. Each computer program uses a somewhat different algorithm. Among the Lagrangian programs, von Neumann-Richtmyer, Lax-Wendroff, and other methods are used, which are second-order accurate for uniform mesh spacings, but which involve first-order errors when non-uniform meshes are used. Extensive one-dimensional testing has shown that while each method may be "tuned" to give better results for some particular problem type, no one method seems to be distinctly superior for a broad spectrum of problems. Eulerian methods similarly are mostly first-order accurate. No attempt has been made to differentiate the programs listed below on the basis of accuracy or efficiency, largely because the comprehensive testing required to establish relative efficiencies on a wide spectrum of problems has not been carried out. It should be mentioned that the quality of the results for a given expenditure of computer time is perhaps more dependent on the choice of computational grid and material description than on the particular choice of numerical algorithm from among the various successful ones implemented in the programs described here.

PROGRAM STRUCTURE

Finite difference methods are based upon representing the partial differential equations expressing conservation of mass, momentum and energy, the strain-displacement relations and the constitutive equations by numerical algorithms. While these algorithms may be developed in a number of different ways, most can be considered in terms of the expansion of partial derivatives in the differential equations in truncated Taylor series. The method is very direct; separate numerical algorithms are obtained for each of the basic governing equations. In most explicit techniques used in the programs described here, these equations can be solved explicitly in sequence at each mesh point in the computational grid. While boundaries and interfaces may require special treatment, data flow for interior meshpoints can often be straightforward, allowing simple ordered access to data stored in peripheral memory devices. Initialisa-

tion can be handled in a separate part of the program, and may be reduced to reading data prepared by a preprocessor. Similarly, output functions may be segregated, and may be reduced to writing data files for subsequent use by postprocessors.

Since separate explicit algorithms are usually obtained for each of the governing equations, it is often not too difficult to change one or more of them without affecting the remainder of the program. This is particularly true of the constitutive routines. In many (but not all) programs, the constitutive equations are coded in a segregated section of the program or in subroutines. The sophisticated user may remove or add a specific constitutive equation or failure criterion without excessive difficulty. A number of users have successfully transplanted coding of a constitutive equation or failure criterion from one program to another. For this reason, constitutive equations are described separately below. In some programs it is possible, but often more difficult, to add different initialization routines, boundary conditions, prescribed heat sources and body forces, and output and display routines.

Stress wave programs, irrespective of the numerical method used, are generally large in terms of computer storage and running time requirements, particularly for multidimensional cases. Existing programs are frequently tailored to a greater or lesser extent to particular computer hardware and software configurations in an attempt to increase efficiency. Most also make compromises in their structuring for the same reason, thus losing some of the flexibility inherent in finite difference techniques. Programs are not always easily transported from one computer to another. Several of the programs described below have been converted to run on several different computers. Since conversions are frequently done by sophisticated users, rather than by the original developer, systematic listings of sources for different versions usually do not exist. The original developer may be able to provide information on the various versions which exist, and where they may be obtained.

CONSTITUTIVE RELATIONS

Various material descriptions which have been used in stress wave programs are discussed in this section. A number of these are incorporated into several programs, although they almost always differ in the precise forms of the material functions which they employ. Some of the material descriptions have been incorporated into only one or two programs. Additional material descriptions are constantly being developed. These are often accompanied by computer subroutines designed for one or another wave propagation program. The sophisticated user of a given program may very well find it desirable to add material descriptions as he tackles a variety of problems. As has been mentioned, this can often be done without prohibitive effort. In view of this flexibility, it has been felt worthwhile to discuss some of the material descriptions separately from a discussion of the computer programs.

Hydrodynamic

Early programs were based on the concept that material shear strength was negligible at very high pressures, and effectively treated solid materials as if they were compressible fluids. This allowed description of stress and strain by scalar quantities, that is, by pressure and density, rather than by tensor quantities, with considerable reduction of complication and computer storage requirements. It has long been recognized that there are very few, if any, situations in which the entire motion may be described in this way, and programs which are limited to scalar stress and strain measures are excluded from consideration in this paper. Nevertheless, fluid equations of state are still useful to describe very high pressure and temperature states of materials, and they are often combined with deviator stress-strain relations for use at less extreme conditions.

The most popular description of the equation of state is based on the Mie-Gruneisen equation, which was first adapted to high pressure states of

solid materials at Los Alamos [21, 22, 23]. Several variations in functional forms are in use. The PUFF [24] and Tillotson [25] equations include modifications which, in the high pressure limit, tend to a perfect gas description. Some care is necessary in the use of the various forms, since the material parameters corresponding to one cannot usually be converted directly to those corresponding to another. Consequently, it is necessary to be sure that material property input parameters are compatible with the particular form implemented in the program.

High velocity impacts, explosive loadings, or pulsed radiation may cause sufficient compression or heating to induce polymorphic, melting, or vaporization phase changes in solid materials. These effects are ignored in the hydrodynamic equations described above, which merely provide a smooth approximation to the equation of state surface. In many cases, it is necessary to provide a realistic description of phase boundaries and mixed phase states. Several descriptions which assume that the phases are in local thermodynamic equilibrium during homogeneous phase changes have been developed. Among those which have been used in a number of codes are the ANEOS [26, 27] and the GRAY [28, 29] descriptions. Both allow solid, liquid and vapor regions together with solid-liquid and liquid-vapor mixed phase states. The former has a somewhat wider range of applicability, and can handle certain solid-solid polymorphic phase transitions as well.

Kinetic non-equilibrium effects in phase changes have also been considered, but to a lesser extent. Andrews [30] has developed a calculational procedure in which the transformation rate in a binary mixture of phases is a function of the difference in the Gibbs potentials of the two phases. A similar procedure has been used for describing chemically reacting mixtures where the reaction rate is governed by Arrhenius kinetics [31].

Detonation of explosives has been described by similar chemically reacting mixture routines, usually regarding the unreacted explosive and the reacted explosion products as components of a binary mixture. Often, simplified reaction rate laws are used; for example, taking the mass fraction of explosion products to vary linearly with specific volume of the mixture [31]. A simpler device, implemented in most programs, is to pre-calculate the detonation time for each mesh in the computational grid containing explosive from the Chapman-Jouget detonation velocity and the distance from the point of initiation. Prior to this time, the explosive is taken to be rigid. After this time, the explosion products are described by one of many gaseous equations of state, varying in complexity from a perfect gas law to the BKW (Becker-Kistiakowsky-Wilson) theoretical equation of state [32] and the empirical JWL (Jones-Wilkins-Lee) equation [33]. Many combinations and variations of these burn laws and equations of state have been used.

Elastic-Plastic

The most common treatment of metal plasticity is that due to Wilkins [34]. The stress and velocity strain (or stretching) are both decomposed into spherical and deviator parts. The spherical parts are, of course, related to the pressure and the rate of change of density. Since plasticity is assumed to be absent from volume changes, the pressure and density, together with thermodynamic variables are assumed to be related by a hydrodynamic equation of state in one of the forms discussed above. For the deviator parts, the velocity strain is divided into elastic and plastic parts. The elastic part is related to a suitably rotationally invariant (objective) stress rate via a shear modulus, which is usually taken to be a variable dependent on the compression and the thermodynamic variables. Upon integration over a time step, the stress is limited by a yield criterion, which may also be a function of the thermodynamic state. Herrmann [35] has discussed the approximations involved in this type of description when the deformation is finite, and has shown that they are compatible with the assumption that the elastic portion of the deviator strain is infinitesimal, as it is for most ductile materials.

Most wave propagation programs incorporate a description of the above

type, usually assuming a von Mises perfectly plastic yield criterion, and a shear modulus obtained from the bulk modulus (which is implicit in the hydrodynamic equation of state) by means of a constant Poisson's ratio. The yield stress is often taken to vary with temperature (or internal energy) and to reduce to a negligible value at the onset of melting. Consequently, it is possible to use a hydrostatic equation of state with a multi-phase description, combined with a plasticity description to span the entire range of stress and temperature states from ambient to vaporization.

Various strain hardening laws have been incorporated into descriptions of this type, including isotropic [36, 37], kinematic [38], and combined multi-element hardening [39]. Strain rate effects have also been incorporated, based on a Maxwellian description [39, 40] and on a Voigt description [41]. Sometimes the description of both strain hardening and strain rate effects has been couched in terms of dislocation mobilities, but the equations are formally identical to those obtained in a continuum description.

Porous

There are a number of descriptions of porous materials. In the simplest, all shear strength is neglected, that is, the material is treated as if it were a fluid which can suffer irreversible volume collapse. At pressure levels where the hydrodynamic description given above is valid, this is a perfectly good approach. The material is given, in the input parameters, an initial density which corresponds to that of the porous material. All other hydrodynamic material parameters are taken to be those of the corresponding compacted solid material. When a disturbance arrives, all voids collapse, whereafter the material acts as an ordinary hydrodynamic fluid. However, numerical or artificial dissipation serves to provide the correct entropy change, which includes that due to irreversible pore collapse, so that the initially porous material has a higher temperature and volume when subjected to a given shock pressure than the corresponding initially non-porous material.

In a sense, the above approach assumes that pore collapse occurs at negligible pressures. To accommodate a finite crush strength into the theory, Herrmann [42, 43, 44] developed the so-called p-α model. This is essentially a plasticity theory [45] in which no shear stresses can be supported, but plastic volume strain can occur. Strain hardening is included to account for the fact that an increasing pressure is needed to reduce porosity as compaction proceeds. Butcher [46] has incorporated a Maxwellian strain rate effect into such a description.

Precisely the same results are obtained in a different way by Seaman et al. [47, 48] in their rate independent description, which relies on the definition of a series of equation of state surfaces to describe elastic volume changes, irreversible pore collapse, and compression of the fully compacted solid.

Shear strength has been added to such hydrodynamic porous descriptions by simply adding an elastic-plastic routine for the deviator stresses. The equivalent yield surface in stress space is then a von Mises cylinder with a flat end cap on the pressure axis which may move to represent hardening due to pore collapse. The von Mises cylinder may also be taken to strain harden. More realistic descriptions of shear strength of porous materials are discussed in the next section, since they were developed to describe earth materials, although they may be useful for other porous materials as well.

Rocks and Soils

There are several descriptions of earth materials which have been implemented in stress wave programs, only two of which will be referred to here. In the simpler of these, applicable to rocks, the deviator stresses are assumed to be related to the deviator strains up to failure, described either by a nonlinear Mohr failure criterion, or by a modified Drucker-Prager failure criterion.

Upon reaching the failure criterion, the material is considered to be broken, and the deviator stresses collapse to a lower Mohr or Drucker-Prager envelope which has zero cohesive strength, appropriate to soils [49, 50]. Soils, of course, begin with the lower failure envelope.

A rather different approach, represented by the soil cap or rock cap descriptions [51, 52, 53], uses the modified Drucker-Prager failure envelope as a yield surface. Upon reaching this envelope, part of the strain is irreversible plastic strain. Since the failure envelope in stress space is not a cylinder, use of an associated flow rule then implies that there is irreversible plastic volume change whenever the failure envelope is reached, that is, the material is dilatant. In order to account for the fact that the material may be initially porous, and compact irreversibly under a hydrostatic pressure, the failure envelope is closed on the pressure axis by a cap (usually an ellipsoid) which may move to account for strain hardening. The soil-cap and rock-cap descriptions have been implemented in several wave propagation programs. They have also been extended to transversely isotropic materials [54].

Variable Moduli

A description which has been used chiefly for earth materials is the so-called variable moduli model [51, 55]. The material is described by an incremental Hooke's law; however, the bulk and shear moduli are allowed to be complicated irreversible functions of various stress and strain measures. Hysteretic behavior in both volume change and shear can be allowed; however, volume and shear behavior are always uncoupled, and this description cannot accommodate dilatancy. While such a description is extremely flexible, it is difficult to restrict the behavior to stable and physically realistic responses.

Viscoelastic Materials

Computational techniques for nonlinear viscoelastic materials have been developed in two different forms. The first is that of a generalized Maxwell material which allows both a nonlinear strain dependence and a nonlinear relaxation function [56, 57]. The other involves the use of the theory of internal state variables, which can accommodate similar nonlinearities [58].

Composites

There are several descriptions of composite materials at different levels of complexity. The simplest involves representing the composite by a homogeneous material with equivalent properties. Such an "equivalent modulus" or mixture theory uses some weighted average of the properties of the components of the composite to represent the properties of the composite. The simplest such approach involves neglect of shear strength, and uses weighted averages of the equations of state of the components [59, 60, 23]. Once the averaged properties are obtained, these are used in a hydrodynamic subroutine without modifications of the routine being required.

Dispersion of stress waves has been accounted for, in the simplest description, by means of a viscoelastic formulation. Barker [61] has provided the rational for calculating the relaxation time from the geometry of the composite, and the properties of the components, at least for simple layered composites. More complex composites, such as fiber composites, require an empirical determination of the relaxation time. Once the material parameters are determined, these are used in a viscoelastic subroutine [56] without further modification. An extension to include shear strength of the components has also been made [62].

Another approach to dispersion has been developed by Drumheller [63]. The composite is modelled as a series of alternating layers with different properties. Using linear elastic theory, layer dimensions and properties are chosen

so that the dispersion and absorption spectra measured for the real composite to be modelled are obtained. The layers of this "equivalent composite" are then modelled in the calculation by assigning to adjacent columns of computational meshes the properties of the layers. By using hydrodynamic, elastic-plastic or viscoelastic descriptions for one or more of the layers, various kinds of nonlinear behavior can be added to the geometric dispersion. Again, no special material subroutines are required beyond those already described previously.

Two descriptions of composites cannot be introduced easily into general stress wave programs; both have been implemented in special purpose codes. The SPADES [64] program utilizes Bloch sinusoidal wave modes, which are found using a single unit cell of the composite, and superposes them by Fourier series in time to calculate the transient response of slabs to surface loads. The TINC [65, 66] program considers a mixture of interacting continua, in which the geometrical arrangement of the various components is not explicitly considered, but is implicit in the selection of the interactions between the constituents. These interactions are modelled as body forces and energy transfer processes influencing the motion of constituents.

Material Failure

Failure descriptions have received relatively less attention, although a number have been incorporated into various programs. There are some features which must be provided in the programs from the outset to allow realistic opening and closing of voids resulting from fracture; these cannot be added later with limited effort. Once these features have been provided, however, the criteria for fracture may be easily changed.

In one-dimensional Lagrangian methods, several programs allow for separation of the material to form new free surfaces when a failure criterion is satisfied. These surfaces may later collide and separate as demanded by the dynamics. Such a feature is referred to as a spall-and-join routine. In multidimensional Lagrangian methods, the use of sliding interfaces in conjunction with rezones may be used to provide free surfaces, with or without tangential friction, when a failure criterion is satisfied. In two or three dimensions, of course, information as to the orientation of the fracture is required, so that the new free surfaces can be oriented correctly. This information may be provided by the failure criterion. Alternatively, the problem may be run beyond the point at which the failure criterion is first exceeded, without providing free surfaces or stress relief. After several computational meshes exceed the failure criterion, the problem is stopped and the operator makes a decision as to where free surfaces are to be introduced. A sliding interface is then introduced, and the problem is restarted from an earlier problem time prior to the failure criterion being exceeded. As the problem proceeds, the fracture develops and may tend to extend. The problem may be stopped periodically, and the position of the unseparated portion of the sliding interface may be adjusted toward the direction of meshes which are exceeding the failure criterion, if the initial choice of location was incorrect. Obviously, the provision of new free surfaces or sliding interfaces affects the basic logic of the computer program, and it is extremely difficult to provide this feature if the originator has not provided it.

More simplified treatments have been used in programs which do not allow the actual opening of voids. The simplest merely limits the maximum hydrostatic tension which the material can sustain. Whenever the constitutive relation would indicate that the hydrostatic pressure should be less than the limit (pressure positive in compression), the pressure is set to the limit, taking care, however, that energy is properly conserved. Those computational meshes which reach this criterion then tend to expand. In a refinement of this method, the tensile strain is monitored. Fracture is regarded to be complete if a critical tensile strain is exceeded. No free surfaces are introduced, and the post-failure behavior may not be represented correctly, but this criterion does provide a rough indication of the integrity of the material following the motion.

A similar criterion involves the use of the maximum tensile principal stress. When this exceeds some fixed value, fracture is considered to have occurred. The maximum tensile principal stress may then be limited to a critical value, usually zero. While considerable computation is required to find the principal stresses, and after fracture has occurred, to find the limited stress components in the computational coordinate system, such a procedure does allow anisotropic modification of material strength upon fracture.

Alternatively, free surfaces can be introduced upon reaching the maximum tensile stress. Sometimes it is considered that the fracture will run normal to the maximum principal stress direction to guide the location of the free surfaces. Variations of this type of critical fracture criterion involve the use of a Mohr-Coulomb, or nonlinear Mohr failure criterion, the direction of the fracture sometimes being taken to be given by the Coulomb fracture angle. Many variations of these critical stress criteria have been used.

Critical stress criteria, such as those above, do not reflect a time dependence of the fracture process. Cumulative damage criteria were developed by Butcher and Tuler [67], in which some function of the stress (in excess of a tensile threshold stress) is integrated in time for each computational mesh. When the integral reaches a critical value, fracture is considered to have occurred. A more complicated theory, in which the rate of damage accumulation at any given time is taken to be a function of the damage which has accumulated up to that time has been developed by Davison and Stevens [68]. An even more complicated theory in which the crack length and orientation distributions are followed in time has been developed by Seaman et al.[69]. Each of these failure criteria requires the provision of one or more internal history variables to describe the development of fracture. The last two approaches also provide the framework for the progressive anisotropic modification of material strength due to developing fractures, prior to complete separation, although they have not been fully implemented in multi dimensional programs.

COMPUTER PROGRAMS

A very large number of finite-difference computer programs, suited to the analysis of transient shock problems, have been developed. Many of these were developed in the nuclear weapons laboratories of the Atomic Energy Commission, or the Department of Defense laboratories and their contractors. Many of these programs were intended for the solution of very specific problems; often user-oriented documentation has been skimpy or absent. Although some of these programs have been, and continue to be, extended to other problems, they are difficult to transmit to and be used by others than their originators. However, a few programs have been developed continuously for a long time, some of them for well over a decade, and have acquired in this time very flexible features and reasonably good documentation so that they can be applied with little or no modification to a broad spectrum of problems. In order to render this review useful, a few of the most widely used of these programs have been selected for inclusion here, principally on the basis of their generality and completeness of documentation. Some other less widely used programs with similar features are mentioned in passing. No attempt has been made at completeness; the selection given here is based on a subjective judgment.

It is quite possible that computer programs exist which will handle some specialized problems more effectively than the general purpose programs which are in Appendix I. However, the general purpose programs are, by virtue of constant use, relatively free of programming errors, and in some cases results obtained by their use have been compared very extensively with exact analytical solutions and experimental data. The decision to search for a special purpose program, or to develop a new one, which is closely matched to the particular problem which the user may have, must be based on economics. The user must often complete debugging, documentation, and experimental validation of the program before he can use it with confidence, the cost of which must be balanced against the possible cost of extra computing time, if any, used in completing the project with a well-developed general purpose program

which, in some instances, may be somewhat less efficient. Obviously, no hard and fast rules can be given; however, the potential user who has not previously engaged extensively in the development of similar programs would perhaps be well advised to gain experience with well-developed software before embarking on an ambitious program modification or development effort.

Most of the general purpose programs referred to below continue to be in active development at the time of this writing. In most instances features which are not described in the documentation of the program have been added by the originator. This is particularly true of material constitutive equations, initialization options, boundary condition routines, prescribed radiative heat source or body force routines, output options, etc. New problems sometimes require a slight modification of the program, particularly for multidimensional cases, and the program developer is constantly making such modifications during the lifetime of the program. It is always advisable to consult the originator if some feature is required which is not described in the published documentation. If the originator has not already developed the desired feature, he may be able to add it with trivial or very modest effort.

Although the technical community is quite fluid, the present names and addresses of individuals responsible for the programs are listed in Appendix II. As well, references to the program documentation are given. The authors, or in their absence, the organizations with whom they were affiliated, can be contacted for further details. Since there have been several important changes in program dissemination channels and restrictions recently, the availability of each of these programs is subject to change. The dissemination of certain programs developed by or on contract to some government agencies are subject to certain restrictions; at least one set of programs is proprietary and their use is merchandized. Again, the authors or their organizations should be consulted in each case regarding current procedures affecting program dissemination.

One-Dimensional Lagrangian Methods

As has been mentioned, most one-dimensional general purpose programs are based on the Lagrangian description, utilizing dissipation, either artificial or numerical, to spread discontinuities into continuous profiles. In order to follow transient shock problems, these methods require several times 10^2 computational meshes, irrespective of their nominal order of accuracy. Frequently, up to 10^3 meshes are used to provide sufficient resolution. Since most programs store between 10 and 20 variables per meshpoint, computer storage requirements are on the order of 10^3 to 10^4 words, excluding instructions. About as many time cycles are required as spatial meshes in order to propagate a disturbance once across the computational grid, irrespective of numerical method if resolution is to be maintained. Consequently, from 10^4 to 10^6 individual mesh point calculations are required. Efficient algorithms used on modern large high speed computers (e.g., CDC 6600) are capable of several times 10^6 mesh point calculations per hour of C.P. time; running times from several minutes to several hours are typical with all of these programs, depending on the complexity of the problem and the length of problem time during which the motion is to be followed.

Programs incorporating material strength (i.e., tensor stress and strain measures) grew from earlier hydrodynamic methods. The earliest documentation of these appears to be that of the KO program [34], in which an elastic-plastic material description was introduced by 1960. While not itself well documented, it influenced many subsequent programs. One of the earliest of these was the WAVE program [70], which subsequently was developed into the WONDY series [71]. Others are the SOC code [49, 72] and the PISCES 1DL code [73], the latter being available commercially. A somewhat similar program is CHART-D [27] which also includes thermal conduction and radiation diffusion effects. Each of these programs has capabilities of treating a very wide variety of initial and boundary descriptions, can handle rectangular, cylindrical and spherical symmetries, and has had a very wide variety of material descriptions implemented.

Another series of programs which grew from earlier hydrodynamic programs is the PUFF series, which had elastic-plastic descriptions added in 1966 [74]. A large number of PUFF programs, all derived from a common ancestor, flourished at one time; some of these remain in use. They were designed primarily for solving problems of dynamic mechanical response excited by radiation or by plate impact; consequently, they are not capable of treating as wide a variety of initial and boundary conditions as the programs referred to above. Independent developments motivated by the same class of problems led to the RIP code [75], and the SRI PUFF code [76], which contain viscoplastic, porous and mixed phase material descriptions.

An important development which offers significant computer time savings for many problems is rezoning. By rearranging the computational grid as the computation proceeds, smaller meshes can be provided in propagating regions of high gradient while larger meshes are used in relatively quiescent regions. For some problems, computer times can be reduced by an order of magnitude without significantly degrading the accuracy of the solution. The WONDY IV [77] program incorporates a rezoning routine based on subdividing or combining computational meshes which requires no knowledge of the direction in which waves are propagating, and can handle any number of waves moving in both directions. Somewhat more restricted rezoning routines are available in the PISCES 1DL and Puff programs.

Most of the remaining very large number of extant one-dimensional Lagrangian programs are either poorly documented, are designed to solve a very specific problem type, or both. The general purpose programs mentioned above can solve almost all of the problems for which the special purpose programs were developed, and consequently, extensive tabulations of the latter will not be made.

Two-Dimensional Lagrangian Methods

Two-dimensional Lagrangian methods are similar to one-dimensional methods in resolution. In order to follow transients, they also require on the order of 10^2 computational meshes in each direction, or on the order of 10^4 meshes total. It is not unusual to run problems with 20,000 or more meshes. With somewhat more variables per mesh point, computer storage requirements may be several times 10^5 words, excluding program instructions. Consequently, well developed programs utilize peripheral storage devices, with ordered access to stored variables and, if necessary, buffering. Since $\sim 10^2$ time cycles are required to propagate a pulse once across the computational grid, the number of individual mesh point calculations may be between 10^6 to 10^7; computer running times (CDC 6600 class) of from 1 to 20 hours per problem are common.

Elastic-plastic two-dimensional Lagrangian programs were developed by the early 1960's, TENSOR [78] and HEMP [33, 79] being among the first. The latter strongly influenced the RAVE [80] code which subsequently developed into the TOODY [81, 82] series. TENSOR was developed originally for calculating ground shock from large explosions and emphasized earth material models. It is less well documented than some others. HEMP and TOODY contain extremely flexible initial and boundary condition specifications, have fully developed (although somewhat different) sliding interface and rezone options as well as interactive and passive graphical displays, and have had a wide variety of material models implemented in them. Both handle rectangular and cylindrical symmetries, and both are reasonably well documented and accessible. A number of other programs have been based upon the HEMP formulation. The best documented of these is PISCES 2DL [83], which is available commercially, and which contains most of the important features described above.

Several other versions exist; SHEP is the version active at Shock Hydrodynamics Division, Whittaker Corp., Sherman Oaks, California, while earlier versions termed CRAM [84], TEDDY [85] and others have existed elsewhere. Most of these are either poorly documented and relatively inaccessible, or are no longer active. A quite separate development at Los Alamos Scientific Laboratory led to the P. MAGEE program, which acquired a material strength descrip-

tion in the late 1960's; however, this program is not documented. The programs described in the previous paragraph are capable of handling virtually all of the problem types which these other codes address, and their superior documentation and availability makes them preferable for the potential user.

Two-Dimensional Eulerian Methods

Two-dimensional Eulerian methods are roughly similar to two-dimensional Lagrangian methods in their demands upon machine time and storage, since these demands are predicated on resolution in discretized space and time to a greater extent than on the particular numerical method being used. Eulerian methods do tend to require more computational meshes than Lagrangian methods for a given resolution, since the computational grid must extend over all of the space into which materials may move. The extra computational time required to handle the greater number of meshes is usually more than offset by the lack of the necessity to rezone due to mesh distortion, required by Lagrangian methods when large distortions occur.

Early Eulerian programs treated materials in the fluid approximation. Apparently the first to achieve a tensorial representation of material strength was the OIL code [86] in versions named RPM [87] and DORF [88]. The appearance of convective terms in the time derivatives of the differential equations makes the numerical algorithms of Eulerian methods more complicated than their Lagrangian counterparts. These codes employed a two-step procedure. The first corresponds to a Lagrangian calculation, while the second handles convection across Eulerian mesh boundaries. Eulerian meshes which straddle material interfaces are considered to contain mixtures of materials, and mass averaging of component properties is used to describe the mixed cell quantities. A program derived from these is the HELP code [89] in which massless tracer particles are used to define material surfaces and interfaces in order to define mass ratios in mixed meshes. A somewhat similar but undocumented program is the SHEZAM code in use at Shock Hydrodynamics Division, Whittaker Corp., Sherman Oaks, California. An independently developed Eulerian program with extended capabilities is the CSQ code, which has recently been fully documented [90]. In this program, mixed meshes are treated by defining volumes of component materials. Meshes may be partially filled by defining void volumes, allowing treatment of porous materials, open interfaces, or crack formation which is difficult or impossible with other methods. The above programs all contain flexible initial condition options, extensive plot and display options, and have had several material models implemented in them.

A program in which the material convective terms are handled with second-order accuracy is the SMITE code [91, 92]. Surfaces and interfaces are treated explicitly as singular surfaces moving through the Eulerian computational grid. Elastic-plastic and explosive material models have been incorporated, although very flexible initial conditions have not yet been fully implemented.

Other Two-Dimensional Methods

Programs which utilize arbitrarily convected coordinates are the AFTON series; AFTON 2A [93] is a two-dimensional cylindrically symmetric code including elastic-plastic material descriptions, while AFTON 2P [94] treats rectangular symmetry. These programs are similar to Eulerian programs in that convective terms are included, but the computational grid may move in any manner prescribed by the user. A program in which consistent nominally second-order accurate algorithms are applied directly to the convective terms is the ADAM code [95]. While these programs, in principle, provide the user great flexibility in optimizing the use of the computational grid during the calculation, algorithms for moving the mesh to utilize this flexibility have not been forthcoming, and these programs have been predominantly used in the Lagrangian or Eulerian modes.

Of various hybrid methods, the only ones which appear to be applicable to

a variety of problem types are the so-called coupling codes which accept output from one code, typically an Eulerian code, and transform the data to input for another type, typically a Lagrangian code. Several of these exist; documented versions are the SHAPE code [17] which transforms output from an Eulerian hydrodynamic code EPIC to input for the SHEP code mentioned above, and the TOTEN code [18] which transforms output from the Eulerian hydrodynamic OIL code to input to the TENSOR code, both of which have been mentioned previously.

Three-Dimensional Methods

Several three-dimensional programs have been written. However, at this stage of development of computer hardware, they are marginally useful for transient stress pulse problems. Irrespective of the numerical method used, the previous arguments on discretization (without shock fitting) imply that at least 10^2 computational meshes are required in each coordinate direction if any resolution is to be retained. This would imply use of $\sim 10^6$ total computational meshes. Corresponding computer storage requirements would be several times 10^7 words, while computer running times (CDC 6600 class) would be on the order of 100 hours. Understandably, three-dimensional codes have been applied chiefly to quasi-static problems where solutions are very smooth and sharp pulses requiring high resolution do not occur, so that an order of magnitude less computational meshes could be used. Alternatively, three-dimensional codes have been applied to very simple initial and boundary problems, with the expectation that relatively crude results would be obtained.

The Lagrangian HEMP code has been extended to three dimensions in HEMP 3D [96]. Similarly the Eulerian OIL code has been extended to three dimensions in TRIOIL [97], although this code has not yet been extended to a full tensorial treatment of material strength. Several other three-dimensional codes have been written for rather specific problems, although they are poorly documented, if at all, and one or two efforts are currently underway to develop additional three-dimensional programs, but these are not yet to the point of having produced user oriented software.

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APPENDIX I

FINITE DIFFERENCE CODES FOR THE TRANSIENT RESPONSE OF SOLID MATERIALS(1)

	Dimensions	Rectangular Symmetry	Cylindrical Symmetry	Spherical Symmetry	Lagrangian, Eulerian	Multiple Materials	Spall and Join	Sliding Interfaces	Rezone	Hydro (2)	Explosives (2)	Elastic-Plastic (2)	Porous (2)	Viscoelastic (2)	Composite (2)	Stress Spall Criterion (2)	Cumulative Damage (2)	Thermal Conduction	Radiation Diffusion
WONDY	1	Y	Y	Y	L	Y	Y	na	Y	Y	Y	Y	Y	Y	Y	Y	Y	N	N
SOC	1	Y	Y	Y	L	Y	Y	na	N	Y	Y	Y	Y	N	N	Y	N	N	N
PISCES 1DL	1	Y	Y	Y	L	Y	Y	na	Y	Y	Y	Y	Y	N	N	Y	N	Y	N
CHART D	1	Y	Y	Y	L	Y	Y	na	N	Y	Y	Y	Y	N	N	Y	N	Y	Y
PUFF IV	1	Y	N	N	L	Y	Y	na	Y	Y	N	Y	Y	N	N	Y	N	N	N
SRI PUFF	1	Y	N	N	L	Y	Y	na	Y	Y	N	Y	Y	N	N	Y	Y	N	N
RIP	1	Y	N	N	L	Y	Y	na	N	Y	N	Y	N	N	N	Y	Y	N	N
TENSOR	2	Y	Y	na	L	Y	Y	Y	Y	Y	N	Y	Y	N	N	Y	N	N	N
HEMP	2	Y	Y	na	L	Y	Y	Y	Y	Y	Y	Y	Y	Y	N	Y	Y	Y	N
TOODY	2	Y	Y	na	L	Y	Y	Y	Y	Y	Y	Y	Y	Y	N	Y	Y	N	N
PISCES 2DL	2	Y	Y	na	L	Y	Y	Y	Y	Y	Y	Y	Y	N	N	Y	N	N	N
DORF	2	Y	Y	na	E	Y	N	na	N	Y	Y	Y	N	N	N	Y	N	N	N
HELP	2	Y	Y	na	E	Y	N	na	N	Y	Y	Y	N	N	N	Y	N	N	N
CSQ	2	Y	Y	na	E	Y	N	na	N	Y	Y	Y	Y	N	N	Y	N	Y	Y
SMITE	2	Y	Y	na	E	Y	N	na	N	Y	Y	Y	N	N	N	Y	N	N	N
AFTON	2	Y	Y	na	C†	Y	N	na	N	Y	Y	Y	N	N	N	Y	N	N	N
HEMP 3D	3	Y	na	na	L	Y	Y	N	Y	Y	Y	Y	Y	Y	N	Y	Y	Y	N
TRIOIL	3	Y	na	na	E	Y	N	na	N	Y	Y	Y	N	N	N	Y	N	N	N

Notes:

(1) Most programs have been adapted to run on several different large scale computers, see text.

(2) Material descriptions in most cases can be modified or new descriptions can be added, see text.

APPENDIX II

PROGRAM AVAILABILITY

WONDY	R. Jeffrey Lawrence Sandia Laboratories Albuquerque, New Mexico 87115
SOC	Charles Snell Lawrence Livermore Laboratory Livermore, California 94550
PISCES 1DL	Dennis Orphal Physics International San Leandro, California 94577
CHART D	Samuel Thompson Sandia Laboratories Albuquerque, New Mexico 87115
PUFF IV	C. D. Newlander Air Force Weapons Laboratory Albuquerque, New Mexico 87117
SRI PUFF	Lynn Seaman Stanford Research Institute Palo Alto, California 94305
RIP	Harold A. Read Systems, Science and Software La Jolla, California 92037
TENSOR	Donald E. Burton Lawrence Livermore Laboratory Livermore, California 94550
HEMP	Mark L. Wilkins Lawrence Livermore Laboratory Livermore, California 94550
TOODY	Rupert Byers Sandia Laboratories Albuquerque, New Mexico 87115
PISCES 2DL	Dennis Orphal Physics International San Leandro, California 94577
DORF	Wallace E. Johnson Computer Code Consultants 527 Glencrest Drive Solana Beach, California 92075
HELP	J. M. Walsh Los Alamos Scientific Laboratory Los Alamos, New Mexico 87544
CSQ	Samuel Thompson Sandia Laboratories Albuquerque, New Mexico 87115

SMITE	Samuel Z. Burstein Courant Institute New York University New York, New York 10003
AFTON	John G. Trulio Applied Theory, Inc. 1010 Westwood Blvd. Santa Monica, California 90406
HEMP 3D	Mark L. Wilkins Lawrence Livermore Laboratory Livermore, California 94550
TRIOIL	Wallace E. Johnson Computer Code Consultants Solana Beach, California 92075

Time Dependent Materials

Yoshiaka Yamada

University of Tokyo

INTRODUCTION

The class of time dependent material properties which is considered in this chapter is the one represented by the Boltzmann superposition law or the Duhamel integral. Due to the hereditary integral form of the pertinent constitutive equations, it is frequently conceived that the solution should depend on all preceding time steps, thereby tracing out the entire past history of the deformation. This is the case when continuous relaxation and/or creep function spectra are assumed. By discretizing the spectra, however, an alternative analysis procedure is evolved which requires merely the information of the state at the current time, i.e. immediately before the computation of the succeeding step. The discretization is equivalent to the representation of the time dependent property by the Prony series or the use of the generalized Maxwell and/or Voigt (Kelvin) mechanical models.

Although almost all of the applications of the mechanical models so far are confined to plastics, recent experimental results suggest that they are appropriate even for metals. In this chapter the mathematical structure of the theory of viscoelasticity is initially reviewed in the discretized form for the uniaxial loading case. The constitutive equation is then extended to the multiaxial stress field. Not only the transient phenomena, but also the harmonic responses are considered, and it is shown that the formulation which assumes that the creep potential can be included in the mechanical model theory is a special case.

Several computer programs have been developed specifically for performing viscoelastic structural analysis and a survey of them was prepared by Gupta et al. in 1974 [1]. As pointed out in the survey, some of the viscoelastic analysis capabilities are "spin-offs" of larger general multipurpose computer programs. As most of the nonlinear analysis programs are still in the development stage, this chapter is inclined to the theoretical aspect of the topic.

NOMENCLATURE

a	= Superscript identifying apparent stress or related quantities
c	= Superscript for creep component
C	= Compliance or reciprocal of Young's modulus
C(t)	= Creep compliance
C(jω)	= Complex compliance
[C(ω)]	= Viscous damping matrix
D	= Stiffness
[D]	= Stress-strain or stiffness matrix
e	= Superscript for elastic component
E	= Young's modulus

$E(t)$ = Relaxation modulus
$E(j\omega)$ = Complex modulus
g = Subscript identifying model constant for the simple Maxwell element in generalized Voigt model
G = Modulus of rigidity, or subscript for shearing component
i = Subscript indicating generic component of mechanical model
$[I]$ = Unit matrix
j = Imaginary unit
$[k]$ = Element stiffness matrix
K = Bulk modulus, or subscript for volumetric component
$[K]$ = Assembled overall stiffness matrix
m = Superscript for volumetric component of stress and strain
$[m]$ = Element mass matrix
$[M]$ = Assembled overall mass matrix
p = Superscript for plastic component
$\{p\}$ = Element nodal force
$\{P\}$ = External load vector
$\{u\}$ = Element nodal displacement
$\{U\}$ = Overall nodal displacement vector
t,τ = Time
T = Relaxation or retardation time, or temperature
$'$ = Superscript for deviatoric component of stress and strain
$*$ = Superscript indicating complex stress and strain amplitude
δ = Delta function
$\epsilon,\{\epsilon\}$ = Strain and strain vector
$\bar{\epsilon}$ = Equivalent strain
$\{\underline{\epsilon}\}$ = Strain vector;underline indicating the shear components are tensor quantities, not the ones of engineering definition
η = Coefficient of viscosity
ξ = Critical damping ratio
$\xi(t)$ = Reduced time of thermo-rheologically simple material
$\sigma,\{\sigma\}$ = Stress and stress vector
$\bar{\sigma}$ = Equivalent stress
$\phi(t)$ = Relaxation function
φ = Shift function
$[\Phi]$ = Matrix composed of column eigenvectors
$[\Phi]^T$ = Transpose of $[\Phi]$
$\psi(t)$ = Creep function
ω = Circular frequency

THEORY OF VISCOELASTICITY

Constitutive Equation in Integral Forms

The principle of Boltzmann superposition or, equivalently, the hereditary integral is expressed as follows [2-5]:

$$\sigma(t) = \int_{-\infty}^{t} \frac{\partial \varepsilon(t)}{\partial \tau} [E_0 + \eta_0 \delta(t-\tau) + \phi(t-\tau)] d\tau \tag{1}$$

or

$$\varepsilon(t) = \int_{-\infty}^{t} \frac{\partial \sigma(\tau)}{\partial \tau} [C_g + \frac{t-\tau}{\eta_g} + \psi(t-\tau)] d\tau \tag{2}$$

When the relaxation function $\phi(t)$ is approximated by the Prony series

$$\phi(t) = \sum_i E_i \exp(-t/T_i) \tag{3}$$

the following relation is obtained for a unit step strain input $\epsilon(t) = 1(t)$

$$\sigma(t) = E(t) = E_0 + \eta_0\delta(t) + \sum_i E_i \exp(-t/T_i) \tag{4}$$

Alternatively, when the retardation or creep function is approximated by

$$\psi(t) = \sum_i C_i[1-\exp(-t/T_i)] \tag{5}$$

the indicial strain response for a unit step stress $\sigma(t) = 1(t)$ is expressed by

$$\epsilon(t) = C(t) = C_g + \frac{t}{\eta_g} + \sum_i C_i[1-\exp(-t/T_i)] \tag{6}$$

$E(t)$ of Eq.(4) and $C(t)$ of Eq.(6) are called the relaxation modulus and creep compliance respectively.

In the mechanics of solids, the viscosity η_0 in Eq.(4) which is connected with the delta function $\delta(t)$ is disregarded, so that the relaxation modulus reduces to

$$E(t) = E_0 + \sum_i E_i \exp(-t/T_i) \tag{7}$$

or Eq.(7) is regarded as the degenerate case of the relaxation modulus

$$E(t) = E_0\exp(-t/T_0) + \sum_i E_i \exp(-t/T_i) \tag{8}$$

when $1/T_0$ tends to zero. It can be verified that Eq.(8) represents the stress relaxation characteristics of the generalized Maxwell model of Fig.1(a). Similarly Eq.(6) represents the creep characteristics of the generalized Voigt model of Fig.1(b).

Differential Forms

When the relaxation and/or creep function spectra are discretized, the differential forms of the constitutive equation for the viscoelastic material can be obtained conveniently by representing the behavior with mechanical models. Thus for the generalized Maxwell model of Fig.1(a)

$$\dot{\sigma}_0 = E_0\dot{\epsilon} - \sigma_0/T_0, \quad \dot{\sigma}_i = E_i\dot{\epsilon} - \sigma_i/T_i$$

Therefore, the total strain and stress rate equation is

$$\dot{\sigma} = \dot{\sigma}_0 + \sum_i \dot{\sigma}_i = (E_0 + \sum_i E_i)\dot{\epsilon} - (\sigma_0/T_0 + \sum_i \sigma_i/T_i) \tag{9}$$

Similarly, for the generalized Voigt material

$$\dot{\epsilon}_g = C_g\dot{\sigma} + \sigma/\eta_g,\ \dot{\epsilon}_i = \sigma/\eta_i - \epsilon_i/T_i$$

The total strain and stress rate equation is

$$\dot{\epsilon} = \dot{\epsilon}_g + \sum_i \dot{\epsilon}_i = C_g\dot{\sigma} + \left(\frac{1}{\eta_g} + \sum_i \frac{1}{\eta_i}\right)\sigma - \sum_i \frac{\epsilon_i}{T_i} \tag{10}$$

Solving for $\dot{\sigma}$

$$\dot{\sigma} = \frac{1}{C_g}\dot{\epsilon} - \left(\frac{1}{T_g} + \frac{1}{C_g}\sum_i \frac{1}{\eta_i}\right)\sigma + \frac{1}{C_g}\sum_i \frac{\epsilon_i}{T_i} \tag{11}$$

Equations (9) and (11) can be put together in the form of

$$\dot{\sigma} = D^e\dot{\epsilon} - \dot{\sigma}^a \tag{12}$$

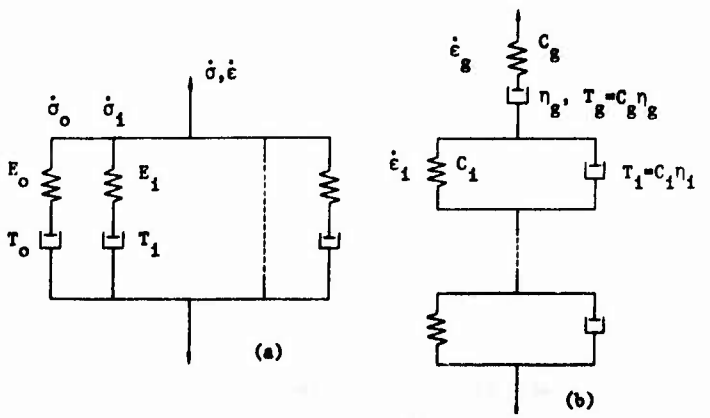

Fig. 1 Mechanical models of viscoelastic materials.
(a) Generalized Maxwell model; (b) Generalized Voigt or Kelvin model

D^e represents the elastic constant and $\dot{\sigma}^a$ is the apparent stress term due to the viscous component in the mechanical models. It should be noted that Eq. (8) and Eq.(6) are the solutions of Eq.(9) and Eq.(10) respectively for the strain and stress input given by the step function.

Harmonic Response

The response of the generalized Maxwell model to sinusoidally varying strain input $\epsilon = \epsilon^* \exp(j\omega t)$, or the steady state solution of Eq.(9) under forced oscillation, can be expressed by

$$E(j\omega) = \frac{\sigma^*}{\epsilon^*} = E_o\frac{\omega^2T_o^2}{1+\omega^2T_o^2} + \sum_i E_i\frac{\omega^2T_i^2}{1+\omega^2T_i^2} + j\left(E_o\frac{\omega T_o}{1+\omega^2T_o^2} + \sum_i E_i\frac{\omega T_i}{1+\omega^2T_i^2}\right) \tag{13}$$

$E(j\omega)$ is called the complex modulus. Similarly the complex compliance of the generalized Voigt model can be obtained as follows:

$$C(j\omega) = \frac{\varepsilon^*}{\sigma^*} = \left(C_g + \sum_i \frac{C_i}{1+\omega^2 T_i^2}\right) - j\left(\frac{C_g}{\omega T_g} + \sum_i C_i \frac{\omega T_i}{1+\omega^2 T_i^2}\right) \tag{14}$$

Generalization to Multi-Axial Stress Field

The extension of the differential constitutive equation Eq.(9) for the uni-axial loading case of the generalized Maxwell material to a three-dimensional stress field is

$$\{\dot{\sigma}\} = ([D_o] + \sum_i [D_i])\{\dot{\varepsilon}\} - ([T_o]^{-1}\{\sigma_o\} + \sum_i [T_i]^{-1}\{\sigma_i\}) \tag{15}$$

where $[T]^{-1}$ denoted by subscript o or i, is given by

$$[T]^{-1} = \frac{1}{3T_G T_K}\begin{bmatrix} T_G+2T_K & & & \text{Sym.} & & \\ T_G-T_K & T_G+2T_K & & & & \\ T_G-T_K & T_G-T_K & T_G+2T_K & & & \\ 0 & 0 & 0 & 3T_K & & \\ 0 & 0 & 0 & 0 & 3T_K & \\ 0 & 0 & 0 & 0 & 0 & 3T_K \end{bmatrix}$$

or

$$[T]^{-1}\{\sigma\} = \frac{1}{T_G}\{\sigma'\} + \frac{1}{T_K}\{\sigma^m\}$$

It is assumed that the material is isotropic. Similar extension of Eq.(11) for the generalized Voigt material to three dimensions is

$$\{\dot{\sigma}\} = [C_g]^{-1}\{\dot{\varepsilon}\} - \left(\frac{1}{T_{Gg}} + \frac{1}{C_{Gg}}\sum_i \frac{1}{\eta_{Gi}}\right)\{\sigma'\} - \left(\frac{1}{T_{Kg}} + \frac{1}{C_{Kg}}\sum_i \frac{1}{\eta_{Ki}}\right)\{\sigma^m\}$$
$$+ \frac{2}{C_{Gg}}\sum_i \frac{1}{T_{Gi}}\{\dot{\varepsilon}_i'\} + \frac{3}{C_{Kg}}\sum_i \frac{1}{T_{Ki}}\{\varepsilon_i^m\} \tag{16}$$

where

$$[C_g]^{-1} = \begin{bmatrix} \frac{1}{C_{Kg}} + \frac{4}{3C_{Gg}} & & & \text{Sym.} & & \\ \frac{1}{C_{Kg}} - \frac{2}{3C_{Gg}} & \frac{1}{C_{Kg}} + \frac{4}{3C_{Gg}} & & & & \\ \frac{1}{C_{Kg}} - \frac{2}{3C_{Gg}} & \frac{1}{C_{Kg}} - \frac{2}{3C_{Gg}} & \frac{1}{C_{Kg}} + \frac{4}{3C_{Gg}} & & & \\ 0 & 0 & 0 & 1/C_{Gg} & & \\ 0 & 0 & 0 & 0 & 1/C_{Gg} & \\ 0 & 0 & 0 & 0 & 0 & 1/C_{Gg} \end{bmatrix}$$

The amplitude $\{\sigma^*\}$ of stress response to sinusoidally varying strain input $\{\varepsilon^*\}\exp(j\omega t)$ is obtained from Eq.(15) for the generalized Maxwell model as

$$\{\sigma^*\} = [D(j\omega)]\{\varepsilon^*\} \tag{17}$$

where

$$[D(j\omega)] = \begin{bmatrix} K(j\omega) + \frac{4}{3}G(j\omega) & & & \text{Sym.} & & \\ K(j\omega) - \frac{2}{3}G(j\omega) & K(j\omega) + \frac{4}{3}G(j\omega) & & & & \\ K(j\omega) - \frac{2}{3}G(j\omega) & K(j\omega) - \frac{2}{3}G(j\omega) & K(j\omega) + \frac{4}{3}G(j\omega) & & & \\ 0 & 0 & 0 & G(j\omega) & & \\ 0 & 0 & 0 & 0 & G(j\omega) & \\ 0 & 0 & 0 & 0 & 0 & G(j\omega) \end{bmatrix}$$

$G(j\omega)$ and $K(j\omega)$ are the shearing and volumetric complex moduli which have the identical form as $E(j\omega)$ in Eq.(13). Similarly, the strain response amplitude $\{\varepsilon^*\}$ to stress input $\{\sigma^*\}\exp(j\omega t)$ is obtainable from Eq.(16) for the generalized Voigt material as

$$\{\varepsilon^*\} = [C(j\omega)]\{\sigma^*\} \tag{18}$$

$$[C(j\omega)]=\frac{1}{18}\begin{bmatrix} 2C_K(j\omega)+6C_G(j\omega) & & & & & \text{Sym.} \\ 2C_K(j\omega)-3C_G(j\omega) & 2C_K(j\omega)+6C_G(j\omega) & & & & \\ 2C_K(j\omega)-3C_G(j\omega) & 2C_K(j\omega)-3C_G(j\omega) & 2C_K(j\omega)+6C_G(j\omega) & & & \\ 0 & 0 & 0 & 18C_G(j\omega) & & \\ 0 & 0 & 0 & 0 & 18C_G(j\omega) & \\ 0 & 0 & 0 & 0 & 0 & 18C_G(j\omega) \end{bmatrix}$$

Relation to Creep Potential Law

The constitutive equation (16) is solved for the strain rate to give

$$\{\dot{\varepsilon}\} = [C_g]\{\dot{\sigma}\} + [\eta_g]^{-1}\{\sigma\} + \sum_i [\eta_i]^{-1}\{\sigma\} - \sum_i [T_i]^{-1}\{\varepsilon_i\} \tag{19}$$

The expression for $[T_i]^{-1}$ is as given in Eq.(15) and $[\eta]^{-1}$, denoted with subscript g or i, is given by

$$[\eta]^{-1}=\frac{1}{18}\begin{bmatrix} \frac{2}{\eta_K}+\frac{6}{\eta_G} & & & & & \text{Sym.} \\ \frac{2}{\eta_K}-\frac{3}{\eta_G} & \frac{2}{\eta_K}+\frac{6}{\eta_G} & & & & \\ \frac{2}{\eta_K}-\frac{3}{\eta_G} & \frac{2}{\eta_K}-\frac{3}{\eta_G} & \frac{2}{\eta_K}+\frac{6}{\eta_G} & & & \\ 0 & 0 & 0 & 18/\eta_G & & \\ 0 & 0 & 0 & 0 & 18/\eta_G & \\ 0 & 0 & 0 & 0 & 0 & 18/\eta_G \end{bmatrix}$$

The creep law which assumes the creep potential [6,7] and employs a strain hardening rule is expressed by

$$\{\dot{\underline{\varepsilon}}^c\} = \frac{3}{2}\,\frac{\dot{\bar{\varepsilon}}(\bar{\sigma},\bar{\varepsilon},T)}{\bar{\sigma}}\{\sigma'\} \tag{20}$$

The type of creep rate of the strain hardening material given by Eq.(20) can be considered as a special case of the second term in the right hand side of Eq.(19), when incompressibility of the creep strain is assumed. In fact, when $\eta_K \to \infty$

$$[\eta_g]^{-1}\{\sigma\} = \frac{1}{2\eta_{Gg}}\begin{Bmatrix} \sigma_x' \\ \sigma_y' \\ \sigma_z' \\ 2\tau_{yz} \\ 2\tau_{zx} \\ 2\tau_{xy} \end{Bmatrix} \tag{21}$$

Eq.(21) is identical to Eq.(20) if the tensor notation is adopted and

$$\frac{1}{2\eta_{Gg}} = \frac{\dot{\bar{\varepsilon}}\,(\bar{\sigma},\bar{\varepsilon},\bar{T})}{2\,\bar{\sigma}}$$

In the conventional equation-of-state approach, Eq.(20) is replaced by the following relation when the time-hardening rule is adopted

$$\{\dot{\varepsilon}^c\} = \frac{3}{2}\,\frac{\dot{\bar{\varepsilon}}(\bar{\sigma},t,T)}{\bar{\sigma}}\{\sigma'\} \tag{22}$$

Primary and secondary creep rates are incorporated in the function $\dot{\bar{\varepsilon}}(\sigma,t,T)$ in this formulation. In contrast to the mechanical model approach, the secondary creep is described by the term $[\eta_g]^{-1}\{\sigma\}$ and the primary creep can be conveniently represented by the components composed of $[\eta_i]^{-1}$ and $[T_i]^{-1}$. Therefore, it is concluded that the mechanical model formulation is self-consistent and more versatile than the creep potential theory.

Temperature Effect: Thermo-Rheologically Simple Material

The viscoelastic material is called thermo-rheologically simple, when the temperature effect is represented by

$$\xi(t) = \int_0^t \varphi[T(\tau)]\,d\tau \tag{23}$$

$\varphi(T)$ is the shift function and Eq.(23) postulates that the behavior of the viscoelastic material subjected to the temperature history T(t) is described by the single parameter $\xi(t)$ which is termed the reduced time. The well-known shift function is the Williams-Landel-Ferry(WLF) equation. When the thermo-rheologically simple law is adopted, all of the viscoelastic equations are written as functions of the reduced time $\xi(t)$ in place of the natural time t in the case of problems with constant temperature.

SOLUTION PROCEDURES

Improved Time Integration Scheme

In the transient dynamic analysis, the differential constitutive equations for the one-dimensional problem (Eqs. (9) and (10)) and for the three-dimensional case (Eqs. (15) and (16)) are replaced by the appropriate finite difference relations. Then, from these relations the relevant stiffness matrices and the apparent stress vectors can be derived. The simplest time integration scheme is to use the Gaussian forward difference equation, e.g. for Eqs. (9) and (11) respectively.

$$\Delta\sigma = (E_o + \sum_i E_i)\Delta\varepsilon - (\sigma_o/T_o + \sum_i \sigma_i/T_i)\Delta t \tag{24}$$

and

$$\Delta\sigma = \frac{1}{C_g}\Delta\varepsilon - \left(\frac{1}{T_g} + \frac{1}{C_g}\sum_i \frac{1}{\eta_i}\right)\sigma\Delta t + \frac{1}{C_g}\sum_i \frac{\varepsilon_i}{T_i}\Delta t \tag{25}$$

An improved time integration scheme can be devised by assuming a linearly

changing strain rate during time interval Δt in the case of Eqs.(9) and (15) for the generalized Maxwell material [8-10]. The resulting difference equation is, for example, for Eq.(9)

$$\Delta\sigma = \sum_i \frac{2E_iT_i}{\Delta t}\{1-[1-\exp(-\frac{\Delta t}{T_i})\frac{T_i}{\Delta t}]\}\Delta\varepsilon - \sum_i [1-\exp(-\frac{\Delta t}{T_i})]\sigma_i(t)$$

$$+ \sum_i E_iT_i\{-[1+\exp(-\frac{\Delta t}{T_i})] + 2[1-\exp(-\frac{\Delta t}{T_i})\frac{T_i}{\Delta t}]\}\dot{\varepsilon}(t) \qquad (26)$$

For simplicity, only the sum for the generic component (i) is described in Eq.(26). When the strain rate is assumed constant during time interval Δt, Eq.(26) reduces to

$$\Delta\sigma = \sum_i \frac{E_iT_i}{\Delta t}[1-\exp(-\frac{\Delta t}{T_i})]\Delta\varepsilon - \sum_i [1-\exp(-\frac{\Delta t}{T_i})]\sigma_i(t) \qquad (27)$$

The corresponding improved time integration scheme for the generalized Voigt material is written [11]

$$\Delta\sigma = \frac{1}{C(t)}\Delta\varepsilon - \frac{1}{C(t)}\Delta\varepsilon^a \qquad (28)$$

where

$$C(t) = C_g + \frac{\Delta t}{2\eta_g} + \sum_i C_i\{1-[1-\exp(-\frac{\Delta t}{T_i})\frac{T_i}{\Delta t}]\}$$

$$\Delta\varepsilon^a = \frac{\Delta t}{\eta_g}\sigma(t) + \sum_i [C_i\sigma(t)-\varepsilon_i(t)][1-\exp(-\frac{\Delta t}{T_i})]$$

In Eq.(28), a linear change of stress during time interval Δt is assumed. When the stress is taken to be constant in the integrand, C(t) degenerates to C_g[12] and it can be shown that the stability of computation often deteriorates. In the program summaries below, the improved temporal integration scheme implies that the program uses the finite difference relations such as given by Eq.(26) and/or Eq.(28).

Response to Sinusoidal External Excitations

In the formulation which adopts the mechanical model or the complex stiffness for time dependent materials, the equation describing the frequency dependence of the generic finite element is expressed by

$$-\omega^2[m]\{u^*\} + [k(j\omega)]\{u^*\} = \{p^*\} \qquad (29)$$

By dividing the nodal displacement and force amplitude $\{u^*\}$, $\{p^*\}$, and the complex stiffness matrix $k[(j\omega)]$ into real and imaginary parts

$$\{u^*\} = \{u_1\} + j\{u_2\},\ \{p^*\} = \{p_1\} + j\{u_2\}$$

and

$$[k(j\omega)] = [k_1(\omega)] + j[k_2(\omega)]$$

the following equation is obtained from Eq.(29)

$$\begin{bmatrix} -\omega^2[m] + [k_1] & -[k_2] \\ [k_2] & -\omega^2[m] + [k_1] \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} P_1 \\ p_2 \end{Bmatrix} \tag{30}$$

The assemblage of Eq.(30) leads to the following overall stiffness equation

$$\begin{bmatrix} -\omega^2[M] + [K_1] & -[K_2] \\ [K_2] & -\omega^2[M] + [K_1] \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix} = \begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix} \tag{31}$$

Solution of Eq.(31) for a range of input frequencies ω gives the harmonic response of the relevant structure. When the sizes or degrees of freedom of the component stiffness matrices $[K_1]$ and $[K_2]$ are n×n, this requires the repeated solution of a 2n×2n simultaneous linear equation for each specified value of ω. An efficient method is applicable, however, if the element complex stiffness matrix $[K(j\omega)]$ has a form such as

$$[K(j\omega)] = (D_1(\omega) + jD_2(\omega))[h] \tag{32}$$

and if the structure is composed of a single material throughout. In this case, the total stiffness equation (31) is written as

$$\begin{bmatrix} -\omega^2[M] + D_1(\omega)[H] & -D_2(\omega)[H] \\ D_2(\omega)[H] & -\omega^2[M] + D_2(\omega)[H] \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix} = \begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix} \tag{33}$$

As pointed out by Yamada et al. [13], the eigensolution of the equation

$$-\omega^2[M]\{U\} + [H]\{U\} = 0 \tag{34}$$

is used conveniently for the frequency response solution of Eq.(33).

Under the conventional assumption of viscous damping, the equation for the frequency response analysis of the structure is obtained as follows [14].

$$\begin{bmatrix} -\omega^2[M] + [K] & -\omega[C] \\ \omega[C] & -\omega^2[M] + [K] \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix} = \begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix} \tag{35}$$

The eigenvector of the equation

$$-\omega^2[M]\{U\} + [K]\{U\} = 0 \tag{36}$$

is represented by $[\Phi]$ and scaled so that it is [M]-orthogonal,i.e.

$$[\phi]^T[M][\phi] = [I], \text{ and } [\phi]^T[K][\phi] = \text{diag}(K_i) \tag{37}$$

Further, modal damping is assumed such that

$$[\phi]^T[C][\phi] = \text{diag}(2\xi_i\omega_i) \tag{38}$$

Then, from Eq.(35)

$$\begin{bmatrix} \text{diag}(A_i) & -\text{diag}(B_i) \\ \text{diag}(B_i) & \text{diag}(A_i) \end{bmatrix} \begin{Bmatrix} V_1 \\ V_2 \end{Bmatrix} = \begin{Bmatrix} Q_1 \\ Q_2 \end{Bmatrix} \tag{39}$$

$$\{U_1\} = [\phi]\{V_1\}, \{U_2\} = [\phi]\{V_2\}$$

where

$$\{Q_1\} = [\phi]^T\{P_1\}, \{Q_2\} = [\phi]^T\{P_2\}$$

and

$$A_i = -\omega^2 + K_i, \; B_i = 2\xi_i\omega_i\omega$$

It can be seen from Eq.(39) that the harmonic response analysis of the structure with the usual viscous damping can be executed by a simple algebraic procedure.

PROGRAM SUMMARIES

In spite of the adequate formulation of finite element analysis procedures for time dependent materials exploited earlier by White [15], Heer and Chen [16], Malone et al. [8-10] and Taylor et al. [17,18] for the generalized Maxwell material and Zienkiewicz et al. [12] and Yamada et al. [11] for the generalized Voigt material, the available computer programs with a suitable incorporation of discrete mechanical models are rather few, except THVISC and VISCEL.

Thermoviscoelastic Stress Analysis (THVISC) [17,18]

Date: Program was issued June 1968, revised April 1969.
Capability: Determines temperatures, deformations, and stresses in solids of revolution loaded axisymmetrically. Elastic bulk modulus and viscoelastic shear modulus specify the properties of isotropic linear viscoelastic materials.
Method: Step forward time integration assuming linear change of strain during each time interval. Variable time steps can be used.
Limitations and Restrictions: Number of nodal points (150 maximum), Number of elements (150 maximum), Number of materials (12 maximum).
Input: Material data for temperature distribution analysis, bulk and shear elastic moduli, and relaxation time for shear. The viscoelastic shear property is represented by the generalized Maxwell model with three term Prony Series.
Language: FORTRAN
Hardware: CDC 6000
Usage: Used by the Navy at China Lake, Calif., for solid propellant studies.

Developer: R. L. Taylor and G. L. Goudreau
University of California
Davis Hall
Berkeley, California 94720
Availability: For a cost of $100/deck is available from the National Information Service - Earthquake Engineering (NISEE)
University of California
Department of Civil Engineering
Berkeley, California 94720
Subjective Comments: This program is an historical one and the original version is listed fully in [17]. A version for incompressible materials (THVINC) is available from the same source.

Viscoelastic Analysis Program (VISCEL) [1,19,20]

Date: Program was issued October 1972 as an updated and extended version of its earlier form [21]
Capability: Solve equilibrium problems associated with one-, two-, or three-dimensional linear thermo-viscoelastic structures. The generalized Maxwell model is used and material properties may be temperature dependent in the thermo-rheologically simple sense.
Method: The step-by-step incremental equation in a recursive form is solved. Program based on the hereditary integral and recomputes the solution results at each stage by going back to all past memories. A provision is, however, incorporated to cover a long time domain by keeping the time intervals constant in the logarithmic scale.
Limitation and Restrictions: Requires up to 195K core memory in a 260K UNIVAC 1108/EXEC 8 machine.
Input: Refer to [19]
Output: Refer to [19]
Language: FORTRAN V
Hardware: UNIVAC 1108 under the EXEC 8 system
Usage: Originally developed in connection with the stress analysis of solid-propellant rocket motor, this program is considered the only existing comprehensive viscoelastic analysis program opened to the public in the U.S.
Developer: K. K. Gupta, F. A. Akyuz, and E. Heer
Jet Propulsion Laboratory
California Institute of Technology
Pasadena, California 91103
Availability: Available from the Computer Software Management and Information Center (COSMIC).
Subjective Comments: Future publication of a detailed and engineer oriented user's manual is desirable.

The creep analysis capabilities in almost all of the general purpose programs are "spin-offs" in the efforts to cater to the user by making the programs suitable for high temperature design of nuclear reactor components. An extensive survey of programs in this area was done by Nickell [22]. In conformity with the constitutive equations recommended by ORNL [23], MARC-CDC [24] and ANSYS [25] incorporate the creep law of the type given by Eqs.(20) and (22). Various formulae as determined by the experiments are proposed to represent the strain and time hardening creep behaviors, and it is particularly interesting that the following creep law has been suggested to apply for types 304 and 316 stainless steel.

$$\bar{\varepsilon}\,(\bar{\sigma},t,T) = \varepsilon_t(\bar{\sigma},T)\{1 - \exp[-r(\bar{\sigma},T)t]\} + \dot{\varepsilon}_m(\bar{\sigma},T)t \qquad (40)$$

Differentiation of Eq.(40) with respect to time yields

$$\dot{\bar{\varepsilon}} = \varepsilon_t(\bar{\sigma},T) \times r(\bar{\sigma},T) \exp[-r(\bar{\sigma},T)t] + \dot{\varepsilon}_m(\bar{\sigma},T) \qquad (41)$$

In the equation-of-state approach, the creep data of Eq.(40) is associated with the time-hardening creep potential law of Eq.(22). It should be pointed out, however, that Eq.(40) has a form identical with the creep component in Eq.(6) for the generalized Voigt model. In fact, in the mechanical model approach, Eq.(40) is obtained as a solution of Eq.(10) or more generally of Eq.(19). These similarities and the desirable properties of the mechanical model formulation have not been fully recognized to date. The general purpose program ASKA [26] adopts the creep potential law, and CREEP-PLAST [27] and EPACA [28] distributed by Oak Ridge National Laboratory to the public are naturally in conformity with their recommendation of time-dependent law for inelastic design analysis [29]. VISCO-3D is the program developed for viscoelasticplastic analysis of three-dimensional solids. This program has features similar to CREEP-PLAST in many respects and is summarized as follows.

Three Dimensional Viscoelastic (VISCO-3D) [30]

Date: Program completed June 1972
Capability: Viscoelastic plastic analysis of three-dimensional initially isotropic solids. Plasticity with kinematic hardening for the von Mises Material. Memory theory of creep is adopted.
Method: Iterative method using linear-displacement tetrahedral elements.
Limitation and Restrictions: 10,000 unknowns, 750 band width.
Input: Internal mesh generation. Input interfacing with plotting programs.
Output: Output interfacing with plotting programs.
Language: FORTRAN IV
Hardware: UNIVAC-1108 (65K Core)
Usage: Numerical result is in preparation, although the program has been fully checked out for two-dimensional solutions. Companion heat conduction program is not incorporated.
Developer: Y. R. Rashid
General Electric Company
Nuclear Energy Division
175 Curtner Avenue M/C
San Jose, California 95125
Availability: Available from the developer.
Subjective Comments: Iterative method of solution may impede the extension of the program to large deformation area or problems with geometric nonlinearities

SAFE-CREEP [31], SAFE-NCREEP [32] and SAFE-GRAFIT [33] are viscoelastic analysis programs for plane and axisymmetric concrete and graphite structures which are the major components of HTGR (High Temperature Gas-Cooled Reactor) designed by General Atomic Company. These programs are based on the hereditary integral theory which is summarized in a recent report [34]. On the other hand, TEPC-2D [35,36] developed by the same company for the thermal elastic-plastic-creep analysis of two-dimensional solids is creep potential oriented. Several SAFE program versions are available from Argonne Code Center. Information about the Argonne Code Center availability for these programs can be supplied upon request to

Dr. Neil Prince
Gulf Computer Sciences, Inc.
Post Office Box 608
San Diego, California 92112

CARB (Predicting the Creep Behavior of Axisymmetric Bodies of Revolution) and CRASH-1 (Creep Analysis) are the programs developed by TRW Systems. TESS and PITT are developed for two-dimensional creep and plasticity problems and integrated in the system BERSAFE of Central Electricity Generating Board (CEGB), Berkeley Nuclear Laboratories in the United Kingdom. The PITT program, although useful in its own right, was used by CEGB to exploit the methods of creep and plasticity analyses which are not incorporated in the TESS program for more general geometries.

Regarding the dynamic analysis for the structures of time dependent materials, almost all of the general purpose computer programs, e.g. MARC-CDC [24], ANSYS [25] and SAP IV [37,38], adopt the viscous damping which is of the type of Eq.(38) and/or proportional damping, although a special purpose program, e.g. VISCOSUPERB of SDRC (Structural Dynamics Research Corporation, 5729 Dragon Way, Cincinnati, Ohio 45227), incorporates the hysteretic damping (loss factor). These are the topics of the chapter by Nelson and Greif [39] on shock and vibration programs with damping in this book. The general purpose programs usually have capabilities for the transient response analysis by mode superposition as well as step-by-step direct integration. As for the harmonic response, ANSYS [25] features the complex eigensolution capability. Recent developments in the solution of eigenvalue problems including complex vectors and modes are summarized by Gupta [40]. The method introduced in this review for harmonic analysis is an alternative one and has been used conveniently to obtain dynamic response of bar and beam structures of time-dependent materials [13].

CONCLUSION

As various finite element computer programs for nonlinear analysis are still in the development stage, the discussions of the present chapter have been focused mainly on the basic formulation of the solution procedure for problems connected with time-dependent material properties. The method which uses a discretized spectrum model is versatile and can be easily introduced into the existing general purpose programs. Although the experimental determination of the model constants or parameters is beyond the scope of the present review, a variety of means are conceivable [41], with the aid of novel material testing facilities as well as strain measurement instruments being developed recently.

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Prediction of Highway Noise

John K. Haviland

University of Virginia

INTRODUCTION

Noise pollution was recognized as an environmental problem on a national scale when the U. S. Environmental Protection Agency (EPA) formed the Office of Noise Abatement and Control. Following this, in 1972, the Congress passed the Noise Control Act.

To meet one of the requirements of this act, EPA has published "Information on Levels of Environmental Noise Requisite to Protect Public Health and Welfare with an Adequate Margin of Safety"[1]. In a supporting document [2], EPA estimates that 78 million people live in residential areas where the outdoor day/night exposure level, L_{dn} [1] exceeds 60 decibels, which is 5 decibels above the recommended level. Of these, 59 million are impacted by urban traffic, and 3.1 million by freeways.

The Federal Highway Administration (FHWA) has recognized this problem for some time, and has sought to limit the exposure of residential areas to noise from federally funded highways by setting standards, the latest of which are included in the Federal Aid Highway Program Manual [3]. Because the determination of the noise level near a highway must be made before it is built they have also approved two prediction methods. The first is described by the design guide contained in the National Cooperative Highway Research Program (NCHRP) Report No. 117 [4], with modifications according to NCHRP Report No. 144 [5], and is used in the MICHIGAN/144 Traffic Noise Level Predictor Computer Program [6]. The second is the Transportation Systems Center (TSC) method by Wesler [7], for which a computer program has also been written. Recently, a six volume report on a 1974 study for NCHRP has been available on loan in which a new design guide is proposed [8]. This contains a computer program listing which includes features of the earlier NCHRP and TSC methodologies, together with new ideas drawn from an extensive study of the overall subject. This new method has not yet been approved by the FHWA. Although the first two methods have been approved by FHWA, all variants of these methods, including computer programs must be submitted for approval also.

Because the MICHIGAN/144 program is written for use on a time-sharing terminal, a batch program referred to as MICNOISE 10 has been prepared for the IBM 370 by the Data Processing Division of the Virginia Department of Highways and Transportation. It only differs from the MICHIGAN/144 program in a minor way so as to predict slightly higher values, and is therefore also acceptable to the FHWA. There is no available publication on MICNOISE 10, however, earlier and similar variants have been evaluated by Haviland, Noble, and Golub [9, 10], and these reports contain listings of MICNOISE 2 and 5, respectively. A summary of the computer programs, their earlier variants, and some previous programs, will be found in Table A1 of the appendix.

In this chapter, some of the descriptors for noise levels are defined, and the noise standards set by FHWA are given, together with the EPA recommended

[1] The derivation of L_{dn} is explained in a later section of this paper.

levels. It will be noted that FHWA standards are based on the one hour equivalent or 10% levels, whereas the EPA recommended levels are based on the day/night or 24 hour equivalent levels. This difference is unfortunate, because it makes direct comparison impossible. However, the current highway noise prediction methodology is perfectly capable of producing 24 hour levels, should the FHWA standards be changed. Also, there is a brief description of the three principal methodologies for highway noise prediction. Essential details of four computer programs are given.

DESCRIPTORS FOR RATING NOISE LEVELS

Environmental sound pressure levels are generally found initially in terms of the A-weighted decibel level LA (also referred to as the overall A-weighted sound pressure level, or OASPL), or in terms of the one-octave or third-octave band levels. The linear level (also referred to as the "overall sound pressure level," or OSPL), or the B-, C-, and D- weighted levels are rarely used.

However, where long-term or daily fluctuations occur, an overall average of the A-weighted level is generally obtained, while, for short-term fluctuations, such as in transportation noise, a statistical value based on the distribution of A-weighted levels is often given.

Average Levels

Equivalent Levels

The H-hr. equivalent level, $L_{eq(H)}$ is the constant level which would have the equivalent 'energy' to the actual time-varying level, thus

$$L_{eq(H)} = 10 \log_{10} \langle p_A^2 \rangle_H + 94 \quad \text{dB} \tag{1}$$

where p_A is the A-weighted acoustical pressure, in N/m^2, obtained by passing the microphone signal through an appropriate filter and $\langle \ \rangle_H$ denotes averaging over H hours.

When $L_{eq(H)}$ is to be obtained by measurement, several techniques are possible. The most common method is to use the averaging circuit of a sound level meter to obtain the mean square pressure $\overline{p_A^2}$ where

$$\overline{p_A^2}\,(t) = (1/RC) \int_{-\infty}^{t} p_A^2(r) \exp\{(r-t)/RC\}\, dr \quad N^2/m^4 \tag{2}$$

and RC is the equivalent averaging time, so that

$$\overline{p_A^2}\,(t) \approx (1/RC) \int_{t-RC}^{t} p_A^2\,(r)\, dr \quad N^2/m^4 \tag{3}$$

Then the A-weighted sound pressure level is

$$LA = OASPL = 10 \log_{10} \overline{p_A^2}\,(t) + 94 \text{ dB(re } 2 \times 10^{-5} N/m^2) \quad \text{dB} \tag{4}$$

The averaging time RC is much less than the overall averaging time H, so that, to a very close approximation, Eqs. (1), and (3) combine to give

$$L_{eq(H)} = 10 \log_{10} \langle \overline{p_A^2} \rangle_H + 94 = 10 \log_{10} \langle 10^{LA/10} \rangle_H \quad \text{dB} \tag{5}$$

Note that, on changing the base of the logarithm

$$L_{eq(H)} = 3.01 \log_2 \langle \overline{p_A^2} \rangle_H + 94 = 3.01 \log_2 \langle 2^{LA/3.01} \rangle_H \quad dB \qquad (6)$$

thus, when the average rms pressure doubles, its logerithm to base 2 increases by one, so that $L_{eq(H)}$ increases by 3. This increase by 3 dB for a doubling of the quantity involved is referred to as the "3 dB law," and is characteristic of the so-called energy or power law.

Note that, in terms of LA, the A-weighted mean square pressure is

$$\overline{p_A^2} = 10^{\frac{LA-94}{10}} = e^{\frac{LA-94}{4.34}} = 2^{\frac{LA-94}{3.01}} \quad N^2/m^4 \qquad (7)$$

however, the reference pressure is arbitrary so that

$$L_{eq(H)} = 10 \log_{10} \langle 10^{\frac{LA-R}{94}} \rangle_H + R \quad dB \qquad (8)$$

where R can take any value.

Generally, H denotes a specific period of the day, so that

$$L_{eq(1)} = HNL = HOURLY\ NOISE\ LEVEL$$

based on the peak hour of the day.

$$L_{eq(15)} = L_d$$

based on daylight hours from 0700-2200

$$L_{eq(9)} = L_n$$

based on the nighttime hours 2200-0700, while

$$L_{eq(24)}$$

is based on the whole 24 hours.

Composite Levels

A composite average which penalizes nighttime noise by a factor of 10 is

$$L_{dn} = DAY\text{-}NIGHT\ LEVEL$$

$$= 10 \log_{10} \left\{ \frac{15}{24} \cdot 10^{Ld/10} + 10 \cdot \frac{9}{24} 10^{Ln/10} \right\} \quad dB \qquad (9)$$

Statistical Levels

These are generally based on the peak hours of the day, and are most common in surface transportation applications.

Excedence

The E% excedence level L_E is

$$L_E = \text{E\% EXCEDENCE LEVEL}$$
$$= \text{Level exceeded E\% of the time during the period specified.}$$

Commonly used values are L_{10}, L_{50}, and L_{90}.

Noise Pollution Level

The noise pollution level NPL, developed by Robinson [11], is defined in terms of L_{eq} and the standard deviation σ_L of L_E as

$$\text{NPL} = L_{NP} = \text{NOISE POLLUTION LEVEL}$$
$$= L_{eq} + 2.56\sigma_L \quad \text{dB} \tag{10}$$

Traffic Noise Index

The traffic noise index is defined in terms of three excedence levels as

$$\text{TNI} = L_{NI} = \text{TRAFFIC NOISE INDEX}$$
$$= 4(L_{10}-L_{90}) + L_{90} - 30 \quad \text{dB} \tag{11}$$

Normal Distribution

By assuming a normal distribution for L_E, the following approximations can be made

$$L_{eq} \sim 10 \log_{10}\left[\frac{1}{\sqrt{2\pi}\,\sigma_L}\int_{-\infty}^{\infty} \exp\left\{\frac{L_E}{4.34} - \frac{1}{2}\left(\frac{L_E-L_{50}}{\sigma_L}\right)^2\right\} dL_E\right]$$

$$= 10 \log_{10} \exp\left\{\frac{L_{50}}{4.34} + \frac{1}{2}\,\frac{\sigma_L}{4.34}\right\}^2$$

$$= L_{50} + 0.115\,\sigma_L^2 \quad \text{dB} \tag{12}$$

The standard deviation σ_L is readily obtained from

$$1.28\,\sigma_L = L_{10} - L_{50} = L_{50} - L_{90} \quad \text{dB} \tag{13}$$

thus the noise pollution level NPL, is obtained by substituting (13) into (10).

$$\text{NPL} = L_{NP} \sim L_{50} + 2.56\,\sigma_L + 0.115\,\sigma_L^2 \quad \text{dB} \tag{14}$$

Sometimes this is written as

$$\text{NPL} \sim L_{50} + (L_{10} - L_{90}) + (L_{10} - L_{90})/60 \quad \text{dB} \tag{15}$$

Also

$$TNI = 7(L_{10} - L_{50}) + L_{50} - 30 \quad dB \tag{16}$$

Other Methods

Many other methods of rating noise levels have been proposed, or are in use. These mainly apply to noise in airport environs, and are not of particular concern in highway noise work.

Subjective Response

The essential intent behind all of the many proposed descriptors for rating noise is to find a measure of noise which correlates well with the responses of those subjected to it. Serendipity Inc. [12], in a review of many such attempts, has pointed out that it is almost impossible to find statistically significant differences between the correlation of ratings based on A-weighting and those based on more sophisticated approaches. The EPA has requested all federal agencies to use L_{dn} as the standard designator for noise level, presumably $L_{eq(24)}$ could be used in its place for applications in which no residential areas are involved. If L_{dn} were adopted universally as the standard descriptor, there would be little if any, impact on the general public, but there would be much less confusion and misunderstanding about the subject of noise, and about what the associated numbers mean.

Table 1 Summary of Noise Levels Identified as Requisite to Protect Public Health and Welfare with an Adequate Margin of Safety [1]

EFFECT	LEVEL	AREA
Hearing Loss	$L_{eq(24)} \leq$ 70 dB	All areas
Outdoor activity interference and annoyance	$L_{dn} \leq$ 55 dB	Outdoors in residential areas and farms and other outdoor areas where people spend widely varying amounts of time and other places in which quiet is a basis for use
	$L_{eq(24)} \leq$ 55 dB	Outdoor areas where people spend limited amounts of time, such as school yards, playgrounds, etc.
Indoor activity interference and annoyance	$L_{dn} \leq$ 45dB	Indoor residential areas
	$L_{eq(24)} \leq$ 45 dB	Other indoor areas with human activities such as schools, etc.

NOTE: EPA has determined that for purposes of hearing conservation alone, a level which is protective of that segment of the population at or below the 96th percentile will protect virtually the entire population. This level has been calculated to be an L_{eq} of 70 dB over a 24-hour day.

TABLE 2 Design Noise Level/Activity Relationships [2]

Activity Category	Design Noise Levels - dBA[1] L_{eq}	L_{10}	Description of Activity Category
A[2]	57 (Exterior)	60 (Exterior)	Tracts of land in which serenity and quiet are of extraordinary significance and serve an important public need, and where the preservation of those qualities is essential if the area is to continue to serve its intended purpose. Such areas could include amphitheaters, particular parts or portions of parks, or open spaces which are dedicated or recognized by appropriate local officials for activities requiring special qualities of serenity and quiet.
B[2]	67 (Exterior)	70 (Exterior)	Residences, motels, hotels, public meeting rooms, schools, churches, libraries, hospitals, picnic areas, recreation areas, playgrounds, active sports areas, and parks.
C	72 (Exterior)	75 (Exterior)	Developed lands, properties or activities not included in categories A and B above.
D	-	-	For requirements on undeveloped lands.
E	52 (Interior)	55 (Interior)	Residences, motels, hotels, public meeting rooms, schools, churches, libraries, hospitals, and auditoriums.

[1]Either L_{10} or L_{eq} design noise levels may be used.

[2]Parks in categories A and B include all such lands (public or private) which are actually used as parks as well as those public lands officially set aside or designated by a governmental agency as parks on the date of public knowledge of the proposed highway project.

PUBLISHED STANDARDS

EPA Recommended Levels

A summary of the levels recommended by EPA is given in Table 1. It will be noted that both L_{dn} and $L_{eq(24)}$ appear in this table, the latter applying to non-residential areas. The values given have no legal force, but set targets which could possibly be incorporated into other standards in the future.

FHWA Standards

A summary of the levels required by FHWA [2] is given in Table 2. Compliance with these standards must be demonstrated by using an approved method of calculation or an approved computer program before federal funds can be committed to a highway project. It will be noted that either L_{eq} or L_{10} values can be used, but that simultaneous compliance with both sets of standards is not required. Either value can be based on the maximum traffic capacity of the highway or on the average of the worst three hours, thus either $L_{eq(1)}$ or $L_{eq(3)}$ is appropriate for what is referred to in this paper as L_{eq}.

METHODS OF PREDICTING HIGHWAY NOISE

Most methods of predicting highway noise that are presently in use are either based on design guides in two reports by the Highway Research Board, NCHRP 117 [4] and 144 [5], or on a method by Wesler [7] of the Transportation Systems Center, and referred to as the TSC method. Several versions of computer programs are available for the NCHRP 117/144 methods, and a program is available for the TSC method. Recently, Kugler, et al. of Bolt, Beranek and Newman have developed a new design guide [8], referred to here as the 1974 NCHRP method, containing a computer program written in ANSI specification FORTRAN. The three methods are reviewed briefly in the following section, and are summarized in the appendix.

The three methods of predicting highway noise essentially perform similar tasks. Starting with a locality in which the influences of several roads might be felt, the user either proceeds to make hand calculations, or prepares input to one of the computer programs, based on the following data.

1. The locations and heights above ground at which levels are to be calculated.
2. The roads which might influence these levels. Curved roads are divided into equivalent straight segments.
3. The presence of barriers or other topographical features which might influence acoustical shielding, also trees, shrubs, or buildings.
4. Other miscellaneous effects, such as grade, road surface, roughness, etc.
5. The density of traffic on the roads.

The NCHRP 117/144 Method

Theoretical Background

The theoretical background of the NCHRP 117 [4] and 144 [5] method is as follows. The equivalent level, L_{eq}, can be given for a line of traffic at a perpendicular distance D from the observer, in terms of the level L_{REF} in A-weighted decibels for one vehicle at a distance D_{REF}, as

$$L_{eq} = L_{REF} - 20 \log_{10} D/D_{REF} + 10 \log_{10} \pi R_L'/V + 10 \log_{10} \Delta\theta/180 \quad \text{dB} \quad (17)$$

where R is the number of vehicles per hour, V is their speed in units consistent with R and D, and $\Delta\theta$ is the subtended angle of the roadway element, in degrees, as shown in Fig. 1. The value obtained for L_{eq} from Eq. (17) does not depend in any way on how the traffic is spaced out, although it does assume that each vehicle is equally noisy. To obtain the statistical level L_E, the level exceeded E% of the time, it is also necessary to assume uniform spacing. This leads to equation

$$L_{50} = L_{eq} + 10 \log_{10} \left\{ \frac{\sinh 2\pi RD/V}{\cosh 2\pi RD/V - \cos \pi E/100} \right\} \quad \text{dB} \quad (18)$$

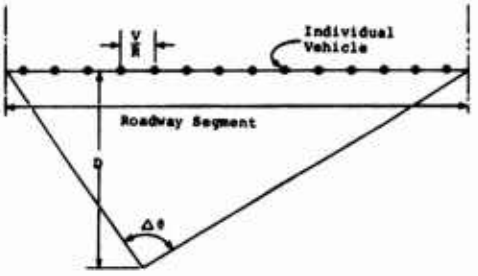

Fig. 1 Traffic on an element of roadway

Application of the Methods

An average automobile passing 100 ft (30.48 m) away at 60 mph (96.6 km/hr) is assumed to emit 64.6 dBA. The acoustical power is assumed to originate at the tires, and is taken to depend on the cube of the speed, so that, for automobiles,

$$L_{REF} = 64.6 + 30 \log_{10} S/60 \quad \text{dBA} \quad (19)$$

where S is the speed mph.

The engine of the average truck passing 100 ft (30.48 m) away is assumed to emit 77.2 dBA, regardless of speed. Thus, for trucks

$$L_{REF} = 77.2 \quad \text{dBA} \quad (20)$$

Substituting Eqs. (19), (20) in turn into Eq. (17), and making appropriate adjustments for the units used, one obtains, for automobiles at 100 ft (30.48 m)

$$L_{eq} = 10 \log_{10} Q_A S_A^2 - 1.0 \text{ dB} \quad (21)$$

and, for trucks at 100 ft (30.48 m)

$$L_{eq} = 10 \log_{10} Q_T/S_T + 65.0 \text{ dB} \quad (22)$$

where subscripts A and T refer to automobiles and trucks, respectively, and Q is the peak hourly traffic rate.

Putting E equal to 50% in Eq. (18), and adjusting for units as before,

$$L_{50}(100 \text{ ft}) = L_{eq}(100 \text{ ft}) + 10 \log_{10} \{\tanh 0.119\, Q/S\} \text{ dB} \quad (23)$$

which applies to automobiles or trucks.

Distance Correction

The distance correction used in the MICHIGAN 144 and MICNOISE 10 programs is DEL1 where

$$DEL1 = -15 \log_{10} D_E/100 \text{ dB} \quad (24)$$

Note the 15 decibel attenuation per tenfold increase in Eq. (24). However NCHRP/144 suggests 10 dB for very flat terrain or for high receiving points. In the above equation, D_E is an effective distance, given by

$$D_E = \sqrt{D_N D_F} \quad (25)$$

in which D_N and D_F are distances from the nearest and furthest lanes, respectively. This correction does not give the same value as would be obtained on a reevaluation of Eqs. (17) and (18). The correction for L_{eq} would be based on -10 dB as opposed to -15 dB, and the correction for L_{50} would be considerably different.

L_{10} Values

The difference between L_{10} and L_{50} is readily found from Eq. (18) to be

$$L_{10} - L_{50} = -10 \log_{10} \{1 - 0.951/\cosh (0.00119\, QD_E/S)\} \text{ dB} \quad (26)$$

However, this does not correlate well with actual measurements for values of QD_E/S over 300 vehicle feet per mile (equivalent to 56.8 vehicle meters per km). Therefore, the curve shown in Fig. 2 has been recommended in the NCHRP 144 report, in which Eq. (26) is faired into an empirical curve. The use of Eq. (26) in conjunction with Eqs. (21) to (25) is not consistent, and must be viewed as partly empirical. The MICHIGAN/144 and MICNOISE 10 programs actually obtain the curve shown in Fig. 2 by interpolation of tables containing the points indicated in the figure.

Roadway Length Correction - DEL2

In accordance with Eq. (17), the correction for roadway length is based on the subtended angle as

$$DEL2 = 10 \log_{10} \{\Delta\theta/180\} \text{ dB} \quad (27)$$

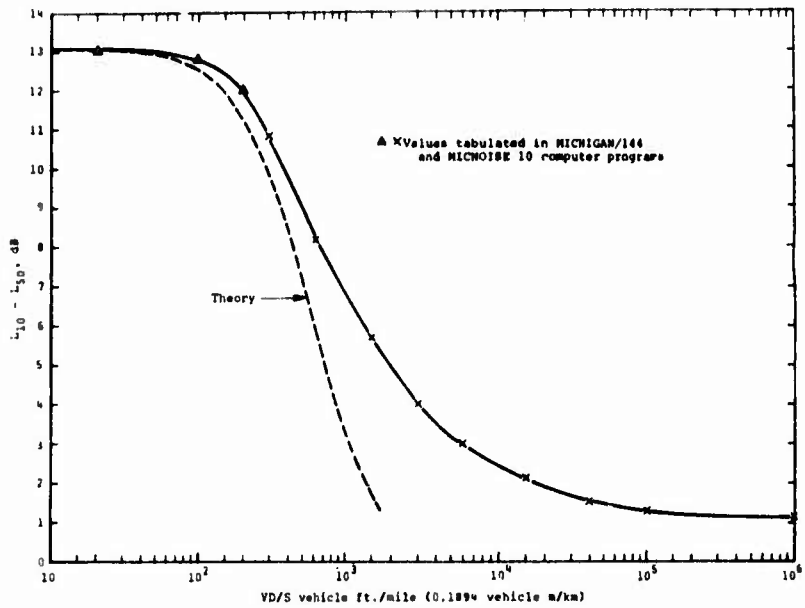

Fig. 2 Values of $L_{10} - L_{50}$ vs. vehicle spacing used in NCHRP methods

Vertical and Barrier Corrections - DEL4, DEL6

When sound from a pure tone point source impinges on the edge of a semi-infinite plane, it is diffracted, so that it may penetrate into the shadow zone. It is possible to obtain exact theoretical solutions for cases in which the geometry is very simple. However, the case of a line source of incoherent broad band noise impinging on the top of a barrier, in the presence of ground effects, is considerably more complex. The corrections recommended in the NCHRP 144 report were based on original work by Maekawa [13], as developed by Kurze and Anderson [14]. The method of application of these corrections is shown in Figs. 3 and 4. It will be seen that the procedure is to find first the deficiency X + Y - Z, and then to read the curve to find the attenuation DEL4 for elevation effects, or DEL6 for barriers.

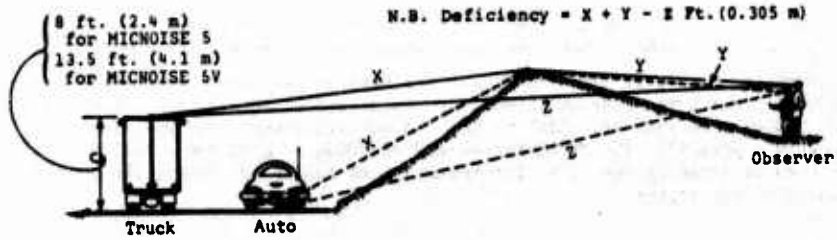

Fig. 3 Definition of deficiency used for elevation corrections in NCHRP methods

In determining the deficiency, truck sources are assumed to be 8 ft (2.4 m) above the road for MICHIGAN/144, as recommended in NCHRP 144, and 13.5 ft (4.1 m) for MICNOISE 10. The curve in Fig. 4 is represented by interpolation of a table in the two programs. There is a further correction to DEL6 for barriers of finite length.

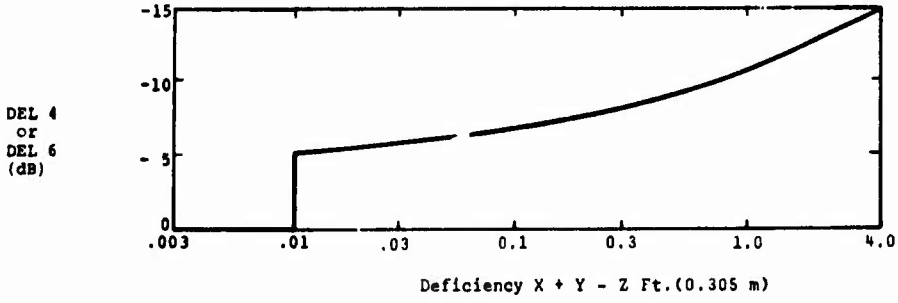

Fig. 4 Elevation corrections used in NCHRP methods

Other Corrections

Several corrections are made at the option of the user, they are:
Gradient Correction - DEL3: From 0 to 4 dB
Roadway Surface Correction - DEL5: From -5 to +5 dB
Structure Correction - DEL7: From -10 to 0 dB for intervening buildings, trees, etc.

Combined Levels

First, the effect of automobile or truck traffic from each element of roadway is determined as follows

$$L_{eq} = L_{eq}(100\ \text{ft}) + \text{DEL1} + \text{DEL2} + \text{DEL3} + \text{DEL4} + \text{DEL5} + \text{DEL6} + \text{DEL7} \quad \text{dB} \quad (28)$$

$$L_{50} = L_{eq} + 10 \log_{10}\{\tanh 0.119\ Q/S\} \quad \text{dB} \quad (29)$$

$$L_{10} = L_{50} + (L_{10} - L_{50}) \quad \text{dB} \quad (30)$$

where $L_{10} - L_{50}$ is obtained as in Eq. (26).

Then levels due to automobiles and trucks from the different roadway elements are combined by the power law, according to which

$$L = 10 \log_{10} \Sigma(j)\ 10^{Lj/10} \quad \text{dB} \quad (31)$$

where L_j represents a typical contribution to the total. The operation described in Eq. (31) is often referred to as 'dB-summing.'

Output of MICHIGAN/144 and MICNOISE 10 Programs

The output of the MICHIGAN/144 and MICNOISE 10 programs includes L_{eq}, L_{50},

L_{10}, L_{NP}, and TNI at points designated by the user. Also, distances from the road to given L_{10} contours are found.

The TSC Model

The Transportation Systems Center model by Wesler [7], often referred to as the TSC model, is based on a more rigorous approach to the statistical problem of predicting L_{10} levels than is used in the NCHRP 117/144 method. In addition to grouping vehicles by type (automobiles, trucks, and a third user supplied vehicle) and to including the effects of different roadway elements, the TSC method provides for up to nine octave bands and for up to five speed groups. Thus, the final summation is over all of these variables. At the option of the user, the summation of octave levels can be omitted, and the overall levels can be used in the calculations with 500Hz frequency assumed for acoustical shielding calculations.

Average Vehicles

Whereas the NCHRP 117/144 method treats a line of vehicles as equally spaced point sources, the TSC method treats it as an incoherent line source with a normal distribution and specified standard deviation. Since the strength of this source is directly proportional to the number of vehicles in a given distance, it is possible to state the reference A-weighted level per vehicle at a given distance from the road. These levels are given in Table 3 for the nine octave bands.

Table 3 Reference A-Weighted Octave Levels Used in TSC Method

Band No.	Octave Center Frequency Hz	L_{REFi} A-weighted level in dB at 50 ft (15.2 m)		
		Autos at 30 mph (48.3 kph)	Autos at 70 mph (112.7 kph)	Trucks at all speeds
18	63	38	48	60
21	125	45	57	73
24	250	47	62	78
27	500	55	66	83
30	1000	58	70	82
33	2000	54	72	79
36	4000	49	63	74
39	8000	42	57	60
	OASPL	61	75	87
	Standard Deviation σ_L	2.5	2.5	3.5

Levels for automobiles at other speeds are obtained by linear interpolation. For comparison of these levels with those used in the NCHRP 117/144 method, it should be noted that the levels in Table 3 are median values, whereas the NCHRP levels are essentially peak values. Thus, for direct comparison, $0.115\ \sigma^2$ must be added to the values in Table 3 where σ_L is the standard deviation (this can be demonstrated by an analysis similar to the one used in the derivation of Eq. (12)). Using the OASPL value, making the correction for the standard deviation, interpolating for 60 mph, and correcting to 100 ft (30.48 m), the comparative values in Table 4 can be obtained.

Table 4 Peak levels in dBA for vehicles at 60 mph (96.6 kph) at 100 ft (30.48 m)

	NCHRP 117/144	TSC	1974 NCHRP
Autos	64.6	66.2	65.3
Medium Trucks	-	-	75.3
Heavy Trucks	77.2	82.4	79.9

Calculation of L_{eq}

Using essentially the same notation as was used in describing the NCHRP method,

$$L_{eq} = 10 \log_{10} \sum_{\substack{\text{road} \\ \text{segments}}} \frac{50^2}{D_E} \quad \frac{\pi\Delta\theta}{180} \sum_{\substack{\text{vehicles} \\ \text{speeds} \\ \text{frequencies}}} \frac{Q}{5280\,S} 10^{DEL_i/10}\, 10^{(L_{REFi} + 0.115\sigma^2)/10} \quad \text{dB} \tag{32}$$

The calculations are carried out approximately as indicated to avoid frequent inefficient "dB-summing." However the contribution of one term could be written as

$$\Delta L_{eq} = L_{REFi} + 0.115\sigma^2 - 10 \log_{10} D_E/100 + 10 \log_{10} Q/S + 10 \log_{10} \Delta\theta/180 + DEL_i - 18.3 \quad \text{dB} \tag{33}$$

It will be noted that the distance correction is based on the 10 dB law. There is no correction for speed here because L_{REFi} already includes a speed correction. Otherwise corrections for traffic flow and roadway length are similar to those in the NCHRP method. Several other corrections are included in the term DEL_i, these are described in the next paragraphs.

Contribution of Atmospheric Attenuation

The contribution to DEL_i from atmospheric attenuation is

$$DEL_i = -10^{-7}\, 4^{\frac{i-15}{3}} \quad \text{dB} \tag{34}$$

where i = 18, 21 . . . 39 is the octave band number

Contribution of Acoustical Shielding

The contribution of acoustical shielding due to barriers and roadway elevation effects is based on the Fresnel angle N_i, where

$$N_i = 2f_i\,(X + Y - Z)/c \tag{35}$$

in which f_i is the octave center frequency, c is the speed of sound, and X + Y - Z is the deficiency, as shown in Fig. 3. In calculating these values, noise sources for automobiles are placed on the road surface, noise sources

for trucks are placed 8 ft (2.4 m) above the road, while the user can select the height of the noise source for the third vehicle.
Then

$$DEL_i = \begin{cases} 0 & \text{for } N_i \leq -0.2 \\ -20 \log_{10}\left[\dfrac{\sqrt{2\pi|N_i}}{\tanh\sqrt{2\pi|N_i}}\right] & \text{for } -0.2 < N_i \leq 0 \\ -20 \log_{10}\left[\dfrac{\sqrt{2\pi N_i}}{\tanh\sqrt{2\pi N_i}}\right] & \text{for } 0 < N_i \leq 12.5 \\ -24 & \text{for } N_i > 12.5 \end{cases} \quad (36)$$

The above contribution is first evaluated for the nearest point on the road, and for points at the ends of the road segments. Whenever a difference of more than 1 dB is obtained, the roadway element is halved, and the calculation is repeated.

Contributions of Reflections off Barriers

The contributions of reflections are combined with the acoustical shielding effects by dB-summing.

Contribution of Ground Cover

Attenuations of up to 30 dB are calculated for the effects of ground cover, including shrubbery, thick grass, and trees.

Output of TSC Program

The TSC program computes L_{eq} directly, and includes a procedure developed by Kurze [14] for the determination of the standard deviation of the sound level σ_L. Then, as in Eqs. (12, 13 and 14)

$$L_{50} = L_{eq} - 0.115\,\sigma_L^2 \quad (37)$$

$$L_{10} = L_{50} + 1.28\,\sigma_L \quad (38)$$

$$L_{NP} = L_{eq} + 2.56\,\sigma_L \quad (39)$$

The 1974 NCHRP Method

The new design guide in the report by Kugler, et al. [8] was developed for the NCHRP in 1974. It contains a short method, using nomographs, and a computer program, written in ANSI standard FORTRAN. The program is in two parts, the first will give listed results, while the second will produce the input to a CALCOMP plotter program.

Theoretical Background

The new guide gives a method leading directly to the calculation of L_{eq}, from which L_{10} can be obtained. The calculation of L_{eq} is made as follows (in most cases, the terminology of the present report has been used in place of that given in [8]):

$$L_{eq} = (EL - 4) + 10 \log_{10} Q/SD + 10 \log_{10} \Delta\theta/180 + (1.2 - 10 \log_{10} r_n/50) + DEL_B + 2 \quad (40)$$

where r_n is the distance from the observer to the nearest part of the road element, DEL_B is the attenuation due to barrier or road elevation, and EL is the emission level, the remaining symbols have the same meaning as in the section on the NCHRP 117/144 method.

Application of the Method

Values for EL are given as follows

Automobiles: $EL_1 = 22 + 30 \log_{10} S$ dB (41)

Medium trucks, a new designation: $EL_2 = 32 + 30 \log_{10} S$ dB (42)

thus the new medium truck is exactly 10 dB noisier than an automobile.

Heavy trucks, i.e., tractor trailers: $EL_3 = 90$ dB (43)

Comparison with NCHRP 117/144

To compare with the older design guide or NCHRP 117/144 methodology, it is best first, to reevaluate Eqs. (21, 22) according to the values in the new guide. These now appear as

Automobiles at 100 ft (30.48 m): $L_{eq} = 10 \log_{10} Q_A S_A^2 - 0.3$ dB (44)

Medium trucks at 100 ft: $L_{eq} = 10 \log_{10} Q_m S_m^2 + 9.7$ dB (45)

Heavy trucks at 100 ft: $L_{eq} = 10 \log_{10} Q_T/_T + 67.7$ dB (46)

Thus the new guide increases the levels of automobiles by 0.7 dB and heavy trucks by 2.7 dB; however, compared to the earlier guide, many trucks would be taken out of the heavy truck category, and placed in the new medium truck category, so that overall noise level predictions may not increase by as much as 2.7 dB. An alternative approach, which permits comparison of all three methods, is to calculate the corresponding dBA values at 60 mph and 100 ft and to enter the results into Table 4. It will now be seen that the new 1974 NCHRP method gives values which are intermediate between the other two.

Distance Correction

The distance correction now becomes

$$DEL1 = -10 \log_{10} (D_g/100) - 5 \log_{10} (r_n/100) \quad (47)$$

This new correction is the same as the old one of Eq. (24) if the roadway element passes the observer, but is different for a distant element.

Vertical and Barrier Corrections

The elevation and barrier corrections are essentially the same as in the TSC method, but with a frequency of 500 Hz. However, a completely new nomograph has been drawn for finite barriers, with corresponding tables stored in the computer program.

L_{10} Values

Values of L_{10} are obtained by adding the values in Table 5 to L_{eq}.

Table 5 Values of $L_{10} - L_{50}$ correction in new 1974 NCHRP design guide

Vehicle density parameter, QD/S		Correction
Vehicle ft/mile	Vehicle m/km	$L_{10}-L_{eq}$ dB
0	0	0
10	1.894	-5
25	4.74	-2
50	9.47	1
200	37.88	3
3000	568.2	2
16000	3030.0	1

The values given in Table 5 are new, being based on a statistical analysis of the overall problem. Values given for QD/S greater than 200 vehicle ft/mile (37.88 vehicle m/km) are stated to be within ± 2 dB.

EVALUATION OF COMPUTER PROGRAMS

As will be seen in Table A1 of the appendix, there are four computer programs which are of especial interest. These are the MICHIGAN/144 and MICNOISE 10 programs based on the NCHRP 177/144 method, the TSC program, and the 1974 NCHRP program. The first three of these are based on methods approved by the FHWA, while the fourth is based on a method which will probably be approved shortly. There are numerous variants of these programs, and there have been a number of other programs which do not, however, have FHWA approval.

Programs Based on NCHRP 117/144 Method

Haviland, Noble and Golub [9, 10] have evaluated MICNOISE 2, 5, and 5V which are earlier variants of MICNOISE 10. Except in the matter of output, MICNOISE 5 is almost identical to MICHIGAN/144, and MICNOISE 5V to MICNOISE 10. The only difference between MICHIGAN/144 and MICNOISE 10 is in the height of noise sources of trucks above the ground. A summary of the referenced evaluation is given in Fig. 5, which shows the 68% confidence bands on errors between calculated and experimental values of L_{10} for five test sites. Details of the test sites at which comparative experimental readings were obtained, and the number of measurements made at each, are shown in Table 6.

Table 6 Details of test sites used in evaluation of MICNOISE 2, 5 and 5V computer programs

No.	Route	Location	Geometry	Traffic per hour	Number of 15 min readings
1	I-495	Springfield, Va.	Depressed Roadway	5,733	16
2	I-495	Alexandria, Va.	Level	4,318	12
3	I-64	Fisherville, Va.	Elevated Roadway	713	8
4	US 29	Ruckersville, Va.	Depressed Roadway	436	7
5	I-95	Doswell, Va.	Elevated Roadway	2,304	8

The 68% confidence band covers the mean range of values of error plus and minus one standard deviation of error. Thus there is a 16% chance that the error will fall below the lowest band, in which case the computer program will underestimate the actual L_{10} level by more than one standard deviation.

It will be seen from Fig. 5 that the lower band is at about -2 dB for MICNOISE 2, that it drops to -3 dB for MICNOISE 5, due to introduction of a new acoustical shielding calculation, and that it increases to -2 dB again for MICNOISE 5V, due to the greater height used for truck noise sources. However, the probability of overestimation is greater for MICNOISE 5V than for the other variants, as is indicated by the relatively higher upper limits.

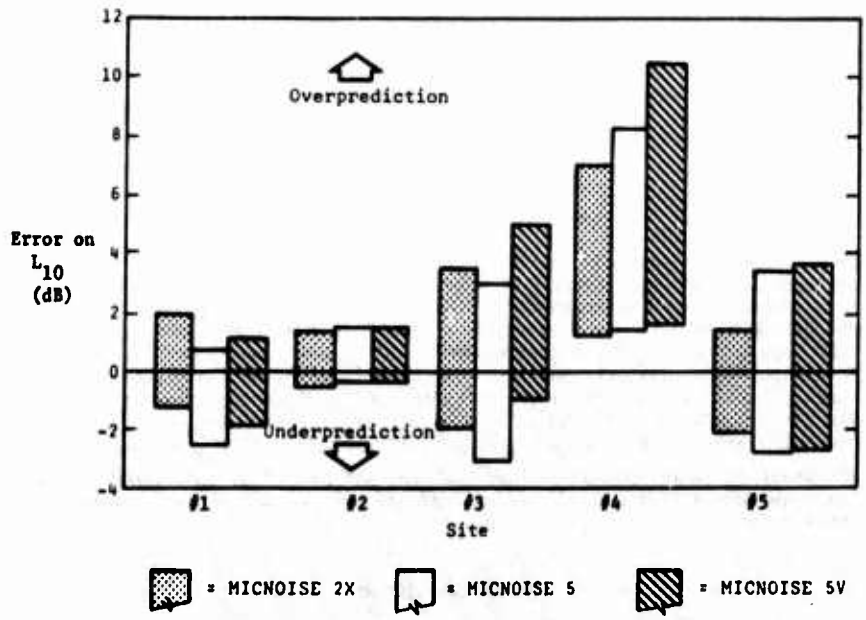

Fig. 5 68% confidence limits for errors on L_{10} for three MICNOISE variants (MICNOISE 5 is equivalent to MICHIGAN/144 and MICNOISE 5V is equivalent to MICNOISE 10)

FHWA does not require better than 50% chance of underestimation, however, according to this study, it would be necessary to add 3 dB to MICHIGAN/144 and 2 dB to the MICNOISE 10 calculations, to reduce the chance of underestimation to 16%.

Summarizing other comments in the above mentioned evaluation, and in the report by Kugler, et al. [8] which contains the new 1974 NCHRP method, the following general observations can be made about the MICHIGAN/144 and MICNOISE 10 programs which are based on the NCHRP 117/144 method:

1. The method of "dB-summing" of L_{10} contributions from different vehicles and road elements is incorrect, and can be readily shown to lead to small inconsistencies.
2. The method fails to recognize the random nature of individual vehicle noise levels and of vehicle spacing along a road.
3. The method is partly based on empirical laws without physical justification. Examples are the 15 dB distance correction and the effects of distance on L_{50} and L_{10}.
4. The method only recognizes an engine noise source on trucks, independent of speed, and a tire noise source on automobiles, dependent on the cube of the speed. More is known about the sources of noise than is indicated by this, and could be readily included in the method.
5. On many roads, the predominant truck is not the tractor trailer, for which the program is designed, but rather it is a medium type of truck, such as those used in truck farming, local delivery, etc. If these medium trucks are counted in the NCHRP 117/144 method, they lead to overprediction of noise levels. Note that the program based on the 1974 NCHRP method includes medium trucks.

The TSC Method

The TSC method has been used by a number of highway engineers and consultants. As might be inferred from Table 4, it predicts higher levels than do the other methods. However, the methodology used appears to be more closely based on physical principles.

In comparison with the NCHRP 117/144 method, probably the most distinct difference that can be noted is that, whereas the TSC method is the more theoretically correct, the NCHRP method is based on many field measurements, and is therefore expected to be the more accurate, on an empirical basis.

Program Based on the 1974 NCHRP Method

In proposing this method, Kugler, et al. [8] performed an exhaustive review of the overall problem, conducted surveys of users of NCHRP 117/144 and TSC methods, and made many field measurements. Therefore, the program based on this method can be expected to be as good as any available. Examining Table 4, one sees that enough conservatism (an increase of 2.7 dB for trucks) has been added to cover the 68% confidence band noted in the discussion of the NCHRP 117/144 methods. Also, the new medium truck category will avoid the problems of overprediction under many conditions. From a cursory review of the method, the only remaining criticisms concern the continued dB-summing of L_{10} levels, and the treatment of vehicle noise sources. At the present time, unexpected delays in the development of the new guide are responsible for a delay in its approval and distribution.

PROGRAM SUMMARIES

MICHIGAN/144 [6]

Capability: Prediction of highway noise levels (L_{50}, L_{10}, L_{eq}, NPL, TNI) at

locations specified by user, or distance from nearest lane to a given L_{10} contour.
Method: NCHRP 117 [4] and 144 [5]
Input: The input is supplied through a teletype keyboard to an interactive program. Therefore some irrelevant input data is deleted through the system logic. All or part of the following data is required for each roadway element, but some may be deleted for a new observer location on a continuation run. Number of lane groups per element; vehicles per hour; percent trucks, speeds of trucks and automobiles; road elevation; distance of observer from near lane; number of lanes per lane group; grade, surface, and structure attenuations; median width; roadway included angle; observer height; distance to roadway shoulder or cut; barrier height and included angle.
Output: L_{10} values for automobiles and trucks, separately and combined for each lane group. Overall values for L_{50}, L_{10}, L_{eq}, NPL, and TNI to nearest decibel. Alternatively, distance from nearest lane to a given L_{10} contour.
Language: FORTRAN IV
Hardware: Burroughs 5500 through time-sharing remote terminal.
Usage: Extensive
Developer: Mr. G. H. Grove, Electronic Engineer
Michigan Department of State Highways and Transportation
Research Laboratory Section
735 E. Saginaw Street
Lansing, Michigan 48906
Availability: FORTRAN IV listing in manual [6]
Subjective Comments: Based on method approved by FHWA.

MICNOISE 10

Capability: Same as for MICHIGAN/144.
Method: Same as MICHIGAN/144.
Input: Same as MICHIGAN/144, but on cards in batch format.
Output: Summary of input data, followed by same output as MICHIGAN/144.
Language: FORTRAN IV for IBM 370.
Hardware: IBM 370 through batch loading, HASP II system.
Usage: Extensive
Developer: Adapted for batch programming from MICHIGAN/144 program by
Ron Heissler
Virginia Department of Highways and Transportation
1221 E. Broad Street
Richmond, Virginia 23230
Availability: Request listing and card deck from Heissler.
Subjective Comments: The only change from MICHIGAN/144 is placement of truck noise sources 13.5 ft above road, in place of 8 ft. Based on method approved by FHWA.

TSC

Capability: Prediction of highway noise levels (L_{50}, L_{10}, L_{90}, L_{eq}, NPL).
Method: TSC [7]
Input: Programmed for batch input using 5 card formats, as follows. Program initialization-receiver height; frequency bands; standard deviations and source heights for passenger cars, trucks, and new (i.e. user designated) vehicles; spectrum for new vehicles. Road and vehicle data - traffic flows by vehicle type and speed group; road element coordinates. Barrier parameters - berrier top coordinates; barrier type. Ground cover parameters - centerline coordinates; width; type. Receiver data - coordinates.
Output: Input summary; octave and A-weighted values for L_{eq}. Overall values for L_{50}, L_{10}, NPL.

Language: FORTRAN IV
Hardware: Originally written for IBM 7094. Has been used on IBM and CDC computers.
Usage: Extensive
Developer: J. Wesler (presently at:)
Office of Noise Abatement
U.S. Department of Transportation
Washington, D. C., 20590
Availability: FORTRAN IV program for IBM 7094 listing is in [7]. Contact J. Wesler for help in locating program for a given computer.
Subjective Comments: Based on method approved by FHWA. There are problems with program as listed in [7], and potential users should refer to J. Wesler before use.

1974 NCHRP [8]

Capability: Prediction of highway noise levels, L_{10} or L_{eq}, comparison with design levels, contours of noise levels.
Method: 1974 NCHRP [8].
Input: Input is by batch loading on four data card formats, with five control card types. RDWY card-coordinates of roadways by element, with information on surface texture and gradient of road. TPAR card-vehicle type, flow volume and speed. BARR card - top point coordinates of barriers, by element; barrier designation as normal, single structure, or vegetation. RCVR card-receiver coordinates. Control cards include provisions for labelling and for adding data for subsequent runs.
Output: The output consists of listings and magnetic tape. Listings include the following: summary printout of input data; L_{10} (or L_{eq} with a small program change) to the nearest 0.5 dB, and amount by which design level is exceeded; summary of points at which levels are exceeded and critical roadway elements responsible. The magnetic tape contains instructions to a CALCOMP plotter for a contour map of noise levels.
Language: FORTRAN IV (ANSI Standard X3.9-1966). Assumes at least 4 alphanumerics to a computer word.
Hardware: Program has been run on a CDC-6400. CALCOMP 30 in. drum plotter required for optional contour plots.
Usage: Has only been used on a trial basis.
Developers: Nicolaus H. Reddingius and Noel Lockwood
Bolt, Beranek, and Newman, Inc.
Waltham, Massachusetts 02154
Availability: FORTRAN IV listing in [8]. Available on loan from:
David K. Witherford (Project Engineer)
National Cooperative Highway Research Program
2101 Constitution Avenue, N.W.
Washington, D.C., 20418
Subjective Comments: The method is presently being evaluated. Will probably be the only method approved by FHWA eventually.

SUMMARY

The methodologies approved by FHWA (or for which approval is anticipated), the four corresponding computer programs, their variants, and other computer programs for highway noise, are summarized in Table A1. The information given in [15] is acknowledged, this includes several programs used in the UK, (ANGER, NOISE, MWAY and CONTOUR) for which inadequate references are available.

Basic information on the four computer programs is summarized in Table A2 in the appendix.

Differences between the four programs are relatively minor, since all lean heavily on empirical laws. The 1974 NCHRP method has been developed as a result of an extensive evaluation of the other two methods, and of the overall problem

of highway noise prediction, combined with a comprehensive program of field measurement. It is strongly oriented towards the problem of designing barriers in an optimum manner. In the light of the current state of the art, it is difficult to see how any prospective user can do better than to adopt and use the new 1974 NCHRP method, once it has been approved.

The problems encountered in coming up with really satisfactory methods of predicting highway noise are typical of those encountered in any systematic approach to acoustical analysis. The form of the acoustical equation, and many of the solutions for simple boundary conditions have been known since the nineteenth century. However, whereas new and powerful methods of analytical solutions have been developed in such fields as fluid dynamics and solid mechanics, the acoustical equation has defied the modern computer, so that methods of predicting attenuations in the presence of obstructions and rough ground are still quite unreliable. As was noted in [9, 10], typical prediction errors have standard deviations of 2 to 3 dB, representing factors of up to two on mean square pressures. There are few fields in which one is happy to come within a factor of two!

If one is to criticize the 1974 NCHRP method, it is in the lack of confidence limits in the output. The whole subject of highway noise prediction is full of statistical variations. To name a few: each vehicle has its own noise level; vehicle spacing is governed by traffic flow laws; actual traffic flows fluctuate considerably; wind and atmosphere conditions vary from hour to hour; effects of vegetation, little understood anyway, vary with the seasons; and as already noted, attenuation near the ground is understood very imperfectly. An attempt to cope with some of these problems was evident in the TSC method, however, although the derivation of the $L_{10} - L_{eq}$ correction was based on statistical considerations in the 1974 NCHRP method, the result was almost a reversion to the NCHRP/117 approach.

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APPENDIX

Table A1 Summary of Methodologies for Highway Noise Prediction, and of the Corresponding Computer Programs

Methodology (FHWA approved)	Current Programs	Other Variants and Previous Programs
NCHRP 117/144 [4,5]	Michigan/144 [6] MICNOISE [10]	Michigan/177 MICNOISE 2, 5, 5V [9,10] NOISE [15]
TSC [7]	TSC	
1974 NCHRP [8]*	1974 NCHRP	
		Rhode Island I-84 [15,16] NOISESIM [9]

* Approval by FHWA is anticipated.

Table A2 Comparison of 4 Computer Programs for Highway Noise Prediction

Name of Program	MICHIGAN/144	MICNOISE 10	TSC	1974 NCHRP
Method	NCHRP 117/ 144 [6,7]	NCHRP 117/ 144 [6,7]	TSC [7]	1974 NCHRP [8]
Computer	Burroughs 5500	IBM 370	-	-
Language	FORTRAN IV	FORTRAN IV	FORTRAN IV	FORTRAN IV
Max. dBA levels at 100 ft at 60 mph				
Auto	64.6	64.6	66.2	65.3
Medium Truck	-	-	-	75.3
Truck	77.2	77.2	82.4	79.9
Dist. Corrections for Tenfold Increase	-15 dB	-15 dB	-10 dB	-15 dB
Max. Barrier Correction	-15 dB	-15 dB	-24.5 dB	-20 dB

Liquid Propellant Dynamics Analysis

Frank M. Bugg

Marshall Space Flight Center

INTRODUCTION

The Saturn IB launch vehicle, as used in the Apollo Soyuz Test Project, contains approximately 1.1 million pounds of liquid propellant at lift-off. The propellant is 87% of the vehicle weight, and this percentage is typical of liquid fueled launch vehicles. Early in this country's space program it was discovered that avoidance of control system/liquid resonance conditions, provision of adequate damping, and, therefore, knowledge of normal modes of liquid oscillation in propellant tanks are essential for successful launch and boost flight of large liquid fueled rockets.

The Apollo moon missions had as part of their flight plans an orbital coast of the Saturn S-IVB stage containing 157,000 lbm of liquid oxygen and hydrogen in its 22-foot-diameter tanks. During the coast period it was required that pressure relief vents be kept free of liquid and that the engine feedline inlets remain submerged in preparation for engine restart, while vehicle orientation was maintained by a system of reaction jets. Future space transportation systems (including Space Shuttle and Tug) will perform satellite deployment, maintenance, and retrieval operations which will include rendezvous and docking maneuvers with large quantities of liquid propellant onboard. Knowledge of liquid location in the tanks and liquid/tank/control system force interactions will also be important in these flights.

Many investigations, both theoretical and experimental, have been conducted in the past 17 years to provide solutions to propellant dynamics problems as typified in the above examples. A few of the resulting computer programs will be discussed in this chapter. The programs are available at no cost through the author, Marshall Space Flight Center.

NOMENCLATURE

a = Acceleration tending to press liquid against its container wall

B = Bond number, $\frac{\rho a R^2}{\sigma}$

FY = Horizontal force exerted by a liquid on its container

FZ = Vertical force exerted by a liquid on its container

g = Standard acceleration due to Earth's gravity, 9.80665 m/sec^2

R = Tank radius

χ = An empirical coefficient

ρ = Liquid density

σ = Liquid surface tension

PROPELLANT BEHAVIOR REGIMES

The Bond number, B, is a dimensionless parameter which gives the ratio of acceleration forces to surface tension forces on a liquid.

$$B = \frac{\rho a R^2}{\sigma},$$

where: ρ = liquid density
a = acceleration
R = tank radius
σ = liquid surface tension

Bond number is generally used to categorize propellant behavior regimes. High Bond number typically exists while engines are thrusting, and may also exist for tanks with a large radius in low earth orbit coast. The low acceleration environment of orbital or interplanetary coast and small tank size result in surface tension dominated or low Bond number propellant dynamics.

Oscillatory liquid motion can occur at either high or low Bond number. Acceleration configures the equilibrium liquid free surface for $B>1$ and provides the restoring force which sustains oscillations about the equilibrium. For $B<1$, surface tension minimizes the free surface area consistent with liquid/tank wall contact angle and liquid volume. Oscillations occur about this curved (nearly spherical) equilibrium.

Transient liquid motion is of concern during the docking of two spacecraft. Propellant seated at one end of a spacecraft's tank may receive a sudden acceleration toward the opposite end as a result of thruster firings or docking impact. Similar liquid motion, flow from one tank end to the other, can also affect the reentry trajectory and debris impact area of spent stages.

Computer programs for analysis of high Bond number oscillatory liquid motion will be considered first.

COMPUTER PROGRAMS

High Bond Number

Lomen Slosh

The equations of motion are linearized and solved for small amplitude oscillations of an ideal, incompressible liquid in a tank described by the revolution of an arbitrary curve about an axis of symmetry. The acceleration must be parallel to the axis. The tank shape generating curve is input as a series of segments which can be straight line, circular, elliptical, or parabolic. Up to 30 ring baffles, in planes perpendicular to the tank axis, can be included. A liquid level measured from the tank bottom defines each case.

The output gives eigenvalues and eigenvectors for up to five slosh modes. The force distribution on the tank wall and the center of pressure are calculated. Instantaneous and average energy dissipation rates are computed if there are baffles in the tank.

Components of a spring-mass model of the oscillating liquid are also computed. In control and stability studies it is convenient to represent the sloshing liquid as a mass attached to the tank wall by a spring and a stationary mass near the tank bottom. The forces and moments exerted on the tank wall and the frequency of the sloshing liquid are matched by the spring-mass model. The moving mass is referred to as the slosh mass. The program computes and plots total liquid mass, slosh frequency for an acceleration of 9.80665 m/sec^2, slosh mass, and slosh mass location as a function of liquid level in the tank for each mode. For a particular problem, the frequency calculated should be multiplied by the square root of the applied acceleration in g's. A symmetric half of the tank is also plotted. An example of these plots is shown in Fig. 1 for the Space Shuttle liquid oxygen tank. The run time for the case plotted was 5 minutes on the UNIVAC 1108. This was for calculation of the first mode at 15 liquid levels.

The accuracy of the program results has been verified by laboratory and flight data. The calculated frequency has been found to be within 0.5% of measured values for most cases. Use of the spring-mass analogy parameters in the Saturn program has proven successful for avoidance of slosh/control system

instability. The parameters have been used to simulate high Bond number slosh during Saturn boost flight ($B = 10^7$) and during low orbit coast ($B = 10^2$). The frequency accuracy was not as high at $B = 100$ (0.0030 Hz measured and 0.0032 Hz calculated) which is a possible indication that some low Bond number effects were involved.

The program is written in NASTRAN for both the IBM 7094 and the UNIVAC 1108. Details of the equations solved, solution method, program input, and program output are given in [1] and [2].

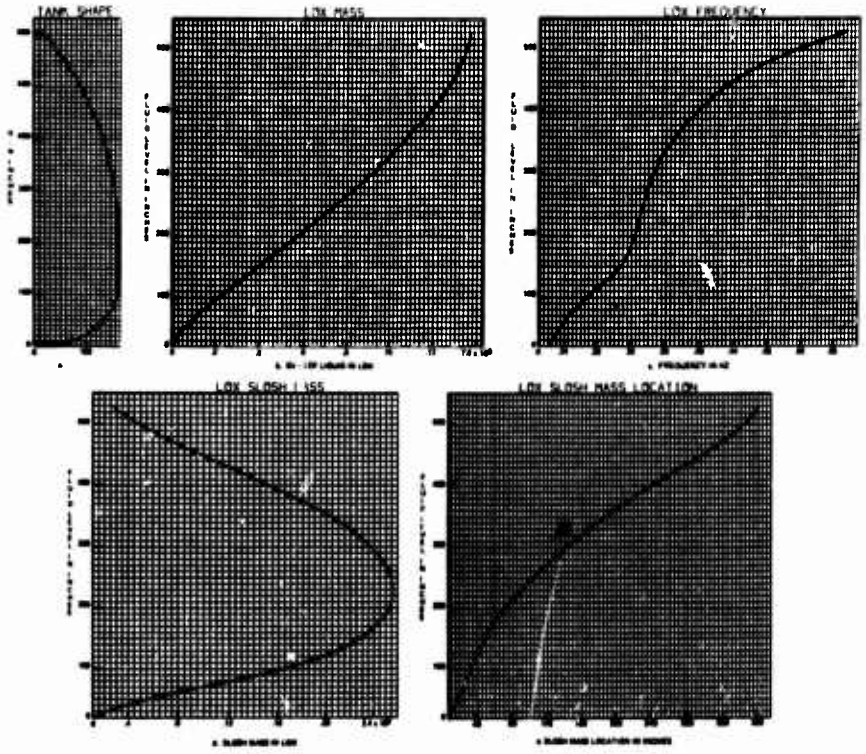

Fig. 1 Spring-mass analogy parameters

SLOSH5

This program uses a finite element approach to fluid sloshing and fluid/structure vibration analysis. Compressible tetrahedral, pentahedral, and hexahedral elements are formulated to represent the liquid. Quadrilateral membrane and plate bending structural elements are used to represent the container wall. Kinetic and potential energy are expressed as functions of modal displacements, making the formulation similar to that for structural elements, except that the fluid can possess gravitational potential and the fluid equations contain no shear coefficients. The structural and fluid elements are designed to be used interchangeably in an efficient sparse matrix computer routine.

The current program is temporary, pending incorporation of the fluid elements into an existing structural analysis program of greater speed, capacity, and generality. Some features of the current program are:

1. Automatic network generators for simplified input.
2. Full matrix eigensolver with automatic strip width size selection to fit core allocation. The solution is by tridiagonalization and Householder reduction.
3. Extensive output options, checks, and plotting capability. All modes up to specified frequency are calculated, but solution data are printed and plotted only for the frequency range selected.

The program input is mainly element geometry, joint locations, joint constraints, selection of frequency range, and option specification. The program output includes the assembled mass and stiffness matrices for all elements, the mode shapes, and the modal frequencies. These are computed for liquid oscillating in tanks of arbitrary shape.

The accuracy of the slosh frequency calculation is a strong function of the configuration of liquid elements in the tank. Quarter models of liquid in a circular cylindrical tank are shown in Fig. 2. Figure 2a shows a computer plot of the first lateral slosh mode. Using the element configuration shown in Fig. 2b, a frequency 7% less than the closed form value was calculated, while

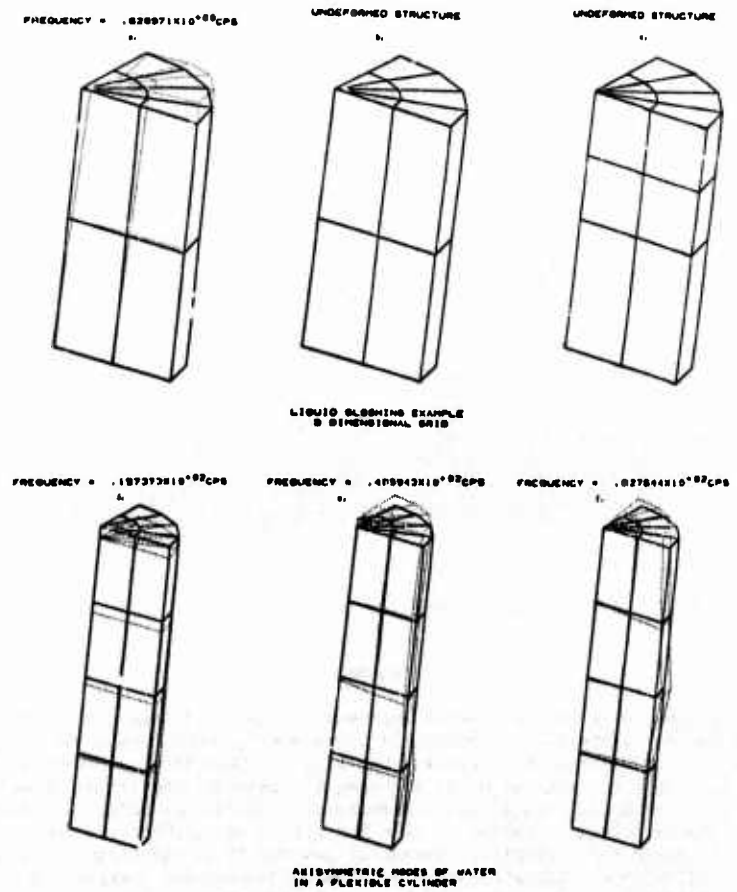

Fig. 2 Finite element quarter models

the liquid representation of Fig. 2c gave a frequency with only 1.5% error. All of the significant liquid motion takes place within one tank radius of the free surface for first lateral mode liquid slosh in a circular cylinder; therefore, greater accuracy was expected for the configuration in Fig. 2c with elements two deep in this region. The configuration in Fig. 2c had 77 unrestrained degrees of freedom and the modal solution required 1 minute and 21 seconds of UNIVAC 1108 cpu time.

Figures 2d, 2e, and 2f show three axisymmetric modes of coupled liquid and flexible cylinder motion. The frequency of the first of these modes was checked and found to be within 1% of the closed form value.

Hydroelastic Slosh

The purpose of this program is to compute mode shapes and frequencies for coupled liquid/tank modes in the Space Shuttle propellant tanks for the range of tank tilt angles expected during Shuttle boost flight. The initial program version is specifically for modes in the Shuttle external liquid oxygen tank.

Finite element representation of liquid and tank is utilized. The basic fluid element is a tetrahedron; pentahedrons and hexahedrons are constructed from these. A linear displacement field is used for the element. The mass and stiffness matrices for the fluid elements are obtained from the kinetic and potential energies, respectively. The potential energy consists of dilational strain energy (compressible liquid) and gravitational potential energy. Quadrilateral plate elements are used to represent the tank.

Two eigensolution methods are used. A Jacobi routine computes modes for systems with 115 or fewer degrees of freedom and an iterative Rayleigh-Ritz method is employed for larger problems. With the Rayleigh-Ritz procedure a frequency of interest is specified and only modes near this frequency are computed (if the significant slosh modes are to be calculated, this procedure is efficient since computation of many extraneous circulation modes is avoided; but for structural analysis where all structural modes up to some frequency must be found, overlapping frequency ranges and the large number of runs required decrease the efficiency).

Automatic generation of finite element data is provided and only basic information such as fluid level, tilt angle, number of vertical tank divisions, wall thickness, and material properties are required as input. An extension of this program to include automatic data generation for user specified tank shape is planned.

Figure 3 shows computer plots of the Shuttle oxygen tank for two fill levels. Figure 3a is the first lateral slosh mode for a liquid level 487 inches above the tank bottom. The frequency, 0.515 Hz, is within 6% of the frequency by the Lomen program shown in Fig. 1c. It is likely that modeling with more fluid elements near the surface (as demonstrated above for SLOSH5) would increase accuracy for the frequency calculation. Figures 3b and 3c show a front and side view of the element arrangement for liquid at the 317 inch level in a tank tilted 13°.

The program is written in FORTRAN for the UNIVAC 1108 computer. Program details are described in [5].

Low Bond Number

The first two programs discussed for low Bond number solve eigenvalue problems and, thus, give information on oscillatory liquid motion. The remaining three programs were developed primarily for transient response studies but will give a first lateral mode frequency for cases which produce oscillatory motion. The transient motion programs can be applied in either high or low Bond regimes but are more accurate for the slow, orderly (minimum surface splashing or rippling) liquid motion which often characterizes large amplitude low Bond flow.

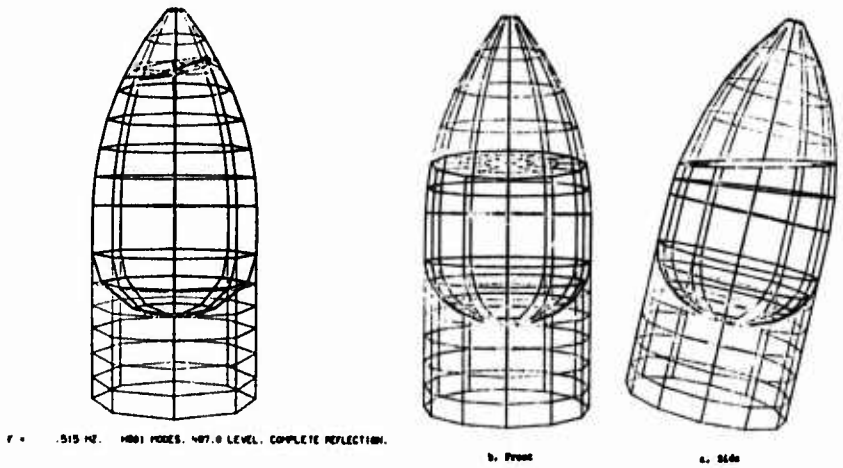

Fig. 3 Finite element Space Shuttle oxygen tank models

LOBOND

A combination of two programs computes the equilibrium free surface shape and the lateral vibration mode shapes and frequencies for liquid in a container of arbitrary axisymmetric shape with surface tension forces the same order of magnitude as acceleration forces (Bond number ~1). A force balance is the basis for the equilibrium free surface shape equations. The input is difficult since it requires coefficients which are related to the free surface defining variables (fill percentage, contact angle, tank shape, surface tension, acceleration), rather than the variables directly. An iterative procedure is used and convergence depends on proper choice of input coefficients. Using this program, overhanging (double valued) surface shapes can be determined.

Mass and stiffness matrices are calculated for the vibration analysis using a finite element approach. The continuous liquid is assumed to be composed of simple elements, such as tetrahedrons, pentahedrons, and hexahedrons for the volumetric fluid elements, and triangles and quadrilaterals for the surface elements (both gravitational and surface tension). The derivation to obtain the finite element mass and stiffness matrices is based on kinetic energy and strain energy principles, respectively.

An iterative Rayleigh-Ritz routine (described in [5]) is used to compute modes and frequencies. Three basic mode types were found; internal circulation modes (no surface movement), compression modes, and slosh modes. The compression mode frequencies are high compared to the others, but it was difficult to distinguish between circulation and slosh modes. For a particular test case, a second run was made with acceleration and surface tension increased by two orders of magnitude. The results showed proportional changes in frequency squared for the slosh modes and little effect on the circulation modes; and in this way the two types were identified. No comparisons with experimental results have yet been made.

LOBOND is written in FORTRAN for the UNIVAC 1108 computer. Details of the program and results for slosh at a Bond number equal to one in a particular tank shape are given in [6].

Mercury Slosh

A spacecraft concept called Solar Electric Propulsion Stage (SEPS) has been studied and considered for transportation of payloads between low Earth orbit and geosynchronous orbits. A reusable vehicle was envisioned, propelled by accelerated mercury ions. A candidate mercury storage system consisted of spherical tanks with a hemispherical neoprene bladder separating the mercury from its pressurizing gas. As the mercury is depleted, the bladder slowly inverts, assuming shapes determined by the mercury volume, bladder stiffness, and acceleration environment. A computer program has been developed to compute equilibrium shapes of the mercury and bladder for various tank fill percentages, and to compute vibration mode shapes and frequencies at these equilibriums.

The program consists of the two parts, as with LOBOND discussed above; the equilibrium shape is computed and used in a vibration analysis. An energy minimization procedure is used to determine equilibrium shape. Important differences between factors determining the equilibrium surface formed by the bladder and those for a free liquid surface at low Bond number are the bladder bending stiffness and constant surface area. Some of the inputs to the static surface shape program are bladder thickness, Young's modulus for the bladder material, fluid density, tank acceleration, ullage pressure, tank radius, and ullage volume.

The bladder is represented by triangular membrane and bending elements, and the mercury is in the form of tetrahedrons, pentahedrons, and hexahedrons for formation of mass and stiffness matrices. Again the Rayleigh-Ritz technique of [5] is used to compute mode shapes and frequencies.

Equilibrium shapes computed by the program are shown in Fig. 4, along with two computer generated mode shape plots. The equilibrium shapes are for a tank

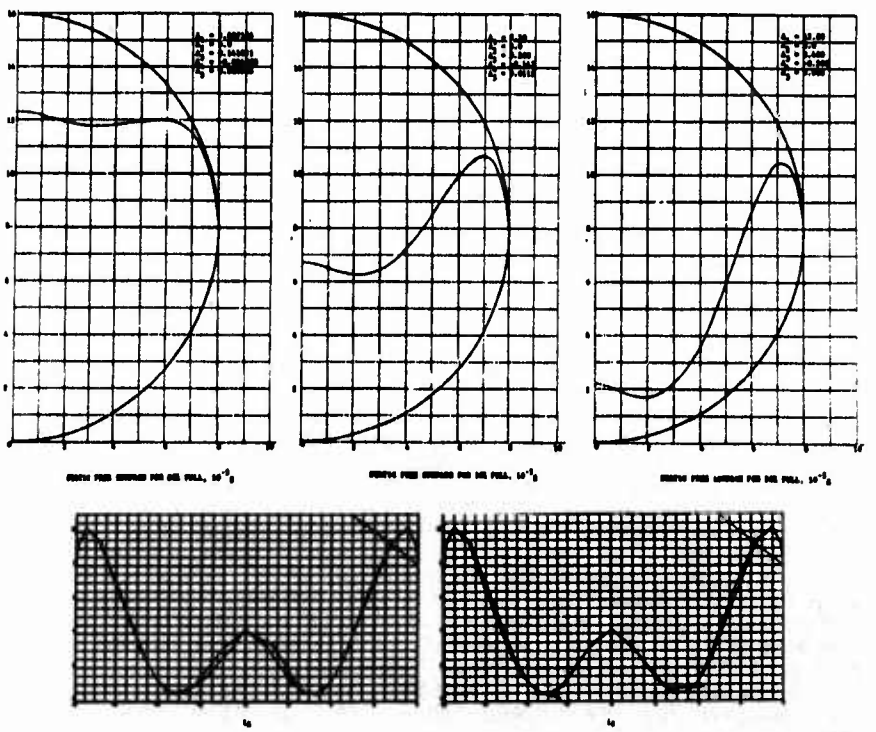

Fig. 4 Mercury/bladder configuration and modes in a spherical tank

acceleration of 10^{-5} g, whereas the modes are at 1 g. The calculations are for a bladder 0.06 inch thick in a 16 inch diameter tank.

The program is in FORTRAN for the UNIVAC 1108 computer, and details are given in [7].

LAMPS

LAMPS is a program to simulate the large amplitude slosh of propellant in a low gravity environment. The simulation is two-dimensional and constrains the fluid to move as a point mass on a surface generated by slowly rotating the tank (analytically) in high gravity and tracing the path the center of mass prescribes. In the simulation this surface is represented by piece-wise continuous elliptical segments. Updating of these segments is performed to insure that the center of mass does not deviate substantially from the constraint surface.

The input includes parameters for definition of tank geometry for a tank with a circular cylindrical middle section and ellipsoidal domed ends. The tank fill percentage, liquid properties, initial liquid location, and acceleration applied to the tank as a function of time are also input. Output includes fluid center of mass location, velocity, and acceleration. The forces and moments exerted on the tank by the liquid are also presented.

Drop tower experiments were conducted in conjunction with development of LAMPS. A tank partially filled with liquid was dropped from various initial orientations. Small horizontal and vertical accelerations were applied to the tank during the fall, and forces and moments exerted by the liquid were measured. The liquid motion was flow up one side of the tank, around the upper dome, and down the other side.

Correlations between LAMPS and experimental results showed similar trends of force versus time, but the calculated forces were generally too large. Figure 5 shows force results for 10% and 25% fills. FY is horizontal and FZ is vertical.

The analytical results were adjustable by the parameter χ which controls the dissipative force contribution. The time match is best with $\chi = 0$, and the forces are closer with χ near 0.01. Improved correlation is probably possible with further study of dissipative force behavior and with fluid center of mass trajectory modification (bulging of the liquid surface was observed in some tests and this is equivalent to movement of the point mass away from the constraint surface).

LAMPS is in FORTRAN for the UNIVAC 1108 computer. Details of the program and the supporting drop tower experiments are in [8].

Sticky Rubber Ball Analogy

This program was written specifically to investigate the effects of residual propellant on the reentry dynamics of the Space Shuttle external tank. The tank is a 75,000 lbm structure which will contain from 0 to 80,000 lbm of unused liquid oxygen and hydrogen when it separates from the Shuttle orbiter and reenters. The program solves the equations of motion for a sphere rolling or sliding on a curved surface. Propellant damping is introduced by rolling and sliding friction of the sphere on the surface. The curved surface shape is related to propellant oscillation frequency and possible center of mass locations. The sphere is given a degree of freedom normal to the slosh surface controlled by a spring, damper, and deflection limit, such that movement of the sphere toward the tank center greater than the limit allows the sphere to break away from the surface. Program results have not been checked by experiment.

The program is operational at NASA/MSFC, but no documentation is available for a bibliography. The language used is FORTRAN and the computer is a UNIVAC 1108.

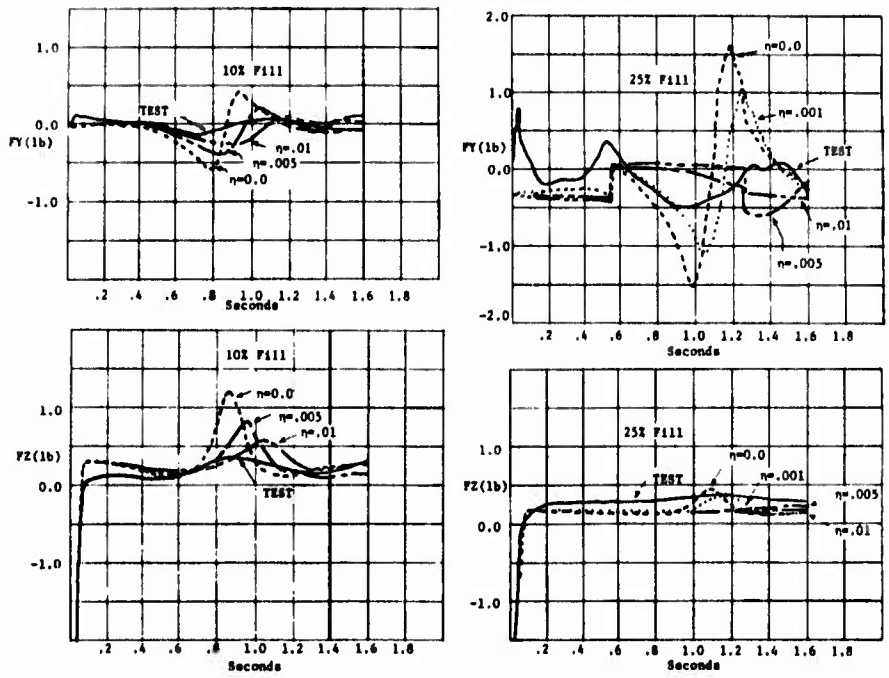

Fig. 5 Correlation with liquid forces during drop test

LHMAC2

LHMAC2 is a modification of an existing marker and cell (MAC) program. It was developed to describe large amplitude motion (either transient or periodic) of liquid in a tank. Motion in two-dimensional or axisymmetric tanks is analyzed. Modifications to the original MAC program include arbitrary curved flow surfaces, surface tension effects, and computation of forces and moments.

Rectangular meshes fixed relative to the tank are employed in writing the finite difference equations of the formulated problem. The velocity components of a fluid are specified at the boundaries of a cell. The pressure and all other quantities representing fluid are specified at the center of a cell. A set of marker particles is assigned for tracking the flow field, and these particles are displaced in accordance with the local velocities after completion of each computing cycle.

A specific problem analyzed with LHMAC2 was the determination of participation in separation dynamics by residual propellant. A change in the number of separation rockets used to separate the depleted S-IC stage from the remainder of the Saturn vehicle was accompanied by a detailed investigation of separation distance versus time. Deceleration of the dry S-IC is easily computed, but deceleration of the residual propellant takes place over an extended period of time, as the residuals flow from one end of the tank to the other. Marker particle plots of liquid oxygen moving toward the forward dome of the S-IC oxygen tank are shown in Fig. 6 for various times after separation and ignition of the separation retro-rockets. Also shown is a plot of force exerted by the oxygen tending to pull the tank bottom upward.

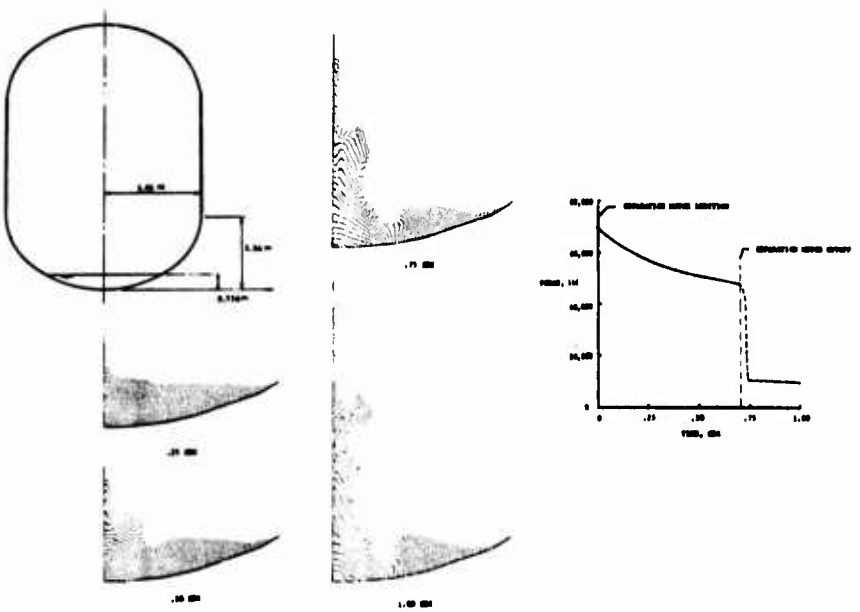

Fig. 6 Dynamics of residual propellant

No comparison between experimental and computed forces has been made. Comparisons of qualitative results, such as flow patterns and wave front shapes, have been made using films from drop tower tests and the results appear reasonable.

Instabilities occurred for low kinematic viscosity and for situations where a thin film of liquid was left at a tank wall. Comparison with test results would be required to determine which instabilities are real physical phenomena and which are characteristics of the computing scheme. Flow of a hypothetical, extremely high viscosity liquid from one end of its curved wall container to the other, including geyser formation at the latter tank end, was successfully simulated without instabilities.

The program is written in FORTRAN for the UNIVAC 1108 computer. Details of LHMAC2 and a limited three-dimensional version, LHMAC3, are given in [9] and [10].

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Optimum Design of Dynamic Mechanical Systems

A. Seireg

University of Wisconsin

INTRODUCTION

The analysis of dynamic mechanical systems has long been an important part of the mathematical, physical, and engineering disciplines. The equations describing the behavior of such systems are usually complex in nature and the influence of the different parameters on the system response is not generally expressible in an explicit manner. Accordingly, designers of such systems usually rely on intuitive or empirical guides for the selection of the main parameters. The system behavior is then checked, and appropriate modifications are introduced whenever necessary to insure the feasibility and safety of the design.

The development of high speed computers and related mathematical sciences opened new horizons for the designers of mechanical systems. In many cases computer-aided design is limited to analytically evaluating, comparing, and modifying different solutions which are obtained from experience and intuitive conception. Recent trends in computer utilization are moving toward completely automating the design process. The designer is expected to develop the procedure by which an optimal system is automatically determined. The control over the design process is exercised by the designer through the establishment of the governing criteria. Many of the judgment aspects of the design can be quantified so that they can be handled by computers. Thus the utilization of the computer can be extended beyond the mere analytical function of predicting the behavior of trial designs. Procedures are devised to allow the computer to continually modify a trial design until the optimum solution is found.

The interest in optimum design is as old as design itself. "Best" answers to many design problems were frequently sought after by exhaustive trials and tests. Orderly and efficient optimization gained a significant boost with the development of differential calculus, which provided an elegant tool for the determining the maxima and minima of differentiable relationships without the need for successive trials. Mathematical optimization procedures continued to develop. Variational techniques progressed from the classical work of Bernoulli [1], Euler [2], Lagrange [3], etc. to the more recent work of Pontryagin [4]. The gradient search algorithm was proposed by Cauchy [5] in 1847.

Recent advances in numerical methods gave rise to numerous useful algorithms. Among them are linear programming [6], dynamic programming [7], geometric programming [8], piecewise linear programming [9], direct search [10], gradient projection [11], and many others. Applications of programming and variational techniques in dynamic systems have been mainly in the area of optimal control, and many contributions can be found in the literature and in the numerous books on the subjects. ([12] to [18] for example.)

By contrast, relatively few applications to the optimum design of dynamic mechanical systems are available.

DESIGN STRATEGY

Rational design procedures require formulating the problem under consideration

in mathematical terms and developing the logic and decision steps necessary to facilitate the search for optimum solutions. It is difficult in many cases to express mathematically a general merit function by which a single optimum solution with the highest possible merit value can be attained. Mechanical systems in general are poorly structured since the search may not necessarily produce one optimum solution but several good solutions.

A design strategy for such systems can be briefly described in the following steps:

a. Defining the independent parameters of the problem.
b. Mathematically defining the constraints which are the limits imposed on each of the system parameters or any particular combination of them.
c. Stating the functional constraints which are the physical laws governing the behavior of the system under consideration.
d. Developing a suitable criterion for decision which mathematically expresses the objective of the designer. It is, therefore, possible that this criterion varies according to the goal set by the individual designer or his judgment.
e. Developing effective search techniques for systematic investigation of feasible solutions for the design with the highest possible merit value.

The first three steps deal with the description of the design domain which includes all feasible solutions. The boundaries of this domain are defined by the constraints. The last two steps define the objective and the search strategy to attain this objective.

This can be formulated as a classical problem in constrained maxima and minima as follows.

Find $$\bar{x} = (x_1, x_2, \ldots x_n)$$

which maximizes or minimizes

$$U = f(\bar{x})$$

subject to the constraints:

$$g_j(\bar{x}) = 0 \qquad j = 1 \to m$$

$$g_k(\bar{x}) \leq 0 \qquad k = (m+1) \to r$$

where $x_i (i=1,\ldots n)$ are the design variables and U is the objective function. The functions g_j are equality constraints and g_k are inequality constraints.

A necessary condition for the existence of a design problem is

$$n > m$$

Accordingly, many alternative solutions are theoretically possible by assigning arbitrary values to any (n-m) of the variables and solving the m equality conditions for the remaining (n-m) variables. A solution is feasible if it satisfies all inequality constraints. The design with the highest merit according to the stated objective is the optimal solution. The ultimate objective may include many factors, some of which may not be easily defined, and the designer may have to resort to limited objectives in order to obtain a practical solution. It is often desirable to transform the constrained minimization into an unconstrained problem by including the constraints in the objective function. Unconstrained optimization problems are usually more convenient to solve.

PROGRAMS FOR OPTIMIZATION OF VIBRATORY SYSTEMS

Several examples of computer programs illustrating the expanding interest in developing strategies for optimizing the design of vibratory systems are listed in the following. Since the majority of the activities in this field are

relatively recent, the listed programs illustrate methods of attack for specific systems and are, for the most part, not completely developed in a form suitable for general application. A brief description of each program is given with the appropriate references for detailed information. The list is by no means complete but nonetheless it identifies some of the published developments in this field. For example, several programs of interest are described in Reference [19] but are not yet available for distribution. A number of papers describing programs of interest are also given in the bibliography.

General Optimization Program for Inequality Constrained Problems

CONMIN

Date: January, 1972
Capability: General optimization program for inequality constrained problems; timeshare inter-active; designer oriented.
Method: The sequence of unconstrained minimizations technique with special features for finite differencing of derivatives and extrapolation for efficiency. Automatically determines a feasible design and then optimizes.
Limitations: 20 design variables, 100 inequality constraints of arbitrary form.
Input: Self explanatory inter-active timeshare mode. Formulas for objective function and constraints must be supplied. These can be in the form of a final expression computationally supported by coding of any degree of complexity.
Output: On-line progress reports during computation at user's option in real time and final design optimum including any level of detail specified by the user.
Language: FORTRAN
Hardware: Timeshare
Usage: The program has been applied successfully by a number of industrial users. There is a user's manual available.
Developers: F. Cinadr and R. Fox, Case Western Reserve University, Cleveland, Ohio. The program was developed privately by the authors.
Availability: The program is available for use through the CHI Corporation 1000 Cedar Avenue, Cleveland, Ohio 44106 on a royalty charge basis. The cost is based on a percentage of computer time used.

A Program for the Analysis and Optimal Design of Mechanical Systems

ADAMS (Automatic Dynamic Analysis of Mechanical Systems)

Date: November, 1973
Capability: The program gives descriptions of mechanical components. Specifically, it gives descriptions of: linkages-from the masses, inertial moments, and a guess of the initial generalized coordinates; joints-by their type and linkage adjacencies; springs and dashpots-by their force coefficients and their attachment points relative to the links; and force and displacement inputs. There is a three-dimensional design capability taking into account static analysis, large displacement (nonlinear) transient analysis, small displacement (linearized) analysis around a static solution or at any solution point in time. These analyses include vibrational analysis, modal analysis, modal sensitivity, and modal optimization.
Method: The algorithms used include modal formulation, a sparse matrix compiler for static and transient analysis and a sparse matrix interpreter for vibrational and modal analysis, gear implicit integration for transient solution, and Muller's method for modal analysis.
Usage: Typical applications of the program are for suspension system design of automotive, aircraft, and other transportation vehicles, three-dimensional dynamic analysis of vehicular motion, analysis of biomechanical systems (e. g., collision analysis), and high speed mechanical device design (e. g.,

fast printers, textile machines).
Language: FORTRAN IV, IBM 360/370 assembly; major revision required to run on a non-IBM machine.
Storage: Phase I- 400,000 bytes, Phase II- 350,000 bytes
Operating system requirements: ADAMS runs on Michigan Terminal System in batch and interactive modes.
Other requirements: Uses sparse matrix programs available separately (see report AFOSR-TR-72-1973).
Developers: N. Orlandea and D. A. Calahan Systems, Eng. Lab. Univ. of Michigan, Ann Arbor, 48104. Available from developers.

General Nonlinear Optimization Programs

POWELL, FLETCH and HOOK

Capability: These programs can be used to minimize an objective function of n variables subject to inequality and equality constraints. The number of variables and constraints can be specified by the user.
Method: These programs use Fiacco-McCormick's penalty function, including both inequality and equality constraints, and Powell's, Davidon-Fletcher-Powell's, and Hook and Jeeves' methods to minimize the unconstrained function, to obtain the optimal solution.
Input: The input requires defining the objective function and constraints within the program and inputting a starting point that satisfies the inequality constraints.
Output: The output includes the values of the objective function, constraints, and variables at each iteration.
Language: FORTRAN IV
Hardware: IBM 360 with at least 128K bytes of core.
Usage: The programs have received a limited amount of in house usage. No users manuals are available.
Developer: K. D. Willmert, Clarkson College, Potsdam, N. Y. 13676, unsponsored.
Availability: Available for the cost of reproduction from the developer.
Other Comments: Type of Program: These are three general nonlinear optimization programs using an interior penalty function and Powells', Davidon-Fletcher-Powell's, and Hook and Jeeves methods to minimize the resulting unconstrained function.

A Computer Augmented Design Engineering Technique

IOWA CADET

Date: Developed, 1968; has been continuously refined and augmented.
Capability: A battery of more than 150 subprograms that perform mechanical analysis, mathematical and statistical tasks upon which can be superposed optimization strategems such that optimal design configuration may be determined. Many of the subprograms implement procedures such as are found in Mechanical Engineering Design, J. Shigley, McGraw Hill, second edition. The documentation scheme is designed to encourage multiple contribution to some package in order that the capability grow in precisely the direction which improves its usefulness. The error messaging procedure is structured to discover logic errors in user's executive programming in order to facilitate use.
Input/output: Subprograms are called, and contain argument lists described in the documentation. Some subprograms are really an aggregate which appear to the user to be a single subprogram. The explicit form of the input and the output is under the user's control for greater flexibility. Convergence monitors and useful tabulations may be suppressed at the user's opti
Language: FORTRAN IV
Hardware: IBM 360
Developer: Charles R. Mischke, Alcoa Foundation Professor of Mechanical

Engineering, Iowa State University, Ames, Iowa, 50010.
Availability: Individual subprograms can be obtained for approximately $50 per program, including FORTRAN deck, test deck, listing of test run and documentation. Large subprograms are proportionately more expensive.

Optimum Design of Special Purpose Dynamic Absorber

Date: 1968
Capability: Many applications require the design of an absorber to perform optimally at a particular excitation frequency or within a narrow band of excitation frequencies. A program is developed for the selection of the optimal absorber spring and damper for any given operating frequencies and constraints on the amplitude of vibration of the absorber.
Language: FORTRAN
Developers: B. Hamad and A. Seireg
Availability: Program listing is available from A. Seireg, Mech. Eng. Dept., Univ. of Wisconsin, Madison, Wisconsin.

Optimum Design of a Three-Rotor Torsional System Subjected to an Impulse

Date: 1966
Capability: The objective of this program is to design the shafting connecting a prime mover (with moment of inertia I_1) to a machine (with moment of inertia I_3) through a coupling (with moment of inertia I_2). The machine in this case is subjected to an impulse causing a sudden change in velocity $\Delta\omega$. It is required to select the diameter of the two connecting shafts so that the total shaft volume is minimum. The allowable torsional stresses for the shafts are given as S_1 and S_2 psi respectively. Inertias may be added to the coupling (within a maximum value of $I_{2(max)}$) if a reduction in shaft size can be expected.
Inputs: I_1, I_3, I_{2min}, I_{2max}, L_1, L_2, S_1, S_2, $\Delta\omega$
Outputs: d_1, d_2, I_2
Language: FORTRAN
Developers: B. Hamad and A. Seireg
Availability: Program listing is available from A. Seireg, Dept. of Mech. Eng. Univ. of Wisc., Madison at reproduction cost.

Optimum Design of a Three-Rotor Torsional System Subjected to Step Load on One Rotor

Date: 1966
Capability: The objective of this program is to design the shafting connecting a prime mover (with moment of inertia I_1) to a machine (with moment of inertia I_3) through a coupling (with moment of inertia I_2). The machine in this case is subjected to a step load T acting on I_3. It is required to select the diameter of the two connecting shafts so that the total shaft volume is minimum. The allowable torsional stresses for the shafts are given as S_1 and S_2 psi respectively. Inertias may be added to the coupling (within a maximum value of $I_{2(max)}$) if a reduction in shaft size can be expected.
Inputs: I_1, I_3, I_{2min}, I_{2max}, L_1, L_2, S_1, S_2, T
Outputs: d_1, d_2, I_2
Language: FORTRAN
Developers: B. Hamad and A. Seireg
Availability: Program listing is available from A. Seireg, Dept. of Mech. Engr., Univ. of Wis., Madison at reproduction cost.

Optimum Design of a Three-Rotor System Subjected to A Sinusoidal Input on One of the Rotors

Date: 1966
Capability: The objective of this program is to design the shafting connecting a prime mover (with moment of inertia I_1) to a machine (with moment of inertia I_3) through a coupling (with moment of inertia I_2). The machine in this case is subjected to an impulse causing a sudden change in velocity $\Delta\omega$. It is required to select the diameter of the two connecting shafts so that the total shaft volume is minimal. The allowable torsional stresses for the shafts are given as S_1 and S_2 psi respectively. Inertias may be added to the coupling (within a maximum value of $I_{2(max)}$) if a reduction in shaft size can be expected.
Inputs: I_1, I_3, I_{2min}, I_{2max}, L_1, L_2, S_1, S_2, T and ω
Outputs: d_1, d_2, I_2
Language: FORTRAN
Developers: B. Hamad and A. Seireg
Availability: Program listing is available from A. Seireg, Dept. of Mech. Eng., Univ. of Wis., Madison at reproduction cost.

Optimum Design of a Three-Rotor System with Rayleigh Damping Subjected to Sinusoidal Excitation on One of the Rotors

Date: 1966
Capability: The objective of this program is to design the shafting connecting a prime mover (with moment of inertia I_1) to a machine (with moment of inertia I_3) through a coupling (with moment of inertia I_2). The machine in this case is subjected to a sinusoidal excitation T sinωt. It is required to select the diameter of the two connecting shafts so that the total shaft volume is minimum. The allowable torsional stresses for the shafts are given as S_1 and S_2 psi respectively. Inertias may be added to the coupling (within a maximum value of $I_{2(max)}$) if a reduction in shaft size can be expected. For convenience, the damping is assumed to be of the Rayleigh type

$$\left(\frac{C_1}{I_1} = \frac{C_2}{I_2} = \frac{C_3}{I_3} = R_1 \quad , \quad \frac{C_{12}}{K_{12}} = \frac{C_{23}}{K_{23}} = R_2\right)$$

Inputs: I_1, I_3, I_{2min}, I_{2max}, L_1, L_2, S_1, S_2, T, and ω, R_1, R_2
Outputs: d_1, d_2, I_2
Language: FORTRAN
Developers: B. Hamad and A. Seireg
Availability: Program listing is available from A. Seireg, Dept. of Mech. Eng., Univ. of Wis., Madison at reproduction cost.

Optimum Design of a Two-Mass Lateral Vibration System

Date: 1968
Capability: The program deals with the design of a two mass flexible rotor supported on two bearings which undergoes lateral vibrations because of the eccentricity of one of the masses. The design objective is to select the shaft diameters which minimize the weight of the shaft for a given allowable stress.
Language: FORTRAN
Developer: B. Hamad and A. Seireg
Availability: Program listing is available from A. Seireg, Dept. of Mech. Eng., Univ. of Wis., Madison.

Computer Programs for Automated Minimum Weight Structural Design

Capability: The objective of the computer programs is the automatic determination of the least weight of a given structural configuration under the action of a multiplicity of external loading configurations under the action of a multiplicity of external loading and thermal conditions, and subjected to limitations on the stresses and deflections of the structure. The possibility of considering overall instability as a criterion is also provided for one class of structures. The range of structures which may be optimized using these programs is only limited by the extent to which a given structure may be satisfactorily idealized using the structural elements available: axial force member, triangular plate in-plane stress, axial force-twist-flexural member, axial force twist-flexural member with instability, rectangular shear panel, rectangular plate in bending in two dimensions, axial force-twist flexural member in two dimensions.

AFFDL-TR-66-180 documents the development of new computer programs for the automated minimum weight design of aerospace structures. Two programs are discussed in detail, an intermediate capacity linear merit function program and a large scale nonlinear program. The intermediate capacity program can be used for the optimization of major structural components of up to approximately 170 degrees of freedom with fixed configuration. The large scale program is capable of handling structures with up to 450 degrees of freedom and includes geometric variables. Detailed descriptions on the preparation of input data are included. Examples are given of the applications of the programs to a wide variety of structures including major airframe components, indicating the weight savings possible through application of the optimization techniques.

Language: FORTRAN IV

Availability: FDL-TDR-64-141 (AD 611 310) AFFDL-TR-66-180 (AD 804 602), Wright-Patterson AFB, Ohio.

Design of Prismatic Bar with Variable Cross Section for Force Amplification or Attenuation in Longitudinal Impact

Capability: The program automatically selects the configuration of stepped prismatic bars placed between a hammer and an anvil to produce maximum force amplification or attenuation. The program incorporates a relatively inexpensive computer scheme and a special search procedure to develop the optimum design of such bars. The program includes provisions for changing segment impedance through material selection.

Language: FORTRAN

Developer: D. Wallace and A. Seireg

Availability: From A. Seireg, Dept. of Mech. Eng., Univ. of Wis., Madison at reproduction cost.

Structural Optimization for Aeroelastic Requirements

SOAR

Date: 1972

Capability: This program will automatically size structural members of a finite element generated lifting surface design to achieve minimum weight without violation of a fixed flutter speed constraint. Zoutendijk's method of feasible directions is utilized to determine resizing steps. Since this method requires gradient information, a method due to Van de Vooren was implemented to perform this calculation.

Limitations: Can be used in preliminary design.

Language: FORTRAN

Hardware: CDC 6600

Developer: L. B. Gwin and S. C. McIntosh, Jr., Stanford University
Availability: R. F. Taylor, Capt. S. M. Batill, and R. A. Andries, Air Force Flight Dynamics Laboratory (FYS), Wright-Patterson AFB, Dayton, Ohio, 45433

WIDOWAC - Wing Design Optimization with Aeroelastic Constraints

Date: 1974
Capability: A pilot computer program developed for the design of minimim mass structures under flutter, strength, and minimum gauge constraints. The wing structure is idealized by finite elements, flutter conditions may be both subsonic and supersonic and mathematical programming methods are used for the optimization.
Limitations: Symmetric wing airfoils, small (∿300 degrees of freedom) finite element models.
Language: FORTRAN
Hardware: CDC 6600, IBM 370/168
Developer: R. T. Haftka
Documentation: NASA TM X - 3071 (184 pages)
Availability: Dr. J. H. Starnes, Jr., NASA Langley Research Center, Hampton, Virginia, 23665.

Optimization of Structures with Constraints on Dynamic and Frequency Characteristics

Capability: The program deals with minimum weight elastic structure designs under dynamic loads with certain constraints imposed on stress, displacement and natural oscillation frequencies. A random search is used. The procedure is applied to the synthesis of a perimeter-hinged, three-layer plate.
Developer: Yu. M. Pochtman
Availability: NASA-TT-F-14540 (Sept 1972).

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Mechanical and Thermal Shock Analysis

Philip Underwood

Lockheed Palo Alto Research Laboratory

INTRODUCTION

This chapter deals with the nuts and bolts of mechanical and thermal shock analysis programs; the lumped parameter programs and shock spectra programs. These analysis techniques were employed by vibrations engineers many years before the computer had any impact on vibration response analysis and they still form the basic design analysis tools for shock environments. For this reason, many of the concepts discussed are well founded and available in standard engineering texts, hence the discussion sections are quite brief and contain only an overview of the theory. Of course the more modern developments in time integrators for nonlinear systems are presented, as these new methods are finding their way into the well maintained computer programs. The sections on computer programs include only those programs that are readily available for a nominal cost (less than $300 with a couple of exceptions). Hopefully, currently used programs are included and there are not too many old and out of date programs. In general, these programs are small or moderate sized and provide efficient computations for good quick design studies.

LUMPED PARAMETER STRUCTURAL DYNAMIC ANALYSIS

Lumped parameter (or lumped mass) structural dynamic computer programs are used by dynamic analysis engineers who view the world as composed of masses connected by weightless springs and dampers. The structure's mass is divided into discrete lumps, each of which is coupled to others by the springs and dampers to produce a dynamic model of the structure. The resulting equations of motion can be written as

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + [K]\{X\} = \{F(t)\}$$

where X is the system displacement, dots represent time derivatives; M, C, K are the mass, damping, and stiffness matrices, respectively, and F(t) is the transient forcing function. The derivation of this equation is included in any vibration reference book that covers multi-degree of freedom structural or mechanical systems, [1, 2, 3], for example.

The main uses for these lumped parameter programs are for the transient or steady state solution of structures that can be modeled by simple torsion, bending or stretching elements. This encompasses a large range of structures, and the complexity of the model is only limited by the engineer's ability. One advantage of these programs is that nonproportional damping is quite easy to include. Many of the programs also include nonlinear stiffness elements. The programs may require a fair amount of thought and time to prepare the input, as a very general model of a structure may involve some very complex coupling of the mass elements. Some of the programs do provide element libraries for beams, shells, and solids that ease the modeling considerably.

The art or science of properly modeling a structure or mechanical system for a lumped parameter dynamic analysis is usually learned by trial and error, but there are some basic guidelines that may get one started along the correct path. For modeling a continuum structure (beam, shell) the mass and stiffness elements should generally be uniform to properly characterize wave propagation in the structure. The fineness or coarseness of the discretization is of course a function of the frequencies one wishes to reproduce. A good example of discrete modeling techniques for an array of linear oscillators is given by Morse and Ingard [4]. For mechanical systems (packaging, machinery) the main idea is to try to build the model of components of relatively similar frequencies. This is not always possible, and reality should always be considered before some fancy modeling is tried, as you don't want to eliminate a possible significant response from your model. But if the model is composed mostly of low frequency elements with one or two high frequency elements, many of the programs will become expensive to run, as the time integrator will take small steps to reproduce the high frequency response. If all the forcing functions are applied to the low frequency portion of the system, the high frequency modeling is not necessary; it should be treated as rigid.

The lumped parameter structural dynamic computer program is the basic design analysis tool of the structural dynamic engineer. During design the engineer can check suspected critical areas on a very quick computer run. The spring or damper elements are easily changed for parametric studies. Indeed, for many structures and most mechanical systems the lumped parameter transient response computer program is all the analyst needs for a complete dynamic design analysis. If not, they will usually provide an indication of critical areas that can be studied in more detail with special purpose programs.

FORCING FUNCTIONS

The kind of a forcing function that is called a shock is not explicitly defined; but in general, any forcing function that produces a transient response that is significant in magnitude compared with the steady state response can be called a shock loading. For structural dynamic analysis these shock loadings are characterized by sudden changes in surface, displacement, or thermal loads.

The first two loads are easily described; for example, surface loads may be due to a blast wave or a sonic boom disturbance; displacement loads may be due to bumps in a road or earthquakes. All lumped parameter programs can easily handle these mechanical loads. Thermal loads are also easily described as they are due to sudden changes in temperature that force a structure to find a new equilibrium position; but implementation of thermal loads is seldom included in the computer programs. Dynamic thermal loads are becoming more important in aerospace and nuclear power applications, so more programs should have provisions for thermal loads in the future. At present there are only a few shell and general structures programs that have thermal forcing function capabilities. There is also a method for using the lumped parameter programs for transient thermal analysis by computing the thermal loads and using them as applied forces. This technique is described below.

Figure 1 illustrates a typical section of a simple lumped parameter model

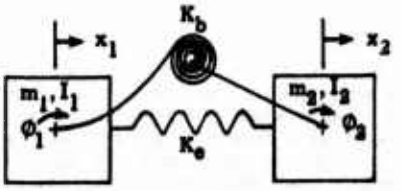

Fig. 1 Typical lumped parameter model

with translational and rotational degrees of freedom. The translational force due to a thermal transient can be expressed as

$$F_e(t) = K_e \alpha \left(T_o - T(t)\right) \ell$$

where α is the thermal expansion coefficient, T_o the reference temperature, $T(t)$ the temperature at time t and ℓ is the length of the discretized element of the structure. Similarly, an applied moment can be calculated as

$$M(t) = K_b \frac{\alpha}{h} \int_h \left(T_o - T(t)\right) z \, dz$$

where h is the thickness and z the thickness coordinate. This is admittedly a crude technique and a brief discussion, but you can make some surprisingly accurate estimations of thermal response using this technique. Remember the actual forces in the springs become the forces due to kinematics of the structure (computed by the program) plus the thermal forces which must be superimposed on the forces due to kinematics. As an illustration of the reality of thermal load as applied to structural dynamics, Fig. 2 illustrates the computed and measured tip deflection for a cantilever beam subjected to a sudden increase in temperature along its length, and varying exponentially through the thickness [5].

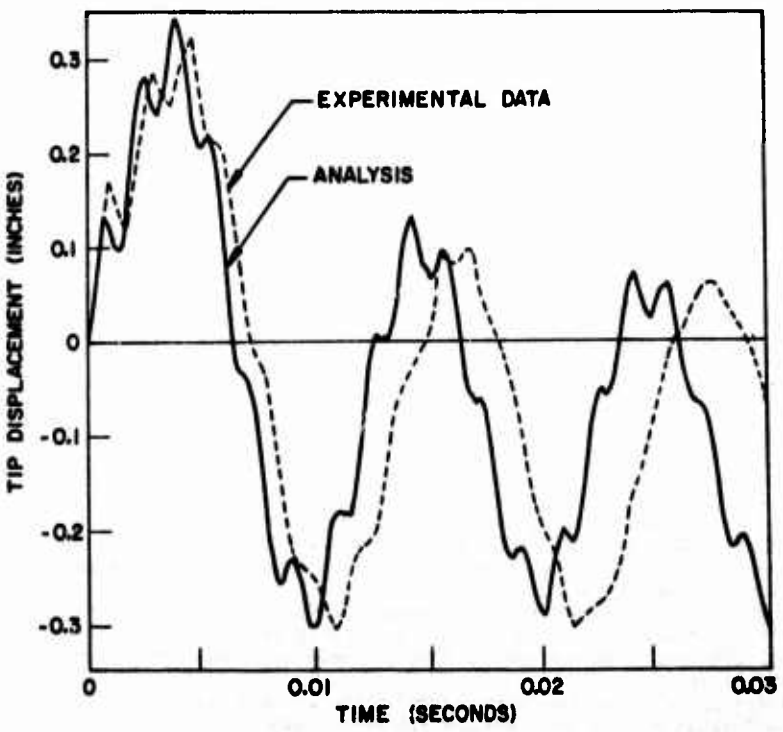

Fig. 2 Tip response of a suddenly heated cantilever beam

SHOCK SPECTRA

The concept of a shock spectrum is a widely used design and test analytical tool for the engineer. The first user of the shock spectrum concept appears to have been Biot [6], who felt the effect of the input transient force was a more important engineering criteria than the actual shape of the input force itself. The shock spectrum of a given forcing function is a plot of some response quantity, displacement, velocity, and/or acceleration, that represents the effect of the forcing function on a single degree of freedom undamped oscillator versus the dimensional or nondimensional frequency or period of the oscillator. When the input force has a definable time characteristic, the nondimensional form is usually used. The shock spectrum is broken into two regions, primary and residual response. The primary response applies to the response quantity obtained while the forcing function is acting, and the residual response is the response quantity obtained after the forcing function has ceased, the "ringing" portion of the response. A third response often plotted is the maximax response which is the envelope of the maximum of the primary and residual response. Two reference books [7, 8] that provide many more details of shock spectrum will provide the interested reader with more insight than possible in this brief discussion; reference [7] has many good additional references pertaining to applications. References [7] and [8] also consider shock spectrum for damped oscillators and nonlinear springs. In addition, Hieber et al. [9] provide a current review of shock response spectrum.

The two uses of shock spectrum are in design and test applications. In design, the shock response spectrum provides a quick method of determining the maximum dynamic loads in a structure. In testing applications, one can test equipment with easily produced pulses by using an equivalent shock response spectrum produced by an actual input transient and/or determine the shock spectrum for various critical components by recording base accelerations.

Computer programs to determine the shock response spectrum of a given input transient basically use two different methods. One is direct numerical integration (see the section on time integrators) of the response of an array of discrete frequency oscillators to the forcing function. Techniques used in writing efficient shock spectrum programs can be found in references [10, 11, 12]. The second method used for generating shock spectrum is the fast Fourier transform (FFT) [13, 14]. The FFT can also be used to synthesize the response spectrum to reproduce the input transient, so one can work back and forth in the frequency and time domain if desired. And although they are not digital computer programs (a deck of cards), digital and analog computer hardware can be obtained to provide shock response spectrum of test data; see any issue of Sound and Vibration.

TIME INTEGRATORS

In general, transient response of structures relies on the direct numerical time-wise integration of the equations of motion. In lumped parameter and shock spectra models, characteristic frequencies can encompass a large range and elements can be nonlinear, requiring a sophisticated and economical integration technique. Some of the most intensive investigation into the characteristics of time integration schemes is currently in progress mainly due to the surging interest in nonlinear structural response and the unpredictable behavior of linear system methods when applied to nonlinear systems. A brief review of the more popular methods and the current research follows; hopefully this will enable a user to wisely select a computer program to match his requirements.

Basically, all the time-integration techniques fall into the category of linear multistep methods that have the following form

$$\sum_{i=0}^{\blacksquare} \alpha_i X_{n+1-i} = \Delta t \sum_{i=0}^{\blacksquare} \beta_i \dot{X}_{n+1-i} \tag{1}$$

or

$$\sum_{i=0}^{\blacksquare} \gamma_i X_{n+1-i} = \Delta t^2 \sum_{i=0}^{\blacksquare} \delta_i \ddot{X}_{n+1-i} \tag{2}$$

where α_i, β_i, γ_i and δ_i are coefficients associated with a particular method. If $\beta_0 \neq 0$, $\delta_0 \neq 0$ the method is implicit, i.e. the value to be computed (left hand side) also appears implicitly on the right hand side of the equation. Otherwise the method is explicit. Equation (1) represents the stiffly stable methods of time integration, see Gear [15], Jensen [16], and Park [17]. Equation (2) represents the classical multistep methods, see Newmark [18], Houbolt [12], and Wilson [20].

Whether to use an explicit or implicit method is a difficult question, but there are some guidelines as long as they are not taken too seriously. The explicit methods are easier to implement and usually require less storage. The biggest saving in implementation is that no matrix inversion or decomposition is required for an explicit technique. Hence you can discretise your structure anyway you wish and not worry about band width and connectivity of the stiffness matrix. The major disadvantage is that the time step for stable computations is inversely proportional to the highest frequency of the system and this can result in a very small time step. This may be uneconomical for cases in which the response mode of interest is of a much lower frequency. For this reason the implicit procedures are many times favored, i.e. they can be constructed to attenuate high frequencies while retaining low frequency accuracy and using a larger time step. These methods require matrix decomposition and the solution of a system of coupled equations at every time step, hence the number of computations per time step can become quite large. If one needs the high frequency response to be well defined an explicit method is probably better, but if one needs the lower frequency response with all the high frequency response the implicit methods are better.

For linear systems the central difference operator [21] is the most popular explicit method and the trapezoidal method is probably the most accurate and best of the implicit methods. For nonlinear systems the issues become somewhat clouded. The central difference method is still the most popular explicit method, but the controversy over the best or even a good implicit method is still raging. The majority of the research on time integration methods for nonlinear structural response has been done by the numerical experiment method, [22, 23, 24, 25]. These studies have not produced unanimous results; but the Houbolt method seems to be highly favored. Recently, Park [17, 26, 27] has done some very fundamental research on the characteristics of time integrators as applied to nonlinear structural response. This work has resulted in a new stiffly stable time integrator [17] that shows much promise, and probably even more important, a method for evaluating the integrators for nonlinear response has been developed. This work [26, 27] has demonstrated the importance of historical derivatives that has heretofore been neglected by previous studies.

Though this section has been sketchy, hopefully the flavor of selecting a time integrator has been presented. For more detailed information the chapter in this book by E. Stein should be consulted plus the more recent review type papers [22, 23, 24, 25, 27]. Also Dahlquist and Björck [28] and Gear [15] provide a goodly amount of background material.

LUMPED PARAMETER COMPUTER PROGRAMS

The list of lumped parameter computer programs in Table 1 is by no means complete; they represent programs that are available and/or unusual. It is unfortunate that some very good lumped parameter codes are not generally available, usually for proprietary reasons or the programs are too inhouse oriented for general usage. Most of the generally available lumped parameter codes,

Table 1 Lumped Parameter Computer Programs

Program	DOF (or Masses)	Time Integrator	Damping	Stiffness	Computer	Comments
INT-DDAM	48 DOF 60 joints	Normal mode (no integrator specified)	?	1) Plane Truss 2) Space Truss 3) Plane Frame Plane or normal load 4) Space Frame	GE	Set up for interactive timesharing operation
ISIP	50 masses	1) Normal mode for linear 2) Unnamed direct for nonlinear	Viscous	50 elements	CDC	1) Primarily for shock isolation 2) Synthesis of shock spectra 3) Fourier analysis (uses WAVSYN)
FUGIT1 (M0678) and (M0266)	30 ?	Modified Euler method	3 types	30 x 30 flexibility or stiffness non-linear	CDC	1) FUGIT1 is described 2) M0266 is another transient response code also available from Argonne code center
OPSHK	197 nodes	Adams-Moulton	Coulomb viscous	Linear 100 isolators 100 isolator types	CDC	Primarily for shock isolation studies
SHS (Shell-shock)	1000 DOF (depends on computer structure)	Newmark (implicit) Taylor series (explicit)	Viscous complex	Linear or non-linear beams, shell, solid elements plus general stiffness	CDC	1) 3-D axisymmetric structures are reduced to 2-D by Fourier decomposition 2) Program optimizes input to provide an efficient solution
SHOCK	100 masses with 1 or 2 DOF	Runge - Kutta or Newmark	Coulomb viscous	200 elements linear and nonlinear	CDC IBM	Has associated postprocessor GRABBAG for steady state and eigenproblems
TDYNE	80 masses	Modified Jennings	1) $\alpha[M]+\beta[K]$ 2) Modal 3) Structural	120 springs non-linear ground to structure coupling	CDC	Primarily for tower structures
XTABS (and TABS)	--	--	--	Building structures, frame, shear panels and floor diaphragm	CDC	A statics code and for earthquake response from acceleration spectrum

especially those with nonlinear elements, could be upgraded by the inclusio' of a better time integration technique. Now that the author has alienated all the computer program developers, it should be stated that these codes are still very versatile and usable and for the small number of degrees of freedom they usually have, are very efficient. A well modeled lumped parameter analysis can usually provide a wealth of analysis for a mere fraction of the cost of a huge general purpose program.

Although the author is not thoroughly familiar with all the programs listed, SHOCK, SHS, and TABS (XTABS) are probably the most modern. In addition NISEE (National Information Service - Earthquake Engineering), see (h) below, provides many computer programs of applicability to earthquake motion. A listing of the programs along with the developers and/or availability is given below:

a) INT-DDAM (An Interactive Time-Sharing Compu'er Program for the Dynamic Design Analysis Method)
   Developer: Richard G. Gauthier
   Electric Boat Division
   General Dynamics Corp.
   Groton, Connecticut
   Available from developer or
   1) Supships 3, USN, Brooklyn, New York 11251
   2) GE - Timesharing System, Teaneck, New Jersey 07666

b) ISIP (System of Shock Isolation Programs)
   Developer: Ralph M. Parsons Co.
   Los Angeles, California
   Available: U. S. Army Corps of Engineers
   Huntsville Division
   P. O. Box 1600 West Station
   Huntsville, Alabama 35805
   Attn: R. Bradshaw

c) MO266 and MO678 (FUGIT1)
   Developer: C. M. Friedrich (FUGIT1)
   Bettis Atomic Power Laboratory
   Westinghouse
   Available: Argonne Code Center
   Argonne National Laboratory
   9700 South Cass Avenue
   Argonne, Illinois 60439

d) OPSHK
   Developer: Dr. David Platus/Mr. Raymond Curtis
   Mechanics Research Incorporated
   Los Angeles, California
   Available: Air Force Weapons Laboratory
   (DEV-S)
   Kirtland AFB, New Mexico 87117

e) SHS (Shell-Shock)
   Developer: Joe E. Grant
   Applied Mechanics Division 8113
   Sandia Laboratories
   Livermore, California 94550
   Available: Developer
   Argonne Code Center
   Argonne National Laboratory
   9700 South Cass Avenue
   Argonne, Illinois 60439

f) SHOCK
   Developer: V. K. Gabrielson/C. S. Hoyle/R. T. Reese
   Sandia Laboratories
   Livermore, California 94550

Available: Developers
Argonne Code Center
Argonne National Laboratory
9700 South Cass Avenue
Argonne, Illinois 60439

g) TDYNE
Developer: R. P. Kennedy
Holmes and Narver, Inc.
400 Orangethorpe Avenue
Anaheim, California 92801
Available: Developer

h) XTABS (TABS) Three-dimensional analysis of building systems
Developer: (TABS)
E. L. Wilson and H. H. Dovey
University of California
Berkeley, California 94720

(XTABS modified version of TABS)
Sexton, Fitzgerald & Kaplan, Engineers
San Francisco, California
Available: NISEE/Computer Applications
Davis Hall
University of California
Berkeley, California 94720

SHOCK SPECTRUM COMPUTER PROGRAMS

The computer programs listed in this section are not the only shock spectrum computer programs available. The list is to provide a sampling of relatively easy to obtain programs and/or programs with unusual features that may make the effort to obtain them worthwhile. No attempt has been made to evaluate these programs as to run time, cost, accuracy, etc., except that these are programs that are in use. Table 2 lists the programs and their basic features. As you can see, wave synthesis programs have also been included, as many times they go hand-in-hand with a shock spectra analysis. Some of the more versatile programs include features such as multi-d.o.f.s and nonlinear stiffness characteristics.

In addition to the table the developer and/or availability of the various computer programs are given below.

a) NPO-10528 (Shock Spectrum Analysis Program)
Developer: Jet Propulsion Laboratory
Available: COSMIC
112 Barrow Hall
University of Georgia
Athens, Georgia 30601

b) PSZQGN (Artificial Generation of Earthquake Accelerograms)
Developer: Patricio Ruiz & Joseph Penzien
University of California
Berkeley, California
Available: NISEE/Computer Applications
Davis Hall
University of California
Berkeley, California 94720

c) SHOCK 3 (Shock Spectrum Analysis Program)
Developer: Jack D. McBryde
Lockheed Electronics Co.
16811 El Camino Real
Houston, Texas 77058
Available: NASA - Johnson Space Center
Houston, Texas

Table 2 Shock Spectra Computer Programs

Program	Computation	Method	Damping	Nonlinearity	Computer	Input
NPO-10528	Acceleration response	Recursive filtering	?	?	IBM	Acceleration
PSEQGN	Ground acceleration, velocity and displacement	Provides a simulation of an earthquake			CDC	Intensity, duration, etc.
SHOCK 3	Shock spectra	FFT	?	?	UNIVAC	Acceleration
SHOCSPEC	Shock spectra Fourier spectra	Fourier Transform	Input data	?	?	Acceleration
SPECANAL	Response spectra, power spectral density	Direct integration	Yes	No	CDC	Ground motion velocity or acceleration
SPECTR	Response spectra	Direct integration	Yes	No	CDC	Acceleration
STEP	2 d.o.f. shock response	Direct integration	Input data	Yes	CDC	Acceleration or velocity histories
SPECEQ/ SPECUQ	Acceleration, velocity and displacement response spectra	Exact analytical solution to linear segments of input	Yes	No	CDC	Acceleration history
VIBANA	Statistical evaluation and wave analysis	?	--	--	IBM	Time history
WAVSYN & MWAVSYN	Time history	Iterative	None	No	CDC	Spectrum Frequencies and response amplitudes

d) SHOCSPEC (Shock Spectra Analysis of a Single Degree of Freedom System)
Developer: TRW Systems Group
& One Space Park
Available: Redondo Beach, California 90278

e) SPECANAL
Developer: R. P. Kennedy
& Holmes & Narver, Inc.
Available: 400 E. Orangethorpe Avenue
Anaheim, California 92801

f) SPECTR (Spectra Response Analysis)
Developer: Dames and Moore
San Francisco, California
Available: NISEE/Computer Applications
Davis Hall
University of California
Berkeley, California 94720

g) STEP
Developer: Dr. Howard F. Korman
& TRW Systems Group
Available: One Space Park
Redondo Beach, California 90278

h) SPECEQ/SPECUQ
Developer: N. C. Nigam & P. C. Jennings
California Institute of Technology
Pasadena, California
Available: NISEE/Computer Applications
Davis Hall
University of California
Berkeley, California 94720

i) VIBANA
Available: COSMIC
112 Barrow Hall
University of Georgia
Athens, Georgia 30601

j) WAVSYN/MWAVSYN
Developers: R. M. Parsons Co.
Los Angeles, California

Sperry Space Support Division
Sperry Rand Corporation
Huntsville, Alabama 35805
Available: U. S. Army Corps of Engineers
Huntsville Division
Huntsville, Alabama 35805

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Random Vibration of Structures

C. Sundararajan

The Lummus Company Canada Limited

INTRODUCTION

Probabilistic dynamics, or random vibration, has become a distinct and well-established branch of structural dynamics. The main impetus for the application of the random vibration theory to practical structural dynamics problems arose first in the aerospace industry, because many aerospace structures are subjected to severe nondeterministic (stochastic) loads such as forces due to turbulent boundary layers, jet exhausts, etc. Today, many structures in the areas of nuclear power generation and offshore oil exploration and production are also designed on the basis of random vibration analysis, because of the stringent safety requirements and severe dynamic loading conditions. Structures such as suspension bridges, suspended roofs and tall buildings are also analysed for wind and earthquake loadings by probabilistic methods.

Because of the size and complexities of modern structures, it is not possible to obtain exact, or simple approximate solutions in most cases. The high-speed digital computer, which has proven to be a powerful tool in the static and deterministic dynamic analysis of structures, has been employed for random vibration analysis also.

PROGRAM SUMMARIES

A brief summary of fifteen programs available from private and public institutions is given in this section. The summaries are based on the information obtained from the developers and users.

NASTRAN [1, 2]

Capability: Random response analysis of linearly elastic structures subjected to stationary random loads. There is no restriction on the type of the structure.
Method: Finite element method, and generalised harmonic analysis.
Language: FORTRAN IV
Hardware: IBM 7044, 7094, 360, 370, CDC 6000 series, 7600 and UNIVAC 1100 series. A minimum core of 50,000 single precision words is required.
Comments: NASTRAN is probably the largest general purpose structural analysis program today. It can perform a variety of static, stability, dynamic and hydroelastic analysis, including the random vibration analysis.
Developers: The program was developed for NASA by the Computer Science Corporation, with MacNeal-Schwendler Corporation, Martin Baltimore, and Bell Aerospace Company as subcontractors.
Availability: NASTRAN can be used from the CDC and other data centers throughout the U.S.A. and Canada by paying a certain percentage of the computer

charges as royalty. The program can also be leased, and installed in the user's computer system. It is available from

1) COSMIC
   112 Barrows Hall
   University of Georgia
   Athens, Georgia 30601
2) Naval Ship Research and Development Center
   Bethesda, Maryland 20034
3) MacNeal-Schwendler Corporation
   7422 N. Figueroa Street
   Los Angeles, California 90041
4) McDonnell Douglas Automation Company
   P. O. Box 516
   St. Louis, Missouri

Cost: $750.00 per month (lease) from McNeal-Schwendler Corporation.

STARDYNE [3, 4]

Capability: Random response analysis of any linearly elastic structure subjected to stationary random forces.
Method: Finite element method, and the method of normal modes.
Language: FORTRAN IV
Hardware: CDC 6600, 7600
Comments: STARDYNE is a general purpose structural analysis program with static, stability, and dynamic analysis capabilities.
Developer: Mechanics Research Incorporated
9841 Airport Boulevard
Los Angeles, California 90045
Availability: STARDYNE can be used from the CDC and other data centers on a royalty basis.

ELAS [5]

Capability: Random response analysis of initially stressed, linearly elastic structures subjected to random excitations.
Method: Finite element method.
Language: FORTRAN IV
Comments: ELAS 75 is a general purpose program for the static analysis of structures. A few new subroutines have been added to it, to handle random vibration problems.
Availability: Listing or deck can be obtained from its developer
Professor S. Utku
Department of Civil Engineering
Duke University
Durham, North Carolina 27708
Cost: $1000.00.

PURDUE PROGRAM [6, 7]

Capability: Estimation of the mean square response of linearly elastic systems subjected to weakly stationary random forces. This program is suitable for medium sized structures, of about 500 degrees of freedom.
Method: An approximate method developed by Kayser and Bogdanoff [6, 7]. It is based on the relaxation technique, and the accuracy of the solution can be improved with successive iterations.
Language: FORTRAN IV
Hardware: Large as well as minicomputers. A minicomputer with a 16K memory will solve a system of about 500 degrees of freedom.
Comments: The mass, damping, and stiffness matrices are to be calculated outside the program, and input into it.

Availability: Listing or deck is available from its developer
Professor J. L. Bogdanoff
School of Aeronautics, Astronautics and Engineering Sciences
Purdue University
West Lafayette, Indiana 47907

RANVIB

Capability: Random response analysis of structures, such as aircraft fuselage walls, due to attached turbulent boundary layers.
Method: Finite element method. A variety of beam and plate elements are used.
Language: FORTRAN
Hardware: CDC 6600
Comments: The program is claimed to be efficient and economical for medium-to-small structures. For a CDC 6600 computer, the size of the stiffness/mass matrix is limited to 300th order.
Availability: The program is available from its developer
Dr. F. L. Gloyna
Boeing Commercial Airplane Company
P. O. Box 3707
Seattle, Washington 98124

RSPC

Capability: Random response analysis of structures subjected to multiple random loadings. The major options for structural representations are conical sections, general shells and wing buffet.
Language: FORTRAN
Hardware: CDC 6600. A 56100 (octal) memory is necessary.
Comments: The mode shapes, natural frequencies, generalized forces, masses and dampings are to be fed in. These may be generated by a user-written preprocessor.
Availability: Cards can be obtained from
AFFDL Computer Program Library
Wright-Patterson Air Force Base
Dayton, Ohio 45433

MSF 371770 (DARC)

Capability: Random response analysis of truncated conical shells subjected to random pressure excitations induced by aerodynamic turbulence.
Method: A normal mode approach, with each mode represented by a second order, linear oscillator is employed. Either theoretical or experimental vibration modal data may be used.
Language: FORTRAN IV
Hardware: IBM 7094
Availability: The program is available from
Marshall Space Flight Center
Huntsville, Alabama 35812

RANDOM [8-10]

Capability: Random response analysis of curved shell panels, reinforced with stiffeners, under the excitation of acoustic pressure environments. The applicable boundary conditions are (i) four sides simply supported, (ii) four sides clamped, (iii) two opposite sides simply supported and

the other two clamped, and (iv) four sides elastically supported.
Method: Method of normal modes.
Language: FORTRAN IV
Hardware: IBM 7094. A 32K memory is required.
Developers: Dr. T. N. Lee and Dr. W. L. Swanson
Engineering Mechanics Department
Chrysler Corporation
Huntsville, Alabama 35805
Availability: Deck, tape or listing can be obtained from its developers or
COSMIC
112 Barrows Hall
University of Georgia
Athens, Georgia 30601
Cost: $400.00 for deck or tape; $100.00 for listing.

CYLRES,PLRSEP,PLGRP

Capability: CYLRES,PLRSEP and PLGRP are part of a series of programs used to calculate the structural response, noise transmission losses, and interior noise levels of an aircraft fuselage excited by random pressure fields. These three programs are used to calculate the structural response of an aircraft fuselage;CYLRES for the low frequency, complete fuselage case; PLRSEP for the high frequency, single panel case; and PLGRP for the intermediate frequency, panel group case.
Language: FORTRAN
Hardware: CDC 6600
Availability: Cards are available from
AFFDL Computer Program Library
Wright-Patterson Air Force Base
Dayton, Ohio 45433

NASA-AMES PROGRAM

Capability: Random response analysis of rectangular panels under the excitation of an attached boundary layer in subsonic flow or an attached or separated boundary layer in supersonic flow.
Method: Method of normal modes.
Language: FORTRAN
Hardware: IBM 360/370
Comments: Empirical formulas for the excitation pressure based on the extensive tests conducted at the NASA-AMES center are incorporated in the program.
Availability: The program can be obtained from
NASA-AMES Research Center
Moffett Field, California 94035

BOXVIB

Capability: Random response analysis of the skin, rib and frame of a nine-cell box structure subjected to random acoustic loading.
Language: FORTRAN
Hardware: CDC 6600. A 17700 (octal) core is required.
Availability: Cards are available from
AFFDL Computer Program Library
Wright-Patterson Air Force Base
Dayton, Ohio 45433

WDGVIB

Capability: Random response analysis of a three-cell wedge structure subjected to random acoustic loading.

Language: FORTRAN
Hardware: CDC 6600
Availability: Cards are available from
AFFDL Computer Program Library
Wright-Patterson Air Force Base
Dayton, Ohio 45433

MSF 362870

Capability: Random response analysis of rectangular, simply supported honeycomb plates subjected to time-random concentrated force or uniform pressure that has a uniform power spectral density.
Language: FORTRAN II
Hardware: IBM 7094
Availability: The program can be obtained from
Marshall Space Flight Center
Huntsville, Alabama 35812

DISCUSSION

The first four programs, namely, NASTRAN, STARDYNE, ELAS and the PURDUE program, differ from the others, in that they are more general, and can be used to analyse any type of linearly elastic structure. NASTRAN, STARDYNE and ELAS use the finite element method, and these programs can analyze large, complex structures. Though user's manuals are available, data preparation is usually time-consuming and involved. All the three programs have a high confidence level. NASTRAN and STARDYNE are available at the CDC and other computer data centers on a royalty basis. These programs are updated and maintained by the developers. Local data center analyst's assistance is available to the users. NASTRAN and STARDYNE can also be leased and installed in the user's computer system. ELAS can be purchased and installed in the user's computer, but the maintenance is mainly the user's responsibility.

The PURDUE program is suitable for medium sized structures of up to 500 degrees-of-freedom. In contrast to NASTRAN, STARDYNE and ELAS, this program requires the mass, damping, and stiffness matrices to be calculated outside the program, and fed in. The advantage of the PURDUE program over the others is that it can be run on minicomputers with about 16K memory.

The remaining eleven programs are developed for specific problems in the aerospace industry. These programs can be obtained as listings, decks or tapes. RANVIB, RSPC, DARC, RANDOM, CYLRES, PLRSKP, and PLGRP are applicable to shells and curved panels subjected to random excitations like aerodynamic forces, fluctuating pressure fields, etc. The NASA-AMES program is used for the response analysis of flat, rectangular plates subjected to boundary layer excitations, BOXVIB and WDGVIB analyzes box-like structures, and MSF36287 computes the response of simply supported, rectangular honeycomb plates.

In general, these programs developed for specific problems are more efficient than the "general-purpose" programs like NASTRAN, STARDYNE and ELAS. However, their applicability is limited, and modifying them for related problems may be much involved.

CONCLUDING REMARKS

A brief summary of fifteen commercially or publicly available computer programs for the random vibration analysis of structures is presented. The relative merits of the programs from the user's point of view and a general guideline on the choice of programs are discussed. Structural analysts and designers in search of random vibration computer software may use this chapter as a starting point, and obtain additional information about the programs of interest from the sources listed in the PROGRAM SUMMARIES section.

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Beams

Andrew Jay

Pratt and Whitney Aircraft

INTRODUCTION

All disciplines of engineering have become increasingly computer oriented in the last several years. Accompanying this increase has been a need to have access to the proper software for a specific task. Hence the engineering emphasis has shifted from being purely technical to slightly more administrative. The engineer of today is faced with a problem to be solved and must determine if the tools (software, etc.) exist to solve the problem economically. Large general purpose programs developed in the last several years often provide the tools, but are not always the economical solution to the problem. These programs are not economically feasible for the analysis of a single beam element. An engineer must frequently rely on either a quick "ball park" hand calculation or the use of a so-called "special purpose" program. A review of available software capabilities for beams is presented to aid the engineer in selecting the most economical for his purposes.

NOMENCLATURE

A	=	Cross-sectional area
A_s	=	A/k_s equivalent shear area
c	=	Moment intensity (force-length/length)
E	=	Young's modulus
G	=	Shear modulus (force/length^2)
I	=	Moment of inertia about the y axis
k_z	=	Modulus of elastic displacement (Winkler) foundation (force/length)
k_θ^z	=	Modulus of elastic rotary foundation (force-length/length)
M	=	Bending moment
N	=	Axial load
q	=	Transverse loading
r_y	=	Radius of gyration of cross-sectional area
t	=	Time
V	=	Shear force
w	=	Transverse displacements
x	=	Axial coordinate
y	=	Coordinate in transverse direction (length)
z	=	Component in downward direction for horizontal beam, the third component of the right-handed x,y,z coordinate system
ρ	=	Mass per unit length
θ	=	Rotation (radians)

EQUATIONS OF MOTION

This section presents the static equations of motion due to mechanical or thermal loadings according to bending theory wherein cross sections are assumed to remain plane, stress is proportional to strain, and slopes due to bending are small.

These equations are given in 1st order form by equilibrium and force-deformation relationships.

$$\frac{\partial w}{\partial x} = -\theta + V/GA_s \tag{1}$$

$$\frac{\partial \theta}{\partial x} = M/EI \tag{2}$$

$$\frac{\partial M}{\partial x} = V + (k_g^* + N)\theta + \rho r_y^2 \frac{\partial^2 \theta}{\partial t^2} - c(x,t) \tag{3}$$

$$\frac{\partial V}{\partial x} = k_g w + \rho \frac{\partial^2 w}{\partial t^2} - q(x,t) \tag{4}$$

These equations represent the Timoshenko beam. By setting $1/GA_s = 0$ these equations represent the Rayleigh beam. If $\rho r_y^2 \, \partial^2\theta/\partial t^2 = 0$ they represent the shear beam, and by setting both of these equations equal to zero the Euler-Bernoulli beam is derived.

Numerical Methods

The most common numerical techniques used for beam analysis are analytical numerical integration, the finite element method, the transfer matrix method, and the finite difference method. On the basis of available programs it would appear that the finite element method and the transfer matrix method lend themselves more easily than the other methods to the solution of beams with complex geometries and loadings. (See Fig. 1 for example.)

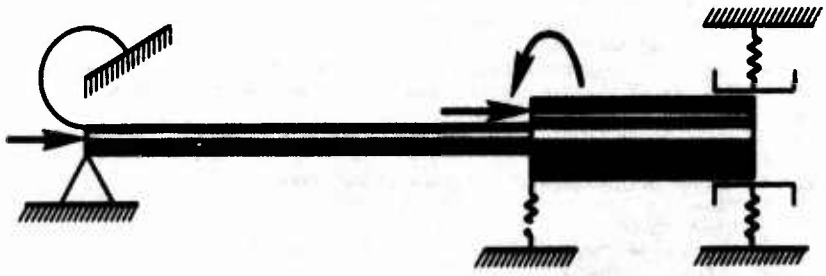

Fig. 1

A large number of the programs in the following section, including the more general beam analysis programs, are transfer matrix method type programs.

PROGRAM SUMMARIES

To solicit information from program developers, a questionnaire was sent out. Responses to this questionnaire, plus information gathered from other sources, provided the information necessary to compile these summaries. Some programs have been omitted because they were very poorly documented; and large general purpose programs with beam elements have also been omitted. All programs mentioned have been confirmed to be available for purchase or use, by a qualified individual. Often an insufficient amount of information was available regarding a specific program's input, output, or usage and then these items were omitted.

SPIN

Date: Developed November 1970.
Capability: This program calculates the critical speeds of rotating shafts and the natural frequencies in bending of multispan beams of arbitrary cross sections. In addition it calculates the response due to sinusoidally applied forces. The deflections, bending moments, shear forces, and stresses created by static forces can also be found by forcing the shaft at zero speed. SPIN uses a distributed mass method for dynamic analysis, but additional mass and rotary inertia can be lumped at points for proper modeling of gears, disks, etc. External springs to ground, both linear and rotary, can be included in the analysis to represent supports or bearings. Two segments of the beam or shaft can also be joined with linear and rotary springs to represent a flexible coupling, gear mesh, or hinge. A forced response can be found due to concentrated forces and moments, distributed loading, rotating unbalance loads, weight loading, or any combination of the above. Both forced static deflections and dynamic frequency response curves may be generated.
Method: Transfer matrices are used for the analysis.
Limitations: Lack of inspan support conditions available. Unstable for higher frequencies.
Input: Span length, support conditions, applied loads, material constants, cross-sectional properties, and analysis type are required. The program has a batch version only, but it is executed on a time-sharing system.
Output: Displacements, forces, and stresses are output, as well as natural frequencies.
Language: FORTRAN IV
Hardware: Honeywell 400 and 600, XDS Sigma 9, and CDC 6000 series.
Usage: User's Manual available [1], technical manual also available.
Developer: Structural Dynamics Research Corp. (SDRC)
5729 Dragon Way
Cincinnati, Ohio 45227
(513) 272-1100
Availability: Use basis from SDRC, contact Edward Carl at the above address, also available on the Acts, Com-Share, Metridata, G.E., and U.S. Steel Systems.
Comments: A problem is encountered with modeling in-span supports as springs. Numerical problems can occur if the spring constant is not properly chosen. Input data is in free format form. The program may also be purchased from the developer for a negotiable price.

BEAM RESPONSE

Date: Developed 1974.
Capability: This is a general beam analysis program. Static, stability, and dynamic analyses can be performed for beams of uniform or variable cross section with arbitrary mechanical or thermal loading. The beams may be

ordinary Euler-Bernoulli beams, or the effects of axial forces, shear deformation, rotary inertia, and gyroscopic moments can be included. The beam can lie on Winkler elastic or higher-order foundations. Any number of in-span supports are acceptable, including extension springs, rotary springs, rigid supports, guides, shear releases, and moment releases.

Method: This is a transfer matrix program taken from [2]. This reference contains the theory and many example problems.

Input: Length of member, support conditions, applied loads, material constants, foundations, cross-section properties and analysis type are required input. An interactive prompting-type preprocessor is available.

Output: Deflection, slope, bending moment, and shear force are output for static and steady-state analyses, the critical load and mode shape for stability, the natural frequencies and mode shapes for transverse vibration. Line printer plot of mode shapes is available.

Language: FORTRAN IV

Hardware: CDC 6000 Series, UNIVAC 1108, IBM 370, PDP 10, Data General, Honeywell.

Usage: Technical manual is available.

Developers: P. Y. Chang
W. D. Pilkey
Dept. of Engr. Sci. & Systems
University of Virginia
Thornton Hall
Charlottesville, Virginia 22901
(804) 924-3291

Availability: Purchase for $150 from:
Structural Members Users Group, Ltd.
P.O. Box 3958
University of Virginia Station
Charlottesville, Virginia 22903
Also, this is available on commercial networks. Contact the Structural Members Users Group for details.

Comments: Every effort has been made to make the documentation as self-contained as possible. Program is very easy to use in a time-sharing format, but a bit awkward to use in a batch mode.

Non-Uniform Beam With Attached Masses(NUBWAM) [3,4]

Date: May 1972.

Capability: This program computes upper and lower bounds to bending frequencies and estimates mode shapes of non-uniform beams with elastically attached masses. The Rayleigh-Ritz procedure is used to obtain the upper bounds and the mode shapes; the lower bounds are obtained using the method of intermediate problems. Upper and lower bounds that bracket the true frequencies are found, whereas other methods give estimates of the frequencies but do not give error bounds.

Method: Rayleigh-Ritz method, and the method of intermediate problems.

Input: Free-field input is used.

Output: Natural frequencies and mode shapes.

Language: PL/1

Hardware: IBM 360/91 - 250K Storage.

Developer: N. Rubinstein or J. T. Stadter
APPLIED PHYSICS LAB
Johns Hopkins University
8621 Georgia Avenue
Silver Spring, Maryland 20910

Availability: Available for no cost from developer, just send tape for exchange.

Comments: The computer program is printed in [3] and [4]. It consists of roughly 2000 source statements.

GBRP

Date: June 1965

Capability: This program performs the free vibration, steady state dynamic and transient response of straight beams, systems of straight beams elastically connected, and systems of beams with attached sprung masses. The cross section may be piecewise constant. Inspan supports may be prescribed. The material must be linear elastic with viscous and structural damping. Loading may be general periodic. For dynamic response the beams are modeled with lumped masses. Concentrated rotary inertia and gyroscopic effects may be included.

Method: The transient dynamics are performed with Fourier analysis. Undamped natural frequencies are found via iterative search. Finite differences are used to generate coefficients for the matrix equations. While in some cases, such as masses connected by springs, the coefficient matrix generated by GBRP does reduce to the sum of mass, damping, and stiffness matrices, this is not always true. For example, in the case of bending vibrations the unknown variable bending moment appears in a vector on the left side of the matrix equation rather than in the force vector on the right side. This results from the form of the difference equations which are derived in the analysis.

Limitations: The total number of beam masses (or section, since masses are concentrated at the centers of respective sections) is actually limited only by available computer core. The degrees of freedom at each mass are deflection, bending moment, and angle of twist about the beam axis depending upon the application. For example the degree of freedom for purely longitudinal or torsional vibration is deflection; for bending or whirling vibrations, the degrees of freedom are deflection and bending moment; for bending coupled with torsion the degrees of freedom are deflection, bending moment and twisting angle. The total degrees of freedom are therefor equal to the number of masses times an appropriate multiplier as determined per application.

Input: The input consists of the mass-elastic parameters for the beams subdivided into intervals for finite differencing of the differential equations - i.e., masses of beam sections, bending rigidities ($\Delta x/EI$), shear rigidities ($\Delta x/KAG$), rotatory inertias ($I_{xx}\,\Delta x$), etc.

Output: Output consists of the response vectors for forced frequency vibration, or critical whirling frequencies, or natural (undamped) frequencies. The response vector consists of the deflection (longitudinal or pure torsional vibrations), or deflection and bending moment (bending vibrations), or deflection, bending moment and angle of twist (coupled bending and torsion), or deflection and moment in the two perpendicular planes of vibrations into which whirling vibration is resolved. See [5, 6,7,8,9,10].

A complete description of the input and output is covered in the documentation in addition to that already noted under that heading "Comments."

Note that GBRP is a unified version of General Bending Response Codes -1, -2, -3 with the added capability for general periodic forced response.

Language: Primarily FORTRAN with some COMPASS in the CDC version.

Hardware: Older versions of the modules of GBRP (GBRC 1, -2, -3) are available on IBM and UNIVAC machines. The current version is available only on CDC machines at NSRDC at the present.

Usage: Program is operational.

Developer: Dr. Elizabeth Cuthill & Mr. Francis Henderson
Naval Ship Research & Development Center
Bethesda, Maryland 20034

Availability: Available from developer to all but foreign commercial firms.

Comments: Plotting capability is available for an SC 4020. Documentation is in the form of NSRDC Reports [8, 9, 10].

OBSV [11, 12, 13]

Date: December 1974.
Capability: This program is designed for the optimization of beams for stability and vibration. The beam may have an elastic foundation. In-span supports are permitted and the beam must be straight. Cross sections can be piecewise constant. Natural frequencies are computed by iterative search.
Method: The finite element displacement method is used for the analysis.
Limitations: Unstable for higher frequencies. A maximum of 128 degrees of freedom are permitted.
Input: The beam cross section and other geometry must be input. Type of analysis and support condition must also be specified.
Output: Buckling load or natural frequencies.
Language: IBM 370/168, UNIVAC 1108.
Usage: In use at two institutions.
Developer: Manohar P. Kamat
230 Norris Hall, VPI & SU
Blacksburg, Va. 24061
Availability: Available from developer for $250. A revised version became available September 1975.
Comments: The program also determines:
1. The distribution of material of a given volume so as to maximize the buckling load or the fundamental frequency of vibration subject to an inequality constraint $A(x) \geq A_{min}$.
2. Frequency closest in absolute modulus to any input frequency.
3. A large number of cross-sectional shapes can be described by relations of the form $I(x) = \rho[A(x)]^n$ where n is any real positive number.

DANAXX4 [14]

Date: Developed March 1968.
Capability: This program calculates the time history of the response of a beam to applied force pulses and applied torque pulses. The beam is represented by a lumped parameter system which is essentially equivalent to a finite difference approximation of the governing equations. In addition to solving the general case of coupled bending and torsion, the program can be used for uncoupled bending and torsion, for torsion alone, or for bending alone. Bilinear external springs may be inserted between the beam and ground. The program allows any combination of hinged, clamped, free or guided flexural boundary conditions. The applied forces and torques are functions of time. The program provides for inelastic behavior by assuming that both the moment curvature and the torque angle of twist relations are of the bilinear type with hysteretic recovery. Shear and rotary inertia are neglected, and no damping is included. The response is determined by a step-by-step integration of the equations of motion using the linear acceleration method.
Method: Finite differences with numerical integration using the linear acceleration method.
Input: Beam element bilinear moment-curvature and torque-twist relations plus the element length, mass and radius of gyration normal to the beam axis. Applied forces and torques are defined as trilinear pulses at each node. Beam initial conditons, boundary conditions and integration time constants are also required.
Output: Time history of the response.
Language: FORTRAN IV.
Hardware: CDC 6600
Developer: T. R. Jackson, Manager
Computer Laboratory
Southwest Research Institute
8500 Culebra Road
P. O. Drawer 28510
San Antonio, Texas 78284

Availability: For sale from developer for a nominal handling charge.

DANAXXO [14]

Date: Developed March 1968.
Capability: The program calculates the frequencies and eigenvectors of a beam with lumped masses. Rotary inertia may be included. The program may also be used to calculate the response due to static loads. The beam is subdivided into segments, within each segment the bending stiffness EI is constant. The program allows the user to include effects of both linear and rotational external elastic springs at one or more joints.
Method: The program uses a stiffness matrix method of analysis.
Input: Element identification, bending stiffness and inertias plus static loads and external spring stiffness.
Output: Frequencies and mode shapes.
Language: FORTRAN IV
Hardware: CDC 6600
Developer: T. R. Jackson, Manager
Computer Laboratory
Southwest Research Institute
8500 Culebra Road
P. O. Drawer 28510
San Antonio, Texas 78284
Availability: For sale from developer for a nominal handling charge.

NOMOD

Date: Unknown.
Capability: This program determines the dynamic response of a multi-degree of freedom system.
Method: The normal mode method of analysis is used to determine the response of a beam modeled as a system of lumped masses connected by linear flexible elements.
Input: Either the flexibility matrix or the equivalent simple beam geometry may be specified. The excitation function can be in the form of constant starting velocity, velocity or acceleration spectrum.
Output: The modal frequencies and associated mode shapes are output. Forces and deflections of each mass point and the forces on the connecting element are also printed.
Language: FORTRAN IV.
Developer: Westinghouse Electric Corporation
Advanced Systems Technology
700 Braddock Avenue
Pittsburgh, Penna. 15112
Availability: Contact developer regarding availability.

TIMOSH

Date: Developed 1974.
Capability: This program performs the stability, free vibration, and steady-state dynamic analysis of a Timoshenko beam. The beam cross section must be piecewise constant. Any type of boundary conditions are permitted. Rigid inspan supports may be included.
Method: The finite element displacement method with continuously distributed mass is used.
Limitations: Unknown.
Input: Beam geometrical and physical properties must be input.
Output: Mode shapes, critical loads, natural frequencies and nodal forces and displacements are output.
Language: FORTRAN IV.

Hardware: IBM and CDC machines.
Developer: Dr. Franklin Y. Cheng
Civil Engineering Dept.
University of Missouri - Rolla
Rolla, Missouri 65401
Availability: Program available from developer.

FAMSUB [15]

Date: March 15, 1971.
Capability: This program determines the transverse natural frequencies and mode shapes of uniform beams subjected to any of the following boundary conditions: simply supported, cantilever, simply supported-free, simply supported-clamped, free-free, and clamped-clamped. Frequencies and mode shapes of these beams are obtained by finding the roots of the frequency function for any of the above boundary conditions, putting the values of the frequencies into the corresponding modal equation, and calculating the relative displacements of evenly placed points along the axis of the beam. The frequency functions and modal equations are derived from the Timoshenko theory of the transverse vibrations of uniform beams. The Timoshenko theory accounts for the effects due to bending, rotary inertia, and shear flexibility. Provisions are made for deleting certain terms from the frequency and modal equations so that other cases accounting for bending only, bending and rotary inertia, or bending and shear flexibility can also be considered.
Method: Roots of frequency function.
Limitations: No inspan supports are allowed.
Input: Type of beam, material properties, cross-sectional properties are input.
Output: Frequencies and mode shapes are output.
Language: FORTRAN IV.
Hardware: IBM 7094/7044
Developer: NASA, Goddard
Goddard Space Flight Center
Greenbelt, Maryland 20771
Availability: Batch program is available for $275 from COSMIC, Prog. #GSC-10429.
Barrow Hall
University of Georgia
Athens, Georgia 30601
Comments: Appears to be a fairly specialized simple program.

Critical Speed and Natural Frequencies

Date: Developed February 4, 1965.
Capability: The purpose of this program is to compute the critical frequency of a rotating shaft with rotary inertia and shear effect and the lateral frequency of a beam vibrating in plane motion. The analysis is identical in both cases. The shaft or beam is divided into an arbitrary number of sections. Each section is selected so that there is a linear relation between the parameters (deflection, slope, moment, shear) at the two ends of each section. The parameters of two adjacent sections are also connected by a linear relation. This leads to a reduced transfer matrix from which the critical speed or lateral frequencies are obtained.
Method: Transfer matrices.
Input: Geometrical and mechanical properties of the shaft must be provided.
Output: Natural frequency and determinate are printed.
Language: FORTRAN IV.
Hardware: IBM 7094.
Developer: Aerojet-General Corp.
P. O. Box 15847
Sacramento, California

Availability: Batch program is available for $275 from:
COSMIC, Prog. #NUC-10090
Barrow Hall
University of Georgia
Athens, Georgia 30601
Comments: This appears to be a fairly specialized simple program.

Critical Speeds

Date: Developed December 1965.
Capability: The purpose of this program is to determine the critical speeds or the lateral vibrations of uniform or nonuniform continuous shafts with any number of supports. The beam is represented by a system of lumped parameters in matrix form using the transfer matrix method. The conditions at one end of the beam are related to those at the other end. The mass of each shaft is assumed to be concentrated at the middle with or without a spring support. The deflection, slope, moment, and shear of the left end are expressed in terms of those at the right end in matrix form. Based on static and dynamic equilibrium conditions, similar matrices can be put together according to the structural system and are reduced one by one to a single four-by-four matrix. Equating the determinant to zero yields the desired lateral frequencies or critical speed of the system.
Method: Transfer matrices.
Input: Geometry and physical properties of beam, boundary condition, and initial frequency guess are input.
Output: Natural frequencies and mode shapes.
Language: FORTRAN IV.
Hardware: IBM 7094.
Developer: Aerojet-General Corp.
P. O. Box 15847
Sacramento, California
Availability: Batch program is available for $275 from:
COSMIC, Prog. #NUC-10091
Barrow Hall
University of Georgia
Athens, Georgia 30601

DEPROSS1 [16]

Date: Developed August 1965.
Capability: This program calculates the dynamic elastic-plastic response of beams that are subjected to an initial impulsive loading. Beams are represented as discrete mass assemblages, and they can be simply supported or clamped. Beam cross sections must be rectangular and uniform.
Method: Finite differences are used for the analysis.
Limitations: No inspan supports. Cross sections must be rectangular.
Input: Unknown.
Output: Unknown.
Language: FORTRAN IV
Hardware: IBM Machines.
Developer: Hans A. Balmer
Dept. of Aeronautics and Astronautics
M. I. T.
Cambridge, Mass. 02139
Availability: For sale for nominal charge from:
T. R. Jackson, Manager
Computer Laboratory
Southwest Research Inst.
8500 Culebra Road

P. O. Drawer 28510
San Antonio, Texas 78284

Response of a Simply Supported Beam (GOLD1)

Date: Unknown.
Capability: An approach is presented by which the displacement, velocity, and acceleration history at points along a nonuniform simply supported beam with time dependent boundary conditions may be determined. Excitation of the structure is in the form of velocity histories at the two supports. These velocity histories at the two supports are not required to be identical.
Method: The stiffness method is used to formulate the finite element problem.
Output: Displacements, velocity, and accelerations are output.
Language: FORTRAN IV.
Hardware: IBM 1130 - 5K of storage.
Developer: Jackie Potts
Naval Ship Research and Development Center
Bethesda, Maryland
Availability: Available only to qualified individuals.

Transverse Modal Data of Nonuniform Beams [17]

Date: Developed September 1967.
Capability: This program calculates the transverse natural frequencies of free-free discontinuous nonuniform beamlike structures including the secondary effects of rotary inertia and shear deformation. In addition, mode shapes, cross-sectional rotations, moments, shear and their first derivatives can be calculated. Total mass, center of gravity, moments of inertia about the center of gravity and about the end of the beam, and generalized mass may also be calculated. A method is also presented for using this program for boundary conditions other than free-free, and the elimination of secondary influences from the solutions.
Method: Recurrence solution.
Output: Natural frequencies and mode shapes are output. Also mass quantities and cross-sectional properties are output.
Language: FORTRAN IV.
Hardware: IBM 7094.
Developer: NASA, Goddard
Goddard Space Flight Center
Greenbelt, Maryland 20771
Availability: Program may be obtained from developer (program no. G GSF 800018) or from COSMIC.
Comments: Program is discussed in [17]. The source deck consists of 2260 cards.

Dynamic Response

Date: Developed 1971.
Capability: The program performs the analysis of skewed girder highway bridges under static and dynamic loadings.
Method: Both finite element method and finite difference techniques are used for problem solution.
Limitations: Specialized for bridge structures.
Input: Geometry and physical properties are input.
Output: Deflections are output.
Language: FORTRAN IV.
Hardware: CDC and IBM equipment.
Developer: U. J. U. Eka
McGill University
Canada

Availability: Order thesis from: National Library of Canada
Ottawa, Canada
Comments: Author's Ph.D. Thesis, McGill University, 1971

OMEGA2

Capability: This program calculates the critical speeds and natural beam mode frequencies of variable circular (hollow or solid) cross section shafts without damping. Mode shapes are calculated for each frequency. The program accounts for shear deflection and rotary inertia effects, the elasticity of shaft, and local "hinge" effects such as flexible (in bending) shaft couplings or flanged joints, and any number of supports of finite radial and bending (slope) stiffness.
Method: Finite element method.
Language: FORTRAN
Hardware: Originally written for a Burroughs B-6700 system.
Developer: Dr. H. Nolle
Department of Mechanical Engineering
Monash University
Clayton, Victoria 3168
Australia
Availability: The program is available commercially, contact the developer.

ISO-DAMAGE

Date: Developed December 1971.
Capability: The programs determines the iso-damage curves for elastic, perfectly plastic simply supported beams which are loaded with blast waves.
Method: A blast wave is modeled as a simple exponentially decaying forcing function. The beam is assumed to respond in its normal modes of vibration during the elastic phase of the response, and as a mechanism after the elastic limit is reached.
Developer: School of Engineering
Air Force Institute of Technology
Wright-Patterson AFB, Ohio
Availability: Contact developer.
Comments: Reference report is AD-737 362.

AX BEAM

Date: Developed May 1970.
Capability: There are two programs for the calculation of static and dynamic characteristics of axial structures (such as beams). Static deflection as well as natural frequencies and mode shapes are determined. Damping is also included.
Input: Physical and material properties must be input.
Output: Natural frequencies, mode shapes, and static deflections are output.
Developer: L. Vanden Noortgate
Louvain University
Belgium
Availability: Contact developer for details.

DYNLAR [18]

Date: Developed 1971
Capability: This program calculates the large flexural vibratory planar response of thin elastic or viscoelastic beams. Nonlinear geometric effects

are included to account for arbitrarily large, contrasted with moderately large, deflections. Presently, the program is geared to the free vibratory response of cantilever and simply-supported beams, however, it may be conveniently adapted to include other boundary, initial and loading conditions.

Method: The analytical basis for the program is the generation of a system of n nonlinear equations of motion for a lumped parameter beam model in which elasticity is concentrated at discrete points while mass remains distributed. Numerical integration is accomplished using the 4th order Runga-Kutta technique.

Limitations: None.

Input: Beam geometric and material properties, boundary conditions (simply supported or damped), the number of degrees of freedom desired, the initial conditions (initial displacement and velocity values for n elements), the time step size and number of time steps to be executed.

Output: Displacement and velocity of the beam elements as a function of time.

Language: FORTRAN IV - 143 cards.

Hardware: WATFOR - 30K

Developer: Dr. Milton O. Critchfield
Code 1735
Naval Ship Research and Dev. Center
Bethesda, Maryland 20084

Dr. Jack C. Wiley
John Deere & Company
Technical Center
3300 River Drive
Moline, Illinois 61265

Availability: Contact developer. The program is available at no cost.

Comments: Running time is approximately 10 minutes on the above mentioned hardware for one response cycle of a beam model with 20 degrees of freedom. Developed as Critchfield's Ph.D. thesis [18].

A-1437

Date: Developed July 1968.

Capability: This program calculates the natural uncoupled modal data of a nonuniform branched beam in transverse, longitudinal or torsional vibration. The beam may be constrained by an elastic foundation. Secondary effects of rotary inertia and shear deformation may also be included.

Method: The four first order beam equations are integrated for transverse vibrations. The two first order beam equations are integrated for longitudinal or torsional vibrations.

Limitations: Each end of the beam or branch may be free, fixed or partially constrained with respect to the ground or to the main member.

Input: Input consists of necessary physical characteristics of the structure and definition of boundary constraints.

Output: Natural frequencies, mode shapes, mode slopes, modal forces and moments, and generalized mass.

Language: FORTRAN IV.

Hardware: CDC 6000 series equipment.

Usage: Used primarily to calculate the modal characteristics of missiles and launch vehicles.

Developer: N. A. S. A.
Langley Research Center
Hampton, Va. 23665

Availability: Contact Robert B. Davis at N. A. S. A. Langley (Mail Stop 317) regarding availability.

Natural Frequencies

Date: Developed 1972.

Capability: Given information about beam loading, moment of inertia, and modulus of elasticity, the program calculates the simple beam deflection for each individual load and determines the natural beam frequency from

these deflections. Beams of any type material may be analyzed by the input of proper material properties. This program will calculate the natural beam frequency for simple beams only. Input loads are limited to dead loads attached to the beam.
Method: See comments below.
Input: Loading, moment of inertia, and modulus of elasticity are input.
Output: Natural frequencies and deflections.
Language: FORTRAN IV-G compiler.
Hardware: IBM 360 OS.
Developer: Bill Frazure (713) 623-3011
Dow Engineering Co.
3636 Richmond Avenue
Houston, Texas 77027
Availability: Batch program is available for $350 from developer.
Comments: Method of analysis is based on "Vibration Analysis for Structural Floor Systems," by Lawrence R. Burkhardt, ASCE, October, 1961.

Rotating, Twisted Beam

Date: Developed 1972.
Capability: This program determines the natural frequencies and normal modes of a lumped parameter model of a rotating, twisted beam, with nonuniform mass and elastic properties. The end of the beam near the center of rotation may have one of four types of boundary conditions which are common to helicopter rotor systems; the outboard end has zero forces and moments, i.e., free boundary conditions. Six types of motion coupling may be modeled: fully coupled torsional-flatwise-edgewise motion; partially coupled torsional-flatwise motion or flatwise-edgewise motion; and uncoupled torsional motion, flatwise motion, or edgewise motion. Three frequency search methods have been implemented including an automated search technique which allows the program to find up to fifteen lowest natural frequencies without the necessity for input estimates of these frequencies by the user.
Method: A modified transfer matrix technique is applied to a lumped parameter model and iterative procedures utilized to obtain the roots of the final governing matrix representing the system.
Input: Unknown.
Output: Natural frequencies and normal modes, also the modes shapes corresponding to a generalized mass of unity for use in a blade loss type of analysis.
Language: FORTRAN IV.
Hardware: CDC 6000 series.
Developer: Rochester Applied Science Assoc., Inc.
140 Allens Creek Rd.
Rochester, New York 14616
Availability: Batch program is available for $350 from
COSMIC: Program #LAR-11461
Barrow Hall
University of Georgia
Athens, Georgia 30601
Comments: Input does not require a frequency estimate for natural frequencies. See NASA-CR-112071 for program description.

Table 1 Capabilities of Several Beam Programs

BEAM PROPERTY \ COMPUTER CODE	SPIN	BEAM-RESPONSE	DYNLAR	NUBEAM	GBRP	OBSV	DANAXIX4	NOWOD	TIMOSH	FAMSUB	COSMIC NUC-10090	COSMIC NUC-10091	DEPROSSI	COLD1	TRANSVERSE MODAL DATA
ANALYSIS TYPE															
Stability		X				X			X						
Free vibration	X	X	X	X	X	X		X	X	X	X	X			X
Steady state dynamic	X	X			X			X	X					X	
Transient					X								X		
Wave propagation															
Random vibration															
SUPPORT CONDITIONS															
Pinned-pinned	X	X	X		X	X	X		X	X	X	X	X	X	X
Pinned-fixed	X	X			X	X	X		X	X	X	X	X		X
Fixed-pinned	X	X			X	X	X		X	X	X	X			X
Fixed-fixed	X	X			X	X	X		X	X	X	X	X		X
Fixed-free	X	X	X		X	X	X	X	X	X	X	X			X
Free-fixed	X	X			X	X	X		X	X	X	X			X
Free-free		X		X	X		X		X	X					X
INSPAN SUPPORTS															
None			X	X			X		X	X			X	X	
Rigid (continuous beam)		X			X	X		X	X		X	X			X
Elastic extension springs	X	X			X	X	X				X	X			X
Elastic rotary springs	X	X				X						X			X
Plastic springs							X								

Table 1 (continued)

BEAM PROPERTY \ COMPUTER CODE	SPIN	BEAM-RESPONSE	DYNLAR	NUBEAM	GBRP	OBSV	DARAXI4	NOMOD	TIMOSH	FANSUB	COSMIC NUC-10090	COSMIC NUC-10091	DEFROSSI	GOLD1	TRANSVERSE MODAL DATA
Dashpots					X										
Guides		X													
Shear releases		X				X									
Moment releases		X				X			X						X
Branches					X										X
FOUNDATIONS															
Winkler (elastic)		X				X					X				X
Shear (rotary)		X													X
Viscoelastic															
Plastic															
MATERIAL															
Linear elastic	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
Nonlinear elastic															
Rigid		X													
Plastic							X						X		
Viscoelastic			X												
Viscous damping					X										
Creep															
Isotropic		X		X	X	X									
Anisotropic															
GEOMETRY															
Straight	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X

Table 1 (continued)

BEAM PROPERTY \ COMPUTER CODE	SPIN	BEAM-RESPONSE	DYNLAR	NUDBAM	GBRP	OBSV	DANAXX4	NONOD	TIMOSH	FAMSUB	COSMIC NUC-10090	COSMIC NUC-10091	DEPROSSI	COLD1	TRANSVERSE MODAL DATA
Curved															
Semi-infinite length						X									
Infinite length									X						
THEORY FOR EQUATIONS OF MOTION															
Engineering theory		X						X							
Euler-Bernoulli beam		X	X	X		X	X			X			X		
Shear beam		X				X				X	X	X		X	X
Rayleigh (rotary inertia) beam		X				X				X					
Timoshenko (shear & rotary inertia) beam	X	X			X	X			X	X	X		X		X
Elasticity		X			X										
Plasticity															
STRESS ANALYSIS															
Postprocessors for stress-analysis															
Automated yield criteria															
ANALYSIS METHOD															
Transfer matrices	X	X									X	X			X
Finite element			X	X		X	X	X	X					X	
Finite differences					X		X						X		
Numerical integration			X				X			X			X		
COMPUTATIONAL STABILITY															
Unstable for long members		X													
Unstable for higher frequencies		X				X									

Table 1 (continued)

BEAM PROPERTY \ COMPUTER CODE	SPIN	BEAM-RESPONSE	DYNLAR	NUBEAM	GBRP	OBSV	DANAXX4	NOMOD	TIMOSH	FAMSUB	COSMIC NUC-10090	COSMIC NUC-10091	DEPROSSI	GOLD1	TRANSVERSE MODAL DATA
LIMITATIONS															
Max. number of degrees of freedom	NONE					128							200		
CROSS SECTION															
Any shape	X	X		X	X	X	X			X		X		X	
Restricted to particular geometry													X		
Thin-walled		X													
Shear center and c.g. coincide		X	X			X		X	X	X					X
SPATIAL VARIATION OF PROPERTIES															
No variation			X					X	X				X		
Cross section can be piecewise constant	X	X			X	X	X				X	X		X	
No restrictions															X
Continuously variable cross section				X											
Foundation constants can vary		X				X									X
Restrictions on number of variations															
MODELING FOR DYNAMIC RESPONSE															
Lumped mass	X	X			X	X	X	X				X	X		
Continuously distributed mass	X	X	X			X			X						
Lumped and/or continuous mass	X	X		X		X					X			X	
Concentrated rotary inertia		X			X										
Distributed rotary inertia		X							X						
Gyroscopic effects for rotating shaft	X	X			X						X	X			

Table 1 (continued)

BEAM PROPERTY \ COMPUTER CODE	SPIN	BEAM-RESPONSE	DYNLAR	RUBHAM	CBRP	ORSV	DANAXX4	NOMOD	TIMOSH	FAMSUB	COSMIC NUC-10090	COSMIC NUC-10091	DEPROSSI	COLD1	TRANSVERSE MODAL DATA
NATURAL FREQUENCIES METHOD															
Iterative search	X	X			X	X				X	X	X			X
Noniterative search															
Invariant imbedding															
STABILITY															
Ordinary buckling		X				X			X						
Lateral instability															

CONCLUSION

Table 1 contains summaries of the capabilities of several beam programs. It should provide a quick reference and comparison of program capabilities. Several programs included in the program description section were not included in the table because too little information was available regarding their capabilities.

Many of the programs mentioned are very poorly documented for easy usage. The single most important feature that should be considered in software selection, other than technical capability, should be the ease of use of the software. If the input is ambiguous or difficult to prepare, many valuable engineer man-hours are wasted. Programs of this type have a questionable usefulness to a practicing engineer.

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Piping Systems

C. Visser and A. J. Molnar

Westinghouse Research Laboratories

INTRODUCTION

The demand for dynamic analysis of piping systems is increasing, especially in the nuclear power industry where great emphasis is placed on safety. Both government and private agencies are generating specifications for the testing and structural analysis of components and structures. Seismic disturbances and hypothetical accidents such as instantaneous pipe breaks are a few of the conditions which must be dynamically analyzed. These types of analyses involve strong ground motions for seismic analysis and large transient forces and displacements in the case of pipe breaks. As a result demands for the more difficult large deformation, large strain, elastic-plastic and fluid-structure interaction analyses are becoming more common.

The assumptions used in the analysis of piping involve the deformations and strain distributions in pipes, bends (elbows), and tees. These components can also have flanges and reinforcements which will affect the deformations and strain distributions. The assumptions commonly used for straight pipes are those used in the various beam theories. A rigorous analysis of bends and tees can only be obtained using shell or three-dimensional theory. With the present state of the art a dynamic analysis of shell or three dimensional structural models is very costly. Additional assumptions are, therefore, used for bends and tees in order to reduce the three-dimensional problem to a reasonable size.

It is the purpose of this chapter to present a survey of the computer programs which are applicable for the shock and vibration analysis of piping systems. Efforts are made to identify the methods on which the majority of computer programs for piping analysis are based, including elastic-plastic behavior and large deformations and strains. The discussion of these methods is limited to finite element techniques and the finite elements used for piping analysis are briefly described. The desirable features for efficient and economical analysis of piping systems are briefly reviewed followed by a description of computer programs currently available for piping analysis.

Information about the computer programs was obtained from the authors' own knowledge of some of the programs and from responses to detailed questionnaires by users and developers. Only the more important aspects of widely available programs will be summarized to avoid the use of large detailed tables. Available addresses and phone numbers for both proprietary and non-proprietary dynamic piping program developers who responded are also given.

NOMENCLATURE

a_1, a_2 = Scalars multiplying mass and stiffness matrices respectively for defining proportional damping matrix
$[C]$ = Viscous damping matrix
$[\bar{C}]$ = Uncoupled modal damping matrix
$[C_c]$ = Condensed damping matrix

c_i = Generalized modal damping coefficient for ith mode
$\{d\}$ = Nodal earthquake direction vector
$[E_e]$ = Array of elastic coefficients relating incremental elastic strains and stresses
$[E_p]$ = Array of coefficients relating incremental plastic strains and stresses
$\{f_c\}$ = Condensed nodal force vector
$\{fe_i\}$ = Element forces of ith element
$\{f(t)\}$ = Applied nodal force vector
$[K]$ = System stiffness matrix
$[K_c]$ = Condensed stiffness matrix
k_i = Generalized stiffness for the ith mode
$[M]$ = System mass matrix
$[M_c]$ = Condensed mass matrix
m_i = Generalized mass of the ith mode
N = Total number of degrees of freedom in the system
$[P]$ = Plastic matrix for defining incremental pseudo-forces from incremental plastic strains
$\{q\}$ = Vector of normal coordinates
q_i = Normal coordinate of the ith mode
S_a = Spectral value of acceleration or pseudo acceleration
S_d = Spectral value of displacement or pseudo displacement
S_v = Spectral value of velocity or pseudo velocity
$[T_i]$ = Transfer matrix
$\{u\}$ = Nodal displacement vector of the total system
$\{\ddot{u}_a\}$ = Nodal acceleration vector with respect to a fixed coordinate system
$\{\ddot{u}_g\}$ = Nodal acceleration vector of ground or base
$\ddot{u}_g$ = Earthquake ground acceleration
$\{u_n\}$ = Displacement vector at $n\Delta t$
$\{\dot{u}_n\}$ = Velocity vector at $n\Delta t$
$\{\ddot{u}_n\}$ = Acceleration vector at $n\Delta t$
$\{\ddot{u}_r\}$ = Relative acceleration vector
$\{u_1\}$ = Vector of retained nodal displacements
$\{u_2\}$ = Vector of eliminated nodal displacements
β = Constant used in Newmark's β-method
$\{\Delta\varepsilon\}$ = Vector of incremental strains
$\{\Delta\varepsilon_e\}$ = Vector of incremental elastic strains
$\{\Delta\varepsilon_p\}$ = Vector of incremental plastic strains
$\{\Delta f_{ps}\}$ = Incremental pseudo-force vector calculated from the incremental plastic strains
Δt = Interval of time integration
$\Delta\sigma$ = Incremental stress vector
θ = Constant used in Wilson's θ-method
λ_i, ω_i^2 = Eigenvalue or square of the natural frequency
ξ_i = Modal damping ratio of the ith mode
$[\phi]$ = Modal matrix
$\{\phi_i\}$ = Eigenvector or ith mode shape
ω_i = Natural frequency of the ith mode

THEORETICAL CONSIDERATIONS

Most of the computer programs for the dynamic analysis of piping systems are based on the finite element method. The following brief discussion of the theory used in the development of dynamic piping programs is, therefore, restricted to finite element formulations.

The governing equations of motion for a finite element model can be expressed in matrix form [1] by

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{f(t)\} \qquad (1)$$

where [M] is the mass matrix, [C] is the viscous damping matrix, [K] is the stiffness matrix, {u} is the nodal displacement vector, and {f(t)} is the applied nodal force vector.

In many finite element programs two types of mass matrices, lumped and consistent, are available. Lumped mass matrices are based on the assumption that the mass of the structure is concentrated at those nodal points at which the translational degrees of freedom are defined. The lumped mass matrix is diagonal and the mass corresponding to a rotational degree of freedom is zero. Consistent mass matrices [2] are evaluated by a procedure which is similar to that used in the formulation of the stiffness matrix. The consistent mass matrix is symmetrical but non-diagonal and has non-zero terms associated with the rotational degrees of freedom.

The damping characteristics of a structure are generally more difficult to determine than its mass or stiffness. In most practical dynamic analyses damping is assumed to be viscous as in Eq. (1) since this leads to the most convenient mathematical form of the equations of motion.

The stiffness matrix [K] can have different forms depending on the type of analysis to be performed. For elastic analysis [K] is symmetric and generally sparse, additional terms occur if the structure is prestressed and the initial stresses must be taken into account. Depending on the method used in elastic-plastic analysis the stiffness matrix can have additional terms due to plasticity; this will be discussed below. Large deformations and strains also require modifications in the stiffness matrix.

Linear Analysis

In the small deformation analysis of elastic structures, the [M], [C], and [K] matrices of Eq. (1) are symmetric and generally sparse. These properties can be used to advantage in computer programs to reduce storage requirements. For large structures, however, long computing times are required for solving dynamic problems. This has led to the development of condensation methods to reduce the dynamic degrees of freedom of a model. Two condensation methods are used. 1) Static condensation eliminates from the stiffness matrix the unwanted degrees of freedom associated with zero mass. The method assumes that no damping is associated with those degrees of freedom which have zero mass. 2) Guyan reduction [3] is a more general method of condensation and will be discussed below in somewhat greater detail. The equation of motion (1) can be written in partitioned form

$$\left[\begin{array}{c|c} M_{11} & M_{12} \\ \hline M_{12}^T & M_{22} \end{array}\right] \left\{\begin{array}{c} \ddot{u}_1 \\ \hline \ddot{u}_2 \end{array}\right\} + \left[\begin{array}{c|c} C_{11} & C_{12} \\ \hline C_{12}^T & C_{22} \end{array}\right] \left\{\begin{array}{c} \dot{u}_1 \\ \hline \dot{u}_2 \end{array}\right\} + \left[\begin{array}{c|c} K_{11} & K_{12} \\ \hline K_{12}^T & K_{22} \end{array}\right] \left\{\begin{array}{c} u_1 \\ \hline u_2 \end{array}\right\} = \left\{\begin{array}{c} f_1(t) \\ \hline f_2(t) \end{array}\right\} \quad (2)$$

where $\{u_1\}$ represents the vector of nodal displacements which are to be retained and $\{u_2\}$ is the corresponding vector of displacements which are to be eliminated from the system of equations. The superscript T denotes the transpose of a vector or matrix. Guyan reduction assumes that Eq. (2) is to be solved subject to the constraint

$$\{u_2\} = -[K_{22}]^{-1} [K_{12}]^T \{u_1\} = [H_{12}] \{u_1\} \quad (3)$$

where the superscript -1 denotes the inverse of a square matrix. This results in the following reduced set of equations

$$[M_c]\{\ddot{u}_1\} + [C_c]\{\dot{u}_1\} + [K_c]\{u_1\} = \{f_c\} \tag{4}$$

where

$$[M_c] = [M_{11} + M_{12}H_{12} + H_{12}^T M_{12}^T + H_{12}^T M_{22} H_{12}]$$

$$[C_c] = [C_{11} + C_{12}H_{12} + H_{12}^T C_{12}^T + H_{12}^T C_{22} H_{12}]$$

$$[K_c] = [K_{11} + K_{12}H_{12}]$$

$$\{f_c\} = \{f_1(t)\} + [H_{12}]^T \{f_2(t)\}$$

Condensation methods generally work well for practical problems provided the larger masses are included in the reduced model and the retained degrees of freedom are uniformly distributed.

Two methods are used in the transient analysis of structural systems, namely, 1) direct integration and, 2) mode superposition. Condensation of the dynamic degrees of freedom can be used with both methods.

Direct Integration Methods

Direct integration methods are based on step-by-step integration of the coupled equations of motion represented by Eqs. (1) or (4). These methods generally use difference formulas relating the nodal displacements, velocities, and accelerations. Two methods are in use for time integration of dynamic equations: implicit integration and explicit integration. The following discussion will be restricted to finite element programs, which most commonly use implicit integration. Several time integration formulas [4] are available, such as Newmark's β-method [5], Wilson's θ-method [6], and Houbolt's method [7].

Newmark's β-method is used in several large general purpose computer codes and will be briefly discussed below.

The application of the Newmark β-method in finite element programs usually takes the form described by Chan, et al. [8]. In Newmark's method the velocity and displacement vectors at time t_{n+1} are expressed by

$$\{\dot{u}_{n+1}\} = \{\dot{u}\}_n + \frac{\Delta t}{2}\{\ddot{u}_n\} + \frac{\Delta t}{2}\{\ddot{u}_{n+1}\}$$

$$\{u_{n+1}\} = \{u_n\} + \Delta t\{\dot{u}_n\} + (1-\beta)\Delta t^2\{\ddot{u}_n\} + \beta\Delta t^2\{\ddot{u}_{n+1}\} \tag{5}$$

where $\{u_n\}$, $\{\dot{u}_n\}$, and $\{\ddot{u}_n\}$ are respectively the nodal displacement, velocity, and acceleration vectors at the end of the nth time interval, Δt is the time interval, and β is a parameter. The value of β can vary and affects the stability of the step-by-step integration.

Combining Eq. (1) at times $t_{n-1} = (n-1)\Delta t$, $t_n = n\Delta t$, and $t_{n+1} = (n+1)\Delta t$ with Eq. (5) the following equation is derived [8]

$$\left[\frac{1}{\Delta t^2} M + \frac{1}{2\Delta t} C + \beta K\right] \{u_{n+1}\} = \beta \{f_{n-1}\} + (1-2\beta) \{f_n\} + \beta \{f_{n+1}\}$$
$$+ \left[\frac{2}{\Delta t^2} M - (1-2\beta) K\right] \{u_n\} - \left[\frac{M}{\Delta t^2} - \frac{1}{2\Delta t} C + \beta K\right] \{u_{n-1}\} \quad (6)$$

Equation (6) is a set of simultaneous equations that is equivalent to the simultaneous differential equations of motion represented by Eq. (1). Once the displacement vectors are known the velocity and acceleration vectors can be obtained from Eqs. (5). The direct method is particularly useful when nonlinear material behavior and large deformations and strains must be taken into account.

Mode Superposition

In mode superposition the physical degrees of freedom of the structural model are replaced by its normal coordinates. In order to accomplish this the natural frequencies and mode shapes of the undamped model are first determined. The eigenvalues λ_i of the equation

$$[K - \lambda M] \{u\} = \{0\} \quad (7)$$

are the squares of the natural frequencies and the corresponding eigenvectors $\{\phi_i\}$ represent the mode shapes. Several methods are used to compute the eigenvalues of structural systems, such as Householder's, Given's, Jacobi's, inverse power with shift, etc. [9]. Several of these methods such as Householder's, Given's, and Jacobi's will obtain all the eigenvalues of the system and are often used in combination with a condensation procedure, while other methods such as inverse power with shifts are suitable to obtain a limited number of eigenvalues and are usually employed for problems with a large number of degrees of freedom or where accurate higher frequencies are required.

Another procedure commonly used to determine the natural frequencies and mode shapes of so called "chain structures" is the transfer matrix technique [10] which evolved from methods described by H. Holzer and later by N. O. Myklestad. For a pipe element the transfer matrix $[T_i]$ between nodes i and j relates the displacements and forces at node j to those of node i

$$\left\{\begin{matrix} u_j \\ \hline fe_j \end{matrix}\right\} = [T_i] \left\{\begin{matrix} u_i \\ \hline fe_i \end{matrix}\right\} \quad (8)$$

where $\{u_i\}$ and $\{u_j\}$ are the generalized nodal displacement vectors for nodes i and j respectively and $\{fe_i\}$ and $\{fe_j\}$ are the nodal force vectors of nodes i and j respectively. Successive multiplication of the transfer matrix for each element until all elements are included yields, for the entire structure,

$$\left\{\begin{matrix} u_m \\ \hline fe_m \end{matrix}\right\} = [T_m] \cdots [T_2] [T_1] \left\{\begin{matrix} u_o \\ \hline fe_o \end{matrix}\right\} \quad (9)$$

where the subscript o denotes the first node, and m denotes the last node

of the system.

Application of the boundary conditions to Eq. (9) leads to a polynomial or transcendental expression from which the natural frequencies, ω_i, can be obtained. Repeated application of Eq. (9) i = 1, 2... etc. with $\omega = \omega_i$ yields the mode shapes and nodal forces for m modes.

Transfer matrix techniques are best suited to "chain structures" with a limited number of branches, thus they are generally applicable to piping system analysis.

Once the eigenvalues and eigenvectors of the system are known the equations of motion can be uncoupled. For this purpose the modc shape matrix

$$[\phi] = [\{\phi_1\}\ \{\phi_2\}---\{\phi_m\}] \tag{10}$$

is formed, where the $\{\phi_i\}$, i = 1, 2, -- m, are the eigenvectors corresponding to the eigenvalues λ_i.

In many cases the number of modes m retained in Eq. (10) is considerably smaller than the number of degrees of freedom N in the system. The matrix $[\phi]$ is generally an Nxm matrix with $m \leq N$. In mode superposition the following transformation is introduced

$$\{u\} = [\phi]\ \{q\} \tag{11}$$

where {q} is a vector of m generalized displacements called the normal coordinates. When Eq. (11) is substituted into Eqs. (1) or (4) the following equations are obtained

$$[\bar{M}]\ \{\ddot{q}\} + [\bar{C}]\ \{\dot{q}\} + [\bar{K}]\ \{q\} = \{\bar{F}(t)\} \tag{12}$$

where

$$\begin{aligned} [\bar{M}] &= [\phi]^T\ [M]\ [\phi] \\ [\bar{C}] &= [\phi]^T\ [C]\ [\phi] \\ [\bar{K}] &= [\phi]^T\ [K]\ [\phi] \\ \{\bar{F}(t)\} &= [\phi]^T\ \{f(t)\} \end{aligned} \tag{13}$$

Due to the orthogonality of the eigenvectors the matrices $[\bar{M}]$ and $[\bar{K}]$ are diagonal. In practical problems it is usually assumed that $[\bar{C}]$ is also diagonal, an assumption which can be shown to hold if the damping matrix is proportional to the mass and/or the stiffness matrix[1], that is,

$$[C] = a_1[M] + a_2[K] \tag{14}$$

In this case, Eq. (12) may be written as m uncoupled equations

[1]More general combinations of [M] and [K] exist which will yield a diagonal matrix $[\bar{C}]$.

$$m_i\ddot{q}_i + c_i\dot{q}_i + k_iq_i = f_i(t),\ i = 1,2,\text{---},\ m$$

or

$$\ddot{q}_i + 2\,\xi_i\omega_i\dot{q}_i + \omega_i^2\,q_i = \frac{1}{m_i}\,f_i(t) \qquad (15)$$

where $\xi_i = \dfrac{c_i}{2\,m_i\omega_i}$ is the damping ratio

and ω_i is the natural frequency

The solution of Eq. (15) will yield the normal coordinates and the physical displacements {u}, velocities {u̇}, and accelerations {ü} can be found using the transformation of Eq. (11).

In cases where the dynamic response can be represented by a limited number of modes, the mode superpositon method is clearly economical. This depends on the frequency content of the loading and the dynamic characteristics of the structure. In general the largest computational effort is required to determine the eigenvalues of the system.

Nonlinear Analysis

There are several nonlinear types of analysis in use for piping systems, such as nonlinear material behavior, large deformations and strains, and contact problems involving gaps (bumpers) and friction. Two methods are used to deal with nonlinearities; the variable stiffness method in which the stiffness of the the structural model is continually modified to take the nonlinear effects into account, and the pseudo-force or constant stiffness method in which the nonlinear effects are treated as pseudo-forces.

Nonlinear dynamic analysis is usually based on the direct integration method. The mode superposition method is, to our knowledge, only used in contact problems by treating the nonlinear contact and friction forces as pseudo-forces.

To explain the two methods, consider a contact problem that often occurs in piping systems, namely when bumpers are used. In such problems, the computer program will monitor the displacements of the nodal points at the bumper. When the displacements indicate that the initial gap between the nodal points at the bumper has disappeared, a fictitious stiff spring is inserted between the nodes. In the pseudo-force method, the overlap of the nodes at the bumper is used to compute the pseudo-force required to decrease the overlap. In the variable stiffness method, the spring stiffness is added to the total stiffness matrix of the structural model, and the computations are continued with the new stiffness matrix. When the displacements of the nodes at the bumper indicate that the gap is opening up, the spring stiffness is set to zero. In the pseudo-force method, this will remove the pseudo-force from the right hand side of the equations, while in the variable stiffness method, the spring stiffness is removed from the total stiffness matrix. Friction is usually treated by means of the pseudo-force method. The pseudo-force method allows the use of modal superposition since with this method the nonlinearities appear on the right side of the equations of motion.

Elastic-plastic procedures in finite element computer programs are based on a yield condition, a flow rule, and a hardening rule. The most commonly used procedures include the yield condition of Von Mises and the associated flow rule, the Prandtl-Reuss equations. Several hardening rules have been developed [11], however, isotropic hardening and kinematic hardening are the rules used most in finite element programs. For dynamic analysis the deformation theory of plasticity is not applicable and the incremental theory must be used. In this theory the increments of strain are separated into elastic and plastic parts

$$\{\Delta\varepsilon\} = \{\Delta\varepsilon_e\} + \{\Delta\varepsilon_p\} \tag{16}$$

where $\{\Delta\varepsilon\}$ is the vector of incremental strains,
$\{\Delta\varepsilon_e\}$ is the vector of the elastic part of $\{\Delta\varepsilon\}$, and
$\{\Delta\varepsilon_p\}$ is the vector of the plastic part of $\{\Delta\varepsilon\}$.

For constant temperatures the Prandtl-Reuss equations (or other suitable flow rule) combined with the hardening rule will yield an expression between the incremental plastic strain and incremental stress

$$\{\Delta\varepsilon_p\} = [E_p]\{\Delta\sigma\} \tag{17}$$

where $[E_p]$ is the material matrix which is based on the current state of stress and hardening of the material.

For isothermal conditions, generalized Hooke's Law gives a relationship between the incremental elastic strains and stresses

$$\{\Delta\varepsilon_e\} = [E_e]\{\Delta\sigma\} \tag{18}$$

where $[E_e]$ is an array of the elastic coefficients.

Combining Eqs. (17) and (18) with Eq. (16) gives a relation between the increments of stress and strain.

$$\{\Delta\varepsilon\} = [E_e + E_p]\{\Delta\sigma\} = [E_{ep}]\{\Delta\sigma\} \tag{19}$$

In the variable stiffness method (in elastic-plastic analysis also referred to as the tangent modulus method), the element stiffness matrices are formed using $[E_{ep}]^{-1}$ for each new increment of loading, and thus depend on the state of stress and hardening at the beginning of each increment.

In the pseudo-force method the elastic stiffness matrices are formed using Eq. (18). The pseudo-force vectors can be obtained by two methods: the initial strain method [12] which uses the known incremental plastic strains at the end of the previous increment, and the initial stress method [13] which uses the known incremental stresses at the end of the previous increment.

For a constant strain element the incremental pseudo-force vector based on the initial strains is given by

$$\{\Delta f_{ps}\} = [P]\{\Delta\varepsilon_p\} \tag{20}$$

where the plastic matrix [P] is formed using the incremental stress-plastic strain relation of Eq. (17), and $\{\Delta\varepsilon_p\}$ is the vector of known plastic strains. If the strains are varying on the element, the element pseudo-force vector is obtained by numerical integration.

By combining Eq. (20) with Eq. (17) the element pseudo-force vector is obtained in terms of the known initial stresses

$$\{\Delta f_{ps}\} = [P][E_p]\{\Delta\sigma\} \tag{21}$$

The total pseudo-force vector is obtained by adding the contributions of all elements. The pseudo-force method has the advantage that the stiffness

matrix stays constant. However, the pseudo-force method can lead to considerable deviation or drift from the stress-strain curve. This can be improved by taking small increments or by iteration during an increment to improve the pseudo-force vector. The influence of temperature on the material properties, such as the modulus of elasticity and yield stress, can also be taken into account in elastic-plastic analysis and has been incorporated into several computer codes. For dynamic analysis the strain rate effects may influence the results both in elastic and elastic-plastic analysis. These effects have not been incorporated in most computer programs.

The dynamic analysis of piping systems with large deformations and strains is feasible with a number of currently available computer programs. As in elastic-plastic analysis, the equations are linearized and the solution is obtained by an incremental procedure [14], which can be conveniently combined with the step-by-step time integration for dynamic analysis.

Two methods are in use for incremental large deformation and strain analysis: the reference configuration is the undeformed configuration of the structure (Lagrangian formulation), and the deformed configuration at the end of the previous increment is the reference configuration (Eulerian formulation) [15]. In the finite element method both methods lead to additional terms in the stiffness matrix. For example, in the Lagrangian formulation [16,17] at least two additional stiffness matrices must be formed and added to the usual stiffness matrix used in small deformation analysis: 1) the initial stress stiffness matrix which takes the effect of the initial stresses at the beginning of the increment into account, 2) the initial displacement matrix which represents the influence on the stiffness of the displacements at the end of the previous increment, and 3) the loading matrix is added to the stiffness matrix if the loading depends on the deformations of the structure. The load vectors are also changed in these procedures to take the effects of large deformations into account.

Considerable experience is required to carry out a dynamic analysis with plasticity and large deformations. The choice of the time step Δt in the direct time integration must be governed not only by the dynamic characteristics of the structure and frequency content of the load but also by the magnitude of the incremental loads, deformations, and the elastic and plastic strains which will result.

Seismic Analysis

Seismic analysis of structures is concerned with the dynamic response of structures due to strong ground motions caused by earthquakes. In finite element analysis the nodal accelerations are usually expressed as follows:

$$\{\ddot{u}_a\} = \{\ddot{u}_r\} + \{\ddot{u}_g\} \tag{22}$$

where $\{\ddot{u}_a\}$ is the nodal acceleration vector with respect to a fixed coordinate system,
$\{\ddot{u}_g\}$ is the nodal earthquake ground acceleration vector, and
$\{\ddot{u}_r\}$ is the relative nodal acceleration vector.

Combining Eq. (22) with Eq. (1) gives

$$[M]\{\ddot{u}_r\} + [C]\{\dot{u}_r\} + [K]\{u_r\} = -[M]\{\ddot{u}_g\} \tag{23}$$

The vector on the right hand side of Eq. (23) represents the known forces caused by the ground motion.

The above formulation has the disadvantage that only one known acceleration, $\{\ddot{u}_g\}$, can be applied to the structure. In the analysis of piping systems it is sometimes desirable to apply two or more known accelerations to the sys-

tem. For example, where a piping system is attached to different points of one or more structures which are subjected to an earthquake ground acceleration, the points of attachment will generally have different accelerations. For those cases, the equations of motions can be expressed in terms of the absolute displacements, which yields

$$[M]\{\ddot{u}_a\} + [C]\{\dot{u}_a\} + [K]\{u_a\} = [C]\{\dot{u}_g\} + [K]\{u_g\} \tag{24}$$

Most of the available computer codes are based on the formulation of Eq. (23). The reason for this is that Eq. (23) is suitable for most structures, such as buildings, and is simpler to program than Eq. (24). In addition, the earthquake motions generally are represented in terms of the ground accelerations.

In seismic analysis of piping systems two methods are used:

1. transient response, which will obtain the time history of the structural response. This type of analysis can be made by modal superposition or direct time integration.
2. the response spectrum method which uses the normal modes of the structural model and is, therefore, restricted to linearly elastic behavior.

The first method as it generally applies to transient response analysis has been described above.

In the response spectrum method the natural frequencies and mode shapes of the structural model are first determined. Using the mode shapes, the equations of motions (Eq. (2)) are uncoupled, resulting in m uncoupled equations similar to Eq. (15).

$$\ddot{q}_i + 2\xi_i\omega_i\dot{q}_i + \omega^2_i q_i = \frac{1}{m_i}\{\phi_i\}^T[M]\{d\}\ddot{u}_g \tag{25}$$

where $\{\phi_i\}$ is the eigenvector (mode shape) corresponding to the i^{th} natural frequency ω_i, $\{\ddot{u}_g\} = \{d\}\ddot{u}_g$,
$\{d\}$ is the nodal earthquake direction vector, the components of which are numbers ranging from zero to one, and
$\ddot{u}_g$ is the earthquake ground acceleration.

The solution of Eq. (25) can be written in terms of the Duhamel integral

$$q_i(t) = \frac{\{\phi_i\}^T[M]\{d\}}{\omega_i m_i}\int_0^t \ddot{u}_g(\tau)\exp[-\xi_i\omega_i(t-\tau)]\sin\omega_i(t-\tau)\,d\tau \tag{26}$$

where the undamped frequency has been used instead of the damped value [18]. Equation (26) shows that the response of the i^{th} normal mode depends on the natural frequency ω_i, the damping ratio ξ_i, and the ground acceleration $\ddot{u}_g$. The spectral value of the velocity relative to ground is defined by the maximum value of the integral of Eq. (26), that is,

$$S_{v_i} = \left[\int_0^t \ddot{u}_g(\tau)\exp[-\xi_i\omega_i(t-\tau)]\sin\omega_i(t-\tau)\,d\tau\right]_{max} \tag{27}$$

The pseudo values of the displacement and acceleration relative to ground are defined by

$$S_{d_i} = \frac{1}{\omega_i} S_{v_i}$$

$$S_{a_i} = \omega_i S_{v_i} \tag{28}$$

A plot of the maximum response (displacement, velocity, or acceleration) for a given earthquake ground motion $\ddot{u}_g$ and a given damping ratio against the natural frequency of vibration ω_i, is called a response spectrum. A typical example is shown in Fig. 1.

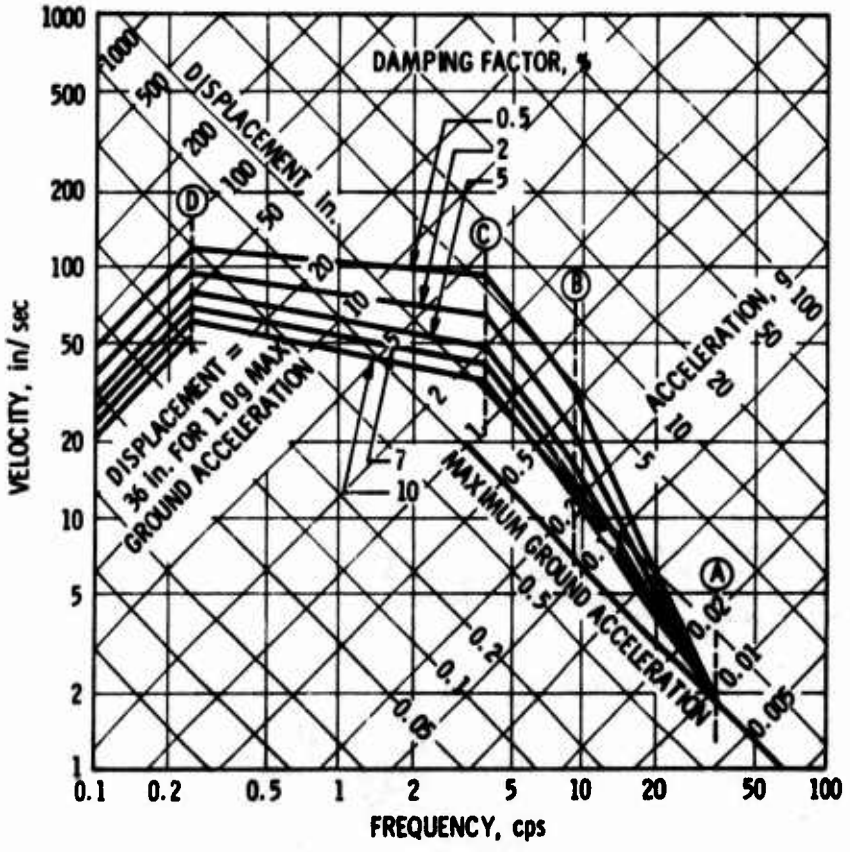

Fig. 1 Vertical design response spectra-scaled to 1g horizontal ground acceleration (from U.S.A.E.C. regulatory guide 1.60)

The maximum normal coordinate displacement of the i^{th} mode is according to Eqs. (26) and (27) given by

$$q_i\Big|_{max} = \frac{\{\phi_i\}\ [M]\ \{d\}}{\omega_i m_i} S_{v_i} \tag{29}$$

The maximum values of the physical nodal displacements corresponding to $q_i\Big|_{max}$ are obtained from

$$\{u\}_i\Big|_{max} = \{\phi_i\}\ q_i\Big|_{max} \tag{30}$$

An estimate of the maximum response of the structure can be computed by three methods:

1. The maximum response is computed as the sum of the absolute values for all modes of Eq. (30)

$$\{u\}\Big|_{max} = \sum_{i=1}^{m} |\{u\}_i|_{max} \tag{31}$$

This value is conservative since the maximum values usually do not occur at the same time.

2. A better estimate of the maximum response is given by a root mean square (RMS) approximation, that is, the values of the components of $\{u\}\Big|_{max}$ are given by

$$u_r\Big|_{max} = \left[\sum_{i=1}^{m} (u_{ri})^2\right]^{\frac{1}{2}} \qquad r=1,---,N \tag{32}$$

where N is the number of degrees of freedom in the system. The stresses and other response values are also obtained by the RMS approximation procedure.

3. Another method for estimating the maximum response is called the NRL sum which was developed by the Naval Research Laboratories. The peak response at node r is defined by

$$u_r\Big|_{max} = \left|u_{rj_{max,max}}\right| + \left(\sum_{\substack{i=1 \\ i\neq j}}^{m} u_{ri}\Big|_{max}^2\right)^{\frac{1}{2}} \tag{33}$$

where $u_{rj_{max,max}}$ is the maximum of the $u_{ri_{max}}$ modal contributions at r and u_{rj} is excluded from the summation. The NRL sum gives results which are less than the absolute sum of Eq. (31) and greater than the RMS values from Eq. (32).

FINITE ELEMENTS FOR PIPING ANALYSIS

The dynamic analysis of piping systems with the finite element method requires a library of finite elements. The extent of a library varies with the program but the following elements are common to most programs:

1. Straight pipe elements with constant thin walled circular cross section. The formulation of these elements is based on beam theory and usually takes shear deformations into account.

2. Straight tapered pipe elements. These elements have a straight centerline but the radius of the cross-section varies linearly along its length. The formulation of the stiffness and load vectors are based on beam theory.

3. Beam elements. These elements are usually straight and can have cross-sections other than those of pipes.

4. Elbow elements. This type of element is essential in most piping analyses. Most elbow elements available are based on smooth elbows. The theory for this element is more difficult than for the other elements. This is due to the fact that the cross-section of thin walled pipes will deform due to bending. This makes an elbow more flexible than predicted by beam theory [19] and also causes the stresses to be higher than computed by beam theory. The ANSI Standard Code for Pressure Piping[2] gives flexibility and stress intensification (concentration) factors for smooth and mitered bends. The flexibility factor is defined as the ratio of angular displacement of a pipe bend and that of a straight pipe subjected to the same end moment. The stress intensification factor is defined as the ratio of maximum stress in a bend and that in a straight pipe under the same load. The flexibility factor can be taken into account in the stiffness matrix of the elbow [20]. The flexibility and stress intensification factors when taken into account in a piping analysis program give reasonable results for most linearly elastic problems. However, the method is unsatisfactory for elastic-plastic and large deformation analysis. Marcal developed a method based on axisymmetric shell elements with additional bending and stretching added [21,22]. A good list of references and several papers dealing with pipe elbows may be found in [23].

5. Tee-elements. Tees with or without reinforcing are used in piping systems. The ANSI piping code provides flexibility and stress intensification factors for several types of tees. Very few computer programs have suitable elements to handle tees, in most programs they can only be modeled as intersecting beams ignoring the added flexibility inherent in a piping tee.

6. Spring elements. These elements are required to model hangers, cable supports, etc. In the case where no compressive forces can be supported by the structural member a nonlinear spring is required. The analysis will then also be nonlinear.

7. Gap and friction elements. These elements are needed at locations where small initial gaps must be modeled for thermal growth and friction where sliding between the pipe elements and the support structure can occur. Both contact and friction require nonlinear analysis techniques.

8. Mass elements. These elements are needed to model large masses which must be taken into account in the analysis.

9. Shell and three-dimensional elements. With the present state of the art detailed stresses in elbows and tees can only be obtained by carrying out a complete shell or three-dimensional analysis. This is particularly true if elastic-plastic behavior must be taken into account. This type of analysis is usually carried out independently of the piping analysis.

PIPING ANALYSIS COMPUTER PROGRAMS

A considerable number of computer programs have been developed for piping analysis. There are essentially two types of programs available, (1) special

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[2]The analysis of nuclear piping systems is governed by the ASME Boiler and Pressure Vessel Code, Section III, Nuclear Power Plant Components.

programs for piping analysis, and (2) general purpose programs that can handle the analysis of many different types of structures including piping systems.

In an evaluation of computer codes for piping systems there are a number of features which are desirable for efficient and economical analysis:

1. Restart capability. A restart capability is desirable for dynamic, elastic-plastic, and large deformation analysis. In modal analysis the eigenvalues and eigenvectors are usually examined before the transient response or response spectrum analysis is carried out. If the examination shows that more eigenvalues are required a restart capability allows these to be computed without recalculating those already obtained. In any incremental analysis it is important to be able to examine the results at certain time intervals. With a restart capability the analysis can then be continued if the intermediate results are considered correct.

2. User oriented input and output. The ease with which the user can apply a computer program to his analysis problem is an important consideration in the selection of a program. Special purpose programs are usually easier to use than the general purpose programs since the user does not have to consider the many different capabilities available to him in the latter programs. The user documentation is very important as far as the ease of use of a program is concerned.

3. Documentation. The documentation is a weak point with most computer programs. Ideally the documentation should consist of: (a) User Manual with examples illustrating the use of the manual. (b) Theoretical Manual including derivation of matrices and vectors. The limitations of the assumptions made in the theory should be pointed out. (c) Verification Manual. Verification of the capabilities of a computer program is essential to assure correct results. Verification should consist of comparing the program results with those obtained with exact solutions, experimental results, other numerical procedures (such as finite differences), and other finite element computer programs. In the authors' opinion, many of the available computer programs have not been adequately verified. (d) Programmers Manual. For those programs in the public domain a manual which describes the organization of the program and its overlays, subroutines, and functions is very desirable.

4. Pre- and postprocessors. Preprocessors are computer programs that can be used to generate the finite element input for two- and three-dimensional problems. Postprocessors are computer programs that can be used to organize the output in such a form that the user can easily digest the data generated by the computer.

Efficient pre- and postprocessors usually have extensive plotting capabilities to graphically display the input and output.

Interactive graphics hardware now exist which permit the interaction between user and computer in real time for the generation of input data and the checking of output results.

5. Code requirements. An important factor in the choice of a piping analysis program is whether the program capabilities satisfy the requirements of the piping codes with regard to the treatment of elbows and tees. The stress output should preferably be in the manner specified by the code, this can to a large extent be accomplished by a postprocessor program.

6. Condensation and substructures. For elastic problems a capability to reduce the dynamic degrees of freedom by condensation is very desirable for reasons of economy.

For very large problems a substructuring capability can result in a considerable reduction in the cost of the analysis. For modal analysis the ability to combine condensation with substructuring is desirable, that is, the dynamic degrees of freedom for each substructure are reduced before the eigenvalues of the total structure are determined. The ability to handle repeated substructures is also a desirable feature. Composite mode synthesis has not been used extensively for large structural problems.

The following descriptions are of programs that are widely available. Proprietary programs that are only used in the developers organization have not been included. Some of the programs listed are in the public domain, others must be purchased or are available for use on a royalty basis. The main features of these programs are shown in Tables 1, 2, and 3.

Table 1 Dynamic Analysis Capabilities for Piping Analysis

Program Name	Conden-sation	Modal (Normal Modes)					Direct Time Integration				
		Natural Frequencies	Modal Time History			Response Spectra	Linear	Non-Lin Elem.	Plasticity	Large Def.	Seismic
			Linear	Non-Lin Elem. (Pseudo-force)	Seismic						
ADLPIPE	Yes	Yes	Yes	No	Yes	Yes	Yes	No	No	No	No
ANSYS	Yes	Yes	Yes	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes
MARC	Yes	Yes	No	No	No	No	Yes	Yes	Yes	Yes	No
NASTRAN	Yes	Yes	Yes	Yes	No	Yes	Yes	Yes	No	No	No
PIPDYN	Yes	Yes	Yes	No	Yes	Yes	Yes	No	No	No	Yes
PIPESD	Yes	Yes	No	No	No	Yes	No	No	No	No	No
SAGS/DAGS	No	Yes	Yes	No	Yes	Yes	No	No	No	No	Yes
SAP IV	No	Yes	Yes	No	Yes	Yes	Yes	No	No	No	Yes
STARDYNE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	No	Yes
WECAN	Yes	Yes	No	No	No	Yes	Yes	Yes	Yes	No	Yes

Table 2 Finite Elements for Piping Analysis

Program Name	Straight Pipe	Straight Tapered Pipe	Straight Beam With Shear	Elbow	Tee	Spring		Gap Elem.	Friction Elem.	Mass		Shell
						Lin.	Non-Lin.			Lumped	Consist	
ADLPIPE	Yes	No	Yes	Yes	Yes	Yes	No	No	No	Yes	No	No
ANSYS	Yes	No	Yes	Yes	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes
MARC	Yes	Yes	Yes	Yes	No	Yes	Yes	Yes	Yes	No	Yes	Yes
NASTRAN	Yes	Yes	Yes	No	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes
NUPIPE	Yes	No	Yes	Yes	Yes	Yes	No	No	No	Yes	No	No
PIPDYN	Yes	No	Yes	Yes	Yes	Yes	No	No	No	Yes	No	No
PIPENUP	Yes	No	Yes	Yes	Yes	Yes	No	No	No	Yes	No	No
PIPESD	Yes	No	No	Yes	Yes	Yes	No	No	No	Yes	No	No
SAGS/DAGS	No	No	Yes	Yes	Yes	Yes	No	No	No	Yes	No	No
SAP IV	Yes	No	Yes	Yes	No	Yes	No	No	No	Yes	No	Yes
STARDYNE	Yes	No	Yes	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes	No
WECAN	Yes	No	Yes	Yes	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Table 3 Additional Capabilities for Piping Analysis

Program Name	Restart	Sub-Structures	Conforms to Piping Code			Mesh Generator	Plotting	Temperature Dependent Material Properties	Documentation	
			Elbows	Tees	Stress Output				User Man.	Theory Man.
ADLPIPE	No	No	Yes	Yes	Yes	Yes	Yes	No	Yes	No
ANSYS	Yes	Yes	Yes	No	Yes	Yes	Yes	Yes	Yes	No
MARC	Yes	No	Yes	No	No	Yes	Yes	Yes	Yes	No
NASTRAN	Yes	Yes	No	No	No	No	Yes	Yes	Yes	Yes
NUPIPE	No	No	Yes	No	No	No	Yes	No	Yes	No
PIPDYN	Yes	No	Yes	Yes	Yes	No	Yes	Yes	Yes	Yes
PIPERUP	No	No	Yes	No	No	No	Yes	No	Yes	No
PIPESD	Yes	No	Yes	Yes	Yes	No	Yes	No	Yes	No
SAGS/DAGS	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No
SAP IV	Yes	No	Yes	No	No	No	No	No	Yes	No
STARDYNE	Yes	Yes	Yes	Yes	No	No	Yes	No	Yes	No
WECAN	Yes	No	Yes	No	No	Yes	Yes	Yes	Yes	No

ADLPIPE

ADLPIPE is a special purpose program for the static and dynamic analysis of complex piping systems.

It is basically an elastic analysis program which uses the method of transfer matrices in its formulation. It has capabilities for modeling straight pipes, springs, elbows (flexibility and intensity factors), linear constraints and also has a general stiffness matrix input capability.

A static condensation scheme is available and eigenvalues and eigenvectors for shock response spectra analysis can be obtained by the Jacobi, Givens, or Householder methods.

Stresses can be computed in accordance with a number of piping codes. Plots of both input geometry and deformed output geometry can be obtained. User manuals and some verification problems are also available.

Developer: Arthur D. Little Inc.
Acorn Park
Cambridge, Massachusetts 02140
Phone (614) 864-5770

Available from: 1. Developer
2. McDonnell Douglas Automation Co.
Box 516
St Louis, Missouri 63166
3. Westinghouse Electric Corporation
Power Systems Co.
Advanced Systems Technology
Building 7L45
700 Braddock Avenue
East Pittsburgh, PA 15112
Phone (412) 256-2853

Computers: CDC 7600 and 6000 series
Cost: User basis, purchase
Program Language: FORTRAN

ANSYS

ANSYS is a general purpose structural finite element program and is used extensively in industry for the linear and nonlinear analysis of structures, including piping systems. It is based on the displacement method of analysis and its dynamic capabilities include Guyan reduction, eigenvalue extraction (Jacobi), frequency response, modal and direct (Houbolt) transient response, seismic time history, and response spectrum analysis. A restart option is available. The elastic-plastic analysis capability is based on the constant stiffness (initial strain) method.

ANSYS has several elements for piping analysis such as straight pipes and beams, elbows (flexibility and stress intensification factors), linear and nonlinear springs, gap and friction elements. The program has a capability for large deformation analysis. A piping network generator, geometry plotting and a piping postprocessor for ASME code evaluations are available.

The documentation consists of a user's manual and an examples manual. User response indicates that the program is user oriented.

Developer: Swanson Analysis Systems, Inc.
870 Pine View Drive
Elizabeth, PA 15037
Phone (412) 751-1940

Available from: 1. Swanson Analysis Systems, Inc.
2. Control Data Corporation
Service Bureaus (Cyber network)

Computers: CDC 6400, 6600, 7600
IBM 370
Univac 1106, 1108

Cost: Use basis
Program Language: FORTRAN

MARC

The MARC program is a general purpose finite element program for the analysis of structures. The program has capabilities for linear static and dynamic analysis but was primarily designed for nonlinear analysis, plasticity, creep, and large deformations.

For dynamic analysis it has capabilities for eigenvalue-eigenvector extraction (inverse power), modal and direct (Newmark Beta, Houbolt, and Central difference operator) transient response analysis. Dynamic nonlinear analyses with elastic-plastic material behavior and large deformations can be obtained with the direct integration procedure.

The elastic-plastic analysis is based on the variable (tangent modulus) stiffness method and the large deformation analysis capability is based on the Lagrangian formulation. A restart option is available.

For piping analysis MARC has straight pipe elements, linear and nonlinear spring elements, gap and friction elements. A special element suitable for elastic-plastic and large deformation analysis of elbows is available. This element allows for deformation of the elbow cross-section. MARC does not have a tee element. The geometry input can be obtained with a special pipe line mesh generator. Plots of the undeformed geometry and the deformed geometry can be obtained. The present version of the program does not have a capability for seismic analysis.

The documentation of MARC consists of three volumes, namely, a user information manual, a program input manual, and a demonstration problems manual. Users report that the program is not easy to use, however, significant improvements have been made in recent versions.

Developer: MARC Analysis Corporation
105 Medway Street
Providence, Rhode Island 02960
Phone (401) 751-9120
Available from: Control Data Corporation
Service Bureaus (Cybernet Service)
Computers: CDC 6600
Cost: Use basis
Program Language: FORTRAN

NASTRAN

NASTRAN is a large general purpose finite element program that was developed for the National Aeronautics and Space Administration. The program was originally developed to meet the needs for structural analysis at the NASA research centers. The program is based on the displacement method and has capabilities for Guyan reduction, eigenvalue extraction (determinant, inverse power, Givens), frequency response, modal and direct (Newmark Beta) transient response analysis. A restart capability is available. Since 1970 NASTRAN has been used extensively by industry and government agencies, and has been verified by many users. NASA maintains the program through its NASTRAN Systems Management Office, which is also responsible for continuing development. For a large general purpose program NASTRAN is easy to use, however, since it has very extensive capabilities it takes considerable time to become acquainted with all its options. The documentation of NASTRAN is extensive and consists of user, theoretical, demonstration, and programmers manuals.

NASTRAN can perform elastic, static, and dynamic analyses of piping models consisting of straight pipe, linear and nonlinear spring, gap, and friction elements with a variety of supports, boundary conditions, and loadings. The program does not have elbow and tee elements or other capabilities designed

especially for piping analysis.

The time history due to ground motion can be obtained with NASTRAN using special techniques but the program has no capabilities for response spectrum analysis. No capabilities exist for large deformation analysis and its capabilities for elastic-plastic analysis are limited. Substructuring can be accomplished via the DMAP (Direct Matrix Abstraction Program) capabilities of NASTRAN. The program can produce plots of the undeformed and deformed geometry.

Developer: NASA, Washington, D.C.
Available from: COSMIC
Barrows Hall, University of Georgia
Athens, GA 36061
Computers: CDC 6600, IBM 370, Univac 1108
Cost: Nominal fee
Program Language: FORTRAN (some machine language routines)

A proprietary enhanced version of NASTRAN is also available.

Available from: The MacNeal-Schwendler Corp.
7442 Figueroa Street
Los Angeles, CA 90041
Phone (213) 254-3456
Also available at: Control Data Corporation
Service Bureaus (Cybernet Service)
Computers: CDC 6600, IBM 370
Cost: Use basis

NUPIPE

NUPIPE is a special purpose program for the analysis of linearly elastic piping systems. It is based on the finite element method and is capable of transient and response spectrum analysis using normal mode theory.

The program can handle a variety of loading conditions and time dependent forces. In addition, ground motion for seismic analysis can be specified as well as several displacement boundary conditions including general linear constraints.

Lumped mass matrices are used, and static condensation is available to reduce the dynamic degrees of freedom of piping systems.

The element library includes straight pipes with constant cross-sections, beams (with shear deformations), elbows (flexibility factor), tees (based on beam intersections), hangers, and concentrated mass elements.

Plots can be obtained of the deformed and undeformed geometry. No information was received on the ease of use of the program.

Developer: Nuclear Services Corporation
1700 Dill Avenue
Campbell, CA 95008
Available from: Developer
Computers: IBM 360, 370
CDC 6000, 7000 series
Cost: Use basis
Program Language: FORTRAN

PIPDYN

PIPDYN is a finite element piping program based on the displacement method. It has a capability for linearly elastic analysis with temperature dependent material properties and proportional viscous damping. It can handle concentrated, pressure, gravity, stationary and transient thermal and sinusoidal types of loading. It is also capable of handling prescribed and time dependent displacement boundary conditions as well as velocity and acceleration inputs. The program has a capability for condensation of the dynamic degrees of freedom.

Using a modal formulation response spectra, harmonic response, transient and reduced modal transient analyses can be obtained. The eigenvalue routines are based on the Givens and Householder QR methods.

A direct transient analysis capability using explicit time integration is also available.

It has straight pipe, elbows (curved beam theory), tees (modified beam intersection), and 6 x 6 general stiffness matrix input as modeling elements.

It can plot deformed and undeformed geometries. A theoretical manual and guides are available from the developer.

Developer: Franklin Institute Research Laboratories
20th Street and Franklin Parkway
Philadelphia, PA 19103
Phone (215) 648-1595

Available from: 1. Developer
2. Utility Network of America
7540 LBJ Freeway, Suite 830
Dallas, TX 75240
3. Westinghouse Electric Corporation
Power Systems Co.
Advanced Systems Technology
Building 7L45
700 Braddock Avenue
East Pittsburgh, PA 15112
Phone (412) 256-2853

Computers: IBM 360, 370, CDC 6500, 6600, 7600 UNIVAC 1108

Cost: Purchase, use basis

Program Language: FORTRAN

PIPERUP

This is a special purpose program for the analysis of piping systems. The program is based on the finite element method and has been written by the developers of NUPIPE. PIPERUP has, therefore, many features in common with NUPIPE.

The finite element library of PIPERUP consists of straight pipes, beams (with shear deformations), elbows (flexibility factor), tees (based on intersecting beams), hangers, concentrated mass, and spring-dampers.

The program can handle time dependent loadings and has special capabilities for the analysis of pipe whip.

The differential equations of motion are solved by direct methods based on explicit time integration.

Lumped mass matrices are used and damping is proportional to the mass and stiffness. The program has a capability for elastic-plastic analysis using isotropic hardening. Although the program claims a capability for pipe whip it is based on small deformations and strains.

Time history plots of forces and displacements can be obtained with PIPERUP.

A user manual which includes a number of verification problems is available.

Developer: Nuclear Service Corporation
1700 Dell Avenue
Campbell, CA 95008

Available from: Developer

Computers: CDC 6000, 7000 series

Cost: Use basis

Program Language: FORTRAN

PIPESD

PIPESD is a finite element program for calculating the dynamic response of piping systems using modal response spectra. A diagonal mass matrix is used and static condensation of the stiffness matrix is available. Natural frequencies and modes are calculated using Householder techniques.

Finite elements which are available are straight pipes, elbows (flexibility factor per ASME), tee elements (flexibility factor per ASME), hangers, and spring-dampers to ground.

Other capabilities include general linear constraints, plotting of deformed and undeformed geometry, restart calculation of ASME class 1 and class 2 stresses, output of complete class 1 fatigue stress reports, and performance of B31.1 power piping stress analyses.

Developer: URS/John A. Blume & Associates, Engineers
130 Jessie Street
San Francisco, CA 94303
Phone (415) 397-2525 (x34)
Available from: Control Data Corporation
Service Bureaus (Cybernet Service)
Computer: CDC 6600/7600
Cost: Use basis
Program Language: FORTRAN

SAGS/DAGS

The SAGS/DAGS programs are beam-truss type structural analysis programs which can be used for the dynamic analysis of linearly elastic piping systems through the use of straight, tapered, and curved beam elements.

Eigenvalues and eigenvectors are obtained by determinant or Sturm sequences techniques for the dynamic modal analysis. Damping can be incorporated based on proportional viscous, structural, hysteresis, or modal damping. Transient, harmonic, and shock response type analyses are available using modal superposition.

A variety of static and time dependent loadings and boundary conditions can be handled including seismic ground motions. A capability exists for plotting various views of deformed and undeformed geometry.

User manuals are available from the developer.

Developer: Structural Dynamics Research Corp.
5729 Dragon Way
Cincinnati, OH 45227
Phone (513) 272-1100
Available from: Developer
Computers: CDC 6000 series, UNIVAC 1108, GE 6080 and Xerox Z9
Cost: Use basis
Program Language: FORTRAN

SAP IV

The SAP IV program is a general purpose finite element program for the linearly elastic analysis of structural systems.

The program has capabilities for static, eigenvalue-eigenvector extraction (determinant method, sub-space iteration method), dynamic modal and direct (Wilson Theta) transient response analysis. In addition, seismic response spectrum and time histories based on relative displacements can be obtained with the program. A restart option is available to the user. The program is used extensively by industry, universities, and consulting firms and has been verified by many users. Users report that the program is user oriented and can be easily modified to suit the needs of a particular organization.

SAP IV can perform the elastic static and dynamic analysis of piping

systems consisting of straight pipe, elbow, spring, and beam elements. The program does not have a tee element. A variety of loadings and boundary conditions can be applied including ground accelerations.

The program manual giving user information, input data instructions, and sample problems is available. The program lacks plotting and mesh generating routines and does not have a condensation or substructuring capability. However, it is not proprietary and can be used as a starting point in the development of a more comprehensive piping system program.

Developer: E. L. Wilson
College of Engineering
University of California
Berkeley, CA 94720
Phone (415) 642-3743

Available from: Earthquake Engineering Research Center
University of California
Berkeley, CA 94720

Computers: CDC 6400, 6000, 7600
IBM 370, Univac 1106, 1108

Cost: Nominal fee

Program Language: FORTRAN

STARDYNE

STARDYNE is a general purpose finite element program for the static and dynamic analysis of elastic structures. It is based on the displacement method of analysis. STARDYNE is used extensively in industry for the analysis of elastic structures. It has capabilities for Guyan reduction, eigenvalue extraction (inverse power, Givens, and Householder), modal and direct (Newmark Beta and Wilson Theta) transient response, frequency response, seismic time history (using relative displacements), and response spectra analysis. The program has a restart capability and permits the use of substructures. User response indicates that the program is easy to use. STARDYNE has several elements for piping analysis such as straight pipes, beams, hangers, linear and nonlinear springs, elbows and tees (using flexibility factors), and gap elements. The program has no capabilities for large deformations or elastic-plastic analysis, nor does it have a special capability for mesh generation of piping systems and stress computations in the manner prescribed by the piping codes. The documentation consists of a user information manual. Plots for the deformed and undeformed geometry can be produced.

Developer: Mechanics Research Inc.
9841 Airport Blvd.
Los Angeles, CA 90045
Phone (213) 670-4650

Available from: Control Data Corporation
Service Bureaus (Cybernet Service)

Computers: CDC 6600

Cost: Use basis

Program Language: FORTRAN

WECAN

The WECAN program is a large general purpose finite element program for linear and nonlinear structural analysis. The program was developed for the analysis of structures used in the nuclear and power generation industry. It is based on the displacement method of analysis. Its dynamic capabilities include Guyan reduction, several routines for eigenvalue extraction (inverse power, Jacobi, Householder QR), frequency response, direct (Houbolt, Newmark Beta) transient response, seismic time history, and response spectrum analysis.

WECAN can perform elastic and elastic-plastic-creep analysis. Piping systems can be modeled with straight pipe and beam elements, elbows (flexibility and stress intensification factors), linear and nonlinear springs, hangers, spring-dampers, gap and friction elements. A variety of boundary and initial conditions can be used with these piping models. The program has a restart capability for linear and nonlinear dynamic analysis.

The elastic-plastic analysis capability is based on the constant stiffness (initial strain) method and can be used in dynamic analysis. The documentation consists of a user's manual, a demonstration manual, a pre- and postprocessor manual and a verification manual. A theoretical manual is under development. User response indicates that the program is user oriented. An extensive plotting capability is available for the generation of the deformed and undeformed geometry.

Developer: Westinghouse Electric Corporation
1310 Beulah Road
Pittsburgh, PA 15235

Available from: Westinghouse Electric Corporation
Power Systems Co.
Advanced Systems Technology
700 Braddock Avenue
East Pittsburgh, PA 15112
Phone (412) 256-2853

Computer: CDC 7600
Cost: Use basis
Program Language: FORTRAN

The descriptions of programs has been limited to those programs where enough information was available for the authors to give a general idea of the dynamic capability for analysis of piping systems. In addition, the program summaries are limited to programs which are currently available either through the developer or through service bureaus outside of the developers organization. For the sake of completeness the following list of programs is given which relate to either proprietary programs or to programs of which the detailed capabilities for the dynamic analysis of piping systems were not known to the authors. These additional programs are:

1. DRIPS (Dynamic Response in Piping Systems)
   Developer's Name: A. P. Gelman
   Affiliation: Rockwell International
   Atomics Internation Div.
   8900 DeSoto Street
   Canoga Park, CA 91304
   Phone (213) 341-1000
2. FESAP
   Developer's Name: D. B. VanFossen
   Affiliation: Babcok and Wilcox R&D
   P. O. Box 835
   Alliance, OH 44601
   Phone (216) 821-9110, ext. 544
3. MAGIC
   Affiliation: Air Force Flight Dynamics Laboratory
   Structures Division
   Wright-Patterson AFB, OH 45433

   or Bell Aerospace Company
   Structural Systems Department
   Post Office Box One
   Buffalo, NY 14240
   Attn: MAGIC System Office
4. Piping Systems Analysis
   Developer's Name: G. Belonogoff

Affiliation: Bechtel Corporation
350 Mission Street
San Francisco, CA 94105
Phone (415) 764-6324

5. SAP4GE (Adaptation of SAP IV)
Developer's Name: C. S. Parker
Affiliation: General Electric Corporation
175 Curtner Avenue
San Jose, CA 95114
Phone (408) 297-3000

6. ICES STRUDL II
Developer's Name: M.I.T. Civil Engineering Systems Laboratory
Affiliation: Massachusetts Institute of Technology
Cambridge, MA 02139

Further investigation int the availability, cost, and the present or future capabilities of these programs is suggested before choosing a particular program for a dynamic piping analysis.

FUTURE DEVELOPMENTS

The present state of the art computer programs for piping analysis have several weaknesses.

The available elbow and tee elements should be improved for elastic-plastic and large deformation and strain analysis. At the present time the complete state of the stress in an elbow or tee can only be obtained with a shell or three-dimensional analysis. This type of analysis is very costly since the number of degrees of freedom will be large. Such improved elements should also account for the influence of flanges and reinforcement.

The computer programs described above are structural analysis programs, all ignore the effects of fluid flow. Future developments should take fluid-structure interaction into account. Such a capability should eventually be able to account for large deformations and strains, plasticity, water hammer, slugging, pipe whip, etc.

Nonlinear (elastic-plastic, large deformations) dynamic analysis with state of the art computer programs is very costly. More efficient gorithms are needed to reduce the long computer running times. The use of explicit integration schemes for finite element analysis should be explored.

The preparation of the input data of the finite element mesh of a piping system is generally very time consuming. Additional work should be done in developing an interactive graphics capability for the generation of finite element meshes of piping systems.

Most computer programs lack the capability to output the stresses and strains in a form suitable for use in evaluations of piping systems in accordance with ANSI and ASME piping codes.

The verification of many computer programs leaves much to be desired. This is particularly true of the nonlinear capabilities. The development and publication of a set of piping problems for which experimental and theoretical results are available would be very useful for purposes of verification.

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Dynamic Buckling of Structures

Robert E. Ball

Naval Postgraduate School

INTRODUCTION

This chapter presents information intended to assist an analyst in the selection and use of a computer program for the dynamic buckling of beam, plate and shell type structures due to deterministic forces that are functions of time. Dynamic buckling is sometimes considered to be a subset of the more general subject of dynamic stability, which also includes oscillatory instabilities and aeroelastic or flutter problems. Several examples of some dynamic stability problems in actual aerospace structures were presented by Hedgepeth in [1] and are given in Tables 1, 2 and 3.[1] Dynamic buckling has also been used to denote an oscillatory instability or parametric resonance [5]. In this context, the problem of dynamic buckling is obtained simply by replacing a static load with one that varies harmonically in time. Hoff has suggested that perhaps the terms "sympathetic excitation" or "sympathetic resonance" would be more descriptive and acceptable than dynamic buckling or parametric resonance [6]. The more general concept of dynamic buckling is taken in the following discussion.

In theory, any computer program that computes the geometrically nonlinear, transient response of a structure can be used to predict dynamic buckling.[2] However, selection of the proper software and a correct interpretation of the results require both an understanding of the nature of the response of the structure to the load and a criterion for the inadmissible response or design failure. For an illustration of the physical aspects of the response that must be considered, consider the problem of the dynamic buckling of a shallow spherical cap subjected to a uniform step pressure load. Will the shell buckle axisymmetrically (snap-buckling) or must asymmetric (orthogonal) motion be considered? What about imperfections in the initial shape of the shell; must they be considered? Will the shell buckle elastically, or must inelastic material effects also be considered? Should damping be considered? Is the shell orthotropic; does it have discrete rings? Additionally, we must decide what constitutes buckling or failure; the maximum stress or the largest deflection? Is the minimum buckling load of interest, or is the structure only required to withstand a particular load? Is the post-buckling response of interest? These questions must be answered before a program can be selected because very few programs have more than a few of these features.

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[1] For an introduction to the stability problems associated with the control and propulsion of launch vehicles refer to [2] and [3] respectively. For a review of computer programs for aeroelastic stability calculations see [4].

[2] A review of some of the programs for the nonlinear, transient response of structures is given by Belytschko in this volume and in [7].

Table 1 Transient or Shock Loading [1]

Dynamic stressing	Dynamic buckling
X-15 landing	Titan II staging
Minuteman launch	Subroc water impact
Apollo impact	Explosive forming
Sprint launch	Energy absorption
Warhead blast	
Other	
Echo I deployment	
Pyrotechnic shock	

Table 2 Oscillatory Instability [1]

Control system coupling	Propulsive system coupling
Vanguard	Thor
X-15	Kiwi
Redstone	Titan II
Atlas	
Scout	
Saturn I	
Other	
Saturn I slosh	
Pilot coupling	

Table 3 Flutter [1]

Control surfaces	Panel
KC-135 fin, rudder & tab	V-2 ogive
Bomarc vanes	X-15 fairing
FJ4 stabilizer	A3J fuselage
P6M T-tail	F4H wing
F3H rudder	
XP5Y stabilizer	
Other	
Electra propeller-whirl	

To add to the analyst's problems is the fact that many computer programs for nonlinear dynamic response are more difficult to use than their linear counterparts. Have enough elements or nodes been used to assure spatial convergence of the solution? Are they distributed properly? Is the time step small enough to give a converged solution; is it small enough to assure numerical stability? Is the time step too small, so that too much computer run time is used? Have enough time steps been taken to assure that buckling will not occur? Is it necessary to iterate at each time step, and if so, how many iterations should be taken? Is it necessary to refactor the governing matrices during the response computation, and if so, how often? Will the method of solution give a solution up to buckling, and into the post-buckling region? Will an iterative procedure fail to converge prior to buckling, or does the failure of an iterative procedure to converge mean buckling has taken place?

How much confidence can be placed in the results; i.e. how well do results from one program agree with those from another program and how well do the solutions agree with experimental results? And last, but certainly not least, is the fact that determining the dynamic buckling load of a structure can be very expensive relative to the cost of obtaining linear dynamic response solutions, which are themselves expensive to obtain.

As a consequence of these difficulties, and the peculiarities of each particular problem, progress in this area has been slow. Most of the computer studies on dynamic buckling that have appeared in the open literature use programs that were written specifically for the problem considered. The developers usually had no intention of making their program available to others. This is understandable since preparing a fool-proof program for dissemination requires a considerable amount of work. Nevertheless, there are a number of general purpose and special purpose programs readily available that can be used to determine the dynamic buckling of structures.

NOMENCLATURE

E = Young's modulus
I = Moment of inertia
L = Column length
m = Mass per unit length
P_E = Euler buckling load
P_o = Axial load
P_1 = Harmonic part of axial load
q_n = Generalized coordinate
t = Time
w = Lateral displacement
w_I = Initial imperfection
w_n = Series coefficient of initial imperfection
x = Axial coordinate
Ω = Frequency of oscillation of axial load
$\bar{V}$ = Nondimensional volume of deformation
P = Nondimensional load factor
λ = Shallowness parameter for spherical shells

EXAMPLES OF DYNAMIC BUCKLING

Dynamic buckling means different things to different people. Consequently, a brief description of several kinds of dynamic buckling is presented here to assist the analyst in his search for the proper computer program and to illustrate some of the features he must consider.

Symmetric (Snap) Buckling

Two examples of dynamic symmetric or snap buckling are the simply supported shallow arch [8] and the clamped shallow spherical cap [9] subjected to a symmetric external load.[3] Dynamic snap buckling is usually characterized by a rapid and significant geometrically nonlinear growth in some response parameter such as the average displacement or the volume of deformation. Asymmetric motion is excluded from consideration in a symmetric buckling analysis. For certain arch and shell geometries pure symmetric buckling occurs naturally and the exclusion of asymmetric motion is inconsequential. If the analyst is

[3]Symmetric refers here to an axis or plane of symmetry. A cylindrical shell subjected to a pressure that is symmetric about a principal plane containing the axis is said to buckle symmetrically if antisymmetric motion about that plane does not influence the buckling.

interested only in whether or not a structure can withstand a specific load then he is required only to make a single geometrically nonlinear run to compute the response (deflections, stresses, etc.) of the structure to that load.[4] An examination of the solution will reveal whether or not the response of the structure is within allowable bounds. On the other hand, if he is interested in the minimum dynamic buckling load then several runs must be made with different magnitudes of the load. For example, Fig. 1, taken from [10], shows a computed time history of the nondimensional volume of deformation $\overline{V}$ of a clamped shallow spherical cap for two values of a nondimensional uniform pressure load P. Note the significant difference in the maximum value of $\overline{V}$ for the two loads. A typical plot of $\overline{V}_{max}$ versus P is shown in Fig. 2, where $\overline{V}_{max}$ is the maximum value of $\overline{V}$ over the response history. For a certain range of P, where the nonlinearities are small, the relationship between $\overline{V}_{max}$ and P is approximately linear. There may exist, however, a certain value of P at which a very small increase in P produces a very large increase in $\overline{V}_{max}$, as illustrated in Fig. 2.[5] This load has been defined as the critical load for symmetric buckling [12].

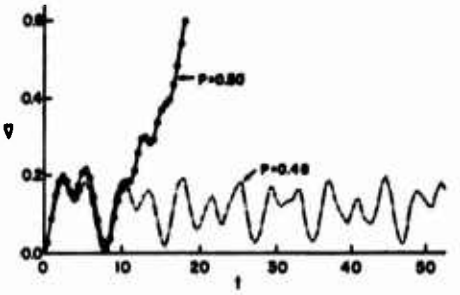

Fig. 1 Time history of $\overline{V}$ for two values of P, [10]

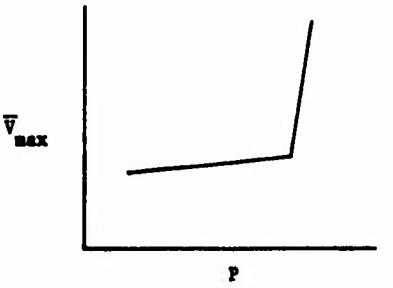

Fig. 2 Typical plot of $\overline{V}_{max}$ versus P

When computing the response of the structure to the load, the computations must be carried out for a sufficiently long period of time to assure that buckling does, or does not, take place since symmetric buckling does not always occur as early as that shown in Fig. 1. A plot taken from [10] of the critical uniform step pressure of clamped shallow spherical shells P_{crit}, computed using several computer programs, versus λ, a shallowness parameter, is given in Fig. 3 to illustrate the degree of agreement between current programs for dynamic snap buckling.

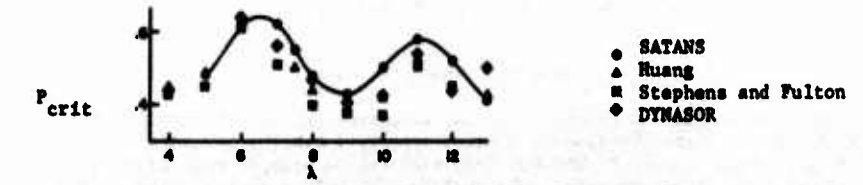

Fig. 3 Critical axisymmetric step-pressure load versus λ, [10]

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[4]Most geometrically nonlinear theories are based upon small strains and moderately small rotations.

[5]Very shallow clamped spherical caps subjected to uniform step pressure loads and clamped arches subjected to impulsive loads do not exhibit a sharp change in $\overline{V}_{max}$ versus the magnitude of the load and the impulse [10, 11]. In these problems there is no well-defined symmetric buckling load.

Asymmetric Buckling

Asymmetric buckling is loosely defined here to be buckling that takes place due to motion in an asymmetric mode or a mode orthogonal to the loaded modes. A simple example of asymmetric buckling is the axially loaded, initially imperfect column. Consider the simply supported, prismatic column with an initial imperfection w_i expressed in the series form

$$w_i = \sum_{n=1}^{\infty} \bar{w}_n \sin\frac{n\pi x}{L} \tag{1}$$

where L is the length of the column and x is the axial coordinate. When the column is subjected to an axial dynamic load, the displacement of the column will consist of an axial component u, corresponding to symmetric motion, and a lateral component w corresponding to asymmetric or orthogonal motion. If the axial motion is approximated by a uniform load over the length of the column, small lateral motion is governed by the linear equation

$$EI\, w'''' + P_o(t)(w'' + w_i'') + m\ddot{w} = 0 \tag{2}$$

where E is Young's modulus, I is the moment of inertia, m is the mass per unit length, and primes and dots denote partial differentiation with respect to x and time t respectively. The solution to Eq. (2) can be taken in the form

$$w = \sum_{n=1}^{\infty} q_n(t) \sin\frac{n\pi x}{L} \tag{3}$$

Substituting Eqs. (1) and (3) into Eq. (2) leads to the equations of motion in the generalized coordinates q_n

$$\ddot{q}_n + \omega_n^2 \left[1 - \frac{P_o(t)}{n^2 P_E}\right] q_n = \omega_n^2 \frac{P_o(t)}{n^2 P_E} \bar{w}_n \qquad n = 1, 2, \ldots . \tag{4}$$

where

$$P_E = \left(\frac{\pi}{L}\right)^2 EI$$

$$\omega_n^2 = \left(\frac{n\pi}{L}\right)^4 \frac{EI}{m}$$

The equations of motion in the generalized coordinates are uncoupled because the static buckling modes are identical to the free vibration modes. When P_o is larger than n^2P_E, the Euler buckling load, the solution to Eq. (4) is hyperbolic in nature, i.e. the displacement w will grow exponentially in time. When P_o is less than n^2P_E the solution will oscillate. For example, if P_o is a step function in time

$$w = \sum_{n=1}^{\infty} \frac{\bar{w}_n}{1-\eta_n} \begin{bmatrix} \cosh \\ \cos \end{bmatrix} \mu_n t - 1 \sin\frac{n\pi x}{L} \tag{5}$$

where

$$\eta_n = n^2 P_E / P_o$$

$$\mu_n = \omega_n \left| (\eta_n - 1)/\eta_n \right|^{\frac{1}{2}}$$

The cosh is used when $\eta_n < 1$ and the cos is used when $\eta_n > 1$. Thus, when $\eta_n > 1$, the computed response will be bounded, but when $\eta_n < 1$ those modes whose static buckling load is less than P_o will grow exponentially in time. This simplified linear theory is valid when the rise time of the end load is relatively short compared to the natural frequencies of the lateral modes and relatively long compared to the time required for an elastic wave to traverse the length of the column. The situation where elastic and plastic dynamic buckling due to a suddenly applied thrust at one end occurs in times so short that an elastic compression wave has traversed only one or two column lengths has been examined in [13].

The cylindrical shell subjected to a dynamic axial load will respond with both axisymmetric motion and asymmetric motion. If the internal compression load in the shell is not assumed to be effected by the deflections, then linear equations of motion similar to Eq. (4) arise in which the generalized coordinates are the axisymmetric and asymmetric modes. However, when consideration is given to the nonlinear effects in the compatibility equations or strain-displacement relations, i.e. when the internal compression load becomes a function of the displacements, the equations of motion become nonlinear. The nonlinear equations are normally solved by assuming a displacement function with several arbitrary parameters that are functions of time and employing a Galerkin type procedure. This leads to a set of coupled ordinary differential equations in time which are usually solved using a Runge-Kutta procedure [14].

Both the uniform step-pressure loaded, simply supported, shallow arch [8] and clamped shallow spherical cap [10] have been found to buckle (at certain geometries) at a load below the critical load for symmetric buckling when asymmetric motion is considered. Motion of the asymmetric modes can be initiated by introducing a very small asymmetric load or imperfection or by numerical roundoff errors. It has been found that this asymmetric motion can grow large and can significantly influence the symmetric response.

Several definitions of asymmetric dynamic buckling have been proposed. In one [8], the buckling load is defined as the minimum load for which a small change in the load produces a relatively large change in the asymmetric response, which is the same criterion as that for symmetric buckling except that it is applied to the asymmetric response. This may be a conservative criterion since the asymmetric motion at the critical load may be relatively small. Another suggestion is that the criterion for symmetric buckling also be used for asymmetric buckling [10]. This criterion is based upon the argument that the symmetric deformation is the only true measure of buckling. The structure can suffer large asymmetric displacements, but these displacements in themselves do not represent a catastrophic buckling failure. This criterion is unconservativ If the symmetric response is not significantly influenced by the asymmetric motion, the buckling has been referred to as direct snapping. If the asymmetric motion reduces the critical load for symmetric buckling, the buckling has been called indirect snapping [14]. Another criterion for asymmetric buckling has been proposed in which the critical load is a bifurcation load. It is determined by examining the symmetric response at each step in time to see if static bifurcation from the symmetric path can take place [16].

Parametric (Sympathetic) Resonance

Parametric resonance refers to an instability that arises as the result of an harmonically oscillating forcing function. The forcing function can be an

external load or it can be due to motion of the structure itself. An example of the former is the initially straight, simply supported prismatic column subjected to a uniform compressive axial load $P = P_o + P_1 \sin \Omega t$. The equation of disturbed lateral motion of the column can be given in the form

$$EIw'''' + (P_o + P_1 \sin \Omega t)w'' + m\ddot{w} = 0 \qquad (6)$$

The essential feature of parametric resonance is the presence of the harmonic coefficient of w'' in Eq. (6). The solution to Eq. (6) can be taken in the form given by Eq. (3), which leads to

$$\ddot{q}_n + (\mu_n^2 - \beta_n^2 \sin \Omega t)q_n = 0 \qquad n = 1, 2, \ldots. \qquad (7)$$

where

$$\beta_n^2 = \omega_n^2 P_1/(n^2 P_E)$$

Equation (7) is the linear Mathieu differential equation. There are two possible solutions to Eq. (7) depending upon the values of μ_n and β_n. Both of them are oscillatory, but one grows unbounded with increasing time. The distinction between a bounded response and an unbounded one is given by the Mathieu stability chart.

In the above example, the mode for static buckling is identical to the mode for free vibration. This fact led to uncoupled equations of motion. In problems where the static buckling modes are not identical to the free vibration modes it has been the custom to use either the modes of free vibration or the static buckling modes as the generalized coordinates in a Galerkin procedure. This leads to coupled second order ordinary differential equations of the Mathieu type. Usually only a few modes are used, with the potential consequence that any unstable behavior may not be adequately described by the limited basis. This feature has been considered in [17] where the finite element method is used to investigate the dynamic stability of rectangular plates subjected to harmonically oscillating edge loads. Use of the finite element method eliminates the necessity of selecting generalized coordinates, but it does not eliminate the potential problem of inadequately describing the unstable behavior since the discretized model may be too coarse or the elements may be poorly sized.

An example of parametric resonance due to the motion of the structure itself is the dynamic response of a complete sphere subjected to a nearly uniform radial impulse [18]. Under the impulsive load the initial growth of the asymmetric modes is governed by the Mathieu equation. The harmonic coefficient in the Mathieu equation is due to the harmonic free vibration of the uniform radial mode. However, due to nonlinear coupling between the modes, the modes undergo beat-like oscillations accompanied by a cyclic exchange of energy. The linear Mathieu equation, which considers only the nonlinear coupling from the symmetric mode to the asymmetric modes and neglects the effects of the asymmetric modes upon each other and upon the symmetric mode, does not account for the energy exchange and predicts instability when in fact there is no instability. In this instance a stress or deflection criterion must be used to define a safe design.

Combined Buckling

For an example of dynamic buckling in which there is both asymmetric buckling and parametric resonance, consider the problem of a simply supported, nearly-circular cylindrical shell subjected to an exponentially decaying uniform radial pressure first considered in [19]. In [19] radial motion was assumed to consist of a uniform radial displacement plus asymmetric modes that satisfy simply supported boundary conditions. The influence of the axisymmetric mode on the asymmetric modes was the only coupling retained; no coupling was retained which might extract energy from the axisymmetric mode, nor was any coupling between asymmetric modes retained. The oscillation of the axisymmetric mode was

neglected. Hence the equation of motion in each asymmetric mode was of the same form as Eq. (4). The maximum displacement of several asymmetric modes was found by numerical integration, and dynamic buckling was assumed to have occurred whenever the maximum displacement of any mode exceeded 1,000 times the assumed initial imperfection in that mode. Actually, the asymmetric response of the cylinder is essentially composed of two types of modes; modes whose static buckling pressure is less than the applied pressure (hyperbolic modes), and modes that are excited by the harmonic oscillation of the axisymmetric mode (Mathieu modes). Modes in the first category grow in time in an exponential manner, while modes in the second category grow in time in an oscillatory manner, as governed by the classical Mathieu equation. Only the former modes were considered in [19]. The writers reasoned that the parametrically excited modes would dissipate energy locally by plastic flow and, therefore, would eventually decay before causing deflections large enough to buckle the shell. In order to examine the validity of the neglect of the Mathieu modes, a computer program was used in [20] to compute the total nonlinear response of the shell to the uniform, exponentially decaying, radial pressure. These results for a cylinder with an initial imperfection of 1×10^{-3} times the thickness in each mode showed that at the lower peak pressures and total impulses the Mathieu modes dominate the response. As the peak pressure and total impulse are increased, the hyperbolic modes become more responsive. Near the dynamic buckling threshold, the hyperbolic modes completely dominate the Mathieu modes. The critical dynamic buckling loads computed using the fully coupled nonlinear analysis agree very well with those of [19]. However, at peak pressures and total impulses well below the dynamic buckling threshold, stresses in excess of the yield stress were found in the shell due to the combined response of the Mathieu and hyperbolic modes.

METHODS OF SOLUTION

In order to determine whether or not a structure has buckled dynamically, the equations of motion of the structure, including geometrically nonlinear effects, must be solved for a sufficiently long period of time. The discrete forms of the equations of motion in most computer programs are obtained in one of the following general ways: (1) by approximating the spatial partial derivatives in either the differential form of the equations of motion or the energy integral with finite differences (the finite difference method), (2) by assembling a number of finite elements to represent the structure (the finite element method), (3) by representing the solution as a series of preselected modes in conjunction with a trial solution method such as Galerkin (the modal method) or (4) by some combination of the above three techniques, such as representing the solution as a series of harmonic modes in one coordinate and using finite differences or finite elements for the other coordinate, as has been done for the shell of revolution. A more detailed presentation of these methods for shell analysis is given in [21].

The timewise integration of the nonlinear equations of motion is usually accomplished using either explicit or implicit integration methods or Runge-Kutta type methods. A general discussion of the first two methods, with their advantages and disadvantages in nonlinear applications, is given in [7]. The Runge-Kutta methods are usually used in conjunction with the modal method when only a few modes are considered. In general, implicit methods require the solution of a system of simultaneous nonlinear algebraic equations at each time step, explicit methods do not. Implicit methods are more numerically stable than explicit methods for linear problems, and hence larger time steps can be used. However, for nonlinear problems the use of large time steps in the implicit methods may degrade the nonlinear solution, due to the presence of inelastic effects and high frequency modes from the geometrically nonlinear coupling. Furthermore, the numerical stability limits of implicit methods for nonlinear problems are unknown at the present time. Hence the time step used in implicit methods for nonlinear problems is usually of the same size as the time step for explicit methods. The importance of choosing the most efficient

method of integration is made vivid in [7] where it is shown that a nonlinear analysis of a 31x62 node plane mesh by an implicit method requires 150 times as many computations per time step as the explicit method, whereas it requires less than twice as many computations per time step as the explicit method for the axisymmetric nonlinear analysis of a 100 node cylindrical shell.

The modal superposition method commonly used in linear problems has recently been proposed as an alternate method of solution to nonlinear problems [22]. The modes and frequencies of the structure would be recomputed at several points in time over the response history. The assumption is made that the response is linear over each time interval between these points. This gives a kind of piece-wise linear transient response. The method appears to be most applicable to those problems in which only a few modes are important.

A finite element program for shells and bodies of revolution that uses convected coordinates has recently been developed in an attempt to reduce the excessive computation time required for a nonlinear transient analysis [23]. Each element is associated with a coordinate system that rotates, but does not deform, with the element. The equations of each element are geometrically linear. The nonlinearities that arise from large rotations are accounted for entirely by transformations between the global and the convected coordinates, and the omission of the rigid body motion in the strain-displacement relations. The resulting finite element equations are considerably simplified.

MODELING GUIDELINES

Correct numerical modeling of a structure for a dynamic buckling analysis is more difficult than for a corresponding linear analysis. This is due to the fact that the critical mode (or modes) of behavior is usually not known prior to the analysis. The dynamic buckling of the pressure loaded cylinder described above is an example of this. When using computer programs that employ the superposition of modes technique, care must be taken to assure that no important mode is omitted. Spatial convergence of the solution must be verified. At the present time there are very few guidelines to assist the user in his selection of modes to include. Use of a program that employs finite differences or finite elements for the spatial discretization does not avoid this problem because it is possible that an insufficient number of nodes or elements might be used or the elements may not be properly sized to adequately represent the buckling behavior. If only a few modes are important then the use of a finite difference or finite element program may be extremely expensive compared to a modal program. A classical example of this is the static asymmetric buckling of an axisymmetrically loaded perfect shell of revolution. Only two modes are important here, the axisymmetric mode and the critical asymmetric mode. For very thin shells the asymmetric mode has many waves around the circumference, and hence a finite difference or finite element program would require many more degrees of freedom to adequately model the asymmetric mode.

Very little printed information concerning the use of public and commercially available computer programs for dynamic buckling has appeared in the literature. Perhaps the most revealing study to date is the comparison of results for the linear and geometrically nonlinear response of an impulsively loaded truncated cone presented in [24]. The codes DYNAPLAS, SATANS, SMERSH, SHORE, and REPSIL used either 30 elements or 31 finite difference stations along the meridian and either 4 harmonics or 19 stations around half of the circumference. All programs used a 2 μsec time step. The five programs gave essentially identical linear results for all response quantities except the meridional strain at the outer surface. Figure 4a, taken from [24], illustrates the agreement between the results from the five codes for the linear analysis. In the nonlinear analysis the impulse was increased by a factor of 25. There is fair agreement between the nonlinear results for the first 750 μsec, as illustrated in Fig. 4b, taken from [24]. However, for later times the solutions, except for DYNAPLAS and SATANS, can be seen to

progressively diverge.[6] Preliminary studies indicate that the spatial discretization is inadequate for the nonlinear analysis. This is a clear illustration of the necessity for including all of the important modes. It also points out that although programs are available to compute dynamic buckling the results must be verified. Much work needs to be done to develop a greater understanding of the important aspects of nonlinear analyses. The run times for 750 steps for the linear analysis varied from 11 to 26 minutes on CDC 6600 and UNIVAC 1108 computers for codes that used a two-dimensional discretization, and between 2.5 to 5 minutes on IBM 360/65 computers for the codes that used a one-dimensional discretization. The large differences in run times between the one-dimensional codes and the two-dimensional codes is due in large part to the fact that only four harmonics were used as opposed to 18 circumferential increments. The run times increased to 26-41.5 minutes and 7.8-9.5 minutes respectively for the nonlinear analysis.

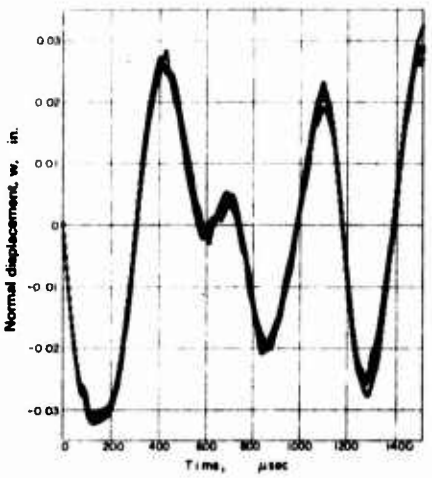

Fig. 4a Normal displacement, s = 6.5 in., θ = 0°, linear analysis, [24]

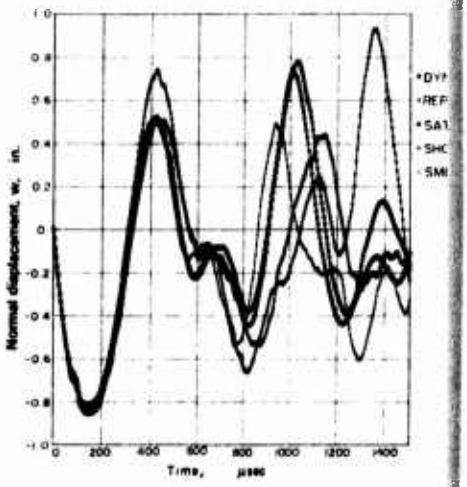

Fig. 4b Normal displacement, s = 6.5 in., θ = 0°, nonlinear analysis, [24]

INITIAL IMPERFECTIONS

Many static buckling studies have shown that initial imperfections in the shape of the structure have a significant effect upon the buckling load. This is especially true for shell buckling problems like the axially loaded cylindrical shell and the pressure loaded spherical cap. In contrast, only a few studies of the effect of imperfections upon the dynamic buckling of structures have been made [14, 25, 26]. Nevertheless, imperfection sensitivity can be expected to be as significant in dynamic buckling as it is in static buckling [27]. Thus, any program that is to be used for a potentially imperfection-sensitive buckling problem should have a capability for including arbitrary imperfections in the shape of the structure, either as a perturbation from a standard shape like a cylinder or sphere or as an arbitrary three-dimensional definition of nodal space coordinates. The difficulty with imperfection studies is the fact

[6] Results from an updated version of SHORE are different than those shown in Fig. 4b. The latest results are in agreement with the other results up to 600 μsec.

that the actual imperfections in the real structure are usually not known at the time of the analysis. Thus, most studies have concentrated upon the effect of some arbitrary imperfection, usually in the critical buckling mode. However, there often is a construction specification, such as the maximum out-of-roundness a shell can have, and this can be used to define an initial imperfection in shape. Much work needs to be done in this area.

A method for determining the dynamic buckling loads of imperfection-sensitive structures that avoids the computation of the nonlinear transient response has been presented in [28], where it is assumed that the perfect structure has a classical static bifurcation buckling load λ_C. Due to very small geometrical imperfections, the imperfect structure buckles at λ_S a load less than the bifurcation load. The assumption is made that the dynamic response can be described adequately in terms of the deformation pattern that occurs when the structure buckles statically. Also, prebuckling inertia is neglected and the degrees of nonlinearity higher than the lowest are disregarded. Thus, parametric resonance is not considered. The results of the method provide an estimate for the ratio of the dynamic buckling load λ_D to λ_S as a function of only the ratio λ_S/λ_C. Although the reliability of these results has not been tested as yet, the available evidence appears to support the conjecture that structures that are imperfection-sensitive under static load will display the same property under dynamic load [14, 25, 26].

AVAILABLE SOFTWARE

In general, programs that compute the geometrically nonlinear, transient response of structures can be used to predict dynamic buckling. Some of the public and commercially available programs are ANSYS [29], DYNAPLAS/DYNASOR [30], JET3 [32], MARC [33], NONLIN2 [34], PETROS3 [35], REPSIL [36], SABOR/DRASTIC6 [37], SATANS [38], SHORE [39], SMERSH [40], STAGS [41], TROCS [42], UNIVALVE [43] and WHAM [23].[7] Information about these programs can be found scattered throughout [46]. The sources for these programs are given in Appendix II.

The above programs are usually classified as either general purpose programs or special purpose programs, depending upon the number of different structures they can treat. The general purpose codes for dynamic buckling are ANSYS and MARC. The special purpose programs are listed in Table 2 according to the kind of structure or element they consider.

A comparison of the capabilities of ANSYS, MARC, NONLIN2, PETROS3, REPSIL, STAGSA1, STAGSB, DYNAPLAS II, DYNASOR II, SABOR/DRASTIC6, SATANS, SHORE, and TROCS is given in Appendix II. The information presented was obtained from the COMSTAIRS Systems, Lockheed Palo Alto Research Laboratory, Palo Alto, California [47].

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[7]The nonlinear dynamic response programs for two-dimensional solids and three-dimensional bodies of revolution, such as AFTON, DYNS, HONDO, NONSAP, PISCES and TOODY will not be discussed here. These codes do not have plate or shell elements; hence, any plate or shell structure requires several elements through the thickness [44]. Descriptions of these elements are in [45].

Table 2 Special Purpose Programs

Beam	Ring	Plate	Shell	Shell of Revolution Axisym.	Shell of Revolution Arbitrary
NONLIN2	JETS	WHAM	IMAN	WHAM	DYNAPLAS/
UNIVALVE	NONLIN2		PETROS3		DYNASOR
WHAM	UNIVALVE		REPSIL		SABOR/
			STAGS		DRASTIC6
			(WHAM)		SATANS
					SMERSH
					SHORE
					TROCS

A brief description of the codes not contained in Appendix B follows:

IMAN: Finite element (curved quadrilateral shell element), Runge-Kutta integration, elastic, FORTRAN V, UNIVAC 1108, EXEC 8, 1800 cards, available from COSMIC.

JET3: Finite element (structural ring, complete or partial, arbitrarily curved with variable thickness), explicit or Houbolt implicit integration, elastic-plastic.

SMERSH: Finite difference, two bonded concentric shells of revolution to arbitrary impulse and internal stress loading, explicit integration, elastic-plastic, FORTRAN IV, limited distribution.

UNIVALVE: Finite difference (beams and rings, arbitrary loads), explicit integration, elastic-plastic, available from COSMIC.

WHAM: Finite element (beam -2D and 3D, flat triangular plate -2D and 3D, axisymmetric shell and body of revolution), explicit integration, elastic-plastic.

SUMMARY

This chapter considers computer programs for the dynamic buckling of beam, plate and shell type structures due to deterministic forces. The material presented is intended to assist the analyst in his selection of the proper software and in the development of a meaningful numerical model for dynamic buckling. Several examples of dynamic buckling are described to illustrate the the various aspects that must be considered such as snap buckling, asymmetric buckling and parametric resonance. The most common methods of solution are briefly discussed and some modeling guidelines are given. Results from several programs are presented for two examples to indicate the degree of agreement that can be expected from current programs. The effects of initial imperfections are briefly considered. Most of the public and commercially available programs for the geometrically nonlinear, dynamic analysis of thin beam, plate and shell structures are described in considerable detail using information from the COMSTAIRS Systems. User evaluations of many of the programs are listed, and the sources for, and references to, the programs are provided.

ACKNOWLEDGEMENT

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APPENDIX I

SOURCES FOR THE COMPUTER PROGRAMS

ANSYS	Swanson Analysis, Inc., 870 Pine View Drive, Elizabeth, PA 15037
DYNAPLAS/ DYNASOR/ SAMMSOR	Prof. W. Haisler, Dept. of Aerospace Eng., Texas A&M Univ., College Station, TX 77843 (COSMIC)
IMAN	K. K. Gupta, Jet Propulsion Lab., Pasadena, CA (COSMIC)
JET3	Prof. E. Witmer, Dept. of Aeronautics and Astronautics, MIT, Cambridge, MA 02139
MARC	Dr. P. Marcal, MARC Analysis Corp., 105 Medway St., Providence, RI 02906 (CDC-CYBERNET)
NONLIN2	J. Anderson, Sargent & Lundy, 1405, Dearborn, Chicago, IL 60603
PETROS3	Dr. N. J. Huffington, Jr., U. S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, MD 21005 (Government users)
REPSIL	See PETROS3
SABOR/DRASTIC6	Dr. S. Klein, Ohio Ford Corp., Ford Rd., Newport Beach, CA 92663
SATANS	Prof. R. Ball, Code 57Bp, Naval Postgraduate School, Monterey, CA 93940 (COSMIC, Ref. LAR-11109)
SHORE	P. Underwood, Lockheed Palo Alto Research Lab., 2351 Hanover St., Palo Alto, CA 94304
SMERSH	Dr. P. Wieselmann, Kaman Sciences Corp., P. O. Box 7463, Colorado Springs, CO 80933
STAGS	B. Almroth, Lockheed Palo Alto Research Lab., 3251 Hanover St., Palo Alto, CA 94304
UNIVALVE	R. Kreig, Analytical Development Div., Sandia Lab., Albuquerque, NM 87115
WHAM	Prof. T. Belytschko, Dept. of Materials Engineering, Univ. of Illinois, Chicago Circle, IL 60680

APPENDIX II

COMPARISONS OF ANSYS, MARC, NONLIN2, PETROS3, REPSIL, STAGSA1, STAGSB, DYNAPLAS II, DYNASOR II, SABOR/DRASTIC6, SATANS, SHORE, AND TROCS FROM THE COMSTAIRS SYSTEMS

This appendix contains a computer printout listing the properties of the programs. The key to reading the columns follows:

column	program described
1	ANSYS
2	MARC
3	NONLIN2
4	STAGSA1
5	STAGSB
6	PETROS3
7	DYNAPLAS II
8	DYNASOR II
9	REPSIL
10	SABOR/DRASTIC6
11	SATANS
12	SHORE
13	TROCS

Geometry and Boundary Conditions

```
                                                          1111
                                                  1234567890123
01 GENERAL STRUCTURE ..........................  XX...........
02 SOLID OF REVOLUTION.........................  XX...........
03 GENERAL SHELL...............................  XX.XXX..X....
04 PART OF SHELL OF REVOLUTION ( L.T. 360 DEG.   XX.XXXX.X..XX
05 SHELL OF REVOLUTION (360 DEG.) .............  XX.XXXXXXXXX
06 PRISMATIC STRUCTURE.........................  XXXXX........
07 SEGMENTED (IN SERIES) ......................  XX.XX....X...
08 BRANCHED ...................................  XX..X....X...
09 SHELL WITH CUTOUTS .........................  XX.XX........
10 THICK SHELL ................................  XX...........
11 FRAMEWORK-SHELL COMBINATIONS ...............  XX.XX........
12 FLAT PLATE OF GENERAL SHAPE ................  XX.XXX..X....
13 OTHER GEOMETRY--SEE SECT.8, 1.13 ..........  XX..........X
14 AXISYMMETRIC IMPERFECTIONS .................  XX.XX....X...
15 GENERAL IMPERFECTIONS ......................  XX.XX....X...
16 AXISYMMETRIC SUPPORT CONDITIONS.............  XX.XX.XX..X..
17 GENERAL SUPPORT CONDITIONS .................  XX.XX.XX.XX..
18 SUPPORTS AT BOUNDARIES ONLY ................  XX...X..X.XXX
19 SUPPORTS AT INTERNAL POINTS ................  XXXXX.XX.X.X.
20 ELASTIC FOUNDATION .........................  XXX.....X.XX
21 STIFFENERS .................................  XX.XX....X...
22 CONTACT.....................................  XX...........
23 FRICTION ...................................  XXX..........
24 SLIDING WITHOUT FRICTION ...................  XX.XX........
25 OTHER DEFORMATION DEPENDENT SUPPORT--SEE SE  .X.........X.
26 JUNCTURE COMPATIBILITY .....................  XX..X........
27 SINGULARITY CONDITIONS .....................  .X....XX.....
28 GENERAL LINEAR CONSTRAINT COND. U(I)= TU(N)  XX...........
```

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                                                                  1111
                                                 1234567890123
29 GENERAL NONLINEAR CONSTRAINT CONDITIONS       XX...........
30 TRANSITION FROM 3-D TO 2-D REGION .....       XX...........
31 TRANSITION FROM 2-D TO 1-D REGION .....       XX...........
32 OTHER CONSTRAINT CONDITIONS--SEE SECT.A. 1.   ...........XX
33 LAGRANGE MULTIPLIER METHOD USED.............  ...XX........
```

Wall Construction and Material Properties

```
                                                                  1111
                                                 1234567890123
01 MONOCOQUE WALL CONSTRUCTION ...........       XX.XX.XX.XXX.
02 LAYERED WALL CONSTRUCTION..................   XX.XXX.....XX
03 SANDWICH WALL CONSTRUCTION ...........        XX........X..
04 COMPOSITE MATERIAL ........................   XX.XX........
05 MERIDIONAL STIFFENERS TREATED AS DISCRETE     XX.XX........
06 CIRCUM. STIFFENERS TREATED AS DISCRETE        XX.XX.X..X...
07 GENERAL STIFFENERS TREATED AS DISCRETE        XX..X........
08 STIFFENERS TREATED AS SMEARED ...........     XX.XX........
09 GENERAL OPEN-SECTION THEORY FOR STIFFENERS    XX.XX.X......
10 CG, SHEAR CENTER OF STIFFENER MUST COINCIDE   XX.XX........
11 OTHER WALL CONSTRUCTION--SEE SECT.A. 2.11     .X.XXX.......
12 ISOTROPIC MATERIAL PROPERTIES ...........     XX.XXXXXXXXX
13 ANISOTROPIC MATERIAL PROPERTIES............   XX.XX....X.XX
14 LINEAR ELASTIC MATERIAL PROPERTIES .....      XX.XXX.XXXXXX
15 NONLINEAR ELASTIC MATERIAL PROPERTIES.....    .X.......X...
16 TEMPERATURE-DEPENDENT MATERIAL PROPERTIES     XX.XXX.....X.
17 RIGID-PERFECTLY PLASTIC MATERIAL .....        .X.XX....X...
18 ELASTIC-PERFECTLY PLASTIC MATERIAL ....       XXXXXX..XX.X.
19 ELASTIC-LINEAR STRAIN HARDENING MATERIAL      XXX......X.XX
20 STRESS-STRAIN CURVE OF MANY LINE SEGMENTS     XX.XXXX.XX...
21 RAMBERG-OSGOOD STRESS-STRAIN LAW .....        .X...........
22 ISOTROPIC STRAIN HARDENING ...........        XX....X..X.XX
23 KINEMATIC STRAIN HARDENING ...........        XX...X..X....
24 WHITE-BESSELING HARDENING LAW ...........     ...XX.X......
25 OTHER POST-YIELD LAW--SEE SECT.B. 2.25        XX...........
26 PRIMARY CREEP..............................   XX...........
27 SECONDARY CREEP ...........................   XX..X........
28 VISCOELASTIC MATERIAL .....................   .X...........
29 STRAIN-RATE EFFECTS INCLUDED ...........      ..X..XX.X....
30 NO SPATIAL VARIATION OF PROPERTIES .....      .....X..X...X
31 AXISYMMETRIC VARIATION OF PROPERTIES ONLY     ......XX..X..
32 ARBITRARY VARIATION OF PROPERTIES .....       XX.XX....X...
33 COUPLING OF STRAINS AND CURVATURE CHANGES     .X.XXX..X..XX
34 COUPLING OF NORMAL AND IN-PLANE SHEAR.....    .X.XX........
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Loading

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01 UNIFORM DISTRIBUTION OF LOADS ...........     X..XX....X...
02 AXISYMMETRIC VARIATION OF LOADS ONLY .....    X............
03 GENERAL VARIATION OF LOADS ...........        XXXXXXXXXXXXX
04 STATIC OR QUASI-STATIC LOAD VARIATION.....    XXXXX....XX..
05 GENERAL TRANSIENT VARIATION OF LOADS .....    XX.XXXXXXXXX
06 PERIODIC (DYNAMIC) VARIATION OF LOADS.....    XXXXX........
07 IMPULSIVE LOADS ..........................    XXXXXXXXXXXXX
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08 LOADS VARYING RANDOMLY IN TIME ..............    XXX.........
09 OTHER TEMPORAL VARIATION OF LOADS--SEE SECT      .X..........
10 LOADS VARYING PROPORTIONALLY DURING CASE         XXXXX.XX..X..
11 SOME LOADS VARY, SOME CONSTANT DURING CASE       XX.XX....X...
12 DIFFERENT LOADS HAVE DIFFERENT TIME HISTORI      XX.......X.XX
13 QUASI-STATIC LOADS VARYING CYCLICLY DURING       XX..........
14 OTHER MANNER OF LOAD VARIATION DURING CASE-      .X..........
15 LIVE (FOLLOWING) LOADS ......................    XX.X........
16 GYROSCOPIC LOADS ............................    .X..........
17 INERTIAL LOADS ..............................    XXX.......X..
18 ELECTROMAGNETIC LOADS .......................    ............
19 FLUID-STRUCTURE INTERACTION .................    ............
20 ACOUSTIC LOADING ............................    ............
21 SOIL-STRUCTURE INTERACTION ..................    XX..........
22 RIGID SOLID-STRUCTURE INTERACTION ...........    .X..........
23 OTHER DEFORMATION-DEPENDENT TYPE OF LOADING     ............X
24 POINT LOADS, MOMENTS ........................    XXXXX.XX.X.X.
25 LINE LOADS, MOMENTS..........................    XX.XX.XX.X.X.
26 NORMAL PRESSURE .............................    XX.XXXXXXX.XX
27 SURFACE TRACTIONS ...........................    XX.XX.XX.XXXX
28 BODY FORCES .................................    XX.XX....X...
29 DEAD-WEIGHT LOADING..........................    XX.XX.XX.....
30 THERMAL LOADING .............................    XXXXXX.X..XX.
31 INITIAL STRESS OR STRAIN ....................    XX.........X.
32 CENTRIFUGAL LOADING..........................    XX..........
33 NON-ZERO DISPLACEMENTS IMPOSED ..............    XXXXX....X...
34 OTHER TYPE OF LOADING--SEE SECT.B, 3,34         ............
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Phenomena

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01 AXISYMMETRIC SMALL DEFLECTIONS ..............    XX.XX....X...
02 GENERAL SMALL DEFLECTIONS....................    XX.XX....X...
03 AXISYMMETRIC LARGE DEFLECTIONS ..............    XX.XX....X...
04 GENERAL LARGE DEFLECTIONS....................    XXXXXXXXXX..
05 AXISYMMETRIC PLASTICITY .....................    XX.XX....X...
06 GENERAL PLASTICITY ..........................    XX.XXX..XX...
07 LARGE STRAINS................................    .....X..X....
08 TRANSVERSE SHEAR DEFORMATIONS ...............    XXX.........
09 THERMAL EFFECTS .............................    XX.XXX.X..X..
10 RADIATION EFFECTS ...........................    XX..........
11 AUTOMATED YIELD CRITERION....................    XX...XX.X..X.
12 AUTOMATED FRACTURE CRITERION ................    ..........X.
13 AUTOMATED FATIGUE CRITERION .................    ............
14 AUTOMATED BUCKLING CRITERION ................    .X..........
15 OTHER BUILT-IN CRITERIA--SEE SECT.B, 4,15       ............
16 NONLINEAR COLLAPSE ANALYSIS .................    XX.XX.....X..
17 POST-BUCKLING PHENOMENA .....................    .X.XX.......
18 BIFURC, BUCKLING, LINEAR AXISYM, PRESTRESS      .X.XX.......
19 BIFURC, BUCKLING, LINEAR GENERAL PRESTRESS      .X.XX.......
20 BIFURC, BUCKLING, NONLIN, AXISYM, PRESTRESS     .X..........
21 BIFURC, BUCKLING, NONLIN, GENERAL PRESTRESS     .X..........
22 BIFURC, BUCKLING, PREBUCKLING ROTATIONS         .X..........
23 BIFURC, BUCKLING, TRANSVERSE SHEAR DEFORMAT     .X..........
24 BIFURCATION BUCKLING WITH OTHER EFFECTS--SE     ............
25 DYNAMIC BUCKLING ............................   .X...XXXXXXX.
26 MODAL VIBRATIONS WITH NO PRESTRESS ..........   XX..........
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27 MODAL VIBRATIONS, LINEAR AXISYM, PRESTRESS         .X.........
28 MODAL VIBRATIONS, LINEAR GENERAL PRESTRESS         .X.........
29 MODAL VIBRATIONS, NONLIN, AXISYM, PRESTRESS        .X.........
30 MODAL VIBRATIONS, NONLIN, GENERAL PRESTRESS        .X.........
31 MODAL VIBRATIONS, TRANSVERSE SHEAR DEFORMAT        ............
32 MODAL VIBRATIONS WITH OTHER EFFECTS--SEE SF        ............
33 NONLINEAR VIBRATIONS ..................            .X.........
34 LINEAR AXISYMMETRIC DYNAMIC RESPONSE ......        XX.XX....X.XX
35 LINEAR GENERAL DYNAMIC RESPONSE.....               XX.XX....X.XX
36 NONLINEAR AXISYMMETRIC DYNAMIC RESPONSE            XX.XX....X.XX
37 NONLINEAR GENERAL DYNAMIC RESPONSE  ......         XXXXXXXXXXXX
38 WAVE PROPAGATION ......................            .X.........
39 DYNAMIC RESPONSE, TRANSVERSE SHEAR DEFORMAT        ..X.......
40 DYNAMIC RESPONSE WITH OTHER EFFECTS INCLUDE        ...........X
41 VISCOUS DAMPING ......................             XX........XX
42 STRUCTURAL DAMPING ...................             XX.XX....X...
43 LINEAR DAMPING .......................             XX.XX......XX
44 NONLINEAR DAMPING ....................             .X.........
45 OTHER KIND OF DAMPING--SEE SECT.8, 4.45            ..X.......X
46 FLUTTER.....................................       ............
47 OTHER DYNAMIC PHENOMENA--SEE SECT.8, 4.47          ............
48 OTHER PHENOMENA OF ANY KIND--SEE SECT.8, 4.        ............
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Discretization

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01 MULTI-SEGMENT FORWARD INTEGRATION  ......          X...........
02 GALERKIN METHOD  .....................             ............
03 RAYLEIGH-RITZ METHOD  ................             .X.........
04 BOUNDARY POINT MATCHING  .............             ............
05 FINITE DIFF. METHOD BASED ON EQUILIBRIUM EQ        .....X..X.XXX
06 FINITE DIFFERENCE ENERGY METHOD.....               ...XX.......
07 1-DIMENSIONAL FINITE-DIFF. DISCRETIZATION          ..........X..
08 ORTHOGONAL 2-DIM. FINITE-DIFFERENCE GRID           ...XX......XX
09 NON-ORTHOGONAL 2-DIM. FINITE-DIFF. GRID            ...XXX..X.....
10 GENERAL QUADRILATERIAL 2-D FINITE-DIFF.GRID        ...XX.......
11 OTHER FINITE-DIFF. DISCRETIZATION SCHEME--         ............
12 FOURIER SERIES IN CIRCUMFERENTIAL DIRECTION        X........XX..
14 ROD ELEMENT  .........................             XXXXX.......
15 STRAIGHT BEAM ELEMENT  ...............             XXX.........
16 CURVED BEAM ELEMENT...................             .XX...X.....
17 CONICAL SHELL ELEMENT  ...............             XX....XX.X...
18 AXISYM. SHELL ELEMENT  ...............             XX....XX.X...
19 FLAT MEMBRANE ELEMENT  ...............             .X.........
20 FLAT MEMBRANE ELEMENT  ...............             XX.XX.......
21 CURVED MEMBRANE ELEMENT  .............             .X.........
22 CURVED MEMBRANE ELEMENT  .............             .X.........
23 FLAT PLATE ELEMENT ...................             XX....XX....
24 FLAT PLATE ELEMENT ...................             ............
25 SHALLOW SHELL ELEMENT  ...............             X...........
26 SHALLOW SHELL ELEMENT  ...............             ......XX....
27 DEEP SHELL ELEMENT ...................             .X....XX....
28 AXISYM. SOLID ELEMENT  ...............             XX.........
29 AXISYM. SOLID ELEMENT  ...............             .X.........
30 THICK PLATE ELEMENT...................             XX.........
31 THICK SHELL ELEMENT...................             XX.........
32 SOLID (3-D) ELEMENT...................             XX.........
33 SOLID (3-D) ELEMENT...................             XX.........
34 SOLID (3-D) ELEMENT...................             ............
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35 CRACK-TIP ELEMENT ..........................  .X...........
36 OTHER TYPE OF ELEMENT ......................  ..X..........
37 FORCE METHOD OR OTHER METHOD ...............  .............
38 INCOMPATIBLE DISPLACEMENT FUNCTIONS USED     X.............
39 SUBSTRUCTURING .............................  X.............
40 REPEATED USE OF IDENTICAL SUBSTRUCTURES      XX............
44 NODES CAN BE INTRODUCED IN ARBITRARY ORDER   X.............
45 AUTOMATIC RENUMBERING OF NODES .............  .X............
46 NON-DIAGONAL MASS MATRIX ...................  XX.....X.X...
47 DIAGONAL MASS MATRIX .......................  .XXXX........
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Solution Methods

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01 FULL-MATRIX EQUATION SOLVER ................  .........X...
02 CONSTANT BANDWIDTH EQUATION SOLVER .........  ..X...XX.XX..
03 SKYLINE METHOD OF SOLUTION USED.............  .X.XX........
04 MATRIX PARTITIONING USED ...................  .........X...
05 WAVE-FRONT METHOD USED FOR SOLUTION ........  XX.......X...
06 OTHER SPARSE MATRIX METHOD USED FOR SOLN--   .............
07 ITERATIVE METHOD FOR SOLUTION OF LINEAR EQU  ..........X..
08 CONJUGATE GRADIENT METHOD FOR SOLN OF LIN.   .............
09 FORWARD INTEGRATION USED FOR SOLN OF LIN.    .............
10 OTHER SOLUTION METHOD FOR LINEAR EQUATIONS   .............
11 CONDENSATION METHOD--SEE SECT.8. 6.11.......  X............
12 LR,QR METHOD FOR EIGENVALUE EXTRACTION       X............
13 LANCZOS METHOD FOR EIGENVALUE EXTRACTION     .............
14 STURM SEQUENCE METHOD FOR EIGENVALUE EXTRAC  .............
15 DETERMINANT SEARCH FOR EIGENVALUE EXTRACTIO  .............
16 SINGLE VECTOR INVERSE PWR ITER. WITH SHIFTS  .X.XX........
17 MULTI-VECTOR (SUBSPACE) ITERATION FOR EIGEN  .X...........
18 OTHER EIGENVALUE EXTRACTION METHOD--SEE SEC  .............
19 FULL NEWTON-RAPHSON METHOD FOR NONLIN. SOLN  .............
20 MODIFIED NEWTON METHOD FOR NONLINEAR SOLN    ...XX........
21 INCREMENTAL METHOD WITH NO EQUILIB. CHECK    X...X........
22 INCREMENTAL METHOD WITH EQUILIBRIUM CHECK    .XX..........
23 RUNGE-KUTTA TYPE OF INCREMENTAL METHOD       .............
24 OTHER TYPE OF INCREMENTAL METHOD--SEE SECT.  .............
25 DIRECT ENERGY SEARCH FOR NONLINEAR SOLUTION  .............
26 DYNAMIC RELAXATION FOR NONLINEAR SOLUTION    ...XX........
27 NONLIN. EQ. SOLVED BY SUCCESSIVE SUBSTITUTI  .X.......XX..
28 OTHER NONLINEAR STRATEGY--  ................  .............
29 LAGRANGIAN FORMULATION .....................  .X.XXX..XX...
30 UPDATED LAGRANGIAN FORMULATION .............  .............
31 EULERIAN FORMULATION .......................  .............
32 TIME INTEGRATION BY MODAL SUPERPOSITION      .X...........
33 TIME INTEGRATION BY EXPLICIT METHOD ........  .X.XXXX.X..XX
34 TIME INTEGRATION BY IMPLICIT METHOD ........  XX.XX.XX.XX..
35 TIME INTEGRATION BY EULER (CONST. ACCEL.)    .............
36 TIME INTEGRATION BY NEWMARK BETA METHOD, SE  .XX.......X...
37 TIME INTEGRATION BY RUNGE-KUTTA OF ORDER--S  .............
38 TIME INTEGRATION BY HOUBOLT METHOD .........  .X....XX..X..
39 TIME INTEGRATION BY WILSON METHOD ..........  .............
40 TIME INTEGRATION BY CENTRAL DIFFERENCE METH  .X...XX.X..XX
41 TIME INTEGRATION BY STIFFLY-STABLE METHOD    ...XX........
42 TIME INTEGRATION BY PREDICTOR-CORRECTOR OR   ...XX........
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43 TIME INTEGRATION BY MULTI-STEP METHOD..... ...XX.........
44 TIME INTEGRATION BY SOME OTHER METHOD--SEE  .............
45 OTHER COMMENTS PERTINENT TO THIS SECTION--S .............
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Computer Program Distribution, Documentation, Organization and Maintenance

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01 SOURCE PROGRAM TAPE AVAILABLE  ..............  ....X.XX.X.XX
02 SOURCE PROGRAM CARDS AVAILABLE ..............  .........X.XX
03 PROGRAM ABSOLUTE ELEMENT  ONLY AVAILABLE      .............
04 PROGRAM COMPLETELY PROPRIETARY ..............  XXX..........
05 PROGRAM PRICE IN DOLLARS--SEE SECT.8, 7.05    X..X..XX.....
06 PROGRAM IS AVAILABLE THROUGH DEVELOPER        XXXXXXXXXX.XX
07 PROGRAM IS AVAILABLE THROUGH COSMIC  ......   .......X..X..
08 PROGRAM AVAILABLE THRU OTHER SOFTWARE CENTE   .X..X.....X..
09 PROGRAM CAN BE EXECUTED THRU SERVICE BUREAU   XX...........
11 THIS PROGRAM HAS BEEN SUPERSEDED BY ANOTHER   .X.X...X.....
12 IMPROVED VERSION BEING PREPARED--SEE SECT.8   .X..X..X.....
13 PROGRAM IS NOT COMPLETED ....................  .............
14 USERS MANUALS ARE SELF-CONTAINED  ......      XX.XXXXX..XXX
15 USERS MANUALS HAVE LIST OF PITFALLS  ......   .X.XX.XX....X
16 USERS MANUALS HAVE TEST CASES  ..............  XX.XXXXX..XXX
17 USERS MANUALS HAVE FLOW CHARTS ..............  XX..X.XX..XXX
18 USERS MANUALS GIVE THEORY.....................  XX.XXXXXX.XXX
19 OTHER QUALITIES OF USERS MANUALS--  ......    .........X..X
20 PROGRAM LANGUAGE IS--SEE SECT.8,7,20 ......   ..........XX
21 PROGRAM RUNS ON IBM.....................      XX...XXX.XX..
22 PROGRAM RUNS ON CDC.....................      XX.XX.XX.XXX.
23 PROGRAM RUNS ON UNIVAC  ..................    XXXXX......XX
24 PROGRAM RUNS ON GE .......................    X........X...
25 PROGRAM RUNS ON COMPUTER IDENTIFIED IN SECT   .....X..X....
28 NO. OF PRIMARY OVERLAYS *.....................  7L..2....4.22
   MAN-YEARS REQUIRED TO DEVELOP PROGRAM *       LL1.L.21.9291
31 MINIMUM CORE SPACE REQUIRED FOR EXECUTION     59,99,S.......
32 PROGRAM RUNS ENTIRELY IN CORE ..............  .....X.XXXX..
33 NO. UNITS OF DIRECT ACCESS MASS STORAGE GIV   15111....L.12
36 RESTART CAPABILITY ..........................  XX.XXXXXXXX.
37 OPTIONS FOR VARIOUS OUTPUT  ..............    XX.XXXXXX.XX
38 DIRECT ACCESS DATA BASE  .....................  .............
39 FREE-FIELD INPUT  ...........................  .............
40 RUNS IN INTERACTIVE MODE .....................  X..........X.
41 AUTOMATIC MESH GENERATOR(S)  ..............    XX.XX.XX.X...
42 AUTOMATIC LOADING GENERATOR  ..............    XX.XX.XX.X.X.
43 SECTION PROPERTIES CALCULATED  ..............  .X.......X...
44 PRE-PROCESSOR.................................  XX.XX........
45 POST-PROCESSOR  ..............................  XX.XX....X...
46 EIGENVALUE SOLVER  ...........................  X............
47 MODAL SUPERPOSITION ROUTINE  ..............     X............
48 OTHER TIME-INTEGRATION ROUTINE ..............   X............
49 OTHER CAPABILITY--SEE SECT.8, 7.49  ......     .............
50 PLOT ROUTINES FOR UNDEFORMED/DEFORMED GEOME   XX...X..XX.X.
51 VARIOUS VIEWS AND SECTIONS PLOTTED  ......     XX...X..X..X.
52 CONTOUR PLOTS.................................  XX.XXX..X....
53 ORDINARY Y = F(X) PLOTS  ....................  XX.XX....X.X.
54 STRESS-ON-STRUCTURE PLOTS.....................  XX...........
55 OTHER KINDS OF PLOTS--SEE SECT.8, 7.55        X....X..X..XX
56 PLOTTING CAPABILITY FOR CALCOMP..............  XX.XXX..XX...
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57	PLOTTING CAPABILITY FOR STROMBERG-CARLSON	XX.XX....X.XX
58	PLOTTING CAPABILITY FOR HARDWARE IDENTIFIED	.....X..X....
59	OPTIONAL DETAILED PRINT OUTPUT	XX.XXXXXXX...
60	EQUILIBRIUM CHECKS	XX..........
61	ALTERNATIVE BRANCHES FOR SAME PROBLEM.....	.X.XX.......
62	MATRIX CONDITIONING ESTIMATION	XX.......X...
63	AUTOMATIC ERROR CONTROL	XX....X.....
64	GRAPHICAL PREPROCESSOR	XX.XX.......
65	APPLIED LOADING CHECKS	.X....XX.X...
66	BOUNDARY CONDITION CHECKS,.....	.X.......X...
67	SECTION PROPERTIES CHECKS,.....	.X.......X...
68	GEOMETRY CHECKS	.X....XX.X...
69	OTHER CHECKING AIDS--SEE SECT.8. 7.69.....	..........X.
70	DEVELOPER WILL HELP USER GET PROGRAM RUNNIN	XX.XX.XX.X...
71	DEVELOPER WILL HELP USER MODELING CASES	XXXXX....X.XX
72	DEVELOPER WILL HELP USER FIND SUSPECTED BUG	XX.XX.XX.X.XX
73	DEVELOPER WILL CIRCULATE NOTICES OF BUGS FO	XX....XX...XX
74	DEVELOPER WILL HELP USER SET UP AND RUN CAS	XXXXX....X.XX
75	DEVELOPER WILL CONDUCT WORKSHOPS	XX.XX....X.XX
76	DEVELOPER WILL MAKE SPECIAL PROGRAM CHANGES	XXXXX....X.XX
77	DEVELOPER WILL KEEP USERS INFORMED OF IMPRO	XX.XX.XX.X..X
78	OTHER ASSISTANCE DEVELOPER WILL GIVE USERS-	....

Limiting Performance of Structural Systems

Paul D. Smith

Los Alamos Scientific Laboratory

Walter D. Pilkey

University of Virginia

INTRODUCTION

Two methods to study the optimal dynamic response of a structure are the direct synthesis method and the limiting performance method. Both approaches can be used to seek to minimize (or maximize) some measure of the system motion response that results from external system excitations.

Several investigators [1 - 6] have developed software for the direct synthesis method. The response of the structure is to be optimized by judicious selection of system parameters which are typically spring constants and damping coefficients. Constraints are usually imposed on the values of these parameters. By repetitively integrating the system equations of motion, one of the gradient following search routines is used to find the optimizing set of system parameters.

An alternative approach to dynamic response optimization is to seek the limiting performance of the system. The structural elements which are to optimize the system response are replaced with generic control forces. The equations of motion are expressed in terms of these control forces and solved. In this fashion, the performance index and constraints on various response quantities can be expressed as linear combinations of the discretized control forces. The control forces can be nonlinear, although the rest of the system must be linear. A linear programming algorithm is then used to solve for the set of discretized control forces which both optimizes the system response and satisfies the constraints imposed. The process is analogous to control system optimization without state feedback. The resulting system response is the best possible, or limiting performance of the system. The time history of each generic controller, along with the relative motion time history across the controller can be translated into terms of spring constants and damping coefficients by use of an identification technique such as least squares curve fitting.

SOFTWARE FOR LIMITING PERFORMANCE ANALYSIS

Three codes are available for limiting performance analysis. These are COSI (Configuration-free Optimum Shock Isolation), PERFORM (Performance Optimizing Computer Program), and SYSLIPEC (Steady-state Limiting Performance Code). All three of these codes are pre- and postprocessors for use with the Control Data Corporation OPTIMA linear programming package. In addition, PERFORM has been modified to operate with the CDC APEX linear programming package, and SYSLIPEC has been modified to run with a small linear programming code from IBM called RSMSUB.

COSI optimizes the response of the system shown in Fig. 1. Discretized time histories of the control forces (u_i, i = 1, 4) are determined such that

the objective function

$$J = \max \left(\max \alpha_1 |\ddot{z}_1|, \max \alpha_2 |\ddot{z}_2|, \max |\ddot{\theta}|\right) \tag{1}$$

is minimized. Constraints are imposed such that

$$\begin{aligned} |x_1 + a\theta| &\le \epsilon_1 \\ |x_1 - a\theta| &\le \epsilon_1 \\ |x_2 - b\theta| &\le \epsilon_2 \\ |x_2 + d\theta| &\le \epsilon_2 \end{aligned} \tag{2}$$

Output consists of tabularized values of (u_i, i = 1, 4) at each time step. The user supplies subroutines to generate horizontal and vertical accelerations of the base of the system.

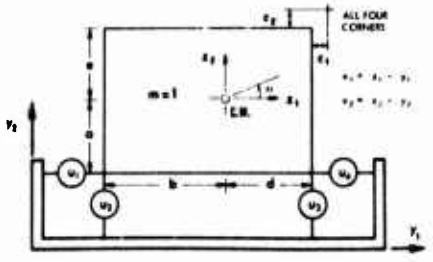

Fig. 1 System optimized by COSI

PERFORM, like COSI, consists of a preprocessor and a postprocessor for the OPTIMA linear programming package. Unlike the fixed system configuration of COSI, PERFORM will optimize any dynamic system of first or second order for which the equations of motion are linear in the state, control, and input functions.

The dynamic system can be described and input to PERFORM using the first or second order equations

$$\dot{\bar{s}} = \underline{A}\bar{s} + \underline{B}\bar{u} + \underline{D}\bar{f}_k$$

$$\underline{M}\ddot{\bar{q}} + \underline{C}\dot{\bar{q}} + \underline{K}\bar{q} + \underline{U}\bar{u} = \underline{F}\bar{r}_k$$

in which $\bar{u}$ is a vector of time varying functions, called control or isolator forces, that have replaced portions of the dynamic system. $\underline{A}$, $\underline{B}$, $\underline{D}$, $\underline{M}$, $\underline{C}$, $\underline{K}$, $\underline{U}$, $\underline{F}$ are coefficient matrices. $\bar{s}$ and $\bar{q}$ are vectors of response variables, e.g., displacements, stresses, accelerations. $\bar{r}_k$ is a forcing function vector where

the subscript k designates the kth set of forcing or loading functions. This allows the system to encounter alternative sets of disturbances which might occur with equal probability.

The acceptable equations of motion appear to be linear. In fact, however, they are "quasilinear" since those portions of the system replaced by $\bar{u}$ can be linear, nonlinear, active, or passive. The remainder of the system must be linear as must the overall kinematics.

The user must place his equations in one of the forms of the above equations. The nonzero elements of the matrices $\underline{A}$, $\underline{B}$, $\underline{D}$ or $\underline{M}$, $\underline{C}$, $\underline{K}$, $\underline{U}$, $\underline{F}$ are then are then entered as inputs. This is accomplished by identifying the matrix, e.g., M MATRIX, and then specifying an element and its value, e.g., i, j, and M_{ij}. Elements not entered are assumed to be zero.

PERFORM finds the characteristics, including $\bar{u}$ and tradeoffs between optimal response variables, of the dynamic system such that bounds on some of the response variables $\bar{s}$ or $\bar{q}$ or control forces $\bar{u}$ are not violated, while the maximum (or minimum) in time of other elements of $\bar{s}$ or $\bar{q}$ are minimized (or maximized).

Regardless of the form (first or second order) used to describe the equations of motion, the formats for the objective function and constraints are the same. In the case of the second order equations, a state variable vector $\bar{s}$ is established as

$$\bar{s} = \begin{bmatrix} \dot{\bar{q}} \\ \hline \bar{q} \end{bmatrix}$$

Any linear combination of state variables, derivatives of state variables, or control forces can be used as an objective function. In the case of the system described by second order equations, these become linear combinations of accelerations, velocities, displacements, and control forces. The objective function is input to PERFORM in the form

$$\underline{PX1}\bar{s} + \underline{PX2}\dot{\bar{s}} + \underline{PX3}\bar{f}_k$$

where $\underline{PX1}$, $\underline{PX2}$ and $\underline{PX3}$ are coefficient matrices. If more than one row of the matrices of this equation contains nonzero elements, then the peak values in time of the vectors resulting from the meaningful rows are to be compared. PERFORM minimizes (maximizes) the maximum (minimum) of the peak values.

Constraints may be placed on state variables, derivatives of state variables, and control forces. The general form, which is again linear, is

$$\overline{YL} \leq \underline{Y1}\bar{s} + \underline{Y2}\bar{u} + \underline{Y3}\bar{f}_k \leq \overline{YU}$$

where $\underline{Y1}$, $\underline{Y2}$, $\underline{Y3}$ are coefficient matrices and $\overline{YL}$, $\overline{YU}$ are lower and upper bound vectors. Constraints can be imposed at every time of the response or at specific times.

For prescribed initial conditions, PERFORM computes the $\bar{u}$ vector such that the max $|\underline{PX1}\bar{s} + \underline{PX2}\bar{u} + \underline{PX3}\bar{f}_k|$ is minimized (or min $|\ \ |$ is maximized) while the above constraints are satisfied. Any linear combination of $\bar{s}$, $\bar{u}$, and $\bar{f}_k$ can be tabulated or plotted versus time. A tradeoff curve between the maximum objective function and any particular constraint can be generated by varying the bounds on that constraint.

SYSLIPEC computes the limiting performance of the steady-state response of a structural system. The system equations of motion, the objective function, and the constraint relationships are of the same form as in PERFORM. Because the steady-state response is of interest, motion responses, excitation forces, and control forces are expressed as phasors in a complex plane which rotates at the circular frequency of the excitation function. Instead of discretizing

motions and forces in time, as in COSI and PERFORM, discretization is done in ωt over the interval $0 \leq \omega t \leq 2\pi$. The solution of SYSLIPEC includes real and imaginary parts of the complex amplitudes of the response motions and the control forces. The ratio of the complex control force and the complex relative displacement across the controller gives the complex impedance required to optimize the response of the system. Since there is no way to impose linear constraints on the phase angles across the controllers, the optimum impedances identified may not be realizable as passive mechanical networks.

AN EXAMPLE LIMITING PERFORMANCE ANALYSIS

In [10], Pilkey and Wang have investigated the limiting performance of shock absorbers in freight car couplings. As shown in Fig. 2, the system consists of a freight car modeled as a mass M_C which contains a load modeled as a damped oscillator M_L, c, and k. The laden car rests freely on a horizontal track and is struck by a second car modeled as a mass M_B. The shock absorber coupling is modeled as a time varying force u. The limiting performance problem is to minimize the peak force transmitted through the spring and dashpot to the load M_L while the relative displacement of the shock absorber coupling is constrained to lie between zero and a given upper bound. For each of five striking velocities, Fig. 3 shows typical results of the limiting performance analysis. The tradeoff between the force transmitted to the load and the deflection of the coupling is clearly shown. These curves provide the designer with information about the best possible shock absorber performance against which he can measure the performance of his candidate absorber designs.

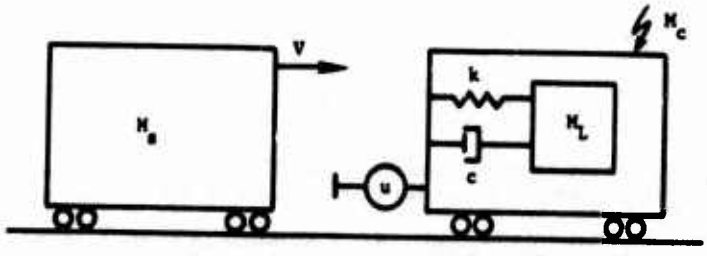

Fig. 2 Limiting Performance Model

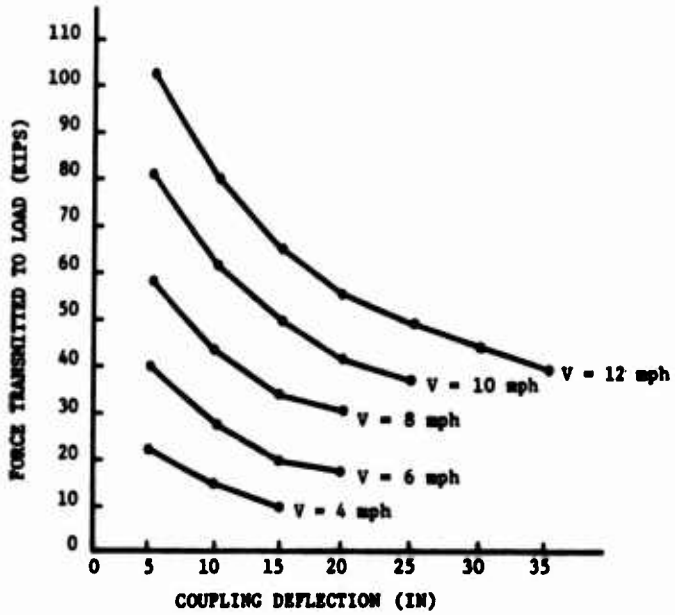

Fig. 3 Typical limiting performance curves

OPERATING DETAILS OF THE CODES

COSI [5, 6]

Capability: Optimizes transient response of base-excited, three degree of freedom system shown in Fig. 1.
Language: FORTRAN IV contained on approximately 900 cards.
Additional Software Required: CDC OPTIMA linear programming package plus user supplied subroutines for horizontal and vertical base accelerations.
Hardware: CDC 6600, CDC 6400
Developers: G. H. Klein, E. I. Axelband, R. E. Parker
Mechanics Research Incorporated
Los Angeles, California
Sponsored by the Defense Atomic Support Agency, Washington, D. C.
Availability: Aerospace Structures Information and Analysis Center (ASIAC)
AFFDL-FBR
Wright-Patterson AFB, Ohio 45433

PERFORM [7, 8]

Capability: Optimizes transient response of any first or second order dynamic system for which the equations of motion are linear in the motion, control, and excitation function.
Language: FORTRAN IV contained on approximately 3300 cards.
Additional Software Required: CDC OPTIMA or CDC APEX linear programming package.
Hardware: CDC 6600, CDC 6400

Developers: W. D. Pilkey and B. P. Wang
University of Virginia
Charlottesville, Virginia 22901
Sponsored by NASA Langley Research Center, Hampton, Virginia
Availability: Computer Software Management and Information Center (COSMIC)
Barrow Hall
University of Georgia
Athens, Georgia 30601

SYSLIPEC [9]

Capability: Optimizes steady-state response of any first or second order system for which the equations of motion are linear in the motion, control, and excitation functions.
Language: FORTRAN IV contained on approximately 550 cards.
Additional Software Required: CDC OPTIMA or IBM RSMSUB linear programming package.
Hardware: CDC 6600, CDC 6400
Developer: B. P. Wang
University of Virginia
Charlottesville, Virginia 22901
Sponsored by NASA Langley Research Center, Hampton, Virginia
Availability: COSMIC
Barrow Hall
University of Georgia
Athens, Georgia 30601

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Grillages

Thein Wah

Texas A&I University

INTRODUCTION

The term grillage is usually understood to mean a network of beams rigidly connected at their intersections. In actual practice such a network usually forms the supporting framework for a plate or envelope which transmits the superimposed load to the grillage. In many structures the grillage forms the primary load carrying component, and the safety of the structure is dependent on its satisfactory performance.

Despite the customary use of the term grillage to mean a plane structure of orthogonally intersecting beams, such a restricted meaning is unwarranted. Grillages may be of any shape, curved or flat, and the network of beams may intersect at any angle. The beams composing the grillage need not be uniform, their variation being dictated by the particular structural application. Ship bottoms and bridge decks are among the most commonly encountered structural configurations that are classified as grillages. But a supporting network for a cylindrical shell may be classified as a grillage provided it is the primary load carrying framework.

For the purposes of this article, it has not been possible to construe the term grillage quite so broadly, if only because this would make the chapter too volumnious. In particular, only the dynamic characteristics of grillages are covered here, the static response having been adequately treated elsewhere [1].

BASIC THEORY

The theory of grillages is usually based on the Bernoulli-Euler theory of elastic beams. At the intersection between two beams compatibility of deflection, slope, and rotation must be ensured. Finally, the solution must satisfy the boundary conditions at the supports. The loads applied to the grillage may be either at the beam intersections or on a beam between two consecutive intersections.

The difficulties in the analysis are usually in incorporating the torsional stiffness of the beams and the distributed mass characteristics. Some formulations omit the torsional stiffness of the beams not only for mathematical simplicity but because in many applications the torsional stiffness of the beams (such as I beams) is indeed small. Another common simplification is to assume that the masses of the beams are lumped at the intersections of the beams.

Apart from the foregoing, many idealizations are possible for the mathematical model. One of long standing is orthotropic or anisotrophic plate theory, which replaces the discretized actual structure by a continuum, thereby bringing it within the scope of a differential equation [2].

A uniform grillage is understood to mean that each set of intersecting beams is identical and equally spaced. Such grillages, provided the boundary

conditions are not too complicated, may be treated exactly by difference equations [3].

Since the grillage is assumed to be the primary load carrying component, the effect of plating is approximated by incorporating it in some manner into the properties of the beams.

FREE VIBRATION AND DYNAMIC RESPONSE

There are two distinct problems in grillage dynamics, namely, the determination of the natural frequencies of the system and the determination of the response to given external forces. The former is, generally speaking, simpler analytically. Computer programs can be developed for either problem.

Considering the importance of grillages in structural applications, it is surprising that more computer programs are not generally available that are aimed specifically at grillage dynamics. The truth probably is that such programs are available within particular industries, but are not disseminated widely nor published in the open literature.

Because of time limitations it has not been possible to seek out software which may be known to, or used only by, limited groups within industries or enterprises. We have confined ourselves therefore to the more readily availalbe programs.

As implied in the previous section, certain mathematical models of the structure are sufficiently simple to permit exact solutions of the problem under certain boundary conditions.

For such cases, it seems appropriate here to present some of the explicit formulas in the hope that, even without related software, computer programs could be written by the user without undue effort.

The simplest formulation for grillages is the anisotropic plate; and once the parameters are determined or approximated in some manner, one could apply classical plate theory.

Rectangular grillage vibration may be represented by the following differential equation

$$D_x \frac{\partial^4 w}{\partial x^4} + 2D_{xy} \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} + \rho \frac{\partial^2 w}{\partial t^2} = 0 \qquad (1)$$

where $$D_x = \frac{B_1}{b_1}, \quad D_y = \frac{B_2}{a_1} \quad 2D_{xy} = \frac{c_1}{b_1} + \frac{c_2}{a_1} \qquad (2)$$

and ρ is the mass of the plate per unit area.

The quantities on the right hand sides of Eqs. (2) are defined as follows:

- a, b = dimensions of grid in the x and y directions, respectively
- a_1, b_1 = distance between beams in the x and y directions, assumed small in comparison with a and b
- B_1, B_2 = flexural rigidities of beams in the x and y directions
- c_1, c_2 = torsional rigidities of beams in the x and y directions

Eq. (1) is not restricted to uniform grids as Eqs. (2) imply. It may be used even if the grillage is not uniform and some "smearing out" process is used to approximate the grillage as a plate.

The square of the natural frequency obtained from Eq. (1) is given by

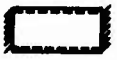
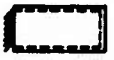
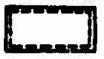
$$\omega^2 = \frac{1}{\rho} \left(\frac{A^4 D_x}{a^4} + \frac{B^4 D_y}{b^4} + \frac{2CD_{xy}}{a^2 b^2} \right)$$

The coefficients A, B, and C are given in Table 1 [4].

Table 1

$$\gamma_0 = m\pi \ , \ \gamma_1 = m + \frac{1}{4}\pi \ , \ \gamma_2 = m + \frac{1}{2}\pi$$

$$\varepsilon_0 = n\pi \ , \ \varepsilon_1 = n + \frac{1}{4}\pi \ , \ \varepsilon_2 = n + \frac{1}{2}\pi$$

Boundary conditions	A	B	C	m	n
	4.730	4.730	151.3	1	1
	4.730	ε_2	$12.30\varepsilon_2(\varepsilon_2-2)$	1	2,3,4,...
	γ_2	4.730	$12.30\gamma_2(\gamma_2-2)$	2,3,4,...	1
	γ_2	ε_2	$\gamma_2\varepsilon_2(\gamma_2-2)(\varepsilon_2-2)$	2,3,4,...	2,3,4,...
	4.730	ε_1	$12.30\varepsilon_1(\varepsilon_1-1)$	1,2,3,...	1
	γ_2	ε_1	$\gamma_2\varepsilon_1(\gamma_2-2)(\varepsilon_1-1)$	1,2,3,...	2,3,4,...
	4.730	ε_0	$12.30\varepsilon_0^2$	1	1,2,3,...
	γ_2	ε_0	$\gamma_2\varepsilon_0^2(\gamma_2-2)$	2,3,4,...	1,2,3,...
	γ_1	ε_1	$\gamma_1\varepsilon_1(\gamma_1-1)(\varepsilon_1-1)$	1,2,3,...	1,2,3,...
	γ_1	ε_0	$\gamma_1\varepsilon_0^2(\gamma_1-1)$	1,2,3,...	1,2,3,...
	γ_0	ε_0	$\gamma_0^2\varepsilon_0^2$	1,2,3,...	1,2,3,...

An exact solution may be obtained for a simply supported gridwork provided it is uniform. Two formulations are possible, one taking into account the distributed mass of the beams, and the other assuming that the mass of the beams is concentrated at their intersections. The latter, while an approximation, gives an explicit expression for the frequencies

$$p^2 = \left(1-\cos\frac{m\pi}{R}\right)\frac{1-\cos\frac{m\pi}{R} + 2\nu\left(1-\cos\frac{n\pi}{S}\right)}{\cos\frac{m\pi}{R} + 2 + \nu - \nu\cos\frac{m\pi}{R}}$$

$$+ \lambda\left(1-\cos\frac{n\pi}{S}\right)\frac{1-\cos\frac{n\pi}{S} + 2\bar{\nu}\left(1-\cos\frac{m\pi}{R}\right)}{\cos\frac{n\pi}{S} + 2 + \bar{\nu} - \bar{\nu}\cos\frac{m\pi}{R}} \qquad (4)$$

where

$p^2 = \frac{\rho\omega^2 a^3}{12EI}$

R, S = number of beams in the x and y directions

$\lambda = \bar{I}a^3/I\bar{a}^3$

$\nu = G\bar{J}a/2EI\bar{a}$

$\bar{\nu} = GJ\bar{a}/2E\bar{I}a$

EI, $E\bar{I}$ = flexural rigidity of beams in the x and y directions

GJ, $G\bar{J}$ = torsional rigidity of bars in the x and y directions

ρ = mass concentrated at the beam intersections

m, n = integers

a, $\bar{a}$ = spacing of beams in the x and y directions

The distributed mass solution is considerably more complicated and does not give an explicit formula. With ρ now as the mass density, the frequency equation assumes the form

$$\frac{3C_4^2\left(\cos^2\frac{m\pi}{R}-1\right)}{2C_1+C_2\cos\frac{m\pi}{R}-\nu\left(C_8\cos\frac{n\pi}{S}-\bar{C}_7\right)}+2C_5-2C_6\cos\frac{m\pi}{R}$$

$$+\lambda\frac{3\bar{C}_4^2\left(\cos^2\frac{n\pi}{S}-1\right)}{2\bar{C}_1+\bar{C}_2\cos\frac{n\pi}{S}-\bar{\nu}\left(\bar{C}_8\cos\frac{m\pi}{R}-\bar{C}_7\right)}+2\bar{C}_5-2\bar{C}_6\cos\frac{n\pi}{S}=0 \qquad (5)$$

in which

$C_1 = (\cosh u \sin u - \sinh u \cos u)/4T$

$C_2 = u(\sinh u - \sin u)/2T$

$C_4 = u^2(\cosh u - \cos u)/6T$

$C_5 = u^3(\cosh u \sin u + \sinh u \cos u)/12T$

$C_6 = u^3(\sin u + \sinh u)/12T$

$C_7 = v \cot v$

$C_8 = v \operatorname{cosec} v$

$u = (\omega^2 A\rho a^4/EI)^{1/4}$

$v = (\omega^2\bar{a}^4\rho I_p/G\bar{J})^{1/2}$

$T = 1 - \cos u \cosh u$

$\bar{u}$ and $\bar{v}$ are defined by interchanging the barred quantities for those without bars and vice versa. $\bar{C}_1 \ldots \bar{C}_8$ are similar functions of $\bar{u}$ and $\bar{v}$. A, $\bar{A}$ are the cross-sectional areas of the beams in the x and y directions and I_p is the polar moment of inertia. Other quantities are as defined under Eq. (4). Equation (5) is readily programmed for the determination of the natural frequencies ω for chosen values of m and n.

COMPUTER PROGRAMS

GRIDSAP

Capability: Solves for the natural frequencies and associated mode shapes of a rigidly jointed, two-dimensional lumped-mass grid. Stiffness matrix alterations can be used to add complex structural elements which cannot be represented by members. The output gives stiffness matrix, natural frequencies, up to 20 mode shapes.

Language: FORTRAN
Hardware: CDC, Honeywell
Availability: STRUPAK
TRW Systems
Redondo Beach, Calif.
Available on CDC commercial networks

Dynamic Analysis of Elastoplastic Beam-Girder Systems (RESI)

Date: 1970.
Capability: Calculates the dynamic response of elastoplastic beam-girder systems subjected to a uniformly distributed arbitrary forcing function. The ends of the beams may be either simply supported or clamped. Each member is assumed to possess a bilinear elastoplastic resistance function. The beams are equally spaced and have identical properties.
Output: The center deflections of the girder and beam relative to the girder, and the dynamic reactions of the beams and the girder, are printed out.
Language: FORTRAN IV
Hardware: IBM 7090
Developer: S. L. Wang and H. P. Gray
Naval Ship Research and Development Center
Washington, D. C. 20034
Availability: An international journal, Computers and Structures, Vol. 2, (1 and 2), Pergamon Press, Feb. 1972, pp. 223-251 presents a listing of the program RESI.

Impulsive Response of a Stiffened Plate Mass Loaded Around the Edges

Capability: This program gives the approximate solution of a cross-stiffened plate with mass loadings around the edges treated as an orthotropic plate. The solution is made of modal expansions. The modes of the plate are approximated by the modes of a beam.
Language: FORTRAN
Hardware: Will run on any time-sharing system
Developer: J. E. Greenspon
J. G. Engineering Research Associates
3831 Menlo Drive
Baltimore, Maryland 21215
Availability: The program is available, along with two example solutions in the report: Impulsive Response of a Stiffened Plate Mass Loaded Around the Edges, Joshua Greenspon, NSRDC - Underwater Explosions Research Division, Contract N00167-71-C-0024, Final Report Jan 1972.

Dynamic Modal Analysis of Large Bar Structures Having up to 4000 Dynamic Degrees of Freedom

Capability: Determination of eigenvalues, periods and eigenvectors as well as dynamic modal analysis leading to the determination of modal participation factors, spectral displacements and maximum probable inertia forces. The structure may have up to 2500 joints, 4,000 members, 4,000 lumped masses and 4,000 dynamic degrees of freedom. The structure's members may be circularly curved in space with a constant section or straight with a constant as well as a variable section and they may also be of different materials. Members' bending, axial and shear deformation are taken into account. The structure may be subjected to 4,000 simultaneous different dynamic forcing functions arbitrarily varying with time and acting at any number of joints. Damping factors and elastic supports are also considered. Fixed input form. Operates in and out of computer core.

Language: FORTRAN
Hardware: UNIVAC 1108, but may be adapted to any computer.
Availability: M. G. Kostro, President
Electronic Calculus, Inc.
468 Park Avenue, South
New York, New York 10019
Also commercially available on UCC computer network.

WDGVIB

Capability: Program computes the natural frequencies and modal amplitudes for the ribs and skin of a three-cell wedge structure.
Language: FORTRAN
Memory Required: 17,500 octal
Hardware: CDC 6600
Source: Program available on cards to Department of Defense Agencies or their contractors. Contact D. L. Smith
R. C. W. Vander Heyde
Aero-Acoustics Branch
AFFDL/FYA
Wright-Patterson Air Force Base
Ohio 45433
Telephone: 513-25-54279

PANAL

Capability: Program computes the natural frequencies, normal mode shapes, modal shear distribution along the center line of a one-dimensional panel array undergoing cylindrical bending.
Language: FORTRAN, program available on cards
Memory Required: 34,600 octal
Hardware: CDC 6600
Source: D. L. Smith
R. C. W. Vander Heyde
Aero-Acoustics Branch
AFFDL/FYA
Wright-Patterson Air Force Base
Ohio 45433

PLTVEB

Capability: Program computes the natural frequencies and normal mode shapes of a nine-bay orthogonally stiffened panel. The structure is assumed to have clamped edges and four orthogonal stiffeners dividing the uniform cover sheet into nine bays. The stiffeners are modeled with a finite element representation of a thin walled open-section beam. The rectangular plate bending element used is based upon a sixteen degree of freedom element made in the form of a clamped-clamped beam fundamental mode.
Language: FORTRAN, program available on cards
Memory Required: 67,200 octal
Hardware: CDC 6600
Source: D. L. Smith
R. C. W. Vander Heyde
Aero-Acoustics Branch
Wright-Patterson Air Force Base
Ohio 45433
Telephone: 513-25-54279

Numerical Study of Free Vibration of Grillages

Capability: Calculation of natural frequencies of grids using either the distributed mass or the lumped mass approximation. Solutions for both simple supports on all sides and with elastic supports on 2 parallel sides are possible. Torsional stiffness may be taken into account.
Developer: Fu-Shan Chen
Hota V. S. Ganga Rao
Department of Civil Engineering
West Virginia University
Morgantown, West Virginia
Availability: The program is printed out and discussed with example problems in Report No. 2021, Civil Engineering Studies, West Virginia University.

BEAMRESPONSE (Computer Programs for Grillage Analysis)

Date: Completed 1974.
Capability: Calculates natural frequencies of grillages assuming mass of transverse members is lumped at the intersections and these members act as elastic springs. Longitudinal members are identical while the transverse members may have differing properties. Longitudinal members' shear deformation and rotary inertia may be taken into account. Girders may be either simply supported and fixed-ended.
Method: Transfer matrix method
Input: Batch or prompting interactive
Output: For the interactive version, user can modify the input on-line as the output appears.
Language: FORTRAN
Hardware: CDC, IBM, UNIVAC, Honeywell, Data General, PDP.
Usage: In use in industry and several universities
Developer: Walter D. Pilkey Pin Yu Chang
Dept. of Engineering Science and Systems
University of Virginia
Charlottesville, Virginia 22901
Availability: Can be purchased or used on national networks. Contact:
Structural Members Users Group
P. O. Box 3958, University of Virginia Station
Charlottesville, Va. 22903
Telephone: (804) 296-4906

RECTANGULARPLATE

Date: Complete 1974.
Capability: Calculates natural frequencies and steady state response for isotropic and orthotropic plates. Gives mode shapes of free vibration, and deflection, slope, bending moments, twisting moments and shears for steady state loads. Two opposite edges (y=0, L_y) must be simply supported, but there may be intermediate supports between x=0 and x=L up to a total of 10. The supports may be rigid, moment release, guided, or shear release type. The supports at x=0 and L may be fixed, simple, free or guided. There may be any number of concentrated parameters (e.g. lumped masses, concentrated forces, line springs) in the x direction. The plate can be formed of uniform panels in the x direction. The thickness, modulus of elasticity, modulus of elastic foundation, Poisson's ratio, thermal expansion coefficient, and density can be varied from panel to panel. Stiffened plates with large stiffeners in both x and y directions can be analyzed
Method: Transfer matrix
Limitations and Restrictions: Uses classical plate theory
Input: Batch or prompting interactive.

Output: For the interactive version, user can modify the input on-line as the output appears.
Language: FORTRAN
Hardware: CDC, IBM, UNIVAC, Honeywell, Data General, PDP
Usage: In use in industry and at several universities
Developer: Walter D. Pilkey Pin Yu Chang
Department of Engineering Science and Systems
University of Virginia
Charlottesville, Virginia 22901
Availability: Can be purchased or used on national networks. Contact:
Structural Members Users Group, Ltd.
P. O. Box 3958, University of Virginia Station
Charlottesville, Va. 22901
Telephone: (804) 296-4906

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Kinematic and Dynamic Design of Mechanism

Roger E. Kaufmann

Massachusetts Institute of Technology

INTRODUCTION

Kinematics is a science with roots in antiquity. Under the constant pressure of exigency, engineers have developed innumerable clever mechanisms to generate motions in space. At the same time, theoreticians interested in the more fundamental properties of time-varying geometric relationships have developed an extensive body of analytical procedures for studying or designing such devices. During the last century, the algebraic theories for mechanisms were developed to a point which surpassed the ability of the typical unaided engineer to manipulate or interpret the mathematics. As a result, a gulf developed between the theory and the practice of mechanism design.

Digital computers have empowered engineers and given them the ability to quickly and easily process vast amounts of data. Further, sophisticated mathematical theories can be embedded in a user-oriented "packaged" program, so that the designer himself can treat the computer as a skilled specialist, or mechanisms consultant. He need only understand how to communicate his problem to the machine and how to interpret the results. He need not have a detailed understanding of the theory embodied in the program or of the intricacies of computer science. As a result, there has been a reawakening of interest in the sophisticated, though heretofore "useless" theories of eminent mathematicians and kinematicians. The theoretical contributions of men such as Sylvester, Lagrange, Burmester, and Chebyshev, though dormant for close to a century, are now being implemented in user-oriented computer programs which hold the promise for revolutionizing the practice of mechanism design in the near future.

Kinematics, as originally defined by Ampere, is the science "in which movements are considered in themselves" (independent of the forces which produce them). Generally, the subject is subdivided into two major areas, analysis and synthesis. Analysis deals with the study of the behavior of an existing mechanism or mechanism design. Synthesis is the inverse process of designing a mechanism so as to obtain a preconceived kinematic behavior. The task of synthesis may be further subdivided into type synthesis and dimension synthesis. In type synthesis, one is concerned with determining the topologies of mechanisms which would be capable of theoretically providing the kind of performance required. That is, one is concerned with such overall questions as "how many links and hinges must a mechanism possess so as to generate an eighth degree coupler curve", or "what kinds of mechanisms can invert a body and then dwell for an instant?" Dimension synthesis, on the other hand, is concerned with establishing specific sizes or proportions for a device of known topology which must satisfy a specified time sequence of positions.

Programs for the kinematic analysis and synthesis of mechanisms are surveyed in this chapter. Coverage has been extended to include programs dealing with nonkinematic properties such as static forces, joint friction, and mechanism dynamics.

Insofar as possible, programs have been selected for their generality and their designer-engineer user orientation. An effort has been made to weed out programs of limited general interest and those which appear to be of purely academic concern. Poorly documented programs and those which require detailed supplementary training to understand their application have also been eliminated. However, since the author has not had first-hand experience with many of the available programs, the choice as to which to include has been somewhat arbitrary, with more concern for including programs of potential interest and utility than for eliminating marginal programs. Inevitably, some programs which should have been scratched have slipped through. In advance, the author would like to apologize to his friends whose programs were eliminated, either by design or by chance.

ANALYNK

Developers: Kaufman, Barnett, Archie, Houck
Affiliation: Department of Mechanical Engineering
Massachusetts Institute of Technology
Cambridge, Mass. 02139
Application: Animation and display of most pin or slider-jointed planar mechanisms. Closed-form animation procedures automatically generated where theoretically possible.
Graphics: Yes. Interactive program. Storage tube or plotter output. Graphical data tablet input currently being incorporated.
Hardware: Storage tube; minicomputer oriented but convertible for time sharing. FORTRAN IV
Availability: From developer
Comments: Being incorporated into KINSYN II. Useful as standalone animation package.

CAMPAC

Developers: Mathews, Tesar
Affiliation: University of Florida
Gainsville, Fla. 32601
Application: Synthesis, analysis, and design of cams. Polynomial and trapezoidal curves of second, third, and fourth order. Parametric error analysis. One and two degree of freedom models.
Graphics: Calcomp plotter.
Availability: From developer.
Comments: Generalized cam design program covering everything but harmonic analysis.

COMMEND I

Developers: Knappe, Kiss
Affiliation: IBM Systems Development Division
Development Laboratory
Rochester, Minn.
Application: An ambitious, generalized mechanical design system incorporating linkage, cam, gear, spring, shaft, and timing-belt design programs, together with related routines for N/C machining, etc. Analysis and synthesis of general planar serial mechanisms. Analysis of three-dimensional mechanisms. Planar mechanism synthesis by Freudenstein's equations or by numerical methods where closed-form techniques were not applicable. Dynamic analysis of mechanical systems.
Hardware: Implemented in 360 FORTRAN IV, PL-1, PLAN/360 and PLAN 1130 versions.

Graphics: 2250 option, plotter.
Availability: COMMEND I is internally available within IBM. Limited subsets of COMMEND have been released to the public as IBM program products. Mechanism Design System-Kinematics and Mechanism Design System-Gears and Springs are available and run under PLAN.
Comments: Unfortunately, only limited portions of COMMEND are available to the public. MDS-Kinematics package is available only for two dimension and three dimension motion analysis. None of the kinematic synthesis systems was released. The version available as a program product provides displacements, together with higher derivatives and plotter output. Apparently, IBM has essentially abandoned the mechanical projects associated with COMMEND I.

DKINAL (formerly DYMAC)

Developer: Paul, Hud
Affiliation: Department of Mechanical Engineering
University of Pennsylvania
Philadelphia, Pennsylvania 19174
Application: Dynamic analysis of machinery. User provides all geometric and inertial properties of mechanisms, together with initial velocities and applied forces. User specifies which members are included in each user-specified independent loop. Program generates and solves the equations of motion and prints out positions, velocities, accelerations, and forces.
Availability: From developer.
Comments: User manual available from B. Paul, Department of Mechanical Engineering and Applied Mechanics, University of Pennsylvania, Philadelphia, Pa., 19174.

DRAM

Developers: Chace, Smith, Rubens, Angell
Affiliation: University of Michigan
Ann Arbor, Mich. 48104
Application: Interactive simulation of two-dimensional dynamic mechanical systems. Open or closed loop, single or multi-degree of freedom systems. Continuous motions or impacts. Lumped parameter systems only. Allows specification of torque-speed characteristics. Provides display of selected positions, velocities, accelerations, or reaction forces.
Hardware: Time-sharing or batch versions. Tektronix 4010 display.
Graphics: Tektronix 4010 storage tube for presentation of results.
Availability: Available through DECAL, 1210 E. Engineering, University of Michigan, Ann Arbor, Mich., 48104, tel. 313-763-0651
Commercially available through Com Share Time-sharing system. Contact: Structural Dynamics Research Corporation, 5729 Dragon Way, Cincinnati, Ohio 45277
Comments: Users Lagrangian multiplier approach with all joint variables as independent variables, force constraints hold joints together. Large system of differential equations. System maintained for commercial use.

DRPL

Developers: Carson, Trummel
Affiliation: University of Iowa
Iowa City, Iowa 52242
Application: Three-dimensional dynamic response of mechanisms. Hartenberg-Denavit transformation matrix notation with least-squares loop closure. Lagrange equations then determined and solved by Runge-Kutta method.

Availability: From developer.
Language: FORTRAN II
Comments: Early system with capabilities similar to those of IMP. No longer maintained.

DYAD

Developers: Pollack, Tesar
Affiliation: University of Florida
Gainsville, Fla. 32601
Application: Dynamic analysis of planar mechanisms composed of interconnected two-link Assur groups. Force analysis and dynamic analysis to second order. Pin or slider joints. Closed-form analysis.
Availability: From developer.
Comments: Handles most common linkages found in industrial applications.

DYSIN

Developers: Muklebust, Tesar
Affiliation: University of Florida
Gainsville, Fla. 32601
Application: Group of Burmester theory based synthesis programs for 4-bars, 6-bars and geared 5-bar linkages. Synthesis for finite or infinitesimal motion properties.
Availability: From developer.
Comments: Effective use of these programs probably requires a fair amount of sophistication on the part of the user to properly interpret and utilize the output. A manual is being prepared to describe the technical details of their operation.

FIVEPOS

Developers: Sandor, et al.
Affiliation: Rensselaer Polytechnic Institute
Troy, New York 12181
Application: Five position Burmester Theory. Synthesis of function, path, and motion generating four-bar linkages.
Hardware: Time-sharing with T4002 Tekronix terminal.
Graphics: Limited, with T4002 storage tube.
Availability: From developer.
Comments: As with Dysin, effective use of this program requires considerable sophistication on part of user. Basic theory has wide application when cleverly employed.

FORCE

Developer: Peterson
Affiliation: General Dynamics, Convair Division
San Diego, Cal.
Application: Two-dimensional kinematic analysis of mechanisms including effects of joint friction. Employs friction circle concept.
Hardware: CDC 6400, FORTRAN extended compiler
Graphics: Yes, with Control Data Cyber display system, CDC 274.
Availability: From developer.
Comments: Control Data system is incompatible with most IBM hardware.

GEAR

Developer: Kersten
Affiliation: University of Nebraska
Lincoln, Nebraska 68508
Application: Package of small programs for cam, gears, and four-bar linkage coupler curves. Primarily intended for student use.
Graphics: CAL-COMP
Availability: From developer.
Comments: Many researchers have similar such programs available. These seem fairly elementary but useful for teaching purposes in undergraduate design courses. Permit visualization of gear tooth undercutting, etc.

IMP, IMP-73

Developer: Uicker
Affiliation: University of Wisconsin
Madison, Wisconsin 53706
Application: Analysis of two- or three-dimensional closed-loop rigid link mechanisms. All types of joints, linear springs, viscous dampers, mass, and gravity effects may be included. Driving forces or motions may be specified as functions of time. Kinematic, static and time response modes of operation. Matrix based system formulates and solves as many differential equations as system has degrees of freedom. Provides positions, velocities, accelerations, static, and dynamic constraint forces, damping ratios, and natural frequencies.
Hardware: Time-sharing or batch. Two machine dependent routines. System runs on most large-scale computers.
Graphics: None at present.
Availability: From developer. About 80 systems now distributed.
Comments: IMP-73 is latest release of IMP and supersedes earlier versions. New releases are planned as developments warrant. System is well documented and supported. Several time-sharing services offer IMP.

IMP-UM

Developer: Sheth
Affiliation: University of Michigan
Ann Arbor, Michigan 48104
Application: Essentially identical to IMP, but with different integration technique. (Predictor corrector method). Provides additional small oscillation mode of analysis yielding frequency domain and transfer function data. Also handles impacts.
Hardware: Time-sharing or batch.
Graphics: Storage Tube.
Availability: Available through DECAL, 1210 E. Engineering, University of Michigan, Ann Arbor, Mich. 48104.
Commercially available: Structural Dynamics Research Corp., 5729 Dragon Way, Cincinnati, Ohio, 45227.
Comments: IMP-UM is currently well supported but may deteriorate since Sheth is leaving Michigan.

KIDYAN

Developer: Brat
Affiliation: Czech Technical University
Prague 2, USSR

Application: Provides kinematic and dynamic analysis of general planar mechanism. Also provides phase plane analysis. Analysis technique based on Assur groups.
Hardware: Written in SLANG language for the MINSK 22 computer.
Availability: Perhaps, in Vladivostok.
Comments: Great, if you own a MINSK 22 computer.

KINAL

Developer: Paul
Affiliation: Department of Mechanical Engineering
University of Pennsylvania
Philadelphia, Pa. 19174
Application: Kinematic analysis of planar, multi-loop single degree of freedom mechanisms. User provides starting estimate of all secondary position variables, together with a small subroutine given the driving variable, its velocity and acceleration, as a function of time.
Availability: From developer.
Comments: User manual is being developed.

KINE

Developers: Gupta, Banerjee, Fox
Affiliation: Case Western Reserve University
Cleveland, Ohio 44106
Application: General program for position, velocity and acceleration analysis of planar machanisms with revolute and prismatic joints, gears, and cams. Systems of multi-degree of freedom. Provides spline fit to cam or curved slide track contour. Uses a modified Newton Raphson scheme.
Hardware: UNIVAC 1108, FORTRAN V
Comments: System is noninteractive, but appears easy to use and versatile. Capability of easily incorporating cams and curved sliders is useful feature.

KINEMAT

Developer: Dratch
Affiliation: Stevens Institute of Technology
Hoboken, N. J.
Application: Planar analysis of machanisms for positions, velocities, and accelerations. Program based on line geometry, uses forward integration scheme.
Hardware: Requires both SNOBOL and FORTRAN IV compilers.
Availability: Not available.
Comments: System is interesting because it is the only one based on line geometry. Also, it uses SNOBOL to derive and formulate the system of analysis equations, then uses FORTRAN as a postprocessor to carry out the numerical solution.

KINSYN

Developers: R. Kaufman, Maurer
Affiliation: Department of Mechanical Engineering
Massachusetts Institute of Technology
Cambridge, Mass. 02139
Application: Synthesis and analysis of pin or slider-jointed planar linkages, particularly four-bars and slider-crank mechanisms. Burmester-based

exact synthesis for four or five positions in motion generation, function generation, or path generation. Yields all theoretically available designs with given constraints. Closed-form. Provides interactive feedback, graphical input and animation. Designer oriented system.
Hardware: Original version 8K IBM 1130, Tektronix 611, storage tube display, "bowling ball" graphical input device.
Graphics: Yes, interactive graphics through use of non-store mode on storage tube.
Comments: System capabilities similar to those of SYNTH, but on a much smaller computer. Unusual for interactive synthesis orientation and for ease of use by non-specialists.

KINSYN II

Developers: Kaufman, Ruegsegger, Rubel, Parsons, Hare, Knutson
Affiliation: Department of Mechanical Engineering
Massachusetts Institute of Technology
Cambridge, Mass. 02139
Application: KINSYN II enables a designer to synthesize for two, three, four, or five positions of pin or slider jointed mechanisms without losing any of the design freedom inherent in the problem. Numerical techniques allow for synthesis of mechanisms of arbitrary topology and for arbitrary performance requirements. Interactive, dynamic system. Synthesized mechanisms can be analyzed, with full display of part shapes and clearance.
Hardware: Imlac display, Shintron data tablet, Interdata Model 80 minicomputer.
Graphics. Yes, dynamic interactive display, with data tablet.
Comments: Dynamic graphics in a user-oriented, interactive system makes system ideal for use by designers with limited experience with computers or with theoretical kinematics. System is self-explanatory using a natural, graphical language.

LSD

Developer: Uncertain.
Affiliation: Boeing Computer Services
Seattle, Washington
Application: Kinematic analysis as special case of large scale structural deformation. Three-dimensional. Analysis by kinetic and potential energy.
Availability: Commercially available from Boeing Computer Services.
Comments: Program was not primarily developed for kinematics.

MARKUS

Developer: Markus
Affiliation: Institute of Machine Mechanics
Slovak Academy of Sciences
Bratislava, Czechoslavakia
Application: Kinematic analysis of planar mechanisms with lower pairs. Interactive solution procedure yields positions, velocities, and accelerations. Input consists of matrices giving the mechanism topology and proportions.
Hardware: Algol 60 required.
Availabiltiy: Not certain.
Comments: System uses numerical approach to solving automatically generated closure equations.

MECHSYN

Developer: Garret, Reed
Affiliation: Purdue University
Lafayette, Ind.
Application: Interactive construction, modification, and animation of pin and slider-jointed mechanisms. Coupler curves, velocity, and acceleration holographs.
Hardware: IMLAC PDS-1 tied to CDC 6500.
Graphics: Interactive dynamic graphics using light pen.
Availability: From developer.
Comments: System is extremely hardware dependent so that conversion to another installation would be very difficult.

MECH 3D

Developer: Peterson
Affiliation: General Dynamics, Convair Division
San Diego, Cal.
Application: Computer graphics front-end for IMP-73. Permits interactive construction of three-dimensional linkages, performs topological dissection of the mechanism and massages the results into form for processing by IMP.
Hardware: CYBER display, Control Data 6400.
Graphics: Yes, with dynamic display and light pen interaction.
Availability: Not at this time.
Comments: MECH 3D hasn't been finished, due to the difficulties of reworking IMP's data management system for use on the CDC 6400.

MEDES

Developer: Bona, Galletti, Lucifredi
Affiliation: Olivetti, Ivrea, Italy, and University of Genoa, Italy
Application: Planar mechanisms analysis and synthesis. Five-position synthesis and numerical optimization, analysis of general mechanisms. Cam and gear design. Stress analysis and tolerance analysis.
Hardware: IBM 1130, 2250.
Graphics: Yes, dynamic light pen interaction with 2250 display.
Availability: Internal to Olivetti.
Comments: Part of ambitious mechanical design system similar to COMMEND I. System interfaces to drawing management system and N/C machining capabilities.

MEDUSA

Developer: Dix
Affiliation: Illinois Institute of Technology
Chicago, Ill. 60616
Application: Dynamic simulation. Formulates equations of motion (newton) and integrates. User codes one subroutine consisting of a series of subroutine calls. This sequence describes the particular mechanisms joints to the system.
Availability: From developer. Manual available.
Comments: System is much slower running than Lagrange-based systems.

RAP

Developer: Rappaport
Affiliation: United Aircraft Research Laboratories
E. Hartford, Conn.
Application: Position analysis of three-dimensional mechanisms. Provides for all conventional joint types, together with gears, racks, and cams acting on cams.
Hardware: IBM/360 with IBM 2250.
Graphics: Yes. Dynamic interaction with 2250 light pen display.
Availability: Internal to United Aircraft.
Comments: Allows display of three-dimensional linkages during animation, by showing two orthogonal views on split screen.

SKETCHPAD, SKETCHPAD-III

Developers: Sutherland, Johnson
Affiliation: Lincoln Laboratory, M.I.T.
Lexington, Mass. 02139
Application: One of the first CAD systems. Sketchpad was capable of animating linkages by satisfying the graphical constraint that the lines not separate. Sketchpad III could do the same in three dimensions. Neither system was intended specifically for kinematics.
Hardware: Original system used Lincoln's TX-2 computer. Sketchpad-like capabilities have since been implemented on other systems using AED Compiler.
Graphics: Yes. Dynamic Interaction with light pen on a special display scope.
Availability: From developer.
Comments: Sketchpad is mentioned mainly for historical perspective, not as a practical kinematic analysis tool.

SYNTH

Developer: Peterson
Affiliation: General Dynamics, Convair Division
San Diego, Cal.
Application: Synthesis of four-bar linkages using Burmester curves. Allows synthesis of motion generating mechanisms with four precision positions. Also allows synthesis of toggle mechanisms having specified dead center positions. Uses dynamic interaction via light pen. Animation allows judgment of clearances, etc.
Hardware: CDC 6400, CDC 274.
Graphics: Yes. CDC 274 display provides dynamic interaction for both input and display.
Availability: From developer.
Comments: A powerful design capability. Many features similar to KINSYN.

TOAD

Developer: Sturges
Affiliation: Charles Stark Draper Laboratories, Inc.
Cambridge, Mass.
Application: Teleoperator arm design program. Symbolically derives the exact differential equations for open-loop manipulators. Includes internal, centripetal, Coriolis, and gravitational effects. Computationally optimizes the equations by factoring and by application of

trigonometric identities, etc.
Hardware: Requires PL/1-FORMAC interpreter.
Availability: From developer.
Comments: Output of TOAD consists of the differential equations, in symbolic form, not the manipulator's behavior. Typically these equations are highly complex and could not be obtained by hand.

VECNET

Developer: Andrews
Affiliation: University of Waterloo,
Waterloo, Ontario, Canada
Application: Rigid body dynamic analysis of three-dimensional mass-spring-dashpot systems. Not specifically applicable to mechanisms at present, though joint constraints are being incorporated in the system. Based on graph theory, program develops its own analysis equations using trees and cut sets starting with a node-node incidence matrix description of the mechanical system.
Availabiltiy: From developer, though no manual currently exists.
Comments: At present, system is not designer oriented and is limited in its ability to handle three-dimensional mechanisms. System is being expanded to handle a wider class of problems.

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Seismic Analyses

Charles A. Miller and Carl J. Costantino

The City College of New York

INTRODUCTION

Lateral loads resulting from a seismic disturbance are an important consideration in the design of structures in many parts of the world. Unlike most other loads acting on the structure, the dynamic effects associated with seismic loads are significant. The energy contained in a seismic disturbance is concentrated in the 1 through 10 cps frequency band so that response amplifications due to the interaction of the disturbance with the primary vibrational modes of the structure are to be expected. This suggests that dynamic analyses are required. Because of the complexities, however, of performing such analyses it has been standard practice [1] to represent seismic loads with "equivalent" static loads. Seismic loads may then be handled in the same manner as any other static loading. Computer software available to treat the static problem is discussed elsewhere [2], and is therefore not considered further in this chapter.

The severity of seismic loadings have required that engineers make some reasonable trade off between safety and economics during the design process. To alleviate this difficulty recent trends in building codes have been in the direction of requiring that a dynamic analysis be performed for seismic loading. For example, all primary components of nuclear power plants constructed in the United States must be the subject of a dynamic analysis to evaluate seismic effects. This is even required for plants which are to be constructed in seismically inactive areas. The objective of this chapter is to discuss computer software that is available to treat the seismic response problem when dynamic effects are included.

The subject area is divided into the following areas of interest:

- free-field
- structure/media interaction
- structural response

In each of the areas the alternative approaches which can be used in generating solutions are first discussed followed by a presentation of computer software which is available. So that each of the above areas may be put into proper perspective, an overview of the seismic design problem is first given.

DESIGN OF STRUCTURES FOR SEISMIC EFFECTS

An overview of the problems involved in the design of facilities for seismic effects is given in Fig. 1. An earthquake originates at the epicenter by a "sudden" fracture of the bedrock. The strain energy stored in the rock is released to the surrounding rock mass in the form of kinetic energy. This energy then radiates out to the location of interest.

The magnitude and shape of the disturbance that finally reaches the site depends of course on the intensity of the energy released at the source, but

also depends on the geology between the source and site. Because of the relative stiffness of the bedrock to that of the overlying soil most of the energy is transmitted through the bedrock. As the disturbance propagates through the bedrock in the vicinity of the facility, energy propagates through the soil overburden to the facility. The disturbance reaching the facility then depends on:

- the characteristics of the faulting at the epicenter
- the geometry, mass, and stiffness properties of the bedrock between the
- epicenter and the site
- the properties of the soil overburden in the area of the facility

The facility itself tends to move with the soil overburden, thereby introducing seismic inertial loadings on the structure. Because of differences in mass and stiffness of the facility from that of the soil it displaces the motion of the facility varies from the motion of the surrounding soil. This effect is termed soil/structure interaction. If the structure is massive then the free-field motion is also altered at considerable distances from the structure as a result of this interaction. This may require that the effects of one structure on a nearby structure must be considered.

Once the motion, or equivalently forces, acting on the structure have been determined its response may be evaluated by standard methods of structural dynamics. It is usual practice to uncouple the equipment from the main structure when this dynamic analysis is performed. This is done to accomodate the usual design process with different engineering groups having responsibility for the structure and equipment. Seismic response of the primary structure is evaluated considering the mass of all equipment to be rigidly attached to the structure. Motion histories at equipment support points are then used as the basis for detailed design of the equipment.

SHOCK SPECTRA

In many parts of the design process outlined above a detailed treatment of the seismic disturbance in terms of a motion time history is unwarranted in view of the uncertainties which exist. A less detailed representation of any disturbance is the shock spectra of the disturbance. The shock spectra represents the peak response of a linear oscillator to the disturbance, thereby characterizing a disturbance by the peak response it causes. Software available for generating spectra from motion histories and motion histories to fit a given spectra are discussed.

Shock Spectra for a Wave Form

The problem is to calculate the peak response of a damped linear oscillator shown in Fig. 2a to the given disturbance for a range of oscillator frequencies. The governing equation is

$$\ddot{z} + \frac{c}{m}\dot{z} + \omega^2 z = -\ddot{y} \qquad (1)$$

where Z = relative displacement of linear oscillator base to mass
c = damping
m = mass
y = disturbance
ω = circular frequency

The usual method of solution is to obtain an analytic solution based upon a piecewise linear description of the disturbance. The maximum response is then sought by searching throughout the total duration of the pulse. A plot of this peak response as shown on Fig. 2b is the shock spectra of y(t). Special provision must be made for extending the pulse so that the final velocity is zero if maximum responses occurring after the pulse duration are to be

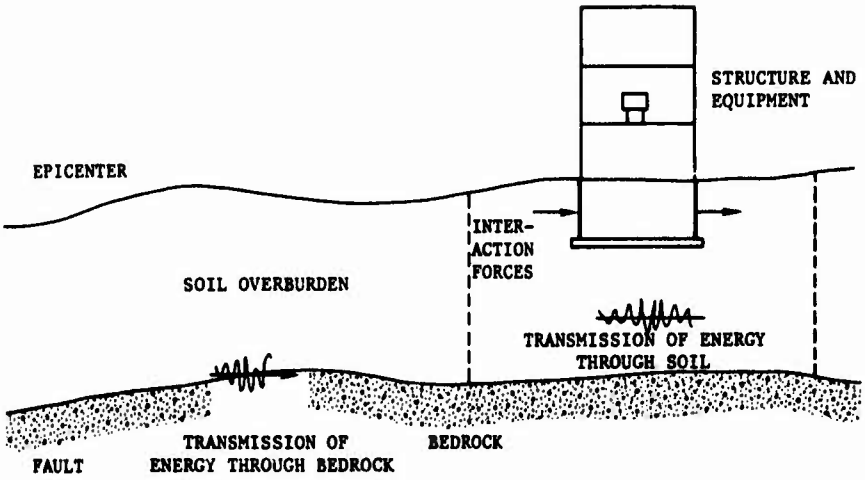
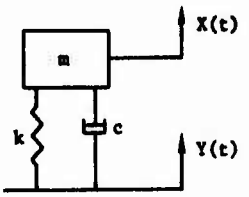
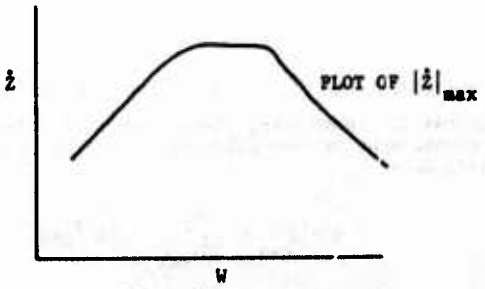


Fig. 1 Seismic design problem



$\omega^2 = k/m$
$Z(t) = X - Y$

(a) Linear Oscillator



(b) Shock Spectra of Y(t)

Fig. 2 Definition of shock spectra

considered. Warnings should be included in the computer code setting frequency limits for the validity of the results. The maximum valid frequency for the spectra is determined by the spacing of data points (Δt) in the disturbance while the minimum valid frequency is set by the duration of the pulse (t_{max}). Reasonable ranges are

$$\frac{T_{min}}{\Delta t} = 10 \tag{2}$$

$$\frac{T_{max}}{t_{max}} = 0.10$$

where T_{max}, T_{min} are maximum and minimum periods of shock spectra.

Many of the dynamic response computer software packages have shock spectra routines built into the code; these will be discussed in conjunction with the response codes. Specific packages available to generate shock spectra are SPECEQ/SPECUQ (1)[1] and SPECTR (2).

Wave Form to Fit a Spectra

The inverse problem of generating a wave form that fits a given spectrum is also of interest. There is no unique solution to this problem however so that some additional criteria are needed to select the wave form best suited to the problem. The usual criteria employed in selecting a wave form to fit a given spectra is that given in Eq. (2) defining the frequency range over which one can expect the wave form to fit the spectra plus a criteria that the acceleration record contain a relatively uniform frequency content (so that all possible modes in the frequency range of interest may be excited).

Wave forms are selected in the form,

$$y(t) = \sum_{n=1}^{N} C_n \sin \lambda_n (t - t_n) \tag{3}$$

$$\lambda_n = 2\Pi / t_{max}$$

where C_n are the magnitudes of the modal components of the pulse and t_n are the modal time phasing components. The range in λ_n must encompass the frequency range of the spectra over which the fit is desired. The required may be evaluated from

$$N = \eta_{max} = \frac{\lambda_n}{2\Pi t_{max}} = 10 \frac{T_{max}}{T_{min}} \tag{4}$$

[1] The number after Computer Code Acronyms refers to a bibliographic listing of codes in Table 2.

SIMEAR (3) is a computer code available through National Information Service Earthquake Engineering (NISEE) which generates the parameters (C_n) based upon a random selection of modal time phasing components (t_n).

FREE FIELD

The seismic analysis of structures is performed with consideration given to the area immediately surrounding the facility. Any effects that the geologic formations located between the site and epicenter may have on the transmission of the disturbance to the site are not considered in detail. The free field could be modeled with finite elements as indicated in Fig. 3a, a dynamic disturbance input at the fault, and the equations of motion integrated to obtain the resultant seismic disturbance at various distances from the source. Many finite element codes (e.g. ANSYS (7), SLAM (6)) could be used to perform this calculation.

Several factors mitigate against such computations however. First, the sites of interest are located far from the source so that the required size of a finite element mesh would lead to computationally intractable problems. Since cylindrical dispersion of the energy is an important consideration, the problem will be three dimensional unless an axisymmetric model is used. But this places unreasonable restriction on the geologic variations used to model the region. This difficulty makes the problem size still further beyond the reach of current codes and computer capabilities. Second, the wave propagation calculation requires detailed material property data for the entire region of interest (i.e. from epicenter to site). As a minimum the elastic and damping properties of the bedrock are needed if any useful data were to be obtained. This data is usually not available and would be costly to obtain.

This is an area where one would expect further development. Probably the initial directions will be to apply existing software to problems associated with unusual geologic formations in the immediate vicinity of the site as indicated in Fig. 3b (e.g. vertically stratified bedrock, domes in the bedrock).

There are several codes available to treat such problems; current seismic design practice however does not require that these analyses be performed. There are codes such as QUAD -4 (5) and SLAM (6) which have been developed for the specific purpose of analyzing the propagation of disturbances through soil/rock media. Both of these codes are based upon a finite element representation of the media and have the capability of incorporating nonlinear media properties. The former code can include material damping. Material (soil) damping is an important factor in assessing the extent to which the input disturbance is amplified by the frequencies of the soil system. SLAM code contains "non-reflecting" boundaries (boundaries of the finite element mesh which absorb the disturbance as if the mesh were extended beyond the boundary). This feature allows relatively small meshes to be used. The large general purpose codes such as ANSYS (7) and NASTRAN (8) can also be used. However the special purpose programs are more useful than the general purpose programs in that they contain features designed to meet the specific needs of the special problems.

The difficulties discussed above are circumvented in practical design problems by specifying a criterion seismic disturbance at the site of interest. Actual earthquake records may be obtained from California Institute of Technology (4). This specification is usually in the form of a shock spectra describing the expected seismic motion at the surface. The magnitude of spectra is deduced from a consideration of: the seismological history of the area; potentially active faults; the local geology; and the soil overburden.

For many analyses the spectral form of the disturbance is inadequate and must be converted to a time history. This may be accomplished as discussed above. The surface time history often must be expanded to include time histories at various levels in the soil overburden.

This may be done based upon the shear beam model of the soil as shown in Fig. 3c. This model is based upon the assumptions that adjacent columns of soil undergo the same displacement as the one under consideration, and that

the deformations of interest are associated with shear strains of the column. The required column properties are the shear modulus (G_i) and the soil damping (D_i). These properties may be varied from layer to layer. While the soil properties usually vary with strain level, most of the analyses are performed assuming constant properties with the actual strains used to select the appropriate property value. This often requires that trial runs be made to assess the strain magnitude.

Two methods of analysis are available, one using a vertical finite element technique and the other making use of the continuous solution technique for one dimensional shear waves travelling vertically upwards through continuous layers.

With the properties of the soil known, the continuous method of analysis follows in a straightforward manner. If, for example, the bedrock motion history (at a particular frequency) is defined in complex form by

$$u_b(t) = U_b\, e^{i\omega t} \tag{5}$$

a simple matrix equation of the form

$$[C]\ \{U\} = \{F\} \tag{6}$$

can be generated to determine the corresponding coefficients at each soil layer interface. In Eq. (6), the vector {F} is a known vector in terms of the specified input coefficient, U_b, and {U} is the vector of corresponding interface coefficients; that is, at the ith interface

$$u_i(t) = U_i\, e^{i\omega t} \tag{7}$$

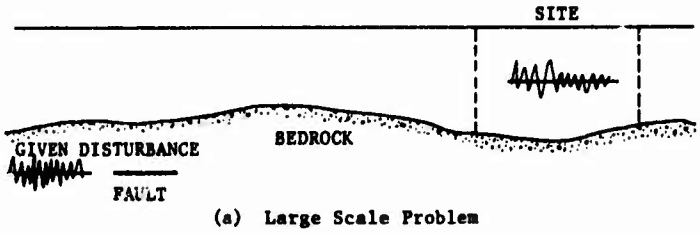
The matrix C is determined by satisfying the soil interface continuity conditions. All terms in Eq. (6) are of course complex so that for N soil layers, Eq. (6) is equivalent to solving a 2N system of linear equations at every frequency of interest, a straightforward task.

If the lumped mass method of analysis is used, the first step in the analysis requires that all the corresponding natural frequencies of the column be generated. Once this is available, the steady state solution is easily obtained. However, for long soil columns or for cases requiring high frequency response, a fine finite element mesh is required leading to a large number of degrees of freedom. Generation of the modes of this system then leads to lengthy computer running times. SHAKE (9) and SLABS (10) are codes available to perform this analysis. Both are based upon the continuum model of the soil column.

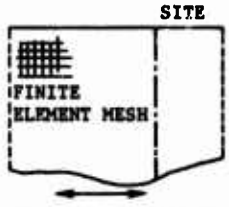
STRUCTURE/MEDIA INTERACTION

The process of converting the free-field seismic disturbance to forces acting on the structure is termed structure/media interaction. This aspect of the overall computation is usually handled as a part of the response calculation so that there is no computer software devoted solely to the interaction problem. Rather various models of the interaction process are embodied within the response codes to be discussed in the next section of the paper. The various approaches to the problem are discussed here.

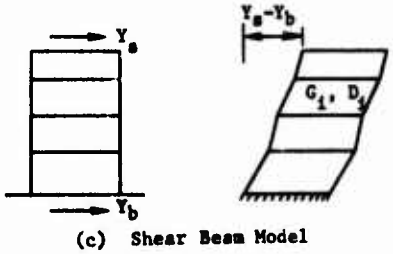
Consider the general problem, shown in Fig. 4, of computing interaction



(a) Large Scale Problem



(b) Local Effects.



(c) Shear Beam Model

Fig. 3 Free field problems

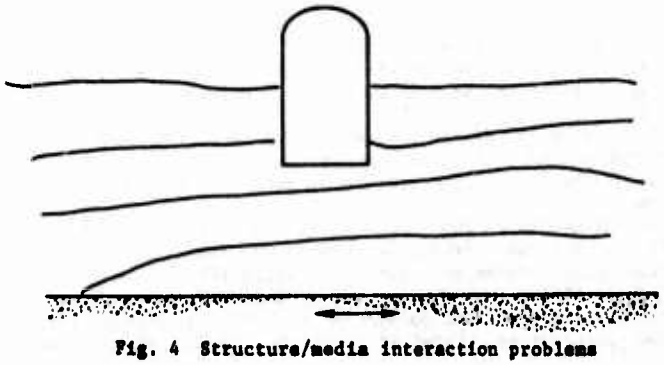


Fig. 4 Structure/media interaction problems

forces acting on the embedded structure given the bedrock seismic disturbance. There are two general approaches used to attack this problem (excluding wholly analytic solutions as untenable for configurations of interest). The first is a finite element model of the entire problem and the second is based upon an approximate interaction law which decouples the structure from the free field.

Finite Element Model

A finite element model of the problem is shown on Fig. 5 wherein finite elements are used to model the free field. The propagation of the disturbance from bedrock to the surface is computed by numerical integration of the equations of motion developed for the mesh. To treat the interaction of the free-field soil/rock with the structure, the finite element mesh can be continued through the structure wall. Thus, no other considerations need be made in the developed code (save for possible separation and sliding effects). However, this approach has serious drawbacks. Many more elements would be required to treat the structure in this manner, increasing computer running times. Of more importance, however, is the following fact. The stiffest material encountered in a typical problem is usually the structural material. In addition, the smallest sized elements in the problem occur through the thin wall of the structure. This combination leads to extremely high frequencies in the mesh, in turn leading to extremely small time steps in the required numerical integration procedure. In fact, this occurs because the mesh in the structural wall is able to transmit the high frequency through-the-thickness waves which would develop.

This refinement in the solution is generally unwarranted. In fact, the usual structural representation in a soil-structure interaction analysis consists of the rigid body modes of the structure together with its lower free-free elastic modes of vibration.

In the finite element approach to the dynamic soil-structure interaction problem, the following aspects are usually included.

a) The free-field is represented by a finite element mesh and treats the wave propagation problem through a layered soil/rock site, each material in the free-field being allowed to have its own nonlinear constitutive relationship.
b) The structure is represented by its rigid body modes together with its lower free-free elastic modes.
c) Potential separation and sliding between the structure and the free-field can be treated by means of a special element (of zero thickness) placed between the structure and the soil.

Approximate Interaction Law

Interaction forces may be computed with an approximate law, the essential feature of which is decoupling of the structure from the free field. The obvious advantage of this approach is that the free field motion time history in the vicinity of the structure may be used as the forcing disturbance of the structure, thereby eliminating the requirement to simultaneously model the free field and structure.

In determining the proper soil-structure model, two aspects of the interaction process may be considered, namely: equivalent stiffness and damping of the soil below the base of the structure; and effects of depth of burial on the interaction.

The determination of the base interaction parameters is made by comparison with analytic solutions obtained for the steady state response of a uniform elastic half-space. These solutions were originally obtained by Bycroft [4] for circular foundations on the surface of the half-space. From these analyses a series of studies was conducted from which equivalent base interaction springs and dampers were determined [5 - 11]. In all cases, it was

found that the base interaction constants were a function of the input steady state frequency. For a typical seismic input, however, the free-field motion consists of energy over a band of frequencies. This implies that the interaction parameters should then be a function of frequency of the input motion. Fortunately, for most of the parameters, this frequency dependence is not severe so that constant spring and dashpot parameters can be used.

Representative parameters are listed below, and shown on Fig. 6.

Horizontal spring constant:

$$K_x = (0.9)\,\frac{32(1-\mu)}{(7-8\mu)}\,\rho V_s^2 a$$

Horizontal dashpot constant:

$$C_x = (0.57)\,\frac{32(1-\mu)}{(7-8\mu)}\,\rho V_s a^2 \tag{8}$$

Rocking spring constant:

$$K_\theta = (0.8)\,\frac{8}{3(1-\mu)}\rho V_s^2 a^3$$

Rocking dashpot constant:

$$C_\theta = (0.15)\,\frac{8}{3(1-\mu)}\rho V_s a^4$$

In Eq. (8), V_s is the shear wave velocity ($=\sqrt{G/\rho}$), G is the shear modulus of the soil, ρ is the soil mass density, a is the base radius, and μ is Poisson's ratio.

In addition to the base stiffness effects listed above, the effects of depth of burial on the interaction parameters must be accounted for. To use the same philosophy as the previous analysis, that is, to represent the interaction process through a series of springs and dashpots, a different formulation technique must be used. In [12], a detailed discussion of such an interaction model is presented. The loading applied to the structure is written as

$$\sigma_r = k(w_o - w_s) + s(\dot{w}_o - \dot{w}_s) \tag{9}$$

where

σ_r = radial stress applied to the structure
w_o = free field radial displacement of the soil
w_s = radial displacement of the structure
$\dot{w}$ = corresponding radial velocities
k = foundation interaction spring
s = foundation interaction damper

This type of interaction law has been used previously for both static and dynamic problems (protective construction problems, culvert design), and was developed on the basis of both analytical and experimental studies on buried

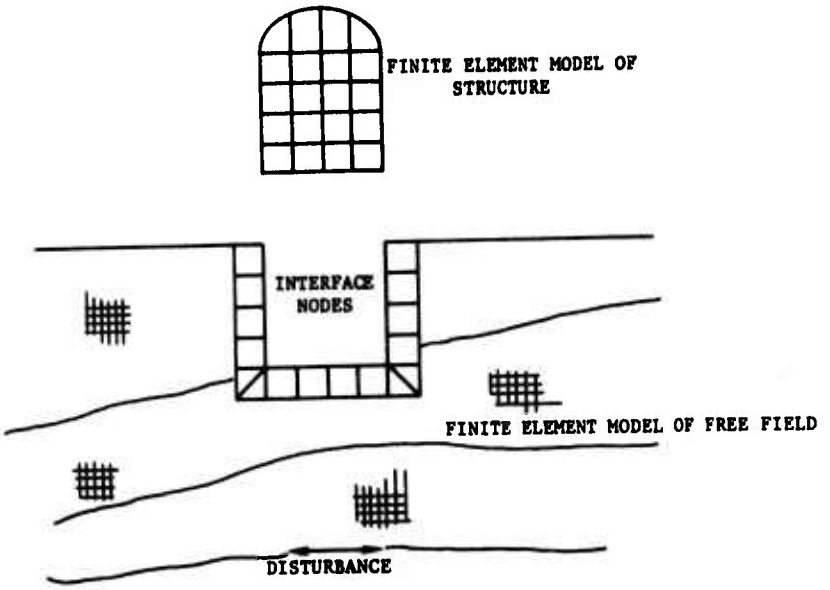
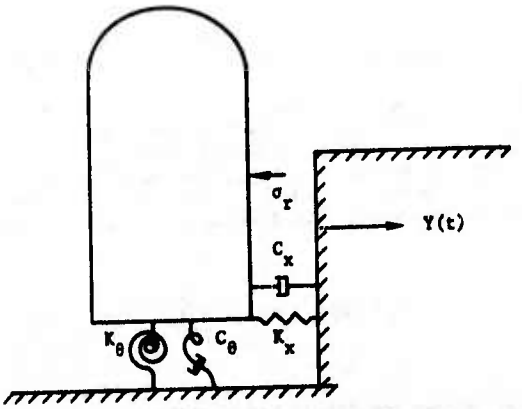


Fig. 5 Finite element model of structure/media interaction



$$K_\theta = .213V_s^2\rho a^3/(1 - \mu)$$

$$C_\theta = .400V_s\rho a^4/(1 - \mu)$$

$$K_x = 27.8V_s^2\rho a(1 - \mu)/(7 - 8\mu)$$

$$C_x = 18.2V_s\rho a^2(1 - \mu)/(7 - 8\mu)$$

$$\sigma_r = k(\Delta w) + s(\dot{\Delta w})$$

Fig. 6 Approximate interaction model

cylindrical structures. A comprehensive list of these references is presented in [12]. From [12], the parameters k and s can be determined from the soil properties through

$$k = E_c/2a$$
$$s = \eta \rho V_c \tag{10}$$

where E_c is the constrained modulus of the soil, ρ is the soil mass, and V_c is the dilatational wave speed of the soil. The parameter η is a damping factor which on the basis of the experimental results of [12] is chosen as 0.5. For an elastic material the constrained modulus is simply

$$E_c = 2G(1-\mu)/(1-2\mu) \tag{11}$$

where G is the shear modulus and μ is Poisson's ratio. The dilatational wave speed is

$$V_c = \sqrt{\frac{E_c}{\rho}} \tag{12}$$

Beredugo and Novak [13] have recently obtained analytic solutions for this problem of the same form as Bycroft [4] did for the foundation. The "constants" of the interaction law are again found to be functions of frequency but for practical problems the selection of these parameters as constants is reasonable.

STRUCTURAL RESPONSE

The problem of primary interest to the engineer is assessing the magnitude of the stresses in the structure and the motion of equipment support points as a result of the seismic disturbance. For a typical structure such as shown in Fig. 7 the governing equations are:

$$M\ddot{x} + C\dot{x} + Kx = F + P_c \tag{13}$$

where

- M = mass matrix of structure
- C = damping matrix of structure
- K = stiffness matrix of structure
- F = interaction forces
- P_c = correction forces to account for nonlinear effects
- x = displacement vector

Selection of the method of solution and the form in which the parameters of Eq. (13) are specified depends to a large extent upon the degree of precision desired. Computer software is available to treat most structural representations; the only real limitation is the cost of computer time to implement the more sophisticated solutions.

The following three methods may be used to solve Eq. (13):

1. Direct Integration - A numerical integration scheme

$$M\ddot{X} + C\dot{X} + KX = F + P_c$$

M = Mass Matrix
C = Damping Matrix
K = Stiffness Matrix
X = Displacement Vector
F = Interaction Forces
P_c = Correction Forces Accounting for Non-Linear Structural Behavior

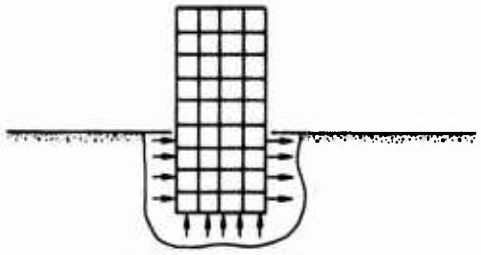


Fig. 7 Structural response problem

(e.g. Runge-Kutta) is used to directly integrate the equations of motion. This has the advantage of allowing for the inclusion of nonlinear effects (P_c) and any form of the damping matrix (C). The disadvantage is the long running times required for problems of typical size.

2. Direct integration of modal equations - The modes of vibration are determined and used to recast Eq. (13) into modal equations. The principal advantage of this method is that the modal equations are uncoupled from each other so that only the modes of interest need to be integrated, thereby greatly reducing the order of the system. The disadvantage of this approach is the restriction placed on the description of the structure. The damping matrix must be of a particular form so that the modal equations are uncoupled and nonlinear effects in the structure cannot be included.

3. Shock spectrum - The modes of vibration of the structure are determined. The peak response in each of the modes is evaluated from the shock spectrum of the free field motion. The total response may then be determined by summing the modal responses. Since all aspects of phasing are lost in this method, only an upper bound to the peak response is found.

For each of these methods there are several options for describing the structure and the structure/media interaction process. The mass and stiffness of the structure may be modeled in detail, resulting in a large number of degrees of freedom, or may be modeled with an equivalent system having a reduced number of degrees of freedom. A detailed model may be required to obtain reliable stress estimates, while a less detailed model would suffice to compute the motion of equipment support points. Nonlinear structural effects normally are not included in the analysis. The increased model complexity and resultant longer computer running times required to include nonlinear effects are unwarranted in view of the uncertainties which still exist in the description of the free-field seismic disturbance.

The interaction process may be modeled as discussed above. Again, the more sophisticated model of the interaction process is often unwarranted. A summary of the types of codes available to treat these response problems is given in Table 1. Each of the structural model types shown in the Table is discussed.

Table 1 Structural Analysis Software

INTERACTION MODEL / STRUCTURE	"EXACT"			APPROXIMATE	
	NONE	WELDED	SLIP	NO DEPTH OF BURIAL	DEPTH OF BURIAL
3D FRAME/PLATE	I	NOT FEASIBLE	NOT FEASIBLE	I	I
2D FRAME/PLATE	I	I	NOT PRACTICAL	I	I
SHELLS	II	NOT PRACTICAL	NOT PRACTICAL	I	I
EQUIVALENT BEAMS	III	I	NOT OF INTEREST	III	III
FOCUS ON INTERACTION PROBLEM - F.E. SOIL MODAL REPRESENTATION OF STRUCTURES (2D)	NOT OF INTEREST	IV	IV	NOT OF INTEREST	
FOCUS ON INTERACTION (3D)	NOT OF INTEREST	V	V	NOT OF INTEREST	

I ANSYS, NASTRAN, STARDYNE, ICES/DYNAL

II KSHEL, SABOR

III SIM

IV SLAM

V FARSS

1. 3D Frame/Plate - When three-dimensional aspects of the structure are to be included, general purpose codes such as ANSYS (7), NASTRAN (8), STARDYNE (11), ICES/DYNAL (12), or SAP IV (13) should be used. Because of bandwidth limitations it is not feasible to include an exact modeling of the interaction process in these problems.

2. 2D Frame/Plate - The same general purpose codes are used for two-dimensional problems. It is feasible to extend these structural models into the free-field so that the exact-welded interaction boundary is included. The soil is modeled with finite elements and soil nodes connected to the structural nodes at the interface. The slip boundary would permit relative motion of the soil and structural nodes located along the interface. Because of computational difficulties it is not practical to incorporate this slip boundary with a detailed model of the structure.

3. Shells - KSHEL (14) and SABOR (15) are two codes which may be used to handle shell type structures. These codes integrate classical shell equations along a meridian with all quantities expanded in a Fourier series in the circumferential direction. Structure/media interaction cannot be included.

Table 2 Bibliographic Listing of Codes

NO.	TITLE	APPLICATIONS	LANGUAGE	SPECIAL MACHINE LIMITATIONS	SOURCE
1	SPECEQ/SPECUQ	Generates shock spectra from pulse digitized at equal or unequal steps.	FORTRAN IV	None	NISEE 729 Davis Hall University of California Berkeley, Calif. 94720
2	SPECTR	Generates shock spectra from pulse digitized at equal steps.	FORTRAN IV	None	NISEE
3	SIMEAR	Generates simulated earthquake to fit shock spectra.	FORTRAN IV	16K storage	NISEE
4	Strong Motion Earthquake Accelerogram Data	Records of actual earthquakes.	—	—	California Institute of Technology E ERL Pasadena, California 91109
5	QUAD - 4	Seismic response of soil deposits - finite element.	FORTRAN IV	CDC 6400, 6600 7600	NISEE
6	SLAM	Finite element solution to soil-structure systems subjected to dynamic loadings.	FORTRAN IV	CDC 6600, 7600 IBM 360-370 series	C. Miller Dept. of Civil Engineering City College of New York New York, New York 10031

Table 2 (continued)

NO.	TITLE	APPLICATIONS	LANGUAGE	SPECIAL MACHINE LIMITATIONS	SOURCE
7	ANSYS	General purpose finite element analysis of structures.	--	CDC 6600, 7600 UNIVAC 1108 IBM 360/165	Swanson Analysis 870 Pine View Dr. Elizabeth, Penna. 15037
8	NASTRAN	General purpose finite element analysis of structures.	--	General	COSMIC University of Georgia Athens, Ga. 30601
9	SHAKE	Computer response of horizontally layered soil system to shear waves.	FORTRAN IV	50K OCTAL STORAGE	NISEE 729 Davis Hall University of California Berkeley, California 94720
10	SLABS	Computer response of horizontally layered soil system to shear waves.	FORTRAN IV	None	CCNY
11	STARDYNE	General purpose finite element analysis of structures.	--	General	Mechanics Research 9841 Airport Blvd. Los Angeles, Calif. 90045
12	ICES/DYNAL	General purpose finite element analysis of structures.	--	General	McDonnell Douglas P. O. Box 516 St. Louis, Missouri 63166
13	SAP IV	General purpose finite element analysis of structures.	--	General	NISEE

Table 2 (continued)

NO.	TITLE	APPLICATIONS	LANGUAGE	SPECIAL MACHINE LIMITATIONS	SOURCE
14	KSHEL	Response of axisymmetric shell structures for arbitrary time-dependent loads.	FORTRAN IV	IBM 360, CDC 6400 UNIVAC 1108	Prof. A. Kalnins Lehigh University Bethlehem, Penna. 18015
15	SABOR	Response of axisymmetruc shell structures for arbitrary time-dependent loads.	FORTRAN IV	IBM 360/65 370/155 CDC 6400/6600	Philco Ford Ford Road Newport Beach, Calif. 92663
16	SIM	Response of equivalent beam type structures.	FORTRAN IV	IBM 360/50 370/155 CDC 6600	CCNY
17	FARSS	Axisymmetric solid with Fourier expansion.	FORTRAN IV	CDC 6600 UNIVAC 1108	Lockheed 3251 Hanover St. Palo Alto, Calif. 94394

If interaction effects are to be included, then the general purpose codes mentioned in 1. must be used with flat plate finite elements used to model the shell.

4. Equivalent beams - For many problems a detailed model of the structure may be too sophisticated. An equivalent system with a reduced number of degrees of freedom is constructed. Masses are lumped at floors or at other points where heavy equipment is located, and elements having stiffnesses equivalent to that of the actual structure are used to connect the masses. SIM (16) is a code which employs beam elements to connect the masses and incorporates the approximate interaction law.

5. Structure/Media Interaction - The last two rows of Table 1 represent applications focused on the interaction problem. Since the structure is usually much more rigid than the soil, it would be desirable to concentrate the degrees of freedom, used to describe the problem, in the soil rather than the structure. SLAM (6) allows for a finite element representation of the soil coupled with a modal representation of the structure. This code is restricted to two-dimensional problems. FARSS (17) allows for a Fourier series representation of parameters in the circumferential direction so that three-dimensional effects could be approximated.

SUMMARY

A review of computer software available to treat the seismic response of structures has been given. The following results were given:

1. Little consideration is given to the effect of the geologic conditions between a fault and the site in performing seismic analysis. A qualitative assessment of the seismic risk leads to the specifications of a free field spectra at the surface of the site. Computer codes are available to fit motion histories to this spectra and to compute consistent time histories at other depths.

2. Structure/media interaction effects may be included in an analysis through an "exact treatment of the boundary conditions at the interface between the structure and soil or through an approximate interaction law which uncouples the free field from the structure. It is usually not possible, because of computer hardware limitations, to use the "exact" interaction model with a detailed model of the structure. The same limitations have restricted the "exact" interaction model to be applied to two-dimensional problems.

3. A large variety of computer codes are available to treat the structural response problem.

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Simulation of Human Body Response to Crash Loads

D. H. Robbins

Highway Safety Research Institute

INTRODUCTION

During the past fifteen years, mathematical simulations of gross human body motions have been developed as research and development aids for studying human response to a dynamic impact environment. The major impetus for this work was the highway safety and automobile occupant protection movement which began in the mid-1960's. More recently, these simulations have been adapted to the study of aircraft occupants, pedestrians, and cyclists. In general, they are applicable to any situation where a human may be subjected to impact. Figure 1 is a schematic diagram showing the occupant linkage, vehicle contacts, and belt restraint system for one such model.

The bulk of this chapter is included in Tables 1 through 5 compare the features of ten models which are available and adaptable to most large digital computers. Table 1 is a general comparison giving sources of documentation, information, and contacts for obtaining the programs. Table 2 compares the parameters defining the crash victim, while Table 3 gives the means by which forces are transmitted to the victim. Table 4 defines the acceleration or other kinematic inputs which force a dynamic interaction between the environment and the victim. Table 5 provides a summary of computer program information.

HUMAN GROSS MOTION SIMULATIONS

Table 1 identifies ten computer programs. In some cases, two programs are lumped together with either an earlier version or a subset of the most widely used version shown in parentheses. Four of these predict crash victim motions and impact forces in a plane, while the other six are three-dimensional. All models except CAL3D have specified numbers of rigid masses connected in a linkage. The number of masses used in CAL3D is variable and can be used to model several occupants in the same exercise. The degrees of freedom in the models are associated with victim motions in most cases, but PROMETHEUS and SOM-LA have additional degrees of freedom associated with finite element deformable seat structures.

The developing organizations include four universities (University of Michigan - HSRI, Wayne State University, Texas A and M University - TTI, University of Cincinnati) and four corporations (General Motors Corporation, Boeing Computer Services, Inc., Ultrasystems, Inc., Calspan Corporation). The key to the initials of each organization is given as Table 6. The name of an individual in the organization who is most likely to have direct knowledge is usually given. References are included for each of the models including both detailed and summary presentations. In most cases, the models are available from more than one source. The preferred source, usually the developer, is given first. Documentation

and information is available on all the models. For some of the models, major users are included in the list. Where reports have been submitted to NTIS, this fact is noted.

OCCUPANT DESCRIPTION

The properties of the occupants which are given in Table 2 include the distribution of mass, the joint characteristics, the means of defining body exterior geometry, and the types of external forces which can be applied to the victim. All models consist of a chain of rigid body masses with the center of mass on a line connecting the joint center with the exception of PROMETHEUS, CAL3D, and UCIN. PROMETHEUS uses point masses at the joint centers, while CAL3D and UCIN use more general representations. In most cases, the joints contain range of motion limits and some means for resisting relative motions between adjacent body segments. The most advanced joint models are included in CAL3D and MVMA2D, both of which can make use of considerable biomechanical input for simulating human dynamics as well as the more simple case of crash test dummies.

All models include some body exterior geometry to sense force interaction with a dynamic environment. With the exception of the simple springs of UCIN and the body offsets of PROMETHEUS, these consist of circles, ellipses, spheres, ellipsoids, and/or cylinders. The position of these victim-mounted geometric descriptors is monitored during execution

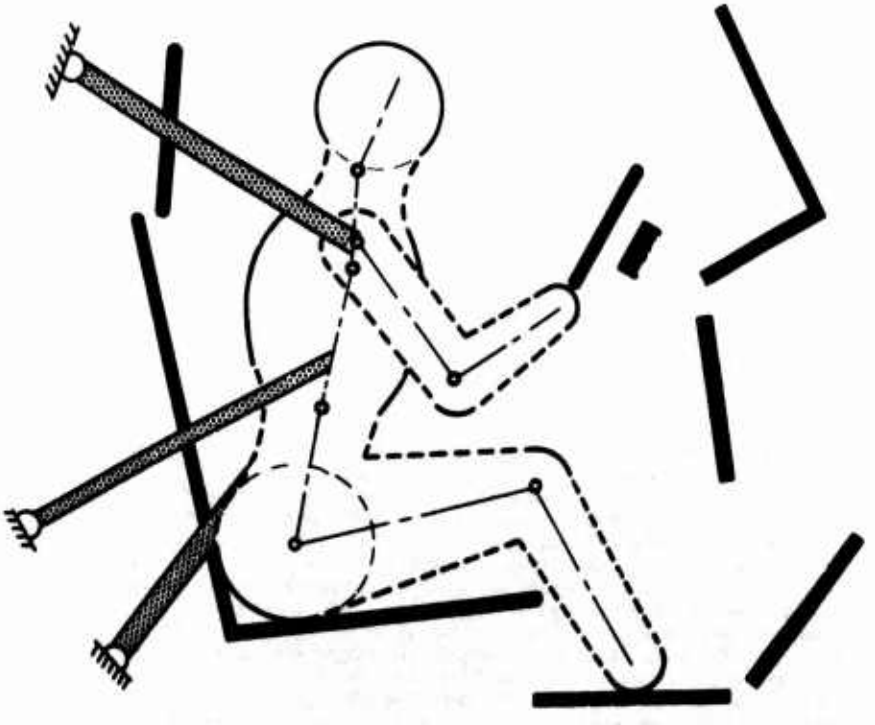

Fig. 1. Schematic for HSRI2D crash victim simulation

of the program to sense when impingement on an environmental contact surface occurs. A resulting force must then be applied to the victim. In addition to contact forces, the victim may be subjected to external forces of restraint belts, airbags, steering columns, and/or time-dependent forces directly applied to the body.

CONTACT SURFACES

Table 3 summarizes the vehicle or environmental contact forces which act on the victim. In all cases except the curved airbags of MVMA2D, MODROS, WSU3D and CAL3D, these surfaces are straight lines or sections of planes. In three of the HSRI models, the planes can be specified to move independently to realistically model vehicle-pedestrian and the ensuing pedestrian-ground impact. Force-deformation curves take many forms, ranging from the complex force loading and unloading tables of HSRI3DE, MVMA2D, MODROS, and CAL3D to the linear springs of UCIN. In seven of the models, lateral frictional forces are generated in the presence of normal forces. The most general set of frictional forces in a restraint belt model is the MVMA2D three-point continuous loop harness. Only three of the models allow shared deflection, that is, deformation of the victim in the presence of deformation of a contact surface. These are HSRI3DE, MVMA2D, and CAL3D.

FORCE INPUT TO MODEL

With the exception of TTI, the models are forced by tabular accelerating-time histories as indicated in Table 5. In TTI, vehicle position is specified as a function of time. Direct time dependent forces can be applied to the body only for the MVMA2D. In the MVMA2D, HSRI3D, HSRI3DE, and CAL3D, surfaces can be specified to move into contact with the victim.

COMPUTER PROGRAM INFORMATION

A summary of computer program information is given in Table 5. All programs are in Fortran IV with the exception of some output sections of MODROS which are in PL1. Most programs have been installed on IBM equipment but all appear to be convertible to other systems, some with very minimal effort. Program sizes ranges from 34K to 143K words. In addition to the usual tabular output in the form of neatly titled tables, most are provided with exportable plot packages, both for printer plots and for solid line plots. These computer programs are large, and the need for program debug information in the case of unsuccessful exercises is obvious. Detailed debugging is possible only with the various HSRI models and the CAL3D which follows the HSRI pattern.

The size of the program is related to the complexity of logic primarily in the generation of contact forces. The largest model is MVMA2D. Maximum core is 104 K words for the execution load module. The input load module is relatively small while the output/graphics module is almost as large as the execution load module. PROMETHEUS and UCIN are relatively small. UCIN is analytically the simplest of the models. PROMETHEUS seems to include a goodly variety of useful options in a relatively small package.

The models all require sophisticated engineering data as input and must be classed as difficult to use. The task is eased somewhat for SOM-LA which bases most occupant parameters on standard anthropometric data which are used and manipulated within the program. Auxiliary programs are available for use with CAL3D in preparing occupant input data. In cases where detailed force-deflection curves are possible (e.g., MVMA2D and CAL3D) the input package can be formidable.

Execution times for the models are difficult to estimate. Comparisons have not been made between models on a standard baseline test exercise. However, the general rule can be stated that the user should choose as few contact interactions as are practible to minimize cost. The parallel rule is that the simplest model containing the required interactions should be used. The two-dimensional models are generally inexpensive and can be considered for parameter and design studies where in excess of 100 exercises may be required. In this regard, caution must be used with the MVMA2D because of the wide range of options. It can be inexpensive or expensive depending on which options are specified. Of the three-dimensional models, only HSRI3D, UCIN, and possibly the HSRI3DE should be considered for large parameter studies. The others (CAL3D, TTI, SOM-LA) are reputed to be expensive to run because of all their options. Their use should be restricted to those specific cases which require the great generality which they possess.

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Table 1 Human Gross Motion Simulations

NAME OF PROGRAM	TYPE OF PROGRAM	NUMBER OF MASSES DESCRIBING HUMAN	NUMBER OF DEGREES OF FREEDOM	DEVELOPING ORGANIZATION	REFERENCES	PROGRAM SOURCE AVAILABLE FROM	DOCUMENTATION AND INFORMATION AVAILABLE FROM
HSRI2D	PLANAR	8	10	HSRI	1	HSRI, NHTSA	HSRI, NHTSA, NTIS
MVMA2D	PLANAR	8	14	HSRI	2,3,4	HSRI, MVMA	HSRI, MVMA, NTIS
MODROS (ROS)	PLANAR	8	10	GMC (Calspan)	5,6	MVMA (Calspan)	MVMA, GMC (Calspan)
PROMETHEUS (SIMULA)	PLANAR	8	Occupant-9 Seat - $0 \leq n \leq 20$ (Occupant-9)	BCS (Ultra-systems)	7,8,9	BCS	BCS, ONR, NHTSA, NTIS
HSRI3D (WSU 3D)	Three-dimensional	3	12	HSRI (WSU)	10,11,12, 13	HSRI, (WSU)	HSRI, NHTSA, (WSU), NTIS
HSRI3DE	Three-dimensional	6	17	HSRI	14,15	HSRI, MVMA	HSRI, MVMA, NTIS
CAL3D (Several versions available)	Three-dimensional	Variable (15 standard)	Variable	Calspan	16,17,18 19	Calspan, MVMA NHTSA, GMC	Calspan, MVMA, NHTSA, GMC, NTIS
TTI	Three-dimensional	12	31	TTI	20,21,22	TTI	TTI, NHTSA, FHWA, NTIS
SOM-LA	Three-dimensional	11	Occupant-28 Seat-16 finite beam elements	Ultrasystems	23,24,25	Ultrasystems	Ultrasystems, FAA, NTIS
UCIN	Three-dimensional	12	42	U. Cincinnati	26,27,28	U. Cincinnati	U. Cincinnati

Table 2 Occupant Description

PROGRAM	MASS/INERTIAL PROPERTIES	JOINT CHARACTERISTICS	BODY EXTERIOR GEOMETRY	FORCE INTERACTION CAPABILITY
HSRI2D	Rigid masses on lines connecting joints	1. Bilinear, nonsymmetric torsional springs. 2. Linear torsional damping. 3. Coulomb friction.	Optional circles on head, chest, pelvis, knee, foot, elbow, and hand.	1. Lap and shoulder belts. 2. Seat cushion nonlinear spring/damper model. 3. Force interaction between body circles and line segments representing vehicle.
MVMA2D	Same as HSRI2D	1. Linear and nonlinear unsymmetric torsional springs. 2. Linear torsional dampers. 3. Coulomb friction. 4. Active muscle tension elements represented as tabular time-dependent functions. 5. Extensible neck and shoulder joints add degrees of freedom.	1. Ellipses located as desired on body segments. 2. Continuous line representation of body for airbag contact.	1. Four belt segments which may interact in various user-specified ways. 2. Three-point continuous loop belt model with deformable vehicle anchorages. 3. Predictive inflatable airbag. 4. Predictive energy-absorbing steering column (2 mass, 3 element, 4 degree of freedom model).
MODROS (ROS)	Same as HSRI2D	Same as MVMA2D without the muscle tension elements.	1. Circles for head, chest, gut, butt, knee, foot, elbow, hand and upper leg. 2. Continuous line body for airbag contact.	1. Lap and shoulder belt independent systems. 2. Force interaction between body circles and line segments representing vehicle. 3. Predictive inflatable airbag.
PROMETHEUS (SIMULA)	Point masses at nodes (joints)	1. Muscle tone torque to maintain initial equilibrium. 2. Coulomb torque opposing motion. 3. Restoring torque when motion exceeds unsymmetric joint stops.	Offsets from body nodes to represent flesh.	1. Seat belt and shoulder harness. 2. Seat cushion and back. 3. Foot-floor contact.

Table 2 Occupant Description (continued)

PROGRAM	MASS/INERTIAL PROPERTIES	JOINT CHARACTERISTICS	BODY EXTERIOR GEOMETRY	FORCE INTERACTION CAPABILITY
MSRI3D (MSU3D)	Same as MSRI2D	Bilinear, unsymmetric torsional springs are different for pitch, roll, and yaw relative motions at each joint.	1. Any number of ellipsoids at any point on the body segments. 2. Parallelograms move with body for airbag contact.	1. Four extensible belt segments attached rigidly to any points on torso. 2. Force interactions between body ellipsoids and parallelograms representing vehicle and/or ground in inertial space. 3. Torso surface can interact with axisymmetric airbag.
MSRI3DE	Same as MSRI2D	Same as MSRI3D plus addition of coupling between pitch, roll, and yaw stops is allowed for combined motions.	Same as MSRI3D except force interaction between body ellipsoids is allowed.	1. Four belt segments can be attached to torso. Belt slipping, breaking and interactions between belts are allowed. 2. Force interactions between body ellipsoids and parallelograms representing vehicle and/or other surfaces.
CAL3D	Rigid body masses may be offset from lines connecting joints. Several sets of connected masses are possible.	1. Torsional and flexural joint torques. 2. Possibility for locking joints. 3. Sliding and rolling constraints to motion at joints. 4. Global-graphic joint stops.	Each body segment has an associated contact ellipsoid.	1. Extensible belts, fixed at one point on body are free to slip over and conform to body ellipsoid. 2. Contact between body ellipsoids and vehicle surfaces. 3. Contact between ellipsoids (Calspan model can simulate multiple occupants). 4. Airbag-occupant contact.
TTI	Same as MSRI2D.	1. Torsional viscous bilinear damper to model muscle reaction and range of motion limits. 2. Linear torsional springs for spinal elasticity.	Spheres at joints and at strategic points on the center lines of body segments.	1. Sphere-vehicle contact force interaction. 2. Lap and shoulder belts loop around pelvic and torso contact spheres.

Table 2 Occupant Description (continued)

PROGRAM	MASS/INERTIAL PROPERTIES	JOINT CHARACTERISTICS	BODY EXTERIOR GEOMETRY	FORCE INTERACTION CAPABILITY
SOM-LA	Rigid masses on lines connecting joints (Mass and body geometry set up within program based on standard anthropometry).	Nonlinear torsional spring and viscous damper to represent dummy coulomb friction joints, viscous damping, muscle tension, and stops.	Contact ellipsoids, spheres, and cylinders located to outline the body.	1. Planar contact surfaces. 2. Finite element deformable seat and support structure. 3. Lap and torso belt with sliding. Belts may slide on pelvic and torso ellipsoids.
UCIN	Rigid masses do not have to be on lines connecting joints.	1. Joint damping. 2. Joint stops with large damping. 3. Neck spring and damping (extensible joint).	Seat cushion and back are oriented at 90°. Contact force applied to mass centers of body elements.	1. Seat and floor contact forces. 2. Up to ten linear spring belt elements.

Table 3 Vehicle (or Environmental) Contact Surfaces

PROGRAM	GEOMETRIC DESCRIPTION	EDGE MODEL	RELATION BETWEEN SURFACES	COORDINATE SYSTEMS	FORCE-DEFLECTION MODEL (LOADING)	FORCE-DEFLECTION MODEL (UNLOADING)	FRICTION MODEL	MUTUAL FORCE-DEFLECTION CHARACTISTICS
HSRI2D	Finite line segments.	Force discontinuous at edge.	Unrelated	Surfaces move with vehicle.	1. Polynomial force-deflection, 2. Non-linear elastic, perfectly plastic model.	Automatic unloading curves computed based on energy-absorbing properties of contact interactions. Reloading allowed.	Friction force proportional to normal force.	Each surface has its own force-deflection curve.
HVHA2D	Finite line segments.	Force linearly continuous as occupant slides off surface.	Unrelated	Surface may be in both inertial and vehicle coordinates.	Polynomial or tabular force-deflection and deflection rate.	Same as HSRI2D or as tabular unloading curves.	Friction force is nonlinear function of normal force.	Force-deformation properties are independent for each body and contact surface.
	Real-line interior composed of finite, connected line segments.		Lines are connected as series of finite elements.	Surface may be in both inertial and vehicle coordinates.	As above for each region.	As above for each region.	Friction force is nonlinear function of normal force.	Force-deformation properties are independent for each body and contact surface.
	Airbag inflates as a circle.		Unrelated	Attached to vehicle.	Depends on thermodynamics, inflation rate, etc. of bag.	Based on pressure and geometry of bag.	None.	Body and bag deform.
	Line segments represent steering rim and hub.	Force discontinuous at edge.	Unrelated	Attached to vehicle.	Depends on geometry and collapse dynamics of column model.	Based on column dynamics.	Same as HSRI2D.	Body is rigid.
MADBOS (ROS)	Same as HSRI-2D.	Same as HSRI-2D.	Unrelated	Same as HSRI-2D.	Tabular force-deflection and deflection rate.	Similar to HSRI-2D.	Similar to HSRI2D.	Similar to HSRI2D.

Table 3 Vehicle (or Environmental) Contact Surfaces (continued)

PROGRAM	GEOMETRIC DESCRIPTION	EDGE MODEL	RELATION BETWEEN SURFACES	COORDINATE SYSTEMS	FORCE-DEFLECTION MODEL (LOADED)	FORCE-DEFLECTION MODEL (UNLOADED)	FRICTION MODEL	MUTUAL FORCE-DEFLECTION CHARACTERISTICS
PROMETHEUS (SIMULA)	Outline of seat formed from nodes of planar finite element structure.	Same as HSRI-2D.	Seat and occupant surfaces are connected.	Seat attachments move with vehicle but seat/occupant surfaces may move as finite element seat structure deforms inelastically.	Polynomial or tabular elastic force-deflection curves. Force applied to occupant over length of contact and then added to equations of motion at nodes.	Linear unloading.	Friction between feet and floor.	Each surface/occupant combination has its own force-deflection curve.
HSRI3D (WSU3D)	Parallelograms.	Same as MVMA-2D.	Unrelated.	Same as MVMA-2D.	Same as HSRI2D.	Same as HSRI2D.	None.	Same as HSRI2D.
HSRI3DE	Parallelograms.	Same as MVMA-2D.	Unrelated.	Same as MVMA-2D.	Same as MVMA2D.	Same as MVMA2D.	Same as MVMA2D.	Same as MVMA2D.
CAL3D	1. Parallelograms. 2. Contact ellipsoids.	Same as HSRI-2D.	Unrelated.	Same as HSRI-2D.	Same as MVMA2D.	Same as MVMA2D.	Linear friction	Same as HSRI2D.
TTI	Node points outlining vehicle are connected to form a vehicle enclosure modeled as planes.	Same as HSRI-2D.	Connected at node points.	Same as HSRI-2D.	Bilinear dissipative spring.	Bilinear dissipative spring (magnitude is percentage of loading curve).	Linear friction	Same as HSRI2D.
SOM-LA	Plane surfaces describe cockpit interior.	Same as HSRI-2D.	Can be connected to finite element seat structure or operate as independent surfaces as in other models.	Same as HSRI-2D.	Tabular force-deflection curves.	Unloads on fourth order curves intersecting origin. Reload on linear curve.	None.	Deformation only in cockpit structure.
UCIN	No surfaces.	None	Seat and back at 90°.	Same as HSRI-2D.	Linear springs.	Linear springs.	None.	None.

Table 4 Force Input to Model

PROGRAM	ACCELERATION INPUTS	DIRECT FORCES	DISPLACEMENT INPUTS
HSRI2D	Tabular piecewise continuous linear deceleration of vehicle coordinate system.	None.	None.
MVMA2D	Tabular piecewise continuous decelerations for the three planar motions of the vehicle coordinate systems.	1. Time-dependent tabular force can be applied to a body segment. 2. Individual contact surfaces can be programmed to move in inertial space.	Linear motion may be specified for one element of the energy-absorbing column.
MODROS (ROS)	Same as MVMA2D.	None.	None.
PROMETHEUS (SIMULA)	Same as HSRI2D.	None.	None.
HSRI3D (WSU3D)	Tabular piecewise continuous deceleration functions for the six possible motions of the vehicle coordinate system.	Individual contact surfaces may be specified to move into contact with victim.	None.
HSRI3DE	"	"	None.
CAL3D	"	Contact surfaces can move into contact with victim.	None.
TTI	None.	None.	Vehicle position in space is given as tabular function of time.
SOM-LA	Same as HSRI3D	None.	None.
UCIN	Same as HSRI3D.	None.	None.

Table 5 Computer Program Information

PROGRAM	LANGUAGE	INSTALLED ON COMPUTERS	CORE MEMORY	INPUT REQUIRED	OUTPUT			
					TABULAR	PRINTER PLOT GRAPHICS	POST-PROCESS GRAPHICS	DEBUG OPTIONS
HSRI2D	Fortran IV	IBM 360 and 370	68000 (238K bytes)	Fixed format card images.	Yes.	Occupant stick-man and time plots of variables.	Stick-man on Computek from file of output.	Error messages and optional deep debugging procedures
MWMA2D	Fortran IV	IBM 360/370 Honeywell 6080	104000 (364K bytes)	Fixed format card images.	Yes.	Occupant stick-man.	No.	"
MODROS (ROS)	Fortran IV and PLI	IBM 360/370	55000 (192K bytes)	Fixed format card images.	Yes.	No.	Yes.	Error messages.
PROMETHEUS (SIMULA)	Fortran IV	CDC 6600	45000 (158K bytes)	Free field format.	Yes.	Same as HSRI-2D.	Stick-man on Tektronix may be used interactively in data setup.	Error messages.
HSRI3D (WSU3D)	Fortran IV	IBM 360/370	88000 (306K bytes)	Fixed format card images.	Yes.	No.	Output tape produced for movie production using Stromberg-Carlson 4020.	Same as HSRI-2D.
HSRI3DE	Fortran IV	IBM 360/370	110000 (386K bytes)	Fixed format card images.	Yes.	No.	No.	Same as HSRI-2D.
CAL3D	Fortran IV	IBM 360/370 CDC 6600	143000 (500K bytes)	Fixed format card images.	Yes.	Occupant stick-man.	Output tape may be produced for movie production	Similar to HSRI2D.

Table 5 Computer Program Information (continued)

PROGRAM	LANGUAGE	INSTALLED ON COMPUTERS	CORE MEMORY	INPUT REQUIRED	OUTPUT			
					TABULAR	PRINTER PLOT GRAPHICS	POST-PROCESS GRAPHICS	DEBUG OPTIONS
TTI	Fortran IV	IBM 360	91000 (320K bytes)	Fixed format card images.	Yes.	No.	Line plots of variables and vehicle/occupant stick-man.	Error messages.
SOM-1A	Fortran IV	Univac 1108 CDC 6600	64000 (224K bytes)	Fixed format card images.	Yes.	Kinematic time histories.	No.	Error messages.
UCIN	Fortran IV	IBM 370	34000 (120K bytes)	Fixed format card images.	Yes.	Kinematic time histories.	System-dependent.	IBM-COMPRINT.

Table 6 Sources for Programs, Information and/or Documentation

INITIALS	SOURCE
BCS	Mr. Richard N. Karnes Boeing Computer Services, Inc. P. O. Box 24346 Seattle, Washington 98124
Calspan	Mr. R. McHenry Transportation Research Department Calspan Corporation Buffalo, New York 14221
FAA	U.S. Department of Transportation Federal Aviation Administration Systems Research and Development Service 800 Independence Ave., S.W. Washington, D.C. 20590
FHWA	U.S. Department of Transportation Federal Highway Administration Nassif Building Seventh and D Streets, S.W. Washington, D.C. 20590
GMC	Mr. John P. Danforth Biomedical Sciences Research Laboratories General Motors Corporation Warren, Michigan 48090
HSRI	Dr. D. H. Robbins Highway Safety Research Institute The University of Michigan Ann Arbor, Michigan 48105
MVMA	Mr. John C. Scowcroft Motor Vehicle Manufacturers Association 320 New Center Building Detroit, Michigan 48202
NHTSA	The following individuals have been involved with one or more of the listed programs: Dr. L. Ovenshire Mr. J. Hofferberth Mr. C. Strother Mr. S. Backaitis Dr. R. Eppinger Their general address is:
NHTSA (cont.)	U.S. Department of Transportation National Highway Traffic Safety Administration Nassif Building Seventh and E Streets, S.W. Washington, D.C. 20590
ONR	Dr. K. Saczalski Navy Department, Structural Mechanics Program Code 439 - 800 N. Quincy Street Arlington, Virginia 22217

Table 6 Sources for Programs, Information and/or Documentation (continued)

INITIALS	SOURCE
TTI	Dr. H. E. Ross Texas Transportation Institute Texas A and M University College Station, Texas 77849
Ultrasystems	Mr. R. Carr Ultrasystems, Inc., Dynamic Sciences Division 1850 W. Pinnacle Peak Rd. Phoenix, Arizona 85027
U. Cincinnati	Dr. R. L. Huston Engineering Analysis Department University of Cincinnati Cincinnati, Ohio 45221
WSU	Dr. Albert I. King Bioengineering Center Wayne State University Detroit, Michigan 48202

Test Data Reduction and Processing

Loren D. Enochson

Time/Data Corporation

INTRODUCTION

This chapter contains a review of computer software and hardware systems employed in shock and vibration test data analysis. Software packages are generally available for large-scale computers and are often written in a high level language (e.g. FORTRAN) making them generally available for a wide class of medium to large scale machines. There is also available a broad class of special purpose hardware systems essentially based on digital circuitry implementations of the fast Fourier transform. A closely related group of systems are minicomputer based, with custom software for the particular minicomputer involved. They tend to appear functionally much the same as the true hardware devices but are fundamentally distinct. All these systems are in daily use in shock and vibration data analysis. Hence, we have included descriptions of all three types of systems, even though they are not all, strictly speaking, software packages.

Closely related applications are discussed in other chapters in this book and will not be covered here. This includes modal testing, in particular, which generally is based on test data results. In this chapter we shall restrict ourselves to what might be termed basic time series data analysis methods including the following:

- Amplitude Domain
  - Mean, variance and rms
  - Probability Density Histogram
- Time Domain
  - Auto and cross covariance (correlation) functions
  - Convolution
  - Recursive Digital Filters
- Frequency Domain
  - Fourier Transforms
  - Power (auto) and cross spectra
  - Frequency Response Functions
  - Coherence Functions
- Miscellaneous
  - Digital Filters

The systems which will be described are summarized in Table 1. Generally we have tried to include all software and hardware packages which incorporate most of the above described functions and which are in wide use. Undoubtedly we have neglected some due to a lack of knowledge of their existence. This is particularly true with respect to the software packages for the large computers.

We have only been able to include those whose documentation has been brought to our attention. Some packages used primarily in other fields, such as seismic data analysis, have been omitted even though in principle they incorporate all the computations of concern. Acoustical excitations are often involved in vibration analysis and we have included descriptions of systems that are slanted toward acoustical data analysis. Certain hardware systems have been omitted purposely since they typically comprise only one element in an overall system rather than the complete package.

We believe the systems described represent a fair cross section of available systems. It has been difficult to isolate a common set of specifications or standards by which the systems can be directly compared. The potential user will have to conduct an evaluation based on his unique requirements. Questions to be answered include: Do I need on line real time results? Do I have a large computer readily available already? Is batch processing adequate? Is the precision of a large computer important? Will I be building up a large data base of results to be stored? Is very high speed and very high frequency analysis important? These and other considerations will direct the user to one of the described systems.

Table 1

Medium to Large Scale Computer Software Systems

MAC/RAN^{tm} III, University Software Systems
DYVAN, NASA Goddard SFC
RAVAN, NASA Marshall SFC
BMD, UCLA
MR WISARD, NOL

Minicomputer Based Software/Hardware Systems

TIME/DATA
HEWLETT-PACKARD
CSPI

Special Purpose Digital Hardware Systems

NICOLET SCIENTIFIC CORPORATION
SPECTRAL DYNAMICS CORPORATION
HONEYWELL/SAICOR

NOMENCLATURE

$x(i)$ = Discrete time history
$X(k)$ = Discrete Fourier transform of $x(i)$
$S_x(k)$ = Power spectral density of $x(i)$
$S_{xy}(k)$ = Cross spectral density of $x(i)$ and $y(i)$
$H_{xy}(k)$ = Frequency response function of $x(i)$ and $y(i)$
$\gamma^2_{xy}(k)$ = Coherence function between $x(i)$ and $y(i)$
$e_{xx}(i)$ = Auto covariance (correlation) function of $x(i)$
$s_{xy}(i)$ = Cross covariance (correlation) function of $x(i)$ and $y(i)$
$c_{xy}(i)$ = Convolution of $x(i)$ and $y(i)$
$h_{xy}(i)$ = impulse response function of $x(i)$ and $y(i)$
$p(x)$ = probability density of $x(i)$
S = Sampling rate
F = Folding (Nyquist) frequency
T = Sampling interval
τ = Time delay
i = Time index
k = Frequency index
n = Number of degrees of freedom

d.f. = Degrees of freedom
$\varphi_{xy}(k)$ = Phase between x(i) and y(i)
f = Continuous frequency in Hz
ω = Continuous frequency in radians/sec
t = Continuous time
Re = Real part
Im = Imaginary part
$C_{xy}(k)$ = Co-spectrum
$Q_{xy}(k)$ = Quadrature-spectrum
j = Imaginary unit $\sqrt{-1}$

TIME SERIES ANALYSIS FUNCTIONS

We shall briefly define the major functions computed by most of the systems. For a detailed discussion of the concepts and computational methods involved see [1]. We assume we have one or two histories denoted by

$$x_i,\ y_i;\ i = 0,1,2,\ldots,N-1$$

where,

$$\begin{aligned} &x_i \equiv x(iT) \\ &T = \text{sampling interval (seconds)} \\ &S = 1/T = \text{sampling rate (samples per second)} \\ &P = NT = \text{period or record length (seconds)} \end{aligned} \tag{1}$$

Fourier Transform

$$X(k) = T\sum_{i=0}^{N-1} x(i)\exp[-j2\pi(ik/N)] \tag{2}$$

$k = 0,1,2,\ldots,N/2$ for N even

$k = 0,1,2,\ldots,(N-1)/2$ for N odd

$X(k) \equiv X(kb)$

$b = 1/NT$ elementary resolution bandwidth

Subsequent frequency indices will always run over the limits in Eq. (2) unless specified otherwise.

The time history is often modified by multiplying it by a data window (or applying a convolution to the Fourier transform). This enhances the leakage characteristics of the transform (suppresses side lobes of the spectral window). However the statistical variability of resulting spectral functions will be increased due to an induced loss in degrees of freedom. See [1] for a discussion of this.

Power and Cross Spectral Density Functions

$$S_x(k) = (1/NT)|X(k)|^2 \tag{3}$$

$$S_{xy}(k) = (1/NT)X(k)^*Y(k) \tag{4}$$

$$= |S_{xy}(k)|e^{j\phi_{xy}(k)} \tag{5}$$

where,

$|S_{xy}(k)|$ = absolute value

$\phi_{xy}(k)$ = phase

The statistical degrees of freedom associated with the above spectra is

$$n = 2B_eP$$

$$= 2bNT = 2 \tag{6}$$

Statistical stability is obtained by either ensemble averaging or frequency band averaging or a combination of the two.

Ensemble Averaging

Assume several spectra, $S_{xi}(k)$, $i = 1,2,\ldots,M$, are available, usually from consecutive segments of a long time history. The average of several such spectra to obtain higher d.f. has become known as the "ensemble averaging" method. In equation form:

$$S_x(k) = \frac{1}{M} \sum_{i=1}^{M} S_{xi}(k) \tag{7}$$

If P_s is the segment length then

$$B_e = 1/P_s \tag{8}$$

is the effective resolution bandwidth and the d.f. are

$$n = 2M \tag{9}$$

The equation for cross spectra would be identical in form:

$$S_{xy}(k) = \frac{1}{M} \sum_{i=1}^{M} S_{xyi}(k) \tag{10}$$

Frequency Band Averaging

Assume we have a single 2 d.f. spectrum computed from N data values denoted by $\tilde{S}_x(k)$. We can obtain high d.f. by averaging together several neighboring frequency points of the spectral function. In equation form

$$S_x(k) = \frac{1}{2M+1} \sum_{i=-M}^{M} \tilde{S}_x(k + i) \tag{11}$$

We must interpret the index module N in order to avoid problems at the end points. The resulting spectral estimates will have resolution

$$B_e = (2M+1)/P$$

and d.f.

$$n = 2(2M + 1) \tag{12}$$

Again, the formula for cross spectra is identical. Since B_e is now much larger than 1/P, the averaged PSD points will be highly correlated and redundant to a large extent. Hence, it often makes sense to decimate $S_x(k)$ by a factor up to 2M + 1. If decimated by 2M + 1, then the resulting estimates will overlap at

roughly the half-power points of their effective inherent filter shape and will be approximately uncorrelated.

Frequency Response and Coherence

We assume we have PSD and CSD estimates of n d.f. and B_e resolution obtained by either ensemble averaging, frequency band averaging or a combination of the two. We then compute the frequency response function estimate from the formula

$$H_{xy}(k) = \frac{S_{xy}(k)}{S_x(k)} \tag{13}$$

In terms of absolute value (gain) and argument (phase) we have

$$H_{xy}(k) = |H_{xy}(k)|e^{j\phi_{xy}(k)} \tag{14}$$

We note that phase of the frequency response function is the same as the phase of the cross spectrum.

The coherence function is defined by the relation

$$\gamma^2_{xy}(k) = \frac{|S_{xy}(k)|^2}{S_x(k)S_y(k)} \tag{15}$$

and it is easily shown that

$$0 \leq \gamma^2_{xy}(k) \leq 1 \tag{16}$$

We remark that coherence may be interpreted as the frequency domain counterpart of the square of the correlation coefficient of basic statistics. The frequency response function is analogous to a regression coefficient.

Multiple Frequency Response and Coherence Functions

We can generalize to the multiple input single output linear system by computing all possible combinations of power and cross spectra. If we have p inputs, we define a p × p spectral matrix, a p × 1 cross spectrum vector and a p × 1 frequency response function vector as follows (omitting the frequency index to simplify notation):

$$\vec{S}_{xx} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1p} \\ S_{21} & S_{22} & \cdots & S_{2p} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ S_{p1} & S_{p2} & \cdots & S_{pp} \end{bmatrix} \quad \vec{S}_{xy} = \begin{bmatrix} S_{1y} \\ S_{2y} \\ \cdot \\ \cdot \\ \cdot \\ S_{py} \end{bmatrix} \tag{17}$$

$$\vec{H}_{xy} = \begin{bmatrix} H_{1y} \\ H_{2y} \\ \cdot \\ \cdot \\ \cdot \\ H_{py} \end{bmatrix} \tag{18}$$

In the above

$$\begin{aligned} S_{11} &\equiv S_{x_1x_1} \equiv S_{x_1} \\ S_{ij} &\equiv S_{x_ix_j} \\ S_{iy} &\equiv S_{x_iy} \\ H_{iy} &\equiv H_{x_iy} \end{aligned} \tag{19}$$

We then have the matrix equation

$$\vec{S}_{xy} = \vec{S}_{xx}\vec{H}_{xy} \tag{20}$$

whose solution is

$$\vec{H}_{xy} = \vec{S}_{xx}^{-1}\vec{S}_{xy} \tag{21}$$

The multiple coherence is given by

$$\gamma^2_{y\cdot x} = 1 - 1/S_{yy}S^{yy} \tag{22}$$

where S^{yy} is given by

$$S^{yy} = [S_{yy} - S_{yx}S_{xx}^{-1}S_{xy}]^{-1} \tag{23}$$

Partial coherence functions can be defined by similar formulas which we omit in the interest of brevity here. Please consult [1], Chap. 9 for a complete description.

Confidence limit computations for the frequency response and coherence functions are quite complicated and will be omitted here. Again the reader may consult [1] for full explanations.

Covariance (Correlation) Functions

The statistical term for the average cross product of two sets of data which have their mean values removed is covariance. The word correlation is reserved for a normalized version of this quantity so that it ranges between plus and minus one. The term correlation function is often applied to any average cross product in time series analysis. We shall employ the statistical terminology here but the reader should understand that many uses of the term correlation may coincide with our use of covariance. The <u>covariance</u> (also called the <u>cross correlation</u>) function is defined as

$$s_{xy}(i) = \frac{1}{N-i-1}\sum_{p=0}^{N-i-1}(x_p - \bar{x})(y_{p+i} - \bar{y}) \tag{24}$$

$$i = -m,-(m-1),\ldots,-1,0,1,\ldots,m-1,m$$

where m is termed the maximum lag value. If $x(i) = y(i)$ then $s_{xx}(i)$ is termed the auto-covariance (auto-correlation) function. In this case

$$s_{xx}(i) = s_{xx}(-i) \tag{25}$$

so that we only need to compute $s_{xx}(i)$ for positive lag values.

Digital Filters

Digital filtering can be accomplished in two different ways.

1. Implementation of a discrete convolution either directly or via fast Fourier transforms by use of the convolution theorem. This has now become known as finite impulse response (FIR) filtering.
2. Implementation of difference equations which are recursive digital filters which are known as infinite impulse response (IIR) filter.

The discussion of the design of these filters is beyond the scope of this book. See Chap. 3 of [1] for a discussion of recursive filters of the Butterworth type. See [2] for a discussion of convolutional filter design. Generally, the IIR filters simulate analog filters and possess a non-zero phase shift characteristic. The phase shift can often be controlled to be suitably linear across the pass band so that only a simple time delay of the data results. These filters are usually the most efficient from a computational standpoint.

Convolution (FIR) filters are usually designed with zero phase shift since one only needs to make this impulse response symmetric to accomplish this. Hence, even though they are usually less efficient, the advantage of zero phase shift sometimes makes them very useful for many applications.

Probability Histograms

The determination of a histogram to describe the average amplitude characteristics of a process is more related to basic statistics than time series analysis, as are most of the other methods employed in shock and vibration data reduction. However, the histogram is no less important and provides important amplitude information.

The amplitude range is subdivided into k intervals termed "class intervals".

$$x_{min} = a_0, a_1, a_2, \ldots, a_k = x_{max} \tag{26}$$

Usually the quantity

$$\Delta x = a(i + 1) - a(i)$$

is a constant but not necessarily so. Data values, x(i), are then compared to the class interval limits. If

$$a(i - 1) < x(i) \leq a(i) \tag{27}$$

then a count is entered into the "pocket" defined by the class interval [a(i - 1), a(i)]. We end up with a set of counts

$$N_1, N_2, \ldots, N_k \tag{28}$$

When normalized to become probability estimates they represent

$$p(x_i)\Delta x = \frac{N_i}{N} \tag{29}$$

where x_i is the midpoint of the ith interval. Then the probability density function estimate is

$$p(xi) = \frac{N_i}{N} \frac{1}{\Delta x} \tag{30}$$

SOFTWARE AND HARDWARE SYSTEM SUMMARIES

The preceding functions constitute the major basic class of computations desirable for shock and vibration data reduction. We shall now summarize systems with the capability to accomplish some or all of these. In various instances, other capabilities are included which will be mentioned but not defined in detail. We shall refer the reader to other texts or papers when such a case arises.

Part I: Medium to Large Scale Computer Software Systems

MAC/RAN^{tm} III [3],[4]

Date: Original version first operational July 1967.

Capability: General purpose time series analysis software system. Consists of an Executive and several data analysis processors which are:

- Calibration
- Data Preparation, including filter design, trend removal, wild point editing and decimation
- Amplitude Statistics
- Time and Frequency Analysis, computes correlation functions and spectra by Fourier transforms of correlation.
- Linear Systems Analysis, including multi-channel frequency response and coherence functions
- Fast Fourier Transform
- Fast Fourier Spectra, including cross spectra, coherence and frequency response
- Convolution and Correlation
- Print and Plot
- Plugboard Simulation, which allows a wide variety of miscellaneous arithmetic and functional operations on time histories and frequency functions

Optional Add-On Processors

- 1/3 Octave Analysis
- Tracking Filter
- Shock Spectrum
- Ensemble Averaging

Method: The computations implemented are essentially all of those reviewed in this chapter and in more detail in [1].

Input Data: All data processed by MAC/RAN must be supplied in a Standard Intermediate Data Tape Format, described in the MAC/RAN Program Reference Manual. This format is used for all data, either read or recorded by the various modules. Finally, the Executive maintains a directory of all data being processed which is produced on punched cards at the end of each computer run. The directory contains relevant statistical and locational information for each channel of data, providing the user with a quick check on the status of analyses being performed and the data being retained. It is possible to enter data in punched cards in various formats. This generally is inefficient for large amounts of data however. All control parameters are supplied on fixed field punched cards in order to minimize setup errors. Parameters supplied to the Executive define the job data flow both as to the data to be processed and the peripheral units to be used. Processor parameters are used in controlling the calculations to be performed. Control input is kept to a minimum by pre-setting as many parameters as possible with standard values. The user must then supply only those parameters which cannot be preset and replacements for any standard values to be overridden. Standard parameters can be preset during installation of the MAC/RAN System to conform to normal user requests.

Output: End user output is both graphical plots and printed results corresponding to the plotted data. In addition, all processors can generate their results on external system I/O units in standard format. Comprehensive plotting capability utilizing manufacturer-supplied plotting software is available with the Executive. By means of this approach, a change in digital plotters requires only that the new manufacturer-supplied software be inserted to replace the existing software. No changes in the processor modules are required. All output is controllable by the user and hence the amount varies according to the job.

Language: The MAC/RAN System is written entirely in a version of ANS FORTRAN IV which is compatible with the major FORTRAN dialects. Care has been exercised to eliminate the use of operations peculiar to any one dialect or computer. This limitation has been imposed with minimal loss in program efficiency.

Hardware: The MAC/RAN System is usable on any medium to large scale digital computer having an appropriate FORTRAN compiler and sufficient memory and peripheral units available.

Word size requirements is to a large extent dependent on the FORTRAN compiler used. Since a majority of the computations are performed in single precision with real variables, the word length used by the compiler must be able to maintain at least 20 binary digits significance. A less stringent requirement imposed by MAC/RAN is that the computer word be of sufficient size to contain a minimum of four characters.

Internal memory requirements will vary with both the FORTRAN compiler and operating system used. A minimum requirement of 32,768 words of memory is a reasonable expectation.

Peripheral units are required to maintain both the data being processed and the MAC/RAN System itself. At least the following units must be available in addition to any units used by the operating system:

- One system overlay unit.
- One unit for maintaining a data directory.
- Two or more data input/output units.
- One plotting output unit (if off-line plotting is used).

These units may be tape drives, disks, or drums, provided the operating system has the capability of using these various units interchangeably. Additional units may be employed if available.

Typical computers acceptable for the MAC/RAN System are:

- IBM 360-40 and above, 370-135 and above.
- CDC 3600, 3800, 6400, 6600, 7600.
- UNIVAC 1107, 1108.
- XEROX SIGMA 5, SIGMA 7.
- HONEYWELL/GE 625, 635, 645, 6000 Series.
- SYSTEMS ENGINEERING LAB, SYSTEMS 85, 86.

Usage: About 30 installations of MAC/RAN exist throughout the U.S., Canada, and Europe. Hence it is a well tested system and has been applied to automotive crash test and emissions data, nuclear reactor noise, vibration and acoustics, aircraft flight test data and a wide variety of other areas.

Developer: Originally MAC/RAN was developed at Measurement Analysis Corporation with the latest version done at the University Software Systems of Agbabian Associates primarily by R.K. Otnes and L. Enochson.

Availability: The MAC/RAN System, including computer installation, on-site instruction and reference manuals, is available to lessees either on a monthly rental plan, or on a single payment plan. The lease benefits the lessee by keeping him informed about system changes and additions. The lessee is granted unlimited use of the MAC/RAN System for one specific computer at the installation identified on the lease contract. A separate or supplemental leasing agreement must be arranged with University Software Systems for use of the system on any additional computer at the same site or at another location.

The one-time license fee at the time of writing was $23,900 with options at $2,900 each. The monthly rental varies from $800 to $1000 per month.

For further information regarding the MAC/RAN System contact: University Software Systems, a subsidiary of Agbabian Associates, 250 North Nash Street, El Segundo, California 90045, (213) 640-0576.

Comments: The MAC/RAN System seems to be the only commercially available software system for medium to large scale computers. It undoubtedly has the widest use in terms of different computer systems and different end users. Hence it is probably the most thoroughly tested. It is also quite well documented.

Its virtues are probably also its failings. It is quite flexible and quite user oriented. It perhaps has incorporated the widest variety of features for time series analysis, either hardware or software. Its flexibility also contributes to certain inefficiencies however. The desire to maintain its availability for a wide variety of computer systems prohibits customizing it for a specific system.

It does not have the advantage of the minicomputer systems in being able to be utilized in an on line real-time mode. It does not incorporate analog to digital conversion hardware and hence this operation must be separately accomplished and the data reformatted into the MAC/RAN standard format for analysis. Other failings by contemporary standards are that it does not have a completely automated data file cataloging and retrieving capability nor is there an interactive version available.

However, in its Version III form, MAC/RAN is probably the most thoroughly tested system in existence and is quite flexible and user oriented.

MR WISARD [5]

Date Issued: January 1969

Capability: MR WISARD (Multi-Record Wave Investigator for Sine and Random Data) utilizes FFT techniques to compute Fourier transforms, power spectra functions on single data records, and cross spectra and correlation functions on multiple records. The program also computes amplitude and peak distribution functions and tests for goodness-of-fit with theoretical functions. In addition to the analysis capability, the program includes procedures for the manipulation and preparation of data for analysis such as: filtering with both analog-simulated recursive filters and with digital filters that have no analog equivalent, and editing and modifying the data in order to remove trends, offsets, and invalid data.

Method: The computations implemented are essentially all of those reviewed in this chapter and in more detail in [1].

Input: Raw data can be entered into the program in three ways. The most common method of entry is from the ADC tape. The data, as they are read from the ADC tape, are in a fixed-point coded form. A floating-point form is required by the MR WISARD routines. In some cases it is the combined (packed) data samples from several channels which were multiplexed (scanned) by the ADC system. Routines which decode and unpack the data, as they come from the tape, are available.

Raw data can also be entered from cards using the CARD option. This technique is useful for the entry of data available in pictorial form, such as might be taken from an oscilloscope and punched onto cards manually. Other uses of this option might include entry of data from an ADC system which uses cards for output.

The third method for raw data entry concerns data which are stored on tape but do not have an acceptable format for MR WISARD. In this case the user must write a FUNCTION routine which reads the data from the EXTRA tape with the required format.

Identification numbers, units, calibration information, and alphanumeric identification information for the data can be entered from cards.

Output: The output from MR WISARD consists of comprehensive printouts and plots generated under user control. In general, almost all output is under control of the user. The plot routines are those for a Calcomp digital incremental plotter.

Language: MR WISARD is written almost entirely in FORTRAN IV language. (Four exceptions are noted below.) The program is presently in operation on an IBM 7090 Computer with an IBSYS operating system and FORTRAN IV language. The exceptions to FORTRAN are as follows:

a. The Calcomp plotting package.

b. The routine for reading variable-length binary records from the ADC tape.

c. A routine which unpacks a standard 36-bit word.

d. A routine which aids in the detection of numbers and words for the free-mode input.

Hardware: MR WISARD is written for the IBM 7094 Computer with 32,768 words of core memory plus about six tape units depending on the user.

Usage: The MR WISARD Program was prepared to fulfill a requirement in the shock and vibration groups at NOL for a comprehensive data manipulation and analysis capability. However it is coded mainly in FORTRAN and hence in principle could be adapted for use on other machines at other locations.

Developer: The majority of the programming for MR WISARD was done by R.S. Reed, the author of the cited report. Some of the routines were taken from the NOL library and SHARE.

Availability: In principle, MR WISARD would be available at very low cost to other government installations. The organization involved would have to be prepared to solve the attendant problems inherent in FORTRAN compiler and operating system incompatibilities. For further information contact R. S. Reed, Environment Simulation Division, U.S. Naval Ordnance Laboratory, White Oak, Maryland.

Comments: The MR WISARD System is a comprehensive system for time series data analysis and is comparable with the MAC/RAN System in overall capability. It does not possess some of the special add-ons nor the multi-input linear system analysis capability of MAC/RAN; however, it does possess additional flexibility in probability density functions.

The author has no hands on usage of the MR WISARD System and cannot vouch for its ease of use. However, its design, and control statements certainly seem well thought out. It should be reasonably efficient since it undoubtedly has been somewhat customized for the IBM 7094. On the other hand, its potential usefulness to other organizations may be limited since it has only been used and tested at NOL.

DYVAN [6]

Date Issued: July 1971

Capability: The DYVAN program is intended to analyze sinusoidal sweep, shock, single and two channel random data. This includes auto and cross correlations, power and cross spectral density function, coherence and probability histograms.

Method: The computational methods are based on the pre-FFT algorithms of Blackman and Tukey [7] but implement most functions discussed here and in [1].

Input Data: The system accepts magnetic tapes produced by a digitizer which outputs a standard IBM-compatible half-inch 7-track tape, recorded in odd parity. The packing density varies from 200 to 800 bits per inch, depending on the sample rate.

The input to control the analysis is on punched cards in a fixed field format much like the MAC/RAN and MR WISARD Systems.

Output: End user output is graphical plots and printed results. Plotted output can be produced on either a CRT type plotter (SC 4020 and SD 4060) or a digital incremental plotter (CALCOMP 565). The user can exercise some control over the deletion of plots and the selection of certain scale options.

Language: The program is 95% coded in ANS FORTRAN IV and is "almost machine-independent."

Hardware: The program is operational under release 18 of the IBM system/360/91 Operating System and version 2.0 of the CDC 3000L Real-Time SCOPE computer system. It is not known how many tape and/or disk drives and other peripheral devices are required for system operation. However, it is guessed that a typical large scale computer complement of peripheral equipment is necessary.

Usage: DYVAN is used at Goddard SFC for various types of shock and vibration analysis. Discussions with two users have indicated general satisfaction with its characteristics and capability.

Developer: The DYVAN system, developed under research and technology operating plan (RTOP) 124-08-14, High Frequency Launch Dynamics, as part of GSFC lead center responsibility in random data analysis and applications, represents the combined effort of several people in the Test and Evaluation Division. Reginald S. Mitchell of the Electronics Test Branch (ETB) designed, assembled, and implemented the DYVAN computer programs; Anthony Villasenor of ETB contributed the sinusoidal analysis capability; Roy Morgan of ETB designed and implemented the analog-to-digital conversion, formatting, and recording equipment; and Robert Dorian of the Structural Dynamics Branch provided operational liaison between the users and the computers and aided in program checkout and documentation.

Availability: The DYVAN system is highly customized for use by GSFC in their special requirements. Although the system could undoubtedly be made available to other government organizations by GSFC, it is unlikely that it would be useful without a substantial investment in manpower to adapt it to the particular computer and A/D conversion system involved. For further information, contact E. J. Kirchinan, Code 321, Structural Dynamics Test Branch, Test and Evaluation Division, Systems Reliability Directorate, NASA Goddard Space Flight Center, Greenbelt, Maryland 20770.

Comments: The DYVAN system does not have the general broad capability of the MAC/RAN and MR WISARD systems. However, considerable effort seems to have been expended in adapting to the capability of the A/D conversion system which plays an integral part in its overall use. It appears to be a well thought out system from that standpoint. Also limited conversations with users indicates a general satisfaction with its capability.

It is unlikely that DYVAN is a very "portable" system in its entirety due to its customized interface with the digitization system. However, portions of it may be, since it is at least operational on two distinct computer systems.

RAVAN [8]

Date Issued: November 17, 1965

Capability: The RAVAN program performs various statistical, spectral, and correlation analyses for vibration, acoustics, and related data.

Method: The computational methods are based on the pre-FFT algorithms of Blackman and Tukey [7] but implement many of the functions discussed here and in [1].

Input Data: The input to the program is normally an 800 bit-per-inch floating binary magnetic tape. It is also possible to input data via punched cards, but this is relatively inefficient. Control parameters for the processing are input via punched cards. Special formatting programs allow substantial flexibility to the input data format.

Output: The primary output from RAVAN is plots generated on the SC 4020 CRT digital plotter. All functions computed are plotted with substantial annotation. Either linear or logarithmic scales are generally available.

Language: The program is written in SHARE Compiler-Assembler-Translator (SCAT) and is designed to operate on the IBM 7094 computer with an IBM 1401 off-line printer and Stromberg-Carlson 4020 plotter as outputs. If a plotter is not available, the program contains a print-plot option. The program is operated under the special operating system, "SPOOK."

Hardware: RAVAN is written for the IBM 7094.

Usage: RAVAN was written to analyze vibration, acoustic, and related data from various missile and space vehicle vibration tests. It and related programs were probably used more, or at least as much, in terms of computer time usage as any other program. In its exact form, it was employed only at Marshall Space Flight Center, but it was the model for other related program packages at other NASA installations.

Developer: The major part of the development of this program package was by Mr. Murl Newberry at MSFC.

Availability: The RAVAN package would only be operable on an IBM 7094. Hence it is not widely available. However, its description is included since it formed the basis for several other NASA vibration and acoustic data reduction systems. For further information, contact: Murl H. Newberry, Computation Laboratory, Marshall Space Flight Center, NASA, Huntsville, Alabama 35812.

Comments: The RAVAN system was one of the earliest developed computer program packages for vibration and acoustic data. Considerable effort was spent in its development. The computations include probability density and distribution histogram, Gaussian and Rayleigh distribution calculations, X^2 goodness of fit test, stationarity test, peak analysis, correlation functions, power and cross spectra, frequency response and coherence.

The package lacks some of the more recently developed digital filtering and multi-spectral analysis capability. In later versions, undoubtedly many of these capabilities are included.

Due to the very restrictive nature of the language in which it was coded, it is not really available for widespread use. However, as mentioned before, RAVAN has formed the basis for other packages adapted to other systems.

BMD [9]

Date Issued: The BMD system is continually changing. The time series capability was issued in about 1964 and updated in about 1968.

Capability: The time series section of BMD includes an original correlation and spectrum computation section based on the Blackman-Tukey method and a later extension (X series) to FFT based spectra including detrending, filtering and multichannel spectral analysis. Hence the BMD package can implement most of the techniques discussed here and in [1].

Method: The computational methods include the Blackman-Tukey methods for correlation and spectra, amplitude and phase estimates via digital filters, and the later versions (X series) are based on the FFT. The spectral computations are generally limited to a core full of data, which may be a severe limitation.

Associated with the BMD package is a complete set of allied programs for data screening, editing, transformation, and a wide variety of statistical computations.

Input Data: The system accepts FORTRAN compatible input data files, from tape and/or disk and cards depending on the computer system. Usually the data would have been processed by the data screening routines prior to input to the time series analysis routines. Generally, if one is dealing with the output of an analog-to-digital converter, a reformatting step would be necessary, similar to the MAC/RAN system. The control of the operation of the programs is via control cards and all input parameters are entered into the program via cards.

Output: Output is generally in the form of printed listings and printer plots. Recent versions of the programs also include a digital incremental plot capability.

Language: The program is almost entirely coded in ANS FORTRAN IV and is nearly machine independent.

Hardware: Most of the BMD modules were coded to operate on the IBM 360/91 under various releases of OS. In principle, the package may be transferred to any system that supports FORTRAN IV. In practice the transfer is

non-trivial, but has been accomplished for almost every large scale computer in existence.

Usage: The BMD package is intended to be a generally available program package. Its emphasis is in basic statistics and it is a well exercised package for those modules. The time series analysis modules may not be as extensively used, but have definitely seen utilization throughout a wide number of organizations in the U.S., Canada, and Europe.

Developer: The BMD package has had many contributors over the years. The main developer of the Blackman-Tukey routines was Mr. Lynn Hayward, and the main developer of the later FFT and multichannel routines was Mr. Robert Generich. Prof. W. J. Dixon of the UCLA Biostatistics Department has maintained cognizance over the ongoing project.

Availability: The BMD package is one of the most widely available packages in existence. Most universities and most computer service bureaus have an operational version of the basic system. In some instances the latest X-series may not be available. The system is also available from the UCLA Health Sciences Computing Facility. The potential user who intends to adapt it to his system must be cautioned that the BMD package is a large system and substantial effort is required to adapt it to a given computer system. For further information, contact: Prof. W. J. Dixon, Health Sciences Computing Facility, UCLA, Los Angeles, CA 90025.

Comments: Overall, the BMD package is a very complete, very large program package. However, its emphasis is basic statistics and not time series analysis. If there is a major criticism to the package, it is that it has grown like "Topsy" and hence, not all modules are as well integrated into the system as they might otherwise be.

The capability of the newest modules for multichannel spectral analysis is considerable. However, its flexibility has been restricted by the fact that the computational algorithms depend on all data being available in high speed storage. Hence the resolution and degrees of freedom of the spectral analysis can encounter serious limitations. However, recent modifications have corrected these limitations although the author has no experience with the upgraded version. Also, digital incremental plotting capability has been added to correct the printer plot limitations of the earlier version.

The BMD package is well documented with a manual available for sale by the UCLA bookstore. In general, it is a continually growing and developing package and improvements are announced regularly.

Part II: Minicomputer Based Software/Hardware Systems

Time/Data Corporation Systems [10, 11, 12]

Date Issued: Original System 1966, Minicomputer System 1970.

Capability: Time Series Analysis systems are based on DEC PDP-11 minicomputer. They include both data analysis systems and vibration test control systems. All systems are FFT software based with a microcoded FFT processor available for higher speeds. All basic functions are implemented:

- Direct/inverse Fast Fourier Transform
- Auto/cross spectrum
- Transfer/coherence function
- Impulse response
- Auto/cross correlation
- Amplitude histogram
- Characteristic functions

Additional functions are available in a special software package called TSL^{tm} (Time Series Language).

The vibration test control systems constitute a special subset of processors which perform more or less standard time series data reduction methods, but are dedicated to the specific application of vibration

testing. These break down further into random vibration, sinusoidal vibration, and shock control systems.

All systems have direct two-channel analog data input capability with real time data acquisition and analysis bandwidths up to 4 kHz. Two channels of data can be handled simultaneously with up to 32 optional.

Certain modal analysis capabilities are also available and are discussed in another chapter of this monograph.

Method: The computations implemented are essentially those reviewed in this chapter and in [1] with the exception of the multi-spectral analysis. The FFT computations are performed in 16-bit block floating arithmetic. Subsequent computations are usually performed in 32-bit floating point.

The vibration control algorithms are patented methods based on random number generation and spectrum shaping and estimating via FFTs.

Input Data: Standard input is analog signals routed directly into 12-bit A/D converters which are a standard part of the system. Disk and tape drives are also available as options for the system so it is possible to input properly formatted data from these devices. Sampling rate capability ranges up to 200,000 sps which outstrips the transfer rate capability of the peripheral devices.

Inputs to set parameters and control the processing sequence come from one of three sources:

1. A panel with various knobs and switches
2. A Teletype or similar typewriter style keyboard
3. A CRT display with Teletype style keyboard.

Output: Data is displayed on a standard 4" x 5" storage CRT. The display is fully calibrated with alphanumeric indications of vertical and horizontal scales, and of the value of the left edge in Hz or seconds as appropriate. Sampled inputs and processed functions are displayed with linear or logarithmic scales in either axis. Coordinate systems are selectable for complex data displays: real, imaginary, Nyquist (phase plane), magnitude, and phase. Data is displayed in points, bars, or continuous trace - and any portion of the display may be expanded to fill the full display. Duplicate displays may be obtained with an optional X-Y plotter.

Larger displays are available on an optional Tektronix CRT with an 8" x 10" screen. Additional flexibility in display scaling and format may be obtained via the TSL software system.

Data may be output to disk or tape assuming these I/O units are on the system.

Language: Almost all software in a T/D system is proprietary and written in assembly language for the PDP-11. Certain special modules are programmed in the proprietary Time Series Language (TSL^{tm}). The system user also has TSL available to him with which to code special application solutions.

Usage: Time/Data systems are in widespread use throughout the U.S., Canada, Europe, Japan and Australia. Approximately 200 systems are in regular use.

Developer: The Time/Data system is a company product developed by the Time/Data staff.

Availability: Since Time/Data systems are commercial products they are immediately available. Typical prices are about $50,000 up for delivered, warranted, checked out systems. For further information on availability contact Time/Data Corporation, 1050 E. Meadow Circle, Palo Alto, CA 94303, (415) 494-7000.

Comments: The Time/Data systems are good, fast, well tested, and flexible machines. As is the case with similar products of Hewlett-Packard, CSPI, and others, they offer certain advantages over large scale computer systems since they are cost effective in an on-line laboratory situation. Since A/D conversion is a built in hardware/software feature, one has immediate results as contrasted to the minutes to hours to days typically encountered in large-scale systems.

The disadvantage of the minicomputer based systems is that they have a limited word length (16 bits), sometimes creating problems with numerical precision, and generally do not have the complement of peripheral storage devices. The advent of less expensive peripheral devices such as flexible (floppy) disks and cassette tapes is rapidly reducing the gap in this area.

A special feature of Time/Data is the Time Series Language. This allows the relatively novice user to program special application solution with a minimum of training. Special display routines make it quite easy to work in an interactive manner.

Hewlett-Packard Fourier Analyzer System [14, 15, 16, 17]

Date Issued: Approximately 1969.

Capability: Time series analysis system based on a HP 2100S minicomputer. Includes both basic data analysis systems, vibration test control system, and special software for rotating machinery analysis. All systems are FFT software-based with a hardware FFT processor as an option for higher speed All basic time series analysis functions are available:

- Direct/inverse FFT
- Power/cross spectrum
- Transfer function/coherence function
- Auto/cross correlation
- Convolution
- Histogram

In addition, the HP system incorporates a programmable pushbutton keyboard which allows the computation of other related functions. Also, differentiation, integration and complex arithmetic are available.

The vibration test control system is a specialized Fourier Analyzer System with a special control panel which augments the keyboard. This system is a dedicated random vibration control system.

All systems have two channel analog input capability with four channel options. Real time data acquisition and analysis bandwidth up to 4-5 kHz is a standard capability. Other options allow up to 32 channels of input data.

Method: The basic computations implemented are essentially those reviewed in this chapter and in [1] with the exception of the multi-spectral analysis. The HP 2100S is microprogrammable and much of the HP standard software is accomplished this way. The basic FFT is done in 16 bit fixed point arithmetic with block floating scaling. Spectrum computations are optionally done in single or double precision.

The Fourier Analysis keyboard controls the software and is itself programmable. Also, user routines may be called from the keyboard.

Input Data: Standard input is analog signals routed directly into 10 bit A/D converters with 12 bit resolution optional. Sampling rate capability is up to 100,000 sps. Disk drives and tape drives are optional features so that properly formatted data may be input from these devices.

Inputs to set analysis parameters and control the processing sequence come from the Fourier Analyzer keyboard. This keyboard controls all system operations with additional input data control being set by switches at the A/D converter. The HP keyboard is also programmable to a limited extent, allowing considerable flexibility in control from the keyboard.

Output: The results of all computations are observed using the 5460A Display unit and 180 Oscilloscope. The refresh rate provides a stable display on the oscilloscope even at the largest blocksizes, eliminating in most cases the need for a storage oscilloscope. A calibrated scale factor for the vertical axis is shown on the display unit digital readout. Also shown are whether the display is in time or frequency domain and log, polar, or rectangular coordinates. The user can select real/imaginary, magnitude/phase, or complex (Nyquist) displays of the data. Larger CRT display devices are optional from Tektronix.

Also, data may be output to disk or tape assuming these units are available with the system.

Language: The HP Fourier Analyzer software is written primarily in HP 2100S assembly language. This minicomputer is microprogrammable so that some of the software is done at that level.

The Fourier Analyzer keyboard is programmable so the on-line system user essentially operates the system by programming the keyboard. User-written software routines may be added to the system to perform specialized tasks. These programs may be written in Assembly or FORTRAN languages and additionally may be microcoded for fast operation. Each program is given a numerical name and is called from the Fourier keyboard by means of the USER PROGRAM key. Any number of user programs may be included in a system, limited by the amount of memory available.

Usage: HP systems are in widespread use throughout the U.S., Canada, and Europe. The author guesses that about 200 systems are operational.

Developer: The Hewlett-Packard Fourier Analyzer System is a company product developed by the HP staff.

Availability: Since HP systems are commercial products, they are immediately and widely available. Typical prices are about $50,000 and up for a delivered, warranted, checked-out system. For further information, contact Hewlett-Packard, 5301 Stevens Creek Blvd., Santa Clara, Calif. 95050.

Comments: The HP Fourier Analyzer systems are good, fast, well tested, and flexible machines. As is the case with Time/Data, CSPI, and others they offer certain advantages over large scale computer systems since they are cost effective in an on-line laboratory situation. Since A/D conversion is a built in hardware/software feature, one has immediate results as contrasted to the minutes to hours to days typically encountered in large scale systems.

The disadvantage of the minicomputer based systems is that they have a limited word length (16 bits) sometimes creating problems with numerical precision and generally do not have the complement of peripheral storage devices. The advent of less expensive peripheral devices such as flexible (floppy) disks and cassette tapes is rapidly reducing the gap in this area.

A major feature of the HP system is the keyboard which implements most standard time series analysis functions in addition to I/O control, complex arithmetic, and various data manipulation keys. This keyboard is in many respects similar to the Time Series Language of the Time/Data systems. TSL probably has more flexibility but the keyboard has more accessibility. A potential user would have to evaluate the relative merits.

Major defects of this type of minicomputer system relate to the display capability. Even though hard copy displays are available, they do not yet match up with the quality and flexibility of large computer systems.

CSPI [18]

Date Issued: Approximately 1970.

Capability: CSPI produces a high-speed minicomputer based Digital Signal Processor. The system is software based and does not necessarily control analog input or displays as standard, but all these are optional features. An array processor is available as an option to provide higher speed processing. The basic signal processing library contains the following:

- Radix-4 FFT
- Radix-3 FFT
- Radix-2 FFT
- FFT related functions--auto/cross spectrum, convolution/correlation, cepstrum
- Zoom FFT
- Complex multiply
- Complex magnitude squared
- Complex magnitude

Cosine/sine table interpolation
Log of the complex magnitude
Base-2 Log (2 approximations)
Complex exponential generator
Recursive filter
Integrate and dump filter
Histogram
Direct correlation
Direct convolution
Hanning weighting
Predictive coding
1/3 octave filtering

Method: The computational methods are all FFT based and essentially implement the basic functions discussed in this chapter and in [1] with the exception of the multichannel spectral analysis. The software is written in assembly language for the CSP-30 system. The usual arithmetic is 16 bit fixed point.

Input Data: The basic CSPI systems have no standard data input device. However 2 channel analog input into 8 to 15 bit A/D converters are standard options. Sampling rate capability is advertised as ranging up to 1 million sps. Also, disk and tape decks are standard options so that properly formatted digital data along with appropriate software allows data to be input from these devices.

The CSPI system standard input device is a Teletype. The parameters and signal processing function control must be accomplished by writing programs to make use of appropriate system subroutines. There is no system input analogous to the panels of Time/Data or keyboard of Hewlett-Packard.

Output: There is no standard display or data output device on CSPI systems other than the Teletype. However, line printers, digital plotters, CRT displays, and related devices are all advertised as options. Similarly, disk and tape drives, if available on a given system, could be utilized for output of processed data.

Language: All standard signal processing software is coded in assembly language for the CSP-30. Cross-assemblers that allow assembly language programs to be coded and checked out for the CSP-30 on a large computer are advertised as available.

Usage: The CSPI machines have seen more limited usage than systems such as are available from Time/Data and HP. Most of the usage known to the author is associated with speech processing and sonar data processing where high speed is of paramount importance. In at least one case, the author is familiar with the use of an earlier version in a special purpose analog-to-digital conversion system where the main type of data was vibration.

In general, the CSPI systems are not in as widespread usage as other similar systems. However, it is a commercially available product and is successfully used by many organizations.

Developer: The CSPI systems are company products developed by company staff.

Availability: Since CSPI systems are commercial products, they are immediately available. Typical prices start about $30,000 but go up as optional components are added. For further information contact: CSPI, 209 Middlesex Turnpike, Burlington, Mass. 01803.

Comments: CSPI systems tend to emphasize speed of processing rather than ease of operation. Also, they do not offer a well tested standard system consisting of I/O gear and an operator panel, keyboard, or the equivalent. However, their processing speed tends to be faster than Time/Data or HP.

Some of the advertised software capability must be taken with a grain of salt. For example, when one speaks of a digital filter capability in the MAC/RAN package, this includes filter design from user specified bandwidths and slopes, plus the implementation of the filtering operation. When CSPI speaks of a digital filtering capability, they mean high speed subroutines to implement the user supplied filter design.

The most recent emphasis at CSPI is on their array processor (MAP). This is a high FFT box with additional multiply-add capability. It can be added onto many computers to provide the basis for a high speed digital time series analysis capability. Hence CSPI seems to be continuing the emphasis of high speed as opposed to an integrated complete system capability.

Part III: Special Purpose Digital Hardware Systems

Nicolet Scientific Corporation [19, 20, 21]

Date: Ubiquitous[tm] system originally developed in middle 1960s. Omniferous[tm] FFT analyzer initially announced in 1973.

Capability: The newer Omniferous Analyzer will be emphasized due to its greater capability. This is essentially a hardwired digital computer. It is capable of inputting two channels of analog data. A system including "typical" options will compute the following:

- Fast Fourier Transform
- Inverse Fast Fourier Transform
- Power Spectrum
- Cross-Spectrum
- Transfer Function
- Coherence Function
- Auto-correlation
- Cross-correlation
- Ensemble Averaging for Any of the Above
- Signal Enhancement or Time-Function Averaging for Two Channels

With the addition of the mini-computer, system operation can be expanded to include octave-band analysis, one-third-octave analysis, frequency equalization, probability density, probability distribution, automatic spectrum peak detection (special calculations optional), and spectrum signature recognition.

Thus the Omniferous Analyzer essentially can compute the functions discussed in this chapter and [1] except for the multichannel spectral computations.

The maximum real time processing bandwidth is 10 kHz for two channels of data.

Method: The Omniferous Analyzer is based on a digital FFT. Instead of pure software, however, it is a hardwired program. The arithmetic is apparently a combination of 16 bit fixed point and 12 bit mantissa, 4 bit characteristic floating point. The basic system calculates an FFT and PSD only. An optional add-on processor is required for the cross-spectrum, etc. A further add-on of a minicomputer brings in more capability.

Input Data: A signal conditioner input unit provides amplitude scaling, filtering, and digitizing of two analog input signals. Sixteen standard analysis ranges, from 1 Hz to 100 kHz, are selectable. Built-in dual sets of phase-matched anti-aliasing filters cover the frequency ranges from 10 Hz to 100 kHz. The maximum sampling frequency for each of the two input channels is 260 kHz. Sampling is controlled either internally (by the setting of the range switch) or externally (to normalize the coverage to some external parameter, such as rpm). Actuating the manual or automatic "HOLD" retains a transient (in digital form) in the memory.

Output: A single-display unit is available which is a 5 inch X-Y oscilloscope (rack mounted) which provides fully calibrated displays of all output functions. The unit is a Tektronix Type 502, slightly modified and fully integrated with the x, y, and z outputs derived from the Analyzer. Permanent records of the display are available by using the Tektronix C-30A camera with adapter (not supplied as part of the standard system). Also available is a dual display unit which consists of two 5 inch oscilloscope display units mounted side by side in a rack adapter. The

analyzer provides separate vertical display selection and scaling controls for each oscilloscope.

Language: The concept of computer program language does not apply to this analyzer since the program is essentially hardwired.

Usage: The Ubiquitous analyzer has much larger usage throughout the world, especially in Navy applications. The newer Omniferous analyzer is not yet in such widespread use; however, many of these units do exist and apparently are in regular use.

Availability: This analyzer and the others are commercial products and immediately available. Typical prices for the Omniferous are in the $60,000 range. The lesser capability Ubiquitous is in the $10,000 and up category. For further information contact: Nicolet Scientific Corp., 245 Livingston Street, Northvale, New Jersey 07647.

Comments: Nicolet emphasizes hands-on, easy to use, high speed laboratory type devices. The penalty paid is in expandability and flexibility. From the standpoint of high speed two channel time series analysis capability, the Nicolet system is very powerful. The displays are not necessarily as well formatted as are available in the minicomputer based systems, but they are adequate for many uses and extremely good for some aspects of real time data analysis displays.

The system can be augmented with a minicomputer to provide additional flexibility but the author is not familiar with this configuration.

The hardwired approach to a system such as this has virtues and faults which must be evaluated by the potential user. One essentially pays the penalty of lack of flexibility in exchange for high speed and laboratory instrument type knobs, dials and switches and their attendant convenience.

Spectral Dynamics Corp. SD360 [22], [23]

Date Issued: Late 1973.

Capability: The all-digital SD360 Digital Signal Processor - the "DSP" - is a stand-alone, hardwired FFT Analyzer that combines capabilities of two Real Time Analyzers, a transfer function analyzer, analog signal conditioners and a computer. It provides a complete signal analysis capability from 0.01 Hz to 150,000 Hz.

The DSP looks and operates like an instrument, not a computer. It performs a dozen different data analysis functions, including:

- Signal Averaging
- Single or Dual Channel FFT
- Cross-Spectrum Analysis
- Inverse Transforms
- Autocorrelation
- Cross-Correlation
- Convolution
- Transfer Function Analysis
- Coherent Output Power
- Probability Density Histograms
- Probability Distribution

Hence the SD360 essentially implements all of the procedures reviewed in this chapter and [1] with the exception of the multichannel spectral analysis. The maximum real time bandwidth for two channel spectral analysis is approximately 30 kHz.

Spectral Dynamics also produces a vibration test control system based on the SD360. In addition options are available for tracking filter and shock spectrum analysis. The system can be interfaced to a PDP-11 minicomputer to provide additional I/O flexibility.

Method: The SD360 is a digital hardwired system based on the FFT. The computations are apparently performed in 16 bit block floating point, a 16 bit mantissa, 7 bit characteristic floating point, and a 7 bit character-

istic, 8-bit mantissa floating point. Presumably the FFT is done in block floating point with subsequent spectrum averaging, etc., done in floating point.

Input Data: Standard input is two channels of analog data routed through anti-aliasing filters and 12 bit A/D converters. The maximum sampling rate capability is 307,200 sps.

Control parameters for the system are entered via switches, knobs and pushbuttons. This includes I/O control and processing function control.

The SD360 also has a digital signal input capability so that it is possible to connect appropriate digital devices to the system.

Output: The only standard output is to LED digital numerical displays. However, in typical systems optional x,y oscilloscopes are provided. Another option is a 3D display control to provide plots of time varing spectra. Also hard copy x,y plotters may be attached to a system. A digital output channel exists so that the capability would be available to drive digital devices.

If the system was interfaced with the PDP-11 minicomputer, then additional output to tape and disk would be possible.

The standard LED numerical display is controlled by a joystick. Under joystick control, the intensity marker (cursor) follows the processed data on the scope to any selected location. Digital values of that specific location are then displayed on the 6 digit LED panel: frequency (in Hz); time (in milliseconds); amplitude (in dB or normalized level); or phase (in degrees). A panel immediately adjacent to the LED's automatically defines the units being displayed and whether they are linear or log values.

The 0 dB reference for the LED readout can be set at any convenient data level, or at the full scale voltage level. For log readouts, the level is displayed as $\pm$dB from this reference level.

The x address of the memory corresponding to the cursor location can also be displayed on the LED's.

Language: The concept of computer program language does not apply to this analyzer since the program is essentially hardwired.

Usage: The SD360 DSP is a relatively new addition to the Spectral Dynamics equipment line. S-D tracking filters, mechanical impedance analyzers, and related analog equipment have been in widespread use for many years however. Therefore, it is assumed that the SD360 is in fairly wide use in vibration data analysis throughout the U.S.

Availability: This analyzer is a commercial product and hence immediately available. Typical minimum prices for a system including displays would be in the $60,000 range. For further information contact: Spectral Dynamics Corporation, P.O. Box 671, San Diego, Ca. 92112.

Comments: Spectral Dynamics, as does Nicolet, emphasizes hands-on, easy to use, high speed laboratory type devices. The penalty paid is in expandability and flexibility. From the standpoint of high speed two channel time series analysis capability, the Spectral Dynamics system is very powerful. Their standard display, the numerical LED, has interesting features with its joystick control, but is very limited. It is unlikely that anyone in the shock and vibration data reduction business would ever be satisfied with just this. When the optional oscilloscope displays are obtained, some very useful optional display features are available. In particular, the time varying spectral display is useful. However, in any case, the displays would not be as well formatted as is possible from the general purpose computer based system.

The system can be augmented with a minicomputer to provide additional flexibility but the author is not familiar with this configuration.

The hardwired approach to a system such as this has virtues and faults which must be evaluated by the potential user. One essentially pays the penalty of lack of flexibility in exchange for high speed and laboratory instrument type knobs, dials and switches and their attendant convenience.

Honeywell/SAICOR [24, 25]

Date Issued: Approximately 1973.

Capability: We are describing the combined capability of a digital correlation (SAI 43A) and digital Fourier transform analyzer (SAI 470). SAICOR has produced various types of analog and hybrid signal processing gear for several years but we restrict our discussion to this particular gear.

The SAI-43A Correlation and Probability Analyzer is an all digital high speed processing instrument which provides an on-line, real time computation in three primary operating modes - Correlation (auto and cross), Enhancement (or signal recovery) and Probability (density and distribution). A 400 point analysis is accomplished in all modes. The SAI-43A provides a minimum Δt of .2 μsec or a 5 MHz sampling rate. Also standard are 800 points of precomputation delay, exponential (RC) averaging, and binary digital outputs.

The SAI-470 Fourier Transform Analyzer (FTA) is a fully digital instrument which performs a Fourier analysis of any function computed by either the SAI-42 or SAI-43 100 and 400 point Correlation and Probability Analyzers. (External digital input data can also be applied to the FTA for transformation.) The combination of these two devices gives:

Auto/cross correlation function
Power/cross spectral density function
Probability density
Probability distribution
Signal averaging

Hence, these instruments implement a subset of the functions described in this chapter and in [1].

Method: The correlation and probability functions are computed directly; the correlation from a sum of products. This data is then input to the Fourier transform analyzer which computes the transform via a "slow" algorithm. The computations are performed rapidly due to the digital circuitry implementation, not the computational algorithm. The combination of these two devices essentially implements the Blackman-Tukey method [7] for digital spectral computations.

Input Data: The input is two channels of analog data to the correlator. This data is digitized at a 5,000,000 sps rate. The output of the correlator is a 400 point digital correlation function which becomes the input to the FTA.

Input data control and processing controls are set by pushbuttons and knobs on both the correlator and spectrum analyzer.

Output: The instruments themselves contain no displays. However scopes and strip chart recorder may be attached to either instrument. Also x,y plotters are optional. The correlator also outputs the correlation function in a format suitable for the FTA. The correlation function or other input data to the FTA may be displayed on optional scopes.

The output of the FTA is a 1000 point transform. This function can be in terms of absolute value and phase or real and imaginary parts with log or linear scales. The display scope is an optional item.

Language: The concept of computer program language does not apply to this analyzer.

Usage: The author is not as specifically familiar with users of the SAICOR gear as compared to the others. However SAICOR has been producing this and other data processing equipment for some time and presumably the correlator and FTA are in regular use.

Availability: The SAICOR Correlator and Fourier Transform Analyzer are commercial products and hence immediately available. Prices begin about $20,000 for a minimum system consisting of both components and peripherals. For further information contact: Honeywell, Signal Analysis Operation (SAICOR), 595 Old Willets Path, Hauppage, N.Y. 11787, (516) 234-5700.

Comments: The SAICOR gear strikes the author as not taking advantage of contemporary computational techniques. The FTA is definitely an add-on afterthought type of approach. However, in spite of this, fast computa-

tional times are attained with highly parallel digital circuitry. There is a definite lack of flexibility in data analysis parameters relative to other systems. For example, 400 lags and 1000 frequency points are more or less fixed. However, the price is relatively low and hence if the parameters available are adequate then the low price can make this equipment attractive. These two basic analyzers must be augmented with display equipment in order to make them really suitable in a vibration data analysis job.

EXPECTED FUTURE DEVELOPMENTS

A substantial evolution in shock and vibration test data analysis has occurred over the past 15 years. Prior to 1960, probably 99% of such analysis was performed in laboratories with analog data analysis equipment such as strip chart recorders, voltmeters, wave analyzers, and analog PSD analyzers. As of 1975 probably 85% of such analysis is accomplished digitally, either on a large scale general purpose computer or on a dedicated minicomputer or hard-wired digital system.

The combination of the FFT algorithm, first widely noticed in 1965, and the continuing advancement of digital computer technology has been responsible. In the past few years it has become possible to cloak a digital computer with knobs, switches, A/D converters, scope displays and the like to make it appear like the analog lab devices of the past.

The cost of digital computing continues to plummet. The peripheral mass storage devices such as tapes and disks are now beginning to follow suit, along with more flexible and less expensive displays. Hence, it should be possible to further specialize and dedicate digital devices to special applications such as shock and vibration test data analysis. The advantage of the large scale computer in terms of speed and extensive complements of peripheral devices should diminish substantially.

It seems unlikely that dramatic changes in software and computational algorithms will occur in the near future. Thus the changes will be in hardware more so than software. Since the software is a major part of the cost of all computer systems, the total cost of data analysis systems will drop but not as much as pure hardware components.

More and more applications, involving multiple channels of high frequency data will become tractable to analysis because of increases in speed and capability of the hardware.

The shock and vibration data analysis system of the near future will probably be based on a small size digital computer, but very fast and with very large storage -- hundreds of thousands of words rather than thousands. It will undoubtedly have multichannel analog or digital input capability (allowing for the forthcoming digital transducers) and have multi-kilohertz A/D conversion and real time analysis bandwidth capability. It should have large screen CRT plot/print displays with hard copy attachments. Much more flexible interactive software for display control will be available. Undoubtedly, this system will have several inexpensive mass storage peripherals such as floppy disks or cassette type tapes.

Since the computer will be large enough to support higher level language compilers such as FORTRAN IV, it is expected that most new software will be coded in FORTRAN. This should somewhat reduce the cost of the analysis software. The computer will undoubtedly have a microprogrammable store so that certain key algorithms can be implemented in high speed microcode. This will further speed and simplify certain types of programming.

In general, it is expected that the trend will be away from the large scale, general purpose, multi-program computer to the smaller dedicated system. This has the tremendous advantage of doing away with the tremendous system software overhead encountered in present day large scale systems.

Probably the advances will not be as great as might otherwise be expected due to the limitations and cost of software development. There exist no forseeable major breakthroughs in this aspect of the overall system.

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Fluid Structure Interaction

A. J. Kalinowski

Naval Underwater Systems Center

INTRODUCTION

The general topic of fluid structure interaction is indeed a particularly broad subject in that it simultaneously brings together all the aspects associated with both solid mechanics and with fluid mechanics. Each of these two areas are complex by themselves; however, when considered together, the coupling (or interaction) between the fluid and solid response compounds the solution methodology. For an in vacuo structure, the surface loading is typically in the form of some known applied force. However, for a structure submerged in a fluid, the surface loading is not known a priori but depends on the surface motion of the structure. The structure's surface motion is in turn viewed as a motion type loading to the fluid field equations, wherein the pressure in the fluid field, including the interface boundary, are unknown functions of space and time that are to be determined. The interaction process can be viewed as a feedback loop [1] as illustrated in the diagram of Fig. 1. The feedback loop shown in Fig. 1 is a general one in that no assumptions are made regarding the type of fluid employed or type of structure being analyzed.

A vast amount of theoretical work at all levels of complexity is in the open literature in both the area of fluid flow theory and structural mechanics (e.g. nonlinear and linear theories alike). A good deal of this work (both linear and nonlinear) has been implemented in the form of computer programs, particularly in the area of structural mechanics. Computerization of the combined problem of fluid structure interaction has not received nearly as much attention as either problem taken separately. Many computer programs exist for solving interaction problems; unfortunately however, more of them are research oriented, lightly documented programs (refer to here as ROP) rather than user oriented, heavily documented programs (refer to here as UOP). Consequently, more consideration is given here to the ROP type than might normally be considered in a survey on a more popular area like structural analysis programs. It is the opinion of this author that a well documented UOP that is capable of solving a particular class problem is worth more to a person with a specific problem to solve than a more efficient ROP that can solve the same problem. This opinion is arrived at from the following considerations: 1. the man hours of labor (and hence cost) required to learn how to use (and install on a particular computer system) a ROP is usually an order of magnitude greater than performing the same task with a UOP. Therefore, from a cost point of view, it is likely that the savings in expected computer time usage of ROP can be greatly exceeded by the man hour cost of implementation. 2. Although a ROP might give a more accurate answer than the UOP for the same discretization (or mesh size), the UOP can often be run at a finer discretization and still arrive at a sufficiently accurate result. 3. Often minor or major changes can be made in the UOP coding to accommodate some special purpose (such as a change in input/output format, or the introduction of some new structural element), whereas doing the same thing to a ROP could involve going as far as engaging the program author under contract to accomplish a modification. 4. The closer the ROP is to being

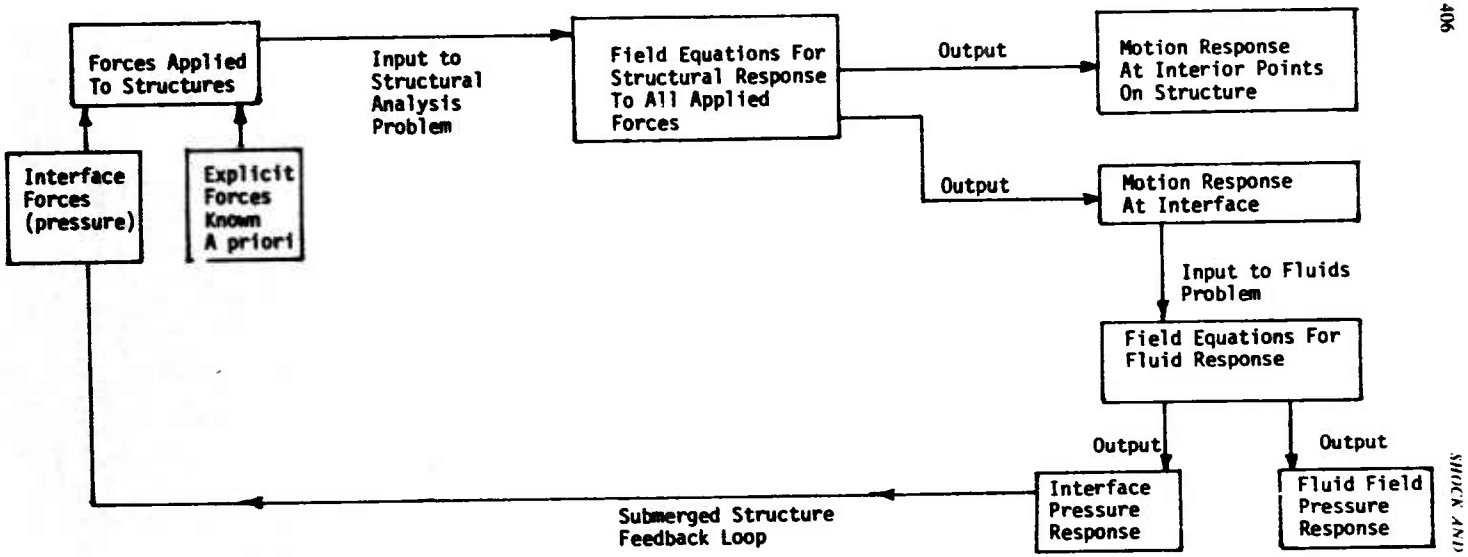


Fig. 1 Fluid structure interaction problems

(1) Physical Phenomenon
- Scattering
- Radiation
- Forced Vibration

(2) Time Variation
- Transient
- Harmonic

(3) Structure Type
- Flexible (finite stiffness)
- Ideal (either zero or infinite stiffness)

(4) Fluid Region Type
- Unbounded
- Bounded
- Partially Bounded

Fig. 2 Computer program categories

a "black box" in terms of understanding the inner workings of the program, the less confidence one tends to have in its results.

Terminology

The types of fluid structure interaction programs considered in this survey (e.g. Fig. 2) can be subdivided into four basic categories; namely, 1. physical phenomenon, 2. time variation, 3. structure type, 4. fluid region type. These four categories are further subdivided and explained in the following more detailed description below. The formation of these categories will further serve to set up the terminology used in defining the capabilities of the various programs reviewed in this survey.

Physical Phenomena

Under this category three types of problems are considered; namely, those of scattering, radiation and forced vibration.

Scattering. A structure submerged in a fluid media is subject to an incident pressure disturbance (e.g. Fig. 3a).

Radiation. A structure submerged in a fluid is acted upon by some forcing functions applied directly to the structure (e.g. Fig. 3b).

Forced vibration. All other types of fluid structure interaction problems not fitting the above scattering or radiation description will be lumped into this more general category. In a broad sense even the radiation and scattering definitions can be called "forced vibration" problems, however, the radiation and scattering problems are encountered so often in this field that they deserve special treatment.

One example of a forced vibration problem not fitting the radiation or scattering category would be a flexible tank containing fluid with a free surface. The base of the structure is excited by some known base acceleration (e.g. Fig. 3b) that could, for example, simulate an earthquake input.

Time Variation

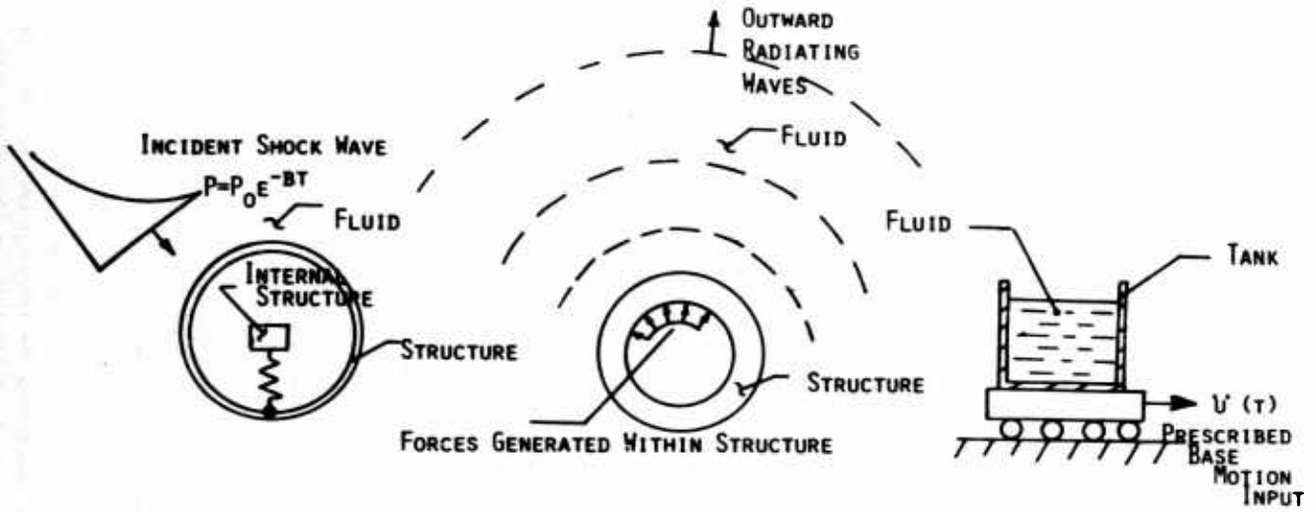
Here the time variation of the input is categorized as either transient or harmonic.

Transient. The governing field equations are treated as initial value problems and are driven by known time history forcing functions in the form of applied force, pressure or motion (or any combination thereof) transients.

Harmonic. The governing field equations are driven by known harmonic forcing functions as in the above case, except here the driving functions are proportional to $e^{i\omega t}$. Further, the usual steady state assumption is made; namely, that the input has been on for $-\infty \leq t \leq +\infty$.

Structure Type

Two types of stiffness characteristics are considered for the structure representation; namely, flexible structures and ideal structures.



a. Scattering b. Radiation c. Forced vibration

Fig. 3 Examples of physical phenomena

Flexible. The structure has a finite amount of stiffness. Here the full fluid structure interaction loop illustrated in Fig. 1 is in effect.

Ideal. In this situation the stiffness of the structure is so large, relative to the fluid stiffness, that the structure can be viewed as being perfectly rigid (undeformable). This shall be referred to as a "hard" structure. The hard structure can be "fixed in space" in which case the fluid structure interaction response depends only on the surface shape of the structure (we refer to this as a hard immovable structure). Alternatively, the structure can be hard, but not fixed in space (we refer to this as a hard movable structure), wherein the solution response depends on both the surface shape and total mass of the structure.

Finally, one more ideal structure is one for which the weight and stiffness are so small that the structure can be viewed as a weightless void (or pocket) in the fluid (we refer to this as a soft structure).

Fluid Region Type

The boundedness of the fluid domain in contact with the structure has a direct bearing on solution methodology; consequently, the categories of unbounded, bounded, and partially bounded are established.

Unbounded. In this case, the extent of the fluid around the structure is so vast that for all practical purposes it can be considered to be infinite in extent. This type of fluid region is usually used in conjunction with radiation or scattering problems (e.g. Fig. 3a and Fig. 3b).

Bounded. Here the fluid has finite boundaries in all directions (e.g. Fig. 3c).

Partially Bounded. This is a combination of the two above cases wherein part of the fluid is infinite in extent in one direction but bounded in another. A mathematical model of a dam would be an example in this case. The water depth is finite but the fluid is assumed to extend to infinity parallel to the lake bottom.

Scope of Survey

The scope of this survey on fluid structure interaction is governed more by what is actually available than by any planned selection procedure. By far, the majority of the computer programs that exist in this field have been sponsored either partially or totally by the United States government and, in particular, are aimed at naval applications. Consequently, practically all of the computer codes employ an acoustical representation for the fluid field governed by the wave equation [3].

$$\nabla^2 p = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} \tag{1}$$

where p denotes the fluid pressure, t time and c the speed of sound in the fluid medium. The only programs considered that employ a more general fluid constitutive equation are the finite difference type programs usually designed for solving "shock wave propagation in solids" type problems [2], [4]. Although theoretical work exists for nonlinear fluids (e.g. allowing for cavitation [56], it has not found its way into any UOP). These programs are primarily designed to solve stress waves through solids problems, however, their constitutive laws

are usually written in a general enough form so that the fluid equations of motion in contact with some structural component can reduce down to Eq. (1), or for that matter, a more general form of Eq. (1), say the full Navier Stokes equations.

The majority of the programs considered in this survey are for structures that can be represented in a continuum by a set of linear partial differential field equations of the form

$$\tilde{D}(\bar{u}) = \bar{F}_I(t, \bar{x}^s, \bar{u}^s) + \bar{F}_A(t, \bar{x}) \tag{2}$$

or in the finite element form by the matrix relations

$$[M]\{\ddot{U}\} + [C]\{\dot{U}\} + [K]\{U\} = -\{F_I\} + \{F_A\} \tag{3}$$

where $\tilde{D}$ is a linear differential operator, $\bar{u}$ is the displacement field in the structure, $\bar{F}_I$ represents the fluid interaction forces applied to the structure, $\bar{F}_A$ represents all other explicitly known driving forces acting on the structure (including incident wave loading if it is present), [M] is the mass matrix of the finite element model, [C] the damping matrix, [K] the stiffness matrix, {U} the displacement matrix, $\{F_I\}$ the column vector of interaction forces, and $\{F_A\}$ the column vector of known applied forces. The only attention given here to nonlinear structures (say for nonlinear operators $\tilde{D}$) is first through the application of the finite difference shock wave in solids programs (e.g. HEMP, PISCES) referred to earlier [2], [4] (i.e. by altering the media constitutive equation to reduce to a fluid, and by using the appropriate nonlinear constitutive equation for the structure), and second, through the application of "mock" fluid finite elements which are adaptable to most any existing nonlinear dynamic structures code. It is emphasized that the term nonlinear dynamic structures code refers to a computer program with no explicit fluid structure interaction rigid format.

The finite difference class of programs referred to above can solve all of the four solution categories given in Fig. 2 except for the harmonic steady state option. As a group, the chief disadvantage of this class program is that they are not readily adaptable to solving fluid structure interaction problems involving shell-like structures. Another disadvantage is that the entire fluid media must be modeled as part of the system finite difference network; this is in contrast to many of the finite element orientated programs which treat the fluid media as a continuum and need only model the structure (and interfacing fluid boundary) discretely. Another disadvantage of these finite difference formulations is that they are geared towards solving highly nonlinear problems (in both the kinematics and constitutive laws). Consequently, when employing them to solve completely linear fluid structure interaction problems, it appears that the scheme of the solution technique is performing a good deal of unnecessary computations to allow for nonlinearities that are not even present. The severity of this last disadvantage is not clear. This author is not aware of any benchmark studies which make a careful comparison of running times in comparing finite difference codes of the type referred to in [2], [4], with finite element orientated fluid structure interaction programs.

The details of the finite difference programs have been covered elsewhere in this book [4], consequently, they will not be repeated here. With one exception, the remaining portion of this survey will be entirely devoted to computer programs employing fluid structure interaction solution techniques other than by the finite difference method.

NOMENCLATURE

c = Wave speed of sound
$[C]$ = Damping matrix
$[E]$ = Material constant array
$\{F\}$ = Force vector
$[K]$ = Stiffness matrix
$[M]$ = Mass matrix
p = Pressure
t = Time
u_1, u_2, u_3 = Continuous displacement component
$\{U\}$ = Column vector of discrete displacements
w_1, w_2, w_3 = Continuous velocity components
$\{W\}$ = Column vector of discrete velocity components
x, y, z = Cartesian coordinates
β = Decay constant
ε = Strains
λ = Lamè constant
μ = Lamè constant
ρ = Mass density/unit volume
σ = Stress or source strength distribution
ω = Steady state driving frequency
$(\dot{})$ = Partial derivative on time
∇^2 = Laplacian operator
$\{\ \}$ = Column vector
$[\ ]$ = Matrix
$i = \sqrt{-1}$

SOLUTION METHODOLOGY

Many different numerical solution techniques are used to solve the various problem types illustrated in Fig. 2. The intent of the survey article is aimed more at presenting what is available in terms of existing computer programs than covering the finer details of solution methodology. Nevertheless, it is felt that some brief outline of the basic solution techniques is still necessary for the sake of completeness. Several programs may employ the same solution method, thus explanations of methodology, program-by-program would involve considerable duplication. Instead, only the more important solution techniques are briefly explained here, and cross references to these techniques are made in the detailed program discussions that appear later in this chapter.

Finite Element Structure-Continuum Fluid

In this category, the fluid field equations are represented by Eq. (1) and the structure response field equations by Eq. (3). The method employed for solving this system of equations simultaneously differs substantially depending on whether one is solving a transient or a harmonic (steady state) type problem.

Transient Methodology (Time Integration)

In this approach, the governing equations of motion are integrated in time. The most often used approach appears to be one which gives an approximate relation between the interaction force (F_I) and the fluid interface motions (U_s); i.e.

$$\tilde{D}((U_s), \{F_I\}) = 0 \tag{4}$$

where $\tilde{D}$ is some linear differential operator. Now, the new system of equations to be solved simultaneously is Eqs. (3) and (4). Next, consider the construction of the system of simultaneous Eqs. (3) and (4) in more detail.

Surface Fluid Structure Interaction Approximations. The earliest and simplest form of Eq. (4) is the plane wave approximation [7]. Here the interaction approximation takes the form

$$\{\overset{*}{F}_I\} = [A]\{\overset{*}{W}_S\} \tag{5}$$

where $\{\overset{*}{F}_I\}$ is the interaction force vector normalized[1] to $\rho c^2 a^2$, the velocity vector $\{W_S\} = \{\dot{U}_S\}$, is normalized to the wave speed c, and the diagonal finite element area matrix [A] is normalized to a^2, where ρ is the fluid mass density, and a is some characteristic length factor. The continuous counterpart of Eq. (5) is simply that the fluid pressure, p, at the interface is related to the normal velocity by $p = \rho c w_n$. The starred superscripts denote nondimensional variables. It is to be noted that Eq. (5) applies to finite elements of the structure that interact with the fluid. Eq. (5) is accurate only during the early time high frequency response. An approximation that is valid at the other end of the transient, i.e. the latter times which are characterized by low frequency free vibration response, is given by the so-called virtual mass approximation [8, 6]

$$\{\overset{*}{F}_I\} = [M_V]\{\overset{*}{W}_S\} \tag{6}$$

where the nondiagonal virtual mass matrix $[M_V]$ is normalized to ρa^3, and may be determined through an analysis of incompressible fluid motion approximate to a distribution of elemental sources on the surface of the structure [9, 10].

The significance of the [A] matrix being diagonal and the $[M_V]$ matrix nondiagonal is that in Eq. (5) the force at a node i depends only on the motion at the same node i; however, in the case of Eq. (6), the force at node i depends on the motion of other structure surface nodes as well.

Reference [6] extended the cylindrical wave approximation of [11] into a three-dimensional form known commonly today as the doubly asymptotic approximation and is given by the relation

$$\{\dot{\overset{*}{W}}_S\} \simeq [A]^{-1}\{\dot{\overset{*}{F}}_I\} + [M_V]^{-1}\{\overset{*}{F}_I\} \tag{7}$$

where the $[\ ]^{-1}$ notation denotes a matrix inverse.

This approximation is accurate at the early time (high frequency) content response (first term in Eq. (7)) and for the slowly varying (low frequency) response (second term in Eq. (7)). Transient fluid structure problems (particularly of the shock input type) are usually characterized by a rapidly applied loading followed by mostly low frequency free vibration response; consequently, Eq. (7) provides an interaction law which has both the ingredients of the early and late time approximations denoted by Eqs. (5) and (6). Although other forms of Eq. (4) exist today in the open literature [12, 13], it appears that Eq. (7) is finding its way into many of the transient computer codes employing approximations of the type defined by Eq. (4).

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[1]The normalization of variables process used here is to keep the write-up consistant with the major source reference [6].

Governing Equations of System. At this point, one might say that upon simultaneously solving Eqs. (3) and (4), the problem is viewed as completely defined short of integrating the equations of motion by some numerical scheme. However, there are still some additional details to be explained before this statement can comfortably be made. It still remains to reduce the equations to a tractable form for extracting the solution numerically. The following development closely follows [6].

For simplicity, the damping matrix [C] will be omitted in the following development. The equations of motion for a scattering type problem result in the following nondimensional form of Eq. (3)

$$[\overset{*}{M}]\{\ddot{\overset{*}{U}}\} + [\overset{*}{K}]\{\overset{*}{U}\} = \{\overset{*}{F}_A\} - \{\overset{*}{F}_I\} \tag{8}$$

where $\{\dot{\overset{*}{U}}\} = \{\overset{*}{W}_I + \overset{*}{W}_S\}$ for surface elements only and the applied force is given by $\{\overset{*}{F}_A\} = \{\overset{*}{F}_E\} - \{\overset{*}{F}_I\}$.

In the Eqs. (8), the nondimensional displacement vector $\{\overset{*}{U}\}$ is normalized to a, the structural mass and the stiffness matrices [M] and [K] are normalized to ρa^3 and $\rho c^2 a$ respectively. The net applied force $\{\overset{*}{F}_A\}$ is composed of the incident wave force vector $\{\overset{*}{F}_I\}$; and the externally applied driving forces applied directly to the structure (should any be present in addition to the incident wave loading) and are denoted by $\{\overset{*}{F}_E\}$. The external force vector $\{\overset{*}{F}_E\}$ is normalized by $\rho c^2 a$. The incident wave velocity, $\{W_I\}$, denotes the prescribed fluid particle velocity taken normal to the structures' surface, and is normalized to c. Here the surface displacement is taken as positive going into the fluid. The incident wave force vector, $\{\overset{*}{F}_I\}$, is given by the relation

$$\{\overset{*}{F}_I\} = [A]\{\overset{*}{P}_I\} \tag{9}$$

where $\{P_I^*\}$ is the pressure of the fluid action on the structure surface and is normalized to ρc^2.

The next step is to partition the N equations of motion into two sets; the first set, J equations, pertaining to the structural elements in contact with the fluid, and K equations (K = N-J) for the remaining elements not in contact with the fluid.

Thus we have

$$\begin{bmatrix} \overset{*}{M}_{JJ} & \overset{*}{M}_{JK} \\ \overset{*}{M}_{KJ} & \overset{*}{M}_{NK} \end{bmatrix} \begin{Bmatrix} \ddot{\overset{*}{U}}_J \\ \ddot{\overset{*}{U}}_K \end{Bmatrix} + \begin{bmatrix} \overset{*}{K}_{JJ} & \overset{*}{K}_{JN} \\ \overset{*}{K}_{KJ} & \overset{*}{K}_{KK} \end{bmatrix} \begin{Bmatrix} \overset{*}{U}_J \\ \overset{*}{U}_K \end{Bmatrix} = \begin{Bmatrix} \overset{*}{F}_{AJ} \\ \overset{*}{F}_{AK} \end{Bmatrix} - \begin{Bmatrix} \overset{*}{F}_{IJ} \\ \overset{*}{F}_{IK} \end{Bmatrix} \tag{10}$$

for the partitioned equations of motion which leads to

$$\underbrace{\begin{Bmatrix} \overset{*}{M}_{JJ} & \overset{*}{M}_{JK} \end{Bmatrix}}_{\overset{*}{M}_J} \{\ddot{\overset{*}{U}}\} + \underbrace{\begin{bmatrix} \overset{*}{K}_{JJ} & \overset{*}{K}_{JK} \end{bmatrix}}_{\overset{*}{K}_J} \{\overset{*}{U}\} = \{\overset{*}{F}_A\}_J - \{\overset{*}{F}_I\}_J \tag{11}$$

in which the force vectors are Jx1 and the rectangular matrices $[\overset{*}{M}]_J$ and $[\overset{*}{K}]_J$ denoted by the braces above) are of size JxN. Combining Eqs. (10), (7), and the second of Eqs. (8) leads to the third order differential equation.

$$[\overset{*}{M}]_J\{\dddot{U}^*\} + \left([A]_J + [A][M_V]^{-1}[\overset{*}{M}]_J\right)\{\ddot{U}^*\}$$

$$+ [\overset{*}{K}]_J\{\dot{U}^*\} + [A][M_V]^{-1}[\overset{*}{K}]_J\{\overset{*}{U}\} = + [A][M_V]^{-1}\left(\{\overset{*}{F}_A\}_J + [A]\{\overset{*}{U}_I\}\right) \tag{12}$$

where $[A]_J$ is a JxN matrix formed by adding K columns of zeros to the JxJ [A] matrix, i.e.

$$\underset{JxN}{[A]_J} = \left[\underset{JxJ}{[A]} \;\vdots\; \underset{JxK}{[0]}\right]$$

Equation (12) represents J equations with N unknowns (i.e. the unknown structure displacements, $\{\overset{*}{U}\}$). The remaining set of K equations can be obtained from the lower partition of matrix Eq. (10) resulting in the relation

$$\left[[\overset{*}{M}]_{KJ} \;\vdots\; [\overset{*}{M}]_{KK}\right]\{\ddot{U}^*\} + \left[[\overset{*}{K}]_{KJ} \;\vdots\; [\overset{*}{K}]_{KK}\right]\{\overset{*}{U}\} = \{\overset{*}{F}_E\}_K \tag{13}$$

The noninteracting elements have no forces other than the Kxl vector, $\{F_E\}$, wherein the $\{\overset{*}{F}_I\}_K$ and $\{\overset{*}{F}_I\}_K$ terms are zero.

Equations (12) and (13) constitute a set of N coupled linear ordinary differential equations with N unknowns (the variable $\{\overset{*}{U}\}$). These equations can be integrated numerically by existing step by step integration schemes.

The problem formulation leading up to Eqs. (12) and (13) considers a general scattering type problem where simultaneously one could have the structure being loaded internally (i.e. the $\{\overset{*}{F}_E\}$ term) as well as loaded by an incident wave (the $\{\overset{*}{F}_I\}$ term). A purely scattering problem is one for which $\{\overset{*}{F}_E\}$ is zero and only $\{\overset{*}{F}_I\}$ acts; a purely radiation problem is one for which $\{\overset{*}{F}_I\}$ is zero and only $\{\overset{*}{F}_E\}$ is present.

The procedure used to reduce system Eqs. (3) and (7) down to the final set of simultaneous equations in the unknown vector $\{\overset{*}{U}\}$ (Eqs. (12) and (13)) can be followed in a similar manner should some other form of the surface fluid structure interaction (i.e. Eq. (4)) be used in place of Eq. (7). In general, the higher the order of the operator, D, in Eq. (4), the higher the order of the final governing equations in the unknown $\{\overset{*}{U}\}$.

Up to now, the solution technique described has been based on a direct approach to the solution of the field equations. An alternative method of solution is the modal approach wherein the structure response, $\{U\}$, is expanded in terms of the structure free vibration modes (in vacuo), thus

$$\{U\} = \sum_{m=1}^{M} Q_m(t)\psi_m(\bar{x}) \tag{14}$$

where $\psi_m(\bar{x})$ are the in vacuo, undamped mode shapes of the structure, and Q_m

are the unknown functions of time (generalized displacement) that must be determined for each of the M modes. Consideration of Eq. (14) in conjunction with Eqs. (3) and (4) results in a system of differential equations for the unknown time variation $Q_m(t)$. In problems for which there is no fluid interaction, i.e. $\{F_I\} = 0$ in Eq. (3), the modal approach is straightforward, in that solutions can be obtained in terms of a set of M uncoupled ordinary differential equations for the variables $Q_m(t)$. The introduction of the $\{F_I\}$ term, which is itself a function of the unknown structure displacements, complicates the problem, in that for a general structure, the governing equations for the unknowns, $Q_m(t)$, are mode coupled simultaneous differential equations.

Although the modal approach has the advantage that the number of degrees of freedom involved in the solution technique are substantially smaller than the direct approach, the direct approach has the advantage that 1. the extension of the solution technique to situations where the structure has certain nonlinearities is substantially easier for the direct approach (the dependence of the modal approach on linear superposition must be overcome), 2) in order to keep the modal solution scheme computationally tractable, a modal selection (omission) process must be made which is often time consuming and dependent on a good deal of physical intuition regarding the particular problem at hand. If one can afford the computer time, and the matrix size can be handled by the computer, one can include all the modes in the analysis without any omissions.

Transient Methodology (Frequency Integration)

The time integration approach to solving transient problems has the feature that it is readily adaptable to nonlinear problems. For the class of problems where the system field equations (e.g. Eqs. (1) and (3)) are linear, the transient solution can be constructed from the system transfer function (steady state solution to harmonic inputs).

Let $\{U(f)\}^*$ be the structural steady state response (say, displacement)[2] to a harmonic forcing function of the form given later by Eq. (19). Then the general time response in Duhamel integral form can be expressed

$$\{U(t)\} = \int_0^t p_i(t-\tau)\{\bar{U}(\tau)\}d\tau \tag{15}$$

where $\{\bar{U}(\tau)\}$ is the impulse response of the system and is related to the steady state response by the expression

$$\{\bar{U}(\tau)\} = \int_{-\infty}^{\infty} \{U(f)\}^* e^{2\pi i f \tau} df$$

where f is related to ω in Eq. (19) by $f = \omega/2\pi$. Note that the standard Fourier transform pair of a general aperiodic function is given by

$$\begin{aligned} \text{Transform:} \quad & B^*(f) = \int_{-\infty}^{\infty} B(t)e^{-2\pi i f t}dt \\ \text{Inverse:} \quad & B(t) = \int_{-\infty}^{\infty} B^*(f)e^{2\pi i f t}df \end{aligned} \tag{16}$$

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[2]The desired response does not necessarily have to be displacement, rather it could be velocity, stress in the structure or pressure in the fluid.

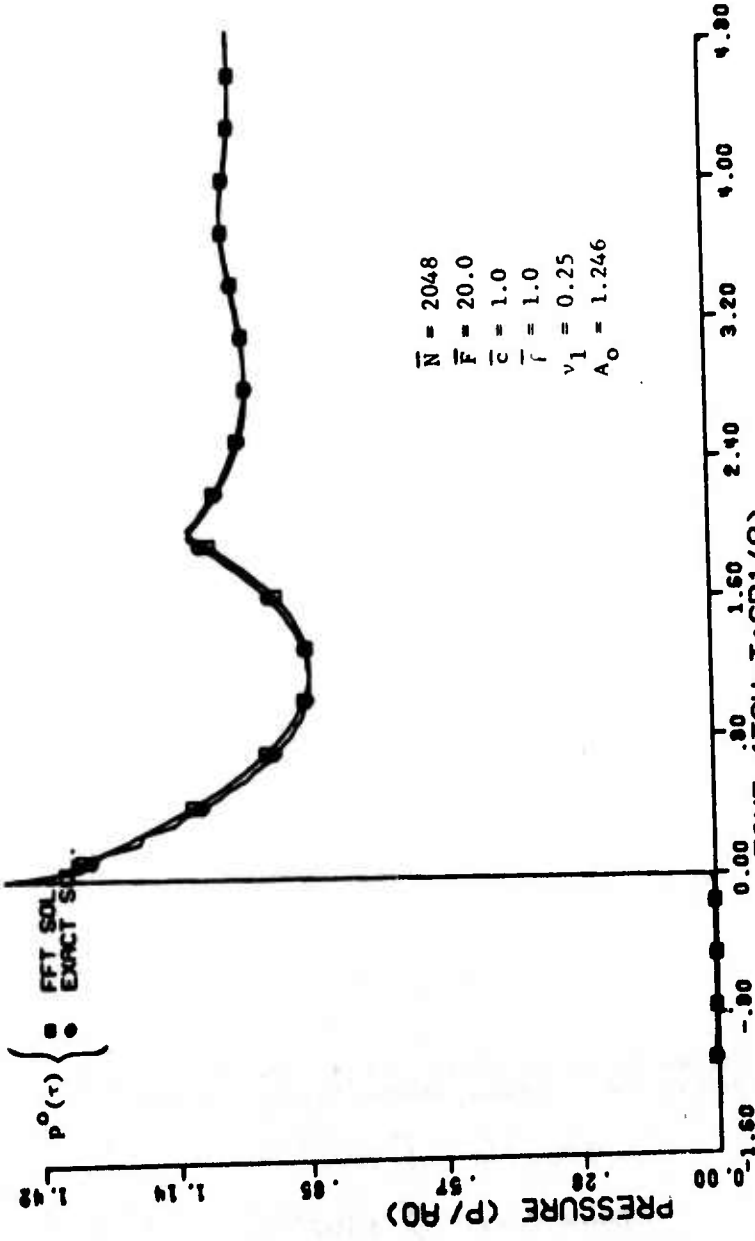

Fig. 4 Convolution FFT - Exact Solution transient response solution comparison

It can be shown formally that upon taking the transform of Eq. (15), and employing the transform pair, Eqs. (16); that

$$\{U(f)\}^{\circ} = p_1^{\circ}(f)\{\bar{U}(f)\}^{\circ} \tag{17}$$

where the variables with frequency agruments denote the transformed variables (superscript o). The corresponding time synthesis of Eq. (17) can be constructed for $\{U(t)\}$, namely

$$\{U(t)\} = \int_{-\infty}^{\infty} p_1^{\circ}(f)\{\bar{U}(f)\}^{\circ} e^{2\pi i f t} df \tag{18}$$

The calculation of the steady state response $\{\bar{U}(f)\}^{\circ}$ is covered in detail in the next subsection. For simple analytic input wave forms, $p_1^{\circ}(f)$ can easily be determined analytically through application of the first of Eqs. (16). For input waves that are not simple analytical forms (e.g. an earthquake record) then $p_1^{\circ}(f)$ must be determined numerically.

One approach to evaluating the improper integral defined by Eq. (18), is to employ some direct quadature evaluation, say, Simpson's rule. This, however, could be costly computationally since the complete integral must be evaluated repeatedly for each time sampling of the desired response and for each desired component of the response vector.

A far more efficient procedure for evaluating Eq. (18) is by the Fast Fourier Transform (FFT) technique [22]. The FFT is in principal equivalent to the finite Fourier transform, and computationally speaking, is simply a more efficient algorithymn for evaluating the finite Fourier transform. Reference [23] has adopted the FFT approach to evaluate convolution integrals of the form of Eq. (15) through the inversion of a harmonic synthesized integral like Eq. (18). Applications of the FFT are primarily used in the area of signal processing, however, it is steadily being employed in the area of shock and vibration. For example, [24] has employed the method for the solving of integral equations in the reverse direction (i.e. being given the output wave form, $\{U(t)\}$, the input wave form, $p_1(t)$, is to be determined) and [21] has applied the FFT in the forward direction (i.e. given $p_1(t)$, compute $\{U(t)\}$). As an illustration of the technique, consider the fluid structure interaction problem of finding the pressure time response in the center of a fluid filled spherical cavity imbedded in an infinite elastic medium subject to a plane dilatational step wave input. The exact elasticity pressure transient response solution has been presented in [24] where it is compared to the solution to the same problem which uses FFT techniques to convert the exact steady state solution into the corresponding time history solution (i.e. the forward solution of Eq. (15)) with pressure rather than displacement as the response quantity being monitored. The comparison of solutions is shown in Fig. (4) where the pressure has been normalized to the static solution, A_o, the time variable TAU is time normalized to a cavity radius transit time (A is the cavity radius and CD1 the dilatational wave speed in the solid elastic medium, T is time). The problem parameters are listed on the graph where $\bar{\rho}$ = solid-to-fluid mass density ratio, c = solid-to-fluid dilatational wave speed ratio, ν_1 = Poisson's ratio of the solid, N = number FFT sampling points, F = FFT frequency sampling range normalized to A/CD1. The computational time for the FFT solution in Fig. 4 was on the order of one minute for execution on an 1108 computer. The exact solution is the result of truncating an analytical series solution after the first 20 terms. The comparison between the exact and FFT solution is very good. As can be seen, the FFT solution forms a very sharp step wave front with very little overshoot. Similarly, good results were obtained using substantially fewer frequency sampling points. This is important when one realizes that if the steady state solution is being gener-

ated numerically by the techniques discussed in the next section, the solution process must be repeated for each frequency argument in the numerical evaluation Eq. (18). When employing the FFT technique in evaluating Eq. (18), care must be taken not to blindly apply the FFT inversion algorithmn. There are certain pitfalls involved with employing the method such as the periodic "wrap around" errors introduced through the finite Fourier transform replacing the infinite Fourier transform, "aliasing errors" introduced by sampling the steady state solution at too coarse a frequency interval and finally, "leakage errors" introduced by the truncation of the time history. A detailed discussion of these pitfalls and how they may be overcome is given in [25]. These errors are not introduced by the mechanics of the FFT algorithmn but are an outgrowth of applying finite Fourier transforms to problems that, strictly speaking, demand the use of infinite Fourier transforms for a periodic time function.

As a final note regarding this method, it is pointed out that the flexibility of the structure appreciably influences the form of the steady state response, $\{\bar{U}(f)\}^*$, for driving frequencies up to some limiting frequency f_m. For $f>f_m$, the response of the structure behaves, for all practical purposes, the same as a rigid movable structure. Thus, the $\{\bar{U}(f)\}$ need only be computed for values of $f\leq f_m$, since $\{\bar{U}(f)\} = o$ for $f>f_m$. In cases where the variable $\{\bar{U}(f)\}$ denotes, say, pressure in the fluid rather than structural displacement as explained above, then $\{\bar{U}(f)\}^*$ will approach a high frequency limit which is predictable from a simple model that considers the structure to be perfectly rigid.

Steady State Methodology (Unbounded Fluid Region)

In this class of problem one considers a structure which is totally submerged and surrounded by an infinite fluid domain. The fact that the fluid domain is unbounded allows one to take advantage of certain integral theorems in acoustic theory which relate the motion (normal velocity) at the fluid structure interface to the interface pressure.

For harmonic steady state problems, the problem loading (or driving term), $\{F_A\}$, in Eq. (3) is of the form

$$\{F_A\} = \{F_A\}^* e^{+i\omega t} \tag{19}$$

Accordingly, the complex frequency response time variation will also be proportional to $e^{+i\omega t}$, thus

$$\{U\} = \{U\}^* e^{+i\omega t}, \quad \{F_I\} = \{F_I\}^* e^{+i\omega t} \tag{20}$$

where ω is the driving frequency, $\{F_A\}^*$, the amplitude of the applied driving forces and $\{U\}^*$ the amplitude of the displacement response and $\{F_I\}^*$ is the amplitude of the interaction force. Generally, $\{U\}^*$ is complex and is a function of ω. Some authors prefer to drive the system with a $-i\omega t$ exponent on e in Eq. (19). Regardless of which choice is made, one can always convert one solution to another by replacing ω with $-\omega$.

As in the case of the transient problem, here, too, all the methods share the requirement that a relation between the interaction force amplitude $\{F_I\}^*$, and fluid interface motions $\{U_S\}^*$ is required in order to tie together the solution of the structure's field equations (3) to the solution of the wave equation (1). Such a relation is denoted by the functional form

$$F(\{U_S\}^*, \{F_I\}^*) = 0 \tag{21}$$

where here F is a function operator denoting a relationship which relates the two arguments. This is in contrast to the transient approach where the counterpart of Eq. (21) (i.e. Eq. (4)) had a linear differential operator in place of F.

Fluid Structure Interaction Approximations (Helmholtz Method). In the solution techniques to be discussed in this subsection, the only approximations used are those involved with converting the continuous pressure distribution existing at the fluid structure interface into a set of discrete nodal forces, $\{F_I\}^*$, applied to the structure surface nodal points.

The most popular way to arrive at Eq. (21) appears to be derived from the Helmholtz integral equation [15] where for any point, x, on the closed surface, S, which interfaces with the fluid, the total pressure $p(\bar{x})$ on the surface is related to the normal velocity, w, on the surface by the integral relation

$$p(\bar{x}) = p^i(\bar{x}) - 2\int_S p(\bar{y})\frac{\partial G(\bar{x},\bar{y})}{\partial n(\bar{y})}\,dS(\bar{y}) + 2i\omega\rho\int_S w(\bar{y})G(\bar{x},\bar{y})dS(\bar{y}) \qquad (22)$$

where $\bar{y}$ is a dummy variable for any position $\bar{x} \in S$, ρ the mass density of the fluid and G is the free space Green's function given by

$$G(\bar{x},\bar{y}) = \frac{\exp(-i\omega|\bar{x}-\bar{y}|/c)}{4\pi|\bar{x}-\bar{y}|} \qquad (23)$$

The development to follow in this subsection on harmonic analysis follows reference [17] (modified for incident pressure by the method given in [16]) for the first part of this subsection on a direct solution to the problem and follows reference [14] for the modal solution to the problem.

The partial derivative of G with respect to $n(\bar{y})$ denotes the rate of change of G in the direction normal to the surface at point $\bar{y}$, and $|\bar{x}-\bar{y}|$ denotes the distance between the $\bar{x}$ and $\bar{y}$ points.

The next step is to obtain a discrete version of Eq. (22) which is accomplished by representing the surface pressure and normal velocity in terms of a linear combination of scalar basis functions ψ_i defined in [17].

$$p(\bar{x}) = \sum_{n=1}^{N} p_n\psi_n(\bar{x})$$
$$w(\bar{x}) = \sum_{n=1}^{N} w_n\psi_n(\bar{x}) \qquad (24)$$

where N denotes the number of surface grid points in contact with the fluid. For example, [14] has used a cubic spacial distribution consistent with the finite element displacement fields for the neighboring structural elements. This is in contrast to [18] which employs a piecewise constant distribution of pressure over the interface zones of the structure. Employing a higher order basis function has the advantage that the same solution accuracy can be achieved with a coarser interface mesh. This fact ultimately results in a computer program that should run more efficiently when employing the higher order distribution basis functions.

Upon substituting Eqs. (24) into Eq. (22) and evaluating Eq. (22) over a discrete set of points ($\bar{x}_j$, j = 1, 2, J) corresponding to the fluid structure interface node points, one obtains

$$[L]\{P\} = [R]\{W^I\} + \{P^i\} \tag{25}$$

where [L] and [R] are JxJ matrices and $\{P^i\}$ is a known Jx1 column vector, and {P} is a column vector of discrete pressure values $p(\bar{x})$; these matrices result from the evaluation of Eq. (22).

Assuming for the moment that the driving frequency, ω, is not at (or very near) certain characteristic wave numbers of the fluid field enclosing the structure, Eq. (25) can be solved for {P}, thus

$$\{P\} = [Z(\omega)]\{W\} + [L]^{-1}\{P_i\} \tag{26}$$

where $[Z(\omega)] \equiv [L]^{-1}[R]$.

where

Reference [19] has presented a method for arriving at Eq. (26) even in situations where ω is at or near one of the characteristic cavity resonance frequencies[3]. Briefly stated, the improved method consists of determining the unique surface pressure, $p(\bar{x})$, that simultaneously satisfied the surface Helmholtz integral Eq. (22) and the interior Helmholtz integral [20]. The interior Helmholtz integral is a relation similar to the form of Eq. (22) except it relates the fact that the fluid pressure for all points in the region of space occupied by the structure is zero. Enforcing this interior Helmholtz integral over a judiciously selected set of M interior points leads to a matrix analogous to Eq. (25) in the form

$$[L']\{P\} = [R']\{W\} \tag{27}$$

Thus, Eqs. (25) and (27) result in a set of (J+M) equations for the J unknowns {P}. Solving the overdetermined set of equations specified by Eqs. (25) and (27) in a least square sense leads to an equation in the same form as Eq. (26) except that Z(ω) is determined in a more involved manner.

Next by employing the principal of virtual work, the total surface pressures can be related to a set of consistent interaction nodal forces, {F}, thus

$$\{F\} = [C^I]\{P\} \tag{28}$$

For harmonic steady state problems, the continuous velocity and displacement amplitudes are related by $w(t) = i\omega u(t)$. Making use of this relation in conjunction with the surface geometry relating normal components of motion into the Cartesian components employed in Eq. (3) results in the expression

$$\{W\} = i\omega[S]\{U\} \tag{29}$$

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[3] This is sometimes referred to as the cavity resonance problem.

Thus combining Eqs. (28), (26) and (29) leads to

$$\{F\} = [T]\{U\} + [C^I][L]^{-1}(P_i\} \tag{30}$$

where $[T] \equiv i\omega[C^I][Z][S]$ is typically a fully populated matrix that relates the interaction forces to the boundary displacement field.

For steady state harmonic motion, all response qualities are proportional to $\exp(+i\omega t)$. The corresponding equation of motion for the structure (Eq. (3)) becomes

$$\left(-\omega^2[M] + i\omega[C] + [K]\right)\{U\}^\circ = -\{F\}^\circ + \{F_E\}^\circ \tag{31}$$

where $e^{i\omega t}$ has been canceled out on both sides of the equation and the forcing terms have been regrouped as $\{F\} = \{F_I\} + \{F_I\}$ where $\{F_E\}$ denotes all driving terms on the structure (except for the incident fluid pressure term). Thus, substituting Eq. (30) into Eq. (31) results in the relation

$$[V]\{U\}^\circ = \{F_A\}^\circ \tag{32}$$

where $[V] \equiv -\omega^2[M] + i\omega[C] + [K] + [T]$ and $\{F_A\}^\circ = \{F_E\}^\circ - [C^I][L]^{-1}\{P_i\}$

It is to be noted that Eq. (31) contains matrices that are the size of the entire structure whereas the matrix size in Eq. (30) is only a size corresponding to the nodes in contact with the fluid. Thus, when substituting Eq. (30) into Eq. (31), allowances must be made in filling out the [T] and product matrix $[C^I][L]^{-1}$ with zeros in the appropriate place to account for the matrix size mismatch.

Formally one may now state the solution to the interaction problem as finding the inverse of the highly populated [V] matrix. Thus,

$$\{U\}^\circ = [V]^{-1}\{F_A\}^\circ \tag{33}$$

Once $\{U\}^\circ$ is determined all other response quantities can be routinely computed. Substituting the solution $\{U\}^\circ$ into Eq. (24) and Eq. (29) and then Eq. (29) into Eq. (26) provides the total pressure, $\{P\}$, at the interface. Then substituting the surface pressure and surface velocity into the exterior form of the Helmholtz integral, [16], the pressure in any far field point in the media can easily be computed. Premultiplying the surface motion, $\{U\}^\circ$, by the individual (unassembled) stiffness matrix for each element produces the individual structural nodal forces which in turn can be converted to element stresses.

For large size problems, the nonsymmetry and highly populated form of the complex [V] matrix makes its inversion somewhat of a problem when [V] is large. In some situations, [V] is ill-conditioned for certain frequency ranges due to the presence of large size [K] terms in the [V] expression in comparison to the rest of the terms comprising [V]. To get around these problems, an alternate modal analysis approach is sometimes taken [14, 21].

For the modal approach, let $[\phi]$ be the NxM matrix of M undamped, in vacuo modes of the structural vibrations having N degrees of freedom, thus

$$[\phi] = \left[\{\psi_1\}, \{\psi_2\}, \ldots \{\psi_M\}\right] \tag{34}$$

where $(\psi_m(\bar{x}))$ is the m^{th} mode column vector which is normalized to the MxM unit identity matrix [I] such that

$$[\phi]^T[M][\phi] = [I] \tag{35}$$

The modes $[\phi]$ have the property that

$$[\phi]^T[K][\phi] = [\lambda] \tag{37}$$

where $[\lambda]$ is a MxM diagonal eigenvalue matrix whose non-zero elements are the squares of the natural frequencies (rad/sec) of the structure. The displacement field can be expressed in terms of the modes by the relation

$$\{U\}^\bullet = [\phi]\{Q\}^\bullet \tag{38}$$

which is simply Eq. (14) in matrix form with $e^{i\omega t}$ factored out. Next, upon substituting Eq. (38) into Eq. (32) and premultiplying the result by $[\phi]^T$ one obtains

$$[\phi]^T[V][\phi]\{Q\}^\bullet = [\phi]^T\{F_A\}^\bullet \tag{39}$$

which can be rewritten in short notation as

$$[\bar{V}]\{Q\}^\bullet = \{F_G\}^\bullet \tag{40}$$

where

$$[\bar{V}] \equiv -\omega^2[I] + [\lambda] + [\phi]^T\left(i\omega[C] + [T]\right)[\phi] \tag{41}$$

and

$$\{F_G\}^\bullet \equiv [\phi]^T\{F_A\}^\bullet \tag{42}$$

Generally the MxM $[\bar{V}]$ matrix is complex, nonsymmetric and only under special situations [1] is the $[\bar{V}]$ matrix fully diagonal (note only the first two contributions to Eq. (41) are diagonal). When $[\bar{V}]$ is fully diagonal, its inversion is trivial, however, the general case must usually be considered where one is faced with the inversion of the $[\bar{V}]$ matrix in order to solve the system of equations defined by Eq. (40). Formally then, the solution to the fluid structure interaction problem can be written as

$$\{Q\}^\bullet = [\bar{V}]^{-1}\{F_G\}^\bullet \tag{43}$$

where we have traded having to invert a NxN [V] matrix in the direct approach for having to invert a MxM $[\bar{V}]$ matrix in the modal approach. Strictly speaking, there is one mode shape for each degree of freedom, consequently if M is set equal to N, one is right back where one started in being faced with the in-

version of a NxN complex matrix. However, one can usually judiciously relate the important modes of vibrations based on certain symmetries of loading or based on the customary omission of the higher modes of vibration. After the selection process, one is usually left with a $[\bar{V}]$ matrix that is substantially smaller in size than the original [V] matrix encountered in the direct approach.

Fluid structure interaction approximations (source method). So far, the development of steady state methodology of unbounded fluid regions has been based on the Helmholtz integral equation. Another approach that is often used is the simple source formulation [28-32]. For simplicity, we consider a radiation problem only; the reader is referred to [29] for an extension of the source method to include scattering. The starting point of this method is to represent the pressure in the fluid region surrounding the structure by the expression

$$p(\bar{x}) = i\omega\rho 4\pi \int_S \sigma(\bar{y})G(\bar{x},\bar{y})dS(\bar{y}) \qquad \bar{x}\epsilon R \tag{44}$$

where $\sigma(\bar{y})$ is the source density function and $G(\bar{x},\bar{y})$ is given by Eq. (23). The term $|\bar{x}-\bar{y}|$ is the distance between a point in the fluid media, $x\epsilon R$, and a point on the fluid structure interface $y\epsilon S$. Properly employing the boundary condition that the normal gradient of pressure is equal to $-i\omega\rho$ times the normal velocity at any point $y\epsilon S$ leads to the following integral equation for point $x\epsilon S$

$$w(\bar{x}) = 2\pi\sigma(\bar{x}) - \int_S \sigma(\bar{y}) \frac{\partial G(\bar{x},\bar{y})}{\partial n(\bar{x})} dS(\bar{y}) \qquad \bar{x}\epsilon S \tag{45}$$

Note that here the gradient on G is with respect to the normal with a live variable $\bar{x}$ as opposed to the Helmholtz surface integral, Eq. (22), where the gradient is on the normal with dummy index $\bar{y}$. The surface integral in Eq. (45) is improper and is evaluated as

$$\lim_{\Delta S \to 0} \int_{S-\Delta s} \sigma(\bar{y}) \frac{\partial G(\bar{x},\bar{y})}{\partial n(\bar{x})} dS(\bar{x})$$

where Δs surrounds the point $\bar{x}\epsilon S$.

An approximate representation of Eq. (45) is often obtained by subdividing the surface into $\tilde{N}$ subdivisions over each of which the source density is assumed to be constant. By allowing $\bar{x}$ in Eq. (45) to take an $\tilde{N}$ different values, $x\epsilon S$, (each of which is a reference point on one of the $\tilde{N}$ subdivisions) Eq. (45) can be written in matrix form by the following set of linear algebraic equations

$$[\tilde{A}]\{\sigma\} = \{W\} \tag{46}$$

where $[\tilde{A}]$ is a known $\tilde{N}x\tilde{N}$ matrix resulting from the evaluation of Eq. (45), $\{\sigma\}$ is an $\tilde{N}x1$ column vector of unknown constant source values, and $\{W\}$ is a $\tilde{N}x1$ column vector of normal surface velocities. Solving Eq. (46) for $\{\sigma\}$

$$\{\sigma\} = [\tilde{A}]^{-1}\{W\} \tag{47}$$

and substituting Eq. (47) into a discretized version of Eq. (44) evaluated for $x \in S$ leads to the result

$$\{P\} = [\tilde{Z}(\omega)]\{W\} \tag{48}$$

where $\{P\}$ is a column vector of discretized pressures, and $[\tilde{Z}(\omega)]$ is a known matrix resulting from the discrete evaluation of Eq. (44). For scattering problems the development is very similar except that a known additional term (resulting from the incident pressure) is added to the right of Eq. (48). In comparing Eq. (26) to Eq. (48), one notes that from this point on, the procedure required to convert the surface pressure-surface velocity relationship given by Eq. (48) into the final matrix equation relating surface displacements to surface nodal forces (Eq. (33) or Eq. (43)) would follow in a similar manner, hence, it will not be repeated here. As pointed out in [19] and [24], the source method fails to have a solution (except for certain special velocity distributions) when the driving frequency ω is at or near one of the characteristic wave numbers of the fluid region alone. One might be fortunate in that the driving frequencies of interest lie in between the forbidden frequencies, however, the likelihood of being near one of the characteristic values gets greater with higher driving frequencies where the spacing between forbidden frequencies becomes very tight. The advantage of using the Helmholtz integral formulation over the source method lies in the fact that at least for the former method, a procedure exists, as discussed in the previous section, for eliminating the problem.

Steady State Methodology (Partially Bounded Region)

When the structure is completely surrounded by an unbounded fluid in all directions, the application of either the Helmholtz integral method or the source distribution method covered in the previous section can be used to solve the fluid structure interaction problem without having to model the fluid region with finite elements. When only part of the region is unbounded, an alternate approach must be taken. A good example of encountering this situation is the problem of finding the fluid structure interaction at a dam interface during an earthquake situation. The fluid region can be considered, for all practical purposes, as being infinite in the horizontal direction but finite in the vertical direction.

The method starts with the basic structural equation (31) in steady state form, however, a different means must be applied for relating the total interaction nodal forces $\{F\}^*$ with the nodal displacement, i.e. the counterpart of Eq. (30) in the earlier bounded region development must be found. For illustrative purposes, consider the interaction problem involved in a dam problem where the part of the dam is excited with a unit horizontal acceleration of the form $1e^{i\omega t}$. The following development is taken from [21]. Employing a modal solution of the type considered in the previous section allows one to express, with the aid of Eq. (38), the absolute horizontal acceleration of the upstream dam face (the fluid-solid interface) in the form

$$\{\ddot{U}(t)\} = \{1\} + [\phi]\{\ddot{Q}\}^*)e^{i\omega t} \tag{49}$$

where we are using the same notation as described in the previous modal analysis section. For the purpose of obtaining an analytical solution for the

field equations, one temporarily rewrites Eq. (49) in its continuous counterpart, namely

$$\ddot{u}(o,y,t) = (1 + \sum \phi_m(y)\{\ddot{Q}\}^\circ_m)e^{i\omega t} \tag{50}$$

where the continuous arguments of the horizontal fluid field displacement are u(x,y,t) where x and y denote a right-handed Cartesian coordinate system centered at the base upstream corner of the dam (all the fluid is located to the left of the vertical y axis). For a dam (with a straight fluid structure interface), the boundary conditions for the Eq. (1) (in 2D) are

$$\begin{gathered} \frac{\partial p}{\partial y}(x,o,t) = 0 \\ p(x,H,t) = 0 \\ \frac{\partial p}{\partial x}(0,y,t) = -\tilde{M}(1 + \sum_{m=1}^{M} \phi_m(y)\{\ddot{Q}\}^\circ_m)e^{i\omega t} \end{gathered} \tag{51}$$

where $\tilde{M}$ is the mass of the dam, and H is the height of the dam. The first boundary condition states that the vertical fluid motion at the interface is zero, the second states that the surface fluid pressure is zero and the third that the horizontal notion of the dam is equal to the horizontal motion of the fluid. The solution to this problem has been obtained exactly which we denote as $p(x,y,\omega)$. Next, by evaluating $p(x,y,\omega)$ at x = 0, and converting it into equivalent nodal forces (analogous to Eq. (28)) one arrives at the relation

$$\{F\} = ([\tilde{T}]\{Q\}^\circ + \{\tilde{B}\})e^{i\omega t} \tag{52}$$

where the $[\tilde{T}]$ and $\{\tilde{B}\}$ matrices are known arrays. Finally, substituting the coefficient of $e^{i\omega t}$ in Eq. (52) into the expression for $\{F\}$ in Eq. (31), substituting Eq. (38) into Eq. (31) and multiplying the resulting equation by $[\phi]^T$ results in an expression for the unknown $\{Q\}^\circ$ of the form

$$[\hat{V}]\{Q\}^\circ = \{\hat{F}_G\}^\circ \tag{53}$$

where $[\hat{V}] = -\omega^2[I] + [\lambda] + [\phi]^T(i\omega[C] + [\tilde{T}])$ and $\{F_G\}^\circ = [\phi]^T(\{F_E\}^\circ - \{\tilde{B}\})$

Upon inverting the $[\hat{V}]$ matrix in Eq. (53) to find the generalized displacement $\{Q\}^\circ$, one can consider the problem as being solved.

The procedure described above depended upon solving the field equations for the fluid exactly. In situations where the fluid-solid interface does not have nice mathematically convenient geometries (e.g. planes, cylinders, spheres) one may not be able to obtain an exact solution. It appears that one should be able to solve the fluid field equations by a numerical technique to obtain the surface pressure response in terms of the surface motion. Upon doing this, one would proceed in a similar manner as described above and strive to obtain a relation like Eq. (52). Development of the final matrix simultaneous equations for the unknown $\{Q\}^\circ$ would follow in exactly the same manner once Eq. (52) is defined.

Finite Element Structure-Finite Element Fluid

With this method, both the fluid and the structure are represented with finite elements. The general area is broken up into two types of solutions, the first of which employs displacements in the structure and pressure in the fluid as the set of unknowns in the problem formulation. The second solution technique employs displacements in both the structure and fluid as the unknowns in the problem formulation. The finite element structure-finite element fluid approach to solving problems is readily adaptable to both steady state or transient type problems. For problems having finite fluid boundaries (e.g. Fig. 3c), application of the proper boundary conditions at the finite fluid boundaries is straightforward. However, for problems involving unbounded fluid domains, one is faced with having to make a mathematical cut in the fluid domain at some finite distance surrounding the structure since, of course, one cannot handle an infinite number of fluid elements. By enforcing the appropriate boundary condition at the mathematical cut, the infinite domain can usually be handled. This point is covered in detail later in this section.

Pressure Finite Elements

The finite element description of the structure is again provided by Eq. (3) where the unknown boundary interaction force, $\{F_I\}$, is to be determined. The next major task is to obtain another set of equations involving $\{F_I\}$ such that when they are considered in conjunction with Eq. (3), a balanced set of simultaneous differential equations can be developed.

Transient methodology. The missing equation needed to complement Eq. (3) can be derived from the wave equation (1). Upon subdividing the fluid region into a set of finite element regions and applying the standard methodology for reducing a continuous partial differential equation into discrete finite element form (e.g. see [34, 35]), one easily arrives at the discrete form of Eq. (1), namely

$$[H^*]\{P\} + [G^*]\ddot{\{P\}} + \{f\} = 0 \tag{54}$$

where $[H^*]$ and $[G^*]$ are matrices resulting from applying the usual finite element procedure (they are functions of the fluid topology and the fluid dilatational wave speed c) and the forcing function vector $\{f\}$ does not contain any contribution from volume integrals (as in the determination of $[H^*]$ and $[G^*]$) but is entirely determined from the fluid boundaries, i.e. boundaries formed by the fluid-solid interface and any mathematical cuts in the fluid field. On the fluid boundaries either p is specified or the boundaries are solid and subject to the applied motion

$$\frac{\partial p}{\partial n} = -\rho \frac{\partial^2}{\partial t^2} \{U_n\} \tag{55}$$

where n is a unit normal to the surface and $\{U_n\}$ is the corresponding normal displacement.

The fluid structure interface motion is prescribed by the movement of the structure, consequently one can express the normal fluid motion in terms of the structural nodal displacement, $\{U\}$, through the relation

$$\{U_n\} = [\tilde{N}]\{U\} \tag{56}$$

where $[\tilde{N}]$ is determined from the appropriate shape functions. The forcing function $\{f\}$ can be related to the structure surface motion using Eqs. (55) and (56) to obtain

$$\{f\} = [S]\{\ddot{U}\} \tag{57}$$

where $[F] = [N]^T \rho \int_{\tilde{S}} [\tilde{N}] df$ and $\{P\} = [N]\{p\}$ where $[N]$ is defined as the relationship between the continuous pressure field within the element and the discrete value of p at the nodal points. Finally, the fluid interface pressure $\{P\}$ can be related to the equivalent interaction nodal force values, $\{F_I\}$, by the principal of virtual work, this leads to the expression

$$\{F_I\} = \int_S [\tilde{N}]^T P dS = \frac{1}{\rho}[S]^T \{P\} \tag{58}$$

Finally, considering Eqs. (3), (54), (57) and (58) simultaneously result in the partitioned matrix equation

$$\left[\begin{array}{c|c} [M] & [o] \\ \hline -[S] & [G^*] \end{array}\right] \left\{\begin{array}{c} \{\ddot{U}\} \\ \hline \{\ddot{P}\} \end{array}\right\} + \left[\begin{array}{c|c} [C] & [o] \\ \hline [o] & [o] \end{array}\right] \left\{\begin{array}{c} \{\dot{U}\} \\ \hline \{\dot{P}\} \end{array}\right\} + \left[\begin{array}{c|c} [K] & \frac{1}{\rho}[S]^T \\ \hline [o] & [H^*] \end{array}\right] \left\{\begin{array}{c} \{U\} \\ \hline \{P\} \end{array}\right\} = \left\{\begin{array}{c} \{F_A\} \\ \hline \{o\} \end{array}\right\} \tag{59}$$

Once the equation is written in the above form, from a mathematical point of view it is possible to view the partitioned unknown vector comprised of $\{U\}$ and $\{P\}$ as a single unknown vector, say, $\{\underline{U}\}$. Thus, Eq. (59) can be rewritten as

$$[\underline{M}]\{\ddot{\underline{U}}\} + [\underline{C}]\{\dot{\underline{U}}\} + [\underline{K}]\{\underline{U}\} = \{\underline{F}_A\} \tag{60}$$

where $\{\underline{U}\} = [\{U\},\{P\}]^T$ and the $[\underline{M}]$, $[\underline{C}]$, $[\underline{K}]$ arrays are the coefficients of $\{\ddot{\underline{U}}\}$, $\{\dot{\underline{U}}\}$ and $\{\underline{U}\}$ in the above Eq. (59) respectively. Unfortunately, the solution to Eq. (59) is not as convenient as the usual structures type problem because in the case of Eq. (60), the coefficient matrices of Eq. (60) are not symmetric and banded. Consequently, any canned solution techniques that depend on these properties cannot be used directly in Eq. (60) unless some sort of transformation can be made to recover these desired matrix properties.

Typically, equations like those in Eq. (60) are integrated by, say, the Newmark-β method [36] (or by similar methods, e.g. see chapter on transient solutions in [2]). Unfortunately, implementation of these techniques result in spurious oscillations of the solution. Methods employing some form of damping have been used to eliminate these oscillations, however, the artificial introduction of damping results in unnatural dissipation of energy and depends heavily on an experience factor when judging the amount of damping to introduce. Reference [37] has applied digital filter techniques in solving second order simultaneous differential equations of the Eq. (60) type and has obtained results which substantially reduce the spurious oscillations. Using the method requires some a priori knowledge regarding the frequency content of the spurious oscillations. In other words, one does not want to accidently filter out some real response oscillation.

There are existing computer programs (to be discussed later) that are set up directly to solve the formulation represented by the solution to Eq. (60). Should these programs be either unavailable, undesirable, or uninstallable on the user's existing computer facility, there is still a method that can be used to implement the approach implied by Eqs. (60). The approach is presented in detail in [38] and will only be briefly discussed here. The philosophy of the method is to dummy the construction of the stiffness and mass matrices of conventional displacement type finite elements so that only one displacement component is active (the remaining ones are zero) and further, the remaining nonzero component plays the role of pressure. This is accomplished by setting

$$u_1 = p \quad u_2, u_3 = 0 \tag{61}$$

$$\text{and } E_{14} = E_{16} = E_{46} = 0; \; E_{11} = E_{44} = E_{66} = \rho c^2 \tag{62}$$

where (u_1, u_2, u_3) are the three displacement components of a node and the E_{ij} values are components of a 6x6 elasticity matrix defined by

$$\{\sigma\} = [E]\{\epsilon\} \tag{63}$$

where $\{\sigma\}$ and $\{\epsilon\}$ are the corresponding stress and strain vectors. The other 15 E_{ij} constants, not defined by Eqs. (62) can be arbitrarily chosen; however, it is convenient to select them so that [E] is invariant under a coordinate system rotation thus making [E] an isotropic matrix. A necessary and sufficient condition for [E] to be isotropic is that it has the general form

$$[E] = \begin{bmatrix} \lambda+2\mu & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda+2\mu & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda+2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix} \tag{64}$$

where λ and μ are the Lamè constants. The only values of λ and μ satisfying both Eq. (64) and Eq. (62) are

$$\lambda = -\rho c^2, \; \mu = \rho c^2 \tag{65}$$

Employing an isotropic [E] frees one from worrying about using finite elements whose material matrix is based on some local element coordinate system. Thus, for ordinary structures programs (not necessarily having any explicit fluid elements) one can construct the diagonal matrices of Eq. (59) directly; the structural matrices [M], [K] and [C] are formed the usual way and [G*], [H*] are formed as though they were solid elements having a material constant matrix [E] and all u_2, u_3 displacement components constrained out. The non-

zero off diagonal matrices of Eq. (59) (i.e. $-[S]$, and $\frac{1}{\rho}[S]^T$) have to be introduced by hand (i.e. most programs, NASTRAN for one, have the generality to introduce additions to the matrices that have been automatically built by the program). The construction of these diagonal matrices come through the enforcement of the fluid structure interface boundary condition denoted by Eq. (55). A discrete evaluation of the continuous expression given by Eq. (55) leads to the evaluation of the off diagonal matrices. The reader is referred to [38] for details; briefly, the $-[S]$ matrix is constructed by inserting $-(\rho c)^2 A^*$ in the row corresponding to each interface {P} variable and the column corresponding to the associated structural normal displacement, where A^* is an area corresponding to the conversion of the 1 direction surface traction into an equivalent lumped force in the 1 direction. Further, the $[S]^T/\rho$ matrix can be constructed by inserting a $+A^*$ in the row corresponding to each surface normal displacement and the column corresponding to the associated {P}. For a problem of any substantial size, one should write a small preprocessing program to automatically generate the input data cards required to install the off diagonal matrices.

Steady State Methodology

Extension of the solution to Eq. (60) for steady state problems is handled in a straightforward manner. All response quantities $\{\underline{U}\}$ and $\{\underline{F}_A\}$ are assumed to vary timewise as

$$\{\underline{U}\} = \{U\}^{\circ} e^{i\omega t} \quad \text{and} \quad \{\underline{F}_A\} = \{F_A\}^{\circ} e^{i\omega t}$$

Substituting the above expression into Eq. (60) leads to

$$[\underline{V}]\{U\}^{\circ} = \{\underline{F}_A\}^{\circ} \tag{67}$$

where $[\underline{V}] = -[\underline{M}]\omega^2 + i\omega[\underline{C}] + [\underline{K}]$

the solution to Eq. (67) is therefore $\{\underline{U}\}^{\circ} = [\underline{V}]^{-1}\{\underline{F}_A\}^{\circ}$

where the complex, nonsymmetric matrix, [V], must be inverted to obtain the final result.

Displacement Finite Elements

All the methods considered up to this point have always worked with the governing equations of motion of the form given by Eq. (3) where the interaction force, $\{F_I\}$, is treated as an unknown force that somehow must eventually be related to the structural response motion $\{U(t)\}$. In this approach, the fluid region surrounding the structure is actually considered to be part of the structure in the sense that when the stiffness matrix is assembled for the entire system (structure and surrounding fluid) no special distinction is made between solid or fluid elements other than the fact that the material constants used to define the element fluid stiffness are selected in a particular manner. Thus, the $\{F_I\}$ term does not appear explicitly in Eq. (3) but rather appears implicitly within the $[K]\{U\}$ expression. For the general 3-D type problem, there are three degrees of freedom per node (3 displacement components) as opposed to the pressure type finite elements considered in the prior section

which had only one unknown per element, namely the pressure at the node. It appears that Miller, Constantino and Fey, all formerly of the IIT Research Institute, were the first to employ this approach to solving fluid interaction problems in 1965. Later, applications employing this approach were made in [5, 40, 44] on shock loading problems.

The development of these type elements is straightforward. One starts by recognizing the fact that the dynamic elasticity field equations [42] will reduce to a displacement form of the wave equation if one lets the shear modulus (or equivalently the Lamè constant μ) go to zero. That is the solid elasticity field equations reduce to

$$c^2\nabla^2 u_i = \ddot{u}_i; \quad c^2 = \lambda/\rho \tag{68}$$

and the pressure is related to the displacement vector by

$$p = -\lambda\nabla \cdot u_i \tag{69}$$

where λ is the Lamé constant and u_i is the displacement vector.

Note that operating on Eq. (68) with the divergence $\nabla.(\ )$ operator and employing Eq. (69) reduces Eq. (68) to the standard pressure wave equation

$$c^2\nabla^2 p = \ddot{p}$$

Actually the displacement elements are used in conjunction with Eq. (68). The relationship Eq. (68) will be enforced provided that in the general stress-strain law, Eq. (64) one lets $\mu \to 0$ and $\lambda \to \rho\, c^2$ where ρ is the fluid density, and c is the fluid wave speed.

The reduction of Eq. (64) described above is equivalent to the approach taken in [5]. We point out that the parallel development in [5] has several sign errors, namely on page 74 of [5], replace -k with +k on the third line from the bottom, replace k with -k in Eq. (3) and finally replace -k with +k at the bottom of page 75. The variable k in [5] is equivalent to the Lamè constant λ here.

Thus, the local element stiffness and mass matrices for each fluid element can be created from ordinary solid (either one, two or three dimensional) linear elastic elements by applying the above limiting value of μ. Elements created in this sort of artificial manner have been referred to in [5] as "mock" fluid finite elements. The stiffness of the remaining structural elements are created in the usual way. Finally, the total stiffness and mass matrices of the entire system of elements are assembled in the usual manner dictated by the finite element method.

With all the fluid structure interaction methods discussed until now, a boundary condition at the fluid structure interface that is used required that the normal motion of the structure equals the normal motion of the fluid. To achieve this affect with the mock elements, a double set of nodes must be applied at the fluid structure interface. One set of nodes belongs to the fluid and the other to the solid. The motion of the two sets of nodes are assumed to be independent of each other in the direction tangental to the surface (i.e. slip is allowed in the tangental direction) but are forced equal in the normal direction. Many existing general prupose programs have multi-point constraint capabilities which easily permit the enforcement of this condition. For those programs that do not have multipoint constraints all one can do (without program modification) is to employ a common set of nodes at the interface (which automatically implies both the tangental and normal motions at the fluid solid interface are set equal). Since the fluid elements cannot sustain a shear stress (by virtue of setting $\mu = 0$) it appears that the tangental motions would not endure any additional tangental forces on the

fluid elements; thus, not strictly enforcing the slip condition may not seriously affect the solution. This author has made trial runs with and without enforcing the slip condition and found only small differences in the results for a limited number of numerical experiments; the generality of this is uncertain.

Transient methodology. Upon forming the stiffness and mass matrices for the structure and mock fluid elements by the procedure described above, one arrives at Eq. (3) (with the omission of the $\{F_I\}$ term) for the equations of motion of the fluid structure system. The equations of motion can be integrated by standard numerical integration techniques [36, 37, 45]. Application of mock elements is straightforward for scattering or radiation problems with finite boundaries. With infinite boundaries, special considerations must be made and are treated is a separate section later.

Steady state methodology. The solution for steady state problems is handled in the usual manner. Substitution of all response and driving quantities proportional to exp(iωt) leads to the form of equations like Eq. (67) except that the lower bars on the variable are omitted (because here pressures are not explicitly part of the unknown solution vector $\{U\}^o$). The problem solution involves inverting a banded, symmetric, complex [V] matrix.

Comparison of Pressure and Mock Elements

The greatest advantage of applying the mock fluid elements is that they can be employed to instantaneously convert an ordinary dynamic structures code (one not necessarily having any direct fluid structure interaction capability but presumably one having ordinary linear elastic solid elements, 2-D or 3-D, as part of the admissible set of finite elements available to the user) into a fluid structure interaction computer program with absolutely no computer program rewriting! In contrast, to implement the pressure element approach, one would have to do one of the following: 1) have an existing program with this capability already programmed; 2) reprogram an existing structures code; 3) employ the dummy displacement vector method of reference [38]. Even if one employs option 3 above, one is still faced with requiring the program to operate on nonsymmetric matrices whereas most structure codes (NASTRAN excluded) have their equation solving tools geared only toward symmetric matrices.

The disadvantage of employing mock elements is that there are i unknowns per node (i = number of displacement degrees of freedom; = 1, 2, or 3) as opposed to 1 in the pressure element method. This disadvantage may not be as negative as it appears when one remembers that when employing mock elements the matrix operations are more efficient for banded symmetric matrices than for highly populated unsymmetric ones. Thus, although there are more unknowns in the method employing mock elements, the final solution running time may be comparable. This writer is not aware of any published work comparing running times of the same problem solved by both approaches.

Finally, one additional advantage of employing the mock fluid elements is that a dynamic nonlinear structures program (e.g. one that considers plasticity in the structure) can easily be adapted to solve fluid structure interaction problems with no program rewriting.

Infinite Region Fluid Boundaries

Special considerations must be taken for problems involving infinite fluid boundaries. The manner in which the infinite boundary is handled depends upon the type of problem being solved.

Temporal truncation. This is the most straightforward approach and is readily adaptable to both the pressure or mock element type transient solutions. In construction of the finite element mesh, one models the fluid surrounding the solid cutout to, say, 2 structure lengths. For scattering or radiation type problems, one can take advantage of the fact that the continuous equations are hyperbolic in nature. Thus for, say, a radiation problem, the solution will be such that the response in front of a radiating wave is zero, thus the problem does not know a boundary to the mesh even exists until the radiating wave actually gets there (due to the discretization of the problem, the governing equations do not exactly behave like hyperbolic equations, but the idea of traveling waves are still roughly approximated by the governing discretized differential equations). Thus, the solution to the problem can be obtained in the same manner as a finite boundary case, except that the solution response must be truncated at the time when the radiated wave reaches the mesh boundary.

Scattering problems can be treated in a similar manner. The free-field incident wave solution starts the problem, i.e. the initial conditions for the problem solution are obtained by setting the response field, behind the incident wave, equal to the free-field solution. The equations of motion are integrated in time in the usual manner but must be truncated when scattered waves off the structure reach the finite fluid mesh boundary.

Absorbing boundary. Another approach to handling the infinite boundary problem is through the introduction of wave absorbing boundaries. This alternate procedure is especially important for steady state problems which do not have the alternative of truncating the time response solution. The idea behind this method is to again model the fluid field out to a certain number of structure lengths (as in the temporal truncation approach), however, in this approach a special boundary condition is applied which relates the problem response variables at the mathematical cut in the fluid field. The exact manner in which this is implemented is different for the pressure and mock element, and thus far must be treated separately.

Pressure Elements (Radiation Problems)

For radiation problems, [35], applies a boundary condition at the mathematical cut given by the relation

$$\frac{\partial p}{\partial n} = -\frac{1}{c}\frac{\partial p}{\partial t} \tag{70}$$

and is strictly valid for plane waves. If the boundary is placed "far enough" away, radiated components will appear to be plane (from an impedance point of view) thus the Eq. (70) can be applied in those situations as well. According to the original source reference, [35], "In the real situation, a test must be made to determine if the infinite boundary has been placed far enough. This is the case if the portion of the boundary is far enough not to affect the results at the focus of interest and, generally, two or more trial solutions should be attempted." Omitting the development details, the enforcement of Eq. (70) in discrete form results in the addition of a term, $[D^*]\{P\}$ in Eq. (54), where $[D^*]$ is assembled from

$$[D^*] = \frac{1}{c}\sum_{s}\int [N][N]dS$$

The final equations of motion are the same as Eq. (59) except that the lower left partition of the $\{\underline{U}\}$ vector has $[D^*]$ in place of $[0]$.

Employing this approach to transient problems allows one to continue the radiation solution beyond the point where the radiation waves interact with the boundary. Employing it to steady state problems automatically gives the proper (within the accuracy of the boundary plane wave approximation) boundary forces simulating the steady state forces that would have been there had the fluid media been extended to infinity.

Mock Elements (Radiation Problems)

The information discussed in the next two sections on mock elements is based on work done by the author at NUSC.

A relationship consistant with Eq. (70) can be written for the boundary mock elements existing at the mathematical cut in the fluid media, namely that the radiated boundary pressure, p, is related to the normal velocity by the relation

$$p = \rho c w_n \tag{71}$$

which is exactly true for plane waves and asymptomatically true for cylindrical and spherical waves. For convenience, the following discussion on wave absorbing boundaries for mock elements will assume that the fluid outer boundaries are straight and parallel with the global Cartesian coordinate system of the problem. Thus, one can drop the normality subscript in Eq. (71) when referring to the outer boundary. The equations of motion for the problem are still given by Eq. (3) with F_I taken to be zero (since the fluid structure interaction is implicitly taken into account in the assembled stiffness) and with

$$\{F\}_A = -\{F_b\} + \{F_a\}$$

where $\{F_b\}$ are the outer mesh boundary forces applied at the outer nodal points (due to the pressure existing at the mathematical cut in the fluid) and $\{F_a\}$ are the known applied driving forces acting directly on the structure. The $\{F_b\}$ term has a meaning similar to $\{F_I\}$ except that these forces exist at the mathematical cut in the fluid, not at the fluid structure interface. Upon rewriting Eq. (71) in discrete form, a relationship for relating to the fluid velocity field can be obtained, namely

$$\{F_b\} = [C_b]\{\dot{U}\} \tag{72}$$

where $[C_b]$ is a diagonal matrix with zero diagonal values for non-outer boundary nodal points and value of $\rho c \Delta A_i$ for normal outer boundary nodal d.o.f. where ΔA_i is the pressure-to-force conversion term and corresponds to a segment of surface area at boundary node i. Reference [43] has applied a similar approach to soil structure interaction problems wherein damping coefficients similar to $[C_b]$ were used to absorb radiated dilatational waves in the soil; also, the $[C_b]$ damping matrix employed in [43] considered shear wave absorbing dashpots which have no counterpart for acoustic orientated fluid problems.

In summary, the equations of motion of the system are given by Eq. (3) with the following modifications: 1) setting $\{F_I\} = 0$; 2) replacing $\{F_A\}$ with $\{F_a\}$;and, 3) replacing $[C]$ with $[C] + [C_b]$.

Mock Elements (Scattering Problems)

Scattering problems are handled like radiation problems except that special attention must be given to the incident wave pressure which eventually finds itself as an applied force on the outer fluid mesh boundary. In the equations of motion (3), let $\{F_I\} = 0$ and the applied force vector $\{F_A\}$ is given by

$$\{F_A\} = -\{F_s\} - \{F_i\} + \{F_a\} \tag{73}$$

where $\{F_s\}$ is the component of the mesh boundary force due to the scattered component of pressure, p_s; $\{F_a\}$ is any driving force applied directly on the structure (it is usually zero in most scattering problems) and $\{F_i\}$ is the component of the mesh boundary fource due to the incident pressure p_i (the total mesh boundary force being $\{F_s\} + \{F_i\}$). In view of Eq. (72), the scattered force can be related to the scattered velocity by the relation

$$\{F_s\} = [C_b]\{\dot{U}_s\} = [C_b]\left(\{\dot{U}\} - \{\dot{U}_i\}_n\right) \tag{74}$$

where $\{\dot{U}_i\}$ is the incident velocity <u>normal</u> to the mesh boundary surface node and $\{\dot{U}\}$ is the total velocity (incident plus scattered component). Substituting Eq. (74) into Eq. (73) one obtains

$$\{F_A\} = (-[C_b]\{\dot{U}\} + \{F_a\}) + (+[C_b]\{\dot{U}_i\}_n - \{F_i\}) \tag{75}$$

Finally, substituting Eq. (75) into Eq. (3) (with $\{F_i\} = 0$) the following result is obtained

$$[M]\{\ddot{U}\} + ([C] + [C_b])\{\dot{U}\} + [K]\{U\} = \{F_a\} + \{F_d\} \tag{76}$$

where

$$\{F_d\} = [C_b]\{\dot{U}_i\}_n - \{F_i\} \tag{77}$$

The value of $\{F_d\}$ on the fluid mesh boundary is perhaps best illustrated with a specific example. The specific example not only helps demonstrate the explicit calculation of the $\{F_d\}$ term, but further serves to illustrate the accuracy of the solution against some known reference solution. Consider the two-dimensional problem of an infinitely long cylindrical void existing in a fluid medium, subject to a plane harmonic input wave. The problem is to compute the total pressure amplitude, p_t, at five angular positions (all positions are at 2.125 cavity radii from the cavity center) for a driving frequency of $ka = 2.122$ where a = radius of the cavity and $k = \omega/c$. The formula for an incident wave is given by [3]

$$p_i = p_o e^{i(\omega t - kx)} \tag{78}$$

where $+x$ is the coordinate of the direction of wave propagation. The force vector $\{F_i\}$ can be related to p_i via the area of the fluid mesh surface. For

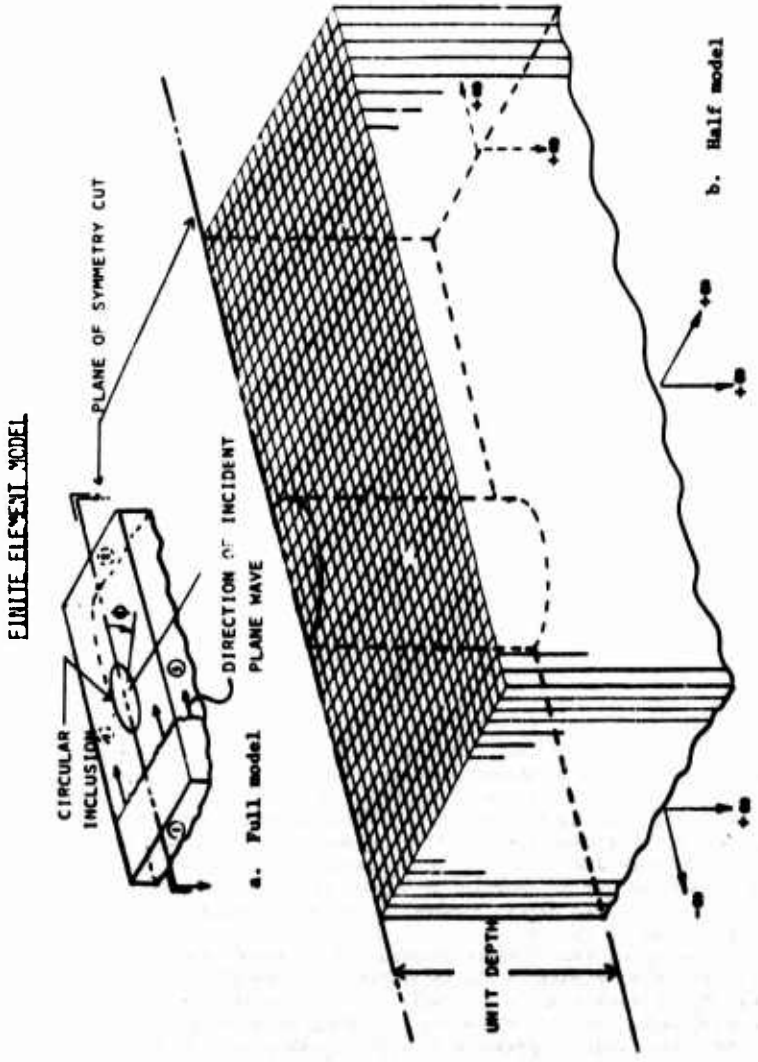

Fig. 5 Finite element model

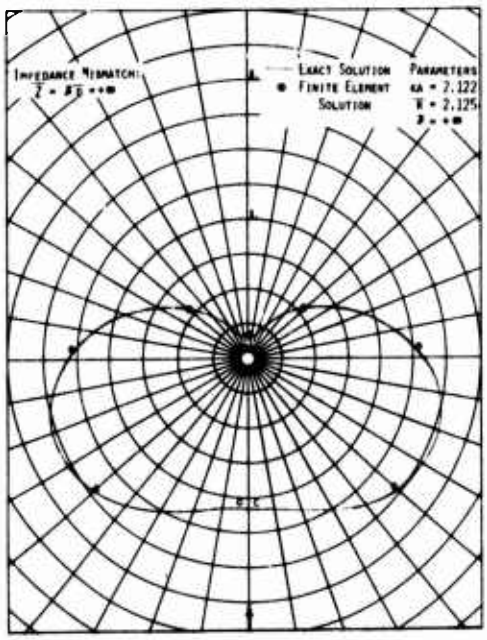

Fig. 6 Total (incident and scattered) steady state pressure response of a cylindrical void in water (intermediate distance from cylinder)

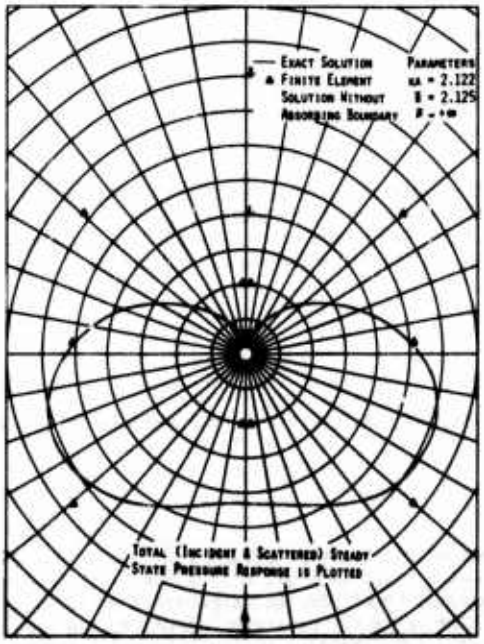

Fig. 7 Comparison of exact and finite element solution without absorbing boundary dashpots (cylindrical void in water)

a unit depth model, the $\{F_i\}$ value can be computed by multiplying p_i times the area, $\Delta L \times 1$, of the mesh where ΔL is the mesh spacing ($\Delta L/2$ is used for the end nodes). This relationship can be used to compute $\{F_i\}$ on all four faces (numbered 1 through 4) of Fig. 5a. Next, it is best to partition Eq. (77) into four segments, one for each of the four faces on the model (do this for six faces for 3D problems), thus, noting that $\{\dot{U}_i\} = 0$ for the component of free-field displacement normal to face 3 and 4, Eq. (77) can be written as

$$\{F_d\} = \begin{Bmatrix} [C_d]^1 \{\dot{U}_i\}_n \\ \hline\hline [C_d]^2 \{\dot{U}_i\}_n \\ \hline\hline \{0\} \\ \hline\hline \{0\} \end{Bmatrix} - \begin{Bmatrix} \{F_i^1\} \\ \hline\hline \{F_i^2\} \\ \hline\hline \{F_i^3\} \\ \hline\hline \{F_i^4\} \end{Bmatrix} \tag{79}$$

But on face 1 $[C_d]^1\{\dot{U}_i\}_n = -\{F_i^1\}$ and on face 2, $[C_d]^1\{\dot{U}_i\}_n = +\{F_i^2\}$ thus Eq. (79) becomes

$$\{F_d\} = -\begin{Bmatrix} 2\{F_i^1\} \\ \hline\hline \{0\} \\ \hline\hline \{F_i^3\} \\ \hline\hline \{F_i^4\} \end{Bmatrix} \tag{80}$$

Upon substituting Eq. (80) into Eq. (77) with $\{F_a\} = 0$ (since there is no load applied other than the incident wave), the governing equations of the system are obtained. Solving the steady state version of Eq. (76) by the steady state methodology described earlier leads to the solution $\{U\}^*$. Converting $\{U\}^*$ into pressure with the unassembled element stiffness matrices provides the solution shown in Fig. 6. The numerical solution is shown in comparison to the exact solution to the wave equation (1). Next consider what happens if the absorbing boundary is omitted and the problem is driven directly with the free field on all four faces. One can obtain completely enormous results as indicated by the solution shown in Fig. 7.

For plane waves, the incident pressure is constant along faces 1 and 2, however it varies spacially in the direction of propagation. An alternate to computing $\{F_i\}$ on face 3 and 4 by simply setting the pressure at a node equal to a constant value over 1/2 the element surface neighboring the node is to convert the actual spacial pressure variation into an equivalent set of "consistent" driving forces by the principal of virtual work. This is a standard technique and can be found in any modern text on finite elements [34]. Still another way is to run the free-field incident pressure through a mesh (with the structure region temporarily filled with fluid) and applying a zero normal displacement condition on faces 3 and 4. The resulting reaction forces at the node will give the proper $\{F_i\}$ value to use when solving the scattering

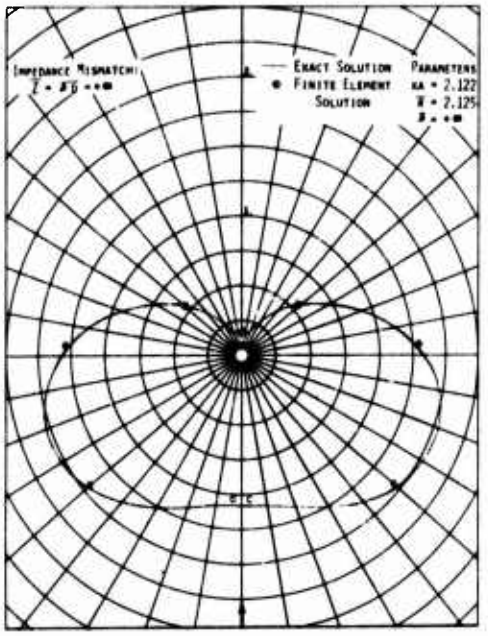

Fig. 6 Total (incident and scattered) steady state pressure response of a cylindrical void in water (intermediate distance from cylinder)

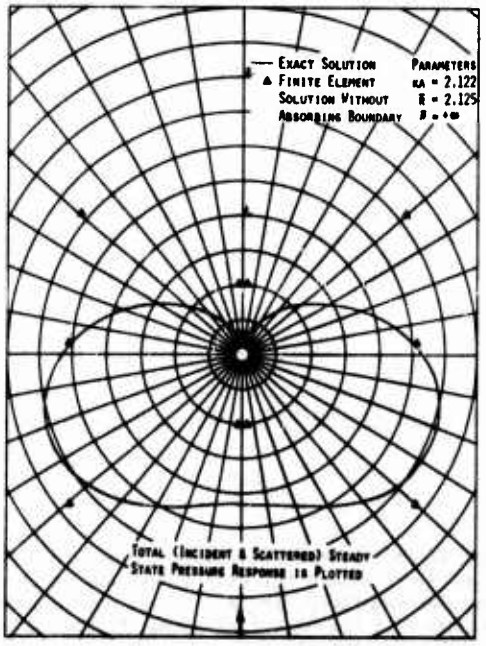

Fig. 7 Comparison of exact and finite element solution without absorbing boundary dashpots (cylindrical void in water)

problems. This latter approach also has the advantage of checking out any potential mesh ringing by comparing the response with the exact solution for a plane wave.

Placement of Outer Fluid Boundary (Steady State Problems)

A rough guideline is needed for determining how far the fluid boundary should be placed in order for the plane wave approximation, Eq. (71), to be valid. A steady state solution can be constructed from some distribution of point sources around the structure fluid boundary. For a 3-D problem, a single source will approach (within 98.6%) a plane wave pressure velocity relationship (like Eq. (71)) after moving one wave length away from the source. The percentage quoted refers to the fact that complex impedance ($z = p/w$) is $.986\rho c$. Moving away 1½ wave lengths, this percentage becomes 99.4%. Thus, we are suggesting that the single source decay information can be used to judge the distance to place the absorbing boundary. For 2D problems, a line source emitting cylindrical waves will approach a plane wave boundary condition to within 99.2% for one wave length away and to within 99.7% for 1½ wave lengths away. Thus, it is suggested to place the boundary a distance $\tilde{D}$ away from the structure where

$$\tilde{D} = \alpha\tilde{\lambda} \tag{81}$$

and α is a proportionality constant (e.g. 1.5 for a 99.4% correct plane wave assumption) related to the degree of the plane wave boundary condition assumption, and $\tilde{\lambda}$ is the wave length of the steady state driving frequency in water (i.e. $\tilde{\lambda} = 2\pi c/\omega$).

Next one must consider the size elements to use so that the elements of the mesh do not artificially "ring" at their natural frequencies. To avoid ringing, there exists a minimum element length, Δ_L, that is related to some fraction, β, of the wave length of the driving frequency in the fluid, thus

$$\Delta L = \beta\tilde{\lambda} \tag{82}$$

The value of β will depend on the type of elements being used. For example, if one employs CQDMEM elements of the NASTRAN program, $\beta \simeq 1/6$ to avoid mesh ringing. Modeling the region from the fluid structure interface out to the mathematical cut in the fluid boundary would result in $\tilde{n}$ elements of length Δ_L, thus

$$\tilde{n} = \frac{\tilde{D}}{\Delta L} \tag{83}$$

Substituting Eqs. (81) and (82) into Eq. (83) results in the expression

$$\tilde{n} = \frac{\alpha}{\beta}$$

which is independent of the driving frequency $\tilde{\omega}$. Thus, employing typical values of $\alpha = 1.5$ and $\beta = 1/6$ into Eq. (83) shows, for example, that regardless of the driving frequency magnitude, it is possible to model the fluid field with as few as 9 elements in the direction normal to the surface. In cases where the surface structure elements are coarse, more elements would be needed to blend in the fine surface elements into the coarser field elements.

In closing this section on infinite fluid boundaries, the method employed by [16, 50, 51] is another way to handle the infinite boundary problem for

computational methodology that considers both the structure and fluid to be comprised of finite elements. In this approach, the mathematical cut in the fluid medium is in the form of a sphere whose diameter, $2R_0$, is slightly larger than the largest length dimension of the structure. Pressure type fluid finite elements fill the region between the structure and the outer boundary of the sphere. With the possible exception of ultra-high driving frequencies, the fluid mesh outer boundary is too close to the structure to be able to use any of the wave absorbing techniques discussed earlier. The next key step in the procedure is to represent the remaining fluid beyond the spherical collection of fluid finite elements (i.e. from the sphere radius R_0 out to infinity) with the continuous wave equation (1). The Helmholtz surface integral, Eq. (22), is then employed in a similar manner as discussed in the methodology section, except that in this case, advantage is taken if the fact that an analytical expression for the solution to Helmholtz's surface integral is known which relates the radiated pressure at the radius R_0 of the sphere to the normal velocity at the sphere. As one increases the radius larger and larger, the relationship eventually reduces to Eq. (71) in the limit of increasing R_0. Upon setting up the final equations of motion for the system, one eventually arrives at a set of equations that look very similar to Eq. (59).

Continuum Structure-Continuum Fluid

Unlike the prior sections, here both the structure and the fluid are treated as continuums wherein primarily analytical solutions are employed extensively. Strictly speaking, a shell structure is not a continuum in the pure elasticity sense, however, here plates and shell-like structures treated from a continuous partial differential equation point of view are viewed as continuum structures. Reference [33] is a good example of problems treated from this point of view wherein mainly classical methods of solution involving separation of variables, Laplace transforms, Fourier series expansions, etc. are employed to obtain solution. This review article is orientated more towards computational techniques, therefore a detailed survey of these methods is beyond the scope of this article (see references [66, 67] for a survey of analytical methods). However, later in the computer program survey section existing computer programs that solve fluid structure interaction problems by simply programming existing analytical solutions will be presented in situations where the program scope has enough generality to be of general use to the engineering community.

Finite Difference Structure-Finite Difference Fluid

The methodology in this area is not as commonly employed as the prior section on finite element structure-continuum fluid, particularly when one is interested in an approach that has applicability to a wide class of structures (broaded than just shells) of arbitrary shape. The solution technique presented by [27] considers a prolate spherical shell submerged in an acoustic medium subject to an arbitrary loading in terms of time and space. Briefly, the procedure starts with expressing the acoustic field equations (1) in spheroidal coordinates while suppressing the circumferential coordinates through a Fourier series expansion. The radial coordinate transformation maps the external infinite fluid region into a finite rectangular domain in which the fluid potential is regular. Central differences applied to the spatial variables and backward differences to the time variable provide an unconditionally stable finite difference approximation. The shell analysis is based on a finite difference solution of shells of revolution. The unknown interaction pressure at the shell-fluid interface is expressed in terms of the velocity potential and subsequently introduced into the normal surface loading term of the finite difference expression of the governing system of second order partial differential equations for the shell. At a given time step the normal deflection of the shell is expressible as the sum of a (given surface loading induced) known part and (an acoustic pressure loading induced) unknown part. Finally, the equations resulting from matching the normal shell velocity to the normal

fluid velocity at the fluid-shell interface are considered simultaneously with the finite difference equations over the acoustic fluid field to provide a sufficient set of equations for the determination of the unknown velocity potential on the fluid-solid interface and within the mapped fluid field. The resulting system of equations are banded and solved by the method of successive overrelaxation. Once having solved for the fluid potential, the surface pressure can be determined which in turn can be used to determine any shell response quantity of interest.

This approach has an advantage of not having to employ an approximate interaction law (viz. [6]) to uncouple the fluid field equations, however, one pays the price of having an rxr grid work (and its associated unknown velocity potential) representing the fluid field. This is in contrast to [6] which needs only the surface pressure unknown in the problem formulation. When using this approach care must be taken to select r large enough to obtain a sufficiently accurate solution. The ease of adaptability of the procedure to a more general type structure with an arbitrary shape is not clear. A more general finite difference scheme for the structure and a more general mapping transformation relating the arbitrary shape to a rectangular region must be employed to extend the procedure.

AVAILABLE COMPUTER PROGRAMS

The previous section on solution methodology gives the reader a broad overview of the solution techniques currently being used in the engineering community to solve fluid-structure interaction problems. In this section a list of currently available computer programs is presented (user oriented programs, UOP, and research oriented programs, ROP) whose category descriptions are defined in the introduction to this survey. Many of these programs, (particularly the ROP type) are in a rather rapid state of change, consequently, it is not practical to give finer details of the program's capability such as the maximum number of nodes and elements the program in question can handle. In most cases it is a good idea to contact the program author to be sure one obtains the most up to date documentation that exists.

The computer program summaries will be broken down into the same categories used to describe the solution methodology. In most cases the solution methodology used in the programs to be reviewed herein has been touched on in the previous section on methodology. In the process of describing the solution technique for each of the computer programs below, reference is made to the methodology most closely fitting the one used in the actual program. Any differences in methodology between what was presented earlier and what is actually done in the program should only be minor. The reader interested in all the finer details should refer to the original source material.

Finite Element Structure-Continuum Fluid Programs

1. NAsa STRuctural ANalysis (NASTRAN) Application-1 [46]

Capability: For transient problems, the program has the ability to accept the application of the doubly asymptotic approach for general structures. Implementation of this method does not exist as a fixed format in the program; however, [38, 48] has shown how to dummy the fixed format input to apply this approach. The final equations to be solved in applying the method prescribed by [38] are in a different form than those given by Eqs. (12) and (13). Using the method of [38] in conjunction with NASTRAN is awkward, especially if one does not write a preprocessing program to prepare some of the off diagonal mass and stiffness matrices.

Availability: NASTRAN is currently being managed by the NASTRAN Systems Management Office at NASA Langley Research Center, and the latest official version, Level 15.5, is available from COSMIC. A level 16.0 is going to be available in the very near future, although its initial distribution

will be on a selected user basis. As of this writing no new fluid structure interaction capabilities are planned to be included in level 16.0.

Subjective Comments: Although the documentation of the NASTRAN computer program is complete, employing the program to solve fluid structure interaction problems by the procedure outlined in [38] should be left only to an experienced NASTRAN user with a good background in structural dynamics and acoustics. NASTRAN is certainly a UOP, however, when used for the above application it should be viewed as a ROP.

2. Earthquake Analysis of gravity Dams in doing Hydrodynamic Interaction (EADHI) [47]

Capability: The program is written specifically for the solution to earthquake oriented problems. The dam can be modeled from a set of 2D quadrilateral elements and the fluid is represented as a continuum (i.e., via the solution to Eq. (1)). The solution technique is like that discussed in the methodology section on partially bounded fluid regions. The dam is loaded by vertical and horizontal earthquake motions. The program has the ability to solve the steady state response directly and the transient response indirectly by the method of Fast Fourier Transforms (the FFT approach is also discussed in the solution methodology section).

Availability: Information regarding the acquisition of the program can be achieved by contacting co-author A. K. Chopra at the College of Engineering, University of California, Berkeley, California.

Subjective Comments: The user's manual [47] is well written and appears to be a UOP. Although a listing of the program, along with sample problems, are included in the manual, no details of the inner workings, flow charts, etc., are presented. The program is relatively new, therefore, it has not had much of a chance to be widely used.

3. Earthquake Analysis of axisymmetric Tower structure Surrounded by Water (EATSW) [49]

Capability: The transient response to submerged solid axisymmetric towers excited at their base by prescribed earthquake motions is determined. The structure is built up from solids of revolution elements having an arbitrary quadrilateral cross section. The fluid is modeled as a continuum, however, compressibility of the fluid is omitted (i.e., the bulk modulus, $\lambda \to \infty$, which implied the wave speed, $c \to \infty$). Thus the wave equation (1) reduces to Laplace's equation wherein the time dependent variable does not explicitly appear in the equation of motion for the fluid. In the case where the tower is a straight cylinder, the finite element representation of the structure is coupled to a continuum representation for the fluid by applying exact solutions for the fluid field equations analogous to what was discussed in the methodology section on partially bounded media. When the shape of the structure is not a right circular cylinder, the fluid field is made up of pressure fluid elements derived from an integral functional equivalent to the boundary value problem associated with Laplace's equation.

Availability: See program no. 2.

Subjective Comments: Same as program no. 2 except reference [49] is the user's manual.

4. Underwater Structures Analysis (USA-2)

Capability: All the programs listed in the survey are currently operational with the exception of the program described herein. The proposed program will have the capability of treating a submerged shell-structure of virtually unrestricted geometry and having an off-center internal structure. The system is subject to transient loadings from any direction. The solu-

technique employs the doubly asymptotic fluid structure interaction approximation discussed in the methodology section.

Availability: The program is not yet operational; information on the program can be obtained by contacting the Structural Mechanics Program at the Office of Navy Research. Use of the program will be limited to users within or associated with the United States government.

Subjective Comments: The program will be a UOP version of the original ROP called USA-1. The validity of the approach to be used in USA-2 has been checked out in detail employing both USA-1 and another ROP called USSOR.

5. ARRAY ANALYSIS (not known by a formal acronym) [53]

Capability: The program solves for the three-dimensional steady-state radiation response emitting from either cylindrical shaped or box shaped structures. Problems with prescribed acoustic velocity, force, or mixed conditions across discrete finite elements over the structure surface are treated. The solution technique follows the solution technique resulting in final equations similar to Eq. (31) except that the source method (resulting in Eq. (48)) is used in place of the Helmoltz surface integral method (Eq. (26)) for relating the interface pressure and velocity. Fourier expansions of the response (in the circumferential direction) are applied to the cylindrical shaped structures when applicable. This reduces the core storage requirements. The authors have a provision for handling the cavity resonance problem.

Availability: Same as Program no. 18.

Subjective Comments: From outward appearances, the documentation for the program [53] looks like there is enough information available to use it; however, contact with individuals who have tried using the code had difficulty exercising it. Part of the problem evolves from the fact that the program is oriented towards employing SAMIS [54]. Although the SAMIS program is a good program from a theoretical and general purpose point of view, it is, in this author's opinion, a rather cumbersome program to use, which requires the user to write a set of "Pseudo-Instructions" comparable to NASTRAN's DMAP operations each and every time one solves a problem.

Conversations with the authors indicate that with minor modifications, the fluid structure interaction program [53] could be reprogrammed to be free of SAMIS and perhaps tie in with a more commonly used structure code such as NASTRAN. At this point in time, this program is not being used heavily. The program is viewed as a ROP.

6. Fluids Interacting with STructures (FIST) [14]

Capability: The program treats the steady state response of arbitrary shape structures of revolution subject to both radiation and/or incident (scattering) type loadings. The ability to solve rib-stiffened shells is automatically included in part of the program scope. The program formulation employs the Helmholtz surface integral in conjunction with necessary alterations for dealing with the cavity resonance problem. The structure characteristics are accepted in the form of direct stiffness and mass matrix inputs when employing the direct solution option (i.e. solution to Eq. (33)) or in the form of Eigenvalue and mode shapes when using the model solution option (i.e., solution to Eq. (43)).

Availability: Inquiries regarding this program should be addressed to A. Carlson, Naval Underwater Systems Center, New London, Conn., 06320.

Subjective Comments: With the possible exception of the form of the input data for reading in the structural data, the program can be viewed as a UOP. Currently, the program is constructed to read this structural data directly from an output tape generated by the 1108 version of the NASTRAN

program. Due to a defect in the 1108 version NASTRAN, the modal data cannot be conveniently written on tape without slightly altering the programming logic of NASTRAN and adding some additional DEMAP instructions. The NUSC version of NASTRAN has been properly altered. By making a minor connection in the read input programming of FIST, it can easily be reprogrammed to handle mode shape data generated by any other general purpose structures program.

Another unique feature of the program is that it is able to employ a rather crude spacing of the surface discretation points by allowing the pressure and velocity to vary as a cubic polynomial variation in the surface coordinates. This is in contrast to all other programs (e.g., Program no. 5 which is of the type that employs the Helmholtz or source method); all these other codes assume a piecewise constant representation (sometimes known as the piston approach) for the element by element surface distribution. The program is currently an "incore size" code; however, a much larger size problem could be solved by employing "out of core" matrix operations which are not currently part of the program.

7. NAsa STRuctural ANalysis (NASTRAN); Application-2 [39]

Capability: Unlike the transient NASTRAN Application-1 discussed earlier, this approach addresses steady state solutions to radiation programs. The solution technique involves the joining of the NASTRAN program [46] and the XWAVE program [39]. Normally, the XWAVE program, described later as program no. 17 is for solving purely steady state radiation problems (i.e., working only with Eq. (1)) by the Helmholtz integral approach (i.e., the solution to Eq. (26) with no incident wave term). Instead of inserting Eq. (26) into the equations of motion for the structure as done in the development leading up to governing Eq. (25) to account for the dynamic effect of fluid pressure on the structure (surface mobility). The modification term to the [L] matrix can be achieved through NASTRAN's rigid formats 8 and 11 for dynamic frequency response.

Availability: See availability for NASTRAN (Program no. 1) and XWAVE (program no. 17) separately.

Subjective Comments: Assuming one is able to extract the proper information from NASTRAN that is necessary for the operation of XWAVE, one is still faced with essentially the same degree problem of being able to properly execute the XWAVE code. Since XWAVE is a ROP, the joining of NASTRAN and XWAVE is also to be viewed as a ROP process. The same subjective comments used to describe XWAVE alone apply here as well. Correcting for some slight differences in reading input data, it appears a small amount of reprogramming would be involved to accept the surface mobility matrix from other general purpose structure programs, thus freeing the user from a strict dependence on NASTRAN.

8. Fluid Structure Interaction of Plates (SI34)

Capability: The transient response of plates in fluid is treated. The plate is represented by its fixed-base normal modes in vacuo. Fluid interactions are accounted for by the improved doubly-asymtotic approximation [6]. Inputs consist of a specified motion of the supports for the plate and a shockwave which is normally incident on one side of the unbaffled plate. Responses of the modes are superposed to find time-history deflections, slopes, curvatures, strains, or stresses. A fourth-order digital filter is used to generate samples from the solution to the fourth-order differential equations representing structural and fluid effects for each mode.

Since the program accepts the in vacuo modes as basic input, these modes could have been generated from any finite element structures

program or for that matter, be inserted from analytical formulae.
Availability: Robert Borts,
Code 8444
Navy Research Lab
Washington, D. C. 20375
Subjective Comments: The computer code is a UOP; although no formal written documentation exists, the program is "self-documenting" on a time-shared terminal. An unusual feature of the program is that it is the only fluid structure interaction code that employs a recursive digital filter to solve the transient equations of motion. An undesirable feature is that the coupling of modes through the fluid is ignored. The impact of the uncoupling on the solution accuracy is not clear.

9. A Computer Program for Predicting Acoustic Radiation of Finite Elastic Shells with Internal Structure [57]

Capability: The steady state radiation from general elastic bodies of revolution are treated. Off axis of revolution structures attached to the inside of the structure are handled. The solution technique is similar to the approach that results in a system of equations whose governing equations are of the form of Eq. (43), where the source method (Eq. (42)) is used to relate the surface pressure and velocity.
Availability: See Program no. 4.
Subjective Comments: The code can be considered a UOP, hoever, due to its generality, its operation can by no means be considered as "push button" as far as exercising it is concerned. Using the program requires a six step process wherein three major sub-programs (BOSOR4, ARAB, and SNAAP) are required to solve a complete problem. Documentation is provided.

10. Acoustic RAdiation from finite elastic Bodies (ARAB) [58]

Capability: The capabilities are essentially the same as Program 9 except that the methodology for treating internal structures is not covered.
Availability: See Program no. 4.
Subjective Comments: The code is a UOP. Documentation program listing and sample problems are provided.

Finite Element Structure-Finite Element Fluid Programs

11. NAsa STRuctural ANalysis (NASTRAN); Application-3 [46]

Capability: The program has built-in pressure type elements (called CFLUIDi elements) and are described through RINGFL, PRESPT and FREEPT type fluid nodes. The elements are designed to operate in contained tanks that may have either rigid or elastic walls. These special elements have the following restrictions:

1. The user may not apply loads, constraints, sequencing or omitted coordinate directly on the fluid nodes involved. Instead, the user supplies information related to the boundaries and NASTRAN internally generates the correct constraints, sequencing and matrix terms.
2. The input data to NASTRAN may include all of the existing operations except the axis symmetric structural element data (e.g., axisymmetric shell elements cannot be used).
3. The fluid must lie within the walls of an open or closed tank.
4. The first 6 rigid formats of NASTRAN may not be used in conjunction with these elements. NASTRAN assumes the walls of the container take rigid for these first 6 rigid formats but allows elasticity for the

remaining 6 (fortunately direct frequency and direct transient response are included in the remaining 6).

5. No means are provided for the direct input of applied loads on the fluid. Loading must come through the motion of the walls.

Availability: See Program no. 1.

Subjective Comments: The list of constraints that are placed on the usage of these elements rather severely limits the range of application, particularly in the case where unbounded fluid regions are of interest. Even within these constraints, [59] however, some rather interesting applications to acoustic noise problems associated with automobiles have been found.

Another unique feature that the NASTRAN program has is the ability to treat the free surface problem and include gravity terms into the fluid equations of motion.

Using NASTRAN in the context of the application described under Program no. 11 puts it in the UOP class.

12. MARTSAM IV [60]

Capability: The MARTSAM computer code is a general purpose code capable of solving a wide variety of structures problems. One option in the program is the ability to solve fluid structure iteration problems for general structures surrounded by pressure-type fluid elements. Provisions are included to handle the wave absorbing boundary problem for unbounded fluid regions.

Availability: Information regarding the acquisition of the program should be addressed to NAVSHIPS, Code PMS 3-2-421, United States Navy.

Subjective Comments: It appears that no rigid format exists for employing the fluid structure interaction option in the program. A good deal of prior experience with the program is needed before one can execute this option. A set of matrix operation controls, (comparable to the DMAP operations in NASTRAN or the Pseudo-Instructions of SAMIS) must be written by the user. For a person not familiar with MARTSAM, it could take weeks before one is able to interpret the manuals and solve a complete fluid structure interaction problem.

13. NAsa STRuctural ANalysis (NASTRAN); Application-4 [46]

Capability: The fluid structure capability referred to here is discussed in the methodology section on mock fluid elements.

Availability: See Program no. 1.

Subjective Comments: NASTRAN should be viewed as a ROP for the purpose of applying the mock fluid element approach. Actually, many dynamic structures programs can be converted to employ mock elements so long as they have the slipping boundary option, the ability to accept an arbitrary stress strain matrix, and have one, two, or three-dimensional solid elements.

14. Finite Element Analysis of Cylinder Transducers [50]

Capability: The radiation steady state response to submerged, finite length cylinders is treated by the application of a series of three auxiliary programs called DATFFC, MRTFFC, and FRQFFC. The finite element representation of these cylinders can be composed of arbitrary solid of revolution elements having an optional piezoelectric term in the constitutive equations if needed. The methodology employs finite elements for both the structure and part of the fluid (out to a spherical region, of radius R_0, which surrounds the structure). The boundary condition at the radius of the sphere is handled by consideration of the exact solution to the

Helmholtz surface integral. These details are discussed in the methodology section under placement of the outer fluid boundary.

Availability: Further information on the program can be obtained by contacting Robert Smith at the Naval Undersea Center, San Diego, Calif., 92132.

Subjective Comments: To date [50] is the only written documentation on the computer program. This reference was never meant to serve as the user's manual, however, it does give an overall idea of how to string the necessary sub-programs together for solving a particular problem. This program is unquestionably a ROP type, however, the availability contact cited above expressed an interest in assisting persons wanting to use the code. The approach has been checked out against theory and experiment [16, 51] and it appears to give excellent results.

The advantage of using the intermediate pressure fluid finite elements out to radius R_0 over the approach of representing the entire fluid field with the continuum approach appears to be a question of computer running time. Both personal contact with the author and statements in [50] imply that the main advantage is avoiding having to "numerically evaluate numerous and expensive integrals" which is the reason the approach runs faster.

The fact that the number of unknowns in the solution is larger for the approach considering intermediate finite elements than the approach representing the entire fluid as a continuum appears to weaken the savings one makes by avoiding having to numerically evaluate the Helmholtz surface integral. The authors did not give any specific numbers on running time savings.

IDEAL STRUCTURE-CONTINUUM FLUID PROGRAMS

This class of programs deals strictly with the wave equation (1) to obtain problem solutions. The structure is ideal in the sense that it is represented by some simple boundary condition on the wave equation (e.g., a rigid surface). Program numbers 5, 6, and 10 already have this capability as an option in the program, therefore will not be discussed in this subsection. Finally, most of the programs discussed under the above heading can be used to obtain the surface pressure velocity relation (e.g., Eq. (26) or Eq. (48)) for use in constructing one's own fluid structure interaction code.

15. CHIEF [61]

Capability: This program computes the pressure field radiated from bodies of arbitrary shape, given the normal velocity distribution on the surface of the body. The method employs the surface Helmholtz integral technique in conjunction with the proper corrections for the cavity resonance problem. The program employs a piecewise constant pressure distribution over the Helmholtz integral discretization.

Availability: See Program no. 14.

Subjective Comments: The lack of formal documentation on this code has to classify it as a ROP. Upon examining [61], it appears that it would be difficult to run the program with this document alone. This is unfortunate since the technical quality of the code is very good.

16. Acoustic Analysis for Large Cylindrical Arrays [52]

Capability: The program is primarily designed to solve the steady state radiation field emitting from a rigid surface having a prescribed velocity field. The program uses the Helmholtz surface integral approach, described in the solution methodology, resulting in Eq. (26) (with the incident wave term set equal to zero). No provision is made to handle the cavity resonance problem (i.e., the enforcement of a relation like Eq. (27)

is not built into the formulation). Also, a piecewise constant distribution of pressure and normal velocity is taken over the surface of the structure.

Availability: Information on obtaining the program can be obtained from J. P. D. Wilkinson, General Electric Corporate Research & Development, Schenectady, New York, 12305.

Subjective Comments: Although the program is meant to be a UOP, the amount of control cards (64 in all) needed to handle all out of core tape and disk operations implies that one is going to have to work closely with a computer systems-type person in order to install the program. The program does not appear to be actively in use at this time. Reference [52] serves as the user's manual which includes sample problems and proper listings.

17. XWAVE [39]

Capability: Same as Program no. 15, except cavity resonance problem not treated.

Availability: For program acquisition, contact F. M. Henderson, Computation and Mathematics Department, Naval Ship Research and Development Center, Bethesda, Maryland 20034.

Subjective Comments: No formal documentation exists for the ROP other than [39], consequently information regarding exercising the program must be obtained directly from the author. The program requires the mobility matrix (i.e., the expression for V in Eq. (31)) with the T and C matrices omitted. The user has the burden of generating the inverse of this matrix, which may become a problem for large size structures. No parallel modal analysis is given. The program is not heavily used as of this writing.

18. Computer Program for Solving Two-Dimensional Radiation Problems (GERAD2) [62]

Capability: The source strength method is used to solve two-dimensional radiation and scattering problems involving bodies of arbitrary shape with known boundary conditions. The program employs a piecewise constant source distribution in evaluating the source integrals. A means for handling the cavity resonance problem is not explicitely included.

Availability: Inquiries regarding this program should be addressed to R. P. Radlinski, Naval Underwater Systems Center, New London, Conn., 06320.

Subjective Comments: The documentation of this UOP exists and is included as part of [62]. The program receives moderate usage, mainly at NUSC.

19. Numerical Calculation of Acoustic Normal Modes Programs [63]

Capabilities: A series of three related computer programs are available, each of which can find the eignevalues and eigenfunctions of acoustic normal modes in the ocean based on solutions to the wave equation (1). The programs are based on an iterative finite difference scheme and a method based on the Wentzel-Kramers-Brillouin (WKB) approximation of quantum mechanics. Free surface and flexible bottom boundary conditions are treated wherein the fluid sound wave can vary with depth.

Availability: The program listings are not very long and are given in the back of [63]. The author was not reached for comments.

Subjective Comments: The finite difference and WKB approaches give comparable accurate results, however, the WKB based program runs faster.

20. Helmholtz Acoustic Wave Program (32Q) [64]

Capabilities: The program employs the source distribution method described in the methodology section, for determining the pressure response to a surface vibrating in a known manner (radiation) or the scattering of an incident acoustic wave by a surface with known values of impedance. The program has the capability of simultaneous calculation of solutions for several different boundary conditions for the same body shape and driving frequency.

Availability: Contact the author or the Naval Ship Systems Command General Hydromechanics Research Program of the Naval Ship Research and Development Center, Bethesda, Maryland.

Subjective Comments: The formal documentation of this UOP [64] is perhaps the most clear and complete of all the programs in this class. The cavity resonance problem is not treated. A piecewise constant pressure distribution of sources is assumed for the surface integral evaluations, thus requiring a fine distribution of surface zones.

21. Calculation of Sound Radiation From A Vibrating Surface of Revolution [65]

Capabilities: The capabilities of this program are like those of Program no. 17, however, this code is specialized to an arbitrary surface of revolution rather than a general three-dimensional solid.

Availability: An eight page listing of the program is located in the back of [65].

Subjective Comments: The documentation for this UOP can be found in [65]. Surface integrals are evaluated by piecewise constant representations for pressure and velocity. No provisions are made to handle the cavity resonance problem.

22. NAsa STRuctural ANalysis (NASTRAN); Application-5 [46]

Capability: The cavity vibration modes of compressible fluids (represented by the wave equation (1) existing in rotationally symmetrical cavities (that optionally could have had sharp corner slots) are treated. This capability was originally implemented to handle solid rocket motor fuel, hence the emphasis on the slots. The shape of the cavity may consist of a circular center volume surrounded by equally spaced narrow radial slots. The width and depth of the slots and the diameter of the center volume may vary along the axis of the cavity. A finite element model is defined, using pressure-type elements, by a set of two-dimensional elements lying on the center plane of one slot and on the corresponding cross section of the center volume. NASTRAN Rigid Format 3 is employed to exercise this capability.

Availability: Same as Program no. 1.

Subjective Comments: NASTRAN is to be viewed as a UOP for this particular application. Reference [59] has employed some interesting applications of the option for automobile noise problems.

CONTINUUM STRUCTURE-CONTINUUM FLUID PROGRAMS

Many POP's exist that treat special fluid structure interaction problems whose physical description is simple enough to permit an analytical solution approach. Since these programs have a less general range of applicability than the previous computational oriented programs, only a brief description of the program will be given.

There are several submerged rib stiffened cylinder ROP type programs that compute the transient response to shock type inputs. The following is a list of programs addressing this class problem.

Program Number	Program Name		Availability Information
23	MASD	A. Harari (NUSC) M. L. Baron (P. Weidlinger Assoc.)	See Program no. 4
24	STOCY	A. Harari (NUSC) M. L. Baron (P. Weidlinger Assoc.)	See Program no. 4
25	FLSSI	R. J. Scavuzzo Dept. of Mechanical Eng. Univ. of Akron Akron, Ohio 44325	Contact program author
26	FLLASH	A. Harari (NUSC) E. Sandman (NUSC)	Program Authors at Naval Underwater Systems Center (NUSC) New London, Conn. 06320
27	HTWHIP[3]	M. Garrelic Cambridge Acoustical Associate 1033 Massachusetts Ave. Cambridge, Mass. 02138	See Program no. 4
28	USSOR	Tom Geers Lockheed Palo Alto Research Lab 3251 Hannover St. Palo Alto, Calif. 94302	See Program no. 4

[3]This treats the shell as a beam in bending.

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Rotating Machinery

William Shapiro

The Franklin Institute Research Laboratories

Garnett C. Horner and Walter D. Pilkey

The University of Virginia

INTRODUCTION

A rotating machine is a system of components and the vibration characteristics of the rotor are a function of many of these components. If one attempts to establish the response or stability of a high speed machine, on fluid-film bearings, and does not accurately represent the bearings, results will probably be grossly in error. The bearings introduce cross-coupled stiffness and damping, can produce whirl instability, and shift critical speeds significantly from where they might be predicted if the bearings are oversimplified. Similarly, floating ring seals, flexible foundations, aerodynamic and hydraulic excitations, etc., are all part of the rotor system. No matter which computer codes are available, if the system is not properly represented vibrational performance cannot be accurately produced. This chapter describes the important considerations of the separate components, and the system, and discusses available computer codes.

NOMENCLATURE

$\{a\}$ = Amplitude vector
C = Radial clearance, in.
$[C]$ = Damping matrix
D_{ij} = Damping in i direction due to velocity in j direction
D = Shaft diameter, in.
DELI = Pitch of shoe I, rad.
ΔF_i = Incremental force in i direction, lbs
FX = Force on shaft in X direction, lbs
FY = Force on shaft in Y direction, lbs
$\{G\}$ = Unbalance force vector
h = Local film thickness, in.
$H = h/c$ = Non-dimensional film thickness
I_p = Polar moment of inertia
I_T = Transverse moment of inertia
K_{ij} = Stiffness in i direction due to displacement in j direction
$[K]$ = Spring stiffness matrix
L = Bearing length, in.
$[M]$ = Mass matrix
MI = Moment on shoe I, in-lbs
m = Journal mass
N' = Shaft speed in rev/sec
N = Shaft speed, rpm
$P = p/P_r$ = Non-dimensional pressure
$\{P\}$ = External load vector
R = Bearing radius, in.
S = Sommerfeld Number = $(\mu N'LD/w)\ (R/C)^2$

$T = U_1^t/2L$ = Non-dimensional time
t = Time, sec
U = Surface tangential velocity, in./sec
$\{U\}$ = Displacement vector
U_1 = Reference velocity, in./sec
v = Surface speed, in./sec
W = Bearing load, lbs
$\{W\}$ = Gravity load vector
X,Y = X,Y displacement and velocities (Table 1) of journal in stiffness and damping matrices respectively
x = Surface coordinate in direction or tangential velocity in Reynolds equation; shaft displacement in x direction
$X = X/L$ = Non-dimensional coordinate
y = Surface coordinate in direction normal to tangential velocity; shaft displacement in Y direction
$Y = y/L$ = Non-dimensional coordinate
α = Real part of eigenvalue, β
β = Complex eigenvalue (real part is growth factor, imaginary part is frequency)
$\Lambda = 6\mu U_1 L/P_r C^2$ = Speed parameter
μ = Absolute viscosity, lb-sec/in^2
ρ = Fluid density, lb-sec^2/in^4
ω = Imaginary part of eigenvalue

FLEXIBLE ROTOR DYNAMICS ANALYSIS

There are a variety of computer codes available for determining performance of a flexible rotor. A flexible rotor may be modeled as a series of disc elements connected by beam elements (see Fig. 1).

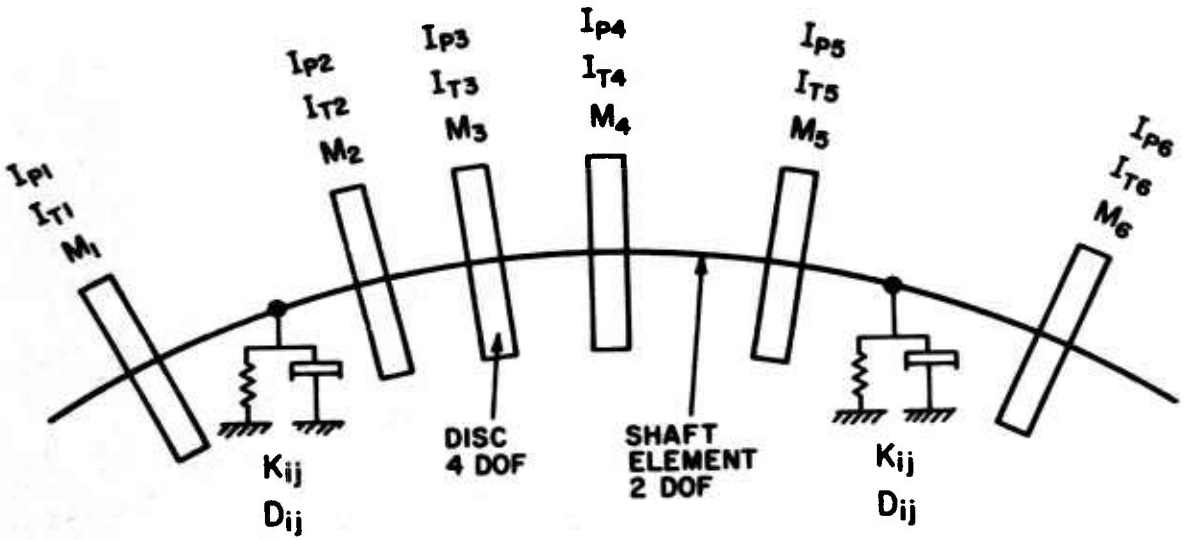
The rotor is supported by bearings which can also be resiliently mounted. In general, the theoretical development starts by examination of a free body that includes a disc element and shaft elements on either side. The moment and shear from adjacent beam elements are usually determined from beam theory equations. In all programs there are certain general assumptions such as:

1. lateral excursions of the shaft are small in magnitude
2. the rotating assembly is axisymmetric

In some programs the shaft elements are lumped at the discrete nodal points selected to represent the shaft system. The rotor is subdivided into finite elements. The mass of each element is divided into two parts and placed at the nodes of the element. Discs such as turbine wheels and impellers that are mounted on the shaft are made to coincide with nodal points, and they are treated as rigid bodies with associated mass and inertias. Thus, the mathematical model constructed to represent the rotating assembly is composed of a series of lumped masses and rigid discs connected by weightless, flexible beams. Inasmuch as the actual rotor is a continuous system with an infinite number of degrees of freedom, the replacement of this system by a model having a finite number of degrees of freedom clearly requires engineering judgement. In general, accuracy of the computed result depends to a large extent on the model chosen to represent the system. However, in practice, a reasonable number of masses will yield accurate results.

Generally, with the lumped mass approach the transverse and polar moments of inertia of the shaft are neglected. A shaft element is thus limited to two degrees of freedom, which are X and Y translations. Disc elements have four degrees of freedom, X and Y translations and rotations about orthogonal axes. Although it is possible to include the angular degrees of freedom for shaft elements, it has generally been found that the additional degrees of freedom do not provide a commensurate increase in the accuracy of the results, and is not worth the increased computional time involved.

When the entire system is considered, a matrix of dynamic equations results, and there are as many equations as there are degrees of freedom in the rotor representation.



K_{ij} = STIFFNESS COEFFICIENT IN j DIRECTION DUE TO DISPLACEMENT IN i DIRECTION

D_{ij} = DAMPING COEFFICIENT IN j DIRECTION DUE TO VELOCITY IN i DIRECTION

DOF = DEGREES OF FREEDOM

Fig. 1 Mathematical model of rotor-bearing system

The equations can take on a variety of forms depending upon the approach used in their generation. In some programs a straightforward Newtonian approach is employed where beam theory equations are applied to obtain moments and shears. In more sophisticated programs the complete rotor bearing system is set up into stiffness and damping matrices including the bearing representations. In so doing, many of the principles of the theory of elasticity are used for deriving stiffness and damping matrices. A typical set of equations can be written as follows:

$$[M]\{\ddot{U}\} + [C]\{\dot{U}\} + [K]\{U\} = \{P\} + \{W\} + \{G\} \tag{1}$$

where $\{U\}$ is a column vector containing all the system degrees of freedom, $\{\dot{U}\}$ and $\{\ddot{U}\}$ the time derivatives of $\{U\}$. $[M]$ is a diagonal mass matrix corresponding to the degrees of freedom in $\{U\}$. Since rotating inertia properties of the shaft are neglected, components of $[M]$ corresponding to those degrees of freedom will be zero except at stations with discs. $[C]$ contains the damping matrix of the bearings and the gyroscopic coefficients of the discs. $[K]$ is an assembly of the stiffness matrices of all shaft elements and those of the bearings. The column vector $\{G\}$ contains the unbalance forces. $\{P\}$ and $\{W\}$ are the external and gravitational load vectors respectively.

Two studies are often conducted. These are labeled stability and response. For stability, the rotor is considered operating quiescently. A small disturbance is applied, and it is then determined whether the response to this disturbance will grow or decay in time. If the response grows then self excited instabilities are present. It should be noted that self excited instabilities are usually caused by cross-coupling influences. This means that a force applied in one direction is not only resisted by a colinear force, but also produces a force orthogonal to the applied force. Thus, there is some motion orthogonal to the direction of the applied force and whirling of the rotor occurs. Prime sources of whirl are fluid-film bearings and aerodynamic forces.

For stability calculations, there are no external forces and moments acting on the system. Equation (1) becomes

$$[M]\{\ddot{U}\} + [C]\{\dot{U}\} + [K]\{U\} = 0 \tag{2}$$

Here, the displacements and rotations are taken to be small excursions from a known equilibrium configuration. Assuming the solution to be of the form

$$\{U\} = \{a\}\, e^{\beta t}$$

where β is a complex quantity Eq. (2) becomes:

$$[M\,\beta^2 + C\beta + K]\{a\} = 0 \tag{3}$$

Equation (3) is a set of linear algebraic equations describing an eigenvalue problem. The determinant of the coefficient matrix must vanish. Expansion of the determinant yields a characteristic polynomial whose order is 2^N, where N equals the number of degrees of freedom or the number of equations. A simplified explanation of the stability theory is discussed in the section dealing with bearing stability.

The polynomial is then solved for the roots of β. The real part of β is the growth or attenuation factor. A positive number implies an instability. The more negative the real part is, the more damped is that particular mode

of vibration. The imaginary part of the root provides the frequency of vibration of any particular mode. After the natural frequencies and growth factors are determined, the roots of β can be substituted back into Eq. (2) and relative amplitudes {a} determined. This will produce the mode shape or deflected shape of the rotor for that particular mode. Thus the information obtainable from the stability analysis is quite considerable. It includes:

1. Information as to whether the system is stable or not, and to what degree (growth factors).
2. The natural frequencies of vibration. This information also provides critical speeds, because if a natural frequency coincides with an operating speed, then a critical speed exists.
3. The stability analysis also provides the shape of the deflected rotor for each mode of vibration.

Figure 2 shows results of a stability analysis and how they are interpreted. Two modes of vibration are traced over a speed range. Both the growth factor and response frequency of each mode are indicated. Mode 1 becomes unstable at about 16,000 rpm, since at this point it begins to go positive. At the threshold of instability the frequency of mode 1 is about 8,000 rpm. Thus, the unstable frequency is about half the operating speed indicative of some half-frequency whirl condition. If the mode shape was a characteristic first bending mode, then that would be the shape of the deflected rotor, as it orbited at approximately half frequency! Mode 2 does not become unstable. However, intersection of the modal frequency with the line in which the operating speed equals the response frequency (45° straight line) is a critical speed. For mode 2 this occurs at a point where the growth factor is not highly negative. Thus a potential unbalance response problem may occur at this point, and the system should be examined to determine the absolute amplitude of unbalance response.

Figure 3 shows a critical speed map for a multi-disc machine, supported by bearings at either end. The rotor will pass through a large variety of different modes.

In some rotors, such as this, horizontal and vertical modes separate very distinctively; on other machines there is only one distinguishable first bending mode. The parenthetic nomenclature on Fig. 3 indicates the primary plane of vibration and where the maximum amplitude occurs. The conical mode is a rigid body type. Two mode shapes are indicated on Figs. 4 and 5. The letter B indicates a bearing location and the reference line is the undeflected rotor. If a mode occurs at a bearing, as shown on Fig. 4, then a bearing change would not be helpful in attenuating this mode. If the mode however, is deflected as shown in Fig. 5, there are large displacements at the bearing locations. In this situation a bearing change could be helpful. In general, there would be two curves for each mode; one in the horizontal plane, and one in the vertical plane. For clarity, only single curves are shown on Figs. 4 and 5.

The second significant type of computation is called a response analysis. Here actual amplitude response is determined as a function of unbalance or other specified external excitations. In other words, a complete solution to Eq. (1) is accomplished. Very often a time-transient scheme is employed in which time is discretized into finite increments. At each time step new displacements are computed using information from the previous time step. A time history of the motions of the rotor at each shaft location is thus produced. There are various methods of describing the mathematical procedures [1, 2]. A typical response plot is shown on Fig. 6. Theoretically, the information obtained from a response case is what could be seen by orthogonal displacement probes that monitor the motions of the rotor at a particular location.

BEARING REPRESENTATION

It is essential that bearings be appropriately represented. This is especially true if fluid-film bearings are used. The cross-coupled spring and damping characteristics of bearings can influence non-synchronous response and critical speeds.

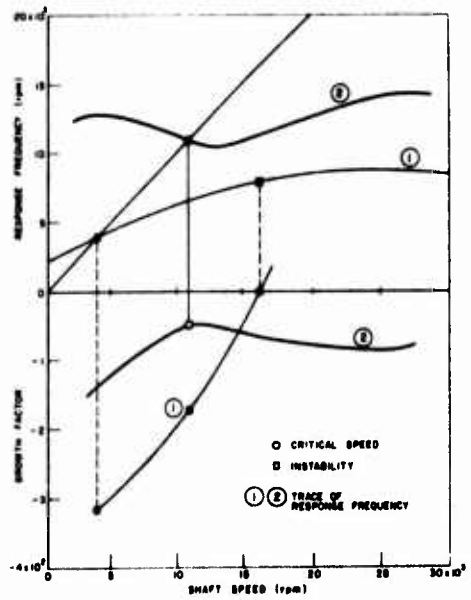


Fig. 2 Stability map - flexible rotor-bearing system

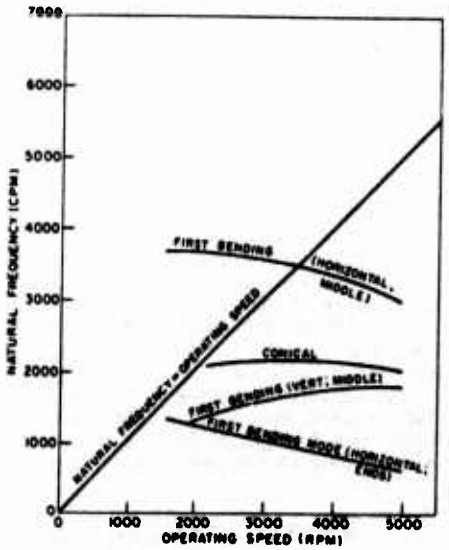


Fig. 3 Critical speed map for a multi-disc machine

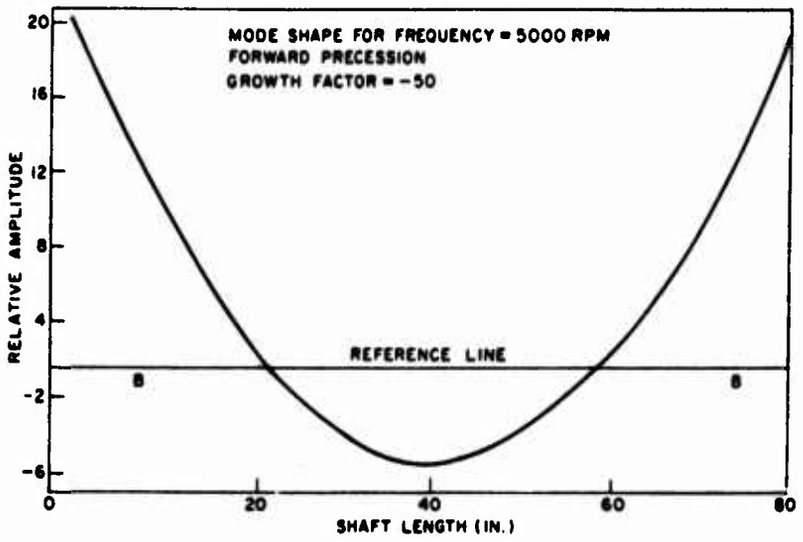


Fig. 4 Mode shape for 5000 rpm

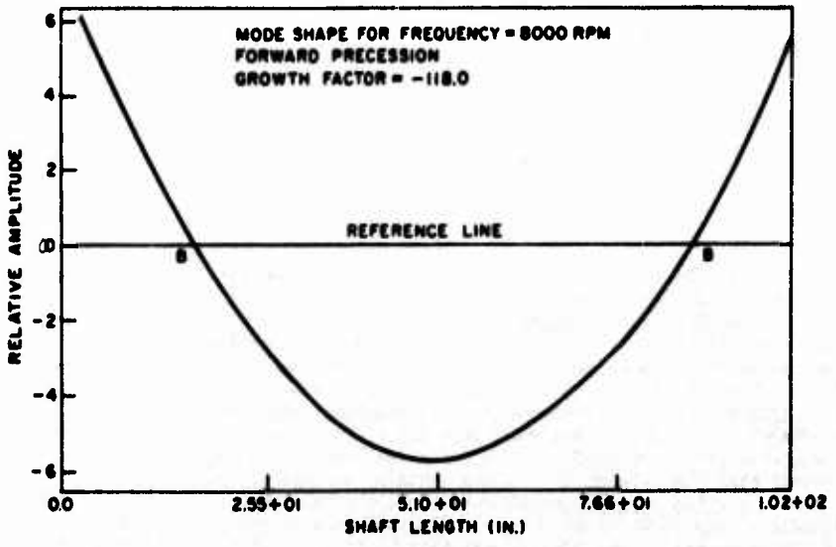


Fig. 5 Bending mode with large bearing motions

The cross-coupled coefficients are generally obtained from separate bearing computer codes (see section on fluid-film bearing programs). It is generally necessary to determine the steady-state operating position of the journal in the bearing and accomplish displacement and velocity pertubations about that position to produce spring and damping coefficients. The spring and damping coefficients are probably the most popular representation in rotor-dynamic analysis. The coefficients apply only to small bearing motions that are still in linear range. They are absolutely accurate for stability analyses. However, if a response analysis orbit indicates large bearing motions, then amplitudes are probably incorrect since excursions have gone beyond the linear range of the coefficients. If bearing excursions are large however, it is not a healthy situation, and modifications to the bearings, rotor or excitation are probably necessary. Some rotor-dynamics computer codes will permit inputing the complete tilting-pad motions, including the cross-coupling between the shoes and rotor (see section on bearings), so that pad motions are included. This is very helpful, since incorrect results are sometimes obtained when the complete matrices are reduced to standard 2 x 2 matrices to accommodate the particular rotor-dynamics program in which they are utilized.

The program ROTDYN has a unique feature, whereby, at a bearing station an arbitrary number of degrees of freedom can be specified, and identified by the terms in the stiffness, damping and mass matrices that are inputted at a bearing station. This permits applying resilient mounted bearings such as shown on Fig. 7, and accepting the complete tilting pad cross-coupled matrices and pad inertias to permit the determination of pad motions.

Incidentally, squeeze film characteristics for damping devices that surround a bearing shell can be obtained from fluid-film bearing computer programs. Floating ring seal characteristics are also obtainable from these programs.

Effects of Gear Mesh

A gear mesh in a rotor can significantly reduce the tendency for the rotor to whirl. Thus, the effects of the gear should be included in a stability and response analysis. One way to accomplish this is to simulate the gear as a flexible foundation at the nodal point where the mesh occurs. The tooth stiffness is considered as a spring, the opposing gear as a mass, and the opposing gear bearings as lumped cross-coupled spring and damping coefficients.

Computer Program Selection

There are a number of factors to be considered when selecting a rotor-dynamics computer code. These include:

1. Non-Synchronous Capability - A major limitation of many programs has been their inability to predict non-synchronous frequencies and response. For many years the primary consideration has been evaluating response to unbalance. Since unbalance is a synchronous excitation, it was natural to limit response to a synchronous phenomenon. The true situation today however, is that subsynchronous frequencies are prevalent and often the most troublesome. This is especially true when fluid-film bearings are employed. Therefore, it is desirable that the program have a non-synchronous capability.

2. Bearing Representation - Accurate bearing representation is a vital element to any rotor response program. If rolling element bearings are used, then cross-coupled capabilities are probably not necessary. If fluid-film bearings are used, it is absolutely essential that the bearing cross-coupling influences are properly represented. It is also important to be able to treat resilient and damper bearing housings, and to consider the vibrations of the damped housings as well as the rotor. For tilting-pad bearings, consideration of the pad motions is desirable.

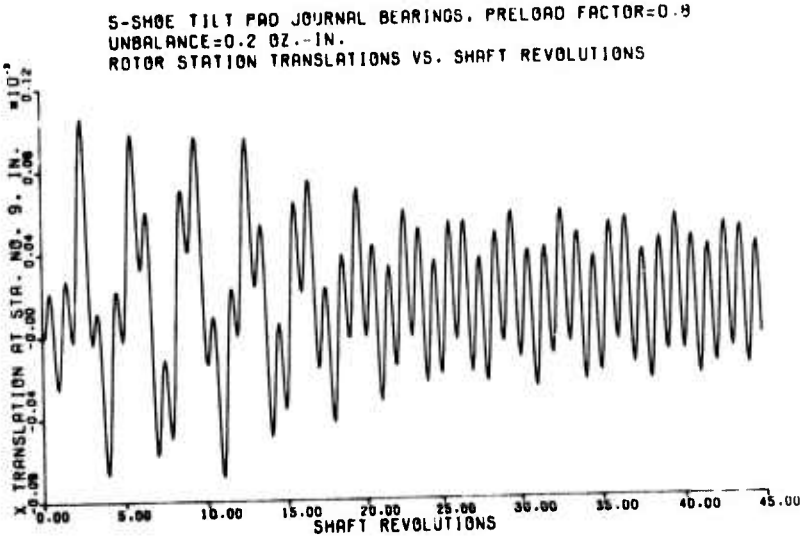


Fig. 6 Typical unbalance response results

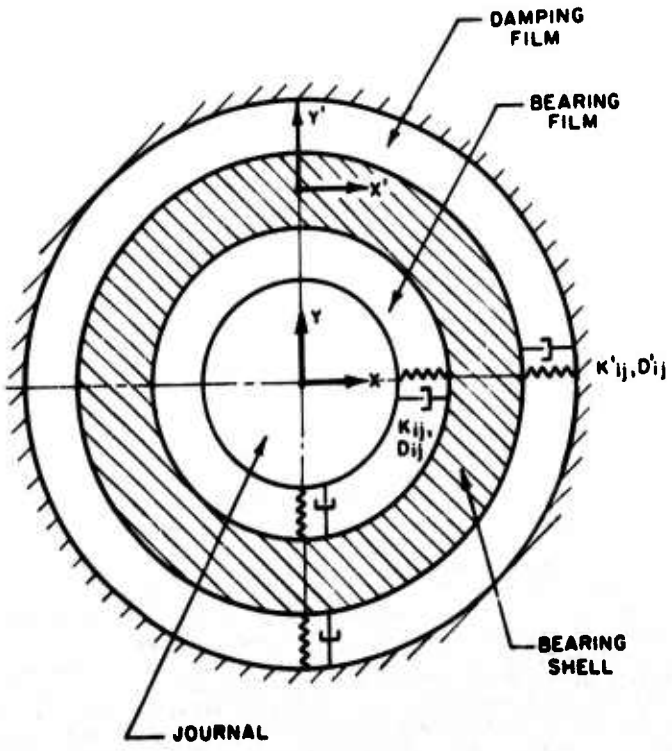


Fig. 7 Schematic of resilient mounted bearing

3. Shear Factors - Most programs will consider shear and moments from bending. For thick rotors it is important to also consider shear deformations.

4. Number of Stations - To accurately represent a long rotor it is essential that the program have the capability to handle a reasonably large number of mass stations. A program should be able to handle at least 15 stations.

5. Gyroscopic Influences - Gyroscopic forces emanating from disc rotation and radial motions should be included. They are especially significant for overhung rotors with large discs attached.

6. Loading - The program should consider unbalance and gravity loading as a minimum. It is desirable if other arbitrary excitation could be applied.

7. Damping - Provisions for damping should be included especially if fluid-film bearings are used. Damping can shift critical speeds, stabilize a system, and reduce orbital amplitudes.

Presented below are those programs that can satisfy the primary criteria. Mention is also made of other programs that would have application to more restricted problems.

COMPUTER PROGRAM SUMMARIES

Non-Synchronous, Flexible Rotor Dynamics Program (ROTDYN)

Purpose: To determine stability and orbit response of flexible rotor bearing systems. The response computed in this program is the true theoretical response inasmuch as the artificial distinction between synchronous and non-synchronous response made in several other analyses is not made. The response computation yields the time history for each degree of freedom of the assembly. In addition, several quantities of interest such as stresses, forces and moments at bearings, are generated for user-specified locations. For each shaft speed the stability analysis yields the rigid body and bending criticals, and their growth or damping factors: and for each of the criticals, the displacement and velocity mode shapes. These results may be put in the form of a stability chart. This analysis is also computationally quite economical.

Applications: General rotating machinery.

Capabilities: Shaft is represented by finite elements based on beam theory with shear factors. Sectional and inertial properties are internally generated for axisymmetric, hollow, or solid elements of revolution. Axial taper of the shaft within each element is also taken into account. Bearings are represented by stiffness and damping coefficients. Foun ia-tions are represented by stiffness and damping coefficients. Foundation inertial properties are also included in the analysis. Arbitrary number of degrees of freedom can be handled at a bearing station. Thus, pad motions of tilting-pad bearings are included, and gear mesh can be computed. Also squeeze film damping and resiliant mounting is permitted. Material damping in the shaft and foundations is also considered. External loads and body forces are included. Force or displacement type excitations may be applied to the foundation. The stability option permits 100 degrees of freedom. The response option permits 400 degrees of freedom.

Method: The mathematical model utilized in the program consists of a rotating, non-uniform, axisymmetric shaft supported by flexible bearings. The bearings are assumed to be supported by flexible or rigid foundations with mass distributed throughout. The shaft, assumed to be rotating at constant speed, is taken as being flexible with distributed mass and with several rigid, axisymmetric discs. Assuming small excursions from an equilibrium configuration, linear dynamical equations for the system are devised. The elastic continuum in the shaft and the foundation is reduced to discrete degrees of freedom by the finite element method. For the shaft, beam theory with shear factors is used; whereas for the foundation, appropriate one-, two- or three-dimensional elasticity theory

may be used depending on the foundation configuration. (The routines for finite element representation of general foundations are external to the ROTDYN program. The program accepts stiffness matrices for the foundation elements.) Bearings are represented by stiffness and damping coefficients with arbitrary number of degrees of freedom to account for arbitrary bearing geometries. (Routine for generating these coefficients are also external to the program.) Material damping is assumed to be proportional to the internal elastic forces in the continuum. (Proportionality factor is an input to the program.) Response of the rotor-foundation system is computed by forward integration in time. An unconditionally stable algorithm due to Newmark is used for this purpose. With this technique, history for five shaft revolutions for eighty-four degrees of freedom requires about twenty-two seconds of Univac 1108 (EXEC 8) time. For stability analysis, the dynamical equations are converted to an eigenvalue problem by assuming small excursions from the equilibrium configuration. In this process, noninertial degrees of freedom are eliminated from the system by a theoretically consistent procedure. The eigenvalue problem is then solved for complex roots by the QR-2 algorithm. For each root of interest, the mode shape is computed by solving the resulting matrix equation by Gaussian elimination.

Input: Number of stations, shaft elements, discs, bearings, shaft materials and time steps. Material properties of shaft elements (moduli of elasticity and shear and weight density). Shaft element geometry (solid, hollow, rectangular, tapered, length, diameters). Disc locations and inertia properties. Unbalance locations, magnitudes and phase angles. External force, locations and time varying magnitudes. Bearing stiffness and damping matrices. Bearing pad and resiliant mount mass inertia matrix--size equal to number of degrees of freedom at each bearing.

Output: Stability analysis (for each speed). Critical frequencies. Growth or damping factors. Displacement mode shapes. Velocity mode shapes. Stability and critical speed diagram. Orbit analysis. Orbital amplitudes at each mass station. Transmitted bearing force. Shaft moments and stresses.

Language: Fortran IV

Hardware: Univac 1108, CDC 6600

Usage: Extensive; over past three years have analyzed approximately 75 rotors.

Developer: Dr. M. M. Reddi
The Franklin Institute Research Laboratories
Philadelphia, Pennsylvania 19103

Availability: For sale

Contact: Mr. W. Shapiro
Manager of Mechanical Engineering Laboratory
Franklin Institute Research Laboratories
Philadelphia, Pennsylvania 19103

Subjective Comments: This program is very comprehensive. It treats both stability and response and satisfies the necessary and desirable criteria mentioned above.

Dynamic Stability of a Flexible Rotor (CADENSE - 25)

General Description: This program computes the damped natural frequencies of a rotor bearing system. For each natural frequency the magnitude and sign of the effective system damping is calculated. Based on this result the dynamic stability of rotor-bearing system may be assessed. The rotor is described in terms of lengths and radii of discrete sections. Concentrated inertias may be placed on the rotor and described by their mass and moments of inertia. Fluid film bearings are described by linear stiffness and damping values.

Analysis: An extension of the Prohl critical speed method is employed to determine the complex eigenvalues of the system dynamic matrix. The imaginary part is the natural frequency and the real part is the amplitude growth exponent. Passive aerodynamic excitation may be accounted for

and it is represented by linearized coefficients based on the torque per stage, vane efficiency, and the wheel to stator clearance-to-vane height ratio. Internal material damping, and damping in shrink fits and friction joints may also be accounted for and are specified in terms of an associated logarithmic decrement. The program uses very efficient root searching techniques based on a generalized Newton-Raphson method.

Features: Rotor of arbitrarily varying cross section. Arbitrary location of bearings, seals, and concentrated inertias. Arbitrarily located sources of aerodynamic excitation. Convenient input of bearing data via nondimensional tables, which are manipulated by the program to minimize the input preparation effort. This program allows for convenient investigation of system sensitivity to variations in bearing and rotor characteristics. This program computes nonsynchronous stability parameters, and appears to be similar to the stability option of ROTDYN. It does not produce nonsynchronous response.

Input: Rotor dimensions. Material properties. Concentrated inertia values. Frequency ranges of interest. Table of nondimensional bearing stiffness and damping coefficients. Bearing dimensions. Operating speed(s) of interest.

Output: All input data. Damped natural frequencies. Logarithmic decrement (indicator of system damping) for each mode. Rotor major and minor mode shapes at frequency. Phase angle of rotor amplitude.

Running time: 1-2 seconds CDC 6600 per damped natural frequency (for a 40 station rotor).

Language: Fortran IV

Availability: Mechanical Technology Incorporated
968 Albany Shaker Road
Latham, New York 12110

Unbalance Response of a Flexible Rotor (CADENSE - 21)

General Description: This program computes the elliptical whirl response of a flexible, elastically mounted rotor to a given unbalance excitation. It incrementally scans a specified speed range and computes the absolute amplitude of vibration (unbalance response) for each rotor station at each speed increment. The rotor mountings are considered as elastic bearings described by their stiffness and damping coefficients. Bearing pedestals may be treated as rigid or elastic. The rotor is described in terms of lengths and radii of discrete sections. Concentrated inertias may be placed on the rotor and described by their mass and moments of inertia. For the case of a rigid pedestal, the program output includes the force transmitted to the bearing housing. For a flexible pedestal, the program provides the pedestal motion and the force transmitted to the foundation. The pedestal is represented by a model having two translational degrees of freedom. An option is available to generate a plot tape to depict amplitude versus speed for selected rotor stations. The program employs an extension of the Prohl critical speed method described by Lund and Orcutt. Prohl's method is modified to account for elastic rotor mountings and for the influence of distributed shaft mass. Prohl's method is described in "A General Method for Calculating Critical Speeds of Flexible Rotors," Journal of Applied Mechanics, Volume 12, Trans. ASME, Vol. 67, 1945, pp. 142-148. Lund and Orcutt's modification is described in "Calculations and Experiments on the Unbalance Response of a Flexible Rotor," ASME Paper 67-Vibr-27.

Features: The program allows the designer of rotating machinery to calculate the magnitude of vibration of a rotor with anisotropic, damped bearings and to establish safe tolerances for residual unbalance. It is a valuable complement to the lateral critical speed program, and the two programs have compatible input requirements.

Input: Rotor dimensions. Material properties. Concentrated inertia values. Speed ranges of interest. Bearing stiffness and damping coefficients at each speed increment. Unbalance values.
Output: All input data. Major and minor axes of rotor orbit at each station. Phase angle of rotor amplitude. Forces transmitted to bearings and pedestals. Plots of amplitude versus speed at selected stations. Printed summary output option for selected stations.
Memory Requirement: 111,000 words
Running Time: 2 to 4 response speeds per CDC 6600 second
Language: Fortran IV
Usage: Used extensively
Availability: Mechanical Technology Incorporated
968 Albany Shaker Road
Latham, New York 12110

Aero Propulsion Laboratory
Wright-Patterson Air Force Base
Dayton, Ohio 45433
Subjective Comments: This unbalance response program produces synchronous response only. It is very useful, if it has been previously determined (by a stability analysis for example) that the primary modes of vibration will be near synchronous.

Reference material is contained in Wright-Patterson AFB Technical Report AFAPL-TR-65-45 or X65-20875. At Lewis Research Center, contact Delmar Drier of Engineering Design Division for IBM 7094, IBM 360, or Univac 1106 versions. The Univac 1106 version requires 35K of memory and typical running time is 4 seconds.

Time Transient Nonsynchronous Rotor Response

Date: 1971
Capability: This program determines the stability of a rotor with film bearings, seals, and pedestal supports. The full nonlinear and cross-coupling characteristics of the film bearings are included. The program may be used to calculate synchronous response. Features which are included in the program are

- Unbalance
- Gravity
- Misalignment of bearings
- Rotor bow
- Fluid dynamical destabilizing forces
- Several bearing models:
  - Circular bore
  - Fired arc (elliptical, axial groove, tilting pad - geometry of each arc is specified independently so that nonsymmetric geometry may be examined)
- Pedestal film dampers
- Flexible stator
- Film includes the effects of turbulence and variable viscosity locally through the film
- Floating ring seals
- Time varying tilting pads
- Hydrostatic pockets in bearings
- Short, long, or finite bearing solutions

Method: The equations of motion are integrated step by step through time including all nonlinearities of fluid film.
Limitations and Restrictions: 40 mass stations, 8 bearings, 6 tilting pads or arcs per bearing
Input: Rotor, bearing, pedestal, seal geometry, and operating conditions.

Output: Plots of mass station trajectories from which stability characteristics are derived. Response of rotor, bearings, and stator versus time.
Language: FORTRAN
Hardware: UNIVAC, CDC
Usage: Well tested.
Developer: Dr. Melbourne F. Giberson
Turbo Research Inc.
1440 Phoenixville Pike
West Chester, Pennsylvania 19380
Availability: This program is for rent. Contact the developer.
Comments: This is a very complete program.

Synchronous Vibratory Response

Date: 1963
Capability: This is a steady state response program which includes the following effects:
- Nonlinear film bearings
- Bearing cross-coupling
- Pedestal squeeze-film damping
- Floating ring seals
- Flexible bearing pedestal
- Random or prescribed mass unbalance distribution

This program may be used to analyze compressors, pumps, or any type of rotating machinery where the rotating forces are large compared to the rotor weight.
Method: An iterative technique (because of nonlinearities) is incorporated with the Prohl-Myklestad method [3]. The results of this analysis give circular orbits about the bearing center line.
Limitations and Restrictions: 300 shaft steps, 60 elastic sections, 18 bearing locations, 10 bearing types, 200 speed points, 10 trials of mass distribution.
Input: Rotor, bearing, pedestal, and seal geometry and operating conditions.
Output: Calcomp plot of each bearing force over the speed range. The resultant radial deflection of the shaft and bearing forces are printed for each speed.
Language: FORTRAN
Hardware: UNIVAC, IBM, CDC
Usage: Well tested.
Developer: Dr. P. R. Trumpler
Turbo Research Inc.
1440 Phoenixville Pike
West Chester, Pennsylvania 19380
Availability: This program may be rented. Contact Dr. Melbourne F. Giberson of Turbo Research Inc. for details.

Damped Critical Speeds (DAMPEDROTOR)

Date: 1975
Capability: This program determines the complex eigenvalue and eigenvector of a damped rotating shaft. The complex eigenvalue gives the stability of the rotor-bearing system and the damped natural frequency. A special feature of this program allows adjacent rotor sections to be connected by a rotary spring and/or a linear spring.
Method: The rotor is modeled as a series of lumped masses with gyroscopic moments and massless beam elements. The program uses a new method of analysis called the Riccati Transfer Matrix Method. A Newton-Raphson iteration is used in determining the complex eigenvalue.
Limitations and Restrictions: This program uses linear theory and isotropic bearings.

Input: Rotor geometry and bearing coefficients.
Output: Damped natural frequency, the "Q" or quality factor, and the damped mode shape. Mode shape plotting is included.
Language: FORTRAN
Hardware: CDC, IBM
Usage: Limited usage
Developer: Walter D. Pilkey
Department of Engineering Science and Systems
University of Virginia
Charlottesville, Virginia 22901
Availability: Deck and user's manual with example problems are available from W. D. Pilkey at a nominal cost.
Comment: This is the only program which will determine an unlimited number of complex eigenvalues and eigenvectors. Due to the Riccati transformation analysis, no numerical difficulties are encountered in computing the eigenvalues.

Computerized Mechanical Design Analysis (BEST I)

Date: 1970
Capability: The steady state response, critical speeds and corresponding mode shapes of a rotor are determined. The rotor may have asynchronous motion where the whirl and spin frequencies are different. Foundations or bearings are represented by isotropic springs.
Method: The rotor is represented as a lumped mass system and the effects of shear and gyroscopic moments are included. The transfer matrix method is used for the analysis.
Limitations and Restrictions: A rotor may be represented by no more than 20 spans and 25 lumped masses per span. No damping may be input, so that the critical speeds are for an undamped rotor. Bearings must be modeled as a simple linear spring.
Input: Span properties, spring values, frequency interval for critical speed search, and loading are required input. No preprocessor is available.
Output: Boundary condition equations, critical speed analysis and mode shapes, kinetic and potential energy of each span, and mode shape plots are output.
Language: FORTRAN
Hardware: Remote Batch
Developer: Structural Dynamics Research Corp.
5729 Dragon Way
Cincinnati, Ohio 45227
Availability: Through developer or several commercial systems.
Comments: The input instructions seem to be too complicated for most design engineers in turbomachinery. For instance, the program requires that moments of inertia be input rather than having an input based upon rotor geometry and letting the program calculate the moments. On the other hand, the form of the input allows designers to account for the flexibility of disks.

Damped Natural Frequencies and Mode Shapes of Multi-Mass Rotor Systems and the Investigation of Rotor Stability (MFIN4)

Date: 1974
Capability: This program calculates the damped natural frequency of a rotor-bearing system. The stability of the system is checked by calculating the exponential growth.
Method: This is a finite element program. The equations of motion are written for each rotor element. The resulting matrix equation is

transformed into an eigenvalue problem and is solved by the Q-R algorithm for complex eigenvalues and eigenvectors.
Limitations and Restrictions: The rotor may be represented by a maximum of 12 elements.
Input: Rotor geometry and bearing coefficients.
Output: The complex eigenvalue and the complex mode shape.
Language: FORTRAN
Hardware: CDC
Usage: Limited usage.
Developer: Dr. E. J. Gunter
Department of Mechanical Engineering
University of Virginia
Charlottesville, Virginia 22901
Availability: Contact E. J. Gunter for details.
Comment: This program has some novel features which should give insight for solving complex eigenvalue problems.

Lateral Vibration (LAVIB)

Capability: The steady state response and natural frequencies of a general shafting system modeled as a series of stations are calculated. A shafting station consists of a massless beam, a lumped mass, springs to ground, and a forcing function.
Method: The Holzer-Myklestad-Prohl method is used to calculate natural frequencies, and modal analysis is used to calculate the lateral vibration response of the shafting system to synchronous and nonsynchronous shaft speed forcing phenomena. Modal damping is used to model dissipative forces.
Hardware: UNIVAC 1108
Developer: Dr. Ronald L. Eshleman
Availability: Cost $500. Includes deck, instruction manual, and examples.
Contact: The Vibration Institute
5401 Katrine
Downers Grove, Illinois 60515
Attn: Dr. Ronald. L. Eshleman, Director

Critical Speeds of a Rotor-Bearing System (CRITSPD)

Date: Program prepared in July 1974
Capability: CRITSPD is a computer program which calculates the undamped critical speeds of a flexible rotor. It gives the amplification factor and the mode shape of the rotor at the critical speed. The stability of the rotor is determined using the modal mass, modal damping and amplification factor. Bearing stiffness and damping coefficients may be speed independent and/or speed dependent.
Method: The transfer matrix method is used to analyze a rotor modeled as lumped masses/discs and/or distributed mass.
Limitations and Restrictions: Maximum number of 150 mass stations, 50 bearing stations. With plot output, maximum number of mass stations is 70.
Input: Rotor geometry, bearing coefficients, and speed range.
Output: Critical speed, mode shape, modal weight, modal damping, and amplification factor. Optional plots of the rotor geometry and mode shapes.
Language: FORTRAN
Hardware: CDC
Usage: Well tested.
Developer: Dr. E. J. Gunter
Department of Mechanical Engineering
University of Virginia
Charlottesville, Virginia 22901
Availability: Contact E. J. Gunter for details.

Unbalanced Response and Critical Speeds (SHAFT)

Date: 1971
Capability: The program SHAFT calculates the unbalanced response and critical speeds of a shaft with no cross-coupling coefficients in the bearings. The critical speeds are found for a rotor with no damping in the bearings. For unbalanced response, the deflection, shape, bending moment, and shear force are calculated with damping in the bearings. The shaft can be formed of lumped or continuous mass segments with foundations, any boundary conditions, and any distribution of unbalanced masses. The user can include any or all of bending, shear deformation, and rotary inertia effects.

Bearing systems can include springs, dampers, and a pedestal mass.

Method: This program uses the transfer matrix method of solution.
Limitations and Restrictions: No bearing cross-coupling coefficients.
Input: The bearing coefficients and rotor properties for each segment are input. An interaction preprocessor is available.
Output: For unbalanced response, the deflection, slope, bending moment, and shear force are printed at each station and each speed. The critical speeds are printed with the corresponding mode shapes.
Language: FORTRAN
Hardware: IBM, CDC, UNIVAC, HONEYWELL, PDP
Usage: Well tested.
Developer: The Structural Members Users Group
Department of Engineering Science and Systems
University of Virginia
Charlottesville, Virginia 22901
Attn: Walter D. Pilkey
Availability: Deck and documentation are available at a nominal cost. Contact W. D. Pilkey. Also available on commercial computers.
Comments: This program has many features which allow any combinations of shaft geometry, boundary conditions, and in-span conditions to be analyzed.

Transient Response of a Rotating Shaft (TRANSIENTSHAFT)

Date: 1975
Capability: TRANSIENTSHAFT is a program for determining the transient response of a rotating shaft. The bearings are isotropic and may include damping.
Method: The method of solution for the transient response is the modal superposition analysis. The complex mode shapes are calculated by the program DAMPEDROTOR.
Input: The input data consists of a description of the rotor, the mode shapes and damped critical speeds, and the loading as a function of time. The type of loadings that are admissible are:

1. Motion of the base of any of the bearings.
2. Applied bending moment on the rotor.
3. Applied shear force on the rotor.

The input requires that the user define the position at which the forcing function is to act, the type of function, and the actual data describing the force. Initial displacements and velocities of points along the shaft may be prescribed.

Output: The deflection, slope, moment, and shear at each station is printed. This is repeated for increments in time.
Language: FORTRAN
Hardware: IBM, CDC
Usage: This is a new program with limited usage.
Developer: Walter D. Pilkey
Department of Engineering Science and Systems
University of Virginia
Charlottesville, Virginia 22901

Availability: Deck and documentation available at a nominal cost. Contact W. D. Pilkey.
Comments: This program should be useful for studying the time-history of a rotor subjected to a wide variety of excitations. It appears to be one of the few programs that will handle arbitrary ground motion fed into a bearing system.

Other useful programs and their sources are briefly described below:

CADENSE: Program No. 26 - Lateral Critical speeds of multi-level rotors. Produces critical speeds of vibration. Bearings are represented by non-cross coupled springs. Damping does not appear to be included.
CADENSE: Program No. 27 - Unbalance Response of a Multi-Level rotor can handle up to three interconnected rotors. Produces circular orbits only. Bearings are represented by direct translational and angular stiffness and damping values. Cross-coupling is not included.
CADENSE: Program No. 20 - Lateral Critical Speeds of Flexible Rotors. Computes lateral critical speeds of single or coupled rotors with flexible bearing supports. Isotropic, linear bearing characteristics are assumed. Damping not included.
Source: Mechanical Technology Incorporated, MTI, Latham, New York 12110
Contact: Mr. Paul Babson

CSPRJT: Fortran IV Computer Program for Calculating Critical Speeds of Rotating Shafts, Roger J. Trivisonno
This program calculates the critical speeds of rotating shafts. The shaft may include bearings, couplings, extra masses (nonshaft mass), and discs for the gyroscopic effect. Shear deflection is also taken into account, and provision is made in the program for sections of the shaft that are tapered. The boundary conditions at the ends of the shaft can be fixed (deflection and slope equals to zero) or free (shear and moment equal to zero). The fixed end condition enables the program to calculate the natural frequencies of cantilever beams. Instead of using the lumped-parameter method, the program uses continuous integration of the differential equations of beam flexure across different shaft sections. The advantages of this method over the usual lumped-parameter method are less data preparation and better approximation of the distribution of the mass of the shaft. A main feature of the program is the nature of the output. The Calcomp plotter is used to produce a drawing of the shaft with superimposed deflection curves at the critical speeds, together with all pertinent information related to the shaft. NASA TN D7385 (with the same title) is a complete report of the program and its use.
Source: Cosmic, Information Services, 112 Barrow Hall, University of Georgia, Athens, Georgia 30602, Reference: LEW-11910.

CRITSPEED: Th performs a critical speeds analysis of rotating shafts. The shaft can have circular or square, solid or hollow segments, each having different material properties. The program will handle concentrated weights, non-aligned shaft segments, and fixed or pinned shaft supports. Lateral and torsional spring constants may be used to approximate arbitrary stiffnesses at a support point. The shaft critical speeds are found by calculating the lateral natural frequencies of a lumped-mass shaft model. The number of critical speeds to be computed is selected by the user. Data can be supplied interactively or from a data file. The program can be recycled to specify new shaft geometry and materials, or new support conditions, or new concentrated weights.
Output: Mass model printout, mass model acceptability check ratio, critical speeds.
Input: 15 shaft segments, 15 support stations, 25 concentrated weight, 75

unconstrained degrees of freedom.
Source: Control Data Corporation, CDC KRONOS Time-Sharing System

SPIN: Static and dynamic in-plane bending analysis of beams and rotating shafts on elastic foundations yielding deflection, slope, bending moment, shear, and bending stress. Finds response to static or harmonic loads. Calculates natural frequencies and mode shapes including critical speeds of rotating shafts. Whirling effects included. Plotted results of deflection, slope, moment, shear, and stress for static or dynamic loads can be generated on incremental plotters or storage tube terminals.
Source: Structural Dynamics Research Corporation, Cincinnati, Ohio

TORSIONAL VIBRATIONS

Torsional vibrational analysis generally follows the philosophies expounded for lateral vibrations of shafts. Some of the key items are discussed below:

1. Critical Speeds: A significant item to be determined in any torsional system are critical speeds. It is quite difficult to apply torsional damping and operation near torsional criticals should probably be avoided.
2. Mode Shapes: The critical speed analysis will generally produce mode shapes which are helpful in determining where the maximum amplitudes in a system are.
3. Response: It is desirable to know what the amplitudes of torsional vibration are as a function of a vibrating applied torque. Response routines are available to accomplish this. The program should be capable of including damping in the system.
4. Branched Systems: A torsional program should be capable of handling multiple branched geared system.

Some examples of computerized torsional vibrational analysis follow.

Figure 8 shows a system with a single branch. Results are indicated for both a critical speed and response analysis. Figure 8 indicates the natural frequencies obtainable from the critical speed analysis. Figure 9 indicates the mode shapes obtainable from this analysis. For the response analysis, the system was excited with a harmonic torque acting at disc #1. The exciting frequency was chosen as a nonresonant one. The results obtained from the computer run are summarized in Fig. 10.

Figure 11 shows a two-branch system. In the computer run, the damping of the system, i.e., damping of the generator, was specified. For response analysis, the system was excited with a harmonic torque acting at the generator. The computer runs covered a number of exciting frequencies including the first and the second natural frequencies of the system. The results are summarized in Figs. 12, 13, and 14. The plots in Fig. 13 show the vibratory amplitudes at the generator when the system is excited with the first natural frequency. Maximum vibratory amplitude vs. exciting frequency are shown in Fig. 14.

Available torsional computer programs follow:

Torsional Critical Speeds of a Geared System (CADENSE-22)

General Description: This program calculates the critical frequencies of a torsional system. The system may include branches, gears, epicyclic gears, and elastic torsional connection to ground. At each critical frequency, the program evaluates the normalized mode shape and corresponding torque distribution.
Analysis: The program employs the Holzer method extended to account more accurately for continuously distributed shaft sections. Effective root methods are employed to calculate all critical frequencies within specified ranges.
Features: The ability to handle epicyclic gear trains. Rapid execution time.
Input: Rotor geometry. Rotor material properties. Additional concentrated inertias. Single reduction gear dimensions. Epicyclic gear train dimensions. Frequency ranges to be searched.

Undamped System

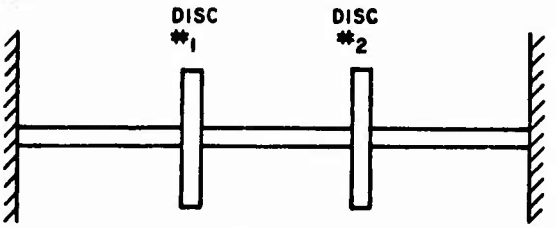


Fig. 8 Single branch model

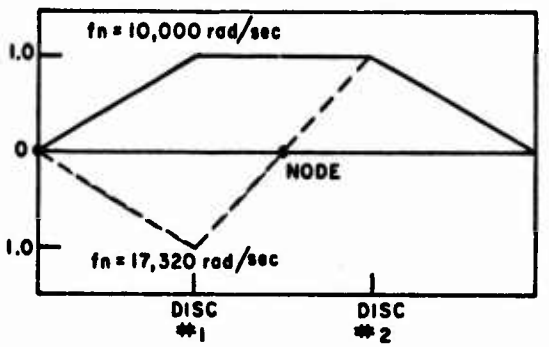


Fig. 9 Natural frequencies and mode shapes of single branch model

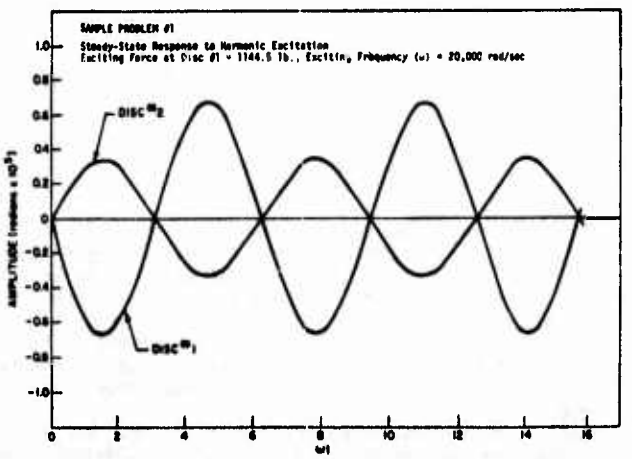


Fig. 10 Amplitude response of single branch model

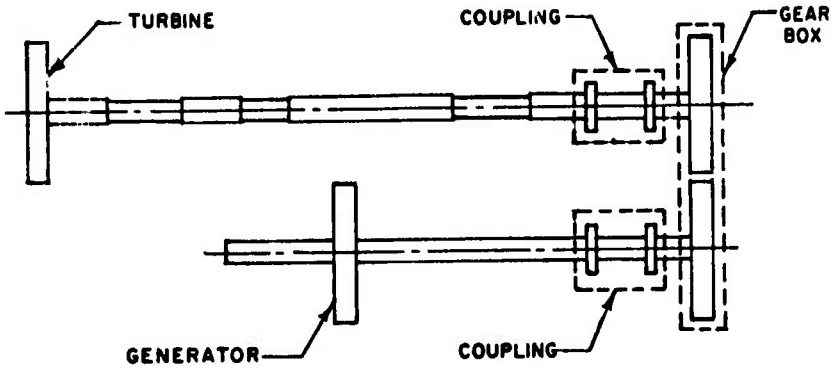


Fig. 11 Double branch model

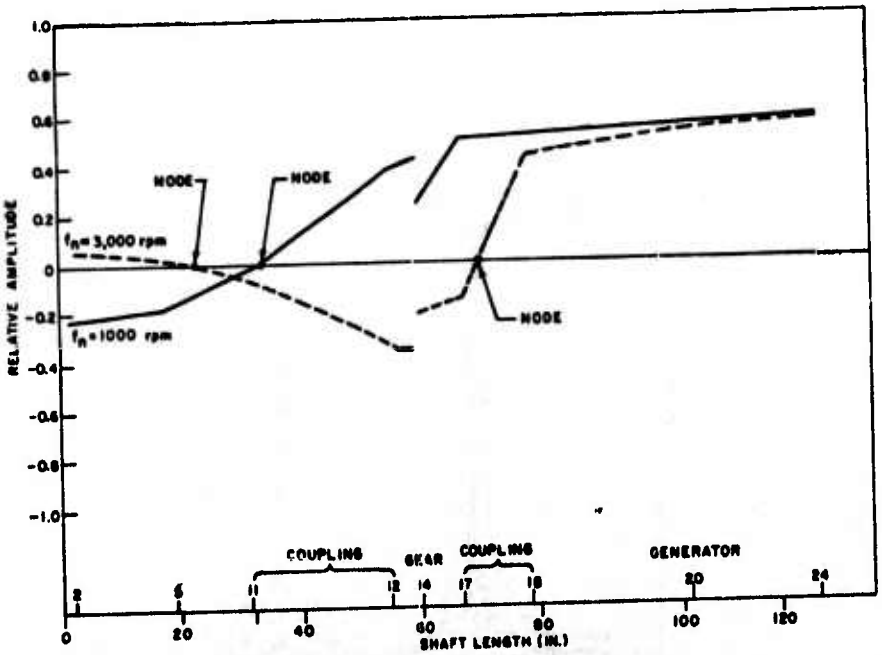


Fig. 12 Mode shapes, double branch system

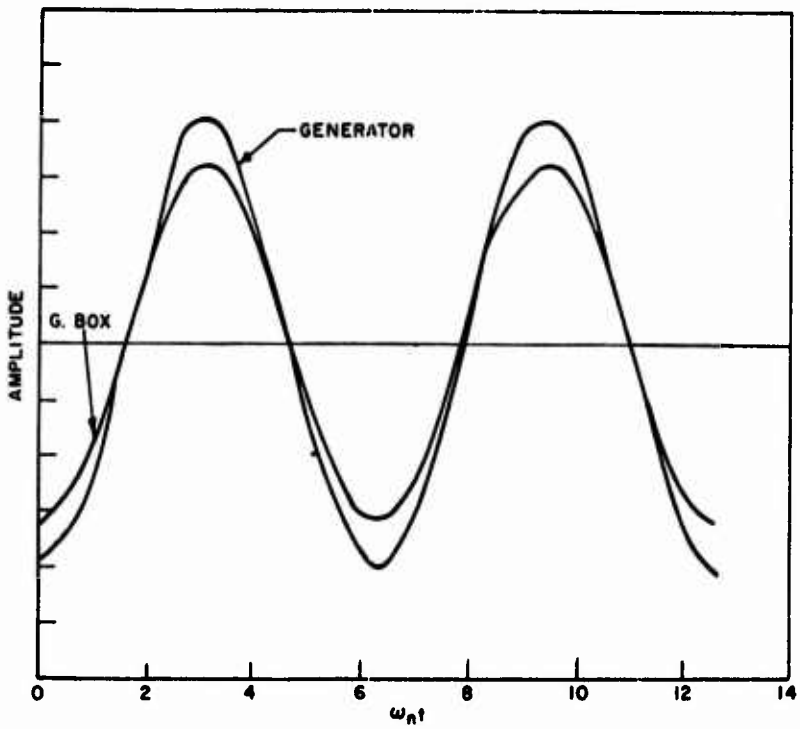


Fig. 13 Steady-state response to harmonic excitation of double-branched system

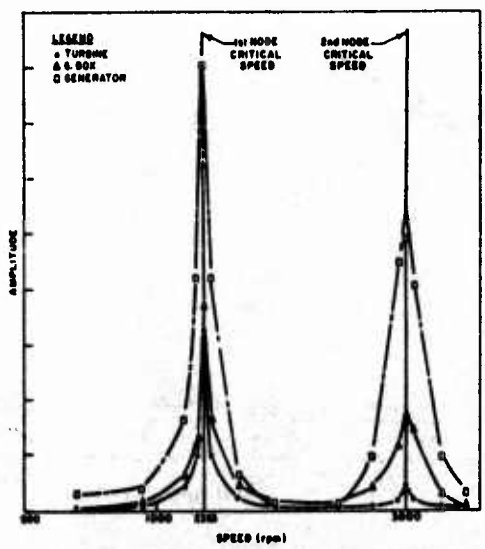


Fig. 14 Forced vibration amplitudes - double branched system

Output: Torque as a function of frequency. Critical frequencies. Mode shape at each critical. Torque distribution at each critical.
Memory Requirement: 20,000 words.
Running Time: Typically 15 seconds for CDC 6600
Language: Fortran IV
Availability: Mechanical Technology Incorporated
968 Albany Shaker Road
Latham, New York 12110
Comment: This program is quite typical of a torsional critical speed program.

Damped Torsional Response of a Geared System (CADENSE-23)

General Description: This program computes the damped torsional response of a system to excitation in the form either of torques, or angular displacements (gear errors). The system may include branches, single reduction gears, and constraints to ground with both stiffness and damping. The gears may be rigidly or elastically mounted. The system amplitudes, torque distribution, and gear tooth meshing forces are calculated by the program, and for elastically mounted gears the transmitted forces and gear tooth displacements are evaluated.
Analysis: A Holzer-type method is used, extended to account more accurately for continuously distributed shaft sections.
Features: The ability to handle epicyclic gear trains. The ability to accept excitation in the form of gear manufacturing errors. An option is available to include gear noise prediction on a long-term lease only basis.
Input: Rotor geometry. Rotor material properties. Additional concentrated inertias. Single reduction gear dimensions. Epicyclic gear train dimensions. Frequency values of interest. Gear error amplitudes. Exciting torque amplitudes.
Output: Rotor station amplitudes at each frequency. System torque distribution. Gear tooth forces. Gear shaft bearing forces (for elastically mounted gears). Axial and lateral tooth forces and displacements (for elastically mounted gears).
Memory Requirements: 40,000 words.
Running Time: Typically 5 seconds CPU for CDC 6600
Language: Fortran IV
Availability: Mechanical Technology, Inc.
968 Albany Shaker Road
Latham, New York 12110
Comment: This is indeed a unique and useful program, since it has the capability to simulate start-up and shut down conditions and backlash in gears.

Torsional Vibration Analysis (TORVIB)

Purpose: To determine natural frequencies and transient response of undamped and damped systems.
Applications: Determination of natural frequencies, vibratory amplitudes, and dynamic stresses for systems such as (i) power transmission system, i.e., combination of internal combustion engines, turbines, pumps and compressor (ii) power generating system, i.e. combination of electric generator, turbine, etc.
Capabilities: Computes critical speeds and mode shapes, and response to torsional excitations, if desired. Each shaft may have a number of masses. Multi-shaft (shafts connected by gears, coupling, etc.) The system may contain up to fifteen branches and there is no restriction on the number of branches at one junction. Couplings: flexible, hydraulic Hooke's coupling. Flexible support; torsional spring, etc. Damping; material, system damping and external damping. System excitation; excitation torque may be an arbitrary function of time, i.e. harmonic, step and

ramp type. System may be excited at a number of locations either with the same or different type of exciting function. The exciting functions may be in phase or out of phase with each other. Dynamic stresses. Gear tooth flexibility.

Method: The method treats the system as a series of masses connected by shafts. For a multi-shaft system, the actual system is replaced by a dynamically equivalent system in which all shafts and masses rotate with the same angular velocity. Using D'Alembert's principle, the equilibrium equations are written for each station of the system. The number of equations is equal to the number of degrees of freedom in the entire rotating assembly. For stability calculations, the set of equations is transformed into standard form of eigenvalue problem. The problem is then solved by the QR-2 algorithm. The solution yields the natural frequencies and the mode shapes of the system. Response of the system is carried out by the method of modal analysis. The solution to the governing equations is written in terms of the convolution integral which is then integrated numerically.

Input: Geometry and inertia properties of rotating system. Rotating speeds of multi-shaft system. Damping; material, system-damping and external. Flexible supports. Data for response: excitation torques, frequencies, phase angles and locations

Output: Natural frequencies. Growth factors. Mode shapes. Vibratory amplitudes at transient and steady state. Dynamic stresses.

Language: FORTRAN IV

Memory: 65,000 words

Typical Running Time: 30 seconds CPU or UNIVAC 1108

Availability: Mr. W. Shapiro
The Franklin Institute Research Laboratories
Philadelphia, Pa.

Torsional Analysis of Shaft Systems (TASS)

Introduction: The TASS computer program calculates the torsional critical frequencies and the forced dynamic response in torsion of undamped shaft systems. The static deflection pattern can also be found by forcing the shaft at 0.0 rpm. This program uses a distributed mass approach. TASS can analyze any single branched gear train system with any number of gear trains. Each gear train is specified by giving the torsional stiffness of the gears and the gear ratio. External forces, lumped inertias and torsional springs to ground can also be included in the analysis.

Language: FORTRAN IV

Availability: Structural Dynamics Research Corporation
Cincinnati, Ohio

Torsional and Longitudinal Natural Frequencies (TORLONG)

Capability: The torsional and longitudinal natural frequencies of a general branched shafting system modeled as a series of stations are calculated. A shafting station consists of a section of distributed parameter modeled shaft, a lumped inertia, and springs to ground.

Method: The Holzer-Myklestad-Prohl method is used to calculate natural frequencies.

Hardward: UNIVAC 1108

Developer: Dr. Ronald L. Eshleman

Availability: Cost $500.00. Includes deck, instruction manual, and examples.
Contact: The Vibration Institute
5401 Katrine
Downers Grove, Ill. 60515

Attn: Dr. Ronald L. Eshleman, Director

Torsional System (TWIST)

Capability: For static and steady state torsional loads it calculates the angle of twist and the twisting moment of a shaft. It also computes the natural frequencies and mode shapes of torsional vibration. The torsion system can be a bar formed of uniform segments with any loading, gears, branches, foundations, and boundary conditions.
Method: Transfer matrix method.
Input: Batch or timesharing. An interactive preprocessor is available.
Language: FORTRAN
Hardware: IBM, CDC, UNIVAC, HONEYWELL, PDP
Usage: Well tested
Developed: The Structural Members Users Group
Dept. of Engineering Science and Systems
University of Virginia
Charlottesville, Va. 22901
Attn: W. D. Pilkey
Availability: Deck and documentation are available at a nominal cost from developer. Also available on commercial systems.

FLUID-FILM BEARINGS

Computer codes are available to treat practically any type of fluid film bearing configuration. Figures 15 and 16 schematically indicate some common bearing types. In some diagrams a fluid resistance is coupled to a recess incorporated into the bearing area. These represent external pressurization that is supplied to the bearing clearance to augment hydrodynamic pressure capacity. The varying configurations are not as difficult to model as they appear to be. It is sometimes sufficient to simply vary a clearance formation routine to change the bearing type and this may only involve several punched cards. For Rayleigh - Step bearings additional measures must be taken to insure pressure and flow continuity at the step junction. Although this chapter is not intended to prescribe what bearings should or should not be used, abbreviated comments are discussed below.

<u>360° Cylindrical (Plain)</u>
- •simple construction
- •high load capacity
- •whirl prone
- •high heat generation
- •can be contamination sensitive
- •comments - should be used if possible, but should be carefully checked for problem areas. Generally used for medium speed, medium to heavy load applications.

<u>360° Cylindrical with Axial Slots</u>
- •a little less simple, and a little less load capacity than the full 360° bearing.
- •a better cooling available (oil feed) through slots and less contamination sensitive
- •comments - should be considered as an alternative to a 360° plain journal if it can correct a reason for elimination of a plain bearing.

<u>Elliptical and Lobe Bearings</u>
- •good whirl resistance
- •reasonably good load capacity

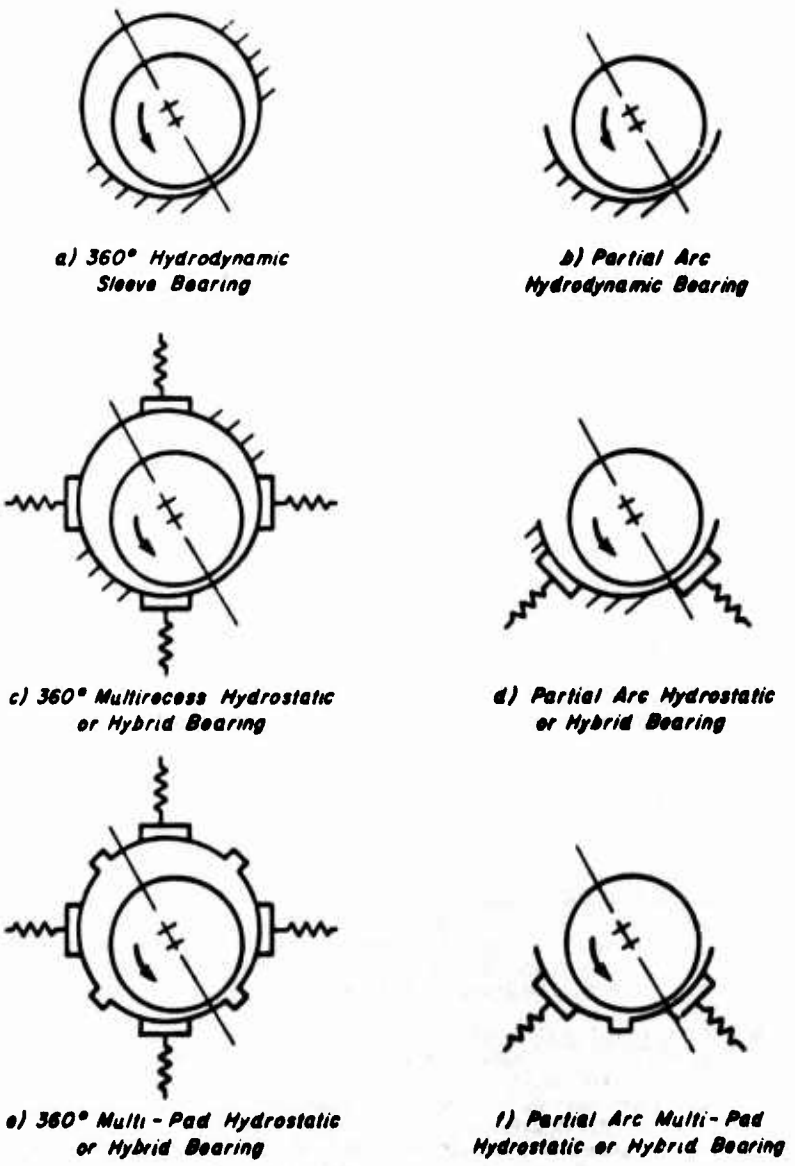
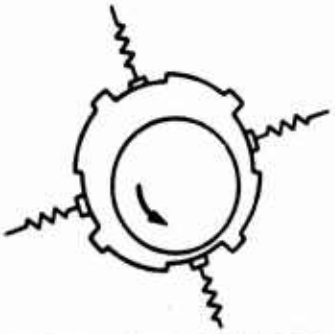


Fig. 15 Simple journal bearing types

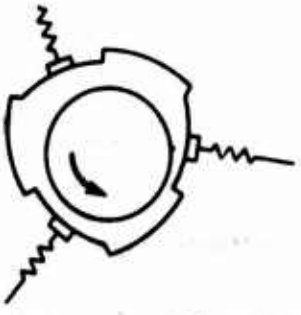
g) Multi or Single Pad Rayleigh Step Hydrodynamic Bearing

h) Multi-Lobe Hydrodynamic Bearing



i) Multi or Single Pad Rayleigh Step Hydrostatic or Hybrid Bearing

j) Combination Lobe and Rayleigh Step Hydrodynamic Bearing



k) Combination Lobe and Rayleigh Step Hydrostatic or Hybrid Bearing

l) Tilting Pad in all Combinations

Fig. 16 Some complex journal bearing types

- complicated to manufacture (except for elliptical)
- clearance and tolerance sensitive
  - comments - elliptical bearings have poor horizontal stiffness canted lobes (complete converging film) are better than symmetrical lobe configurations
    General use is for high speed, and for low load applications where whirl is a problem.

Rayleigh Step Bearings

- excellent load and whirl free characteristics
- expensive to manufacture and tolerance sensitive
  - comments - only used in special applications because of difficulty to manufacture.

Tilting Pad Bearings

- most whirl free
- not as clearance sensitive as most bearings
- generally provide less damping
- expensive
- fretting corrosion of pivots for high speed applications
  - comments - because of whirl free characteristics these bearings are used extensively. Generally to facilitate manufacture they are designed and made with insufficient pre-load and excessive clearance

Externally Pressurized or Hybrid

- very high load capacity
- extreme stiffness capability
- flow through clearance region can have a cooling effect
- clearance and tolerance generally more liberal than hydrodynamic bearings
- requires external fluid supply system
  - comments - applied when there is insufficient hydrodynamic generating speed, or where very high load capacity and stiffness are required. Sometimes applied to prevent whirl, but they are not as whirl-free as one might intuitively think. Rotational speed can offset recess pressures in a way to promote whirl.

The basic governing equation for fluid-film bearings is Reynold's Lubrication Equation

$$\frac{\partial}{\partial x}\left(\frac{h^3}{\mu}\frac{\partial P}{\partial X}\right) + \frac{\partial}{\partial Y}\left(\frac{h^3}{\mu}\frac{\partial P}{\partial Y}\right) = 6U\frac{\partial h}{\partial X} + 12\frac{\partial h}{\partial t} \qquad (4)$$

Variation in equations occur, under conditions of turbulence but can be accounted for. Turbulence occurs when local Reynolds No, Re $\geq$ 1000

$$Re = \frac{\rho \mu h}{\mu}$$

A non-dimensional form of Reynolds equation applicable to both laminar and turbulent lubrication is as follows:

$$\frac{\partial}{\partial X}\left[12G_x H^3\right]\frac{\partial P}{\partial X} + \frac{\partial}{\partial Y}\left[12G_y H^3\frac{\partial P}{\partial Y}\right] = \Lambda\left[\frac{\partial h}{\partial X}\frac{U}{U_1} + \frac{\partial H}{\partial T}\right] \qquad (5)$$

G_x and G_y are local plain eddy viscosity coefficients which are used for turbulence corrections. They are dependent upon a local Reynolds number. If the Reynolds' number indicates a laminar flow, then the G's are given the appropriate laminar coefficient values [4].

Solution Methods

There are a number of solution techniques. The first and most common employs the formation of the differential equations into finite difference equations. Within this category lie many numerical variations from the forward and direct explicit formulations to the more sophisticated implicit schemes [5,6]. Reference [7] is an excellent compilation of many common types of finite difference techniques indicating advantages and preferences.

Another basic formulation stems from a variational integral approach and is termed the finite-element method. This technique has a long history in the field of elasticity, but has general applicability to a variety of disciplines. Reddi has applied this method to both compressible and incompressible lubrication problems [8, 9].

A typical grid network for a finite difference scheme is shown in Fig. 17. Alongside the grid network is shown the pad bearing representation. It consists of a pad with two recesses per pad. Each grid point is given a code number that clues the computer. For instance, a 0 means that the pressure is specified at that particular point with zero gage pressure (incompressible bearing) at the boundary points. The 11 and 12 indicate the positions of the recess and the computer is notified that the recess pressures are to be determined there in conjunction with the feed circuitry and restrictor compensating types. The index 8 means it is a point where the pressure is to be computed from Reynolds' equation. The code representation offers great flexibility because recesses, grooves, etc. can be arbitrarily located in a simple fashion. Other code numbers exist for positions of Rayleigh steps, lines of symmetry and joined boundaries.

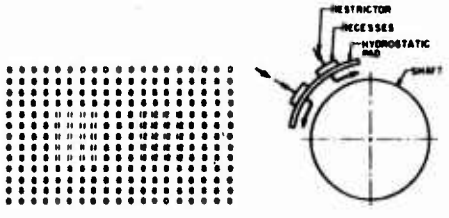


Fig. 17 Grid network finite difference scheme

Figure 18 shows a typical bearing representation of a hydrostatic pad for a finite element analysis. Either quadrilateral or triangular elements of arbitrary size and orientation can be used to represent the bearing land area. The blank spaces are recesses. There are no restrictions with regard to coordinate reference frames, such as the case for finite-difference, and fillets, radii tapers, etc. present no special difficulty. Although not shown on Fig. 18, it is necessary to number each element and identify each nodal point. The greater flexibility of finite element schemes is somewhat counter-balanced by more difficult input preparation.

Typical program output includes:

- pressure distribution throughout grid network
- load capacity
- side leakage and carryover flows
- viscous power losses
- righting moments due to misalignments
- attitude angles

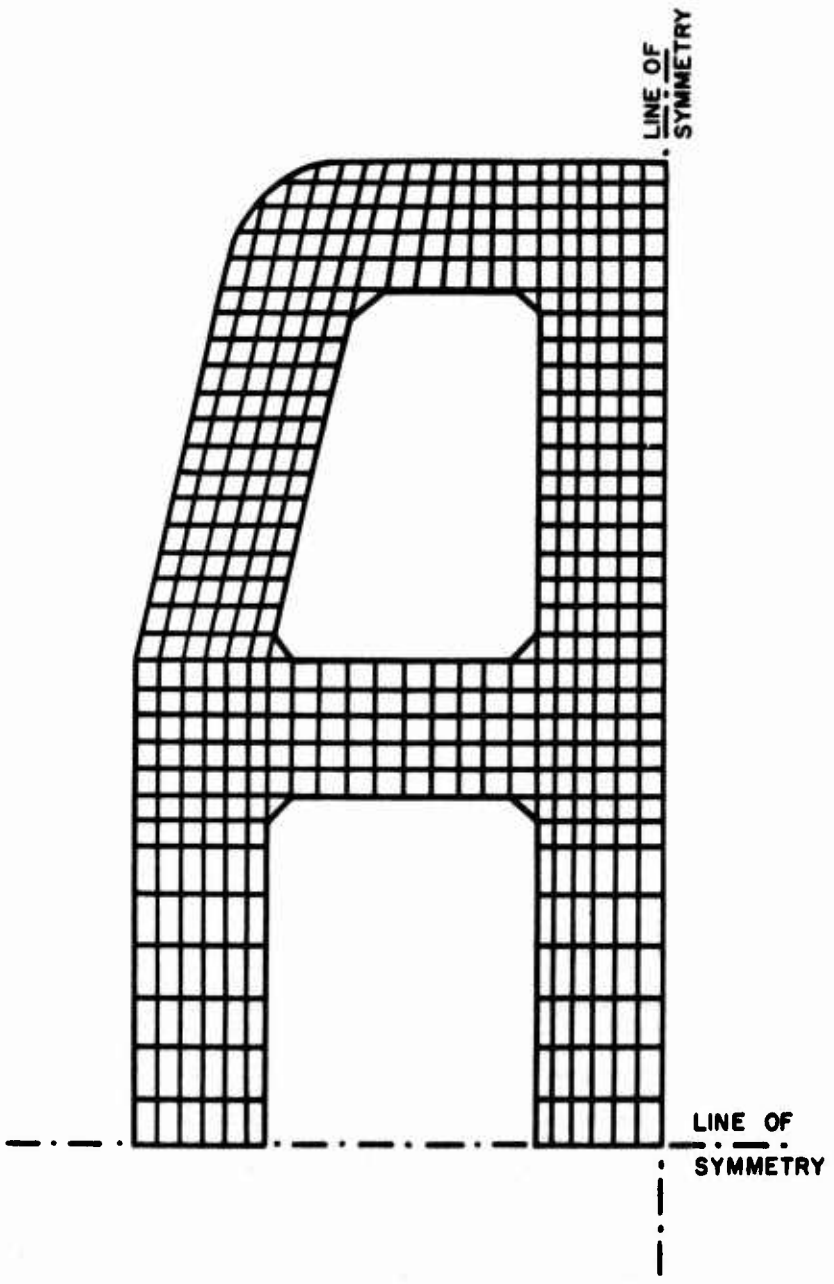


Fig. 18 Grid network finite element

•cross-coupled spring and damping coefficients due to displacement and velocity perturbations of the journal center. (These coefficients are necessary for rotor dynamic analyses).

Experimental Verification

Validation of codes has been accomplished by comparison with experimental data and information in the literature. Harry Rippel of the Franklin Institute conducted controlled tests on hybrid oil bearings with the express purpose of validating computer predictions. He tested a four recess hybrid bearing in which the recesses were symmetrically spaced. Results are shown in Fig. 19 and 20. These figures show dimensionless load capacity and attitude angle vs. eccentricity ratio for a pure hydrostatic bearing in which rotation was included. Through the use of dimensionless quantities, results are independent of supply pressure. The results indicate extremely good correlation.

Load Direction

Recent additions to computer codes permit application of arbitrary load direction. The program will reposition the journal until applied and resisting loads are colinear.

Tilting-Pad Bearings

To obtain steady state data for tilting-pad bearings, the pads are pivoted until moments due to fluid film pressures are nulled. Then the pad is in the proper position. It has been found to be advantageous to determine characteristics as a function of pivot-film thickness for a single pad. Then, for multiple pad bearings it is only necessary to interpolate single pad data points to establish individual pad contributions to steady-state performance. Total bearing performance is obtained by accumulating the individual pad results.

Dynamic Characteristics

Cross-Coupled Spring and Damping Coefficients - Obtained from Computer Analysis
Cross-coupled spring and damping coefficients are prominently employed in rotor dynamics analysis and in bearing stability analysis. The general method is as follows:

1. Obtain equilibrium operating position by steady-state analysis.
2. Perturb X and Y displacements independently.
3. Compute $K_{xx} = \Delta F_x/\Delta x$ $K_{yx} = \Delta F_y/\Delta x$
4. Return to equilibrium and apply velocity perturbations. $\dot{x}$ and $\dot{y}$
5. Compute $D_{xx} = \Delta F_x/\dot{x}$, $D_{xy} = \Delta F_x/\dot{y}$, etc.

These are a total of eight coefficients, four spring and four damping, for most fixed bearing configurations. (See Fig. 21)

Tilt-pad bearings are special. There is cross-coupling between pitch modes of shoe and displacements of journal. For a 5-shoe bearing there are 7 degrees of freedom and thus the spring and damping matrices are each 7x7.

Bearing Stability Analysis

Cross-coupled coefficients are used in establishing whether a bearing is prone to self-excited instabilities. Consider a journal of mass m operating in a bearing. The journal can be considered to have two degrees of freedom, x and y. The governing equations are

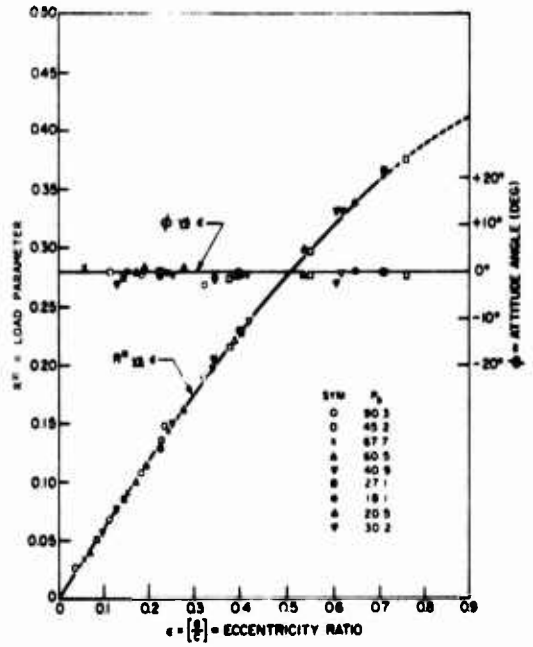


Fig. 19 Experimental verification of computer program INCYL

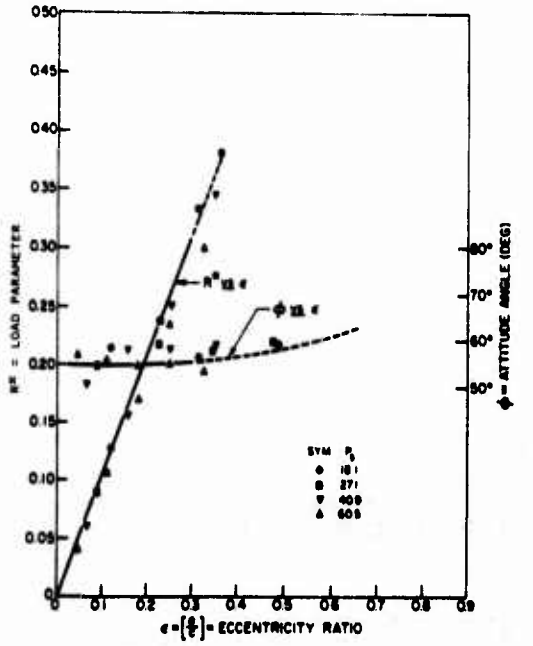


Fig. 20 Experimental verification of computer program INCYL,

$$m\ddot{x} + k_{xx}x + k_{xy}y + d_{xx}\dot{x} + d_{xy}\dot{y} = 0$$
$$m\ddot{y} + k_{yx}x + k_{yy}y + d_{yx}\dot{x} + d_{yy}\dot{y} = 0 \qquad (6)$$

Assume a small disturbance

$$x = x_0 e^{\beta t}$$
$$y = y_0 e^{\beta t} \qquad (7)$$

where β is a complex variable. Substituting into the equations of motions produces the following

$$\begin{bmatrix} (m\beta^2 + D_{xx}\beta + K_{xx}) & (d_{xy}\beta + K_{xy}) \\ (\beta d_{yx} + k_{yx}) & (m\beta^2 + \beta d_{yy} + k_{yy}) \end{bmatrix} \begin{Bmatrix} x_0 \\ y_0 \end{Bmatrix} = \{0\} \qquad (8)$$

The determinant of the coefficient matrix must vanish. Expansion produces a polynomial in β that can be solved for the roots of β. The real parts are growth or attenuation factors, and the imaginary parts are the frequencies. A positive real part implies an instability (see Fig. 22). A typical stability map is shown on Fig. 23. It shows a critical mass parameter as a function of Sommerfeld Number for a four sectored axial-grooved journal bearings.

The following are the more significant and useful programs.

Bearing and Seal System (INCYL)

Purpose: To determine steady-state performance, spring and damping coefficients and stability of cylindrical bearings in laminar regime.

Applications: General determination of journal bearing performance.

Capabilities: Bearing configurations include cylindrical, grooved, symmetrical and canted lobes (up to six lobes), Rayleigh step, pressure dam. Hydrodynamic, hydrostatic or hybrid bearing configurations. Cavitation. Symmetry boundary conditions to improve accuracy or speed of computation. As many as ten recesses, each with arbitrary specified capillary, orifice or flow control valve compensation. Recess pressures specified or computed in conjunction with feed circuit problem. Arbitrary placement of grooving and three other known pressure conditions. Production of dynamic spring and damping constants due to normal relative velocity of opposed surfaces. Nondimensional or dimensional input/output. Determination of stability where the shaft journal is considered as a point mass.

Method: The bearing area is subdivided into a grid pattern and the pressure at each point is determined by numerical solution of the Reynold's lubrication equation. For externally pressurized systems, continuity of flow through the supply circuit and bearing film is also satisfied. The Reynolds' equation is solved by the implicity, non-iterative matrix column inversion method [5]. Advantage is taken of the linearity of the equation by employing component solutions (influence coefficients) to solve the feed circuit problem [5].

Input: Geometry and orientation. Misalignment, eccentricity. Specified film thickness, optional. Location of origin for moments and tilts. Rotative speed. Normal velocity. Restrictor coefficient and size. Cavitation vapor pressure. Supply circuit pressures (or flows). Supply system feed circuitry. Perturbation for spring and damping coefficients.

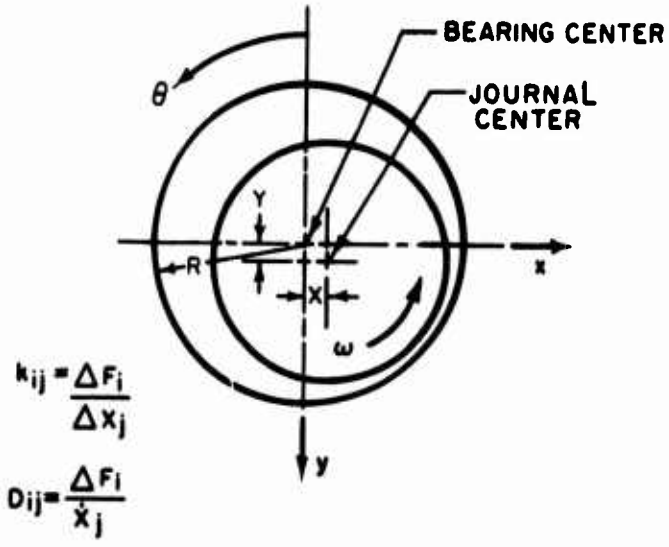


Fig. 21 Cross coupled spring and damping coefficients

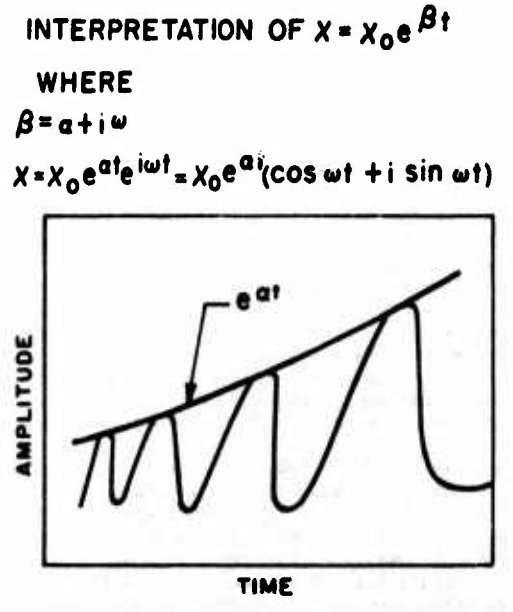


Fig. 22 Interpretation of stability parameters

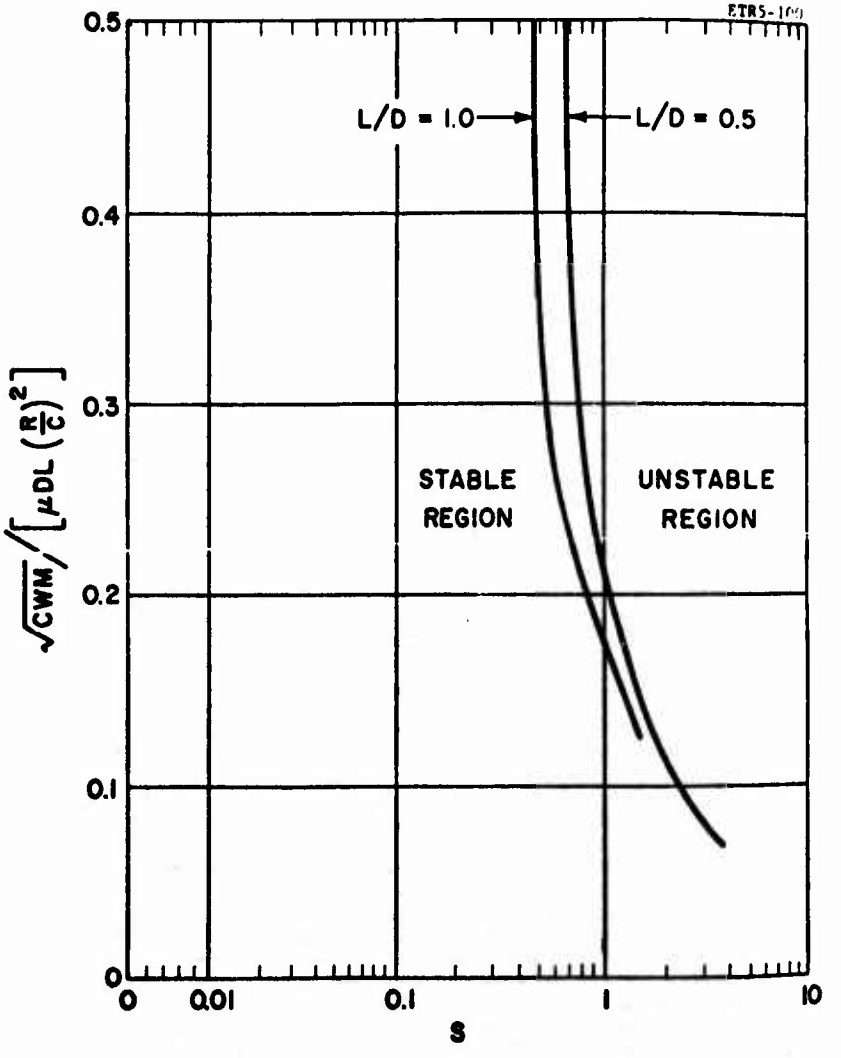


Fig. 23 Critical rotor mass factor at onset of instability as a function of the Sommerfeld Number (S) for four-sector axially grooved journal bearings (from [9])

Dimensional data for computation of stability.
Output: Pressure distribution. Load capacity. Flow (individual recess and total). Recess pressures. Viscous power loss. Center of pressure. Righting moments about orthogonal axes through origin. Minimum film thickness. Component solutions for individual recesses and velocities. Side leakage. Supply circuit flows (or pressures). Attitude angles. Cross-coupled spring and damping coefficients. Stability parameters.
Language: FORTRAN IV
Memory: 65,000 words
Running time: 10-20 seconds per case UNIVAC 1108
Author: W. Shapiro
Availability: The Franklin Institute Research Laboratories
Philadelphia, Pa. 19103
Attn: Mr. W. Shapiro, Manager of the Mechanical Engineering Laboratory

Tilting Pad Journal Bearing Performance (TPJB)

Purpose: To determine steady state and dynamic characteristics of tilting pad journal bearings.
Applications: Design of tilting pad journal bearings incompressible laminar and turbulent regime.
Capabilities: Option for non-dimensional or dimensional input. Computation of steady state performance of multi-pad bearings. The pivot position is pre-specified. Option for computation of cross-coupled spring and damping coefficients of multi-pad bearing. Complete cross-coupled spring and damping matrices are produced in a square matrix. The size of the matrix is equivalent to the number of degrees of freedom.
For example, for pads that pitch only, the number of degrees of freedom is equivalent to the number of pads plus 2, which are the journal displacements. In addition to the stiffness and damping matrices, equivalent spring and damping coefficients are determined for use in synchronous unbalance response studies. The equivalent matrix is the 4 x 4 matrix reduced from the complete matrix and is a function of the complete matrix coefficients, the shoe inertia properties, and the vibrational frequency which is assumed synchronous. Option for computing whirl and flutter stability. The stability information is produced as responses to small disturbances in terms of growth factors and frequency. A positive growth factor implies a growing response or instability. The resulting frequencies are useful because they represent natural frequencies of the system. Option for completing a heat balance. The heat balance produces inlet side leakage and carry-over temperatures as well as the average temperature for viscosity determinations and steady state performance. Options for conducting a time transient or nonlinear orbit analysis. The shoes are permitted degrees of freedom in pitch and radial translations while the journal is permitted X and Y translations. Option for producing either dimensional or nondimensional output.
Method: Steady-State Performance - steady state performance of each of the individual shoes is obtained by an iterative scheme whereby the shoes are pitched until moments about the pivot position are nulled. Bearing performance is accomplished by accumulating the individual pad data.
Spring and Damping Coefficients - these are obtained by perturbing the journal in X and Y displacements and velocities while holding the pads in their equilibrium positions. The pads are then given pitch displacements and velocities while the shaft is held in its equilibrium position. Equivalent spring and damping coefficients are a function of the stiffness and damping matrices plus the frequency of vibration. Synchronous frequency is assumed.
Stability - utilizing spring and damping coefficient matrices, whirl stability of the journal and flutter stability of the individual pads are established by analyzing the complete coupled system of pads and journal.

The stability information is produced as responses to small disturbances in terms of a growth factor and frequency.

Heat Transfer - specified inlet flow is mixed with the carry-over flow from the preceding pads to produce an inlet temperature at the leading edge. Viscous heat generation is then added to the fluid flowing through the pads which permits computation of a side leakage and exit flow temperatures. All heat transfer is between fluids.

Time Transient Analysis - at each interval of time, Reynolds' equation is solved throughout the grid network representing the pads. Fluid film forces are added to external forces and the dynamics equations establish accelerations, velocities and displacements for the next time interval. The displacements produce a new clearance distribution in the bearing which requires solution of Reynolds' equation and the process continues.

Input: Initial pitch displacements, shaft displacements, pitch velocities and shaft velocities. Geometry of the bearing system. Pivot locations, pivot film thickness. Operating speeds and external forces. Various indicators for options.

Output: Individual pad forces and accumulated forces. Individual pitch positions of the pads. Individual flows and accumulated flows. Individual friction viscous drags and accumulated viscous friction. Pressure distribution in all pads. Stiffness and damping matrices. Stability data.

Language: FORTRAN IV

Memory Requirements: 65,000 words

Receiving time: 10-20 seconds per case on UNIVAC 1108

Availability: The Franklin Institute Research Laboratories
Philadelphia, Pa. 19103
Attention: Mr. W. Shapiro - Manager of the Mechanical Engineering Laboratory.

Turbulent-Laminar Hybrid, Cylindrical Bearings (ITURB)

Purpose: To determine steady-state performance characteristics of hybrid bearings operating with cavitated regions in the laminar or turbulent, incompressible flow regime.

Application: Primarily applicable to hydrodynamic, hydrostatic, and hybrid bearings which operate with turbulence, either pressure induced or rotation induced.

Capabilities: Variable grid. Eccentricity, misalignment, and deformation to specify arbitrary clearance distributions. Cavitation for hydrodynamic and hybrid modes. 360 degrees or partial arc cylindrical bearings. Symmetry boundary conditions on edges to improve accuracy or speed of computation. Maximum of twenty recesses per bearing with capillary compensation fed from a constant pressure supply or constant flow to each recess. Recess pressures specified or computed in conjunction with feed circuit problems. Arbitrary placement of grooving and three other known pressure conditions. Consideration of inertia effects at recess and groove boundaries. Dynamic spring and damping constants due to normal relative velocity of opposed surfaces. Dimensionless parameters for increased generality.

Method: The bearing area is subdivided into a rectangular grid pattern and the pressure at each grid point determined by simultaneous numerical solution of modified turbulence mass flow and momentum from that point. Turbulence effects on the fluid flow are considered for each grid domain. The fluid film flow results are coupled to the external supply by flow continuity to solve for recess pressures and flow requirements [6]. An iterative scheme is employed to solve the problem for unknown recess pressures [6].

Input/Output: Geometry and orientation. Misalignment, eccentricity, structural deformation. Specified film thickness. Lubricant properties. Location of origin for moments and tilts. Rotative speed. Axial flow

pressure gradient parameter. Normal velocity. Restrictor coefficient and size. Cavitation vapor pressure. Supply circuit pressures (or flow).

Output: Pressure distribution. Load capacity and attitude angle. Flow (individual recess and total). Recess pressures. Viscous power loss. Center of pressure. Righting moments about orthogonal axes through origin. Minimum film thickness. Side leakage. Supply circuit flows (or pressures).

Language: FORTRAN IV

Memory Requirements: 65, 000 words

Running time: 10-20 seconds a case on UNIVAC 1108

Availability: Franklin Institute Research Laboratories
Philadelphia, Pa. 19103
Attention: Mr. W. Shapiro, Manager of the Mechanical Engineering Laboratory.

Performance of Liquid Lubricated Journal Bearings (CADENSE-30)

General Description: This program computes the normalized film forces, flow rates, friction factor, and stiffness and damping coefficients for multiple pad (with or without preloading) and single arc liquid lubricated journal bearings. The single arc data is suitable for assembly into tilting pad bearing data. The program accounts for cavitation of the lubricant, and is applicable to both laminar and turbulent lubricant regimes. The program allows specification of journal location relative to the pads, or is capable of iteratively determining the correct journal attitude angle for a specified load direction. The critical mass for bearing stability is also calculated.

Analysis: A finite difference solution of the two-dimensional incompressible lubrication equation is used to calculate the bearing pressure distribution for a particular journal location. If the load direction is specified relative to the bearing pads, the program initially assumes a journal location and integrates the corresponding pressure distribution to yield bearing film force components and the direction of their resultant. The journal location is then iteratively adjusted until the bearing forces act vertically within a specified convergence limit, at which stage the stiffness and damping coefficients are calculated by dynamic perturbation of the pressures.

Features: Provides data in dimensionless form, applicable to a wide range of operating conditions.

Input: Bearing L/D ratio. Reynolds number (if turbulent). Bearing pad geometry.

Output: Bearing film forces. Bearing eccentricity ratio. Friction. Flow rate. Attitude angle. Stiffness coefficients. Damping coefficients. Critical journal mass coefficients.

Memory Requiresment: 32,000 words

Running Time: Typically six seconds CPU for CDC 6600/Bearing eccentricity.

Language: FORTRAN IV

Availability: Mechanical Technology, Inc.
968 Albany Shaker Rd.
Latham, N.Y. 12110

Design of Liquid Lubricated Axial Groove Journal Bearings (CADENSE-31)

General Description: This program calculates dimensional static and dynamic performance data for liquid lubricated axial groove journal bearing. It will, at the user's option, perform a heat balance upon the power loss, lubricant flow, and lubricant characteristics to yield an effective, mean operating viscosity, and corresponding performance data. The program

operates on basic dimensionless data to generate dimensional bearing data. Dimensionless data are obtained from Cadense Program CAD-30. Sets of CAD-30 output data may be purchased individually. One set is initially provided with CAD No. 31.

Analysis: The basic data consists of dimensionless performance parameter values at a series of discrete values of journal eccentricity ratio. Dimensional design and performance data are calculated by the program for specified bearing size operating speed, applied load, and lubricant properties, using interpolation to establish continuously varying performance quantities in terms of the discrete basic data points.

Language: FORTRAN IV

Availability: Mechanical Technology, Inc.
968 Albany Shaker Rd.
Latham, N.Y. 12110

Design of Liquid Lubricated Tilting Pad Journal Bearings (CADENSE-32)

General Description: This program calculates dimensional static and dynamic performance data for liquid lubricated tilting pad journal bearings. It performs a heat balance based upon power loss, lubricant flow and lubricant characteristics to yield an effective mean operating viscosity and corresponding performance data.

Analysis: The program operates on basic dimensionless pad data to generate dimensional bearing data. Dimensionless data are obtained from Cadense Program No. CAD-30. Sets of data may be purchased individually (see price list). One is initially provided with CAD-32. The basic pad data consists of dimensionless performance parameter values for a single bearing pad at a series of discrete values of journal eccentricity ratio. The program initially assembles the single pad data to yield dimensionless performance parameters for a journal bearing with a specified number of pads, which may be preloaded or nominally concentric with the journal. The assembled bearing data is calculated and stored by the program for a range of discrete bearing eccentricity ratios. Dimensional performance data is then calculated by the program for specified bearing size, operating speed, applied load, and lubricant properties, using interpolation to establish continuously varying performance quantities in terms of the discrete assembled data points. The heat balance is performed assuming all heat is carried away by the lubricant (a conservative assumption). The user is free to specify the fraction of overall temperature rise to be applied in determining the effective mean operating viscosity.

Input: Bearing geometry. Lubricant characteristics. Speeds of interest. Applied loads of interest. Dimensionless bearing data.

Output: Bearing eccentricity ratio. Lubricant flow. Power loss. Stiffness and damping values. Bearing temperature rise.

Memory Requirement: 20,000 words.

Running time: Typically 3 seconds CPU for CDC 6600

Language: FORTRAN IV

Availability: Mechanical Technology, Inc.
968 Albany Shaker Rd.
Latham, N.Y. 12110

Performance of Pressure Dam Journal Bearings (CADENSE-36)

General Description: This program computes the pressure distribution, normalized film forces, flow rate, friction factor, and stiffness and damping coefficients for the pressure dam journal bearing. The program accounts for cavitation of the lubricant, and is applicable to both laminar and turbulent lubrication regimes. The program allows specification of journal arcs, or is capable of iteratively determining the correct journal attitude angle for a specified load direction. The critical mass

for bearing stability is also calculated.
Analysis: A finite difference solution of the two-dimensional incompressible lubrication equation is used to calculate the complete bearing pressure distribution for a particular journal location. The solution method accounts for surface discontinuities in the upper pad. If the load direction is specified relative to the bearing arc, the program initially assumes a journal location and integrates the corresponding pressure distribution to yield bearing film force components and the direction of their resultant. The journal location is then iteratively adjusted until the bearing forces act vertically within a specified convergence limit, at which stage the stiffness and damping coefficients are calculated by dynamic perturbation of the pressures.
Features: Provides data in dimensionless form, applicable to a wide range of operating conditions.
Input: Upper arc L/D ratio. Lower arc L/D ratio. Reynolds No. (if turbulent). Angular extent of upper and lower arcs. Location of arcs relative to load direction. Upper arc pocket width, length, depth ratios.
Output: All input. Load capacity. Friction. Flow rate. Attitude angle. Stiffness coefficients. Damping coefficients. Critical journal mass parameter.
Memory Requirement: 64,000 words (can be tailored to fit smaller capacities).
Running time: Typically 10 seconds CPU for CDC 6600/Bearing eccentricity ratio.
Language: FORTRAN IV
Availability: Mechanical Technology, Inc.
968 Albany Shaker Rd.
Latham, N.Y. 12110

Performance of Deep Pocket Hydrostatic Journal Bearings (CADENSE-38)

General Description: This program computes the pressure distribution, film forces, flow rates, and stiffness and damping coefficients for the multiple pocketed hydrostatic journal bearing, with or without axial drain grooves. The program accounts for independent orifice restriction to each pocket. The program is applicable both to turbulent or laminar operation.
Analysis: A finite difference solution of the two-dimensional incompressible, isoviscous, lubrication equation is used to calculate the bearing pressure distribution. The nonlinear "Ng-Elrod" theory of Poiseuille flow induced turbulence is employed. A simple "Bernoulli" pressure drop at exit from the pockets is imposed - based on the mean film velocity. The combined nonlinearity of turbulence, Bernoulli pressure drop and orifice flow balance are handled by a generalized Newton-Raphson iterative algorithm.
Features: Dimensionless or dimensional usage. Point by point and tabular summary output. Orifice size may be specified directly, or obtained from calculation by specifying a particular pocket pressure ratio under zero eccentricity conditions.
Input: Bearing geometry. Pocket geometry. Orifice geometry. Lubricant density and viscosity, or Poiseuille flow Reynolds No.
Output: All input. Bearing load. Bearing flow. Bearing stiffness and damping. Pad pressures and forces.
Memory Requirements: 64,000 words
Running time: Typically 20 seconds per pad per eccentricity ratio
Language: FORTRAN IV
Availability: Mechanical Technology, Inc.
968 Albany Shaker Rd.
Latham, N.Y. 12110

Journal Bearing Design Program

Capability: This program is the same as the Time Transient Nonsynchronous Rotor Response Program without the rotor. Stiffness and damping properties are calculated by this program.
Availability: This program is for rent:
Dr. M. F. Giberson
Turbo Research Inc.
1440 Phoenixville Pike
West Chester, Pa.
should be contacted for details.

Calculation of Bearing Coefficients and Stability of Short Journal Bearing, L/D<1 (JBRGCOF)

Date: 1974
Capability: The stiffness and damping coefficients for a pressurized plain journal bearing are calculated. The eccentricity and attitude angle at operating speed are given and the stability of the bearing system is checked. Handles horizontal and vertical rotor bearings.
Method: The Reynolds equation for a short bearing is integrated to give the fluid film pressure. Cavitation is considered by setting negative pressure equal to zero. Bearing stiffness and damping coefficients are determined from the numerical integration of the pressure.
Limitations and Restrictions: This program applies to plain journal bearing with a length to diameter ratio less than one. The fluid film is assumed incompressible and isothermal.
Input: Requires bearing geometry and fluid film properties. No preprocessor.
Output: Gives stiffness and damping coefficients and the stability of the system.
Language: FORTRAN IV
Hardware: CDC 6400 with a 25 k words core
Usage: This program has had extensive usage. A technical manual is available [11].
Developer: E. J. Gunter
Dept. of Mechanical Engineering
University of Virginia
Charlottesville, Va. 22901
Availability: Deck and documentation available from developer. Price on request.
Subjective Comments: The documentation is brief but adequate. The cost per run is approximately $1.20.

Calculation of Bearing Coefficients and Equilibrium Position of a Long Journal Bearing (LJBFEM)

Date: 1974
Capability: The stiffness and damping coefficients of a long journal bearing are calculated for horizontal or vertical rotor bearings. Used as input to MFIN4 and CRITSPD.
Method: The fluid film is modeled by finite elements and the pressure and film thickness are approximated by the Reynolds equation. Cavitation is considered by setting negative pressures equal to zero. A leakage correction is also used.
Limitations and Restrictions: Applies to plain journal bearings with a length to diameter ratio greater than one. The fluid film is assumed to be incompressible and isothermal
Input: Bearing geometry and fluid film properties. No preprocessor.
Output: At the equilibrium position, the fluid film thickness and pressure

are given. Intermediate iteration results and final bearing coefficients, forces, equilibrium position, eccentricity, attitude angle, and Sommerfeld number are printed.
Language: FORTRAN IV
Hardware: CDC 6400 with 25 k core
Usage: This program has had extensive usage.
Developer: Dr. E. J. Gunter
Dept. of Mechanical Engineering
University of Virginia
Charlottesville, Va. 22901
Availability: Deck and documentation available from developer. Price on request.
Subjective Comments: The documentation is brief and some input was not fully described. Cost is approximately $1.20 per run.

Finite Journal Bearings and Centrally Loaded Partial Arc Bearings (FJBFEM1) [12]

Date: 1975
Capability: This program determines the direct and cross-coupling coefficients of finite length journal and partial arc bearings with an incompressible fluid film. The pressure profile, load carrying capacity, and friction loss for a horizontal or vertical bearing is also determined. Cavitation and variable viscosity is considered.
Method: Solves the equations of motion using a finite element technique.
Limitation and Restrictions: Journal bearings must have a length to diameter ratio of less than 1 and the partial arc bearing must have a length to diameter ratio less than 1/2.
Input: Bearing geometry, fluid properties, and operating conditions.
Output: Equilibrium position, bearing coefficients, load carrying capacity, friction loss, attitude angle, and eccentricity ratio.
Language: FORTRAN IV
Hardware: CDC
Usage: Limited usage.
Developer: Dr. E. J. Gunter
Department of Mechanical Engineering
University of Virginia
Charlottesville, Virginia 22901
Availability: Contact E. J. Gunter for details.
Subjective Comments: Well documented program with validation and example problems.

Multilobed Bearing Program (BRGCML 2) [13]

Date: 1975
Capability: This program determines the direct and cross-coupling stiffness and damping coefficients of multilobed, axial groove, and partial arc bearings with an incompressible fluid film. The stability of the bearing is determined. Vertical or horizontal bearings may be analyzed.
Method: Integrates the equations of motion and use a Newton-Raphson iteration to search for the equilibrium position.
Limitations and Restrictions: Constant loading and finite length.
Input: Bearing geometry, fluid properties and operating conditions.
Output: Equilibrium position, bearing coefficients, and stability of the bearing.
Language: FORTRAN IV
Hardware: CDC
Usage: Well tested.
Developer: Dr. E. J. Gunter
Dept. of Mechanical Engineering

University of Virginia
Charlottesville, Va. 22901
Availability: Contact Dr. E. J. Gunter for detail.
Subjective Comments: Well documented program with validation and example problems.

Calculation of Bearing Coefficients and Stability of Multi-Lobe Axial Groove Bearings (STABIL)

Date: Program prepared in July 1974.
Capability: Same as BRGCML 2
Method: Same as BRGCML 2
Limitations and Restrictions: Same as BRGCML 2 except it applies to increasing vertical loading of horizontal bearings.

All remaining comments are the same as BRGCML 2.

ACKNOWLEDGMENT

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Aircraft Noise Prediction

Jimmy Cawthorn and Ronnie Gillian

NASA Langley Research Center

INTRODUCTION

Computer programs for the purpose of predicting aircraft noise represent a new technology area which is currently undergoing improvement and new development. NASA's activities associated with this improvement and development are discussed in [1]. The requirement for this predictive ability stems from the need to assess the benefits of new aircraft/engine designs or proposed modifications to existing aircraft. These benefits can be assessed in terms of the noise signature of a single engine or aircraft. Or, the assessment can be made in terms of sensitivity studies of airport community impact due to changes in aircraft/airport operations procedures, types of aircraft, and fleet mixes.

The types of computer programs needed to satisfy these requirements are source noise modeling programs for the single aircraft signature and community exposure modeling programs for the sensitivity studies. The source noise modeling programs are typically used by engineering and research specialists in aircraft noise, while the community exposure programs are typically used by noise control engineers, civil engineers, or community planners.

The programs which are currently available for the prediction of aircraft noise are principally empirical and largely data base dependent. The programs operate at different functional levels of computational sophistication with the source noise programs being higher in level of sophistication (treatment of acoustical detail) than the community exposure programs. There is a desire to improve the analytical modeling capability and thus decrease the dependency on data bases or empiricisms, particularly for the higher level programs.

The source noise modeling programs are used to predict the noise generated by a single aircraft event, and the computational results are typically given in scales of A-level or Effective Perceived Noise Level in units of dB(A) and EPNdB, respectively. The community exposure programs are used to predict noise from multiple aircraft events and the most commonly used indices are Noise Exposure Forecast (NEF) in dB-like units and Day-Night Level (L_{dn}) in units of dB.

The purpose of this paper is to describe some aircraft noise prediction programs which are generally available and which have been incorporated into an interim system at the Langley Research Center. A background discussion including levels of acoustical sophistication and units of measure is also presented as an aid to understanding the requirements of an aircraft noise prediction system.

SYMBOLS

Symbol	Unit	
EPNL	EPNdB	Effective Perceived Noise Level
L_A	dB(A)	A-weighted sound pressure level

L_D	dB(D)	D-weighted sound pressure level
L_{dn}	dB	Day-Night Level
NEF	dB like	Noise Exposure Level
PNL	PNdB	Perceived Noise Level
PNLT	PNdB	Tone Corrected Perceived Noise Level

(For detailed definitions see [2])

BACKGROUND

Levels of Predictive Capability and Their Users

As an aid in understanding the present state of the art and future requirements of aircraft noise prediction, the following concept of levels of sophistication is presented. There is envisioned a hierarchy of four functional levels of computational sophistication required to satisfy the needs of various user groups as is shown in Fig. 1 which is taken from [1]. Level I represents the simplest conceptual mode of computational capability. This level would produce measures of community noise environment by the use of time-integrated flyover (Noise, Thrust, Altitude) data or from an integration of the overall noise level of Level II.

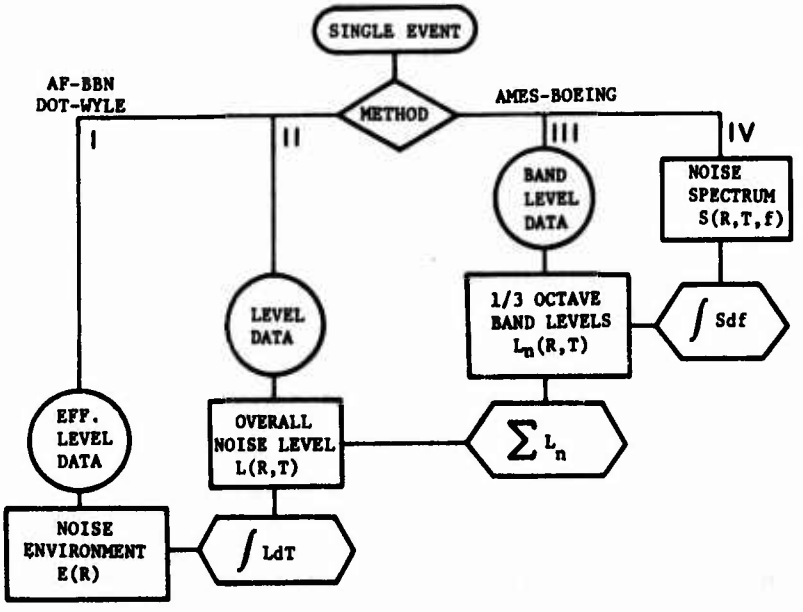

Fig. 1 Aircraft noise prediction functional levels

Level II should produce an overall (type) noise level, as a function of distance and time, based on the use of input parameters and computed or measured values of noise levels such as PNdB and dB(A) which vary during the aircraft flyover. The results produced by this level could also be obtained from a summation of the one-third octave band levels of Level III.

Level III should be used to predict the time and distance dependent one-third octave band noise spectrum from an analysis of the aircraft component noise sources and an analysis of the aircraft flightpath. This prediction could also be obtained from an integration of the spectrum of Level IV as a function of frequency. Level III is conceived to be based primarily on empirical formulas for the noise of different aircraft source components and would be suitable for making detailed systems studies of aircraft/engine configurations.

At the highest level, (IV), analytical modeling techniques are utilized to compute a continuous noise spectrum as a function of distance, time, and frequency from inputs of engine performance and aircraft operating parameters. Level IV should be the repository for the most advanced acoustical technology and should be used in an experimental sense for technology validation and improvement or for detailed designs of advanced low-noise components for aircraft.

Levels III and IV, since they represent higher levels of sophistication, are intended to be used by engineering and research specialists in aircraft noise. Level II is intended to serve aeronautical engineers in making systems studies involving general aircraft types as well as the noise control engineer who requires greater knowledge of the community aircraft noise exposure than the time-integrated estimates provide. And, Level I is intended to serve civil engineers and community planners who have minimal knowledge of the complex technology of aircraft noise prediction.

Scales and Indices

There are two types of calculated quantities available form computer programs which are used for the prediction of aircraft noise - single-event scales of individual loudness and annoyance ratings and multiple-event indices of community response ratings.

At the present time, there are no universally accepted scales and indices for use as ratings of aircraft noise; however, there are a number which are in contention for adoption as standards by various Government agencies and other organizations. Some of the leading contenders which are currently in the forefront of usage or which have been recommended as standards are A-weighted Sound Pressure Level (L_A), D-weighted Sound Pressure Level (L_D), and Effective Perceived Noise Level (EPNL) which contains Perceived Noise Level (PNL) and Tone Corrected Perceived Noise Level (PNLT). One multiple-event index which is based on L_A is Day-Night Level (L_{dn}) which can also be computed using L_D as the base scale. Noise Exposure Forecast (NEF) is based on EPNL. These scales and indices are shown in Fig. 2, which also depicts their interrelation with each other. The arrows indicate the computational flow of calculating a community response index from a measured one-third octave band time history. (For example, one-third octave band spectra are used to compute PNL, which is used to compute PNLT, which is used to compute EPNL, which is the base for computing NEF.) A detailed description of each of the scales and indices is contained in [2].

From the previous section, the multiple-event indices (NEF and L_{dn}) are the product of Level I, while Level II will produce overall type scales such as L_A and L_D. Levels III and IV will produce any of the single-event scales - L_A, L_D, EPNL, or the one-third octave band spectra from which these scales are derived.

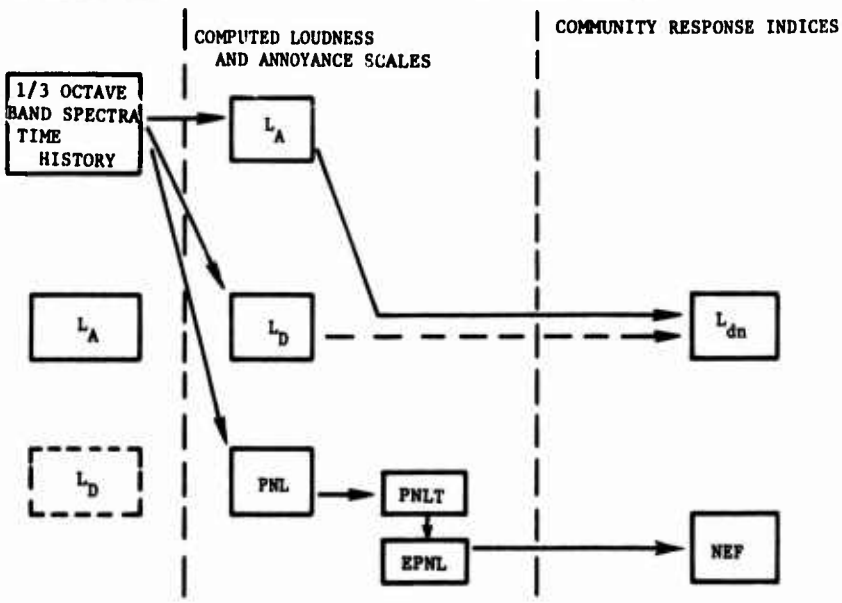

Fig. 2 Some examples of direct measures and calculated quantities used for the quantification of community exposure to aircraft noise

CURRENTLY AVAILABLE PROGRAMS

Source Noise Program (Ames/Boeing)

The Boeing Company developed an aircraft source noise modeling program under contract to the NASA Ames Research Center [3, 4]. This is an empirically based program which operates at Level III described in the preceding section. As is illustrated in Fig. 3, the noise from each of the following appropriate components is computed separately: primary jet, primary and secondary jet, core and turbine, compressor and inlet fan, exit fan, augmentor wing, blown flaps, lift fan, ejector-suppressor, propeller, helicopter, measured data input. The individual component noises are then summed to produce the total predicted aircraft noise. Corrections are then added as indicated in the figure.

The inputs to the program are in a NAMELIST format and consist of engine performance data and aircraft flight performance data. A variety of outputs are available and include the following:

1. Component and total noise directivity patterns in terms of one-third octave band spectra and PNL as a function of directivity angle (10° - 170°)
2. Peak PNL and PNLT
3. EPNL values at a given location
4. EPNL contours
5. Plot of EPNL at a given sideline distance along flight track

The program, which runs at the Ames Research Center on an IBM 360 with a standard IBM random access capability, has also been installed at the Langley Research Center on a CDC 6600 with a standard CDC random access capability. This program is available through COSMIC, 112 Barrow Hall, University of Georgia, Athens, Georgia 30601, Reference: ARC-10880.

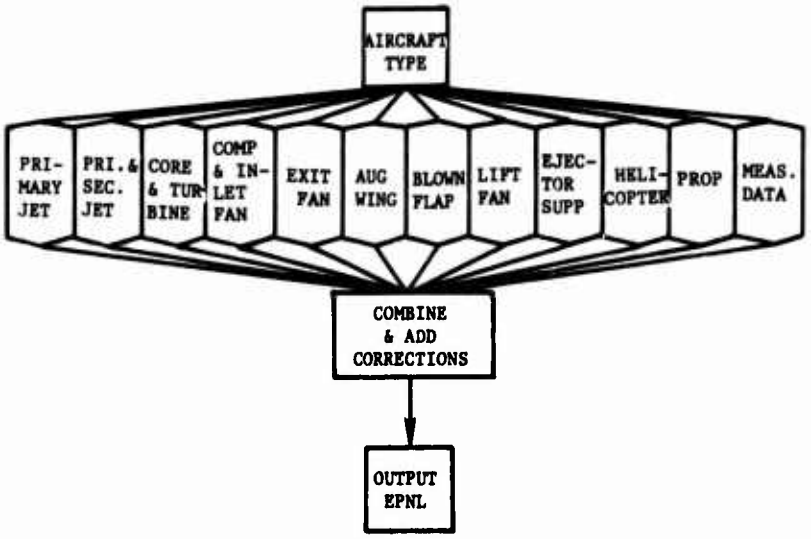

Fig. 3 Components of source noise program developed by the Boeing Company for Ames Research Center

Community Exposure Programs

There are several programs currently in existence for the purpose of performing analyses of community exposure to aircraft noise. The most prominent are the Noise Exposure Forecast (NEF) programs which Bolt, Beranek, and Newman (BBN) developed for the U.S. Air Force [5, 6, 7, 8], and which Wyle Laboratories developed for the Department of Transportation.

The BBN and Wyle programs are very similar in nature in that they both require the same type of inputs and produce the same type of output. A sample output in the form of NEF contours is illustrated in Fig. 4. Some of the computational details differ, but for a given input both programs produce similar results. Both programs are data base dependent - with the BBN program the data base is user supplied while the Wyle program has the data base built in. In both cases, the data base consists of noise versus distance data (in the form of EPNL - Slant Range curves) for specific aircraft types.

The inputs required are:

1. Types of aircraft
2. Numbers of takeoff and landing operations for each aircraft type on a daily basis
3. Aircraft performance data - flightpath and flight track information
4. Airport description - runway orientation and utilization

The BBN program is relatively easy to input since there is only one data format for the entire program. However, the Wyle program is more difficult since there are many different data formats - each input parameter requires a different format.

The BBN program is installed at two locations - Langley Research Center and CDC Cybernet in Los Angeles (access controlled by BBN). The program is written in FORTRAN IV for a CDC 6600. For each aircraft flyover the program computes NEF values for a 100 x 100 grid. For large airports with many flights to compute this procedure requires large amounts of CPU time.

Fig. 4 NEF contours; sample output of community exposure programs

The Wyle program is operational at Langley and Wyle Laboratories at El Segundo, California. The Langley version has been converted to operate on the Langley CDC 6600 while the Wyle version operates on a Univac 1108. The Wyle program computes the noise at finite points along the aircraft flight track and, thus, requires less CPU time than the grid point computation technique of the BBN program.

Two outputs are available from each of the above programs: an NEF contour plot and a noise grid (NEF) printout.

Data Base Programs

The Federal Aviation Agency (FAA) contracted with each of the three major commercial airplane manufacturers in the United States (Boeing, McDonnell Douglas, and Lockheed) to produce computer programs documenting the noise and performance characteristics of the airplanes of their own manufacture. These programs are described in [9, 10, 11]. Thus, the complete commercial airplane data base is comprised of three separate programs. The three programs are similar in nature and each produces the same type of output with the same general inputs.

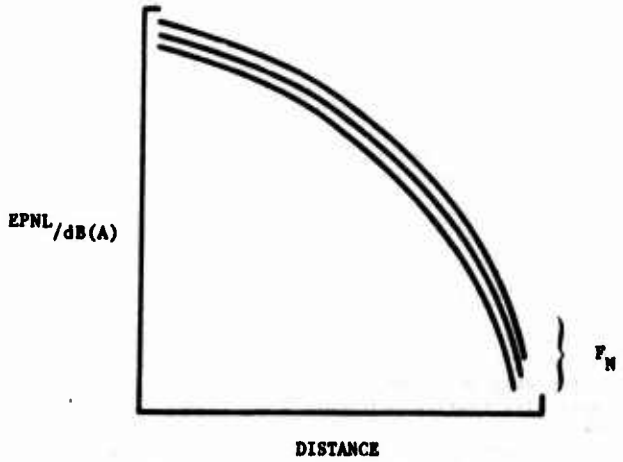

Fig. 5 Noise versus distance functions; sample output of FAA sponsored data base programs

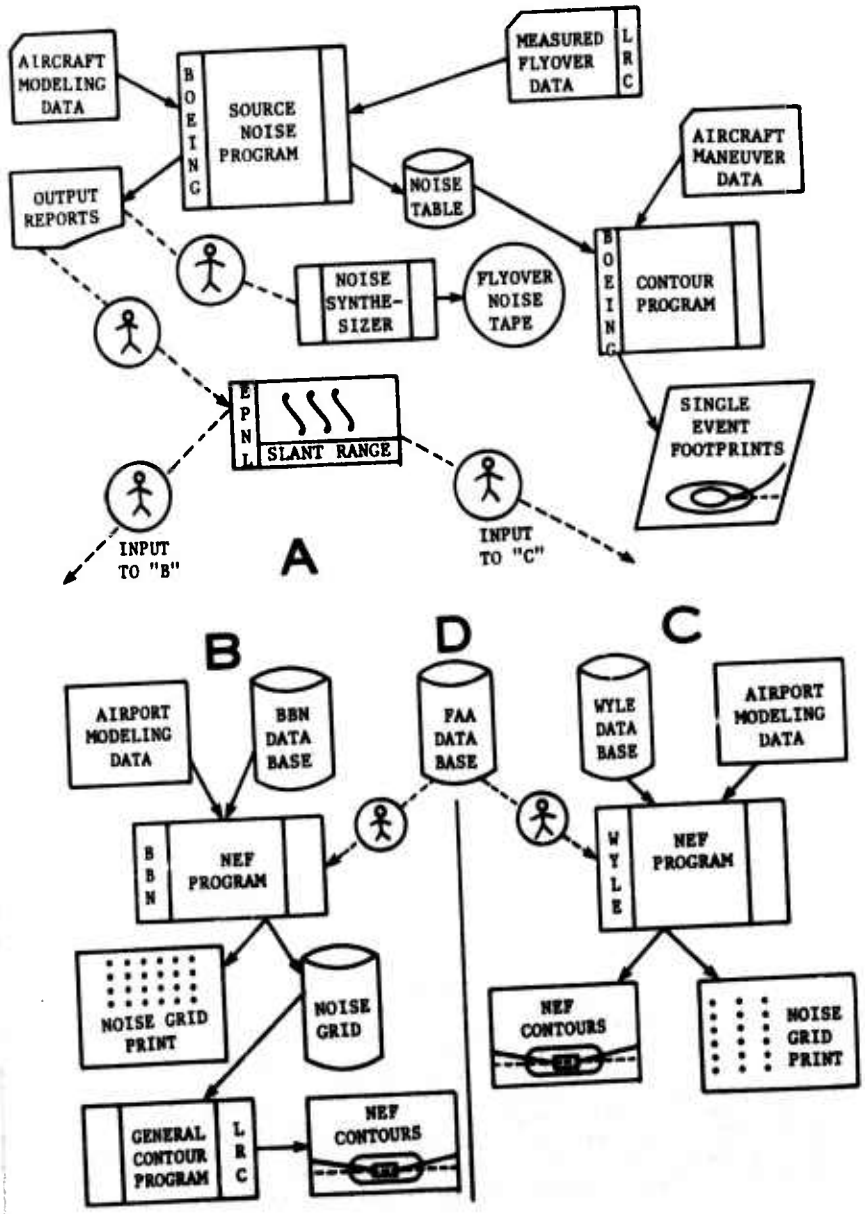

Fig. 6 NASA Langley Research Center interim program for aircraft noise prediction

The inputs to the programs consist of such information as aircraft type and weight and parameters associated with the airport such as altitude, temperature, and wind conditions. The programs consist of two parts: a performance routine which computes an altitude distance profile for takeoff or landing and a noise routine which computes noise under the flightpath in units of dB(A) or EPNL. A typical output of the programs is illustrated in Fig. 5 which plots noise level (dB(A) or EPNL) as a function of distance and and aircraft thrust (Fn).

The aircraft contained in the data base programs are given in the following table:

Table 1

	Data Base Program		
	Boeing	McDonnell-Douglas	Lockheed
Aircraft	707 720 727 737 747	DC-8 DC-9 DC-10	L-1011

The Boeing program (written in Fortran IV) is currently installed at two locations, the Boeing Computer Services on an IBM 360 and at NASA Langley on the CDC 6600.

The McDonnell-Douglas program is installed only at McDonnell-Douglas, Long Beach, California, and the Lockheed program is installed only at Lockheed, Burbank, California; both operate on IBM 360 computers.

NASA LANGLEY RESEARCH CENTER INTERIM PROGRAM

The Aircraft Noise Prediction Office (ANOPO) at NASA Langley has developed an interim aircraft noise prediction program through the acquisition, installation, and modification of several of the independently developed programs described in the previous section. This interim system is depicted schematically in Fig. 6. The separate parts of the system are:

1. The Ames/Boeing source noise modeling program
2. USAF/BBN NEF contour program
3. The DOT/Wyle NEF contour program
4. The Boeing Company portion of the FAA data base

Some modifications have been made by ANOPO to these programs as follows:

1. The Ames/Boeing program was originally written to accept measured data inputs in the format of a directivity pattern at a reference distance of 1 meter. This program has been modified to accept measured data as a directivity pattern at any specified radial distance, as sideline measurements as a function of angle, or flyover measurements wherein the time history has been translated into an angular directivity pattern.

2. The DOD/BBN program has been overlain to reduce the core requirement and the program has been interfaced with an existing Langley contour program. These programs have been installed and are operational at Langley on the CDC 6600 in Fortran IV. They comprise a relatively comprehensive aircraft noise prediction interim system; however, this system was intended to serve as a research tool. It is, therefore, not available for distribution. Since the system is a research tool, the programs have not been completely interfaced - the stick figures in Fig. 6 represent manual interface requirements.

CONCLUDING REMARKS

The state of the art of aircraft noise prediction is in its infancy, but is currently undergoing extensive development and is changing rapidly. A

number of special purpose programs consisting of aircraft source noise modeling programs, community noise exposure modeling programs, and aircraft fleet noise data base programs have been developed for use by and are available from the USAF, DOT, and NASA Ames.

These programs, as modified and configured for the Langley Research Center computer complex, comprise a relatively comprehensive interim aircraft noise prediction system for use as a research tool and are, therefore, not available for general distribution.

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Shell Analysis

Arturs Kalnins

Lehigh University

Lawrence I. Weingarten

Sandia Laboratories

INTRODUCTION

This chapter is concerned with both the theoretical background of shell dynamic response and the description and characteristics of the more widely-used structural software packages for shell dynamic computation. In the first part of the chapter, the dynamic response problem is defined in terms of the spatial distribution of the loading and the frequency representation of particular shell theories. Also, the solution procedures for transient analysis are discussed. The second part of the chapter deals directly with structural software for the dynamic response problem, discussing the various shell theories that have been implemented computationally and the characteristics and special features of current programs.

THE DYNAMIC RESPONSE PROBLEM

An overview of shell dynamic response problems reveals different methods of analysis for different time distributions of loading. In order to simplify the discussion, these loading distributions will be assumed to be of the form $p(x,t) = P(x)f(t)$, where $P(x)$ describes the space and $f(t)$ the time distribution. The classification of time distributions is: 1. periodic; 2. long duration, such that a steady-state response is sought; and 3. pulse-shaped, for which a transient response is sought.

The simplest response problems arise for finite shells when the time distribution of the loads is periodic and the steady-state response is desired. This means that the loads are assumed to have been acting for a long time, many reflections have occurred, and any transient motions that were produced by the starting of the excitation (progressing wave fronts) have died out. The space distribution of the loads is arbitrary. The solutions to such problems are obtained by first representing the periodic time distributions by Fourier series and then obtaining the response to each harmonic Fourier component separately. For finite systems, such problems are solved with the same methods as those used for static loads. The only difference is that the inertia terms with harmonic time variations are added. Because the mathematical problem is the same as that in the static case, most of the automated computer programs that are available for static shell analysis can also be used to calculate the harmonic response.

The response problems to periodic loads can be extended further to those cases when the loads are applied on a small portion of the shell surface, or on one edge, and when other edges are sufficiently far away. Then, before any reflections have occurred, the motion of the shell resembles continuous wave motion, with the nodal lines moving away from the location where the loads are again represented by their Fourier series, and the solution is obtained first for each harmonic, and then the harmonic waves are summed. The interesting feature of these problems is that at a given frequency only definite types of

waves can be propagated; and, depending on the space distribution of the loads, it is possible that in some low-frequency interval no wave motion at all can be propagated. These problems are usually formulated for systems that can be imagined to have infinite extent, such as beams, plates, and shells of revolution. Solutions of these problems are valid at times well after the initial wave front has passed and the transients caused by the starting of the excitation have died out. Examples of these types of problems have been worked out for beams, infinite plates, unlimited shallow spherical shells (which are represented mathematically by unlimited parabolic shells of revolution), and for cylindrical shells, as shown in the following two papers.

In [1, 2], D. C. Gazis considers continuous, harmonic waves in an infinite cylindrical shell that are propagated in the axial direction. The waves can be imagined as being generated by sources that are distributed in one section of the shell (say, $z = 0$), and they impart some disturbance to the shell that is harmonic in time, with a frequency ω. Such excitation produces continuous, harmonic waves that travel along the infinite shell in both directions from $z = 0$. They are superpositions of spherical waves which emanate from the points within the section $z = 0$, where the sources are applied, and then reflect from the two cylindrical bounding surfaces of the shell. In summation, they establish standing waves, with fixed nodes in the radial and circumferential directions, but produce wave motions in the axial direction.

The object of Gazis' papers is to predict the axial wavelengths and velocities (phase velocities) and the radial and circumferential standing-wave patterns that can be propagated from given sources at $z = 0$. Such information is revealed by the dispersion curves shown in Figs. 3 - 10 of [2]. For example, referring to Fig. 8, no waves with a circumferential wave number $n = 2$ can be propagated at a frequency $\omega = \omega_s/2$. What happens is that at that frequency the harmonic-in-time motion has an axial distribution that decays exponentially when going away from $z = 0$. (This motion is sometimes called an "evanescent wave".) At $\omega = 2\omega_s$, three types of waves can be propagated. Their wavelengths can be read off Fig. 8, and their radial standing-wave patterns are described by the code numbers 1, 2, and 3, attached to the curves, and are given by the r-dependent functions of the solution. It must be emphasized that only these waves can be propagated at each of the given frequencies, regardless of the radial distribution of the sources. For a general radial distribution of the sources, the other expected components are simply nonpropagating (evanescent).

Gazis' results are based on the three-dimensional theory of elasticity, and they include an infinite number of radial standing-wave patterns. At a given frequency, the patterns having up to a number of cylindrical nodal surfaces are propagating and the higher patterns are not. These conclusions provide a logical basis for a mathematical model of a "shell" theory, which would be applicable only for frequencies that are below a limiting frequency. This means that only standing-wave patterns with a low number of radial nodes would be propagated. Therefore, if the limitation to low frequencies is acceptable, it would make sense to limit the theory at the outset to those radial standing-wave patterns that are propagated and leave out all the nonpropagating patterns with higher number of radial nodes. The usual mathematical models for shells assume displacement components with at most one node across the thickness of the shell. Then, for a thin shell, at most six modes can be propagated. Starting from such a shell model, further approximations can be made. If the normal displacement is restricted to zero radial nodes, which eliminates the lowest thickness-stretch mode, then at most five modes are propagated. If the model is simplified further by assuming that the transverse shear strains are zero (classical shell theory), which eliminates the lowest thickness-shear modes, then at most three modes can be propagated. This information can be obtained from a number of papers that consider the same problem as Gazis, except that they use a mathematical model of shell theory (for references, see those listed in Gazis' papers). Such results as those in Fig. 6 of Gazis' paper can be used to estimate the ranges of valid frequencies for each of the shell models.

Once it is recognized that the mathematical models of beam, plate, and shell theories are valid only within a certain low-frequency interval, then an

important conclusion can be made with regard to the class of time distributions that can be modeled meaningfully by these models. The conclusion is that, as far as the predictions of the behavior of the physical systems are concerned, differences in the responses have no meaning when they are produced by two time distributions of loads that differ only in those Fourier components that have frequencies outside the low-frequency interval of validity.

It should be remembered that Gazis' results apply only after the initial wave front has passed and a steady-state motion is reached. When reading Gazis' paper, it is important to distinguish between the constant wave velocity with which the spherical waves emanate from the source points into a three-dimensional medium and the phase velocity of the summed disturbance , consisting of the reflections from the cylindrical surfaces, that is described by Gazis. As seen from Fig. 11, the phase velocity can range from infinity to zero, while the velocity of spherical waves has a fixed value. Excellent discussions of the meaning of phase velocity are given in [3, 4]. In order to give a physical interpretation of phase velocity, [3] draws the analogy of a continuous harmonic wave in water, striking a wall at an angle. The point of intersection of one crest of the wave with the wall travels along the wall at the phase velocity which is different from that at which the straight crests of the waves themselves propagate. The water wave is analogous to the spherical wave produced by a source point and the wall is analogous to the two cylindrical surfaces.

It is clear from the analogy that the phase velocity can approach infinity when the wave approaches normal incidence with respect to the wall. Also, the energy of a single harmonic wave is propagated with the so-called group velocity (see [4], p. 211) which is zero when the phase velocity is infinite. So, an infinite phase velocity does not imply any physical contradiction.

When the time distribution of the loads is not periodic, the Fourier series representation must be replaced by the Fourier integral (see [4], p. 39). Then the harmonic solutions can be regarded as solutions in the Fourier transform plane, and the response can be obtained by using them in the integral of the inverse Fourier transform. In the evaluation of this integral, it must be again remembered that in the high frequency range the predictions of the mathematical model are unreliable.

SOLUTION METHODS

The very process of inverting the Fourier transform can be a formidable task and not well suited for problems for which the time distribution of the loads is zero before, say, $t = 0$, at which point the excitation begins. There are four alternate methods that are commonly used for such cases: 1. modal method, 2. numerical integration in time, 3. Laplace transform in time, and 4. the method of characteristics. It should be understood that all four methods can be said to apply to all time-dependent problems; and, theoretically, they all will give identical results. However, in order to achieve that, one would have to use an infinite number of modes and an infinitesimal time step, invert the Laplace transform exactly, and trace and sum all individual reflecting waves. Because of such practical limitations, each of the methods is better suited for certain classes of problems. In general, it can be said that the first two methods are applicable to finite systems only. The Laplace transform method is intended for semi-infinite or infinite systems, and the method of characteristics is intended for the study of an individual wave front as it enters a region of quiet.

Modal Method

In order to discuss the limitations of the modal method, its main features, when applied to a shell, will be indicated here. As before, the loads will be assumed in the form $p(x,t) = P(x)f(t)$, where $p(x,t)$ and $P(x)$ can be regarded as vectors. In the modal method, $P(x)$ is represented by the series

$$P(x) = \sum_i c_i \rho h u_i(x) \tag{1}$$

where

$$c_i = \int_S P(x) \cdot u_i(x)\, dS \Big/ \int_S \rho h u_i(x) \cdot u_i(x)\, dS \tag{2}$$

and $u_i(x)$ is the displacement vector of the ith mode of free vibration, x stands for any number of space coordinates, S is the reference surface of the shell, ρ is mass density, and h is thickness of shell wall. The solution is given by

$$V(x,t) = \sum_i V_i(x) c_i D_i(t) \tag{3}$$

where $V(x,t)$ is any variable in the solution, $V_i(x)$ is its value in the solution of the ith mode of free vibration, and, in the absence of damping and with zero initial conditions,

$$D_i(t) = \frac{1}{\omega_i} \int_0^t f(\tau) \sin\omega_i(t-\tau)\, d\tau \tag{4}$$

where ω_i is the natural frequency of free vibration in the ith mode.

In the modal method, each term in the load series (Eq. (1)) gives the corresponding term in the solution series (Eq. (3)) as an exact solution of the governing equations. The only error, then, comes from having an inexact space distribution of the load. The difficulty of representing the load distribution by the modal series is the same as that in any other Fourier series. It is clear that sharp, localized load distributions, which are zero over a part of the shell surface, will be modeled particularly inaccurately; and the zero values of the loads will never be achieved. Therefore, whatever the actual load distribution, the modal method will model it as if it were covering the whole shell surface. For the same reason, the solution will also be unable to predict accurately any discontinuities or regions of quiet.

It can be seen from this discussion that the modal method is best suited for space distributions of the loads which are smooth and cover the whole of the shell surface. When sharp variations in the space distributions occur, the number of modes required for the series will increase drastically; and, if a smaller number is used, the sharp variations will be simply modeled inaccurately.

It must be pointed out here, however, that this inaccuracy of the modal method refers to the solution of the mathematical model. Since the sole purpose of the mathematical model is to represent the behavior of a physical system, such inaccuracies must be viewed within the scope of the limitations of the model itself. It makes no sense to strive toward additional accuracy in solutions in which the model is unable to transfer to the physical system. Fortunately, this argument removes much of the significance of the inability of the modal method to represent sharp space variations of the loads, because a shell model cannot recognize any statically equivalent redistributions of loads over areas that can be enclosed within circles with minimum radii that are comparable to, or less than, the thickness. This means that a sinusoidal space distribution of a load, with wavelength comparable to thickness, or smaller, can be replaced by a zero load without affecting meaningfully the prediction of the

behavior of the shell. Since the wavelengths of the displacement distributions, $u_i(x)$, in the modal series become shorter as i increases, it is clear that a shell theory can digest beneficially only a finite number of terms of the modal series and, consequently, that it is worth including only such load distributions that can be made up of this finite number of terms. Addition of further terms simply does not filter through the model to the system.

A good example of this argument is provided by a point load on a shell. First of all, the shell model will be incapable of predicting the difference of the behavior of the shell when subjected to a point load or a statically equivalent load that is distributed over an area with the diameter of the order of the magnitude of the thickness, or less. When such a distributed load is represented by the modal series, it will not be meaningful to differentiate between a series that is truncated up to the terms which have wavelengths of the order of the thickness, or a series that is truncated to higher order terms. Clearly, such a truncated series will no longer give an exact "region of quiet" at the moment when the load is applied; but, as argued in the foregoing, the differences in the predictions between short-wavelength and zero-load distributions are physically meaningless.

It should be added that it is impossible to say how many terms in the modal series are relevant for a given shell theory. It can only be said that theoretical predictions for load distributions that can be represented by terms with longer wavelengths will agree better with the behavior of the physical system and that the classical shell theory will be limited to fewer terms than the higher order shell theories which account for transverse shear and normal strains.

Another comment is in order here. From Fig. 1 of [5], the node count (represented by n in the Legendre function and related to wavelength) for each of the three infinite sequences of modes (bending, stretching, and shearing) begins with zero. This shows that the modal series should be written separately for the bending, stretching, and shearing modes of a shell, because the limitation of their ability to represent loads is applied to each of the mode types separately. Whether or not the sequence of a mode type is relevant in an application will depend on the character of the load, which is revealed by the numerator of c_i in Eq. (2). For axisymmetric deformation of a spherical shell, the integrand of the numerator is given by

$$pw + p_\phi u_\phi + m_\phi \beta_\phi$$

where p, w and p_ϕ, u_ϕ are the normal and meridional (tangential) surface loads and displacement components, respectively; m_ϕ is a distributed surface couple and β_ϕ is the rotation of the normal in the meridional direction. In general, it can be said that w determines the wavelengths in bending modes, u_ϕ in stretching modes, and β_ϕ in shearing modes. It follows, then, that in the presence of normal surface loads (e.g., pressure) the bending mode sequence is most relevant; for tangential loads, the stretching modes are most relevant; and for surface couples, the shear modes are most relevant. The last situation is very seldom encountered in applications, which explains the reason why the classical shell theory, which is incapable of predicting the shearing modes, is sufficient in most applications.

As far as the decision on which relevant terms are to be included in the modal series is concerned, it should be viewed at first as a matter of the load representation in space, as given by the series of Eq. (1). It is important to realize that the best load representation may not be achieved by summing blindly consecutive modes of the series, starting from the lowest, because of certain properties of the load distribution. For example, the loads may have certain symmetries that rule out some modes; and, for a thin shell, normal surface loads will rule out most of the stretching modes and all shearing modes. After a selected list of modes for the load distribution has been arrived at, the decision on the number of modes from this list that is actually used must be

made on the decreasing magnitudes of the terms in the solution series, as given by Eq. (3). Once the point has been reached where the change in the solution by the addition of a further term is within an acceptable error, then the series can be declared converged.

Examination of the solution series reveals that its convergency may differ from that of the load series because of the additional factor $D_i(t)$, which means that the time distribution of the loads and the natural frequencies of the system will play their parts in the decision on the number of modes to be included. In order to see their role, it is useful to consider the integral $D_i(t)$ for some time inputs $f(t)$. Expressions of this integral for various $f(t)$ can be found in [6].

For example, when $f(t)$ is a single rectangular pulse with height Q and width t_1 then

$$D_i(t) = \frac{Q}{\omega_i^2}(1 - \cos\omega_i t) \text{ for } 0 \le t \le t_1 \tag{5}$$

and

$$D_i(t) = A \sin\omega_i\left(t - \frac{t_1}{2}\right) \text{ for } t_1 \le t \tag{6}$$

with the amplitude

$$A = \frac{2Q}{\omega_i^2} \sin\pi \frac{t_1}{T_i} \tag{7}$$

and

$$T_i = \frac{2\pi}{\omega_i}$$

where T_i is the natural period of free vibration.

It can be seen that, for a given t, the modal series converges in general as $(1/\omega_i)^2$ but that the convergence can be affected also by the duration of the pulse t_1. Equation (7) shows that for a pulse that is much shorter than the lowest natural period, the amplitude of $D_i(t)$ may not have the largest value in the first term of the modal expansion. Depending on the increasing values of the natural frequencies, the amplitude may not reach a maximum until a number of modes have been added to the response. This illustration provides the reason why for some time distributions the convergence can be slower than for others, even though the space distributions of the loads remain the same.

When using the modal method, it is important to recognize the two properties of the loads which affect the convergence of the modal series. The space distribution affects the decay of the numerator of c_i in Eq. (2) and the time distribution affects the decay of $D_i(t)$. The fastest convergence will occur when the space distribution of the load resembles that of the lowest mode, the natural frequencies are widely spaced, and the duration of pulses is about one-half of the lowest period. The slowest convergence will occur for a point load, when the natural frequencies are closely spaced and when the time input is that of an impulse. As long as the sources of convergence of the modal series are understood, the user of this method should be able to make sound decisions with regard to the terms that must be considered.

Numerical Integration

The numerical integration method in time replaces the time derivatives in the inertial terms by finite differences and expresses the governing equations at discrete points in time. The problem at each time step is solved in the same way as in the static case.

The numerical integration method shifts the difficulty of load representation to the static solution at each time step. Since a static solution is felt instantaneously everywhere in the shell, the method is incapable of predicting discontinuities at wave fronts and zones of quiet. For this reason, it is suited best for load distributions that are smooth and cover the whole shell, just like the modal method. Its main difficulty lies in selecting the time steps that give an acceptable solution and knowing when the solution is acceptable. This difficulty corresponds to that in the modal method in deciding where to truncate the modal series. However, in the modal method a definite criterion is found in the comparison of each successive term with the sum of the preceding terms, while the only accuracy assessment in the numerical integration method is provided by the repetition of the integration with a smaller time step. The modal method requires some judgement on the part of the analyst in examining the loads and the mode shapes and in deciding what types of modes to include in the series. The bulk of its computational work lies in the determination of the modal solutions throughout the system. Usually, it is a simple matter to calculate the coefficients c_i, as given by Eq. (2), together with the modal solutions. Once that is done, it only remains to evaluate the integral for $D_i(t)$ for a given time input, as required by Eq. (3). Each of the terms can be subjected to close scrutiny and its relevancy to the solution ascertained. The modal method possesses the exciting possibility that for some special space and time distributions of the loads the analyst can construct an acceptable solution with only a few terms, or even one term, of the series.

The numerical integration method bypasses all that and relies on the computer to produce answers. While having acquired more automation in its execution, it has obscured the makeup of the solution from which its accuracy could be assessed. It has also given up the possibility of producing simpler solutions to special problems because it obtains all solutions in the same way, by stepping away in time. Since the errors at progressive time steps are accumulative, the numerical integration method is most effective in obtaining a short-time response. Also, the numerical integration method can be readily extended to nonlinear problems, while this is not the case with the modal method.

Application of the Methods

Reference [7] uses the straightforward modal approach applied to shells. They do not start with Eq. (1) but arrive at its equivalent by their Eq. (6). Starting with the modal expansion of the load, Eq. (1) brings out more clearly the fact that each load term gives the corresponding solution term as an exact solution. This is obscured in the procedure used in [7].

In [8], a variant of the modal method is described which makes use of a known static solution to an analogous problem obtained by omitting the time variation from all the load terms that are applied either on the shell surface or on the edges. (Instead of $P(x)f(t)$, the static solution is obtained by using only $P(x)$.) This variant is often called Williams' method. Wilkinson calls it the "mode acceleration" method because in its corresponding Duhamel integral (Eq. (4)) the second derivative of $f(t)$ (acceleration) appears. However, after deriving the integral, Wilkinson has integrated it by parts twice and obtained his Eq. (13).

The idea in Williams' method is that the modal expansion is added to the static solution. As shown by W. Ramberg [9], for some problems it can lead to

faster convergence than the ordinary modal method. J. Sheng [10] claims that Williams' method will converge faster during the time when the loads are applied. A great advantage of Williams' method is that it can be applied to find the response to time-dependent edge loads while the usual modal method cannot. For general shells, such edge loads were considered by Kalnins [11]. A disadvantage is that a static solution must also be calculated, which is not needed in the ordinary modal method.

A very good presentation of the numerical integration method can be found in [12]. The application was to a cylindrical shell and was motivated by the many new applications arising in the U.S. space program in the early 1960's. It provided the groundwork for many further papers on this topic. An excellent comparison of the available computer programs for shells of revolution that use the numerical integration method in time is included in [13]. Unfortunately, the analysis for the same problem by the modal method was not included in this paper. The inclusion of the corresponding modal analysis would have provided a very valuable comparison. Perhaps someone will carry that out in the future.

Laplace Transform and Method of Characteristics

The remaining two methods for obtaining a dynamic response of a shell, those of Laplace transform and characteristics, have not yet reached the state of automation that has been enjoyed by the modal and numerical integration methods for some time. Nevertheless, they offer powerful techniques for the extraction of specific information of the response.

COMPUTATIONAL CONSIDERATIONS

The concepts discussed in the first two sections of this chapter have applications to both analytical and numerical solutions. For example, the user of an elastodynamic computer program can, and often does, treat the program as a "black-box", assuming that reasonable output implies a correct solution. A better approach is to apply the principles discussed in the previous sections to both the numerical model (input) and the output. Some of these principles are more readily adapted to the computer than others. For instance, long wavelength calculations are more easily obtained from the majority of shell programs than are calculations where thickness effects are deemed to be important. More than likely, different computer programs would be used in separate spatial and temporal regimes for the same problem.

It seems efficient to briefly discuss the relevant features of that which would normally determine the choice of a particular program separate and prior to the computer codes themselves. The topics to be discussed which are important to some, if not all, dynamic shell problems are: shell theories, shell description, methods of spatial discretization, eigenvalue analysis, methods of time integration, and special features (including geometric nonlinearities, viscoelasticity and plasticity). The inclusion and/or manner of including the above features renders an individual shell computer code distinctive from others. A basic understanding of the approaches available in the development of a dynamic shell code should aid the potential user in the selection of the appropriate computer program.

Shell Theories

It should be recognized that most specialized shell codes employ theories which claim to be derived from the Love-Kirchhoff (or equivalent) assumptions. Essentially these assumptions are:

1. $h/L \ll 1$ where h is the shell thickness and L is any characteristic length of the shell.
2. Strains and displacements are small everywhere although Koiter [14] shows that large deflections may be permitted.

3. Transverse normal stress is small and hence is neglected with respect to the other normal stresses.

4. Straight lines initially perpendicular to the middle surface remain straight and perpendicular to that surface during deformation and remain unstretched.

Shell theories conforming to these assumptions are commonly called "first-order" in that the state of the shell following deformation can be completely characterized by the deformation of the middle surface. However, there exist about as many first-order shell theories as there are competent researchers in the field of elastic shells. Following the adoption of the Love-Kirchhoff assumptions, each theoretician would embark on dissimilar courses of derivation, making various approximations and arriving at different strain-displacement and stress-resultant-strain relations. Even though there may be little difference in the analysis of thin shells, if one were to compare the results of a sample problem using different first-order theories, the problem remains in which is the "best". The terminology "best" can probably be defined in that the theory can be expressed in general tensor notation and the theory is consistent with the order of the Love-Kirchhoff assumptions, i.e., no additional approximations are made which are of the same order of magnitude as the original assumptions. These concepts of "best" and consistent were pursued by Koiter [14], Sanders [15], and Budiansky and Sanders [16] arriving at what are now commonly denoted as the Sanders or Koiter-Sanders shell equations. The features of this shell theory are contained in Budiansky and Sanders [16] and are summarized below:

1. General tensor form

2. State of deformed shell is represented by six stresses which satisfy the equilibrium equations and six strains, three of which are the conventional membrane strains; the others are composed of the geometrical curvature-change tensor and terms which are bilinear in components of the curvature and membrane strain tensors.

3. A principle of virtual work is exact for displacements obeying the Kirchhoff hypothesis.

4. An exact static-geometric analogy.

5. Stresses and strains reduce to generally-used measures when applied to symmetrical bending of shells of revolution.

6. Stresses and strains obey uncoupled Love constitutive relations.

No other "first-order" shell theory can satisfy 1 through 6. This does not imply that other "first-order" theories should be considered insufficient for shell analysis. Most analysts are bound to use certain theories since they are more comfortable with them. This may be due to educational background or the experience gained by colleagues. Two excellent surveys of other "first-order" theories can be found in the text by Kraus [17] and the monograph by Leissa [18].

The effects of large deflections on the resultant deformation state are quite more dependent on exactly which shell theory it is based upon. Three of the most popular nonlinear shell theories are due to Reissner [19], Naghdi and Nordgren [20], and Sanders [21], the latter two being the more general. The theory due to Sanders is an extension of the linear theory detailed previously. Additional information on the nonlinear response of structures can be found elsewhere in this monograph by Belytschko.

Higher order shell theories in computer codes have mainly been included through the use of the isoparametric element or one of its forms. This element has been used by several researchers to analyze problems which vary in scope from thin shells through thick shells to continua [22]. A problem inherent to the more general isoparametric elements is that excessive shear-strain energy can be retained unless properly accounted for. Also, if applied to thin shells, numerical problems may arise from ill-conditioned equations. This is due to high stiffnesses in the thickness direction as opposed to the low stiffnesses in directions orthogonal to the thickness. This problem can be avoided in thinner shells by using the degenerate super-parametric element. Here straight lines initially perpendicular to the middle surface remain straight after deformation but are only approximately normal to the middle. Also, strain energy associated with stresses perpendicular to the middle surface is neglected. This is

comparable to what shell theoreticians call a "second-order" or shear deformation theory. This element can be simplified further to produce the discretized Kirchhoff element [23]. In this element the rotations are forced to produce zero transverse shear strains at the nodal points and the remaining transverse shear strain energy between the points is neglected. This is comparable to "first-order" theories previously discussed. Another disadvantage of the isoparametric element (other than the super-parametric) is the fact that the purist finds it hard to state exactly which higher-order theory he is using. An excellent source which discusses thick shell representation in computer codes is the chapter by Bathe and Wilson [24].

Shell Description

Shells can be broadly placed in one of two categories: arbitrary shells or rotationally symmetric shells. The computational analysis of arbitrary shells is necessarily more complex. It takes up more computer core, runs longer resulting in fewer codes that can perform calculations on this class of shells. Most of the codes that can do general shell problems are the large general-purpose codes. In fact, there are only a few special-purpose codes that can do the arbitrary shell. One example is SLADE-D [23] and that is limited to shells for which the reference surface is a portion of a surface of revolution. The simplification to a shell of revolution leads to a much more tractable analysis. If finite elements are used, then the element will be "one-dimensional". Often, it is advantageous to the user to reformulate a general shell problem as one where he can use the results of a shell of revolution code.

Another type of shell which can fall into both the above categories is a stiffened shell. If the shell stiffeners are relatively close together and information on the general shell response is desired, the stiffeners can be "smeared out". "Smearing" should not be done if the analyst wishes information on responses which have a wavelength similar to or less than the stiffener spacing. If the stiffener is treated as a discrete structure, it is usually allowed to rotate and translate as the shell does to which it is attached. If a general-purpose code is to be used, it is important that the user verify that beams and other structures or masses can be attached. If so, many practical problems involving combinations of shells or shells with other structures may be amenable to analysis.

Another subset of shells is that which is referred to as shallow shells. Shallow shells are defined by the shallowness parameter (maximum height of the middle surface above base plane/base plane diameter) being less than approximately 1/5 to 1/10. This simplification from the full shell theory can make computations more tractable. For instance, a shallow spherical shell has an identical mathematical behavior to that of a circular plate on an elastic foundation.

Methods of Spatial Discretization

The three principal methods of discretizing shells in general-purpose codes or special-purpose shell codes are: finite differences, forward integration, and finite elements. The finite element method was the latest of the three to mature, but the great majority of the present codes which treat shells use this method.

The finite difference method can be further subdivided into what might be called the conventional finite difference method and the finite difference energy method. In the conventional method, the differential equations of motion are converted to algebraic equations through finite difference discretization of the differential operators [25]. Principal difficulties of this method include difficulties with general boundary conditions and numerical problems associated with the lack of positive definiteness of the system. In the finite difference energy method, the strains in the expression for total energy are represented

by their appropriate finite difference formulas [26]. The expression for energy is now obtained through a numerical integration over the shell. This is then minimized with respect to the displacement components. The algebraic forms for the energy are easily programmable, the difference expressions are of a lower degree than the conventional method, and boundary conditions are easier to satisfy. However, even with the development of the energy method, present finite difference codes cannot solve the more general structural problems handled by the finite element methods.

In the forward integration method, the shell is first divided into segments in the meridional direction after removing the circumferential coordinate by a Fourier decomposition. The equations are integrated over each segment starting from one of the boundaries. The solution procedure is similar to that of a two point boundary value problem. There are only a few special-purpose shell codes which employ this method, but they are widely used.

Use of the finite element method is initiated by subdividing the shell structure into a finite number of subregions or elements. The displacements within the element are then expressed in terms of the generalized coordinates of the element by polynomial functions. If interpolation functions are used, the method works with nodal displacements and not generalized displacements. Using one of these formulations, the element's strains and stresses are now written. The element stiffness matrix and load vector can now be derived by the direct or variational method. If one-dimensional elements are used, the approach is probably equivalent, but for general structures, the variational formulation is superior. In the variational approach, the potential energy is formed and then minimized to yield the element characteristics. The procedure is now straightforward to determine element displacements now that the stiffness matrix and load vector are known. Excellent references on the finite element method can be found in the books by Zienkiewicz [22] and Desai and Abel [27].

Eigenvalue Analysis

The eigenvalues or natural frequencies are important to the shell analyst for subsequent use in a transient response calculation by mode superposition. They are also important so that the shell is designed in a manner such that its natural frequencies are removed from possible excitation frequencies.

A difficult situation for eigenvalue extraction codes when applied to shells is that eigenvalues associated with predominantly membrane and bending behavior are far removed from each other and often cluster within small intervals. If all of these eigenvalues are important, the eigenvalue routine must be able to separate and extract most of the eigenvalues. Also, in mode superposition transient shell response problems, many frequencies may be excited and hence must be obtained.

Eigenvalue extraction routines may be broadly categorized to be one of two types: those which calculate all the system eigenvalues and those which only determine a small percentage. The direct methods such as those of Householder or Jacobi will obtain all of the eigenvalues with a lower portion of them being accurate. One disadvantage is that only relatively low degree of freedom shell systems can be analyzed (300–400 degrees of freedom on most computers) because the matrix is full and is stored in core. Computation times may be restrictive since they grow with the cube of the matrix order being analyzed.

If the analyst is interested in only a relatively small portion of the shell eigenvalues, inverse power iteration is the most widely used technique. Eigenvalues of systems at least an order of magnitude greater in size can be analyzed. The method takes advantage of the sparseness of the stiffness. A trial eigenvector is assumed and the system of resultant inhomogeneous equations is iterated upon until there is satisfactory convergence. If higher eigenvalues are desired, a new trial eigenvector is assumed orthogonal to all previously calculated eigenvectors. A spectral shift is performed and a new eigenvalue is obtained nearest the shift. Disadvantages include the low

number of eigenvalues easily obtained, the possibilities of missing eigenvalues, and problems with clustered eigenvalues. A similar technique known as subspace iteration uses a set of orthogonal trial eigenvectors instead of one eigenvector in the procedure just described. A chapter on eigenvalue extraction by Stricklin and Wilkie is in this volume.

Method of Time Integration

Of the four methods described in the first part of this chapter which treat the time variable, only the first two are widely employed in shell codes. These are the mode superposition and direct (numerical) integration techniques.

Mode superposition techniques are efficient for relatively simple shells with few modes. However, if both bending and membrane responses are important, many modes are needed, as pointed out previously. The modes which significantly add to the response are not always those associated with the lowest eigenvalues in ascending order. For instance, the lowest bending and membrane modes are the most salient modes in the response. However, some of the eigenvalues associated with higher bending modes are less than the first membrane eigenvalue. A good analyst computing the transient response should recognize this and eliminate those modes from the calculation.

There are two types of schemes used in the direct integration approach in dynamic shell codes. These are the implicit and explicit integration procedures. Usually one of these methods should be used if the analyst contemplates that a large number of modes will be excited. The solution of the eigenvalue problem is not needed in the use of this method.

In the implicit method, the solution everywhere in the shell proceeds forward in time simultaneously. At each time increment, a set of algebraic equations must be solved. In the case of nonlinear transient problems, these are necessarily nonlinear. The implicit approach can be either conditionally or unconditionally stable. The latter schemes include the Newmark β ($\beta \geq 1/4$), Houbolt, and Wilson θ methods. The only sources of error in these schemes are truncation and round-off errors. These errors are inherent in all direct integration methods.

The important feature of the explicit methods is that the solutions advance in time at each nodal point independent of the other nodal points. Systems of equations are not solved at each time step. If nonlinearities are present, this is an advantage over the implicit schemes since solutions of nonlinear equations are expensive and time consuming to solve. The principal disadvantage of the explicit approach is that all the operators are conditionally stable. The time step must always be less than a certain limit which is inversely proportional to the highest frequency present. In linear problems, this can be quite time consuming when compared to the unconditionally stable implicit methods which can use larger time steps.

The convergence of the implicit and explicit methods is influenced by the form of the mass matrix. Krieg and Key [28] show that for best convergence the mass matrix should be diagonal for explicit schemes and consistent for the implicit methods.

Special Features

The discussion to this point has emphasized the computer analysis of isotropic, homogeneous, linear elastic shell structures with small deflections. Nonlinearities due to large deflections have been briefly discussed in the sections on shell theories and time integration.

Codes that treat composite (layered, filamentary, or sandwich) shell structures may include more complex shell theories that include shear deformation. This mandates higher-order finite elements which will increase computer time.

The inclusions of plasticity and viscoelasticity into shell codes should be handled with great care. This is due to the fact that these features are extremely material dependent. The ideal code should provide a wide variety of

the plasticity or viscoelasticity models that engineering materials can follow. Otherwise, the code might be unduly restrictive.

COMPUTER CODES

The purpose of this section is to enumerate and briefly describe the dynamic shell codes which are widely employed or show potential for extensive usage. If shell code development continues at the same pace in the future as it has in the past five years, some of the codes listed may be obsolete or may not be maintained. However, the information contained here will be useful in selecting the appropriate new or improved computer code.

Selection of a code is heavily a function of the user's particular situation. There is an advantage to the user in choosing a general-purpose code, such as NASTRAN. In these types of codes there is usually more than one shell element in addition to other types of elements, e.g., continuum and beam. The acquisition of such a code could be quite cost effective if the user's facility has a continued use for the variety or problems that can be solved. However, these programs often require much time to learn how to use and may need a person devoting most of his time to maintain.

For the individual in a small facility, the answer is probably in the use of one of the more specialized shell codes, such as SATANS or the SOR family.

ANSYS

ANSYS is a general-purpose finite element code with an extensive element library (about 70). The structural element library includes pipe, solid, beam, flat plate, and axisymmetric shell elements. Triangular or quadrilateral elements may be used as general shell elements. There are also axisymmetric shell elements. The shell elements use a thin shell theory. Eigenvalues are extracted via the Jacobi method. Large displacement and large rotation capabilities exist within the code. A number of the elements include geometric stiffening as an option. Transient response calculations are accomplished through an implicit method. Stiffeners can be included in a discrete or smeared sense. Branched shells can be treated. There is a plasticity capability. Material behavior is characterized in one of several ways. These are: virgin stress-strain curve, kinematic hardening, isotropic hardening, and 10^{th} cycle empirical hardening. Viscoelastic equations are of the power type for creep strains in metals. Both primary and secondary creep equations are available. Shells can be layered and the material orthotropic.

Address of Availability: Swanson Analysis Systems
870 Pine View Drive
Elizabeth, Pennsylvania 15037

ASTRA

ASTRA is a general-purpose finite element code. Elements include a beam, spar, rod, and flat-plate element. Membrane and bending are combined in this flat-plate element forming a general quadrilateral element. Axisymmetric solid, conical shell, and cylindrical shell elements are also included. The eigenvalue extraction scheme uses a Householder decomposition scheme. There is no capability for time integration. It can treat general shells with discrete stiffeners and branches. The shells can be of an orthotropic material.

Address of Availability: Boeing Computer Services, Inc.
P. O. Box 24346
Seattle, Washington 98124

BOSOR 4

BOSOR 4 is based on the finite difference energy method. For dynamic problems it can only treat an eigenvalue analysis of axisymmetric shells. The eigenvalue method of extraction used is inverse power iteration with spectral shifts and deflation. The shell theory used is that attributable to Sanders. It can solve problems with prestress including large deformation. Stiffeners can be treated either as discrete or smeared. Since this code is restricted to shells of revolution, meridional stiffeners must be smeared. The code will treat layered and orthotropic shells. It will also analyze branched shells.

Address of Availability: Dr. David Bushnell
Lockheed Missiles and Space Company
3251 Hanover Street
Palo Alto, California 94304

ICES STRUDL

ICES STRUDL is a general-purpose finite element code whose element library includes many plane stress-plane strain, plate bending, shallow shell, and solid elements. It also has a bending-membrane flat-plate element. Eigenvalue extraction can be performed by Householder tridiagonalization or inverse iteration. Time integration is accomplished via the Newmark β implicit method. The code can analyze a general shell geometry with discrete stiffeners and branches. There is no plastic or viscoelastic capability. No geometric nonlinearities are allowed in the dynamic version. The shell material may be orthotropic. A layered shell is analyzed by laying elements on top of one another.

Address of Availability: ICES Users Group
P. O. Box 8243
Cranston, Rhode Island 02920

KSHEL

KSHEL is a forward integration code which can treat branched shells of revolution. The shell theory employed is that of Love-Reissner. Eigenvalues can be extracted by either inverse iteration with shifts or determinant plotting. Transient response is performed with the mode superposition method. Ring stiffeners are simulated by short cylindrical shells. The shell may be layered and orthotropic. At present there is no plastic or viscoelastic capability.

Address of Availability: Dr. Arturs Kalnins
Lehigh University
Department of Mechanical Engineering and Mechanics
Bethlehem, Pennsylvania 18015

MARC

MARC is a general-purpose finite element code which contains a wide variety of 1D, 2D, and 3D solid elements. These include several beam elements and axisymmetric and doubly-curved shell elements. The shell elements use the Koiter-Sanders shell theory. Eigenvalues are extracted using inverse power iteration. There are several types of time integration included. There are both mode superposition and direct integration capabilities. The direct methods are implicit Newmark β and Houbolt techniques and the central difference explicit technique. Recently included is an "exact" explicit operator attributable to R. Melosh. Geometric nonlinearities are included in the sense of the full Koiter-Sanders theory. There are several ways of including plasticity (perfectly plastic, isotropic hardening, kinematic hardening). There is a viscoelastic capability through the use of nonlinear Kelvin and Maxwell models. These can be used in series. The code has general shell and shell of revolution

elements and can treat stiffened and branched shells. It has an orthotropic capability and a thick shell element.
Address of Availability: MARC Analysis Corporation
105 Medway Street
Providence, Rhode Island 02906

MINIELAS

MINIELAS is a general-purpose finite element code which can treat general shells through a variety of beam and surface (flat-plate, shell) elements. There is no solid element at the present time. Its dynamic shell capability extends only as far as eigenvalue extraction and random excitation. Eigenvalues are extracted using inverse iterations with shifts. Only small deformations are allowed.
Address of Availability: Dr. S. Utku
Duke University
School of Engineering
Durham, North Carolina 27706

NASTRAN (MSC)

NASTRAN is a general-purpose finite element code. The element library includes beam, plate, and shell elements. Most shell analysis is performed with the flat-plate element. It also has an element that accounts for transverse shear deformation. Eigenvalue analysis is accomplished by one of three methods: inverse iteration, Given's method, determinant search. Time integration is performed via the Newmark β implicit integration scheme. In general, there is no plasticity or viscoelasticity capability. Large deflections can be handled as long as you can use geometric stiffness. General shells with branches and discrete stiffeners can be handled. The material can be orthotropic and the shell of layered construction.
Address of Availability: The MacNeal-Schwendler Corporation
7442 N. Figueroa Street
Los Angeles, California 90041

NONSAP

NONSAP is a general-purpose finite element which uses one, two, and three-dimensional isoparametric elements. These are variable number node elements. Eigenvalue extraction is performed via determinant search. Transient response is obtained through the implicit Newmark β or Wilson θ techniques. There exist a large displacement and rotation capability in the one and two-dimensional elements. The code will handle general shell geometry with stiffeners and branches. Plasticity can be included in the two-dimensional element. This can be von Mises isotropic hardening or elastic-perfectly plastic. There is no viscoelastic capability. An extension of this code, ADINA, is being developed by Klaus Bathe at MIT.
Address of Availability: Dr. Edward L. Wilson
Department of Civil Engineering
University of California
Berkeley, California 94720

REPSIL

REPSIL is a general-shell code which uses the conventional finite difference method. The code uses a thin shell theory. There is no eigenvalue analysis capability. Time integration in transient response problems is handled by an explicit central difference scheme. Plasticity can be handled in the following ways. The material can be elastic-perfectly plastic or elastic with strain

hardening. The actual stress-strain curve can be input in straight line segments. A mechanical sublayer model is used. A power law strain-rate material constitutive model capability exists. The code treats orthotropic shells; however, it may have problems when plasticity is included with such shells. There is no viscoelastic capability and it cannot treat layered shells or shells with stiffeners or branches. An energy check is made during the calculations which can be used to verify the calculations.

Address of Availability: N. J. Huffington, Jr.
U.S. Army BRL
Aberdeen Proving Ground, Maryland 21005

SATANS

SATANS employs conventional finite differences after a Fourier decomposition in the circumferential direction. Eigenvalue analysis is not included in the version that is presently disseminated; however, there is on-going work in this area. The shell theory employed is that due to Sanders with small strains and moderate rotations. The shells must be axisymmetric. Time integration is handled by the implicit Houbolt method. It can handle layered shells, though each layer must be isotropic. No orthotropic materials can be handled. The shell may not have stiffeners or branches and there is no plasticity or viscoelasticity capability.

Address of Availability: R. E. Ball
Code 57Bp
Naval Postgraduate School
Monterey, California 93940

SHORE

SHORE is a conventional finite difference code in which the equilibrium equations are written in terms of the stress resultants. Time integration is handled by explicit central differences. There is no eigenvalue extraction capability. Large deflections and moderate rotations are allowed. The middle surface of the shell must be axisymmetric, though thickness and material properties can change arbitrarily. The material can have the option of being elastic-perfectly plastic or elastic with linear strain hardening. Springs can be attached at any point in the three coordinate directions. Also, masses can be attached to these nodal points. There is a layered and orthotropic shell capability. The code contains no viscoelastic nor branched shell capability.

Address of Availability: Philip Underwood
Lockheed Missiles and Space Company
3251 Hanover Street
Palo Alto, California 94304

SLADE D

SLADE D is a finite element code which treats shells whose reference surface is a portion of an axisymmetric surface. The surface can include cutouts and it could be incomplete. There is no eigenvalue extraction capability. The shell theory is a Kirchhoff-type thin shell theory. The time integration is performed by an explicit central difference scheme. However, unlike many other codes, the time step is computed internally. The shell may contain stiffeners and branches such that the reference surface remains a portion of an axisymmetric surface. No geometric nonlinearities, plasticity, or viscoelasticity are allowed. It can handle orthotropic shells. Layered shells are treated through an equivalent modulus method.

Address of Availability: Dr. Samuel W. Key
Sandia Laboratories
Division 1541
Albuquerque, New Mexico 87115

SOR FAMILY

The SOR family of codes is a series of finite element codes which treat shells of revolution. These currently are SAMMSOR IV which evaluates the stiffness matrix, SNASOR II which does static problems, FAMSOR which extracts eigenvalues, and DYNAPLAS II which performs the transient response. Eigenvalues are extracted by inverse iteration with shifts and deflation. Time integration is either by the implicit Houbolt or explicit central difference method. The code can handle geometric nonlinearities, orthotropic materials, and beam-type ring stiffeners. If plasticity is included, the material should be isotropic. Material hardening is either isotropic or mechanical sublayer. It can handle strain-rate effects. No viscoelasticity is included and it will not treat branched shells. DYNAPLAS III (soon to be released) will handle layered shells, thermal loading, and temperature dependent materials.

Address of Availability: Dr. Walter E. Haisler
Department of Aerospace Engineering
Texas A&M University
College Station, Texas 77843

STAGS

STAGS is based on the finite difference energy method. The present version of the code can do both an eigenvalue and transient response analysis. There are two eigenvalue extraction schemes. The user can employ either single vector inverse iteration or subspace iteration. The user has the choice of explicit central difference or implicit time integrators. The shell theory used is attributable to Flügge-Marlowe. This is similar to the nonlinear Sanders theory except for some curvature terms. The code treats general branched shells with stiffeners. The stiffeners can be either discrete or smeared. Large deflection effects can be included. Plasticity capability exists using a mechanical sublayer model. The code can treat orthotropic shells. No viscoelastic capability exists generally, though some versions floating around have it.

Address of Availability: Dr. B. Almroth
Lockheed Missiles and Space Company
3251 Hanover Street
Palo Alto, California 94304

STARDYNE

STARDYNE is a general-purpose finite element code whose element library includes beams, plates, and shells. Eigenvalue analysis is accomplished by either single vector inverse iteration or Householder tridiagonalization followed by QR transformation. Time integration can be achieved by either mode superposition or implicit (Newmark β or Wilson θ) integration. The code can handle general shells with discrete stiffeners and branches. It has an orthotropic material capability and can analyze layered shells.

Address of Availability: Dr. Richard Rosen
Mechanics Research Inc.
9841 Airport Boulevard
Los Angeles, California 90045

STARS

STARS is a forward (multi-segment forward) integration code. It can extract eigenvalues via either a determinant search or a Householder-Givens technique. Time integration is accomplished with an implicit Houbolt technique. The code can treat stiffened and branched shells. The shell theory used is that due to Love-Reissner. Orthotropic materials can be used in the shell analysis.

Address of Availability: V. Svalbonas
The Franklin Institute
Philadelphia, Pennsylvania 19103

CONCLUSION

It is expected that this chapter will acquaint the analyst with response problems of thin shells, the features of computational dynamic shell analysis, and the present widely used computer codes in this field.

ACKNOWLEDGEMENT

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II. Capabilities and Routines Within Programs

Summary of General Purpose Programs

Suresh K. Desai

Commonwealth Associates Inc.

INTRODUCTION

General purpose programs have been a topic of considerable interest ever since the power of the finite element method became generally recognized. The first developments came in the field of linear static analysis, although, as Clough has pointed out, the initial impetus to the method came in the field of dynamic response [1]. The dynamic capabilities of most general purpose programs were developed later.

Comprehensive reviews of structural mechanics software have been published in [2], with capabilities of, and user reaction to, general purpose programs discussed in the context of transient analysis, non-linear continua, and plastic analysis. Also, more detailed reviews of a few general purpose programs are in [3]. An excellent overview of general purpose programs is provided in [4].

The theoretical aspects of structural dynamics have been adequately treated in many textbooks [5], and will not be discussed here.

SCOPE

In order to define the scope of this paper, some broad requirements for a general purpose program are laid down as follows:

1. At least one of the methods of analysis is the displacement method using finite elements.
2. The program should have proven capability for both linear static and linear dynamic response, including computation of natural frequencies. Desirable, but not essential, capabilities include (a) nonlinear static response, (b) nonlinear dynamic response, (c) stability, both static and dynamic, and (d) thermal loading.
3. A good library of elements for both planar and three-dimensional problems should be available. It should include:
    a. One-dimensional straight elements for axial stress,
    b. One-dimensional elements, both straight and curved, subjected to forces and moments in three dimensions,
    c. Plate and shell elements of various shapes, both for membrane forces and/or bending,
    d. Three-dimensional elements of various shapes, and
    e. Preferably, axisymmetric elements and axisymmetric shells, thick and thin.
4. The program should be of modular design with the facility for easy modification or extension.

Two more requirements are added by this writer from the user's point of view:

5. The program should be readily available and should have well documented User's Manual and Theoretical Manual.

6. Competent technical assistance must be readily available, either from the computer center offering the program, or from the developers, or from a user's group.

GENERAL CHARACTERISTICS

Details of dynamic capabilities of general purpose programs are summarized in the next section. However, the following characteristics are common to most programs:

1. Coding language is FORTRAN IV or some other level of FORTRAN. Some programs have small sections of code written in an assembler language because of special considerations.
2. Consistent inertia matrix is available.
3. Guyan reduction for retaining selected "dynamic" degrees of freedom for eigenvalue extraction is available.
4. A maximum of six degrees of freedom per node are permitted: three displacements and three rotations referred to three orthogonal axes.
5. Modal damping is viscous damping specified as a distinct fraction of the critical damping for each mode. Proportional damping is also viscous damping specified as a linear combination of the global inertia and stiffness matrices.
6. Selected input may be generated or prepared in advance for later use by the program.
7. Transient analysis includes seismic analysis.
8. Good plotting capability is available.
9. Restart capability is available.
10. Interface with user routines available, although this is generally not easy.
11. Input is formatted.
12. Consistent units must be used.
13. Dynamic allocation of working storage.
14. Specification of selective output.

PROGRAM DETAILS

ANSYS

Developer: Swanson Analysis Systems, Inc.
870 Pine View Drive
Elizabeth, Pennsylvania 15037
Documentation: ANSYS User's Manual & Examples Manual
Availability: CDC Cybernet
Westinghouse Telecomputer Center, Pittsburgh, Pa.
Swanson Analysis Systems, Inc.
Machine: CDC 6600/7600, UNIVAC 1108, IBM 360/370
Analyses: Modal, harmonic, spectrum, transient - both modal and direct
Elements: Extensive library of one-, two- and three-dimensional elements, superelements
Nonlinearity: Material plasticity, creep and swelling, geometric nonlinearity gaps
Damping: Proportional, modal
Data Generation: Good mesh generation capability
Notable Items: (a) Linear equations of constraint between multiple degrees of freedom. (b) Polar, cylindrical, and spherical systems in addition to cartesian. (c) Only Jacobi iteration available for the eigenproblem.
Comments: ANSYS is quite a versatile program, particularly for nonlinear problems. This writer recently used the program in a comparative study of nonlinear transient analysis of pipe breaks and received excellent support from the developers.

ASKA

Developer: ASKA - Group
Pfaffenwaldring 27
Stuttgart-80, West Germany
Documentation: ASKA User's Reference Manual ISD-Rep. No. 73
DYNAN User's Reference Manual IDS-Rep. No. 97
Availability: A few private computer centers in U. S.
Machine: CDC 6600, UNIVAC 1108, IBM 360/370
Analyses: Modal, Harmonic, Spectrum, Transient (modal), Complex eigenvalue
Elements: Extensive library
Nonlinearity: No
Damping: Proportional, modal, nonproportional (for complex eigenproblem only)
Data Generation: Good mesh generation capability
Notable items: (a) A variety of procedures available for both the real and the complex eigenproblem. (b) Extensive and effective use of substructuring. (c) Up to 32 degrees of freedom per node. (d) A very large system requiring considerable experience. (e) Analysis of non-proportionally damped systems is done entirely in the core, a feature which limits the size of problem which may be handled. (f) Free field input.
Comments: DYNAN, as the dynamic part of ASKA is called, was developed much later than the static part and is not nearly as versatile. As far as this writer is aware, ASKA is one of the few packages developed in Europe which is being used in the U. S., although it is not offered by any major computer network in the U. S.

COSA

Developer: Dornier GmbH
799 Friedrichshafen, Postfach 317
West Germany
Documentation: COSA User's Manual
Availability: Dornier Computer Center, Friedrichshafen, Germany
CDC Computer Center, Frankfurt, Germany
Machine: CDC 6600, IBM 360/370
Elements: Good library
Nonlinearity: No
Damping: Proportional, modal, nonproportional, structural, combination of viscous & structural
Data Generation: Information not available
Comments: DYNAME is the name of the part of COSA which performs dynamic analysis.

MARC-CDC

Developer: Marc Analysis Research Corporation
105 Madway Street
Providence, Phode Island 02906
Documentation: MARC-CDC User Information Manual Vols. 1-3
Availability: CDC Cybernet
Machine: CDC 6600/7600
Analyses: Modal, Transient - modal and direct
Elements: Good library for one- and two-dimensional elements only. Only one three-dimensional element available
Nonlinearity: Material plasticity, creep and swelling, geometric nonlinearity
Damping: Modal
Data Generation: Built in routines for mesh generation.
Notable Items: (a) Dynamic capabilities are very limited. (b) No Guyan reduction. (c) The program appears to be difficult to use and rather poorly documented.

Comments: The program offers the most advanced technology for nonlinear static analysis [2]. However, the dynamic capabilities are still quite limited. When the dynamic capabilities are brought on a par with the static capabilities the program may become a very powerful tool for nonlinear dynamic analysis, especially since this would include effects of dynamic instability.

MINIELAS

Developer: Computer Structural Analysis Fund
Department of Civil Engineering
Duke University
Durham, North Carolina 27706
Documentation: MINIELAS User's Manual
Availability: Duke University
Machine: IBM 360/370
Analyses: Modal, transient (modal)
Elements: Limited library
Nonlinearity: No
Damping: Modal
Comments: MINIELAS is a part of the general purpose programs called ELAS.

NASTRAN

Developer: Computer Sciences Corporation, MacNeal Schwendler, Martin Baltimore, and Bell Aero Systems under contract to NASA
Documentation: The NASTRAN Theoretical Manual, NASA SP-221
The NASTRAN User's Manual, NASA SP-222
The NASTRAN Programmer's Manual, NASA SP-223
Availability: COSMIC, CDC Cybernet, McAuto Computer Network, Westinghouse Telecomputer Center, Pittsburgh, and others
Machine: CDC 6600/7600, UNIVAC 1108, IBM 360/370
Analyses: Modal, harmonic, spectrum, transient - both modal and direct, complex eigenvalue
Elements: Good library, general element
Non-linearity: Limited
Damping: Proportional, modal, nonproportional
Data Generation: No
Notable Items: (a) A very large versatile system requiring considerable experience. (b) Limited choices for eigenproblems. (c) Linear equations of constraint between multiple degrees of freedom. (d) Cylindrical and spherical systems in addition to cartesian. (e) Very expensive for small problems.
Comments: NASTRAN has been the subject of more discussion than any other piece of software. Its documentation is the most thorough this writer has come across, but its volume is likely to scare many potential users. NASTRAN is available on most major computer networks. This makes it the most easily available package.

NISA

Developer: Engineering Mechanics Research Corporation
24001 Southfield Road
Southfield, Michigan 48075
Documentation: NISA User's Manual
Availability: Engineering Mechanics Research Corporation
McAuto Computer Network
Machine: CDC, IBM, UNIVAC, Honeywell
Analyses: Modal, Harmonic, Transient (modal)
Elements: Good library, strong in isoparametric elements, also thick and

thin shells
Nonlinearity: No
Damping: Modal
Data Generation: Good mesh generation capability
Notable Items: (a) QR Householder and subspace iteration for eigenproblem. (b) Lumped mass matrix option also available. (c) Well developed plotting capability. (d) Numerous pre- and postprocessors.

SAP IV

Developer: Department of Civil Engineering
University of California
Berkeley, California 94720
Documentation: SAP IV - Bathe, Wilson & Peterson
Availability: Prof. Edward L. Wilson
Department of Civil Engineering
University of California, Berkeley, California 94720
Machine: CDC 6600/7600, UNIVAC 1108, IBM 360/370
Analyses: Modal, Spectrum, Transient - both modal and direct
Elements: Good library
Nonlinearity: No
Damping: Modal, proportional
Data Generation: Nodes, Elements
Notable Items: (a) No Guyan reduction. (b) Only lumped inertia matrix. (c) Only subspace iteration for eigenproblem. (d) Only one value of modal damping is available for transient analysis.
Comments: The unique feature of SAP IV is that it is the only general purpose program that is simple enough so that users have tried to modify it for their needs. Hence there are many offshoots of the original SAP program, none of which are listed here.

SESAM-69

Developer: Det Norske Veritas
P. O. Box 6060
Etterstad, Oslo, 6
Norway
Documentation: SESAM-69 User's Manual
Availability: Det Norske Veritas and at some computer centers in U. S.
Machine: CDC 6600, UNIVAC 1108
Analyses: Modal, harmonic, transient (modal)
Elements: Good library, superelements
Nonlinearity: No
Damping: Modal
Data Generation: Nodes, elements
Comments: The package is oriented to extensive use of superelements.

STARDYNE

Developer: Mechanic Research, Inc.
9841 Airport Blvd.
Los Angeles, California 90045
Documentation: MRI/STARDYNE User Information Manual
Availability: CDC Cybernet
Machine: CDC 6600
Analyses: Modal, harmonic, spectrum, transient (modal)
Elements: Good library
Nonlinearity: No
Damping: Modal, composite modal

Data Generation: Nodal coordinates, element connectivity, loadings
Notable Items: (a) Cylindrical system in addition to cartesian. (b) Only lumped inertia matrix. (c) Built-in bandwidth optimizer. (d) Program is fairly well documented, easy to learn, and well supported by MRI/CDC Cybernet.
Comments: Although the program capabilities are limited, it has received a very high rating from users for its reliability and economy. Implementation of Guyan reduction in early 1975 has eliminated the rather awkward handling of "dynamic" degrees of freedom.

STRUDL II

Developer: Structures Division & Civil Eng. Systems Laboratory
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139
Documentation: ICES - STRUDL II Engineering User's Manual, Vols. 1 - 3
Availability: ICES - USER'S GROUP INC.
P. O. Box 8243
Cranston, Rhode Island 02920
Machine: IBM 360/370
Analyses: Modal, spectrum, transient (modal)
Elements: Limited library
Nonlinearity: No
Damping: Modal
Data Generation: Limited to element properties and loadings.
Notable Items: (a) Free field input. (b) No plotting capability. (c) Great flexibility for units of various parameters in input and output, all conversions being handled internally. (d) Easy additions, deletions and modifications of problem parameters at any stage of program execution. (e) Lumped inertia matrix option also available. (f) Householder method and iteration are available for the eigenproblem. (g) Coding language is ICETRAN, which is very similar to FORTRAN with the added capability of dynamic storage allocation. (h) Flexibility in the labeling of nodes and elements. (i) Tables for properties of rolled shapes built into the program.
Comments: STRUDL II, which is a part of the ICES system was developed at M.I.T. during the mid-sixties and was, until recently, a very popular program in the building industry. However, after further developments on the program were stopped due to lack of support, and McDonnell Douglas developed and marketed its proprietary version, the use of STRUDL II appears to be declining.

STRUDL DYNAL

Developer: See comments below.
Documentation: ICES - STRUDL II Engineering User's Manual, vols. 1 - 3
ICES STUDL DYNAL User's Manual
Availability: McDonnell Douglas Automation Company
Computer Network
Machine: IBM 360/370
Analyses: Modal, harmonic, spectrum, transient (modal)
Elements: Good library, general element
Nonlinearity: No
Damping: Modal
Data Generation: Limited to nodal coordinates and element properties
Notable Items: (a) Free field input. (b) Great flexibility for units of various parameters in input and output, all conversions being handled internally. (c) Easy additions, deletions and modifications of problem parameters at any stage of program execution. (d) Lumped inertia matrix option also available. (e) Householder method, Jacobi iteration and QR method available for eigenproblem. (f) Coding language is ICETRAN which

is very similar to FORTRAN with the added capability of dynamic storage allocation. (g) Flexibility in the labeling of nodes and elements. (h) Tables for properties of rolled shapes built into the program.

Comments: STRUDL DYNAL is the proprietary version of ICES STRUDL II. McDonnell Douglas Automation Company and Engineering Computer International have made extensive modifications and additions to the M.I.T. version of STRUDL II. The dynamic part DYNAL is completely new. The new STRUDL DYNAL is now serviced and supported by McAuto. As a result, many of the original bugs have been removed and the program is much more streamlined and efficient.

However, the program still has some deficiencies. To mention a couple of items, STRUDL DYNAL has no transient analysis using direct numerical integration, nor does it have the incompatible plane stress rectangle.

SELECTION OF SOFTWARE

The essential and desirable characteristics of a general purpose program listed above are met to a great extent by the programs detailed above. However, for further discussion, the field of programs is now narrowed down to the following:

Proprietary programs:

1. ANSYS
2. MARC-CDC
3. STARDYNE
4. STRUDL DYNAL

Public programs:

1. NASTRAN
2. SAP IV

The overall suitability of the above programs for different classes of problems is given below. The selection represents the personal views of this writer.

1. Routine small and medium size problems of linear dynamics - STARDYNE, STRUDL DYNAL, SAP IV, ANSYS.
2. Very large problems in linear dynamics - NASTRAN.
3. Non-linear dynamics - ANSYS, MARC-CDC.
4. Problems with special requirements; research type problems requiring considerable interface with user routines - SAP IV.
5. Special problems may be solved on other special purpose programs. For example, for axisymmetric shells or solids, use of special programs - based on finite element or other methods - should be definitely considered.

Based on his experience, this writer feels that no general purpose program is suitable to be used under all situations. Any institution, where a significant amount of diverse forms of dynamic analysis is done, should be using more than one program for the most effective results.

FUTURE TRENDS

The field of linear dynamics is now well covered, and new developments are to be expected in the field of nonlinear dynamics. Nonlinearity arises in one or more of three ways: (a) material behavior, (b) large displacements causing geometric nonlinearity and (c) nonlinear boundary conditions, for example, gaps. Both ANSYS and MARC-CDC have already addressed themselves to some of these problems. More work in this direction may also provide tools for dynamic stability analysis.

ACKNOWLEDGMENTS

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Nonlinear Analysis Description and Numerical Stability

Ted Belytschko

University of Illinois at Chicago Circle

INTRODUCTION

This chapter is concerned with two aspects of the nonlinear transient problem: the kinetic, kinematic, constitutive, and mesh descriptions, and the stability of the numerical integration schemes. The aim is to present the methods that are currently used in solid and structural mechanics software, and to evaluate and compare these methods.

We are concerned here with problems nonlinear in both geometry and materials, with the emphasis on the former. Geometrically nonlinear problems are those in which deformation and/or rotation are sufficiently large so that the initial shape of the solid must be distinguished from the deformed shape. In small deflection theory, matters of descriptions are of no consequence, even when the material is nonlinear.

We will divide our discussion of descriptions into two parts, the first concerned with solid continua, the second with structures. The choices and difficulties for these two classes of problems are quite different, so it is quite confusing to treat them simultaneously.

Structures are here defined to be slender, solid members in which the deformation is governed by the deformation of the midsurface or midline. In transient analysis, the choice of a continuum or structural approach depends both on the geometry of the solid and on the loading and response that is of interest. If the wave propagation through the thickness of the structure is of importance, a continuum approach is appropriate. Otherwise, it is convenient to make assumptions such as those of Kirchhoff or Reissner to define the motion of the structure relative to the midplane, and thus reduce the number of degrees of freedom. More will be said on this point later.

In these discussions, when a need arises for examining the details, we will often refer to the finite element method. However, it is now widely accepted that the finite element method and many finite difference methods (especially those applicable to arbitrary meshes) are basically very similar in that both can be viewed as solutions of the weak form of the governing partial differential equations. In both cases, the dependent variables are defined in elements or zones and difference equations are obtained for the discrete dependent variables. Although there are substantial differences in the mathematical accoutrements, in many cases the discrete equations for the finite element and finite difference methods are identical; the reader is referred to Refs. [1, 2, 3] for examples and further details. In the same spirit, we will not distinguish between nodes and mesh points or elements and zones, but use these terms synonymously.

The discussion of numerical stability is necessarily brief, for this aspect of the problem is not well understood. The major aim is to present a synopsis of current practice and to point out some pitfalls.

NOMENCLATURE

b_i	=	Body forces
$C_{ijk\ell}$	=	Linear elastic constants
$\underset{\sim}{d}$	=	Velocity strain (rate-of-deformation) tensor
$\underset{\sim}{e}$	=	Alamansi (Eulerian) strain tensor
$\underset{\sim}{E}$	=	Green (Lagrangian) strain tensor
$\underset{\sim}{F}$	=	Deformation gradient tensor
$\underset{\sim}{L}$	=	Velocity gradient tensor
M_{IJ}	=	Mass matrix
P_{Ii}	=	Nodal forces
R	=	radius of curvature
s	=	Arc length or line segment
$\underset{\sim}{S}$	=	Second Piola Kirchhoff (Lagrangian) stress tensor (also called Kirchhoff stress tensor in text)
t	=	Time and time step
u, u_i	=	Displacements and their Cartesian components
v	=	Tangent displacement for beam
$V, V^{(e)}$	=	Domain of problem and element or zone, respectively
w	=	Transverse (normal) displacement for beam
x_i	=	Spatial (or Eulerian) coordinates
X_i	=	Lagrangian coordinates
δ_{ij}	=	Kronecker delta or unit tensor
Δ	=	Increment
$\hat{\underset{\sim}{\varepsilon}}$	=	Corotational (or rigid-convected) strain tensor
$\underset{\sim}{\varepsilon}$	=	Small strain in flexural theories
κ	=	Curvature
ρ	=	Density
ϕ_I	=	Shape functions
$\underset{\sim}{\sigma}$	=	Cauchy (Eulerian) stress tensor
$\overset{\nabla}{\underset{\sim}{\sigma}}$	=	Frame invariant rate of Cauchy stress tensor
ω	=	Vorticity tensor
$\dot{}$	=	Superscript dot denotes time derivative
o	=	Subscript nought denotes original value of variable
i	=	Lower case subscripts designate Cartesian components
I	=	Upper case subscripts denote node numbers

DESCRIPTION OF SOLID CONTINUA

We will first sketch a few of the basic equations and concepts of the finite element method for future reference: these equations are not essential to the discussion, but useful for an understanding of the fine points. In the finite element method, the domain of the problem (see Ref. [4] for a definitive treatment) is subdivided into elements $V^{(e)}$. In each of these elements, the dependent variables u are approximated by shape functions $\phi_I(s)$ in the form

$$u = u_I^{(e)}(t)\phi_I(s) \tag{1}$$

where s are the independent variables, and $u_I^{(e)}$ are discrete dependent variables, usually defined at nodes. A relation between discrete element variables and the discrete variables of the system is provided by the connectivity matrix.

If the system of governing partial differential equations is written in the form

$$L(u) = 0 \tag{2}$$

then the discrete governing equations are provided by a Galerkin method yielding

$$\langle \phi_I(s), \quad L\ (u_I^{(e)}(t)\phi_I(s)) \rangle = 0 \tag{3}$$

where the brackets denote a scalar produce, which in discrete field problems usually corresponds to an integration over an element. The above is called the weak form of the partial differential equation; in transient problems they represent a set of ordinary differential equations, which are then integrated in time.

We will now describe the choice of the independent variables, s, and the state variables, such as stress and strain, in the analysis of solid continua.

Coordinates

In a solid, the complete deformation history may influence the stress, so both the original and current coordinates of any particle must be known in some sense. We denote the original Cartesian coordinates of a particle by X_i, (i = 1, 2 and 3 denoting X,Y, and Z, respectively) and the current coordinate by x_i, i = 1 to 3; reduction to one or two dimensions is accomplished by reducing the range of the indices. The components of the displacement vector u_i are then given by

$$u_i = x_i - X_i \tag{4}$$

The coordinates X_i are called Lagrangian coordinates. The coordinates x_i are known by two names: Eulerian or space coordinates; the latter designation indicates that they are fixed in space. We may also describe the space by attaching a continuously varying set of labels to material particles: these are called material coordinates. If we choose these labels to be the initial positions of the particles in space, then the material and Lagrangian coordinates coincide. This practice is customary in most numerical applications to continua, so the nomenclature material and Lagrangian coordinates are often interchanged.

The lines of constant X_i also form a set of curvilinear coordinates in the deformed solid: these are called convected coordinates. For an initially Cartesian system, the equations in the convected and Lagrangian coordinates are the same and need not be distinguished; the differences arise only when the initial configuration is described by a set of curvilinear coordinates.

Meshes

In discretizing a continuum by a finite element or finite difference mesh, two types of meshes are commonly used. If we attach each mesh point to a material particle and move the mesh point with the material, the mesh is called Lagrangian: the Lagrangian coordinates X_i of any mesh point in a Lagrangian mesh do not change with time. Alternatively, we can fix each mesh point in space, so that the Eulerian coordinate x_i is invariant in time; the resulting mesh is called Eulerian.

In solid mechanics problems, Lagrangian meshes have been used almost exclusively. Lagrangian meshes are prefered for solid mechanics problems for two reasons: (1) as a solid deforms, its boundaries (if they initially coincided with an Eulerian mesh), will no longer coincide with the Eulerian mesh lines, so complex procedures are needed at the boundaries and (2) the stress-strain behavior and history of a solid is associated with material points, so that it is most convenient to associate material zones in some manner with mesh zones.

In this respect there has been some confusion in the literature on the notions of "updated" mesh points (or coordinates). In using a Lagrangian

mesh, whether the mesh coordinates need to be updated depends on the kinematic and kinetic descriptions that are used. As long as each node is associated in some sense with a material point throughout the deformation, the mesh is Lagrangian. For an Eulerian mesh, the nodal coordinates are not updated.

A comparison of Eulerian and Lagrangian meshes is illustrated in Fig. 1. Though both meshes are shown to be initially rectangular this is not necessary in either case. The difficulties of treating boundaries with Eulerian meshes can be seen immediately; the boundaries fail to coincide with mesh lines after deformation. Furthermore, material points may move between zones (or elements), which makes the definition of their history very difficult. These problems are overcome in a Lagrangian mesh. The major difficulty with Lagrangian meshes is that very large deformations result in so much mesh distortion that the solution can not proceed; this difficulty appears regardless of whether or not the coordinates are "updated".

Kinematic Description

The term kinematic description here refers to how the deformation is measured, which determines the strain-displacement equations. We will here use the nomenclature of modern continuum mechanics for these measures and the other state variables, and the reader not familiar with it should be warned that it is quite confusing. To provide some help, in the next three sections, we will use upper case symbols for variables with a material (Lagrangian) character, and lower case symbols for variables with a spatial (Eulerian) character.

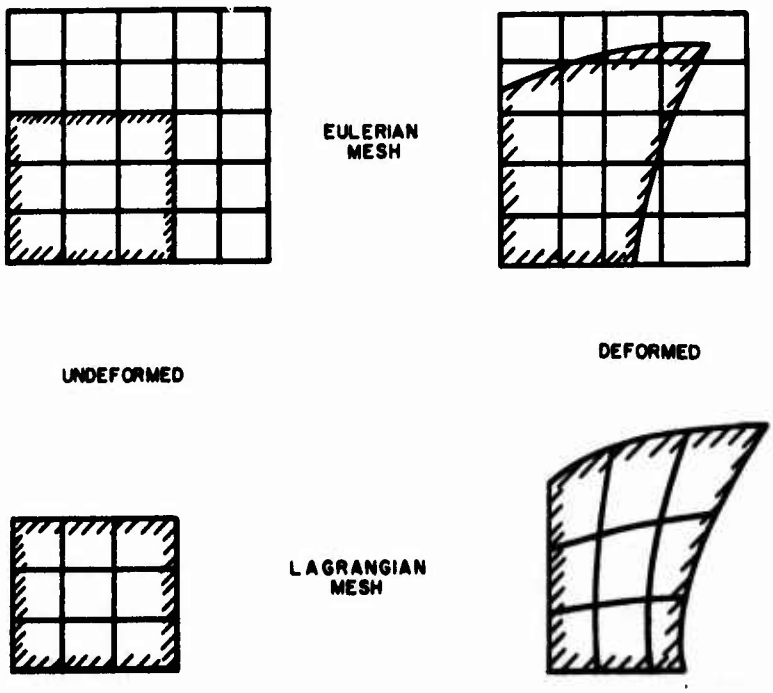

Fig. 1 Eulerian (spatial) and Lagrangian (material) meshes before and after deformations

The three most frequently used strain measures are: the Green (Lagrangian) strain tensor[1]

$$E_{ij} = \frac{1}{2}\left[\frac{\partial u_i}{\partial X_j} + \frac{\partial u_j}{\partial X_i} + \frac{\partial u_k}{\partial X_i}\frac{\partial u_k}{\partial X_j}\right] \tag{5}$$

the Alamansi (Eulerian) strain tensor

$$e_{ij} = \frac{1}{2}\left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{\partial u_k}{\partial x_i}\frac{\partial u_k}{\partial x_j}\right] \tag{6}$$

the velocity strain (rate-of-deformation) tensor

$$d_{ij} = \frac{1}{2}\left(\frac{\partial \dot{u}_i}{\partial x_j} + \frac{\partial \dot{u}_j}{\partial x_i}\right) \tag{7}$$

The names in the parentheses are alternative names: other names are also used.

The first two, E_{ij} and e_{ij}, measure the deformation by the change in the lengths of line segments. This can be seen by considering their definitions

$$2E_{ij}\,dX_i dX_j = dx_i dx_i - dX_i dX_i = d\ell^2 - d\ell_0^2 \tag{8}$$

$$2e_{ij}\,dx_i dx_i = dx_i dx_i - dX_i dX_i = d\ell^2 - d\ell_0^2 \tag{9}$$

Here $d\ell$ and $d\ell_0$ are the current and original lengths of a line segment. If we consider a bar in a uniaxial state of strain, then

$$2E_{11} = \left(\frac{d\ell}{d\ell_0}\right)^2 - 1 \qquad 2e_{11} = 1 - \left(\frac{d\ell_0}{d\ell}\right)^2 \tag{10}$$

The two strains can always be related as shown in Table 1.

The velocity strain measures the rate of change of the length (squared) of a line segment, as can be seen from the following alternative definition

$$\frac{d}{dt}\left[d\ell^2\right] = 2\,d_{ij}dx_i dx_j \tag{11}$$

In finite element programs, the Green and Alamansi strains are used most often, while in finite difference programs, the velocity strain is most common. The two types of strains are strikingly different in their retention of the history, i.e., the original shape. Both $\underset{\sim}{E}$ and $\underset{\sim}{e}$ contain information about the original shape, whereas $\underset{\sim}{d}$ contains information only about the current rate of deformation. Some information can be obtained by integrating $\underset{\sim}{d}$, but while in one dimension, the integral of $\underset{\sim}{d}$ for a material point yields the logarithmic strain, in two or three dimensions the interpretation of a material integral of $\underset{\sim}{d}$ is not clear. Therefore, to obtain a complete history of the deformation, the velocity strain would have to be transformed to a Green or Alamansi strain rate (or $\dot{\underset{\sim}{E}}$) by one of the formulas given in Table 1; this rate could then be integrated to yield a measure of the total deformation. The velocity strain

[1]Because of the expository nature of this discussion, we will not include original references. A lucid textbook account along with original references may be found in [5].

Table 1 Transformation of Strain Measures

	$\underline{E}$	$\underline{\varepsilon}$	$\underline{\varepsilon}$	$\underline{d}$
E_{ij} Green strain		$\underline{E} = \underline{F}^T \cdot \underline{\varepsilon} \cdot \underline{F}$	$\underline{E} = \underline{\varepsilon} + \frac{1}{2}\underline{\varepsilon}^T \cdot \underline{\varepsilon}$	$\dot{\underline{E}} = \underline{F}^T \cdot \underline{d} \cdot \underline{F}$
e_{ij} Alamansi strain	$\underline{e} = \underline{F}^{-1T} \cdot \underline{E} \cdot \underline{F}^{-1}$		$\underline{e} = (\underline{F}^{-1})^T \cdot (\underline{\varepsilon} + \frac{1}{2}\underline{\varepsilon}^T \cdot \underline{\varepsilon}) \cdot \underline{F}^{-1}$	$\dot{\underline{e}} = \underline{d} - (\underline{e}^T \cdot \underline{L} + \underline{L}^T \cdot \underline{e})$
$\hat{\varepsilon}_{ij}$ Corotational or convected strain	*	*		*
d_{ij} Velocity strain	$\underline{d} = (\underline{F}^{-1})^T \cdot \dot{\underline{E}} \cdot (\underline{F}^{-1})$	*	*	

Key: $F_{ij} = \frac{\partial x_i}{\partial X_j}$, $L_{ij} = \frac{\partial \dot{u}_i}{\partial x_j}$

has therefore been used in analysis primarily for materials in which the history of deformation does not influence the stress, such as viscous fluids and viscoplastic materials. For materials where the stress is a function of the original shape, the Green and Alamansi strains are preferred. Of course, in elastic-plastic materials, the elastic part is strongly history dependent, while the plastic part depends only on the current rate of deformation, so an appropriate choice of deformation measure is not clear.

Whether or not the mesh coordinates should be updated for the purpose of the strain-displacement computation depends on the choice of the strain measure. If Eqs. (6) or (7) are used, the shape functions, as given in Eq. (1), must be expressed in terms of spatial coordinates x, so the mesh coordinates must be updated. If the Green strain tensor, Eq. (5), is used, the shape functions have to be expressed in terms of the Lagrangian coordinates X, and therefore the shape functions need not be updated for the strain computation.

Kinetic Description

The term kinetic description here refers to the equations of motion and the types of stress description. Two types of stress measures are most commonly used: the Cauchy stress $\underline{\sigma}$ and the second Piola-Kirchhoff stress $\underline{S}$. The first of these, $\underline{\sigma}$, refers to the forces on surfaces of the current configuration, that is, its components are defined as current forces divided by the current areas. The second Piola-Kirchhoff stress is defined in terms of forces (transformed by the local deformation) per unit of undeformed area. The relationship between the two stresses is given by

$$\sigma_{ij} = \frac{\rho}{\rho_0} \frac{\partial x_i}{\partial X_k} \frac{\partial x_j}{\partial X_\ell} S_{k\ell} \tag{12}$$

The first Piola-Kirchhoff stress is defined directly in terms of the forces per unit of undeformed area (the forces are not transformed); it is seldom used because it is not symmetric. We therefore will drop the prefixes, and simply call $\underset{\sim}{S}$ the Kirchhoff stress. The stress $\underset{\sim}{S}$ is advantageous when Green strain, $\underset{\sim}{E}$, is used because of their complementary character: both $\underset{\sim}{S}$ and $\underset{\sim}{E}$ are referred completely to the Lagrangian coordinates X_i, and the two are conjugate in the sense that the scalar $S_{ij}E_{ij}$ gives the rate of work per unit of the original volume.

A second advantage of the Kirchhoff stress $\underset{\sim}{S}$ is that its rate is frame invariant, so that incremental constitutive equations may be written directly in terms of $\Delta\underset{\sim}{S}$. The time derivative of the Cauchy stress is not frame invariant, so a modified rate such as the Jaumann rate must be used. This rate, $\overset{\nabla}{\underset{\sim}{\sigma}}$, is given by

$$\overset{\nabla}{\sigma}_{ij} = \dot{\sigma}_{ij} - \sigma_{ip}\omega_{pj} - \sigma_{jp}\omega_{pi} \tag{13}$$

$$\omega_{pj} = \frac{1}{2}\left(\frac{\partial \dot{u}_p}{\partial x_j} - \frac{\partial \dot{u}_j}{\partial x_p}\right) \tag{14}$$

Here $\dot{\sigma}$ is the change in the stress due to the response of the material; the second terms arise out of the rotation of the material. The need for the additional terms in Eq. (13) can easily be appreciated by considering a bar in a state of prestress, σ_{11}, with $\sigma_{22} = 0$, oriented along the x_1 axis. If the bar were to rotate 90^0 to the x_2 axis without any deformation, σ_{22} would become the prestress and σ_{11} would vanish, yet $\dot{\sigma}_{ij}$ would vanish throughout the process.

The additional terms in the Cauchy stress rate do not pose a major problem, but they do require additional computations and they introduce another truncation error when used to obtain an increment in stress. In explicitly integrated solutions, this truncation error is probably of little concern, but if the rotation is large during an increment, its effect may be significant.

The advantage of the Cauchy stress tensor is that it yields somewhat simpler equations of motion. The equations of motion in terms of the Cauchy and Kirchhoff stress tensors are, respectively

$$\frac{\partial \sigma_{ij}}{\partial x_j} + \rho b_i = \rho\, \ddot{u}_i \tag{15}$$

$$\frac{\partial}{\partial X_j}\left(S_{\ell j}\frac{\partial x_i}{\partial X_\ell}\right) + \rho_0 b_{oi} = \rho_0 \ddot{u}_i \tag{16}$$

The corresponding finite element equations of motion, which are the counterpart of Eq. (3) are,

$$M_{IJ}\ddot{u}_{Ji} + \int_V \frac{\partial \phi_I}{\partial x_j}\,\sigma_{ij}\,dV = P_{Ii}^{ext} \tag{17}$$

$$M_{IJ}\ddot{u}_{Ji} + \int_{V_o}\left(\frac{\partial \phi_I}{\partial X_j}S_{ij} + u_{Ji}\frac{\partial \phi_J}{\partial X_\ell}\frac{\partial \phi_I}{\partial X_j}S_{j\ell}\right)dV_o = P_{Ii}^{ext} \tag{18}$$

The Kirchhoff (or Lagrangian) form of the equations of motion is always more complex, so computationally, a Cauchy stress formulation may be advantageous.

It is of interest to note here that the use of Eq. (18) requires original

mesh coordinates, whereas in Eq. (17) the shape functions must be expressed in terms of current mesh coordinates. Although Eq. (17) is in terms of Cauchy (Eulerian) stress, this does not imply that the procedure is completely Eulerian, for in solids Eq. (14) is generally used with a Lagrangian mesh. As noted by Key [6], in fluid mechanics and hydrodynamics, Cauchy stress and velocity strains have been used exclusively, and computer programs are called Lagrangian or Eulerian depending on the type of mesh. On the other hand, in solid mechanics, meshes are always Lagrangian, and the classification of the computer programs seems to depend on the kinematics and kinetics: ($\underset{\sim}{\sigma}$ - $\underset{\sim}{e}$) programs are called Eulerian, ($\underset{\sim}{S}$ - $\underset{\sim}{E}$) programs are called Lagrangian.

Constitutive Equations and Interchangeability

In viewing the above, it can be seen that basically three methods of description have been discussed: Kirchhoff stress - Green strain ($\underset{\sim}{S}$ - $\underset{\sim}{E}$), Cauchy stress - Alamansi strain ($\underset{\sim}{\sigma}$ - $\underset{\sim}{e}$), and Cauchy stress (or frame indifferent rate) - velocity strain ($\underset{\sim}{\sigma}$ - $\underset{\sim}{d}$). In solid mechanics, a Lagrangian mesh is used for all of these. Moreover, as can be seen from Table 1 and Eq. (12), these three classes of description are interchangeable in that transformation formulae are available between the stresses and the strain measures (or their rates). The kinematics and kinetics of each of these procedures is consistent. Furthermore it is perfectly possible to devise appropriate and consistent procedures that involve alternative combinations, such as $\underset{\sim}{\sigma}$ - $\underset{\sim}{E}$. It is interesting to observe that in a $\underset{\sim}{\sigma}$ - $\underset{\sim}{E}$ formulation, the nodes would be updated for the kinetics but not for the kinematics.

From a theoretical point of view, the choice of descriptions for the kinematics and kinetics is not of great importance. As a matter of practicality, the issues are not as simple. First of all, it is preferable that the kinematics and kinetics use the same stresses and strains as the constitutive equations, for although the transformations are available, they are quite time consuming. In this respect it may also be noted that the Green and Alamansi strains can be interchanged more readily with each other than with the velocity strain: the computation of the Green or Alamansi strain from the velocity strain requires a transformation and an integration, plus the storage of auxiliary variables, such as F; see Table 1. The integration also introduces a truncation error. On the other hand, the ($\underset{\sim}{\sigma}$ - $\underset{\sim}{d}$) formulations are computationally more efficient because the kinetic and kinematic equations, Eqs. (7) and (16), are considerably simpler. Therefore, when a constitutive equation is expressed in terms of $\underset{\sim}{\sigma}$ and $\underset{\sim}{d}$, a very efficient computational scheme can be developed.

Let us now review some of the constitutive theories for elastic and elastic-plastic large strain behavior. For elastic materials, the constitutive theory of materials is well developed. Truesdell [7] has classified elastic materials as follows:

(1) Elastic materials, in which there exists a one-to-one correspondence between $\underset{\sim}{\sigma}$ and $\underset{\sim}{e}$; this relationship can always be transformed to an $\underset{\sim}{S}$ - $\underset{\sim}{E}$ relationship.

(2) Hyperelastic materials, for which a strain energy exists.

(3) Hypoelastic materials, in which a frame invariant rate is linearly related to the velocity strain tensor.

$$\overset{\nabla}{\sigma}_{ij} = C_{ijk\ell}\, d_{k\ell} \tag{19}$$

Any elastic or hyperelastic relationship can be transformed to a hypoelastic relationship, but the inverse is not true: given a hypoelastic law, a path-independent integral which corresponds to a strain energy can not necessarily be found. In engineering, the elastic law most frequently used for large strain problems is the Mooney-Rivlin material which is a hyperelastic law applicable to rubber. Linear elastic laws are frequently used for small-strain, large rotation problems of metallic structures.

In elasto-plasticity, the state of the art is much less definitive. A major difficulty in extending small strain, elastic-plastic constitutive theories to large strains is that the additive decomposition into elastic and plastic strains no longer applies. This was pointed out by Lee [8], who developed a large strain theory in terms of the Cauchy stress and velocity strain. A somewhat different theory in terms of Kirchhoff stress and Green strain has been described by Green and Naghdi [9]. There are numerous other papers on this topic which can not be included here. In any case, none of these theories has reached the stage of development necessary for implementation in software. Currently, most finite element programs use elastic-plastic constitutive laws which are simple extrapolations of small strain laws in terms of the Kirchhoff stress and Green strain. MARC, HONDO and NONSAP use this approach. When correctly executed, such relations hold for large rotation, small strain problems. They do not hold for large strain problems because an additive decomposition of elastic and plastic Green strains is made.

A somewhat different approach has been used in finite difference programs, such as Wilkins [10]. In these programs, the constitutive equations are expressed in terms of a rate of the Cauchy stress and a velocity strain (in many of the equations in the text of [10], a frame invariant rate is not specified, but it is included in the difference equations). The elastic relations are hypoelastic and of the form of Eq. (19). In the elastic-plastic law, an additive decomposition into the elastic and plastic parts of the velocity strain is assumed. No rigorous justification of the theory is given, but it is interesting to note that Lee [8] has shown the validity of an additive decomposition of the elastic and plastic velocity strains, provided the elastic part of the deformation is small. Thus, these approaches appear to be reasonable. The major difficulties would probably occur in the analysis of anisotropic materials, where the material coefficients would have to be updated to reflect the rotation of the material.

In summary, there appears to be no consensus as to the formulation of large strain elastic-plastic laws. Thus, in the development of more powerful and expensive software packages, it appears to be wise to include several different modes of description, preferably programmed in parallel so as to minimize the computational penalties. HONDO and NONSAP are two programs in which alternative descriptions are already available.

It should be clear, however, that programmers manuals must specify the type of description used and the user must tailor his stress-strain law accordingly. In large strain problems, there is a large difference in the uniaxial behavior for the different modes of description. This is illustrated in Table 2, which compares various strains in a uniaxial case. Yet, although these differences are well known, many user's manuals fail to include this information or bury it so that it is difficult to retrieve.

Table 2 Comparison of Strain Measures for Uniaxial Strain

Elongation $\Delta\ell/\ell_0$	0.1	0.5
Green strain $E_{11} = \frac{1}{2}\left(\frac{\ell}{\ell_0}\right)^2 - 1$	0.105	0.626
Alamansi strain $e_{11} = \frac{1}{2}\left(1-\left(\frac{\ell}{\ell_0}\right)^2\right)$	0.0868	0.278
Corotational (rigid convected) strain $\hat{\epsilon}_{11} = \frac{\Delta\ell}{\ell_0}$	0.1	0.5
Logarithmic strain $\int d_{11}dt = \ln\left(\frac{\ell}{\ell_0}\right)$	0.0953	0.405

Alternative Descriptions

Corotational (or Rigid-Convected) Description

Several types of descriptions have been developed specifically for application in discrete element methods, with the aim of exploiting the simple form of the discrete element displacement fields for computational efficiency. One of these methods is the convected coordinate method in which a local Cartesian coordinate system is embedded in the element so that it rotates but does not deform with the element; in this sense, the formulation may also be called corotational.

A complete formulation of the corotational finite element method is given in [11]; we will only outline it here. The displacements of each element are first decomposed into rigid body displacements and deformation displacements in the form

$$u_i = u_i^{rig} + u_i^{def} \tag{20}$$

where u_i^{rig} is found from the motion of the element's coordinates. The deformation of the element is measured by the strain

$$\hat{\varepsilon}_{ij} = \frac{1}{2}\left(\frac{\partial \hat{u}_i^{def}}{\partial X_j} + \frac{\partial \hat{u}_j^{def}}{\partial X_i}\right) \tag{21}$$

The conjugate stress is $\hat{\underset{\sim}{\sigma}}$ which can be expressed in terms of $\underset{\sim}{S}$ by

$$\hat{\sigma}_{ij} = \frac{1}{2}\left[S_{ik}(\delta_{kj} + \hat{\varepsilon}_{kj}) + S_{jk}(\delta_{jk} + \hat{\varepsilon}_{ki})\right] \tag{22}$$

Both the stress and strain as defined here are of a material character. The convected strain is the stretch tensor minus the unit tensor, so any relations between $\hat{\underset{\sim}{\sigma}}$ and $\hat{\underset{\sim}{\varepsilon}}$ may easily be interchanged with $\underset{\sim}{S}$ - $\underset{\sim}{E}$ relationships. The advantage of a corotational formulation is the simplification in the strain-displacement and kinetic relationships (see Ref. [11] for details), which improve computational efficiency. Also, both the stress rate and the strain rate, $\dot{\hat{\underset{\sim}{\sigma}}}$ and $\dot{\hat{\underset{\sim}{\varepsilon}}}$ are frame indifferent, so both can be used in incremental stress-strain laws directly.

"Updated" Lagrangian Formulation

Another method of description which has generated some interest is the "updated" Lagrangian formulation described in [12], which is one of the options of NONSAP.

In this description, a Lagrangian mesh is used: the equations at any step I+1 of the computations are referred to the configuration at the previous step I. The state at step I is described by the Cauchy stress and Alamansi strain; the increment in state variables is described by the Kirchhoff stress and Alamansi strain relative to the configuration at I.

The value of this formulation lies principally in its development of a consistent set of incremental equations, which are useful in implicit transient and static solutions where deformations may be large during a step. Broadly speaking, this formulation is identical to a Cauchy stress, Alamansi strain formulation with a Lagrangian mesh.

DESCRIPTIONS AND FORMULATIONS FOR STRUCTURES

In the analysis of structures, the choices and difficulties are quite different from those in solid continua. The choice of Eulerian or Lagrangian meshes or stresses is realistically no longer pertinent: the mesh and stress must both be of a Lagrangian type. A mesh fixed in space is obviously inappropriate. An Eulerian description of stress is not suitable because certain stresses, such as the ones through the thickness of a plate, are assumed to vanish in structural elements. These conditions can not be expressed in a straightforward fashion unless the stresses are related to the deformed structure in a specific way, regardless of the extent of rotation and deformation. Hence the stresses should preferably be of a Lagrangian type.

In numerical analysis the major difficulty in shell formulations lie in the development of strain displacement equations, (and in finite element methods, the choice of good shape functions). Although relations can be written for the strains in terms of the geometry of the deformed surface, the development of reasonably concise and workable equations for the strains in terms of displacements is possible only if restrictions are made on the magnitudes of rotations.

For the purpose of providing a simple example, let us consider a beam in the x,y plane which is initially of curvature 1/R as shown in Fig. 2. Although many of the difficulties in nonlinear shell theory cannot be illustrated in a beam, some of the difficulties will become apparent. We will make the usual Kirchhoff assumption (called the Euler-Bernoulli assumptions for a beam) that normals to the midline remain straight and normal. The axial strain is then given by

$$\varepsilon = \varepsilon_{mid} + \zeta(\kappa - \frac{1}{R}) \tag{23a}$$

$$\kappa = (x_{,ss}^2 + y_{,ss}^2)^{\frac{1}{2}} \tag{23b}$$

where ε_{mid} is the midline strain, ζ is a coordinate perpendicular to the midline, κ is the current curvature, and a comma denotes a differentiation with respect to the subsequent subscripts. In writing Eq. (23), we have assumed that the midline strains (or membrane strains) are small, so that an additive decomposition into membrane and bending strains is possible. This assumption is made in almost all present day shell theories.

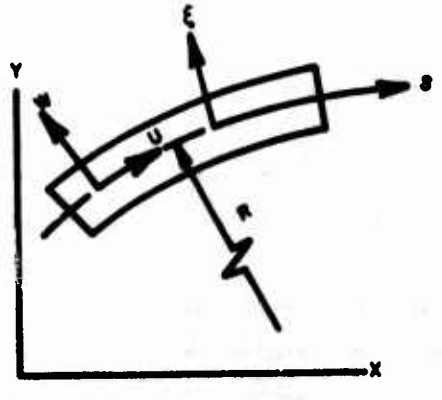

Fig. 2 Nomenclature for beam

Moderately Small Rotation Equations

One of the difficulties in shell theory lies in expressing the current curvature in terms of the displacements. The curvature expression, Eq. (23b), is very complex, and once it is simplified the effect of displacements normal to the midline differs from that of the tangential displacements. If the rotations are large, this obviously presents problems, since the identification of a normal displacement is difficult.

Therefore most strain displacement equations are restricted to moderately small rotations. The most popular of these are the equations of Sanders [13], which are used in SATANS, SHORE and several other programs. Koiter [14] derived a similar set of equations by a different approach, so these equations are sometimes also called the Koiter-Sanders equations. In these equations it is assumed that the rotations are of order ε, where ε is small compared to unity (in radians), and that the midplane strains are of order ε^2. The accuracy of these equations is quite good for rotations of up to 5°, but progressively deteriorates from that point.

For the beam, the Sanders counterpart of Eq. (23) is

$$\varepsilon = u_{,s} + \frac{w}{R} + \frac{1}{2}\left(w_{,s} + \frac{u}{R}\right)^2 + \zeta\left[\frac{u_{,s}}{R} - w_{,ss}\right] \tag{24}$$

The effect of the terms u/R and $u_{,s}/R$ in Eq. (24) is quite small if the radius of curvature is large. If the corresponding terms are dropped in the shell equations, which implies a quasi-shallow shell, the result is the Donnell-Mushtari-Vlasov equations, which were first given by Donnell [15] for cylindrical shells; see Brush and Almroth [16] for a more complete discussion. These equations are of simpler form and in many cases as accurate as the Sanders equations.

If the beam is shallow, the curvature can be approximated by

$$\frac{1}{R} = Y_{,XX} \tag{25}$$

where X is a coordinate along the chord of the beam, and Y is normal to X. This approximation yields Maguerre's shallow shell equations [17]. These equations only hold for very small curvatures where the angle between the chord X and the surface of the shell is less than 10°. These equations have been used by Vos [18] and Bergan and Clough [19], but do not appear directly in any of the software reviewed here.

All of these equations are restricted to moderately small rotations. In a straight beam, these equations introduce ficticious membrane strains of order 0.06% when the rotation is 2°, and of order 1.5% when the rotation is 10°. Thus if the actual strain is 1%, the errors in the strain computation are 6% and 150%, respectively. Obviously, the error for 10° of rotation is unacceptable.

These equations are very useful for the study of dynamic instability problems (see [20]) where a structure is usually considered to have failed when rotations of order 5° occur. On the other hand, for crashworthiness and safety studies, where the response and energy absorbing capacity of the structure for much larger rotations is of importance, these equations are not appropriate.

Large Rotation Equations

Two approaches to developing large rotation formulations are used in computer software. One method is to take the strain equations written in terms of the current geometry, such as Eq. (23), and to obtain an incremental form

$$\Delta\varepsilon = \Delta\varepsilon_{mid} + \zeta\Delta\kappa \tag{26}$$

This approach is used by Morino, et al., in PETROS [21]. The incremental equations are quadratic in the incremental displacements so that the resulting equations are quite simple. This approach may be used for arbitrarily large rotations. The only drawback lies in the truncation error: these equations should probably be used only with explicit integration schemes where the incremental displacement is small.

A second approach is to use a system of corotational coordinates associated with each element [11]. The rigid-connected coordinate system is embedded in the element so that it rotates with the element. If we consider an element that is initially straight, such as shown in Fig. 3, the deformation displacements may be considered $\hat{u}$ and $\hat{w}$, and the strains are given by

$$\hat{e} = \frac{\partial \hat{u}^{def}}{\partial \hat{x}} + \zeta \frac{\partial^2 \hat{w}^{def}}{\partial \hat{x}^2} + \frac{1}{2}\left(\frac{\partial \hat{w}^{def}}{\partial \hat{x}}\right)^2 \tag{27}$$

The second term is included only when the variation of rigid body rotation within an element is moderate, i.e. when $(\partial \hat{w}/\partial \hat{x})^2$ is significant compared to $\partial \hat{u}_x^{def}/\partial \hat{x}$. Similarly, for initially curved elements, Maguerre's shallow shell equations may be used in the corotational coordinates. As can be seen from Fig. 3, the transverse displacement $\hat{w}^{def}$ is always normal to the coordinate $\hat{x}$.

This approach is valid for arbitrarily large rotations and is used by Argyris, et al. [22], by Murray and Wilson [23] and in [11]. It is interesting to observe that this approach may also be viewed as a direct application of Eq. (23) with the arc length s approximated by straight line segments.

Nonlinear Membrane Strain Equations

Many structural mechanics programs use a complete Lagrangian theory for the membrane strains and ignore the second order effects on bending strains. Among these programs are DYNAPLAS and the flat plate elements in MARC. In this approach, the displacement normal to the midplane is taken to be the displacement normal to the original midplane, and the bending strains are obtained by a linear theory. In addition, the second order terms in the mem-

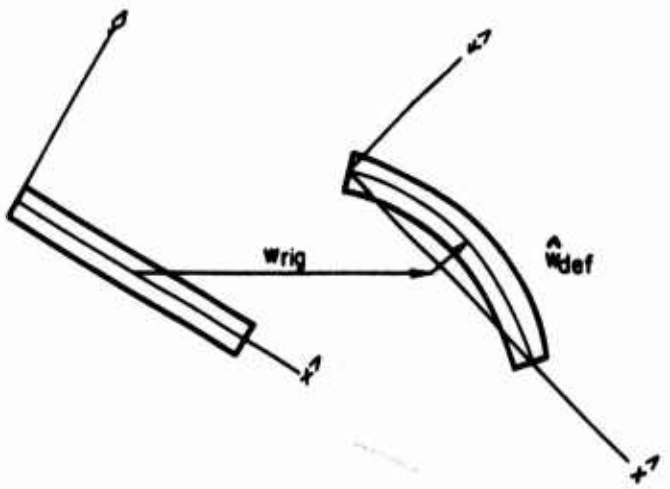

Fig. 3 Decomposition of displacements of beam element into rigid body and deformation displacements

brane displacement gradient are often dropped. The beam equation for this approach is

$$\varepsilon = u_{,X} + \tfrac{1}{2}w_{,X}^{2} - \zeta w_{,XX} \tag{28}$$

By letting $R \to \infty$ and $X = s$ in the Sanders' type equations, Eq. (24), the above equation is also obtained. Similarly, the reduction of the Sanders equations to a flat plate yields the vonKarman plate equations, which include the nonlinearities only in the membrane terms. These equations are valid only for moderately small rotations.

NUMERICAL STABILITY

In the integration of the discrete equations of motion, if the time step is too large, the computed response may grow in an unbounded manner. This phenomenon is called a numerical instability. It has no physical significance and is entirely a numerical problem.

In order to discuss numerical stability, it is useful to separate explicit and implicit integration schemes. The relative advantages have been discussed in [24, 25]. The basic difference between the two methods relates to how the displacements at the end of the time step are obtained: in implicit methods, the displacements are found directly by the solution of the governing equations; in explicit methods, the accelerations are found first and integrated to yield the displacements. Implicit methods, as a rule, involve greater computational effort per time step.

For linear problems, a rigorous understanding of numerical instabilities has been developed. It has been proven and is well known that in a central difference, explicit integration, the computation will be stable if the time step is less than

$$\Delta t \leq \frac{2}{\lambda} \tag{29a}$$

or equivalently for homogeneous-strain elements

$$\Delta t \leq \frac{\ell}{c} \tag{29b}$$

where λ is the maximum frequency of the discrete system, c the acoustic wave speed, and ℓ the minimum element dimension. Implicit integration schemes, such as the Newmark β-method with $\beta \geq \frac{1}{4}$, are unconditionally stable for linear problems: there are no limitations on the time step; see Newmark [26] or Nickell [27].

For nonlinear problems, the situation is quite unclear. For many years, continuum problems have been solved by using the criterion of Eq. (29b) with c taken as the current local wave speed in the material, and this criterion has been found to be adequate.

Oden and Frost [28] recently proved that Eq. (29b) is sufficient for energy stability for a one-dimensional, linear displacement, finite element mesh. This proof was restricted to materials in which the stress is a monotonically increasing function of strain, and did not include geometric nonlinearities. These restrictions were motivated to a large extent by the proof of convergence, which was also included. From the empirical evidence cited above, and the Oden and Frost paper, it appears that rigorous proofs of this stability criterion for arbitrary meshes and arbitrary nonlinear materials will soon be forthcoming.

In the analysis of solid continua the effect of geometric nonlinearities is usually quite small. However, it is not clear whether Eq. (29b) also applies to geometrically nonlinear structural problems. Richtmyer and Morton [29] include stability limits for a string under uniform tension. These stability limits are smaller than Eq. (29b) and depend on the tension

(initial stress) so evidently in a structure undergoing large rotations, the limits on the time step may be more restrictive than Eq. (29b).

In most structural software, such as WHAM and PETROS a trial and error approach, with Eq.(29) as a guidepost, has been recommended. It is tacitly assumed that when the computations are unstable, the response is so large that the error is obvious to the user. In addition to its inconvenience, this approach has its pitfalls. Thus, as mentioned in [24] and [25], if the material is elastic-plastic, it may dissipate the energy due to a numerical instability and thus arrest it. The resulting computations look quite reasonable in spite of their large errors.

These errors in nonlinear analysis, and many others, can be detected by energy balance checks. The occurence of an arrested instability is always associated with a spurious growth of the internal energy. Energy balance checks have been a standard feature of many finite difference programs for years, but are still absent in some of the finite element programs. When an energy computation is not available, it is sometimes wise to rerun the problem with a smaller time step.

Implicit Integration

Whereas implicit integrations are unconditionally stable for linear problems, this is not true for nonlinear problems. Belytschko and Schoeberle [30] have recently shown that an implicit procedure combined with iterations is unconditionally stable provided the error in the energy is controlled by a tolerance in each time step. This proof applied only to nonlinear materials. Results given in [30] for geometrically nonlinear problems also exhibited stability for very large time steps, but an analysis of the proof shows that numerical stability probably requires bounds on the rotation during a time step.

Again, most finite element programs with implicit integration, such as DYNAPLAS or MARC, have not specified a stability limit for the time step: it must be obtained by trial and error. An energy balance check is obviously of value.

Another problem that arises in implicit integration is that even if stability is maintained, a large time step may give bad results because the stress-strain path is not followed adequately. No guidance is presently available for the user, either before or after the computation, as t whether the time step allowed the stress history to be tracked with sufficient accuracy. The only recourse open to the user is to repeat the computation with a different time step.

COMPUTER PROGRAMS

We will here classify the programs in the same manner as our treatment of descriptions: continuum programs and structural programs. The programs, along with their capabilities and features are summarized in Tables 3 and 4, respectively. Structural programs are those which treat elements such as plates, shells and beams, so in a broad sense they involve certain restrictions on the variation of displacements such as the Reissner or Kirchhoff assumptions. Continuum programs are those in which the field equations are treated directly with no simplifying assumptions.

Structural programs are most suitable for the analysis of slender or moderately thick elements when the nature of the loading or desired response is such that the wave propagation through the thickness is unimportant. Continuum programs are often used for structural analysis, but unless the wave propagation through the thickness is important, such applications are needlessly uneconomical. To illustrate this point, it should be noted that an adequate treatment of a structural element requires 3 to 5 constant strain zones through the thickness. Thus, when continuum programs are used

for structural analysis, not only are a large number of elements needed, but because the zones are very thin, the time step in explicit integration procedures must be very small.

Most continuum programs are restricted to two-dimensional or axisymmetric three-dimensional problems. Programs for true three-dimensional analysis are still rare, not because of any theoretical difficulties, but because it is almost impossible for present-day computers to economically handle a sufficient number of degrees of freedom to achieve adequate resolution in three-dimensional problems.

A third category which may be considered are those programs with both structural and continuum elements. These programs are included in both tables and denoted by asterisks; the entries in Tables 3 and 4 pertain only to the continuum and structural elements, respectively.

The combination of structural and continuum elements is very advantageous for structure-media interaction problems because the structure then need not be modelled by continuum elements and its attendant disadvantages are avoided.

In an earlier survey, Belytschko [24] classified transient analysis programs as general purpose and special purpose. Only two programs with nonlinear capabilities were included in the general purpose category: MARC and ANSYS. Even this classification was rather optimistic, since the nonlinear capabilities of these two programs are quite limited compared with the range of capabilities required for nonlinear analysis. Therefore, this classification is not pursued herein.

Continuum Analysis Programs

The continuum analysis programs are listed in Table 3. They are further subdivided in this discussion into finite element and finite difference programs. Programs that also include structural elements are denoted by asterisks.

Finite Element Programs

ANSYS is a finite element program with a large variety of elements: beam, plane elements, flat plate (shell) elements, and solid three-dimensional elements. The equations of motion are solved by an implicit procedure: the type of difference operator is not specified in the documentation, nor are the mesh, kinematic or kinetic descriptions. No iterations are used during a time step to reduce error forces, so even in small deflection problems, the implicit method is not unconditionally stable. Geometric stiffness matrices are included in only some of the elements. An elastic-plastic material with the von Mises yield surface and either kinematic or isotropic hardening is used. In the survey reported in [24], the user reaction to this program was very favorable, but most uses of this program appear to involve at most material nonlinearities. The capabilities for completely nonlinear problems are continuously being improved, but are still rather limited.

MARC is a similar program with a somewhat larger library of elements. Additional elements are: higher order isoparametric solid elements, curved beam elements and curved shell elements. Materials included are the standard von Mises, linear hardening, elastic-plastic law, with both isotropic and kinematic hardening, and a class of Mohr-Coulomb laws, which are useful for soils. The meshes are Lagrangian, and the solid elements use the second Kirchhoff-Piola stress and Green-strain tensor.

Integration of the equations of motion is by an implicit procedure using the Newmark difference equations. Residual force calculations are made but it is not clear whether iterations are controlled by a tolerance (no tolerance is input). An explicit integration option has recently been added.

Table 3. Programs for Solid Continua

Program	Format	Mesh	Stress	Strain	Geometry Types	Material Laws EP: elastic plastic M: Mises yield kin: kinematic hardening iso: isotropic hardening	Time Integration	Automatic Time Step	Energy Balance or Residual Iter.	Other Features
ANSYS*	F.Ele.	L	$\underset{\sim}{\sigma}$	$\underset{\sim}{\varepsilon}$	2D; 3D	EP, M, kin or iso	imp.	no	no	Fourier superposition in 3D axi
HONDO	F.Ele.	L	$\underset{\sim}{\sigma}$ (or $\underset{\sim}{s}$)	$\underset{\sim}{E}$ (or $\underset{\sim}{d}$)	3D axi.; 2D	Kirchhoff elastic; Mooney-Rivlin elastic; viscoelastic; soil or crushable foam; EP, combined kin and iso, M	exp. C.D.	yes	no	artificial and antihourglass viscosity description is user option
MARC*	F.Ele.	L	$\underset{\sim}{S}$	$\underset{\sim}{E}$	2D; 3D; 3D axi.	EP, M or Mohr-Coulomb, iso or kin	imp. N or exp.	no	yes	isoparametric elements
NONSAP	F.Ele.	L	$\underset{\sim}{S}$ or $\underset{\sim}{\sigma}$	$\underset{\sim}{E}$ or $\underset{\sim}{e}$	2D; 3D; 3D axi	EP, M or Drucker-Prager, iso(2D only); Mooney-Rivlin elastic (2D only); variable modulus	imp. N or imp. W	no	yes	isoparametric elements description is user option
WHAM*	F.Ele.	L	$\underset{\sim}{\hat{\sigma}}$	$\underset{\sim}{\hat{\varepsilon}}$	2D; 3D axi	EP, M, iso	exp. C.D.	no	yes	artificial viscosity sliding interface
HEMP	F.Dif.	L	$\underset{\sim}{\sigma}$	$\underset{\sim}{d}$	2D; 3D axi	EP, M, iso; hydrodynamic equation of state, others	exp. C.D.	yes	yes	artificial and antihourglass viscosity; sliding interface
PISCES	F.Dif.	L	$\underset{\sim}{\sigma}$	$\underset{\sim}{d}$	2D; 3D axi	EP, M or Mohr-Coulomb, iso; hydrodynamic equation of state with change of phase, spalling	exp. C.D.	yes	yes	artificial and antihourglass viscosity; sliding interface
TOODY	F.Dif.	L	$\underset{\sim}{\sigma}$	$\underset{\sim}{d}$	2D; 3D axi	EP, M; hydrodynamic equation of state, others	exp. C.D.	yes	yes	artificial and antihourglass viscosity; sliding interface

*also includes structural elements, see Nomenclature for identification of symbols; Abbreviations: xi - axisymmetric; C.D. - central difference; exp. - explicit; F.Ele. - finite element; F.Dif. - finite difference; L Lagrangian; N - Newmark β-method; W - Wilson θ-method; SS - Stiffly-stable; R - Runge-Kutta

Table 4. Structural Element Programs

Program	Format	Types of Structures	Strain Displacement Equations	Rotation Limits	Material Laws (see Table 3 for abbreviations)	Time Integration	Automatic Time Step	Energy Balance
ANSYS*	FE	space frame; plate-shell; thick-shell; axi-shell, arb. load	not specified	large in some elements	see Table 3	imp.	no	no
DYFLAS	FE	plate-shell	nonlinear memb. corotational	large	EP, M, iso.	exp. C.D.	no	no
MARC*	FE	plate-shell; curved shell; space-frame; axi. shell	nonlinear memb. (flat plates); Sanders (curved shells)	moderate	see Table 3	imp. N or exp.	no	no
PETROS	FD	curved shell	incremental strain curvature	large	EP, M, White-Besseling hard.	exp. C.D.	no	yes
SABOR/ DRASTIC 6A	FE	axi. shell-arb. load			EP, M, iso.	imp. N	no	no
SAMMSOR/ DYNAPLAS	FE	axi. shell - arb.load	nonlinear memb.	moderate	EP, M, iso or White-Besseling	imp. H or exp.	no	no
SATANS	FD	axi. shell-arb. load	Sanders	moderate	elastic	exp. R.	no	no
SHORE	FD	axi. shell-arb. load	Sanders	moderate	EP, M, iso.	exp. C.D.	no	no
STAGS	FD	curved shell, bars	similar to Sanders	moderate	EP, M, White-Besseling hard.	exp. C.D. or imp. S.S.	no	
WHAM*	FE	plate-shell; space frame; 3D frame; axi shell	corotational	large	EP, M, iso.	exp. C.D.	no	yes

NONSAP is a readily available and widely used finite element program. The user of this program has the choice of either a Lagrangian or what is called an updated Lagrangian formulation. The first formulation is completely Lagrangian: it uses a Lagrangian mesh, the Kirchhoff-Piola stress, and the Green strain. The second formulation also uses a Lagrangian mesh, but it uses a Cauchy stress and the Alamansi strain (so that the shape functions must be computed in terms of the current or "updated" nodal coordinates).

The element library consists primarily of isoparametrics: a spring element, an 8 node, curved-side, two-dimensional plane stress, plane strain, and axisymmetric element, and a 20 node, three-dimensional isoparametric. The latter is available for nonlinear materials, but not large displacement problems. An extensive library of material laws is included: elastic-plastic materials, variable tangent modulus models, and a Mooney-Rivlin material.

Integration of the equations of motion is by an implicit procedure based on the Newmark or Wilson difference equations (user option), and residual force iteration is included. Overall it is one of the better programs available for nonlinear problems under moderately rapid rates of loading (as distinct from shock). Its major drawbacks are the absence of structural elements and explicit integration.

HONDO is a finite element program developed for shock and wave propagation problems that duplicates many of the features of finite difference programs discussed later (HEMP, TOODY, etc.). The major difference is in the use of a finite element format and a different kinematic and kinetic description. A Lagrangian mesh is used, and the equations of motion are expressed in terms of the Cauchy stress. Both the velocity-strain and Green (actually the deformation tensor $\partial x_i/\partial X_j$) strain are computed; so that either type of constitutive equation may be used. When the stress-strain law is expressed in terms of the second Kirchhoff-Piola stress vs. Green strain, provisions are included for effecting Eq. (2), the transformation to Cauchy stress, which is used in the equations of motion. An extensive library of materials is included: an elastic-plastic strain hardening material, a soil or crushable foam model, a viscoelastic material, a continuum rubber, and a "Kirchhoff" elastic law. Only one element is included: the bilinear displacement quadrilateral applicable to axisymmetric, three-dimensional problems (plane problems are treated by letting the radius be very large).

Temporal integration is by a central difference, explicit procedure. A lumped mass matrix is used. The time step is variable and computed internally, but energy balance checks are not made. Artificial viscosities for shock spreading and hourglass control are included.

WHAM II is for two-dimensional plane and axisymmetric problems. A rigid-convected (corotational) formulation is employed. The mesh is Lagrangian, while the stress is corotational. Deformation is measured by a convected strain $\hat{\epsilon}$, which is the stretch tensor minus the unit matrix. The element library consists of: beam, conical axisymmetric shell, constant strain axisymmetric or plane triangle. Only elastic and elastic-plastic materials with bilinear, isotropic strain hardening are included.

Integration is by a central difference, explicit procedure, with constant time step. Energy balance checks are made, and the energy in any element or group of elements may be output. Artificial viscosity and sliding interfaces are included. The program is quite efficient: on an IBM 370/195, a 5000 element mesh takes 1 second per time step. An updated version with quadrilaterals, isoparametrics and an implicit integration option (over part of or the whole mesh) is soon to be completed. A three-dimensional structural program described later, WHAM III, is part of the system and uses the same input and output routines.

Finite Difference Programs

The three most widely used finite difference programs are HEMP, PISCES and TOODY. All three of these programs use similar difference equations initially described by Wilkins [10], the author of HEMP. These programs can treat two-dimensional plane and three-dimensional axisymmetric geometries. The mesh is Lagrangian, the deformation is measured by the rate-of-deformation tensor, and the kinetics are Eulerian. The rate-of-deformation tensor is not integrated in time, so all constitutive equations are incremental.

These programs use central-difference, explicit integration in time and lumped masses. Sophisticated formulae are included for computing a stable time step, and the size of the time step is varied during the computation.

The constitutive equations in these three programs are oriented towards problems of high-pressure impact. HEMP, in its original form, includes only an elastic-plastic law and an equation of state (pressure-volume relationship) for energy deposition problems, such as used in the analysis of detonations. In the elastic-plastic law, an additive decomposition into plastic and elastic velocity rates is assumed, and a frame invariant Cauchy stress rate is expressed in terms of the velocity strain tensor; the pressure is computed from a cubic function of the specific volume, the Hugoniot equation of state. It may be noted, that according to Lee [8], the additive decomposition into plastic and elastic velocity strains is only valid when the elastic strains are small. Both perfect plasticity and isotropic strain hardening are included; for the latter the yield function is taken to be a function of the strain energy. The elastic portion of this stress-strain law also relates the Jaumann rate of Cauchy stress to the rate-of-deformation, so it is a hypoelastic relationship.

PISCES and TOODY are very similar to HEMP in all aspects: kinematics, kinetics and constitutive equations, differing only in details. An interesting feature of TOODY is a sophisticated implementation of access to external storage that provides it with almost unlimited size capability.

All of these programs have large libraries of constitutive equations. Their elastic-plastic laws have the option of various yield surfaces: pure hydrodynamic, where the yield is a function of only the pressure; the von Mises yield function; the Mohr-Coulomb yield function. A constant hardening modulus can be used in conjunction with isotropic hardening. In addition, some compaction models and spalling models and complex equations of state with phase changes and chemical reactions are included.

In all of these programs, the shape of the mesh is quite unrestricted. The zones are all quadrilaterals and are generated automatically by an I-J system. No triangular zones are included. Instead, an interface between regions of different zone sizes can be treated as a special case of a sliding interface. The interfaces may be either rigid, sliding, or sliding and debonding. However, in spite of this generality, some complex geometries found in engineering problems cannot be treated because there are restrictions on the meshes; for example, in TOODY, sliding interfaces must be lines of constant I. Moreover, the techniques of applying the I-J system to complicated geometries is difficult to learn from the user's manual.

The finite difference governing equations are very similar to certain finite-element equations. Thus, for example, Belytschko et al. [3] have shown that the finite element equations for a quadrilateral mesh with a single point per element quadrature for the hydrodynamics are identical to the equations given by Wilkins. However, at the present time, for problems of shock and high impact, only solutions obtained by finite difference programs have been extensively reported in the literature.

Some other programs with similar difference equations are REXCO [31], AFTON [32] and CRAM [33]. REXCO includes a finite difference formulation of shell equations, but there are restrictions on its placement and shape relative to the continuum.

Many of these programs are restricted in distribution to government and agency users. PISCES is a proprietary program available to CYBERNET, its use

is quite expensive: a 75 element mesh for a time step takes one ARU unit of CDC 6600 time. TOODY is available from Argonne Code Center.

Structural Programs

We will group the structural programs as follows: axisymmetric shell programs (both finite element and finite difference), finite element space frame and shell programs, and finite difference arbitrary shell programs. The greatest variety of programs are available in the first group, and many of these have reached a high degree of development, both technically and in usability. The second group, which is of great importance in many engineering applications, is quite sparse. Only finite element programs have been developed for the nonlinear analysis of space frames, some of these programs also include plate bending or shell elements.

Axisymmetric Shells Programs

This class of programs is applicable to rotationally symmetric shells. Except for a few of the programs, the load need not be rotationally symmetric.

SAMMSOR/DYNAPLAS is a finite element program for this class of problems. The shell equations formulation is nonlinear only in the membrane strains, so it is restricted to moderately small rotations. Loads that are not rotationally symmetric are treated by a Fourier superposition in the circumferential direction. An elastic plastic stress strain law with either isotropic strain hardening or a White-Besseling hardening law is included.

Temporal integration is by a Houbolt implicit method, with the nonlinearities treated as pseudo forces. An explicit integration option is available. Neither equilibrium iteration nor energy balance checks are included. The program is very nicely documented and user-oriented, and it has been used by outside users with little difficulty.

SABOR/DRASTIC 6A and STARS are similar programs. STARS uses a multisegment forward integration procedure to solve the shell equations; it is applicable to geometrically nonlinear problems only for axisymmetric loads.

SATANS is a finite difference program for axisymmetric shells under arbitrary loads. Its shell equations are of the Sanders type, applicable to moderate rotations. A Houbolt method is used to integrate the equations of motion, with the nonlinearities appearing as pseudo-forces. Only elastic materials can be treated. The program is intended primarily for the study of dynamic instabilities, and many such studies with this program have been reported; (see [20]).

SHORE uses a somewhat different finite difference approach to this problem class: although the program is restricted to axisymmetric shells, the equations are formulated on a two-dimensional mesh. This approach may be more effective than Fourier superposition for loads that are applied on very small sectors about the circumference.

Space Frame and Shell Programs

For structural applications, ANSYS has both beam and flat plate elements; the same comments made in the section on continuum programs apply. MARC also includes these elements, and in addition, a curved shell element. The curved shell element is based on a Koiter-Sanders formulation. The flat plate elements use a complete Lagrangian formulation for the membrane strains, with the bending terms treated in a small displacement sense: moderately small rotations can be treated.

DYPLAS is a recently developed program with plate, shell of revolution, and plane elements. Although the program uses a set of corotational coordinates, it also includes the second order terms in the membrane strain

relative to the corotational coordinates. The strain is measured by an "incremental" Lagrangian strain, (see [34]), while in the equations of motion the Cauchy stress is used. The stress-strain relations involve an increment of Kirchhoff stress and the "incremental" Lagrangian strain.

STAGS has been under development for several years and it is now a highly developed program. Its shell formulation is very similar to the Sanders equations. The program is applicable to arbitrary shell geometries, although the original surface must be described by a mathematical expression. The program has provisions for external storage access, so problem size limitations are almost entirely based on running time.

The program includes both explicit and implicit temporal integration; the latter is done by a stiffly-stable method of the Gear-Jensen type. Elastic-plastic materials are treated by a sublayer (White-Besseling) model. An interesting feature of this program is the inclusion of a rod finite element which is linked to the shell finite difference formulation.

PETROS is another powerful finite difference, arbitrary shell program which uses an approach quite different from STAGS. In PETROS, the strain-displacement equations are developed from relations for the strain in terms of the current and original geometry. The strains are computed incrementally and the program is applicable to arbitrarily large rotations.

Whereas STAGS is oriented primarily toward nonlinear stability problems, PETROS is oriented toward problems with very large deflections and elastic-plastic behavior. The elastic-plastic law is a sublayer (White-Besseling) type, and uses a piecewise linear plastic modulus; as is customary, the law is a straightforward extrapolation of a small strain law.

Temporal integration is by a central difference, explicit procedure; precise stability limits are not available, but an energy balance check is made in PETROS 3.

A third finite difference program, which is similar to PETROS, is REPSIL [35]. Its shell equations are of a simpler form, and it includes a damping option to obtain static configurations. Otherwise, the specifications of PETROS apply to REPSIL.

WHAM includes both beam and flat-plate elements; the latter can be used to represent arbitrary shells. Rigid bodies and linkages are also included. The orientation of nodes in three-dimensional space is described by unit vectors and a corotational formulation is used for the elements, so the program is applicable to arbitrarily large rotations. Elastic-plastic materials with isotropic strain hardening may be treated. Only explicit integration is presently available; time steps are not computed internally but energy balance checks are made.

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APPENDIX

Program Sources

1 ANSYS:
CDC DYBERNET System or
Dr. John A. Swanson
Swanson Analysis Incorporated
870 Pine View Drive
Elizabeth, Pennsylvania 15037
Documentation: Ref. [36]
Cost: Royalty surcharge on computer time or negotiable

2 DYPLAS:
Dr. Z. Zudans
Franklin Institute Research Laboratories
Benjamin Franklin Parkway
Philadelphia, Pennsylvania 19103

3 HEMP:
Dr. Mark Wilkins
Lawrence Livermore Laboratories
P.O. Box 808
Livermore, California 94550

4 HONDO:
Argonne Code Center
Argonne National Laboratory
9700 South Cass Avenue
Argonne, Illinois 60439
Documentation: Ref. [37]
Cost: available to ERDA contractors and others who have arrangements with the Center

5 MARC:
CDC CYBERNET System or
Dr. Pedro Marcal
MARC Analysis Research Corporation
105 Medway Street
Providence, Rhode Island 02906
Documentation: Ref. [38]

6 NONSAP:
NISEE
729 Davis Hall
University of California
Berkeley, California
Documentation: Ref. [39]
Cost: $200.00

7 PETROS:
N.J. Huffington, Jr.
U.S. Army Ballistics Research Laboratories
Aberdeen Proving Ground, Maryland 21005
Cost: Government users

8 PISCES:
CDC CYBERNET System

9 SABOR/ DRASTIC
Dr. Stanley Klein
Philco-Ford Corporation
Ford Head
Newport Beach, California 92663

10	SAMMSOR/ DYNAPLAS:	Dr. Walter E. Haisler Department of Aerospace Engineering Texas A&M University College Station, Texas or COSMIC Barrow Hall University of Georgia Athens, Georgia 30601 Documentation: Ref. [40]
11	SATANS:	COSMIC (see 10)
12	SHORE:	P. Underwood Lockheed Palo Alto 3251 Hanover Street Palo Alto, California
13	STAGS	F.A. Brogan (see 12 for address) Cost: about $2,000.00
14	TOODY:	Argonne Code Center (see 4)
15	WHAM:	Argonne Code Center (see 4) or from author Documentation: Ref. [41]

Fracture and Fragmentation Under Shock Loading

Lynn Seaman

Stanford Research Institute

INTRODUCTION

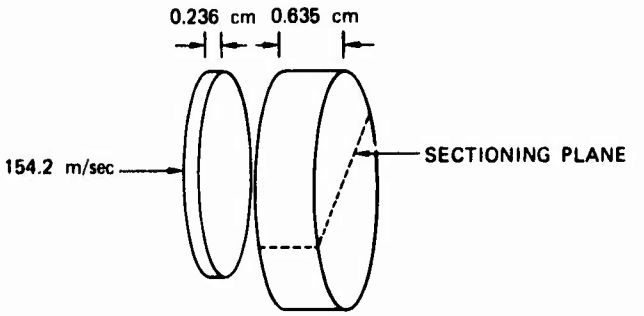
Fracture and fragmentation may occur under shock loading caused by impact, air shock, deposition of intense thermal radiation, or the detonation of explosives. Simulation of such processes requires a wave propagation computer program (except for very simple problems) and criteria for the occurrence of fracture and fragmentation under shock conditions. This chapter outlines such damage criteria, describes means for selecting an appropriate criterion, and indicates how to implement the criteria in wave propagation codes. Several popular one- and two-dimensional wave propagation codes that may include these criteria are listed at the end of the chapter. No review is given here of these codes; rather the emphasis is on the fracture models that represent one small feature of such codes.

To provide a common basis for considering damage criteria, some experimental data are presented to illustrate damage phenomena. Figure 1 is a cross section of an aluminum target plate (0.635 cm thick) that has undergone a planar impact at 154.2 m/sec by another aluminum plate (0.236 cm thick). Following the compression waves resulting from the impact, rarefaction waves have intersected near the middle of the target plate to cause damage in the form of nearly spherical voids. The damage is spread over a significant portion of the target plate (about two-thirds of the plate thickness appears in the section of Fig. 1), but the heaviest damage is in a narrow zone. Both the number and size of voids decrease with distance from this zone. This type of damage is termed ductile fracture because of the high ductility (ability to flow) required of the plate material.

An example of brittle fracture is shown in Fig. 2. An Armco iron target was impacted by a flyer plate, which was tapered on the back to provide a varying tensile wave duration across the plate. The damage, which appears as randomly oriented microcracks, varies in proportion to the tensile wave duration. In this chapter, the damage in Fig. 2 is termed "brittle," although the crack growth is much slower than elastic crack velocities, indicating considerable plastic flow at crack tips.

When cracks coalesce, fragmentation occurs, as illustrated in Figs. 3 and 4. Figure 3 is a cross section of a 1.27 cm diameter target of Arkansas novaculite (fine-grained quartz) encased in an aluminum box. The target was impacted by a 0.079-cm-thick lucite plate at 49.5 m/sec. An identical plate was impacted at 48.9 m/sec, but instead of sectioning it, we performed a sieve analysis on the fragments. The resulting fragment groups are shown in Fig. 4. All the fragments are bulky, having six to eight major surfaces. By comparing the results of Figs. 3 and 4, we may deduce that a family of cracks of various lengths and orientations has led to a family of fragments of various sizes.

A sample of full separation is shown in Fig. 5. An aluminum target impacted by a flat plate in a configuration similar to that in Fig. 1 has been damaged to the extent that full separation occurred near the center of the target. This full separation appears as a macrocrack running through heavily



(a) IMPACT CONFIGURATION FOR 1145 ALUMINUM SPECIMEN NO. 872

(b) SECTION THROUGH TARGET SHOWING DUCTILE FRACTURE DAMAGE

Fig. 1 Configuration for a flat plate impact experiment in 1145 aluminum and observed damage on a cross section

damaged material. Dvorak [1] has pointed out that brittle fracture in polycrystalline material also occurs by the coalescence of small microcracks ahead of the observed microcrack. The microcracks cleave individual grains; these small cracks widen and join to form the main crack.

From the review of the typical fracture cases above and from other observations, we have deduced the following general features:

- A range of damage is possible; there is no instantaneous jump from undamaged to fully separated.
- Damage grows as a function of time and the applied stress. Hence a single stress or strain at any time cannot be expected to characterize the dynamic fracture process. At the least some time-integral quantity (such as impulse) must be used to represent the dynamic strength.
- As the damage occurs, the stiffness of the material decreases; hence, the wave propagation character changes. If the developing damage is not permitted to alter the wave processes in a computational procedure, then subsequent stress histories and damage must be invalid.
- Even incipient damage levels are important, because, while the voids or cracks are difficult to observe, they may seriously weaken a structure.

While the foregoing features represent experimental observations well enough, it may be possible to simplify or eliminate some of these features for computational purposes.

In the remainder of the chapter several computational models are introduced. An effort is made to indicate under what circumstances each might correctly characterize fracture. First, the very simplest models are given and later more complex ones that more nearly represent the experimental observations.

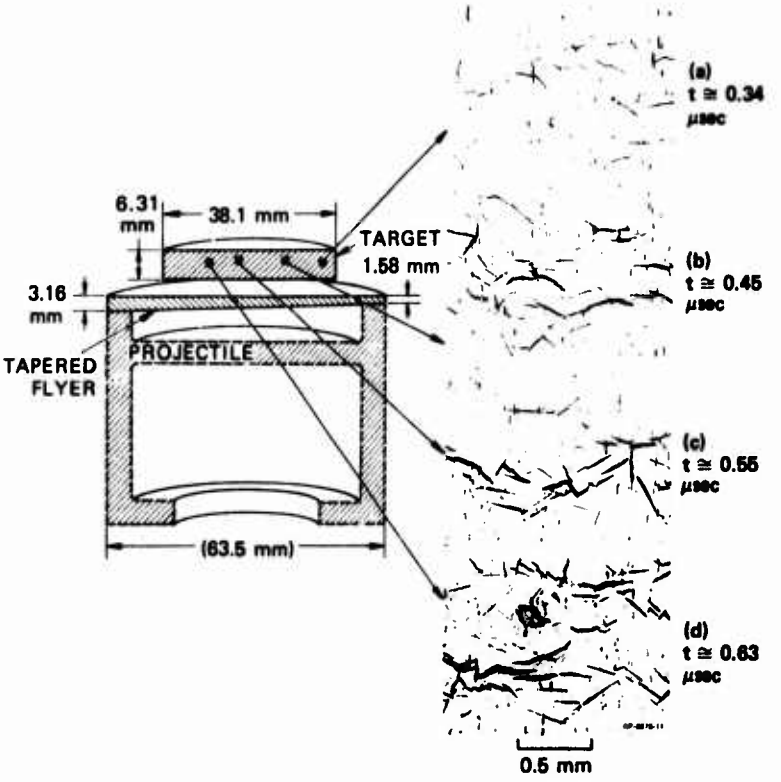


Fig. 2 Configuration of a tapered flyer impact experiment in Armco iron (Shot S1) and observed damage on a cross section of the target

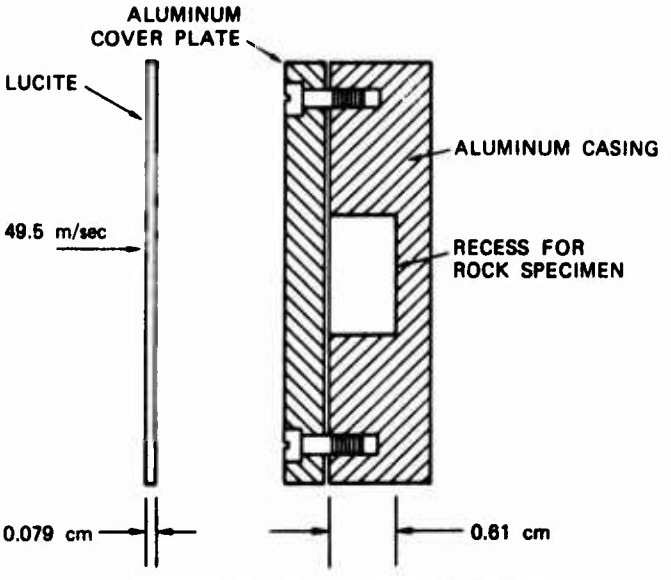
STATIC CRITERIA

Under static conditions it is often assumed that fracture occurs when a peak stress is reached. For multidimensional problems, the peak stress may be replaced by an effective shear stress based on a Mises, Tresca, or Coulomb criterion. Alternatively, a critical tensile strain or some "effective" strain criteria may be used also under wave propagation conditions.

Recently, Bertholf [2] used the critical tensile stress criterion in the two-dimensional wave-propagation codes TOODY and CSQ to determine the occurrence of spall in a target impacted by a hypervelocity pellet. The critical stress value was derived from plate impact experiments that had evidenced full spall. The resulting simulations by Bertholf modeled the experimentally observed spall quite well. Cherry [3] implemented a similar criterion in the two-dimensional code TENSOR to study the collapse of the overburden above an explosion at some depth in the earth. In Cherry's case, tensile failure in one direction did not alter strength in the orthogonal directions. Thus, an anisotropy of damage was permitted.

Under what conditions can such a simple peak stress criterion be used with some justification? Probably the following conditions should be met:

• The experimental data on which the criterion is based (the plate impacts in Bertholf's case) should exhibit the same stress levels, durations, and wave shapes as those in the problem to be simulated.



(a) CONFIGURATION FOR IMPACT EXPERIMENT

(b) CROSS SECTION OF NOVACULITE TARGET FOLLOWING IMPACT

Fig. 3 Configuration of impact experiment and resulting damage on cross section of Arkansas novaculite

• Damage should occur abruptly - within the rise time of a wave, for example.

• Subsequent stress histories and damage are not of interest. The second and third conditions both deal with the waves that emanate from the region where damage is occurring. Because such waves are not treated correctly in the computations, effects from these waves cannot be interpreted from the computer results.

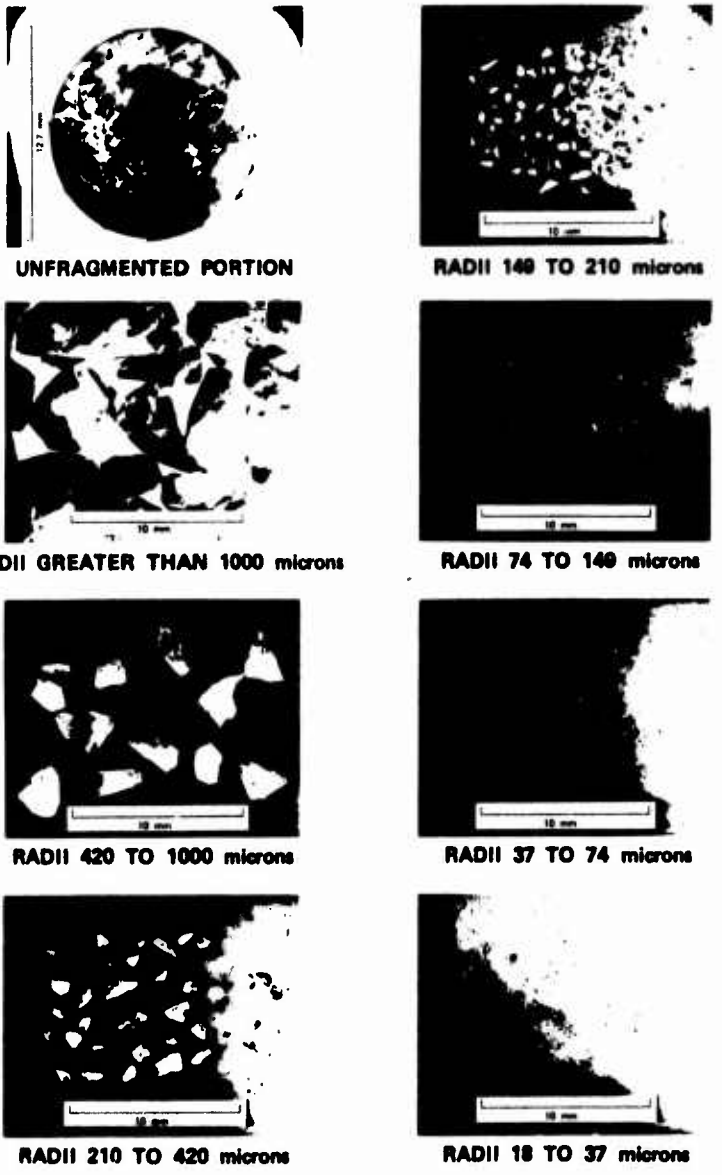


Fig. 4 Photomicrographs of various sized fragments from experiment 53

Addition of a peak stress (or other static) damage criterion to a wave propagation code is usually a small task. Such criteria are often present already in the codes. If an isotropic fracture criterion is used, then one new constant is required for each material and one indicator for each cell of the material. Following the stress computation at each cycle for each cell, the three principal stresses are compared with the criterion. If fracture

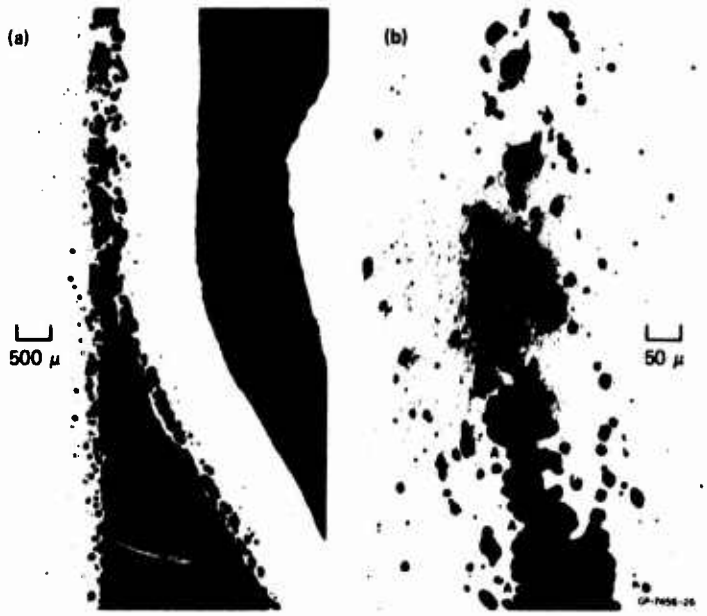


Fig. 5 Ductile cracks. (a) Ductile crack propagation by void coalescence. (b) Tip of ductile crack shown in (a) at higher magnification. Material failure by void coalescence due to necking of the regions separating the voids is apparent near points A

occurs, the stresses are zeroed and the added indicator is set. Thereafter, the stresses are computed as usual but they are zeroed unless they are compressive. For an anisotropic criterion such as Cherry's, a tensile strain (or tensile opening) should be stored for each principal direction for each cell. This strain quantity can be used both as an indicator and also to determine when recompression occurs.

DYNAMIC-PASSIVE CRITERIA

Dynamic fracture criteria account explicitly for the gradual (time-dependent) growth of damage. Passive criteria monitor the development of damage but do not modify the wave propagation calculations to account for this damage.

An example of this dynamic type of criterion is the one introduced by Tuler and Butcher [4]. They represented the damage K by a time integral of the tensile stress σ above some threshold stress σ_o.

$$K = \int_o^{t_1} (\sigma - \sigma_o)^\lambda \, dt \qquad (1)$$

where integration is only over times when $\sigma - \sigma_o$ is tensile. Here λ is an exponent, usually about 1 or 2.

The critical damage level is presumed to be a material constant K_{cr}. This critical level may refer to full separation, incipient damage, or any other defined level (if an appropriately quantitative definition of damage

level can be constructed). This model requires three constants: K_{cr}, λ, and σ_o. With the parameter λ set to 1, Eq. 1 is an impulse criterion; at 2, it is an energy criterion. When Eq. 1 is used to fit experimental data, λ generally has a non-integral value, and K_{cr} has very odd units.

Dynamic experiments must be performed to obtain the three material constants for the Tuler-Butcher model. The experiments should span the stresses and durations expected in the calculations. A typical set of experiments might produce the results in Fig. 6. In this figure, the well-chosen data points just span the amount of damage K_{cr} that is of interest. Usually there will be more experimental points just below the damage threshold: these aid in determining σ_o. With such a set of data, we can determine the three constants by trial. Note that both duration and stress level must be varied in the experiments to provide a basis for the determination.

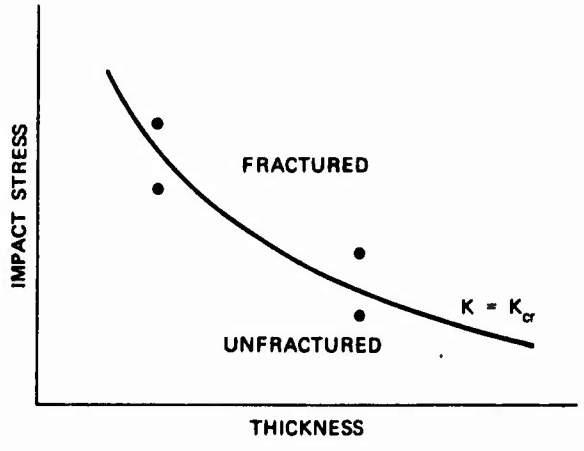


Fig. 6 Fracture data from a series of flat plate impacts to obtain the damage criterion for the Tuler-Butcher model

The Tuler-Butcher criterion may be readily inserted into finite difference wave propagation codes of the Lagrangian type. (The code should have Lagrangian features so that the computed damage quantity K follows a particular material particle.) For each cell the quantity K is stored, in addition to the three constants for each material. The criterion was designed for one-dimensional impact problems: if a three-dimensional problem such as Cherry's were considered, the criterion could be extended to three K values at each cell, one for each principal direction. The criterion is inserted immediately following the stress calculation. Two computations are needed. First compute K for the J^{th} cell

$$K(J) = K(J) + \left(\frac{\sigma_2 + \sigma_1}{2} - \sigma_o\right)^{\lambda} \Delta t \qquad (2)$$

where $(\sigma_2 + \sigma_1)/2$ is the average stress in the time interval Δt. Then compare K(J) with K_{cr}. If K(J) exceeds K_{cr}, the program may be halted, a message may be printed, or the stresses may be zeroed to show that the j^{th} cell has spalled.

The Tuler-Butcher criterion has several advantages over the simpler peak stress criteria. The integral represents more realistically the time-dependent growth of observed damage. Because the criterion is not so critically dependent on a peak stress, larger finite difference cells may be used and still accurately determine the criterion. Such a time-integral criterion is appropriate for extending fracture data obtained with a nearly square

pulse shape to more general pulse shapes such as sinusoidal or multiple.

This criterion has the disadvantage of requiring several well-designed experiments plus computer simulations of the experiments. Because there is no physical model from which λ and K_{cr} can be obtained, "reasonable" values cannot be selected intuitively without such experimental data. Another disadvantage is that the developing damage does not have an effect on the wave propagation processes; hence, subsequent stress histories and computer damage must be invalid.

DYNAMIC-ACTIVE CRITERIA

The dynamic-active criteria follow the gradual development of damage during the wave-propagation process and permit the damage to alter that process. Arbitrary levels of damage may be treated by these methods: an initial designation of a damage level such as full spall is not necessary.

Two approaches have been attempted: The first is a macroscopic approach developed by Wilkins [5] and his coworkers. The second, in which microscopic flaws are treated, was developed at SRI and is discussed in a later section.

In the macroscopic approach of Wilkins the usual stress-strain relations are used for each cell until a critical plastic strain is reached. Then a single crack is permitted to grow across the cell. This growth occurs at a prescribed velocity and in a direction governed by the orientation of the applied tensile stresses. As the cracks grow, the cell weakens, reaching zero strength as the crack severes the cell. As a cell breaks, nearby cells must take up the load. They may also begin to crack and so the crack appears to extend. Because the direction of crack growth is arbitrary, both within a cell and from cell to cell, fairly complex and realistic cracking patterns may be treated. The two principal material constants--critical plastic strain for crack initiation and the growth velocity--are derived from experiments with notched specimens. These tests are simulated with two-dimensional wave propagation code with trial material constants until the experimental results are satisfactorily represented by the computed results.

The main advantage of this macrocrack approach is that the large, observed cracks are treated directly. In addition these cracks may wander across the computational grid; neither presence nor trajectory needs to be known in advance. The stress-strain relations for a damaged cell are anisotropic, thus tending to direct the growth of the crack. Another advantage of the model is that material constants have a clear physical meaning.

This model has several significant drawbacks. The coding is much more complex than for the earlier models considered. Variables must be stored to define the orientation and length of the crack in each cell, plus the accumulated plastic strain. The anisotropic stress-strain relation must be developed and used. In addition, the material constants are derived from a combination of notch test experiments and multiple simulation calculations; these two-dimensional calculations tend to be lengthy and expensive.

NUCLEATION AND GROWTH MODELS

The nulceation-and-growth (NAG) models developed at SRI [6] and studied by Kreer [7] and by Stevens, Davison, and Warren [8] deal with the nucleation of microscopic flaws, their growth and coalescence, and the formation of fragments. Two models have been constructed: one for ductile and one for brittle fracture. Nucleation may occur physically by widening of inherent flaws in the material, cracking of hard inclusions, separating along grain boundaries, or by other mechanisms. In the models, however, nucleation means the appearance of the void or crack at an observable and easily identifiable size on photomicrographs at a scale of about 100X. This nucleation occurs in the model as a function of stress and stress duration. Following nucleation, the voids or cracks grow at a rate that is dependent on the stress level, duration of loading, and the size of the void or crack. The models also account

for the stress reduction that accompanies the development of damage. When the number and size of cracks meet a coalescence criterion in the brittle model, the cracks begin to join and form isolated fragments. With continued loading, all the material forms fragments and complete separation may occur.

The computational models of ductile and brittle fracture are implemented in subroutines that may readily be inserted into one- and two-dimensional Lagrangian wave propagation computer codes. While the material is undergoing fracture, these subroutines are called instead of the usual equation-of-state subroutines.

Basic to the development of the NAG models is the quantitative determination of the damage observed on cross sections of impacted samples. In ductile fracture, the damage occurs as nearly spherical voids (as shown in Figs. 1 and 5), which grow by a viscous growth law. In brittle fracture, the flaws are cracks, like those in Fig. 2. Both ductile and brittle fracture are treated in the NAG method, but only the brittle version (the more complex case) is discussed here.

The brittle fracture damage is usually derived from experiments like that shown in Fig. 1 in which a thin flyer plate is propelled against a thicker target plate. Then the target is sectioned as shown in the inset in Fig. 7, and the length, orientation, and distance from the impact plane of all cracks are measured. For convenience in organizing the data, zones may be marked off on the cross section as shown in Fig. 7. In the uppermost zone in the inset, near the impact plane, there is little damage. The amount of damage increases as we proceed down through the zones until a plane of maximum damage is reached (zone 3 here); the damage then decreases as we continue to the rear surface of the target. The peak tensile stress was approximately the same throughout the region of damage, but the duration of that stress varied in approximately the same way as the damage, having a maximum at the plane of maximum damage. The observed cracks in each zone are organized into groups according to size intervals and orientation angle intervals. These surface distributions are then transformed statistically to volumetric distributions in size and angle by a method analogous to Scheil's [9]. The transformation is handled by the BABS2 computer program [10]. In this transformation it is assumed that the cracks are penny-shaped and that the distribution is axisymmetric around the direction of propagation. A sample set of volumetric crack distributions is shown in Fig. 7 for the zones shown in the inset. Here the angular variation has been suppressed so the ordinate is the total number of cracks larger than the indicated radius. Comparable damage appeared in zones 2, 3, and 4 from the top but lesser damage occurred in zones above and below.

These crack size distributions are all approximated by the equation

$$N_g(R) = N_o \exp(-R/R_1) \tag{3}$$

where N_g is the cumulative number/cm^3 of cracks with radii larger than R, N_o is the total number/cm^3, and R_1 is a distribution shape parameter.

Nucleation in the model occurs as the addition of new cracks to the existing set. These new cracks are presumed to occur in a range of sizes with a size distribution like Eq. (3). At nucleation, the parameter R_1 equals R_n, the nucleation size parameter (a material constant). The number of cracks nucleated is governed by a nucleation rate function:

$$\dot{N} = \dot{N}_o \exp\left(\frac{\sigma_{\varphi\psi} - \sigma_{no}}{\sigma_1}\right) \tag{4}$$

where $\dot{N}_o$, σ_{no}, and σ_1 are fracture parameters and $\sigma_{\varphi\psi}$ is stress normal to the plane of the cracks.

The growth law, derived from experimental data on both ductile and brittle fracture, is [6, 11]:

$$\frac{dR}{dt} = T_1 (\sigma - \sigma_{go})R \tag{5}$$

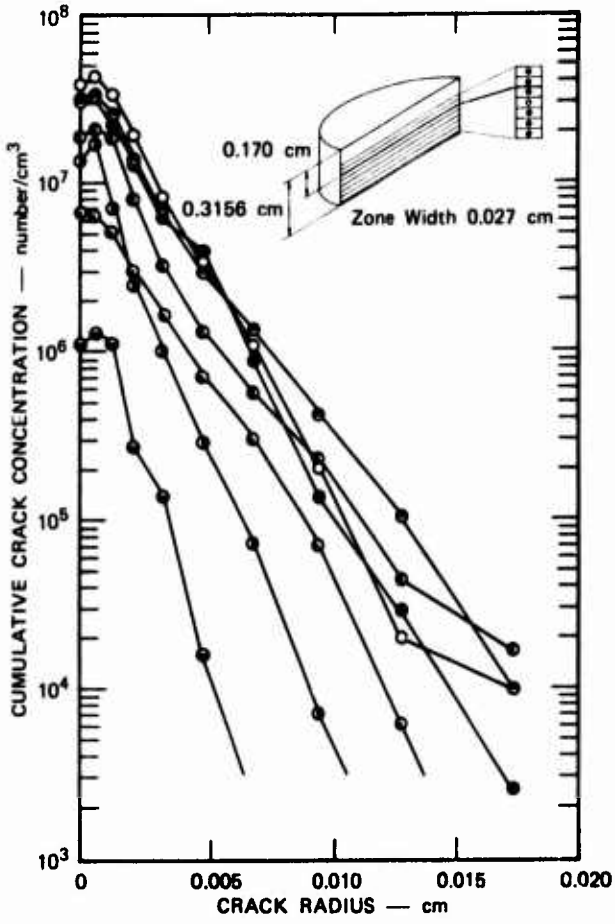


Fig. 7 Crack size distribution in zones near the spall plane in an Armco iron target after a one-dimensional impact: Shot S25

where T_1 is a growth coefficient and σ_{go} is the growth threshold stress. Here σ_{go} is usually treated as a constant material parameter.

All the stresses are computed from the strains in the solid material, not the gross strain. This gross strain is presumed to be composed of a solid strain and a change in crack volume associated with growth and nucleation:

$$\Delta\varepsilon = \Delta\varepsilon_s + \Delta\varepsilon_g + \Delta\varepsilon_n \tag{6}$$

For the model calculations, only $\Delta\varepsilon$ is known. $\Delta\varepsilon_s$ is related to the applied stresses through the usual elastic-plastic relations. $\Delta\varepsilon_g$ represents the increase in strain associated with opening of the existing and growing cracks. $\Delta\varepsilon_n$ represents the increase in strain associated with opening of new cracks. Appropriate stresses are found by requiring that Eq. (6) be satisfied. Each term on the right is a nonlinear function of stress; therefore the solution is conducted by an iteration procedure.

With nucleation, growth, and stress-strain relations available, model calculations may be conducted to simulate damage data such as that in Fig. 7. First, estimates are made of the six nucleation and growth parameters. These estimates are made from computed peak stress and duration in the experiment (from a no-damage calculation) and from the observed damage: the method is given in Ref. [12]. Then simulation calculations are begun. The parameters are varied and simulations are repeated until the damage data are satisfactorily reproduced.

The fragmentation process envisioned in the model under discussion here is a natural extension of the fracture process. The fracture process is presumed to occur until some point at which interaction of the cracks becomes significant and coalescence begins. At this point the gross stress on the cross section is still approximately equal to the stress in the solid material. The growth and nucleation processes are presumed to continue beyond the beginning of coalescence. Eventually, isolated fragments will occur within the solid material as shown in Fig. 8. As these fragments separate, they form voids that cannot support the applied stress. To maintain the external tensile stress, the stress in the remaining solid material must increase. The increased stress in the solid will lead to more growth and nucleation of cracks; hence, the final stages before complete fragmentation are cataclysmic and will tend to continue with very little outside encouragement. This crescendo of damage that occurs in the model just preceding full separation is probably very similar to the damage that actually occurs. This process will tend to emphasize damage in a few regions while allowing adjacent regions to be only slightly fractured.

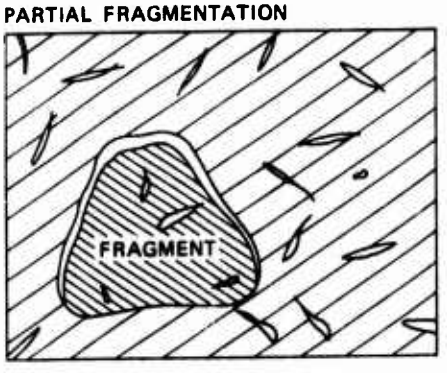


Fig. 8 Schematic depiction of the fragment formation process

A relationship between the fragment sizes and the numbers and sizes of the preceding cracks is required to model the fragmentation process. From our observations in both rock and metals it appears that the fragments are typically chunky objects with an average of six to eight sides, each side probably being produced by one crack. Thus, for large fragments, the crack radius may approximate the fragment radius and the number of fragments may be one-third to one-fourth the number of cracks.

Both the ductile and brittle fracture models have been inserted into one- and two-dimensional wave propagation computer programs at Stanford Research Institute. The subroutines containing these models (DFRACT and BFRACT) act as equation-of-state routines. Such routines are provided the strain and must compute the stresses. DFRACT and BFRACT compute stress but also compute the current damage. In the wave propagation calculations, the usual equation-of-state routine is called until damage begins. Thereafter, the appropriate fracture routine is called. At any time in the calculation a complete listing of the fracture damage present in a cell may be obtained.

Because of the complexity of the NAG calculations, it is suggested that the subroutines be obtained from the originator at Stanford Research Institute for incorporation.

The main advantage of the NAG approach is that damage is obtained in such detail that precise comparisons with observed damage can be made. Residual strength properties can be assessed from the computed damage. The stress-strain relations are modified to account for developing damage so that subsequent waves are handled with some accuracy. The NAG computations are independent of stress pulse shape and source: thus the same parameters describe fracture under plate impact, explosive loading, or sudden radiant heating. The NAG approach also has the advantage that there is more material information involved in it and hence it is more likely to represent reality. Some of the parameters are estimable from static fracture data, some from shock front thickness, others from microscopic observation; hence, the parameters appear to have physical significance.

The increased memory required for the fracture quantities and the increased computer time are the main drawbacks of the NAG models. The fracture parameters must be derived from several plate impact experiments plus a series of one-dimensional simulation calculations.

AVAILABLE PROGRAMS

A great variety of wave propagation computer codes are available. One-dimensional codes are Lagrangian and are based on the leapfrog and artificial viscosity method. Versions of the code termed PUFF have been developed at Air Force Weapons Laboratory, Albuquerque, New Mexico [13]; Kaman Sciences, Colorado Springs, Colorado [14]; AVCO Corporation, Wilmington, Mass. [15]; and SRI, Menlo Park, California [16]. WONDY [17] was written at Sandia Laboratories, Albuquerque, New Mexico. RIP [8] is a similar code written at Systems, Science and Software, La Jolla, California.

A few of the two-dimensional codes are HEMP [19] from Lawrence Livermore Laboratory, Livermore, California, TOODY [20] and CSQ from Sandia Laboratories, Albuquerque, and HELP [21] from Systems, Science and Software, La Jolla, California. HEMP and TOODY are Lagrangian while CSQ is Eulerian. HELP is Eulerian with Lagrangian tracer particles.

All the foregoing programs are written in FORTRAN and are generally running on CDC machines. Conversion to machines with similar word length (60 bit) and core size (64,000 words or larger) is usually not difficult, but extensive changes are needed to use shorter words or a small core. Costs for a single computation are typically $10 to $100 for one-dimensional problems and $100 to $1,000 for two-dimensional. These costs depend largely on the nature of the problem and on variables under the control of the user, not on the particular code selected. Currently the best documented codes are RIP, WONDY, and TOODY.

NAG fracture and fragmentation models are currently available in SRI PUFF. Simpler fracture models are present in all the codes. Wilkins' model has been used with HEMP but is not yet documented. NAG has been inserted into HEMP and TOODY at several institutions, but is not a standard part of any two-dimensional code.

Sources for obtaining the computer programs discussed in this chapter are provided in the Appendix.

PROJECTIONS

Fracture and fragmentation capabilities have improved greatly in the last ten years, and similar improvements can be expected in the future. Future work should provide more detailed and accurate fracture information. Currently less than a dozen materials have been well-characterized for fracture. It is expected that a large number of materials will be tested and characterized. Possibly some guidelines will be derived for estimating fracture parameters from usual engineering data.

Future fracture models will probably be readily separable from the rest of the wave propagation program. Then they can be easily added to whichever code is appropriate for the problem at hand. The models will also be extended to three-dimensional codes.

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APPENDIX

SOURCES OF COMPUTER PROGRAMS

PUFF 66R	Richard Scammon and David Newlander (DYV) Air Force Weapons Laboratory Kirtland Air Force Base, New Mexico 87117
PUFF V	Ed Walsh Kaman Sciences Garden of the Gods Road Colorado Springs, Colorado 80907
PUFF AV	William Bade AVCO Corporation 201 Lowell Street Wilmington, Massachusetts 01887
SRI PUFF, DFRACT, BFRACT	L. Seaman Stanford Research Institute 333 Ravenswood Avenue Menlo Park, California 94025
WONDY	R. J. Lawrence, Division 5162 Sandia Laboratories Alburquerque, New Mexico 87115
RIP	Harold E. Read Systems, Science and Software P. O. Box 1620 La Jolla, California 92037
HEMP	Mark Wilkins Lawrence Livermore Laboratory P. O. Box 808 Livermore, California 94550
TOODY	L. D. Bertholf, Division 5162 or R. K. Byers, Division 5166 Sandia Laboratories Albuquerque, New Mexico 87115
CSQ	Samuel Thompson, Division 5162 Sandia Laboratories Albuquerque, New Mexico 87115
HELP	L. J. Hageman and J. M. Walsh Systems, Science and Software P. O. Box 1620 La Jolla, California 92037

Eigenvalue Extraction

Robert A. Wilke and James A. Stricklin

Texas A & M University

INTRODUCTION

This chapter is concerned with the eigenvalue and eigenvector extraction routines that are available to shock and vibration analysts. The analyst will be most often concerned with the calculation of eigenvalues and eigenvectors in buckling and vibration problems. For the buckling problem the lowest eigenvalue is needed, whereas for vibration problems several or all of them may be required. The specific number required will depend upon the problem and the type of analysis being performed. The problem being considered and the character of the matrices in the basic relations will be factors in the particular eigenvalue solution routine that is used.

EIGENVALUE FORMS

Buckling and vibration problems may be divided into four categories depending upon the form of their eigenvalue relation.

Most structural analyses of shock and vibration problems involve the solution of matrix relations of the form:

$$Aq_1 + Bq = Q \tag{1}$$

where A,B = real, symmetric matrices

For dynamic analysis Eq.(1) is written

$$A\ddot{q} + Bq = Q \tag{2}$$

where A = mass matrix, B = stiffness matrix, $\ddot{q}$ = acceleration vector, q = displacement vector, Q = loads vector

In order to determine the natural frequencies and modes of the system the free vibration case is considered and the substitution $\ddot{q} = -\omega^2 q$ is made to obtain the general eigenvalue problem:

$$(A - \lambda B)q = 0 \tag{3}$$

We know from matrix algebra that the nontrivial solution to this expression is the solution to:

$$\det(A - \lambda B) = 0 \tag{4}$$

The eigenvalues, λ, give the natural frequencies of the system and the eigenvectors are the corresponding vibrational modes.

$$\lambda = 1/\omega^2$$
ω = natural frequency

The corresponding equation for linear buckling analysis of a structure is given by:

$$(A + \lambda B)q = 0 \tag{5}$$

where

A = elastic stiffness matrix
B = geometric stiffness matrix
λ = buckling load parameter (eigenvalue)
q = displacement vector

This is also an eigenvalue problem of the same form as equation (3).

The matrices A and B in both cases described thus far are real, symmetric, banded matrices [1]. A real symmetric matrix has real eigenvalues and a full orthonormal n-set of eigenvectors [2,3]. The mass and stiffness matrices are of the same band width when the consistent mass matrix is used. The lumped mass matrix is a diagonal matrix; this has the effect of making a structure softer and reduces the magnitudes of the eigenvalues. This results in a loss in accuracy [4]. When the lumped mass matrix is used the general eigenvalue problem $(A - \lambda B)q = 0$ quickly reduces to the standard eigenvalue problem $(C - \lambda I)x = 0$. The mathematical form of the dynamic problem with a consistent mass matrix is identical to that of the buckling problem [5]; therefore, in the discussion of the routines in the next section it will be assumed, unless otherwise stated, that the general eigenvalue problem $(A - \lambda B)q = 0$ is to be solved with B being a consistent mass matrix.

A second form of the structural problem is that of a quadratic matrix equation. It occurs in vibration analysis when the mass and stiffness matrices are frequency dependent [8]. The basic eigenvalue relation is

$$[K - \omega^2 B_o - \omega^4 C]q = 0 \tag{6}$$

K = stiffness matrix (symmetric, positive definite)
B_o = static inertia matrix (symmetric, positive definite)
C = dynamic correction matrix (symmetric, positive definite)
ω = natural frequency
q = displacement

The corresponding buckling equation is present when the second order stiffness terms are retained. The basic relation is:

$$[K_E + \lambda K_1 + \lambda^2 K_2]q = 0 \tag{7}$$

K_E = elastic stiffness matrix
K_1 = geometric stiffness matrix
K_2 = stiffness matrix embodying 4th order effects
λ = buckling load parameter

Since the matrices in both cases are symmetric and positive definite the eigenvalues and eigenvectors will be real. In order to efficiently solve the quadratic matrix equation the matrices must be rearranged to place it in either general eigenvalue or standard eigenvalue form. This rearrangement will, depending upon the problem, change the bandedness and sparsity of the A and B matrices, and requires that much care be taken in selecting the eigenvalue extraction routine.

A third form of the structural equations includes a matrix that may not be symmetric and may not be positive definite. The general form of the equations is

$$A\ddot{q} + C\dot{q} + Bq = 0 \tag{8}$$

For the case of free vibration analysis of spinning structures the matrices are [8]:

A = inertia matrix
C = coriolis matrix, a function of the angular motion Ω
B = $K_E + K_C + K_G$
K_E = elastic stiffness matrix
K_C = centrifugal force effects matrix, a function of Ω^2
K_G = geometric stiffness matrix, a function of Ω^2

Matrices A and B are symmetric and positive definite (for small Ω) but the coriolis effects matrix, C, is skew-symmetric.

The vibration of damped structures involves the following matrix definitions [8]:

A = mass matrix
B = stiffness matrix
C = viscous damping matrix (symmetric)

In both the free vibration and damped vibration cases the eigenvalue problem defined by Eq. (8) is rearranged and rewritten such that it is of the general eigenvalue form. The solution procedure then should retain the banded form of the matrices.

The last form to be considered occurs in the solution of flutter problems. One case of this form, studied by Gupta [39], is as follows:

$$[K_E + K_G + \ell A - (w^2 + wg_a)M]q = 0 \tag{9}$$

K_E = elastic stiffness matrix
K_G = geometric stiffness matrix
ℓ = dynamic pressure parameter
A = aerodynamic load matrix
w = eigenvalue
M = inertia matrix
g_a = aerodynamic damping parameter

This case is of particular interest because while K_E, K_G, and M are symmetric, matrix A is unsymmetric, which causes the existence of complex eigenvalues (ℓ>0). This requires that additional care be taken in the selection of the eigenvalue extraction routine. Many of the routines presented in this paper will compute complex eigenvalues and, further, most industrial firms have in their numerical libraries codes that solve for either real or complex eigenvalues.

EIGENVALUE SOLUTION METHODS

Shock and vibration problems with relatively full matrices (not sparsely populated) and of small size (less than 200-400 elements) are amenable to solution by use of direct methods of eigenvalue extraction such as those of Givens and Householder. The main advantages of these methods are that they are in-core, computational time is less than for iterative methods, all eigenvalues can be calculated in ascending order, and the opportunity for round-off error is at a minimum. When the system size becomes large the program will exceed the available core thus limiting the size of problem that can efficiently be solved by a direct method [6].

When the matrices are sparsely populated and strongly banded the best method of solution is an iterative method. A major proportion of eigenvalue techniques are iterative; therefore, the choice of method depends upon the problem being solved, program availability, and user's preference. There is no one best routine that is applicable to all problems. Factors to be considered in the choice of eigenvalue routine are problem size, number of eigenvalues and eigenvectors desired, band width of the matrices, consistent mass matrix or lumped mass, problem type, machine precision and storage capability.

The computation of eigenvalues and eigenvectors of the general form $(A - \lambda B)x = 0$ may take one of two forms:

a) Operations are carried out on both A and B matrices simultaneously. This preserves the form of the matrices and allows the exploitation of their banded, sparse population characteristics.

b) The problem is reduced to standard form $(C - \lambda I)x = 0$ through matrix decomposition and multiplication. The C matrix becomes full and the special configuration is lost.

The choice of the form to be used is dependent upon the solution algorithm used, the makeup of the A and B matrices, and whether the user prefers to retain the characteristics of the matrices.

Theorems of matrix analysis show that two real symmetric matrices may be simultaneously reduced to upper triangular, to tridiagonal form, or to diagonal form without altering the eigenvalues [3,6]. Most eigenvalue extraction routines reduce the matrices to upper triangular or tridiagonal then apply one or more of the following algorithms to compute the eigenvalues and eigenvectors: Inverse Iteration, LR Iteration, QR Iteration, Rayleigh Quotient Iteration, and Jacobi Iteration. Three excellent references for discussion of algorithm selection and description of solution techniques are the works by Wilkinson [6], Bathe and Wilson [7] and Gupta [8].

EIGENVALUE AND EIGENVECTOR EXTRACTION ROUTINES

The eigenvalue extraction routines known by the authors to be in use at this time are presented in this section. Knowledge of the existence of the routines and their use was gained through personal experience, an extensive literature search, and a survey of researchers, program authors, and program users. Certain standard information is given for each program along with a paragraph of special comments.

Engineering ANalysis SYStem (ANSYS) [9]

Capability: A general purpose finite element computer program that contains eigenvalue extraction capability.
Method: Jacobi iteration with Guyan reduction for eigenvalue and eigenvector computation.
Language: FORTRAN
Hardware: IBM, UNIVAC, CDC and Honeywell
Working tapes required: Disk files only

Developer: J. A. Swanson
Swanson Analysis Systems Inc.
870 Pine View Dr.
Elizabeth, PA 15037
Availability: Available from developer with the cost negotiable. Users manual is available.
Comments: Program is a complete engineering analysis system. CP time for a complete problem of order ≈200 is 7 minutes on a CDC 6600.

Eigenvalues and Eigenvectors of a Complex Matrix (ALLMAT) [10]

Capability: Computation of the eigensystem of the standard eigenvalue equation directly in terms of complex arithmetic.
Method: A general (non-Hermitian) complex matrix is transformed to a Hessenberg form then the eigenvalues are determined using the complex QR method. Eigenvectors are computed using inverse iteration.
Language: FORTRAN
Hardware: CDC, IBM, UNIVAC
Developer: G. Fair
Lewis Research Center
Cleveland, OH 44135
Availability: Program is available and is listed in the users manual, [10].
Comments: This routine computes eigenvalues of the standard form so the general form must be transformed to the standard linear form prior to use of the routine. The routine is as general as possible; it contains a perturbation method for use when the eigenvalues are either degenerate or nearly of equal value. It also provides error and convergence information.

Eigenvalue Routine for Symmetric Band Matrices (BANEIG) [11]

Capability: Determination of selected number of eigenvalues and eigenvectors of real, banded matrices with the smallest values determined first.
Method: Rayleigh quotient iteration and inverse power iteration to find the first eigenvalue and eigenvector. Matrix deflation and spectral shift before repeating iterations to find succeeding values.
Language: FORTRAN
Hardware: CDC
Developer: C. A. Felippa
University of California at Berkeley
Berkeley, CA 94700
Availability: Program available from developer or COSMIC. Users manual is available, [11].
Comments: Program is most economical when matrices A and B are large.

REAL SYMMETRIC MATRIX EIGENSYSTEM SOLVER (BIGMAT)

Capability: Determination of eigenvalues and eigenvectors of real symmetric matrix.
Method: Matrix is reduced to tridiagonal form by Given's method. The eigenvalues are found through the use of Sturm sequences and the eigenvectors by the Wielandt inverse power method.
Language: FORTRAN
Hardware: IBM
Developer: M. Elson and R. E. Funderlic
Oak Ridge Central Data Processing Facility
Oak Ridge, TN 37830

Availability: Program available from developer.
Comments: Structural general eigenvalue equation must be transformed to standard form prior to use of this routine.

CXLNESM [12]

Capability: Computation of eigenvalues and eigenvectors of the general linear eigenvalue problem with complex square matrices, one of which is invertible.
Method: Eigenvalues of a general (non-Hermitian) complex matrix using complex QR iteration and computing eigenvectors using inverse iteration.
Language: FORTRAN
Hardware: CDC, IBM, UNIVAC
Developer: D. A. Gignac and Dr. Y. Liu
Computation and Mathematics Department
Naval Ship Research and Development Center
Bethesda, MD 20034
Availability: Program is available from developer. Users manual is available, [12].
Comments: Program is an extension of the ALLMAT routine from standard eigenvalue form to general linear eigenvalue form.

CXQDESM [12]

Capability: Computation of eigenvalues and eigenvectors of the quadratic eigensystem problem with complex square matrices.
Method: Eigensystem is reduced to standard (complex) eigenvalue form then calculates the eigenvalues and eigenvectors through complex QR iteration and inverse iteration.
Language: FORTRAN
Hardware: CDC, IBM, UNIVAC
Developer: D. A. Gignac and Dr. Y. Liu
Computation and Mathematics Department
Naval Ship Research and Development Center
Bethesda, MD 20034
Availability: Program is available from developer. Users manual is available, [12].
Comments: The quadratic eigenvalue equation $(w^2A + wB + C)Z = 0$ used in flutter analysis is solved when A, B and C are complex matrices and either A or C is invertible.

Free Vibration Analysis of DAMPed Structures (DAMP) [13]

Capability: Performs free vibration analysis of spinning and non-spinning structures with or without damping (viscous, structural). Program computes the desired roots and vectors without computing more than those needed.
Method: The desired roots are isolated by the Sturm sequence method then are accurately located by a special inverse iteration scheme. Eigenvectors are also computed through the inverse iteration method.
Language: FORTRAN
Hardware: UNIVAC
Developer: K. K. Gupta
M/S 144-218
Jet Propulsion Laboratory
4800 Oak Grove Dr.
Pasadena, CA 91103

Availibility: Program is available (approx 1300 instructions) from COSMIC. Users manual available [13].
Comments: Program determines roots lying within a specified spectrum for the eigenvalue problem $A\ddot{q} + c\dot{q} + Bq = 0$. Input matrices are the mass matrix A, (complex) stiffness matrix B, and viscous damping matrix C. Matrix bandedness and sparsity are exploited to effect best economy in core storage and computation time. This program is one of six (DAMP, EASI, QMES, SPIN, STURM, SUPFLUT) developed by Dr. Gupta to solve all of the forms of the eigenvalue problem outlined in the previous section. An out of core version is presently under development.

Dynamic-Transformation Adapted to Modal Synthesis Using Stiffness-coupling (DAMUS) [14]

Capability: Program improves the computational economy for the vibration analysis of complex structures while still considering substructure modes. A dynamic transformation is used to reduce the mass and stiffness matrices. The reduced eigenvalue problem is solved and used in a new solution.
Method: Modal synthesis methods are used to determine the low frequency modes.
Limitations: Total number of modes allowed is 300 with a maximum eigenvalue size of 100 and 200 co-ordinates reduced by the dynamic transformation.
Language: FORTRAN
Hardware: UNIVAC
Working tapes required: Twelve (12) with the number used at any time variable but never all twelve simultaneously.
Developer: E. J. Kuhar, Jr., and C. V. Stahle, Jr.
General Electric Company
P. O. Box 8555
Philadelphia, PA 19101
NASA Technical Monitor
Dr. J. R. Admire
George C. Marshall Space Flight Center
Availability: Program is available from COSMIC. Users manual [14] available from NTIS or COSMIC.
Comments: The program begins with the entry of substructure mass and stiffness matrices. The eigenproblem for the individual substructure is solved and the substructures are coupled together by coupling springs. The dynamic transformation is used to reduce the size of the eigenproblem. The coupled system eigenvalues and eigenvectors are determined and, at the users option, may be used to assess the accuracy of the results and obtain new transformation. This may be repeated or a new set of co-ordinates and transformations obtained for another group of modes.

DANUTA [15]

Capability: Complete structural analysis capability with eigenvalue extraction routines imbedded in the program.
Method: Structural matrices transformed and eigenvalues and eigenvectors extracted through the use of Householder's method.
Language: FORTRAN
Hardware: IBM, UNIVAC, CDC
Developer: S. Chacour
Allis Chalmers Co.
Box 712
York, PA 17405

Availability: Program usage is marketed in the U.S. by McDonnell Douglas Automation Co., Box 516, St. Louis, MO 63166. Users manual available with program usage.
Comments: Solution time for complete problem of order 216 on a UNIVAC 1108, double precision, with all eigenvalues and 20 eigenvectors extracted required 48 minutes.

DYNamic AnaLysis (STRUDL DYNAL) [16]

Capability: Program is a full dynamic structural analysis including harmonic analysis, transient analysis, and shock (response) analysis.
Method: Eigenvalues and eigenvectors are calculated through the use of Jacobi iteration, Sturm sequence property, Householder method, QR iteration and Wielandt inverse iteration.
Language: FORTRAN
Hardware: IBM
Developer: McDonnell Douglas Automation/ECI Systems
Box 516
St. Louis, MO 63166
Availability: Program and users manual are available through developer for approximately $27,000. Program license is in conjunction with STRUDL and ICES Executive.
Comments: STRUDL DYNAL is based on the modal superposition method where the reduced co-ordinates are mathematically related to the structural frequencies and modes of vibration. The eigenvalue extraction routine contains a special orthogonalization feature to ensure orthogonal eigenvectors when close or equal eigenvalues exist.

Eigenvalue Algorithm based on Sturm Sequence and Inverse Iteration (EASI) [16]

Capability: Free vibration analysis of large structural systems using finite element method. Computes only desired roots and vectors.
Method: Desired eigenvalues are isolated by the Sturm sequence method then accurately calculated by a special inverse method. Eigenvectors are also determined by the inverse method.
Language: FORTRAN
Hardware: UNIVAC, CDC
Developer: Mr. K. K. Gupta
M/S 144-218
Jet Propulsion Laboratory
4800 Oak Grove Dr.
Pasadena, CA 91103
Availability: Program is available (approximately 900 instructions) from COSMIC. Users manual available, [16].
Comments: Program efficiently determines the roots lying within a specified spectrum for the general eigenvalue problem. Matrix bandedness and sparsity are exploited to effect best economy of storage and computation time. An out of core version is presently under development.

General Axisymmetric Stiffened Shells (BOSOR4-EBAND2) [17]

Capability: Program BOSOR4 performs stress, stability and vibration analysis of general axisymmetric, stiffened, elastic shells of revolution. Routine EBAND2 calculates the eigenvalues and eigenvectors for the stability and vibration analyses.
Method: Eigenvalues and eigenvectors are extracted by the inverse iteration method with spectral shifts.

Output: Random access high speed auxiliary storage, tape or disk, is required.
Language: FORTRAN
Hardware: IBM, CDC, UNIVAC
Developer: Frank Brogan (EBAND2)
Dept. 52-33, BLDG. 205
Lockheed Missiles and Space Co.
3251 Hanover St.
Palo Alto, CA 94304
Availability: Routine EBAND2 is imbedded in BOSOR4 so is available from developer or COSMIC as part of the entire program. Users manual available [17].
Comments: BOSOR4 is the latest in a series of computer programs that perform stress, stability and vibration analyses of segmented, stiffened, elastic shells of revolution. The analysis has the capability of treating branched shells, variable mesh point spacing within each shell segment and reformulation of the buckling eigenvalue problem to account for the pre-buckling shape change of the shell in linear buckling analyses.

Eigenvalues and Vectors by Iteration (EIGIT3) [18]

Capability: Calculation of frequencies and mode shapes of multidegree linear systems.
Method: Eigenvalues and eigenvectors calculated by inverse iteration.
Language: BASIC
Developer: W. Weaver, Jr.
Stanford University
Stanford, CA 94305
Availability: Program and user information are contained in [18].
Comments: Recurrance formulas are used to determine the dominant eigenvalue and corresponding eigenvector. Sweeping matrices are used to eliminate certain modes. All eigenvalues and eigenvectors are determined in the routine. Lumped mass matrix is assumed in solution of the standard eigenvalue problem.

Eigenvalues of the Generalized Eigenvalue Problem (EQZVEF) [19, 20]

Capability: Calculate the eigenvalues and eigenvectors of the general matrix eigenvalue problem.
Method: Input matrices A, B are transformed to upper Hessenberg and upper triangular form. Then matrix A is transformed to quasi-triangular. Finally the eigenvalues and eigenvectors are determined through the use of a modified Moler-Stewart algorithm.
Language: FORTRAN
Hardware: IBM, UNIVAC, CDC
Developer: Int'l Mathematics and Statistics Library (IMSL)
6200 Hillcroft, Suite 510
Houston, TX 77036
Availability: Program and users manuals are available from developer.
Comments: Generalized eigenvalue problem may be input in real or complex form. Output is in complex form. Program does not exploit matrix bandedness or sparsity but is efficient in operation.

Frequency and Modes of Shells of Revolution (FAMSOR) [21]

Capability: Calculates eigenvalues and eigenvectors of the standard eigenvalue problem for stiffness and mass of a shell of revolution.

Method: First eigenvalue and eigenvector are determined using inverse power iteration. Matrix deflation is accomplished and succeeding eigenvalues and eigenvectors are calculated.
Working tapes or discs required: 4
Language: FORTRAN
Hardware: IBM
Developer: L. B. McWhorter and W. E. Haisler
Aerospace Engineering Department
Texas A&M University
College Station, TX 77843
Availability: Program and users manual are available from COSMIC or developer.
Comments: Program converts the input consistent mass matrix to a lumped mass matrix and solves the standard eigenvalue problem. The eigenvalue extraction routine is based upon the BANEIG routine written by C. A. Felipa [11].

FARSS

Capability: Static and dynamic structural analysis of solids and shells of revolution with isotropic or anisotripic materials.
Method: Eigenvalues and eigenvectors are determined through the use of the inverse iteration with reduced subspace.
Working tapes required: 1 (disc)
Language: FORTRAN
Hardware: UNIVAC, CDC
Developer: M. B. Marlowe
Lockheed Missiles and Space Co.
Palo Alto, CA 94304
Availability: Program and users manual are available from developer.
Comments: Program will analyze bifurcation buckling of shells with linear axisymmetric prestress and modal vibrations with no prestress. Cases of viscous damping, structural damping and linear damping may be considered. Mass matrix used is of the lumped mass form.

Free Vibration and Stability Analysis of Shells of Revolution (KSHEL) [22, 23]

Capability: Free vibration and stability analysis of shells of revolution subjected to axisymmetric (program KSHEL2) or nonsymmetric (program KSHEL3) pre-stress.
Method: Calculation of eigenvalues and eigenvectors in KSHEL2 by determinant search. In KSHEL3 calculation is by inverse iteration.
Language: FORTRAN
Hardware: IBM, UNIVAC, CDC and any computer with a FORTRAN IV compiler.
Developer: A. Kalnins
Mechanical Engineering Department
Lehigh University
Bethleham, PA 18000
Availability: Program and users manual are available from developer with cost negotiable.
Comments: Two computer codes have been written - KSHEL2 and KSHEL3. Programs use a multisegment itegration method to analyze the shells of revolution. The static and axisymmetric eigenvalue routines can admit discontinuities in the slope of the meridian and permit branches. The nonsymmetric routine permits neither.

MARC [24, 25]

Capability: General purpose finite element program for linear and nonlinear

analysis of structures and soils. Structural analysis capability includes rods, beams, membranes, plates, shells, solid 3D and crack tips.
Method: Eigenvalues and eigenvectors determined by inverse iteration and subspace iteration.
Working tapes required: Up to 3
Language: FORTRAN
Hardware: IBM, CDC
Developer: Pedro V. Marcal
MARC Analysis Research Corporation
105 Medway St.
Providence, RI 02906
Availability: Program is not for sale. Use of it obtained through developer or CDC Data Centers.
Comments: Program has capability for considering plasticity, large deflections, anisotropic materials, nonlinear collapse analysis and bifurcation with linear or nonlinear prestress.

MODES [26]

Capability: Calculation of eigenvalues and eigenvectors of the general eigenvalue problem.
Method: For small order problems the matrices are treated as one block. For one block (NBLOCK=1) eigenvalues and eigenvectors are calculated by determinant search. For large order systems inverse subspace iteration is employed (NBLOCK>1)
Working tapes required: 6
Language: FORTRAN
Hardware: CDC, IBM, UNIVAC, GE, TELEFUNKEN
Developer: K. J. Bathe
Dept. of Mechanical Engineering
Massachusetts Institute of Technology
Cambridge, MA 02139
Availability: Program is available from COSMIC. Users manual and program listing is [26].
Comments: Program Modes is used in NONSAP and SAP IV structural analysis programs and is one of a series of three routines written by Dr. Bathe to solve the large general eigenvalue problem. The subspace iteration algorithm uses Raleigh coefficient shifts and secant iteration with extrapolation. Mass matrix is input as a diagonal (lumped) non-negative definite matrix.

NROOT-EIGEN [27]

Capability: Calculation of eigenvalues and eigenvectors of the general eigenvalue problem.
Method: Jacobi iteration.
Language: FORTRAN
Hardware: IBM
Developer: IBM Corporation
Availability: Program is available from IBM. Users manual is [27].
Comments: Two routines, NROOT and EIGEN, are used in solution of the general eigenvalue problem. The main program NROOT transforms the general eigenvalue problem to standard form and uses routine EIGEN to compute the eigenvalues and eigenvectors. Input matrices must be real, symmetric. Matrix sparsity and bandedness are not exploited.

Quadratic Matrix Equation Solver (QMES) [28]

Capability: Solution of the quadratic matrix eigenvalue problem.
Method: Inverse iteration and the Sturm sequence property are used to find the eigenvalues and eigenvectors of the quadratic problem. The desired roots are first isolated by the Sturm sequence method then accurately located by an interpolation bisection method.
Language: FORTRAN
Hardware: UNIVAC
Developer: K. K. Gupta
M/S 144-218
Jet Propulsion Laboratory
4800 Oak Grove Dr.
Pasadena, CA 91103
Availability: Program is available (approximately 927 instructions) from COSMIC. Users manual available from COSMIC or NTIS.
Comments: QMES is an efficient routine for computation of the desired roots and associated vectors of the quadratic matrix equation $(A - w^2B - w^4C)q=0$; A is the static stiffness matrix, B is the static mass matrix and C is the dynamic correction matrix. The routine exploits matrix bandedness and sparsity.

A Lanczos Algorithm Subroutine for $(A - \lambda B)X=0$ (REDUCB1) [29, 30]

Capability: Calculation of eigenvalues and eigenvectors of the real, general eigenvalue problem form.
Method: General eigenvalue problem is reduced to standard form with the matrix becoming tridiagonal. Tridiagonalization is accomplished through the Lanczos algorithm. Eigenvalues are computed by the method of bisection. Eigenvectors are computed through the use of inverse iteration.
Language: FORTRAN
Hardware: CDC
Developer: D. A. Gignac
Computation and Mathematics Dept.
Naval Ship Research and Development Center
Bethesda, MD 20034
Availability: Program and users manual available from Head, Computation and Mathematics Dept., Naval Ship Research and Development Center, Bethesda, MD.
Comments: REDUCB1 is a specialized FORTRAN version of REDUCB [30] an ALGOL procedure written by Golub, Underwood and Wilkinson. REDUCB1 exploits the property of matrix bandedness and sparsity.

Structural Analysis and Matrix Interpretive System (SAMIS) [31, 32]

Capability: Linear, elastic, transient analysis of shell structures.
Method: Eigenvalues and eigenvectors are computed by Jacobi iteration and Sturm sequence method.
Working tapes required: 2
Language: FORTRAN
Hardware: CDC, UNIVAC, GE
Developer: R. J. Melosh
MARC Analysis Research Corporation
314 Court House Plaza
260 Sheridan Ave.
Palo Alto, CA 94306
Availability: Program and users manual are available from developer.
Comments: Eigenvalue extraction routine is imbedded in the complete SAMIS program.

SECANT [26]

Capability: Calculation of eigenvalues and eigenvectors of the large general eigenvalue problem.
Method: Calculation of eigenvalues and eigenvectors through the use of determinant search, Sturm sequence property, and inverse iteration.
Working tapes required: 1
Language: FORTRAN
Hardware: CDC, IBM, UNIVAC, GE, TELEFUNKEN
Developer: K. J. Bathe
Dept. of Mechanical Engineering
Massachusetts Institute of Technology
Cambridge, MA 02139
Availability: Program is available from COSMIC. Users manual and program listing is [26].
Comments: Program SECANT is used in NONSAP and IV structural analysis programs. It is one of a series of three routines written by Dr. Bathe. Mass matrix may be either banded positive definite or diagonal non-negative definite.

Spinning Structures Eigenproblem Solver (SESI) [33]

Capability: Solution of the eigenvalue problem $M\ddot{q} + C\dot{q} + Kq = 0$ for free vibration analysis of large spinning structures.
Method: The desired eigenvalues are first isolated by the Sturm sequence then accurately computed by a special inverse iteration method. Eigenvectors are found by the same inverse iteration method.
Language: FORTRAN
Hardware: UNIVAC
Developer: K. K. Gupta
M/S 144-218
Jet Propulsion Laboratory
4800 Oak Grove Dr.
Pasadena, CA 91103
Availability: Program (approximately 1100 instructions) and users manual are available from COSMIC. Users manual and program listing also in [33].
Comments: Input matrices are M, the mass matrix, K, the stiffness matrix including geometric and centrifugal effects and C, the coriolis matrix. Program fully exploits the bandedness and sparsity of the mass and stiffness matrices. Only the desired number of roots are computed. Program SESI is 3 times faster than program SPIN (see following program description) and requires slightly more core storage.

Free Vibration analysis of SPINning Structures (SPIN) [34]

Capability: Performs free vibration analysis of large spinning structures through solution of eigenproblem $M\ddot{q} + C\dot{q} + Kq = 0$.
Method: Desired roots are first isolated by the Sturm sequence method then accurately determined by an interpolation/bisection method. Eigenvectors are found through the inverse iteration method.
Language: FORTRAN
Hardware: UNIVAC
Developer: K. K. Gupta
M/S 144-218
Jet Propulsion Laboratory
4800 Oak Grove Dr.
Pasadena, CA 91103
Availability: Program (approximately 700 instructions) and users manual available from COSMIC. Users manual also included in [34].

Comments: Input matrices are the mass matrix M, stiffness matrix K and Coriolis matrix C. Program enables the user to determine only desired roots and vectors lying within the prescribed spectrum. Matrix bandedness and sparsity are exploited.

SSPACE [26]

Capability: Calculation of eigenvalues and eigenvectors of the large general eigenvalue problem.
Method: Subspace iteration with Jacobi iteration in the subspace for the eigenvalues.
Working tapes required: 2
Language: FORTRAN
Hardware: CDC
Developer: K. J. Bathe
Dept. of Mechanical Engineering
Massachusetts Institute of Technology
Cambridge, MA 02139
Availability: Program is available from COSMIC. Users manual and program listing in [26].
Comments: SSPACE is one of three routines written by Dr. Bathe for solution of the large generalized eigenvalue problem. Mass matrix may be input either as banded positive definite or diagonal non-negative definite.

Buckling of linearized asymmetric equilibrium states (SRA101) [35, 36]

Capability: Program calculates the bifurcation buckling modes of linearized asymmetric prebuckling states.
Method: Eigenvalues are computed using the Rayleigh quotient for an initial estimate then applying inverse iteration to the linearized eigenvalue equations.
Language: FORTRAN
Hardware: CDC, UNIVAC
Developer: G. A. Cohen
Structures Research Associates
Laguna Beach, CA 92651
Availability: Program is available from developer and COSMIC. Users manual is available, [36].
Comments: This routine is part of a series of six compatible programs for six different modes of response of stiffened axisymmetric shells. Three of the programs contain eigenvalue extraction routines (SRA101, SRA201, SRA300) with the eigenvalue problem in a different structural form for each one. The developer plans to integrate the programs into a single program with improved input format, problem size independence through dynamic storage, general material anisotropy, transverse shear deformations and field method of solution (invariant imbedding).

Buckling of nonlinear axisymmetric equilibrium states (SRA201) [35, 36]

Capability: Program calculates the bifurcation buckling modes of nonlinear (or linear) axisymmetric torsionless prebuckling states.
Method: Initial estimates for the eigenvalues are obtained according to the Rayleigh quotient. The inverse iteration method is then applied to the linearized eigenvalue equations.
Language: FORTRAN
Hardware: CDC, UNIVAC

Developer: G. A. Cohen
Structures Research Associates
Laguna Beach, CA 92651
Availability: Program is available from developer and COSMIC. Users manual is available, [36].
Comments: See SRA101 comments.

Vibrations About Axisymmetric Equilibrium States (SRA300) [35, 36]

Capability: Program calculates free vibration modes about nonlinear axisymmetric torsionless equilibrium states.
Method: The eigenvalue equations for the square frequency of harmonic vibrations about an equilibrium state are calculated in the same manner as for the buckling in the vicinity of the same equilibrium state. As in SRA101 and SRA201 the initial estimates for the eigenvalues are obtained by use of the Rayleigh quotients. The final eigenvalues and the eigenvectors are then obtained through the inverse iteration method.
Language: FORTRAN
Hardware: CDC, UNIVAC
Developer: G. A. Cohen
Structures Research Associates
Laguna Beach, CA 92651
Availability: Program is available from developer and COSMIC. Users manual is available, [36].
Comments: See SRA 101 comments.

STARDYNE [37]

Capability: Two-dimensional dynamic analysis of shells of revolution using the finite element method.
Method: Eigenvalues and eigenvectors are computed through the application of Householder's method, QR iteration and inverse iteration,
Language: FORTRAN
Hardware: CDC
Developer: R. Rosen
Mechanics Research Inc.
El Sequendo, CA 92651
Availability: Program is available at Control Data Corporation Data Centers. Users manual available from developer.
Comments: Eigenvalue extraction routine is imbedded in the structural analysis program STARDYNE. Routine uses inverse iteration with shift points and is fully automated to select shift point and the number of iterations needed to minimize the required computer time. Eigenvectors for systems with 15,000 dynamic DOF have been computed.

Solution of Eigenvalue Problems by STURM Sequence Method (STURM) [38]

Capability: Solution of the real, general eigenvalue problem for large structural systems.
Method: The desired roots are first isolated by the Sturm sequence method then they are accurately determined by an interpolation/bisection technique. Eigenvectors are computed through the inverse iteration method.
Language: FORTRAN
Hardware: UNIVAC

Developer: K. K. Gupta
M/S 144-218
Jet Propulsion Laboratory
4800 Oak Grove Dr.
Pasadena, CA 91103

Availability: Program (approximately 860 instructions) and users manual are available from COSMIC. Program listing and users manual are also included in [38].

Comments: Program exploits the input matrix bandedness and sparsity characteristics. Only the desired roots within a prescribed spectrum are computed. Routine may be used in vibration analysis (input mass and stiffness matrices), in structural stability analysis (input matrices geometric stiffness and elastic stiffness), and in vibration analysis of stretched structures (input matrices mass and corrected stiffness, $K_E - K_G$).

SUBCHEB

Capability: Calculation of smallest eigenvalues and eigenvectors of the general eigenvalue problem.

Method: Subspace iteration and Sturm sequence checks are made for the lowest eigenvalues then an acceleration procedure, using Chebyshev polynomial properties and spectral shifts, is used to quickly iterate to an accurate eigenvalue. Eigenvectors are also determined by subspace iteration.

Language: FORTRAN

Hardware: HAITAC 8700/8800, IBM

Developer: Y. Yamamoto and H. Ohtsubo
Dept. of Naval Architecture
Faculty of Engineering
University of Tokyo
Bunkyo-ku, Tokyo, Japan

Availability: Program and users manual are available from the Computer Center, University of Tokyo.

Comments: This program modifies Bathe's subspace method (See SSPACE) by applying the acceleration procedure to the initial subspace iteration and Sturm sequence checks. The appropriate Chebyshev constants and origin shift are determined automatically in the program. The acceleration procedure reduces the computational time by approximately 75 per cent. Program is applicable to both free vibration and buckling analyses.

Eigenproblem Solution of Supersonic Panel Flutter Problem (SUPFLUT) [39]

Capability: Calculation of the solution to the supersonic panel flutter eigenvalue problem and determination of flutter boundary occuring at any point in the eigenspectrum.

Method: Eigenvalue problem is reduced to general form then the eigenvalues and eigenvectors are determined by Sturm sequence method, bisection and inverse iteration.

Language: FORTRAN

Hardware: UNIVAC

Developer: K. K. Gupta
M/S 144-218
Jet Propulsion Laboratory
4800 Oak Grove Dr.
Pasadena, CA 91103

Availability: Program is available from COSMIC. Users manual available from developer.

Comments: Input matrices for the matrix equation $(K + K_G + \lambda A + w^2 M + wD)q=0$ are the elastic stiffness matrix K, geometric stiffness matrix K_G, aerodynamic load matrix A, mass matrix M and aerodynamic damping matrix D. Program exploits matrix bandedness and sparsity characteristics. Flutter boundary may be determined for any point in the eigenspectrum. The routine reduces the matrix equation to general eigenvalue form and solves for the complex roots and vectors.

Structural Analysis of General Shells (STAGSB) [40, 41]

Capability: Structural analysis of arbitrary shells that may exhibit inelastic material behavior.
Method: Eigenvalues and eigenvectors are computed by the inverse iteration method with reduced subspace and spectral shifts.
Language: FORTRAN
Hardware: CDC, UNIVAC
Developer: B. O. Almroth and F. A. Brogan
Lockheed Missiles and Space Company
Palo Alto, CA 94304
Availability: Program and users manual are available from developer at a cost of $2,000.
Comments: Eigenvalue extraction routine is imbedded in program STAGSB.

Eigensystem Programs for General Real Matrices (VARAH1) [42, 43]

Capability: Calculation of the eigenvalues and eigenvectors of the standard eigenvalue problem (VARAH1). Refinement of the computed eigenvalues and eigenvectors and determination of the error bounds (VARAH2).
Method: Matrix is first transformed to upper Hessenberg form then the eigenvalues are computed through double QR iteration. Eigenvectors are computed through inverse iteration.
Language: FORTRAN
Hardware: CDC
Developer: D. A. Gignac
Dept. of the Navy
Naval Ship Research and Development Center
Washington, D.C. 20034
Availability: Unknown
Comments: Complete eigensystem solution involves two programs, VARAH1 and VARAH2, which are FORTRAN adaptations and extensions of J. M. Varah's ALGOL programs, EIGENVALUESANDEIGENVECTORS and EIGENSYSTEMBOUNDS [54]. Program VARAH1 reduces the general eigenvalue problem to standard form and computes the eigenvalues and eigenvectors of the resulting matrix.

DYNA

Capability: Calculation of eigenvalues and eigenvectors of the general eigenvalue problem.
Method: Eigenvalues and eigenvectors are computed through the use of the Inverse Power Method with determinant and origin shift.
Language: FORTRAN
Hardware: CDC
Developer: H. A. Kamel
Aerospace and Mechanical Engineering Department
The University of Arizona
Tucson, Arizona 85721
Availability: Program is available from developer. User's manual is not available.

Comments: Mass matrix may be input in consistent or lumped form. Matrices are stored in core in sparse representation.

PLFREQ [46]

Capability: Calculation of nodal frequencies for plate vibration with clamped or simply supported boundaries.
Method: Rayleigh-Ritz equations are solved to form the standard eigenvalue problem. The eigenvalues are obtained through Jacobi iteration using IBM SHARE Library routine EIGEN [27].
Language: FORTRAN
Hardware: IBM
Developers: Ralph C. Leibowitz (Code 194)
Delores R. Wallace (Code 1844)
Naval Ship Research and Development Center
Bethesda, MD 20084
Availability: Program is available from the developers.

SUNFRE [46]

Capability: Calculation of eigenvalues for plate vibration with clamped or simply supported boundaries.
Method: Rayleigh-Ritz equations are solved by Gaussian quadrature and the standard eigenvalue matrix is formed. The eigenvalues are computed by Jacobi iteration using IBM SHARE Library routine EIGEN [27].
Language: FORTRAN
Hardware: IBM
Developers: Delores R. Wallace
Code 1844
Naval Ship Research & Development Center
Bethesda, MD 20084
B. C. Sun
Newark College of Engr.
Newark, N.J.
Availability: Program and Users manual [46] are available from Ralph Leibowitz, Code 194, Naval Ship Research and Development Center, Bethesda, MD.

There are several eigenvalue extraction routines that the authors know to be in use but for which complete information is not available. They are listed in abbreviated form in the following descriptions:

Bifurcation Buckling Analysis (BALOR) [44]

Capability: Finite Difference, one-dimensional, linear, static, bifurcation buckling analysis due to asymmetric loading of shell structures.
Method: Eigenvalues determined by inverse iteration.
Developer: R. E. Fulton
NASA/Langley Research Center
Newport News, VA 23600
Comments: Eigenvalue routine is included in the buckling analysis program, BALOR. Eigenvalue routine contains spectral shift capability so that all eigenvalues can be computed, if desired.

Eigenvector of a Complex Matrix (CXVEC) [45]

Capability: Eigenvalue and eigenvector extraction for complex matrices.

Developer: W. L. Frank
Space Technology Laboratories, Inc.
Redondo Beach, CA 92651
Availability: Unknown
Comments: The original version of this routine, developed by Mr. Frank, applied to real matrices. Personnel at the Naval Weapons Center, China Lake, CA. later modified it for complex matrices. General eigenvalue problem must be transformed to standard form prior to use of this routine.

Normal Mode Computer Analysis of Structure (DYNASHOR) [45]

Capability: Program computes the frequency mode shape, participation factor, effective mass and weight, (mass x mode shape) squared and (weight x mode shape x participation factor) for each mode of oscillation.
Language: FORTRAN
Hardware: IBM, CDC
Developer: J. Avila
DTMB (Code 247)
Philadelphia Naval Shipyard
Philadelphia, PA 19101
Comments: Original program, written by developer for an IBM 7090 computer, has been modified for use on a CDC 1604.

Eigenvalues of Complex Matrices (EIG4) [45]

Capability: Calculation of the eigenvalues of the general complex eigenvalue problem.
Method: Eigenvalues are computed through the use of the Laguerre method.
Limitations: Computes eigenvalues only.
Language: FORTRAN
Hardware: IBM
Developer: B. N. Parlett
AEC Computing and Applied Mathematics Center
New York University
New York City, NY 10003
Availability: Unknown
Comments: Program was originally developed by Mr. B. N. Parlett and later rewritten in FORTRAN IV by personnel of the Naval Weapons Center, China Lake, California. The general eigenvalue problem must be transformed to standard form prior to use of this routine.

Eigenvalues of Real Matrices (EIG5) [45]

Capability: Routine solves for the eigenvalues of a general real matrix.
Method: The general real matrix is reduced to lower Hessenberg form by similiarity transformations. Hyman's method is used to obtain the characteristic polynomial and its derivatives; then Laguerre's method is used to iterate for the eigenvalues.
Limitations: Computes eigenvalues only.
Language: FORTRAN
Hardware: IBM
Developer: B. N. Parlett
AEC Computing and Applied Mathematics Center
New York University
New York City, NY 10003
Availability: Unknown
Comments: See comments for EIG4

Eigenvalues - Eigenvectors of a Real Symmetric Matrix (JACB) [45]

Capability: Routine solves for the eigenvalues and eigenvectors of a real, symmetric matrix.
Method: The Jacobi method is used to reduce the matrix to a diagonal matrix of eigenvalues. The eigenvectors may be computed by the Jacobi method if the user desires. If computed, the eigenvectors are normalized.
Language: FORTRAN
Hardware: IBM
Developer: F. J. Corbato
MIT Technology Computing Laboratory
Cambridge, MA 02139
Availability: Unknown
Comments: Program was originally written by Mr. Corbato then later rewritten in FORTRAN IV by personnel at the Naval Weapons Center, China Lake, CA. The general eigenvalue problem must be transformed to standard form prior to use of the program.

Eigenvalues - Eigenvectors of a Real, Symmetric Matrix (MLEW) [45]

Capability: Program computes the eigenvalues and eigenvectors of a real symmetric matrix.
Method: Householder's method is used to reduce the matrix to tridiagonal form. The eigenvalues are isolated through Sturm sequence use and the eigenvectors are found by using Wilkinson's method.
Language: FAP, FORTRAN
Hardware: IBM
Developer: S. Greenspan
AEC Computing and Applied Mathematics Center
New York University
New York City, NY 10003
Availability: Unknown
Comments: Program was originally written by Mr. Greenspan then later translated to Fortran IV by personnel of the Naval Weapons Center, China Lake, California. The general eigenvalue problem must be transformed to standard form prior to program use.

Eigenvector of a Real Matrix (RLVEL) [45]

Capability: Routine calculates the eigenvectors of a general real matrix.
Method: Gaussian elimination and a variation of Wilkinson's method are used to determine the eigenvectors.
Language: FORTRAN
Hardware: IBM
Developer: W. L. Frank
Space Technology Laboratories Inc.
Redondo Beach, CA 92651
Availability: Unknown
Comments: Program was originally written by Mr. Frank then later translated into FORTRAN IV by personnel of the Naval Weapons Center, China Lake, CA.

Modal Vibration of Shells of Revolution (VALOR) [44]

Capability: Vibration analysis of shells of revolution using modal analysis method.
Method: Eigenvalues determined by inverse iteration.
Developer: R. E. Fulton
NASA/Langley Research Center
Newport News, VA 23600

Comments: Program, VALOR, has a free vibration analysis capability and contains the eigenvalue and eigenvector extraction routines. The inverse iteration procedure includes a spectral shift capability so that all eigenvalues and eigenvectors can be calculated.

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Damping

Frederick C. Nelson and Rober Greif

Tufts University

INTRODUCTION

The purpose of this chapter is to survey and assess the ways in which damping has been incorporated into general purpose shock and vibration computer programs. The authors believe that a review of this topic will be useful because they have encountered confusion over the various types of damping models which are available, and confusion over the compromises which must be made between physical reality and mathematical convenience in the choice of a damping model.

The chapter will begin with a review of damping forces followed by a comprehensive discussion of damping in multi-degree-of-freedom systems. It is hoped that these sections will clarify the nature of damping mechanisms and illustrate the trade-offs among mechanisms. The final section of this chapter reports the results of a survey aimed at establishing which damping hypotheses are available in various computer programs and the relative success of these programs in solving problems of damped response of discrete, multi-degree-of-freedom models. In addition, this final section will suggest areas in which future research could be fruitfully directed.

THE NATURE OF DAMPING FORCES

Oscillatory mechanical systems have a reservoir of inertial energy, a reservoir of elastic energy and a means of dissipating energy. The forces associated with inertia and elasticity can be calculated by formulas from theoretical mechanics; no recourse is necessary to experiment or experience. This is not the case for damping forces. In this sense, damping is irrational and requires the analyst to adopt a hypothesis before calculating the damping force or the energy dissipated per cycle. Also, the motivation for the hypothesis about damping can vary: the motivation can be primarily physical as in the cases of viscous, hysteretic, or Coulomb damping; or primarily to achieve a simple mathematical formulation as in the cases of proportional damping, equivalent viscous modal damping, or complex stiffness. The challenge for each particular problem is to strike the appropriate balance between physical reality and mathematical tractability.

Since there is a great deal of confusion over the various types of damping hypotheses and their associated terminology, a description of the more common choices will be given first. For clarity, the discussion will be limited to a damped, single-degree-of-freedom oscillator; additional discussion can be found in [1], [2] and [3]. Multi-degree-of-freedom systems are treated in subsequent sections of this chapter.

Viscous Damping

In linear viscous damping, energy is dissipated by laminar fluid friction. An example is energy dissipation due to flow of a Newtonian fluid between a tightly fitting piston and cylinder, i.e., the classical dashpot. The damping force is proportional to the relative velocity and in antiphase with this velocity. For sinusoidal response of a single-degree-of-freedom system we therefore have

$$\underline{F} = - c\dot{\underline{X}} \tag{1}$$

where

$\underline{F}$ is the damping phasor

$\dot{\underline{X}}$ is the velocity phasor

c is the damping coefficient.

Introducing the damping ratio, β, defined as

$$\beta = \frac{c}{c_{cr}} = \frac{c}{2m\omega}$$

where

c_{cr} is the critical damping ratio

m is the mass

ω is the undamped natural (circular) frequency,

Eq. (1) becomes

$$\underline{F} = -2m\omega\beta\dot{\underline{X}} \tag{2}$$

Linear viscous damping is mathematically convenient and, when laminar fluid flow is present, physically reasonable. Its use for the damping of metals or plastics is based more on its mathematical convenience than its physical reality.

The energy dissipation per cycle (D) associated with linear viscous damping can be shown to be

$$D = c\,\Omega\,\pi\,X^2 \tag{3}$$

where

X is the magnitude of the displacement phasor, $\underline{X}$

Ω is the forcing frequency

Equation (3) implies that for viscous damping, $D \propto \Omega$.

Hysteretic Damping

Hysteretic damping is sometimes referred to as material damping. The energy dissipation is due to various forms of internal friction as well as small-scale thermal and electrical effects. More details can be found in [4] and [5]. On a macroscopic scale, the damping force is usually approximated by a force which is in antiphase with the velocity and proportional to the displacement. For sinusoidal response of a single-degree-of-freedom

oscillator, one then has

$$\underline{F} = - i\underline{X}$$

where i is $\sqrt{-1}$. If the constant of proportionality is taken as ηk where k is the oscillator stiffness and η is defined as the loss factor, we may rewrite this as

$$\underline{F} = - ik\eta\underline{X} \tag{4}$$

Since for sinusoidal response $\dot{\underline{X}} = i\Omega\underline{X}$

$$\underline{F} = - \frac{k\eta}{\Omega}\dot{\underline{X}} \tag{5}$$

Comparing Eq. (5) to Eq. (1) we may define the equivalent viscous damping coefficient as

$$c_{eq} = \frac{k\eta}{\Omega} \tag{6}$$

Comparing Eq. (5) to Eq. (2) we may write a relation between loss factor and damping ratio

$$\eta = 2\left(\frac{\Omega}{\omega}\right)\beta \tag{7}$$

where $\omega^2 = k/m$. Note that at resonance ($\Omega=\omega$), Eq. (7) gives $\eta = 2\beta$.

The energy dissipated per cycle associated with this formulation of hysteretic damping is

$$D = k\eta\pi X^2 \tag{8}$$

and implies that D is independent of Ω . It was this experimental observation that led Kimball and Lovell to suggest the hysteretic damping hypothesis in 1927 [6].

The use of the above model of hysteretic damping for free vibration or multiple-frequency forcing (such as force pulses) is open to question ([2] and [7]).

It is helpful to distinguish between hysteretic damping as described above, which occurs on the microscopic scale, and energy dissipation due to the generation of elastic-plastic hysteresis loops on a macroscopic scale. It is certainly debatable whether macroscopic hysteresis should be considered as damping or as nonlinear restoring force associated with elastic-plastic behavior; both affect the response in about the same way and are roughly additive in their effects. One's attitude depends on the context of the problem being studied. We choose to emphasize the nonlinear nature of macroscopic hysteresis and exclude it from the present discussion which is focused principally on linear damping mechanisms.

Energy will also be dissipated if the material is time dependent. For a discussion of this phenomenon see the chapter by Yamada in this volume.

Coulomb Damping

The energy dissipation in this case is due to dry friction between sliding surfaces. The normal force (N) and frictional force (F) between the sliding surfaces are related by Coulomb's law

$$F = \mu N$$

The direction of F is opposite to the relative velocity. The damping force is thus a nonlinear function of the velocity and may be represented mathematically by means of the signum function[1]

$$F(t) = -\mu N \operatorname{sgn}[\dot{x}(t)] \tag{9}$$

It can be shown that for sinusoidal response

$$c_{eq} = \frac{4\mu N}{\pi \Omega X} \tag{10}$$

and

$$D = 4\mu N X \tag{11}$$

This and other forms of nonlinear damping forces (in particular, $F \propto (\dot{x})^n$) are discussed in [8] and [9].

Coulomb damping is often used to represent the energy dissipation per cycle in riveted or bolted joints. However, care must be used since the dependence of D on μ for a fully slipped joint, Eq. (11), is not the same as that for a partially slipped joint. In a partially slipped joint, D is inversely proportional to μ, see [10].

The above three damping hypotheses are based on physical conceptions of damping processes. The choice of one physical damping process over another often makes a neglible difference in the dynamic response of a system, see [11]. For this reason, damping hypotheses have been introduced which emphasize mathematical convenience more than physical process and we shall discuss these next.

Proportional Damping

Proportional damping is sometimes called Rayleigh damping. For a single-degree-of-freedom system the damping force is assumed to be a linear function of the mass and stiffness, i.e.

$$F(t) = -(a_0 m + a_1 k)\dot{x}(t) \tag{12}$$

where $\dot{x}(t)$ is the velocity.

---

[1] $\operatorname{sgn}(\dot{x}) = +1$ for $\dot{x} > 0$; $\operatorname{sgn}(\dot{x}) = -1$ for $\dot{x} < 0$;
$\operatorname{sgn}(\dot{x}) = 0$ for $\dot{x} = 0$

This can be derived by assuming a dissipation function which is quadratic in velocity:

$$R = \frac{1}{2} a_o m\dot{x}^2 + \frac{1}{2} a_1 k\dot{x}^2$$
$$= \frac{1}{2} m\dot{x}^2[a_o + a_1\omega^2] \tag{13}$$

The damping force is given by $F = -dR/d\dot{x}$. The dissipation function was first introduced by Rayleigh, [12]; the term proportional is perhaps derived from the proportionality between R and the system kinetic energy.

Proportional damping is popular in multi-degree-of-freedom systems because it leads to a damping matrix which is diagonalized by the same transformation which diagonalizes the mass and stiffness matrices. This is discussed in a later section of this chapter.

Equivalent Viscous Modal Damping

Modal analysis is a common method of solving multi-degree-of-freedom systems. For damped systems this method requires that the governing equations of motion be written in an uncoupled form for each mode. The simplest way of accomplishing this is to replace the physical damping process by equivalent viscous dampers, one for each normal mode of the system.

Of course, one needs a rationale for selecting an equivalent viscous damping ratio for each mode. This can be as simple as choosing them from experience or as involved as measuring the frequency dependence of the damping of a particular mode shape and then choosing the equivalent viscous damping to match the actual damping at the natural frequency of that mode.

Guidance on representative values of equivalent viscous modal damping ratios is not easy to find in the literature. For the case of nuclear reactor structures [13] is helpful and, for high rise buildings, [14] can be consulted.

Complex Stiffness

Using Eq. (4) for the damping force, the equation of motion of a single-degree-of-freedom system subjected to harmonic forcing can be written

$$m\ddot{\underline{X}} + ik\eta\underline{X} + k\underline{X} = \underline{F} \tag{14}$$

where $(\underline{\ })$ denotes the complex phasor. Clearly, Eq. (14) can be written

$$m\ddot{\underline{X}} + k(1 + i\eta)\,\underline{X} = \underline{F} \tag{15}$$

or

$$m\ddot{\underline{X}} + \underline{k}\,\underline{X} = \underline{F}$$

where $\underline{k}$ is the complex stiffness. The physical solution to Eq. (15) is the real part of $\underline{X}e^{i\Omega t}$.

This method of representing the dynamic stiffness and damping is widely used for metals and polymers. For example, see [1] and [15]. Since it is a mathematical variant of hysteretic damping, it also must be used with caution in the cases of free vibration and multi-frequency forcing, especially when the forcing function has a prominent zero frequency component.

In terms of materials, it is the complex modulus which is of interest. One writes

$$\underline{E} = E_1 + iE_2$$

$$\eta_M = E_2/E_1$$

where

E_1 is the storage modulus in direct stress

E_2 is the loss modulus in direct stress

η_M is the material loss factor in direct stress.

These material properties are functions of temperature and frequency. For values of E_1 and η_M for some materials and their dependence on temperature and frequency, see [15] and [16].

The hypothesis of a complex stiffness or modulus provides a very simple way of solving damped vibration problems analytically. Namely, first solve the undamped problem, e.g., for single-degree-of-freedom sinusoidally forced oscillator

$$x(t) = \frac{F\cos\Omega t}{k-m\Omega^2}$$

Then obtain the damped response by replacing k with $\underline{k} = k(1 + i\eta)$ and taking the real part:

$$x(t) = \mathrm{Re}\left[\frac{Fe^{i\Omega t}}{\underline{k}-m\Omega^2}\right]$$

$$= \frac{F\cos(\Omega t + \phi)}{\sqrt{(k-m\Omega^2)^2 + (k\eta)^2}}$$

As is shown in [17], this method is equivalent to neglecting the imaginary part of the complex eigenvector associated with a damped system. In other words, one assumes that the eigenvector of the undamped mode is the same as that for the damped mode. The error introduced by this technique should be small unless the damping is large.

MATHEMATICAL ANALYSIS OF MULTI-DEGREE-OF-FREEDOM SYSTEMS

Modern problems in shock and vibration involve systems with many degrees of freedom and eventual computer solution. For these problems, the concept of damping for a single-degree-of-freedom, formulated in the previous section, must be expanded to the concept of a damping matrix. Quite often this damping matrix is not constructed directly from the material damping properties of the various components but is developed for mathematical convenience. Because

of this situation, the engineer is often required to exercise considerable judgment in formulating and solving problems with damping. In this section a mathematical analysis of the linear damping problem is given and in the next section a numerical example is worked out in detail.

The basic equation discussed in this work is

$$[m]\{\ddot{x}\} + [c]\{\dot{x}\} + [k]\{x\} = \{f(t)\} \tag{16}$$

in which [m], [c], [k] denote the n x n mass, damping and stiffness matrices, respectively, while {x} and {f(t)} are the n x 1 displacement and forcing function column matrices. Equation (16) represents n linear, coupled equations for the n degrees of freedom. In order to derive this equation, it is assumed that the analyst has modeled the original system to the accuracy desired. Typical modeling aids are the finite difference and finite element methods.

A common approach to the solution of this problem involves modal superposition. If the system is lightly damped, the eigenvalues and eigenvectors are determined from the solution to the free vibration undamped system in the form

$$[k]\{\phi\} = \omega^2[m]\{\phi\} \tag{17}$$

Equation (17) may be transformed to the canonical form

$$[A]\{\psi\} = \omega^2\{\psi\} \tag{18}$$

where [A] is symmetric. An effective way of performing this transformation is the Cholesky decomposition. The eigenvalues, ω_i, can then be determined by any of a number of techniques, such as the Jacobi method, the Givens-Householder method or the Householder-QR method. This latter method is particularly popular. A discussion of all these methods is given in [18]. The eigenvector $\{\phi\}_{(i)}$ associated with the ith mode is obtained by a transformation involving $\{\psi\}_{(i)}$. Due to the homogeneous nature of the eigenvalue problem, the elements of $\{\phi\}_{(i)}$ can only be determined to within an arbitrary multiplicative constant. For convenience of solution, a weighted modal matrix $[\phi]$ is constructed from the modal column matrices $\{\phi\}_{(i)}$ by dividing each column by the square root of the generalized mass for that mode M_i,

$$[\phi] = \left[\frac{1}{\sqrt{M_1}}\{\phi\}_{(1)} \;\vdots\; \frac{1}{\sqrt{M_2}}\{\phi\}_{(2)} \;\vdots\; \cdots \;\vdots\; \frac{1}{\sqrt{M_n}}\{\phi\}_{(n)}\right] \tag{19}$$

$$M_i = \{\phi\}_{(i)}^T [m]\{\phi\}_{(i)}$$

The orthogonality relationships then become

$$[\phi]^T[m][\phi] = [\,I\,], \quad [\phi]^T[k][\phi] = [\omega^2] \tag{20}$$

The uncoupling of the damped equations of motion is done by first transforming from the physical coordinates {x} to the generalized coordinates {q},

$$\{x\} = [\phi]\{q\} \tag{21}$$

Substitution into Eq. (16) and premultiplication by $[\phi]^T$ then yields

$$\{\ddot{q}\} + [\phi]^T[c][\phi]\{\dot{q}\} + \lceil\omega^2\rfloor\{q\} = [\phi]^T\{f(t)\} \tag{22}$$

The crucial step involves the diagonalization of the damping term $[\phi]^T[c][\phi]$. If this diagonalization cannot be done, there are other approximate and exact methods that may be used for the solution. These are discussed in a later section of this chapter.

Assuming that the diagonalization can be made by an orthogonality relationship of the form

$$[\phi]^T[c][\phi] = \lceil\tilde{c}\rfloor \tag{23}$$

then Eq. (22) becomes

$$\{\ddot{q}\} + \lceil\tilde{c}\rfloor\{\dot{q}\} + \lceil\omega^2\rfloor\{q\} = [\phi]^T\{f(t)\} \tag{24}$$

It is convenient at this stage to relate this equation to the standard form of the single-degree-of-freedom system [19]

$$\ddot{q} + 2\beta\omega\dot{q} + \omega^2 q = p(t) \tag{25}$$

in which β is the percent of critical damping (or damping ratio). The individual diagonal elements of $\lceil\tilde{c}\rfloor$ may then be written as

$$\tilde{c}_i = 2\beta_i\omega_i \qquad i = 1,2,\ldots,n \tag{26}$$

There has been a great deal of theoretical work done to ensure that uncoupled equations of this form are indeed obtained. Using the results of Eq. (20) it is obvious that uncoupling can be obtained if the damping matrix is proportional to the stiffness and/or mass matrix

$$[c] = a_o[m] + a_1[k] \tag{27}$$

This particular form of damping is the matrix form of proportional damping or Rayleigh damping discussed in the first section. Following the previous uncoupling procedure, Eq. (24) becomes

$$\{\ddot{q}\} + (a_o[\,I\,] + a_1\lceil\omega^2\rfloor)\{\dot{q}\} + \lceil\omega^2\rfloor\{q\} = [\phi]^T\{f(t)\} \tag{28}$$

Comparing this to an equivalent uncoupled case

$$\{\ddot{q}\} + 2\lceil\beta\omega\rfloor\{\dot{q}\} + \lceil\omega^2\rfloor\{q\} = [\phi]^T f(t) \tag{29}$$

it is found by equating the $\{\dot{q}\}$ coefficients that

$$2[\beta\omega] = a_o[I] + a_1[\omega^2] \tag{30}$$

Thus for any mode i, the damping ratio may be written

$$\beta_i = \frac{a_o}{2\omega_i} + \frac{a_1\omega_i}{2} \qquad i = 1,\ldots,n \tag{31}$$

It should be noted that since there are only two arbitrary constants a_o, a_1 the damping ratio, β_i, can only be specified in two modes. Values of β_i for other modes are then fixed by Eq. (31). The modes used to calculate a_o and a_1 are usually determined from the physics of the problem. Choosing the first two modes for this purpose (as is often done) tends to filter out the effects of the high frequency components.

A relationship for [c] that is more general than Eq. (27), but which retains the crucial orthogonality condition, was derived by Caughey in 1960, [20]. A convenient form of this Caughey series is given in [21]

$$\begin{aligned}[c] &= [m]\sum_{b=0}^{n-1} a_b([m]^{-1}[k])^b \\ &= a_o[m] + a_1[k] + \ldots\end{aligned} \tag{32}$$

Using the orthogonality relationship from Eq. (23) gives

$$[\tilde{c}] = \begin{bmatrix} \sum_{b=0}^{n-1} a_b\omega^{2b} \end{bmatrix} \tag{33}$$

From Eq. (26) it then follows that

$$\beta_i = \frac{1}{2}\left(\frac{a_o}{\omega_i} + a_1\omega_i + a_2\omega_i^3 + \ldots + a_{n-1}\omega_i^{2n-3}\right) \quad i = 1,2,..n \tag{34}$$

It is interesting to note that the first two terms of this series correspond to the special case of Rayleigh damping, Eq. (31). The damping ratios in all the modes can now be controlled, with the constants a_i appropriately determined from the solution of the simultaneous equations dictated by Eq. (34). This may lead to numerical difficulties at high frequency because of the large numerical values of the natural frequency terms. An alternative method that eliminates some of these numerical problems is discussed in [21]. It should be noted that quite commonly the damping ratio is only specified in the first k modes that are the most important in the solution. The constants in Eq. (34) corresponding to the remaining terms are then set equal to zero. The damping ratios in these modes $i = k + 1,\ldots n$ are not zero and are still calculated from Eq. (34). Once again, this will tend to filter out the effects of the high frequency components.

If the damping matrix is not diagonalized by the transformation of

Eq. (23), then the damping matrix is termed nonproportional and the transformed equations of motion (22) remain coupled. Because of the desirability of using modal superposition based on the vibration characteristics of the undamped system, a number of engineering approximations have been developed. The simplest approach is to perform the transformation of Eq. (23) and neglect the off-diagonal terms

$$\lceil \tilde{c} \rfloor = [\phi]^T[c][\phi] \quad \begin{matrix}\text{NEGLECT OFF-} \\ \text{DIAGONAL TERMS}\end{matrix} \tag{35}$$

It is of interest to consider the physical meaning of neglecting these off-diagonal terms. A term on the diagonal represents the damping force of a mode due to motion in that mode. A term off the diagonal is associated with the damping force on a mode due to motion in another mode, in effect coupling the modes. The usual case of small damping involves dominant terms along the diagonal and small terms off the diagonal. Therefore, by neglecting these off-diagonal terms it is implied that the damping is small enough to make the coupling a second order effect. Recently, the results of Thomson, Calkins and Caravani [11] have indicated that this technique is quite accurate for engineering purposes for the solution of specified lumped mass systems to sinusoidal forcing functions.

Another technique that has been successful in dealing with nonproportional damping matrices involves comparing the steady state sinusoidal response of the actual system containing the nonproportional damping matrix to the response of the system with an assumed diagonal damping matrix. Typically this comparison is made at the various peak amplitudes of the response. The equivalent modal damping ratio can then be found by iterative solution of the resulting simultaneous algebraic equations as discussed by Tsai in [22]. Although this technique is apparently more accurate [11] than the preceding method of simply ignoring the off-diagonal terms in the transformed damping matrix, it does have some obvious limitations. First of all, it is necessary to solve the coupled problem, which may be a time-consuming task. Secondly, there is the question of which location in the system to use for matching the coupled and modal solutions. In [22] Tsai suggests the location that is most sensitive to the damping value; for ground motion input to an idealized building, it is suggested that the top mass be used for comparison purposes. Finally, it would appear that the method is most efficient for natural frequencies that are widely spaced.

Another method that is useful for nonproportional problems is the strain energy weighted modal rule first proposed by Biggs [23]. Although partly derived on intuitive grounds, the method is related to the neglect of the off-diagonal terms expressed in Eq. (35). The equivalent modal damping ratio in the ith mode, β_i, is expressed in terms of the critical damping ratio for each element of the system with a weighting factor in the form of the maximum elastic strain energy of each element j in the ith mode of vibration $(SE)^{(i)}_{j_{max}}$. For a system in which all the damping is of a hysteretic type, the equivalent modal damping ratio may be conveniently written in terms of the loss factor for each element η_j,

$$\beta_i = \frac{\sum_j (\eta_j/2)(SE)^{(i)}_{j_{max}}}{\sum_j (SE)^{(i)}_{j_{max}}} \tag{36}$$

It is interesting to note that if all the elements have a constant loss factor, η_j = constant, then the modal damping ratio β_i is a constant for all the modes of vibration. In [24], dealing with a soil-structure interaction problem, this strain energy weighted technique is extended to include the effects of hysteretic and viscous damping elements acting simultaneously.

The previous paragraphs describe approximate methods for uncoupling the equation of motion when the damping matrix is nonproportional. The methods should be reasonably accurate if the damping is light. However, it is possible to exactly uncouple the equations of motion for nonproportional damping by admitting the complex eigenvalues and eigenvectors associated with the damped system. One adds to Eq. (16) the trivial equation $[m]\{\dot{x}\} - [m]\{\dot{x}\} = 0$, [25], and partitions these to form

$$[A]\{\dot{y}\} + [B]\{y\} = \{F(t)\} \tag{37}$$

where

$$[A] = \begin{bmatrix} [0] & [m] \\ [m] & [c] \end{bmatrix} \quad ; \quad [B] = \begin{bmatrix} -[m] & [0] \\ [0] & [k] \end{bmatrix}$$

$$\{y\} = \begin{Bmatrix} \{\dot{x}\} \\ \{x\} \end{Bmatrix} \quad ; \{F(t)\} = \begin{Bmatrix} \{0\} \\ \{f(t)\} \end{Bmatrix} \tag{38}$$

The standard eigenvalue problem is obtained by setting $\{f(t)\} = 0$ and introducing $\{y\} = \{y_i\} \exp \lambda_i t$,

$$\begin{bmatrix} -[m] & [0] \\ [0] & [k] \end{bmatrix} \begin{Bmatrix} \lambda_i \{x\}_i \\ \{x\}_i \end{Bmatrix} = -\lambda_i \begin{bmatrix} [0] & [m] \\ [m] & [c] \end{bmatrix} \begin{Bmatrix} \lambda_i \{x\}_i \\ \{x\}_i \end{Bmatrix} \tag{39}$$

The matrices in Eq. (39) are symmetric but not positive-definite and, in general, the eigenvalues and eigenvectors will be complex. There are 2n eigenvalues and eigenvectors both of which occur in conjugate pairs. If a modal matrix is formed from the eigenvectors of Eq. (39), it provides a coordinate transformation of the form of Eq. (21) which uncouples the damped equation of motion, [26]. It then follows that the equations of motion for a forced vibration problem may be solved by modal superposition techniques. Although this uncoupling technique for nonproportional damping is exact, it is not often used. For lightly damped systems, the slight additional accuracy of this exact procedure is difficult to justify. However, if the damping is heavy or if the damping varies widely in a system, the gains in accuracy may be worth the added complexity of the analysis. Another factor to be considered is the large uncertainty which is usually associated with damping values for structures and equipment.

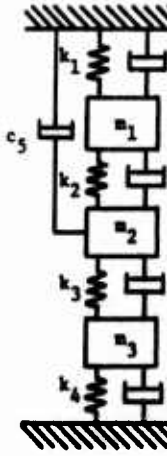
So far, most of this discussion has been based on uncoupling the equations of motion with the solution then proceeding by modal superposition. However, an alternative and widely used method is direct numerical integration of the equations of motion. Two recent reviews of the relative advantages and disadvantages of these two transient analysis techniques are given

by Belytschko, [27], and Chopra, [28]. One obvious advantage of direct numerical integration is that the full damping matrix [c] can be utilized. The implicit and explicit integration methods used for step-by-step solution in the time domain are not hampered by the introduction of the damping terms. Although both modal superposition and direct numerical integration may be used to solve linear problems, for nonlinear problems the direct integration technique tends to be a more convenient and powerful tool since [k] and [c] may be altered at various stages of the numerical procedure. Among the general disadvantages of direct integration are that it essentially provides no information about the modal characteristics; also for problems in which the contribution from the first few modes predominate, the method tends to be relatively expensive to use.

There are certain classes of problems for which the damping terms are functions of frequency and it is not adequate to replace these terms by average values that are independent of frequency [7], [29]. For problems of this type it is useful to transform the equations to the frequency domain and then incorporate the correct relationship between damping and frequency. After solution in the frequency domain, an inverse Fourier transform is used to return to the time domain. Until recently, this procedure was not computationally efficient even when using discrete Fourier transforms. However, the development of the fast Fourier transform (FFT), an accurate and efficient algorithm for computing discrete Fourier transforms, has radically changed this entire situation, [30], [31]. Many special purpose FFT programs have been written to facilitate this procedure and the technique appears to be competitive with direct numerical time integration for certain problems.

NUMERICAL EXAMPLE

In order to provide more physical insight into the previous theoretical discussion the system shown in Fig. 1 will be investigated in detail. For identical mass of m lb-sec^2/ft and identical springs of stiffness k lb/ft, the following matrices can be derived



$$[m] = \begin{bmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{bmatrix} = m \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix}$$

$$[k] = \begin{bmatrix} (k_1+k_2) & -k_2 & 0 \\ -k_2 & (k_2+k_3) & -k_3 \\ 0 & -k_3 & (k_3+k_4) \end{bmatrix} = k \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

$$[c] = \begin{bmatrix} (c_1+c_2) & -c_2 & 0 \\ -c_2 & (c_2+c_3+c_5) & -c_3 \\ 0 & -c_3 & (c_3+c_4) \end{bmatrix} \quad ; \quad \{x\} = \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} \qquad (40)$$

Fig. 1: Three-Degree-of-Freedom System

Using standard methods to solve Eq. (17), the undamped free vibration characteristics of the system are

$$\omega_1 = 0.76537\sqrt{k/m}, \quad \omega_2 = 1.41421\sqrt{k/m}, \quad \omega_3 = 1.84776\sqrt{k/m}\ \frac{\text{rad}}{\text{sec}} \tag{41}$$

$$\{\phi\}_{(1)} = \begin{Bmatrix} .7071 \\ 1.0 \\ .7071 \end{Bmatrix}, \quad \{\phi\}_{(2)} = \begin{Bmatrix} 1.0 \\ 0 \\ -1.0 \end{Bmatrix}, \quad \{\phi\}_{(3)} = \begin{Bmatrix} .7071 \\ -1.0 \\ .7071 \end{Bmatrix}$$

Following Eq. (19) the weighted modal matrix is formed

$$[\phi] = \left[\begin{array}{cc:cc:cc} & .7071 & & 1.0 & & .7071 \\ \frac{1}{\sqrt{2m}} & 1.0 & \frac{1}{\sqrt{2m}} & 0 & \frac{1}{\sqrt{2m}} & -1.0 \\ & .7071 & & -1.0 & & .7071 \end{array}\right] \tag{42}$$

Several calculations will now be done illustrating methods of controlling damping in specified modes and the resulting effects in other modes. Yeh, [32], has presented several numerical calculations of damping ratio with application to nuclear reactor containment vessels. As an initial example, consider the case of proportional damping in the specialized form of $[c] = a_1[k]$. Then from Eq. (20) it follows that

$$[\phi]^T[c][\phi] = a_1 \lceil \omega^2 \rfloor \tag{43}$$

Comparison to the equivalent uncoupled system of Eq. (29)

$$\{\ddot{q}\} + 2\lceil \beta\omega \rfloor\{\dot{q}\} + \lceil \omega^2 \rfloor\{q\} = [\phi]^T\{f(t)\} \tag{44}$$

leads to

$$2\lceil \beta\omega \rfloor = a_1\lceil \omega^2 \rfloor$$

$$\beta_1 = \frac{a_1}{2}\omega_1, \quad \beta_2 = \frac{a_1}{2}\omega_2, \quad \beta_3 = \frac{a_1}{2}\omega_3 \tag{45}$$

This shows quite clearly the increase in modal damping ratio with natural frequency as predicted by Eq. (31) (for $a_0 = 0$). Since there is one arbitrary constant involved, a_1, damping can be specified in just one mode with the remaining values dictated by Eq. (45). Picking this to be 5% in the first mode leads to

$$a_1 = .13066\sqrt{m/k}$$

$$\beta_1 = .05 \ , \quad \beta_2 = .092 \ , \quad \beta_3 = .121$$

$$[c] = \sqrt{km} \begin{bmatrix} .26132 & -.13066 & 0 \\ -.13066 & .26132 & -.13066 \\ 0 & -.13066 & .26132 \end{bmatrix} \tag{46}$$

Comparison of this damping matrix with Eq. (40) leads to the following values of the component damping coefficients $c_1 = c_2 = c_3 = c_4 = .13066\sqrt{km}$, $c_5 = 0$. If, instead of fixing the damping in the first mode, the constant a_1 were chosen to fix the damping ratio at 5% in the second mode, then the following values are found

$$a_1 = .07071\sqrt{m/k}$$

$$\beta_1 = .027 \ , \quad \beta_2 = .05 \ , \quad \beta_3 = .065$$

$$[c] = \sqrt{km} \begin{bmatrix} .14142 & -.07071 & 0 \\ -.07071 & .14142 & -.07071 \\ 0 & -.07071 & .14142 \end{bmatrix} \tag{47}$$

Consider the same system with the dissipative mechanism associated with material or hysteretic damping. Assuming that each element has the same loss factor η associated with it, the equivalent damping matrix extension of Eq. (6) that is proportional to the stiffness matrix may be written in the form

$$[c] = \frac{\eta}{\Omega} [k] \tag{48}$$

This leads to

$$2[\beta\omega] = \frac{\eta}{\Omega} [\omega^2]$$

$$\beta_1 = \frac{\eta}{2}\frac{\omega_1}{\Omega} \ , \quad \beta_2 = \frac{\eta}{2}\frac{\omega_2}{\Omega} \ , \quad \beta_3 = \frac{\eta}{2}\frac{\omega_3}{\Omega} \tag{49}$$

For small damping the frequency Ω may be approximated by the value of the natural frequency ω_i associated with the ith mode [7]. Thus

$$\beta_1 = \beta_2 = \beta_3 = \eta/2 \tag{50}$$

This shows that if the damping in the system is light, uniform and hysteretic, the damping ratio is a constant for all the modes and is identical to the

component damping ratio at resonance. If the damping ratio is specified as 5% in the first mode, it remains 5% in all the modes. These results for hysteretic damping may also be deduced from the strain energy damping rule expressed in Eq. (36).

For the case of Rayleigh or proportional damping the damping matrix takes the form $[c] = a_0[m] + a_1[k]$. From Eq. (31) the relation between damping ratio and the constants a_0, a_1 is

$$\beta_i = \frac{a_0}{2\omega_i} + \frac{a_1\omega_i}{2} \qquad i = 1,2,3 \tag{51}$$

Assuming that the damping ratio is fixed at 5% in the first two modes, the constants a_0 and a_1 may be calculated from Eq. (51) as $a_0 = .04966\sqrt{k/m}$, $a_1 = .04588\sqrt{m/k}$. This leads to

$$\beta_1 = .05 \ , \quad \beta_2 = .05 \ , \quad \beta_3 = .056$$

$$[c] = \sqrt{km}\begin{bmatrix} .14142 & -.04588 & 0 \\ -.04588 & .14142 & -.04588 \\ 0 & -.04588 & .14142 \end{bmatrix} \tag{52}$$

It is interesting to note that the damping ratio in the third mode is quite close to the 5% value of the first two modes. Comparison of the damping matrices of Eq. (52) and Eq. (40) leads to $c_1 = c_4 = .09554\sqrt{km}$, $c_2 = c_3 = .04588\sqrt{km}$, $c_5 = .04966\sqrt{km}$. The damping matrices of Eq. (52) and Eq. (47) show a close comparison (particularly with respect to the dominant diagonal elements) even though in the latter case the damping was only fixed in the second mode. However, the damping ratios in the two cases differ importantly for the first mode. In addition, the damping component from m_2 to ground is zero from Eq. (47) while in the present proportional damping case it is found to have the value $c_5 = .04966\sqrt{km}$.

The damping ratio can be controlled in all three modes by using the Caughey series approach outlined in Eqs. (17)-(19). Fixing the damping ratio at 5% in each mode, the constants a_0, a_1, a_2 are calculated from the three simultaneous equations associated with

$$\beta_i = \frac{1}{2}\left(\frac{a_0}{\omega_i} + a_1\omega_i + a_2\omega_i^3\right) \qquad i = 1,2,3 \tag{53}$$

This produces $a_0 = .04335\sqrt{k/m}$, $a_1 = .05980\sqrt{m/k}$, $a_2 = -.00538(m/k)^{3/2}$. Reasonable comparison is expected for the approaches based on the Caughey series and proportional damping since this method led to a damping ratio in the third modes only 12% above the desired value of $\beta_3 = .05$. The actual damping matrix may now be found from

$$[c] = [m] \sum_{b=0}^{2} a_b ([m]^{-1}[k])^b$$

$$[c] = \sqrt{km} \begin{bmatrix} .13605 & -.03828 & -.00538 \\ -.03828 & .13067 & -.03828 \\ -.00538 & -.03828 & .13605 \end{bmatrix} \tag{54}$$

From comparison with Eq. (40), it is immediately apparent that the present matrix is no longer tri-diagonal; there now exists a component connecting m_1 to m_3 (see Fig. 1) with damping value $.00538\sqrt{km}$. For all the previous calculations this component had zero damping coefficient. However, in order to control damping in all three modes, a non-zero value for this far-coupled term is now required. The rest of the damping constants are $c_1 = c_4 = .09239\sqrt{km}$, $c_2 = c_3 = .03828\sqrt{km}$, $c_5 = .05411\sqrt{km}$. The diagonalization of the transformed damping matrix follows from Eq. (23)

$$[\phi]^T[c][\phi] = \sqrt{k/m} \begin{bmatrix} .0765 & & \\ & .1414 & \\ & & .1848 \end{bmatrix} \tag{55}$$

From Eqs. (41) and (44) it follows that $\beta_1 = \beta_2 = \beta_3 = 5\%$.

For non-proportional damping the transformed matrix is no longer diagonal. Typically non-proportional damping occurs when the damping matrix is composed directly from the individual damping values of the components. For example, consider the previous case involving the matrix of Eq. (54) but now arbitrarily double the value of the damping constant c_5 associated with the component from m_2 to ground. The damping matrix is now slightly different from that of Eq. (54) since the (2,2) element is altered, i.e.

$$[c] = \sqrt{km} \begin{bmatrix} .13605 & -.03828 & -.00538 \\ -.03828 & .18478 & -.03828 \\ -.00538 & -.03828 & .13605 \end{bmatrix} \tag{56}$$

The transformation of the damping matrix then leads to the non-diagonal form

$$[\phi]^T[c][\phi] = \sqrt{k/m} \begin{bmatrix} .1036 & 0 & -.0271 \\ 0 & .1414 & 0 \\ -.0271 & 0 & .2119 \end{bmatrix} \tag{57}$$

To continue with a solution based on modal superposition, the usual assumption as expressed by Eq. (35), is to neglect the off-diagonal terms and define equivalent modal damping values based on the diagonal terms of Eq. (57). For many different types of forcing functions of practical importance this technique would produce adequate answers for dynamic response since the off-diagonal terms are much smaller than the diagonal terms. The damping ratios

based on this approximation are $\beta_1 = .068$, $\beta_2 = .05$, $\beta_3 = .057$; the damping ratio in the second mode remains the same value as that found from the Caughey series approach leading to Eq. (55). This is expected on physical grounds since in the second mode of vibration the middle mass remains stationary. Therefore, increasing the damping of the dissipative element from m_2 to ground should have no influence on β_2. However, as expected, β_1 and β_3 are increased since extra damping has been added to the overall system.

It is of interest to investigate the form $[c]'$ of the approximate diagonalized damping matrix when it is transformed back to the original physical system. Letting $[\tilde{c}]$ be the diagonalized form of Eq. (57), it follows that

$$[c]' = ([\phi]^{-1})^T [\tilde{c}] [\phi]^{-1}$$

$$[c]' = \sqrt{km} \begin{bmatrix} .1496 & -.0383 & .0082 \\ -.0383 & .1577 & -.0383 \\ .0082 & -.0383 & .1496 \end{bmatrix} \tag{58}$$

The damping matrices of Eq. (56) and Eq. (58) differ, as expected. In particular, from Eq. (58) it follows that the damper between m_1 and m_3 now has a negative value of damping coefficient.

COMPUTER PROGRAMS

The authors surveyed both users and developers of general purpose computer programs in shock and vibration via a mailed questionnaire to ascertain the types of damping models which are available and the success of these programs in predicting the damped response of discrete, multi-degree-of-freedom systems. The results of this survey are reported below with most of the remarks limited to the damping aspects of the programs. Further general information on these programs is available in [27].

A major caveat of the reported comments is the confusion generated by simple labels. Many programs with the same acronym appear to have both public and private versions; accordingly, the capabilities and user reactions may vary depending on the version involved. In addition, a number of special purpose programs were reported but it was decided not to list all of these because of uncertainty about limitation of program availability and capability.

ANSYS - This program is available from Swanson Analysis Systems, Inc., 870 Pine View Drive, Elizabeth, PA 15037. It can accept discrete viscous, hysteretic and Coulomb dampers as well as proportional damping matrices and complex stiffness matrices. It has been successfully used for damped response under conditions of free vibration, harmonic forcing, force transients, seismic excitation, and random input. The method of solution is direct integration and both linear and nonlinear responses may be calculated. The program will compute equivalent modal damping parameters based on the supplied material damping properties.

ASKA II - This program is available from the Institut fuer Statik und Dynamik, Pfaffenwaldring 27, Stuttgart, 80, Germany. It accepts either proportional damping matrices or equivalent viscous modal damping. The method of solution is modal superposition. Successful solutions have been obtained for free vibration, harmonic and transient forcing, and seismic and random excitation.

ASTRE - This program is available from the Institut National des Sciences Appliquées, Laboratoire de Mécanique des Structures, Avenue Albert Einstein, Villeurbanne, France. It accepts a complex stiffness matrix and the solution is found by modal superposition. It has been successfully used to solve the response of thick structures damped by viscoelastic layers and subjected to harmonic forcing. The loss factor and storage modulus can be functions of frequency.

DAMP - This program is available from COSMIC. Any structure can be analyzed provided the stiffness, mass and damping matrices are computed from a finite element program. The method of solution is modal superposition. DAMP computes complex eigenvalues and eigenvectors. It has been successful in the solution of practical problems via complex frequencies and mode shapes.

KSHEL - This program is available from Prof. A. Kalnins, Department of Mechanical Engineering, Lehigh University, Bethlehem, PA 18015. It is available in two versions, KSHEL 1-D and KSHEL 3-D. Its principal use is shell dynamics. KSHEL 1-D is used for harmonic forcing and incorporates damping by means of a complex stiffness matrix. KSHEL 3-D accepts discrete viscous or hysteretic dampers and is used for free vibration or transient forcing functions. The damped transient analysis uses modal superposition based on the undamped modes. However, KSHEL 3-D can also compute the complex eigenvalues and eigenvectors. These programs are new but have been successful in cases of free vibration, harmonic forcing and general transient response.

MINI-ELAS - This program is available from Prof. S. Utku and/or Dr. I. B. Alpay, Department of Civil Engineering, Duke University, Durham, NC 27706. It uses modal superposition and equivalent viscous modal damping. It has been successful with random input.

MARC - This program is available from the CDC Cybernet Service and through MARC Analysis Research Corp., 105 Madway Street, Providence, RI. It will accept all the damping formats except complex stiffness and all forcing functions except random. The program provides the option of modal superposition or direct integration. Many forms of structures have been successfully analyzed including stiffened and layered shells for impact and transient loading. Acceptable comparisons with measured responses have been made for nonlinear systems.

NASTRAN - The public version of this program is available from COSMIC. If damped harmonic response is computed by modal superposition, one must use equivalent viscous modal damping. If damped harmonic response is computed by direct integration, one can use discrete viscous or hysteretic dampers or a complex stiffness matrix; the complex stiffness matrix can allow each finite element to have a different loss factor. Transient response with damping can be solved by direct integration or modal superposition; Coulomb damping can be used in conjunction with direct integration. NASTRAN can compute the complex eigenvalues and eigenvectors. In addition, it will accept a user-supplied damping matrix. Success was reported for problems involving periodic and shock pulse forcing functions.

SAMIS - This is a general purpose program available from COSMIC. The method of solution is modal superposition with equivalent viscous modal damping and/or discrete viscous dampers. SAMIS can compute complex eigenvalues and eigenvectors and success was reported in using these to solve a practical problem.

SAP IV - This program is available from the National Information Service Earthquake Engineering Computer Program Applications, Davis Hall, University of California, Berkeley, CA 94720. If modal superposition is used, proportional damping matrices or equivalent viscous modal damping are required. Solution can also be obtained by direct integration for nonproportional damping matrices. Success was reported on structures subjected to general force transients and seismic excitation.

STARDYNE - This program is available from the CDC Cybernet Service and from Dr. R. Rosen, Mechanics Research, Inc., 9841 Airport Blvd., Los Angeles, CA 90045. The methods of solution are direct integration or modal superposition. For the latter, proportional damping matrices or equivalent viscous modal damping values are required; for the former, discrete viscous dampers may be used. Success was reported for a wide variety of forcing functions including seismic and random. The program can use the Biggs strain energy weighting method to compute equivalent viscous modal damping for situations where different elements of the structure have different damping values.

STRUDL - This program is available from the ICES Users Group, Inc., Box 8243, Cranston, RI 02920. In addition, it exists in many private versions. It proceeds by modal superposition and accepts either proportional or equivalent viscous modal damping. Success was reported for free vibration, transient forcing and seismic excitation of damped systems.

Reviewing the above responses as a whole, several common features can be discerned. If modal superposition is a desirable solution method (e.g., the response is expected to be dominated by a few modes), equivalent viscous modal damping ratios or a proportional damping matrix are used. The use of modal superposition with complex eigenvalues and complex eigenvectors is less common.

If, on the other hand, discrete viscous or hysteretic dashpots are desired, a nonproportional damping matrix is generated and solution typically proceeds by step-by-step direct numerical integration. In those cases where the forcing is harmonic, several programs choose to incorporate damping via a complex stiffness matrix.

It appears from the survey that there are many general programs available for solving damped response problems; nevertheless, there are several areas in which additional research would be helpful. One of the main difficulties is ascertaining the correct damping values to be inserted into the program. It is clear that the establishment of a compendium of damping values for a wide variety of structures and materials would be a useful contribution. Further research is needed in modeling systems to account for different values of damping in different elements of the system. Further work is also needed for systems in which different types of damping mechanisms (e.g., viscous and hysteretic) are acting simultaneously. Recent work [33], has shown the utility of damped forced modes in the analysis of damped systems. These modes are essentially complex but they uncouple the equations of motion for arbitrary hysteretic damping. They are a natural way of solving for damped response by modal superposition. It would be feasible for some programs to incorporate this technique into their modal superposition routines.

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Inertia Matrices for Finite Elements

Vernon H. Neubert

The Pennsylvania State University

INTRODUCTION

The representation of the mass distribution of a structure has received considerable attention from the developers of some computer programs. The purpose of this chapter is to discuss some of the programs and the mass matrices used. Attention is given to those which are of particular interest because of schemes used. To represent elements with uniformly distributed mass, the two main choices are: a lumped mass, with a diagonal mass matrix; or a consistent mass, with a nondiagonal mass matrix. The chapter is divided into four parts: theoretical background, computer program capabilities, choice of program or method, and anticipated improvements. The chapter deals primarily with computer programs which were developed for structural analysis. Two programs for the calculation of inertial and damping contributions of a fluid are briefly discussed.

NOMENCLATURE

A = Cross-sectional area
$c = \sqrt{E/\rho}$ = Speed of longitudinal wave in bar
E = Young's modulus
I = Area moment of inertia
J = Mass moment of inertia
$[k]$ = Stiffness matrix
ℓ = Segment length; total length of bar or beam
$[m]$ = Mass matrix
N = Number of cells per wavelength [9]; Number of segments [18]
p = Fluid pressure
$R = [I/A]^{1/2}$ = radius of gyration
SEM = Shock effective mass
t = Time
$\{x\}$ = Displacement matrix for structure
α = Dimensionless parameter in mass moment of inertia J
$\beta = EIk'/GA\ell^2$
γ = Influence function
$\mu = A\rho$ = Mass per unit length
ρ = Mass density
τ = Time variable
$\{\phi\}$ = Modal vector
ω' = Circular natural frequency from finite element model
ω = Exact circular natural frequency

THEORETICAL BACKGROUND

Mass matrices may be needed to represent inertia as follows:

1. Rigid body inertia. A 6x6 matrix is needed to represent mass, moments of inertia, and products of inertia.
2. Centrifugal and gyroscopic loading.
3. Distributed inertia of a flexible body.
4. Accessions to mass of a structure due to motion of contacting fluid or soil.

The first two are part of classical mechanics and are not discussed here. Number 3. is of primary interest for the present discussion.

For elements of flexible bodies, Newton's laws would indicate that masses be lumped at the center of mass. This is not efficient because it would require the location of an additional degree of freedom or junction point at the center of mass, so it is most common to lump masses at the ends or corners of clements. In the 1950's the "direct stiffness" matrix method evolved from the pre-computer slope-deflection method. The concept that stiffnesses were forces due to displacements at element junction points was quickly extended by Archer [1] to the concept that masses were inertia forces due to accelerations. He used the term consistent mass matrix, the term consistent meaning that the same displacement function was used to develop the nondiagonal mass matrix as for the stiffness matrix. The form presented by Archer was:

$$[m]\{\ddot{x}\} + [k]\{x\} = \{0\} \tag{1}$$

At about the same time Leckie and Lindberg [2] presented the same matrix in a somewhat less useful form, that is, imbedded in a dynamic stiffness matrix

$$[k(\omega)] = [[k] - [m]\omega^2] \tag{2}$$

For later reference, the non-diagonal consistent mass matrix is derived in detail for a bar.

Consistent Mass Matrix for a Bar

Direct Derivation

The notation for a bar is shown in Fig. 1.

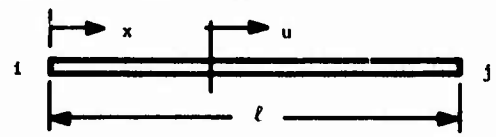


Fig. 1 Uniform Bar

Archer defined $\gamma_i(x)$ as the influence line for force at i. For the bar shown

$$\gamma_i = 1 - \frac{x}{\ell} \text{ and } \gamma_j = \frac{x}{\ell} \tag{3}$$

$$\text{Then } m_{ii} = \rho A \int_0^{\ell} (1 - \frac{x}{\ell})^2 \, dx \tag{4}$$

$$= \frac{\rho A \ell}{3}$$

$$m_{jj} = \rho A \int_0^{\ell} \left(\frac{x}{\ell}\right)^2 dx = \frac{\rho A \ell}{3} \tag{5}$$

$$\text{and } m_{ij} = \rho A \int_0^{\ell} \left(1 - \frac{x}{\ell}\right) \frac{x}{\ell} dx = \frac{\rho A \ell}{6} \tag{6}$$

The consistent mass matrix is:

$$[m] = \rho A \ell \begin{bmatrix} \frac{1}{3} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{3} \end{bmatrix} \tag{7}$$

From Series for Impedance

There are other ways to derive the terms in the consistent mass matrix. One way is to expand the impedance in an infinite series [3]. The exact, thin bar solution is obtained from the wave equation in one dimension.

$$E \frac{\partial^2 u}{\partial t^2} = \rho \frac{\partial^2 u}{\partial t^2} \tag{8}$$

$$\text{with } c^2 = \frac{E}{\rho} \tag{9}$$

Steady state sinusoidal motion is assumed. The boundary conditions are taken such that boundary motion is specified.

$$u(0,t) = a_o e^{i\omega t} \text{ and } u(\ell,t) = 0 \tag{10}$$

The solution of (8) subject to (10) is:

$$u(x,t) = \left[\cos \frac{\omega x}{c} - \cot \frac{\omega \ell}{c} \sin \frac{\omega x}{c}\right] a_o e^{i\omega t} \tag{11}$$

The force at any point x is:

$$AE \frac{\partial u}{\partial x}(x,t) = -\frac{AE\omega}{c} \left[\sin \frac{\omega x}{c} + \cot \frac{\omega \ell}{c} \cos \frac{\omega x}{c}\right] a_o e^{i\omega t} \tag{12}$$

The force is needed at the boundaries. At x = 0

$$AE \frac{\partial u}{\partial x}(0,t) = -\frac{AE\omega}{c} \left[\cot \frac{\omega \ell}{c}\right] a_o e^{i\omega t} \tag{13}$$

$$= \left(\frac{-AE}{\ell} + \frac{A\rho\ell}{3} \omega^2 + \frac{A\rho^2\ell^3}{45E} \omega^4 + \ldots\right) a_o e^{i\omega t} \tag{14}$$

and at $x = \ell$: $$AE \frac{\partial u}{\partial x}(\ell,t) = -\frac{AE\omega}{c}\left[\sin\frac{\omega\ell}{c}\right]^{-1} a_o e^{i\omega t} \tag{15}$$

$$= \left(-\frac{AE}{\ell} - \frac{A\rho\ell}{6}\omega^2 - \frac{7}{360}\frac{A\rho^2\ell^3}{E}\omega^4 - .\right) a_o e^{i\omega t} \tag{16}$$

The first term in each series is the static stiffness. The second term has dimensions of mass times acceleration. The masses are the same as Archer's. The third term is mixed.

Consistent Mass Matrix for a Beam

Archer gave the consistent mass matrix for Bernoulli-Euler beam segment:

$$[m]\{\ddot{x}\} = \frac{A\rho\ell}{420}\begin{bmatrix} 156 & -22\ell & 54 & 13\ell \\ -22\ell & 4\ell^2 & -13\ell & -3\ell^2 \\ 54 & -13\ell & 156 & 22\ell \\ 13\ell & -3\ell^2 & 22\ell & 4\ell^2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{\theta}_1 \\ \ddot{x}_2 \\ \ddot{\theta}_2 \end{Bmatrix} \tag{17}$$

It is not easy to have a physical understanding of the terms in the matrix. However, representation of rigid body motion is quickly checked for three different cases. The kinetic energy T is:

$$T = \frac{1}{2}\{\dot{x}\}^T[m]\{\dot{x}\} \tag{18}$$

Case 1. Translation $$\{\dot{x}\}^T = (\dot{x}\ 0\ \dot{x}\ 0) \tag{19}$$

$$T = \frac{1}{2}(A\rho\ell)\dot{x}^2 \tag{20}$$

This is expected for rigid body translation, since $A\rho\ell$ is the beam mass,

Case 2. Rotation about center of segment

$$\{\dot{x}\}^T = \left(\frac{\ell}{2}\ 1\ -\frac{\ell}{2}\ 1\right)\dot{\theta} \tag{21}$$

$$T = \frac{A\rho\ell}{2}\left(\frac{1}{12}\ell^2\right)\dot{\theta}^2 \tag{22}$$

The $\frac{1}{12}(A\rho\ell)\ell^2$ is the mass moment of inertia of a rigid bar about its center.

Case 3. Rotation about end of segment

$$\{\dot{x}\}^T = (0\ 1\ -\ell\ 1)\dot{\theta} \tag{23}$$

$$T = \frac{1}{2}A\rho\ell\left(\frac{1}{3}\ell^2\right)\dot{\theta}^2 \tag{24}$$

One of the problems that arises is loss of mass at a clamped end. If $\dot{x}_1 = 0$ and $\dot{\theta}_1 = 0$ for a beam clamped at the left end, it is common to zero out the mass matrix for those rows and columns.

$$[m] = \frac{A\rho\ell}{420}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 156 & 22\ell \\ 0 & 0 & 22\ell & 4\ell^2 \end{bmatrix} \tag{25}$$

For pure translation, the kinetic energy is then (special case 1):

$$T = \frac{1}{2}\left(\frac{A\rho\ell}{420}\right)156\,\dot{x}^2 \tag{26}$$

More than one half the mass is lost if this procedure is used and the element is deficient in predicting shear force.

Consistent Mass Matrix for Beam Including Shear Deformations and Rotational Matrix

McCalley [9,5] appears to be the first to derive the consistent mass matrix for a Timoshenko beam segment, considering shear deformation and rotary inertia of the cross section. He was given credit by Archer [6], who derived the matrix separately. McCalley presented the mass matrix as the sum of two parts, a translational matrix $[m_t]$ and rotational $[m_r]$ with $[m] = [m_t] + [m_r]$. The complete matrices are given by Przemieniecki [7]. One term of each matrix is given here to show the form of the variables.

$$[m_t] = \frac{A\rho\ell}{(1+12\beta)^2}\begin{bmatrix} \frac{13}{35} + \frac{42}{5}\beta + 48\beta^2 & - & - & - \\ - & - & - & - \\ - & - & - & - \\ - & - & - & - \end{bmatrix} \tag{27}$$

$$[m_r] = \frac{A\rho\ell\,(R/\ell)^2}{(1+12\beta)^2}\begin{bmatrix} \frac{6}{5} & (\frac{1}{10} - 6\beta)\ell & - & - \\ - & - & - & - \\ - & - & - & - \\ - & - & - & - \end{bmatrix} \tag{28}$$

Here $\beta = \frac{EIK'}{GA\ell^2}$ and $R = (I_{\ }/A)^{1/2}$ = radius of gyration. If the rotary inertia of the cross section is neglected, $R = 0$ and $[m_r] = 0$. If the shear modulus $G \to \infty$, $\beta \to 0$, and $[m_t]$ approaches the consistent mass matrix for the Bernoulli-Euler beam.

Accuracy of Natural Frequencies

It is well known now, in using Rayleigh's method to estimate the lowest natural frequency of a system, that the natural frequency will always be greater than the exact frequency, provided that the assumed displacement function satisfies essential boundary conditions. This has been carried over to finite element systems if consistent mass matrices and compatible elements are used. For diagonal mass matrices, the natural frequencies usually tend to be lower than the exact, but not always, and no general rule is known. MacNeal [9] showed that the natural frequencies of a uniform bar represented by lumped mass matrices and stiffness matrices, based on a linear displacement function, as follows

$$[m_\ell] = \frac{\mu\ell}{2}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \quad \text{and} \quad [k] = \frac{EA}{\ell}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} \tag{29}$$

will be given by

$$\left(\frac{\omega'}{\omega}\right)_\ell = 1 - \frac{1}{6}\left(\frac{\pi}{N}\right)^2 + 0\left(\frac{\pi}{N}\right)^4 \tag{30}$$

where N is the number of cells per wavelengh. Using the consistent mass matrix [9]

$$[m_c] = \frac{\mu\ell}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix} \tag{31}$$

$$\left(\frac{\omega'}{\omega}\right)_c = 1 + \frac{1}{6}\left(\frac{\pi}{N}\right)^2 + 0\left(\frac{\pi}{N}\right)^4 \tag{32}$$

For a Bernoulli-Euler lumped-mass hinged-hinged beam MacNeal [4] gives

$$\left(\frac{\omega'}{\omega}\right)_\ell = 1 - \frac{1}{90}\left(\frac{\pi}{N}\right)^4 + 0\left(\frac{\pi}{N}\right)^6 \tag{33}$$

Shock Effective Mass

Justification for using the consistent matrix is usually made based on the increase in accuracy of the natural frequencies. Unfortunately much less attention has been given to the accuracy of predicted displacements and internal forces. In a discussion of inertia, it is appropriate to consider "shock effective mass" as developed at the Naval Research Laboratory and applied to ground shock problems by Belsheim and O'Hara [10].

To introduce notation consider a system under translational base motion in one of three translation directions:

$$[m]\{\ddot{x}\} + [k]\{x\} = -[m]\{\ddot{x}_0\} \tag{34}$$

If the normal modes of free vibration are $[\phi]$, and a change in coordinates is made,

$$\{x\} = [\phi]\{q(t)\} \tag{35}$$

and equation (34) becomes

$$[M_n]^D\{\ddot{q}\} + [\omega_n^2 M_n]^D\{q\} = -[\phi]^T[m]\{\ddot{x}_0\} \tag{36}$$

Expressing q(t) using the Duhamel integral:

$$[M_n\omega_n]^D\{q(t)\} = -\left[\int \ddot{x}_0(\tau)\sin\omega_n(t-\tau)\,d\tau\right]^D[\phi]^T[m]\{1\} \tag{37}$$

The matrix {1} would have ones in the positions corresponding to the direction of shock and zeroes in the remaining positions.

From (34), the inertia forces are:

$$[m]\{\{\ddot{x}\} + \{\ddot{x}_0\}\} = -[k]\{x\} \tag{38}$$

Since $$[\phi]^T [k] [\phi] = [M_n \omega_n^2]^D \tag{39}$$

and $$[k] [\phi] = [\phi]^{T^{-1}} [M_n \omega_n^2]^D = [m] [\phi] [\omega_n^2]^D \tag{40}$$

the inertia forces can be written

$$\begin{aligned}[m]\{\ddot{x}\} + [m]\{\ddot{x}_0\} &= -[k][\phi][q] \\ &= -[m][\phi][M_n]^{-1} [\int \omega_n \ddot{x}_0(\tau) \sin \omega_n(t - \tau)\, d\tau]^D [\phi]^T [m]\{1\}\end{aligned} \tag{41}$$

The integral has dimensions of acceleration for each mode. The right side gives the inertia force at each point due to the base motion $\{x_0\}$ plus elastic deformation $\{x\}$, relative to the base. Inertia forces can be calculated for individual modes by using the column matrix $\{\phi_n\}$ for the n^{th} mode. The total shear force in the direction of the shock would be

$$- \{1\}^T [k] [\phi] \{q\} = \{1\}^T [m] \{\ddot{x} + \ddot{x}_0\} \tag{42}$$

The modal shock effective mass is

$$SEM_n = \{1\}^T [m] \{\phi_n\} \{\phi_n\}^T [m] \{1\}/M_n \tag{43}$$

$$\text{where } M_n = \{\phi_n\}^T [m] \{\phi_n\} = 1 \tag{44}$$

in most computer programs. O'Hara [11] showed that the sum of the modal effective masses equals the total mass in the mass of the structure, excluding that at the supports.

Example Problem: Clamped-free Bernoulli-Euler Beam

For later reference, the solution to a cantilever beam is presented. The four-segment beam is shown in Fig. 2.

Fig. 2 Four segment cantilever beam

The problem was solved using a diagonal mass matrix as follows:

$$[m_d] = A\rho\ell \begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{\alpha\ell^3}{420} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{\alpha\ell^3}{420} \end{bmatrix} \tag{45}$$

Three values of α were used, namely $\alpha = 1.75$, 17.5 and 35. The largest value, $\alpha = 35$ corresponds to rigid body moment of inertia. This value is usually too high, and smaller values are of interest.

In Table 1, natural frequencies are compared to the exact values for Bernoulli-Euler theory. As expected, the values from the consistent mass solution are the most accurate and are all higher than those from the exact solution. For the lumped model, the value of α has the greatest effect on the upper four frequencies, or upper branch of the spectrum. The best value of α would appear to be between 1.75 and 17.5. It is of particular interest that the lower branch frequencies are all too low, but the upper branch may be high or low, depending on the value of α.

Table 2 shows shock effective mass. The accuracy of the consistent mass representation is comparatively poor here. In fact, for the first four modes, a lumped mass solution with $\alpha = 1.75$ or less is the most accurate. Similar trends have been noted for bending moments.

For both the consistent and lumped mass models, mass is lost at the supports. This could be corrected by putting a short, massless segment at the support in both solutions.

Mass Matrices for Plates and Shells

To summarize all the finite elements and associated inertia matrices is beyond the scope of the present paper. Reference [7] has an excellent introductory chapter on inertia properties which includes mass matrices for some typical structural elements. The book by Zienkiewicz [12] has chapters dealing with curved elements. The terms isoparametric, super-parametric and subparametric are defined there. For the present discussion, we are primarily concerned with whether the elements are compatible or not. The order of the polynominal used to develop the consistent mass matrix is directly related to accuracy of representation. Some properties are discussed in the next section under specific computer programs.

Accessions to Inertia due to Fluid-Structure Interaction

Such interaction is dealt with in another chapter. Only brief mention is made here of two computer programs dealing with accessions to inertia due to rigid body and elastic motions of a body in water.

A review of literature dealing with the oscillation of rigid cylinders in water was given by Frank [13]. He developed a solution for cylinders oscillating in or below the free surface of very deep fluids, as a boundary value problem by distributing source singularities over the submerged portion of the cylinders. He calculated curves of added mass and damping during heave, sway, and roll. A computer program YFA4 was written based on this report.

Chen and associates [14] developed one of the first computer programs for finding the interaction between an elastic structure of arbitrary shape, in contact with an infinite fluid. The mobilities of the structure are determined on mesh points at the interface. The surface of the contacting fluid is represented by a series of triangles and the mobilities found for the fluid surface. The steady-state velocities can then be determined due to specified sinusoidal excitation and the far-field sound pressure determined, one drive frequency at a time.

Zienkiewicz [12] formulated the equations for the coupled motion of an elastic structure and a fluid and arrived at the following coupled differential equations:

$$[m]\{\ddot{x}\} + [c]\{\dot{x}\} + [k]\{x\} + \frac{1}{\rho}[S]^T\{\rho\} + \{R\} = 0 \qquad (46)$$

$$[G]\{\ddot{\rho}\} - [S]\{\ddot{x}\} + [H]\{\rho\} = 0 \qquad (47)$$

Table 1 Clamped-free Beam Natural Frequencies
$\omega_n(\mu\ell^4/EI)^{1/2}$

n	Uniform Beam	Lumped Mass - 4 segment α = 1.75	α = 17.5	α = 35	Consistent Mass
1	3.5160	3.416	3.398	3.379	3.516
2	22.034	20.022	19.434	18.834	22.060
3	61.697	52.894	50.218	47.481	62.175
4	120.902	92.424	89.067	83.757	122.658
5	199.860	515.215	166.524	122.476	228.137
6	298.555	633.788	210.240	156.960	366.390
7	416.991	768.032	250.259	181.744	580.849
8	555.165	849.644	275.280	201.436	953.051

Table 2 Clamped-free Shock Effective Mass

n	Uniform Beam	Lumped Mass - 4 segment α = 1.75	α = 17.5	α = 35	Consistent Mass
1	0.6131	0.5982	0.5917	0.5847	0.6077
2	0.1883	0.1895	0.1870	0.1845	0.1729
3	0.0647	0.0635	0.0617	0.0592	0.0451
4	0.0331	0.0232	0.0266	0.0270	0.0132
5	0.0200	0.0002	0.0031	0.0127	0.0030
6	0.0134	0.0004	0.0035	0.0054	0.0008
7	0.0096	0.0002	0.0012	0.0015	0.0001
8	0.0074	0.0000	0.0000	0.0000	0.0000
Total	0.9494	0.8752	0.8748	0.8750	0.8428

Here ρ = fluid density and $[S] = \int_S [N]^T \rho\, [\bar{N}]\, dS$ with S the fluid-structure interface.

Also, $U_n = [\bar{N}]\,\{S\}$, the normal component of structural displacement. The matrix N represents the shape functions defining the pressure distribution. References are given in [12] in which particular problems have been solved. Of special interest is that of an incompressible fluid, for which (47) becomes

$$\{\rho\} = [H]^{-1}\,[S]\,\{\ddot{x}\} \tag{48}$$

substitution into (46) gives an added mass matrix for the structure

$$\frac{1}{\rho}\,[S]^T\,[H]^{-1}\,[S] \tag{49}$$

INERTIA MATRIX CAPABILITIES OF COMPUTER PROGRAMS

Some typical computer programs are discussed with regard to the types of inertia matrices generated, the reasons given for choice of matrices, and some experiences influenced by choice of matrices.

STRUDL II [15,16]

All of the finite elements have routines which calculate lumped or consistent mass matrices. Additional concentrated masses may be applied at nodes. When isoparametric elements are used, the lumped or diagonal mass matrix is calculated by first calculating the consistent mass matrix and then summing appropriate matrix elements. As indicated in the discussion of SLADE D, this procedure gives the correct dimensions but not necessarily the best magnitude for elements in the diagonal mass matrix.

If the number of static degrees of freedom is greater than the number of dynamic degrees of freedom, or if some of the masses are small enough to be neglected, matrices are condensed. The matrices are partitioned and it is assumed that m_{12} and m_{22} are small.

$$\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} \tag{50}$$

Then $x_2 \doteq -k_{22}^{-1}\,k_{12}^{T}\,x_1$ and condensed matrices are

$$m_{11}^{*} = m_{11} - k_{12}\,k_{22}^{-1}\,m_{21} - m_{12}\,k_{22}^{-1}\,k_{12} + k_{11}\,k_{22}^{-1}\,m_{22}\,k_{22}^{-1}\,k_{21} \tag{51}$$

$$k_{11}^{*} = k_{11} - k_{12}\,k_{22}^{-1}\,k_{21} \tag{52}$$

$$F_1^{*} = F_1 - k_{12}\,k_{22}^{-1}\,F_2 \tag{53}$$

When static condensation is specified in STRUDL, k_{11}^{*} is computed, but m_{11}^{*} is approximated by m_{11}. When kinematic condensation is specified, both k_{11}^{*} and m_{11}^{*} are calculated.

Jones and Gilkey [17], reported that both condensations were operative in STRUDL and presented results for several problems. Those for a clamped-free Bernoulli-Euler beam are of particular interest, because static condensations were used. Their natural frequencies were for a particular beam with $[EI/A\rho\ell^4]^{1/2} = 1.8?16078$. Given frequencies were divided by that number to arrive at the values in Table 3.

It should be noted that when static condensation is used, the natural frequencies by the consistent mass method are not necessarily higher than the exact values. When no condensation is used, twice as many consistent mass frequencies are calculated as recorded here. The values by the lumped mass method agree well with those in Table 1 for $\alpha = 35$.

Table 3 Clamped-Free Beam Natural Frequencies

n	Exact Frequency	STATIC CONDENSATION [17] Two Segments L.M.	Two Segments C.M.	Four Segments L.M.	Four Segments C.M.	NO CONDENSATION* Four Segments L.M.	Four Segments C.M.
1	3.516	3.16	3.36	3.42	3.48	3.38	3.516
2	22.034	16.26	21.20	20.09	22.46	18.48	22.06
3	61.697			53.21	66.11	47.49	62.15
4	120.902			92.74	126.34	83.77	122.55

*Received by telephone from T. J. Jones, May 28, 1975.

NASTRAN [9]

In the NASTRAN theoretical manual, inertia properties have two different kinds of applications in linearized structural analysis: as generators of applied loads in static analysis and as generators of matrix coefficients in dynamic analysis. The former includes gravity loads, centrifugal loads, and inertia relief effects. The latter includes the matrix of ordinary mass coefficients, and also, in problems of rotating coordinate systems, masses of Coriolis damping coefficients and centrifugal stiffness coefficients. Treatment of dynamic inertia effects in rotating coordinate systems is contemplated, but not yet completed, in NASTRAN.

Both lumped mass and consistent mass matrices are used for finite elements. A list of elements and the matrices used is given in Table 4.

Equations (29) through (33) are given in the manual. Based on the difference in sign of the second term in the frequency of the bar frequency Eqs. (30) and (32), the manual suggests an average of the lumped mass and consistent mass matrices be used.

$$[m_a] = \frac{\mu\ell}{12}\begin{bmatrix} 5 & 1 \\ 1 & 5 \end{bmatrix} \tag{54}$$

The error in this case is considerably less and is given by

$$\left(\frac{\omega'}{\omega}\right)_a = 1 + \frac{1}{30}\left(\frac{\pi}{N}\right)^4 + 0\left(\frac{\pi}{N}\right)^6 \tag{55}$$

Equation (54) has been adopted in NASTRAN as the coupled mass matrix for rods and bars, and will be used for torsion of beams, if distributed torsional inertia is considered for beam elements.

For plates, uniformly distributed mass consists of two parts: the mass of the structural material, and nonstructural mass, the surface density of which is specified by the user. In the lumped mass method, one-third of the mass of a triangular element is placed at each of its vertices, which preserves the location of the center of gravity. A quadrilaterial is treated as a set of four overlapping triangles, whose masses are transferred separately to the surrounding grid points.

The manual presents derivations of mass matrices for various elements, which are not repeated here. It points out how continuity between plate elements may be obtained by using, for example, the Clough triangle. Maintaining continuity may be interpreted as applying constraints, which means that if consistent mass is used, natural frequencies will be too high. It is stated in [9] that "it does not follow, however, that elements with displacement continuity give better results than all other elements".

Table 4 NASTRAN Mass Matrices for Finite Elements

	Consistent Mass	Lumped Mass
Bars	✓ (special)	✓
Beams	✓	
Plates (Lateral)	✓	
Plates (In Plane)		✓
Doubly Curved Shell	✓	
Solid of Revolution	✓	
Shear Panels		✓
Twist Panels		✓
Conical Shell Element		✓
Constant Strain Solid Element		✓
Isoparametric Quadrilateral Membrane Element QDMEM 1		✓
QDMEM 2 Element		✓

For the in-plane deformation of plates, one-third the mass of each element is placed at each of its vertices, an arrangement which preserves the location of the center of gravity. Consistent mass methods are not applied to shear and twist panels because they are incomplete physical objects. The elements for membrane action of plates use lumped mass matrices because the models built from such elements tend to be too stiff and the use of any consistent mass method would tend to further increase the calculated frequency.

The NASTRAN program includes the problem of compressible fluids in axisymmetric tanks and compressible fluids in rotationally symmetric cavities with slots. For the first problem, the equations of motion are linearized by assuming fluid motions are small compared to the dimensions of the container. The shape of the container must be axisymmetric to simplify analysis of the fluid, but the container structure need not have axisymmetric mechanical properties. Effects of gravity on a free surface are included. The solution for fluids in rotationally symmetric cavities with slots does not include interaction with the surrounding structure.

SLADE D [18]

This program is of particular interest because of the attention given to the effect of the choice of a mass matrix on natural frequencies and dynamic deformations, velocities, stresses, and stress resultants. The program uses a double curved arbitrary quadrilateral element, which is based on a minimum potential energy principle for thin shells. An explicit difference scheme is used. The program allows up to five separate layers and up to five separate elastic anisotropic materials with temperature-dependent properties. A diagonal mass matrix is used.

A consistent mass matrix is derived for a bar, or a one-directional membrane, using the Hermite interpolation form of the cubic polynomial. The result is the same as that given by Archer for the lateral bending of a beam, Eq. (17). As expected, the stiffness matrices are different for the bar and beam using the third order polynomial. The form of the 4 x 4 diagonal mass matrix is then obtained by considering first only nodal displacements.

For pure translation, the off-diagonal masses were added to the diagonal:

$$m_{11} + m_{13} = \left(\frac{13}{35} + \frac{9}{70}\right) \mu\ell = \frac{\mu\ell}{2}$$
$$m_{31} + m_{33} = \left(\frac{9}{70} + \frac{13}{35}\right) \mu\ell = \frac{\mu\ell}{2} \qquad (56)$$

For the purpose of determining dimensions, the off-diagonal gradient

terms are added to the diagonal inertias:

$$m_{22} + m_{24} = \mu\ell^3 \left(\frac{1}{105} - \frac{1}{140}\right) = \frac{\mu\ell^3}{420}$$
$$m_{42} + m_{22} = \mu\ell^3 \left(-\frac{1}{140} + \frac{1}{105}\right) = \frac{\mu\ell^3}{420} \tag{57}$$

Recognizing that this is not the best value, they use a multiplier α and the diagonal mass matrix for a bar or beam based on a cubic polynomial is given as

$$[m_d] = \mu\ell \begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{\alpha\ell^2}{420} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{\alpha\ell^2}{420} \end{bmatrix} \tag{58}$$

In Fig. 3, the ratio of $\bar{\omega}$ to the exact frequency is plotted versus n/N, where n is the n^{th} frequency and N is the number of elements. Note that there is a discontinuity between the lower branch and upper branch of each curve. The trend in frequencies follows that of Table 1, for the clamped-free beam. For the 36 x 36 mass matrix for the shell element, the diagonal mass matrix is obtained by adding four terms of the 36 x 36 consistent mass matrix, for example:

$$(m_{k,k})_d = m_{k,1} + m_{k,10} + m_{k,19} + m_{k,28} \quad k = 1,10,19,28$$
$$(m_{k,k})_d = \alpha_m (m_{k,2} + m_{k,11} + m_{k,20} + m_{k,29}) \quad k = 2,11,20,29 \tag{59}$$
$$(m_{k,k})_d = \alpha_b (m_{k,8} + m_{k,17} + m_{k,26} + m_{k,35}) \quad k = 8,17,26,35$$

A criterion is then given to relate the time interval Δt to λ, the eigenvalues of the $m^{-1}k$ matrix.

$$\text{Require } \Delta t \leq \frac{4}{\lambda_{max}} \tag{60}$$

$$\text{Membrane } \lambda_{max} = 70\, c_m^2/\alpha\, \ell^2$$
$$\text{Bending } \lambda_{max} = 2520\, c_b^2/\alpha\, \ell^4 \tag{61}$$

Here $c_m = \sqrt{\frac{E}{\rho}}$ and $c_b = \sqrt{\frac{Eh^2}{12\rho}}$ for membrane and bending deformations.

The result given is

$$\Delta t \leq 0.239 \frac{\ell}{c_m} \sqrt{\alpha_m} \qquad 0 < \alpha_m < 14.6$$
$$\Delta t \leq 0.0398 \frac{\ell^2}{c_b} \sqrt{\alpha_b} \qquad 0 < \alpha_b < 52.5 \tag{62}$$

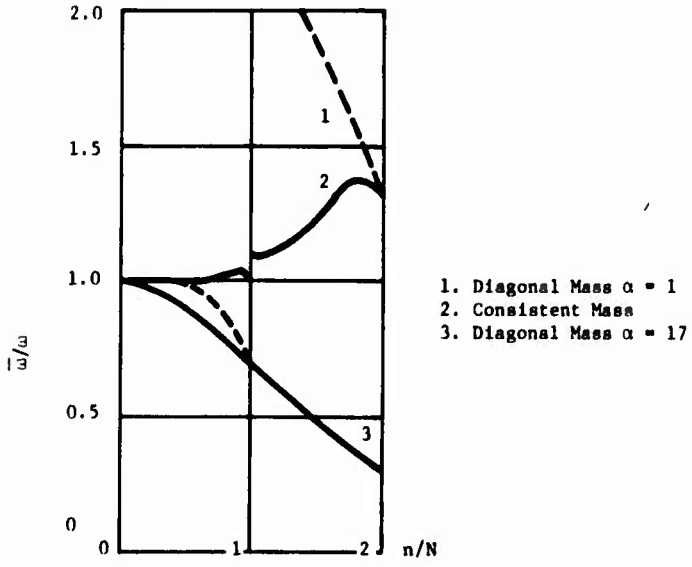


Fig. 3 Frequency ratio versus n/N for both consistent mass and diagonal mass discretization of the pin-pin beam equation using a cubic displacement function. Ref. [18]. p. 35

The non-exact mass matrices cause a distortion of the natural frequency spectrum ω'/ω. When the dynamic equations are integrated numerically, a further distortion occurs. Taking this into account values of $\alpha_m = 14.6$ and $\alpha_b = 17.5$ are recommended, since they produce a frequency spectrum $\bar{\omega}/\omega$ without a jump between the upper and lower brances, at n = 1.

Several example problems are solved in [18] and some results compare favorably with those from other computer programs. One example is for the membrane behavior of a cylindrical shell of very large radius-to-thickness ratio and zero Poisson's ratio. A unit step in pressure is applied to the end of the cylinder and should travel down the cylinder without dispersion. The solution for a fixed point on the cylinder is shown in Fig. 4 for lumped mass and consistent mass representations. The authors prefer the lumped mass results over those from the consistent mass calculations. The following points of difference are mentioned in [18]:

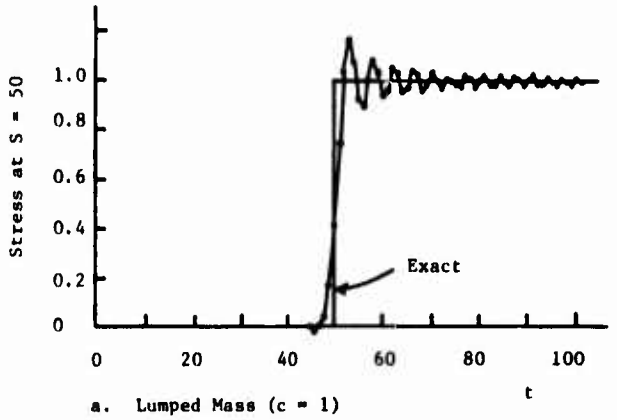
The overshoots in stress are less for the lumped mass results.

The consistent mass required twice as much computation time.

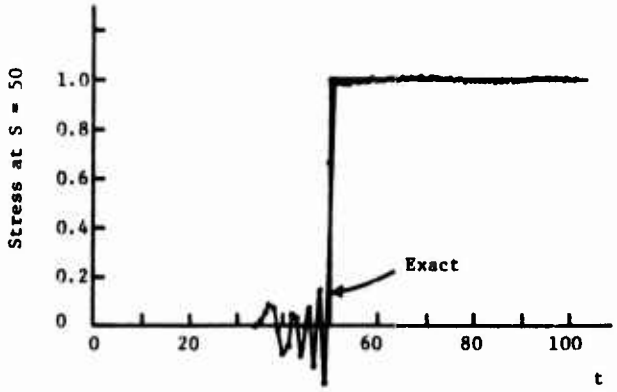
The early arrival time of a significant amount of information in the consistent mass calculations cannot be tolerated in more complicated problems. The following two paragraphs are quoted from page 41, [18].

"It is clear that a diagonal mass representation is superior to a consistent mass representation from a work standpoint. Another facet of their respective behaviors is also important. For the uniform bar, the wave equation gives a nondispersive resultant and is used as the standard (in two figures). The consistent mass representation gives a dispersive result where the higher frequencies travel too fast. The result is a wave traveling faster than the velocity c_m. The diagonal mass representation also gives a dispersive result but, for larger values of the gradient inertia parameter (α), the higher frequencies travel slower than c_m. As a result, stress pulses will not travel faster than c_m, a more realistic result.

For the uniform beam, the biharmonic equation gives a dispersive result where higher and higher frequencies travel at faster and faster velocities and is used as the standard (in two other figures in the report). This is physically unrealistic behavior. The consistent mass representation gives an even



a. Lumped Mass (c = 1)



b. Consistent Mass (c = 1)

Fig. 4 Stress at point on cylindrical shell due to step function input. Ref. [18], p. 134.

greater dispersion at higher frequencies, resulting in an even less realistic behavior. The diagonal mass representation is also dispersive, but at the higher frequencies falls off towards what is the more realistic behavior of the Timoshenko beam."

SAP IV [19]

The program uses only diagonal mass matrices. The justification is based on reference [20]. The following discussion summarizes some relevant comments from that reference. In [20] it is stated that it is evident that the consistent mass matrix provides a better approximation of the inertia forces acting in the structure than is obtained for the elastic forces from the corresponding (consistent) stiffness matrix. This is said to be because the mass matrix is derived from assumed displacement functions, while the stiffness matrix involves derivatives of the displacement functions, and derivatives are less

accurate than the displacements. For this reason it is suggested that it would be reasonable to employ a lower order interpolation function in formulating the mass matrix. The only simplified mass matrix of practical significance is the lumped mass matrix. An example is then discussed in [20] for an arched dam and its solid rock foundation in which 469 degrees of freedom were used. The lumped mass analysis required 63 seconds and the consistent mass 76 seconds. Based on this small difference in cost, it was decided to use only consistent mass on repeat runs of that structure. It is stated that the results do not indicate whether the consistent mass frequencies are more accurate than the lumped mass frequencies.

Element library in SAP IV includes:

- Three-dimensional truss element
- Three-dimensional beam element including torsion, bending, axial and shearing deformations
- Plane stress, plane strain and axisymmetric elements. Element is based on an isoparametric formulation. Incompatible displacement modes can be included to improve the bending properties of the element
- Three-dimensional solid element, which employs incompatible modes
- Variable-number-nodes thick shell and three-dimensional element
- Thin plate and shell element
- Pipe element

ASKA II [21]

The ASKA, Part II, package for dynamic analysis is called DYNAN. The program continues to be developed and will probably be one of the best-planned programs with a great many options. Of interest here is that consistent mass matrices are programmed for every element. Over forty elements are available. Static and dynamic condensation may be used. An excellent review, including a listing of available elements, was presented by Meijers [22], including results of experience of Part I, the static ASKA package.

SNAP [23]

The SNAP program was designed to solve rapidly extremely large finite element problems. This writer's knowledge of the program is limited to information obtained from a brief brochure. The program is said to be much less expensive than other programs for solving dynamic problems. Some of the cost saving apparently is due to the use of a generalization of Stodola's method for solving for lower modes.

There are three options for representation of inertias: (1) the lumped mass method, with lumping of distributed structural and non-structural mass done automatically by the program, (2) the consistent mass matrix method, which is available only for beams and certain membrane elements and (3) a "pseudo-consistent" mass matrix method which is said to yield lower computer costs than the consistent mass method, and is less accurate. The "pseudo-consistent" mass matrix is not presented, but it is apparently derived by using a linear, or first order, displacement function. This results in a non-diagonal mass matrix. Most of the time saved would be in generating the matrix for each element.

ATLAS [24]

ATLAS is a general purpose computer program for the analysis of large complex structures. It is an example of a program with special features for design and analysis of air vehicles. The following analysis capabilities are incorporated: static response to concentrated and distributed loads and enforced

displacements; determination of real eigenvalues and eigenvectors for use in vibration analysis; computation of steady state air loads for use in static analysis of aircraft; computation of unsteady air loads for use in flutter prediction analysis of aircraft; and a substructure capability for static interaction of structural subcomponents.

The elements listed in [24] are: point mass, rod element, plate element, spar element, and cover element. Mass data is computed from densities and dimensions of elements. Diagonal mass matrices are frequently mentioned in [24], but it is understood that consistent mass matrices are also available for some elements. In addition, mass data is generated for payloads and payload management systems and for fuel and fuel management systems. The fuel is treated as rigid mass. An option is available for calculating fuel mass by identifying the tank and specifying percentage fullness of the tank, which indicates the extent to which calculation of masses and weights has been automated. No discussion regarding preference of mass matrix was available in the portion of [24] dealing with mass representation.

ARAB [25]

ARAB is an example of a computer program developed to predict sound radiation from an arbitrary elastic body. The theory is presented in [14] and more details were outlined in [25]. Although far-field sound pressures may be calculated, one of the unique features of the program is the solution for the interaction between a fluid and a structure. The velocities of grid points on the structural side of the interface are matched with the velocities of triangular pistons radiating into the fluid. These velocities are initially unknown, in general, since the specified inputs are usually forces on interior or interface points on the structure. As an option, interface velocities may be specified.

Many examples have been calculated under Chen's direction. Two examples were presented in [14]. The first is for a piston in a sphere using 80 triangular pistons to represent the spherical surface for one computer run and 320 for another. Predicted sound pressure is compared with that from an exact solution. The second example is for a cylindrical shell excited by a point radial sinusoidal force. On the cylinder 120 discrete masses were used. The radiation pattern is associated with low order shell modes. Experimental values are compared with theoretical values at the one drive frequency.

One of the main drawbacks of solving the interaction problem is cost, since many points are usually used to represent a practical structure. In addition, one computer run gives a solution for only one drive frequency. As the frequency is increased, the grid spacing must be decreased. For the fluid side, the grid spacing should be less than one-fourth the wave length in the fluid.

For the structure, only diagonal mass matrices are used. The pistons which excite the fluid are plane, rigid triangles. The fluid is assumed to be of infinite extent. There are no provisions to account for free surfaces.

An important use for the program is to identify the terms which represent virtual mass and damping of the fluid. These could be added to the structure matrices and the structure solved without doing the interaction problem on subsequent runs. Transient problems could be solved if average values of virtual mass and damping could be used over particular frequency ranges.

YFA4 [26]

Computer program YFA4 is based on the theory and program originally developed by W. Frank [13]. It provides the pressures, added mass, and damping of a rigid horizontal cylinder oscillating in heave, sway, or roll while located in or below the free surface.

The theory deals with a cylinder, of infinite length, which is symmetric about its vertical centerplane. The velocity potential is represented by a

distribution of sources over the submerged cross section. The unknown strengths of the distributed sources are obtained by satisfying the kinematic boundary conditions on the cylinder. To solve the integral equation, the cylinder contour is represented by a finite number of straight-line segments along each of which the source density is assumed to be constant.

Reference [26] gives some limitations of the program. For example, wrong results may be produced if the cylinder has horizontal lines as part of its top or bottom contour. Also, two points cannot have identical vertical coordinates. There is difficulty with irregular frequencies at which the matrix of influence coefficients becomes singular. This is usually out of the range of interest for practical ship dimensions. It causes discontinuities in plots of computed values versus frequency. Values away from the discontinuities should be reliable .

In [13], examples of added mass and damping versus frequency curves are given for various motions. Rectangular and circular cylinders are considered, as well as a ship cross section.

CHOICE OF PROGRAM AND INERTIA MATRIX

An excellent summary of computer programs for structural analysis was made by Bushnell [27], who compared over 50 computer programs. The items listed for each program, with 324 sub-items, are as follows: geometry, boundary conditions; wall construction; material properties; loading; phenomena; discretization; solution methods; computer program distribution, documentation, organization, and maintenance. In Table 5, seven items or capabilities for seventeen computer programs are listed. Those programs which are listed in [27] are indicated by +, and the reader is referred there for more details, and for more programs.

The main choice in inertia matrices for finite elements at present is between diagonal mass and consistent mass matrices. There are many programs which offer both, so one would not choose the program primarily because of the mass matrix used. Other factors, such as cost, availability of program, finite element catalogue, and experience would be important. If one were developing a program, a good general purpose program should have a choice of mass matrices; a specialized program need only have the best matrix for the particular problem to be solved. With regard to inertia matrices, the following factors may affect choice between consistent and diagonal mass matrices.

1. The consistent mass matrix is more expensive to run than a diagonal mass matrix. Sometimes the difference in cost is minimal, however.

2. The consistent mass matrix usually gives frequencies which are more accurate for the same number of elements.

3. The consistent mass matrix may produce stress results which are less accurate. Not enough study has been made of stress and deformation predictions.

4. If compatible elements are used with a consistent mass matrix, the natural frequencies will all be too high. Sometimes it is an advantage to know this, because an upper bound is established for frequencies. On the other hand, it may be undesirable to have high frequencies in wave propagation or transient problems, because some waves will travel too fast [18]. With diagonal mass matrices, some waves may travel too slow.

5. If condensation methods are used in which the consistency of the mass matrix is not maintained for compatible elements, the frequencies may be too high or too low.

6. Clough [20] suggested that the consistent mass matrix is more accurate than the associated stiffness matrix. Such accuracy may not be justified. He suggested that lumped masses may be sufficiently accurate. The idea in one option of the SNAP program to use a higher order polynomial for the stiffness matrix than the mass matrix may have merit - resulting in a non-diagonal, non-consistent mass matrix.

7. In some programs, a diagonal mass matrix is obtained by adding appropriate off-diagonal terms of a consistent mass matrix. This practice does not yield the best diagonal mass matrix. The values of terms having dimensions of rotary inertia may not be the best to use. In fact, if diagonal mass matri-

ces are used, it is often better to use more translational dynamic degrees of freedom and to omit the rotary inertia associated with the length of the element.

Table 5 Capabilities of Computer Programs

	ANSYS	ARAB	ASAS	ASEF	ASKA	ATLAS	MARC	NASTRAN	PAFEC 70	SAPIV	SESAM 69C	SLADE	SNAP	SPADAS	STARDYNE	STRUDL EII	YFA4
Gyroscopic Loads			✓				✓				✓						
Centrifugal Loading	✓		✓				✓	✓	✓		✓						
Fluid-Structure Interaction		✓		✓				✓			✓						✓
Time Integration	✓		✓	✓			✓	✓	✓	✓	✓	✓				✓	
Modal Vibrations	✓	✓	✓	✓	✓		✓	✓	✓	✓	✓		✓	✓	✓	✓	
Non-Diagonal Mass Matrix	✓		✓	✓	✓		✓	✓	✓		✓		✓	✓		✓	
Diagonal Mass Matrix		✓			✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	
*Discussed herein		*			*	*		*		*		*	*			*	*
+Listed in [28]	+		+	+	+		+	+	+	+	+	+	+	+	+	+	

ANTICIPATED IMPROVEMENTS

Some improvements in representation of inertia properties may result from improvements in finite elements for solution of static problems. Activity in the development of better finite elements is summarized in the review papers in the publication edited by Fenves, et al. [28]. Wilson, et al. [29], place emphasis on the importance of incompatible displacement models in improving element accuracy. They present example problems in which static stresses as well as displacements are compared for a cantilever beam and a cylindrical shell.

There have been suggestions to improve accuracy for dynamic analysis by including higher order terms such as those in the infinite series for the force/input relationship for a bar in Eqs. (14) and (16). Neubert and associates [30], [31] have developed macro-finite elements for bars and beams in which a diagonal mass matrix results and where both forces and displacements are accurate at the boundaries in the low frequency range. The process involves using exact relationships, such as in Eqs. (13) and (15) to develop the macro-finite elements. Emphasis is placed on improved accuracy using fewer degrees of freedom, in predicting displacements and internal forces in ground shock problems.

Elias [32] used a force method to arrive at an additional "elastic-inertial" flexibility matrix. His matrix for a pin-ended truss member element includes terms of the same magnitude and dimensions as the mixed third terms in the series in Eqs. (14) and (16). Examples using the force method and a displacement method are presented, including natural frequencies, displacements, and chosen mode shapes. Only bar and beam elements are considered. For any improvements to be incorporated into general purpose computer programs, it is desirable that they apply to a wide range of finite elements including plates and shells.

Some economy may be gained by using non-diagonal mass matrices based on lower order functions than those used in deriving the stiffness matrices, resulting in a nonconsistent mass matrix.

There is need for improvement in the accuracy of the prediction of dynamic deformations and stresses. The efficiency also could be increased in dynamic

analysis. For most problems, 50% or more of the modes are discarded because they are inaccurate.

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APPENDIX

PROGRAM SUMMARIES

ARAB [25]

Date: Original version 1962; latest version 1969
Capability: Solves for interaction between structure and fluid and for the sound pressure in fluid at a single drive frequency
Method: Numerical evaluation of Helmholtz integral using triangular surfaces, of initially unknown source strength, radiating into fluid at interface [14]
Input: Material and fluid properties. Finite element data for structure
Output: Far-field sound pressures. Forces and velocities at fluid-structure interface
Language: FORTRAN
Hardware: UNIVAC 1106 (31,000)
Usage: To predict sound radiation from an arbitrary elastic body
Developer: Dr. L. H. Chen and associates
General Dynamics/Electric Boat
Groton, Ct. 06340
Availability: Contact developer.

ASKA II [21]

Date: 1974
Capability: Static and dynamic, linear, finite-element analysis. Undamped, proportionally and nonproportionally damped systems
Method: Undamped systems by a Householder reduction followed by the QR-algorithm or the bisection method
Input: Nodal point coordinates, loads, prescribed displacements, element properties, material properties
Output: Stresses, forces, displacements, eigenvalues and eigenvectors
Language: FORTRAN IV
Hardware: UNIVAC 1108, CDC 6600, IBM 360/370 (512 kbytes core; 10,000,000 word direct access backing)
Usage: Linear, elastic structures
Developer: Prof. J. H. Argyris and associates
University of Stuttgart
7 Stuttgart 80
Pfaffenwaldring 27, Germany
Availability: From developer
Other Comment: Also called DYNAN

ATLAS [24]

Date: February 1973
Capability: Static and dynamic analysis of large, complex structures
Input: Nodal, mass, payload, fuel, boundary, panel, material, and load data
Output: Real eigenvalues and eigenvectors, stresses, deflections, forces
Language: FORTRAN (50,000-125,000 octal)
Hardware: CDC 6600
Usage: Being used for air vehicles and other linear, elastic structures

Developer: ATLAS Task Force
The Boeing Company
Commercial Airplane Division
Renton, Wa. 98055
Availability: Limited availability thru 1976, then generally available
Mr. Ralph E. Miller, Jr.
The Boeing Company
Commercial Airplane Division
Renton, Wa. 98055

NASTRAN [9]

Date: Completed in 1968; revised version 1972
Capability: Static and dynamic structural analysis by the finite element approach. Buckling analysis. Compressible fluids in tanks and cavities. Heat transfer analysis
Method: Direct solution of dynamic equations and modal methods for structural dynamics problems.
Input: Material properties, initial and boundary conditions, finite element coordinates
Output, Dynamic Analysis: Normal modes and frequencies, complex eigenvalues, random response, transient response
Hardware: IBM 360/370, UNIVAC 1108, CDC 6000 series
Usage: Broad range of application
Developer: NASA, Goddard Space Flight Center (Management Supervision)
Computer Sciences Corporation
Baltimore Division of Martin-Marietta
The MacNeal-Schwendler Corporation
Availability: COSMIC (Computer Software Management and Information Center)
University of Georgia
Athens, Ga. 30601

SAP IV [19]

Date: June 1973
Capability: Static and dynamic analysis of linear structural systems. Proportional damping
Method: Natural frequencies by a determinant search technique or a subspace iteration solution. Direct integration by Wilson θ-Method
Input: Material properties, element data, loads, concentrated mass data.
Output: Frequencies and mode shapes. Displacements, forces and stresses
Language: FORTRAN IV
Hardware: CDC 6400, 6600, and 7600
Usage: Determination of mode shapes and natural frequencies. Dynamic response by mode superposition for time dependent loads, response spectrum analysis, and timewise response using step-by-step direct integration.
Developer: Klaus-Jürgen Bathe, Edward L. Wilson and Fred E. Peterson
Earthquake Engineering Research Center
University of California
Berkeley, Ca., 94720
Availability: NISEE
University of California
Berkeley, Ca. 94720
Attn: K. Wong

SLADE D [18]

Date: January 1973

Capability: Dynamic Analysis of Thin Shells
Method: Double curved quadrilateral element. Timewise integration using explicit difference scheme, Kirchoff shell theory
Input: Material properties, nodal coordinates, initial conditions, boundary conditions, surface loads
Output: Displacements, velocities, stresses, and stress resultants versus time
Language: FORTRAN
Hardware: CDC 6600 (49,000 words)
Usage: For dynamic analysis of shells having up to five different layers
Developer: Samuel W. Key
Zelma E. Beisinger
Sandia Laboratories
Albuquerque, N.M. 87115
Availability: Dr. Margaret K. Butler
Argonne Code Center
9700 S. Cass Avenue
Argonne, Il. 60439
Other Comment: Example problems presented in [18]

SNAP

Date: 1972 User's Manual
Capability: Undamped modes and frequencies of large highly interconnected, finite element networks
Method: Generalization of Stodola's method
Input: Material Properties. Finite element data
Output: Natural frequencies and mode shapes
Language: FORTRAN
Hardware: CDC 6600 (32,000-60,000) UNIVAC 1108
Usage: Modal analysis of elastic structures
Developer: Lockheed-Huntsville Research & Engineering Center
P.O. Box 1103
Huntsville, Al. 35807
Availability: W. D. Whetstone
Lockheed Missle & Space Center
Palo Alto, Ca. 94300
Other Comment: Spar is newer version

STRUDL II [15, 16]

Date: Initial Release, 1967; Dynamics option (DYNAL), 1974.
Capability: Static and dynamic analysis of structures
Method: Finite element. Householder and Jacobi methods for eigenvalues
Input: Material and structural properties. Timewise loading. Shock spectrum
Output: Mode shapes, natural frequencies. Displacements, shears, and moments versus time.
Language: FORTRAN, ASSEMBLER
Hardware: IBM 360/370 (60,000 words)
Usage: Dynamic analysis of trusses, frames, and structures made up of two-dimensional membrane and bending elements and three-dimensional elements
Developers: Massachusetts Institute of Technology
ECI Systems, Inc.
1050 Massachusetts Avenue
Cambridge, Ma. 02138
McDonnell Douglas Automation Co,
St. Louis, Mo. 63100
Availability: McDonnell Douglas Automotion Company
P.O. Box 516
St. Louis, Mo. 63100
Attn: Product Marketing

Other Comments: Stresses compared with codes: AISC, AIJS (Japan), API, and CSAS 16 (Canadian)

YFA4 [13, 26]

Date: March 1971
Capability: Calculation of added mass and damping coefficients of rigid cylinders oscillating in or below a free surface
Method: Integral-equation method
Input: Frequencies at which values are to be calculated, depth of submergence, coordinates of cross-section
Output: Pressures in phase with velocity and accelerations
Language: FORTRAN IV
Hardware: IBM 7090
Usage: For ship cross-sections
Developer: J. W. Bedel, C. M. Lee, [W. Frank, original program]
Naval Ship Research and Development Center
Washington, D.C. 20034
Availability: Program is given in [26]

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