U.S. DEPARTMENT OF COMMERCE
National Technical Information Service

AD-A029 450

The Analysis of Dominance Matrices: Extraction of Unidimensional Orders within a Multidimensional Context

University of Southern California

June 1976

UNIVERSITY OF SOUTHERN CALIFORNIA
UNIVERSITY PARK
LOS ANGELES, CALIFORNIA 90007

DEPARTMENT OF PSYCHOLOGY

August 25, 1976

Dear Colleague,

This office recently senc you our third Technical Report entitled, "The Analysis of Dominance Matrices: Extraction of Unidimensional Orders Within a Multidimensional Context," by Thomas J. Reynolds.

We ask that you make a correction on page 37. Line 16 of that page should read, "value from 0 to 1,..."

Thank You,

Norm Cliff

Norman Cliff

Contract No.: N00014-75-C-0684

Program element, project, task area and work unit numbers:
611153N, RR042-04, RR042-04-01, NR 150-373

THE ANALYSIS OF DOMINANCE MATRICES: EXTRACTION OF UNIDIMENSIONAL ORDERS WITHIN A MULTIDIMENSIONAL CONTEXT

Thomas J. Reynolds

Technical Report No. 3

Department of Psychology
University of Southern California
Los Angeles, California 90007

June, 19̣̣̣̣̣7̣6

Prepared under contract No. N00014-75-C-0684
NR No. 150-373, with the Personnel and
Training Research Programs, Psychological Sciences Division

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: Technical Report No. 3

2. GOVT ACCESSION NO.:

3. RECIPIENT'S CATALOG NUMBER:

4. TITLE (and Subtitle): THE ANALYSIS OF DOMINANCE MATRICES: EXTRACTION OF UNIDIMENSIONAL ORDERS WITHIN A MULTIDIMENSIONAL CONTEXT

5. TYPE OF REPORT & PERIOD COVERED: Technical

6. PERFORMING ORG. REPORT NUMBER:

7. AUTHOR(s): Thomas J. Reynolds

8. CONTRACT OR GRANT NUMBER(s): N00014-75-C-0684

9. PERFORMING ORGANIZATION NAME AND ADDRESS: Department of Psychology, University of Southern California, Los Angeles, California 90007

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 61153N, RR042-04, RR042-04-01, NR 150-373

11. CONTROLLING OFFICE NAME AND ADDRESS: Personnel and Training Research Programs, Office of Naval Research (Code 458), Arlington, Virginia 22217

12. REPORT DATE: June 1976

13. NUMBER OF PAGES: ~~69~~ 73

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office):

15. SECURITY CLASS. (of this report): Unclassified

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE:

16. DISTRIBUTION STATEMENT (of this Report): Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report):

18. SUPPLEMENTARY NOTES:

19. KEY WORDS (Continue on reverse side if necessary and identify by block number): (U) Factor Analysis; (U) Multidimensional Scaling; (U) Test Theory; (U) Item Dominance Relations; (U) Social Distance Scale; (U) Binary Matrix

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A method of factor extraction specific to a binary matrix, illustrated here as a person-by-item response matrix, is presented. The extraction procedure, termed ERGO, differs from the more commonly implemented dimensionalizing techniques, factor analysis and multidimensional scaling, by taking into consideration item difficulty. Utilized in the ERGO procedure is the calculation of a dominance matrix which, for either persons or items, has the important attribute of allowing directionality to be inferred between relations.

DD FORM 1473, 1 JAN 73 EDITION OF 1 NOV 65 IS OBSOLETE

S/N 0102 LF 014 6601

PRICES SUBJECT TO CHANGE

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

The theory underlying ERGO is founded in ordering theory (Airasian & Bart, 1972), with its interpretation of dominance relations following logical implicatives similar to Boolean algebra. The redefinition of dimensionality using both the notion of dominance relations and that of logical prerequisities can more aptly be identified with the definition of a Guttman order, thereby placing emphasis on the developmental aspects of recovered sets of dimensions. It is this interpretation that allows for the duality of relationships between persons and items. The resulting placement of both persons and items on the same unidimensional construct presents the researcher with the opportunity to observe direct relations between the two.

A preliminary attempt to utilize the apparent advantages associated with the extraction procedure based on dominance relations, order analysis (Krus, Bart, & Airasian, 1975) is used. This is done both to further explicate the implications of ordering theory as well as to point out the issues with which a dimensionalizing procedure of this type must concern itself. In this discussion, the procedural shortcomings of order analysis are presented to acquaint the reader with the obstacles that an alternative approach must overcome. Premier among these is the failure of order analysis to consider the true nature of multidimensionality in a dominance matrix context. This appears in the order analytic assumption that counter dominance relations are merely a product of error, rather than being manifestations of the multidimensional nature of the data. The alternative procedure (ERGO) is developed by dealing with this essential point.

The key to the dimension extraction problem of ERGO rests in the formulation of an index of dimension consistency that is comparable to classical measures such as the Kuder-Richardson formulae (1937) and the Loevinger homogeneity indices (1947). Cliff (1975b), by demonstrating the relation between these classical indices and their redefinition in a dominance matrix context, lays the foundation for the development of an alternative procedure. Thus, by adopting a consistency measure developed there, ERGO iteratively adds items together, resulting in the construction of various sets of implicative chains representing dimensions. Having constructed these chains, the ERGO procedure orders the chains in terms of maximal number of items contributed. The chain evaluation procedure can best be explained as an attempt to maximize the number of items accounted for in a given dimensional solution.

To give additional understanding of both the ERGO process and the potential advantages a procedure of this type offers, an empirical example which utilizes social distance items (Bogardus, 1925) paired individually with three ethnic groups was analyzed for respondents representing four ethnic groups. Emphasized in the solution was the duality of relationships inherent in a procedure such as this, that is based upon the principles underlying Guttman orders. The results demonstrated the ability of ERGO to (1) group items referring to the same ethnic group; (2) uncover hierarchically graded orders within each chain (3) select the three chains that corresponded to the three ethnic groups; and (4) cluster individuals by ethnic group according to their scores.

INTRODUCTION

The extraction of factors or dimensions from a data matrix has preoccupied many a psychometrician, and methods developed to accomplish this task have taken many forms. From their fundamental beginnings in factor analytic theory (Spearman, 1904) to the more recent multidimensional scaling procedures (MDS) (Shepard, 1962), to the most recent, ordering theory (Airasian & Bart, 1972), all methods have the common concern of the identification of unidimensional structures within a postulated multidimensional context. To date, more traditional methods of factor analysis and multidimensional scaling have fallen short in attacking the dimensionality problem specific to the binary matrix (Horst, 1965). Isolating unidimensional hierarchies within a binary structure has recently undergone revision based upon a unique theoretical conceptualization known as order analysis (Krus, Bart, & Airasian, 1975).

Instead of creating "artificial" measures of association, e.g., a common correlation coefficient or distance measures, order analysis utilizes a logic model. It attempts to isolate the logical orders among variables, and thus produces unidimensional components commonly known as Guttman scales (Guttman, 1944). Using the terminology of Horst (1965) and Lazarsfeld (1958), order analysis can be described broadly as the process of isolating the underlying structures of latent entities and attributes within a given response set.

Logical procedures for uncovering hierarchical graded orders via the construction of unidimensional scales have both practical as well as theoretical significance. The controversial issue of examining the binary relations in a test item by person response matrix serves to illustrate this problem. Though notably hampered by distributional assumptions and also by the choice of an inter-item measure of association, the use of classical factor analysis persists as the principal type of dimensionalizing procedure. Far more serious than the above mentioned drawbacks, however, is the failure of the factor analytic procedures to take into account the difficulty order of the items. Because of the fundamental role of item difficulty in test theory, this failure excludes factor analysis as a desired alternative, and suggests the use of a dimensionalizing system that takes into account the item difficulties. The present article suggests such a procedure, based upon sound measurement principles underlying the Guttman simplex.

The value of relying on such a fundamental notion as Guttman-type scales offers another, and potentially even more significant, advantage. Instead of regarding items, particularly attitudinal items, in a non-theoretical manner as would be the case with factor analysis, the possibility of inferring a qualitative structure among variables is appealing. This possibility, stemming from the developmental notion upon which Guttman scales rely, differs from the compensatory theory of behavior upon which factor analysis necessarily rests. Thus, Guttman-type scales allow for the consideration of various logic models of behavior, i.e., those of a conjunctive or disjunctive nature (Levy, 1973). Logic models such as these allow for the identification of the developmental prerequisites to attitudes, and, at the same time, allow for different developmental orders.

Recently, the development of a dimensionalizing system that works with the logical relationship of a Guttman simplex--ordering theory--has been proposed (Krus, Bart, & Airasian, 1975). Its acceptance, however, has been forestalled by a number of procedural shortcomings. Foremost among these is the failure to develop consistency indices which relate to other more common consistency descriptives, such as the familiar Kuder-Richardson formulae (1937) and the Loevinger homogeneity indices (1947). A solution to this problem has been formalized by Cliff (1975b) in the development of a series of measures constructed from the item-by-item dominance matrices. Importantly, these measures of consistency constructed from dominance matrices parallel their counterpart in classical test theory. The application of these consistency measures offers an alternative methodology, based on sound measurement principles, for the identification of unidimensional structure within an item-person context. In the present study, a new factor extraction method founded in the logic of ordering theory while also incorporating Cliff's (1975b) consistency indices will be presented. An empirical example using Guttman-like social distance attitude items will be examined in an attempt to evaluate how well the model performs.

Elements of Ordering Theory

Simple Orders

The construction of isomorphic number systems is the central issue of any structured psychological research. An isomorphism refers to a similarity in pattern, viz., a situation where a one-to-one relationship exists between an object and its numerical representation. To illustrate, consider the relationships among real numbers which are actually meant to be representative of the interrelations among a set of items or objects. One foundation of this real number system is that it can be linearly ordered. Thus, the following three properties may be said to be the axioms upon which this ordering is dependent (Coombs, Dawes, & Tversky, 1970, pp. 366-368).

asymmetric property - aRb implies b$\bar{R}$a where $\bar{R}$ means not R

transitive property - aRb and bRc implies aRc

connected property - either aRb or bRa

These axioms hold where R indicates the logical relationship typified by "greater than," and a, b, and c are entities in the system.

An example of these three fundamental properties may be illustrated for the one-set dominance case (Coombs, 1964) more commonly equated with a simple preference ordering. A simple order can be defined in terms of the connecting relations that exist between all pairs of the member stimuli. A connecting relation is represented by a 1 in the row/column designate of an otherwise null matrix. The matrix of connections, commonly known as an adjacency matrix, represents a digraph (directed graph) where the arcs or connections between the vertices or stimuli are represented in the form of a binary score matrix (Harary, Norman, & Cartwright, 1965). The adjacency matrix in Figure 1 represents a simple ordering between three items. In the preference context, the connective 1 implies that the row stimulus is preferred to the column stimulus. The property of asymmetry is shown by the absence of symmetric 1's. And, the lack of any logical contradictions, such as aRb, bRc, or cRa, necessarily suggests transivity. Thus a simple order, aRbRc, can be said to exist.

As these essential axioms are the foundation for defining a simple order for members of the same set, so also do they hold for relations between two different sets. The two-set dominance classification (Coombs, 1964), in this case, refers to a set of items and a set of persons. The persons by items matrix seen in Figure 2 can be seen to yield a simple difficulty ordering for items as well as an ability ordering for persons. This dual relationship is the basic concept underlying Guttman scales, which is represented by Figure 2, a perfect Guttman scale or simplex. Not only does there exist an item difficulty and person ability ordering, but a joint person-item order, as discussed by Cliff (1975a), can also be constructed as seen in Figure 3. This joint ordering can also be considered a simple order, thus operating under the same axioms.

As noted, these fundamental properties of relations between real numbers and the objects they represent (be they items, persons, or a combination of both) give rise to defined orders. These proper-

	a	b	c
a	0	1	1
b	0	0	1
c	0	0	0

Figure 1. Adjacency matrix representative of a simple order aRbRc.

Items

Persons		a	b	c	d
	1	1	1	1	1
	2	0	1	1	1
	3	0	0	1	1
	4	0	0	0	1
	5	0	0	0	0

Figure 2. Persons by items response matrix representing a perfect Guttman simplex.

	1	a	2	b	3	c	4	d	5
1	0	1	0	1	0	1	0	1	0
a		0	1	0	1	0	1	0	1
2			0	1	0	1	0	1	0
b				0	1	0	1	0	1
3					0	1	0	1	0
c						0	1	0	1
4							0	1	0
d								0	1
5									0

Figure 3. Rearranged joint ordering of persons and items yielding a simple order.

ties, known as order relations, provide the basic justification for the matrix manipulative procedures developed below that attempt to utilize logic structures as a solution to the dimensionality problem.

Logic Structures

Syllogistic reasoning, as originally formulated by Aristotle, demonstrates the use of simple logic and its cognitive counterpart, the reflection of thought processes. The most basic of the traditional syllogisms is the conjunction of implicative relations, i.e., as $A \rightarrow B$ and $B \rightarrow C$ then $\cdots$. This implicative chaining present in syllogistic reasoning can also be considered as the development of a straight-line dimensional relationship congruent with the notion of simple order. An order, created by implicative relations, can be defined as a condition of logical arrangement among certain specifically related elements in a given set of items.

For small sets of elements, say, a, b, c, it is possible to analyze the relationships between all possible response patterns (a plenum), which can be separated into individual response patterns (see Table 1).

Table 1 was arranged upon considering all possible response patterns of values for each of the three elements, a, b, and c, as seen in step 1. Steps 2 and 3 are essentially using a syllogistic notation noting if the implication exists, "1," or doesn't exist, "0." In step 4, the conjunctive logic function, representing the logical truth of the joining of steps 2 and 3, is again indicated by a

Table 1

Three-dimensional Plenum

Possible Response Patterns			Logical Structure			Compatible Response Patterns		
A	B	C	(A → B)	&	(B → C)	A	B	C
1	1	1	1	1	1	1	1	1
1	1	0	1	0	0		-	
1	0	1	0	0	1		-	
1	0	0	0	0	1		-	
0	1	1	1	1	1	0	1	1
0	1	0	1	0	0		-	
0	0	1	1	1	1	0	0	1
0	0	0	1	1	1	0	0	0
Step 1			Step 2	Step 4	Step 3	Step 5		

Note. A three-dimensional plenum of three variables was constructed in Step 1. Its one dimension, recorded in Step 5, was extracted in Steps, 2, 3, and 4. (Taken from Krus, 1974, p. 46.)

1-truth, 0-false schema. This plenum of response patterns can be seen to be reduced to a Guttman scale (Guttman, 1944) in step 5. This scale has the property of separating the individual response patterns into a unidimensional ordering.

There are many different methods to logically search for relationships within a given data set. Each logical relation, in turn, offers a rationale of inferences or non-inferences that may have theoretical merit. Within this system of logical constants, various interrelations resulting from logical connectives such as "and," "if and only if," "either/or" may be scrutinized. Appropriate utilization of these types of logical implications results in ordered hierarchies or unidimensional components. The implicative functions which lead to these ordered hierarchies, then, may be seen as the crux of the dimensionality issue.

The implicative functions (Table 2) are: (1) ($\leftarrow$) "is a prerequisite to," (2) ($\rightarrow$) "implies," (3) ($\not\leftarrow$) "is not a prerequisite to," and (4) ($\not\rightarrow$) "does not imply." Employing these functions, one can move from one function to another by reflecting variables within the system. In the binary case, this is simply a matter of creating a function's converse. Investigation of what happens when these functions are interchanged reveals that the (1,0) or (0,1) changes (which indicate a reverse in the direction of implication) are variance-generative (Krus, 1974, p. 10). This change can also be seen as an indicator that information becomes available. Such tuples differ from the (1,1) and (0,0) pairs, which are important for defining the

Table 2

Aliorelative Order-dependent Class of Propositional Functions Used by Order Analysis to Logically Search for Relationships within a Given Data Set

A	B	$A \rightarrow B$	$A \leftarrow B$	$A \nrightarrow B$	$A \nleftarrow B$
1	1	1	1	0	0
1	0	0	1	1	0
0	1	1	0	0	1
0	0	1	1	0	0
a		b	c	d	e

Note. Column a--Plenum of response for the two arguments, A and B.

b--Implication

c--Converse of Implication

d--Negative Implication

e--Converse of Negative Implication

within-order structure. While the (1,1) and (0,0) tuples determine the order within the already determined structure, the variance-generative tuples outline the structure of a dimension.

In the construction of such an implicative logico-mathematical system, variables are not differentially weighted. That is, no attempt is made to optimize or focus upon any one set of relationships. In such cases, the most appropriate logic functions are those of negative implication and its converse. The reason for this is that they differ only in their (1,0) and (0,1) tuples (as seen in Table 2). The (1,0) tuple refers to a confirmatory response pattern, and the (0,1) tuple to a disconfirmatory response pattern. These patterns of confirmatory and disconfirmatory response tuples have the essential property of structuring a particular domain of response patterns in a logical manner.

As shown by Krus (1974) and Cliff (1975b), the frequencies of negative implication and its converse, computed from the elements of a binary data matrix, may be used to derive a dominance matrix. Involved in the creation of the dominance matrix is the comparison of all possible row/column tuples. The result of all these comparisons is a dominance matrix with integer values in its row/column designates. These designates represent the frequency of domination of a particular row over a particular column. This comparison of all possible tuples--yielding a dominance matrix of frequencies--is identical to the process of matrix multiplication. However, to properly compare the appropriate (1,0) and (0,1) types, the matrix multiplication is performed on the transposed original data matrix and its logical negation or converse. The dominance matrix produced from this procedure is similar to a correlation or proximity matrix, in the same manner expressed by Coombs (1964). These two types of matrices, however, differ in one important respect--the preservation of directionality.

The value of obtaining a matrix of dominance type, rather than one proximal in nature, centers around the fact that a dominance matrix allows for the preservation of directionality between its elements while the proximity matrix does not. The importance of this distinction relies upon the fact that causal relations cannot be appropriately inferred from a correlative or proximity type solution. Because of the preservation of the directionality in dominance relations, however, the possibility of causal inferences associated with the developmental aspects of Guttman scales becomes a reality.

Difference Relations

The matrix of magnitudes generated from the multiplication of the transposed data matrix times its complement may be considered a dominance summary across all elements in the original data matrix. The magnitude in a given cell of the dominance matrix corresponds to the number of times an element dominates some other element. Conceptually, this magnitude can be thought of as the total number of (1,0) relations existing between the two vectors. Those (1,0) changes may also be thought of as the variance between any row vectors. This definition of variance is unique, and thus warrants further explanation.

Variance in ordering theory differs from the common psychometric interpretation (Krus, 1974, p. 7). The distinction between the psychometric notion and that of the order analytic approach lies within the philosophical distinction between magnitude and quantity (cf. Guilford, 1954, p. 7; Torgerson, 1958, p. 26). This distinction results from the fact that magnitude can only be defined by logical arguments, e.g., true-false relations, thus excluding any of the commonly used quantitative numerical indices. The building of a magnitude model for variance entails a frequency count of the different true-false logical relations. This reinterpretation of variance into magnitudes allows for the reflection of the existing difference relations. In addition, it potentially offers several advantages over the more classical notions. The amount of information contained in a given matrix, defined as the number of one-zero changes, can be directly calculated by simple summing. Compared to the relatively complex formulation of covariance, such an additive model is very appealing. In addition to the previously mentioned order analytic asset of preservation of the directionality of variation then, there is also the advantage of simplicity.

Implication Prerequisite Process

Most psychological data can be arranged in a matrix format, e.g., subjects by items or responses. Ordering theory attempts to identify the latent structures within a data matrix by observing the joint hierarchical relationships that exist among the items and persons. The underlying process is an attempt to organize and evaluate the common structure of the data in some systematic manner. Moreover, it should be noted that the joint nature of Guttman scales necessarily implies that given the structure of either persons or items, the remaining one is also determined. For order analysis, the search for underlying structure utilizes the observable hierarchical structure and bases its operations on those structures upon logical principles.

Within the logic system, various types of logical connectives, such as "and," "if and only if," "either," and "either/or," can be seen to have desirable properties when the goal is to organize data in logical substratums. This family of implicative functions has the ability to separate data into its component parts. As suggested earlier, it can be said to be dimension-generative, meaning that these functions possess the ability to systematically organize the data into independent dimensions.

The logical connectives that are the axiomatic components of the implications (as seen in Table 2) are "is a prerequisite to," "is not a prerequisite," "implies," and "does not imply." Again, it is possible to move from one of these implicative functions to another, simply by reflecting the variable values within the system. Based on this conceptualization of reflection, an understanding of how the (1,0) and (0,1) tuples can be generated should take on new meaning. By performing a series of reflections, e.g., from "implies" to "does not imply," a systematic set of relations are generated separately across the different rows.

What is of initial importance, of course, is the process of identification and separation of the existing subsystems before any internal structuring is undertaken. To be utilized in this prior case are functions that deal solely with the variance-generative tuples of (1,0) and (0,1). The simplest logic function that is suitable for delineating this type of change in relation is that of negative implication and its logical converse. Upon examination, it may be seen that the only difference existing between these tuples is the direction of change: 1 to 0 or 0 to 1. The name assigned to the (1,0) confirmatory response patterns is "prerequisite to." The (0,1) change, or disconfirmatory response patterns, is "is not a prerequisite to."

To summarize, the conditions of asymmetry, transitivity, and connectiveness are the foundation of ordering and produce the composite definition of an order relation. The conceptual product of asymmetry and transitivity conditions is the necessary higher-order notion of connectivity. When relations are transitive in nature, connectivity between the first and last elements in a hierarchy is implied. This property, upon which the notion of prerequisites is based, is the essence of a simple order which ultimately results in a unidimensional construct. Within a given data matrix, a set of these simple orders is said to exist. Therefore, uncovering these latent unidimensional structures involves the identification of the simple orders which in turn define dimensionality. Given a data matrix, the dimensions as commonly defined are the development of a set of orders (Russell, 1919, p. 29).

Order Analysis: An Overview

Order analysis, the prototype measurement model of ordering theory due primarily to Krus and Bart (1973), begins by generating an item dominance matrix which indicates the frequency of both the (1,0) and (0,1) item response patterns. The construction of the dominance matrix, $\underline{N}$, from the person by item response matrix $\underline{S}$ and its transposed complement, $\bar{\underline{S}}'$, may be represented as:

$$N = \bar{S}' \quad S \tag{1}$$

where an element $\underline{n}_{kj}$ is equal to the number of persons who get $\underline{k}$ wrong and $\underline{j}$ right, which is to say, the number of times item $\underline{k}$ dominates item $\underline{j}$. Thus, element $\underline{n}_{kj}$ represents the number of (0,1) disconfirmatory response patterns while its symmetric counterpart, $\underline{n}_{jk}$, represents the number of confirmatory or (1,0) response patterns. This matrix multiplication yields a square matrix of integer values indicating, as stated before, the number of times a row element dominates a corresponding column element. As in a correlation matrix, measurement error may also infiltrate the dominance matrix, in the form of intransitivities. To take this uncertainty into account, order analysis utilizes a probabilistic algorithm designed to measure the relative pureness of each particular pair of dominance relations. This is done by the construction of a $\underline{z}$-ratio (McNemar, 1947) between the symmetric entries of the dominance matrix. In effect, this z-ratio measures the degree of dominance that exists between two items by the formula:

$$z_{jk} = \frac{n_{jk} - n_{kj}}{\sqrt{n_{jk} + n_{kj}}} \ ; \ j \neq k \qquad (2)$$

The probabilistic interpretation of dominance matrices is based upon the assumption of equiprobability between the symmetric counterparts in a dominance matrix. A z_{jk} is calculated for each symmetric pair as well as z_{kj}, with each value being placed on its appropriate side of the diagonal, where the n_{jk} and n_{kj} components of the formula are the magnitudes contained in the original dominance matrix. For example, where $n_{jk} = 7$ and $n_{kj} = 2$, the existence of this apparent intransitivity can be evaluated by the z-test.

$$z_{jk} = \frac{7 - 2}{\sqrt{7 + 2}} = 1.67 \ .$$

As is apparent from the constant sum in the denominator, the symmetric entries in the Z-matrix are identical except for their signs, thus $z_{kj} = -1.67$. Though no direct evaluation is undertaken at this point, an obvious interpretation of the transitive strength between two items, in probabilistic terms, is possible.

By the construction of the Z-matrix comprising all possible relationships, the selection of a cutoff criterion z-value (here termed z-level) can then be implemented to consider only those relationships greater than or equal to a given strength. The z's below the designated minimum criterion $\underline{z}$-level, not being strong enough to warrant consideration, are set to "0," while for the relations greater than or equal to the $\underline{z}$-level criterion, a "1" is placed in the binary matrix, $\underline{M}$. Thus, the creation of the manifest or latent structure matrix, $\underline{M}$, can be represented as:

$$M = Z \geq z\text{-level} \tag{3}$$

Of importance is that a 1 can never be placed in symmetric elements of the $\underline{M}$-matrix because of the sign reversal in the symmetric entries of the $\underline{Z}$-matrix, thus $\underline{M}$ contains no intransitivities.

The extraction of the implicative chains from the binary manifest structure matrix, $\underline{M}$, also involves what can be considered a probabilistic approach. The procedure begins with both a row and column reordering of matrix $\underline{M}$, on the basis of the number of "1's" or transitive dominances. Once reordered, an implication chain of prerequisites is extracted starting with the first item and searching for the closest item that it dominates (is prerequisite to). Thus, the extraction process beginning with the first "1" in the first row is undertaken.

For clarity of description, this procedural overview concerns itself with the extraction of item chains, though the use of person dominance matrix yielding person chains is an equally viable alternative. Given, say, that item one dominates item three, they are combined into the first chain. The same procedure of looking for the closest item (in terms of total dominances) that item three dominates, and so on, is continued until the last item added has no dominances. With the inclusion of each item in the chain, the entire row and column of remaining dominances for that item are set to zero, thus not allowing it to be used in other chains. The yet unused items are reordered, and the search for prerequisite dominations continues until all items have been placed into a chain. The probabilistic nature of this procedure is founded in the assumption that an optimum solution consists of both the minimum number of chains to account for all the items and, more importantly, that the most appropriate grouping of items will emerge. Obviously, this need not and, because of the lack of any internal restrictions aimed at optimizing these relationships, probably will not occur. However, before these shortcomings of the probabilistic order analysis model are elaborated more fully, the description of the model in its entirety will first be presented.

Having extracted the implicative non-overlapping item chains representative of underlying Guttman scales, the total number of person dominances accounted for by each chain are calculated. The person dominance matrix, $\underline{X}_v$, for chain $\underline{v}$ can be calculated by:

$$X_v = S_v \bar{S}'_v \tag{4}$$

where $\underline{S}_v$ is the submatrix of persons by the items in chain $\underline{v}$. An element, $\underline{x}_{ih}$, is the person dominance matrix, $\underline{X}_v$, contains the number of times person $\underline{i}$ dominates person $\underline{h}$, i.e., the number of items in this reduced set that person $\underline{i}$ dominates that $\underline{h}$ does not.

The intransitivities that exist when the items in the chain do not form a simplex makes it misleading to simply calculate the rowwise marginals of $\underline{X}_v$. To adjust for these intransitivities, the $\underline{z}$-test is performed on the person dominance matrix, creating again a totally transitive binary dominance matrix, $\underline{M}_v$. The scalar notation of (5) denotes that the $\underline{ih}$ element of the

$$z_{ihv} = \frac{x_{ihv} - x_{hiv}}{\sqrt{x_{ihv} + x_{hiv}}} \; ; \; x_{ih.} \neq x_{hi.} \tag{5}$$

person dominance matrix $\underline{X}_v$, is converted to $\underline{z}$ values in matrix $\underline{Z}_v$. And (6) represents the logical comparison of all $\underline{z}_{ihv}$s to $\underline{z}$-level, thereby yielding the transitive dominance matrix, $\underline{M}_v$.

$$M_v = Z_v \geq \text{z-level} \tag{6}$$

The total number of dominances are calculated (7) for each person and are placed in an order loading matrix, $\underline{L}_{.v}$.

$$L_{.v} = 1'M_v \tag{7}$$

Thus, an integer value for each person equalling the number of persons dominated for a given chain of items representing a dimension is calculated. In factor analytic terminology, the matrix of order loadings is analogous to factor scores, while the row marginals are communalities. It is this similarity that prompts the rotation to simple structure of the order loading matrix (Krus, 1975, pp. 60-61).

Order Analysis: A Logical Paradox

A hypothetical example utilizing the six person by nine item response matrix seen in Figure 4 will be solved for its implicative chains using the probabilistic order analysis method. This exercise should both clarify the procedural steps as well as demonstrate the inherent shortcomings of this approach. Having pointed out the drawbacks relative to order analysis, suggestions upon which alternative methodology may be based will be presented.

In the first step, the construction of the item dominance matrix (N) in order analysis, is denoted

$$N = \bar{S}'S \tag{1}$$

where S is the hypothetical six person by nine item response matrix. The indicated matrix multiplicative results in the square matrix of order six, with integer values in its n elements. As suggested, order analysis assumes that the counter dominances appearing in the dominance matrix, presented in Table 3, are merely a function of error. The procedure for probabilistically evaluating these intransitive errorful relations, McNemar's (1947) z-test, is performed:

$$z_{jk} = \frac{n_{jk} - n_{kj}}{\sqrt{n_{jk} + n_{jk}}} \; ; \; j \neq k \tag{2}$$

All z's (Table 4) are then compared to the tolerance criterion, in this case, z-level = 1.0. For the z values exceeding the criteria z-level, a "1" is placed in the manifest structure matrix, M, theo-

Items

Persons		A	B	C	D	E	F
	1	1	0	0	0	0	0
	2	1	1	0	0	0	0
	3	1	1	1	1	0	0
	4	1	1	0	0	1	1
	5	1	0	0	0	1	0
	6	0	0	0	0	0	0
	7	0	1	1	1	1	0
	8	0	1	1	1	0	0
	9	0	1	1	0	0	0

Figure 4. Binary data matrix representing nine person response patterns on six items.

Table 3

Item Dominance Matrix N

	A	B	C	D	E	F
A	0	2	4	4	3	4
B	3	0	2	3	4	5
C	3	0	0	1	3	4
D	2	0	0	0	2	3
E	1	1	2	2	0	2
F	0	0	1	1	0	0

Note. Calculated by premultiplying the transposed complement of the original data matrix by the original data matrix.

Table 4

Z-matrix

	A	B	C	D	E	F
A	0	-.447	.378	.816	1	2
B	.447	0	1.41	1.73	1.34	2.24
C	-.378	-1.41	0	1	.447	1.34
D	-.816	-1.73	-1	0	0	1
E	-1	-1.34	-.447	0	0	1.41
F	-2	-2.24	-1.34	-1	-1.41	0

Note. Matrix of *z* values calculated from item dominance matrix.

retically representing the "true" dominance. The binary $\underline{M}$ matrix appears in Table 5.

$$M = Z \geq \left(\text{z-level} = (1.0)\right) \tag{3}$$

The assumptions and procedures presented to this point, designed to isolate the latent dimensions, appear reasonable, yet on closer inspection are paradoxical. The assumption that counter dominance or intransivities are simply brought about by error is clearly antithetical to the issue of multidimensionality. For counter dominance could actually represent the existence of multiple factors within the data unless, of course, the data are simply unidimensional. The paradox, obviously, is that by cancelling out the effect of the counter dominance in the multidimensional case secondary factors are obscured, leaving only a primary first factor. Order analysis by restricting its definition of dominance limits itself to the consideration of the most prominent unidimensional scale. This apparent breakdown at the basis of the order analytic method warrants a rethinking of the entire conceptualization of multidimensionality specific to a dominance matrix context. However, the further elaboration of other related procedural flaws will also be of considerable value, particularly in the consideration of an alternative procedure.

Given the manifest structure matrix, the next step of the order analysis procedure is the extraction of the dimensional chains. This process begins with the reordering of rows and columns of the $\underline{M}$ matrix, from most dominances to least, as has already been seen in

Table 5

Manifest Structure Matrix M

	B	C	A	D	E	F
B	0	1	0	1	1	1
C	0	0	0	1	0	1
A	0	0	0	0	1	1
D	0	0	0	0	0	1
E	0	0	0	0	0	1
F	0	0	0	0	0	0

Note. Reordered manifest structure matrix, _M_, using a _z_-level = 1.0.

Table 5. Commencing with the first row, the extraction procedure searches for the first 1, representative of the connection between the respective elements. In this example, item B dominates or is prerequisite to item C. Then item C dominates item D, and item D dominates item F, completing the hierarchy of the first chain. Thus, the chain of connections B → C → D → F created by the pairwise relations between the adjacent items in the chain defines the first unidimensional structure. Having completed the chain, the remaining relations of its member items are deleted. The construction of the next chain then is approached in an identical manner. In the present example, this yields the A → E chain. As all items are accounted for, a two-dimensional solution emerges.

Examination of this type of extraction procedure reveals two separate but related theoretical flaws. First, it can easily be seen that such a procedure does not guarantee that all chains present in the _M_ matrix are extracted. While the present example is not large enough to give a clearer illustration of this, the existence of the B → E → F and A → E → F chains does suggest this possibility. Once the existence of other chains is acknowledged, however, a more important question arises: Have the optimal chains been selected? Optimal, in this context, may refer to a number of criteria, such as the longest, the most Guttman-like, or the most orthogonal set of chains. In any case, the failure of the procedure to systematically consider any of these criterion standards seriously reflects on its credibility.

The second shortcoming of the extraction procedure is related to the broader issue of intradimension consistency and may be gleaned from Figure 5. In this figure, a simple reordering of the items in the submatrix of chains I and II reveals that one inconsistent relation exists in each chain. For chain I, item F for person 5 is not consistent, and similarly for chain II, person 7's correct response to item E is inconsistent. Because of the lack of any goodness-of-fit statistics measuring the chains' consistency relative to the perfect simplex, a potential user of this procedure cannot compare solutions at different levels of internal consistency. Obviously, such goodness-of-fit indices are crucial to any soundly based measurement procedure. Further, any descriptive statistic developed with this purpose in mind must be comparable to other measures of dimension construction, the most common being measures of variance.

Having already selected the chains, the next step in order analysis is to obtain order loadings for persons on each item chain. On a given chain of items, the person-order loadings represent the number of persons that a particular individual has consistently outscored.

The method for obtaining the order loading matrix begins with the calculation of a person dominance matrix, $\underline{X}_v$, from the submatrix of items, $\underline{S}_v$, from chain $\underline{v}$.

$$X_v = S_v \bar{S}'_v \tag{4}$$

		B →	C →	D →	F
	3	1	1	1	0
	7	1	1	1	0
	8	1	1	1	0
	9	1	1	0	0
Person	4	1	0	0	$\underline{1}$
	2	1	0	0	0
	1	0	0	0	0
	5	0	0	0	0
	6	0	0	0	0

		A →	E
	4	1	1
	5	1	1
	1	1	0
	2	1	0
Person	3	1	0
	7	0	$\underline{1}$
	6	0	0
	8	0	0
	9	0	0

Figure 5. Reordered hypothetical data matrix. Persons reordered within chains to illustrate the inconsistent responses, which are underlined.

As before, a $\underline{z}$-test is performed on the symmetric elements, $\underline{x}_{..v}$, of the dominance matrix. This time, however, it is person dominances rather than item dominances which are sought.

$$Z_{ihv} = \frac{x_{ihv} - x_{hiv}}{\sqrt{x_{ihv} + x_{hiv}}} \; ; \; x_{ihv} \neq x_{hiv} \qquad (5)$$

This $\underline{Z}$-matrix is then compared to the $\underline{z}$-level criterion value, which remains the same as the first test's $\underline{z}$-level = 1.0.

$$M_v \geq \text{z-level} = (1.0) \qquad (6)$$

The resulting matrix, $\underline{M}$, of transitive person dominance is summed and the marginal totals represent the

$$L_{.v} = 1'M_v \qquad (7)$$

number of persons an individual dominates. The earlier mention of consistent wins refers to a consistency inferred through use of the $\underline{z}$-test. The order loading matrix, $\underline{L}$, constructed for the two chains is presented in Table 6. Again, the integer values are interpreted as the number of persons that a particular individual outscored, given the consistent items he got correct. To complete the description of order analysis, the matrix of order loadings is standardized by converting the integer loadings into proportions, and then rotated to simple structure by varimax (Kaiser, 1958).

Though no extensive criterion of the person dominance interpretation of order loadings will be presented here, the methodology

Table 6

Order Loading Matrix O

Persons	CHAIN I $B \rightarrow C \rightarrow D \rightarrow F$	CHAIN II $A \rightarrow E$
1	0	3
2	3	3
3	5	3
4	4	7
5	0	7
6	0	0
7	5	3
8	5	0
9	4	0

Note. Implicative chains extracted from reordered manifest structure matrix with order loadings constructed using second z-level = 1.0.

upon which it is based is nonetheless subject to question (e.g., the *z*-test criterion). And if the extracted chains represent unidimensional scales underlying the data matrix, the need for rotation is unclear. Given the serious procedural shortcomings that have been discussed already, rotation may be nothing more than an attempt to sift through the structure in search of meaning. Any procedure that identifies the true unidimensional components, as order analysis purports to do, should have no need for rotation.

In summary, an example of the probabilistic version of order analysis has been traced through, noting its procedural shortcomings. Two problems emerge that, if resolved, could lead to a theoretically sound procedure for the extraction of multiple Guttman scales. First, the internal consistency of all elements in a chain, rather than just its adjacent members, is crucial. A solution to this problem would, in effect, also resolve the logical paradox of multidimensional counter dominances or intransivities. Internal consistency redefines the multidimensionality of the dominance matrix, allowing for an appropriate appraisal of the existing counter dominance. The second problem involves the development of standard procedures for selecting the optimal chains. Necessary to the selection of the optimal chains, however, is the consideration of all chains. Thus, the factor extraction methodology must first extract all chains before the selection procedure can be implemented.

MODEL AND THEORETICAL CONSTRUCTS

ERGO: A Procedure for Extracting Reliable Guttman Orders

An alternative approach that avoids the pitfalls of order analysis must redefine internal consistency in terms of its counterpart in classical test theory reliability. Cliff (1975b) suggests a series of indices, intended for a testing context, that establish a relation between dominance matrices and classical measurement. Among the indices described by Cliff (1975b) is a measure of internal consistency calculated from a dominance matrix that functions like the standard Kuder-Richardson formulae (KR) (1937). In conjunction with a methodology for defining an optimal, representative set of factors, the application of internal consistency presented by Cliff will be utilized in a new model termed ERGO. This alternative model attempts to resolve the paradoxes common to order analysis, while still associating itself with certain elements of ordering theory.

Internal Consistency

The index proposed by Cliff (1975b) is based upon two parameters and yields a numerical value which represents the internal consistency of a set of dominance relations. The first parameter is the relation of an obtained dominance matrix to a perfect Guttman simplex, and the second parameter is the relation of an obtained dominance matrix to a theoretically random set of dominances. Before the assumptions underlying the development of the index are presented, it should be noted that the identical operations hold for both person or item dominance matrices. However, to be illustratively consistent with the preceding example, the item dominance matrix will be used.

Given the item dominance matrix $\underline{N}$, the total number of relations, $\underline{u}$, is denoted in equation 8.

$$u = \Sigma\Sigma n_{jk} \tag{8}$$

The matrix notation for this summation, when $\underline{S}$ is the binary response matrix and $(S\bar{S}')$ its dominance matrix, is seen in equation 9.

$$u = 1'(\bar{S}'S)1 \tag{9}$$

If the rows and columns of the item dominance matrix are reordered in a descending fashion and the data are perfectly consistent, all the dominance relations will be contained in the upper triangle. Thus for perfectly consistent data the number of dominances in the upper triangle, $\underline{u}_m$, would equal the total, $\underline{u}$.

$$u_m = \sum_j \sum_{k>j} n_{jk} \tag{10}$$

By equating perfectly consistent data with a Guttman simplex, inconsistency can thus be evaluated in terms of dominances that fall below the upper triangular portion of the reordered dominance matrix. To put these relationships into a proper perspective, however, the consideration of a second parameter, a probabilistic distribution of dominances, must also be considered.

The assumption of no order, in the context of a dominance matrix, necessarily suggests an equal number of dominances for each n_{jk}. In the case of equally distributed dominances, n_{jk} and n_{kj} can be viewed as both estimates of the same quantity, v_{jk}. Thus, it follows that by averaging the symmetric entries an expected minimum, u_m, is produced.

$$u_{m'} = \sum_j \sum_{k>j} \left[n_{jk} - \tfrac{1}{2}(n_{jk} + n_{kj}) \right] \tag{11}$$

Distributing the sums, a maximum of $\frac{1}{2}u$ is realized.

$$u_{m'} = u_m - \tfrac{1}{2}u \tag{12}$$

Thus, a consistency index, c, relating the actual number of dominances in the upper triangular portion to that expected by chance, can be constructed. A one is subtracted from the upper triangular "good" dominance-to-chance proportion in order to distribute the consistency value from -1 to +1, thereby yielding

$$c = \frac{u_m}{\frac{1}{2}u} - 1 \tag{13}$$

By simply ridding the denominator of the fraction

$$c = \frac{2u_m}{u} - 1$$

we have Cliff's index of item consistency, c. By essentially the same rationale, Cliff (1975b) redefines the Kuder-Richardson formulae (1937) and the Loevinger index (1947) by considering the obtained upper triangular dominance, the maximum possible number of dominances, and those expected by chance. The foundation of the consistency index, considering its utilization of the same parameter that underlies such classical reliability coefficients as KR20 and KR21, makes it a most appropriate alternative for evaluating order consistency over the entire dominance matrix. What remains, having established the suitability of c, is the methodology through which it may be implemented.

Selecting Optimal Chains: Internal Procedure

Optimal chains employ both internal (within chains) and external (among chains) procedures that are directed toward selecting the most appropriate set of item combinations to represent the data. With the restriction that consistency across all member items remains as high as possible, the internal procedure concerns itself with the chaining of certain items. This contrasts with the order analysis procedure that operationalizes the chaining by considering only the adjacent connections. Once the unidimensional chains are constructed, the external optimization procedures attempt to order the chains in terms of their relative contribution in explaining the dimensionality of the data structure. Necessarily, the evaluation of relative contributions across chains has as a prerequisite the extraction of all chains.

The initial consideration, viz., the combining of items into chains, involves an iterative approach. For a given item chain, the most consistent item as determined by the highest consistency, c_p, where p represents the joint subset of items, is joined to the initial chain. If a tie in consistencies exists, the item closest, in terms of difficulty level, is given priority. Thus, for each item, k, a consistency is calculated, c_{p+k}, combining the new item with the items already in the chain.

$$c_{p'} = \max c_{p+k} \tag{14}$$

The iterative procedure of sequentially adding items to chains on the basis of the overall consistency of the chain is operationalized for all items by allowing each item to initialize its own chain. In matrix terms, the rows become representative of chains while the columns remain representative of items. The ij entries of the final consistency matrix, F, correspond to the consistency level at which the item, j, was added to the chain, i. An illustration of this procedure for the hypothetical example presented previously is seen in Table 7.

To identify member items, an element-by-element comparison of matrix F is made to a subjectively determined consistency cutoff value, cv. For example, by setting cv at any value greater than .84, the resulting binary membership matrix, B, is produced (see Table 8).

$$B = F \geq cv$$

Table 7

Final Consistency Matrix $\underline{F}$

Items

Chains		B	A	C	D	E	F
	I	1.000	.516	1.000	1.000	.611	.800
	II	.680	1.000	.484	.571	.800	1.000
	III	1.000	.516	1.000	1.000	.611	.800
	IV	1.000	.516	1.000	1.000	.611	.800
	V	.833	.680	.516	.571	1.000	1.000
	VI	.833	.680	.516	.571	1.000	1.000

Note. Rows represent chains and entries in reordered columns represent consistency at which item $\underline{j}$ was added to chain $\underline{i}$.

Table 8

Binary Item Membership Matrix B

Chains		Items B	A	C	D	E	F
	I	1	0	1	1	0	0
	II	0	1	0	0	0	1
	III	1	0	1	1	0	0
	IV	1	0	1	1	0	0
	V	0	0	0	0	1	1
	VI	0	0	0	0	1	1

Note. Binary item membership matrix, B, resulting from any consistency cut off value > .84.

Removing the duplication, three chains are revealed, namely, B → C → D (I), A → F (II), and E → F (III). Because each chain has a consistency value of one, as may be seen in Table 7, there are no inconsistent relations. Figure 6 breaks down the chains into their respective submatrices, confirming the existing simplex for each chain.

Having completed the outline of procedures involved with internal consistency, the procedures utilized in evaluating the contributions of the extracted chains will be presented. However, before the details of the considerations used in evaluation are brought forth, the scoring procedure implemented in ERGO needs to be discussed. Instead of defining scores as person dominance as is done in order analysis, a straightforward summary of consistent relations (see Figure 6) for an individual for a given chain defines score. The redefinition of score using marginal sums offers a convenience of interpretation which will be demonstrated in the empirical example to be presented later.

Selecting Optimal Chains: External Procedure

The decision concerning the optimal solution and ordering of chains, like the ordering of factors in factor analysis (FA) or dimensions in multidimensional scaling (MDS), must be related to the overall epistemic contribution of the dimensions. However, the distinction between the structure of the dimensions recovered with the ERGO procedure and those from either FA or MDS requires a redefining of contribution. With FA and MDS, the variables or stimuli are assigned

	I					II				IV		
	B →	C →	D	(Σ)		A →	F	(Σ)		E →	F	(Σ)
(3)	1	1	1	3	(4)	1	1	2	(4)	1	1	2
(7)	1	1	1	3	(1)	1	0	1	(5)	1	0	1
(8)	1	1	1	3	(2)	1	0	1	(7)	1	0	1
(9)	1	1	0	2	(3)	1	0	1	(1)	0	0	0
(2)	1	0	0	1	(5)	1	0	1	(2)	0	0	0
(4)	1	0	0	1	(6)	0	0	0	(3)	0	0	0
(1)	0	0	0	0	(7)	0	0	0	(6)	0	0	0
(5)	0	0	0	0	(8)	0	0	0	(8)	0	0	0
(6)	0	0	0	0	(9)	0	0	0	(9)	0	0	0

Persons

Figure 6. Reordered data for ERGO solution. Extraction of three chains from hypothetical data as determined by ERGO procedure. Person numbers in parentheses.

weights on all factors or dimensions, but in ERGO weights are assigned to dimensions which are actually composed of subsets of items. Thus, a procedure for optimally combining the dimensional chains to account for the maximum number of items appears reasonable. When additional chains are being considered for selection, the maximum number of items refers to unique, or yet unaccounted for, items.

A procedure for ordering the extracted chains in terms of their maximum number of unique items added appears straightforward. Complications from ties arise, however, making additional considerations necessary. For a given set of chains, the selection procedure first calculates for each pair of chains the total number of unique items. Given that one such pair of chains has more than any other, the selection is greatly simplified. The chain containing the most items is put first, the remaining chain second, with additional chains being added corresponding to their number of unique (yet unaccounted for) items. In the case of a tie of unique items, the chain having the least overlap (items in common) with the already accounted for items is chosen. When pairs of pairs are tied in both number of unique and number of overlapping items, a still different procedure is called for. This is to take the pair of chains that, within the pair, demonstrates the largest difference in terms of their resulting orders (person orders). The largest difference is defined as the largest number of inversions in their corresponding person orders. To amplify, a single inversion in order exists between any pair when, say, aRb in one ranking is compared to bRa in the other. Thus, by totalling the number of inversions and the number of agreements and then adjusting for the number of possible agreements, an index reflecting the degree of dissimilarity of the two orders is developed. The index suggested parallels the procedure utilized in the calculating of Kendall's tau a (Kendall, 1962). It differs, however, in that the most appropriate selection (having the most inversions) is the lowest tau value, as tau is a measure of agreement rather than disagreement.

To best illustrate the process by which chains are ordered, the hypothetical example will again be referred to, beginning with the item membership matrix, $\underline{B}$. Chains III, IV, and VI in matrix $\underline{B}$ will be removed because of their obvious redundancy, leaving for consideration chains I, II, and V. The heuristics upon which the subsequent chain selection procedures rely are founded in Boolean arithmetic, briefly summarized here:

$$0+0=0;\ 0+1=1;\ 1+1=1;\ 1\times1=1;\ 1\times0=0;\ 0\times0=0$$

A summary of unique items between chains $\underline{i}$ and $\underline{j}$ is computed from the $\underline{B}$ matrix and placed in the appropriate upper triangular $\underline{ij}$ element of matrix $\underline{O}$ (see Table 9). Thus,

$$o_{ij} = \sum_{i} \sum_{j>i} (b_i + b_j) \qquad (16)$$

where "+" indicates Boolean arithmetic, and $\underline{b}_i$ and $\underline{b}_j$ represent rows corresponding to chains in the nonredundant item membership matrix $\underline{B}$. The lower triangular $\underline{ji}$ elements of matrix $\underline{O}$ are the number of overlapping items between chain $\underline{i}$ and chain $\underline{j}$ denoted as

Table 9

Matrix $\underline{O}$

Chains		I	II	V
Chains	I	-	5	5
	II	0	-	3
	V	0	1	-

Note. Upper triangular portion summarizing all pairwise uniquenesses while lower triangular portion summarizes the pairwise overlap.

$$o_{ji} = \sum_{j} \sum_{i>j} (b_j \times b_i) \tag{17}$$

where "x" again indicates Boolean arithmetic.

Inspection of Table 9 reveals that chain I is tied with chain II and chain V with five unique items. The overlap criterion cannot break the tie, as neither chain has any elements in common with chain I. In this case, the next step is the correlating of the person scores derived from their respective chains (I with II and I with V) so as to determine the most dissimilar pair. The resulting taus as seen in the upper half of Table 10 are -.139 and .0278, respectively. On this basis, the I-II pair is selected. Having not accounted for all the items (viz., item E), chain V is added, resulting in the final order of I, II, and V.

Other situations not represented in this example need to be mentioned. First, given that all items are accounted for, any remaining chains are dropped. Second, the converse situation, where additional chains add only a relatively small number of items, thereby having little substantive value, suggests the implementing of a scree-type procedure to discount the smaller chains. And third, where an attempt for orthogonality of recovered dimensions is desired, the removal of items contained in more than one chain is suggested. To allow for the evaluation of the above mentioned considerations, a summary matrix for each solution as is seen in Table 6 is constructed. The values in the upper triangular portion, as already mentioned, refer to the taus between chains. The values in the lower triangular portion

Table 10

ERGO Summary Matrix

		Chains		
		I	II	V
Chains	I	3	-.139	.0278
	II	-.139	2	.389
	V	-.0278	.611	1

Note. Summary matrix with tau a values in upper triangular portion, tau a discounting all overlapping elements in lower triangular portion, and number unique items added by that chain in diagonal.

refer to the tau a correlations discounting common items, and the integer values in the diagonal denote the number of unique elements added to the solution by the inclusion of that chain.

In summary, having an index that corresponds directly to such classical indices as the Kuder-Richardson formulae (1937) like the internal consistency index proposed by Cliff (1975b) provides an unambiguous procedure for combining items into chains. One possible improvement, however, is a weighting system that adjusts more fully for item difficulty, rather than a total reliance on item consistency. Unfortunately, the external selection process, not being grounded in such fundamentally sound principles, cannot be considered as favorably. The shortcomings become manifest as the dimensionality increases, thus allowing more chance for an erroneous selection. It may be seen that until indices are developed that maximize specific relationships, preferably in both the item and person dominance contexts, the entire extraction procedure may remain suspect. At any rate, a more sophisticated definition of chains relating directly to the duality that exists between item and person dominances is definitely called for. At this time, having not resolved this issue, the selection procedures as described will be implemented in the dimensionalizing of an empirically derived data matrix.

AN EMPIRICAL EXAMPLE

Method

To demonstrate the order extracting procedure in a practical context, an investigation was designed to allow for maximum empirical validation. Selected as a representative, well-known Guttman scale was a Bogardus-type social distance scale (Bogardus, 1925). A questionnaire was constructed that incorporated seven social distance items in a binary choice format (see Appendix). All of these were then paired with three ethnic groups: Black, Mexican-American, and Oriental. By having members of the three ethnic minority groups, in addition to Anglos, responding to the questionnaire, it was felt that the ordering of the items would not only group together items referring to the same ethnic group, but would also serve to cluster the individuals with regard to ethnic group membership.

The 21-item social distance questionnaire was administered to 84 undergraduates at the University of Southern California, who participated in the fulfillment of course requirements. Prior to the administration, subjects were asked to consider the general image of ethnic groups other than their own. To assure compliance with this request, subjects were asked to construct a written outline listing several key descriptors of each group. Once this preliminary task was completed, the subjects were instructed to keep in mind the images rather than a specific individual when responding to the social distance items. This was done to maximize the number of resulting response patterns. Of the 84 respondents, 60 gave non-duplicate response patterns for the 21 items. Ethnic composition of the 60 respondents was as follows: six Mexican-Americans, eight Blacks, fourteen Orientals, and thirty-two Anglos.

Results

The dominance matrix, final consistency matrix, and reduced chain by item membership matrix calculated at a minimum consistency of .95 are presented in the Appendix. The thirteen nonredundant chains were subjected to the chain selection procedure, which reduced to seven the number of chains necessary to account for all the items. The reordering of the seven chains followed the previously described procedural steps of first maximizing the number of unique items and in the case of a tie selecting the chain with the fewest number of overlapping items. The summary matrix for the reordered set of seven chains containing the number of unique items added in the diagonal as well as to their rank order intercorrelations (see Table 11).

As suggested, the issue of limiting the number of chains or dimensions to those considered "significant" is resolved by the application of a scree-type procedure to the respective number of unique items added. In doing so, the apparent cutoff is the third chain, as the fourth chain adds only 2 items to the 16 already accounted for by

Table 11

Summary Matrix for 60 Persons by 21 Item Social Distance Data

Chains	X	XI	I	VIII	XIV	XVIII	IX
X	7	.18	.14	.16	.76	.70	.17
XI	.20	5	.13	.72	.18	.19	.12
I	.12	.16	4	.12	.14	.22	.89
VII	.17	.45	.24	2	.10	.15	.12
XIV	.48	.07	.21	.16	1	.74	.16
XVII	.49	.11	.25	.26	.53	1	.24
IX	.18	.10	.53	.34	.42	.39	1

Note. Tau a values between complete item-chains are in upper triangular portion. In lower triangular portion are tau a values for scores computed from number of unique items added by that chain, which appears in the diagonal.

the first three chains. The items of which the three chains are composed are listed in hierarchical order in Table 12.

As seen in Table 12, the consistency of the ethnic group referenced within the three chains, with a few exceptions, namely items N and S in chain X and item U in chain XI, is apparent. Chain X is composed of items illustrating a social distance scale for Mexican-Americans, as are chain XI for Blacks and chain I for Orientals. Thus, the correspondence of the three item chains to each of the three ethnic groups reflects favorably on the chain selection procedures. However, the existence of the exceptions within chains X and XI does not allow for a clear definition of an individual's social distance specific to an ethnic group. In an attempt to resolve this situation, the overlapping items, that is, items contained in more than one chain, are eliminated. The remaining fourteen unique items (as denoted by an asterisk [*] in Table 12) still contain one inconsistent item, item U in chain XI.

The appropriateness of the resulting solution can be illustrated by comparing the recovered hierarchical groupings of items (Table 12) to the proposed hypothetical ordering of social distance items. Except, of course, for the one inconsistent item, U, in chain XI, the ordering of the unique items within each chain corresponds closely with the hypothetical ordering. In fact, the only exception is the reversal of items 06 and 07 in chain I. Therefore, aside from a few minor flaws, both the homogeneity of scales and the ordering of items within scales resulting from the ERGO procedure would appear quite reasonable.

Table 12

Items in Chains X, XI, and I

	Hypothetical Order	Alphabetical No.	Question	Ethnic Group
(X)	(O6)	N.	Would work in same office	Oriental
	(B7)	B.	Would have as speaking acquaintances	Black
	(M7)	*H.	Would have as speaking acquaintances	Mex-Amer
	(M6)	*C.	Would work in same office	Mex-Amer
	(M4)	*P.	Would invite for dinner	Mex-Amer
	(M2)	*R.	Would have as close friends	Mex-Amer
	(M1)	*J.	Would marry into group	Mex-Amer
(XI)	(B7)	B.	Would have as speaking acquaintances	Black
	(O3)	*U.	Would have as next door neighbors	Oriental
	(B5)	*F.	Would consider as friends	Black
	(B4)	*A.	Would invite for dinner	Black
	(B2)	*Q.	Would have as close friends	Black
	(B1)	*M.	Would marry into group	Black
(I)	(O6)	N.	Would work in same office	Oriental
	(O7)	*L.	Would have as speaking acquaintances	Oriental
	(O5)	*O.	Would consider as friends	Oriental
	(O4)	*D.	Would invite for dinner	Oriental
	(O1)	*K.	Would marry into group	Oriental

Note. Hierarchically ordered items comprising first three dimensions. Hypothetical ethnic distance coding is in parentheses. Asterisk (*) refers to items contained in only one chain.

The further evaluation of the ERGO procedure, the clustering of individuals into their appropriate ethnic groups, is realized in terms of their scores (see Appendix). As would be expected, every ethnic group member endorsed all the items referring to his group. More important, however, is the direct correspondence of an individual's score to his relative position on the unidimensional constructs, thereby permitting ease of interpretation. This fact, combined with the developmental interpretation stemming from the notion of logical prerequisites underlying Guttman orders, adds further clarity to the substantive interpretability of person scores.

To illustrate, the scores for the first Anglo (1 3 2) can be directly interpreted as the subject's social distance relative to the three ethnic groups. Thus, the score of 1 for the first chain (X) composed of the Mexican-American items corresponds to item M7 (would have as speaking acquaintances). Similarly, the score of 3 on the chain referring to Blacks (XI) indicates item B4 (would invite for dinner), while the score of 2 on the Oriental item chain (I) corresponds to item O5 (would consider as friends). The endorsement of the items below the score level designated is assumed, thereby giving a more precise meaning to the scores. The developmental notion of prerequisites corresponds to the previously suggested positioning of people and items on the same unidimensional scale. This dual positioning allows for both persons and items to be considered in relation to each other, yielding an increase in the number of relationships that are directly observable.

However, in the case of inconsistency, which appears as the endorsement of an item without the endorsement of its prerequisite, the question of what score level is most appropriate may be raised. In this example, it was assumed that the failure to positively endorse an item precluded consideration of other endorsements further along in the hierarchy. This highly simplistic approach to scoring (for these particular data) did not suffer from multiple errors, which are defined as the occurrence of endorsements of more than one item without the necessary endorsement of some prerequisite. For data involving instances where multiple errors do exist, more sophisticated types of scoring procedures involving probabilistic evaluation of the individual's response pattern need to be developed.

The overall evaluation of the results appears favorable. The identification of hierarchically graded orders within the three ethnic groups would verify this. In addition to the resulting ethnic-item hierarchies, the case of interpretation of person scores along the recovered ethnic dimensions suggests ERGO to be a viable method for recovering dimensions in dichotomous items. The implications of combining persons and items on the same scale, thereby permitting the direct evaluation of person-item relationships, present the researcher with many interesting possibilities, especially those involving developmental relationships. Moreover, it is this knowledge of both the person and item relations that has practical as well as theoretical importance.

SUMMARY

A method of factor extraction specific to a binary matrix, illustrated here as a person-by-item response matrix, has been presented. The extraction procedure, termed ERGO, differs from the more commonly implemented dimensionalizing techniques, factor analysis and multidimensional scaling, by taking into consideration item difficulty. Utilized in the ERGO procedure is the calculation of a dominance matrix which, for either persons or items, has the important attribute of allowing directionality to be inferred between relations.

The theory underlying ERGO is founded in ordering theory (Airasian & Bart, 1972), with its interpretation of dominance relations following logical implicatives similar to Boolean algebra. The redefinition of dimensionality using both the notion of dominance relations and that of logical prerequisites can more aptly be identified with the definition of a Guttman order, thereby placing emphasis on the developmental aspects of recovered sets of dimensions. It is this interpretation that allows for the duality of relationships between persons and items. The resulting placement of both persons and items on the same unidimensional construct presents the researcher with the opportunity to observe direct relations between the two.

A preliminary attempt to utilize the apparent advantages associated with the extraction procedure based on dominance relations, order analysis (Krus, Bart, & Airasian, 1975) is used. This is done both to further explicate the implications of ordering theory as well as to point out the issues with which a dimensionalizing procedure of this type must concern itself. In this discussion, the procedural shortcomings of order analysis are presented to acquaint the reader with the obstacles that an alternative approach must overcome. Premier among these is the failure of order analysis to consider the true nature of multidimensionality in a dominance matrix context. This appears in the order analytic assumption that counter dominance relations are merely a product of error, rather than being manifestations of the multidimensional nature of the data. The alternative procedure (ERGO) is developed by dealing with this essential point.

The key to the dimension extraction problem of ERGO rests in the formulation of an index of dimension consistency that is comparable to classical measures such as the Kuder-Richardson formulae (1937) and the Loevinger homogeneity indices (1947). Cliff (1975b), by demonstrating the relation between these classical indices and their redefinition in a dominance matrix context, lays the foundation for the development of an alternative procedure. Thus, by adopting a consistency measure developed there, ERGO iteratively adds items together, resulting in the construction of various sets of implicative chains representing dimensions. Having constructed these chains, the ERGO procedure orders the chains in terms of maximal number of items contributed. The chain evaluation procedure can best be explained as an attempt to maximize the number of items accounted for in a given dimensional solution.

To give additional understanding of both the ERGO process and the potential advantages a procedure of this type offers, an empirical example which utilizes social distance items (Bogardus, 1925) paired individually with three ethnic groups was analyzed for respondents representing four ethnic groups. Emphasized in the solution was the duality of relationships inherent in a procedure such as this, that is based upon the principles underlying Guttman orders. The results demonstrated the ability of ERGO to (1) group items referring to the same ethnic group; (2) uncover hierarchically graded orders within each chain; (3) select the three chains that corresponded to the three ethnic groups; and (4) cluster individuals by ethnic group according to their scores.

In summary, the ERGO procedure, based on the uncovering of logical relationships within the context of a dominance relation and postulated in ordering theory (Airasian & Bart, 1972), has been proposed. The rationale, upon which a dimension extraction procedure specific to a binary matrix is based, is accomplished by demonstrating the shortcomings of currently implemented procedures. Given the shortcomings and a definition of the problems confronting a procedure whose goal is to analyze the dimensionality of a dominance matrix, an alternative procedure, ERGO, is presented. In applying the ERGO procedure to well-known social distance type items (Bogardus, 1925), empirical validation of the procedure was attained.

REFERENCES

REFERENCES

Airasian, P., & Bart, W. Ordering theory: A new and useful measurement model, Educational Technology, 1972, 13, 56-60.

Bogardus, E. S. Measuring social distances. Journal of Applied Sociology, 1925, 9, 383-393.

Coombs, C. H. A theory of data. New York: Wiley, 1964.

Coombs, C. H., Dowes, R. M., & Tversky, A. Mathematical psychology: An elementary introduction. Englewood Cliffs, N.J.: Prentice-Hall, 1970.

Cliff, N. Complete orders from incomplete data: Interactive ordering and tailored testing. Psychological Bulletin, 1975, 82(2), 289-302. (a)

Cliff, N. A basic test theory generalizable to tailored testing (NR No. 150-373, Tech. Rep. No. 1). Los Angeles: University of Southern California, 1975. (b)

Guilford, J. P. Psychometric methods (2nd ed.). New York: McGraw-Hill, 1954.

Guttman, L. A. A basis for scaling qualitative data. American Sociological Review, 1944, 9, 139-150.

Harary, F., Norman, R., & Cartwright, D. Structural models: An introduction to the theory of directed graphs. New York: Wiley, 1965.

Horst, P. Factor analysis of data matrices. New York: Holt, 1965.

Kaiser, H. F. The varimax criterion for analytic rotation in factor analysis. Psychometrika, 1958, 23, 187-200.

Kendall, M. G. Rank correlation methods. London: Griffin & Company, 1962.

Krus, D. J. Order analysis of binary data matrices. Los Angeles: Theta Press, 1974.

Krus, D. J., & Bart, W. M. An ordering-theoretic method of multidimensional scaling of items. Educational and Psychological Measurement, 1974, 34, 525-535.

Krus, D. J., Bart, W. M., & Airasian, P. W. Ordering theory and methods. Los Angeles: Theta Press, 1975.

Kuder, G. F., & Richardson, M. W. The theory of the estimation of test reliability. Psychometrika, 1937, 2, 151-160.

Lazerfeld, P. F. Latent structure analysis. In S. Koch (Ed.), Psychology: A study of a science (Vol. 3). New York: McGraw, 1958.

Levy, P. On the relation between test theory and psychology. In P. Kline (Ed.), New approaches in psychological measurement. New York: Wiley, 1973.

Loevinger, J. A systematic approach to the construction and evaluation of test of ability. Psychological Monographs, 1947, 61(4, Whole No. 285).

McNemar, Q. Note on the sampling error of the difference between correlated proportions or percentages. Psychometrika, 1947, 12, 153-157.

Russell, B. Introduction to mathematical psychology (1919). New York: Simon & Schuster, 1971.

Shepard, R. N. The analysis of proximities: Multidimensional scaling with an unknown distance function, I. Psychometrika, 1962, 27, 125-140.

Spearman, C. General intelligence objectively determined and measured. American Journal of Psychology, 1904, 15, 201-292.

Torgerson, W. S. Theory and methods of scaling. New York: Wiley, 1958.

APPENDIX

SOCIAL DISTANCE QUESTIONNAIRE

Age:________	Sex:________
Ethnic Background:____________	

Instructions

1. Fill in Identification box at upper left. Note that your name is in no way referenced.
2. Circle the appropriate response for each item.
3. Please remember to give your FIRST REACTION for every group.
4. Remember to give your reaction to your IMAGE of each GROUP as a whole-- NOT an INDIVIDUAL.

		Question	Ethnic Group	Responses
(B4)	A.	Would invite for dinner	Black	Yes No
(B7)	B.	Would have as speaking acquaintances	Black	Yes No
(M6)	C.	Would work in same office	Mex-Amer	Yes No
(O4)	D.	Would invite for dinner	Oriental	Yes No
(M3)	E.	Would have as next door neighbors	Mex-Amer	Yes No
(B5)	F.	Would consider as friends	Black	Yes No
(M5)	G.	Would consider as friends	Mex-Amer	Yes No
(M7)	H.	Would have as speaking acquaintances	Mex-Amer	Yes No
(B3)	I.	Would have as next door neighbors	Black	Yes No
(M1)	J.	Would marry into group	Mex-Amer	Yes No
(O1)	K.	Would marry into group	Oriental	Yes No
(O7)	L.	Would have as speaking acquaintances	Oriental	Yes No
(B1)	M.	Would marry into group	Black	Yes No
(O6)	N.	Would work in same office	Oriental	Yes No
(O5)	O.	Would consider as friends	Oriental	Yes No
(M4)	P.	Would invite for dinner	Mex-Amer	Yes No
(B2)	Q.	Would have as close friends	Black	Yes No
(M2)	R.	Would have as close friends	Mex-Amer	Yes No
(B6)	S.	Would work in same office	Black	Yes No
(O2)	T.	Would have as close friends	Oriental	Yes No
(O3)	U.	Would have as next door neighbors	Oriental	Yes No

Note: Coding within parentheses indicate hypothetical order of social distances for each ethnic group. Their designated codings did not appear on the questionnaire.

DOMINANCE MATRIX

	N	L	B	O	U	D	S	H	T	C	F	A	P	G	I	K	E	Q	R	J	M
N	0	2	3	3	5	8	9	10	10	13	15	25	26	27	27	25	31	32	40	48	48
L	1	0	3	2	4	7	9	10	9	13	15	24	25	25	26	27	29	31	38	46	47
B	1	2	0	4	3	9	7	7	10	11	12	22	23	24	24	27	29	29	37	45	45
O	0	0	3	0	4	5	9	10	7	12	15	24	24	25	26	25	28	31	37	45	47
U	1	1	1	3	0	7	8	6	9	10	12	21	22	23	25	25	26	27	35	43	43
D	0	0	3	0	3	0	8	9	5	10	15	22	21	23	24	20	26	27	34	42	42
S	1	2	1	4	4	8	0	7	8	8	11	17	20	23	18	23	26	23	33	40	39
H	1	2	0	4	1	8	6	0	9	6	10	18	16	18	20	24	22	24	30	33	38
T	0	0	2	0	3	3	6	8	0	8	12	21	19	20	21	19	24	26	32	39	42
C	1	2	1	3	2	6	4	3	6	0	11	19	14	15	20	21	20	24	27	35	38
F	1	2	0	4	2	9	5	5	8	9	0	11	17	19	14	20	23	17	28	34	33
A	1	1	0	3	1	6	1	3	7	7	1	0	12	16	9	14	18	9	20	26	23
P	1	1	0	2	1	4	3	0	4	1	6	11	0	7	13	13	9	16	14	22	26
G	1	1	0	2	1	5	5	1	4	1	6	14	6	0	16	15	10	20	14	22	29
I	1	1	0	3	1	6	0	3	5	6	2	7	12	15	0	13	15	8	22	26	21
K	0	0	1	0	1	0	3	5	1	5	6	10	10	13	11	10	14	15	19	25	25
E	1	0	1	1	0	4	4	1	4	2	7	12	4	6	11	12	0	16	10	17	23
Q	1	1	0	3	0	4	0	2	5	5	0	2	10	15	3	12	15	0	17	21	16
R	1	0	0	1	0	3	2	0	3	0	3	5	0	1	9	8	1	9	0	8	15
J	1	0	0	1	0	3	1	0	2	0	1	3	0	1	5	6	0	5	0	0	9
M	1	1	0	3	0	3	0	0	5	3	0	0	4	8	0	6	6	0	7	9	0

Note: Dominance Matrix constructed from Social Distance Questionnaire for 60 subjects. Items have been reordered in descending fashion on the basis of their dominances.

FINAL CONSISTENCY MATRIX

	W	L	B	O	U	D	S	H	T	C	F	A	P	G	I	K	E	Q	R	J	N
I	1	.984	.885	1	.875	1	0	0	.945	0	0	0	0	0	0	1	0	0	.898	.918	0
II	.984	1	.885	1	.875	1	0	0	.945	0	0	0	0	0	0	1	0	0	.898	.918	0
III	.92	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
IV	.984	1	.885	1	.875	1	0	0	.945	0	0	0	0	0	0	1	0	0	.898	.918	0
V	.943	0	0	0	1	0	0	0	0	0	0	0	0	0	0	.939	0	0	0	0	0
VI	.984	1	.885	1	.875	1	0	0	.945	0	0	0	0	0	0	1	0	0	.898	.918	0
VII	.95	.917	.987	0	.932	0	1	0	0	0	.889	.901	0	0	1	0	0	.966	0	.882	1
VIII	1	.949	1	.905	.963	0	0	1	0	.926	0	0	1	.889	0	0	.94	0	.973	.967	0
IX	.97	1	.885	1	.875	.945	0	0	1	0	0	0	0	0	0	.978	0	0	.898	.918	0
X	1	.932	.963	.905	.945	0	0	.955	0	1	0	0	.962	.889	0	0	.926	0	.966	.959	0
XI	.944	.927	1	0	.975	0	0	.897	0	0	1	.953	0	0	0	0	0	1	.882	.905	1
XII	1	.927	.964	0	.943	0	0	.897	0	0	.959	1	0	0	0	0	0	.954	.882	.965	.959
XIII	1	.949	.973	.905	.963	0	0	1	0	.926	0	0	1	.889	0	0	.94	0	1	.969	0
XIV	.952	.94	1	.889	.967	0	0	.974	0	.916	0	0	.904	1	0	0	.929	0	.976	.976	0
XV	.95	.917	.987	0	.932	0	1	0	0	0	.889	.901	0	0	1	0	0	.966	0	.862	1
XVI	.9	.894	1	.897	.986	1	0	0	.875	0	0	0	0	0	0	1	0	0	.893	.839	0
XVII	1	.966	.953	.905	.964	0	0	.947	0	.926	0	0	.94	.889	0	0	1	0	.964	1	0
XVIII	.944	.927	1	0	.975	0	0	.897	0	0	1	.955	0	0	0	0	0	1	.882	.908	1
XIX	1	.949	.97	.905	.963	0	0	.973	0	.926	0	0	1	.889	0	0	.94	0	1	1	0
XX	1	.968	.953	.905	.964	0	0	.947	0	.926	0	0	.94	.889	0	0	.964	0	1	1	0
XXI	.934	.921	.966	0	.936	0	.876	.899	0	0	.969	.945	0	0	0	0	0	1	.887	.909	1

Note: Items were added to chains until the chain's consistency reached .875. For items not added at this level a zero was placed in its row/column designate.

REDUCED CHAIN BY ITEM MEMBERSHIP MATRIX

Items

CHAINS	N	L	B	O	U	D	S	H	T	C	F	A	P	G	I	K	E	Q	R	J	M
X	1	0	1	0	0	0	0	1	0	1	0	0	1	0	0	0	0	0	1	1	0
XI	0	0	1	0	1	0	0	0	0	0	1	1	0	0	0	0	0	1	0	0	1
I	1	1	0	1	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
VII	0	0	1	0	0	0	1	0	0	0	0	0	0	0	1	0	0	1	0	0	1
XIV	1	0	1	0	1	0	0	1	0	0	0	0	0	1	0	0	0	0	1	1	0
XVII	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	0
IX	1	1	0	1	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0
VIII	1	0	1	0	1	0	0	1	0	0	0	0	1	0	0	0	0	0	1	1	0
XII	1	0	1	0	0	0	0	0	0	0	1	1	0	0	0	0	1	0	0	0	1
XXI	1	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	1
XVI	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
III	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
V	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	(O6)	(O7)	(B7)	(O5)	(O3)	(O4)	(B6)	(M7)	(O2)	(M6)	(B5)	(B4)	(M4)	(M5)	(B3)	(O1)	(M3)	(B2)	(M2)	(M1)	(B1)

Note. Thirteen nonredundant chains are ordered on basis of chain selection procedure of accounting for most unique items. Coding in parentheses indicates hypothetical ethnic distances.

SCORE MATRIX

Ethnic Group	Chain X	Chain XI	Chain I
M	5	4	3
M	5	4	4
M	5	3	3
M	5	1	3
M	5	3	2
M	5	2	2
B	5	5	4
B	4	5	3
B	2	5	0
B	1	5	1
B	3	5	3
B	5	5	1
B	3	5	4
B	2	5	3
O	2	1	4
O	4	4	4
O	0	2	4
O	0	1	4
O	3	2	4
O	3	4	4
O	3	3	4
O	2	1	4
O	0	4	4
O	3	4	4
O	4	4	4
O	4	2	4
O	4	3	4
O	2	4	4

Chain X

M7 → M6 → M4 → M2 → M1

Chain XI

O3 → B5 → B4 → B2 → B1

Chain I

O7 → O5 → O4 → O1

Ethnic Group	Chain X	Chain XI	Chain I
A	1	3	2
A	0	4	3
A	4	1	3
A	3	3	4
A	1	5	4
A	2	4	3
A	3	4	4
A	0	1	3
A	2	4	4
A	5	4	4
A	2	2	3
A	2	1	3
A	2	4	3
A	5	5	4
A	0	0	2
A	1	2	3
A	1	1	3
A	0	0	3
A	1	5	4
A	3	2	3
A	4	1	4
A	5	3	4
A	0	0	4
A	4	5	4
A	3	1	3
A	0	2	4
A	5	2	4
A	3	1	4
A	3	1	3
A	0	0	3
A	3	0	0
A	2	2	2

Note: Matrix of scores with M, B, O, and A representing ethnic backgrounds of 60 respondents.