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A NEW METHOD FOR TEST AND ANALYSIS OF DYNAMIC STABILITY AND CONTROL

AFFTC

MAY 1976

FINAL REPORT

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AIR FORCE FLIGHT TEST CENTER
EDWARDS AIR FORCE BASE, CALIFORNIA
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE

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This report presents a new method of dynamic stability and control test and analysis which uses several computer programs to perform derivative extraction and characteristic analysis. The report covers the advantages of this method and the elements (test conditions, instrumentation, preferred maneuvers) required for success. In addition complete user guide information is presented to allow setup and execution of these programs. Finally, a section is included on interpreting and evaluating the results.

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SUMMARY

This report is written to cover two major purposes. The first is to familiarize test program managers and stability and control engineers with the philosophy behind the derivative extraction and characteristic analysis approach. The advantages of this procedure, the necessary prerequisites, and the reasons why these prerequisites are important are discussed in the first few sections. The second primary purpose is to provide a handbook for operating the digital programs required for analysis and for understanding the results. Setup, operation and output is discussed for both the derivative extraction program and the characteristic analysis program. In addition, a section is included on interpreting and evaluating the results. It is intended that all the information required to conduct a successful stability and control test program using this approach be included in this report.

PREFACE

This report is written to acquaint the flying qualities engineer with the advantages and techniques of derivative extraction and subsequent data analysis. The techniques and practices included herein represent ten years of experience at the Air Force Flight Test Center and NASA Flight Research Center.

The author wishes to acknowledge the work of Mr. Kenneth W. Iliff and Mr. Richard E. Maine of NASA-FRC for their development of the Modified Maximum Likelihood Estimator derivative extraction program. The Control characteristics analysis program is largely the effort of Mr. John Edwards, also of NASA-FRC. It should be emphasized that the original work on these programs was done by these men, and only the necessary interfacing with AFFTC computers and computer programs was done by the author. Mr. Robert G. Hoey and Mr. Paul W. Kirsten also contributed significantly to the information contained in this report.

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table of contents

Page No.

LIST OF ILLUSTRATIONS 6

LIST OF TABLES 7

INTRODUCTION 11

- Deficiencies of Conventional Testing 11
- Definition of a Derivative 11
- Derivative Extraction Method 17
- Advantages of Derivative Analysis 19

PREPARATIONS FOR EXTRACTING DERIVATIVES 23

- Flight Conditions 23
- Data Requirements 25
- Flight Maneuvers 31

DERIVATIVE EXTRACTION 35

- Preparing the Flight Data 35
- MMLE Program 42
  - Equations 42
  - Algorithm 43
  - Input Data 46
  - Output 61
  - Dl Determination 63
  - A Priori Determination 65
  - Evaluating the Results 67
- Hybrid Matching 68
  - Equations 68
  - The Process 69
- Comparison of the Two Methods 69
  - Time Requirements 69
  - Equations 70

Page No.

Weighting Functions 70

Derivative Accuracy 70

DERIVATIVE ANALYSIS 75

CONTROL 75

CONTROL Algorithm and System Definition 75

Input Data 79

Multiple Cases 95

Alternate Modes of Operation 95

READ Options 95

SYSTEM Options 95

Without the MIXED Option 97

Frequency Response 99

Transient Response 99

Control Output 99

CONSIDERATIONS FOR QUICK RESPONSE DERIVATIVE EXTRACTION 103

Flight Conditions 103

Data Requirements 103

Flight Maneuvers 103

Preparing the Flight Data 103

Executing MMLE 103

Executing CONTROL 104

OVERVIEW 105

REFERENCES 107

APPENDIX A - ADEX 109

APPENDIX B - MMLE 118

APPENDIX C - CONTROL 151

APPENDIX D - SCOPE 3.4 CONTROL CARDS 183

Page No.

APPENDIX E - EQUATIONS OF MOTION ____ 192

APPENDIX F - ROOT LOCUS DISCUSSION ____ 209

APPENDIX G - SAMPLE USER WRITTEN ADEX READ ROUTINES ____ 214

APPENDIX H - SAMPLE USER WRITTEN CONTROL ROUTINES ____ 220

LIST OF ABBREVIATIONS AND SYMBOLS ____ 226

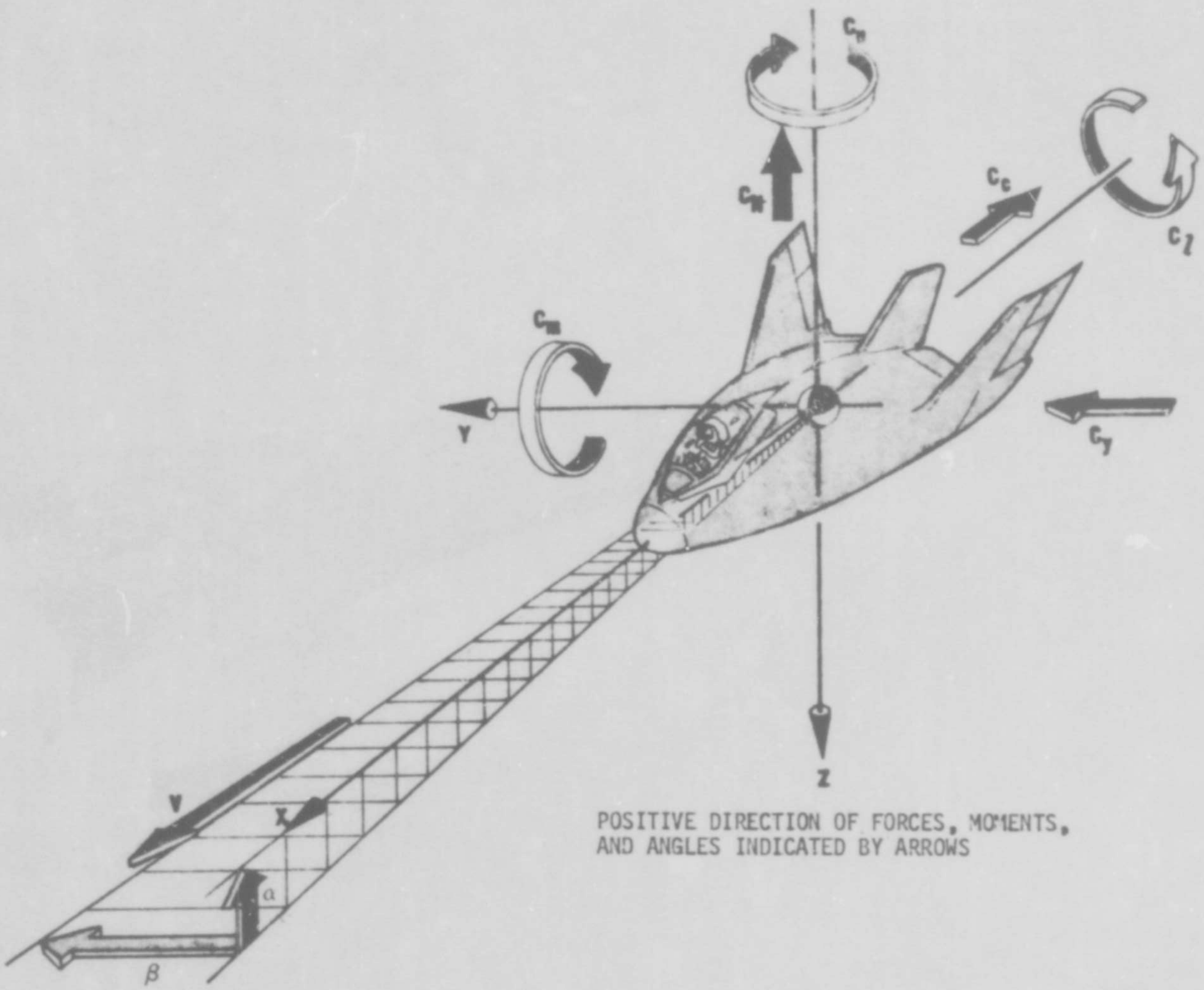
list of illustrations

Figure No.	Title	Page No.
1	Axis System and Sign Conventions	9
2	Derivative Definition	12
3	Math Model Concept	16
4	Mach-alpha Plot	22
5	Airspeed-alpha Plot	22
6	Two Longitudinal Doublets	30
7	Lateral-Directional Doublets	32
8	Effect of Starting Values	57
9	Effect of WMAPR on Derivative Values	64
10	Possible Results of the A Priori Weighting Analysis	66
11	CONTROL Capability	74
12	Flight Control System Block Diagram	86
13	Flight Control System Block Diagram	96
	Appendix F	
F-1	Root Locus Presentation	211
F-2	Impulse Response Portrayed on a Root Locus Presentation	213

list of tables

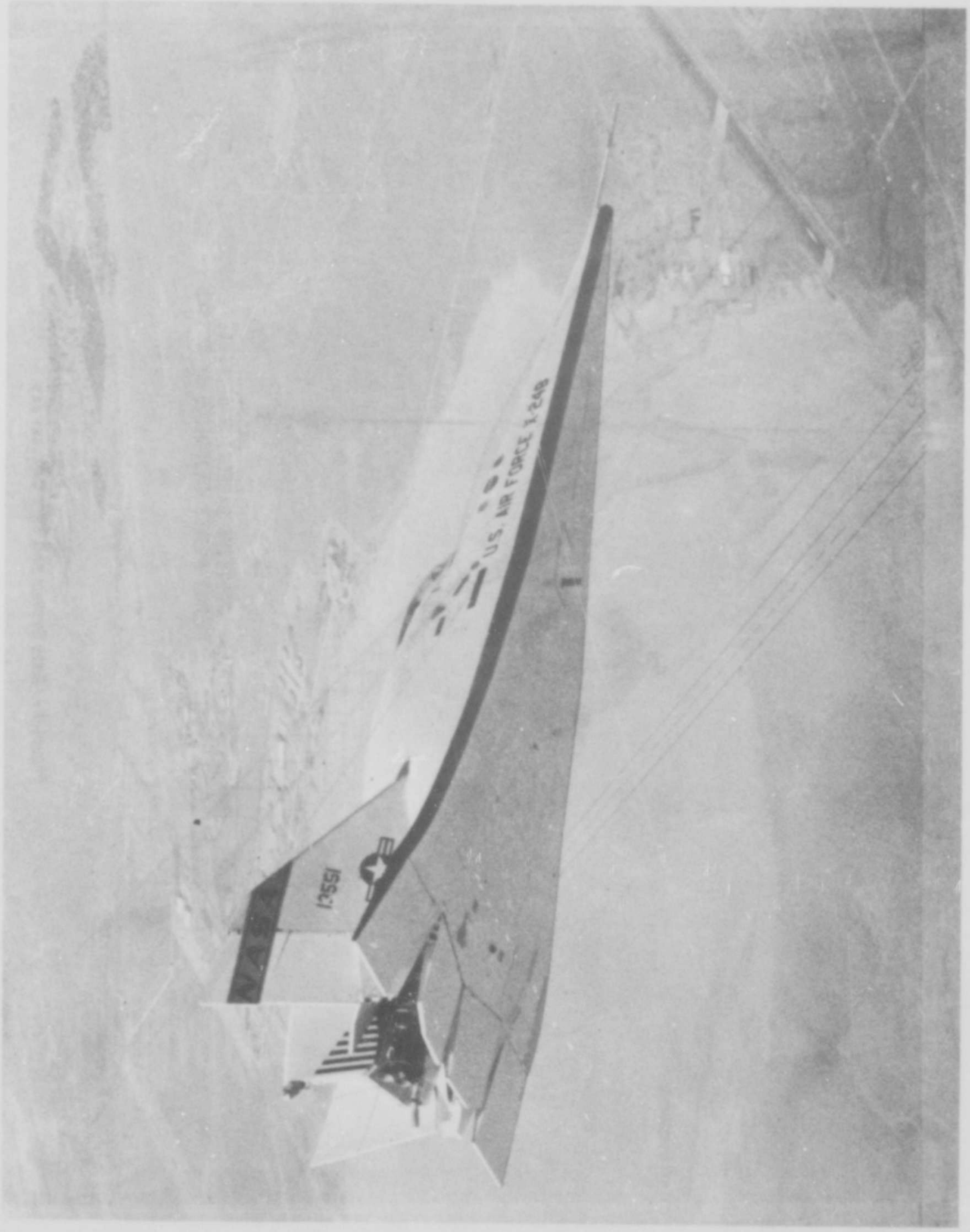
Table No.	Title	Page No.
1	Common Derivatives	14
2	Derivative Classification	18
3	Influence of Test Conditions on Stability Derivatives	24
4	Flight Data Requirement for Derivative Extraction Programs	26
5	Parameter Range and Resolution Requirements	28
6	INPUT Namelist Parameters	47
7	Derivative Accuracy	71
8	CONTROL System Equations	78
9	CODE Namelist Parameters	80
10	Transfer Function Standard Forms	90
11	Milspec Parameters	101
	Appendix A	
A1	ADEX Program Options	110
A2	ADEX Program Description	117
	Appendix B	
B1	INPUT Namelist	119
B2	LOOKUP Data Information	127
B3	Constants Information	130
B4	MMLE Plot Information	133
B5	MMLE Program Description	149
	Appendix C	
C1	CONTROL Data Requirements	152
C2	CODE Namelist	153
C3	CONTROL System Models and Matrix Requirements	157

Table No.	Title	Page No.
C4	Transfer Function Standard Forms	165
C5	CONTROL Program Description	181
	Appendix D	
D1	Program Permanent Files	184



POSITIVE DIRECTION OF FORCES, MOMENTS,
AND ANGLES INDICATED BY ARROWS

Figure 1 - Axis System and Sign Conventions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INTRODUCTION

The purpose of this report is to introduce a method of performing dynamic stability and control testing which has several major analysis advantages over conventional testing. Briefly, this method consists of extracting stability derivatives from flight test data to form a math model of the aircraft. These derivatives are then combined with a model of the aircraft flight control system in a digital computer program to give frequency, damping, and other parameters required by MIL-F-8785B.[1] The advantages of this method and the procedures involved will follow.

Deficiencies of Conventional Testing

Accuracy, ease, and speed of testing and analysis are three primary areas where conventional testing could be improved. Conventional stability and control testing usually requires a specific maneuver for a given parameter on a one-to-one basis. Stick pulses are done to determine frequency and damping, and steady state sideslips are performed to measure aileron and rudder requirements. A method which could calculate several parameters from one maneuver would have an obvious improvement in the reduction of test and analysis time. In many cases, a high gain augmentation system will prevent aircraft motions from being analyzed at all. This is especially true for frequency, damping ratio, and other dynamic MIL-F-8785B (MILSPEC) parameters. In cases where aircraft motions can be analyzed, the procedure is tedious and time-consuming, and the accuracy is often questionable. If testing at two or more variations of the same flight conditions, (i.e., augmentation system on and off, high and low interconnect schedule, etc.) is required, two complete and separate sets of tests must be conducted. If further gain and/or schedule optimization is required, a great deal of time-consuming flight testing must be done at each set of gains and schedules. A method which tested a basic condition and allowed extrapolation to some degree would be desirable. A final consideration comes from a flight safety viewpoint. A high gain augmentation system may mask a serious handling qualities deficiency until the system is saturated with potentially catastrophic results. Extrapolation of these trends is often difficult if only conventional parameters are studied.

Flight test time is becoming more costly for today's sophisticated aircraft. A test concept based on math modeling of both the aircraft and the flight control system would provide a potential for a significant improvement in these aspects of stability and control testing.

Definitions of a Derivative

To understand the method and the potential for improvement, it is necessary to comprehend the concept of stability derivatives. A stability derivative, as the name implies, is the partial derivative

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[1] Reference 1: Military Specification, Flying Qualities of Piloted Airplanes, MIL-F-8785B (ASG), 7 August 1969.

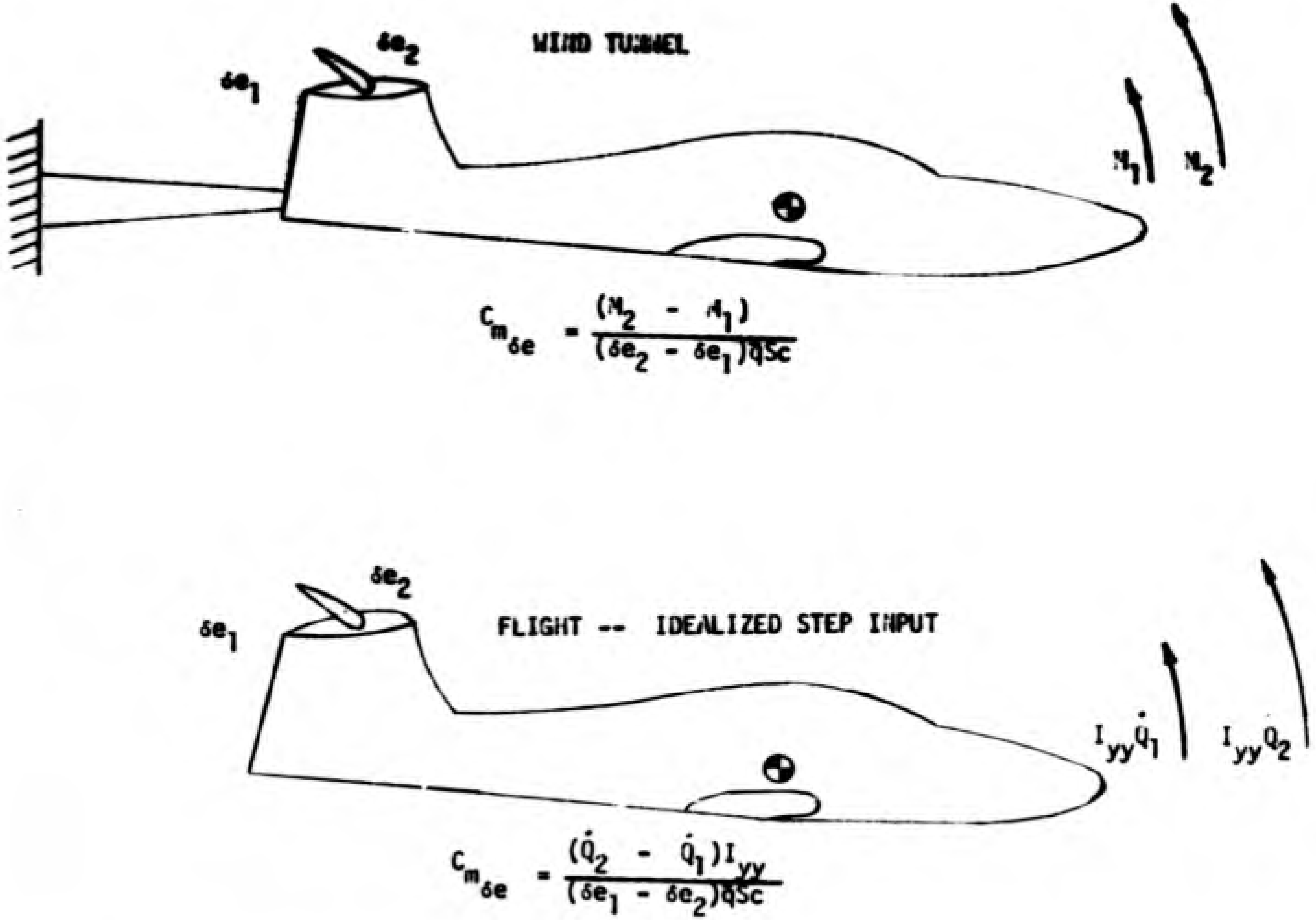

Figure 2 - Derivative Definition

of one parameter with respect to another; that is, the change of one parameter with respect to one other parameter with everything else held constant. An example will help to clarify this definition. Consider an aircraft at some initial flight condition (Figure 2). With the initial elevator deflection, there will be some pitching moment on the aircraft. (It may or may not equal zero.) Now let us change the elevator setting some small amount, holding angle of attack, pitch rate, and all other flight conditions constant. The change in pitching moment divided by the change in elevator approximates $\frac{\partial M}{\partial \delta e}$, which is the derivative $M_{\delta e}$. This derivative may be non-dimensionalized by taking out dynamic pressure, inertias, and reference constants, and the nondimensional coefficient derivative, $C_{m_{\delta e}}$, results. In this case we have measured the change of one parameter, ΔM, with respect to another parameter, $\Delta\delta e$, with everything else fixed. This procedure can be accomplished in a wind tunnel, and indeed it is the procedure by which wind tunnels measure derivatives. In actual flight, this test is not possible since the angle of attack and pitch rate will change as soon as the elevator has moved; hence the condition of all other parameters remaining the same has been violated. These conditions may be closely approximated in flight however, if the parameter which creates the change (such as a control surface) is moving rapidly. Table 1 lists most of the common derivatives and some comments about each one. Basically any derivative may be formed to account for any noticeable aerodynamic effect ($C_{m_{\delta a}}$, $C_{N_{\alpha}}$, etc.).

Now let us look at the three categories into which derivatives may be classified. The first is called stability derivatives where "stability" is defined in a narrower sense that it has been used previously. Stability derivatives are those derivatives which are taken with respect to angle of attack or angle of sideslip. $C_{m_{\alpha}}$ and $C_{n_{\beta}}$ are two examples. Stability derivatives may be further divided into derivatives which describe the natural tendency of an aircraft to return to trim conditions when disturbed ($C_{m_{\alpha}}$, $C_{\ell_{\beta}}$, $C_{n_{\beta}}$), and force derivatives which describe the forces on the aircraft and contribute to vehicle damping ($C_{n_{\alpha}}$, $C_{y_{\beta}}$). The second class is called control derivatives. These are derivatives which are taken with respect to control surface deflections, i.e.,

Table 1

COMMON DERIVATIVES

Lateral-Directional Derivatives	Definition[2]	Vehicle Response Effectiveness[3]	Comments
C_{ℓ_β}	Dihedral effect	Very effective	Contributes to dutch roll stability especially at high angles of attack
$C_{\ell_{\delta a}}$	Aileron control	Very effective	
$C_{\ell_{\delta r}}$	Roll due to rudder	Somewhat effective	Not to be confused with C_{ℓ_β}
C_{ℓ_p}	Roll damping	Effective	
C_{ℓ_R}	Roll due to yaw rate	Not very effective	
C_{n_β}	Yaw stability	Very effective	Primary dutch roll stability
$C_{n_{\delta a}}$	Yaw due to aileron	Effective	
$C_{n_{\delta r}}$	Rudder control	Very effective	
C_{n_p}	Yaw due to roll rate	Somewhat effective	
C_{n_R}	Basic yaw damping	Effective	
C_{y_β}	Sideforce due to sideslip	Effective	Contributes to dutch roll damping
$C_{y_{\delta a}}$	Sideforce due to aileron	Somewhat effective	
$C_{y_{\delta r}}$	Sideforce due to rudder	Somewhat effective	
C_{y_p}	Sideforce due to roll rate	Negligible effect	

---

[2] A good reference for a discussion of common derivatives is Reference 2: Dynamics of the Airframe, Report AE-61-4II, Northrop Aircraft Inc., September 1952.

[3] It is difficult to define an effectiveness to cover all sizes, shapes, and types of aircraft. The response effectiveness shown here apply primarily to light, maneuverable aircraft.

Table 1 (Concluded)

Lateral-Directional Derivatives	Definition[2]	Vehicle Response Effectiveness[3]	Comments
C_{Y_R}	Sideforce due to yaw rate	Negligible effect	
C_{m_α}	Pitch stability	Very effective	Primary contributor to short period stability
$C_{m_{\delta e}}$	Elevator control	Very effective	
C_{m_Q}	Pitch damping	Effective	
C_{N_α}	Lift curve slope	Effective	Provides some pitch damping
$C_{N_{\delta e}}$	Elevator lift	Somewhat effective	
C_{N_Q}	Lift due to pitch rate	Not very effective	
C_{c_α}	Drag due to angle of attack	Effective	
$C_{c_{\delta e}}$	Drag due to elevator	Not very effective	
C_{c_Q}	Drag due to pitch rate	Negligible effect	

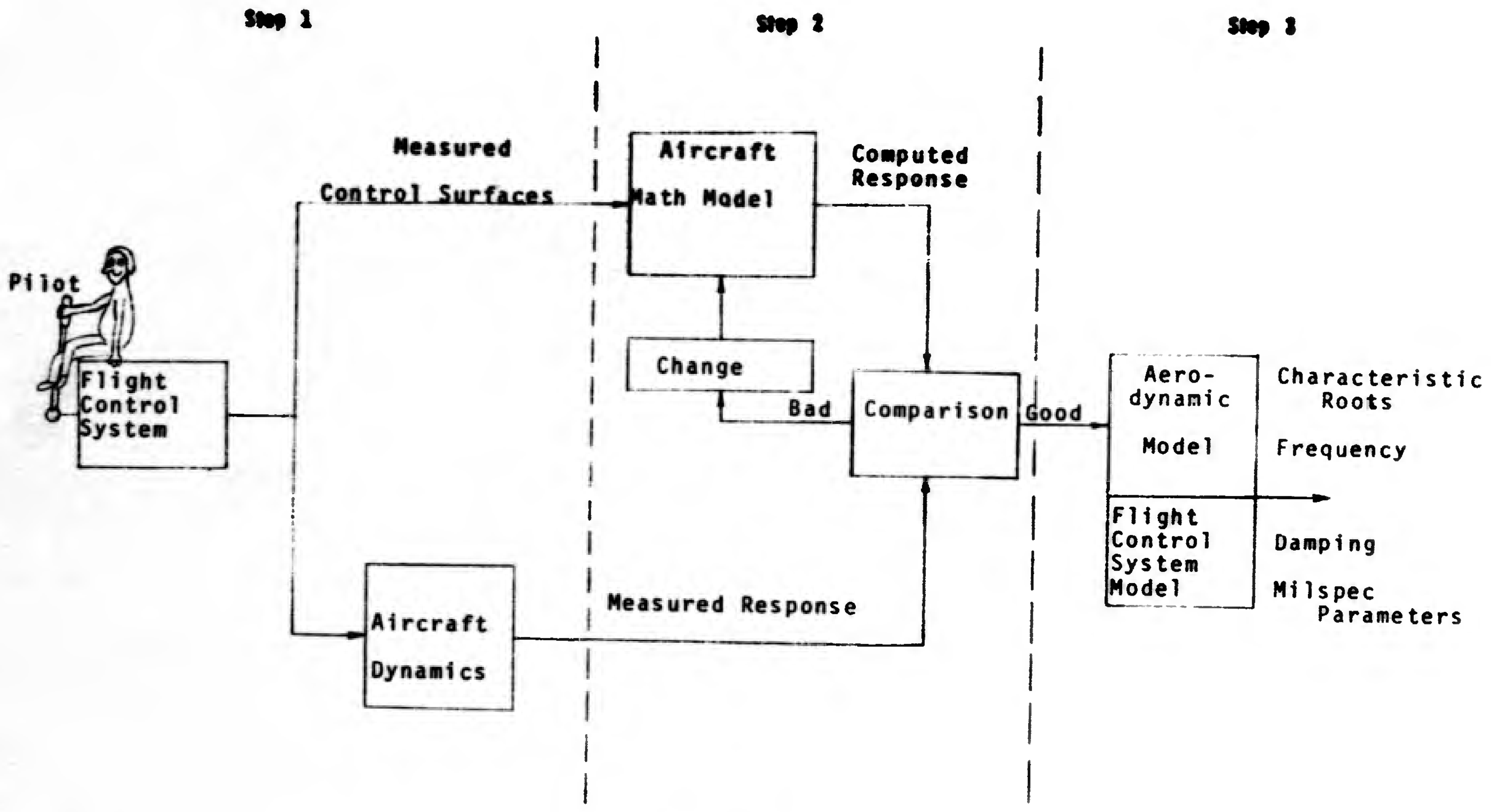

Figure 3 - Math Model Concept

$C_{m_{\delta e}}$, $C_{\ell_{\delta a}}$, $C_{n_{\delta r}}$. The final set is known as damping, rotary, or rate derivatives.[4] C_{m_Q}, C_{ℓ_P}, C_{n_R}, are examples of the classification. The common derivatives are summarized in Table 2.

A word of clarification might be added about derivative non-linearities. Quite often a derivative may have several values at the flight conditions where a single maneuver is conducted. For example, C_{m_α} is quite often a function of angle of attack. Since the angle of attack varies during the maneuver from which C_{m_α} is extracted, the value of C_{m_α} will be the average value of that derivative over the range of angle of attack covered. This value of C_{m_α} is known as a "local slope" derivative. Another example is C_{n_β} as a function of sideslip. If sideslip non-linearities are present, the value of C_{n_β} may be a significant function of the maneuver amplitude (what range of sideslip was encountered). All derivatives extracted from flight test data are local slope derivatives.

Derivative Extraction Method

The general method for extracting derivatives from flight data consists of three basic steps (Figure 3). First the pilot performs some input maneuvers which are conducive to derivative extraction. These maneuvers are designed to create a significant vehicle response with respect to the desired derivative parameters. The second step is to process the measured response time histories, along with measured control surface time histories, in a derivative extraction computer program. The program then determines the best coefficients of a predetermined math model (equations of motion) by varying the coefficients (derivatives) to obtain the best match of the response time histories. The third step is to merge this "best" aerodynamic math model with a math model of the

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[4] It should be noted that both wind tunnel and flight test derivatives combine the terms $\dot{\alpha}$ and $\dot{\beta}$ into pitch rate and yaw rate respectively. While some distinctions have been made analytically, the results of separating the derivatives in actual testing have been poor. All derivatives discussed in this report are in the combined form:

$$C_{m_Q} + C_{m_{\dot{\alpha}}} \rightarrow C_{m_Q}$$

$$C_{\ell_R} + C_{\ell_{\dot{\beta}}} \rightarrow C_{\ell_R}$$

$$C_{n_R} + C_{n_{\dot{\beta}}} \rightarrow C_{n_R}$$

Table 2

DERIVATIVE CLASSIFICATION

	Stability	Control	Damping
Longitudinal	C_{m_α} --------- C_{N_α}, C_{C_α}	$C_{m_{\delta e}}$, $C_{N_{\delta e}}$, $C_{c_{\delta e}}$	C_{m_Q}, C_{N_Q}, C_{c_Q}
Lateral-Directional	C_{ℓ_β}, C_{n_β} --------- C_{y_β}	$C_{\ell_{\delta a}}$, $C_{\ell_{\delta r}}$, $C_{n_{\delta a}}$, $C_{n_{\delta r}}$, $C_{y_{\delta a}}$, $C_{y_{\delta r}}$	C_{ℓ_P}, C_{ℓ_R}, C_{n_P} C_{n_R}, C_{y_P}, C_{y_R}

flight control system. From this combined math model, frequency, damping, steady state sideslip parameters, static margin, and other MILSPEC parameters can be computed.

Advantages of Derivative Analysis

The advantages of derivative analysis are somewhat more encompassing than merely overcoming the deficiencies of conventional testing methods. There are about six major advantages and each will be discussed here with some detail.

The first advantage concerns the added insight which derivatives provide, and is very important when there is a handling qualities problem or a discrepancy between predicted and actual flight test data. Assume for example that during testing a considerable discrepancy is found in trim curves (δe vs. Mach or α). A trim curve is primarily a relation between C_{m_α}, $C_{m_{\delta e}}$, and C_{m_o}. If the trim curve is not right, which derivative is wrong? A knowledge of the individual flight derivatives themselves will pinpoint the problem immediately and help to determine what corrective action, if any, should be taken. A similar case is rudder and aileron required for steady state sideslip. The relations (for zero rates) are:

$$\delta r = \frac{C_{\ell_\beta} C_{n_{\delta a}} - C_{\ell_{\delta a}} C_{n_\beta}}{C_{\ell_{\delta a}} C_{n_{\delta r}} - C_{\ell_{\delta r}} C_{n_{\delta a}}} \beta$$

$$\delta a = \frac{C_{\ell_{\delta r}} C_{n_\beta} - C_{n_{\delta r}} C_{\ell_\beta}}{C_{\ell_{\delta a}} C_{n_{\delta r}} - C_{\ell_{\delta r}} C_{n_{\delta a}}} \beta$$

These determine the amount of aileron and rudder required to maintain a particular steady sideslip. If the measured aileron or rudder are different from predicted, only a knowledge of each derivative will show the cause of the discrepancy and if a problem really does exist.

A second advantage concerns the extrapolation of data over angle of attack and Mach number areas. This is especially helpful in high angle of attack testing. A curve of derivatives may be extrapolated with much more confidence than a curve of frequency and/or damping ratios. This is especially true in cases where one or more derivatives are changing rapidly with respect to Mach number or angle of attack and their deterioration is masked by an overriding and effective flight control system. With a math model approach the suspect flight condition can be "flown" in the digital program and then verified by actual flight. This represents a much safer approach to envelope expansion and high angle of attack testing.

Another advantage is the concept of standardization. Many corrections are possible to raw flight results to enhance poorly flown test data or facilitate direct comparisons with other test data or other aircraft. A good example is a longitudinal trim curve. Most static longitudinal accelerations are done over a range of weight, cg and altitude. Load factor is not always one g nor is pitch rate always zero. An exact trim curve requires a correction for each of the following parameters:

1. cg
2. Weight
3. Load Factor
4. Altitude
5. Pitch rate

We need to calculate the ΔC_m due to each of the variations and divide by $C_{m_{\delta e}}$ to get a $\Delta\delta e$ correction.

Select:

1. Standard cg
2. Standard weight
3. Standard load factor (1g)
4. Standard altitude
5. Standard pitch rate (0)

Now correct for weight, load factor (1g), and altitude

$$\Delta C_N = \frac{n_t W_t}{\bar{q}_t S} - \frac{n_s W_s}{\bar{q}_s S} = \frac{n_t W_t}{\bar{q}_t S} - \frac{(1) W_s}{\bar{q}_s S} \qquad \bar{q}_s = \frac{\rho_s V^2}{2}$$

$$\Delta C_{m_{(C_N)}} = \frac{\partial C_m}{\partial C_N} \Delta C_N = \left(\frac{C_{m_\alpha}}{C_{N_\alpha}}\right) (\Delta C_N)$$

To correct for cg

$$\Delta C_{m_{(cg)}} = \frac{\Delta x}{c} C_N = C_N \frac{(cg_s - cg_t)}{100}$$

To correct for pitch rate ($Q_s = 0$)

$$\Delta C_{m_{(Q)}} = \frac{(Q_t - Q_s)c}{2V} C_{m_Q} = \frac{Q_t c}{2V} C_{m_Q}$$

Then

$$\Delta\delta e = \frac{\Delta C_{m_{(C_N)}} + \Delta C_{m_{(cg)}} + \Delta C_{m_{(Q)}}}{C_{m_{\delta e}}}$$

The required derivatives for this exercise were C_{m_α}, $C_{m_{\delta e}}$, C_{m_Q}, C_{N_α}. Each individual data point has now been corrected to a standard weight, cg, altitude, 1 b flight and zero pitch rate. A fairing of the corrected data points will be easier to make and more meaningful. This type of procedure can be repeated on other test data.

A fourth advantage of derivative analysis is that, given a set of flight derivatives for the basic aircraft, a large control system gain and schedule matrix can be run by computer with very little, if any, additional flight time. Again the final gain settings may be verified by actual flight.

The fifth advantage is the ability to update a simulator with derivatives from flight test data. A simulator will not yield accurate results if the derivative data is inaccurate, and major discrepancies between wind tunnel data and flight test data are often found. Trial and error or qualitative/pilot methods for updating simulators have proven to be inadequate and usually lead to a lack of confidence in simulators by pilots. Updated aerodynamic data provides a better analysis tool and pilot training device.

A final advantage is the overall saving of flight test time. Derivative extraction usually requires pulse or doublet type maneuvers, and these are similar to those used for determination of dynamic characteristics. Since identical information can be derived from the derivatives, much of the remaining testing can be minimized and reoriented to confirmation of analysis data.

One other indirect advantage should be considered. This is the potential for feedback to wind tunnel engineers on which of their techniques gave acceptable results and which techniques failed. If reasons can be found for poor agreement between wind tunnel and flight test data, it is only logical to assume that wind tunnel predictions will improve.

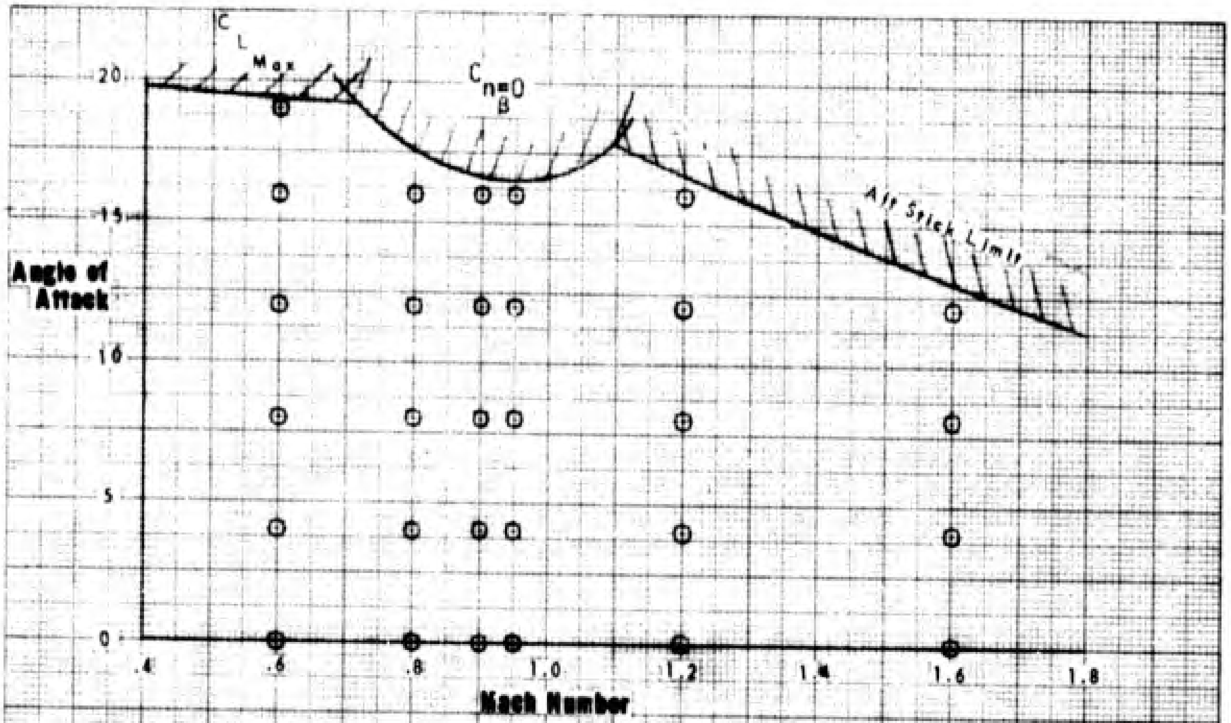


Figure 4 - Mach - Alpha Plot

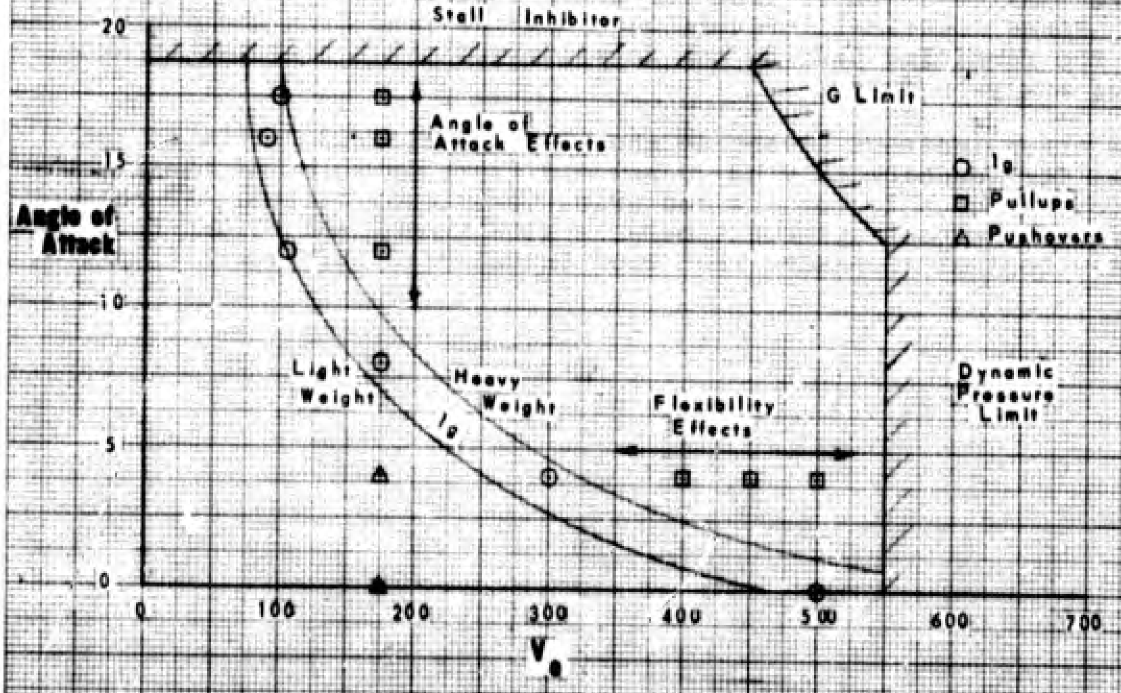


Figure 5 - Airspeed - Alpha Plot (Mach=Constant)*

PREPARATIONS FOR EXTRACTING DERIVATIVES

Preparing the groundwork for derivative extraction is not easy for those who have no access to experience. It is intended that this report will help to fill that need. The following three sections are an accumulation of approximately ten years of experience at the Air Force Flight Test Center and NASA Flight Research Center. Failure to heed these recommendations may result in a great deal of wasted time and worthless data. Particular attention should be paid to the Data Requirements and Flight Maneuvers section. Note that the Data Requirements section, dealing primarily with instrumentation and data reduction, must be addressed long before first flight.

Flight Conditions

To intelligently select flight conditions for extracting derivatives, a knowledge of what flight parameters influence derivatives is necessary. Most derivatives are strong functions of Mach number and angle of attack. There are, however, a number of other parameters which may have some effect on derivative values. Table 3 summarizes these effects and their degree of influence on the derivatives. It should be noted that contrary to most contractor's aerodynamic reports, derivatives are not a function of altitude. The intention, of course, is to allow for flexibility effects. It is strongly recommended that flexibility effects be tabulated as a function of dynamic pressure (which is a measure of air loads) to maintain the linear relationship that usually exists. Aircraft configuration (e.g., wingsweep, flaps, landing gear, stores, etc.) will also effect the derivatives. In addition, there may be some other subtle effects. For example, $C_{n_{\delta a}}$ may be a function of elevator position, i.e., if lateral control is from a rolling tail. $C_{m_{\alpha}}$ may be a function of sideslip. A control surface effectiveness may be non-linear. These effects, however, are extremely difficult to isolate without specialized testing, and therefore are more likely to cause scatter in the data.

It can be seen from Table 3 that for a rigid aircraft, Mach number and angle of attack are the primary factors which influence the derivatives. For the purpose of selecting flight conditions a Mach-alpha plot is useful (Figure 4). Conditions which provide adequate coverage on this plot should be selected. Consideration should be given to testing at Mach numbers where wind tunnel data is available for the purpose of comparison. Mach numbers should also be more closely spaced in the transonic regime since aerodynamic flow, and hence derivatives, are subject to rapid change here. In addition, wind tunnels are not as accurate in this region. Angle of attack spacing should be adequate to define trends. Four degrees is probably a maximum for fighter type aircraft. Two degrees gives better coverage, allows for removal of an occasional bad point without retesting, and requires more test time. It should be noted that, with judicious selection of altitude, most of the Mach-alpha plot can be covered in 1g flight.

Flexible aircraft derivatives are influenced by another parameter, dynamic pressure. For a flexible aircraft, an alpha-equivalent airspeed plot is useful (Figure 5). For each Mach number, angle of attack effects are shown by test points taken along vertical lines.

Table 3

INFLUENCE OF TEST CONDITIONS ON STABILITY DERIVATIVES

Selectable Parameter	Rigid Airplane	Flexible Airplane
Angle of Attack	Strong influence on all derivatives	Strong influence on all derivatives
Mach	Strong influence on all derivatives	Strong influence on all derivatives
Dynamic Pressure, $\overline{q}$ (or Equivalent Airspeed V_e)	No influence	Significant influence, usually linear with $\overline{\sigma}$
Weight & Inertia	No influence	Internal load distribution affects Dyanamic Pressure influence
Center of Gravity	Small influence on some derivatives (correctable)[5]	Small influence on some derivatives (correctable)[5]
Load Factor	No influence	Indirect influence through internal load distribution
Altitude	No influence	No influence
Power Setting	Small or no influence	Small or no influence
Lift Coefficient	No influence	Indirect influence through internal load distribution

[5] The center of gravity correction on C_{m_α} can be quite large depending on the aircraft type and cg range.

Flexibility effects are determined by points taken along horizontal lines. If the purpose of the test plan is only to determine if flexibility effects exist, flexibility testing at a single angle of attack may suffice. If definition of flexibility characteristics is desired, more coverage will be needed. The same Mach number and angle of attack considerations which apply to rigid aircraft are applicable here. Note that some maneuvers will have to be done in pullups (or turns) or pushovers. Also some care should be taken to avoid the corners of the flight envelope for safety of flight reasons.

It is worth a paragraph at this point to dispel the popular idea that derivatives are dependent upon the mode of the flight control system. While that mode may affect the ease with which derivatives are extracted, the aerodynamic derivatives are not, in general, a function of what the flight control system is or is not doing. Testing of aircraft with augmentation system on and off will not increase the size of the test matrix from a derivative extraction standpoint.

Data Requirements

One of the more important steps in getting good derivatives is getting good data. Having the right parameters is important, as is having the right kind of data. Table 4 shows the required parameters for two different extraction programs in use at the Air Force Flight Test Center. Control surfaces should be measured at the surface itself. If two surfaces are involved, as is the case with ailerons, both should be measured. Weights and inertias should be calculated as accurately as possible since an error in an inertia will manifest itself directly as an error in the derivative. Inertias should be measured whenever possible by swinging the aircraft.[6,7] Contractor-provided curves for a "production configuration" should be corrected for known differences between the test and production aircraft such as nose boom, ballast, gun removal, test instrumentation etc. Center of gravity is not used directly in the identification procedure but is needed to correct accelerometers to the center of gravity and to correct derivatives to a standard cg. The angular accelerations are not necessary to the extraction procedure but should be used if they are measured independently of the angular rates. Differentiation of measured aircraft rates should not be used since no new information is added and the extraction program may be misled by a poor differentiation procedure.

---

[6] Reference 3: Engineer's Handbook for Aircraft Performance and Flying Qualities Flight Testing, Performance and Flying Qualities Branch, Flight Test Engineering Division, Edwards AFB, California, May 1971.

[7] Reference 4: Wolowicz, Chester H. and Yancey, Roxanah B., Experimental Determination of Airplane Mass and Inertial Characteristics, NASA TR R-433, NASA Flight Research Center, Edwards, California, October 1974.

Table 4

FLIGHT DATA REQUIREMENTS FOR DERIVATIVE EXTRACTION PROGRAMS

PARAMETER	COMMENTS
Time (hrs, min, sec)	
All Control Surface Inputs (Measured at Surface)	Variable Time History
Sideslip Angle	Variable Time History
Angle of Attack	Variable Time History
Bank Angle	Variable Time History
Pitch Angle	Variable Time History
Pitch Rate	Variable Time History
Roll Rate	Variable Time History
Yaw Rate	Variable Time History
Longitudinal Acceleration[1]	Variable Time History
Normal Acceleration	Variable Time History
Lateral Acceleration	Variable Time History
Pitch Angular Acceleration[2]	Variable Time History
Roll Angular Acceleration[2]	Variable Time History
Yaw Angular Acceleration[2]	Variable Time History
Dynamic Pressure[3]	Constant
Velocity	Constant
Weight	Constant
Inertias	Constant
cg[4]	Constant

[1] Optional. Required only if drag derivatives are to be determined.

[2] Optional. These should be used if available and measured independently.

[3] A variable in hybrid matching program, but constant in MMLE.

[4] Flight cg is needed to correct some derivatives to reference cg and to correct measured linear accelerations.

Sampling rates are not as critical as other considerations as long as the time interval is constant between samples. A maximum sampling rate is probably determined by data storage capacity more than anything else and data can always be "thinned". A minimum sampling rate for fighter type aircraft is about 20 samples per second for no information loss. Larger aircraft may be able to use a lower rate, but usually the speed of the control surfaces is the primary factor in determining at what rate some information will be lost. Some cases as low as four samples per second have been matched, but convergence to a set of derivatives is hampered.

Range and resolution of the recorded parameters is important. If the range is too small, the peaks of some response parameters will be cut off with a devastating effect on the control derivatives. If the resolution is not good enough the responses will tend to move in steps. While this is accurate enough to determine primary derivatives, information on secondary derivatives ($C_{N_{\delta e}}$, $C_{\ell_{\delta r}}$, $C_{n_{\delta a}}$) will be lost. Range and resolution are interdependent parameters for most instruments and the engineer may not be able to have both. One suitable solution is to use two measuring devices: a fine instrument to provide the resolution for low amplitude maneuvers and a coarse instrument which takes over when the fine channel is saturated. This is especially applicable to roll rate. Some suggested ranges and resolutions are given in Table 5.

Filtering to remove high frequency noise can, and quite often does, produce problems. Filters produce phase lag, and if the phase lag is appreciable (greater than .03 seconds at aircraft frequencies) poor derivatives may result. The program susceptibility to phase lag is highest for the rates and control surfaces, followed by the angles and accelerations. Some "anti-filters" (correcting filtered parameters in subsequent data reduction) have been used with a reasonable amount of success. These require a knowledge of the phase lag as a function of frequency and, of course, the necessary programming to make the corrections. Appreciable phase lag on aircraft rates and/or control surfaces will result in a lack of program convergence and, hence, no derivatives at all.[●]

Wild points can cause convergence problems especially if they are in the control surfaces input data. Since wild point editing is a fairly simple procedure, it should be part of any data reduction.

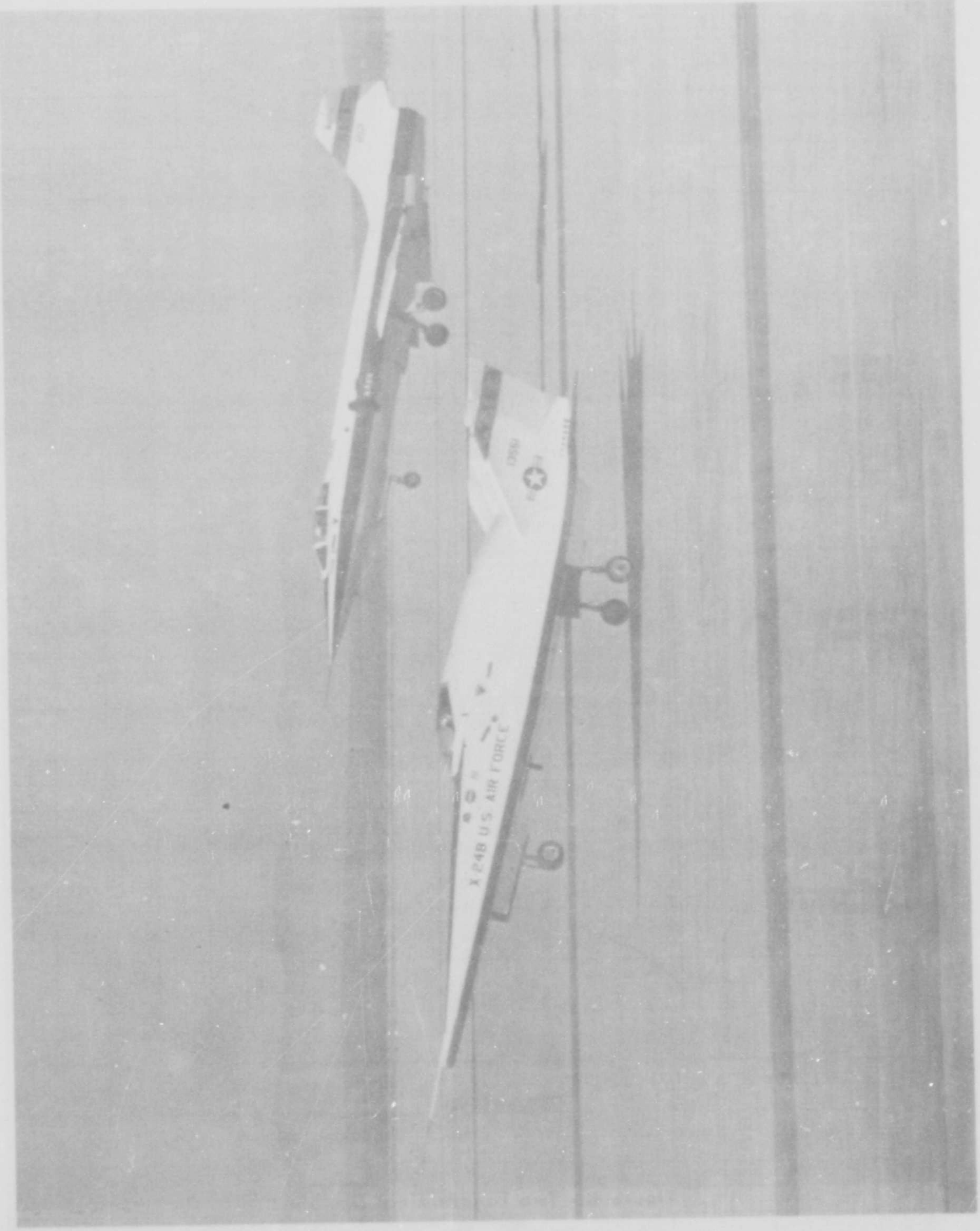
[●] Reference 5: Steers, Sandra T. and Iliff, Kenneth W., Effects of Time Shifted Data on Flight Determined Stability and Control Derivatives, NASA TN D-7803, NASA Flight Research Center, Edwards, California, March 1975.

Table 5

PARAMETER RANGE AND RESOLUTION REQUIREMENTS

PARAMETER	RANGE*	RESOLUTION
Time		Milliseconds
Control Surfaces	± Full Deflection	0.1°
Angle of Attack	-10° → +40°	0.1°
Angle of Sideslip	±10°	0.1°
Bank Angle	±180°	0.5°
Pitch Angle	±90°	0.5°
Pitch Rate	±50°/sec	0.25°
Roll Rate	±100°/sec	0.25°
Yaw Rate	±50°/sec	0.25°
Normal Accel.	-3g → +7g	0.1g
Longitudinal Accel.	±2g	0.1g
Lateral Accel.	±1g	0.1g
Pitch Accel.	±100°/sec^2	1.0°/sec^2
Roll Accel.	±200°/sec^2	1.0°/sec^2
Yaw Accel.	±100°/sec^2	1.0°/sec^2
Dynamic Pressure	0 - aircraft $\bar{q}$ limit	
True Velocity		

*Adequate for pulse maneuvers on a typical fighter aircraft. Other aspects of stability and control testing may require higher ranges.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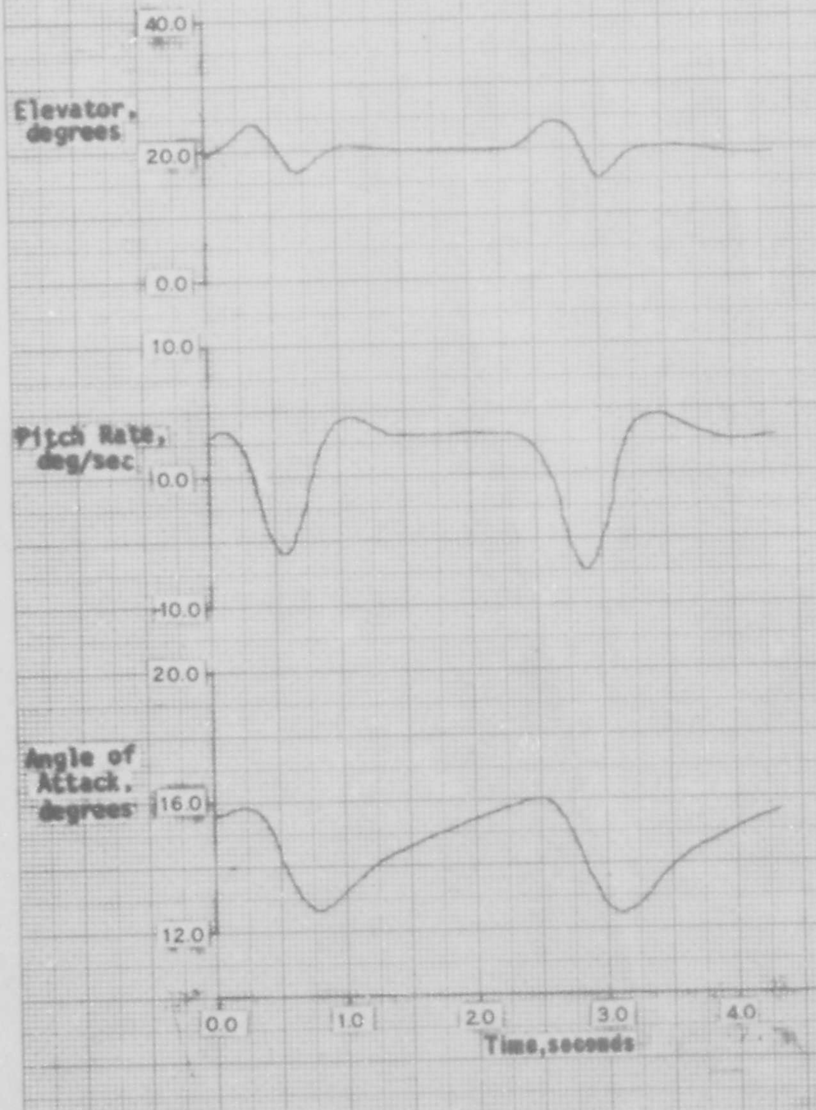


Figure 6 - Two Longitudinal Doublets

Flight Maneuvers

The type of flight maneuver used is the single most important factor in the success of the derivative extraction process, and some considerations for each axis will be given. The overriding criterion for any axis is the ability to separate the effects of stability, control, and damping derivatives. To this end, the recommended maneuver is a rapid doublet followed by a period of stick free oscillation. A rapid doublet most accurately simulates the wind tunnel step input. With this maneuver, the control derivatives can be determined from the stick input, and the stability and damping derivatives from the free oscillation. Augmentation-system-off maneuvers are easier to work with for two reasons. First, the lack of augmentation allows more vehicle response, thereby giving the computer program more information to work with. Second, with the augmentation system on, control surface inputs are mixed in with the stick free response and now two factors are contributing to damping (i.e., for pitch C_{m_Q} and $(K_Q)\cdot(C_{m_{\delta e}})$). The program may have a hard time distinguishing between them. Augmentation-system-on maneuvers can be, and have been, used successfully but, since aerodynamic damping is usually secondary to augmentation damping, increased scatter in the damping derivatives will be noted. Control system interconnects may produce similar effects.

Longitudinal maneuvers usually take from five to ten seconds, and lateral-directional maneuvers usually last ten to fifteen seconds. It is important to maintain reasonable trim conditions during this time. The digital extraction program approximates dynamic pressure and velocity with constant values, and for most maneuvers, the time is short enough that this assumption is valid. High drag configurations may require more power to make this assumption valid. (The dynamic pressure limitation does not apply to the hybrid program.) Since derivatives are a function of angle of attack and Mach number, it is important to keep these parameters constant. Of course the angle of attack will change during an elevator pulse, but the derivatives will be valid for the average angle of attack during the maneuver. The desire to maintain trim conditions is the primary reason for performing doublets rather than pulses. One g flight conditions are the easiest for getting derivatives. Higher g maneuvers can be analyzed successfully, but the pilot workload to attain and maintain trim conditions is increased.

The maneuver for extracting longitudinal derivatives is a rapid elevator doublet. The doublet should be fast enough so that it is completed by the time large angle of attack and pitch rate excursions occur, but not so fast that little or no aircraft response is noted. An example is shown in Figure 6. The magnitude of pitch rate excursions should be on the order of ten to twenty degrees per second, and the angle of attack should vary within four or five degrees. Program convergence is aided if the bank angle variation from the trim bank angle is kept small. Trim bank angle does not need to be zero.

The lateral-directional maneuver is a rudder doublet followed by three or four seconds of stick free oscillation and terminated by an aileron doublet (Figure 7). The rudder doublet is performed first since the angle of attack and bank angle transients from a rudder input are usually smaller than from the aileron. Again, maintaining a constant

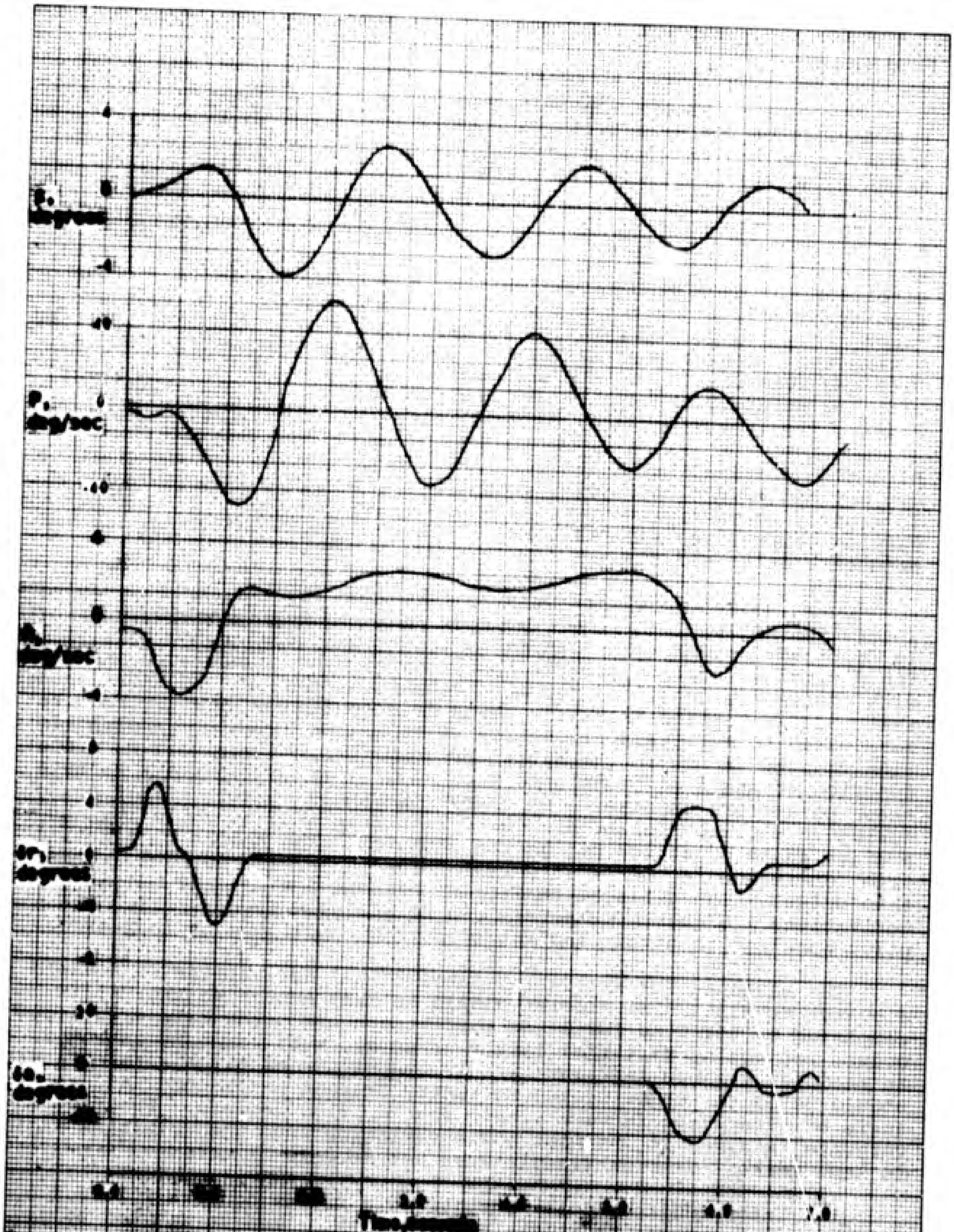


Figure 7 - Lateral-Directional [illegible]

angle of attack is important. The time of the stick free oscillation should be long enough to allow for a cycle or two of aircraft response and no longer. It is not necessary for the response to die out completely. Prolonged time between the two doublets prevents the simultaneous analysis of both, and complicates the extraction process.[9]

Experience has shown that a doublet (either longitudinal or lateral-directional) which is quick enough for derivative extraction may not excite the aircraft enough to permit pilot evaluation of a short period or dutch roll oscillation. If this problem occurs, it is suggested that one doublet for each purpose be done, since a compromise on the rapidity of the doublet may be very detrimental to the matching process. For a fighter aircraft doublets can usually be accomplished in one to one-and-a-half seconds, and control surface rate limiting is a common occurrence. Simulator training is very helpful in familiarizing pilots with these maneuvers.

The advent of recent aircraft with two or more sets of rolling surfaces has complicated the extraction process. Obviously if an aileron and a differential tail are acting in unison to produce roll rate, the program cannot identify how much each is contributing. Some success has been achieved where the augmentation system drives only one surface, but even here the derivatives show more scatter than normal. If there is a constant relation between the two surfaces a "total aileron" may be defined.[10] Wind tunnel "total" derivatives may be calculated in the same way, and comparisons with flight test data using this method have been good.

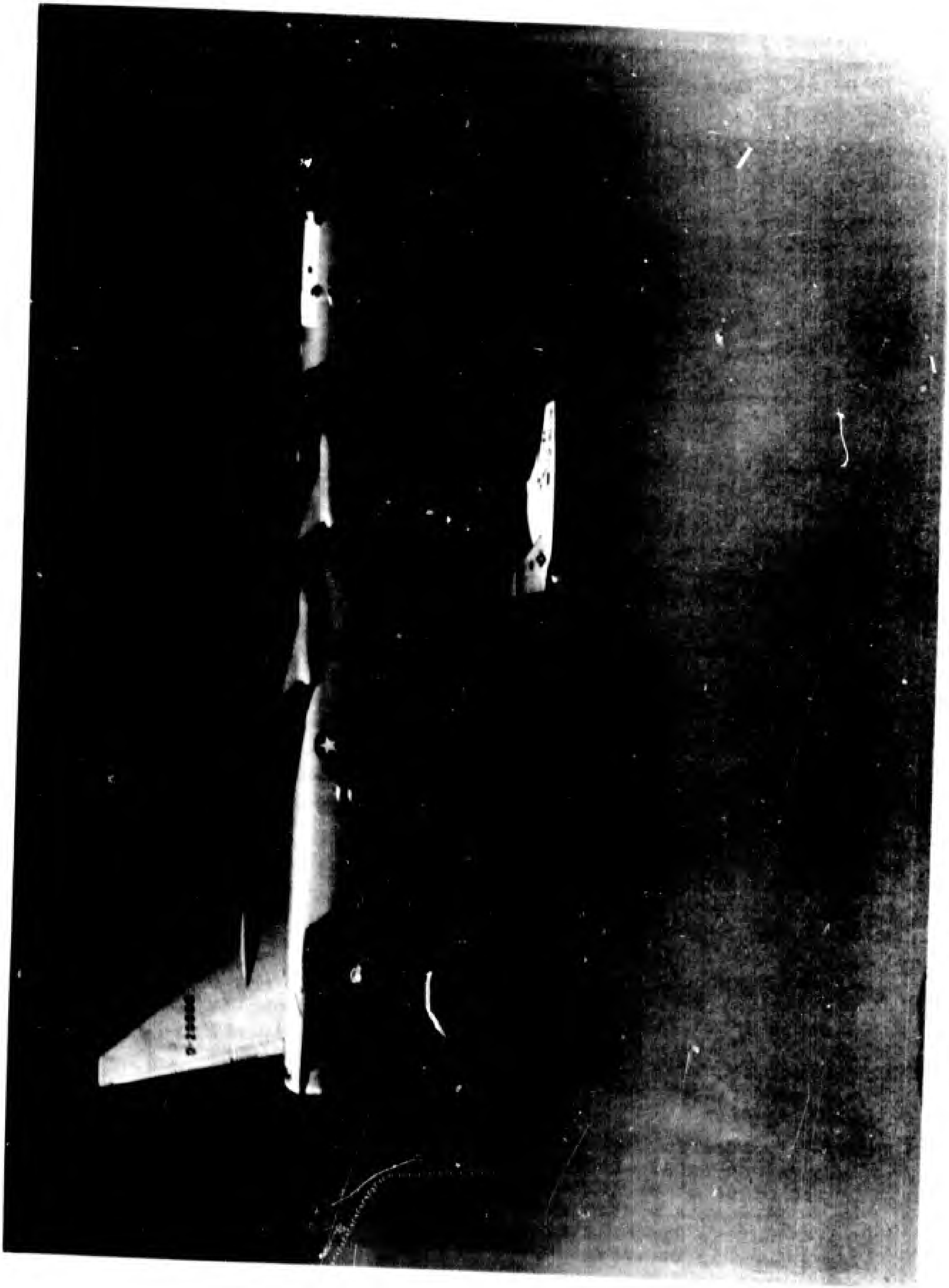
Other maneuvers than doublets have been used with varied success. One maneuver is a high-frequency, continuous sinusoidal input. This input was used successfully on the M2 lifting body program.[11] Another input which is under study is a sinusoidal input which sweeps a range of frequencies. Still a third maneuver is a computed input which is held until a certain response is measured and then reversed. These may or may not provide better results, but all lack the simplicity of a doublet as a general maneuver for extracting all derivatives. Some different maneuver or sequence may be more applicable if information is being sought on one specific derivative.

---

[9] The digital derivative extraction program has the capability to analyze up to fifteen maneuvers to determine one set of derivatives, and this feature may be used to an advantage in some cases. Accuracy, however requires that the several maneuvers be done at almost exactly the same flight conditions, and this is where problems arise.

[10] It is suggested that the two types of ailerons be added to form an equivalent total aileron. For example, if δs is supposed to be one third of δa at all times, the summation of $\delta a + \delta s$ is better than 1.333 δa. Even in a system where the relation is supposed to be constant, rate limits, hysterisis, or system malfunctions can cause the dynamic relationship to vary.

[11] Reference 6: Sim, Alex G., Flight Determined Stability and Control Characteristics of the M2-F3 Lifting Body Vehicle, NASA TN D-75-11, NASA Flight Research Center, Edwards, California, December 1973.



DERIVATIVE EXTRACTION

There are currently two operational methods to extract derivatives at AFFTC. One, MMLE, is an all digital program which is run on the CDC 6500. The other is a hybrid matching program which uses a digital computer for input data storage and analog equipment for equation solving. Although both programs are capable of extracting accurate derivatives, the digital program is more suited to high speed production processing, and the hybrid program is more suited to maneuvers where inertial coupling is significant or where coupling has occurred between the longitudinal and lateral-directional axes. The setup and operation of each program will be discussed and the differences between the two programs will be readily apparent.

Preparing the Flight Data

The final step prior to using an extraction program is preparing the time history file. Only one existing program, ADEX,[12] is suitable for preparing data for the hybrid matching program. Several programs exist to prepare data for the digital program. The initial step in either case is converting the telemetry data or flight recorded data into an engineering units tape (usually an ADAS tape at AFFTC). This report will not deal with that step. Once an engineering units tape has been procured, the next step is to convert the data into a time history file. The time history file for the digital program will be discussed first.

Essentially any program which reads the input data and writes a specific file will work. The time history file should be an unformatted binary file constructed in the following manner:[13]

Header record
Time record #1
Time record #2
.
.
.
Time record #n

$\left.\right\}$ Case number one (n time points)

---

[12] Words in the text written with all capital letters will usually refer to computer names, variables, or programs.

[13] The digital program has the capability of reading files which are not exactly in this format. Reordering and record length specification for the time records may be done using the ORDER, NREC, and BOTH parameters (INPUT namelist, MMLE input data section). The information shown here is the default file for MMLE. This type of file requires the least amount of input information.

Header record
Time record #1
.
.
.
Time record #m
etc.

Case number two (m time points)

A header record is defined as the following:

N - Case number (integer)
W - Gross Weight, pounds (real)
DCG - Increment of test cg from reference, per cent (real)
IX - I_{xx}, slug-ft^2 (real)
IY - I_{yy}, slug-ft^2 (real)
IZ - I_{zz}, slug-ft^2 (real)
IXZ - I_{xz}, slug-ft^2 (real)
C - Reference chord feet (real)
B - Reference span, feet (real)
S - Reference area, square feet (real)
M - Average Mach number (real)
Q - Average dynamic pressure lb/ft^2 (real)
V - Average true velocity ft/sec (real)
ALFA - Average angle of attack, degrees (real)
START - Start time, total seconds (real)
STOP - Stop time, total seconds (real)

All the time records within a case must be the same, i.e., either lateral-directional or longitudinal. A lateral-directional time record is defined as the following:

TH - Time - hours (integer)
TM - Time - minutes (integer)
TS - Time - seconds (integer)
TMS - Time - milliseconds (integer)
BETA - Sideslip, degrees (real)
P - Roll rate, deg/sec (real)
R - Yaw rate, deg/sec (real)
PHI - Bank angle, degrees (real)
NY - Sideforce, g's (real)

PDOT - Roll acceleration, deg/sec^2. Leave zero if not measured. (real)

RDOT - Yaw acceleration, deg/sec^2. Leave zero if not measured. (real)

DC1 - First control surface, usually aileron, degrees (real)

DC2 - Second control surface, usually rudder, degrees (real)

DC3 - Third control surface, degrees (real)

DC4 - Fourth control surface, degrees (real)

ALFA - Angle of attack, degrees (real)

V - Velocity, ft/sec (real)

MACH - Mach number (real)

QBAR - Dynamic pressure, lb/ft^2 (real)

Optional

Finally, a longitudinal time record is:

TH - Time, hours (integer)

TM - Time, minutes (integer)

TS - Time, seconds (integer)

TM - Time, milliseconds (integer)

ALFA - Angle of attack, degrees, (real)

Q - Pitch rate, deg/sec (real)

V - True velocity, ft/sec (real)

THETA - Pitch angle, degrees (real)

NZ - Normal acceleration, g's (real)

QDOT - Pitch acceleration, deg/sec^2. Leave zero if not measured.

NX - Longitudinal acceleration, g's. Leave zero if not measured. (real)

DE1 - First control surface, usually elevator, degrees (real)

DE2 - Second control surface, degrees (real)

DE3 - Third control surface, degrees (real)

DE4 - Fourth control surface, degrees (real)

PHI - Bank angle, degrees, (real)

ALT - Altitude, feet (real)

MACH - Mach number (real)

QBAR - Dynamic pressure, lb/ft^2 (real)

Optional

Some considerations for ease in operating the program are these:

1. DC2 should be rudder if possible

2. If a total is to be used:

   DC1 - should be total aileron

   DC3 - should be the first aileron

   DC4 - should be the second aileron

3. DE1 should be the primary pitch control surface

4. Any surfaces which are not used at all should be set to zero.

5. V and NX may be constant or zero if a time history is not available.

6. As noted the last three parameters in each time record are optional. Their use will be explained later.

Several programs to prepare a data file are already in existence. The first is LINK8 of the Uniform Flight Test Analysis System (UFTAS). If LINK8 is already being used, this method is by far the easiest since all the information is there already. Documentation on how to accomplish this may be found in the UFTAS manual.

The second method for creating the file is to use the ADEX program. The ADEX program reads data off the engineering units tape, searches for the time segment, merges the data with some input card data, and writes in onto a file suitable for the digital program. Since each user may have a differently formatted engineering units tape, it is necessary for the user to write a project-specific subroutine to read his own tape. This can be and should be done prior to first flight. Several routines to read CDAS tapes are already in existence.

The input data for ADEX consists of four cards per case plus two extra cards per run. The cards should be assembled in the following manner:

Card 1 - (Format (10x, I10)

NSEQ - Number of sequences to be processed

Card 2 - (Format (A5, I5, 5A10)

AIRCFT - Name of aircraft, i.e., F-111, YF-16, etc.

INSEQ - Sequence number

MODE - Use LONG or LATDR (left justified)

IFTAPE - Use BIN (left justified) for digital extraction

IFWILD - Use YES (left justified) for wild point search of control parameters

IFLIST - Use INPUT (left justified) to obtain listing of input data

- Use AFTER (left justified) to obtain listing of output data

- Use YES (left justified) to obtain listing of both

IFDIF - Use YES (left justified) for differentiation of rates - normally not required

Card 3 - (Format (6F10.0)

W - Gross Weight, lbs

DCG - Distance of test cg from reference, percent

XXI - I_{xx}, slug-ft^2

YYI - I_{yy}, slug-ft^2

ZZI - I_{zz}, slug-ft^2

XZI - I_{xz}, slug-ft^2

Card 4 - Format (7F10.0)

AMACH - Mach number

Q - Average dynamic pressure, lb/ft^2

VT - Average true velocity, ft/sec

ALFA - Average angle of attack, degrees

CH - Reference chord, ft

B - Reference span, ft

S - Reference area, ft

Card 5 - FORMAT (3F10.0, 10x, 3F10.0)

SH - Start time, hours

SM - Start time, minutes

SS - Start time, seconds

EH - End time, hours

EM - End time, minutes

ES - End time, seconds

Card 6

Same as Card 2 for second case.

.

.

.

Last card - Format (A5)

STOPS - Use STOP (left justified) to end processing.

Some examples of data input, setup cards, and data output can be found in Appendix A.[14]

The output of ADEX is fairly simple and is generally of not much interest to the user. The input information is printed out along with the number of points on each time history and whatever the user-written subroutine might output. An example output is shown in Appendix A.

Finally the time histories may be input on cards. The parameters listed in the time history file are punched on data cards, two cards per data point (Format 312, 14, 7F10.4/8F10.4). Putting the time history onto cards has the inherent advantage that the data is readily accessible, and a wild point can be removed simply by changing a card. Experience has shown, however, that if more than a few cases are to be run, the user soon becomes inundated with boxes of cards. In addition, the deck required to run the program is considerably larger and more cumbersome than the deck which uses a permanent file or magnetic tape.

Preparing the time histories for the hybrid extraction program requires the use of ADEX. The data cards are identical with those used to create the binary file for the digital program with the exception of the IFTAPE parameter. It should be set to BCD (left justified) for this job. The SCOPE 3.4 control cards are considerably changed, and they may by found in Appendix D.

The file written for the hybrid program is considerably different than that written for the digital program. The time segment must be ten seconds in length and be sampled at fifty samples per second, thus generating exactly five hundred time points.[15] The file is written onto a seven track magnetic tape in binary coded decimal form. The tape density is 556 BPI. Information is passed on 501 groups of three 96 character records. The first group contains three records with title information. These records are usually used to pass inertias, flight conditions, etc. The next three records constitute all the parameters for the first time point.

---

[14] Appendices A, B, and C will give detailed "user guide" information on the setup of ADEX, MMLE, and CONTROL respectively. In addition an example will be included for all three programs showing deck setup and program output.

[15] A linear interpolation program, RESAMPL, is available from the author to convert data sampled at a general rate into fifty-sample-per-second data.

Record 1 - Format (8F12.5)

Q - Pitch rate, degrees per second
P - Roll rate, degrees per second
α - Angle of attack, degrees
β - Angle of sideslip, degrees
N_y - Lateral acceleration, g's
R - Yaw rate, degrees per second
ϕ - Bank angle, degrees
δe - Elevator, degrees

Record 2 - Format (8F12.5)

θ - Pitch angle, degrees
δa - Aileron, degrees
δr - Rudder, degrees
$\bar{q}$ - Dynamic pressure, lb/ft^2
N_z - Normal acceleration, g's
V - True velocity, ft/second
T - Time, total seconds
$\dot{P}$ - Roll acceleration, degrees/sec^2

Record 3 - Format (8F12.5)

$\dot{Q}$ - Pitch acceleration, deg/sec^2
$\dot{R}$ - Yaw accleration deg/sec^2
δsp - Extra control surface, degrees
N_x - Longitudinal acceleration, g's
H - Altitude, feet
δd - Extra control surface, degrees

δc - Extra control surface, degrees

δx - Extra control surface, degrees

This pattern continues until all five hundred time points have been exhausted.

MMLE Program

The digital program is called MMLE, Modified Maximum Likelihood Estimator. This is a maximum likelihood estimation program which uses a modified Newton-Raphson algorithm for convergence. The program assumes a linear, time-invariant, three-degree-of-freedom model,[16] and cases are either longitudinal or lateral-directional.

Equations:

The equations of motion for MMLE are simplified, three-degree-of-freedom equations. The derivation of the full five-degree-of-freedom equations and the simplifications required for linearization are given in Appendix E. The three-degree-of-freedom, linear equations are shown here.

Longitudinal (Two-degree-of-freedom)

$$\dot{Q} = \frac{\bar{q}sc}{I_{yy}} (C_{m_\alpha} \cdot \alpha + C_{m_{\delta e}} \cdot \delta e) + \frac{\bar{q}sc^2}{2VI_{yy}} (C_{m_Q} \cdot Q)$$

$$\dot{\alpha} = Q + \frac{g}{V} \frac{\cos \phi \cos \theta}{\cos \alpha} - \frac{\bar{q}s}{mV} \cos \alpha (C_{N_\alpha} \cdot \alpha + C_{N_{\delta e}} \cdot \delta e)$$

Lateral-Directional

$$\dot{P} = \frac{I_{xz}}{I_{xx}} \dot{R} + \frac{\bar{q}sb}{I_{xx}} (C_{\ell_\beta} \cdot \beta + C_{\ell_{\delta a}} \cdot \delta a + C_{\ell_{\delta r}} \cdot \delta r)$$

$$+ \frac{\bar{q}sb^2}{2VI_{xx}} (C_{\ell_P} \cdot P + C_{\ell_R} \cdot R)$$

$$\dot{R} = \frac{I_{xz}}{I_{zz}} \dot{P} + \frac{\bar{q}sb}{I_{zz}} (C_{n_\beta} \cdot \beta + C_{n_{\delta a}} \cdot \delta a + C_{n_{\delta r}} \cdot \delta r)$$

$$+ \frac{\bar{q}sb^2}{2VI_{zz}} (C_{n_P} \cdot P + C_{n_R} \cdot R)$$

---

[16] Some non-linearities can be programmed into the model, but this must be done on a case by case basis and does not lend itself to production processing.

$$\dot{\beta} = P \sin \alpha - R \cos \alpha + \frac{g}{V} \cos \theta \cdot \phi + \frac{\bar{q}s}{mV} (C_{y_{\beta}} \cdot \beta$$

$$+ C_{y_{\delta a}} \cdot \delta a + C_{y_{\delta r}} \cdot \delta r)$$

Since thrust is not always easily determined and since drag derivatives are usually obtained from different sources the longitudinal mode is usually run in two degrees-of-freedom, omitting the $\dot{u}$ equation:

To preserve the linearity of the equations, only some of the terms are allowed to vary with time. These include:

Longitudinal - α, Q, θ, δe (Two degrees-of-freedom)

Lateral-Directional - β, P, $\dot{P}$, R, $\dot{R}$, ϕ, δa, δr

Thus, for example, in the $\dot{\beta}$ equation the sin α and cos α terms will be treated as constants. Dynamic pressure and velocity are held constant for terms in all of the equations. Hence, the necessity of maintaining angle of attack and trim conditions during the length of the maneuver is apparent.

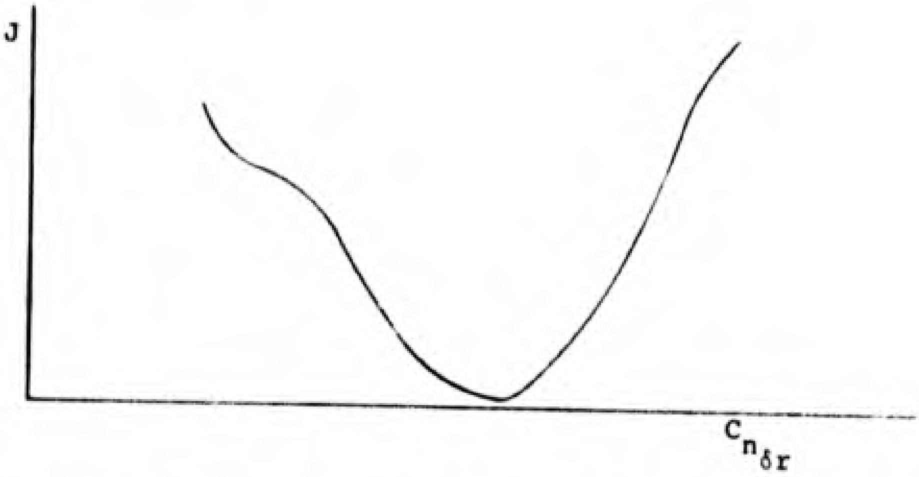
The assumptions (See Appendix E) made in linearizing the MMLE equations of motion are reasonable ones for most maneuvers described in this report. Even in some cases where the assumptions are violated, convergence can be attained and good derivatives found. This is especially true for lateral-directional maneuvers done with a steady state load factor greater than one g ($Q \neq 0$). The assumptions, however, are the primary reason that MMLE is not adept at matching coupled maneuvers, departures, and similar nontrim maneuvers.

Algorithm

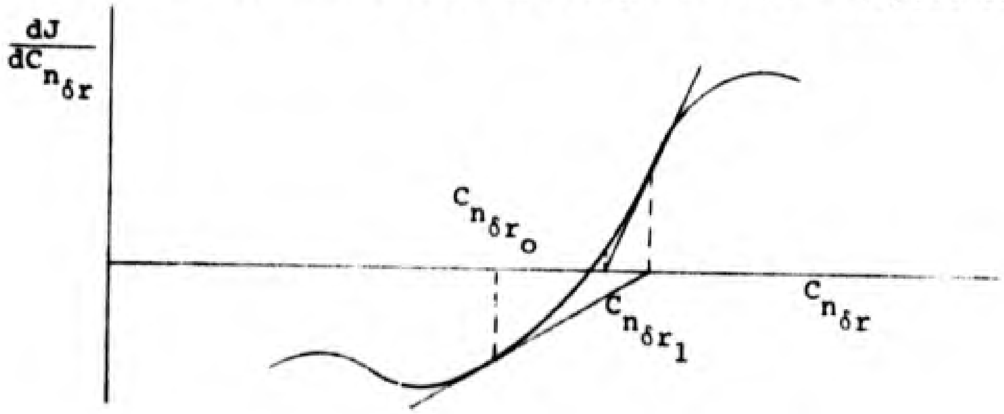
Although the mathematics behind the modified Newton-Raphson algorithm are fairly complex, the basic idea is simple and can be illustrated by a one-degree-of-freedom system. Assume we have a measured time history of yaw rate, R_m (output) and rudder (input). Using the starting value of $C_{n_{\delta r}}$ and the rudder time history, the program will compute a yaw rate time history, R_c. Now define a "cost function", J as

$$J = \int_0^T (R_m (t) - R_c (t))^2 \, dt \quad (T = \text{Total time to be matched})$$

The squaring of the difference is merely to account for errors of either sign between measured and computer values. Obviously we would like to minimize J as this would give us the best match. If we plot J vs $C_{n_{\delta r}}$, we get



To find the minimum we take the slope of J with respect to $C_{n_{\delta r}}$.



Using the starting value of $C_{n_{\delta r}}$ (shown as $C_{n_{\delta r_o}}$) the program again takes a local slope and projects to a zero value of $\frac{dJ}{dC_{n_{\delta r}}}$. The new value of $C_{n_{\delta r}}$ (shown as $C_{n_{\delta r_1}}$) becomes the first iteration value. This process is continued until convergence at the cost function minimum is attained.

The process can now be expanded to more general case. For a lateral-directional case we redefine the cost function as:

$$J = \int_0^T [(P_m - P_c)^2 + (R_m - R_c)^2 + (\beta_m - \beta_c)^2 + (\phi_m - \phi_c)^2$$
$$+ (N_{y_m} - N_{y_c})^2 + (\dot{P}_m - \dot{P}_c)^2 + (\dot{R}_m - \dot{R}_c)^2]\, dt$$

Now J has become a vector, and to find the slope it is necessary to take the gradient of J with respect to all the derivatives to be identified. The iterative process, however, remains analogous to that of the one dimensional case.

It is worthwhile at this point to discuss two of the program's features and the effect they have on the process. The first is called the Dl weighting matrix and is effectively the inverse of the noise covariance matrix. This feature simply allows us to weight some measurement variables more heavily than others. It can be used to deemphasize noisy parameters, such as N_y, or to eliminate a parameter which is not known, such as $\dot{P}$ or $\dot{R}$. This is accomplished by multiplying each parameter of the cost function by weighting value. Thus J becomes

$$J = \int_0^T [Dl_P\ (P_m - P_c)^2 + Dl_R\ (R_m - R_c)^2 + Dl_\beta\ (\beta_m - \beta_c)^2 + Dl_\phi\ (\phi_m - \phi_c) + Dl_{N_y}\ (N_{y_m} - N_{y_c})^2 + Dl_{\dot{P}}\ (\dot{P}_m - \dot{P}_c) + Dl_{\dot{R}}\ (\dot{R}_m - \dot{R}_c)]\ dt$$

and any term can be eliminated from the cost function by setting its Dl term to zero. The magnitude of the Dl weightings are discussed in the "Dl Determination" section.

The second feature is called the a priori feature and allows derivative values to be weighted toward a priori values from another source such as calculations, wind tunnel or previous flight test results. This feature is used when there is known to be little information in a maneuver about a given derivative, i.e., $C_{\ell_{\delta a}}$ in a SAS off rudder pulse. Because of some extraneous input, the program may deduce that it can improve the match slightly by increasing $C_{\ell_{\delta a}}$ one hundredfold. Obviously this solution is wrong, and we would like to be able to hold a derivative at a starting value if there is little or no information about it. To do this there is an additional term added to the cost function such that

$$J = \int_0^T [Dl_P\ (P_m - P_c)^2 + Dl_R\ (R_m - R_c)^2 + Dl_\beta\ (\beta_m - \beta_c)^2 + Dl_\phi\ (\phi_m - \phi_c)^2 + Dl_{N_y}\ (N_{y_m} - N_{y_c})^2 + Dl_{\dot{P}}\ (\dot{P}_m - \dot{P}_c) + Dl_{\dot{R}}\ (\dot{R}_m - \dot{R}_c)]\ dt + APRA_{C_{\ell_\beta}}\ (C_{\ell_\beta} - C_{\ell_{\beta_o}}) + APRB_{C_{\ell_{\delta a}}}\ (C_{\ell_{\delta a}} - C_{\ell_{\delta a_o}}) + \ldots \text{ etc.}$$

where the subscript o indicates a starting value. The APRA and APRB terms determine how much of a penalty will be assessed for deviating from the a priori value. Determination of weighting values will be discussed in the "A Priori Weighting" section. Care must be taken in determining these values since too high a weighting will hamper the convergence process and result in incorrect derivative values. It should be noted that use of a priori inherently increases the error sum and deteriorates the match to some degree. However, better derivative values may result. Most high quality test maneuvers can be run without using the a priori feature.

Input Data :

The input data for MMLE can be broken down into three types. These are the time history file, the input data cards, and a wind tunnel curve file (optional).

The time history curve file has already been discussed. To use it simply attach it as THIST (See Appendix D) prior to execution of the MMLE program. A word might be said here about conciseness of the start and stop time for the time history. Both too much and too little data can be passed. The start time should be about one half second before the initial pulse or doublet. The stop time should be when one of the following occurs.

1. The oscillations cease.
2. The flight conditions (Mn, α, $\bar{q}$) change significantly.
3. Longitudinal or Lateral-directional modes couple.
4. Unplanned inputs occur. (Gusts, heavy turbulence, etc.)

Too much time after one of the above conditions occurs will cause increased convergence difficulty and probably affect the derivatives somewhat.

There are four sections of data required on the input data cards. It should be pointed out that there is a good deal of flexibility in how the data can be set up. The information given here is "default" information: Default implies that the program will set the normal value of a parameter and the user may reset it if required. Much of the information in this section is copied from Ken Iliff's and Richard Maine's work.[17]

Optional inputs will be pointed out.

Section 1

Card 1 (Format 20A4)
Title Card

Section 2

Cards 2-n (Namelist Format)
Namelist "INPUT"

The possible parameters for the INPUT namelist are described in Table 6. Detailed options may be found in Appendix B.

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[17] Maine, Richard E. and Iliff, Kenneth W., A User's Guide for Three Fortran Computer Program to Determine Aircraft Stability and Control Derivatives from Flight Data, NASA TN D-7831, NASA Flight Research Center, Edwards, California, April 1975.

Table 6

INPUT NAMELIST PARAMETERS

	PARAMETER	DESCRIPTION
1.	LONG, LATR	The mode of operation, longitudinal or lateral-directional.
2.	CARD, TAPE	The mode of the input time history file.
3.	SPS	Control sampling rate of time history file.
4.	THIN	Allows thinning of input data.
5.	NCASE	Number of case to be matched simultaneously.
6.	SCALE	Scale factors for observation parameter.
7.	FIXED	Biases for observation parameters.
8.	DC	Biases for control surfaces
9.	NREC	Number of parameters in each time history record.
10.	ORDER	Order of the signals on the input tape.
11.	BOTH	An option for combining input of longitudinal and lateral-directional case.
12.	PLOTEM	Switch for plotting routine.
13.	PLTMAX	Error sum above which plots will be killed.
14.	INCH	Switch for inch or centimeter plotting paper.
15.	ZMIN, ZMAX	Minimum and maximum plotting values for observation parameters.
16.	DCMIN, DCMAX	Minimum and maximum plotting values for control surfaces.
17.	NCPLOT	Number of control and extra signals to be plotted.
18.	TIMESC	Time scale for plots.
19.	PRINT	Switch to print out time histories.
20.	TEST	Allows intermediate printout for debugging.
21.	NOITER	Number of iterations.
22.	ERRMAX	Error sum above which computation stops.

Table 6 (Continued)

	PARAMETER	DESCRIPTION
23.	BOUND	Convergence bound.
24.	PUNCH	Allows punched output for plotting routine.
25.	PUNCHC	Allows punched output for CONTROL program.
26.	PUNCHD	Allows punched output for restarting the program.
27.	NEAT	Number of time halving in the computation of the transition matrix.
28.	METRIC	Determines whether input data will be in metric or English units.
29.	GROSWT	Gross weight.
30.	IX	Roll inertia.
31.	IY	Pitch inertia.
32.	IZ	Yaw inertia.
33.	IXZ	Cross product of inertia.
34.	SPAN	Reference wing span.
35.	CBAR	Reference aerodynamic chord.
36.	S	Reference wing area.
37.	CG	Difference between test and reference cg.
38.	MACH	Mach number.
39.	ALPHA	Angle of attack.
40.	Q	Dynamic pressure.
41.	V	True velocity.
42.	PARAM	Identification parameter.
43.	XB	Beta vane correction.
44.	ZB	Beta vane correction.
45.	XALF	Angle of attack vane correction.
46.	XAY	N_y accelerometer correction.
47.	ZAY	N_y accelerometer correction.

Table 6 (Concluded)

PARAMETER	DESCRIPTION
48. XAN	N_z accelerometer correction.
49. ZAX	N_x accelerometer correction.
50. VAR	Determines a bias factor on the last three observation parameters.
51. ZERO	Allows initial conditions of observation parameters to vary.
52. WMAPR	Weight factor for a priori use.
53. NAPR, WFAC	Variables used for a priori determination.
54. ND1, D1RLX, D1TOL	Variables used for D1 weighting determination.
55. PRNTPLT	Switch for line printer plots.
56. GAMMA	Flight path angle.

Section 3

Time card(s) (Format 3I2, I3, 1X, 3I2, I3, 1X, I10

SH - Start time, hours
SM - Start time, minutes
SS - Start time, seconds
SMS - Start time, milliseconds
EH - Stop time, hours
EM - Stop time, minutes
ES - Stop time, seconds
EMS - Stop time, milliseconds
SEARCH - Activates search mode. Use case number or leave blank.

One time card is necessary for each of the NCASE (Item 5, INPUT namelist) maneuvers to be processed. If tape input is used, the start and/or stop times may be changed from the header record by inserting new values on these cards. If the times on the input tape are satisfactory the times on the cards may be left blank. The SEARCH parameter allows the program to locate a maneuver if one or more preceding maneuvers are to be skipped. The program, while searching for the correct maneuver, will skip cases until the case number from the header record matches the SEARCH number on the card. If the SEARCH number is left blank, the program will assume the next case on the tape file is the correct one. Hence, if the maneuvers on the tape file are to be processed sequentially, the SEARCH values may be left blank. The program does not have the capability to rewind the tape file, so maneuvers that have been processed or skipped cannot be recalled.

Section 4

The program depends on matrices for much of the input data and equations. A total of twelve matrices must be defined for the program to operate. Fortunately, most of these are defaulted adequately, and input for the remaining matrices has been simplified. For all matrices entered, the format is:

Header card (Format A8, I2, I10)

Matrix name - A, B, AA, BB, AR, BR, APRA, APRB, AP, BP, D1, R
(Left justified)

Number of rows - 4 for all cases except D1, AP, and BP

Number of columns - 4 to 8 depending on the matix

Matrix cards - Format (8F10.4)

Matrix values - one card for each row

If the D1 matrix is diagonal (which is usually true) the number of columns may be set equal to zero and all the diagonal values read in on the first card after the header card.

The twelve matrices and their defaults will now be discussed. They may be entered in any order.

A (4X4) - Starting values of stability and damping derivatives. The matrix should be set to

$$\begin{bmatrix} -N_{\alpha} & 1.0 & -N_{V} & -\sin\theta\cos\phi\,\frac{g}{V} \\ M_{\alpha} & M_{Q} & M_{V} & 0.0 \\ -C_{\alpha} & 0.0 & -C_{V} & -g\cos\theta \\ 0.0 & \cos\phi & 0.0 & 0.0 \end{bmatrix}$$

for the three-degree-of-freedom longitudinal case. A two-degree-of-freedom case may be run by setting the third row and the third column to zero. For a lateral-directional case the matrix is

$$\begin{bmatrix} Y_{\beta} & \sin\alpha & -\cos\alpha & \frac{g}{V}\cos\theta\cos\phi \\ L_{\beta} & L_{P} & L_{R} & 0.0 \\ N_{\beta} & N_{P} & N_{R} & 0.0 \\ 0.0 & 1.0 & \cos\phi\tan\theta & 0.0 \end{bmatrix}$$

Note that all derivative values are in dimensionalized form with units of per radian.

B (4X5 to 4X8) - Starting values for the control derivatives and aerodynamic biases. The control derivatives must be in columns 1-4 while the aerodynamic bias for the first four of the NCASE cases are in columns 5-8. The bias values are cumulative such that for the second case to be analyzed biases 1 and 2 (columns 5 and 6) will be added. If NCASE = 1 the matrix should be read in as a 4X5 matrix and generally the starting bias values will be zero.

The matrices are:

Longitudinal

$$\begin{bmatrix} -N_{\delta e_1} & -N_{\delta e_2} & -N_{\delta e_3} & -N_{\delta e_4} & -N_{o_1} & -N_{o_2} & -N_{o_3} & -N_{o_4} \\ M_{\delta e_1} & M_{\delta e_2} & M_{\delta e_3} & M_{\delta e_4} & M_{o_1} & M_{o_2} & M_{o_3} & M_{o_4} \\ -C_{\delta e_1} & -C_{\delta e_2} & -C_{\delta e_3} & -C_{\delta e_4} & -C_{o_1} & -C_{o_2} & -C_{o_3} & -C_{o_4} \\ 0.0 & 0.0 & 0.0 & 0.0 & \dot{\theta}_{o_1} & \dot{\theta}_{o_2} & \dot{\theta}_{o_3} & \dot{\theta}_{o_4} \end{bmatrix}$$

Lateral-directional

$$\begin{bmatrix} Y_{\delta c_1} & Y_{\delta c_2} & Y_{\delta c_3} & Y_{\delta c_4} & Y_{o_1} & Y_{o_2} & Y_{o_3} & Y_{o_4} \\ L_{\delta c_1} & L_{\delta c_2} & L_{\delta c_3} & L_{\delta c_4} & L_{o_1} & L_{o_2} & L_{o_3} & L_{o_4} \\ N_{\delta c_1} & N_{\delta c_2} & N_{\delta c_3} & N_{\delta c_4} & N_{o_1} & N_{o_2} & N_{o_3} & N_{o_4} \\ 0.0 & 0.0 & 0.0 & 0.0 & \dot{\phi}_{o_1} & \dot{\phi}_{o_2} & \dot{\phi}_{o_3} & \dot{\phi}_{o_4} \end{bmatrix}$$

AA (4X4) - **Determines** which of the corresponding terms in the A matrix will be allowed to vary. Values of zero or one should be used. A one indicates the term will be allowed to vary. Generally, any meaningful derivative may be allowed to vary. In addition, the sin α term for the lateral-directional matrix is usually allowed to vary to better the match on sideslip. Default matrices are:

Longitudinal

$$\begin{bmatrix} 1.0 & 0.0 & 0.0 & 0.0 \\ 1.0 & 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix}$$

Lateral-directional

$$\begin{bmatrix} 1.0 & 1.0 & 0.0 & 0.0 \\ 1.0 & 1.0 & 1.0 & 0.0 \\ 1.0 & 1.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix}$$

BB (4X5) - **Determines** which of the corresponding terms of the B matrix will be allowed to vary. Similar to the AA matrix. In addition to varying the control derivatives the aerodynamic bias terms (column 5-8) should be allowed to vary. Default matrices are:

Longitudinal

$$\begin{bmatrix} 1.0 & 0.0 & 0.0 & 0.0 & 1.0 \\ 1.0 & 0.0 & 0.0 & 0.0 & 1.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}$$

Lateral-directional

$$\begin{bmatrix} 1.0 & 1.0 & 0.0 & 0.0 & 1.0 \\ 1.0 & 1.0 & 0.0 & 0.0 & 1.0 \\ 1.0 & 1.0 & 0.0 & 0.0 & 1.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}$$

AR (4X4) - A priori values corresponding to the A matrix. Usually the A matrix and the AR matrix would be identical. The default AR matrix is the A matrix.

BR (4X5 to 4X8) - A priori values corresponding to the B matrix. Comments and default are similar to the AR matrix.

APRA (4X4) - A priori weighting values for corresponding terms of the A or AR matrix. (See MMLE, Algorithm section.) These values are multipled by the WMAPR factor (Item 52, INPUT namelist). Default matrices are:

Longitudinal

$$\begin{bmatrix} 13000. & 0.0 & 0.0 & 0.0 \\ 15. & 800. & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix}$$

Lateral-directional

$$\begin{bmatrix} 13000. & 13000. & 13000. & 0.0 \\ .15 & 500. & 5. & 0.0 \\ 15. & 800. & 800. & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix}$$

APRB (4X5 to 4X8) - A priori weighting values for corresponding terms of the B or BR matrices. These values are multiplied by WMAPR (Item 52, INPUT namelist). Default matrices are:

Longitudinal

$$\begin{bmatrix} 13000. & 13000. & 13000. & 13000. & 0.0 & 0.0 & 0.0 & 0.0 \\ 15. & 15. & 15. & 15. & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix}$$

Lateral-directional

$$\begin{bmatrix} 13000. & 13000. & 13000. & 13000. & 0.0 & 0.0 & 0.0 & 0.0 \\ .15 & .15 & .15 & .15 & 0.0 & 0.0 & 0.0 & 0.0 \\ 15. & 15. & 15. & 15. & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix}$$

AP (3X4) - Used in defining the equations of motion for the last three terms of the observation vector. Normally these matrices (AP and BP) are left defaulted. If AP or BP is read in, both must be read in. Default matrices are:

Longitudinal

$$\begin{bmatrix} -\frac{V}{g} & 0.0 & 0.0 & 0.0 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ \frac{1}{g} & 0.0 & 0.0 & 0.0 \end{bmatrix}$$

Lateral-directional

$$\begin{bmatrix} \frac{V}{g} & 0.0 & 0.0 & 0.0 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 1.0 & 1.0 & 1.0 & 1.0 \end{bmatrix}$$

BP (3X5 to 3X8) - Same comments as AP. Default matrices are:

Longitudinal

$$\begin{bmatrix} -\frac{V}{g} & -\frac{V}{g} & -\frac{V}{g} & -\frac{V}{g} & -\frac{V}{g} & -\frac{V}{g} & -\frac{V}{g} & -\frac{V}{g} \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ \frac{1}{g} & \frac{1}{g} & \frac{1}{g} & \frac{1}{g} & \frac{1}{g} & \frac{1}{g} & \frac{1}{g} & \frac{1}{g} \end{bmatrix}$$

Lateral-directional

$$\begin{bmatrix} \frac{V}{g} & \frac{V}{g} & \frac{V}{g} & \frac{V}{g} & \frac{V}{g} & \frac{V}{g} & \frac{V}{g} & \frac{V}{g} \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \end{bmatrix}$$

R (4X4) - Acceleration transformation matrix. Provides the transormation to principle axis. Default matrices are:

Longitudinal - unit matrix

Lateral-directional

$$\begin{bmatrix} 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & -\frac{I_{xz}}{I_x} & 0.0 \\ 0.0 & -\frac{I_{xz}}{I_z} & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}$$

D1 (5X5 to 7X7) - Provides weighting for each signal (see MMLE, Algorithm section). The size of the matrix determines the number of observation parameters to be used. Since $\dot{P}$, $\dot{Q}$, $\dot{R}$, and N_x are generally not available or not used, a five parameter vector should be used. The omission of the last two parameters (and hence the smaller matrices) results in a considerable saving of computer time. If the matrix is diagonal it should be entered with zero number of columns and the values all on one card as explained earlier. Default matrices are diagonal and values are:

Longitudinal

$$\begin{bmatrix} 30000. & 20000. & 0.0 & 100000. & 2000. \end{bmatrix}$$

Lateral-directional

$$\begin{bmatrix} 500000. & 1500. & 1000000. & 30000. & 5000. \end{bmatrix}$$

The last card of each deck should have an ENDCASE starting in column one if there are more cases to follow. If it is the last case, an END should be substituted. If the time history is to be read in on cards (CARD = .TRUE.) the time history cards would follow.

In the past, the formation of the A and B matrices has been one of the most time consuming portions of the setup procedure, since derivatives had to be obtained from wind tunnel books and then dimensionalized. Two new procedures have been implemented to overcome this delay. The first uses a three dimensional lookup routine. Derivative values are cataloged by the ORIGIN program in a format compatible with UFTAS. Derivatives may be a function of Mach number, angle of attack and/or dynamic pressure. Units, curve numbers, and tape information are contained in Appendix B, and information on how to use ORIGIN may be found in an UFTAS manual.[18] This curve file should be attached as CURVES prior

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[18] Reference 8: Documentation of the Uniform Flight Test Analysis System (UFTAS), Volumes 1 and 2, Air Force Flight Test Center, Edwards AFB, California, June 1973.

to execution of the program. In addition, a CURVIN namelist utilizing KF(I) is used to change the default curve numbers or to eliminate lookup of some derivatives. An example would read $CURVIN KF(3)=0, KF(4)=0, KF(5)=8000, $END. A curve number of zero for a given derivative stops the program from looking up that derivative and assigns it a value of zero. The CURVIN namelist replaces the A and B matrices. An additional file, attached as NCURVE, must be used to define the number of curves for each derivative and the order of the input parameters. The number of curves is defined by the number of third variable breakpoints. This information is contained on two cards (Format 34I1) which may be read in and cataloged as another cycle of the curve file. More information on setting up this file is contained in Appendix B. Using this information, we can set up an example of the NCURVE file. Assume all derivatives are stored as a function of angle of attack, Mach number and dynamic pressure except for the damping derivatives which are a function of Mach number and angle of attack. The number of dynamic pressures in each case is two except for the angle of attack and sideslip derivatives where it is three. Then the number of curves for C_{m_α} (#1) is three, $C_{m_{\delta e_1}}$ (#2) is two, $C_{m_{\delta e_2}}$ (#3) is two, $C_{m_{\delta e_3}}$ (#4) is two, C_{m_Q} (#5) is one, etc. The order number for each derivative will be one, except for the damping derivative (#5, #10, #15, #26, #27, #33, #34) where it is three. The first card (number of curves) is:

3222132221322213222232222113222211

and the second card (order of parameters) is:

1111311113111131111111111133111113 3

The second method of obtaining starting values is to use the predefined constants which are in the program. These constants are an approximate value for each derivative and the assumption has been made that the constants are close enough to allow program convergence. This option is especially valuable if wind tunnel data is not available or if time and/or manpower are not available to create a predicted data curve file. These constants and their units are listed in Appendix B. Any constants may be changed with a DERIVIN namelist, simply by setting the literal name to the new value, i.e., $DERIVIN CLB = -.0015, CLDC1 = .0023, $END. It is important to note that the constant mode works best on good maneuvers. Derivatives are not a function of starting value IF there is information about each derivative in the maneuver. This fact is shown in Figure 8. These plots show the derivative value as the program iterates to a final value for four different starting values. Note

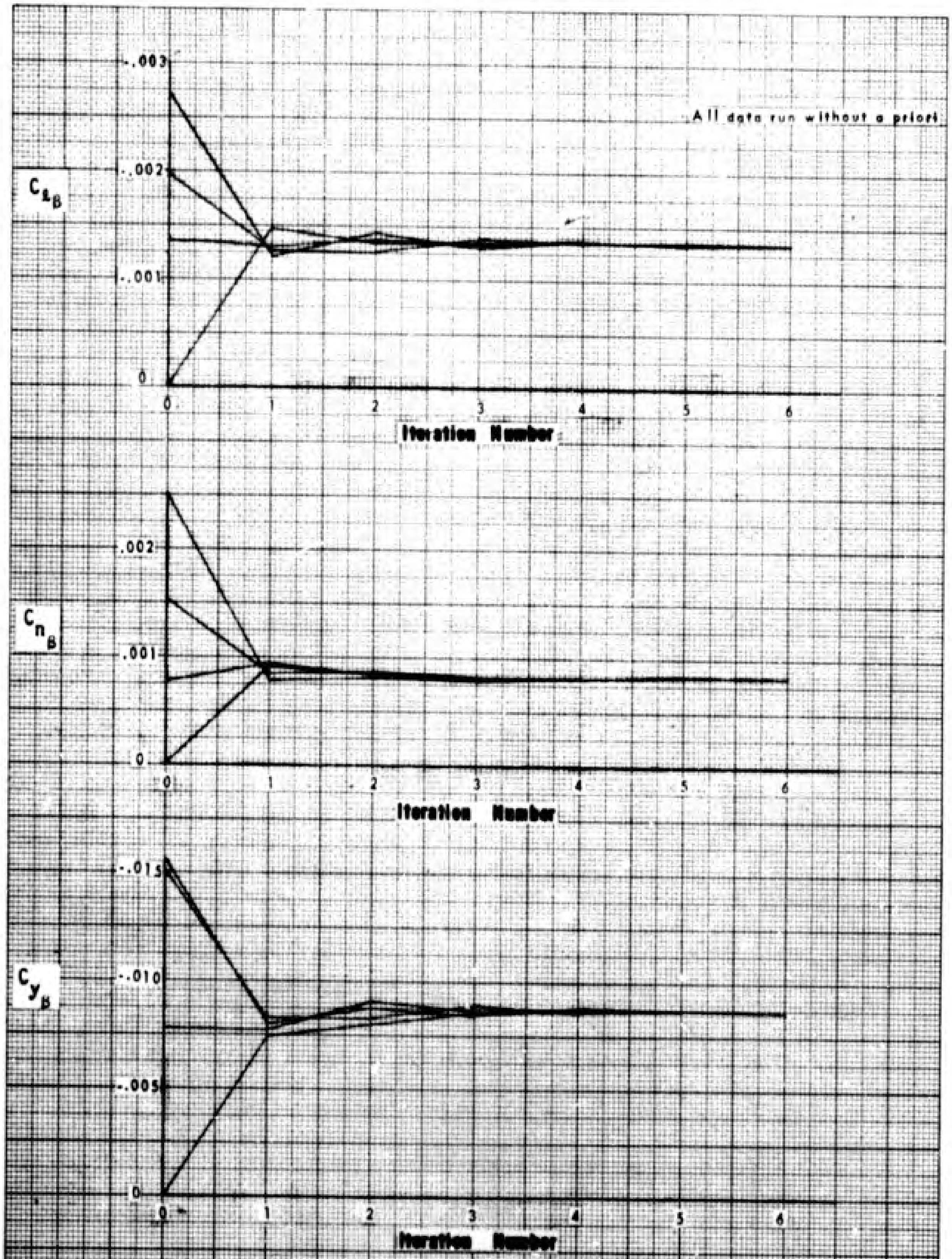

Figure 8 - Effect of Starting Value

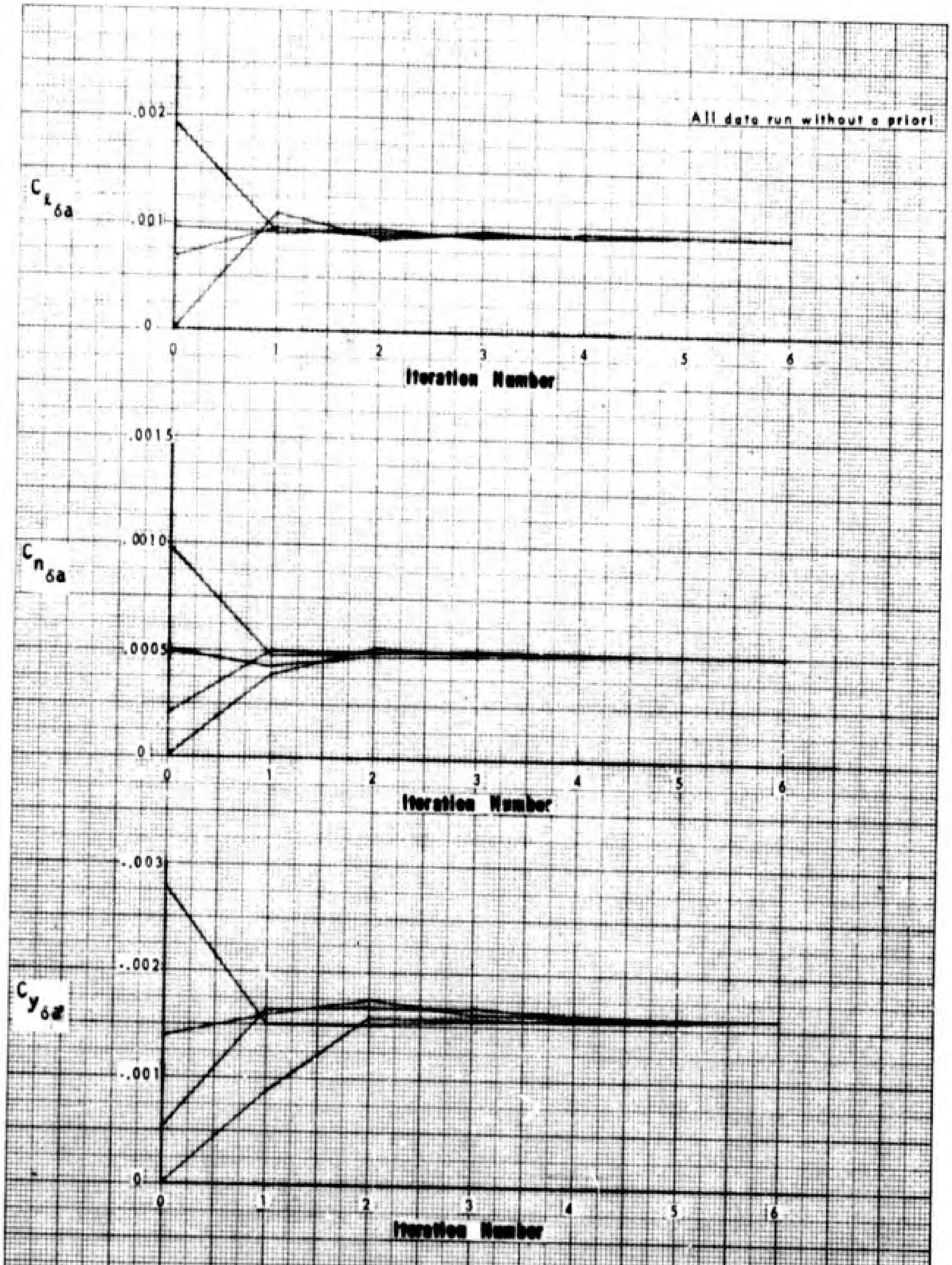

Figure 8- Effect of Starting Values (cont.)

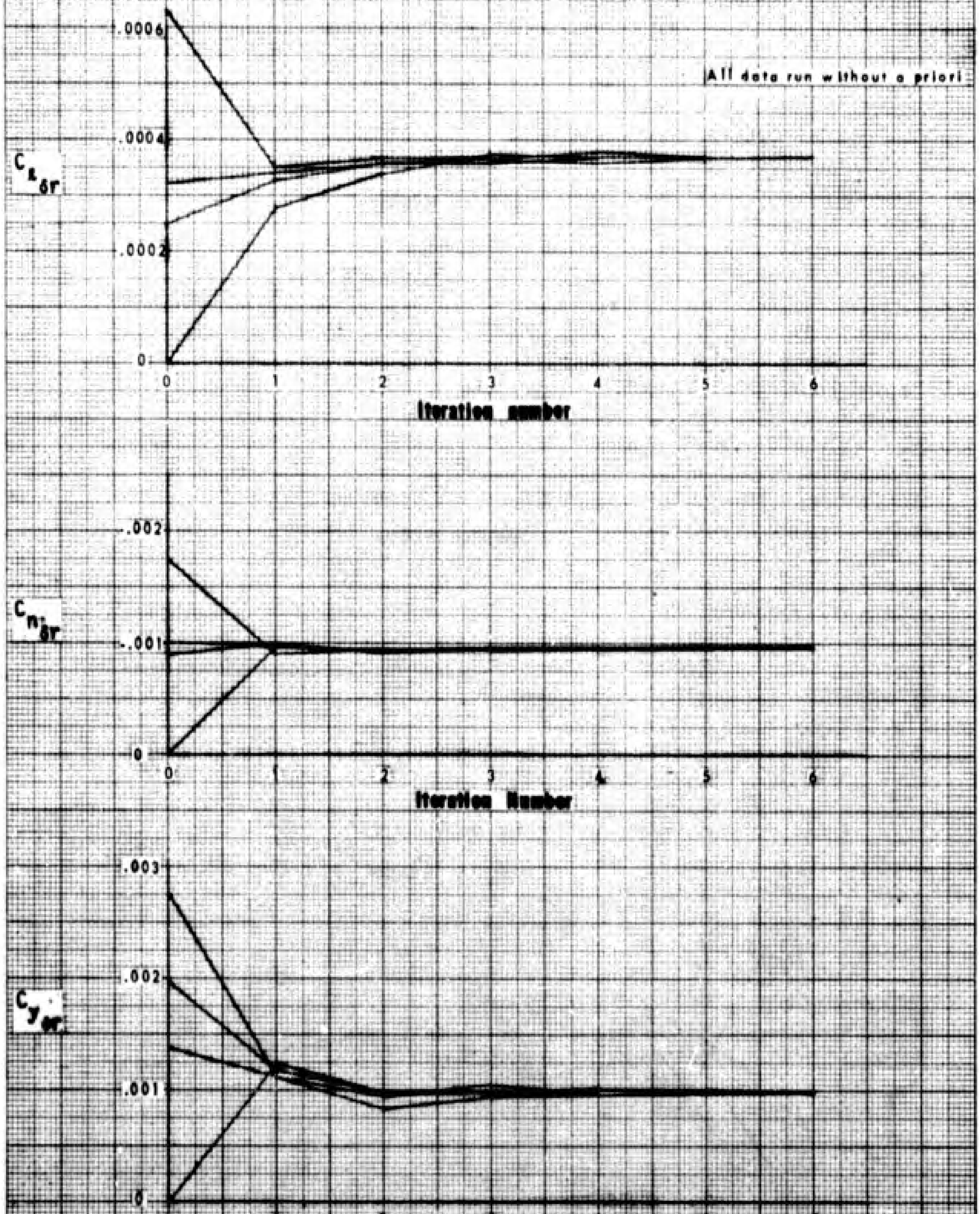

Figure 8 - Effect of Starting-Values (cont.)

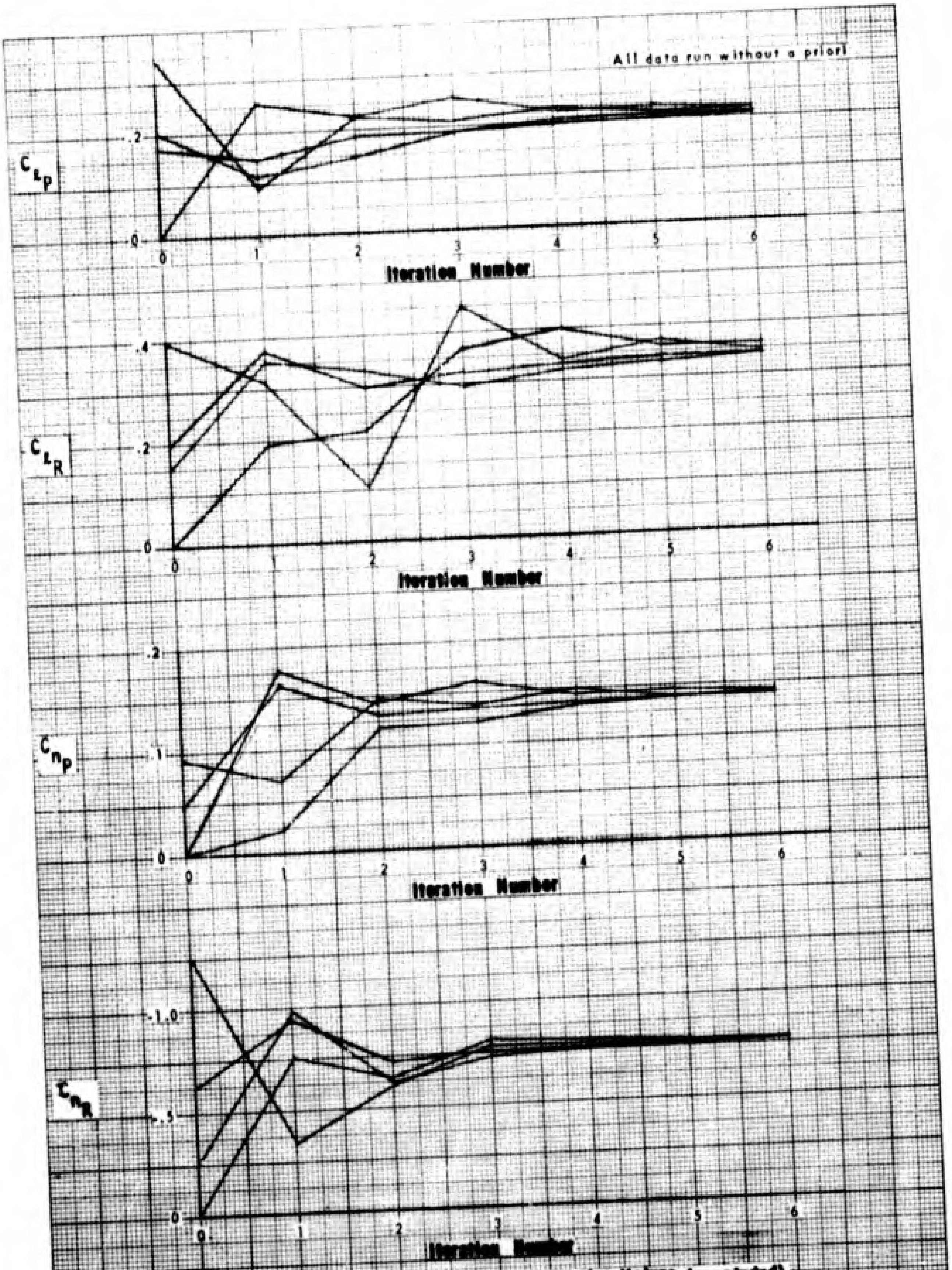

Figure 8 - Effects of Starting Values (concluded)

that the final value in each case is the same. However, this will only work for high quality maneuvers. If the constant mode is used, derivatives to be held fixed during the matching process should be redefined using the DERIVIN namelist. Also derivatives for surfaces which are not used, (DE3, DE4, DC4, etc.) should be set to zero. Obviously, the constants should not be used for a priori operation.

The intent of the lookup and constant modes is to eliminate the A and B matrices. Hence the CURVIN or DERIVIN namelist should replace those matrices. If more than one option is entered (CURVIN or DERIVIN or A or B) the one occurring last in the list of matrices will be used.

Output:

Three types of output, are available from the MMLE program. A sample of the printed output is shown in Appendix B. The program will also generate plots and punch cards. The printed output section may be identified by the numbers written on the sample output.

1. Program options and input data. This section merely tells the user what the program thinks it is supposed to do. The flight conditions and vehicle characteristics are also given. When trouble is encountered in operating the program, this page should be checked first.

2. Input matrices. The matrices read in by the user are shown here. They should be checked for errors. Also, if the lookup or constant mode has been chosen, a message to that effect is given.

3. Time histories. If the PRINT option has been chosen, the input time histories will appear next. These are often valuable for finding sign inversions, wild points, misplaced channels, and other anomalies. The total number of points received is also given. This number should be equal to the sampling rate times the time interval.

4. Starting values. The program prints out the starting values in dimensional and non-dimensional form. The values have the units of per degree except for the damping (rotary) derivatives which are in per radian. An asterisk following the derivative value indicates that derivative is to be held fixed during the matching process.

For numbers 5, 6 & 7 note that no change of the starting derivatives occurs on the first iteration. Hence, iteration number one is really a zero iteration to see how the time histories computed from the starting values match the flight time histories.

5. A and B matrices. The dimensionalized forms of the A and B matrices are shown for each iteration. These generally are not of interest unless a problem occurs. Sometimes a problem can be localized by looking at the first change of a derivative. For example, a change of sign in $C_{\ell_{\delta c_1}}$, $(C_{\ell_{\delta a}})$ might indicate a

reversed sign on δc_1, (δa) or P. If the $C_{n_{\delta c_2}}$ ($C_{n_{\delta r}}$) derivative increases or decreases markedly on the first change, the units of δc_2 (δr) and R might be inspected.

6. Error and error sum. The errors and weighted errors are given for each matched observation trace on each iteration. Also the weighted error sum is shown. The errors are an indication of the contribution of each observation time history to the cost function. The weighted errors are simply the D1 weightings times, the errors, and the weighted error sum is simply the sum of the weighted errors.

   For detailed information on using the errors see the section entitled "D1 Determination."

7. Uncertainty levels. Uncertainty levels are given for each term the program has matched. These uncertainty levels are a measure of the amount of information in the maneuver about each derivative. Since they are a measure of information rather than a derivative value, they must be multiplied by a scale factor in order to apply them as ranges of approximate derivative error. This scale factor has been empirically determined to be on the order of five to ten. More will be said about uncertainty levels in the section "Evaluating the Results".

8. Final values. The final derivative values are given in both dimensional and non-dimensional forms. The δ_o at the end of each line is an aerodynamic bias determined by the program and should generally be small (<< 1.0) for the non-dimensional case.

9. Computed time histories. If the PRINT option has been set, the computed time histories follow.

If the D1 or WMAPR determination option is used, the output will continue as these parameters are identified.

Plotted output is important in determining the quality of the match. A plot may be made by requesting that a magnetic tape be mounted as TAPE13 before program execution (see example in Appendix D). After the plot tape has been created a plotter job request card must be submitted to get plots. The card should contain the following information:

Number of plots - 2 per case
Time per plot - 1 minute
Plot number - Start - 1
End - 999
Pen position - 2
Pen type - Wet
Pen point size - 5
Paper size - Either 201 or 202

The plots show the flight and computed time histories. In addition to showing how well the program matched the flight data, plots greatly facilitate finding incorrect signs, phase lag, and magnitude problems. A good match will overlay the flight data very closely.

Some data can be salvaged from the plots even if there is not a good match. If the magnitude of the rates (P, Q, R) match immediately after the pulses, the control derivatives are probably good. If the frequency of the computed time history matches that of flight time history, the stability derivatives (C_{m_α}, C_{ℓ_β} and C_{n_β}) are probably close.

Some care must be exhibited in setting up the plot program to insure staying within the plotter capability. Plotter matrices are dimensioned to hold up to a thousand data points. Thus, if T is the length of the time segment to be plotted:

$$(SPS\ (T) \leq 1000$$

In addition, the size of the plot paper is limited to 10 inches. If T is large, the TIMESC parameter from the INPUT namelist must be corresponding large. The following relationship may be used:

$$\frac{T}{(2)\ (TIMESC)} \leq 10$$

MMLE is capable of generating line printer plots. This is done by setting the PRNTPLT parameter (item 55, INPUT namelist) to a value of 1 or 2. These plots have the inherent advantage that they may be obtained much faster than Calcomp plots. They have two distinct disadvantages, however. If continuous plots are used (PRNTPLT=1) a large amount of paper is used. If page plots are generated (PRNTPLT=2) resolution of data points is poor. An example of both a continous plot and a page plot is shown in Appendix B.

Punch cards are the final type of output, and three sets of them may be obtained for any maneuver. The PUNCH option (item 24, INPUT namelist) will generate cards designed as input for the follow-on plotting program, SUMARY. The PUNCHD option (item 26, INPUT namelist) gives cards with the dimensionalized A and B matrices on them. These may be used for restarting the MMLE program if desired. The third type, given by the PUNCHC option (item 25, INPUT namelist) punches five cards containing the flight conditions, weights and inertias, and the final non-dimensional derivatives. These cards are designed to be read by the CONTROL program for follow-on characteristic analysis and Milspec computation.

D1 Determination:

It is not always possible or desirable to use the default D1 matrix, and determination of an aircraft-peculiar D1 matrix may be necessary.

Proper D1 values will drive the weighted errors to equal values as the program converges. Since the particular value is entirely relative, the value one is usually chosen as a convenient standard. Then the D1 weighting may be obtained as the inverse of the corresponding error. If the D1 matrix is set to give weighted errors of one, then the final weighted errors sum will be equal to the number of non-zero D1 parameters.

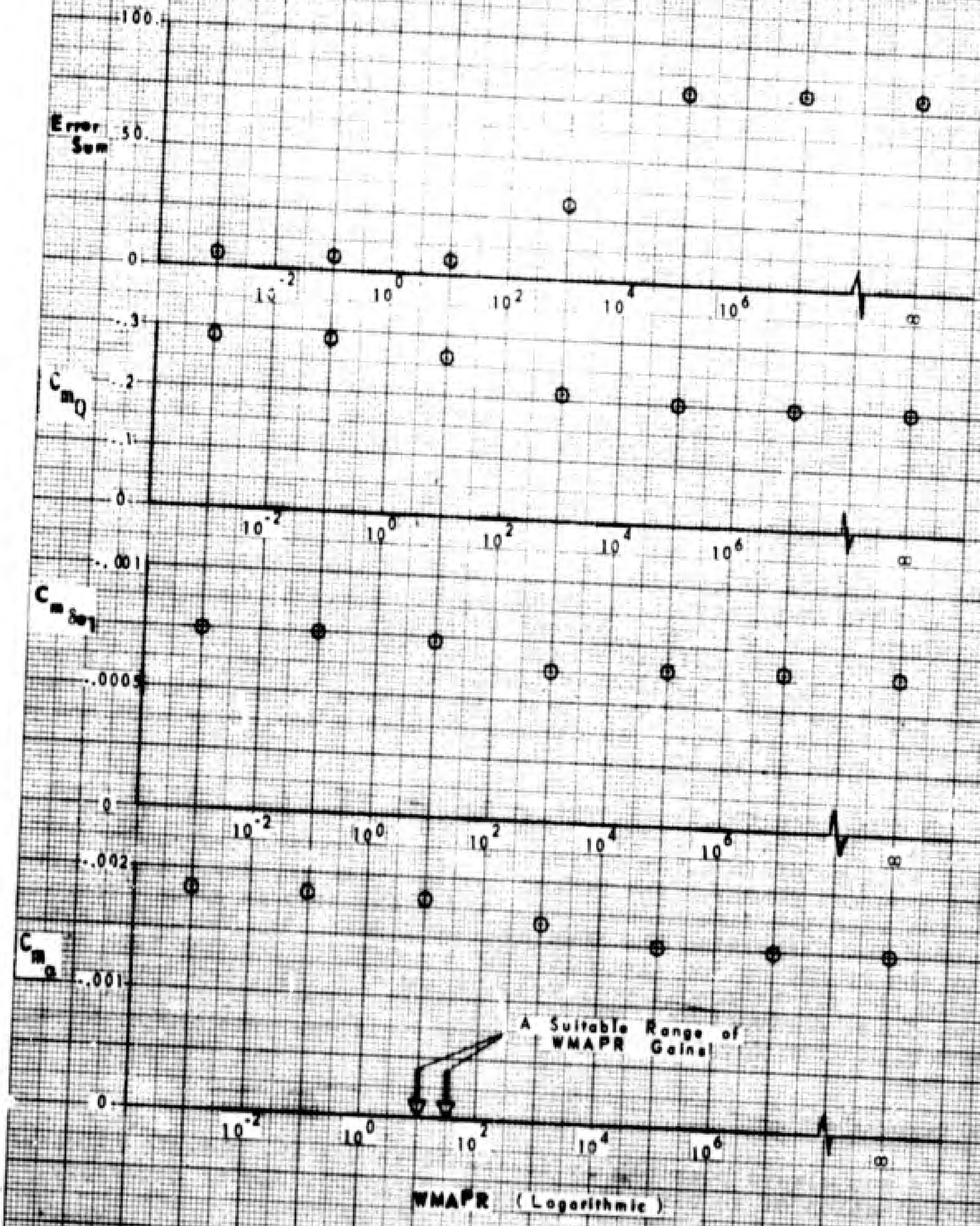

Figure 9 - Effect of WMAPR on Derivative Values

The program has the capability to calculate the D1 matrix by setting the ND1 parameter (item 54, INPUT namelist) to some non-zero value (usually 3-5). The default D1 matrix will serve as an excellent starting point. This should be done with several good maneuvers, and the most consistent D1 results should be used. While a new D1 may be determined for each case, this is usually not necessary and the increased computer time (a factor of ND1) makes this procedure undesirable. In addition, a D1 weighting that is dependent on the maneuver may mask a poor maneuver and provide misleading results. Usually the D1 weighting is a function of instrumentation only, however some experience has shown that the D1 weighting may change for severe flight conditions, i.e., very high or very low dynamic pressure. Given good instrumentation and good maneuvers, the final derivatives are a weak function of D1 weighting and this balancing process is not as important. One entire test program has been run using the default D1 values with good results. However, as the quality of the instrumentation and/or the maneuvers decreases, increased attention must be paid to getting the D1 matrix right. Again it should be pointed out that proper attention to instrumentation and pilot maneuvers makes extensive work with the D1 matrix unnecessary.

A Priori Determination :

Data which yield poor convergence properties and/or poor derivative estimates may sometimes be salvaged using the a priori feature. The a priori option holds a derivative close to its starting value if there is insufficient information to determine a new value. The degree of immobility is determined by the a priori matrices APRA and APRB, and the weighting factor WMAPR. These numbers must therefore be determined before the option can be used. The default matrices for APRA and APRB are a good initial guess. As an aid in determining the overall gain, the program is equipped to give plots of the derivatives as a function of WMAPR (Figure 9). This may be done by setting the NAPR (item 54, INPUT namelist) to some non-zero value. The value of WMAPR should be chosen so that effective derivatives are free to seek an accurate value. Often the value of WMAPR required to double the non-a priori error sum is chosen as a convenient standard. Note that the total weighting is for a derivative equal to WMAPR times the corresponding term in the APRA or APRB matrices. Thus either value may be changed to vary the weighting.

A check to insure the correct weighting may be made by running a number of cases at the same Mach number but at different angles of attack. If these cases are run with and without a priori, the effect of the weighting may be determined (Figure 10). If there is appreciable scatter in the non-a priori-weighted values, and the use of the a priori option does not reduce the scatter significantly, the weighting is too light. If there is little scatter but significant differences between the derivative values run with and without a priori, the weighting is too heavy. If there is little scatter and good agreement between the two sets of data, the weighting may be correct, but a further test is necessary. The cases should be rerun with the a priori option using starting values which have been doubled. If the new values are approximately the same as the original a priori-weighted values, the weighting is correct. Uncertainty levels (discussed below) can also be used as a general guide to indicate correct weighting.

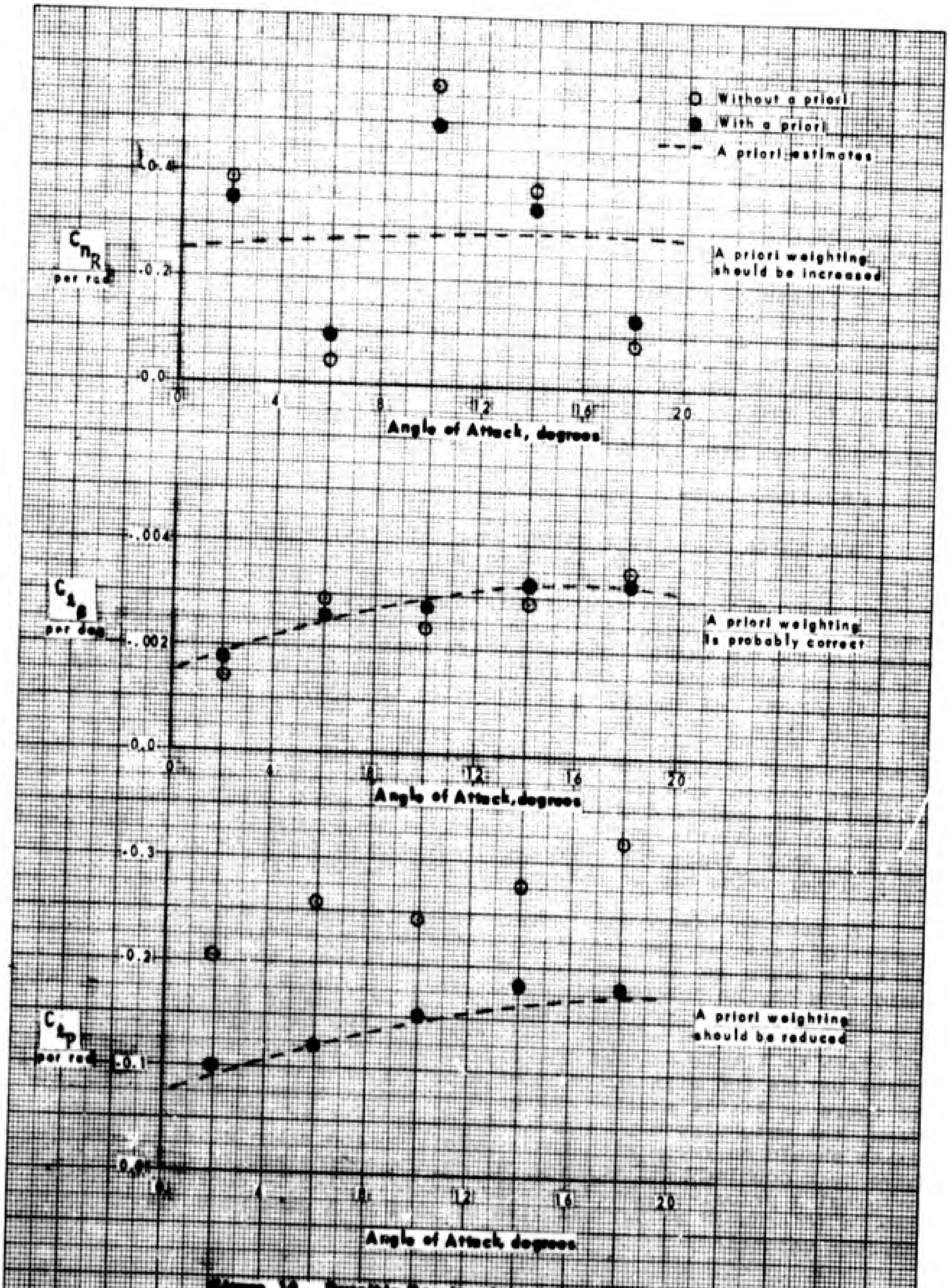

Figure 14 - Possible Results of the A Priori Weighting Analysis

There are several pitfalls to using the a priori feature, even with correct weightings. The first is that there should be some confidence in the starting values, and hence, the pre-defined constants should not be used. The second is that if a derivative falls consistently on the wind tunnel value, it does not necessarily mean that the wind tunnel result is correct. It may be that there is little information about that derivative and it is being held at wind tunnel values by a priori. This is especially true of damping derivatives. (Note that this is still the best estimate available.) In general a priori should be used with care and only by those who understand both its advantages and pitfalls.

Evaluating the Results :

Although the mechanical procedure of processing data through the available computer programs is not difficult, a proper understanding of the resulting output data requires some level of experience. There are many things to be considered in determining how accurate a derivative value is. In general, the more points analyzed, the easier it is to evaluate the results; it is very hard to draw conclusions from one or two points at each Mach number. If an acceptable number of cases are available the following factors should be weighed:

1. Match - First and foremost, did the computer time histories match the flight time histories? This is indicated by a low error sum and the equivalence of the two time histories on the plots. This condition is necessary for accuracy but <u>NOT</u> sufficient.

2. Uncertainty levels - Uncertainty levels are given for each derivative determined. By multiplying these values by a scale factor (empirically determined to be five to ten), they may be used as a range of possible derivative variation: the lower the value of the uncertainty levels, the more accurate the derivative is. Studies on these uncertainty levels indicate that a high uncertainty level (high range of values) usually means that the data point is bad. A low range of values means that the point is probably good, but not necessarily so. Uncertainty levels, when used in conjunction with the other factors described in this section, should be a valuable aid in determining the accuracy of individual derivatives.

3. Scatter - Good data will usually exhibit a low degree of scatter when plotted as a function of angle of attack. The one exception would be, as discussed previously, when a particular derivative stays near its starting value when using a priori due to a lack of information or too high a weighting on that derivative. A large amount of scatter may mean one of two things. The program may be having difficulty in determining a value. If this is the case, it means that the derivative is not very effective (in that maneuver) or the derivative contribution is being masked by some other effect. A second possibility is that the derivative actually is scattered, i.e., C_{n_β} may vary greatly near a vertical tail separation boundary, or an effect such as pulse magnitude, flexibility effects, or aircraft configuration has not been accounted for.

4. Maneuver - As stated previously, the type of maneuver done is the single most important factor in determining whether the derivative values are good. A rapid doublet maneuver (where the effects of individual derivatives are isolated) is much more likely to give accurate results than a match of a pilot induced oscillation (where the control surfaces are driving the frequency and masking the effects of the stability derivatives). It should be pointed out that the time history match of a maneuver like a pilot induced oscillation will probably be very good. The derivatives may not be accurate, however, since it is quite likely that significant trade-offs have occurred between control and stability derivatives. The quality of the maneuver should be determined by evaluating the maneuver with the criteria set forth in the Maneuver Section.

5. Derivative effectiveness - Derivatives which strongly influence aircraft response may be determined much more accurately than noninfluential ones. Table 7 is an attempt to classify the derivatives and show which derivatives are easiest to get and most accurately determined.

6. Agreement with other data - Derivative values should be evaluated with respect to other flight results. For example if the pitch short period frequency is less than predicted, it might be confirmed by a lower-than-predicted C_{m_α}. Theoretical steady state sideslip values may be computed from the derivatives and compared against flight obtained values.

Hybrid Matching

The second method of obtaining derivatives is a hybrid matching program called STABDIV. This program makes use of a Hydac-2400 digital computer to store time histories and an EAI 231R to solve the equations of motion. The program is run in a repetitive operation mode and equations are solved at fifty times real time. Thus, the flight time history and the computed time history appear together as standing waves on an eleven by fourteen inch scope.

Equations:

The equations of motion are the same as those derived in Appendix E for the full five-degree-of-freedom case.

These are full five-degree-of-freedom equations with time invariant coefficients. The assumptions which have been made in deriving them are minor for almost all cases. No linearization or small angle assumptions are made. The dynamic pressure is allowed to vary during the maneuver; velocity is held fixed.[19] It can be seen that for maneuvers, where linearity assumptions have been violated, the hybrid matching program offers a superior math model.

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[19] The constant velocity restriction only affects damping derivatives, whereas the constant dynamic pressure restriction affects all derivatives.

The Process :

The flight time histories are read into the digital computer via a seven track BCD tape. The data time must be ten seconds long and be sampled at fifty samples per second. Exact formatting requirements are given in the section "Preparing the Flight Data". After the digital data has been stored, the analog portion must be set up. Internal potentiometers, which set up scale factors, inertias, etc., must be set or checked. Initial estimates of derivatives must be set on the external potentiometers. Each external potentiometer corresponds to one derivative. After these tasks have been completed, the matching process is ready to start. The process is started in the uncoupled mode. This means that flight time histories for the rates, angles, and control surfaces are used. The only unknowns in the equations are the derivatives themselves. Both the measured and the computed time histories appear as standing waves on a repetitive operation screen. By adjusting the external potentiometers (changing the derivatives) the operator may change the computed time histories until a good match is obtained. After a good match is obtained, the computer is switched to the coupled mode. In the coupled mode some of the rates and angles which are outputs to the equations of motion are fed back as inputs to replace the flight time histories. Now "fine tuning" may be done. When the match is the best obtainable, the potentiometer values are noted, and the derivatives may be obtained by the application of a simple scale factor. More information may be found in a report by Paul W. Kirsten.[20]

Comparison of the Two Methods

While the methods of extracting derivatives are radically different, a great many of the comments which were made about derivatives in the MMLE section are applicable to derivatives obtained from the hybrid program. Both programs are capable of giving accurate derivatives for most maneuvers (Reference 9). There are, however, some fundamental differences which make MMLE more suitable for one mode of operation and STABDIV more suitable for another. These basic differences will be discussed.

Time Requirements:

The time requirements for the two programs dictate that MMLE be used for any production processing of data. While the time requirements vary with the type and number of cases, it may be said that MMLE setup and processing requires about fifteen minutes of time per case by relatively unexperienced personnel. STABDIV, on the other hand, usually requires about an hour of work by a highly trained individual for each case. These estimates are for engineering personnel and do not reflect the manpower required to operate the computer in either case.

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[20] Reference 9: Kirsten, Paul W. and Ash, Lawrence G., A Comparison and Evaluation of Two Methods of Extracting Stability Derivatives from Flight Test Data, AFFTC-TD-73-5, Air Force Flight Test Center, Edwards AFB, California, May 1974.

Equations

The math model is considerably different for the two programs, and this difference suggests that STABDIV be used for non-linear maneuvers. Because of the linearization required, inertial coupling and/or excessive Euler angle variation cannot be easily modelled by the MMLE program (see footnote 16). These are taken into account by the hybrid program. A comparison of the two sets of equations of motion will show that the equations for the hybrid program are much more complete.

Weighting Functions:

The a priori and D1 weighting functions which are in MMLE can be performed adequately by the operator in the hybrid matching program. The STABDIV operator does supply an extra weighting function which is a time weighting. We would like to be able to weight the control derivatives more heavily at the time of the control input. In the free response we would like to weight the stability and damping derivatives. The operator in the STABDIV program can make this judgment. While MMLE does not have this feature, it is done inherently if the separation of inputs criteria is followed. If the control input is rapid, and takes place before the aircraft begins to oscillate, the time weighting will be automatic since most of the information on the control derivative is given at the time of the input.

Derivative Accuracy

The question always arises "How accurately can you get derivatives?" There is no easy answer to this, as all of the factors discussed in this paper will affect the accuracy to some extent. A review of these considerations would include instrumentation, pilot maneuvers, the maintaining of trim conditions, the derivative itself, accuracy of inertias, and the particular aircraft to be analyzed. With these in mind, and assuming that the suggestions in this report have been followed, Table 7 is presented as some estimates of derivative accuracy. Note that since these accuracies are given in per cent, they should not be used as the actual derivative approaches zero.

Table 7

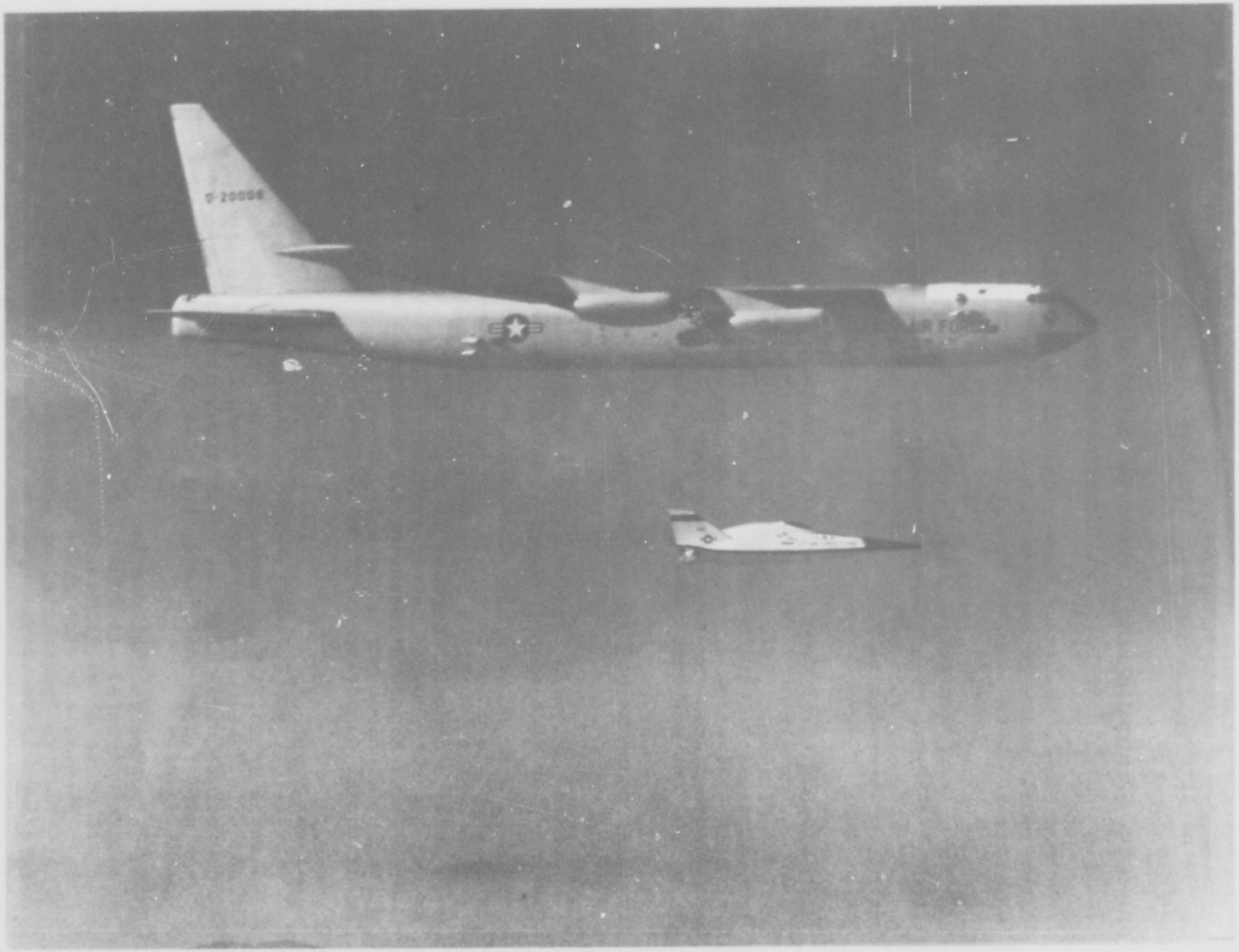
DERIVATIVE ACCURACY

DERIVATIVE	EXPECTED ACCURACY, %	COMMENTS
Primary		
$C_{m_{\alpha}}$	± 7.5	
$C_{m_{\delta e}}$	± 7.5	
$C_{N_{\alpha}}$	± 7.5	
$C_{\ell_{\beta}}$	± 7.5	
$C_{\ell_{\delta a}}$	± 7.5	
$C_{n_{\beta}}$	± 7.5	
$C_{n_{\delta r}}$	± 7.5	
Secondary		
$C_{m_{Q}}$	± 15	SAS off, double for SAS on
$C_{\ell_{P}}$	± 15	SAS off, double for SAS on
$C_{n_{\delta a}}$	± 15	Aircraft dependent
$C_{y_{\beta}}$	± 15	Depends on accelerometer placement
Transitional (Aircraft dependent)		
$C_{N_{\delta e}}$	± 25	
$C_{\ell_{\delta r}}$	± 25	Very aircraft dependent
$C_{n_{P}}$	± 50	Maneuver dependent
$C_{n_{R}}$	± 50	Maneuver dependent
$C_{y_{\delta r}}$	± 25	Depends on accelerometer placement

Table 7 (Concluded)

DERIVATIVE ACCURACY

DERIVATIVE	**EXPECTED ACCURACY, %**	**COMMENTS**
Ineffective		
C_{ℓ_R}	± 200	
$C_{Y_{\delta a}}$	± 100	Depends on accelerometer placement


0-20008

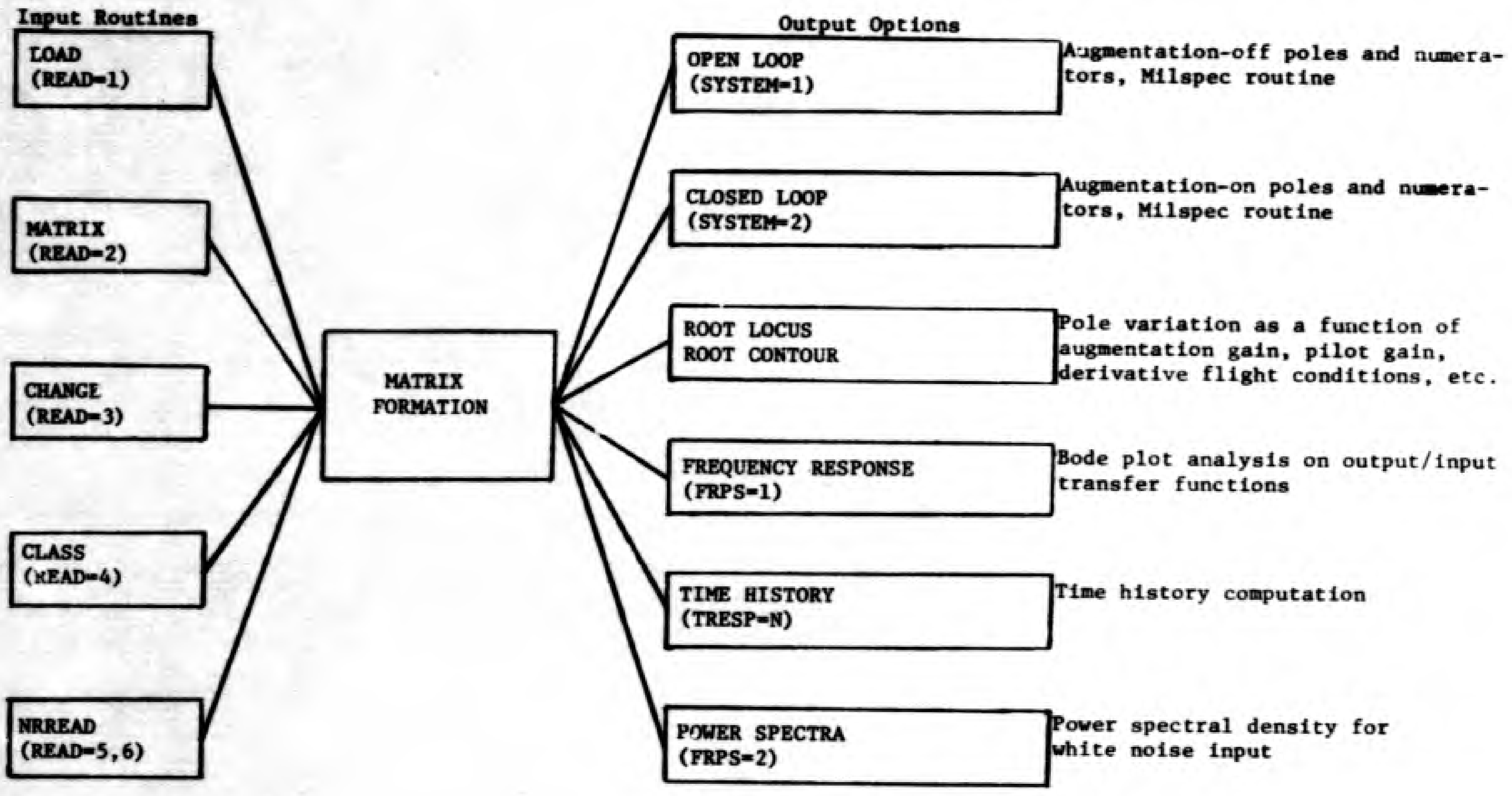


Figure 11 - Control Capability

DERIVATIVE ANALYSIS

A knowledge of the flight derivatives is a very valuable thing, but a great deal more information can be gleaned by coupling a model of the flight control system with the model of the aerodynamics. A digital program, CONTROL, has been developed to do this. The input and output of this program will be discussed in this section.

CONTROL

The CONTROL program is primarily a characteristic analysis program. In addition the program can do frequency response, transient response, and power spectrum analysis (Figure 11). The program is capable of executing in many different modes, and to explain them all is beyond the scope of this report and the knowledge of the author. Thus, the primary mode of analysis, that of getting characteristic roots and Mil-spec data, will be discussed first. Then several other modes and options which are useful will be explained. Power spectral density computation, sampled-data analysis, and digital system analysis will not be discussed in this report.

The CONTROL program is a method of coupling the aerodynamics of the aircraft with the flight control system to obtain vehicle handling qualities characteristics for the entire system. The aerodynamics, of course, are defined by wind tunnel testing initially and will be modified or verified by derivative extraction results as the flight test program proceeds. Definition of the flight control system requires an accurate knowledge of system description, gains and transfer functions and, sometimes, actual component locations.[21] These are usually furnished by the aircraft contractor and are the result of end-to-end response checks. One of the advantages of dividing the aircraft characteristics into components like this is the extrapolation which may be performed on each component. For example, derivative data may be standardized at a particular dynamic pressure to remove that effect from frequency and damping values. Proposed flight control system changes may be analyzed by the CONTROL program much faster and with much less expense than flight testing. Final results should be verified by actual flying, however.

CONTROL Algorithm and System Definition

The concept of state variables is that given the present state of the system, a knowledge of the inputs, and a description of the system, the future state of the system may be predicted. Define the state vector, $\hat{x}$, and the input vector, $\hat{u}$, to be the following:

$$\hat{x} = \begin{bmatrix} P \\ R \\ B \\ \text{Ø} \end{bmatrix} \qquad u = \begin{bmatrix} \delta a \\ \delta r \end{bmatrix} \qquad \text{(Lateral-directional)}$$

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[21] This applies most often to cg corrections for feedback accelerometers.

Then the matrix equation for predicting the future state is:

$$\hat{C} \cdot \dot{\hat{x}} = \hat{A} \cdot \hat{x} + \hat{B} \cdot \hat{u}$$

Here the $\hat{A}$, $\hat{B}$, $\hat{C}$ matrices describe the system. Now let us recall the simplified lateral-directional equations of motion (Appendix E).

$$\dot{P} - \frac{I_{xz}}{I_x} \dot{R} = L_P \cdot P + L_R \cdot R + L_\beta \cdot \beta + L_{\delta a} \cdot \delta a + L_{\delta r} \cdot \delta r$$

$$\dot{R} - \frac{I_{xz}}{I_z} \dot{P} = N_P \cdot P + N_R \cdot R + N_\beta \cdot \beta + N_{\delta a} \cdot \delta a + N_{\delta r} \cdot \delta r$$

$$\dot{\beta} = P \sin \alpha - R \cos \alpha + Y_\beta \cdot \beta + \frac{g}{V} \cos \theta \cdot \phi + Y_{\delta a} \cdot \delta a + Y_{\delta r} \cdot \delta r$$

$$\dot{\phi} = P$$

With a knowledge of matrix multiplication we may formulate the terms of the $\hat{A}$, $\hat{B}$, and $\hat{C}$ matrices.

$$\overset{C}{\begin{bmatrix} 1.0 & -\frac{I_{xz}}{I_x} & 0.0 & 0.0 \\ -\frac{I_{xz}}{I_z} & 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}} \cdot \overset{\dot{\hat{x}}}{\begin{bmatrix} \dot{P} \\ \dot{R} \\ \dot{\beta} \\ \dot{\phi} \end{bmatrix}} = \overset{\hat{A}}{\begin{bmatrix} L_P & L_R & L_\beta & 0.0 \\ N_P & N_R & N_\beta & 0.0 \\ \sin \alpha & -\cos \alpha & Y_\beta & \frac{g}{V} \cos \theta \\ 1.0 & 0.0 & 0.0 & 0.0 \end{bmatrix}} \cdot \overset{\hat{x}}{\begin{bmatrix} P \\ R \\ \beta \\ \phi \end{bmatrix}} + \overset{\hat{B}}{\begin{bmatrix} L_{\delta a} & N_{\delta r} \\ N_{\delta a} & N_{\delta r} \\ Y_{\delta a} & Y_{\delta r} \\ 0.0 & 0.0 \end{bmatrix}} \cdot \overset{\hat{u}}{\begin{bmatrix} \delta a \\ \delta r \end{bmatrix}}$$

The two degree-of-freedom longitudinal matrix equation is:

$$\overset{\hat{C}}{\begin{bmatrix} 1.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 1.0 \end{bmatrix}} \cdot \overset{\dot{\hat{x}}}{\begin{bmatrix} \dot{Q} \\ \dot{\alpha} \\ \dot{\theta} \end{bmatrix}} = \overset{\hat{A}}{\begin{bmatrix} M_Q & M_\alpha & -\frac{g}{V} \sin \theta \\ (1.0 - N_Q) & N_\alpha & 0.0 \\ 1.0 & 0.0 & 0.0 \end{bmatrix}} \cdot \overset{\hat{x}}{\begin{bmatrix} Q \\ \alpha \\ \theta \end{bmatrix}} + \overset{\hat{B}}{\begin{bmatrix} M_{\delta e} \\ N_{\delta e} \\ 0.0 \end{bmatrix}} \cdot \overset{\hat{u}}{\begin{bmatrix} \delta e \end{bmatrix}}$$

The eigenvalues of the $\hat{A}$ matrix (modified by $\hat{C}^{-1}$) constitute the characteristic roots of the open loop system (basic aircraft -SAS off). This is the open loop option of the program.

The augmentation-system-on aircraft characteristics are described by closed loop analysis. To provide closed loop response it is necessary to provide an additional equation for the $\hat{u}$ vector. The equation defined in the program for closed loop is:

$$\hat{u} = \hat{K1} \cdot \hat{x} + \hat{K2} \cdot \hat{\dot{x}} + \hat{D} \cdot \hat{u}_{com}$$

Here $\hat{u}_{com}$ is usually a pilot commanded input and $\hat{K1}$ and $\hat{K2}$ define which parameters from the $\hat{x}$ and $\hat{\dot{x}}$ vectors will be fed back. $\hat{K1}$, $\hat{K2}$, and $\hat{D}$ may be defined by the user or by the program using the MIXED option.

For the root locus option the $\hat{u}$ vector is modified to allow for two independent feedback loops.

$$\hat{u} = (\hat{K1} \cdot \hat{x} + \hat{K2} \cdot \hat{\dot{x}}) + (\hat{K3} \cdot \hat{x} + \hat{K4} \cdot \hat{\dot{x}})$$

Any member of the $\hat{x}$ or $\hat{\dot{x}}$ vectors may be fed back into the system. The root locus option is useful for combining a SAS feedback (loop 1) and a pilot model (loop 2). Unlike the closed loop option, the root locus option provides for a user-selected number of iterations of each loop with arithmetically or geometrically increasing gains.

In addition to the state and input vectors a third vector, called an output vector, is defined by the equation.

$$\hat{y} = \hat{H} \cdot \hat{x} + \hat{G} \cdot \hat{\dot{x}} + \hat{F} \cdot \hat{u}$$

By suitable selection of the terms in the $\hat{H}$, $\hat{G}$, and $\hat{F}$ matrices, any parameter may be selected or created from the x, $\dot{x}$, and u vectors for inclusion in the output vector. For example, assume we wish to create the y vector.

$$\hat{Y} = \begin{bmatrix} P \\ R \\ N_y \end{bmatrix}$$

We have P and R from the $\hat{x}$ vector and the equation for N_y in g's is:[22]

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[22] An alternate equation using β and subtracting the unwanted terms would have worked just as well.

Table 8

CONTROL SYSTEM EQUATIONS

Vectors

$\hat{x}$ - state vector

$\hat{y}$ - output vector

$\hat{u}$ - input vector

System Models

Open Loop

$$\hat{C} \cdot \dot{\hat{x}} = \hat{A} \cdot \hat{x} + \hat{B} \cdot \hat{u}$$

$$\hat{y} = \hat{H} \cdot \hat{x} + \hat{G} \cdot \dot{\hat{x}} + \hat{F} \cdot \hat{u}$$

Closed Loop

$$\hat{C} \cdot \dot{\hat{x}} = \hat{A} \cdot \hat{x} + \hat{B} \cdot \hat{u}$$

$$\hat{y} = \hat{H} \cdot \hat{x} + \hat{G} \cdot \dot{\hat{x}} + \hat{F} \cdot \hat{u}$$

$$\hat{u} = \widehat{K1} \cdot \hat{x} + \widehat{K2} \cdot \hat{x} + \hat{D} \cdot \hat{u}_{com}$$

Root Locus

$$\hat{C} \cdot \dot{\hat{x}} = \hat{A} \cdot \hat{x} + \hat{B} \cdot \hat{u}$$

$$\hat{u} = (\widehat{K1} \cdot \hat{x} + \widehat{K2} \cdot \dot{\hat{x}}) + (\widehat{K3} \cdot \hat{x} + \widehat{K4} \cdot \dot{\hat{x}})$$

$$N_y = \frac{\bar{q}s}{W} C_{Y_\beta} \cdot \beta + \frac{\bar{q}s}{W} C_{Y_{\delta a}} \cdot \delta a + \frac{\bar{q}s}{W} C_{Y_{\delta r}} \cdot \delta r$$

The H, G, and F matrices are defined as:

$$\hat{y} = \hat{H} \cdot \hat{x} + \hat{F} \cdot \hat{u}$$

$$\begin{bmatrix} P \\ R \\ N_y \end{bmatrix} = \begin{bmatrix} 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & \frac{\bar{q}s}{W} C_{Y_\beta} & 0.0 \end{bmatrix} \cdot \begin{bmatrix} P \\ R \\ \beta \\ \phi \end{bmatrix} + \begin{bmatrix} 0.0 & 0.0 \\ 0.0 & 0.0 \\ \frac{\bar{q}s}{W} C_{Y_{\delta a}} & \frac{\bar{q}s}{W} C_{Y_{\delta r}} \end{bmatrix} \begin{bmatrix} \delta a \\ \delta r \end{bmatrix}$$

For this particular $\hat{y}$ vector the $\hat{G}$ matrix was not required, since no terms from the $\hat{x}$ vector were used.

A summary of vectors and system equations is shown in Table 8.

Input Data:

Data for CONTROL may be subdivided into three sections. The first section tells the program what to do and is required for all cases (unless IFLAG=1). The second section describes the aircraft or "plant" equations to the computer, and the third section outlines the flight control system. For some cases either the plant equations or the flight control system description may be omitted.

Controlling CONTROL

The mode in which the program executes is determined by the first section of data cards. These include the following:

Card 1 - Title, Case number (9A8,I8)
Card 2-n - CODE namelist (namelist format)
Card n+1 - OUTPUT labels (10A8)
Card n+2 - INPUT labels (IOA8)

The title may be any message in the first seventy-two columns. The case number should be the same as that written by MMLE, if data from that program is being used. The sign convention established by MMLE to identify the mode (positive for lateral-directional and negative for longitudinal) is not applicable here, and the case number is always positive. If the case numbers from MMLE and the title card do not match, a nonfatal warning message is given. The CODE namelist follows beginning with the second card. The code namelist defines program options, system definition parameters, etc. The parameters in Table 9 are available for use. The actual numbers for each option may be found in Appendix C. Any options where zero is desired may be omitted. The OUTPUT and INPUT labels are merely user-selected, literal names for parameters in $\hat{y}$ and $\hat{u}$ vectors, respectively.

Table 9

CODE NAMELIST PARAMETERS

NAME	DESCRIPTION
READ	Determines the form of the input data for the plant equations.
SYSTEM	Determines whether open loop, closed loop, or root locus will be done.
OUTPUT	Determines the form and matrices required for the output equation.
MIXED	Determines if a flight control system will be coupled to the plant equations.
DIGITL	Determines whether continuous, sampled-data, or discrete system analysis will be done.
FRPS	Determines whether frequency response will be done and for what type of system.
NUMERS	Determines whether numerator transfer functions will be calculated.
TRESP	Determines the number (if any) of transient responses to be calculated.
NX, NY, NU	Determine the size of the plant system vectors, $\hat{x}$, $\hat{y}$, and $\hat{u}$.
NXC, NUC	Determines the size of the state and input vectors for a sampled-data system.
ZOH	For sampled-data systems, the number of inputs to the plant which are outputs of zero-order-hold devices.
N1, N2	Determines the number of iterations of the two feedback loops for root locus systems.
CONTUR	Determines whether parameter variation studies will be done.
MULTRT	For sampled-data systems, determines how many (if any) transient response points will be computed for each sample period.
MODEL	Determines whether model following will be used.
NSCALE	Determines whether the state vector will be numerically conditioned.
CMAT	Determines whether the $\hat{C}$ matrix is necessary.

Table 9 (Concluded)

NAME	DESCRIPTION
NK2	Determines whether the $\hat{K}2$ and $\hat{K}4$ matrices are necessary.
FORM	Determines whether plots will be produced. (Inoperative)
IPT	Provides extra printout for debugging.
IGO	Determines whether flight control system information will be saved.
SAV	Determines whether data matrices will be saved.
IFLAG	Determines whether title, namelist, labels, and input data will be saved.
READ3	Determines whether CHANGE subroutine will be used.
DELT	Defines time increment for transient responses or sample period for sampled-data systems.
FINALT	Final time for transient responses.
IFREQ, FREQ DELFREQ	Defines initial, final, and incremental frequencies for frequency response.
M	Code for modified z-transfer function computation for sampled data systems.
GAIN 1, GAIN 2	Defines gain increments for the two feedback loops in the root locus system.

The second section of input data describes the aircraft to the program. There are several ways to do this but the easiest is to use the NRREAD subroutine (READ=5 or 6). This subroutine accepts nondimensional derivatives and flight parameters and creates the $\hat{A}$, $\hat{B}$, $\hat{C}$, $\hat{H}$, $\hat{G}$ and/or $\hat{F}$ matrices. These data may be broken into two subsections. The first is composed of the following five cards (READ=5):

Card 1 - Format (5I5,5X,5F10.4)

NSEQ - Sequence number. Should be the same as on the title card, use positive for lateral-direction, negative for longitudinal.

SH - Start time in hours. May be left blank.

SM - Start time in minutes. May be left blank.

SS - Start time in seconds. May be left blank.

SMS - Start time in milliseconds. May be left blank.

S - Reference area, ft^2

b - Reference span, ft

C - Reference chord, ft

ALFA - Angle of attack, degrees

GAMMA - Flightpath angle, degrees

Card 2 - Format (8F10.2)

W - Weight, lbs

IX - Roll inertia, slug-ft^2

IY - Pitch inertia, slug-ft^2

IZ - Yaw inertia, slug-ft^2

IXZ - Product of inertia, slug-ft^2

MACH - Mach number

Q - Dynamic pressure - lb/ft^2

V - True speed, ft/sec

Card 3 - Format (7F10.6)

C_{Y_β} or-C_{N_α} - Per radian

C_{y_P} or-C_{N_Q} - Per radian

C_{y_R} or-C_{N_V} - Per radian or dimensionless

$C_{y_{\delta c_1}}$ or-$C_{N_{\delta e_1}}$ - Per radian

$C_{y_{\delta c_2}}$ or-$C_{N_{\delta e_2}}$ - Per radian

$C_{y_{\delta c_3}}$ or-$C_{N_{\delta e_3}}$ - Per radian

$C_{y_{\delta c_4}}$ or-$C_{N_{\delta e_4}}$ - Per radian

Card 4 - Format (7F10.6)

C_{ℓ_β} or C_{m_α} - Per radian

C_{ℓ_P} or C_{m_Q} - Per radian

C_{ℓ_R} or C_{m_V} - Per radian

$C_{\ell_{\delta c_1}}$ or $C_{m_{\delta e_1}}$ - Per radian

$C_{\ell_{\delta c_2}}$ or $C_{m_{\delta e_2}}$ - Per radian

$C_{\ell_{\delta c_3}}$ or $C_{m_{\delta e_3}}$ - Per radian

$C_{\ell_{\delta c_4}}$ or $C_{m_{\delta e_4}}$ - Per radian

Card 5 - Format (7F10.6)

C_{n_β} or C_{c_α} - Per radian

C_{n_P} or C_{c_Q} - Per radian

C_{n_R} or C_{c_V} - Per radian or dimensionless

$C_{n_{\delta c_1}}$ or $C_{c_{\delta e_1}}$ - Per radian

$C_{n_{\delta c_2}}$ or $C_{c_{\delta e_2}}$ - Per radian

$C_{n_{\delta c_3}}$ or $C_{c_{\delta e_3}}$ - Per radian

$C_{n_{\delta c_4}}$ or $C_{c_{\delta e_4}}$ - Per radian

These five cards may be punched by the MMLE program by setting the PUNCHC option. In addition this information may be read from a file (DERIVS) created by MMLE or a tape created by STABDIV. This is done by setting READ=6, and in this case the five cards may be omitted. This data will allow the program to construct the $\hat{A}$, $\hat{B}$, and $\hat{C}$ matrices. The second subsection creates the $\hat{H}$, $\hat{G}$, and $\hat{F}$ matrices. Usually, these are read in from cards in the following format.

Card 6 - Format (A2,I8,I10)

TYPE - Matrix name - H, G or F

NROW - Number of rows in matrix

NCOL - Number of columns in matrix

Card 7 - Format (8F10.4)

First row of matrix

Card 8 - Format (8F10.4)

Second row of matrix

Card 9 -

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The matrices must be loaded in the order in which they are listed in Appendix C. This method of specifying matrices for acceleration feedback is simple, but often requires many cards with zeroes or redundant information. To minimize this input, three special but often used cases have been defined. If the TYPE on the first card of the first matrix is left blank, the program will assume that the output vector is identical to the state vector, and it will setup the matrices as needed. If the TYPE is set equal to NZ or NY the program will assume the output vector is equal to the state vector plus a NZ or NY acceleration. If NZ or NY is used it should be done in the following manner.

Card 6 - Format (A2,8X,2F10.4)

TYPE - either NZ or NY

DELX - x component of the vector from the cg to the accelerometer, positive forward

DELZ - z component of the vector from the cg to the accelerometer, positive down.

No other information on matrices is needed. Note that the value of OUTPUT must be set in accordance with the above information. In the first case (TYPE=blank) OUTPUT should be 1. If TYPE=NZ or NY, OUTPUT should be 3. If, in addition, off-cg components are to be added, OUTPUT should be 4.

The third section of data provides the description of the flight control system. Again, two options are possible. The first card is the same for both options and contains the following information.

Card 1 - (Format 2I5)

NBLOCK - Number of blocks in the flight control system.

NIT - Index to which option is to be used.

The first option (NIT=0) involves setting up a number of matrices, and the second (NIT=1) involves setting parameters in predefined block types. The first is more versatile but requires many more cards. Since the first option provides a basic understanding of the procedure, it will be discussed first. Additional format information may be found in Appendix C.

The first step in constructing the flight control model is obtaining a good block diagram. Assume we have been given the block diagram shown in Figure 12. Arbitrarily number each block (up to twenty blocks may be used). The numbers of these block form the basis of all connections and feedback loops, and once the system is established, it must be adhered to. The first matrix to be filled is GRAPH, an NBLOCK by 5 matrix, where NBLOCK is the number of blocks in the system. In this case NBLOCK=10. Each row of GRAPH will be filled as follows:

Column 1 - Block number

Column 2 - First internal input

Column 3 - Second internal input

Column 4 - Third internal input

Column 5 - External input

For the purpose of defining the flight control system, internal inputs are those generated within the flight control system (i.e. from other blocks). External inputs come from outside the system. For example, roll rate is generated by the aircraft which is outside the flight control system, and, hence, is an external input. Block number one on the diagram has no internal inputs and one external input which we have arbitrarily labeled V_1. Thus the first row of GRAPH becomes:

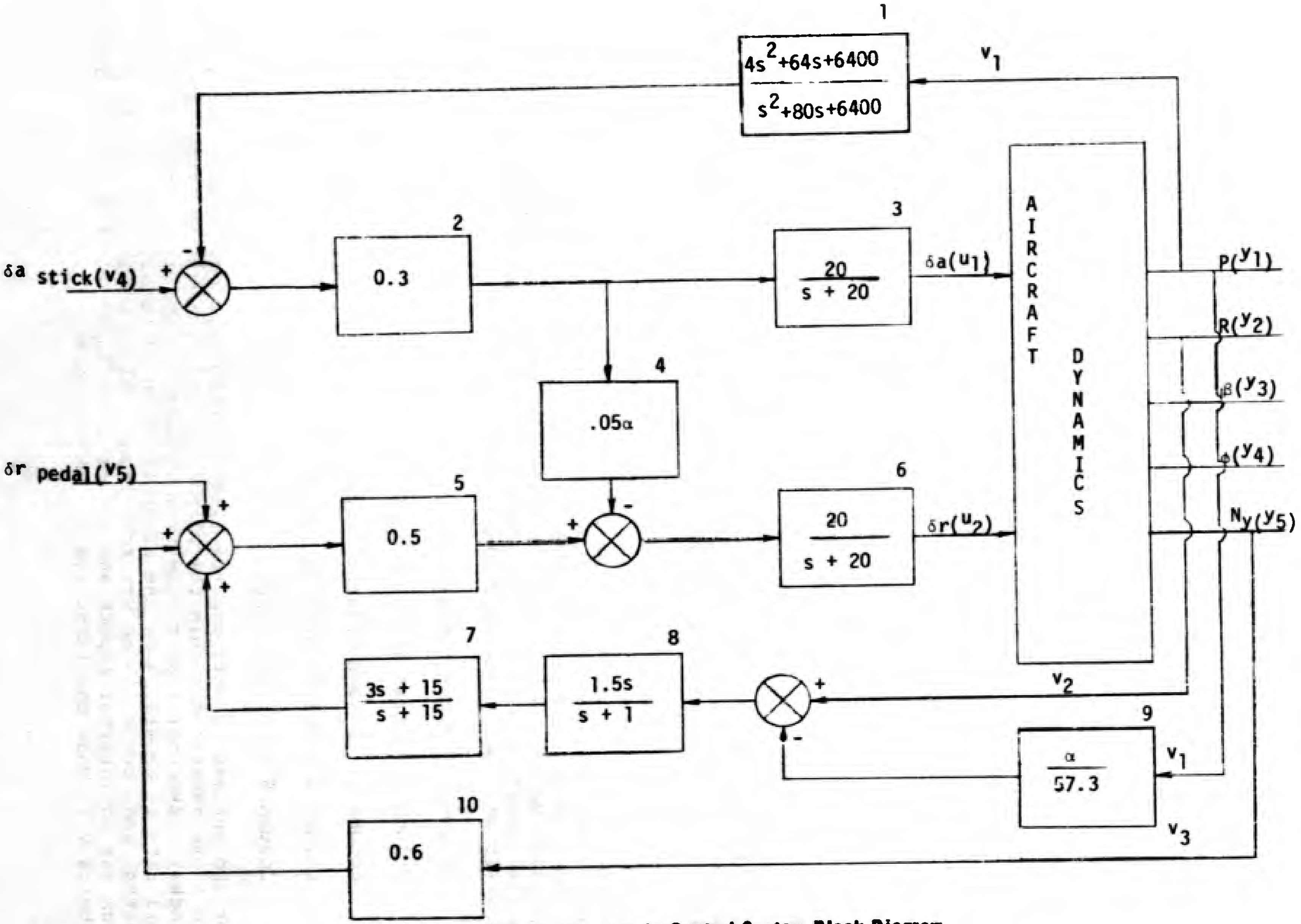


Figure 12 - Flight Control System Block Diagram

$$\text{GRAPH} = \begin{array}{cccccc} & \text{Block Number} & \multicolumn{3}{c}{\text{Internal}} & \text{External} \\ & & \#1 & \#2 & \#3 & \\ & 1 & 0 & 0 & 0 & 1 \end{array}$$

Block number two has an input from block one and an external input V_4. Note, however, that the internal input is negative. This is indicated by a minus sign in from of the input block number. Then the second row is:

$$\text{GRAPH} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 2 & -1 & 0 & 0 & 4 \end{bmatrix}$$

Similarly blocks three and four may be entered.

$$\text{GRAPH} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 2 & -1 & 0 & 0 & 4 \\ 3 & 2 & 0 & 0 & 0 \\ 4 & 2 & 0 & 0 & 0 \end{bmatrix}$$

Block five has inputs from blocks seven and ten and an external input, V_5. The fifth row will be:

$$\text{GRAPH} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 2 & -1 & 0 & 0 & 4 \\ 3 & 2 & 0 & 0 & 0 \\ 4 & 2 & 0 & 0 & 0 \\ 5 & 7 & 10 & 0 & 5 \end{bmatrix}$$

The final GRAPH matrix is then:

$$GRAPH = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 2 & -1 & 0 & 0 & 4 \\ 3 & 2 & 0 & 0 & 0 \\ 4 & 2 & 0 & 0 & 0 \\ 5 & 7 & 10 & 0 & 5 \\ 6 & -4 & 5 & 0 & 0 \\ 7 & 8 & 0 & 0 & 0 \\ 8 & -9 & 0 & 0 & 2 \\ 9 & 0 & 0 & 0 & 1 \\ 10 & 0 & 0 & 0 & 3 \end{bmatrix}$$

Note that the external input numbers for block one and nine are both one since it is the same feedback parameter, roll rate. The second matrix is the BLOCK matrix. The BLOCK matrix is an NBLOCK by 3 matrix as follows:

Column 1 - Block number

Column 2 - Number of numerator coefficients

Column 3 - Number of denominator coefficients

The number of coefficients needed to describe a polynomial is always the order plus one. The polynomial 1.5s require two coefficients, 1.5 and 0.0. Thus BLOCK may be filled out as:

$$BLOCK = \begin{bmatrix} 1 & 3 & 3 \\ 2 & 1 & 1 \\ 3 & 1 & 2 \\ 4 & 1 & 1 \\ 5 & 1 & 1 \\ 6 & 1 & 2 \\ 7 & 2 & 2 \\ 8 & 2 & 2 \\ 9 & 1 & 1 \\ 10 & 1 & 1 \end{bmatrix}$$

Constants (as in block two) are treated as:

$$0.3 = 0.3\ (\frac{1.0}{1.0})$$

After we have defined the order of each polynomial, the actual coefficients must be given. This is accomplished in NUMER and DENOM. Each row of NUMER gives the numerator coefficients of the corresponding block number starting with the lowest power of s (the constant). In this case:

$$\text{NUMER} = \begin{bmatrix} 6400.0 & 64.0 & 4.0 & 0.0 & 0.0 \\ 1.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 20.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 1.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 1.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 20.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 15.0 & 3.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 1.5 & 0.0 & 0.0 & 0.0 \\ 1.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 1.0 & 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix}$$

$$\text{DENOM} = \begin{bmatrix} 6400.0 & 80.0 & 1.0 & 0.0 & 0.0 \\ 1.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 20.0 & 1.0 & 0.0 & 0.0 & 0.0 \\ 1.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 1.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 20.0 & 1.0 & 0.0 & 0.0 & 0.0 \\ 15.0 & 1.0 & 0.0 & 0.0 & 0.0 \\ 1.0 & 1.0 & 0.0 & 0.0 & 0.0 \\ 1.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 1.0 & 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix}$$

Table 10

TRANSFER FUNCTION STANDARD FORMS (NIT=1)

TYPE	BLOCK TRANSFER FUNCTIONS	PARAM (I) I=1	I=2	I=3	I=4	I=5
1	K	K				
2	Ks	K				
3	$\frac{K}{s}$	K				
4	$\frac{K}{1 + s/_{a}}$	K	a			
5	$\frac{K(1+{}^{s}/b)}{(1+{}^{s}/a)}$	K	a	b		
6	$\frac{Ks}{(s+a)}$	K	a			
7	$\frac{K}{(1+{}^{s}/_{a}) \quad (1+{}^{s}/b)}$	K	a	b		
8	$\frac{K}{1 + \frac{2\zeta}{\omega} s + {}^{s^2}/_{\omega^2}}$	K	ω	ζ		
9	$\frac{K(1+\frac{2\zeta_2}{\omega_2} s + {}^{s^2}/_{\omega_2^2})}{1+\frac{2\zeta_1}{\omega_1} s + {}^{s^2}/\omega_1^2}$	K	ω_1	ζ_1	ω_2	ζ_2
10	$\frac{K\ (1 + {}^{s}/_{a})}{(1 + \frac{2\zeta}{\omega} s + {}^{s^2}/_{\omega^2})}$	K	ω	ζ	a	
11	$\frac{Ks}{(1 + \frac{2\zeta}{\omega} s + {}^{s^2}/_{\omega^2})}$	K	ω	ζ		

MOD DIGITL	0	1	2
0	G(s)	-	-
1	G(s), G(z)	G(s)	G(ω)
2	G(z)	G(s)	G(ω)

The final matrix required is the GAIN matrix. This is a 1 by NBLOCK matrix which simply lists the gain of the corresponding block.[23]

GAIN = [1.0 0.3 1.0 .05α 0.5 1.0 1.0 1.0 α/57.3 0.6]

In several cases the gains and numerator coefficients could have been interchanged. For example in block eight.

$$1.0 \left(\frac{1.5s}{s+1}\right) = 1.5 \left(\frac{s}{s+1}\right)$$

It makes no difference to the program where the 1.5 coefficient occurs as long as it only occurs once. However, a provision to run multiple cases requires that all coefficients which change from case to case be included in the GAIN matrix. Thus the 1.5 could occur in either NUMER or GAIN, but the .05α (since it will change on the next case) is required to go in the GAIN matrix.

After the above matrices are tabulated, they may be punched on cards, one card for each matrix row. The format is (16I5) for integers and (8F10.4) for real values. The number of cards for each matrix is determined by the value of NBLOCK read in previously. Appendix C contains more details on card formats. There is no delineation between matrices.

The second option involves fitting each block into one of several predetermined formats. The allowable formats are shown in Table 10. In this option, each block is represented by one card with the following information. (Format I2,I3,5I5,5F10.4).

NUM - Block number

TYPE - Block type

$$\text{CONNEC (I)} \quad I = 1, 4 \left\{ \begin{array}{l} \text{- First internal input block number} \\ \text{- Second internal input block number} \\ \text{- Third internal input block number} \\ \text{- External input number} \end{array} \right.$$

---

[23] Proper care must be given to insure that the gains are in the correct units. The gain value for the CONTROL program may not be the same as that listed on the block diagram. Units for the input and output vector parameters defined by READ=5,6 are radians, radians per second, and g's.

MOD - Domain specification (zero if DIGITL=0)

PARAM (I)
I=1, 5
- First descriptive parameter
- Second descriptive parameter
- Third descriptive parameter
- Fourth descriptive parameter
- Fifth descriptive parameter

The cards follow the first card (NBLOCK, NIT) of the preceeding section and replace all of the control system matrices. The CONNEC numbers are identical to the last four columns of the GRAPH matrix and may be found in the same way. The MOD parameter specified s, z, or w domains if DIGITL≠0 (see Table 10). Using Table 10 and the block diagram we can define the following parameters.

NUM	TYPE	CONNEC				MOD	PARAM				
		1	2	3	4		1	2	3	4	5
1	9	0	0	0	1	0	1.0	80.0	0.5	40.0	0.2
2	1	-1	0	0	4	0	0.3				
3	4	2	0	0	0	0	1.0	20.0			
4	1	2	0	0	0	0	0.5α				
5	1	7	10	0	5	0	0.5				
6	4	-4	5	0	0	0	1.0	20.0			
7	4	8	0	0	0	0	1.0	15.0	5.0		
8	6	-9	0	0	2	0	1.5	1.0			
9	1	0	0	0	1	0	α/57.3				
10	1	0	0	0	3	0	0.6				

The last portion of flight control system information is the same regardless of the option selected for block input. The format for each required card is (16I5). The first two items are ITHINY and ITHINU. When the control system is added to the original aircraft, the output and input vectors are expanded to include the output and input of each of the ten blocks. The expanded ŷ and û vectors become

$$\hat{y} = \begin{bmatrix} P \\ R \\ \beta \\ \phi \\ Ny \\ OB1 \\ OB2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ OB10 \end{bmatrix} \qquad \hat{u} = \begin{bmatrix} \delta a \\ \delta r \\ EI1 \\ EI2 \\ \cdot \\ \cdot \\ \cdot \\ EI10 \end{bmatrix}$$

Where OB1 = Output of block 1, etc.

EI1 = External input 1, etc.

Usually it is not desirable to keep all of these outputs and inputs. ITHINY and ITHINU perform the job of "thinning" the output and input vectors. ITHINY consists of the numbers of the expanded y vector parameter we would like to keep. For example, to keep P, R, β, ϕ, and N_y we would set

ITHINY = [1 2 3 4 5]

If, in addition, we wanted save OB2,

ITHINY = [1 2 3 4 5 7]

The setup of ITHINU is identical. It should be pointed out that there are approximately NY+NBLOCK possible ITHINY members and NU plus all external inputs possible ITHINU members. When these are being read in, enough cards must be allowed for all the possible members. For example, if NY+NBLOCK=24, at sixteen members per card, two cards are required even if we only want to save five members.

The rest of the data connects the aircraft parameters with the flight control system. When the flight control system was setup, numbers were arbitrarily assigned to inputs coming from the aircraft. We assigned roll rate as v_1, yaw rate as v_2 and N_y as v_3. The YTOV matrix tells which y parameters are connected to which v parameters. YTOV is an NYTOV by 2 matrix, and since there are three connections, NYTOV is equal to three.

$$\text{YTOV} = \begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 5 & 3 \end{bmatrix}$$

This matrix says that y_1 (the first y parameter) is the same as v_1. Similarly y_2 is the same as v_2, and y_5 is the same as v_3. The ZTOU matrix connects flight control system outputs (δα and δr) to the aircraft. Here NZTOU is equal to two and:

$$\text{ZTOU} = \begin{bmatrix} 3 & 1 \\ 6 & 2 \end{bmatrix}$$

The output of block three now goes to u_1 (δα), and the output of block six goes to u_2 (δr). A third matrix YZTOK may be used for root locus, but this will be discussed later.

The YTOV, ZTOU, and YZTOK matrices are added in order (one card per row) following an initial card which specifies NYTOU, NZTOU, and NYZTOK. All formats are 16I5 and again there is no delineation between matrices. This example setup is detailed in Appendix C.

All of these inputs are summarized in Appendix C and greater detail on formatting may be found there.

Multiple Cases:

The IGO parameter facilitates the running of multiple cases using the MIXED option. Normally the flight control system structure does not change from case to case, although the system gains may. If this is true, it is redundant to load the same flight control system each time. An option in CLASS allows the user to retain the flight control system data by setting IGO to one. The CLASS subroutine will then read only a new GAIN matrix. Thus, one to three GAIN cards replace all the cards previously setup for the control system. IGO should be left at zero on the first case and set to one on succeeding cases. To efficiently use this option all the cases of the same mode (longitudinal or lateral-directional) should be run together so the control system will not have to be changed.

Alternate Modes of Operation:

The program is capable of operation in other modes than previously described. Some of these alternate modes will now be discussed. Some samples of user written routines are included in Appendix H.

READ Options.

If READ is set equal to one, the program defaults to the LOAD input routine. The format then is the same as that required for loading the $\hat{H}$, $\hat{G}$, and $\hat{F}$ matrices described previously, and all required matrices must be loaded in this manner. The required matrices for any given case may be found in Appendix C. If the LOAD input routine is used, the Milspec option may not be called.

The second READ option (READ=2) calls the MATRIX input routine. MATRIX is a user written routine which is loaded with the input data using the SCOPE 3.4 COPYL routine. Since it is user written, input data may be loaded in any format, and any required matrices may be defined. MATRIX may be made compatible with the Milspec option and the MIXED option. A sample of SCOPE control cards using COPYL is located in Appendix D.

CHANGE is the routine called when READ is set equal to three. As the name implies, CHANGE is a user written subroutine which changes aircraft or flight control system parameters for succeeding cases. CHANGE is used in conjunction with READ3, IFLAG and SAV, and is also loaded with the COPYL routine.

If READ is set equal to four, the CLASS subroutine is called. If READ=4, all data must be in block diagram form. This option is typically used to compute frequency responses for blocks or combinations of blocks in a flight control system. The setup to describe the blocks is identical to that described for the flight control system previously.

SYSTEM Options.

Open loop analysis is run by setting the SYSTEM parameter equal to one. In this case, only the aircraft matrices are needed; no flight control system description is required. If the control description is

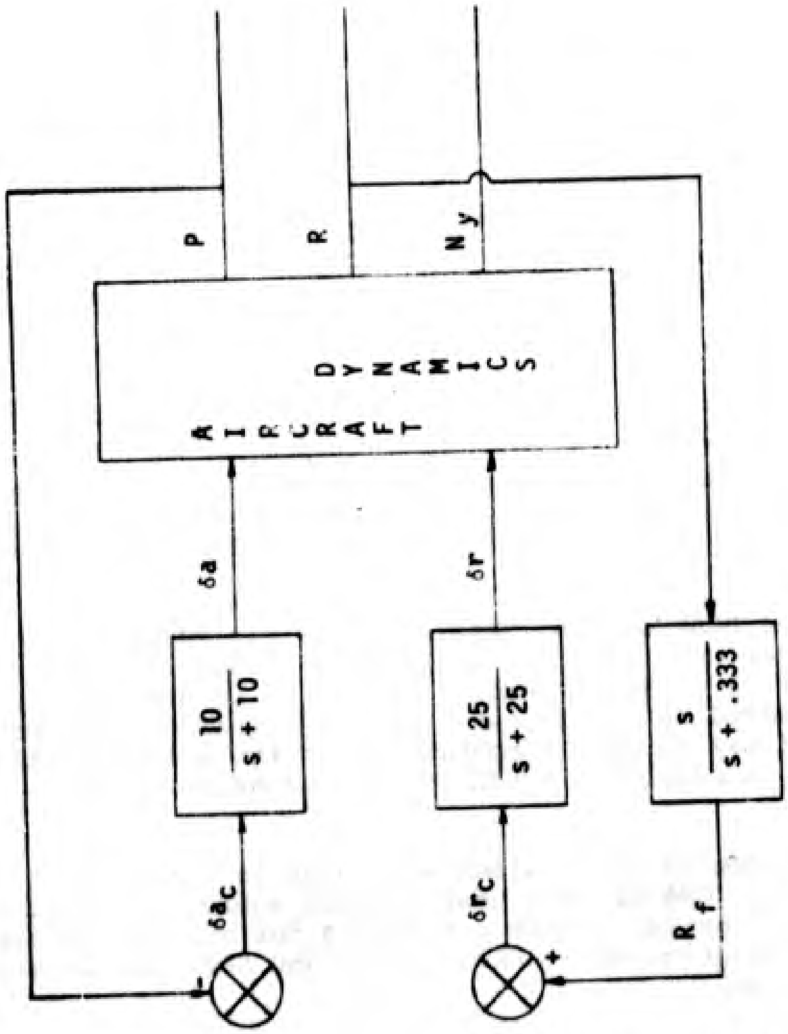


Figure 13. Flight Control System Block Diagram

already there, it may be voided by setting all gains to zero. An open loop case will be run as a closed loop case if MIXED equals one.

The closed loop option (SYSTEM=2) is the one most commonly used. The system matrices must contain information on both the basic aircraft and the flight control system. It should be noted, however, that it is entirely possible to run a closed loop case without using the MIXED option. This will be discussed in greater detail later.

The root locus option is best utilized when a large gain matrix is to be studied. The gains of two independently controlled loops may be varied. This is accomplished using N1, N2, GAIN1 and GAIN2. If the MIXED option is not used, N1 and GAIN1 are applied to the K1 and K2 matrices, and N2 and GAIN2 are applied to the K3 and K4 matrices. If the MIXED option is used, the feedback loops are defined in the YZTOK matrix in a similar manner as the YTOV matrix. The connections are now made, however, from the augmented and thinned $\hat{y}$ vector to the augmented and thinned $\hat{u}$ vector. N1 and GAIN1 apply to the first row of the YZTOK matrix, and N2 and GAIN2 apply to the second row. The root locus option may also be used to determine the migration of a root locus pole from the open loop position to its closed loop position. This is done by connecting the entire augmentation system through the YZTOK matrix and allowing the gain to vary slowly from zero to its nominal value.

Without the MIXED Option

The use of the MIXED option is by far the easiest way to setup the flight control system. For root locus, however, it is not always the best way since the YZTOK loops are a little more restrictive. It is possible to setup a flight control system (at least a simple one) without using MIXED equal to one. Consider the block diagram shown in Figure 13. In addition to the aircraft equations of motion, we may define three additional differential equations.

$$\delta a = \frac{10}{s + 10} \delta a_{com}$$

$$\delta r = \frac{25}{s + 25} \delta r_{com}$$

$$R_f = \frac{s}{s + .333} R$$

or

$$s\delta a + 10\delta a = 10\delta a_{com}$$

$$s\delta r + 25\delta r = 25\delta r_{com}$$

$$sR_f + .333R_f = sR$$

Using the Laplace definition of the s operator, we can rewrite these equations as:

$$\dot{\delta}a = 10\delta a_{com} - 10\delta a$$

$$\dot{\delta}r = 25\delta r_{com} - 25\delta r$$

$$\dot{R}_f - \dot{R} = -.333\ R_f$$

We can include the new equations by expanding the original state vector.

$$\hat{x} = \begin{bmatrix} P \\ R \\ B \\ \emptyset \end{bmatrix} \Rightarrow \hat{x} = \begin{bmatrix} P \\ R \\ B \\ \emptyset \\ \delta a \\ \delta r \\ R_f \end{bmatrix}$$

We can now include this simple flight control system into our basic aircraft model. The state equation then becomes:

$$\overset{\hat{C}}{\begin{bmatrix} 1.0 & \frac{-I_{xz}}{I_x} & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ \frac{-I_{xz}}{I_z} & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & -1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}} \overset{\dot{\hat{x}}}{\begin{bmatrix} \dot{P} \\ \dot{R} \\ \dot{\beta} \\ \dot{\emptyset} \\ \dot{\delta}a \\ \dot{\delta}r \\ \dot{R}_f \end{bmatrix}} = \overset{\hat{A}}{\begin{bmatrix} L_p & L_R & L_\beta & 0.0 & L_{\delta a} & L_{\delta r} & 0.0 \\ N_p & N_R & N_\beta & 0.0 & N_{\delta a} & N_{\delta r} & 0.0 \\ \sin\alpha & \cos\alpha & Y_\beta & \frac{g}{V}\cos\theta & Y_{\delta a} & Y_{\delta r} & 0.0 \\ 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & -10.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -25.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -.333 \end{bmatrix}} \overset{\hat{x}}{\begin{bmatrix} P \\ R \\ \beta \\ \emptyset \\ \delta a \\ \delta r \\ R_f \end{bmatrix}} + \overset{\hat{B}}{\begin{bmatrix} 0.0 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \\ 10.0 & 0.0 \\ 0.0 & 25.0 \\ 0.0 & 0.0 \end{bmatrix}} \overset{\hat{u}}{\begin{bmatrix} \delta a_{com} \\ \delta r_{com} \end{bmatrix}}$$

The feedback loops then feedback roll rate to δa_{com} and filtered yaw rate, R_f, into δr_{com}. This type of setup allows more versatile use of the root locus but becomes very difficult for a control system with any complexity.

Frequency Response

Frequency responses may be generated using the FRPS option. A frequency response will be calculated for each possible pair of output and input parameters which have been saved. For this reason it is important to define the $\hat{y}$ vector sparingly and thin the $\hat{y}$ and $\hat{u}$ vectors liberally. The range and spacing of frequencies is controlled by IFREQ, FFREQ, and DELFRQ. If IFREQ is not set, the program will default to a set of frequencies distributed geometrically between 0.1 and 150 rad/sec.

Transient Response

Transient responses may be run with or without the flight control system by setting the TRESP option and programming an input into a user written INPUTV subroutine. This is done by setting the $\hat{u}$ matrix to a step, ramp, sine wave or other function of time. The available inputs are the $\hat{u}$ matrix after it has been expanded by the flight control system and thinned by ITHINU. Therefore, any input which has been saved may be programmed. Programmed inputs are superimposed on normal inputs. Thus if elevator is programmed as a step input the pitch feedback loops will still be closed. As with the READ subroutines, INPUTV may be changed using the SCOPE 3.4 COPYL routine (Appendix D). A sample INPUTV subroutine to generate a sine wave input is contained in Appendix H.

CONTROL Output

A sample output from the CONTROL program can be found in Appendix C. The amount of output will, of course, vary with the number of analysis options chosen. The first several pages essentially print out all of the input information. This is followed by the matrices of the "reduced" system. These matrices are the ones from which CONTROL gets its information. Three types of output may follow. There are eigenvalues, Milspec parameters, and numerator transfer functions. In addition frequency responses or time histories may be generated, but their output is simple enough not to warrant further comment.

The eigenvalues will appear first for whatever system has been defined. These eigenvalues describe the transient response of a system to a given input. They are, in fact, the denominator roots of the input-output transfer functions. Used in conjunction with the numerator roots, they define a transfer function which gives total system response to any given input. Usually the eigenvalues are plotted on a root locus plot as a means of evaluating aircraft handling qualities and flight control system performance. A discussion of root locus plots and what they mean may be found in Appendix F.

If the Milspec option is set, the Milspec parameter will follow. These parameters are calculated based on the derivatives and eigenvalues to relate aircraft response to the user in terms he is probably more familiar with. A summary of these parameters may be found in Table 11. It is envisioned that the list of Milspec parameters will grow in response to user requests and further equations development. Note that the calculations for Nz/α and $\delta e/g$ assume that the aircraft is being trimmed with Cmδe_1 only. Also note that steady state sideslip information assumes that aileron is δc_1 and rudder is δc_2. Frequency and damping (or a time constant) is calculated for each pair of conjugate roots (or root). Since the program has no way of distinguishing an actuator mode from a dutch roll pole, this determination is left to the user.[24]

---

[24] An open loop case will generate three or four poles depending on the value of NX. When a flight control system is added, several other poles from the flight control system will be added. Often these can be identified from their frequency or time constant. Occasionally, however, a set of poles with no obvious dutch roll or short period mode exists. Sometimes the following relationships for open loop aircraft will help

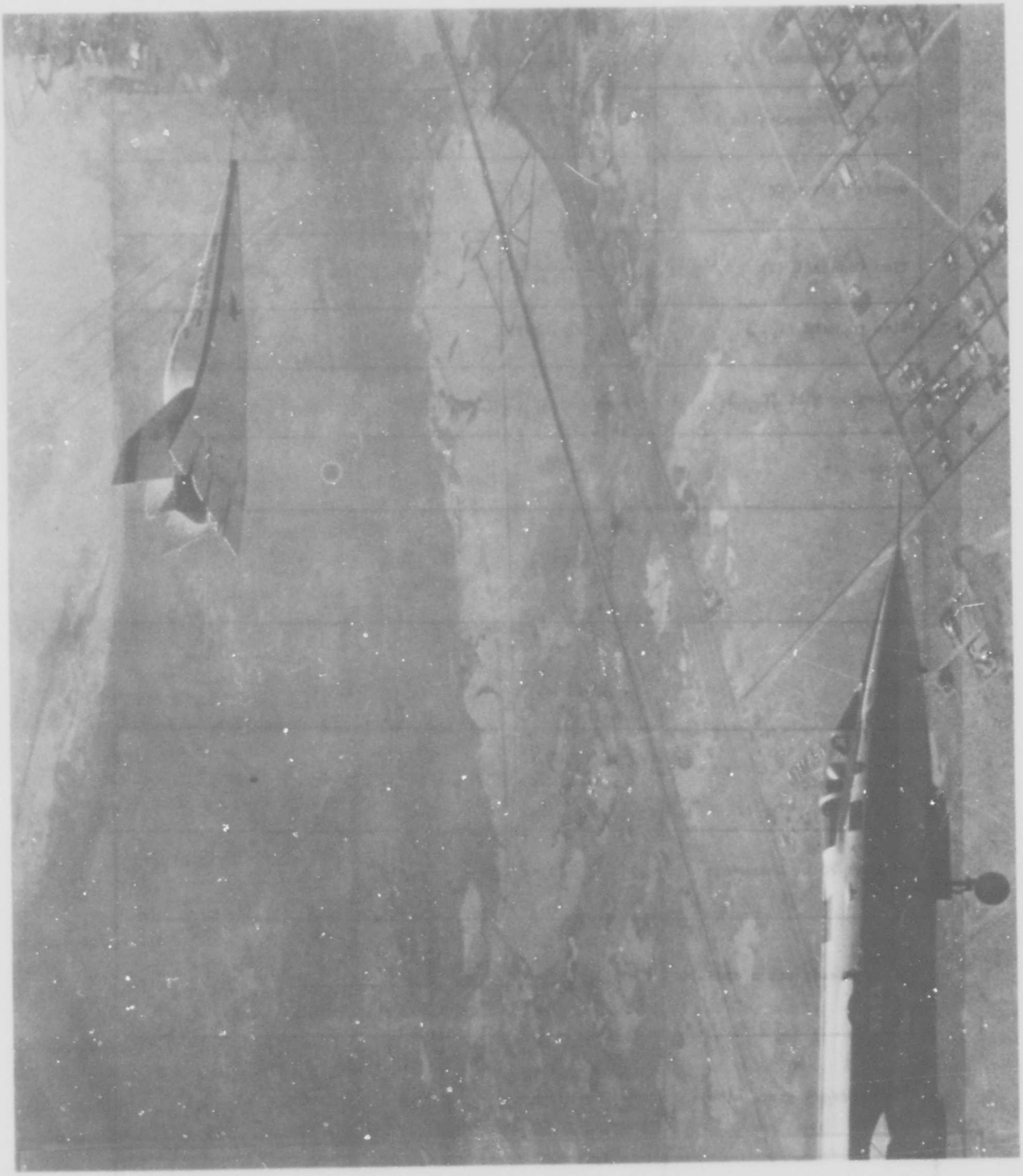
$$\omega_n \text{ (Dutch roll) } = \sqrt{\frac{\bar{q}sb}{I_z} C_{n_\beta}}\ ^*$$

$$\omega_n \text{ (short period } = \sqrt{\frac{\bar{q}sc}{I_y} C_{m_\alpha}}$$

Table 11

MILSPEC PARAMETERS

PARAMETER	**EQUATION**
Actual Frequency (ω_d)	ω_d = (Imaginary Part)
Natural Frequency (ω_n)	$\omega_n = \sqrt{(\text{Real Part})^2 + (\text{Imaginary Part})^2}$
Damping Ratio (ζ)	$\zeta = - \frac{(\text{Real Part})}{\omega_n}$
Time Constant (T)	$T = - \frac{1.0}{(\text{Real Part})}$
Time to Half ($T_{1/2}$)	$T_{1/2} = -.69315T$
Cycles to Half ($C_{1/2}$)	$C_{1/2} = - \frac{(\text{Imaginary Part})}{9.06\ (\text{Real Part})}$
Period (T)	$T = \frac{6.28318}{\omega_d}$
$N_{z/\alpha}$	$N_{z/\alpha} = \frac{\bar{q}s}{W} \left(C_{n_\alpha} - C_{n_{\delta e_1}} \frac{C_{m_\alpha}}{C_{m_{\delta e_1}}}\right)$
$\delta e/g$	$\delta e/g = \frac{57.3\ C_{m_\alpha}}{(N_{z/\alpha})\ (C_{m_{\delta e_1}})}$
Static Margin	$S.M. = \frac{100\ C_{m_\alpha}}{C_{n_\alpha}}$
Maneuver Point Increment	$M.P.I. = S.M. - \frac{1608.7\ \bar{q}\ sc}{WV^2} C_{m_Q}$
($\delta r/\beta$) steady state sideslip	$\delta r/\beta = \frac{(C_{\ell_{\delta c_1}})\ (C_{n_\beta}) - (C_{\ell_\beta})\ (C_{n_{\delta c_1}})}{(C_{\ell_{\delta c_2}})\ (C_{n_{\delta c_1}}) - (C_{\ell_{\delta c_1}})\ (C_{n_{\delta c_2}})}$
($\delta a/\beta$) steady state sideslip	$\delta a/\beta = \frac{(C_{n_{\delta c_2}})\ (C_{\ell_\beta}) - (C_{n_\beta})\ (C_{\ell_{\delta c_2}})}{(C_{\ell_{\delta c_2}})\ (C_{n_{\delta c_1}}) - (C_{\ell_{\delta c_1}})\ (C_{n_{\delta c_2}})}$
(ϕ/β) steady state sideslip	$\phi/\beta = \sin^{-1} [(\frac{\bar{q}s}{W\cos\theta} C_{y_\beta} + C_{y_{\delta c_1}} \cdot \delta a/\beta + C_{y_{\delta c_2}} \cdot \delta r/\beta)]$
$C^*_{n_\beta}$ (Dynamic C_{n_β})	$C^*_{n_\beta} = C_{n_\beta}\ (\cos\alpha) - (\frac{I_z}{I_x}) C_{\ell_\beta}\ (\sin\alpha) + \frac{I_{xz}}{I_x}(C_{\ell_\beta} \cdot \cos\alpha - C_{n_\beta} \cdot \sin\alpha)$



CONSIDERATIONS FOR QUICK RESPONSE DERIVATIVE EXTRACTION

The need often arises for techniques to provide aircraft derivatives and Milspec parameters in a minimum amount of time. This section of the report will spotlight the quickest and easiest program modes and procedures. Compromises in the data which cause only minimum loss of information will also be discussed. Some of these should be made only after considering the ramifications involved, and consultation with someone with derivative extraction experience would be well advised.

Flight Conditions

The large Mach-alpha-dynamic pressure matrices previously discussed are intended to give a complete math model of the aircraft. Usually when derivatives are required quickly it is in response to a particular handling qualities problem, and the flight regime which requires testing is much smaller. The area to be tested can usually be covered easily in one flight.

Data Requirements

Some compromises can be made in this area with minimum loss of information. Some parameters can be deleted from the list of required inputs without sacrificing primary derivative accuracy. These include the aircraft angles, θ and ϕ, and in some cases, β. Range or resolution deficiencies can be compensated for by performing small or large pulses, respectively. Parameter filtering on angle of attack, sideslip and Euler angles will not have a major effect on derivative accuracy.

Flight Maneuvers

Unfortunately there are no shortcuts to producing good maneuvers. Indeed the ability to use most of the simplifying program defaults depends heavily on having a good maneuver. Good maneuvers are the key to extracting derivatives with minimum effort.

Preparing the Flight Data

The conversion of an engineering units tape into a time history file has long been one of the major problems in derivative extraction. Ostensibly, it is not a difficult task, but the combination of programming errors and computer frailties often make this a formidable roadblock. There exists some experience in reading CDAS and ADAS tapes; stranger tapes usually cause the most problems. It is usually easier to use one of the existing programs, either ADEX or UFTAS.

Executing MMLE

The MMLE input will normally consist of five cards. The title card and namelist "INPUT" are as previously defined. Usually only a few of the INPUT namelist parameters need to be defined. These include SPS, TIMESC, PUNCHC, YAY, ZAY, and XAN. If the initial start and stop times are selected carefully the time card may be left blank. The predefined constants are the easiest start up values to use, and they will work admirably for a good maneuver. No matrices need to be defined with the possible exception of the BB matrix. A priori does not need to be used and default D1 weighting is adequate if the maneuver is good.

Executing CONTROL

There are three steps to minimizing the number of input cards required by CONTROL. The first is the use of the PUNCHC option in MMLE. This eliminates the need to punch derivatives on cards. An alternative is to use the derivative file, DERIVS. The use of special case output vectors will greatly reduce the setup work and time. Approximately one-half to two-thirds of a CONTROL deck is the H, G, and F matrices. The third step is to use the predefined transfer functions to input the flight control system. Setting the IGO parameter on subsequent cases will eliminate the need to redefine the control system on each case.

OVERVIEW

While this report does not require the normal conclusions and recommendations which are customary to a report containing test results, a section which summarizes the most important considerations for obtaining good results is useful. It is intended that this section highlight the important points of the report without requiring the reader to wade through the many pages of the entire volume.

Conventional testing techniques for dynamic stability and control have several deficiencies which may be corrected or improved upon with a derivative extraction and characteristic analysis approach. Derivative extraction provides a far better insight into the mechanics of stability and control than is available with conventional techniques. Accurate derivatives are mandatory for standardization and simulation techniques, and wind tunnels have often been in error on major derivatives. Finally, with careful test planning and a knowledge of the results that are possible from characteristic analysis, a potential for reducing the flight test time exists.

Flight conditions, instrumentation and test maneuvers are the items which must be considered in developing the test plan. Test conditions must be selected to identify those parameters which influence the derivatives. Experience has shown that these parameters are usually configuration, Mach number, angle of attack, and, sometimes, dynamic pressure. For this purpose a Mach-alpha plot or equivalent airspeed-alpha plot is useful. Instrumentation can cause problems if parameter ranges, resolutions, and phase lags are not considered. Sometimes instrumentation frailties can be overcome by proper maneuver sizes or downstream processing of the flight data. The flight maneuver is the most important aspect contributing to a successful program. A good maneuver will overcome much of the "art" in derivative extraction, and a poor maneuver will compound the effort required to process the maneuver and contribute significantly to degraded results. A good maneuver is provided by a rapid elevator doublet for longitudinal maneuvers and sequenced, rapid rudder and aileron doublets for a lateral-directional maneuver.

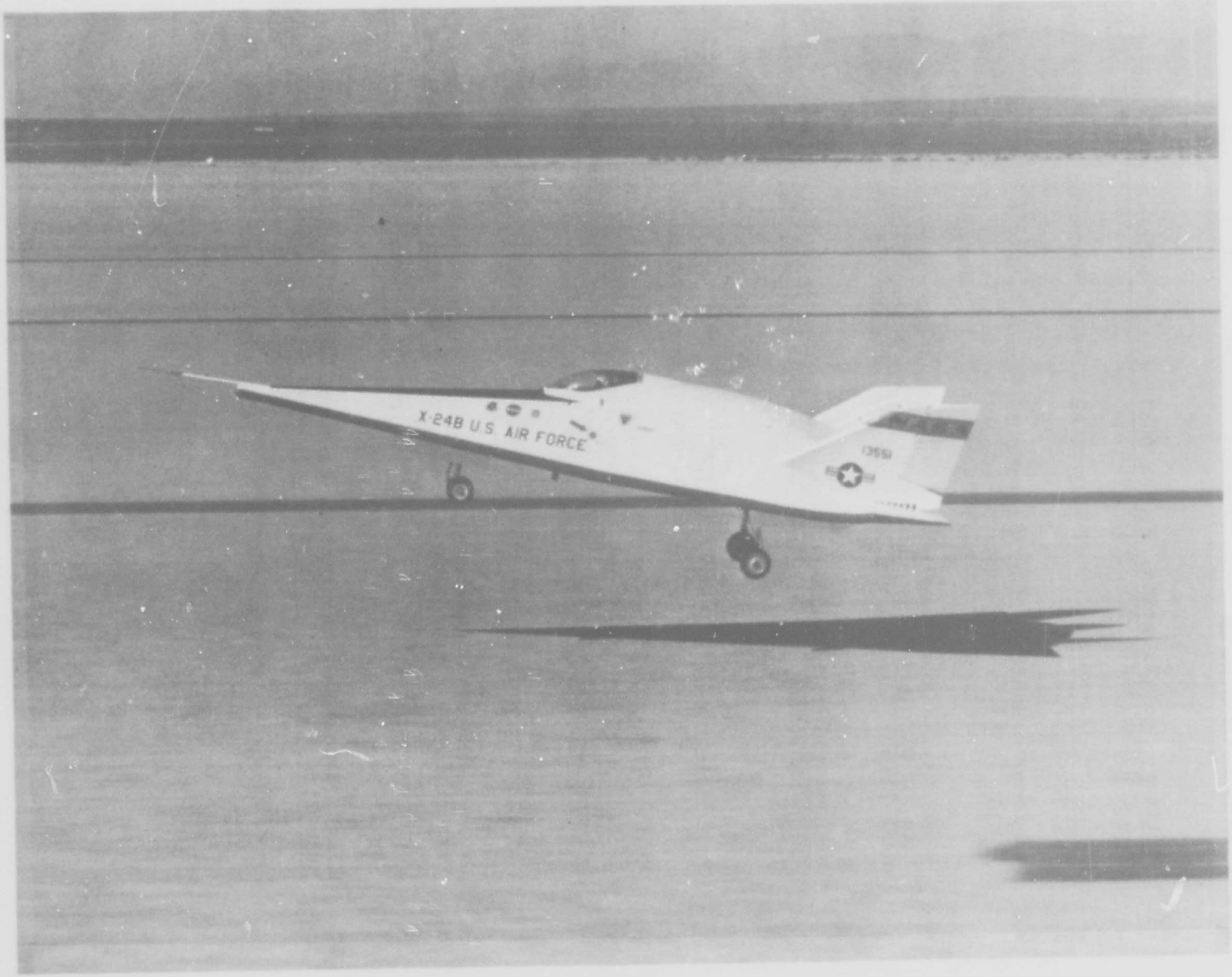
Prior to executing the derivative extraction program the data must be reformatted. Several programs are currently available to edit the parameters and put them into the right order and units. If the UFTAS program is used for data processing, LINK8 provides the fastest method and requires the least amount of work for the engineer.

For the most part, execution of the MMLE derivative extraction program may be accomplished by an aide. Initial setup of the weighting matrices should be done by the engineer, but the program is designed to facilitate this process. The majority of work involved in running the program comes from developing starting values. Three modes for this are currently available. The original method of looking up derivatives from a book and providing them in dimensionalized matrices is time consuming and cumbersome. A set of pre-defined constants may be used, and these will work admirably for most maneuvers, but they are not suitable for use with the a priori feature. The most accurate method is to use the program look-up capability, but this requires that a data table be developed from wind tunnel and/or previously determined derivatives. The results, of course, should be analyzed and interpreted by the engineer.

In addition to determining characteristic roots, the CONTROL characteristic program is capable of several other tasks, and these may prove beneficial, especially in the early phases of the flight test program. For primary reporting purposes however, frequency, damping and other Mil-spec parameters are important, and these are generated from the closed loop option of the program. Total aircraft/flight control system response is determined by merging the math models of the aircraft (derivatives) with a math model of the flight control system. Some care must be taken to insure that linearity and coupling assumptions are not violated, but with this model the results of many stability and control tests can be reconstructed with improved accuracy and a significant decrease in flight test time.

REFERENCES

1. *Military Specification, Flying Qualities of Piloted Airplanes*, MIL-F-8785B(ASG), 7 August 1969.
2. *Dynamics of the Airframe*, Report AE-61-4II, Northrop Aircraft Inc., September 1952.
3. *Engineer's Handbook for Aircraft Performance and Flying Qualities Flight Testing*, Performance and Flying Qualities Branch, Flight Test Engineering Division, Edwards AFB, California, May 1971.
4. Wolowicz, Chester H. and Yancey, Roxanah B., *Experimental Determination of Airplane Mass and Inertial Characteristics*, NASA TR R-433, NASA Flight Research Center, Edwards, California, October 1974.
5. Steers, Sandra T. and Iliff, Kenneth W., *Effects of Time Shifted Data on Flight Determined Stability and Control Derivatives*, NASA TN D-7803, NASA Flight Research Center, Edwards, California, March 1975.
6. Sim, Alex G., *Flight-Determined Stability and Control Characteristics of the M2-F3 Lifting Body Vehicle*, NASA TN D-7511, NASA Flight Research Center, Edwards, California, December 1973.
7. Maine, Richard E. and Iliff, Kenneth W., *A User's Guide for Three Fortran Computer Programs to Determine Aircraft Stability and Control Derivatives from Flight Data*, NASA TN D-7831, NASA Flight Research Center, Edwards, California, April 1975.
8. *Documentation of the Uniform Flight Test Analysis System (UFTAS)*, Volumes 1 and 2, Air Force Flight Test Center, Edwards AFB, California, June 1973.
9. Kirsten, Paul W, and Ash, Lawrence G., *A Comparison and Evaluation of Two Methods of Extracting Stability Derivatives from Flight Test Data*, AFFTC-TD-73-5, Air Force Flight Test Center, Edwards AFB, California, May 1974.
10. Iliff, Kenneth W. and Taylor, Lawrence W. Jr., *Determination of Stability Derivatives from Flight Data Using a Newton-Raphson Minimization Technique*, NASA TN D-6579, NASA Flight Research Center, Edwards, California, 1972.


X-24B U.S. AIR FORCE
13551

APPENDIX A

ADEX

Table A1

ADEX PROGRAM OPTIONS

PARAMETER	AVAILABLE OPTIONS	RESULTS
1. AIRCFT	A-9	Reads A-9 CDAS tape.
	A-10	Reads A-10 CDAS tape.
	F-15B	Reads F-15 CDAS tape.
	X-24B	Reads X-24B user tape.
	YF-16	Reads YF-16 CDAS tape.
2. MODE	LONG	Processes longitudinal case.
	LATDR	Processes lateral-directional case.
3. IFTAPE	BIN	Produces binary tape for MMLE.
	BCD	Produces BCD tape for STABDIV.
	blank	No action.
4. IFWILD	YES	Performs wild point search on control surfaces.
	blank	No action.
5. IFLIST	INPUT	Produces listing of input time histories.
	AFTER	Produces listing of output time histories.
	YES	Produces listing of both time histories.
	blank	No action.
6. IFDIF	YES	Differentiates P, Q, and R.
	blank	No action.

GENERAL PURPOSE PROGRAMMING DATA

PROGRAM NAME: ADEX DATA CARDS

DECK NR: 8115

LINE ITEM:

RETURN TO (Name/Organization/Phone):

PAGE 1 OF 1 PAGES

DATE:

```
cc  5    10   15   20   25   30   35   40   45   50   55   60   65   70 72 73   80
                NSEQ
AIRCFTINSEQMODE     IFTAPE    IFWILD    IFLIST    IFDIF                 CARD 01
    W        DCG       XXI       YYI       ZZI       XZI                      02
 AMACH         Q        VT      ALFA        CK        B         S             03
    SH        SM        SS                  EH        EM        ES            04
STOPS                                                                         05
                                                                              06

                              SAMPLE DECK

                   2
X-24B   51LONG      BIN       YES                                     CARD 01
  8567.      0.0       2646.    23704.    24119.     620.                     02
  1.49       145.      1450.     5.0       37.5      19.1      330.5          03
   10.        17.       52.0                10.       17.       59.0          04
X-24B    1LATDR     BIN       YES                                             05
  8567.      0.0       2646.    23709.    24119.     620.                     06
  1.13       181.      1090.     4.0       37.5      19.1      330.5          07
   10.        18.       25.5                10.       18.       31.0          08
STOP                                                                          09
                                                                              10
```

AFFTC FORM 39 SEP 67 PREVIOUS EDITION OF THIS FORM WILL BE USED UNTIL STOCK IS EXHAUSTED

```
EN123,CM70000,CH99830081I5P,T776,NT1.
VSN,TAPE=1461.
ATTACH,ADEX,AMC,ID=CNAGY,CY=1,MR=1.
REQUEST,TAPE,E,PE.
REQUEST,THIST,*PF.
ADEX.
CATALOG,THIST,X24B,ID=CNAGY,CY=4.
0000000000000000000000
                        2
X-24B     51LONG         BIN        YES
  8567.      0.0           2646.      23704.      24119.      620.
  1.49       145.          1450.      5.0         37.5        19.1        330.5
  10.        17.           52.0                   10.         17.         59.0
X-24B     1LATDR         BIN        YES
  8567.      0.0           2646.      23704.      24119.      620.
  1.13       181.          1090.      4.0         37.5        19.1        330.5
  10.        18.           25.5                   10.         18.         31.0
STOP
0000000000000000000000
0000000000000000000000
```

```
INPUT DATA FOR SET NUMBER   1 OF   2

    X-24B SEQUENCE NUMBER    51. MODE IS LONG
    MACH= 1.49      DYN. PRESS.= 145.00  VELOCITY=1450.00    ALPHA=  5.00        C= 37.50       B= 19.10      S= 330.50
       W=  8567.0          DELCG=  0.00           IXX=   2646.0    IYY=  23704.0  IZZ=  24119.0  IXZ=   620.0
    START TIME = 10:17:52.000            STOP TIME = 10:17:59.000


OPTIONS REQUESTED AS FOLLOWS -


        WILD POINT CHECKING ON CONTROL PARAMETERS


        NO DIFFERENTIATION WILL BE PERFORMED ON RATES.


        A BINARY TAPE WILL BE PRODUCED FOR MMLE.
```

```
DATE OF COMPUTER RUN  05/24/75 DATE OF FLIGHT05/22/75
             1           0           0          350
        ENTER WILD
        ENTER OUT
        ENTER OUTRIN        350
```

```
INPUT DATA FOR SET NUMBER    2 OF    2

     X-24B SEQUENCE NUMBER      1. MODE IS LATOR
     MACH= 1.13     DYN. PRESS.= 181.00  VELOCITY=1090.00     ALPHA=  4.00        C= 37.50        B= 19.10      S= 330.50
        W=  8567.0           DELCG=  0.00          IXX=    2646.0    IYY=  23704.0  IZZ=  24119.0  IXZ=    620.0
     START TIME = 10:18:25.500            STOP TIME = 10:18:31.000


OPTIONS REQUESTED AS FOLLOWS -


          WILD POINT CHECKING ON CONTROL PARAMETERS


          NO DIFFERENTIATION WILL BE PERFORMED ON RATES.


          A BINARY TAPE WILL BE PRODUCED FOR MMLE.
                    1          0          0          275
          ENTER WILD
          ENTER OUT
          ENTER OUTRIN        275
```

```
ALL SEQUENCES HAVE BEEN PROCESSED.
```

Table A2

ADEX PROGRAM DESCRIPTION

MAIN	- Executive routine for entire program. Reads card input and calls proper options.
LIST	- Listing routine. Writes time history data to line printer.
READ	- Executive routine for read subroutines. Directs program to proper read routine.
LABSOR	- Utility label sorting routine. Matches tape labels with correct slots in read sequence.
RDCDAS	- Read routine for A-9 and A-10. Reads input CDAS tape.
READX24	- Read routine for X-24B. Reads input stranger tape.
RDF15	- Read routine for F-15. Reads input CDAS tape.
RDF16	- Read routine for YF-16. Reads input CDAS tape.
WILD	- Wild point routine. Calls DUZ2 to replace wild points on control surfaces.
DUZ2	- Wild point routine. Replaces wild points on control surfaces.
PREDIF	- Rate differentiation routine. Calls DIRSIT to differentiate P, Q, and R.
DIRSIT	- Rate differentiation routine. Differentiates P, Q, and R.
OUT	- Executive routine for tape output subroutines. Calls proper tape output routine.
OUTBCD	- BCD tape writing routine. Writes a BCD tape for STABDIV.
OUTBIN	- Binary tape writing routine. Writes a binary tape for MMLE.

APPENDIX B

MMLE

Table B1

INPUT NAMELIST

PARAMETER	DESCRIPTION	DEFAULT
1. LONG, LATR (Logical)	The mode of operation, longitudinal or lateral-direction is determined by which one is set to TRUE, i.e., LONG=.TRUE. If neither is set the program will attempt to determine the mode based on the A matrix. LONG=.TRUE. if A (1,2) >.5, LATR=.TRUE. otherwise. This feature will not work unless starting values are provided in dimensionalized form.	

Items 2-11 are dependent upon the time history file which is made up of three vectors. These are shown below.

	OBSERVATION	CONTROL[25]	EXTRA
Lateral-Directional	β, P, R, ϕ, N_y, $\dot{P}$, $\dot{R}$	δc_1, δc_2, δc_3, δc_4	α, V, Mn, $\bar{q}$
Longitudinal	α, Q, V, θ, N_z, $\dot{Q}$, N_x	δe_1, δe_2, δe_3, δe_4,	ϕ, h, Mn, q

Of the extra parameters, only the first in each mode is actually used by the program and is therefore required. The default mode assumes only the first extra parameter is on the time history record. The last three parameters may be used by resetting some namelist items.

PARAMETER	DESCRIPTION	DEFAULT
2. CARD, TAPE (Logical)	Time history file source. Set CARD=.TRUE. for cards or TAPE=.TRUE. for tape.	TAPE=.TRUE.

---

[25] The MMLE program is equipped to handle up to four control surfaces in each of the longitudinal or lateral-directional axes. Since each aircraft uses different control surfaces, the program refers to these as $\delta c_{1,2,3,4}$ for the lateral-directional axis and $\delta e_{1,2,3,4}$ for the longitudinal axis. Assignment of a control surface may be made to any one of the four optional inputs, but the Milspec option of CONTROL requires the following:

Total aileron be assigned to δc_1

Rudder be assigned to δc_2

Primary pitch control surface be assigned to δe_1

Table B1 (Continued)

PARAMETER	DESCRIPTION	DEFAULT
3. SPS (Real)	Data rate for time history file in samples per second. Default is determined by taking the difference from the first two data points and rounding off to the nearest 5 milliseconds. SPS is then the reciptocal of this number.	
4. THIN (Integer)	Thinning factor for data. If THIN=1 all points are used. If THIN=2 every other point is used, etc.	1
5. NCASE (Integer)	Number of time intervals to be processed for a given set of derivatives. The program has the capability of processing up to fifteen time segments if they are at approximately the same flight conditions to produce one set of derivatives. Each interval is weighted by the number of points in the interval.	1
6. SCALE (Real, 7 words)	Scale factors for observations. Scale factors may account for sign or magnitude errors in the input data.	7 * 1.0
7. FIXED (Real, 7 words)	Fixed biases for observations. Known biases applied to the input data after scaling.	7 * 0.0
8. DC (Real, 4 words)	Fixed biases applied to the controls.	4 * 0.0
9. NREC (Integer)	Number of parameters in each time history record not counting the times. NREC has no meaning for card input.	12
10. ORDER (Integer, 15 words)	The order of the signals on the input tape. ORDER can be used for rearranging the tape parameters. The value of ORDER tells the program where in the data list the needed parameter is. For example, if β, which is the first parameter required, were the seventh parameter on the tape record, then ORDER (1)=7. The time records are not counted in the numbering process. ORDER has no meaning if CARD=.TRUE.	ORDER (I)=I
11. BOTH (Logical)	Both longitudinal and lateral-direction data may be included on the same time history record in the order α, Q, V, θ, N_z, $\dot{Q}$, N_x, δe_1, δe_2, δe_3, δe_4, ϕ, h, Mn, $\bar{q}$, β, P, R, N_y, $\dot{P}$, $\dot{R}$, δc_1, δc_2, δc_3, δc_4. If BOTH=.TRUE., NREC is set to 25 and ORDER is set to pick up the right signals for the type of case to be processed. Due to the increased computer core required to store the time history file, it is recommended that the BOTH option not be used.	.FALSE.

Table B1 (Continued)

Item 12-18 affect plotted output:

PARAMETER	DESCRIPTION	DEFAULT
12. PLOTEM (Logical)	Generates plotted output if=.TRUE., either time histories or a priori plots (Item 52).	.TRUE.
13. PLTMAX (Real)	Weighted error sum above which plots are no longer desired. PLTMAX prevents plotted output if the program fails to converge.	$1.X10^5$
14. INCH (Logical)	Plots scaled to inch paper if INCH=.TRUE., centimeter paper otherwise.	.TRUE.
15. ZMIN, ZMAX (Real, 7 words each)	Minimum and maximum values for each of the seven observation parameters. Each axis is 2 inches long (4 cm if INCH=.FALSE.) If minimum and maximum values are 0.0, Calcomp automatic scaling will be used.	7 * 0.0, 7 * 0.0
16. DCMIN, DCMAX (Real, 8 words each)	Minimum and maximum values for the four controls and four extra signals. Zero values give automatic scaling. If automatic scaling is used, signals with no non-zero points will not be plotted.	8 * 0.0, 8 * 0.0
17. NCPLOT (Integer)	Number of controls and extra signals to be plotted. Only the first NCPLOT signals after the observation signals appear.	5
18. TIMESC (Real)	Time scale for plots in seconds per half inch (seconds per cm if INCH=.FALSE.)	0.5
19. PRINT (Logical)	Prints input and computed time histories if PRINT=.TRUE.	.FALSE.
20. TEST (Logical)	Prints intermediate program output for debugging.	.FALSE.
21. NOITER (Integer)	Number of iterations to be used. NOITER=0 may be used to compute time histories using starting (or wind tunnel) derivatives. Input time history is always printed if NOITER=0.	6
22. ERRMAX (Real)	The error sum at which the program stops. If ERRMAX is exceeded, the program stops iterating and prints input time histories.	$1.0X10^{20}$
23. BOUND (Real)	Convergence bound. Used to stop the program if further improvement is insignificant. If the change in the error divided by the error is less than BOUND the programs stops and prints the final values.	.001

Table B1 (Continued)

PARAMETER	DESCRIPTION	DEFAULT
24. PUNCH (Logical)	Generates punched output of final derivatives and confidence levels suitable for a follow-on plotting program.	.FALSE.
25. PUNCHC (Logical)	Generates punched output of nondimensional final derivatives and flight conditions suitable for input to the CONTROL program.	.FALSE.
26. PUNCHD (Logical)	Generates punched output of dimensionalized derivatives suitable for restarting the program.	.FALSE.
27. NEAT (Integer)	Number of time halvings in the computation of the transition matrix. The program uses the transition matrix to determine computed time histories. If the sampling rate is low the time interval ($= \frac{1}{SPS}$) becomes too large for stable computation. Usually applicable for SPS < 10. Note that this does not insure that some information will not be lost due to the low sampling rate.	0

Items 28-42 concern the geometry of the aircraft and specify the flight conditions. These values do not need to be defined if TAPE=.TRUE., as they will be overridden by the values on the time history file.

PARAMETER	DESCRIPTION	DEFAULT
28. METRIC (Logical)	Determines the units of input data. If METRIC=.TRUE. all units are in the metric system.	.FALSE.
29. GROSWT (Real)	Aircraft gross weight (lbs or newtons)	$1.0X10^9$
30. IX (Real)	Moment of inertia about the x-axis (slug-ft^2 or kg-m^2)	$1.0X10^9$
31. IY (Real)	Moment of inertia about the y-axis (slug-ft^2 or kg-m^2)	$1.0X10^9$
32. IZ (Real)	Moment of inertia about the z-axis (slug-ft^2 or kg-m^2)	$1.0X10^9$
33. IXZ (Real)	Cross product of inertia between the x and z axes (slug-ft^2 or kg-m^2)	$1.0X10^9$
34. SPAN (Real)	Reference wing span (ft or m)	.001
35. CBAR (Real)	Reference aerodynamic chord (ft or m)	.001

Table B1 (Continued)

PARAMETER	DESCRIPTION	DEFAULT
36. S (Real)	Reference wing area (ft^2 of m^2)	.001
37. CG (Real)	Difference between test cg, and reference cg, in per cent chord. Test cg, aft of reference is positive. Used only for correcting derivatives from lookup curve file to test cg.	0.0
38. MACH (Real)	Mach number. Used only as an argument in lookup routine, and for labeling purposes.	0.0
39. ALPHA (Real)	Average angle of attack. (deg) The program has the capability to get an average number but if ALPHA is not defined here or on the time history header record neither the lookup or constant startup routines will work.	999.
40. Q (Real)	Dynamic pressure (lb/ft^2 or Newton/m^2) Same comment as 39.	0.0
41. V (Real)	Velocity (ft/sec, or m/sec). Same comment as 39.	0.0
42. PARAM (Integer)	Any other parameter that might be used to define a flight case, i.e., wing sweep	0

Items 43-49 give instrument offsets from the cg. Alpha and beta vane readings are corrected to the cg, using angular rates. The accelerometer effects are included in the system model, thus time histories of the angular accelerations ($\dot{P}$, $\dot{Q}$, $\dot{R}$) are not necessary to correct the accelerometers. If accelerations and angles have been corrected to the cg already, do not correct them again here. X direction offsets are positive for instruments forward of the cg; Z direction offsets are positive for instruments below the cg.

PARAMETER	DESCRIPTION	DEFAULT
43. XB (Real)	x direction offset of beta vane from cg, (ft or m)	0.0
44. ZB (Real)	z direction offset of beta vane from cg, (ft or m)	0.0
45. XALF (Real)	x direction offset of alpha vane from cg, (ft or m)	0.0
46. XAY (Real)	x direction offset of N_y accelerometer from cg, (ft or m)	0.0
47. ZAY (Real)	z direction offset of N_y accelerometer from cg, (ft or m)	0.0

Table B1 (Continued)

PARAMETER	DESCRIPTION	DEFAULT
48. XAN (Real)	x direction offset of N_z accelerometer from cg, (ft or m)	0.0
49. ZAX (Real)	z direction offset of N_x accelerometer from cg, (ft or m)	0.0
50. VAR (Logical, 3 words)	The last three observations (N_z, $\dot{Q}$, N_x or N_y, $\dot{P}$, $\dot{R}$) have an unknown bias included in the system model. These biases are determined if the corresponding element of VAR is .TRUE. Initial values of these biases are 0.0 except for the N_z bias which is 1.0. The bias on a signal that has a D1 weighting of zero cannot be determined, and any attempt to do so will be overridden by the program.	3* .TRUE.
51. ZERO (4 Word Logical Vector)	For each element of ZERO that is .TRUE., the corresponding state has a variable initial condition determined. If variable initial condition is used with NCASE > 1 (item 5), the same increment from the measured value is used for the initial condition for each maneuver in the case.	4* .FALSE.

Items 52-53 concern the a priori feature.

PARAMETER	DESCRIPTION	DEFAULT
52. WMAPR (Real)	An overall weighting factor for the a priori information. Each element in the a priori weighting matrices APRA and APRB (see input matrices description) is multiplied by WMAPR before use. A value of 0.0 implies that the a priori feature is not used in the estimation process.	0.0
53. NAPR, WFAC (Integer, Real)	These two variables control the a priori variation option which puts the program into a drastically different mode with changed output. For each aircraft analyzed, a set of a priori weighting matrices should be determined at the beginning of the flight program. In the determination of the best a priori weighting matrices it is useful to run the same case with several values of WMAPR (Item 52). The option to accomplish this is activated if NAPR is greater than 0. The program then runs the entire case a total of NAPR times with different values	WFAC=10. NAPR=0

Table B1 (Continued)

PARAMETER	DESCRIPTION	**DEFAULT**
53. continued	of WMAPR. The first pass is with WMAPR=0. and the second pass with the value specified for WMAPR by item 51 (if 0. was specified, -.001 is used instead). For each subsequent run, the value of WMAPR used is WFAC times the value used on the previous run. Time history plots are never produced when this option is used; instead, if PLOTEM=.TRUE. (Item 12), the final estimates of each of the derivatives are plotted versus WMAPR on a logarithmic scale. The a priori estimates, which may be considered as the estimates obtained with WMAPR=infinity, are also plotted to the right of the other estimates. These plots may then be used as described in Ken Iliff's and Larry Taylor's report to estimate the best values to use for the a priori weightings.[26]	
54. ND1, D1RLX, D1TOL (Integer, Real, Real)	**These three variables control the diagonal D1 determination option, which puts the program into a different mode of operation. A D1 weighting matrix (see matrix input section also should be determined for each aircraft at the beginning of its flight program. This option automatically determines the D1 based on a particular case and is activated if ND1 > 0. One pass is done with the initial D1 matrix input below. Then a simple iterative algorithm is applied ND1 times to determine the proper D1 matrix. Each iteration of this algorithm involves another pass through the estimation loop to obtain a set of weighted relative errors (E_i). The intent of the algorithm is to find a D1 matrix that results in the weighted error being approximately 1.0 on each signal being used (as indicated by a non-zero initial guess of the corresponding D1 element). The revised estimate of each diagonal element of the D1 matrix is then produced by multiplying the previous estimate by a factor that depends on the previous weighted error of that signal (E_i) and, a relaxation**	**ND1=0, DIRLX=1.2, DITOL=1.4**

---

[26] Reference 10: Iliff, Kenneth W. and Taylor, Lawrence W. Jr., Determination of Stability Derivatives from Flight Data Using a Newton-Raphson Minimization Technique, NASA TN D-6579, NASA Flight Research Center, Edwards, California, 1972

Table B1 (Concluded)

PARAMETER	DESCRIPTION	DEFAULT
54. continued	factor (DIRLX). If $E_i > 1.0$, the factor is $\frac{1.0}{(E_i - 1) * D1RLX + 1.0}$ and if $E_i < 1.$, the factor is $(\frac{1}{E_i - 1.}) * D1RLX + 1.)$ D1TOL will stop this process if it has converged before ND1 iterations. If, after any iteration, none of the weighted errors are greater than DITOL or less than $\frac{1}{DITOL}$, one final iteration will be run, and the process will be stopped. WMAPR (Item 52) will be set to 0.0 if this option is used, regardless of the input value. If plotting was specified (Item 12), only time history using the final D1 will be plotted. If both D1 determination and a priori variation (Item 53) are activated, the D1 determination will be done first and the a priori variation will use the final D1 matrix.	
55. PRNTPLT (Integer)	If PRNTPLT is set to 1 or 2, line printer plots will be generated at the end of the normal output section. A value of 1 will generate line printer plots with one character per time point. A value of 2 will condense the each time history to a single page, but resolution is greatly reduced. It should be emphasized that this option does not replace the Calcomp plots but is intended only to produce time histories matches in a more timely manner than is generally available with the Calcomp plotters.	0
56. GAMMA (Real)	Flight path angle. Used for calculation of pitch angle (θ) in matrix calculation.	0.0

Table B2

LOOKUP DATA INFORMATION

Derivative	Number	Default First Curve Number	Units
C_{m_α}	1	8110	per radian
$C_{m_{\delta e_1}}$	2	8120	per degree
$C_{m_{\delta e_2}}$	3	8130	per degree
$C_{m_{\delta e_3}}$	4	8140	per degree
C_{m_Q}	5	8150	per radian
C_{N_α}	6	8210	per radian
$C_{N_{\delta e_1}}$	7	8220	per degree
$C_{N_{\delta e_2}}$	8	8230	per degree
$C_{N_{\delta e_3}}$	9	8240	per degree
C_{N_Q}	10	0	per radian
C_{c_α}	11	0	per radian
$C_{c_{\delta e_1}}$	12	0	per degree
$C_{c_{\delta e_2}}$	13	0	per degree
$C_{c_{\delta e_3}}$	14	0	per degree
C_{c_Q}	15	0	per radian

Table B2 (Continued)

Derivative	Number	Default First Curve Number	Units
C_{Y_β}	16	8400	per radian
$C_{Y_{\delta c_1}}$	17	8410	per degree
$C_{Y_{\delta c_2}}$	18	8420	per degree
$C_{Y_{\delta c_3}}$	19	8430	per degree
$C_{Y_{\delta c_4}}$	20	8440	per degree
C_{ℓ_β}	21	8500	per radian
$C_{\ell_{\delta c_1}}$	22	8510	per degree
$C_{\ell_{\delta c_2}}$	23	8520	per degree
$C_{\ell_{\delta c_3}}$	24	8530	per degree
$C_{\ell_{\delta c_4}}$	25	8540	per degree
C_{ℓ_P}	26	8550	per radian
$C_{\ell R}$	27	8560	per radian
C_{n_β}	28	8600	per radian
$C_{n_{\delta c_1}}$	29	8610	per degree
$C_{n_{\delta c_2}}$	30	8620	per degree

Table B2 (Concluded)

Derivative	Number	Default First Curve Number	Units
$C_{n_{\delta c_3}}$	31	8630	per degree
$C_{n_{\delta c_4}}$	32	8640	per degree
C_{n_P}	33	8650	per radian
C_{n_R}	34	8660	per radian

Number of Curves

Usually dynamic pressure is the third lookup parameter, i.e., one curve for each different dynamic pressure up to ten. If rigid data is desired the number of curves for that derivative should be equal to one.

Order of Input Parameters

Given the function $Y = f(x,z,w)$

Order =

1 for x=angle of attack	z=Mach No.	w=dynamic pressure
2 for x=angle of attack	z=dynamic pressure	w=Mach No.
3 for x=Mach No.	z=angle of attack	w=dynamic pressure
4 for x=Mach No.	z=dynamic pressure	w=angle of attack
5 for x=dynamic pressure	z=angle of attack	w=Mach No.
6 for x=dynamic pressure	z=Mach No.	w=angle of attack

Table B3

CONSTANTS INFORMATION

Derivative	Literal Name	Default Value	Units
C_{m_α}	CMA	-.001	per degree
$C_{m_{\delta e_1}}$	CMDE1	-.001	per degree
$C_{m_{\delta e_2}}$	CMDE2	-.001	per degree
$C_{m_{\delta e_3}}$	CMDE3	-.001	per degree
$C_{m_{\delta e_4}}$	CMDE4	-.001	per degree
C_{m_Q}	CMQ	-1.0	per radian
C_{m_V}	CMV	0.0	dimensionless
C_{N_α}	CNA	.01	per degree
$C_{N_{\delta e_1}}$	CNDE1	.01	per degree
$C_{N_{\delta e_2}}$	CNDE2	.01	per degree
$C_{N_{\delta e_3}}$	CNDE3	.01	per degree
$C_{N_{\delta e_4}}$	CNDE4	.01	per degree
C_{N_Q}	CNQ	0.0	per radian
C_{N_V}	CNV	0.0	dimensionless
C_{c_α}	CCA	0.0	per degree
$C_{c_{\delta e_1}}$	CCDE1	0.0	per degree

Table B3 (Continued)

Derivative	Literal Name	Default Value	Units
$C_{c_{\delta e_2}}$	CCDE2	0.0	per degree
$C_{c_{\delta e_3}}$	CCDE3	0.0	per degree
$C_{c_{\delta e_4}}$	CCDE4	0.0	per degree
C_{c_Q}	CCQ	0.0	per radian
C_{c_V}	CCV	0.0	dimensionless
C_{ℓ_β}	CLB	-.002	per degree
$C_{\ell_{\delta c_1}}$	CLDC1	.0007	per degree
$C_{\ell_{\delta c_2}}$	CLDC2	.00025	per degree
$C_{\ell_{\delta c_3}}$	CLDC3	.0007	per degree
$C_{\ell_{\delta c_4}}$	CLDC4	.0007	per degree
C_{ℓ_P}	CLP	-.2	per radian
C_{ℓ_R}	CLR	.15	per radian
C_{n_β}	CNB	.0025	per degree
$C_{n_{\delta c_1}}$	CNDC1	.0002	per degree
$C_{n_{\delta c_2}}$	CNDC2	-.001	per degree

Table B3 (Concluded)

Derivative	Literal Name	Default Value	Units
$C_{n_{\delta c_3}}$	CNDC3	.0002	per degree
$C_{n_{\delta c_4}}$	CNDC4	.0002	per degree
C_{n_p}	CNP	0.0	per radian
C_{n_r}	CNR	-.25	per radian
C_{y_β}	CYB	-.015	per degree
$C_{y_{\delta c_1}}$	CYDC1	-.0005	per degree
$C_{y_{\delta c_2}}$	CYDC2	.002	per degree
$C_{y_{\delta c_3}}$	CYDC3	-.0005	per degree
$C_{y_{\delta c_4}}$	CYDC4	-.005	per degree

Table B4

MMLE PLOT INFORMATION

Plot Checklist

1. Magnetic tape requested
2. Sample rate X time interval ≤ 1000
3. $\frac{\text{Time interval}}{\text{Time scale}} \times \frac{1}{2} \leq 10$

Plot Tape

Local file name - TAPE13

Plotter Job Request Card

Time Histories

Number of plots - 2 per case
Time per plot - 1 minute
Plot number, start - 1
end - 999
Pen position - 2
Type pen - wet
Point size - 4 or 5
Paper size - 201 or 202

WMAPR Plots

Number of plots - Number of derivatives + 2
Time per plot - 15 seconds
Plot number, start - 1
end - 999
Pen position - 1
Type pen - wet
Points size - 4 or 5
Paper size - 201 or 202

GENERAL PURPOSE PROGRAMMING DATA

PAGE 1 OF 2 PAGES

PROGRAM NAME: MMLE DATA CARDS

DECK NR: 4532

LINE ITEM:

RETURN TO (Name/Organization/Phone):

DATE:

```
                        TITLE                                          SECTION 1
                        INPUT  NAMELIST                                        2
SHSMSSSMS EHEMESEMS     SEARCH                                                 3
      CURVIN NAMELIST,  DERIVIN NAMELIST  OR  MATRICES                         4
                       END  OR  ENDCASE


                        SAMPLE  DECK

 X-24B CASE 23-51, M=1.49 ALFA=5.0 POWER ON , SAS OFF
$INPUT LONG=.T.,XAN=-4.27,$END

$DERIVIN CMDE2=0.0,CMDE3=0.0,CMDE4=0.0,CNDE2=0.0,CNDE3=0.0,CNDE4=0.0,$END
ENDCASE
 X-24B CASE 23-1, M=1.13, ALFA=4.0 POWER OFF, SAS ON
$INPUT LATR=.T.,XAY=-4.27,ZAY=1.02,WMAPR=0.25,$END

A         4        4
 -.106     .070     -.998    .029
 0.0      -.265     .378     0.0
 6.11      .042     -.195    0.0
 0.0       1.0      .07      0.0
```

GENERAL FURPOSE PROGRAMMING DATA

PAGE 2 OF 2 PAGES

PROGRAM NAME: MMLE DATA CARDS

DECK NR: 4532

LINE ITEM:

RETURN TO (Name/Organization/Phone):

DATE:

cc	5	10	15 20	25 30	35 40	45 50
B		4	5			
	-.019		.008	0.0	0.0	0.0
	21.03		7.42	0.0	0.0	0.0
	1.629		-1.764	0.0	0.0	0.0
	0.0		0.0	0.0	0.0	0.0
D	1	5	0			
	136000.		1530.	90000.	1500.	10600.
A	PRA	4	4			
	.100E+06		0.0	.100E+06	0.0	
	.289E+01		.104E+05	.104E+03	0.0	
	.289E+03		.163E+05	.163E+05	0.0	
	0.0		0.0	0.0	0.0	
A	PRB	4	5			
	.100E+06		.100E+06	0.0	0.0	0.0
	.289E+01		.289E+01	0.0	0.0	0.0
	.289E+03		.289E+03	0.0	0.0	0.0
	0.0		0.0	0.0	0.0	0.0
E	ND					

APPTC FORM SEP 67 30 PREVIOUS EDITION OF THIS FORM WILL BE USED UNTIL STOCK IS EXHAUSTED

```
EN223,CM72000,CH9983004532P,T776,NT1.
VSN,TAPE13=5814.
ATTACH,THIST,X24B,ID=CNAGY,CY=4,MR=1.
ATTACH,MMLE,AMC,ID=CNAGY,CY=2,MR=1.
REQUEST,TAPE13,PE,RING.
REQUEST,DERIVS,*PF.
MMLE.
CATALOG,DERIVS,X24B,ID=CNAGY,CY=5.
0000000000000000000000
  X-24B  CASE 23-51  M=1.49  ALFA=5.0  POWER ON   SAS OFF
 $INPUT LONG=.T.,XAN=-4.27,$END

 $DERIVIN CMDE2=0.0,CMDE3=0.0,CMDE4=0.0,CNDE2=0.0,CNDE3=0.0,CNDE4=0.0,$END
ENDCASE
  X-24B  CASE 23-1  M=1.13  ALFA=4.0  POWER OFF   SAS ON
 $INPUT LATR=.T.,XAY=-4.27,ZAY=1.00,WMAPR=.25,$END
A           4          4
  -.106       .070       -.998      .029
  0.0         -.265      .378       0.0
  6.11        .042       -.195      0.0
  0.0         1.0        .07        0.0
B           4          5
  -.018       .008       0.0        0.0        0.0
  21.03       7.42       0.0        0.0        0.0
  1.629       -1.764     0.0        0.0        0.0
  0.0         0.0        0.0        0.0        0.0
D1          5          0
  136000.     1530.      90000.     1500.      10600.
APRA        4          4
  .100E+06    0.0        .100E+06   0.0
  .289E+01    .104E+05   .104E+03   0.0
  .289E+03    .163E+05   .163E+05   0.0
  0.0         0.0        0.0        0.0
APRB        4          5
  .100E+06    .100E+06   0.0        0.0        0.0
  .289E+01    .289E+01   0.0        0.0        0.0
  .289E+03    .289E+03   0.0        0.0        0.0
  0.0         0.0        0.0        0.0        0.0
END
0000000000000000000000
0000000000000000000000
```

```
                    X-240   CASE 23-51   M=1.49   ALFA=5.0   POWER ON    SAS OFF                         07/30/75
                                         NEWTON-RAPHSON DIGITAL DERIVATIVE MATCHING
                                                         1 APR 1974

INPUT DATA (T INDICATES TRUE OR YES, F INDICATES FALSE OR NO)

     LONGITUDINAL CASE
     DATA SOURCE    CARD? F         TAPE? T
     DATA RATE IS   0. SAMPLES/SECOND ON SOURCE FILE (IF 0, DETERMINED FROM TIMES ON THE SOURCE FILE)
          DIVIDED BY THINNING FACTOR OF  1
     ON INPUT TAPE:  12 DATA WORDS PER RECORD.    SPECIAL SIGNAL ORDER DEFAULT? F

PROGRAM OPTIONS

     APRIORI WEIGHTING =0.                          0 TIME HALVINGS IN EAT.
     ITERATIONS =  6  (ITERATION WILL STOP IF ERROR SUM CHANGES BY LESS THAN A FACTOR OF  .10E-02)
     CASE WILL BE STOPPED IF ERROR SUM IS GREATER THAN  .10E+21

OUTPUT

     PLOTS? T  (NO PLOTS UNLESS FINAL ERROR SUM IS LESS THAN .100E+06)
          NUMBER OF CONTROLS AND EXTRA SIGNALS TO BE PLOTTED =  5
          SECONDS PER CENTIMETER =  .50
     PRINTED FLIGHT AND FINAL COMPUTED TIME HISTORIES? F
     EXTRA OUTPUT OF INTERMEDIATE STEPS FOR A DIAGNOSTIC AID? F
     PUNCHED FINAL NON-DIMENSIONAL DERIVATIVES AND CONFIDENCE LEVELS? F
     PUNCHED FINAL DIMENSIONAL MATRICES? F

FLIGHT CONDITION AND VEHICLE CHACTERISTICS (0. INDICATES VALUE OBTAINED FROM TIME HISTORY ON QBAR,V OR MACH)
                                            (MACH,ALPHA,CG AND PARAM ARE FOR REFERENCE ONLY, NOT USED IN PROGRAM)
     METRIC UNITS? F
     DYNAMIC PRESSURE =       145.0       VELOCITY = 1450.0
     MACH = 1.490                         ALPHA =    5.00  (IF 999. , OBTAINED FROM TIME HISTORY)
     CENTER OF GRAVITY = 0.000            OTHER IDENTIFYING PARAMETER = 0.
     WING AREA =   330.5                  SPAN =   19.10      CHORD = 37.50
     IX =    2646.0                       IY =    23704.0     IZ =    24119.0     IXZ =    620.0
     WEIGHT =    8567.0
     INSTRUMENT OFFSETS FROM CG
          X-DIRECTION OFFSETS (+ = INSTRUMENT IS FORWARD OF CG)
               ALPHA    0.000  AN   -4.270
               BETA     0.000  AY    0.000
          Z-DIRECTION OFFSETS ( + = INSTRUMENT IS BELOW CG)
               BETA     0.000  AY    0.000

SIGNAL SCALING AND BIASES
 SIGNALS        ALFA     Q        V        THET     AN       QDOT     AX       DE1      DE2      DE3      DE4      PHI      ALT      MACH     QBAR
 VAR BIAS                                           T        T        T
 VAR I.C.        F       F        F        F
 FIXED BIAS     0.00     0.00     0.00     0.00     0.00     0.00     0.00     0.00     0.00     0.00     0.00
 SCALE FACT     1.00     1.00     1.00     1.00     1.00     1.00     1.00
 PLOT LIMITS
    MINIMUM     0.00     0.00     0.00     0.00     0.00     0.00     0.00     0.00     0.00     0.00     0.00     0.00     0.00     0.00     0.00
    MAXIMUM     0.00     0.00     0.00     0.00     0.00     0.00     0.00     0.00     0.00     0.00     0.00     0.00     0.00     0.00     0.00

 MANEUVER    1   START TIME    10    17    52    18   STOP TIME    10    17    58   998
```

①

```
                     X-24B  CASE 23-51  M=1.49  ALFA=5.0  POWER ON   SAS OFF                         07/30/75

INPUT MATRICES :                                                                       ②
A AND B MATRICIES ARE FROM LONCON
                                     TOTAL NUMBER OF POINTS FOR MANEUVER  1 =   350   ③
```

```
                X-248   CASE 23-51   M=1.49  ALFA=5.0  POWER ON   SAS OFF                                  07/30/75
        STARTING VALUES      MACH = 1.490      ALPHA =    5.00      PARAM =      0.0000      CG = 0.000
DIMENSIONAL DERIVATIVES / SEC / SEC**2
          ALFA             Q              V              DE1            DE2            DE3            DE4          DELTA-O
 N      .711219        -1.000000*      0.000000*        .071122       0.000000*      0.000000*      0.000000*     -0.000000
 M    -4.344139        -.980353        0.000000*      -4.344139       0.000000*      0.000000*      0.000000*      0.000000
 A     0.000000*       -0.000000*      0.000000*       0.000000*      0.000000*      0.000000*      0.000000*     -0.000000*

NON-DIMENSIONAL DERIVATIVES / DEG (ROTARY /RAD)
          ALFA             Q              V              DE1            DE2            DE3            DE4          DELTA-O
 CN     .100014         0.000000*      0.000000*        .010001       0.000000*      0.000000*      0.000000*      0.000000
 CM    -.001000        -1.000000       0.000000*       -.001000       0.000000*      0.000000*      0.000000*      0.000000
 CA     0.000000*       -0.000000*      0.000000*       0.000000*      0.000000*      0.000000*      0.000000*      0.000000*
                              ( * ) INDICATES DERIVATIVE HELD FIXED DURING MATCH          NUMBER OF UNKNOWNS =  9
```

④

```
                    ENTERING ITERATION LOOP
 DIMENSIONAL DERIVATIVE MATRICES PER RAD.AN. BIASES IN RADIANS.

A                                      4  BY  4
               -.7112E+00    .1000E+01 -0.            -.1934E-02
               -.4344E+01   -.9804E+00  0.            0.
              -0.           0.         -0.            -.3205E+02
               0.            .9999E+00  0.            0.
B                                      4  BY  5
               -.7112E-01 -0.          -0.            -0.            0.
               -.4344E+01  0.           0.             0.            0.
              -0.          -0.         -0.            -0.            0.
               0.           0.          0.             0.            0.
                   AN
VARIABLE BIAS     .1000E+01
                                                                    WEIGHTED ERROR SUM =    .2032E+06
ERRORS
     .1253E+00    .5981E-01    .2137E+05    .8132E+00    .5307E+02
WEIGHTED ERRORS
     .3758E+04    .1104E+05  0.             .8132E+05    .1061E+06
```

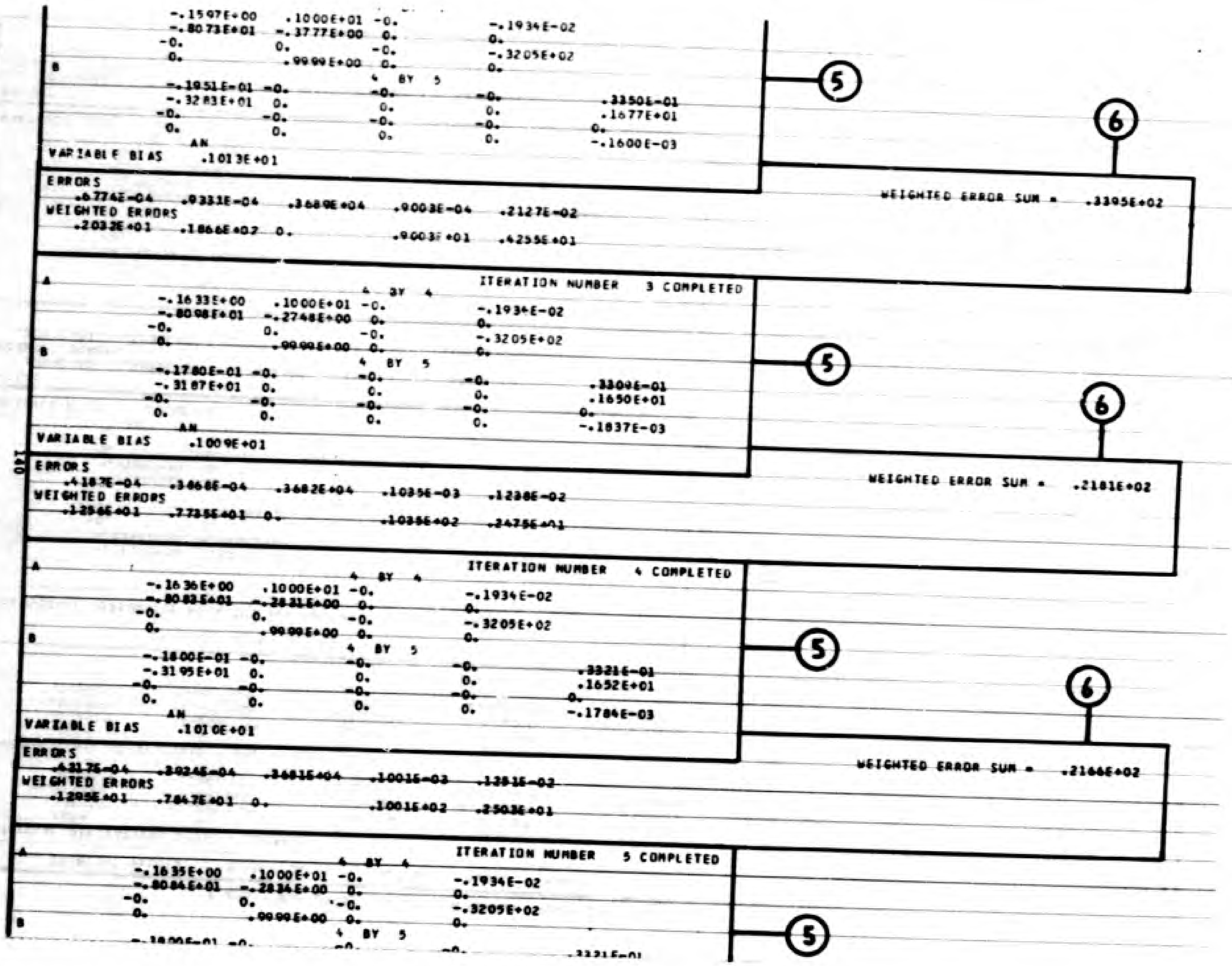
⑤

⑥

```
                                             ITERATION NUMBER    1 COMPLETED
A                                      4  BY  4
               -.2627E+00    .1000E+01 -0.            -.1934E-02
               -.7456E+01   -.2230E+00  0.            0.
              -0.           0.         -0.            -.3205E+02
               0.            .9999E+00  0.            0.
B                                      4  BY  5
               -.2256E-01 -0.          -0.            -0.            .4086E-01
               -.2963E+01  0.           0.             0.            .1520E+01
              -0.          -0.         -0.            -0.           0.
               0.           0.          0.             0.           -.9437E-03
                   AN
VARIABLE BIAS     .9911E+00
                                                                    WEIGHTED ERROR SUM =    .2009E+03
ERRORS
     .1529E-03    .7434E-03    .3667E+04    .3429E-04    .2208E-01
WEIGHTED ERRORS
     .4588E+01    .1487E+03  0.             .3429E+01    .4416E+02
                                             ITERATION NUMBER    2 COMPLETED
A                                      4  BY  4
```

⑤

⑥



```
         -.1597E+00    .1000E+01 -0.               -.1934E-02
         -.8073E+01   -.3777E+00  0.                0.
        -0.            0.        -0.               -.3205E+02
         0.             .9999E+00 0.                0.
B                                 4   BY   5
         -.1951E-01 -0.          -0.               -0.              .3350E-01
         -.3283E+01  0.           0.                0.              .1677E+01
        -0.         -0.          -0.               -0.             0.
         0.          0.           0.                0.             -.1600E-03
              AN
VARIABLE BIAS     .1013E+01
ERRORS
     .6774E-04    .9331E-04    .3689E+04    .9003E-04    .2127E-02
WEIGHTED ERRORS
     .2032E+01    .1866E+02  0.                .9003E+01    .4255E+01        WEIGHTED ERROR SUM =    .3395E+02
A                                                  ITERATION NUMBER    3 COMPLETED
                                  4   BY   4
         -.1633E+00    .1000E+01 -0.               -.1934E-02
         -.8098E+01   -.2748E+00  0.                0.
        -0.            0.        -0.               -.3205E+02
         0.             .9999E+00 0.                0.
B                                 4   BY   5
         -.1780E-01 -0.          -0.               -0.              .3309E-01
         -.3187E+01  0.           0.                0.              .1650E+01
        -0.         -0.          -0.               -0.             0.
         0.          0.           0.                0.             -.1837E-03
              AN
VARIABLE BIAS     .1009E+01
ERRORS
     .4187E-04    .3468E-04    .3682E+04    .1035E-03    .1238E-02
WEIGHTED ERRORS
     .1256E+01    .7735E+01  0.                .1035E+02    .2475E+01        WEIGHTED ERROR SUM =    .2181E+02
A                                                  ITERATION NUMBER    4 COMPLETED
                                  4   BY   4
         -.1636E+00    .1000E+01 -0.               -.1934E-02
         -.8082E+01   -.2831E+00  0.                0.
        -0.            0.        -0.               -.3205E+02
         0.             .9999E+00 0.                0.
B                                 4   BY   5
         -.1800E-01 -0.          -0.               -0.              .3321E-01
         -.3195E+01  0.           0.                0.              .1652E+01
        -0.         -0.          -0.               -0.             0.
         0.          0.           0.                0.             -.1784E-03
              AN
VARIABLE BIAS     .1010E+01
ERRORS
     .4317E-04    .3024E-04    .3681E+04    .1001E-03    .1251E-02
WEIGHTED ERRORS
     .1295E+01    .7847E+01  0.                .1001E+02    .2503E+01        WEIGHTED ERROR SUM =    .2166E+02
A                                                  ITERATION NUMBER    5 COMPLETED
                                  4   BY   4
         -.1635E+00    .1000E+01 -0.               -.1934E-02
         -.8084E+01   -.2834E+00  0.                0.
        -0.            0.        -0.               -.3205E+02
         0.             .9999E+00 0.                0.
B                                 4   BY   5
         -.1800E-01 -0.          -0.               -0.              .3321E-01
```

```
        -.3195E+01  0.          0.          0.          .1653E+01
       -0.         -0.         -0.         -0.         0.
        0.          0.          0.          0.         -.1781E-03
              AN
VARIABLE BIAS     .1010E+01
```

```
ERRORS
    .4323E-04   .3018E-04   .3601E+04   .1002E-03   .1252E-02              WEIGHTED ERROR SUM =   .2144E+02
WEIGHTED ERRORS
    .1297E+01   .7834E+01  0.           .1002E+02   .2504E+01
                                                                                       ⑥
                                                  ITERATION NUMBER   6 COMPLETED

ITERATION TERMINATING, ERROR WITHIN   .001000   BOUNC.
```

```
CONFIDENCE LEVELS FOR NEXT TO LAST ITERATION
    (DIMENSIONAL)

AC                                           3  BY  3
              .2122E-02   0.           0.
              .8646E-02   .4641E-02    0.
             0.           0.           0.
BC                                           3  BY  5
              .1676E-02   0.           0.           0.           .6518E-03
              .1830E-01   0.           0.           0.           .5380E-02        ⑦
             0.           0.           0.           0.          0.

    (NON-DIMENSIONAL)
AC                                           3  BY  3
              .2983E-03   0.           0.
              .1005E-05   .4734E-02    0.
             0.           0.           0.
BC                                           3  BY  5
              .2356E-03   0.           0.           0.           .5252E-02
              .4213E-05   0.           0.           0.           .7096E-04
             0.           0.           0.           0.          0.
```

```
                       X-248   CASE 23-51   M=1.49   ALFA=5.0   POWER ON    SAS OFF                                        07/30/75

          FINAL VALUES     MACH = 1.490      ALPHA =    5.00      PARAM =    0.0000      CG = 0.000

DIMENSIONAL DERIVATIVES / SEC / SEC**2
          ALFA             Q                V                 DE1               DE2               DE3               DE4               DELTA-O
 N        .163538        -1.000000*        0.000000*         .018000          0.000000*         0.000000*         0.000000*         -.033206
 M      -8.084474        -.283361          0.000000*       -3.195201          0.000000*         0.000000*         0.000000*         1.652523
 A       0.000000*       -0.000000*        0.000000*        0.000000*         0.000000*         0.000000*         0.000000*        -0.000000*

NON-DIMENSIONAL DERIVATIVES / DEG (ROTARY /RAD)
          ALFA             Q                V                 DE1               DE2               DE3               DE4               DELTA-O
CN        .022907         0.000000*        0.000000*         .002531          0.000000*         0.000000*         0.000000*         -.267548
CM       -.001861        -.289040          0.000000*        -.000736          0.000000*         0.000000*         0.000000*          .021797
CA       0.000000*       -0.000000*        0.000000*        0.000000*         0.000000*         0.000000*         0.000000*         0.000000*

                                           ( * ) INDICATES DERIVATIVE HELD FIXED DURING MATCH
VARIABLE BIAS     .1010E+01
FINAL DIMENSIONAL MATRICES
A                                            4  BY  4
                  -.1635E+00   .1000E+01 -0.            -.1934E-02
                  -.8084E+01  -.2834E+00  0.             0.
                 -0.           0.        -0.            -.3205E+02
                  0.           .9999E+00  0.             0.
B                                            4  BY  5
                  -.1800E-01 -0.         -0.            -0.              .3321E-01
                  -.3195E+01  0.          0.             0.              .1653E+01
                 -0.          -0.        -0.            -0.             0.
                  0.           0.         0.             0.             -.1782E-03

DEGREES            AN
VARIABLE BIAS     .1010E+01
ERRORS
    .4322E-04    .3916E-04    .3691E+04    .1002E-03    .1252E-02
WEIGHTED ERRORS
    .1297E+01    .7836E+01   0.            .1002E+02    .2504E+01

                                                         ERRORS
 203179.89     200.85      33.95      21.81      21.66      21.66      21.66

                                                                     WEIGHTED ERROR SUM =    .2166E+02
```

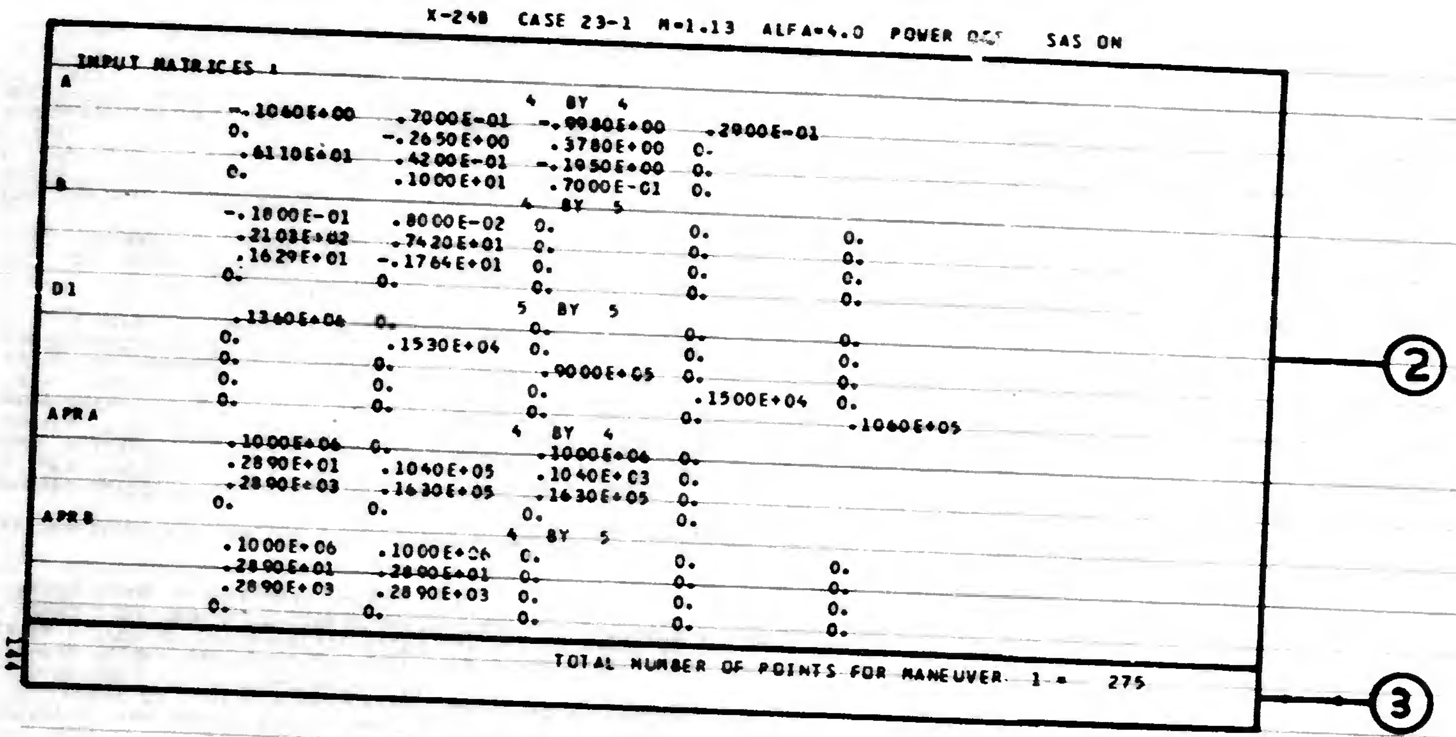
⑧

⑤

⑥

⑨

```
                X-248  CASE 23-1  M=1.13  ALFA=4.0  POWER OFF   SAS ON
                            NEWTON-RAPHSON DIGITAL DERIVATIVE MATCHING                                07/30/75
                                          1 APR 1974
INPUT DATA (T INDICATES TRUE OR YES, F INDICATES FALSE OR NO)

    LATERAL       CASE                                                                                      ①
    DATA SOURCE     CARD? F        TAPE? T
    DATA RATE IS    0. SAMPLES/SECOND ON SOURCE FILE (IF 0, DETERMINED FROM TIMES ON THE SOURCE FILE)
          DIVIDED BY THINNING FACTOR OF  1
    ON INPUT TAPE:  12 DATA WORDS PER RECORD.    SPECIAL SIGNAL ORDER DEFAULT? F

PROGRAM OPTIONS

    APRIORI WEIGHTING = .25E+00              0 TIME HALVINGS IN GAT.
    ITERATIONS =  6  (ITERATION WILL STOP IF ERROR SUM CHANGES BY LESS THAN A FACTOR OF  .10E-02)
    CASE WILL BE STOPPED IF ERROR SUM IS GREATER THAN  .10E+21

OUTPUT

    PLOTS? T  (NO PLOTS UNLESS FINAL ERROR SUM IS LESS THAN .100E+06)
         NUMBER OF CONTROLS AND EXTRA SIGNALS TO BE PLOTTED =  5
         SECONDS PER CENTIMETER =   .50
    PRINTED FLIGHT AND FINAL COMPUTED TIME HISTORIES? F
    EXTRA OUTPUT OF INTERMEDIATE STEPS FOR A DIAGNOSTIC AID? F
    PUNCHED FINAL NON-DIMENSIONAL DERIVATIVES AND CONFIDENCE LEVELS? F
    PUNCHED FINAL DIMENSIONAL MATRICES? F

FLIGHT CONDITION AND VEHICLE CHACTERISTICS (0. INDICATES VALUE OBTAINED FROM TIME HISTORY ON QBAR,V OR MACH)
                                           (MACH,ALPHA,CG AND PARAM ARE FOR REFERENCE ONLY, NOT USED IN PROGRAM)
    METRIC UNITS? F
    DYNAMIC PRESSURE =        181.0       VELOCITY = 1090.0
    MACH = 1.130                          ALPHA =    4.00  (IF 999. , OBTAINED FROM TIME HISTORY)
    CENTER OF GRAVITY = 0.000             OTHER IDENTIFYING PARAMETER = 0.
    WING AREA =   330.5                   SPAN =   19.10      CHORD = 37.50
    IX =    2646.0                        IY =    23704.0     IZ =    24119.0     IXZ =    620.0
    WEIGHT =     8567.0
    INSTRUMENT OFFSETS FROM CG
         X-DIRECTION OFFSETS (+ = INSTRUMENT IS FORWARD OF CG)
              ALPHA    0.000  AN    0.000
              BETA     0.000  AY   -4.270
         Z-DIRECTION OFFSETS ( + = INSTRUMENT IS BELOW CG)
              BETA     0.000  AY    1.000

SIGNAL SCALING AND BIASES
 SIGNALS      BETA     P        R        PHI      AY       PDOT     RDOT     DC1      DC2      DC3      DC4      ALFA     V        MACH     QBAR
 VAR BIAS                                         T        T        T
 VAR I.C.      F       F        F        F
 FIXED BIAS   0.00     0.00     0.00     0.00     0.00     0.00     0.00     0.00     0.00     0.00     0.00
 SCALE FACT   1.00     1.00     1.00     1.00     1.00     1.00     1.00
 PLOT LIMITS
    MINIMUM   0.00     0.00     0.00     0.00     0.00     0.00     0.00     0.00     0.00     0.00     0.00     0.00     0.00     0.00     0.00
    MAXIMUM   0.00     0.00     0.00     0.00     0.00     0.00     0.00     0.00     0.00     0.00     0.00     0.00     0.00     0.00     0.00

MANEUVER   1   START TIME   10   18   25  517   STOP TIME   10   18   30   997
```



```
                         X-24B  CASE 23-1  M=1.13  ALFA=4.0  POWER OFF   SAS ON                                07/30/75
 INPUT MATRICES 1
A
                                        4   BY   4
        -.1040E+00   .7000E-01   -.9980E+00   .2900E-01
        0.           -.2650E+00   .3780E+00   0.
        -.6110E+01   .4200E-01   -.1950E+00   0.
        0.           .1000E+01    .7000E-01   0.
B
                                        4   BY   5
        -.1800E-01   .8000E-02   0.           0.           0.
        .2103E+02    .7420E+01   0.           0.           0.
        .1629E+01    -.1764E+01  0.           0.           0.
        0.           0.          0.           0.           0.
D1
                                        5   BY   5
        .1340E+04    0.          0.           0.           0.
        0.           .1530E+04   0.           0.           0.
        0.           0.          .9000E+05    0.           0.
        0.           0.          0.           .1500E+04    0.
        0.           0.          0.           0.           .1060E+05
APRA
                                        4   BY   4
        .1000E+06    0.          .1000E+06    0.
        .2890E+01    .1040E+05   .1040E+03    0.
        .2890E+03    .1630E+05   .1630E+05    0.
        0.           0.          0.           0.
APRB
                                        4   BY   5
        .1000E+06    .1000E+06   0.           0.           0.
        .2890E+01    .2890E+01   0.           0.           0.
        .2890E+03    .2890E+03   0.           0.           0.
        0.           0.          0.           0.           0.

                              TOTAL NUMBER OF POINTS FOR MANEUVER  1 =    275
```

```
                         X-24B   CASE 23-1   M=1.13   ALFA=4.0   POWER OFF    SAS ON                                              07/30/75
         STARTING VALUES        MACH = 1.130       ALPHA =    4.00      PARAM =      0.0000      CG = 0.000
                                                                                                                                          ④
DIMENSIONAL DERIVA'IVES / SEC / SEC**2
          BETA                    P                     R                    DC1                  DC2                  DC3                  DC4                  DELTA-C
 Y      -.106000               .070000             -.998000*            -.018000              .008000             0.000000*            0.000000*             0.000000
 L      0.000000              -.265000              .378000             21.030000             7.420000             0.000000*            0.000000*             0.000000
 N      6.110000               .042000             -.195000              1.629000            -1.764000             0.000000*            0.000000*             0.000000
NON-DIMENSIONAL DERIVATIVES / DEG (ROTARY /RAD)
          BETA                    P                     R                    DC1                  DC2                  DC3                  DC4                  DELTA-C
CY      -.008977              0.000000             0.000000*            -.001524              .000677             0.000000*            0.000000*             0.000000
CL      0.000000              -.070045              .099913              .000850              .000300             0.000000*            0.000000*             0.000000
CN       .002251               .101192             -.469822              .000600             -.000650             0.000000*            0.000000*             0.000000
                                                  ( * ) INDICATES DERIVATIVE HELD FIXED DURING MATCH                NUMBER OF UNKNOWNS = 19
                    ENTERING ITERATION LOOP
 DIMENSIONAL DERIVATIVE MATRICES PER RADIAN. BIASES IN RADIANS.
A                                                    4   BY   4
                  -.1060E+00    .7000E-01   -.9980E+00    .2898E-01
                  0.           -.2650E+00    .3780E+00   0.                               ⑤
                   .6110E+01    .4200E-01   -.1950E+00   0.
                  0.            .1000E+01    .7000E-01   0.
B                                                    4   BY   5
                  -.1800E-01    .8000E-02   0.              0.             0.                                ⑥
                   .2103E+02    .7470E+01   0.              0.             0.
                   .1629E+01   -.1744E+01   0.              0.             0.
                  0.           0.           0.              0.             0.
                      AY
VARIABLE BIAS     0.
ERRORS                                                                                          WEIGHTED ERROR SUM =    .7439E+02
      .1002E-03    .2143E-02    .3291E-03    .6551E-02    .1702E-02
WEIGHTED ERRORS
      .1363E+02    .3279E+01    .2962E+02    .9824E+01    .1804E+02
                                                      ITERATION NUMBER    1 COMPLETED
A                                                    4   BY   4
                  -.1196E+00   -.7759E-02   -.9980E+00    .2898E-01
                   .2341E+01   -.2156E+00   -.5170E+00   0.
                   .7050E+01   -.4059E-01   -.1256E+00   0.                               ⑤
                  0.            .1000E+01    .7000E-01   0.
B                                                    4   BY   5
                  -.1097E-01    .8362E-02   0.              0.            -.4315E-02                         ⑥
                   .2339E+02    .7425E+01   0.              0.             .4196E-02
                   .1512E+01   -.1945E+01   0.              0.             .2900E-01
                  0.           0.           0.              0.             .2892E-03
                      AY
VARIABLE BIAS      .1463E+00
ERRORS                                                                                          WEIGHTED ERROR SUM =    .1688E+02
      .2461E-04    .4825E-03    .6408E-04    .4635E-03    .5975E-03
WEIGHTED ERRORS
      .3347E+01    .7382E+00    .5767E+01    .6952E+00    .6334E+01
                                                      ITERATION NUMBER    2 COMPLETED
A                                                    4   BY   4
```

```
             -.1195E+00   .9838E-02  -.9980E+00   .2898E-01
              .3403E+01  -.2188E+00  -.7231E-01  0.
              .7137E+01  -.3438E-01  -.1378E+00  0.
             0.           .1000E+01   .7000E-01  0.                              ⑤
B                                   4   BY   5
             -.1156E-01   .8583E-02  0.          0.           -.4540E-02                        ⑥
              .2321E+02   .7587E+01  0.          0.           -.4672E-02
              .1658E+01  -.1931E+01  0.          0.            .3708E-01
             0.          0.          0.          0.           -.5725E-03
               AY
VARIABLE BIAS    .1550E+00
ERRORS                                                                              WEIGHTED ERROR SUM =   .1415E+02
   .2527E-04   .2781E-03   .4108E-04   .2633E-03   .5841E-03
WEIGHTED ERRORS
   .3436E+01   .4254E+00   .3697E+01   .3949E+00   .6191E+01

                                                  ITERATION NUMBER   3 COMPLETED
A                                   4   BY   4
             -.1199E+00  -.6511E-02  -.9980E+00   .2898E-01
              .3080E+01  -.2088E+00  -.7640E-01  0.
              .7187E+01  -.3702E-01  -.1298E+00  0.
             0.           .1000E+01   .7000E-01  0.                              ⑤
B                                   4   BY   5
             -.1099E-01   .8493E-02  0.          0.           -.4341E-02                        ⑥
              .2323E+02   .7559E+01  0.          0.           -.3973E-02
              .1478E+01  -.1945E+01  0.          0.            .3670E-01
             0.          0.          0.          0.           -.1352E-03
               AY
VARIABLE BIAS    .1484E+00
ERRORS                                                                              WEIGHTED ERROR SUM =   .1394E+02
   .2602E-04   .2955E-03   .3705E-04   .2516E-03   .5886E-03
WEIGHTED ERRORS
   .3538E+01   .4521E+00   .3334E+01   .3774E+00   .6239E+01

                                                  ITERATION NUMBER   4 COMPLETED
A                                   4   BY   4
             -.1193E+00  -.1169E-02  -.9980E+00   .2898E-01
              .3320E+01  -.2179E+00  -.2672E-01  0.
              .7164E+01  -.3810E-01  -.1323E+00  0.
             0.           .1000E+01   .7000E-01  0.                              ⑤
B                                   4   BY   5
             -.1125E-01   .8579E-02  0.          0.           -.4378E-02                        ⑥
              .2320E+02   .7596E+01  0.          0.           -.4920E-02
              .1528E+01  -.1934E+01  0.          0.            .3686E-01
             0.          0.          0.          0.           -.8461E-03
               AY
VARIABLE BIAS    .1496E+00
ERRORS                                                                              WEIGHTED ERROR SUM =   .1397E+02
   .2569E-04   .2834E-04   .3789E-04   .2407E-03   .5887E-03
WEIGHTED ERRORS
   .3494E+01   .4335E+00   .3410E+01   .3911E+00   .6241E+01

                                                  ITERATION NUMBER   5 COMPLETED
A                                   4   BY   4
             -.1196E+00  -.5060E-02  -.9980E+00   .2898E-01
              .3227E+01  -.2145E+00  -.3997E-01  0.
              .7178E+01  -.3738E-01  -.1308E+00  0.
             0.           .1000E+01   .7000E-01  0.                              ⑤
B                                   4   BY   5
             -.1109E-01   .8533E-02  0.          0.           -.4355E-02
```

```
           .2320E+02    .7579E+01   0.            0.            
           .1400E+01   -.1042E+01   0.            0.           -.4573E-02
          0.           0.           0.            0.            .3679E-01
               AY                   0.            0.           -.5420E-03
VARIABLE BIAS      .1488E+00
ERRORS
     .2597E-02   .2873E-03    .3705E-04    .2567E-03    .5897E-03              WEIGHTED ERROR SUM =    .1394E+02
WEIGHTED ERRORS
     .3532E+01   .4395E+00    .3334E+01    .3851E+00    .6251E+01

                                                                                                      ⑥
                                             ITERATION NUMBER   6 COMPLETED
CONFIDENCE LEVELS FOR NEXT TO LAST ITERATION
    (DIMENSIONAL)
AC
                                           3   BY   3
           .2065F-02    .6042E-02   0.
           .3806E+00    .8872E-01    .1319E+00
           .3689E-01    .9042E-02    .1025E-01
BC
                                           3   BY   5
           .1330E-02    .7849E-03   0.
           .5700E+00    .2363E+00   0.            0.            .3204E-03
           .8449E-01    .3091E-01   0.            0.            .3540E-02
                                                  0.            .1773E-02      ⑦
    (NON-DIMENSIONAL)
AC
                                           3   BY   3
           .1749E-03    .6042E-02   0.
           .1538E-04    .2345E-01    .3487E-01
           .1359E-04    .2179E-01    .2469E-01
BC
                                           3   BY   5
           .1126E-03    .6647E-04   0.            0.            
           .2344E-04    .9553E-05   0.            0.            .1555E-02
           .3113E-04    .1139E-04   0.            0.            .8199E-05
                                                                .3743E-04
```

```
                 X-24B  CASE 23-1  M=1.13  ALFA=4.0  POWER OFF   SAS ON                                   07/30/75

                                                                                                              ⑧
       FINAL VALUES      MACH = 1.130      ALPHA =    4.00      PARAM =     0.0000      CG = 0.000

DIMENSIONAL DERIVATIVES / SEC / SEC**2
           BETA              P               R             DC1             DC2            DC3             DC4            DELTA-O
  Y     -.110416         -.003390        -.998000*       -.011164         .008557      0.000000*       0.000000*        -.004366
  L     3.289173         -.217008        -.028801       23.194186        7.589643      0.000000*       0.000000*        -.004821
  N     7.171020         -.037716        -.131442        1.505863       -1.939369      0.000000*       0.000000*         .036825

NON-DIMENSIONAL DERIVATIVES / DEG (ROTARY /RAD)
           BETA              P               R             DC1             DC2            DC3             DC4            DELTA-O
 CY     -.010113         0.000000        0.000000*       -.000945         .000725      0.000000*       0.000000*        -.021183
 CL      .000133         -.057360        -.007613         .000937         .000307      0.000000*       0.000000*        -.000011
 CN      .062443         -.090475        -.316688         .000555        -.000715      0.000000*       0.000000*         .000777

                                         ( * ) INDICATES DERIVATIVE HELD FIXED DURING MATCH

VARIABLE BIAS     .1492E+00
FINAL DIMENSIONAL MATRICES
A                                      4   BY   4
             -.1104E+00  -.3390E-02  -.9980E+00   .2898E-01
              .3289E+01  -.2170E+00  -.2880E-01  0.
              .7171E+01  -.3772E-01  -.1314E+00  0.
             0.           .1000E+01   .7000E-01  0.                          ⑤
B                                      4   BY   5
             -.1116E-01   .8557E-02  0.          0.          -.4366E-02                    ⑥
              .2319E+02   .7590E+01  0.          0.          -.4821E-02
              .1506E+01  -.1939E+01  0.          0.           .3682E-01
             0.          0.          0.          0.          -.7519E-03

DEGREES            AY
VARIABLE BIAS     .1492E+00
                                                                           WEIGHTED ERROR SUM =   .1395E+02
ERRORS
     .2585E-04   .2846E-03   .3734E-04   .2594E-03   .5896E-03
WEIGHTED ERRORS
     .3514E+01   .4354E+00   .3362E+01   .3891E+00   .6249E+01

                                                      ERRORS
     74.39       16.88       14.15       13.94       13.97       13.94       13.95

                                                                                                              ⑨
```

Table B5

MMLE PROGRAM DESCRIPTION

NR - Executive routine for program. Calls subprograms in proper order.

AADD - Utility subroutine to add two matrices.

AEAT - Transition matrix subroutine. Uses transition matrix to develop computed time histories.

AMAKE - Utility subroutine to copy a one dimensional matrix to another.

AMULT - Utility subroutine to multiply two matrices.

APRPLT - A priori plotting subroutine. Plots derivatives versus a priori weighting.

ASPIT - Utility subroutine to write out a matrix.

AZOT - Utility subroutine to zero a matrix.

DERIV - Derivative printing routine. Prints dimensional and non-dimensional derivatives and performs non-dimensionalizing process.

LINES - Utility plotting routine to plot time histories.

PLTDAT - Utility plotting routine to plot date and time on Calcomp plots.

SCALES - Utility routine to determine scale factors for plotting routine.

EDIT - Program initialization routine. Reads card input and performs data initialization.

CORDR - Reordering routine for curve lookup. Reorders input arguments based on IORDER parameters.

CREAD - Read routine for curve lookup. Reads curve file for derivative lookup.

FIND - Time history repositioning routine. Repositions read pointer at the next reader record.

FOURD - Four dimensional interpolation routine. Performs four dimensional lookup of curve file.

INV - Matrix inversion routine. Performs inversion of a general matrix.

LATCON - Lateral-directional constants routine. Defines constants and performs dimensionalizing process.

LONCON - Longitudinal constants routine. Defines constants and performs dimensionalizing process.

Table B5 (Concluded)

LOOKUP - Curve lookup executive routine. Calls FOURD to look up derivatives and performs dimensionalizing process.

MAK - Utility subroutine to copy a two dimensional matrix to another.

MATLD - Matrix read routine. Reads matrices from data cards.

TABENT - Three dimensional lookup routine. Used in conjunction with FOURD to look up derivatives.

DATA - Time history input routine. Reads time histories and makes corrections to them.

AGIRL - Main iterative core routine. Iterates to obtain best match derivatives.

CRAMER - Uncertainty level routine. Computes Cramer-Rao bounds and uncertainty levels.

DIAGIN - Finds diagonal elements of inverted matrix.

DMULT - Performs D1 weighting matrix multiplication.

REDUCE - Symmetric matrix reduction routine.

SOLVE -

SUMULT -

OUTPUT - Output routine. Performs printing of output time histories and other parameters.

PLOP - Punch output routine. Punches dimensional and non-dimensional A and B matrices.

THPLOT - Time history plot routine. Plots measured and computed time histories.

PLOTA - Printer plot routine. Generates plots on line printer.

APPENDIX C
CONTROL

Table C1

CONTROL DATA REQUIREMENTS

1.	Title card, case number	Required always	Format (9A8, I8)
2.	Code namelist	Required unless IFLAG=1 for previous case	Format (Namelist)
3.	Output, input labels	Required unless IFLAG=1 on previous case	Format (10A8)
4.	Derivative data cards	Required for READ=5	Format (5I5,5X,5F10.2) (First card) Format (8F10.2) (Second card) Format (7F10.6)
5.	System matrices	Required for READ=1 always and for READ=5,6 unless special inputs are used	Format (A2,I8,I10) (First card) Format (8F10.4)
6.	NBLOCK, NIT card	Required if READ=4 or MIXED=1	Format (2I5)
7.	Block description cards	Required if READ=4 or if (MIXED=1 and NIT=1)	Format (I2,I3,5I5,F10.4)
8.	Block description cards matrices GRAPH, BLOCK, NUMER, DEMON, GAIN	Required if READ=4 or if (MIXED=1 and NIT=0)	Format (515)(Integer) Format (8F10.4)(Real)
9.	ITHINY	Required if READ=4 or MIXED=1	Format (16I5)
10.	ITHINU	Required if MIXED=1 or if (READ=4 and SYSTEM=3)	Format (16I5)
11.	NYTOV, NZTOU, NYZTOK	Required if MIXED=1 or if (READ=4 and SYSTEM=3)	Format (3I5)
12.	YTOV, ZTOU, NYZTOK	Required if MIXED=1 or if (READ=4 and SYSTEM=3)	Format (2I5)

Table C2

CODE NAMELIST

The condition codes and input data are contained in the namelist code and are listed below. All of the codes and data are initialized to zero at the start of each case unless the SAV option is set

Condition Codes (Integer Variables)

READ,SYSTEM,OUTPUT,MIXED,DIGITL,FRPS,NUMERS,TRESP,NX,NY,NU,NXC,NUC, ZOH,N1,N2,CONTUR,MULTRT,MODEL,NSCALE,CMAT,NK2.FORM,IPT,IGO,SAV, IFLAG,READ3,MILSPEC

Input Data (Real Variables)

DELT,FINALT,IFREQ,FFREQ,DELFRQ,M,GAIN1,GAIN2

Condition Code Description (Integer Variables)

READ	1	Data matrices input through LOAD subroutine
	2	Data matrices constructed in user written MATRIX subroutine
	3	Data from previous case altered in user written CHANGE subroutine
	4	Data matrices constructed from block diagram information in CLASS subroutine
	5	Data matrices constructed in NRREAD from input cards
	6	Data matrices constructed in NRREAD from input file
SYSTEM	1	Open loop system analysis
	2	Closed loop system analysis
	3	Root locus analysis
OUTPUT	1	$\hat{y} = \hat{H}\hat{x}$
	2	$\hat{y} = \hat{H}\hat{x} + \hat{G}\hat{\dot{x}}$
	3	$\hat{y} = \hat{H}\hat{x} + \hat{F}\hat{u}$
	4	$\hat{y} = \hat{H}\hat{x} + \hat{G}\hat{\dot{x}} + \hat{F}\hat{u}$
MIXED	0	No action
	1	Mixed system analysis. The MIXED option allows the merger of the aircraft matrices and the flight control system description.
DIGITL	0	Continuous system analysis
	1	Sampled-data system analysis
	2	Discrete system analysis
		If DIGITL ≠ 0, DELT specifies the sample period of the discrete or sampled-data system.
FRPS	0	Not applicable
	1	Frequency response calculated for each transfer function s-Plane If DIGITL = 0 w-Plane If DIGITL = 1,2 (DELT required)
	-1	s-Plane frequency responses calculated from Z-transfer functions with DIGITL = 1,2 (DELT required)
	2	s-Plane power spectra calculated (DIGITL=0)

Table C2 (Continued)

NUMERS	0	Numerator zeroes of s- or z-transfer functions calculated
	1	Numerator zeroes not calculated
		CONTROL will compute transfer function numerator zeroes for all input-output pairs defined by the input and output vectors. For MIXED system analysis, the ITHINU and ITHINY options allow unwanted transfer functions to be eliminated.
TRESP	0	No action
	N	N transient responses calculated. DELT specifies integration step size.
NX,NY,NU		Dimensions of $\hat{x}$, $\hat{y}$, and $\hat{u}$ vectors. If MIXED = 1,NX,NY, and NU specify dimensions of the open loop plant (aircraft). States added in the MIXED option automatically increment NX, NY, and NU.
NXC,NUC		Dimensions of state and input vectors corresponding to the continuous subsystem (plant) of a sampled-data system. The plant must be partitioned in the upper left position of the system matrices (A,B,H,F,etc.) $NXC \leq NX$, $NUC \leq NU$
ZOH		For sampled-data systems, the number of inputs to the plant which are outputs of zero-order-hold devices. These must be the first ZOH components of the input vector, $\hat{u}$.
N1, N2		The root locus option allows two feedback gains to be specified. N1 is the number of iterations of the first variable $(\hat{K}1,\hat{K}2)$ and N2 is the number of iterations of the second variable $(\hat{K}3,\hat{K}4)$. (Commonly, N2 = 0). If N1 > 0, gain increments are arithmetic (0,1,2,3,..) If N1 < 0, gain increments are geometric (0,1,2,4,8,.)
CONTUR	0	Not applicable
	1	Root contour option for parameter variation studies CONTROL determines only system eigenvalues and returns to top of program for next variation. Continues until CONTUR set to zero. (Used with IFLAG,READ3,SAV, and CHANGE)
MULTRT		For sampled-data systems, computes MULTRT transient response points for each sample period so that intersample response may be investigated. Only transient responses are calculated if MULTRT is set.
MODEL	0	Not applicable
	1	Model following on consecutive frequency responses
NSCALE	0	Not applicable
	1	State vector transformed to improve numerical conditioning in determination of eigenvalues. $\hat{A}$ matrix scaled by a diagonal similarity transformation.

Table C2 (Continued)

CMAT	0	$\hat{C}$ Matrix is the identity matrix ($\hat{C}$ not required)	
	1	$\hat{C} \neq$ Identify matrix ($\hat{C}$ required usually required if $I_{xz} \neq 0$)	
NK2	0	$\hat{K}2 = 0$, $\hat{K}4 = 0$	($\hat{K}2$,$\hat{K}4$ not required)
	1	$\hat{K}2 \neq 0$ or $\hat{K}4 \neq 0$	($\hat{K}2$,$\hat{K}4$ required)
FORM	0	Print only for output	
	1	Print and plot output	Not currently operational
	2	Plot only for output	
IPT		Code for extra printout for debugging	
	0	No extra printing	
	1,2	Extra printing	
IGO		Code for data required by CLASS subroutine	
	0	Input data required by CLASS	
	1	CLASS uses data from previous case plus new gains	
SAV	0	Data matrices not saved	
	1	Data matrices saved for subsequent cases. If MIXED = 1, CONTROL saves matrices defined for plant equations. (Class input data, flight control system description, is not destroyed and is available for subsequent cases).	
IFLAG	0	On subsequent case the condition codes and input data are zeroed before the call to CARD. CARD reads title namelist, output labels, and input label cards.	
	1	On subsequent cases, the condition codes and input data of the present case will be used. CARD reads only a title card for all subsequent cases. (The option may be cancelled by setting IFLAG = 0 or by end of data deck).	
READ3	0	No action	
	1	On subsequent cases, READ defaults to 3 to force program to the CHANGE subroutine. The option is used with IFLAG for parameter variation studies.	
MILSPEC	0	No action	
	1	Computes milspec parameters for lateral-directional case.	
	-1	Compute milspec parameters for longitudinal case.	
ISUBNAM	0	No action	
	2	Subroutine names will be printed to allow the program's path to be traced.	
DELT		Time increment for transient responses and/or sample period for sampled-data systems, seconds	
FINALT		Final time for transient responses, seconds	

Table C2 (Concluded)

IFREQ,FFREQ,DELFRQ	Initial, final, and incremental frequencies for frequency responses or power spectra. DELFRQ = 1.1 is good for most applications. Frequencies must be specified in (DELFRQ cannot equal 1.0) Radians/sec (s-plane) even for discrete and sampled-data systems. If (IFREQ = 0., program defaults to an internal set of frequency points spaced between .1 and 150. rad/sec. for sampled-data frequency responses CONTROL defaults in the following manner, If DIGITL≠0 and FRPS ≠-1 and IFREQ=0 IFREQ = TAN (.1*DELT*.5) FFREQ = TAN (.9*3.14*.5) If DIGITL≠0 and FRPS ≠-1 and IFREQ≠0 IFREQ = TAN (IFREQ*DELT*.5) FFREQ = TAN (FFREQ*DELT*.5)
M	Code for modified Z-transfer function computation for sampled-data systems. M is the fractional sample period delay and is in the range $0. \le M \le 1$. M = 1.gives the standard Z-transform if the signal has no jump discontinuity at the sample instant. M = 0. gives the Z-transform with a one sample period delay. However, numerical errors limit M to $M \ge .2$. Therefore, if M=0., the program defaults to standard Z-transform analysis. Only open loop calculations (modified Z-transfer functions and frequency responses) may be performed with this option.
GAIN1,GAIN2	Root locus gain increments for the two feedback gain variables allowed with the root locus option. If not set, program defaults to GAIN1 = 1.0, GAIN2 = 1.0.

Table C3

CONTROL SYSTEMS MODELS AND MATRIX REQUIREMENTS

SYSTEM MODELS (Continuous System)

Open Loop

$$\hat{C}\dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}\hat{u}$$
$$\hat{y} = \hat{H}\hat{x} + \hat{G}\dot{\hat{x}} + \hat{F}\hat{u}$$

Closed Loop

$$\hat{C}\dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}\hat{u}$$
$$\hat{u} = \hat{K}1\hat{x} + \hat{K}2\dot{\hat{x}} + \hat{D}\hat{u}_{com}$$
$$\hat{y} = \hat{H}\hat{x} + \hat{G}\dot{\hat{x}} + \hat{F}\hat{u}$$

Root Locus

$$\hat{C}\dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}\hat{u}$$
$$\hat{u} = (\hat{K}1\hat{x} + \hat{K}2\dot{\hat{x}}) + (\hat{K}3\hat{x} + \hat{K}4\dot{\hat{x}})$$

MATRIX REQUIREMENTS

System	Description	Required Matrices
1	Open Loop	$\hat{A}$, $\hat{B}$, $\hat{C}$, $\hat{H}$, $\hat{G}$, $\hat{F}$
2	Closed Loop	$\hat{A}$, $\hat{B}$, $\hat{C}$, $\hat{H}$, $\hat{G}$, $\hat{F}$, $\hat{K}1$, $\hat{K}2$, $\hat{D}$
3	Root Locus	$\hat{A}$, $\hat{B}$, $\hat{C}$, $\hat{K}1$, $\hat{K}2$, $\hat{K}3$, $\hat{K}4$

If CMAT = 0, eliminate $\hat{C}$
If NK2 = 0, eliminate $\hat{K}2$, $\hat{K}4$
If OUTPUT = 1, eliminate $\hat{G}$, $\hat{F}$
If OUTPUT = 2, eliminate $\hat{F}$
If OUTPUT = 3, eliminate $\hat{G}$
If MIXED = 1, eliminate $\hat{K}1$, $\hat{K}2$, $\hat{K}3$, $\hat{K}4$, $\hat{D}$
If NRREAD = 5, 6, eliminate $\hat{A}$, $\hat{B}$, $\hat{C}$
If N2 = 0, eliminate $\hat{K}1$, $\hat{K}2$

GENERAL PURPOSE PROGRAMMING DATA

PAGE 1 OF 5 PAGES

PROGRAM NAME: Control Data Cards

DECK NR: 8230

LINE ITEM:

RETURN TO (Name/Organization/Phone):

DATE:

```
cc  5    10   15   20   25   30   35   40   45   50   55   60   65   70 72|73     80
                        T I T L E                                          NCASE
                      CODE NAMELIST
                      OUTPUT LABELS
                      INPUT LABELS
       DERIVATIVE DATA CARDS AND/OR MATRICES  (IF REQUIRED)
                FLIGHT CONTROL SYSTEM  (IF REQUIRED)


                        SAMPLE DECK

  X-24B CASE 23-51  M=1.49  ALFA=5.0  POWER ON  SAS ON                          51
 $CODE READ=6,SYSTEM=2,OUTPUT=4,MIXED=1,NX=3,NY=4,NU=1,MILSPEC=-1,$END
      Q   THETA         NZ
ELEVATOR
N2          -4.27
    4    1
 1  4    2    0    0    0    0  1.0       20.0      0.6
 2  8    3    4    0    0    0  0.5       40.0      0.6
 3  6    0    0    0    1    0  1.0       3.0
 4  1    0    0    0    2    0  0.56
    1    3    4            (ITHINY)
    1                      (ITHINU)
```

AFFTC FORM SEP 67 30 PREVIOUS EDITION OF THIS FORM WILL BE USED UNTIL STOCK IS EXHAUSTED

GENERAL PURPOSE PROGRAMMING DATA

PAGE 2 OF 5 PAGES

PROGRAM NAME: CONTROL DATA CARDS

DECK NR: 8230

LINE ITEM:

RETURN TO (Name/Organization/Phone):

DATE:

```
CC       5    10   15   20   25   30   35   40   45   50   55   60   65   70 72 73      80
         2    1    0       (NYTOV, N2TOU, NYZTOK)
         1    1                (YTOV)
         4    2                (YTOV)
         1    1                (2TOU)
 Y-24B CASE 23-1  M=1.13  ALFA=4.0  POWER OFF   SAS ON                                     1
$CODE READ=5,SYSTEM=2,OUTPUT=3,MIXED=1,NX=4,NY=5,NU=2,CMAT=1,MILSPEC=1,$END
         P         R      BETA       PHI        NY
DA        DR
         1   10   18   25  500       330.5     19.1      37.5      4.0        0.0
 8567.     2646.     23704.    24119.    620.      1.13      1.81             1090.
 -.579     0.0       0.0       -.054     .0415     0.0       0.0
 .0076     -.0574    -.0076    .0537     .0176     0.0       0.0
 .1514     -.0909    -.3167    .0318     -.0410    0.0       0.0
H         5         4
 1.0       0.0       0.0       0.0
 0.0       1.0       0.0       0.0
 0.0       0.0       1.0       0.0
 0.0       0.0       0.0       1.0
 0.0       0.0       -4.04     0.0
F         5         2
 0.0       0.0
 0.0       0.0
```

AFFTC FORM 30 SEP 67 PREVIOUS EDITION OF THIS FORM WILL BE USED UNTIL STOCK IS EXHAUSTED

GENERAL PURPOSE PROGRAMMING DATA

PAGE 3 OF 5 PAGES

DATE

PROGRAM NAME: CONTROL DATA CARDS

DECK NR: 8230

LINE ITEM:

RETURN TO (Name/Organization/Phone):

0.0	0.0				
0.0	0.0				
-.377	.290				
1.0	0				
1	0	0	0	1	(GRAPH)
2	-1	0	0	4	
3	2	0	0	0	
4	2	0	0	0	
5	7	1.0	0	5	
6	-4	5	0	0	
7	4	0	0	0	
8	-9	0	0	2	
9	0	0	0	1	
1.0	0	0	0	3	
1	3	3			(BLOCK)
2	1	1			
3	1	2			
4	1	1			
5	1	1			
6	1	2			
7	2	2			
8	2	2			

AFFTC FORM 30 SEP 67 PREVIOUS EDITION OF THIS FORM WILL BE USED UNTIL STOCK IS EXHAUSTED

GENERAL PURPOSE PROGRAMMING DATA

PROGRAM NAME: CONTROL DATA CARDS

DECK NR: 8230

LINE ITEM:

RETURN TO (Name/Organization/Phone):

PAGE 4 OF 5 PAGES

DATE:

CC 5 10 15 20 25 30 35 40 45 50 55 60 65 70 72 73 80

9	1	1	(BLOCK)		
10	1	1			
6400.0	64.0	4.0	0.0	0.0	(NUMER)
1.0	0.0	0.0	0.0	0.0	
20.0	0.0	0.0	0.0	0.0	
1.0	0.0	0.0	0.0	0.0	
1.0	0.0	0.0	0.0	0.0	
20.0	0.0	0.0	0.0	0.0	
15.0	3.0	0.0	0.0	0.0	
0.0	1.5	0.2	0.0	0.0	
1.0	0.0	0.0	0.0	0.0	
1.0	0.0	0.0	0.0	0.0	
6400.0	80.0	1.0	0.0	0.0	(DENOM)
1.0	0.0	0.0	0.0	0.0	
20.0	1.0	0.0	0.0	0.0	
1.0	0.0	0.0	0.0	0.0	
1.0	0.0	0.0	0.0	0.0	
20.0	1.0	0.0	0.0	0.0	
15.0	1.0	0.0	0.0	0.0	
1.0	1.0	0.0	0.0	0.0	
1.0	0.0	0.0	0.0	0.0	
1.0	0.0	0.0	0.0	0.0	

AFFTC FORM SEP 67 30 PREVIOUS EDITION OF THIS FORM WILL BE USED UNTIL STOCK IS EXHAUSTED

GENERAL PURPOSE PROGRAMMING DATA

PAGE 5 OF 5 PAGES

DATE

PROGRAM NAME: CONTROL DATA CARDS

DECK NR: 8230

LINE ITEM

RETURN TO (Name/Organization/Phone)

```
CC   5    10   15   20   25   30   35   40   45   50   55   60   65   70 72 73      80
    1.0       0.3       1.0       0.2       0.5       1.0       1.0         1.0
    .07       0.6                  ←(GAIN)
       1    2    3    4    5       (ITHINY)
       1    2                      (ITHINU)
       3    2    0          (NYTOV, NZTOU, NYZTOK)
       1    1                      (YTOV)
       2    2                        ↓
       5    3                        ↓
       3    1                      (ZTOU)
       6    2                        ↓
```

AFFTC FORM 30 SEP 67 PREVIOUS EDITION OF THIS FORM WILL BE USED UNTIL STOCK IS EXHAUSTED

```
EN323,CM120000,CH9983008230P,T177.
ATTACH,DERIVS,X24B,ID=CNAGY,CY=5.
ATTACH,CONTROL,AMC,ID=CNAGY,CY=3,MR=1.
CONTROL.
0000000000000000000000
   X-24B  CASE 23-51  M=1.49  ALFA=5.0  POWER ON   SAS ON                             51
 $CODE READ=6,SYSTEM=2,OUTPUT=4,MIXED=1,NX=3,NY=4,NU=1,MILSPEC=-1,$END
       Q    THETA        NZ
ELEVATOR
NZ               -4.27
    4     1
 1  4     2     0     0     0     0  1.0        20.0
 2  8     3     4     0     6     0  0.5        40.0       0.6
 3  6     0     0     0     1     0  1.0        3.0
 4  1     0     0     0     2     0  0.56
    1     3     4
    1
    2     1     0
    1     1
    4     2
    1     1
  X-24B  CASE 23-1  M=1.13  ALFA=4.0  POWER OFF   SAS ON                              1
 $CODE READ=5,SYSTEM=2,OUTPUT=3,MIXED=1,NX=4,NY=5,NU=2,CMAT=1,MILSPEC=1,$END
      P         R     BETA       PHI        NY
 DA        DR
    1   10    18   25  500        330.5      19.1       37.5       4.0        0.0
  8567.      2646.     23704.     24119.     620.       1.13       181.       1090.
  -.579      0.0       0.0        -.054      .0415      0.0        0.0
  .0076      -.0574    -.0076      .0537     .0176      0.0        0.0
  .1514      -.0909    -.3167     .0318      -.0410     0.0        0.0
H         5          4
  1.0        0.0        0.0        0.0
  0.0        1.0        0.0        0.0
  0.0        0.0        1.0        0.0
  0.0        0.0        0.0        1.0
  0.0        0.0        -4.04      0.0
F         5          2
  0.0        0.0
  0.0        0.0
  0.0        0.0
  0.0        0.0
  -.377      .290
   10    0
    1    0    0    0    1
    2   -1    0    0    4
    3    2    0    0    0
    4    2    0    0    0
    5    7   10    0    5
    6   -4    5    0    0
    7    8    0    0    0
    8   -9    0    0    2
    9    0    0    0    1
   10    0    0    0    3
```

(CONTINUED NEXT PAGE)

```
        1    3    3
        2    1    1
        3    1    2
        4    1    1
        5    1    1
        6    1    2
        7    2    2
        8    2    2
        9    1    1
       10    1    1
     6400.0     64.0      4.0       0.0       0.0
     1.0        0.0       0.0       0.0       0.0
     20.0       0.0       0.0       0.0       0.0
     1.0        0.0       0.0       0.0       0.0
     1.0        0.0       0.0       0.0       0.0
     20.0       0.0       0.0       0.0       0.0
     15.0       3.0       0.0       0.0       0.0
     0.0        1.5       0.0       0.0       0.0
     1.0        0.0       0.0       0.0       0.0
     1.0        0.0       0.0       0.0       0.0
     6400.0     80.0      1.0       0.0       0.0
     1.0        0.0       0.0       0.0       0.0
     20.0       1.0       0.0       0.0       0.0
     1.0        0.0       0.0       0.0       0.0
     1.0        0.0       0.0       0.0       0.0
     20.0        1.0      0.0       0.0       0.0
     15.0        1.0      0.0       0.0       0.0
     1.0         1.0      0.0       0.0       0.0
     1.0        0.0       0.0       0.0       0.0
     1.0        0.0       0.0       0.0       0.0
     1.0        0.3       1.0       0.2       0.5
     .070       0.6                                     1.0       1.0       1.0
       1    2    3    4    5
       1    2
       3    2    0
       1    1
       2    2
       5    3
       3    1
       6    2
0000000000000000000000
0000000000000000000000
```

Table C-4

MILSPEC PARAMETERS

PARAMETER	EQUATION
Actual Frequency (ω_d)	ω_d = (Imaginary Part)
Natural Frequency (ω_n)	$\omega_n = \sqrt{(\text{Real Part})^2 + (\text{Imaginary Part})^2}$
Damping Ratio (ζ)	$\zeta = -\frac{(\text{Real Part})}{\omega_n}$
Time Constant (T)	$T = -\frac{1.0}{(\text{Real Part})}$
Time to Half ($T_{1/2}$)	$T_{1/2} = -.69315T$
Cycles to Half ($C_{1/2}$)	$C_{1/2} = -\frac{(\text{Imaginary Part})}{9.06\ (\text{Real Part})}$
Period (T)	$T = \frac{6.28318}{\omega_d}$
$N_{z/\alpha}$	$N_{z/\alpha} = \frac{\bar{q}s}{w}\left(C_{n_\alpha} - C_{n_{\delta e_1}} \frac{C_{m_\alpha}}{C_{m_{\delta e_1}}}\right)$
$\delta e/g$	$\delta e/g = \frac{57.3\ C_{m_\alpha}}{(N_{z/\alpha})\ (C_{m_{\delta e_1}})}$
Static Margin	$S.M. = \frac{100.\ C_{m_\alpha}}{C_{n_\alpha}}$
Maneuver Point Increment	$M.P.I. = S.M. - \frac{1608.7\ \bar{q}\ sc}{WV^2} C_{m_Q}$
($\delta r/\beta$) steady state sideslip	$\delta r/\beta = \frac{(C_{\ell_{\delta c_1}})\ (C_{n_\beta}) - (C_{\ell_\beta})\ (C_{n_{\delta c_1}})}{(C_{\ell_{\delta c_2}})\ (C_{n_{\delta c_1}}) - (C_{\ell_{\delta c_1}})\ (C_{n_{\delta c_2}})}$
($\delta a/\beta$) steady state sideslip	$\delta a/\beta = \frac{(C_{n_{\delta c_2}})\ (C_{\ell_\beta}) - (C_{n_\beta})\ (C_{\ell_{\delta c_2}})}{(C_{\ell_{\delta c_2}})\ (C_{n_{\delta c_1}}) - (C_{\ell_{\delta c_1}})\ (C_{n_{\delta c_2}})}$
(ϕ/β) steady state sideslip	$\phi/\beta = \sin^{-1}\left[\left(\frac{\bar{q}s}{W\cos\theta} C_{y_\beta} + C_{y_{\delta c_1}} \cdot \delta a/\beta + C_{y_{\delta c_2}} \cdot \delta r/\beta\right)\right]$
$C^*_{n_\beta}$ (Dynamic C_{n_β})	$C^*_{n_\beta} = C_{n_\beta}(\cos\alpha) - \left(\frac{I_z}{I_x}\right) C_{\ell_\beta}(\sin\alpha) + \frac{I_{xz}}{I_x}(C_{\ell_\beta} \cdot \cos\alpha - C_{n_\beta} \cdot \sin\alpha)$

```
                                                                      51
X-248  CASE 23-51  M=1.49  ALFA=5.0  POWER ON   SAS ON
       CONTINUOUS SYSTEM
       MIXED OPTION
       CLOSED LOOP
       NRREAD ROUTINE INPUT
       TRANSFER FUNCTIONS

                                                                     DELT   =  0.000
       NX  =   3        READ    =   5      TRESP  =   0      CMAT   =  0      FINALT =  0.000
       NY  =   4        SYSTEM  =   2      FRPS   =   0      NK2    =  0      IFREQ  =  0.000
       NU  =   1        MIXED   =   1      NUMERS =   0      IFLAG  =  0      DELFRQ =  0.000
       NXC =   0        OUTPUT  =   4      FORM   =   0      IGO    =  0      FFREQ  =  0.000
       NUC =   0        DIGITL  =   0      CONTUR =   0      HEAD3  =  0      GAIN1  =  0.000
       ZOH =   0        IPT     =   0      MULTRT =   0      SAV    =  0      GAIN2  =  0.000
       N1  =   0        KOUNT   =   1      MODEL  =   0      NSCALE =  0      M      =  0.000
       N2  =   0        MILSPEC=  -1

        DATA  FOR  CASE  NUMBER   51          START  TIME  0: 0: 0.  0
  MACH= 1.49      DYN. PRESS.= 145.00    VELOCITY=1450.00      ALPHA=  5.00       GAMMA=  0.00
WEIGHT=   8567.0            IXX=   2646.0         IYY=   23704.0     IZZ=   24119.0     IXZ=   620.0
CMA=  -.001861   CMDE1=  -.000736   CMDE2=  0.000000   CMDE3=  0.000000   CMDE4=  0.000000   CMO=  -.2890   CMV=  0.0000
CNA=   .022996   CNDE1=   .002531   CNDE2=  0.000000   CNDE3=  0.000000   CNDE4=  0.000000   CNO= -0.0000   CNV= -0.0000
CCA= 0.000000    CCDE1=  0.000000   CCDE2=  0.000000   CCDE3=  0.000000   CCDE4=  0.000000   CCO=  0.0000   CCV=  0.0000

           THE A MATRIX IS

         3          3

 -.2834E+00  -.4084E+01  0.
  .1000E+01  -.1635E+00 -0.
  .9962E+00  0.          0.

           THE B MATRIX IS

         3          1

 -.3195E+01
 -.1800E-01
 0.

           THE H MATRIX IS

         4          3

  .1000E+01  0.          0.
 0.           .1000E+01  0.
 0.          0.           .1000E+01
 0.           .7371E+01  0.

           THE G MATRIX IS

         4          3

 0.          0.          0.
 0.          0.          0.
 0.          0.          0.
 -.1327E+00  0.          0.
```

```
        THE F MATRIX IS
        4         1
  0.
  0.
  0.
   .8113E+00

        THE K1 MATRIX IS
        1         3
  0.         0.          0.

        THE D MATRIX IS
        1         1
  0.

BLOCK DIAGRAM INPUT PARAMETERS

NO.  TYPE      CONNEC     MOD                    PARAM

  1    4    2    0    0    0    0    1.0000   20.0000   -0.0000   -0.0000   -0.0000
  2    8    3    4    0    0    0     .5000   40.0000     .6000   -0.0000   -0.0000
  3    6    0    0    0    1    0    1.0000    3.0000   -0.0000   -0.0000   -0.0000
  4    1    0    0    6    2    0     .5600   -0.0000   -0.0000   -0.0000   -0.0000

        ITHINY

  1    3    4   -0   -0   -0   -0   -0

        ITHINU

  1   -0   -0

        YTOV

  1    1
  4    2

        ZTOU

  1    1
```

```
          THE FINAL REDUCED SYSTEM IS


         THE A MATRIX IS

         7          7

  -.2834E+00  -.8084E+01  0.
   .1000E+01  -.1635E+00  0.           -.6390E+02  0.
   .9962E+00  0.          0.           -.3600E+00  0.          0.           0.
  0.          0.          0.           0.          0.          0.           0.
  0.          0.          0.           -.2000E+02  0.          0.           0.
   .1021E+01   .4728E+01  0.           0.           .8000E+03  0.           0.
   .1000E+01  0.          0.             .1384E+02 0.           .1000E+01  0.
                          0.           0.          -.1600E+04  -.4800E+02  -.3000E+01
                                                   0.          0.          -.3000E+01

        THE B MATRIX IS

        7          1

  -.3195E+01
  -.1800E-01
  0.
  0.
  0.
   .6910E+00
  0.


        THE H MATRIX IS

       3          7

   .1000E+01  0.
  0.          0.          0.           0.          0.          0.          0.
   .3761E-01   .8444E+01  0.  .1000E+01  0.        0.          0.          0.
                                        .2471E+02  0.          0.          0.
                                                               0.          0.
        THE F MATRIX IS

       3          1

  0.
  0.
   .1235E+01


       THE EIGEN VALUES OF THE SYSTEM ARE
                 REAL PART
                                           IMAGINARY PART
 ROOT( 1)
 ROOT( 2)       0.
 ROOT( 3)       -.29056770E+02           0.
 ROOT( 4)       -.29056770E+02           -.33390308E+02
 ROOT( 5)       -.60569784E+01            .33390308E+02
 ROOT( 6)       -.60569784E+01           -.14266852E+01
 ROOT( 7)       -.60970570E+00            .14266852E+01
                -.60970570E+00           -.39921885E+01
                                          .39921885E+01
```

```
                                                                CYCLES TO HALF =     .127               TIME CONSTANT =    .0344
ROOT( 3)     TIME TO HALF =    .024 SEC                         NATURAL FREQ.  =   44.263 RAD/SEC            ZETA     =    .6565
             ACTUAL FREQ. =  33.390 RAD/SEC                                    =    7.045 CYC/SEC            PERIOD   =    .1882
                          =   5.314 CYC/SEC
                                                                CYCLES TO HALF =     .026               TIME CONSTANT =    .1651
ROOT( 5)     TIME TO HALF =    .114 SEC                         NATURAL FREQ.  =    6.223 RAD/SEC            ZETA     =    .9734
             ACTUAL FREQ. =   1.427 RAD/SEC                                    =     .990 CYC/SEC            PERIOD   =   4.4040
                          =    .227 CYC/SEC
                                                                CYCLES TO HALF =     .723               TIME CONSTANT =   1.6401
ROOT( 7)     TIME TO HALF =   1.137 SEC                         NATURAL FREQ.  =    4.038 RAD/SEC            ZETA     =    .1510
             ACTUAL FREQ. =   3.992 RAD/SEC                                    =     .643 CYC/SEC            PERIOD   =   1.5739
                          =    .635 CYC/SEC

FOR THIS CONFIGURATION, THE FOLLOWING INFORMATION IS APPLICARLE
1. NZ/ALPHA =    5.32 G"S PER RADIAN
2. ELEVATOR/G =  27.26 DEGREES PFR G
3. NEUTRAL POINT = X-CG +   8.09 (IN PER CENT MAC)
4. MANUEVER POINT = X-CG +   8.14 (IN PER CENT MAC)

        THE COEFFICIENTS OF THE CHARACTERISTIC FOUATION ORDERED FROM THE LOWEST POWER OF S

       0.
        .12373106E+07
        .51629332E+06
        .15161699E+06
        .30424177E+05
        .28038614E+04
        .71446909E+02
        .10000000E+01

        THE          Q /ELEVATOR NUMERATOR GAIN IS  -.3195F+01

  THE ZEROES OF THE TRANSFER FUNCTION ARE

                          REAL PART                    IMAGINARY PART

                                                     0.
   ROOT( 1)              0.                          0.
   ROOT( 2)             -.20000000E+02              -.32000000F+02
   ROOT( 3)             -.24000000E+02               .32000000E+02
   ROOT( 4)             -.24600000F+02              0.
   ROOT( 5)             -.11600241E+00              0.
   ROOT( 6)             -.30000000F+01

        THE CUEFFICIENTS OF THE NUMERATOR  POLYNOMIAL ORDERED FROM THE LOWEST POWFR OF S

       0.
        .11329231F+05
        .10068234E+06
        .40006159E+05
        .27723782F+04
        .71118002E+02
        .10000000E+01

        THE    THETA /ELEVATOR NUMERATOR GAIN IS  -.3183F+01

   THE ZEROES OF THE TRANSFER FUNCTION ARE

                          REAL PART                    IMAGINARY PART
```

```
ROOT( 1)          -.24000000E+02      -.32000000E+02
ROOT( 2)          -.24000000E+02       .32000000E+02
ROOT( 3)          -.20000000E+02      0.
ROOT( 4)          -.11000241E+00      0.
ROOT( 5)          -.30000000E+01      0.

      THE COEFFICIENTS OF THE NUMERATOR  POLYNOMIAL OHDERED FROM THE LOWEST POWER OF S

     .11320231E+05
     .10060234E+06
     .40006159E+05
     .27723702E+04
     .71110002E+02
     .10000000E+01

      THE       NZ /ELEVATOR NUMERATOR GAIN IS   .1235E+01

THE ZEROES OF THE TRANSFER FUNCTION ARE

                  REAL PART                IMAGINARY PART

ROOT( 1)          0.                  0.
ROOT( 2)          -.24000000E+02      -.32000000E+02
ROOT( 3)          -.24000000E+02       .32000000E+02
ROOT( 4)          -.20000000E+02      0.
ROOT( 5)           .35972694E+01      0.
ROOT( 6)          -.30000000E+01      0.
ROOT( 7)          -.38230666E+01      0.

      THE COEFFICIENTS OF THE NUMERATOR  POLYNOMIAL ORDERED FROM THE LOWEST POWER OF S

     0.
     -.13205259E+07
     -.52406405E+06
      .66971235E+05
      .39329676E+05
      .27663329E+04
      .71226597E+02
      .10000000E+01
```

```
 X-24B  CASE 23-1  M=1.13  ALFA=4.0  POWER OFF    SAS ON                        1
        CONTINUOUS SYSTEM
        MIXED OPTION
        CLOSED LOOP
        NRREAD ROUTINE INPUT
        TRANSFER FUNCTIONS

        NX  =  4            READ    =   5        TRESP  =   0        CMAT   =   1        DELT   =  0.000
        NY  =  5            SYSTEM  =   2        FRPS   =   0        NK2    =   0        FINALT =  0.000
        NU  =  2            MIXED   =   1        NUMERS =   0        IFLAG  =   0        IFREQ  =  0.000
        NXC =  0            OUTPUT  =   3        FORM   =   0        IGO    =   0        DELFRQ =  0.000
        NUC =  0            DIGITL  =   0        CONTUR =   0        READ3  =   0        FFREQ  =  0.000
        ZOH =  0            IPT     =   0        MULTRT =   0        SAV    =   0        GAIN1  =  0.000
        N1  =  0            KOUNT   =   2        MODEL  =   0        NSCALE =   0        GAIN2  =  0.000
        N2  =  0            MILSPEC=    1                                               M      =  0.000
         DATA  FOR  CASE  NUMBER    1            START  TIME 10:18:25.500
  MACH= 1.13       DYN. PRESS.= 181.00    VELOCITY=1090.00     ALPHA=  4.00       GAMMA=  0.00
WEIGHT=   8567.6          IXX=   2646.0         IYY=   23704.0    IZZ=   24119.0    IXZ=    620.0
CLB=   .000133   CLDC1=   .000937   CLDC2=   .000307   CLDC3=  0.000000   CLDC4=  0.000000   CLP=  -.0574   CLR=  -.0074
CNB=   .002642   CNDC1=   .000555   CNDC2=  -.000716   CNDC3=  0.000000   CNDC4=  0.000000   CNP=  -.0909   CNR=  -.3167
CYB=  -.010105   CYDC1=  -.000942   CYDC2=   .000724   CYDC3=  0.000000   CYDC4=  0.000000   CYP=  0.0000   CYR=  0.0000

        THE A MATRIX IS

     4        4

 -.2172E+00  -.2875E-01   .3282E+01  0.
 -.3773E-01  -.1314E+00   .7172E+01  0.
  .6975E-01  -.9976E+00  -.1193E+00   .2945E-01
  .1000E+01   .6992E-01  0.          0.

        THE B MATRIX IS

     4        2

  .2319E+02   .7600E+01
  .1506E+01  -.1942E+01
 -.1113E-01   .8554E-02
 0.          0.

        THE C MATRIX IS

     4        4

  .1000E+01  -.2343E+00  0.          0.
 -.2571E-01   .1000E+01  0.          0.
 0.          0.           .1000E+01  0.
 0.          0.          0.           .1000E+01

        THE H MATRIX IS

     5        4

  .1000E+01  0.          0.          0.
 0.           .1000E+01  0.          0.
```

```
 0.          0.          .1000E+01  0.
 0.          0.         0.          .1000E+01
 0.          0.         -.4040F+01  0.

         THE F MATRIX IS

          5         2

 0.          0.
 0.          0.
 0.          0.
 0.          0.
-.3779E+00   .2906E+00

         THE KI MATRIX IS

         2          4

 0.          0.          0.          0.
 0.          0.          0.          0.

         THE D MATRIX IS

         2          2

 0.          0.
 0.          0.

BLOCK DIAGRAM INPUT PARAMETERS ARE

             GRAPH

     1     0     0     0     1
     2    -1     0     0     4
     3     2     0     0     0
     4     2     0     0     0
     5     7    10     0     5
     6    -4     5     0     0
     7     0     0     0     0
     8    -9     0     0     2
     9     0     0     0     1
    10     0     0     0     3

            BLOCK

     1     3     3
     2     1     1
     3     1     2
     4     1     1
     5     1     1
     6     1     2
     7     2     2
     8     2     2
     9     1     1
    10     1     1

            NUMER

6400.0000    64.0000     4.0000     0.0000     0.0000
   1.0000     0.0000     0.0000     0.0000     0.0000
  20.0000     0.0000     0.0000     0.0000     0.0000
```

```
   1.0000    0.0000    0.0000    0.0000    0.0000
   1.0000    0.0000    0.0000    0.0000    0.0000
  20.0000    0.0000    0.0000    0.0000    0.0000
  15.0000    3.0000    0.0000    0.0000    0.0000
   0.0000    1.5000    0.0000    0.0000    0.0000
   1.0000    0.0000    0.0000    0.0000    0.0000
   1.0000    0.0000    0.0000    0.0000    0.0000

      DENOM

6400.0000   80.0000    1.0000    0.0000    0.0000
   1.0000    0.0000    0.0000    0.0000    0.0000
  20.0000    1.0000    0.0000    0.0000    0.0000
   1.0000    0.0000    0.0000    0.0000    0.0000
   1.0000    0.0000    0.0000    0.0000    0.0000
  20.0000    1.0000    0.0000    0.0000    0.0000
  15.0000    1.0000    0.0000    0.0000    0.0000
   1.0000    1.0000    0.0000    0.0000    0.0000
   1.0000    0.0000    0.0000    0.0000    0.0000
   1.0000    0.0000    0.0000    0.0000    0.0000

      GAIN

   1.0000     .3000    1.0000     .2000     .5000    1.0000    1.0000    1.0000
    .0700     .6000

      ITHINY

   1   2   3   4   5  -0  -0  -0  -0  -0  -0  -0  -0  -0  -0

      ITHINU

   1   2  -0  -0  -0  -0  -0

      YTOV

   1   1
   2   2
   5   3

      ZTOU

   3   1
   6   2
```

```
        THE FINAL REDUCED SYSTEM IS


      THE A MATRIX IS

    10       10

-.2274E+00 -.5991E-01  .4992E+01 0.         0.         0.          .4737E+03  .1438E+03 0.         0.
-.4357E-01 -.1330E+00  .7300E+01 0.         0.         0.          .4231E+02 -.3515E+02 0.         0.
 .6975E-01 -.9976E+00 -.1193E+00  .2945E-01 0.         0.         -.2226E+00  .1711E+00 0.         0.
 .1000E+01  .4992E-01 0.         0.         9.         0.         0.         0.         0.         0.
0.         0.         0.         0.         0.          .1000E+01 0.         0.         0.         0.
 .1000E+01 0.         0.         0.        -.6400E+04 -.8000E+02 0.         0.         0.         0.
-.1200E+01 0.         0.         0.         .5760E+04  .7680E+02 -.2000E+02 0.         0.         0.
 .8250E-01  .2250E+01 -.1212E+01 0.        -.1152E+04 -.1536E+02 -.2262E+01 -.1826E+02 -.1500E+02 -.2250E+01
-.1050E+00  .1500E+01 0.         0.         0.         0.         0.         0.        -.1500E+02 -.1500E+01
-.7000E-01  .1000E+01 0.         0.         0.         0.         0.         0.         0.        -.1000E+01

      THE H MATRIX IS

    10       2

 .2368E+02  .7188E+01
 .2115E+01 -.1757E+01
-.1113E-01  .8554E-02
0.         0.
0.         0.
0.         0.
0.         0.
-.1131E+00  .8700E-01
0.         0.
0.         0.

      THE H MATRIX IS

    5        10

 .1000E+01 0.         0.         0.         0.         0.         0.         0.         0.         0.
0.          .1000E+01 0.         0.         0.         0.         0.         0.         0.         0.
0.         0.          .1000E+01 0.         0.         0.         0.         0.         0.         0.
0.         0.         0.          .1000E+01 0.         0.         0.         0.         0.         0.
0.         0.        -.4040E+01 0.         0.         0.        -.7540E+01  .5800E+01 0.         0.

      THE F MATRIX IS

    5        2

0.         0.
0.         0.
0.         0.
0.         0.
-.3770E+00  .2900E+00

      THE EIGEN VALUES OF THE SYSTEM ARE

               REAL PART                 IMAGINARY PART
```

```
                 REAL PART               IMAGINARY PART

ROOT( 1)        -.40906912E+02        -.72434682E+02
ROOT( 2)        -.40906912E+02         .72434682E+02
ROOT( 3)        -.16515710E+02        -.94901685E+01
ROOT( 4)        -.16515710E+02         .94901685E+01
ROOT( 5)        -.84654101E+01        -.62806024E+01
ROOT( 6)        -.84654101E+01         .62806024E+01
ROOT( 7)        -.87224345E+00        -.24584784E+01
ROOT( 8)        -.87224345E+00         .24584784E+01
ROOT( 9)        -.12207181E+01        0.
ROOT(10)         .15733931E-02        0.

ROOT( 2)    TIME TO HALF =    .017 SEC            CYCLES TO HALF =    .195              TIME CONSTANT =    .0244
            ACTUAL FREQ. =  72.435 RAD/SEC        NATURAL FREQ.  =  83.187 RAD/SEC            ZETA    =    .4917
                         =  11.528 CYC/SEC                       =  13.240 CYC/SEC            PERIOD  =    .0867

ROOT( 4)    TIME TO HALF =    .042 SEC            CYCLES TO HALF =    .063              TIME CONSTANT =    .0605
            ACTUAL FREQ. =   9.490 RAD/SEC        NATURAL FREQ.  =  19.048 RAD/SEC            ZETA    =    .8671
                         =   1.510 CYC/SEC                       =   3.032 CYC/SEC            PERIOD  =    .6621

ROOT( 6)    TIME TO HALF =    .082 SEC            CYCLES TO HALF =    .082              TIME CONSTANT =    .1181
            ACTUAL FREQ. =   6.281 RAD/SEC        NATURAL FREQ.  =  10.541 RAD/SEC            ZETA    =    .8031
                         =   1.000 CYC/SEC                       =   1.678 CYC/SEC            PERIOD  =   1.0004

ROOT( 8)    TIME TO HALF =    .795 SEC            CYCLES TO HALF =    .311              TIME CONSTANT =   1.1465
            ACTUAL FREQ. =   2.458 RAD/SEC        NATURAL FREQ.  =   2.609 RAD/SEC            ZETA    =    .3344
                         =    .391 CYC/SEC                       =    .415 CYC/SEC            PERIOD  =   2.5557

ROOT( 9)    TIME TO HALF =    .568 SEC            CYCLES TO HALF =  -0.000              TIME CONSTANT =    .8192

ROOT(10)    TIME TO HALF =-440.545 SEC            CYCLES TO HALF =  -0.000              TIME CONSTANT =-635.5691

FOR THIS CONFIGURATION, THE FOLLOWING INFORMATION IS APPLICABLE

STEADY STATE SIDESLIP INFORMATION
             DEGREE  PER DEGREE      MEETS MILSPEC 8785A
                OF SIDESLIP            REQUIREMENTS*
RUDDER          2.86                       YES
AILERON        -1.08                       NO
BANK ANGLE      2.82                       YES
*BASED ON THE FOLLOWING SIGN CONVENTIONS
    NOSE LEFT SIDESLIP IS POSITIVE
    RIGHT BANK ANGLE IS POSITIVE
    LEFT RUDDER PEDAL GIVES POSITIVE RUDDER
    RIGHT AILERON STICK GIVES POSITIVE AILERON

DYNAMIC CNB(CNBSTAR)=  .002539

     THE COEFFICIENTS OF THE CHARACTERISTIC EQUATION ORDERED FROM THE LOWEST POWER OF S

   -.36462575E+07
    .23125961E+10
    .30816231E+10
    .15289302E+10
    .56503457E+09
    .98924765E+08
    .94058815E+07
    .47695897E+06
    .12443416E+05
    .13473970E+03
    .10000000E+01
```

```
     THE        P / DA      NUMERATOR GAIN IS    .2368E+02

THE ZEROES OF THE TRANSFER FUNCTION ARE

                  REAL PART                IMAGINARY PART

ROOT( 1)          -.40000000E+02      -.69282032E+02
ROOT( 2)          -.40000000E+02       .69282032F+02
ROOT( 3)          -.20000000E+02      0.
ROOT( 4)          -.15522913E+02      -.91564326F+01
ROOT( 5)          -.15522913E+02       .91564326E+01
ROOT( 6)          -.77053076E+00      -.24744417E+01
ROOT( 7)          -.77053076E+00       .24744417E+01
ROOT( 8)          -.12172830E+01      0.
ROOT( 9)           .20602073E-02      0.

      THE COEFFICIENTS OF THE NUMERATOR  POLYNOMIAL ORDERED FROM THE LOWEST POWER OF S

   -.70150229E+06
    .33969006E+09
    .41202624E+09
    .17524194E+09
    .63411068E+08
    .70052905E+07
    .44166982E+06
    .11801299E+05
    .13381811E+03
    .10000000E+01

     THE        R / DA      NUMERATOR GAIN IS    .2115E+01

THE ZEROES OF THE TRANSFER F~~CTION ARE

                  REAL PART                IMAGINARY PART

ROOT( 1)          -.20000000E+02      0.
ROOT( 2)          -.397631S7E+02      -.68503421E+02
ROOT( 3)          -.39763157E+02       .68503421E+02
ROOT( 4)          -.17886670E+02      -.71157545E+01
ROOT( 5)          -.17886670E+02       .71157545E+01
ROOT( 6)           .46171169E+00      -.22615154E+01
ROOT( 7)           .46171169E+00       .22615154E+01
ROOT( 8)          -.37534853E+00      0.
ROOT( 9)          -.12082579E+01      0.

      THE COEFFICIENTS OF THE NUMERATOR  POLYNOMIAL ORDERED FROM THE LOWEST POWER OF S

    .11234599E+09
    .39070721E+09
    .26124335F+09
    .66254959F+08
    .53395904E+08
    .77479311E+07
    .45207131E+06
    .11888950E+05
    .13595984E+03
    .10000000E+01
```

```
      THE    BETA / DA      NUMERATOR GAIN IS  -.1113E-01

  THE ZEROES OF THE TRANSFER FUNCTION ARE

                      REAL PART                   IMAGINARY PART

 ROOT( 1)             -.20000000E+02       0.
 ROOT( 2)             -.41656360E+02      -.67937572E+02
 ROOT( 3)             -.41656360E+02       .67937572E+02
 ROOT( 4)             -.32200230E+02      -.22901597E+02
 ROOT( 5)             -.32200230E+02       .22901597E+02
 ROOT( 6)             -.14512067E+02       0.
 ROOT( 7)              .59156276E+01       0.
 ROOT( 8)             -.11980763E+01       0.
 ROOT( 9)             -.11813985E-01       0.

      THE COEFFICIENTS OF THE NUMERATOR  POLYNOMIAL ORDERED FROM THE LOWEST POWER OF S

     -.24096855E+09
     -.20599210E+11
     -.17107624E+11
      .29998795E+09
      .33889743E+09
      .27345693E+08
      .95104017E+06
      .17800963E+05
      .17751951E+03
      .10000000E+01

      THE    PHI / DA       NUMERATOR GAIN IS   .2383E+02

  THE ZEROES OF THE TRANSFER FUNCTION ARE

                      REAL PART                   IMAGINARY PART

 ROOT( 1)             -.20000000E+02       0.
 ROOT( 2)             -.39998740E+02      -.69277186E+02
 ROOT( 3)             -.39998740E+02       .69277186E+02
 ROOT( 4)             -.15538164E+02      -.91454346E+01
 ROOT( 5)             -.15538164E+02       .91454346E+01
 ROOT( 6)             -.77018272E+00      -.24740124E+01
 ROOT( 7)             -.77018272E+00       .24740124E+01
 ROOT( 8)             -.12172288E+01       0.

      THE COEFFICIENTS OF THE NUMERATOR  POLYNOMIAL ORDERED FROM THE LOWEST POWER OF S

      .34000667E+09
      .41109047E+09
      .17456556E+09
      .63349708E+08
      .78049345E+07
      .44173438E+06
      .11801843E+05
      .13383140E+03
      .10000000E+01

      THE        NY / DA      NUMERATOR GAIN IS  -.3770E+00
```

```
THE ZEROES OF THE TRANSFER FUNCTION ARE

                    REAL PART               IMAGINARY PART

ROOT( 1)            -.20000000E+02       0.
ROOT( 2)            -.39774371E+02       -.68543453E+02
ROOT( 3)            -.39774371E+02        .68543453E+02
ROOT( 4)            -.17792495E+02       -.72865091E+01
ROOT( 5)            -.17792495E+02        .72865091E+01
ROOT( 6)             .15961428E+01       -.29339135E+01
ROOT( 7)             .15961428E+01        .29339135E+01
ROOT( 8)            -.32042476E+01       0.
ROOT( 9)            -.11977362E+01       0.
ROOT(10)            -.16995553E-01       0.

          THE COEFFICIENTS OF THE NUMERATOR  POLYNOMIAL ORDERED FROM THE LOWEST POWER OF S


       .33785348E+08
       .20223347E+10
       .20328515E+10
       .33595874E+09
       .80880140E+08
       .56349569E+08
       .79384972E+07
       .45712107E+06
       .11950017E+05
       .13636042E+03
       .10000000E+01

          THE        P / DR      NUMERATOR GAIN IS   .7188E+01

THE ZEROES OF THE TRANSFER FUNCTION ARE

                    REAL PART               IMAGINARY PART

ROOT( 1)            -.15000000E+02       0.
ROOT( 2)            -.20000000E+02       0.
ROOT( 3)            -.40000000E+02       -.69282032E+02
ROOT( 4)            -.40000000E+02        .69282032E+02
ROOT( 5)            -.20000000E+02       0.
ROOT( 6)            -.13748699E+00       -.29154645E+01
ROOT( 7)            -.13748599E+00        .29154645E+01
ROOT( 8)            -.10000000E+01       0.
ROOT( 9)             .20594395E-02       0.

          THE COEFFICIENTS OF THE NUMERATOR  POLYNOMIAL ORDERED FROM THE LOWEST POWER OF S


      -.67369066E+06
       .32630717E+09
       .39605879E+09
       .11308630E+09
       .51108497E+08
       .75424201E+07
       .45421570E+06
       .11980635E+05
       .13627291E+03
       .10000000E+01
```

```
      THE       R / DR       NUMERATOR GAIN IS  -.1757F+01

THE ZEROES OF THE TRANSFER FUNCTION ARE

                  PFAL PART                 IMAGINARY PART


ROOT( 1)            -.15000000E+02       0.
ROOT( 2)            -.20000000F+02       0.
ROOT( 3)            -.41202817E+02      -.73854973E+02
ROOT( 4)            -.41202817E+02       .73854973E+02
ROOT( 5)            -.91169532E+01      -.99940479E+01
ROOT( 6)            -.91169532E+01       .99940479E+01
ROOT( 7)            -.25049358F+00       0.
ROOT( 8)             .40064405E+00       0.
ROOT( 9)            -.10000000F+01       0.

        THE COEFFICIENTS OF THE NUMERATOR  POLYNOMIAL ORDERED FROM THE LOWEST POWER OF S

   -.39406571E+08
   -.10734210E+09
    .31038083E+09
    .46636642E+09
    .97028434E+08
    .94614626E+07
    .49573352E+06
    .12775212E+05
    .13648939E+03
    .10000000E+01

       THE     BETA / DR       NUMERATOR GAIN IS   .8554E-02

THE ZEROES OF THE TRANSFER FUNCTION ARE

                  REAL PART                 IMAGINARY PART


ROOT( 1)            -.15000000E+02       0.
ROOT( 2)            -.20000000E+02       0.
ROOT( 3)            -.26256175E+03       0.
ROOT( 4)            -.41345033E+02      -.72680929E+02
ROOT( 5)            -.41345033E+02       .72680929E+02
ROOT( 6)            -.93460374E+01      -.77130608E+01
ROOT( 7)            -.93460374E+01       .77130608E+01
ROOT( 8)            -.10000000E+01       0.
ROOT( 9)             .29535897E-03       0.

        THE COEFFICIENTS OF THE NUMERATOR  POLYNOMIAL ORDERED FROM THE LOWEST POWER OF S

   -.23806001E+08
    .80040990E+11
    .10185813E+12
    .23338232E+11
    .24789135E+10
    .13769379E+09
    .38161642E+07
    .48740351E+05
    .39994359E+03
    .10000000E+01
```

```
        THE      PHI / DR       NUMERATOR GAIN IS    .7065E+01
   THE ZEROES OF THE TRANSFER FUNCTION ARE
                    REAL PART                     IMAGINARY PART

   ROOT( 1)         -.15000000E+02        0.
   ROOT( 2)         -.20000000E+02        0.
   ROOT( 3)         -.39975146E+02        -.69201720E+02
   ROOT( 4)         -.39975146E+02         .69201720E+02
   ROOT( 5)         -.20214153E+02        0.
   ROOT( 6)         -.52352516E-01        -.29354408E+01
   ROOT( 7)         -.52352516E-01         .29354408E+01
   ROOT( 8)         -.10000000E+01        0.

        THE COEFFICIENTS OF THE NUMERATOR  POLYNOMIAL ORDERED FROM THE LOWEST POWER OF S

      .33304974E+09
      .39754901E+09
      .10694169E+09
      .50309800E+08
      .75090417E+07
      .45349357E+06
      .11966814E+05
      .13626915E+03
      .10000000E+01

        THE      NY / DR       NUMERATOR GAIN IS    .2900E+00
   THE ZEROES OF THE TRANSFER FUNCTION ARE
                    REAL PART                     IMAGINARY PART

   ROOT( 1)         -.15000000E+02        0.
   ROOT( 2)         -.20000000E+02        0.
   ROOT( 3)         -.41216979E+02        -.73946450E+02
   ROOT( 4)         -.41216979E+02         .73946450E+02
   ROOT( 5)         -.92500451E+01        -.10077793E+02
   ROOT( 6)         -.92500451E+01         .10077793E+02
   ROOT( 7)         -.38343021E+01        0.
   ROOT( 8)          .44082941E+01        0.
   ROOT( 9)         -.10000000E+01        0.
   ROOT(10)         -.47981738E-03        0.

        THE COEFFICIENTS OF THE NUMERATOR  POLYNOMIAL ORDERED FROM THE LOWEST POWER OF S

     -.32630134E+07
     -.68046466E+10
     -.85761744E+10
     -.15001075E+10
      .27249180E+09
      .86498468E+08
      .91694499E+07
      .49209086E+06
      .12752267E+05
      .13636054E+03
      .10000000E+01
```

Table C5

CONTROL PROGRAM DESCRIPTION

MAIN - Entrance program. Main overlay.

ADD - Utility subroutine to add two matrices.

CARD - Card input routine. Reads first four data cards. Prints options chosen.

CHANGE - User written input routine.

CLASS - Converts block diagram information in system matrices.

CNTRLR - Executive routine for the program. Calls subroutines in order depending on options chosen.

CPMT -

EAT - Transition matrix routine. Computes transition matrix for time histories.

EIGEN - Eigenvalue routine. Finds the eigenvalues of a matrix.

HESSEN - Used by EIGEN to find eigenvalues.

INVR - Matrix inversion routine. Computes the inverse of a matrix.

LOAD - Input read routine. Reads input matrices.

LOAD1 - Used by LOAD to read matrices.

MAKE - Utility subroutine to copy one matrix to another.

MATRIX - User written input routine.

MIL - Milspec computation routine. Computes Milspec parameters.

MULT - Utility subroutine to multiply two matrices.

NRREAD - Input read routine. Reads data produced by MMLE.

QREIG -

QRT -

RDISC - Disc input routine. Recalls data from a previous case if required.

Table C5 (Concluded)

RDISC1 - Used by RDISC to recall data.

REDUCE - Matrix reduction routine. Determines irreducible submatrices of a matrix.

ROOT - Root locus routine. Determines root locus points.

SCALE - Matrix scaling routine. Performs a diagonal similarity transformation on ill-conditioned matrices.

SPIT - Matrix print routine. Prints matrices.

SPIT1 - Used by SPIT to print matrices.

SWZ - NIT conversion routine. Converts predetermined format data (NIT=1) to block diagram form.

WDISC - Disc write routine. Writes data to disc to be used by subsequent case.

WDISC1 - Used by WDISC to write data to disc.

ZOT - Matrix initialization routine. Utility routine to zero matrices.

ZOT1 - Used by ZOT to zero matrices.

NMRATR - Numerator routine. Determines the numerators of system transfer functions.

BLKDAT - Predefined frequency distribution for frequency response analysis.

FRQRSP - Frequency response routine. Calculates frequency response for system transfer functions.

PSD - Power spectrum routine. Computes power spectrum of system transfer functions.

TANG - Subfunction.

ZTOW - Converts z-plane transfer functions to x-plane transfer functions.

SETUP - Matrix reduction routine. Reduces and couples all input matrices and information into a complete system matrix.

THIST - Time history routine. Calculates time histories in response to input from INPUTV.

INPUTV - User written routine to define system inputs.

APPENDIX D

SCOPE 3.4 CONTROL CARDS

Table D1

PROGRAM PERMANENT FILES

Program	Local File Name	Permanent File Name	I.D.	Cycle	Type
ADEX	Any (OLDPL)	AMCUPDATE	CNAGY	1	Update
	ADEX	AMC	CNAGY	1	Absolute
MMLE	Any (OLDPL)	AMCUPDATE	CNAGY	2	Update
	MMLE	AMC	CNAGY	2	Absolute
CONTROL	Any (OLDPL)	AMCUPDATE	CNAGY	3	Update
	CONTROL	AMC	CNAGY	3	Absolute

ADEX CONTROL CARDS

CONTROL CARDS REQUIRED TO CREATE A BINARY FILE FOR MMLE

```
XXXXX,CM70000,CHXXXXXX8115P,T776,NT1.                                          CARD 1
VSN,TAPE=XXXXX.                                                                CARD 2
ATTACH,ADEX,AMC,ID=CNAGY,CY=1,MR=1.                                            CARD 3
REQUEST,TAPE,E,PE.                                                             CARD 4
REQUEST,THIST,*PF.                                                             CARD 5
ADEX.                                                                          CARD 6
CATALOG,THIST,XXX,ID=XXXX,CY=X.                                                CARD 7
7/8/9                                                                          CARD 8
   - ADEX DATA -
7/8/9
6/7/8/9
```

NOTES

CARD 1, XXXXX=USER BANNER TITLE. THE TIME ESTIMATE WILL VARY DEPENDING ON THE ENGINEERING UNITS TAPE, BUT 200 OCTAL SECONDS PER CASE IS A GOOD INITIAL GUESS. XXXXXX=USER JOB ORDER NUMBER.

CARD 2, XXXXX=ENGINEERING UNITS TAPE NUMBER.

CARD 4, PARAMETERS ON THIS CARD MAY VARY DEPENDING ON THE ENGINEERING UNITS TAPE.

CARD 7, CATALOG PARAMETERS FILLED IN BY THE USER.

ADEX CONTROL CARDS

CONTROL CARDS REQUIRED TO CREATE A BCD TAPE FOR STABDIV

```
XXXXX,CM70000,CHXXXXXX8115P,T776,NT1,MT1.                        CARD 1
VSN,TAPE=XXXXX.                                                  CARD 2
VSN,THIST=XXXXX.                                                 CARD 3
ATTACH,OLDPL,AMCUPDATE,ID=CNAGY,CY=1,MR=1.                       CARD 4
REWIND,DUMMY.                                                    CARD 5
UPDATE,F,N,I=DUMMY.                                              CARD 6
FTN,L=0,R=0,I=COMPILE.                                           CARD 7
REQUEST,TAPE,E,PE.                                               CARD 8
REQUEST,THIST,HI,S,RING.                                         CARD 9
MAP,OFF.                                                         CARD10
FILE(THIST,BT=K,RB=1,MBL=100)                                    CARD11
LDSET(FILES=THIST)                                               CARD12
LGO.                                                             CARD13
7/8/9                                                            CARD14
   - ADEX DATA -
7/8/9
6/7/8/9
```

NOTES

CARD 1, XXXXX=USER BANNER TITLE. THE TIME ESTIMATE WILL VARY DEPENDING ON THE ENGINEERING UNITS TAPE, BUT 200 OCTAL SECONDS PER CASE IS A GOOD INITIAL GUESS. XXXXXX=USER JOB ORDER NUMBER.

CARD 2, XXXXX=ENGINEERING UNITS TAPE NUMBER.

CARD 3, XXXXX=STABDIV TAPE NUMBER.

CARD 8, PARAMETERS ON THIS CARD MAY VARY DEPENDING ON THE ENGINEERING UNITS TAPE.

MMLE CONTROL CARDS

CONTROLS CARDS REQUIRED TO EXECUTE MMLE

```
XXXXX,CM72000,CHXXXXXX4532P,T776,NT1.                                   CARD 1
VSN,TAPE13=XXXXX.                                                       CARD 2
ATTACH,CURVES,XXX,ID=XXXX,CY=X.                                         CARD 3
ATTACH,NCURVE,XXX,ID=XXXX,CY=X.                                         CARD 4
ATTACH,THIST,XXX,ID=XXXX,CY=X.                                          CARD 5
ATTACH,MMLE,AMC,ID=CNAGY,CY=2,MR=1.                                     CARD 6
REQUEST,TAPE13,PE,RING.                                                 CARD 7
REQUEST,DERIVS,*PF.                                                     CARD 8
MMLE.                                                                   CARD 9
CATALOG,DERIVS,XXX,ID=XXXX,CY=X.                                        CARD10
7/8/9                                                                   CARD11
  - MMLE DATA -
7/8/9
6/7/8/9
```

NOTES

CARD 1,	XXXXX=USER BANNER TITLE. TIME ESTIMATE SHOULD BE APPROXIMATELY 100 OCTAL SECONDS PER CASE. IF NO PLOT TAPE IS REQUIRED, THE NT1 MAY BE DELETED. XXXXXX=USER JOB ORDER NUMBER.
CARD 2,	XXXXX=PLOT TAPE NUMBER. IF NO PLOTS ARE TO BE GENERATED, THIS CARD MAY BE DELETED.
CARD 3,	PARAMETERS DEPEND ON WHERE THE DERIVATIVE CURVE FILE IS STORED. IF NO CURVE FILE IS TO BE USED, THIS CARD MAY BE DELETED.
CARD 4,	PARAMETERS DEPEND ON WHERE THE CURVE FILE DATA ARE STORED. IF NO CURVE FILE IS TO BE USED, THIS CARD MAY BE DELETED.
CARD 5,	PARAMETERS DEPEND ON WHERE THE TIME HISTORY FILE IS STORED.
CARD 7,	IF NO PLOTS ARE TO BE GENERATED, THIS CARD MAY BE DELETED.
CARD 8,	IF NO DERIVATIVES ARE TO BE SAVED FOR CONTROL, THIS CARD MAY BE DELETED.
CARD10,	CATALOG PARAMETERS FILLED IN BY USER. IF NO DERIVATIVES ARE TO BE SAVED FOR CONTROL, THIS CARD MAY BE DELETED.

CONTROL CONTROL CARDS

CONTROL CARDS REQUIRED TO EXECUTE CONTROL

```
XXXXX,CM120000,CHXXXXXX8230P,T776.                              CARD 1
ATTACH,DERIVS,XXX,ID=XXXX,CY=X.                                 CARD 2
ATTACH,CONTROL,AMC,ID=CNAGY,CY=3,MR=1.                          CARD 3
CONTROL.                                                        CARD 4
7/8/9                                                           CARD 5
   - CONTROL DATA -
7/8/9
6/7/8/9
```

NOTES

CARD 1, XXXXX=USER BANNER TITLE. TIME ESTIMATE SHOULD BE APPROXIMATELY 20 OCTAL SECONDS PER CASE. XXXXXX=USER JOB ORDER NUMBER.

CARD 2, PARAMETERS DEPEND ON WHERE THE DERIVATIVES ARE STORED. IF NO DERIVATIVE FILE IS TO BE USED, THIS CARD MAY BE DELETED.

CONTROL CONTROL CARDS

CONTROL CARDS REQUIRED TO EXECUTE CONTROL WITH COPYL DECK SUBSTITUTION

```
XXXXX,CM125000,CHXXXXXX8230P,T776.                    CARD 1
FTN,L=0,R=0,B=SUB.                                    CARD 2
ATTACH,OLDPL,AMCUPDATE,ID=CNAGY,CY=3,MR=1.            CARD 3
REWIND,DUMMY.                                         CARD 4
UPDATE,F,N,I=DUMMY.                                   CARD 5
FTN,L=0,R=0,I=COMPILE,B=MAIN.                         CARD 6
COPYL,MAIN,SUB,LGO.                                   CARD 7
REWIND,LGO.                                           CARD 8
ATTACH,DERIVS,XXX,ID=XXXX,CY=X.                       CARD 9
MAP,OFF.                                              CARD10
LGO.                                                  CARD11
7/8/9                                                 CARD12
   - FORTRAN SUBSTITUTION DECKS -
7/8/9
   - CONTROL DATA -
7/8/9
6/7/8/9
```

NOTES

CARD 1. XXXXX=USER BANNER TITLE. TIME ESTIMATE SHOULD BE APPROXIMATELY 20 OCTAL SECONDS PER CASE. XXXXXX=USER JOB ORDER NUMBER.

CARD 9. PARAMETERS DEPEND ON WHERE THE DERIVATIVES ARE STORED. IF NO DERIVATIVE FILE IS TO BE USED, THIS CARD MAY BE DELETED.

COMBINATION CONTROL CARDS

CONTROL CARDS REQUIRED TO EXECUTE ADEX, MMLE, AND CONTROL IN SEQUENCE

```
XXXXX,CM120000,CHXXXXXX8230P,T776,NT1.
VSN,TAPE=XXXXX.
VSN,TAPE13=XXXXX.
ATTACH,ADEX,AMC,ID=CNAGY,CY=1,MR=1.                            CARD 2
REQUEST,TAPE,E,PE.                                             CARD 3
ADEX.                                                          CARD 4
UNLOAD,TAPE.                                                   CARD 5
ATTACH,CURVES,XXX,ID=XXXX,CY=X.                                CARD 6
ATTACH,NCURVE,XXX,ID=XXXX,CY=X.                                CARD 7
ATTACH,MMLE,AMC,ID=CNAGY,CY=2,MR=1.                            CARD 8
REQUEST,TAPE13,PE,RING.                                        CARD 9
MMLE.                                                          CARD10
RETURN,TAPE13,LGO.                                             CARD11
ATTACH,CONTROL,AMC,ID=CNAGY,CY=3,MR=1.                         CARD12
CONTROL.                                                       CARD13
7/8/9                                                          CARD14
  - ADEX DATA -                                                CARD15
7/8/9                                                          CARD16
  - MMLE DATA -
7/8/9
  - CONTROL DATA -
7/8/9
6/7/8/9
```

NOTES

CARD 1, XXXXX=USER BANNER TITLE. TIME ESTIMATE SHOULD BE APPROXIMATELY THE SUM OF THE THREE PROGRAMS RUN SEPARATELY. XXXXXX=USER JOB ORDER NUMBER.

CARD 2, XXXXX=ENGINEERING UNITS TAPE NUMBER.

CARD 3, XXXXX=PLOT TAPE NUMBER.

CARD 5, PARAMETERS ON THIS CARD MAY VARY DEPENDING ON THE ENGINEERING UNITS TAPE.

CARD 8, PARAMETERS DEPEND ON WHERE THE DERIVATIVE CURVE FILE IS STORED. IF NO CURVE FILE IS TO BE USED, THIS CARD MAY BE DELETED.

CARD 9, PARAMETERS DEPEND ON WHERE THE CURVE FILE DATA ARE STORED. IF NO CURVE FILE IS TO BE USED, THIS CARD MAY BE DELETED.

LISTING CONTROL CARDS

CONTROL CARDS REQUIRED TO OBTAIN A PROGRAM LISTING

DUE TO THE LENGTH OF THE PROGRAMS, NO LISTINGS ARE CONTAINED IN THIS REPORT. A LISTING OF ANY PROGRAM MAY BE OBTAINED FROM THE FOLLOWING SET OF CARDS.

```
XXXXX,CM65000,CHXXXXXX8230P,T177.                                        CARD 1
ATTACH,OLDPL,AMCUPDATE,ID=CNAGY,CY=X.                                    CARD 2
REWIND,DUMMY.                                                            CARD 3
UPDATE,F,N,I=DUMMY.                                                      CARD 4
FTN,R=X,I=COMPILE.                                                       CARD 5
7/8/9                                                                    CARD 6
6/7/8/9                                                                  CARD 7
```

NOTES

```
   CARD 1.   XXXXX=USER BANNER TITLE.  XXXXXX=USER JOB ORDER NUMBER.
   CARD 2,   X=1, 2, OR 3 FOR ADEX, MMLE, OR CONTROL RESPECTIVELY.
   CARD 5,   X=0, 1, 2, OR 3 DEPENDING ON HOW EXTENSIVE A REFERENCE MAP IS
             REQUIRED.
```

APPENDIX E

EQUATIONS OF MOTION

EQUATIONS OF MOTION

This appendix will be presented in two sections. The first will derive the full five degree of freedom equations which are in general use at the Air Force Flight Test Center. The second section will show how those equations evolve into the linear matrix equations used in MMLE and CONTROL and what assumptions are made in the linearization process.

Five degree of freedom aircraft analyses are usually concerned with short periods of time. During this period of time the change in aircraft mass due to fuel flow is usually negligible. Hence, the aircraft mass (and inertias) are usually assumed to be constant. Then Newton's law

$$F = \frac{d(mV)}{dt}$$

may be written as

$$F = m\frac{dV}{dt} = ma$$

This last equation will be our starting point in this derivation.

SECTION I

Translational Equations

Let us start with Newton's law

$$F = ma \text{ (mass)(acceleration)}$$

Since $a = \frac{dV}{dt}$,

$$F = m \frac{dV}{dt}$$

Now define the components of V,

$$V = ui + vj + wk$$

$$F = m \frac{dV}{dt} = m \frac{d}{dt} (ui + vj + wk)$$

Also define the components of F

$$F = Xi + Yj + Zk$$

Then

$$Xi + Yj + Zk = m \frac{d}{dt} (ui + vj + wk)$$

$$= m (\dot{u}i + u\dot{i} + \dot{v}j + v\dot{j} + \dot{w}k + w\dot{k})$$

$$= m (\dot{u}i + \dot{v}j + \dot{w}k + u\dot{i} + v\dot{j} + w\dot{k})$$

Now recall that

$$\dot{i} = \omega \times i$$

$$\dot{j} = \omega \times j$$

$$\dot{k} = \omega \times k$$

where x represents the vector cross product. Then, substituting for $\dot{i}$, $\dot{j}$, $\dot{k}$

$$Xi + Yj + Zk = m\ (\dot{u}i + \dot{v}j + \dot{w}k + u\dot{i} + v\dot{j} + w\dot{k})$$

$$= m\ (\dot{u}i + \dot{v}j + \dot{w}k + u\ (\omega \times i) + v\ (\omega \times j) + w\ (\omega \times k))$$

$$= m\ (\dot{u}i + \dot{v}j + \dot{w}k + (\omega \times V))$$

Now define the components of ω

$$\omega = Pi + Qj + Rk$$

To evaluate $\omega \times V$

$$\omega \times V = \begin{vmatrix} i & j & k \\ P & Q & R \\ u & v & w \end{vmatrix}$$

$$\omega \times V = i\ (Qw - Rv) + j\ (Ru - Pw) + k\ (Pv + Qu)$$

Now substituting the $\omega \times V$ term

$$Xi + Yj + Zk = m\ (\dot{u}i + \dot{v}j + \dot{w}k + \omega \times V)$$

$$= m\ (\dot{u}i + \dot{v}j + \dot{w}k + i\ (Qw - Rv) + j\ (Ru - Pw) + k\ (Pv - Qu))$$

Equating components, we get

$$X = m\ (\dot{u} + Qw - Rv)$$

$$Y = m\ (\dot{v} + Ru - Pw)$$

$$Z = m\ (\dot{w} + Pv - Qu)$$

Using the definitions of α and β

$$\alpha = \tan^{-1} \frac{w}{u}$$

$$\beta = \sin^{-1} \frac{v}{V}$$

and knowing that

$$u = V \cos \alpha \cos \beta$$

we can rewrite u, v and w

$$u = V \cos \alpha \cos \beta$$

$$v = V \sin \beta$$

$$w = V \sin \alpha \cos \beta$$

Differentiating u, v and w

$$\dot{u} = \dot{V} \cos \alpha \cos \beta + V (-\dot{\beta} \cos \alpha \sin \beta - \dot{\alpha} \sin \alpha \cos \beta)$$

$$\dot{v} = \dot{V} \sin \beta + V \dot{\beta} \cos \beta$$

$$\dot{w} = \dot{V} \sin \alpha \cos \beta + V (-\dot{\beta} \sin \alpha \sin \beta + \dot{\alpha} \cos \alpha \cos \beta)$$

For five degrees of freedom, we make assumption number one, $\dot{V} = 0$. Then

$$\dot{u} = -V (\dot{\beta} \cos \alpha \sin \beta + \dot{\alpha} \sin \alpha \cos \beta)$$

$$\dot{v} = V \dot{\beta} \cos \beta$$

$$\dot{w} = V (-\dot{\beta} \sin \alpha \sin \beta + \dot{\alpha} \cos \alpha \cos \beta)$$

Using the Y force component equation and substituting velocity component terms

$$Y = m (\dot{v} + Ru - Pw)$$

$$\frac{Y}{m} = \dot{v} + Ru - Pw$$

$$= V \dot{\beta} \cos \beta + R (V \cos \alpha \cos \beta) - P (V \sin \alpha \cos \beta)$$

$$= V \cos \beta (\dot{\beta} + R \cos \alpha - P \sin \alpha)$$

Rearranging terms and solving for $\dot{\beta}$

$$\frac{Y}{m} = V \cos \beta (\dot{\beta} + R \cos \alpha - P \sin \alpha)$$

$$\dot{\beta} + R \cos \alpha - P \sin \alpha = \frac{Y}{m V \cos \beta}$$

$$\dot{\beta} = P \sin \alpha - R \cos \alpha + \frac{Y}{m\ V \cos \beta}$$

The Y force component is composed of the gravity components and the aerodynamic sideforce

$$Y = mg\ (\sin \phi \cos \theta - \sin \theta \sin \beta) + \bar{q}\ S\ C_y$$

Therefore

$$\dot{\beta} = P \sin \alpha - R \cos \alpha + \frac{Y}{m\ V \cos \beta}$$

$$= P \sin \alpha - R \cos \alpha + \frac{g}{V} \frac{\sin \phi \cos \theta}{\cos \beta} - \frac{g}{V} \frac{\sin \theta \sin \beta}{\cos \beta} + \frac{\bar{q}\ S}{m\ V \cos \beta} (C_y)$$

Finally, expanding the C_y from

$$\dot{\beta} = P \sin \alpha - R \cos \alpha + \frac{g}{V} \frac{\sin \phi \cos \theta}{\cos \beta} - \frac{g}{V} \sin \theta \tan \beta$$

$$+ \frac{\bar{q}\ S}{m\ V \cos \beta} (C_{y_\beta} \cdot \beta + C_{y_{\delta a}} \cdot \delta a + C_{y_{\delta r}} \cdot \delta r) \tag{1}$$

To determine the $\dot{\alpha}$ equation, recall the Z force component and substitute for component velocities.

$$Z = m\ (\dot{w} + Pv - Qu)$$

$$\frac{Z}{m} = \dot{w} + Pv - Qu$$

$$= V\ (\dot{\alpha} \cos \alpha \cos \beta - \dot{\beta} \sin \alpha \sin \beta) + P\ (V \sin \beta) - Q\ (V \cos \alpha \cos \beta)$$

$$= V\ (\dot{\alpha} \cos \alpha \cos \beta - \dot{\beta} \sin \alpha \sin \beta + P \sin \beta - Q \cos \alpha \cos \beta)$$

or $\frac{Z}{mV} = \dot{\alpha} \cos \alpha \cos \beta - \dot{\beta} \sin \alpha \sin \beta + P \sin \beta - Q \cos \alpha \cos \beta$

Substituting for $\dot{\beta}$

$$\frac{Z}{mV} = \dot{\alpha} \cos \alpha \cos \beta + P \sin \beta - Q \cos \alpha \cos \beta - \sin \beta \sin \alpha \ (P \sin \alpha - R \cos \alpha$$

$$+ \frac{Y}{m\ V \cos \beta})$$

$$= \dot{\alpha} \cos \alpha \cos \beta + P \sin \beta - Q \cos \alpha \cos \beta - \frac{Y \sin \beta \sin \alpha}{m V \cos \beta} - P \sin^2 \alpha \sin \beta$$

$$+ R \sin \beta \sin \alpha \cos \alpha$$

$$= \dot{\alpha} \cos \alpha \cos \beta + P (\sin \beta - \sin^2 \alpha \sin \beta) + R \sin \beta \sin \alpha \cos \alpha$$

$$- Q \cos \beta \cos \alpha - \frac{Y \sin \beta \sin \alpha}{m V \cos \beta}$$

Using the following sine-cosine relationship

$$\cos^2 \alpha = 1 - \sin^2 \alpha$$

and

$$P (\sin \beta - \sin^2 \alpha \sin \beta) = P \sin \beta (1 - \sin^2 \alpha) = P \sin \beta \cos^2 \alpha$$

We have

$$\frac{Z}{mV} = \dot{\alpha} \cos \alpha \cos \beta + P \sin \beta \cos^2 \alpha + R \sin \alpha \cos \alpha \sin \beta - Q \cos \alpha \cos \beta$$

$$- \frac{Y \sin \alpha \tan \beta}{m V}$$

Rearranging terms and solving for $\dot{\alpha}$

$$\dot{\alpha} = \frac{Z}{m V \cos \alpha \cos \beta} - P \frac{\sin \beta \cos^2 \alpha}{\cos \beta \cos \alpha} - R \frac{\sin \alpha \cos \alpha \sin \beta}{\cos \beta \cos \alpha} + Q \frac{\cos \alpha \cos \beta}{\cos \alpha \cos \beta}$$

$$+ \frac{Y \sin \alpha \tan \beta}{m V \cos \alpha \cos \beta}$$

$$= \frac{Z}{m V \cos \alpha \cos \beta} - P \tan \beta \cos \alpha - R \sin \alpha \tan \beta + Q + \frac{Y \tan \alpha \tan \beta}{m V \cos \beta}$$

Now the second assumption must be made. <u>Assume that the product of a sine or tangent of two small angles is negligible</u>. In this case.

$$R \sin \alpha \tan \beta \approx 0$$

$$\frac{Y \tan \alpha \tan \beta}{m V \cos \beta} = 0$$

Then the $\dot{\alpha}$ equation simplifies to

$$\dot{\alpha} = Q - P \tan \beta \cos \alpha + \frac{Z}{m V \cos \alpha \cos \beta}$$

The Z component of force may be written as

$$Z = mg \cos \theta \cos \phi - \bar{q} S C_N$$

Substituting into the $\dot{\alpha}$ equation and expanding C_N

$$\dot{\alpha} = Q - P \tan \beta \cos \alpha + \frac{g}{V} \frac{\cos \theta \cos \phi}{\cos \alpha \cos \beta} - \frac{\bar{q} S}{m V \cos \alpha \cos \beta} (C_{N_0} + C_{N_\alpha} \cdot \alpha + C_{N_{\delta e}} \cdot \delta e) \quad (2)$$

Rotational Equations

Start with the rotational analog of Newton's law

$T = \frac{dH}{dt}$ (Torque = change in angular momentum with respect to time.)

The momentum of each particle of mass is

Mass momentum = V_m dm

and the angular momentum of each particle is

$dH = r \times V_m \, dm$ where r = radius of the mass.

The velocity of each mass may be written as

$$V_m = V + \omega \times r$$

Then substituting

$$\begin{aligned} dH &= r \times V_m \, dm \\ &= r \times (V + \omega \times r) \, dm \\ &= r \times dm \, (\omega \times r + V) \end{aligned}$$

$$= dm\,[r \times (\omega \times r + V)]$$

$$= dm\,[r \times (\omega \times r)] + dm\,(r \times V)$$

The total angular moment is the summation of all the particles.

$$H = \Sigma\, dH$$

$$= \Sigma\,[r \times (\omega \times r)]\, dm + \Sigma\,(r \times V)\, dm$$

$$= \Sigma\,[r \times (\omega \times r)]\, dm + \Sigma\, rdm \times V$$

By the definition of the center of gravity

$$\Sigma\, rdm = 0$$

Then

$$H = \Sigma[r \times (\omega \times r)]\, dm$$

Using the components of r and ω

$$r = xi + yj + zk$$

$$\omega = Pi + Qj + Rk$$

we can evaluate $\omega \times r$

$$\omega \times r = \begin{vmatrix} i & j & k \\ P & Q & R \\ x & y & z \end{vmatrix}$$

$$\omega \times r = i\,(Qz - Ry) + j\,(Rx - Pz) + k\,(Py - Qx)$$

To evaluate $r \times (\omega \times r)$

$$r \times (\omega \times r) = \begin{vmatrix} i & j & k \\ x & y & z \\ (Qz - Ry) & (Rx - Pz) & (Py - Qx) \end{vmatrix}$$

$$r \times (\omega \times r) = i\,[y\,(Py - Qx) - z\,(Rx - Pz)] + j\,[z\,(Qz - Ry) - x\,(Py - Qx)]$$
$$+ k\,[x\,(Rx - Pz) - y\,(Qz - Ry)]$$

$$r \times (\omega \times r) = i\ (Py^2 - Qyx - Rzx + Pz^2) + j\ (Qz^2 - Rzy - Pxy + Qx^2)$$
$$+ k\ (Rx^2 - Pxz - Qyz + Ry^2)$$

Now the angular momentum may be broken into its three components

$$H_x = \Sigma\ (Py^2 - Qyx - Rzx - Pz^2)\ dm$$
$$= \Sigma\ [P(y^2 + z^2) - Qxy - Rxz]\ dm$$
$$+ P\ \Sigma\ (y^2 + z^2)\ dm - Q\ \Sigma\ xy\ dm - R\ \Sigma\ xz\ dm$$

$$H_y = \Sigma\ (Qz^2 - Ryz - Pxy + Qx^2)\ dm$$
$$= \Sigma\ [Q(x^2 + z^2) - Pxy - Ryz]\ dm$$
$$= Q\ \Sigma\ (x^2 + z^2)\ dm - P\ \Sigma\ xy\ dm - R\ \Sigma\ yz\ dm$$

$$H_z = \Sigma\ (Rx^2 - Pxz - Qyz + Ry^2)\ dm$$
$$= \Sigma\ [R(x^2 + y^2) - Pxz - Qyz]\ dm$$
$$= R\ \Sigma\ (x^2 + y^2)\ dm - P\ \Sigma\ xz\ dm - Q\ \Sigma\ yz\ dm$$

Using the definition of inertias, we can rewrite these equations as

$$H_x = I_{xx}\ P - I_{xy}\ Q - I_{xz}\ R$$

$$H_y = I_{yy}\ Q - I_{xy}\ P - I_{yz}\ R$$

$$H_z = I_{zz}\ R - I_{xz}\ P - I_{yz}\ Q$$

Recall now the original torque equation

$$T = \frac{dH}{dt}$$

$$= \frac{d}{dt}\ (H_x\ i + H_y\ j + H_z\ k)$$

$$= \dot{H}_x\ i + H_x\ \dot{i} + \dot{H}_y\ j + H_y\ \dot{j} + \dot{H}_z\ k + H_z\ \dot{k}$$

$$= \dot{H}_x\ i + \dot{H}_y\ j + \dot{H}_z\ k + H_x\ \dot{i} + H_y\ \dot{j} + H_z\ \dot{k}$$

Assuming constant inertias

$$\dot{H}_x = I_{xx}\,\dot{P} - I_{xy}\,\dot{Q} - I_{xz}\,\dot{R}$$

$$\dot{H}_y = I_{yy}\,\dot{Q} - I_{xy}\,\dot{P} - I_{yz}\,\dot{R}$$

$$\dot{H}_z = I_{zz}\,\dot{R} - I_{xz}\,\dot{P} - I_{yz}\,\dot{Q}$$

Also recall that

$$\dot{i} = \omega \times i$$

$$\dot{j} = \omega \times j$$

$$\dot{k} = \omega \times k$$

Then

$$\begin{aligned} H_x\,\dot{i} + H_y\,\dot{j} + H_z\,\dot{k} &= H_z\,(\omega \times i) + H_y\,(\omega \times j) + H_z\,(\omega \times k) \\ &= \omega \times (H_x\,i + H_y\,j + H_z\,k) \\ &= \omega \times H \end{aligned}$$

To evaluate $\omega \times H$

$$\omega \times H = \begin{vmatrix} i & j & k \\ P & Q & R \\ H_z & H_y & H_z \end{vmatrix}$$

$$\omega \times H = i\,(Q\,H_z - RH_y) + j\,(R\,H_x - P\,H_z) + k\,(P\,H_y - Q\,H_x)$$

Substituting for terms in the torque equation

$$\begin{aligned} T &= \dot{H}_x\,i + \dot{H}_y\,j + \dot{H}_z\,k + H_x\,\dot{i} + H_y\,\dot{j} + H_z\,\dot{k} \\ &= (I_{xx}\,\dot{P} - I_{xy}\,\dot{Q} - I_{xz}\,\dot{R})\,i + (I_{yy}\,\dot{Q} - I_{xy}\,\dot{P} - I_{yz}\,\dot{R})\,j \\ &\quad + (I_{zz}\,\dot{R} - I_{xz}\,\dot{P} - I_{yz}\,\dot{Q})\,k + i\,(Q\,H_z - R\,H_y) + j\,(R\,H_x - P\,H_z) \\ &\quad + k\,(P\,H_y - Q\,H_x) \end{aligned}$$

$$
\begin{aligned}
T = {} & (I_{xx} \dot{P} - I_{xy} \dot{Q} - I_{xz} \dot{R})\, i + (I_{yy} \dot{Q} - I_{xy} \dot{P} - I_{yz} \dot{R})\, j \\
& + (I_{zz} \dot{R} - I_{xz} \dot{P} - I_{yz} \dot{Q})\, k + i\, [Q\, (I_{xx} P - I_{xy} Q - I_{xz} R) \\
& - R\, (I_{yy} Q - I_{xy} P - I_{yz} R)] + j\, [R\, (I_{xx} P - I_{xy} Q - I_{xz} R) \\
& - P\, (I_{zz} R - I_{xz} P - I_{yz} Q)] + k\, [P\, (I_{yy} Q - I_{xy} P - I_{yz} R) \\
& - Q\, (I_{xx} P - I_{xy} Q - I_{xz} R)]
\end{aligned}
$$

By dividing the torque into its components

$$T = Li + Mj + Nk$$

and collecting terms we can define the $\dot{P}$, $\dot{Q}$ and $\dot{R}$ equations.

$$
\begin{aligned}
L &= I_{xx} \dot{P} - I_{xy} \dot{Q} - I_{xz} \dot{R} + Q\, (I_{zz} R - I_{xz} P - I_{yz} Q) - R\, (I_{yy} Q - I_{xy} P - I_{yz} R) \\
&= I_{xx} \dot{P} - I_{xy} \dot{Q} - I_{xz} \dot{R} + I_{zz} QR - I_{xz} PQ - I_{yz} Q^2 - I_{yy} QR + I_{xy} PR + I_{yz} R^2 \\
&= I_{xx} \dot{P} + (I_{zz} - I_{yy})\, QR + I_{xy} (PR - \dot{Q}) + I_{xz} (-\dot{R} - PQ) + I_{yz} (R^2 - Q^2)
\end{aligned}
$$

$$
\begin{aligned}
M &= I_{yy} \dot{Q} - I_{xy} \dot{P} - I_{yz} \dot{R} + R\, (I_{xx} P - I_{xy} Q - I_{xz} R) - P\, (I_{zz} R - I_{xz} P - I_{yz} Q) \\
&= I_{yy} \dot{Q} - I_{xy} \dot{P} - I_{yz} \dot{R} + I_{xx} PR - I_{xy} QR - I_{xz} R^2 - I_{zz} PR + I_{xz} P^2 + I_{yz} PQ \\
&= I_{yy} \dot{Q} + (I_{xx} - I_{zz})\, PR + I_{xy} (-\dot{P} - QR) + I_{xz} (P^2 - R^2) + I_{yz} (PQ - \dot{R})
\end{aligned}
$$

$$
\begin{aligned}
N &= I_{zz} \dot{R} - I_{xz} \dot{P} - I_{yz} \dot{Q} + P\, (I_{yy} Q - I_{xy} P - I_{yz} R) - Q\, (I_{xx} P - I_{xy} Q - I_{xz} R) \\
&= I_{zz} \dot{Q} - I_{xz} \dot{P} - I_{yz} \dot{Q} + I_{yy} PQ - I_{xy} P^2 - I_{yz} PR - I_{xx} PQ + I_{xy} Q^2 + I_{xz} QR \\
&= I_{zz} \dot{Q} + (I_{yy} - I_{xx})\, PQ + I_{xy} (Q^2 - P^2) + I_{xz} (QR - \dot{P}) + I_{yz} (-PR - \dot{Q})
\end{aligned}
$$

$$\dot{P} = \frac{1}{I_{xx}} [(I_{yy} - I_{zz})\, QR + I_{xy} (\dot{Q} - PR) + I_{xz} (\dot{R} + PQ) + I_{yz} (Q^2 - R^2) + L]$$

$$\dot{Q} = \frac{1}{I_{yy}} [(I_{zz} - I_{xx}) PR + I_{xy} (\dot{P} + QR) + I_{xz} (R^2 - P^2) + I_{yz} (\dot{R} - PQ) + M]$$

$$\dot{R} = \frac{1}{I_{zz}} [(I_{xx} - I_{yy}) PQ + I_{xy} (P^2 - Q^2) + I_{xz} (\dot{P} - QR) + I_{yz} (\dot{Q} + PR) + N]$$

Expanding the L, M and N terms, we can write the complete $\dot{P}$, $\dot{Q}$ and $\dot{R}$ equations.

$$\dot{P} = \frac{1}{I_{xx}} [(I_{yy} - I_{zz}) QR + I_{xy} (\dot{Q} - PR) + I_{xz} (\dot{R} + PQ) + I_{yz} (Q^2 - R^2) + \bar{q}Sb (C_{\ell_\beta} \cdot \beta + C_{\ell_{\delta a}} \cdot \delta a + C_{\ell_{\delta r}} \cdot \delta r + \frac{b}{2V} (C_{\ell_p} \cdot P + C_{\ell_R} \cdot R))] \quad (3)$$

$$\dot{Q} = \frac{1}{I_{yy}} [(I_{zz} - I_{xx}) PR + I_{xy} (\dot{P} + QR) + I_{xz} (R^2 - P^2) + I_{yz} (\dot{R} - PQ) + \bar{q}Sc (C_{m_o} + C_{m_\alpha} \cdot \alpha + C_{m_{\delta e}} \cdot \delta e + \frac{c}{2V} (C_{m_Q} \cdot Q))] \quad (4)$$

$$\dot{R} = \frac{1}{I_{zz}} [(I_{xx} - I_{yy}) PQ + I_{xy} (P^2 - Q^2) + I_{xz} (\dot{P} - QR) + I_{yz} (\dot{Q} + PR) + \bar{q}Sb (C_{n_\beta} \cdot \beta + C_{n_{\delta a}} \cdot \delta a + C_{n_{\delta r}} \cdot \delta r + \frac{b}{2V} (C_{n_p} \cdot P + C_{n_R} \cdot R))] \quad (5)$$

Summary of Equations and Assumptions

1. $$\dot{\beta} = P \sin \alpha - R \cos \alpha + \frac{g \sin \phi \cos \theta}{V \cos \beta} - \frac{g}{V} \sin \theta \tan \beta + \frac{\bar{q}\ S}{m\ V \cos \beta} (C_{y_\beta} \cdot \beta + C_{y_{\delta a}} \cdot \delta a + C_{y_{\delta r}} \cdot \delta r)$$

2. $$\dot{\alpha} = Q - P \tan \beta \cos \alpha + \frac{g \cos \theta \cos \phi}{V \cos \alpha \cos \beta} - \frac{\bar{q}\ S}{m\ V \cos \alpha \cos \beta} (C_{N_o} + C_{N_\alpha} \cdot \alpha + C_{N_{\delta e}} \cdot \delta e)$$

3. $$\dot{P} = \frac{1}{I_{xx}} [(I_{yy} - I_{zz})\ QR + I_{xy}\ (\dot{Q} - PR) + I_{xz}\ (\dot{R} + PQ) + I_{yz}\ (Q^2 - R^2)$$
$$+ \bar{q}Sb\ (C_{\ell_\beta} \cdot \beta + C_{\ell_{\delta a}} \cdot \delta a + C_{\ell_{\delta r}} \cdot \delta r + \frac{b}{2V}\ (C_{\ell_P} \cdot P + C_{\ell_R} \cdot R))]$$

4. $$\dot{Q} = \frac{1}{I_{yy}} [(I_{zz} - I_{xx})\ PR + I_{xy}\ (\dot{P} + QR) + I_{xz}\ (R^2 - P^2) + I_{yz}\ (\dot{R} - PQ)$$
$$+ \bar{q}Sc\ (C_{m_o} + C_{m_\alpha} \cdot \alpha + C_{m_{\delta e}} \cdot \delta e + \frac{c}{2V}\ (C_{m_Q} \cdot Q))]$$

5. $$\dot{R} = \frac{1}{I_{zz}} [(I_{xx} - I_{yy})\ PQ + I_{xy}\ (P^2 - Q^2) + I_{xz}\ (\dot{P} - QR) + I_{yz}\ (\dot{Q} + PR)$$
$$+ \bar{q}Sb\ (C_{n_\beta} \cdot \beta + C_{n_{\delta a}} \cdot \delta a + C_{n_{\delta r}} \cdot \delta r + \frac{b}{2V}\ (C_{n_P} \cdot P + C_{n_R} \cdot R))]$$

Assumptions

1. $\dot{V} = 0$ ($\dot{\alpha}$ and $\dot{\beta}$ equations)
2. $\sin\alpha \tan\beta \simeq 0$ ($\dot{\alpha}$ equation)
3. $\tan\alpha \tan\beta \approx 0$ ($\dot{\alpha}$ equation)
4. Constant inertias ($\dot{P}$, $\dot{Q}$, $\dot{R}$ equations)
5. Constant mass ($\dot{\alpha}$, $\dot{\beta}$ equations)

SECTION II

Longitudinal Equations of Motion

$$\dot{Q} = \frac{1}{I_{yy}} [(I_{zz} - I_{xx}) PR + I_{xy} (\dot{P} + QR) + I_{xz} (R^2 - P^2) + I_{yz} (\dot{R} - PQ)$$

$$\bar{q}Sc (C_{m_o} + C_{m_\alpha} \cdot \alpha + C_{m_{\delta e}} \cdot \delta e + \frac{c}{2V} (C_{m_Q} \cdot Q))]$$

$$\dot{\alpha} = Q - P \tan \beta \cos \alpha + \frac{g \cos \theta \cos \phi}{V \cos \alpha \cos \beta} - \frac{\bar{q} S}{m V \cos \alpha \cos \beta} (C_{N_o} + C_{N_\alpha} \cdot \alpha$$

$$+ C_{N_{\delta e}} \cdot \delta e)$$

These are the longitudinal equations for a two degree of freedom analysis. Usually thrust and drag derivatives are available from other sources and are not worth the extra effort to obtain them again. This results in the omission of the third longitudinal equation, the $\dot{U}$ equation. The equations in their present form are not suitable for MMLE or CONTROL because they are non-linear and make use of terms which are not available. The simplifying assumptions or approximations which must be made are these.

1. There is no coupling between axes. For a longitudinal pulse, $P = R = 0$.
2. $I_{xy} = I_{yz} = 0$
3. $\cos \beta = 1$
4. The equations are valid around a perturbation point. This allows deletion of C_{m_o} and C_{N_o}, and means that local slopes or derivatives are all we will get.

If these assumptions are used, the equations simplify to the following.

$$\dot{Q} = \frac{\bar{q}Sc}{I_{yy}} [C_{m_\alpha} \cdot \alpha + C_{m_{\delta e}} \cdot \delta e + \frac{c}{2V} (C_{m_Q} \cdot Q)]^{27}$$

$$\dot{\alpha} = Q + \frac{g}{V} \frac{\cos \theta \cos \phi}{\cos \alpha} - \frac{\bar{q} S}{m V \cos \alpha} (C_{N_\alpha} \cdot \alpha + C_{N_{\delta e}} \cdot \delta e)$$

---

[27] Note that α and δe in the $\dot{Q}$ and $\dot{\alpha}$ equations are now perturbations from trim. Both programs do this automatically.

Not all of the terms in these equations are variable. Variables include α, δe and Q. All other parameters, as well as trigonometric functions will be held constant during the length of the match.

Lateral Directional Equations of Motion

$$\dot{P} = \frac{1}{I_{xx}} [(I_{yy} - I_{zz})\, QR + I_{xy}\, (\dot{Q} - PR) + I_{xz}\, (\dot{R} + PQ) + I_{yz}\, (Q^2 - R^2)$$

$$+ \bar{q}Sb\, (C_{\ell_\beta} \cdot \beta + C_{\ell_{\delta a}} \cdot \delta a + C_{\ell_{\delta r}} \cdot \delta r + \frac{b}{2V} (C_{\ell_p} \cdot P + C_{\ell_R} \cdot R))]$$

$$\dot{R} = \frac{1}{I_{zz}} [(I_{xx} - I_{yy})\, PQ + I_{xy}\, (P^2 - Q^2) + I_{xz}\, (\dot{P} - QR) + I_{yz}\, (\dot{Q} - PR)$$

$$+ \bar{q}Sb\, (C_{n_\beta} \cdot \beta + C_{n_{\delta a}} \cdot \delta a + C_{n_{\delta r}} \cdot \delta r + \frac{b}{2V} (C_{n_p} \cdot P + C_{n_R} \cdot R))]$$

$$\dot{\beta} = P \sin \alpha - R \cos \alpha + \frac{g \cos \theta \sin \phi}{V \cos \beta} - \frac{g \sin \theta \sin \beta}{V \cos \beta}$$

$$+ \frac{\bar{q}\, S}{m\, V \cos \beta}\, (C_{y_\beta} \cdot \beta + C_{y_{\delta a}} \cdot \delta a + C_{y_{\delta r}} \cdot \delta r)$$

These are the lateral-directional equations for three degrees of freedom. To linearize and uncouple them we need the following assumptions.

1. There is no coupling between axes. For a lateral-directional case, Q = 0
2. $I_{xy} = I_{yz} = 0$
3. $\cos \beta = 1$
4. $\sin \theta \sin \beta = 0$
5. $\sin \phi = \phi$. This condition is not completely necessary for a good match. It is important that the bank angle excursions from the trim bank angle be within small angle approximation range (±20°). The non-zero trim bank angle is accounted for by modification of the remainder of the term.

These assumptions allow us to write the following equations.

$$\dot{P} = \frac{I_{xz}}{I_{xx}} \dot{R} + \frac{\bar{q}Sb}{I_{xx}} [C_{\ell_\beta} \cdot \beta + C_{\ell_{\delta a}} \cdot \delta a + C_{\ell_{\delta r}} \cdot \delta r + \frac{b}{2V} (C_{\ell_p} \cdot P + C_{\ell_R} \cdot R)]$$

$$\dot{R} = \frac{I_{xz}}{I_{zz}} \dot{P} + \frac{\bar{q}Sb}{I_{zz}} [C_{n_\beta} \cdot \beta + C_{n_{\delta a}} \cdot \delta a + C_{n_{\delta r}} \cdot \delta r + \frac{b}{2V} (C_{n_p} \cdot P + C_{n_R} \cdot R)]$$

$$\dot{\beta} = P \sin \alpha - R \cos \alpha + \frac{g \cos \theta}{V} \phi + \frac{\bar{q}S}{mV} (C_{y_\beta} \cdot \beta + C_{y_{\delta a}} \cdot \delta a + C_{y_{\delta r}} \cdot \delta r)$$

The variable terms in the above equations are P, R, $\dot{P}$, $\dot{R}$, β, ϕ, δa and δr.

APPENDIX F
ROOT LOCUS DISCUSSION

A root locus plot can be a very useful tool for analyzing the response of dynamic systems. To better understand this tool, some fundamentals of the analysis method are presented here.

Any dynamic system can be represented by a set of differential equations which describe the motion at some future time. As a simple case, consider a spring-mass-damper system. The differential equation for this system (no input force) is:

$$m \frac{d^2x}{dt^2} + f \frac{dx}{dt} + kx = 0$$

where

x = displacement from reference

m = mass

f = friction

k = spring constant

Using Laplace transforms this equation can be rewritten as

$$x(s) = \frac{(mx + f)\ x_o}{ms^2 + fs + k}$$

The denominator of this polynomial is called the characteristic equation for this system, and the roots determine the nature of the time response to a given input. This roots may be real or complex. In the case of complex roots, the real portion of the equal to $\zeta\omega_n$ where

ζ = damping ratio

ω_n = undamped natural frequency

The imaginary part of the root is equal to $j\omega_d$, the damped frequency.

For a real root, the system response is nonoscillatory (that is, $j\omega_d = 0$), and the root is equal to 1/T where T is a time constant of the system.

This type of analysis works very well for a single-degree-of-freedom system with only one differential equation. For a more complex system such as an aircraft, a method known as state variables is used. A matrix (analogous to the characteristic equation) can be formulated and the eigenvalues of this matrix are the roots of the system. These roots, as before, are the values of $\zeta\omega_n$ and ω_d.

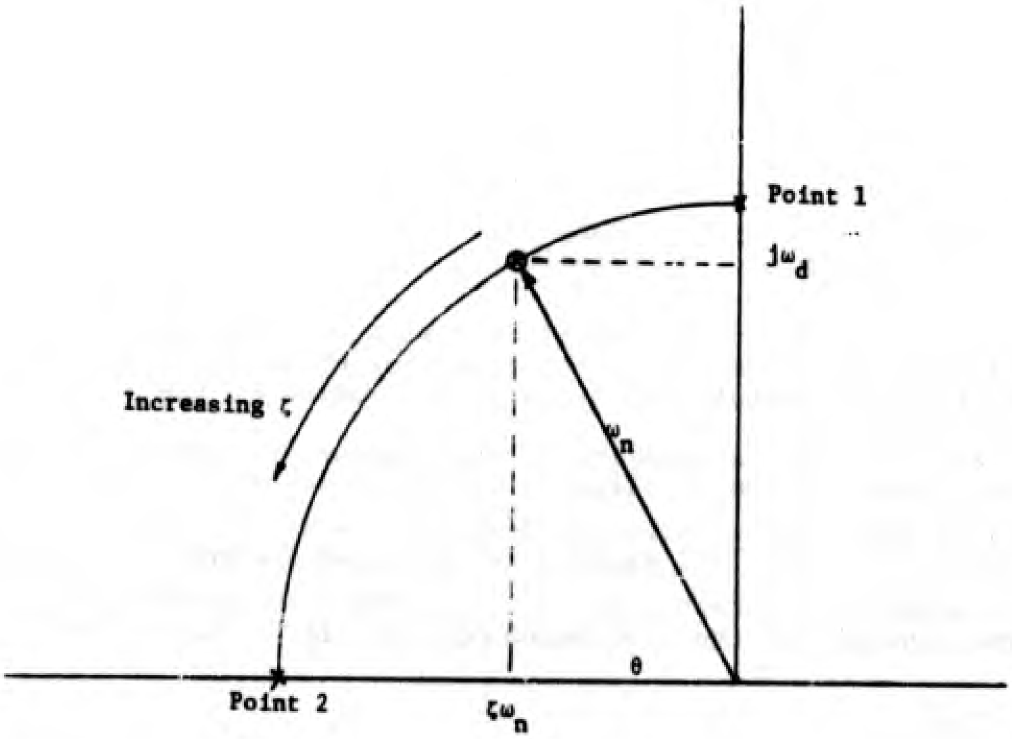


Figure F1 - Root Locus Presentation

The dynamic characteristics for any given mode of oscillation can be represented by the two parameters $\zeta\omega_n$ and ω_d. From these, ζ and ω_n may be calculated:

$$\omega_n = \sqrt{(\zeta\omega_n)^2 + (\omega_d)^2}$$

$$\zeta = \frac{\zeta\omega_n}{\omega_n}$$

The equation for ω_n is in the form of a circle from which these parameters may be displayed. It can be seen from Figure F1 that once the root locus point is plotted, a circle can be drawn through it, the radius of which is ω_n, the undamped natural frequency. Given ω_n, increasing ζ (from zero) will result in the movement of the root locus point from the ω_d axis (point 1), along the perimeter of the circle, until it reaches the $\zeta\omega_n$ axis (point 2). Point 2 corresponds to critical damping ($\zeta = 1$) where the oscillation becomes a periodic ($\omega_d = 0$).

The damping ratio ζ is equal to cosine θ. Thus for $\zeta = 0.707$, θ will equal 45 degrees. For a given ζ, θ remains constant, and varying the natural frequency results in changing the radius of the circle.

The three types of damping may be seen on the root locus plot. Underdamping is seen in the oscillatory points above the $\zeta\omega_n$ axis. As the damping is increased to critical damping ($\zeta = 1$) the root locus point will move to the $\zeta\omega_n$ axis. If damping is increased further, overdamping occurs. This is shown by the root locus points splitting and moving along the axis. The time constant of the overdamped mode can be calculated as

$$T = \frac{1}{\zeta\omega_n}$$

The many types of motion which can be portrayed and identified on a root locus plot are shown in Figure F2.

The limitations of this method should be pointed out. This is a linear model of the system, and as such does not take into consideration control surface rates, SAS authorities, turbulence, or pitch coupling. However, good results have been obtained with this method, as long as gains and motions are limited to reasonable bounds.

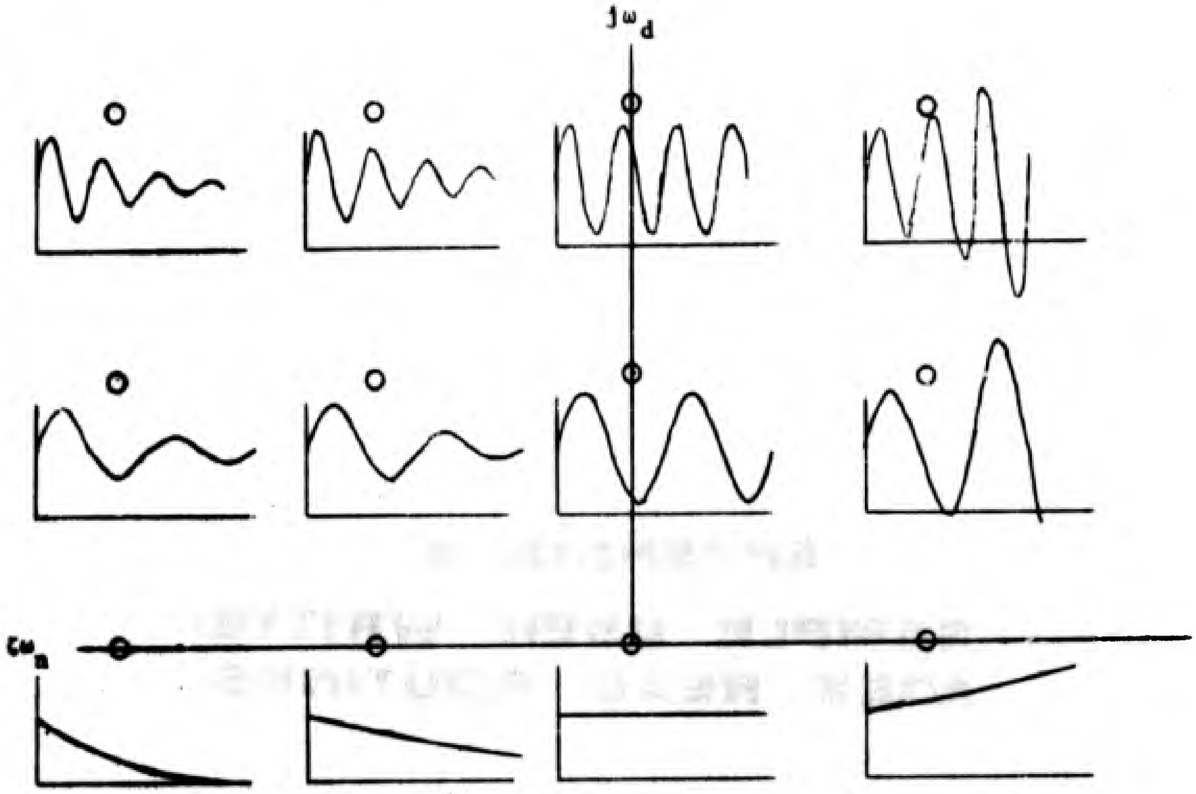


Figure F2 - Impulse Response Portrayed on a Root Locus Presentation

APPENDIX G

SAMPLE USER WRITTEN ADEX READ ROUTINES

SAMPLE STRANGER TAPE READ ROUTINE

THE FOLLOWING IS A SAMPLE PROGRAM WRITTEN TO READ AN X-24B STRANGER TAPE.

```
      PROGRAM  READX24
      COMMON/ARRAY/A(510.24)
      COMMON/SAVE/MODE.LIC.TAIL.ITEST.FLT.DFLT.IRUN.IPOINT.NMANE.ISFQ.
     1    KOUNT.NUMSEQ.INSEQ.INMANE.START.BREAK1.BREAK2.STOP.ISTART.
     2    IB1.IB2.ISTOP.IFTAPE.AIRCFT
      DIMENSION Z(184)
      REAL MILSEC
      I=1
      IF(INMANE.GT.0) GO TO 110
C     READ HEADER RECORD
      READ(15) HID.CDATE.FDATE.DUM.DUM.PTNOA.DUM.PTNOB
      WRITE(6.1009) CDATE.FDATE
 1009 FORMAT(1H1.*DATE OF COMPUTER RUN*.A10.1X.*DATE OF FLIGHT*.A10)
      GO TO 9999
C     READ DATA RECORD
  110 READ(15) ID.TIME.DATE.MILSEC.(Z(I).I=1.184)
      MILSEC=MILSEC/1000.
      IF(MILSEC.LT.START) GO TO 110
      IF(MILSEC.GT.STOP) GO TO 500
      IF(ABS(MILSEC-BREAK1).LE.0.02) IB1=I
      IF(ABS(MILSEC-BREAK2).LE.0.02) IB2=I
      IF(I.LT.510) GO TO 120
      I=510
      GO TO 500
  120 IF (ID .LT. 0) GO TO 110
C     ASSUME 50 SPS
      A(I.1)=Z(74)
      A(I.2)=Z(16)
      A(I.3)=Z(10)
      A(I.4)=Z(11)
      A(I.5)=Z(14)
      A(I.6)=Z(76)
      A(I.7)=Z(136)
      A(I.8)=Z(17)
      A(I.9)=Z(135)
      A(I.10)=Z(18)
      A(I.11)=Z(21)
      A(I.12)=Z(4)
      A(I.13)=Z(12)
      A(I.14)=Z(7)
      A(I.15)=MILSEC
      A(I.16)=0.0
      A(I.17)=0.0
      A(I.18)=0.0
      A(I.19)=Z(20)
      A(I.20)=Z(15)
      A(I.21)=Z(2)
```

(CONTINUED NEXT PAGE)

```
      A(I,22)=0.0
      A(I,23)=0.0
      A(I,24)=0.0
      I=I+1
      GO TO 110
  500 BACKSPACE 15
      ISTOP=I-1
      ISTART=1
 9999 CONTINUE
      END
```

SAMPLF CDAS TAPE READ ROUTINE

THE FOLLOWING IS SAMPLE SUBROUTINE WRITTEN TO READ A YF-16 CDAS TAPE.

```
      PROGRAM RDF16
C
C
C
C           THIS SUBROUTINE READS AN CDAS EDIT TAPE FOR THE AFFTC
C           DERIVATIVE EXTRACTION (ADEX) INTE FACE PROGRAM
      INTEGER CHECKD,CHECKC,REMARK
      DIMENSION NUM(100),PLAB1(100),PLAB2(100),PNAM1(24),PNAM2(24),
     -  REMARK(10),TITLE(10),Z(300)
      COMMON/ARRAY/A(510,24)
      COMMON/OLY1/NUMBER(50)
      COMMON/SAVE/MODE,LIC,TAIL,ITEST,FLT,DFLT,IRUN,IPOINT,NMANE,ISEQ,
     1    KOUNT,NUMSEQ,INSEQ,INMANE,START,BREAK1,BREAK2,STOP,ISTART,
     2    IB1,IB2,ISTOP,IFTAPE,AIRCFT
      DATA PNAM1/1HQ,1HP,6HALFAMN,5HBETAM,4HXNYM,1HR,4HBANK,
     -  5HDELEV,5HPITCH,10.,4HDRUD,5HQBART,4HXNZM,3HVTT,15.,16.,17.,
     -  18.,4HDAIL,20.,3HHCT,3HDTT,4HDLEF,24./
      DATA PNAM2/24*10H          /
      WRITE(6,900)
  900 FORMAT(1H1,*ENTERED SUBROUTINE RDF16*)
      I=1
      IF (INMANE .GT. 0) GO TO 100
C          READ CDAS LABEL RECORD
C
      READ(15) CHECKC
      BACKSPACE 15
      IF (CHECKC.EQ.1HC) GO TO 20
      WRITE(6,910)
  910 FORMAT(1H1,*CDAS LABEL CAN NOT BE FOUNDED*)
      GO TO 9999
   20 READ(15)CHECKC,LIC,ITAIL,TITLE,ITEST,FLT,DFLT,DREQ,DECOM
      WRITE(6,920)LIC,ITAIL,TITLE,ITEST,FLT,DFLT,DREQ,DECOM
  920 FORMAT(1H1,*LIC=*,I5,6X,*ITAIL=*,I4,
     1   //1X,*TITLE=*,5X,10A6
     2           //*ITEST=*,I7,6X,*FLT=*,A5,6X,*DFLT=*,A6,6X,
     3    //1X,*DREQ=*,A8,6X,*DECOM=*,A6)
      GO TO 9999
C     READ BDAS LABEL RECORD
  100 READ(15) CHECKD
      BACKSPACE 15
      IF(CHECKD.EQ.5HLABEL) GO TO 105
      IF (CHECKD.NE.0) GO TO 300
C          NOT EQUAL TO D AND EQUAL TO ZERO - END OF BDAS
C     NOT EQUAL TO LABEL AND EQUAL TO ZERO-END OF BDAS
C          CHECK FOR END OF CDAS
      READ(15) CHECKD
      BACKSPACE 15
      IF(CHECKD.EQ.5HLABEL) GO TO 105
      WRITE(6,1000)
```

(CONTINUED NEXT PAGE)

```
1000 FORMAT(10X,*END OF DATA*)
     GO TO 400
 105 READ(15)CHECKD,NRSECT,NREM,(REMARK(J),J=1,NREM),NLAB,
    1         (PLAB1(J),PLAB2(J),NUM(J),J=1,NLAB)
     WRITE(6,1005)
1005 FORMAT(1X,*THE FOLLOWING BDAS LABEL IS BEING PROCESSED*)
     WRITE(6,1010)CHECKD,NRSECT,NREM,
    1   (REMARK(J),J=1,NREM)
1010 FORMAT(1H1,10X,*LABEL=*,A5,6X,
    1     *NRSECT=*,I4,6X,
    2     *NREM=*,I2,
    3     //10X,*REMARK=*,10X,(A10))
     WRITE(6,1015)NLAB,(PLAB1(J),PLAB2(J),NUM(J),J=1,NLAB)
1015 FORMAT (//10X,*NLAB=*,I3,
    1         /(10X,*PLAB1=*,A6,6X,
    2         *PLAB2=*,A6,6X,*NUM=*,I3))
     CALL LABSOR(PNAM1,PNAM2,PLAB1,PLAB2,NUM,NUMBER,24,NLAB)
 300 DO 310 J=1,300
 310 Z(J)=0.0
C    READ DATA RECORDS
 320 READ (15)NPAR,TIME,(Z(J),J=1,NPAR)
     IF(TIME.EQ.0.0) GO TO 100
     IF(TIME.LT.START) GO TO 320
     IF(ABS(TIME-BREAK1).LE.0.01) IB1=I
     IF(ABS(TIME-BREAK2).LE.0.01) IB2=I
     IF(I.EQ.1) SAMPLE=TIME
     IF(I.GT.510) GO TO 400
     IF(TIME.LE.STOP) GO TO 500
 400 ISTOP=I-1
     GO TO 9999
 500 IF(TIME.LT.SAMPLE) GO TO 320
     Z(NUMBER(1))=Z(NUMBER(1))*57.29
     Z(NUMBER(2))=Z(NUMBER(2))*57.29
     Z(NUMBER(3))=Z(NUMBER(3))*57.29
     Z(NUMBER(4))=Z(NUMBER(4))*57.29
     Z(NUMBER(6))=Z(NUMBER(6))*57.29
     Z(NUMBER(7))=Z(NUMBER(7))*57.29
     Z(NUMBER(9))=Z(NUMBER(9))*57.29
     A(I,1) =Z (NUMBER(1))
     A(I,2) =Z (NUMBER(2))
     A(I,3)=Z(NUMBER(3))
     A(I,4)=Z(NUMBER(4))
     A(I,5) =Z (NUMBER(5))
     A(I,6) =Z (NUMBER(6))
     A(I,7) =Z (NUMBER(7))
     A(I,8)=Z(NUMBER(8))
     A(I,9) =Z (NUMBER(9))
     A(I,10)=-2.0*(Z(NUMBER(19))+Z(NUMBER(22)))
     A(I,11)=Z(NUMBER(11))
     A(I,12)=Z(NUMBER(12))
     A(I,13) =Z (NUMBER(13))
     A(I,14)=Z(NUMBER(14))
     A(I,15)=TIME
     A(I,16)=Z(NUMBER(16))
```

(CONTINUED NEXT PAGE)

```
      A(I,17)=Z(NUMBER(17))
      A(I,18)=Z(NUMBER(18))
      A(I,19)=Z(NUMBER(19))
      A(I,20)=Z(NUMBER(20))
      A(I,21)=Z(NUMBER(21))
      A(I,22)=Z(NUMBER(22))
      A(I,23)=Z(NUMBER(23))
      A(I,24)=Z(NUMBER(24))
      I=I+1
      SAMPLE=SAMPLE+0.02
      GO TO 320
 9999 CONTINUE
      ISTART=1
      END
```

APPENDIX H
SAMPLE USER WRITTEN CONTROL ROUTINES

SAMPLE MATRIX SUBROUTINE

THE FOLLOWING IS A SAMPLE MATRIX SUBROUTINE WRITTEN TO SET UP THE AIRCRAFT MATRICES.

```
      SUBROUTINE MATRIX (A,B,C,H,G,F,K1,K2,K3,K4,D,W1,W2,W3,
     1MX,MY,MU,MS,MAT1,MAT2,MAT3,MAT4,MAT5,MAT6)
      INTEGER READ,SYSTEM,OUTPUT,FORM,CONTUR,SAV,CMAT,FRPS,TRESP,READ3
      INTEGER DIGITL, SCAPLT, ZOH
      REAL IX,IY,IZ,IXZ,IFREQ,KP,KQ,KR,KRA,K1,K2,K3,K4
      DIMENSION A(MX,MX),B(MX,MU),C(MX,MX),H(MY,MX),G(MY,MX),F(MY,MU),
     1K1(MU,MX),K2(MU,MX),K3(MU,MX),K4(MU,MX),D(MU,MU),
     2W1(MX,MX),W2(MX,MX),W3(MX,MX)
      COMMON/AC/WATE,IX,IY,IZ,IXZ,SAREA,BSPAN,CHORD,QBAR,VTRUE,ALPHA,
     1GAMMA,KRA
      COMMON/ACOND/ DELT,FINALT,IFREQ,FFREQ,DELFRQ,GAIN1,GAIN2,MM
      COMMON /COND/ READ,SYSTEM,OUTPUT,NX,NY,NU,NXC,NUC,N1,N2,DIGITL,
     -  CONTUR,NUMERS,FRPS,TRESP,MODEL,NSCALE,SAV,CMAT,NK2,IFLAG,IGO,
     -  FORM,IPT,READ3,MIXED,MULTRT,SCAPLT,ZOH,KOUNT,MILSPEC
      COMMON /DERIV/ CMA,CMDE1,CMDE2,CMDE3,CMDE4,CMQ,CNA,CNDE1,CNDE2,
     -    CNDE3,CNDE4,CNQ,CCA,CCDE1,CCDE2,CCDE3,CCDE4,CCQ,CLB,CLDC1,
     -    CLDC2,CLDC3,CLDC4,CLP,CLR,CNB,CNDC1,CNDC2,CNDC3,CNDC4,CNP,CNR,
     -    CYB,CYDC1,CYDC2,CYDC3,CYDC4
      COMMON /SUBWRIT/ ISUBNAM,ISEQ,NREP
      IF (ISUBNAM .EQ. 2) WRITE (3,990)
  990 FORMAT(1X,*MATRIX SUB 2*)
C     MATRIX IS A USER WRITTEN SUBROUTINE WHICH IS PROJECT SPECIFIC.
C     IT SHOULD BE LOADED AND COMPILED USING THE "COPYL" ROUTINE.
C
  100 FORMAT(8F10.4)
      READ(1,100) AMACH,Q,V,ALFA,GAMMA,S,B,C
      READ(1,100) W,IX,IY,IZ,IXZ,KP,KR,KRA
      READ(1,100) CLB,CLDA,CLDR,CLP,CLR
      READ(1,100) CNB,CNDA,CNDR,CNP,CNR
      READ(1,100) CYB,CYDA,CYDR
      QSB=Q*S*B
      QSBB=Q*S*B*B
      QSMV=Q*S*32.174/W/V
      A(1,1)=QSBB*CLP/IX
      A(1,2)=QSBB*CLR/IX
      A(1,3)=QSB*CLB/IX
      A(2,1)=QSBB*CNP/IZ
      A(2,2)=QSBB*CNR/IZ
      A(2,3)=QSB*CNB/IZ
      A(3,1)=SIN(ALFA/57.3)
      A(3,2)=-COS(ALFA/57.3)
      A(3,3)=QSMV*CYB
      A(3,4)=32.174/V
      A(4,1)=1.0
      B(6,1)=QSB*CLDA/IX
      B(6,2)=QSB*CLDR/IX
      B(7,1)=QSB*CNDA/IZ
```

(CONTINUED NEXT PAGE)

```
      B(7,2)=QSH*CNDR/IZ
      DO 1 I=1,4
    1 C(I,I)=1.0
      C(1,2)=-IXZ/IX
      C(2,1)=-IXZ/IZ
      D(1,1)=1.0
      D(2,2)=1.0
      DO 2 I=1,4
    2 H(I,I)=1.0
      K1(1,1)=0.1
      K1(2,1)=-KRA
      K3(1,1)=KP
      K3(2,5)=KR
      RETURN
      END
```

SAMPLE CHANGE SUBROUTINE

THE FOLLOWING IS A SAMPLE CHANGE SUBROUTINE WRITTEN TO CHANGE PARAMETERS AS A FUNCTION OF THE KOUNT PARAMETER.

```
      SUBROUTINE CHANGE (A,B,C,H,G,F,K1,K2,K3,K4,D,W1,W2,W3,       DA    10
     1MX,MY,MU,MS,MAT1,MAT2,MAT3,MAT4,MAT5,MAT6)                   DA    20
      INTEGER READ,SYSTEM,OUTPUT,FORM,CONTUR,SAV,CMAT,READ3, FRPS,TRESP DA 70
      INTEGER DIGITL,SCAPLT,ZOH,GRAPH,BLOCK,STATE,YTOV,ZTOU,YZTOK
      REAL K1,K2,K3,K4,IFREQ,M,NUMER
      DIMENSION W1(MX,MX),W2(MX,MX),W3(MX,MX)                      DA    30
      DIMENSION A(MX,MX),B(MX,MU),C(MX,MX),H(MY,MX),G(MY,MX),F(MY,MU), DA 110
     1K1(MU,MX),K2(MU,MX),K3(MU,MX),K4(MU,MX),D(MU,MU)             DA   120
      DIMENSION GRAPH(20,5),BLOCK(20,3),NUMER(20,5),DENOM(20,5),GAIN(20)
     -  ,STATE(20,4),ITHINY(25),ITHINU(20),YTOV(20,2),ZTOU(20,2),NXYU(8)
     -  ,YZTOK(20,2)
      COMMON/ACOND/DELT,FINALT,IFREQ,FFREQ,DELFREQ,GAIN1,GAIN2,M   DA    90
      COMMON/BLKDAT/NUMER,DENOM,GAIN,GRAPH,BLOCK,STATE,YTOV,ZTOU,YZTOK, DA 130
     1ITHINY,ITHINU,NBLOCK,NYTOV,NZTOU,NXYU,NYZTOK,NXT,NYT,NUT,NY1,NU1 DA 140
      COMMON /COND/ READ,SYSTEM,OUTPUT,NX,NY,NU,NXC,NUC,N1,N2,DIGITL,
     -  CONTUR,NUMERS,FRPS,TRESP,MODEL,NSCALE,SAV,CMAT,NK2,IFLAG,IGO,
     -  FORM,IPT,READ3,MIXED,MULTRT,SCAPLT,ZOH,KOUNT,MILSPEC
      COMMON /SUBWRIT/ ISUBNAM,ISEQ,NREP                           DA   200
C                                                                  DA   210
C     USER WRITTEN SUBROUTINE TO CHANGE SYSTEM PARAMETERS SET UP IN DA   220
C     PREVIOUS CASE                                                DA   230
C
      IF(ISUBNAM.GE.2) WRITE(3,990)                                DA   260
  990 FORMAT(1X,*CHANGE*)                                          DA   290
      IF (KOUNT.GE.32) GO TO 60                                    DA   300
      IF (KOUNT.GE.21) GO TO 50                                    DA   310
      IF (KOUNT.GE.19) GO TO 40                                    DA   320
      IF (KOUNT.GE.17) GO TO 30                                    DA   330
      IF (KOUNT.GE.6) GO TO 20                                     DA   340
      IF (KOUNT.GE.4) GO TO 10                                     DA   350
C                                                                  DA   360
C EXAMPLE 1  OPEN LOOP                                             DA   370
C                                                                  DA   380
      IF (KOUNT.EQ.3) IFLAG=0                                      DA   390
      NUMERS=2                                                     DA   400
      READ (1,1) A(1,3)                                            DA   410
      IF (EOF(1).NE.0) STOP                                        DA   420
    1 FORMAT (7F10.4)                                              DA   430
      RETURN                                                       DA   440
C                                                                  DA   450
C  EXAMPLE 2  ROOT LOCUS                                           DA   460
C                                                                  DA   470
   10 READ  (1,1) A(2,3)                                           DA   480
      IF (EOF(1).NE.0) STOP                                        DA   490
      IF (KOUNT.EQ.5) IFLAG=0                                      DA   500
      RETURN
```

(CONTINUED NEXT PAGE)

```
C
C EXAMPLE 3  ROOT CONTOUR
C
   20 IF (KOUNT.EQ.16) IFLAG=0
      READ  (1,1) A(2,3)
      IF (EOF(1).NE.0) STOP
      RETURN
C
C EXAMPLE 4  CLASS CHECKCASE
C
   30 IF (KOUNT.EQ.18) IFLAG=0
      READ (1,1) GAIN(2)
      IF (EOF(1).NE.0) STOP
      IGO=1
      CALL CLASS (A,B,C,H,G,F,D,W1,W2,W3,
     1MX,MY,MU,MS,MAT1,MAT2,MAT3,MAT4,MAT5,MAT6)
      RETURN
C
C EXAMPLE 5  MIXED SYSTEM
C
   40 IF (KOUNT.EQ.20) IFLAG=0
      READ (1,1) GAIN(1)
      IF (EOF(1).NE.0) STOP
      IGO=1
      RETURN
C
C BENDING MODES ROOT LOCUS (CONTOUR)
C
   50 IF (KOUNT.EQ.31) IFLAG=0
      GAIN(6)=GAIN(6)+.1
      IGO=0
      RETURN
   60 CONTINUE
      RETURN
      END
```

SAMPLE INPUTV SUBROUTINE

THE FOLLOWING IS A SAMPLE INPUTV SUBROUTINE WRITTEN TO SET UP A STEP INPUT.

```
      SUBROUTINE INPUTV(DELT,T,U,
     1MX,MY,MU,MS,MAT1,MAT2,MAT3,MAT4,MAT5,MAT6)
C                                                                       9520
C     USER WRITTEN SUBROUTINE CONSTRUCTING INPUT VECTOR FOR TRANSIENT   9530
C     RESPONSE.                                                         9540
C                                                                       9550
      INTEGER READ,SYSTEM,OUTPUT,FORM,CONTUR,SAV,CMAT,READ3, FRPS,TRESP  9480
      INTEGER DIGITL,SCAPLT,ZOH
      DIMENSION U(MX)                                                   9510
      COMMON /COND/ READ,SYSTEM,OUTPUT,NX,NY,NU,NXC,NUC,N1,N2,DIGITL,
     -  CONTUR,NUMERS,FRPS,TRESP,MODEL,NSCALE,SAV,CMAT,NK2,IFLAG,IGO,
     -  FORM,IPT,READ3,MIXED,MULTRT,SCAPLT,ZOH,KOUNT,MILSPEC
      COMMON /SUBWRIT/ ISUBNAM,ISEQ,NREP
      IF (ISUBNAM .GE. 2) WRITE (3,990)
  990 FORMAT(1X,*INPUTV*)
      IF (T.GT.0.0) RETURN                                              9560
      READ (1,1) (U(I),I=1,NU)                                          9570
      IF (EOF(1).NE.0) STOP                                         EAF80772
    1 FORMAT (8F10.4)                                                   9580
      RETURN                                                            9590
      END                                                               9600
```

list of abbreviations and symbols

Symbol	Definition	Units
a	Acceleration	- - -
$\hat{A}$	Matrix containing stability and damping derivatives	- - -
$\hat{A}A$	Matrix to determine which of the corresponding terms in the $\hat{A}$ matrix will be allowed to vary	- - -
$\hat{A}P$	Matrix to define the observation vector in MMLE	- - -
$AP\hat{R}A$	A prior weighting values for corresponding terms in the $\hat{A}R$ matrix	- - -
$AP\hat{R}B$	A priori weighting values for corresponding terms in the $\hat{B}R$ matrix	- - -
$\hat{A}R$	A priori starting values of stability and damping derivatives	- - -
b	Reference span	ft
B	Matrix containing the control derivatives	- - -
$\hat{B}B$	Matrix to determine which of the corresponding terms in the $\hat{B}$ matrix will be allowed to vary	- - -
$\hat{B}P$	Matrix to define the observation vector in MMLE	- - -
$\hat{B}R$	A priori starting values of the control derivatives	- - -
c	Reference chord	ft
$\hat{C}$	Acceleration matrix in CONTROL	- - -
$C_{o_{1,2,3,4}}$	Chord force aerodynamic biases for the first four of NCASE maneuvers	g's
$C_{1/2}$	Number of cycles to damp to half amplitude	dimensionless
C_c	Chord force coefficient	dimensionless

Symbol	Definition	Units
C_{c_o}	Chord force coefficient bias	dimensionless
C_{c_Q}	$\partial C_c/\partial(Qc/2V)$	per rad, per deg
C_{c_V}	$\frac{V}{2}\ (\partial C_c/\partial V)$	dimensionless
C_{c_α}	$\partial C_c/\partial\alpha$	per rad, per deg
$C_{c_{\delta e}}$	$\partial C_c/\partial\delta e$	per rad, per deg
$C_{c_{\delta e_1}}$	$\partial C_c/\partial\delta e_1$	per rad, per deg
$C_{c_{\delta e_2}}$	$\partial C_c/\partial\delta e_2$	per rad, per deg
$C_{c_{\delta e_3}}$	$\partial C_c/\partial\delta e_3$	per rad, per deg
$C_{c_{\delta e_4}}$	$\partial C_c/\partial\delta e_4$	per rad, per deg
cg	Center of gravity	percent c
C_ℓ	Rolling moment coefficient	dimensionless
C_{ℓ_P}	$\partial C_\ell/\partial(Pb/2V)$	per rad, per deg
C_{ℓ_R}	$\partial C_\ell/\partial(Rb/2V)$	per rad, per deg
C_{ℓ_β}	$\partial C_\ell/\partial\beta$	per rad, per deg
$C_{\ell_{\delta a}}$	$\partial C_\ell/\partial\delta a$	per rad, per deg
$C_{\ell_{\delta c_1}}$	$\partial C_\ell/\partial\delta c_1$	per rad, per deg
$C_{\ell_{\delta c_2}}$	$\partial C_\ell/\partial\delta c_2$	per rad, per deg
$C_{\ell_{\delta c_3}}$	$\partial C_\ell/\partial\delta c_3$	per rad, per deg

Symbol	Definition	Units
$C_{\ell_{\delta c_4}}$	$\partial C_\ell/\partial \delta c_4$	per rad, per deg
$C_{\ell_{\delta r}}$	$\partial C_\ell/\partial \delta r$	per rad, per deg
C_m	Pitching Moment coefficient	dimensionless
C_{m_o}	Pitching moment coefficient bias when $\alpha = \delta e = 0$	dimensionless
C_{m_Q}	$\partial C_m/\partial (Qc/2V)$	per rad, per deg
C_{m_V}	$\frac{V}{2}\ (\partial C_m/\partial V)$	dimensionless
C_{m_α}	$\partial C_m/\partial \alpha$	per rad, per deg
$C_{m_{\delta e}}$	$\partial C_m/\partial \delta e$	per rad, per deg
$C_{m_{\delta e_1}}$	$\partial C_m/\partial \delta e_1$	per rad, per deg
$C_{m_{\delta e_2}}$	$\partial C_m/\partial \delta e_2$	per rad, per deg
$C_{m_{\delta e_3}}$	$\partial C_m/\partial \delta e_3$	per rad, per deg
$C_{m_{\delta e_4}}$	$\partial C_m/\partial \delta e_4$	per rad, per deg
C_n	Yawing moment coefficient	dimensionless
C_{n_P}	$\partial C_n/\partial (Pb/2V)$	per rad, per deg
C_{n_R}	$\partial C_n/\partial (Rb/2V)$	per rad, per deg
C_{n_β}	$\partial C_n/\partial \beta$	per rad, per deg
$C_{n_\beta^*}$	Dynamic C_{n_β}	per rad, per deg
$C_{n_{\delta a}}$	$\partial C_n/\partial \delta a$	per rad, per deg

Symbol	Definition	Units
$C_{n_{\delta c_1}}$	$\partial C_n/\partial \delta c_1$	per rad, per deg
$C_{n_{\delta c_2}}$	$\partial C_n/\partial \delta c_2$	per rad, per deg
$C_{n_{\delta c_3}}$	$\partial C_n/\partial \delta c_3$	per rad, per deg
$C_{n_{\delta c_4}}$	$\partial C_n/\partial \delta c_4$	per rad, per deg
$C_{n_{\delta r}}$	$\partial C_n/\partial \delta r$	per rad, per deg
C_N	Total normal force coefficient	dimensionless
C_{N_o}	Normal force coefficient bias when $\alpha = \delta e = 0$	dimensionless
C_{N_Q}	$\partial C_N/\partial (Qc/2V)$	per rad, per deg
C_{N_V}	$\frac{V}{2}\ (\partial C_N/\partial V)$	dimensionless
C_{N_α}	$\partial C_N/\partial \alpha$	per rad, per deg
$C_{N_{\delta e}}$	$\partial C_N/\partial \delta e$	per rad, per deg
$C_{N_{\delta e_1}}$	$\partial C_N/\partial \delta e_1$	per rad, per deg
$C_{N_{\delta e_2}}$	$\partial C_N/\partial \delta e_2$	per rad, per deg
$C_{N_{\delta e_3}}$	$\partial C_N/\partial \delta e_3$	per rad, per deg
$C_{N_{\delta e_4}}$	$\partial C_N/\partial \delta e_4$	per rad, per deg
C_Q	$\frac{\bar{q}Sc}{2VW}\ C_{c_Q}$	g's/$\frac{\text{rad}}{\text{sec}}$, g's/$\frac{\text{deg}}{\text{sec}}$
C_V	$\frac{2\bar{q}S}{VW}\ C_{c_V}$	g's/$\frac{\text{ft}}{\text{sec}}$

Symbol	Definition	Units
C_y	Total side force coefficient	dimensionless
C_{y_P}	$\partial C_y/\partial(Pb/2V)$	per rad, per deg
C_{y_R}	$\partial C_y/\partial(Rb/2V)$	per rad, per deg
C_{y_β}	$\partial C_y/\partial\beta$	per rad, per deg
$C_{y_{\delta a}}$	$\partial C_y/\partial\delta a$	per rad, per deg
$C_{y_{\delta c_1}}$	$\partial C_y/\partial\delta c_1$	per rad, per deg
$C_{y_{\delta c_2}}$	$\partial C_y/\partial\delta c_2$	per rad, per deg
$C_{y_{\delta c_3}}$	$\partial C_y/\partial\delta c_3$	per rad, per deg
$C_{y_{\delta c_4}}$	$\partial C_y/\partial\delta c_4$	per rad, per deg
$C_{y_{\delta r}}$	$\partial C_y/\partial\delta r$	per rad, per deg
C_α	$\frac{\bar{q}S}{W} C_{c_\alpha}$	g's/rad, g's/deg
$C_{\delta e}$	$\frac{\bar{q}S}{W} C_{c_{\delta e}}$	g's/rad, g's/deg
$C_{\delta e_1}$	$\frac{\bar{q}S}{W} C_{c_{\delta e_1}}$	g's/rad, g's/deg
$C_{\delta e_2}$	$\frac{\bar{q}S}{W} C_{c_{\delta e_2}}$	g's/rad, g's/deg
$C_{\delta e_3}$	$\frac{\bar{q}S}{W} C_{c_{\delta e_3}}$	g's/rad, g's/deg
$C_{\delta e_4}$	$\frac{\bar{q}S}{W} C_{c_{\delta e_4}}$	g's/rad, g's/deg

Symbol	Definition	Units
d	Differential operator	- - -
$\hat{D}$	Input vector definition matrix in CONTROL	- - -
$\hat{D}1$	Signal noise weighting matrix in MMLE	- - -
E_i	Weighted relative errors in MMLE	- - -
f	Friction	- - -
F	Total resultant force	lbs
Fn	Net thrust	lbs
g	Acceleration of gravity, 32.17495 ft/sec^2	ft/sec^2
$\hat{G}$	Output vector definition matrix in CONTROL	- - -
h	Altitude	ft
H	Angular momentum	$slug\text{-}ft^2/sec$
$\hat{H}$	Output vector definition matrix in CONTROL	- - -
i	Unit vector along the x-axis	- - -
I_{xx}	Moment of inertia about the x-axis	$slug\text{-}ft^2$
I_{xy}	Product of inertia about the x- and y-axes	$slug\text{-}ft^2$
I_{xz}	Product of inertia about the x- and z-axes	$slug\text{-}ft^2$
I_{yy}	Moment of inertia about the y-axis	$slug\text{-}ft^2$
I_{yz}	Product of inertia about the y- and z-axes	$slug\text{-}ft^2$
I_{zz}	Moment of inertia about the z-axis	$slug\text{-}ft^2$
j	Unit vector along the y-azis or imaginary axis	- - -
J	Cost function	- - -
k	Unit vector along the z-axis or spring constant	- - -

Symbol	Definition	Units
K	Transfer function dc gain	- - -
KAR	Aileron to rudder interconnect gain	deg/deg
K_P	Roll SAS gain	deg/deg/sec
K_Q	Pitch SAS gain	deg/deg/sec
K_R	Yaw SAS gain	deg/deg/sec
$\hat{K}1$	Input vector feedback matrix in CONTROL	- - -
$\hat{K}2$	Input vector feedback matrix in CONTROL	- - -
$\hat{K}3$	Input vector feedback matrix in CONTROL	- - -
$\hat{K}4$	Input vector feedback matrix in CONTROL	- - -
L	Total rolling moment	ft-lb
L_P	$\frac{\bar{q}Sb^2}{2VI_{xx}} C_{\ell_P}$	$\frac{rad}{sec^2}/\frac{rad}{sec}$, $\frac{rad}{sec^2}/\frac{rad}{sec}$
L_R	$\frac{\bar{q}Sb^2}{2VI_{xx}} C_{\ell_R}$	$\frac{rad}{sec^2}/\frac{rad}{sec}$, $\frac{rad}{sec^2}/\frac{deg}{sec}$
$L_{o_{1,2,3,4}}$	Rolling moment aerodynamic biases for the first four of NCASE maneuvers	rad/sec^2
L_β	$\frac{\bar{q}Sb}{I_{xx}} C_{\ell_\beta}$	$\frac{rad}{sec^2}$/rad, $\frac{rad}{sec^2}$/deg
$L_{\delta a}$	$\frac{\bar{q}Sb}{I_{xx}} C_{\ell_{\delta a}}$	$\frac{rad}{sec^2}$/rad, $\frac{rad}{sec^2}$/deg
$L_{\delta c_1}$	$\frac{\bar{q}Sb}{I_{xx}} C_{\ell_{\delta c_1}}$	$\frac{rad}{sec^2}$/rad, $\frac{rad}{sec^2}$/deg
$L_{\delta c_2}$	$\frac{\bar{q}Sb}{I_{xx}} C_{\ell_{\delta c_2}}$	$\frac{rad}{sec^2}$/rad, $\frac{rad}{sec^2}$/deg

Symbol	Definition	Units
$L_{\delta c_3}$	$\frac{\bar{q}Sb}{I_{xx}} C_{\ell_{\delta c_3}}$	$\frac{rad}{sec^2}/rad$, $\frac{rad}{sec^2}/deg$
$L_{\delta c_4}$	$\frac{\bar{q}Sb}{I_{xx}} C_{\ell_{\delta c_4}}$	$\frac{rad}{sec^2}/rad$, $\frac{rad}{sec^2}/deg$
$L_{\delta r}$	$\frac{\bar{q}Sb}{I_{xx}} C_{\ell_{\delta r}}$	$\frac{rad}{sec^2}/rad$, $\frac{rad}{sec^2}/deg$
m	Mass	slugs
M	Total pitching moment	ft-lb
Mn	Mach number	dimensionless
M_Q	$\frac{\bar{q}Sc^2}{2VI_{yy}} C_{m_Q}$	$\frac{rad}{sec^2}/\frac{rad}{sec}$, $\frac{rad}{sec^2}/\frac{deg}{sec}$
M_V	$\frac{2\bar{q}Sb}{VI_{yy}} C_{m_V}$	$\frac{rad}{sec^2}/\frac{ft}{sec}$
$M_{o_{1,2,3,4}}$	Pitching moment aerodynamic biases for the first four of NCASE maneuvers	rad/sec^2
M_α	$\frac{\bar{q}Sc}{I_{yy}} C_{m_\alpha}$	$\frac{rad}{sec^2}/rad$, $\frac{rad}{sec^2}/deg$
$M_{\delta e}$	$\frac{\bar{q}Sc}{I_{yy}} C_{m_{\delta e}}$	$\frac{rad}{sec^2}/rad$, $\frac{rad}{sec^2}/deg$
$M_{\delta e_1}$	$\frac{\bar{q}Sc}{I_{yy}} C_{m_{\delta e_1}}$	$\frac{rad}{sec^2}/rad$, $\frac{rad}{sec^2}/deg$
$M_{\delta e_2}$	$\frac{\bar{q}Sc}{I_{yy}} C_{m_{\delta e_2}}$	$\frac{rad}{sec^2}/rad$, $\frac{rad}{sec^2}/deg$
$M_{\delta e_3}$	$\frac{\bar{q}Sc}{I_{yy}} C_{m_{\delta e_3}}$	$\frac{rad}{sec^2}/rad$, $\frac{rad}{sec^2}/deg$
$M_{\delta e_4}$	$\frac{\bar{q}Sc}{I_{yy}} C_{m_{\delta e_4}}$	$\frac{rad}{sec^2}/rad$, $\frac{rad}{sec^2}/deg$
n	Load factor	g's

Symbol	Definition	Units
N	Total yawing moment	dimensionless
N_P	$\frac{\bar{q}Sb^2}{2VI_{zz}} C_{n_P}$	$\frac{rad}{sec^2}/\frac{rad}{sec}$, $\frac{rad}{sec^2}/\frac{deg}{sec^2}$
N_Q	$\frac{\bar{q}Sc}{2mV^2} C_{N_Q}$	$\frac{rad}{sec^2}/\frac{rad}{sec}$, $\frac{rad}{sec^2}/\frac{deg}{sec^2}$
N_R	$\frac{\bar{q}Sb^2}{2VI_{zz}} C_{n_R}$	$\frac{rad}{sec^2}/\frac{rad}{sec}$, $\frac{rad}{sec^2}/\frac{deg}{sec^2}$
N_V	$\frac{2\bar{q}S}{mV} C_{N_V}$	$\frac{rad}{sec^2}/\frac{ft}{sec}$
N_x	Longitudinal acceleration	g's
N_y	Lateral acceleration	g's
N_z	Normal acceleration	g's
$N_{o_{1,2,3,4}}$	Yawing moment aerodynamic biases for the first four of NCASE maneuvers in a lateral-direction matrix	$\frac{rad}{sec^2}$
$N_{o_{1,2,3,4}}$	$\dot{\alpha}$ aerodynamic biases for the first four of NCASE maneuvers in a longitudinal matrix	rad/sec
N_α	$\frac{\bar{q}S}{mV} C_{N_\alpha}$	$\frac{rad}{sec}/\frac{rad}{sec}$, $\frac{rad}{sec}/\frac{deg}{sec}$
N_β	$\frac{\bar{q}Sb}{I_{zz}} C_{n_\beta}$	$\frac{rad}{sec^2}$/rad, $\frac{rad}{sec^2}$/deg
$N_{\delta a}$	$\frac{\bar{q}Sb}{I_{zz}} C_{n_{\delta a}}$	$\frac{rad}{sec^2}$/rad, $\frac{rad}{sec^2}$/deg
$N_{\delta c_1}$	$\frac{\bar{q}Sb}{I_{zz}} C_{n_{\delta c_1}}$	$\frac{rad}{sec^2}$/rad, $\frac{rad}{sec^2}$/deg
$N_{\delta c_2}$	$\frac{\bar{q}Sb}{I_{zz}} C_{n_{\delta c_2}}$	$\frac{rad}{sec^2}$/rad, $\frac{rad}{sec^2}$/deg
$N_{\delta c_3}$	$\frac{\bar{q}Sb}{I_{zz}} C_{n_{\delta c_3}}$	$\frac{rad}{sec^2}$/rad, $\frac{rad}{sec^2}$/deg

Symbol	Definition	Units
$N_{\delta c_3}$	$\frac{\bar{q}Sb}{I_{zz}} C_{n_{\delta c_3}}$	$\frac{\text{rad}}{\text{sec}^2}$/rad, $\frac{\text{rad}}{\text{sec}^2}$/deg
$N_{\delta c_4}$	$\frac{\bar{q}Sb}{I_{zz}} C_{n_{\delta c_4}}$	$\frac{\text{rad}}{\text{sec}^2}$/rad, $\frac{\text{rad}}{\text{sec}^2}$/deg
$N_{\delta e}$	$\frac{\bar{q}S}{mV} C_{N_{\delta e}}$	$\frac{\text{rad}}{\text{sec}}$/rad, $\frac{\text{rad}}{\text{sec}}$/deg
$N_{\delta e_1}$	$\frac{\bar{q}S}{mV} C_{N_{\delta e_1}}$	$\frac{\text{rad}}{\text{sec}}$/rad, $\frac{\text{rad}}{\text{sec}}$/deg
$N_{\delta e_2}$	$\frac{\bar{q}S}{mV} C_{N_{\delta e_2}}$	$\frac{\text{rad}}{\text{sec}}$/rad, $\frac{\text{rad}}{\text{sec}}$/deg
$N_{\delta e_3}$	$\frac{\bar{q}S}{mV} C_{N_{\delta e_3}}$	$\frac{\text{rad}}{\text{sec}}$/rad, $\frac{\text{rad}}{\text{sec}}$/deg
$N_{\delta e_4}$	$\frac{\bar{q}S}{mV} C_{N_{\delta e_4}}$	$\frac{\text{rad}}{\text{sec}}$/rad, $\frac{\text{rad}}{\text{sec}}$/deg
N_{δ_r}	$\frac{\bar{q}Sb}{I_{zz}} C_{N_{\delta r}}$	$\frac{\text{rad}}{\text{sec}^2}$/rad, $\frac{\text{rad}}{\text{sec}^2}$/deg
P	Roll rate	rad/sec, deg/sec
$\bar{q}$	Dynamic pressure	lb/ft^2
Q	Pitch rate	rad/sec, deg/sec
r	Distance from cg to a incremental particle	- - -
R	Yaw rate	rad/sec, deg/sec
$\hat{R}$	Acceleration matrix in MMLE	- - -
s	Laplace transform variable	- - -
S	Reference area	ft^2
SAS	Control and/or stability augmentation system	- - -
SM	Static margin	percent C

Symbol	Definition	Units
t	Time	sec
T	Time constant for a nonoscillatory mode or the period for an oscillatory one	sec
$T_{1/2}$	Time to damp to one half amplitude	sec
u	Velocity component along the x-axis	ft/sec
$\hat{u}$	Input vector	- - -
v	Velocity component along the y-axis	ft/sec
$\hat{v}$	Transfer function block input vector	- - -
V	True velocity	ft/sec
V_e	Equivalent velocity	kts
w	Velocity component along the z-axis or w-transform variable	ft/sec
W	Gross weight	lbs
x	Distance along the x-axis	- - -
$\hat{x}$	State vector	- - -
X	Force component along the x-axis	lbs
y	Distance along the y-axis	- - -
Y	Force component along the y-axis	lbs
Y_p	$\frac{\bar{q}Sb}{2mV^2} C_{Y_p}$	$\frac{rad}{sec}/\frac{rad}{sec}$ $\frac{rad}{sec}/\frac{deg}{sec}$
Y_R	$\frac{\bar{q}Sb}{2mV^2} C_{Y_R}$	$\frac{rad}{sec}/\frac{rad}{sec}$ $\frac{rad}{sec}/\frac{deg}{sec}$
$Y_{o_{1,2,3,4}}$	$\dot{\beta}$ aerodynamic biases for the first four of NCASE maneuvers	rad/sec
Y_β	$\frac{\bar{q}S}{mV} C_{Y_\beta}$	$\frac{rad}{sec}$/rad, $\frac{rad}{sec}$/deg
$Y_{\delta a}$	$\frac{\bar{q}S}{mV} C_{Y_{\delta a}}$	$\frac{rad}{sec}$/rad, $\frac{rad}{sec}$/deg

Symbol	Definition	Units
$Y_{\delta c_1}$	$\frac{\bar{q}S}{mV} C_{Y_{\delta c_1}}$	$\frac{\text{rad}}{\text{sec}}$/rad, $\frac{\text{rad}}{\text{sec}}$/deg
$Y_{\delta c_2}$	$\frac{\bar{q}S}{mV} C_{Y_{\delta c_2}}$	$\frac{\text{rad}}{\text{sec}}$/rad, $\frac{\text{rad}}{\text{sec}}$/deg
$Y_{\delta c_3}$	$\frac{\bar{q}S}{mV} C_{Y_{\delta c_3}}$	$\frac{\text{rad}}{\text{sec}}$/rad, $\frac{\text{rad}}{\text{sec}}$/deg
$Y_{\delta c_4}$	$\frac{\bar{q}S}{mV} C_{Y_{\delta c_4}}$	$\frac{\text{rad}}{\text{sec}}$/rad, $\frac{\text{rad}}{\text{sec}}$/deg
$Y_{\delta r}$	$\frac{\bar{q}S}{mV} C_{Y_{\delta r}}$	$\frac{\text{rad}}{\text{sec}}$/rad, $\frac{\text{rad}}{\text{sec}}$/deg
z	Distance along the z-axis	- - -
$\hat{z}$	Transfer function block output vector	- - -
Z	Force component along the z-axis	lbs
α	Angle of attack	rad, deg
β	Angle of sideslip	rad, deg
γ	Flightpath angle	deg
δa	Aileron deflection	rad, deg
δc_1, δc_2, δc_3, δc_r	Four available lateral-directional control surfaces in MMLE and CONTROL	rad, deg
δe	Elevator	rad, deg
δe_1, δe_2, δe_3, δe_4	Four available longitudinal control surfaces in MMLE and CONTROL	rad, deg
δr	Rudder deflection	rad, deg
δs	Rolling tail deflection	rad, deg
Δ	Prefix meaning increment	- - -

Symbol	Definition	Units
ζ	Damping ratio	dimensionless
θ	Pitch angle	rad, deg
$\dot{\theta}_{o_{1,2,3,4}}$	$\dot{\theta}$ aerodynamic biases for the first four of NCASE maneuvers	rad/sec
ρ	Density of air	slug/ft^3
ϕ	Bank angle	rad, deg
$\dot{\phi}$	$\dot{\phi}$ aerodynamic biases for the first four of NCASE maneuvers	rad/sec
ω	Frequency for an oscillatory root or general expression for rotation	rad/sec, Hz
∂	Partial differentiation operator	- - -
SUBSCRIPTS		
c	computed	- - -
com	command	- - -
d	damped	- - -
f	filtered	- - -
m	measured	- - -
n	natural, undamped	- - -
s	standard, reference	- - -
sss	steady state sideslip	- - -
t	test	- - -
x	x component	- - -
y	y component	- - -
z	z component	- - -
SUPERSCRIPTS		
•	Denotes a differentiation with respect to time	- - -
^	Denotes a matrix	- - -

NOTE: All symbols, equations, notation, etc., are in the aircraft body axis.