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Investigation of subharmonic generation by finite-amplitude waves in a rigid-walled tube

Donnelly, John Jay

Monterey, California. Naval Postgraduate School

INVESTIGATION OF SUBHARMONIC GENERATION BY FINITE-AMPLITUDE WAVES IN A RIGID-WALLED TUBE

John Jay Donnelly

NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

INVESTIGATION OF SUBHARMONIC GENERATION
BY FINITE-AMPLITUDE WAVES
IN A RIGID-WALLED TUBE

by

John Jay Donnelly

June 1976

Thesis Advisor: J. V. Sanders

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by

John Jay Donnelly
Ensign, United States Navy
B.S., United States Naval Academy, 1975

Submitted in partial fulfillment of the
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Finite-amplitude standing waves contained within an air filled rigid-walled tube at ambient temperature and pressure were investigated experimentally. The pressure waveform in the tube was analyzed for subharmonic content for comparison with several existing theoretical models. Due to limitations of the experimental apparatus, the only model for which the predicted threshold for subharmonic generation could be exceeded was that of Coppens which predicts subharmonic generation when the strength parameter exceeds 2.0. Strength parameter is defined as $M_2\beta Q_1$ where M_2 is the Mach number of the driving frequency, $\beta = 1.2$ in air, and Q_1 is the quality factor associated with the subharmonic resonance. Strength parameters up to 2.89 were investigated and no threshold effect was observed.

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I. INTRODUCTION

The problem of large amplitude acoustic waves in dissipative media has been extensively studied. It is well known that an initially sinusoidal signal of large amplitude can distort so that the resulting waveform contains frequency components other than the original frequency. Usually, these additional frequency components are integer multiples (harmonics) of the original frequency. However, under certain conditions, components at frequencies lower than the original signal frequency may be generated. These lower frequency components are known as subharmonics.

The objective of this research is to investigate possible subharmonic generation with finite-amplitude standing waves in air at ambient temperatures in a rigid walled tube for comparison with several existing theoretical models.

II. BACKGROUND AND THEORY

In a nondissipative medium the hydrodynamic equations predict that an elastic wave will change waveform as it propagates. In real absorptive media experience tells us that only those waves of relatively high amplitude experience a change in waveform. Finite-amplitude waves are those waves which undergo harmonic distortion due to self-interaction.

The study of intense waves began in 1868 when Kirchoff [1] used the nonviscous hydrodynamic equations to predict a change in waveform for a traveling plane wave. The investigations of finite-amplitude effects in traveling waves continued [2, 3, 4, 5] but it was not until 1954 that Keller [6] developed solutions for standing waves excited by a piston in a closed tube. Keller assumed a nondissipative medium which resulted in infinite amplitude at resonance; however, his results were still useful for frequencies close to resonance. In 1960, Keck and Beyer [7] developed a non-linear solution for progressive waves in a viscous medium involving terms up to the sixth order by using pertubation methods and Fourier representation of the waveform. Coppens and Sanders [8] extended the Keck-Beyer pertubation approach to include both the bulk losses and wall losses as predicted by Rayleigh-Kirchoff [9] and showed excellent agreement with experiments conducted with standing waves in a rigid-walled tube at low levels of nonlinear interaction. The experimental work by Beech [10] with supporting calculations by Ruff [11] showed that at high excitation levels a significant difference developed between theory and experiment. Winn [12] experimentally demonstrated that the Rayleigh-Kirchoff loss mechanisms failed to describe the phase relationships observed in the distorted waveform. Coppens and Sanders [13] revised their model to utilize empirically determined losses and resonant frequencies, thus extending the excellent agreement between theory and experiment to higher excitation levels. A further extension of their model to two- and three-dimensions was experimentally investigated by Lane [14] and found to successfully predict the major features of the harmonic content for finite amplitude standing waves in a rigid walled cavity. De Vall [15] found that if degeneracies existed, the model failed to account for the experimentally observed excitation of non-family modes. The model was again revised and experimentally tested by Slocum [16] and was found to accurately predict the major features of the harmonic content of the finite amplitude standing wave whether or not degeneracies were present.

Investigation into the nonlinear generation of subharmonic frequencies began in 1965 when Korpel and Adler [17] and Breazeale and McCluney [18] independently observed the effects of acoustic subharmonic generation on optical diffraction patterns. Cook [19] attempted a mathematical explanation of this phenomenon based on the finite-amplitude plane wave interaction in a nondissipative medium. A spectrum analyzer was used to investigate subharmonic generation in water and characteristic frequency doublets which were split by about half the driving frequency (f_d) such that $f_1 + f_2 = f_d$ were observed. [20, 21]. Adler and Breazeale [22] used Mathieu's equation as applied to the case of a cavity with a moving boundary to arrive at a threshold value for subharmonic generation. This threshold, expressed in terms of physical parameters, is

$$h > c\alpha/\omega$$

where $h = \frac{A}{L_0} = \frac{\text{peak boundary amplitude}}{\text{mean length of cavity}}$

c = speed of sound in the medium

α = total absorption coefficient

$\omega = \omega_d/2$ = subharmonic angular frequency.

Yen [23] used an acoustic interferometer driven at 1.5 MHz with water as the medium to investigate the generation of a subharmonic doublet in two lower order modes. His threshold for subharmonic generation is

$$P_{TH} = \frac{8\rho c^2}{1+B/A}\left(\frac{1}{Q_1 Q_2} + \frac{\delta_1 \delta_2}{\omega_{01} \omega_{02}}\right)^{1/2}$$

where P_{TH} = threshold pressure for subharmonic generation in a one-dimensional closed region

ρ = density of the fluid

c = speed of sound

B/A = parameter of nonlinearity.

The Q's are the quality factors associated with the subharmonic modes. The δ's are detuning factors such that

$$\omega_{01} = \omega_1 + \delta_1$$

$$\omega_{02} = \omega_2 + \delta_2$$

$$\omega_{03} = \omega_3 + \delta_3$$

where ω_{01} and ω_{02} are the angular frequencies of the subharmonic doublet, ω_{03} is the driving frequency and the ω_i's are the angular frequencies of the resonant modes.

It is assumed that although the application of Yen's threshold to the case in which a single subharmonic component is generated in air at exactly half the driving frequency may not be entirely valid, the error introduced is not more than a factor of about two. By also assuming that the detuning factors are negligibly small, $Q_1 = Q_2$, $B/A = 2(\beta - 1)$, and Yen's threshold can be expressed as

$$P_{TH} = \frac{8\rho c^2}{Q_1}\left[\frac{1}{2\beta-1}\right]$$

$$= \frac{\rho c^2}{Q_1\beta}[\,6.86\,]$$

where $\beta = 1.2$ in air. Recognizing that the Mach number of the driven mode is

$$M_2 = \frac{U_2}{c} = \frac{P_2}{\rho c^2},$$

the threshold can be expressed as

$$M_2\beta Q_1 = 6.86.$$

The quantity $M_2 \beta Q_1$ characterizes the degree of the finite amplitude interaction and is known as the strength parameter. Expression of Yen's threshold in this form allows for direct comparison to the theoretically predicted threshold of Coppens [24] which is based on similar mechanisms.

Coppens' model begins with the solution to the one dimensional nonlinear wave equation. It is assumed that the harmonic components of the distorted driven waveform are not coupled with the subharmonic and are therefore negligible. The model predicts that above a certain threshold the driving level will be sufficient to stimulate a subharmonic response at exactly half the driving frequency. This threshold occurs when the strength parameter is 2.0.

This investigation was conducted with the objective to experimentally test the theory of Coppens and others in the area of finite amplitude generation of subharmonic frequencies.

III. EXPERIMENTAL CONSIDERATIONS

A. APPARATUS

A block diagram of the experimental system is shown in Figure 1. The resonant tube, containing air at ambient pressure and temperature, is six ft (1.83 m) in length with an inside diameter of 2.250 in. (5.72 cm) and has steel walls 1.125 in. (2.86 cm) thick.

Acoustic oscillations were excited by means of a piston motion at one end of the tube. Two pistons were used during the course of the investigation. Initially, a piston having a total mass of 0.528 kg and an internally mounted Endevco Model 2215 accelerometer was used. The output voltage of the accelerometer was measured on a Hewlett Packard Model 400D Vacuum Tube Voltmeter. The output of the voltmeter was displayed on a Tectronic Model 565 Dual-Beam Oscilloscope. Piston accelerations of up to 42 g were obtained in this configuration. To achieve the highest possible piston accelerations, a light-weight piston having a mass of 0.073 kg was subsequently used. The accelerometer could not be employed in the light-piston configuration. However, traveling microscope observations of the piston in open air indicated that peak accelerations up to 92 g were obtainable. Each piston had a single Vaseline-lubricated O-ring recessed into the piston wall to enable a good seal with the tube.

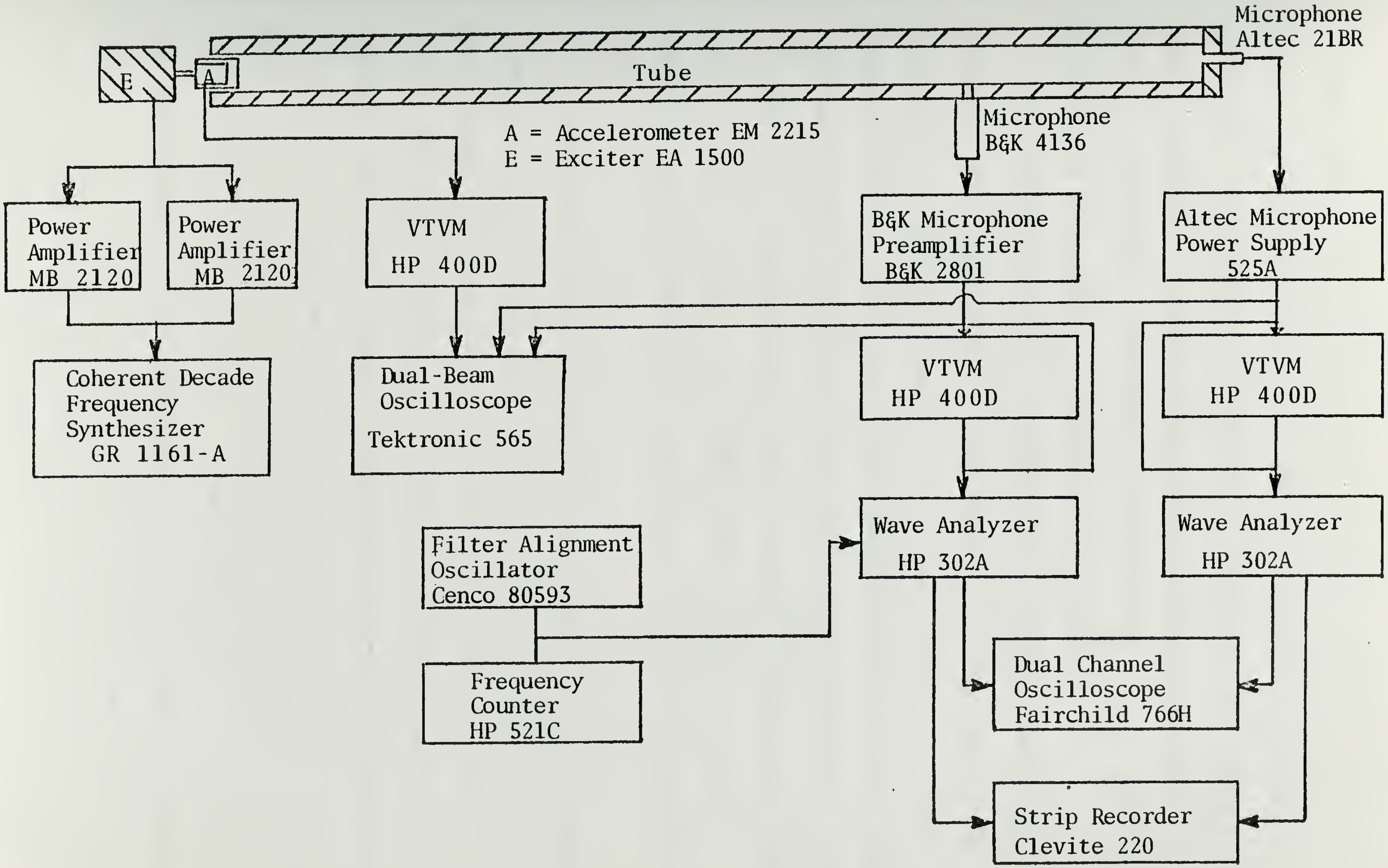

Figure 1. Apparatus Block Diagram

The piston was driven by an M-B Electronics Model 1500 Exciter powered by two M-B Electronics Model 2120 MB power amplifiers connected in parallel. The driving signal was generated with a General Radio 1161-A Coherent Decade Frequency Synthesizer. With this synthesizer it was possible to select a frequency to within 0.001 Hertz.

The pressure waveform within the tube was sensed by two condenser microphones. At the end of the tube an Altec 21-BR microphone having a diameter of one-half inch (1.27 cm) was threaded into a 1.25 inch thick aluminum end cap with the diaphragm of the microphone flush with the inside face of the end cap. The end cap was tightly bolted to the end of the tube with five studs. This Altec microphone was used primarily to determine the sound pressure level within the tube.

Because the apparatus was to be driven at the second harmonic of the tube while looking for a nonlinear generated first harmonic expected to be very much weaker than the signal at the driving frequency, a second microphone was mounted at a point where the standing pressure wave of the driven mode has a node. There are two such points; one at L/4 and the other at 3L/4 where L is taken to be the distance from the piston face to the end cap. (See Figure 2). At these points the magnitude of the subharmonic is 0.707 times its peak value. The second microphone was therefore mounted at the optimum point which was furthest from the driver. A Bruel and Kjaer Type 4136 Microphone was chosen for its small

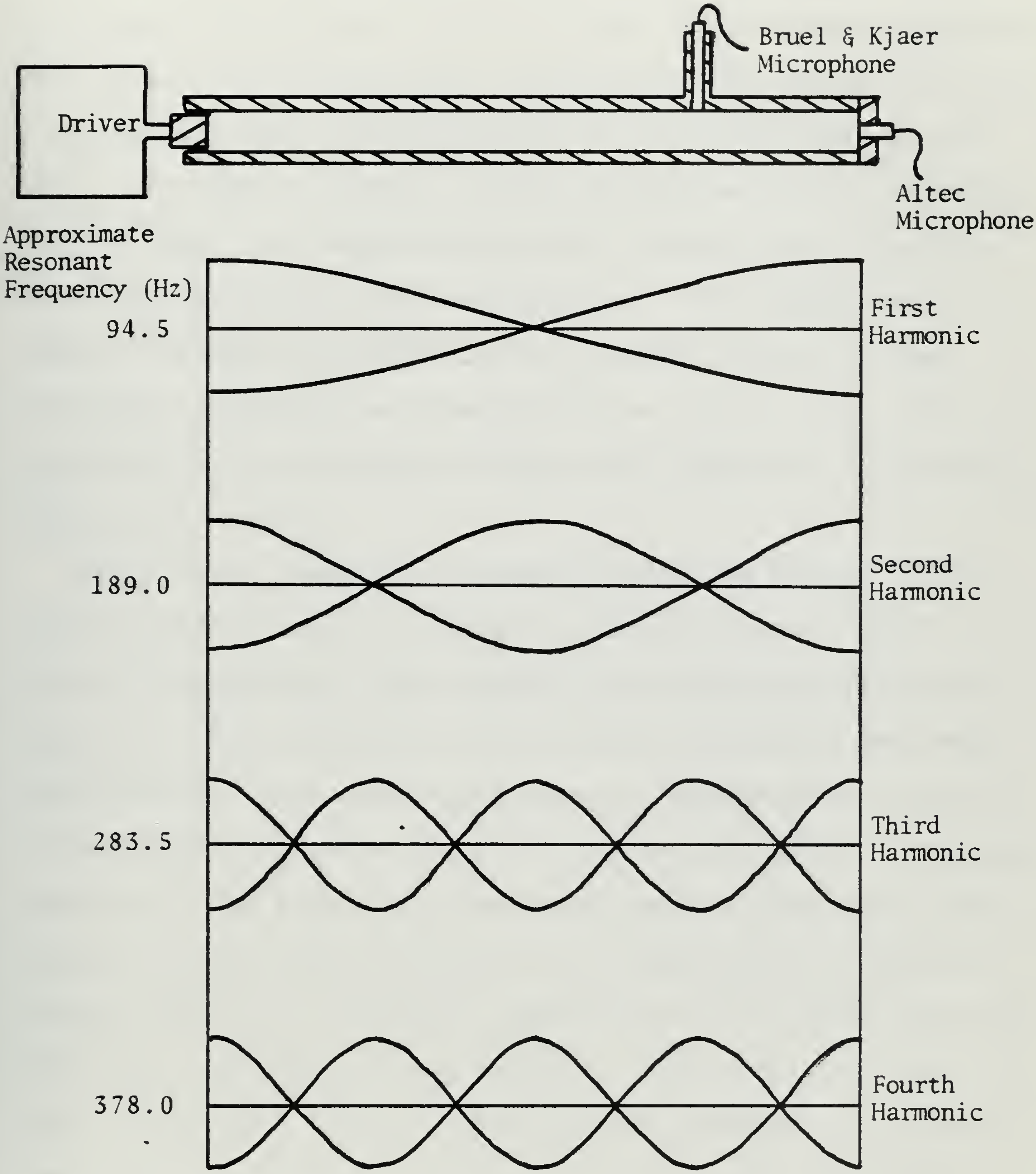

Figure 2. Standing Wave Diagram for Resonant Tube

diameter (only 1/4 inch) and good frequency response (flat to within ± 0.5 dB from 80 Hz to 20 KHz). The small diameter was helpful in reducing the irregularity produced by mounting a microphone with a flat diaphragm in the curved wall of the tube. In order to obtain a tight seal between the microphone and the tube, an aluminum casing was constructed to securely house the microphone with O-ring seals. The casing was tightly threaded into the tube so that the center of the microphone diaphragm was flush with the inside wall. This placement of the microphone reduced the signal at the driving frequency by 40 dB.

After their respective preamplifiers (see Figure 1), the two microphone output voltages were monitored on a pair of Hewlett Packard Model 400D Vacuum Tube Voltmeters and on the second and third inputs of the Tektronic Model 565 Dual Beam Oscilloscope. Each microphone was also connected to a Hewlett Packard Model 302A Wave Analyzer. Whenever a precise frequency analysis of the Bruel and Kjaer wall-mounted microphone was taken, a Ceneco 80593 oscillator was connected to a Hewlett Packard Model 521C Frequency Counter and to the wave analyzer which was to be used in the analysis. The oscillator was used to align the filter to the desired frequency before each measurement and was disconnected while the data was being taken. (See section on run procedures.)

A Fairchild Model 766H Dual-Channel Oscilloscope was used to monitor the filtered output of each wave analyzer. A Clevite Model 220 strip recorder was connected to the recorder output of each filter to obtain a permanent record of the data run.

B. CALIBRATION OF EQUIPMENT

Determination of the strength parameter and the relative strength of the subharmonic required the absolute calibration of both microphone systems. The calibration of the accelerometer was useful in determining the theoretical strength parameter achievable by the system.

Calibration of the microphone systems included the calibration of the microphones, their respective preamplifiers, and the VTVM's. The Bruel and Kjaer microphone system was calibrated using a Bruel and Kjaer Model 4240 Pistonphone to produce a known acoustic pressure level of (124 ± 0.2) dB re 0.0002 μ bar at a frequency of 250 Hz. The sensitivity was found to be

$$S_M = (-76.1 \pm 0.2)\ \text{dB re 1 volt}/\mu\ \text{b.}$$
$$= (1.56 \pm 0.04) \times 10^{-3}\ \text{volt/Nt}/\text{m}^2).$$

The Altec microphone was fitted with three different condenser heads during the course of the investigation. Calibration of the Altec system was performed with a modified General Radio Model 1307-A Transistor Oscillator which produces a known acoustic pressure level of (120 ± 1) dB re 0.0002 μb at a frequency of 400 Hz. The sensitivities for the Altec 21 BR microphone with the three condenser heads were found to be:

21 BR 150: $S_M = (-62 \pm 1)$dB re 1V/μb

$= (7.9 \pm 0.9) \times 10^{-3}$ V/(Nt/m²)

21 BR 200-3: $S_M = (-87 \pm 1)$dB re 1V/μb

$= (4.5 \pm 0.5) \times 10^{-4}$ V(Nt/m²)

21 BR 220-3: $S_M = (-102 \pm 1)$dB re 1V/μb

$= (7.9 \pm 0.9) \times 10^{-5}$ V/(Nt/m²)

As a check of the relative calibrations of the two microphones, the tube was excited in its third overtone at approximately 378 Hz. As shown in Figure 2, the third overtone has a pressure antinode at both microphones so that the acoustic pressures measured by each microphone should be equal. It was found that the two microphones gave acoustic pressure levels that were within 0.2 dB of being equal. This is well within the calibration uncertainty.

The accelerometer was calibrated by direct measurement of the displacement amplitude of the piston face using a traveling microscope and a strobe. Ten measurements at a frequency of 190.000 Hz and an accelerometer output of 0.120 volts (rms) resulted in an average peak displacement of $(3.49 \pm 0.03) \times 10^{-4}$m and a sensitivity of

$$S_A = (6.82 \pm 0.05) \times 10^{-4} \text{ V/(m/sec}^2\text{)}.$$

The traveling microscope and strobe were also used to measure the maximum obtainable acceleration of the lighter piston. While driving in open air at a frequency of 190.000 Hz a peak to peak displacement amplitude of 0.126 cm was measured. This corresponds to a peak acceleration of 91.6 g.

C. SYSTEM ALIGNMENT

Because the amount of subharmonic was expected to be very small, it was necessary to carefully align the piston before each run. The system was designed so that the resonant tube was fixed while the driver assembly was mounted on a cradle with elevation screws that allowed for three-dimensional adjustment. The entire cradle assembly rested on a 5/16 inch pad of stiff acoustic rubber to reduce creep at high driving levels. The resonant tube was supported on a separate table by a pair of foam rubber lined wooden blocks.

The alignment procedure consisted of exciting the tube at a low level while monitoring the accelerometer waveform on the dual beam oscilloscope. A sine wave from the synthesizer was displayed on a separate trace. The cradle elevation screws were adjusted until the two waveforms appeared to be identical. Measurements of the harmonic distortion indicated that this visual alignment technique yields an alignment for which the second harmonic distortion in the piston motion is less than 0.3% of the fundamental and the third harmonic distortion is less than 0.5% of the fundamental. These values are comparable to those obtained by Beech and Winn for a similar apparatus. Any subharmonic distortion was below the noise level of the accelerometer and was measured to be less than 0.05% of the fundamental.

Because the accelerometer was not used in the light piston configuration, alignment could only be accomplished by adjusting the cradle elevation screws until the tone produced by the piston sounded pure. Measurements which were taken while using the heavier piston and accelerometer indicated that alignment by ear alone results in a piston motion with less than 0.5% second harmonic distortion and less than 1.0% third harmonic distortion. Subharmonic distortion is less than 0.05% of the fundamental.

D. MICROPHONE OUTPUT AND STRENGTH PARAMETER

The strength parameter is defined as

$$SP = M_2 \beta Q_1$$

where

$$M_2 = P_2/\rho c^2$$

is the Mach number of the driven mode, a quantity which characterizes the degree of the finite amplitude interaction, Q_1 is the quality factor of the subharmonic mode and β is 1.2 for air. P_2, the acoustic pressure of the driven mode, can be calculated from the microphone output if the microphone sensitivity, S_n, is known. Using the relationships

$$S_n = V_n/P_{2\ rms}$$

$$P_{2\ rms} = P_2/\sqrt{2}$$

the strength parameter may be rewritten as

$$SP = (1.19 \times 10^{-5})\ V_n\ Q_1/S_n$$

where V_n is the rms voltage of the microphone filtered at the driving frequency and S_n is expressed in units of volt/(NT/m²).

The strength parameter can also be calculated from the piston acceleration (for infinitesimal waves at resonance) since

$$M_2 = A_o/\omega\alpha Lc ,$$

where

$$\alpha = \pi f/Q_2 c$$

is the absorption coefficient for the driven mode, L is the tube length, and A_o is the acceleration amplitude of the piston. A_o is determined from the accelerometer output voltage and sensitivity from the relation

$$A_o = V_A/S_A$$

where

$$V_A = \sqrt{2}\ V_{A(rms)}.$$

The theoretically predicted Mach number is therefore

$$M_2 = 57.4\ V_{A(rms)}Q_2/f^2$$

where $V_{A(rms)}$ is the rms voltage output of the accelerometer and f is the driving frequency. This gives a theoretical strength parameter of

$$SP = 68.9\ V_{A(rms)}Q_1Q_2/f^2$$

which should be valid as long as finite amplitude effects are negligible.

All quality factors were determined from the relation

$$Q = \frac{f_u + f_2}{2(f_u - f_2)}$$

where f_u and f_2 are the frequencies of the half power points above and below resonance.

IV. DATA COLLECTION PROCEDURES

A. PRERUN PROCEDURES

Beech reported that the resonant frequency of the tube changed by 0.16 Hz for each degree centigrade of temperature variation. Winn found that by allowing a warm-up period of several hours and by taking his data in the evening when room temperature was relatively constant, the resonant frequency drift was of the order of 0.01 Hz per hour. Measurements for this investigation were taken in the evening following a six hour or longer warm-up period in which all equipment was energized and the tube was excited at its second harmonic. Resonant frequency drifts were found to be consistent with Winn's data and were considered insignificant over the duration of the data run.

Piston alignment was accomplished during the warm-up period as explained in Section III.C. The wave analyzers were calibrated to agree with the VTVM's for the same pure tone input.

Immediately preceding the run sequence the tube was excited at very low levels such that the strength parameter was less than 0.20 while the half power frequencies needed to determine the quality factors and absorption coefficients of the first and second harmonics were measured.

B. RUN SEQUENCE

The run sequence involved the collection of data which were necessary to obtain a spectral analysis of the Bruel and Kjaer microphone output while the tube was excited at various strength parameters for frequencies near the resonance frequency of the first overtone of the tube. The first wave analyzer, in its Automatic Frequency Control (AFC) mode, was used to measure the Altec microphone output at the driving frequency. This allowed determination of the strength parameter of the driven mode. A second wave analyzer, in its normal mode, was used to measure the frequency spectrum of the Bruel and Kjaer microphone. Data points were taken in five Hertz increments from 30 to 220 Hz. Precise selection of the center frequency of the seven Hertz bandpass filter was accomplished by maximizing the filter response to a calibration tone produced by the Ceneco oscillator (see Figure 1). The filter was aligned in this manner for each frequency increment. he oscillator output was disconnected from the filter while data were taken. At each frequency increment the voltage level of the Bruel and Kjaer microphone was recorded from the wave analyzer for various strength parameters. Near the frequency of the subharmonic, where the voltage level was quite variable, a strip recorder was connected to the wave analyzer recorder output to produce a permanent record of the fluctuations as a function of time. The strip recordings were later analyzed to determine average and peak voltage levels.

V. RESULTS

Table I lists characteristic values for resonance frequency and the associated quality factor and absorption coefficient for the resonant tube at various modes in both the heavy- and light-piston configurations. Lower values of α, which were determined by Beech and Lane for the same tube, were duplicated by replacing the 1.25 inch thick end cap with the two-inch thick cap which was used in earlier investigations to house the Bruel and Kjaer microphone.

Figure 3 illustrates the relationship between strength parameter and accelerometer output voltage as predicted from first order theory and as measured with the microphone. It is important to note the increasing divergence between the predicted and measured curves as the piston acceleration is increased. The decrease in slope of the measured curve represents the progressive increase of energy contained in the harmonic components of the distorted waveform. These components are alternately in and out of phase with the driving frequency, thereby resulting in a decrease in pressure at the microphone from what would be present if all the energy were in the driven mode. Figure 3 also illustrates that in the heavy-piston configuration strength parameters greater than 2.0 cannot be obtained. Greater piston accelerations were achieved using the lighter piston and a maximum strength parameter of 2.89 was measured. This exceeds the threshold for subharmonic generation as postulated by Coppens. (SP $\geq$ 2.0). However, it is considerably below the threshold which was extracted from the theory of Yen. (SP $\geq$ 6.86). Since the peak displacement amplitude of the light piston was measured to be 0.063 cm at the maximum driving level, the apparatus is operating below the threshold put forth by Adler and Breazeale which requires

$$X_0 > \frac{L_0 c \alpha}{\omega_1}$$

where X_0 = peak displacement amplitude of piston

L_0 = tube length

ω_1 = angular frequency of subharmonic.

For this apparatus Adler and Breazeale's threshold reduces to $X_0 > 1.92$ cm.

Figures 4 through 10 illustrate the frequency spectrum of the Bruel and Kjaer microphone output for various strength parameters ranging from zero to 2.89. The three major features of Figures 5 through 10 include the large peak at the driving frequency, a much smaller and variable peak at half the driving frequency, and a curious peak at 70 Hz. This 70 Hz peak is thought to be related to 60 Hz noise and in Figure 4 (strength parameter = 0) it does occur at 60 Hz. The mechanism for the translation of this peak to 70 Hz for strength parameters greater than zero is not known.

The voltage level at half the driving frequency varied continuously regardless of the strength parameter. All data points at 95 Hz indicate the average and maximum voltage levels as determined from strip recordings of the data. It is interesting to note from Figure 4 that even when the tube was not being excited by the piston, the peak at 95 Hz was present and quite variable. A possible explanation of this observation is because 95 Hz is the approximate frequency of the first resonant mode of the tube, noise-like vibrations in the laboratory may have stimulated a weak response in that mode.

Although the placement of the Bruel and Kjaer microphone resulted in a 40 dB attenuation of the driven mode, the peak at 190 Hz is by far the dominant feature in Figures 5 through 10. Table II shows the acoustic pressure at 95 Hz as measured by the Bruel and Kjaer microphone, the acoustic pressure at the driving frequency as sensed by the Altec microphone and the ratios of the two pressures for each of the strength parameters which were investigated. These data do not indicate a marked threshold effect for subharmonic generation nor do they indicate a linear relationship between the magnitude of the response at 95 Hz and the strength parameter.

Heavy Piston - 1.25" End Cap		
f_o(Hz)	Q	α (m^{-1} x 100)
94.720	73.48	1.18
189.831	114.22	1.52
284.964	137.93	1.89
380.209	158.03	2.20
475.560	181.72	2.39
571.004	200.63	2.60

Heavy Piston - 2.00 in. End Cap Hole in Tube Wall Plugged			α_{Beech}
f_o(Hz)	Q	α (m^{-1} x 100)	(100/m)
94.904	73.28	1.18	1.1
189.194	118.95	1.46	1.4
285.548	142.85	1.83	1.8
380.977	163.51	2.13	2.1
476.642	186.33	2.34	2.3
572.338	203.03	2.57	2.8

Light Piston - 1.25 in. End Cap		
f_o(Hz)	Q	α (m^{-1} x 100)
94.410	56.10	1.54
189.252	96.05	1.80

Table I. Resonant Frequencies, Quality Factors, and Absorption Coefficients for Resonant Tube

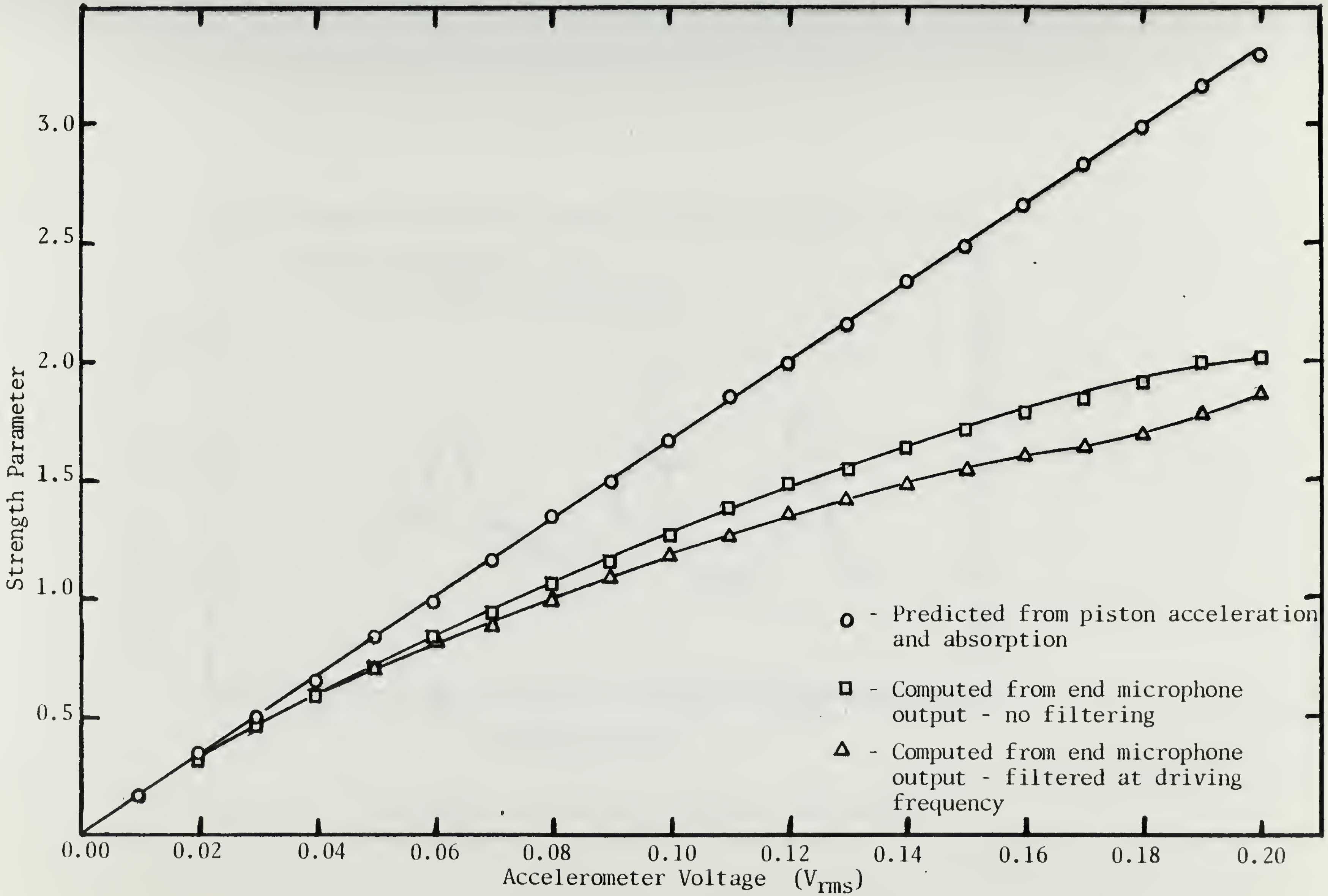

Figure 3. Strength Parameter vs. Accelerometer Voltage

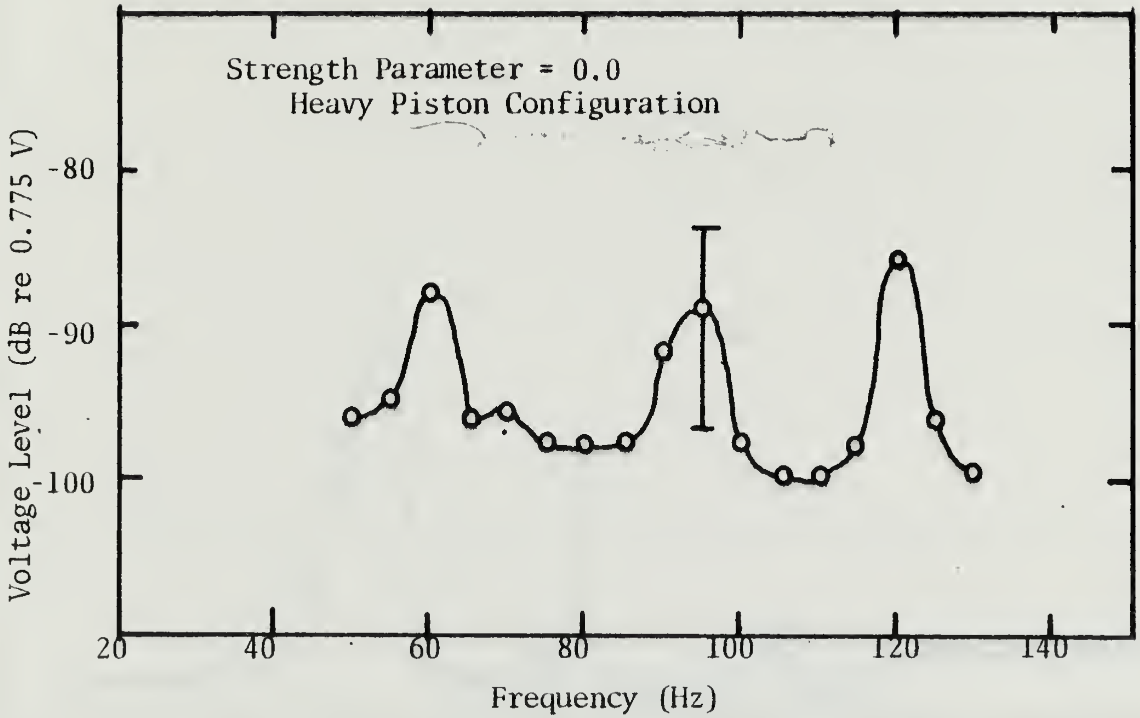

Figure 4. Frequency Spectrum of Bruel and Kjaer microphone.

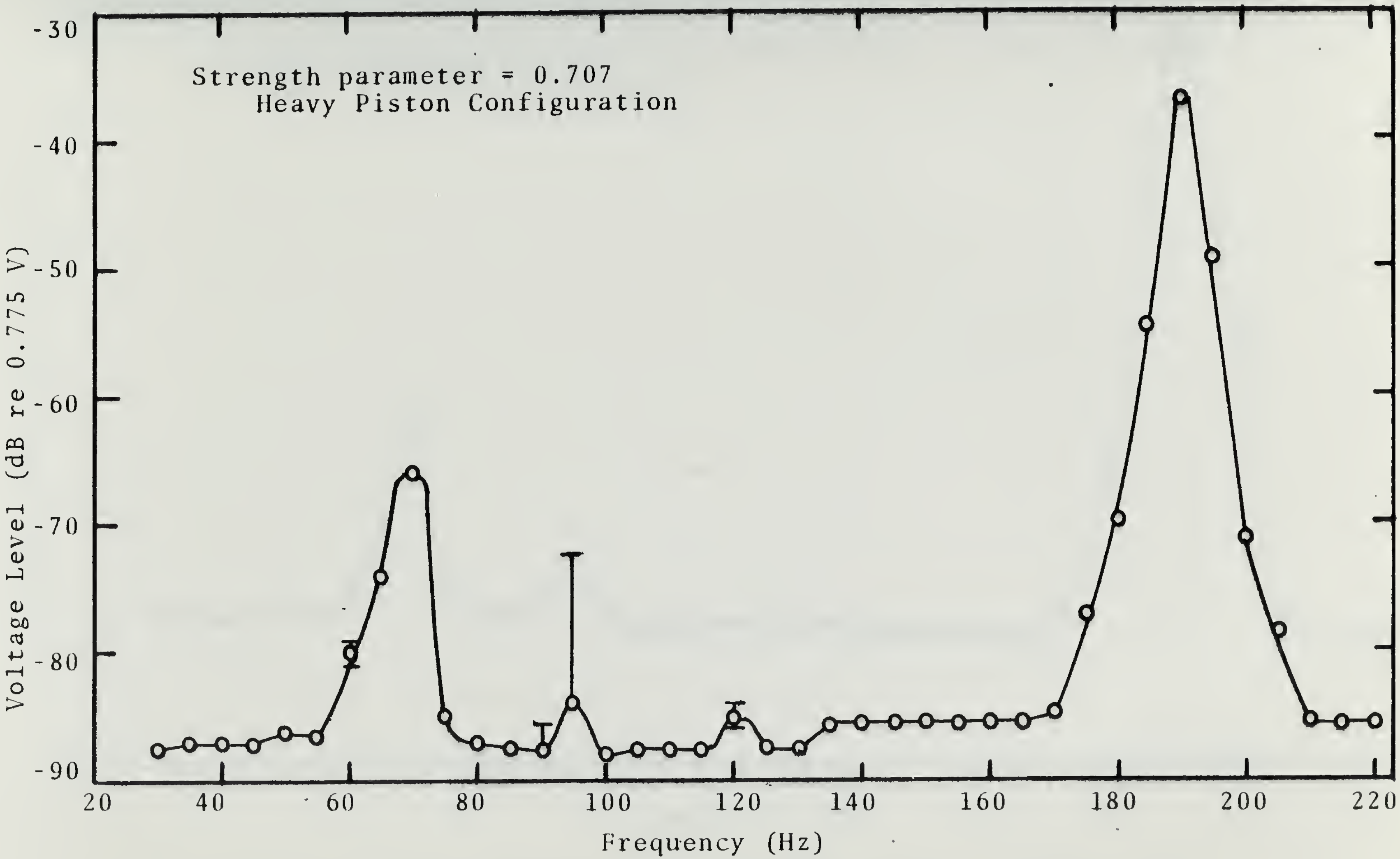

Figure 5. Frequency Snectrum of Bruel and Kjaer Microphone.

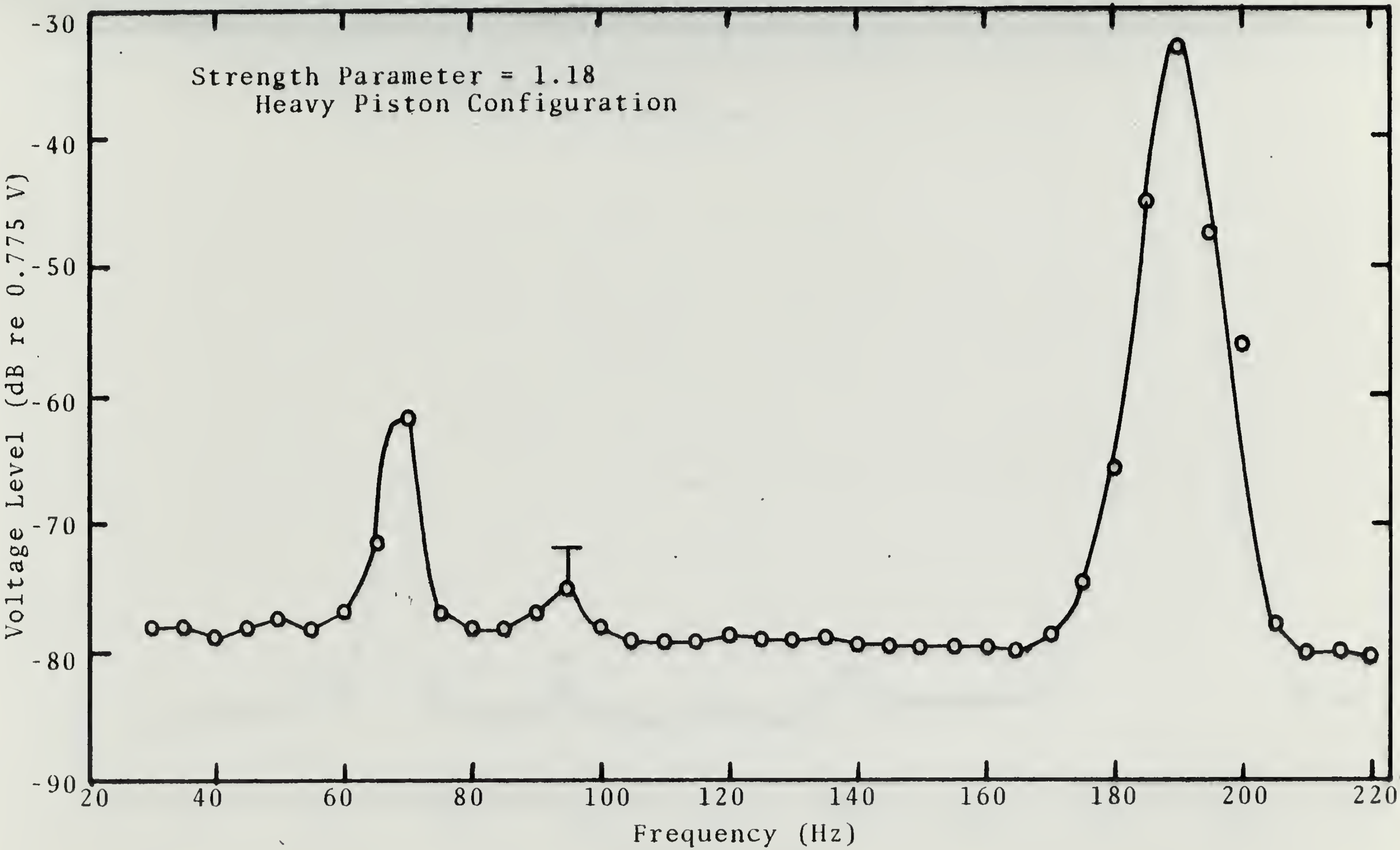

Figure 6. Frequency Spectrum of Bruel and Kjaer Microphone.

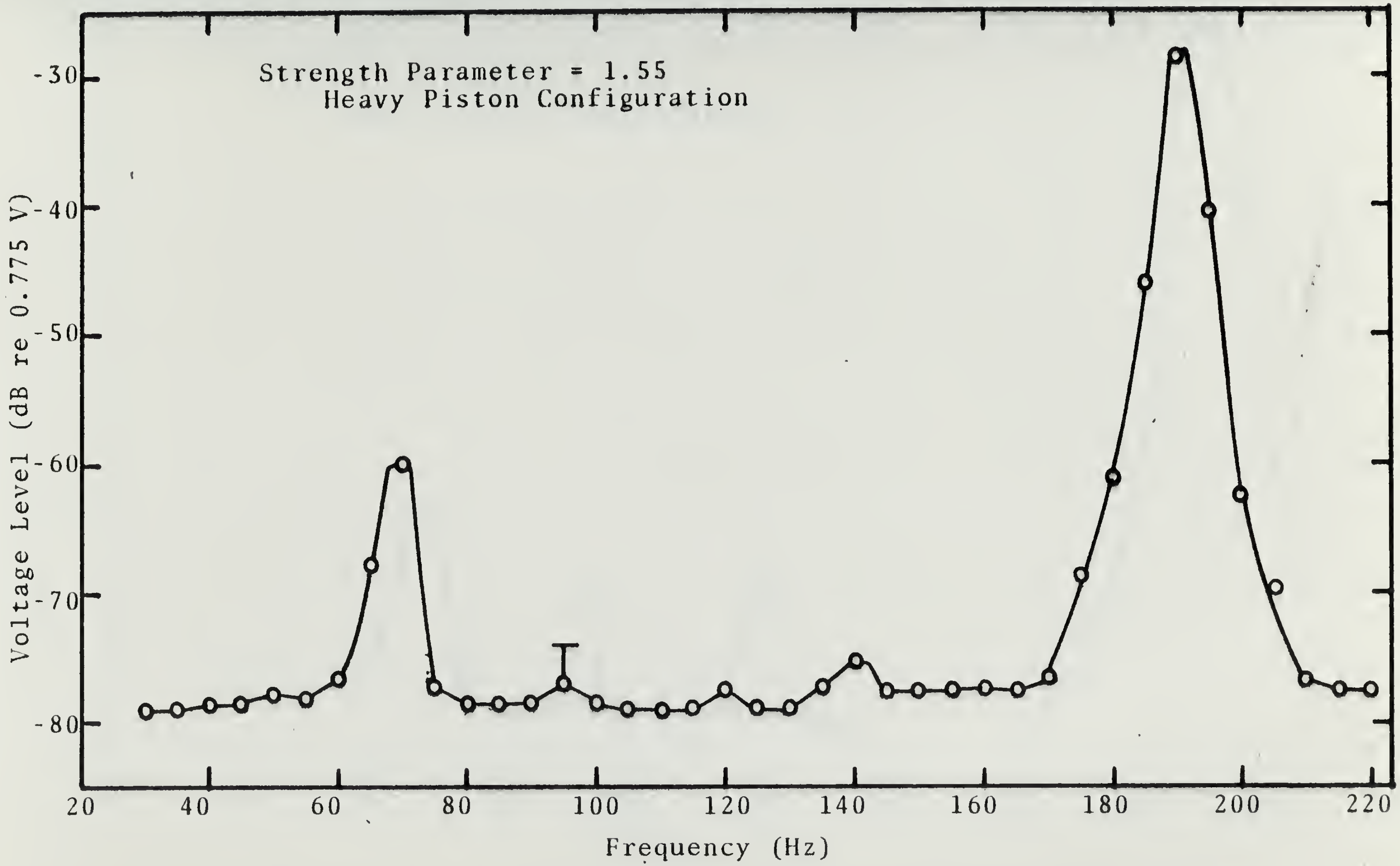

Figure 7. Frequency Spectrum of Bruel and Kjaer Microphone.

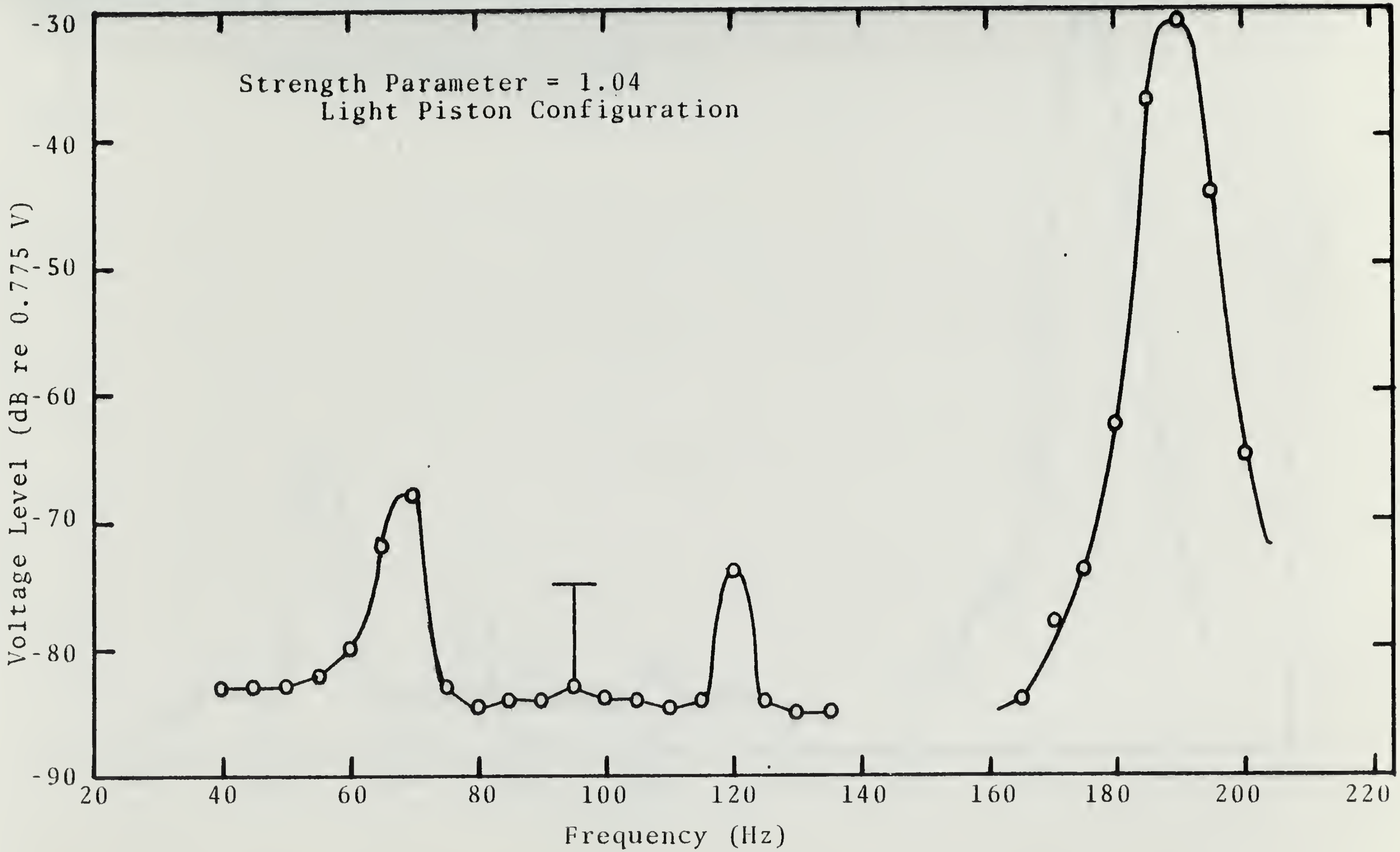

Figure 8. Frequency Spectrum of Bruel and Kjaer Microphone

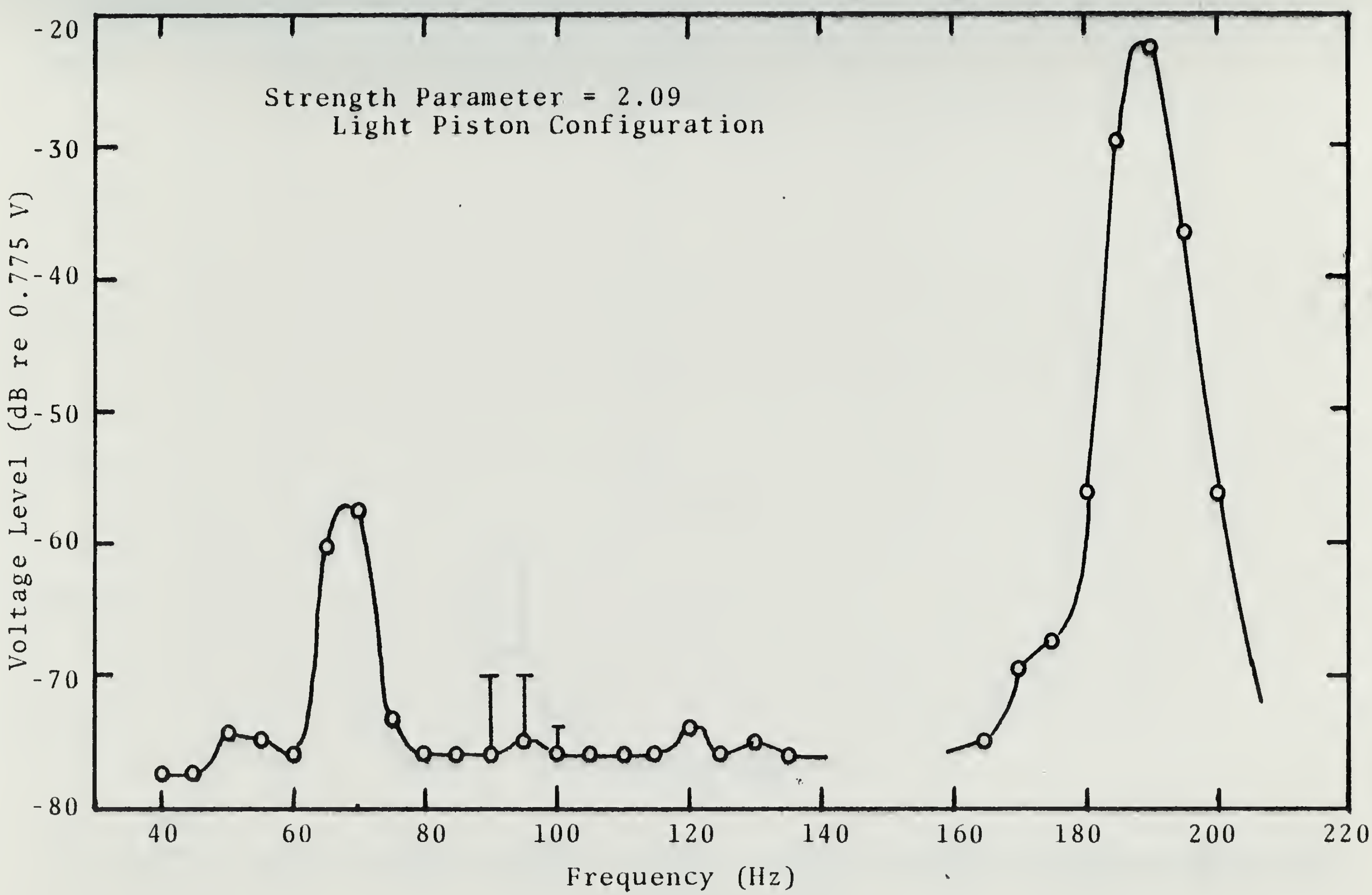

Figure 9. Frequency Spectrum of Bruel and Kjaer Microphone

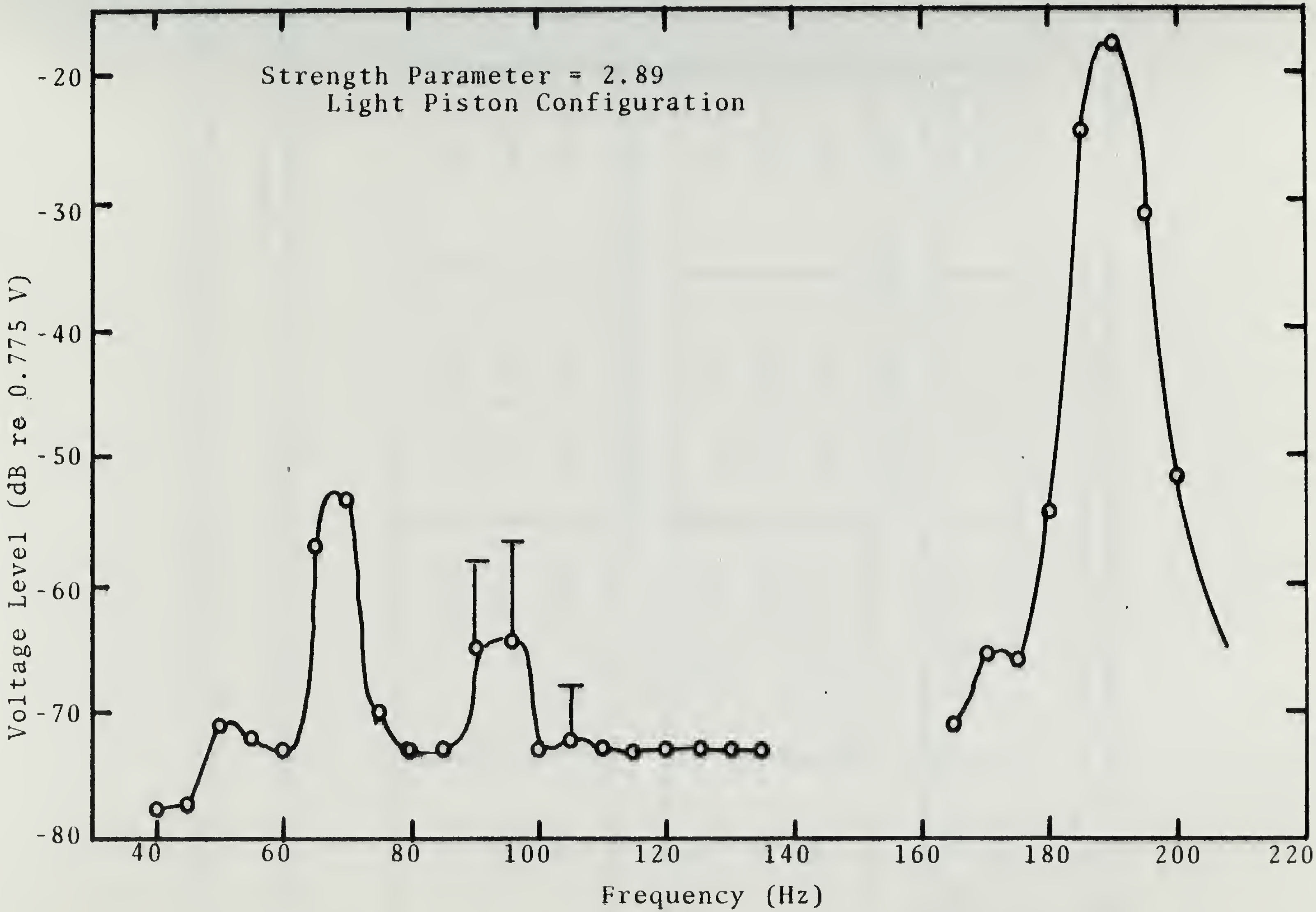

Figure 10. Frequency Spectrum of Bruel and Kjaer Microphone

Table II. Comparison of Acoustic Pressures at 190 Hz and 95 Hz for Various Strength Parameters

Strength Parameter	P_{190} (Nt/m^2)	P_{95} (Nt/m^2)	(dB) $20 \log \left(\frac{P_{95}}{P_{190}}\right)$
HEAVY PISTON CONFIGURATION			
0.707	1.14×10^3	4.43×10^{-2}	-88.2
1.18	1.91×10^3	0.125	-83.7
1.55	2.51×10^3	9.05×10^{-2}	-88.8
LIGHT PISTON CONFIGURATION			
1.04	2.21×10^3	0.126	-84.9
2.09	4.43×10^3	0.157	-89.0
2.89	6.13×10^3	0.433	-83.0

P_{190} = Pressure at 190 Hz as computed from filtered output of Altec microphone.

P_{95} = Average pressure at 95 Hz as computed from filtered output of Bruer and Kjaer microphone.

VI. CONCLUSIONS

Detailed measurements of the pressure waveform within the resonant tube indicate that strength parameters up to 2.89 were obtained. Although the driving levels were not high enough to test the theories of Adler and Breazeale and Yen, the theoretical threshold for subharmonic generation, as developed by Coppens, was exceeded. Experimental observations indicate that there is no noticeable threshold effect in the range of strength parameters achievable by the system. Therefore, Coppens' theory was not verified. Any subharmonic which may have been present was more than 83 dB below the acoustic pressure level of the driven mode.

BIBLIOGRAPHY

1. Kirchoff, G., Ann. Phys. Leipzig, V. 134, p. 177-193, 1868.

2. Lamb, H., Dynamical Theory of Sound, 2d ed., Chap. VI, Edward Arnold and Co., London, England, 1925.

3. Fay, R. D., "Successful Method of Attack on Progressive Finite Waves," J. Acoust. Soc. Am., V. 28, p. 910-914, September 1956.

4. Fay, R. D., "Plane Sound Waves of Finite-Amplitude," J. Acoust. Soc. Am., V. 3, p. 222-241, 1931.

5. Blackstock, D. T., "Propagation of Plane Sound Waves of Finite Amplitude in Nondissipative Fluids," J. Acoust. Soc. Am., V. 34, p. 9-30, January 1962.

6. Keller, J. B., "Finite Amplitude Sound Produced by a Piston in a Closed Tube," J. Acoust. Soc. Am., V. 26, p. 253-254, 1954.

7. Keck, W., and Beyer, R. T., "Frequency Spectrum of Finite Amplitude Ultrasonic Waves in Liquids," Phys. Fluids, V. 3, p. 346-352, 1960.

8. Coppens, A. B., and Sanders, J.V., "Finite-Amplitude Standing Waves in Rigid Walled Tubes," J. Acoust. Soc. Am., V. 43, p. 516-529, March 1968.

9. Weston, D. E., Proc. Phys. Soc., (London), B66, p. 695-709, 1953.

10. Beech, W. L., Finite-Amplitude Standing Waves in Rigid-Walled Tubes, Thesis, Naval Postgraduate School, Monterey, California, 1967.

11. Ruff, P. G., Finite-Amplitude Standing Waves in Rigid-Walled Cavities, Thesis, Naval Postgraduate School, Monterey, California, 1967.

12. Winn, J. R., Fourier Analysis of Experimental Finite-Amplitude Standing Waves, Thesis, Naval Postgraduate School, Monterey, California, 1971.

13. Coppens, A. B., and Sanders, J. V., "Finite-Amplitude Standing Waves in Real Cavities," J. Acoust. Soc. Am., V. 58, p. 1133-1140, December 1975.

14. Lane, C., Finite-Amplitude Waves in a Rigid Walled Cavity, Thesis, Naval Postgraduate School, Monterey, California, 1972.

15. DeVall, R. R., Finite-Amplitude Waves in Imperfect Cavities, Thesis, Naval Postgraduate School, Monterey, California, 1973.

16. Slocum, W. S., Finite-Amplitude Standing Waves in Real Cavities Containing Degenerate Modes, Thesis, Naval Postgraduate School, Monterey, California, 1975.

17. Korpel, A., and Adler, R., "Parametric Phenomena Observed on Ultrasonic Waves in Water," Appl. Phys. Lett. 7, p. 106-108, 1965.

18. Breazeale, M. A., and McCluney, W. R., "Subharmonic Generation in an Ultrasonic Resonant Cavity," J. Acoust. Soc. Am., V. 40, p. 1262(A), November 1966.

19. Cook, B. D., "Parametric Application of Ultrasonic Waves," J. Acoust. Soc. Am., V. 39, p. 1236(A), June 1966.

20. Bamberg, J. A., and Cook, B. D., "Subharmonic Generation in an Acoustic Interferameter," J. Acoust. Soc. Am., V. 41, p. 1584(A), April 1967.

21. Breazeale, M. A., and Chang, C. T., "Spectrum Produced by an Ultrasonic Resonant Cavity," J. Acoust. Soc. Am., V. 42, p. 1152(A), November 1967.

22. Adler, L. and Breazeale, M. A., "Generation of Fractional Harmonics in a Resonant Ultrasonic Wave System," J. Acoust. Soc. Am., V. 48, p. 1077-1083, November 1970.

23. Yen, N., "Experimental Investigation of Subharmonic Generation in an Acoustic Interferameter," J. Acoust. Soc. Am., V. 57, p. 1357-1362, June 1975.

24. Coppens, A. B., personal communication.

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