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# An Adaptive Transform Coding Algorithm

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This technical report has been reviewed and is approved for publication. Publication of this report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

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# AN ADAPTIVE TRANSFORM CODING ALGORITHM

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## Summary

An adaptive transform coding algorithm based on a recursive procedure in the transform domain has been developed. Both the quantization parameters and bit assignment are dynamically determined and thus are closely matched to the actual image structure. Overhead requirements are minimal.

## Introduction

Transform coding techniques for data compression have been considered in numerous current studies. In particular, the recently published proceedings of a specialized conference on image coding included a significant number of papers on image transform coding.<sup>1</sup>

Transform techniques have been demonstrated to be effective although significantly more demanding in terms of potential hardware implementation than simpler procedures operating in the spatial domain. Consequently, most studies concentrated primarily on procedures that would result in simplified implementation even at the expense of performance.

In contrast, the procedure discussed in this paper places performance as its primary objective. Although algorithm complexity is not ignored, it is only of secondary consideration.

## Reasons for Adaptivity

Implicit in all efficient data compression procedures is the fact that only "new" information is transmitted to the receiver via the communication channel. Equivalently, information that is known to the receiver should not be transmitted. Generally, the latter type of information represents some type of statistical characterization of the source (that is, the image).

In the following discussion, we will consider statistical characterization of the transform domain. A transform coding system may be represented by the schematic diagram shown in Figure 1. Here, the

modeling concept refers to information known to both the transmitter and the receiver. Most transform coding algorithms assume the separable correlation function associated with the Markov model that also presupposes image stationarity. This model represents a nonadaptive-type coding approach where image subsections are allocated a fraction of the available bandwidth in linear proportion to the subimage size.

Even casual observation of various images suggests that techniques that permit a more flexible (non-stationary) model are likely to result in superior performance. Although this philosophy is not new, solutions to this modeling approach have been few and were restricted to the case where the image was partitioned into a few classes to be separately encoded.<sup>2</sup>

## An Improved Model

Implicit in all transform coding procedures is the primary transform property that produces approximately uncorrelated coefficients. Assumption of an uncorrelated transform domain and the Markov model determines the optimum coding procedure. Although we also utilize the useful property of transformation to produce (almost) uncorrelated coefficients, we further make the observation that the standard deviations of adjacent coefficients are approximately the same. A more formal statement for this case is: Although the coefficients are uncorrelated, if we also treat the coefficient variances as random variables, this second set of random variables is highly correlated. This transform domain property is significant since it permits additional performance gains over the simpler Markov model discussed above.

## Recursive Coding

The assumption of a strong correlation of transform coefficient variances permits implementation of dynamically determined quantization models. A recursive procedure requires a one-dimensional ordering of

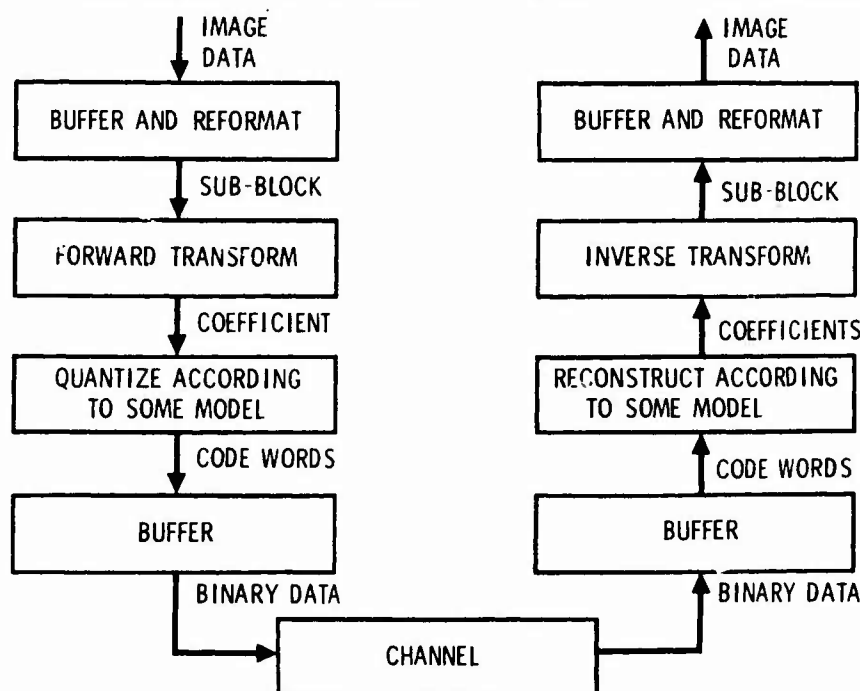


Figure 1. Schematics of Transform Coding

the two-dimensional transform domain. This ordering can be performed several different ways. We chose the procedure indicated in Figure 2. Here, the square lattice represents a "conventional" two-dimensional transform of an image sub-block (we used the two-dimensional cosine transform over  $16 \times 16$  and  $32 \times 32$  sub-blocks). The continuous "zig-zag" pattern scans the transform domain from the large through the small variance values. A typical example of this procedure demonstrates the validity of the technique (Figure 3[a]). This illustration indicates that the particular scanning approach is reasonable and that strong correlation exists among adjacent coefficient variances. The latter observation becomes even more evident when many sub-blocks are averaged together (Figure 3[b]).

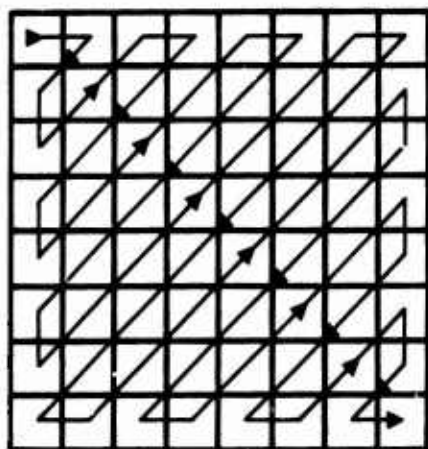


Figure 2. Ordering of the Frequency Domain

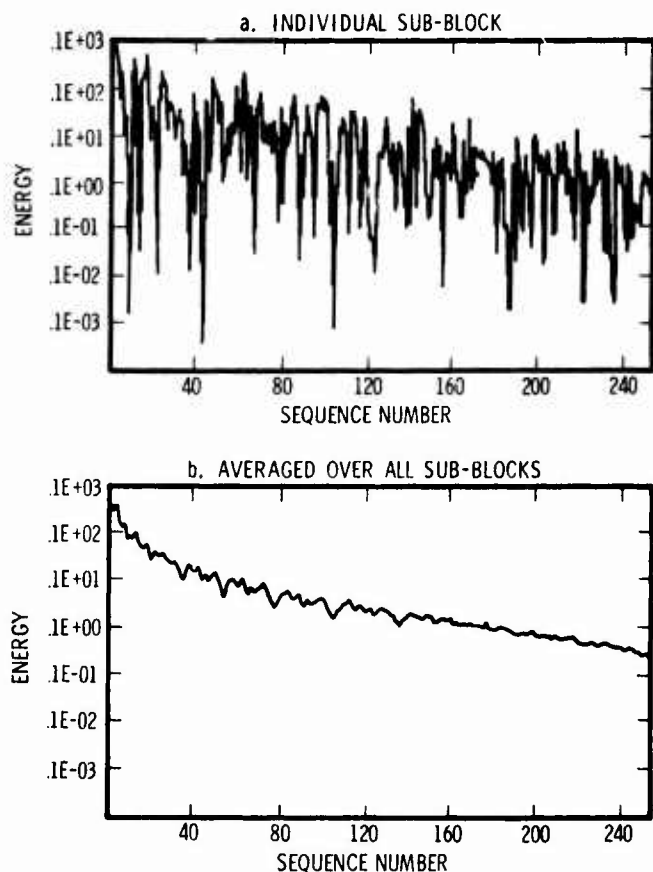


Figure 3. Frequency Domain Fluctuations

For the one-dimensional ordering of the transform domain, the schematic of the coding is shown in Figure 4. This diagram represents the following steps:

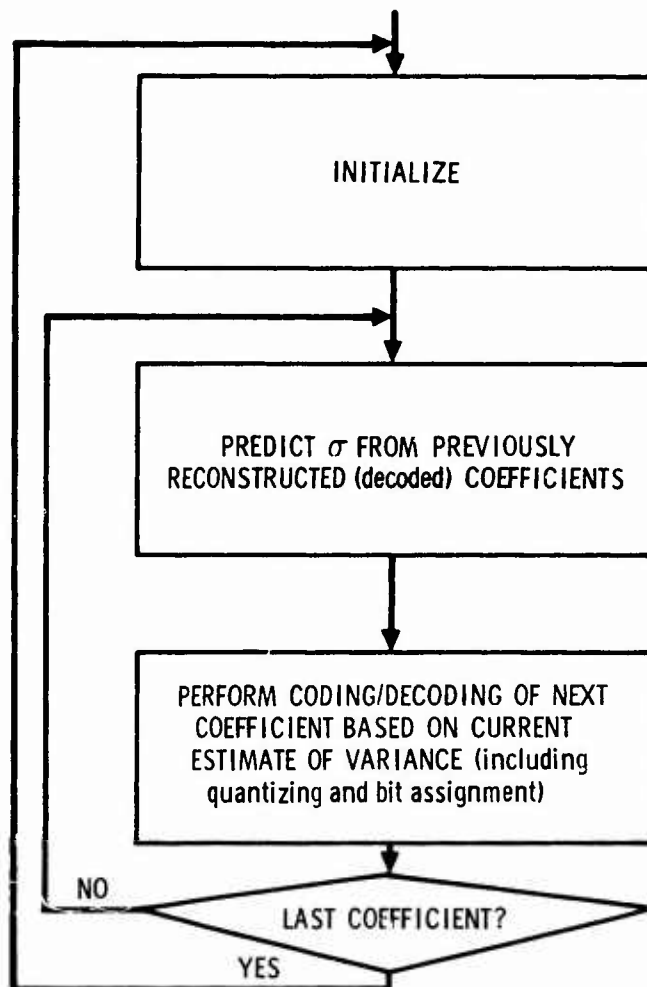


Figure 4. Transform Domain Adaptive Coding/Decoding

Let the one-dimensional ordering be represented by the sequence  $\{x_i\}$  where  $x_i$  is the  $i^{\text{th}}$  coefficient for a particular ordering, the corresponding decoded sequence is  $\{\hat{x}_i\}$ , and the dynamically determined variances are represented by  $\{\hat{\sigma}_i\}$ . In principle, at least, the actual coding/decoding procedure can be specified by the following equation.

$$\hat{x}_i = Q(\hat{\sigma}_i, x_i) \quad (1)$$

We have found that a simple linear predictor is quite sufficient to estimate the coefficient variance.

$$\hat{\sigma}_{i+1}^2 = 0.75 \hat{\sigma}_i^2 + 0.25 \hat{x}_i^2 \quad (2)$$

The actual recursive procedure begins by specifying the first variance estimate by

$$\hat{\sigma}_1^2 = \frac{(x_1^2 + x_2^2 + x_3^2 + x_4^2)}{4} \quad (3)$$

The dc term is  $x_0$  and it is coded separately. The "initial value"  $\hat{\sigma}_1^2$  must be transmitted over the communication channel. Thus, it represents overhead. However, it is the only one for the coding procedure under discussion. Equation (1) represents the quantizing as well as the reconstruction steps. The bit assignment procedure is also implicit in this equation via the conventional expression

$$Z_i = \frac{1}{2} \log_2 \frac{\hat{\sigma}_i^2}{D} + \frac{1}{2} \quad (4)$$

where

$n_i$  = the integer value of  $Z_i$ , for  $Z_i \geq 0$

$n_i = 0$  , for  $Z_i < 0$

The actual value of  $D$  determines the bit rate.

#### Amplitude and Phase Representation

Previously discussed generalized phase concepts<sup>3</sup> are also applicable and were utilized. Instead of the direct coding of the cosine coefficients, pairs of coefficients were mapped into amplitude and phase terms that were then encoded.

Let  $x_{2i}$  and  $x_{2i+1}$  be the appropriate coefficients. Then the mapping into amplitude and phase is given by

$$\begin{aligned} \rho_i &= x_{2i}^2 + x_{2i+1}^2 \\ \phi_i &= \tan^{-1} (x_{2i+1} / x_{2i}) \end{aligned} \quad (5)$$

Although this mapping is similar to the method associated with the Fourier transform the structural properties associated with the latter are neither applicable nor required. The advantage of the generalized amplitude and phase decomposition lies in the fact that the phase information dominates and the appropriate optimum quantizer is uniform.

It can be shown<sup>3</sup> that the amplitude can be coded approximately by 1 bit less than the phase when the mean-square error is the image quality measure. The recursive coding technique, described by Eqs. (1) through (4), is directly applicable to the amplitude terms and was utilized for the actual experiments. Dynamically determined amplitude bit assignment values for several typical  $16 \times 16$  sub-blocks are shown in Figure 5. Two significant observations can be made from the various curves. The first is that the actual image nonstationarity can result in significantly different bit assignment for different image sub-blocks. The second observation is that the recursive coding is "self-terminating." The latter expression refers to the fact that, whenever the predicted code word is less than 1 (see Eq. [4]), the coding procedure is terminated for that sub-block.

#### Results

We have successfully experimented with various types of imagery. It was found that the assumption of image nonstationarity is necessary for efficient encoding at low rates. Using the recursive technique over the  $16 \times 16$  and  $32 \times 32$  sub-blocks, the algorithm can operate at 0.25 bit/pixel with little visual degradation. In special cases it can operate as low as 0.1 bit/pixel. The appropriate performance values are shown in Figure 6, and pictorial demonstrations are shown in Figures 7 and 8. In all cases, the cosine transformation was utilized. The companding quantizer was implemented for the amplitude, and the appropriate probability density function was assumed to be Rayleigh.

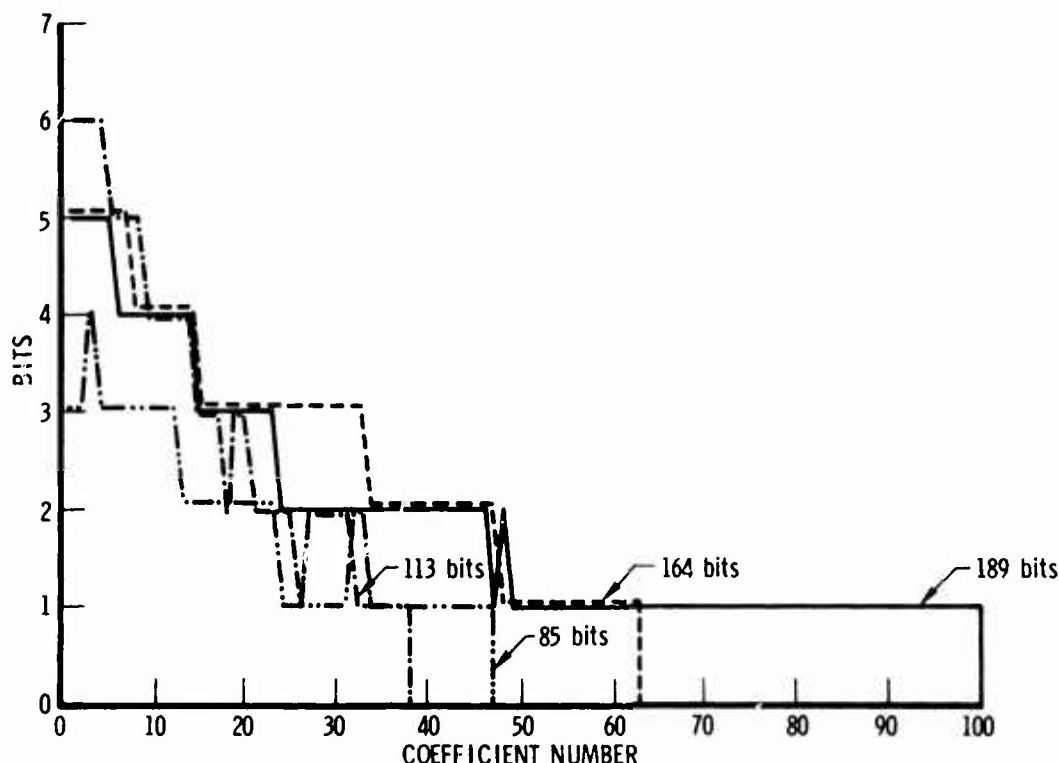


Figure 5. Adaptive Bit Assignment Examples



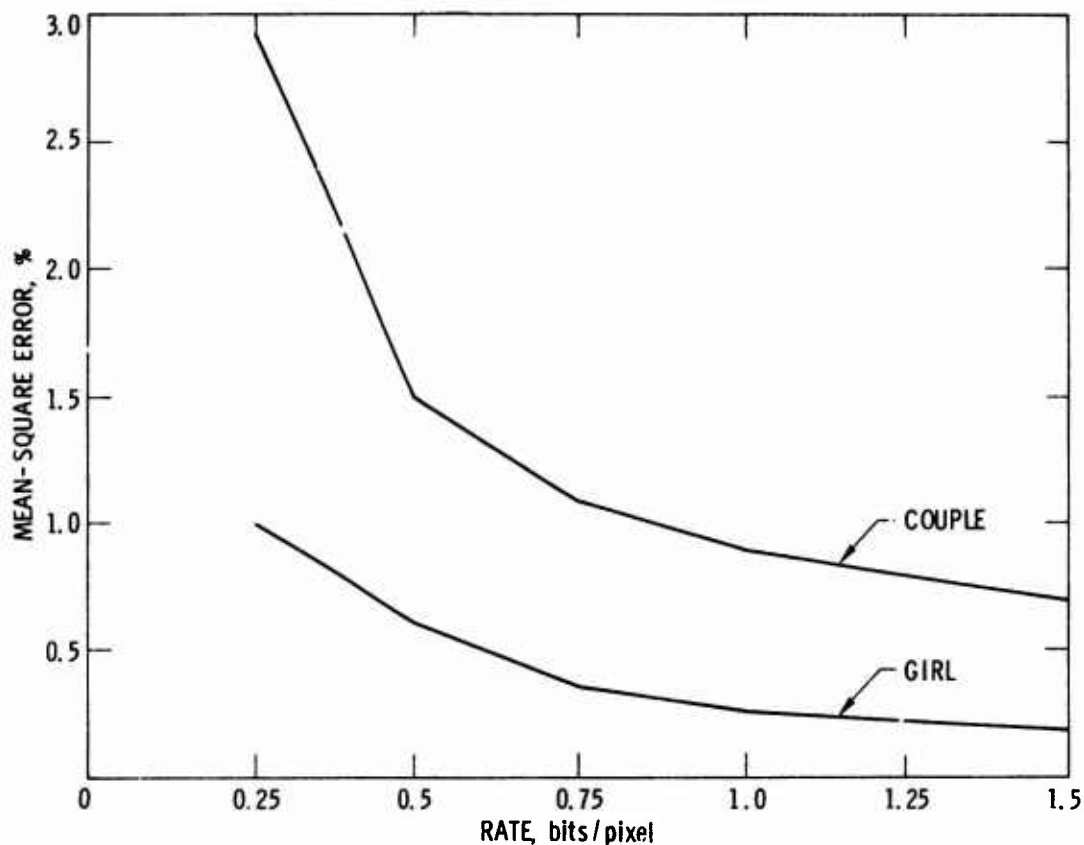


Figure 6. Performance Values Associated with Coding Examples

#### Conclusions

An attempt has been made to allow for image nonstationarity in an adaptive transform coding procedure. The actual image nonstationarity was demonstrated, and the actual compression technique can operate at 0.25 bit/pixel with little degradation.

Although adaptive techniques are more complex and require additional system considerations such as buffering, they are likely to outperform the equivalent nonadaptive techniques.

#### References

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Figure 7. Adaptive Coding of Girl



UNCOMPRESSED



2 bits / pixel



1.5 bits / pixel



1 bit / pixel



0.75 bits / pixel



0.5 bits / pixel



0.25 bits / pixel

Figure 8. Adaptive Coding of Couple