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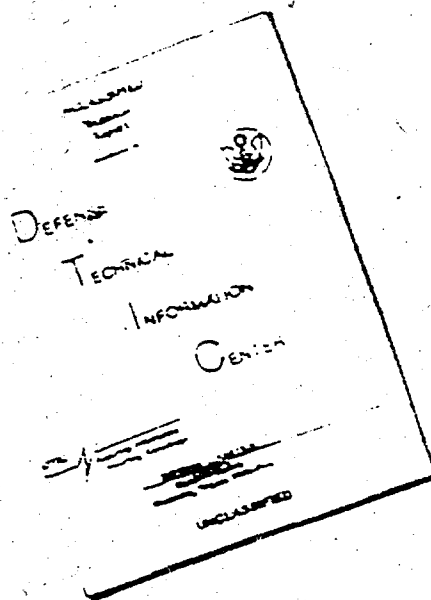
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)  
Results are presented in the areas of signal design bounds, spread spectrum communications, the partial correlation properties of gold codes, the design of special signal sets for good correlation, signal design for totally phase-incoherent communications, the detection of moderately fluctuating Rayleigh targets, and coded channels with memory.

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Final Report

"Research in Communication Theory"

August 4, 1976

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## RESEARCH SUMMARY

### Signal Design Bounds

In the area of digital signal design, some important results were obtained on bounds on the performance of signal sets. Reference 1 reports on a new and relatively tight lower bound on the maximum cross correlation of a signal set.

Abstract: Some communication systems require sets of signals with impulse-like autocorrelation functions and small cross correlation. There is considerable literature on signals with impulse-like autocorrelation functions but little on sets of signals with small cross correlation. A possible reason is that designers put too severe a restriction on cross correlation magnitudes. This correspondence establishes lower bounds on how small the cross-correlation and autocorrelation can simultaneously be.

This bound showed that in many cases signal designers were setting themselves an impossible problem by searching for signal sets with very low cross correlation. Furthermore, for many choices of dimensionality and number of signals, sets were already known with performance within  $\sqrt{2}$  (1.5 dB) of the bound. These sets are therefore nearly optimal. The techniques used in proving this result evolved into tools of analysis of error-correcting codes. This investigator cooperated with several mathematicians at NASA/JPL to obtain new distance bounds on error-correcting codes. One of these bounds is tighter than all previous known bounds and at low information rates gives an upper

bound for codes considerably closer to the Varsharmov-Gilbert existence bound than previous bounds.

### Spread Spectrum Communications

Our interest in signal sets with good correlation properties has motivated our study of spread spectrum communication systems. In particular we have attempted to determine exactly what properties of spread spectrum codes are important to system performance. A semi-tutorial paper<sup>(5)</sup> with the following abstract describes these results.

Abstract: This paper describes an idealized spread-spectrum communication system. The processing gain concept is developed as a measure of a well-designed system's robust performance against independent wide-sense stationary interference. Multipath and repeater jammer rejection, partial correlation problems, and security requirements are related to spread spectrum code properties, and a short list of well-known spread spectrum designs is presented.

Most spread-spectrum code designs are based on full-period signal correlation values since these are mathematically-tractable quantities. The results of this study indicate that in many problems partial-period auto- and cross-correlation properties are important parameters. Our efforts on partial period-correlation problems are discussed in the next section of this report.

### Partial Correlation Properties

We are investigating the partial correlation properties of Gold codes. It is known that the Gold codes of length  $N$  have normalized correlation values on the order of  $-1/N$ ,  $\pm\sqrt{2}/\sqrt{N+1}$  when correlated over a complete period. In this sense they are near ideal as predicted in reference 1. However, in actual communications systems using such signals, correlation time is a small fraction of the period. The distribution of these partial-period correlation values is much broader. The problem is to analytically express the distribution of partial correlation values for the Gold codes. One approach to this problem is to apply the generalized Tchebychev inequalities. These are strong generalizations of the well known distribution bound in terms of a single moment. If  $2k$  moments of the partial correlation distribution are known, an algorithm exists for developing tight upper and lower bounds on the distribution function. The investigation has proceeded on two fronts. First is the investigation of the moments of the distribution of partial correlation values of a Gold code with fixed correlation time. Properties of certain Gold codes enable computation of the first five moments and it is hoped that further moments can be computed. Second, the algorithm for developing bounds was surprisingly numerically unstable on the tails of the distribution. We now have a modified algorithm which seems much more stable. Putting these two lines of investigation together should produce bounds on the distribution.

### Signal Set Design for Good Correlation

Several problems in the design of special signal sets were considered. First, consider sequences with complex roots of unity as terms. The problem is to find sets of such sequences with small cross and auto-correlation. A particularly nice set of signals can be expressed in terms of group characteristics. If the length of the sequences is  $N$  then there are two relevant groups a) the group of addition modulo  $N$ , and b) the group of residue classes relatively prime to  $N$  under multiplication. Each group has a set of group characters and when properly combined, give a set of signals which comes within a factor of  $\sqrt{2}$  of the lower bound reported in reference 1 provided  $N$  is prime. It was found that this case ( $N$  prime) was reported previously by others so it was not published. Generalizations to the non-prime case have been carried out and some preliminary calculations were done on their ambiguity functions but no general results on ambiguity function properties have yet been obtained.

Another problem is to find minimal-correlation, balanced binary sequences. If the length of a binary ( $\pm 1$ ) sequence is divisible by 4 (but strictly greater than 4) and the sequence has an equal number of +1's and -1's (is balanced), it is not possible for its cyclic correlation function to be identically zero for all the out-of-phase values. In the best possible designs the out-of-phase values are restricted to  $0, \pm 4$ . A number of special cases and families of these "minimal-

correlation" sequences are known, but there are many lengths for which no minimal-correlation sequences are known.

A student and one of the principal investigators have attacked this problem in the following way. By finding all sequences (modulo symmetries) of a given length which have the special property, it may be possible to find some additional structure which allows a generalization to a large number of lengths. A major goal of this study has been achieved, namely all  $1836 \times 32 \times 16$  sequences of length 32 with the special property have been found. Since none of these sequences has any symmetries, we are now examining these sequences for other structure.

#### Signal Design for Totally Phase-Incoherent Communications

One interesting wideband signal set for digital communications contains waveforms which are linear combinations of tones. Our investigation of these signal sets, which revealed a definite bias against square law receivers, was reported in the open literature<sup>(2, 3)</sup>.

Abstract: A set of  $M$  equally-likely equal-energy transmittable signals is considered, each of which consists of a linear combination of tones from  $D$  free running oscillators ( $D \leq M$ ). The oscillator tones are assumed sufficiently disjoint to be orthogonal. The design problem consists of finding the optimal receiver and signal set for various  $M$  and  $D$ . For the additive white Gaussian noise channel, the optimal receiver first forms the sufficient statistic which consists of noncoherently detecting the energy



in each of the  $D$  tones. Unlike previous designs of digital transmitters based on minimization of the probability of error, when noncoherent oscillations are employed, the optimal receiver and signal set are dependent on the signal-to-noise ratio. The imposed constraints restrict the signal vectors to the all-positive subspace of the surface of a  $D$ -dimensional sphere. The optimal receiver, signal set, and resulting probability of error and channel capacity are determined for  $M \geq D = 2$  for low and high signal-to-noise ratios. Severe performance constraints imposed by using a suboptimal square-law receiver are discussed. Preliminary results have been obtained for the general case  $M \geq D > 2$ . As was noted in this work, there is a significant capacity loss associated with any increase in the number  $D$  of random-phased oscillators used in the design, relative to a coherent communication system having signal set dimension  $D$ . This raises an interesting unsolved research question: If the free-running oscillators in the transmitter are stable for sufficiently large periods of time, is it possible for the receiver to lock oscillators to each of the intermittent tones which it will observe? If this is possible, it seems that a significant increase in capacity may be possible. We may pursue this problem in a future grant.

#### The Detection of Moderately Fluctuating Rayleigh Targets

The ingredients for the solution of a somewhat classical problem in detection theory have been present for several years. Reference<sup>(4)</sup>

reports our solution to the moderately-fluctuating target problem.

Abstract: Pulse train detection of fluctuating targets whose coherence time is long compared to the time between pulses and short compared to the pulse train duration is compared for two systems (1) a recursive digital optimal receiver operating in conjunction with a coherent pulse train transmitter, and (2) a good (but not optimal) receiver operating with a block coherent frequency-hopping transmitter. The equivalence of this type of problem to noise-in-noise detection problems is demonstrated the performance relations for both systems are derived and comparisons are made.

The significant result of this work is the design and performance evaluation of an optimal, recursive, digital receiver for fluctuating targets illuminated by a coherent radar.

#### Channels with Memory

We have initiated a study of coded channels with memory, in particular, binary input, output channels in which the states of the channel can be described statistically by a Markov chain. Other than the intersymbol interference problem for which a large literature exists, perhaps the most simply stated problem concerns a two state burst channel, which in the quiet state behaves like a binary symmetric channel (BSC) with low error rate and in the noisy state behaves like a binary symmetric channel (BSC) with low error rate

and in the noisy state behaves like a BSC with high error rate. Through this unsophisticated model we hope to determine the main problems in using channels with memory.

Using a convolutionally encoded source signal, we have designed a Viterbi-like decoder which tracks both the encoder state and the channel state using simply recursive computations. The problems here are numerical: The number of decoder states  $N2^k$  grows exponentially with code constraint length  $k$  (as usual) and in addition linearly with the number of channel states  $N$ . In addition the computational effort required for each state in the decoding algorithm grows linearly with  $N$ .

Using flowgraph techniques we have been able to upper bound the probabilities of the several types of errors which can occur in this decoder, but these bounds presented two problems. First, bound evaluation requires the accurate inversion of a matrix whose dimension is  $N^2(2^k+1)$ . The second and more serious problem is that the bound is weak (as are most convolutional code bounds).

Several aspects of this basic problem are being carried over to a follow-on grant. (1) We are now trying to devise error enumeration procedures which will lead us to a better estimate of decoding algorithm performance. (2) In an effort to decrease the number of decoder states, we are studying interleaved codes in conjunction with a decoder in which channel state information is passed from one interleaved decoder to the next.

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