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WEIBULL TOLERANCE INTERVALS ASSOCIATED WITH SMALL SURVIVAL PROPORTIONS

FOR USE IN A NEW FORMULATION OF LANCHESTER COMBAT THEORY

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Weibull Tolerance Intervals Associated with Small Survival Proportions; For Use in a New Formulation of Lanchester Combat Theory*

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Given herein is an easily implemented method for obtaining from complete or censored data, approximate tolerance intervals associated with the upper tail of a Weibull distribution. These approximate intervals are based on point estimators that make essentially most efficient use of sample data. They agree extremely well with exact intervals (obtained by Monte Carlo simulation procedures) for sample sizes of about 10 or larger when specified survival proportions are sufficiently small. Ranges over which the error in the approximation is within 2 percent are determined.

The motivation for investigation of the methodology for obtaining the approximate tolerance intervals was provided by the new formulation of Lanchester Combat Theory by Grubbs and Shuford,[3] which suggests a Weibull assumption for time-to-incapacitation of key targets. With the procedures investigated herein, one can use (censored) data from battle simulations to obtain confidence intervals on battle times associated with given low survivor proportions of key targets belonging to either specified side in a future battle. It is also possible to calculate confidence intervals on a survival proportion of key targets corresponding to a given battle duration time.

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I. INTRODUCTION

In the more usual analyses of life data, the parameters typically of greatest interest are distribution percentiles in the lower tail of the associated life distributions. These percentiles, in reliability analysis, are values of *reliable life* corresponding to specified high survival probabilities. In contrast, if a probability distribution is assumed for *time-to-incapacitation* of key targets as in the new formulation of Lanchester Combat Theory by Grubbs and Shuford,[3] then it is very often the upper-tail percentiles of the distribution that are of primary concern. There are, doubtless, other life-time distributions in which times associated with low survival probabilities are often of concern. For example, this might be true of distributions of particular types of medical data or distributions of data collected to demonstrate inferior quality in a material or piece of hardware.

Here we assume a two-parameter Weibull model for time-to-failure or time-to-incapacitation, as suggested by Grubbs and Shuford,[3] and we consider the problem of obtaining confidence intervals for distribution percentiles associated with the upper tail of the distribution, or specified low survival proportions. Alternatively a "mission time" might be specified and a confidence interval desired for a corresponding (low) survival proportion. For Lanchester Combat Theory such confidence intervals could be obtained from data generated during computer simulation of a sample engagement that is, as Grubbs and Shuford[3] remark, "representative of the hypothesized general characteristics of many battles in the supposed environment."

"In particular, for example, we may be interested in running a sample simulation of a combat situation in order to see whether or not it is likely that our choice of weapons, the tactics employed in using them, and certain command-and-control principles would overwhelm and defeat an enemy with somewhat different weapons capabilities in the same hypothesized battle environment."

If a Weibull model is assumed for time-to-incapacitation, or time-to-neutralize, for key targets of each of the two opposing sides in a fixed combat situation, then the simulated "observed" times of incapacitation of targets on each of the two sides can be used to estimate the two sets of distribution parameters. If a survival proportion is specified for either side, then a time corresponding to this proportion can be estimated as a function of the parameter estimates. If, alternatively, a "mission time," or "battle duration time," is specified, corresponding survival proportions can be estimated for both sides from the two sets of parameter estimates.

In the following, an approximation that can be used in conjunction with the parameter estimates for obtaining confidence intervals on survival proportions corresponding to specified battle duration times or on duration times corresponding to specified survival probabilities is described.

II. A WEIBULL MODEL FOR TIME TO INCAPACITATION

It is assumed here that the random variate *time-to-incapacitation* T is such that

$$P(T < t) = \begin{cases} 1 - \exp[-(t/\delta)^{\beta}] & , \quad t \geq 0 \\ 0 & , \quad \text{otherwise,} \end{cases} \tag{1}$$

where $\delta, \beta > 0$. For $X = \ell nT$, $\eta = \ell n\delta$ and $\xi = \beta^{-1} > 0$, this is equivalent to

$$P(X < x) = 1 - \exp\left\{-\exp[(x-\eta)/\xi]\right\} \quad . \tag{2}$$

The random variate T has a Weibull distribution with shape parameter β and scale parameter (*characteristic time-to-incapacitation*) δ. (The parameters β and δ will of course tend to be different for the two opposing sides in a battle.)

The random variate X has the first asymptotic distribution of the smallest extreme (the *extreme-value* distribution). The parameter η is the mode and $\pi\xi/\sqrt{6}$ is the standard deviation of the distribution of X. The linear estimates discussed in the following are based on observations on X ordered from smallest to largest.

Grubbs and Shuford[3] reference three papers containing tabulations from which one can calculate confidence intervals, for each of the two opposing sides, on the distribution parameters, on high survival probabilities of key targets, or on lower-tail distribution percentiles. These are based on (iteratively obtained) maximum-likelihood estimates, best linear invariant (BLI) estimates, or approximations of BLI estimates. One of these three papers (Billman, Antle and Bain[1]) applies only to censored samples. The tables given are extensions of those published by Thoman, Bain and Antle[17] applying to maximum-likelihood estimates obtained from complete samples. The paper referenced in Grubbs and Shuford with tables for obtaining Weibull confidence intervals from best linear invariant estimates of Mann[7,8] is by Mann and Fertig.[11] The tabulations given therein apply to samples of sizes 2 through 25, either complete or with all possible right-hand censorings. Tabulations of Johns and Lieberman[4] pertain to asymptotic approximations of best linear invariant estimates and four right-hand censorings for each of six sample sizes ranging from 10 to 100. These published tabulations have all been generated by means of Monte Carlo simulation techniques since the exact distributions of the estimators cannot be determined. The estimators to which they apply are all asymptotically fully efficient; that is, as sample size n approaches infinity the variances of the (asymptotically unbiased) estimators approach the Cramér-Rao lower bounds for variances of regular unbiased estimators.

The parameter of primary interest in the present context is $t_p = \exp(x_p)$, the $100P$th percentile of the distribution, where P is proportion of key targets incapacitated by time t_p. The proportion P incapacitated will, of course, tend to be different for each of the two opposing sides for fixed t_p. Note, from (2), that $x_p = \eta + \xi \ell n[-\ell n(1-P)]$. If P is specified, then essentially optimum point estimates of t_p in terms of mean squared error in the log space can be obtained for targets on each of the two sides as $\exp\{\tilde{\eta} + \tilde{\xi}\ell n[-\ell n(1-P)]\}$ or $\exp\{\hat{\eta} + \hat{\xi}\ell n[-\ell n(1-P)]\}$, where for each side $\tilde{\eta}$ and $\tilde{\xi}$ are best linear invariant estimates and $\hat{\eta}$ and $\hat{\xi}$ are maximum-likelihood estimates of η and ξ, respectively, and $\ell n[-\ell n(1-P)]$ is the $100P$th percentile of $(X-\eta)/\xi$. If a battle duration time $t_0 = \exp(x_0)$ is specified, then $\exp\{-\exp[(x_0 - \tilde{\eta})/\tilde{\xi}]\}$ or $\exp\{-\exp[(x_0 - \hat{\eta})/\hat{\xi}]\}$ provide efficient point estimates of $1-P(t_0)$, the proportion of targets on each side surviving incapacitation until at least time t_0.

As noted, the tabulations mentioned earlier apply only to calculation of confidence bounds on lower-tail distribution percentiles or on high survival probabilities. Because of this and because these tabulations of exact values were necessarily generated by Monte Carlo simulation procedures, one would expect that similar simulation procedures would be necessary in order to find values to be used in calculating confidence bounds on upper-tail distribution percentiles or low survival proportions. We show now that such is not the case.

III. APPROXIMATIONS FOR CONFIDENCE INTERVALS ON BATTLE DURATION TIME

Recently, Engelhardt and Bain[2] and Mann and Fertig[12] have shown that efficient linear estimators of ξ (those with smallest or nearly smallest mean squared error) are approximately proportional to chi-square variates. Thus, using the two-moment fit of Patnaik,[15] one sees that if $\xi^*_{r,n}$ is an unbiased efficient linear estimator of ξ, based on the r smallest observations from a sample of size n, with variance $C_{r,n}\xi^2$, then $2\xi^*_{r,n}/(C_{r,n}\xi)$ is approximately a chi-square with $2/C_{r,n}$ degrees of freedom.

If $\xi^*_{r,n}$ is the unique best (uniformly minimum variance) linear unbiased estimator of ξ, then $\tilde{\xi}_{r,n} = \xi^*_{r,n}/(1+C_{r,n})$ is the best linear invariant estimator of ξ. (The estimator $\tilde{\xi}_{r,n}$ has mean squared error independent of η and uniformly smaller mean squared error than that of $\xi^*_{r,n}$.) As can be seen from the results of Lawless and Mann,[6] $\tilde{\xi}_{r,n}$ is very nearly equivalent to the iteratively obtained maximum-likelihood estimator $\hat{\xi}_{r,n}$ of ξ. This near equivalence can be seen, too, if one compares tabulations of distribution percentiles of $\tilde{\xi}_{r,n}/\xi$ and of $\xi/\hat{\xi}_{r,n}$ appearing in Mann and Fertig[11] and Thoman, Bain and Antle,[16] respectively, keeping in mind that both sets of Monte Carlo generated tabulations are correct to within about a unit in the second decimal place for the values of n compared. Hence $2(1+C_{r,n})\tilde{\xi}_{r,n}/(C_{r,n}\xi)$ and $2(1+C_{r,n})\hat{\xi}_{r,n}/(C_{r,n}\xi)$ are both approximate chi-square variates with $2/C_{r,n}$ degrees of freedom. This fact allows one to calculate confidence bounds for ξ by using either iteratively obtained maximum-likelihood estimates or best linear unbiased or best linear invariant estimates (or efficient approximations thereof) in conjunction with values of $C_{r,n}$ obtainable from tabulations in, for example, Mann,[7,8] Engelhardt and Bain,[2] Mann, Schafer and Singpurwalla,[14] and Mann and Fertig.[12] Engelhardt and Bain also provide, for various values of r/n, approximate expressions for $C_{r,n}$ that are quadratics in 1/n.

Mann, Schafer and Singpurwalla[14] use the chi-square property of $2\xi^*_{r,n}/(C_{r,n}\xi)$ to construct an approximately F-distributed statistic for obtaining confidence intervals on t_R (reliable life, or in this context, battle duration time). First, consider best (or approximately best) linear unbiased estimators $\eta^*_{r,n}$ and $\xi^*_{r,n}$ of η and ξ, respectively, with variances $A_{r,n}\xi^2$ and $C_{r,n}\xi^2$ and covariance $B_{r,n}\xi^2$. Best (or approximately best) linear invariant estimators of η and ξ are then $\tilde{\eta}_{r,n} = \eta^*_{r,n} - \xi^*_{r,n}B_{r,n}/(1+C_{r,n})$ and $\tilde{\xi}_{r,n} = \xi^*_{r,n}/(1+C_{r,n})$.

Define $\tilde{x}^*_{r,n}$ as $[\tilde{\eta}^*_{r,n} - (B_{r,n}/C_{r,n})\xi^*_{r,n}] \equiv [\tilde{\eta}_{r,n} - (B_{r,n}/C_{r,n})\tilde{\xi}_{r,n}]$ with variance $\xi^2(A_{r,n} - B^2_{r,n}/C_{r,n})$. The covariance of $X^*_{r,n}$ and $\xi^*_{r,n}$ is given by $[B_{r,n} - (B_{r,n}/C_{r,n})C_{r,n}]\xi^2 \equiv 0$. Mann, Schafer and Singpurwalla[14] give examples of comparisons with exact, Monte Carlo-generated, previously-tabulated values to show that for small values of P one can consider

$$(X^*_{r,n} - x_p)/\left\{\{-B_{r,n}/C_{r,n} - \ell n[-\ell n(1-P)]\}\xi^*_{r,n}\right\} \qquad (3)$$

to have an F distribution with

$$\nu_1 = 2\left\{B_{r,n}/C_{r,n} + \ell n[-\ell n(1-P)]\right\}^2/(A_{r,n} - B^2_{r,n}/C_{r,n}) \qquad (4)$$

and

$$\nu_2 = 2/C_{r,n} \qquad (5)$$

degrees of freedom. Note that both $\xi^*_{r,n}$ and $(X^*_{r,n} - x_p)/\{-B_{r,n}/C_{r,n} - \ell n[-\ell n(1-P)]\}$ are unbiased estimators of ξ; and with a two-moment fit to chi-square, each is under the proper conditions, an approximate chi-square over its degrees of freedom. The value of ν_1 is, like the value of ν_2, obtained from the two-moment chi-square fit [$2m(X^* - x_p)/v$ is approximately chi-square with $2m^2/v$ degrees of freedom, for $E(X^* - x_p) = m$ and $var(X^* - x_p) = v$] and can be calculated from tabulated values. Values of $A_{r,n}$ and $B_{r,n}$ can be found with tabulations of $C_{r,n}$. Additional values of $A_{r,n}$, for large n, are given by Mann, et al,[14] p. 252.

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Mann[10] gives the rationale for the chi-square fit for the numerator and gives ranges in terms of ν_1 and ν_2 for which values for obtaining confidence bounds on x_p from the F-approximation (3) are in error by two percent or less for P = .01, .05 and .10. In most cases the approximation is excellent, even for n or r as small as 4 unless censoring is extreme. Lawless[5] shows that lower confidence bounds on x_p based on (3) with $X^*_{r,n}$ replaced by $\hat{\eta}_{r,n} - (B_{r,n}/C_{r,n})\hat{\xi}_{r,n}$, a function of the maximum-likelihood estimators $\hat{\eta}_{r,n}$, and $\hat{\xi}_{r,n}$, and $\xi^*_{r,n}$ replaced by $(1 + C_{r,n})\hat{\xi}_{r,n}$ agree to two or more significant figures with exact values for sample sizes 25 through 60 unless nearly 90 percent of the sample is censored; i.e., $r/n \leq .10$. Mann[9] demonstrates the excellence of a similar F-approximation in obtaining prediction intervals for future samples, or lots to be manufactured in the future.

Thus, for a specified *large* survival proportion P applying to either of the opposing sides in a battle one might conclude from (3) that an approximate lower $(1-\alpha)$-level confidence bound on the corresponding battle duration time $t_p = \exp(x_p)$ can be calculated as

$$\exp\left\{X^*_{r,n} + F_\alpha(\nu_1,\nu_2)\{B_{r,n}/C_{r,n} + \ell n[-\ell n(1-P)]\}\xi^*\right\} \quad , \tag{6}$$

where $F_\alpha(\nu_1,\nu_2)$ is the *100*αth percentile of F with ν_1 and ν_2 degrees of freedom and ν_1 and ν_2 defined by (4) and (5), respectively. An upper $(1-\alpha)$-level confidence bound on t_p is given by

$$\exp\left\{X^*_{r,n} + F_{1-\alpha}(\nu_1,\nu_2)\{B_{r,n}/C_{r,n} + \ell n[-\ell n(1-P)]\}\xi^*\right\} \tag{7}$$

A two-sided confidence interval on t_p at confidence level $(1-2\alpha)$ is an interval with lower and upper bounds given by (6) and (7), respectively.

Recall, for use of F tables, that $F_{\alpha}(\nu_1,\nu_2) = 1/F_{1-\alpha}(\nu_2,\nu_1)$. In general, however, ν_1 and ν_2 will not be integers so that one can interpolate in tables of percentiles of F, or can use the approximation suggested by Mann and Grubbs[13] to evaluate $F_{\gamma}(\nu_1,\nu_2)$ for given γ.

It should be clear from inspection of the expression (3), (4) and (5) that if x_p is specified and a confidence interval is required for P (or equivalently, for $\ell n[-\ell n(1-P)]$), it must be computed iteratively since P occurs in both (3) and (4). In this case we find that a lower confidence bound for P at level $1-\alpha$ for specified $x_p = \ell n(t_p)$ is given by $1 - \exp\left\{-\exp\left\{(x_p - \overset{*}{x}_{r,n})/[\overset{*}{\xi}_{r,n}F_{1-\alpha}(\nu_1,\nu_2)] - B_{r,n}/C_{r,n}\right\}\right\}$, where ν_1 is a function of the value of P which is being determined.

IV. PRECISION OF THE APPROXIMATION

A Monte Carlo simulation study was undertaken so that the precision of the F-approximation (3) could be determined for values of P of interest. The values of P considered were .75(.05).95 and .99 and the percentiles of the appropriate F-distributions tabulated and compared with Monte Carlo values were .01, .02, .05(.05).95, .98, .99. The values of n and r considered were n = 8, r = 4,8 and n = 15, r = 5,10,15. For r/n fixed, one would expect precision to increase with increasing sample size n since the basis of the approximation is an asymptotic result. Monte Carlo sample size for the study was 10,000.

It was found that the precision of the approximation increased with increasing P over the range considered and with a limited amount of increased censoring for a fixed sample size. In both cases the increase in precision resulted apparently from an increase in ν_1 relative to ν_2. Results indicate that agreement of exact percentiles of (3) and those based on the F-approximation is to within about 2 percent over the range of percentiles from .01 to .99 for

$$\nu_1 \geq .03\nu_2 + 20.0 \quad , \quad \nu_2 \geq 8.0$$

and over the range of percentiles from .10 to .99 for

$$\nu_1 \geq 0.3\nu_2 + 4.0 \quad , \quad \nu_2 \geq 8.0 \quad .$$

These ranges are similar in spirit to those given by Mann[10] for small values of P. Shown in Table I are the results of six of the 30 independent simulations performed

V. EXAMPLE

An example of simulated-battle data is given in Table I of Grubbs and Shuford,[3] and a description of the simulated battle is given preceding their table. For each of the two sides in the engagement, the number of key targets (tanks, in this case) is 20. The data consist of times-to-incapacitation in minutes for four CBT's (chief battle tanks) and for five R10 tanks on the opposing side. Extensions of tables of Mann[8] have been used to provide estimates $\tilde{\eta}_{4,20} = 5.827$ and $\tilde{\xi}_{4,20} = 1.002$ from the R10 data.

To use (7) to obtain a confidence interval on t_p for the R10 tanks for a specified P of, say, .95, one needs to calculate $\ln[-\ln(1-P)] = 1.0972$ and to look up tabulated values from which values of $A_{5,20}$, $B_{5,20}$ and $C_{5,20}$ can be determined in the report that provides the coefficients (or weights) for calculating the estimates of η and ξ. The tabulated values from which the necessary constants can be calculated are $E_{5,20}(LU) = E_{5,20}[(\tilde{\eta}-\eta)^2]/\xi^2$, $E_{5,20}(LB) = E_{5,20}[(\tilde{\xi}-\xi)^2]/\xi^2$ and $E_{5,20}(CP) = E[(\tilde{\eta}-\eta)(\tilde{\xi}-\xi)]/\xi^2$. To determine $A_{5,20}$, $B_{5,20}$ and $C_{5,20}$ one needs to know the relationships:

$$C_{5,20} = E_{5,20}(LB)/[1 - E_{5,20}(LB)]$$

$$B_{5,20} = E_{5,20}(CP)/[1 - E_{5,20}(LB)]$$

Table I. Approximate and Monte Carlo (M.C.) Values of 100γth Distribution Percentiles of the Approximate F-Variate (3)

	n = 15, r = 10, P = .85 ν_1 = 15.6 ν_2 = 22.4		n = 15, r = 5, P = .85 ν_1 = 23.2 ν_2 = 8.8		n = 15, r = 5, P = .75 ν_1 = 19.1 ν_2 = 8.8	
γ	M.C. Percentile	Approximate Percentile	M.C. Percentile	Approximate Percentile	M.C. Percentile	Approximate Percentile
0.01	0.223	0.310	0.315	0.319	0.268	0.283
0.02	0.262	0.358	0.360	0.364	0.318	0.329
0.05	0.409	0.442	0.445	0.446	0.406	0.412
0.10	0.518	0.530	0.535	0.535	0.491	0.504
0.25	0.718	0.713	0.728	0.732	0.697	0.707
0.40	0.886	0.873	0.921	0.917	0.891	0.899
0.50	1.00	0.984	1.06	1.05	1.04	1.04
0.60	1.13	1.11	1.22	1.22	1.21	1.21
0.75	1.37	1.35	1.58	1.56	1.55	1.57
0.90	1.81	1.81	2.34	2.29	2.30	2.33
0.95	2.16	2.15	3.00	2.93	2.96	3.01
0.98	2.65	2.62	4.08	3.97	4.18	4.10
0.99	3.04	3.00	5.08	4.92	5.14	5.10

	n = 15, r = 15, P = .99 ν_1 = 40.4 ν_2 = 44.1		n = 8, r = 4, P = .95 ν_1 = 23.0 ν_2 = 7.0		n = 8, r = 8, P = .95 ν_1 = 9.9 ν_2 = 21.5	
γ	M.C. Percentile	Approximate Percentile	M.C. Percentile	Approximate Percentile	M.C. Percentile	Approximate Percentile
0.01	0.472	0.482	0.284	0.282	0.059	0.233
0.02	0.510	0.525	0.322	0.327	0.147	0.278
0.05	0.588	0.598	0.407	0.410	0.297	0.363
0.10	0.665	0.670	0.498	0.502	0.414	0.454
0.25	0.808	0.809	0.700	0.711	0.652	0.652
0.40	0.922	0.922	0.914	0.915	0.849	0.831
0.50	0.996	0.998	1.07	1.07	0.971	0.958
0.60	1.07	1.08	1.26	1.26	1.12	1.10
0.75	1.23	1.23	1.71	1.67	1.40	1.10
0.90	1.49	1.49	2.63	2.59	1.95	1.93
0.95	1.68	1.67	3.55	3.46	2.37	2.36
0.98	1.93	1.90	5.43	4.96	2.99	2.95
0.99	2.10	2.07	7.20	6.44	3.61	3.42

and

$$A_{5,20} = E_{5,20}(LU) + [E_{5,20}(CP)]^2/[1 - E_{5,20}(LB)] \quad .$$

From the tables and these relationships, we find $A_{5,20} = 0.70308$, $B_{5,20} = 0.33548$ and $C_{5,20} = 0.23662$. (For n = 25(5)∞, p = r/n = 0.10(0.10)1.00, values of $A_{r,n}$, $B_{r,n}$ and $C_{r,n}$ (or $\ell_{r,n}$) are tabulated directly by Mann, Schafer and Singpurwalla,[14] p. 252 and p. 244.)

Next, from (4) and (5), we find for the R10 data,

$$\nu_1 = 2(1.4178 + 1.0972)^2/(0.2274) = 55.62$$

and

$$\nu_2 = 2/.23662 = 8.45 \quad ,$$

correct to two decimal places. Note that $\nu_2 > 8.0$ and $\nu_1 > .3(\nu_2) + 20 = 22.53$, so we might expect the F-approximation to give very nearly the correct percentile of the distribution of (3) over the percentile range of .01 through .99. Using the method of Mann and Grubbs[13] to evaluate, say, the 10th percentile of F(55.62,8.45), we obtain 0.567. Then a 90 percent lower confidence bound on $t_{.95}$ is, from (6), given by

$$\exp[5.040 - 1.4178(1.002) + 0.567(1.4178 + 1.0972)(1.23662)(1.002)] = 218 \text{ minutes}$$

If a lower confidence bound on δ is desired, as on page 938 of Grubbs and Shuford,[3] then the specified value of P is $1 - \exp(-1) = .63$ and $\ell n[-\ell n(1-P)] = 0$.

so that $\nu_1 = 2(1.4178)^2/(.2274) = 17.68$ and $\nu_2 = 8.45$, as before. The value of $F_{.05}(17.68,8.45)$ is approximately 0.408 so that a 95 percent lower confidence bound on $\delta = \exp(\eta)$ is therefore given by

$$\exp[5.040 - 1.4178(1.002) + .408(1.4178)(1.23662)(1.002)] = 76.4 \text{ minutes} .$$

This agrees well with the lower 95 percent confidence bound 74.4 minutes shown in the example of Grubbs and Shuford and calculated as $\exp(\tilde{\eta} - w_{.95}\tilde{\xi})$, where $w_{.95}$ is a percentile of the distribution of $(\tilde{\eta}-\eta)/\tilde{\xi}$ computed by Monte Carlo simulation procedures and published in the report by Mann, Fertig and Scheuer with tabulations supplemental to those of Mann and Fertig.[11] A lower confidence bound on δ, incidentally, will correspond to a lower confidence bound on *mean-time-to-incapacitation* for $\beta = \xi = 1$, since the exponential distribution is a special case of the Weibull with shape parameter equal to 1.

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