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Kalman Filter Configurations for Multiple Radar Systems

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KALMAN FILTER CONFIGURATIONS FOR MULTIPLE RADAR SYSTEMS

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Group 32

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ABSTRACT

The purpose of this report is to examine several Kalman filter algorithms that can be used for state estimation with a multiple sensor system. In a synchronous data collection system, the statistically independent data blocks can be processed in parallel or sequentially, or similar data can be compressed before processing; in the linear case these three filter types are optimum and their results are identical. In multilateration radar tracking applications, the data compression method is shown to be computationally most efficient, followed by the sequential processing, the parallel processing is least efficient. These algorithms are described in detail and their results are compared with a suboptimum tracking algorithm which processes only multiple range measurements. A state estimate compression algorithm is also described. Various radar measurement transformation formulas are listed. Algorithms for a nonsynchronous data collection system are not examined in detail but possible approaches are suggested.

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1. INTRODUCTION

Recent studies (Ref. 1,2) have generated renewed interest in multistatic and multilateral radar systems. These systems can improve target tracking accuracy using range measurements from multiple radars rather than range and angle measurements from a single radar. Preliminary simulated multi-radar estimates (Ref. 3) have shown promising improvement when compared to corresponding single radar results.

The purpose of this note is to formulate the Kalman filter configurations that can be applied to multiple netted-radar measurement systems; this report also addresses the general filtering problem for measurement systems with many simultaneous measurements. In Section 2 the problem is described in more detail. The two main tools for this report are reviewed; the extended Kalman filter for nonsynchronously collected measurements from different locations in Section 3 and the transformation of one measurement system to another in Section 4. In Section 5 the results of Sections 3, 4 are combined and the filter configurations for various measurement systems are derived - some of their advantages and disadvantages are discussed. Emphasis is on the examination of the parallel filter - all measurements are processed simultaneously (parallel), the sequential filter-processing blocks of uncorrelated measurements sequentially, and data compression-compressing the data before processing. Estimate compression combines the filter outputs - comparable to data

compression. In Section 6 some numerical results are presented. Four appendices are attached. They present radar measurement transformation formulas, derivation of data and estimate compression equations, proof of filter equivalence, and computational counts of various filter configurations.

2. PROBLEM STATEMENT

In the measurement system under consideration several radars at different locations make measurements of the same RV. The accuracies of the individual radars are known, their sampling times may or may not be synchronized or they may be random. Figure 2.1 shows a schematic of such a measurement system; the measurement vector of each individual sensor i is subscripted. The individual radars may be active (i.e., transmit and receive) or passive (receive only). A system is defined multilateral if all radars are active, multistatic if there is one active and several passive radars. Special cases under consideration are trilateration (as in RMP-74) with 3 active radars, or a bistatic measurement systems with one active and several passive radars.

There does not exist an extensive literature for the multiple measurement system as for the single observer. This report describes and evaluates possible filter configurations. Particular emphasis is given to the multiple radar siting system in the context of BMD for synchronized, non-synchronized, and random measurement times. Some of the economics of implementing the various filters will be discussed.

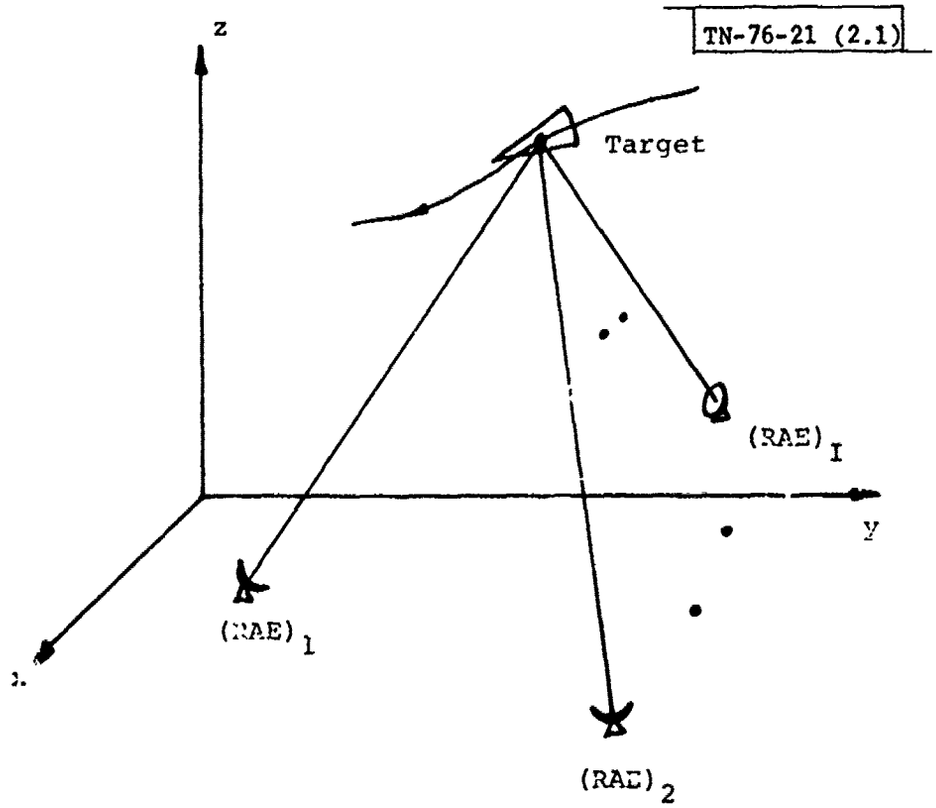

Figure 2.1 Schematic program of a multilateration radar system.

Simplifying assumptions have been made including no wake contamination, no association problem (occurring in a multitarget environment), no sidelobe problem, etc. These problems will have to be solved before any such system can be successfully implemented.

3. THE EXTENDED KALMAN FILTER FOR TARGET TRACKING

In this section the formulation of the extended Kalman filter is reviewed. Both nonlinear and linear cases are outlined. Consider the RV dynamics that can be described by the n-dimensional vector nonlinear differential equation

$$\dot{x}(t) = f(x(t)) + n(t) \quad ; \quad x(0) = x_o \tag{3.1}$$

where $n(t)$ is a zero-mean Gaussian white noise with covariance $Q(t)$ and x_o is Gaussian with mean x_o and covariance P_o. The measurements are collected (randomly) at discrete times in the form

$$z(t_k) = z_k = h(x_k) + v_k \quad ; \quad x_k = x(t_k) \tag{3.2}$$

where v_k is an m-dimensional zero-mean Gaussian white noise sequence with covariance R_K.

It is assumed that (3.1) has a unique solution and can be expressed in the discrete form associated with (3.2)

$$x_k = f_{k-1}(x_{k-1}, \Delta t_{k-1}) + w_{k-1} \qquad k=1,2, \ldots$$

where $\{w_k\}$ is an n-dimensional zero-mean Gaussian white noise sequence with covariance $Q_k = Q_k(t_{k+1}, t_k)$ and $\Delta t_{k-1} = t_k - t_{k-1}$.

The extended Kalman filter associated with (3.1) and (3.2) is stated below:

PREDICT CYCLE:

(State) $$\hat{x}_{k+1/k} = f_k(\hat{x}_{k/k}, \Delta t_k) \quad ; \quad \hat{x}_{o/o} = \hat{x}_o \tag{3.3}$$

or $$\hat{x}_{k+1/k} = \hat{x}_{k/k} + \int_{t_k}^{t_{k+1}} f(x(\tau), \tau) d\tau \tag{3.3a}$$

(Covariance) $$P_{k+1/k} = A_k P_{k/k} A_k^T + Q_k \quad ; \quad P_{o/o} = P_o \tag{3.4}$$

where $\hat{x}_{k/j}$ denotes the estimate of x at time t_k based upon all the data up to time t_j and $P_{k/j}$ denotes the covariance of $\hat{x}_{k/j}$. A_k is the Jacobian matrix of f_k at $\hat{x}_{k/k}$ and Δt_k.

$$A_k = \frac{\partial f}{\partial x} \Big|_{x=\hat{x}_{k/k}} \tag{3.5}$$

UPDATE CYCLE:

(State) $$\hat{x}_{k+1/k+1} = \hat{x}_{k+1/k} + K_{k+1}(z_{k+1} - h(\hat{x}_{k+1/k})) \tag{3.6}$$

(Gain) $$K_{k+1} = P_{k+1/k} H_{k+1}^T (H_{k+1} P_{k+1/k} H^T{}_{k+1} + R_{k+1})^{-1} \tag{3.7}$$

or $$K_{k+1} = P_{k+1/k+1} H^T_{k+1} R^{-1}_{k+1} \tag{3.7a}$$

(Covariance) $$P_{k+1/k+1} = (I - K_{k+1} H_{k+1}) P_{k+1/k} \tag{3.8}$$

or $$P_{k+1/k+1} = (P^{-1}_{k+1/k} + H^T_{k+1} R^{-1}_{k+1} H_{k+1})^{-1} \tag{3.8a}$$

if $P^{-1}_{k+1/k}$ exists

where H_{k+1} is the Jacobian matrix of h at $\hat{x}_{k+1/k}$.

$$H_{k+1} = \frac{\partial h}{\partial x}\bigg|_{x=\hat{x}_{k+1/k}} \tag{3.9}$$

If the measurements are linear with respect to x_k, then Eqs. (3.2) and (3.6) become

$$z_k = H_k x_k + v_k \tag{3.2'}$$

and

$$\hat{x}_{k+1/k+1} = \hat{x}_{k+1/k} + K_{k+1}(z_{k+1} - H_{k+1} \hat{x}_{k+1/k}) \tag{3.6'}$$

respectively.

4. TRANSFORMATION OF MEASUREMENTS

The need for a transformation of measurement arises, e.g., when the filter coordinate system is different from the

measurement coordinate system. This is the case in particular when several radars at different locations make measurements of the same object.

In Section 3 the nonlinear measurement was presented of the form

$$z_k = h(x_k) + v_k \tag{4.1}$$

Here we are concerned with a radar measurement at a different location of the form

$$z_k^* = h^*(x_k) + v_k^* \tag{4.2}$$

where $\{v_k^*\}$ is an m-dimensional zero-mean Gaussian white noise sequence with covariance R_k^*. To use Eq. (4.2) in a filter designed for Eq. (4.1) the measurement has to be transformed. Assuming that the measurement can be transformed into the form of Eq. (4.1) - in the deterministic case - z_k computes to:

$$z_k = g(z_k^*) \tag{4.3}$$

For the stochastic case, Eq. (4.3) can be approximated as

$$z_k = g(z_k^*) \approx h(x_k) + v_k \tag{4.4}$$

where $\{v_k\}$ is an m-dimensional zero mean Gaussian white noise sequence with covariance

$$R_k = G_k \; R_k^* \; G_k^T \tag{4.5}$$

$$G_k = \frac{\partial g}{\partial z^*}\Big|_{z^* = h^*(\hat{x}_{k/k-1})} \tag{4.6}$$

and $\hat{x}_{k/k-1}$ is generated as in the previous section Eq. (3.3) at t_k.

After making the transformation of Eq. (4.3) the filter of Eq. (3.2) can be used by correcting R_k as shown in Eq. (4.5). The H matrix is computed via Eq. (3.9) - as indicated in Section 3 - or in two steps as,

$$H = \left(\frac{\partial g}{\partial z^*}\Big|_{z^* = h^*(\hat{x}_{k/k-1})}\right)\left(\frac{\partial h^*}{\partial x}\Big|_{x = \hat{x}_{k/k-1}}\right) \tag{4.7}$$

It becomes clear, that the above transformation is independent of the RV dynamics. In general, the approximation of Eq. (4.4) is satisfactory only if the nonlinearities are small.

Three basic measurement transformations are considered in this report:

I) $R_1\ A_1\ E_1$ to $R_o\ A_o\ E_o$

II) $R_1\ R_2\ R_3$ to $R_1\ A_1\ E_1$

III) $R_1\ R_2\ R_3$ to $R_o\ A_o\ E_o$

Transformation I is used to convert the (R, A, E) data collected from one radar site to another radar site or the origin. Transformation II can be used to convert 3 range measurements from three radars into RAE measurements - either to use the data for existing RAE-filter inputs, or e.g., to evaluate the elevation and azimuth accuracies of the R_1, R_2, R_3 measuremenrs as compared to the R, A, E measurement of a single radar. Transformation III is a combination of I and II. The transformation formula and the associated G matrices (of Eq. (4.6)) are derived in Appendix A for the three transformations.

5. FILTER ALGORITHMS FOR MULTIPLE SENSORS AND LOCATIONS

5.1 Introduction

In this section various Kalman filter algorithms for sensors at multiple locations are presented. The major advantage of such a multilaterated measurement system is the possibility of obtaining more accurate data for the tracking filter. Using the radar measurements from several different locations may result in a much smaller uncertainty volume. With the proper geometry the angle measurements may become redundant - in the sense, that the processing of the angle measurements does not improve the estimation accuracy by much. Neglecting them in such cases results in a considerable saving in computer resources while sacrificing little in filter performance. Because of the redundancy in this type of measurement system it also is less vulnerable against outages (forced or otherwise.)

Various cases of data collection and filter configurations will be considered. Both the synchronous (all sensors collect data at the same time) and the nonsynchronous case (each sensor works independently of the others) are evaluated. In multilateration radar tracking system the collection could be either synchronized or nonsynchronized. In a system of bistatic radars only the data collection is necessarily synchronized.

The radars at different locations have different measurement coordinate systems with respect to a fixed state (or state estimate) coordinate system. The Kalman filter can be designed to accommodate all measurement coordinate systems - or the measurements must be transformed to fit a particular filter design as discussed in Section 4. Both types of filter configurations will be discussed.

In Section 5.2 three optimal (in the linear case) and one suboptimal filter for the synchronous data collection case are suggested and investigated; the nonsynchronous case is treated in Section 5.3. Also described is the possibility of preprocessing the data to reduce the computational requirements. In Section 5.4 the filter performance is evaluated when only a subset of the data is processed and compared to the optimal case for which all the data are processed.

5.2 Synchronously Collected Data

Let $z_{k+1,i}$ denote the measurement taken at time t_{k+1} from i-th radar with a total of I radars, then

$$z_{k+1,i} = h_i(x_{k+1}) + v_{k+1,i} \qquad \forall\ i=1, \ldots\ldots, I \qquad (5.1)$$

where $\{v_{k+1,i}\}$ is a white Gaussian noise sequence with zero mean and covariance $R_{k+1,i}$. There are four options in processing these measurements by a Kalman filter. They are discussed individually below.

5.2.1 Parallel Filter

All measurment vectors may be used to form a new measurement vector z_{k+1}.

$$z_{k+1} = \begin{bmatrix} z_{k+1,1} \\ z_{k+1,2} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ z_{k+1,I} \end{bmatrix} \qquad (5.2)$$

If each $z_{k+1,i}$ is an m-vector, then z_{k+1} is an M=mxI vector (otherwise $M = \sum_{i=1}^{I} m_i$). If the measurement noise for different

radars are uncorrelated, the covariance of z_{k+1}, R_{k+1}, is

$$R_{k+1} = \begin{bmatrix} R_{k+1,1} & 0 & \cdots & 0 \\ 0 & R_{k+1,2} & & \\ \vdots & & & \\ & & & R_{k+1,I} \end{bmatrix} \tag{5.3}$$

Using (5.2) and (5.3) in the filter update equation and after a few manipulations, we obtain

(State) $$\hat{x}_{k+1/k+1} = \hat{x}_{k+1/k} + \sum_{i=1}^{I} K_{k+1,i}(z_{k+1,i} - h_i(\hat{x}_{k+1/k})) \tag{5.4}$$

(Gain) $$K_{k+1,i} = P_{k+1/k+1} H^T_{k+1,i} R^{-1}_{k+1,i} \tag{5.5}$$

(Covariance) $$P^{-1}_{k+1/k+1} = P^{-1}_{k+1/k} + \sum_{i=1}^{I} (H^T_{k+1,i} R^{-1}_{K+1,i} H_{k+1,i}) \tag{5.6}$$

where $H_{k+1,i}$ is the Jacobian matrix of $h_i(x_{k+1})$ at $\hat{x}_{k+1/k}$. Notice that the inverse covariance matrix equation is used in (5.6). This form is more convenient for discussing filter equivalence. This algorithm is depicted in Figure 5.1.

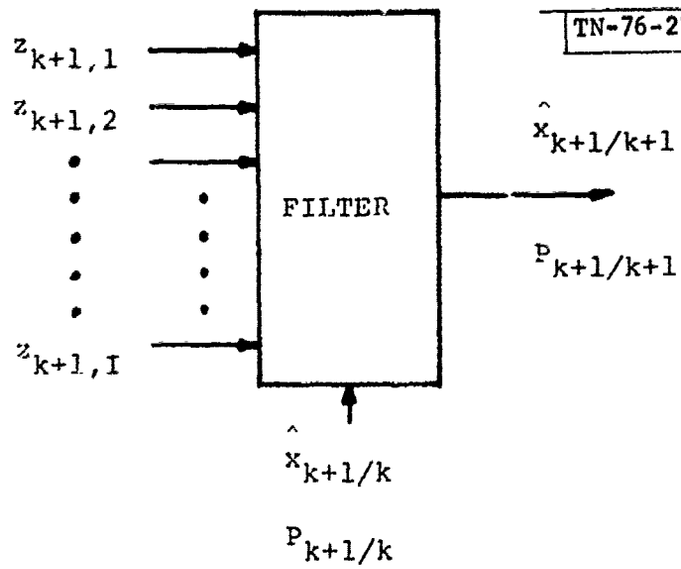

Figure 5.1 Parallel filter.

5.2.2 Sequential Filter

Each measurement may be treated as a new measurement with zero prediction time for $i>1$. The estimates may then be updated sequentially. The update algorithm becomes

(State) $$\hat{x}_{k+1/k+1} = \hat{x}_{k+1/k} + \sum_{i=1}^{I} K_{k+1,i} \left(z_{k+1,i} - h_i(\hat{x}_{k+1/k+1,i-1})\right) \tag{5.7}$$

$$\hat{x}_{k+1/k+1,0} = \hat{x}_{k+1/k}, \qquad P_{k+1/k+1,0} = P_{k+1/k} \tag{5.8}$$

$$\hat{x}_{k+1/k+1,i} = \hat{x}_{k+1/k+1,i-1} + K_{k+1,i} \left(z_{k+1,i} - h_i(\hat{x}_{k+1/k+1,i-1})\right) \tag{5.9}$$

$$i = 1, \ldots, I$$

(Gain) $$K_{k+1,i} = P_{k+1/k+1,i-1} H^T_{k+1,i} \left(H_{k+1,i} P_{k+1/k+1,i-1} H^T_{k+1,i} + R_{k+1,i}\right)^{-1}$$

or $$K_{k+1,i} = P_{k+1/k+1,i} H^T_{k+1,i} R^{-1}_{k+1,i}$$

$$i = 1,2, \ldots, I \tag{5.10}$$

(Covariance) $P_{k+1/k+1,i} = P_{k+1/k+1,i-1} - K_{k+1,i} H_{k+1,i} P_{k+1/k+1,i-1}$ (5.11)

or $P^{-1}_{k+1/k+1,i} = P^{-1}_{k+1/k+1,i-1} + H^T_{k+1,i} R^{-1}_{k+1,i} H_{k+1,i}$

$i = 1,2, \ldots, I$ (5.11a)

$\hat{x}_{k+1/k+1} = \hat{x}_{k+1/k+1,I}$ $P_{k+1/k+1} = P_{k+1/k+1,I}$

(5.12)

Notice that the i-th measurement is used to update the state estimate at the i-th step. This algorithm is illustrated in Figure 5.2.

5.2.3 Data Compression

All measurements may first be combined to form a pseudo-measurement (data compression). In this case, the filter only needs to be updated once. If a weighted least square criterion or a Bayesian estimation formulation is used, the combined measurement z_{k+1} and covariance R_{k+1} are*

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* For derivation, see Appendix B.

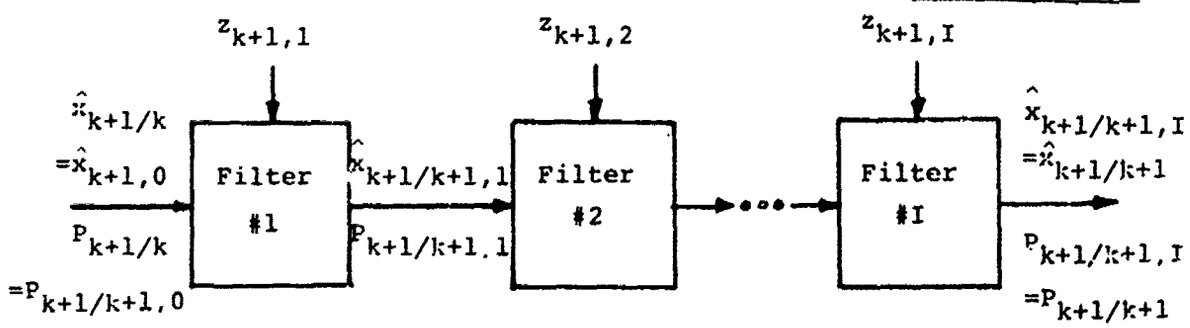


Figure 5.2 Sequential filter.

$$z_{k+1} = R_{k+1} \left(\sum_{i=1}^{I} R_{k+1,i}^{-1} z_{k+1,i} \right) \tag{5.13}$$

$$R_{k+1}^{-1} = \sum_{i=1}^{I} R_{k+1,i}^{-1} \tag{5.14}$$

In order to use (5.13) and (5.14) all measurement vectors have to be transformed to a common coordinate system. The transformation procedure is discussed in Section 4. Using the above results, the update equations are unchanged as stated in (3.5), (3.6), and (3.7). This algorithm is depicted in Figure 5.3.

5.2.4 Estimate Compression

Each radar may have its own filter and process its own measurement. The resulting estimates are then combined (as outlined in Appendix B). However, since the P_{ij} (P_{ij} = correlation of the estimates from the i-th and j-th filter) for $i \neq j$ are generally not available any compressed estimate is suboptimal and no correct estimate of the covariance matrix exists. An algorithm for estimate compression is illustrated in Fig. 5.4.

5.2.5 Algorithm Comparison

Four algorithms have been discussed above. In the case of a linear system it can be shown (see Appendix C) that the resulting estimates of parallel filter, sequential filter, and data

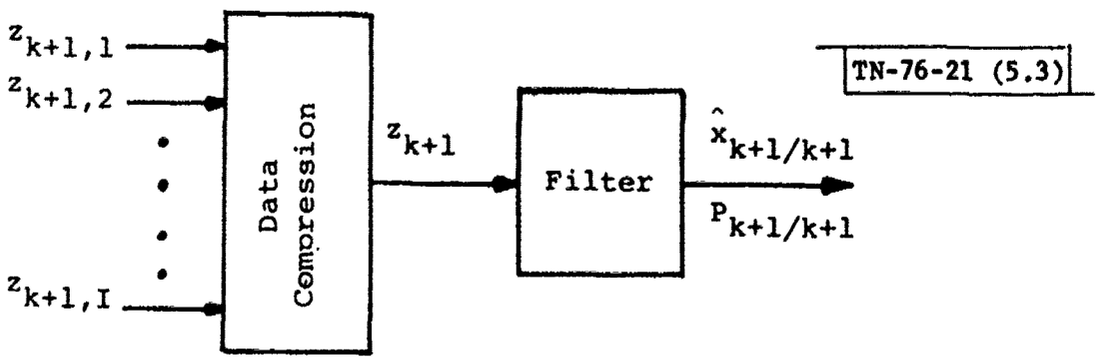


Figure 5.3 Data compression.

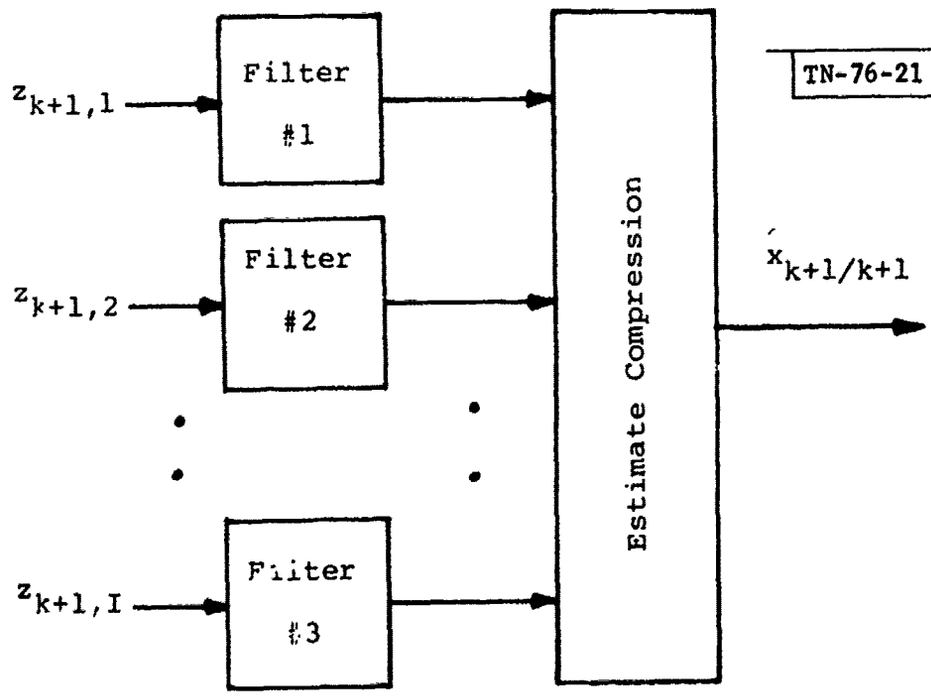


Figure 5.4 Estimate compression.

compression are identical, and optimal. In Table 5.1 a cost comparison is shown (in terms of multiplications per step).* The estimate compression method requires the most computations, it also does not result in a least square estimate and is not optimal.

The data compression method is computationally more efficient than all the others. Although it requires that all measurements be transformed to a common coordinate, the filter needs only to be updated once. The computation requirements between parallel filter and sequential filter depend upon the dimension of the state and the total number of measurements. Let n denote the dimension of the state vector, m the dimension of the measurement vector, and I the total number of measurements, it can be seen from Appendix D that the sequential filter is more efficient than the parallel filter.

The comparison of all algorithms is demonstrated in Table 5.1 for a particular example (n=7, m=9).

5.3 Randomly Collected Data

The filter prediction and update process is carried out according to the availablility of new data set. Suppose at time t_{k+1} that the only available data is from radar i and let it be denoted by $z_{k+1,i}$ and $R_{k+1,i}$, the update is performed based upon this available data.

---

* The number of multiplications is derived in Appendix D.

Table 5.1 Comparison of algorithm efficiency for synchronously collected data (i.e., number of multiplications).

	Parallel Filter	Sequent. Filter	Data Comp.	Estimate Comp.
Computational Requirements	moderate-high	moderate	low	very high
	Pred. 588 Update 2181 2769* Via Eq. (3.7a-3.8a) Pred. 588 Update 1330 1918*	Pred. 588 Update 945 1533*	Pred. 588 Data Comp. 208 Update 394 1190*	Pred. 3 x 588 = 1764 Update 945 Est. Comp. 1092 [illegible]01*
Performance	Optimal			Suboptimal

* Linear Case n=7, m=9 (3 radars RAE each, independent measurements)

(State) $$\hat{x}_{k+1/k+1} = \hat{x}_{k+1} + K_{k+1,i}(z_{k+1,i} - h_i(\hat{x}_{k+1/k}))$$

(5.17)

(Gain) $$K_{k+1,i} = P_{k+1/k+1} H^T_{k+1,i} R^{-1}_{k+1,i} \quad (5.18)$$

(Covariance) $$P^{-1}_{k+1/k+1} = P^{-1}_{k+1/k} + H^T_{k+1,i} R^{-1}_{k+1,i} H_{k+1,i} \quad (5.19)$$

If the filter is restricted to accept data only in a fixed measurement coordinate system $z_{k+1,i}$ and $R_{k+1,i}$ must be first transformed into that coordinate system. The resulting update equations are the same as (3.5), (3.6), and (3.7). These two cases are illustrated in Fig. 5.5.

The draw back of a nonsynchronous data collection system is in its high computational requirements. This is caused by the fact that the filter must be updated sequentially, and the data compression scheme can not be applied.

Two alternatives exist. The first one is simply to insist on a synchronous data collection system. This is possible if bistatic radars are used or if sufficient communication exists between transmitters so that data can be collected synchronously. The second alternative is to preprocess the data for time alignment. A polynomial data smoother could be used as a data processor for data-time-alignment, similar to the one discussed in (Ref. 4).

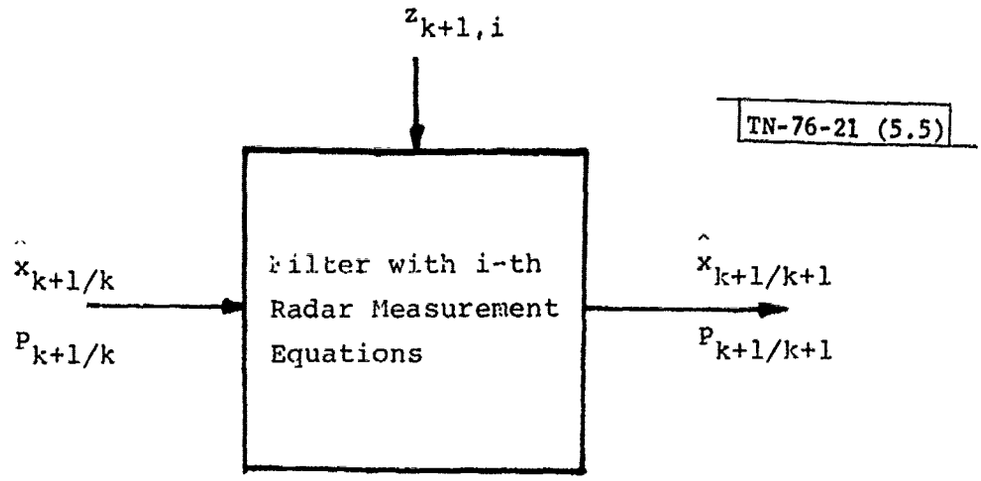


Figure 5.5-a Filter equipped with all measurement equations for randomly collected data.

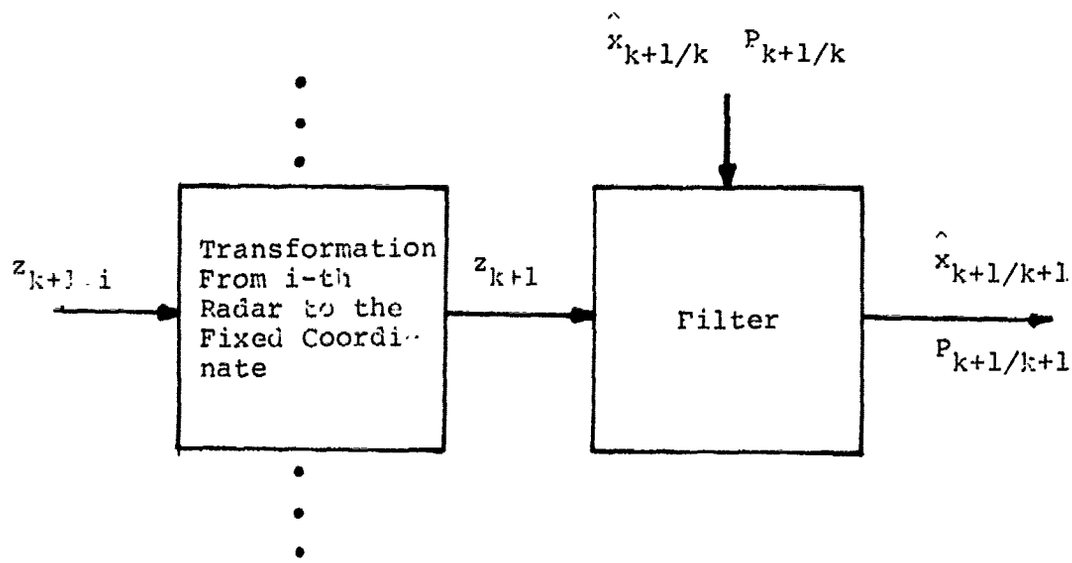


Figure 5.5-b Filter with a fixed measurement coordinate for randomly collected data.

5.4 Options of Processing all or Part of the Measurements

In a data collection system, part of the measurements may be of poorer quality (low SNR) than the others. If the remaining (high SNR) measurement still constitutes an observable system, the noisy measurements may be neglected with trade-offs in computation and performance.

This situation is particularly true in a multilateration tracking system. The range measurement accuracy is usually better than that of the angle (cross-range) measurement for a single radar. Several radars looking from different locations may result in much improved uncertainty volume even if only range measurements alone are used. When only range measurements are processed in the filter, the computation requirements are reduced over even the data compression method - which was the most efficient filter in terms of computation. Two methods may be used in applying multiple range measurements to a tracking filter.

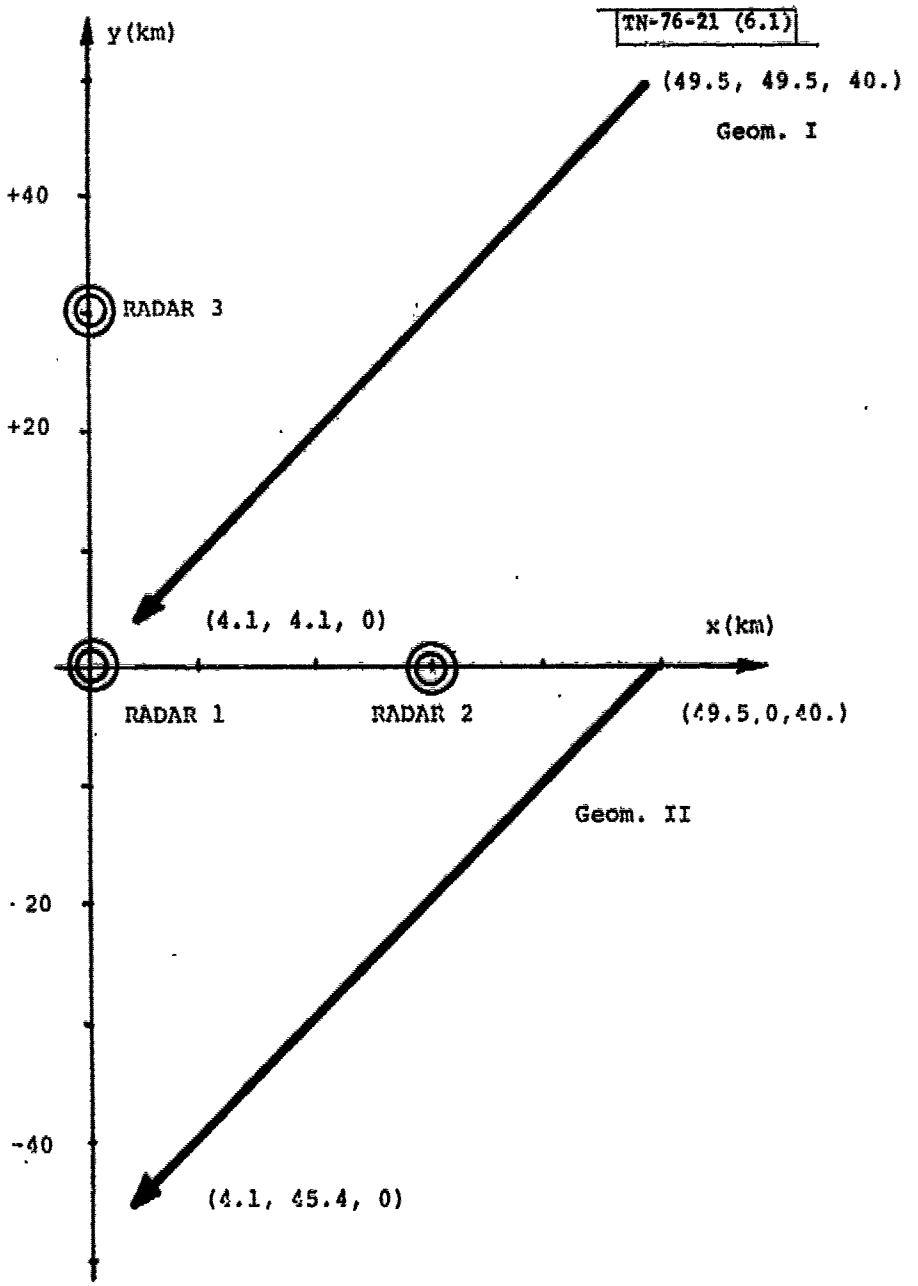
(a) Range measurements from several measurement locations (at least three) may be used to form a set of pseudo-measurements (range and angles) for a "virtual" radar. For proper geometry the effective angle measurement standard deviations can be considerably smaller than those obtained in a conventional radar system (of the order of 10^{-4} radian vs 10^{-3} radian).

(b) The range measurements may be directly processed by the tracking filter without going through the transformation illustrated above. This method uses less computation than the method described in (a). A filter which accepts range and angles will have to be modified to accept several ranges simultaneously. In the case of a linear system and measurements, it can be shown that there is no difference in performance in using either method. In the case of a nonlinear system such as the RV tracking system, it is expected that both methods will achieve close performance.

It will be shown in the numerical results that with proper geometry,processing range measurements alone can achieve virtually the same performance as processing all the measurements.

6. NUMERICAL RESULTS

The parallel, sequential and data compression filters were tested in a RV simulation. The reentry geometry is shown in Fig. 6.1. The estimation results for these various nonlinear filters are extremely close - for linear filters in other runs they were shown to be equal. The statistics for the nonlinear filters are identical.



(for all radars: σ_R = 3m $\sigma_A = \sigma_E$ = 1 mR)

Figure 6.1 Reentry geometries I, II.

In Figs. 6.2 - 6.5 the RMS position and velocity errors are shown for 2 geometries for the following measurement configurations:

Radar 1: R,A,E

Radars 1,2,3: R_i ; i=1,2,3

Radars 1,2,3: $(R,A,E)_i$; i=1,2,3

At low altitudes (<20 km) geometry I has smaller RMS errors; at higher altitudes (>20 km) geometry II seems favorable.

For the radar measurement accuracies used, the trilateration results (3 radars-range only) are better than the typical single radar results (by a factor 2-5). The trilateration results for 3 radars $(R,A,E)_i$; i=1,2,3 show only marginal improvements when compared to the 3 radar - range only - results. It should be pointed out that the improvement due to trilateration will be much greater if the range accuracy is improved or if the angle accuracy of the single radar is reduced.

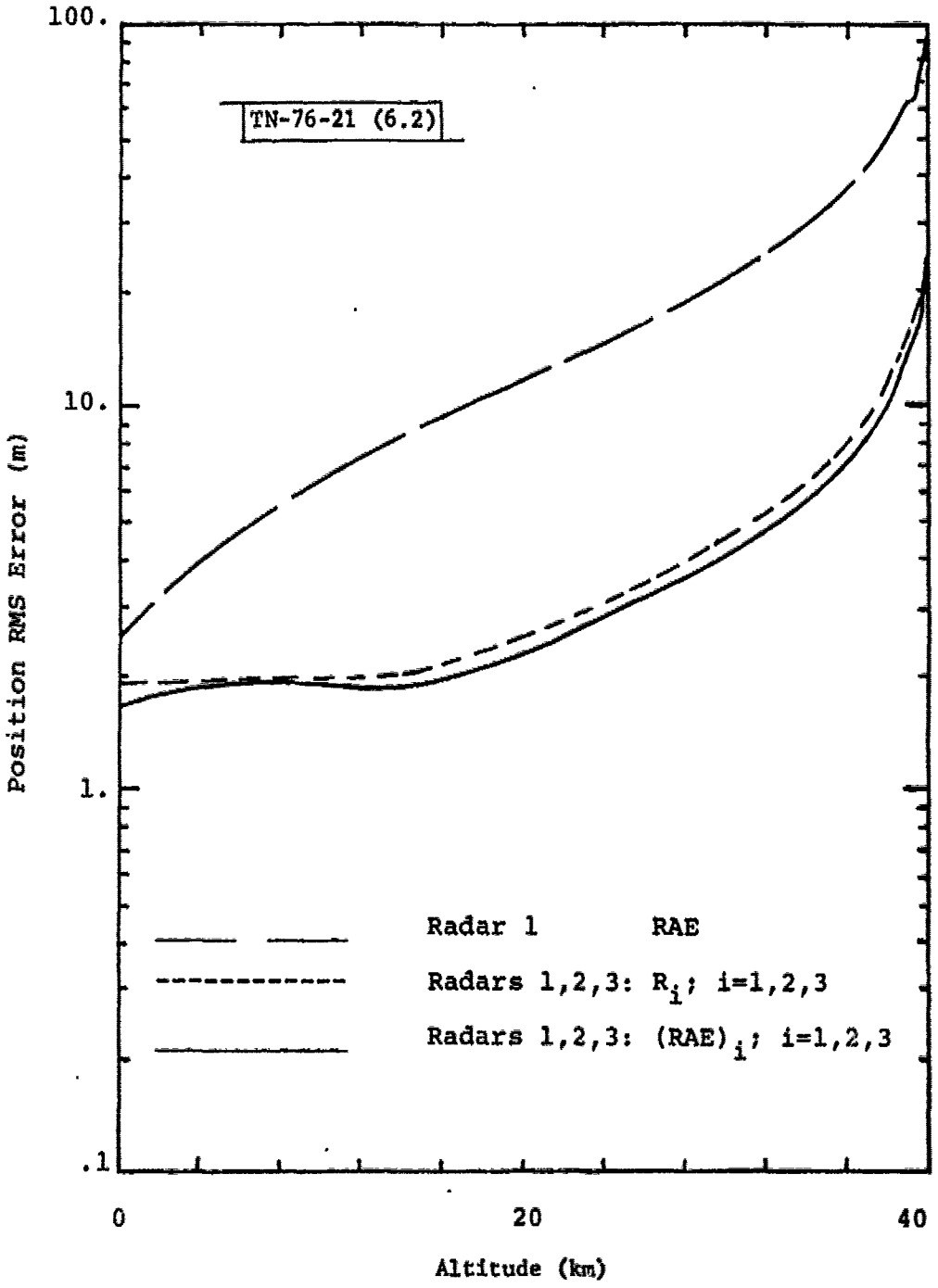


Figure 6.2 Position RMS error vs. altitude for geometry I.

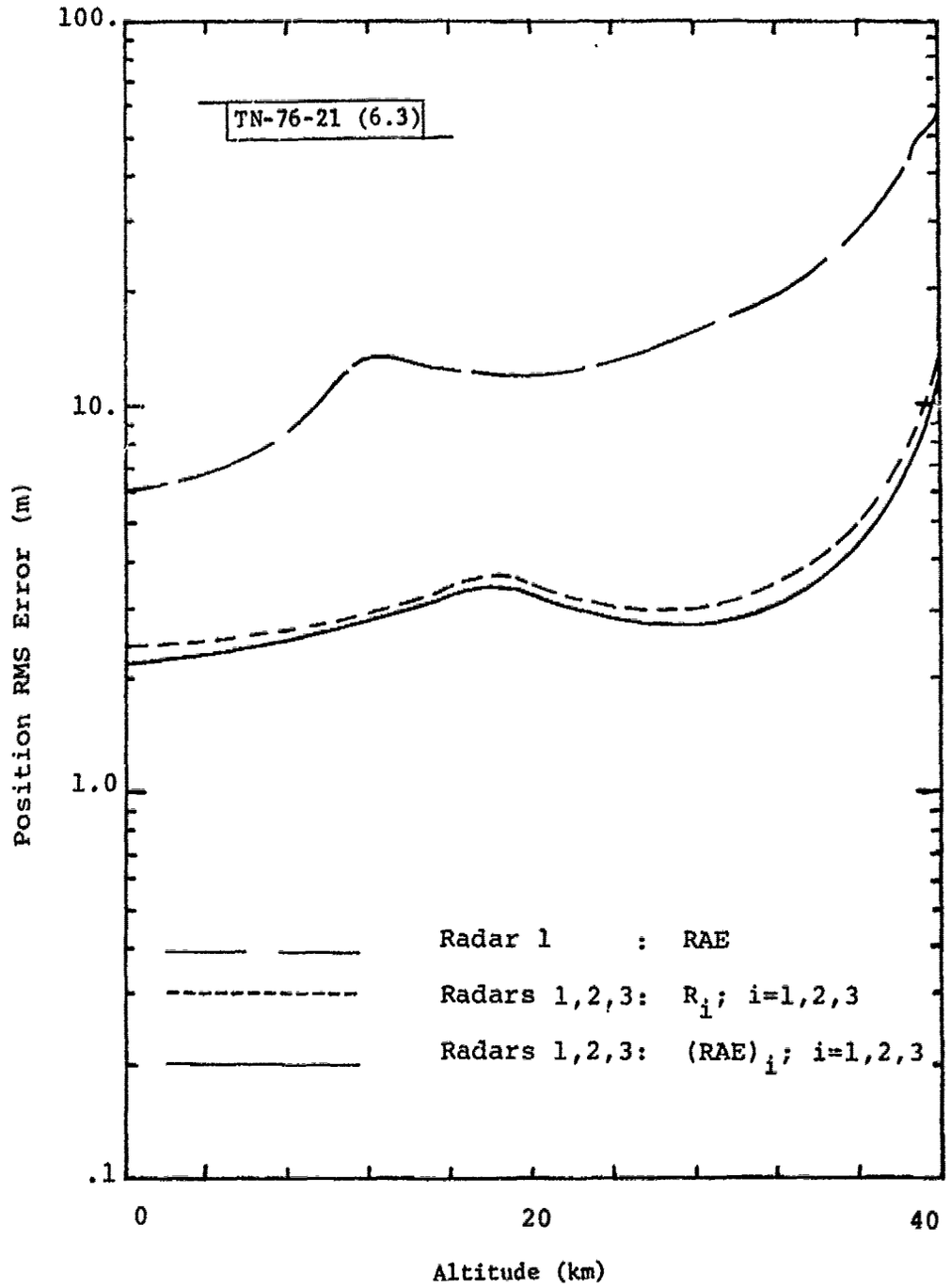


Figure 6.3 Position RMS error vs. altitude for geometry II.

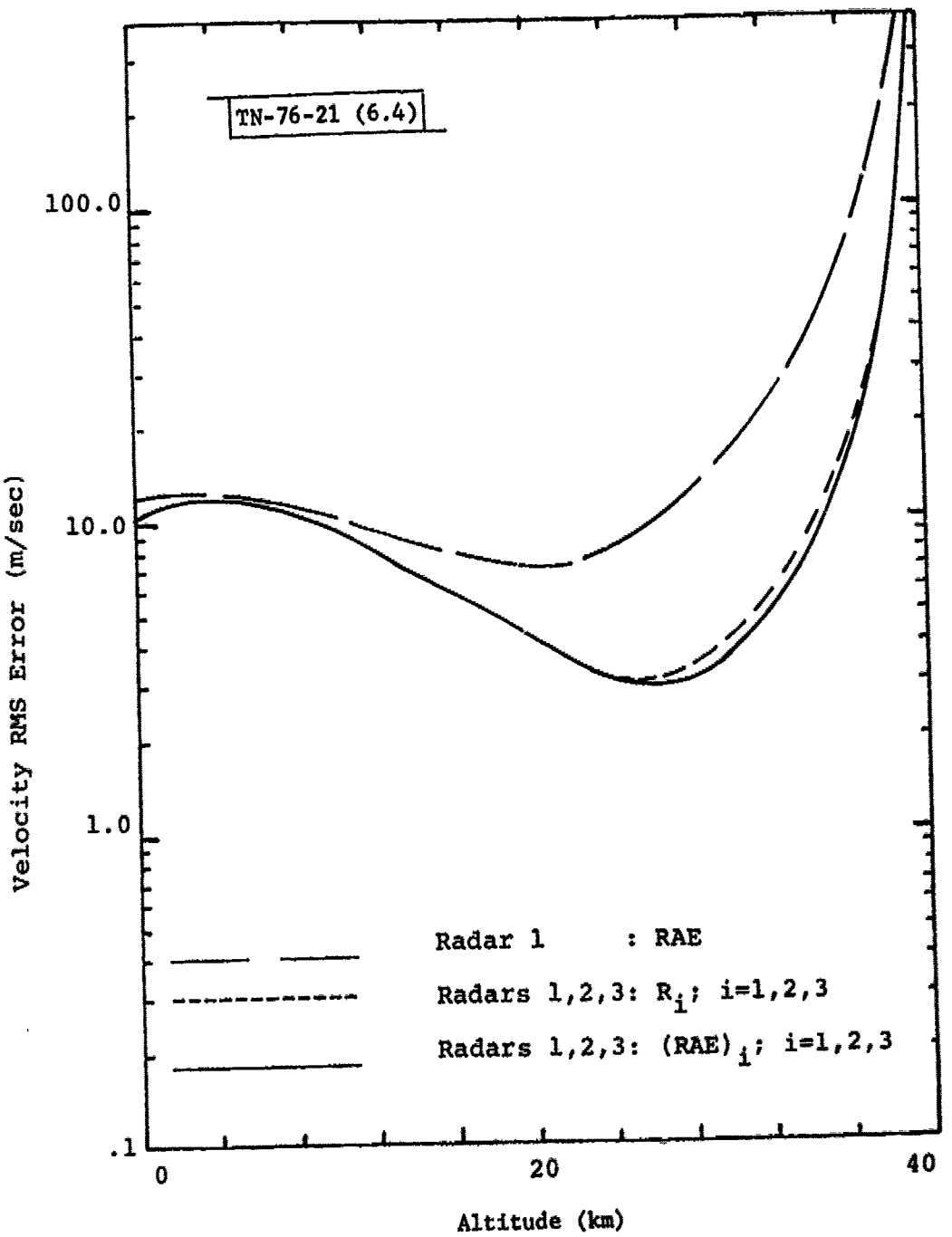


Figure 6.4 Velocity RMS error vs. altitude for geometry I.

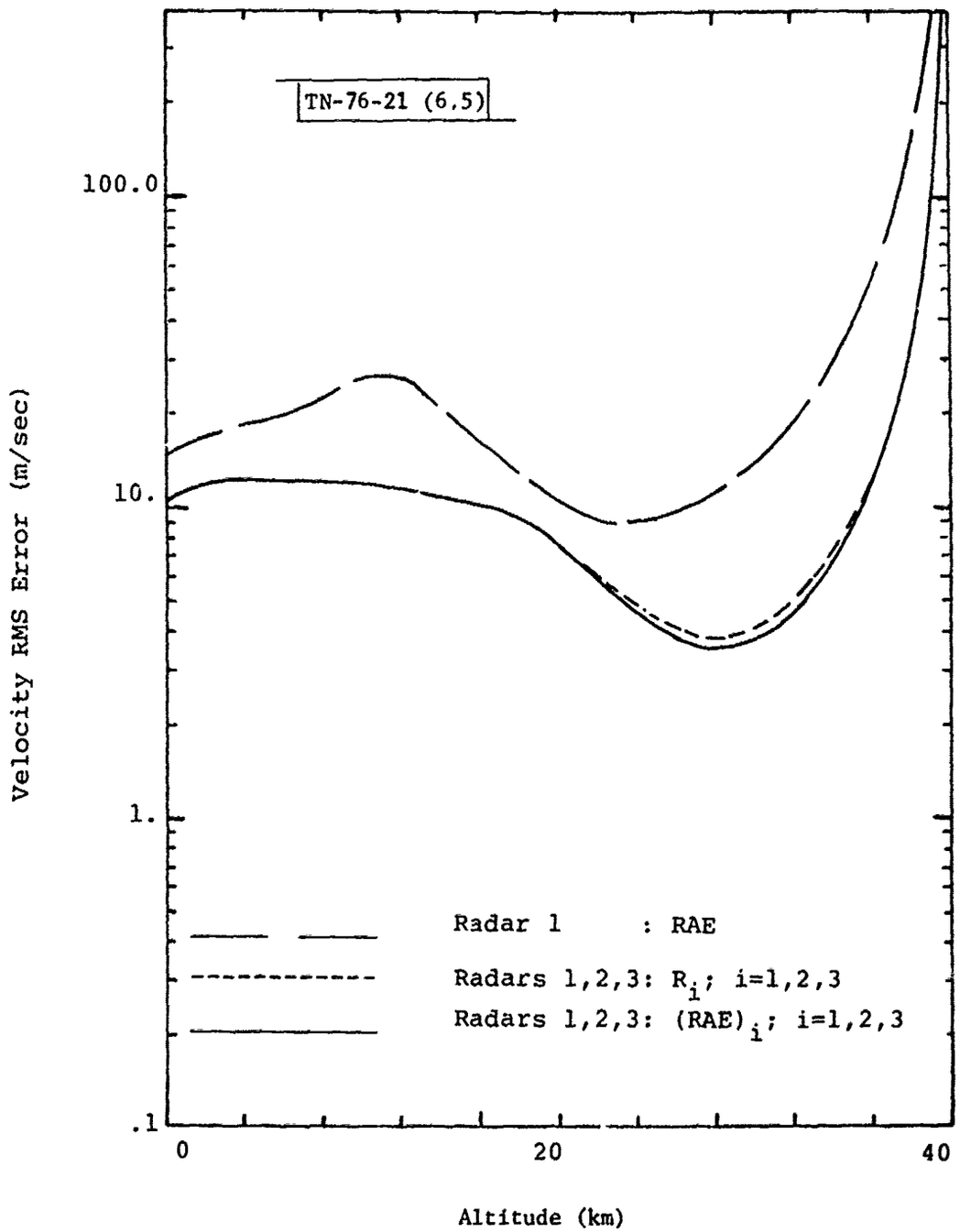


Figure 6.5 Velocity RMS error vs. altitude for geometry II.

REFERENCES

1. I. Kupiec, "Assessment of Miss-Distance Achievable by a Trilateration Radar System," Project Report RMP-74, Lincoln Laboratory, M.I.T. (in preparation).

2. J. E. Salah, private communications.

3. C. B. Chang and D. Willner, "Multilateration Tracking with Monostatic and Bistatic Measurements," Project Report RMP-86, Lincoln Laboratory, M.I.T. (5 April 1976).

4. Y. Bar-Shalom, "Redundancy and Data Compression in Recursive Estimation," IEEE Trans. Automatic Control AC-18, 684-689, (1973).

APPENDIX A

The Three Basic Measurement Transformations

(I) Transformation I: From (R_1, A_1, E_1) to (R_o, A_o, E_o)

Given (See Figure A.1):

Radar I at $(x_1, y_1, 0)$ measures (R_1, A_1, E_1)

Radar 0 at $(0,0,0)$ measures (R_o, A_o, E_o)

Transformation Formula:

$$R_o = \left[R_1^2 + x_1^2 + y_1^2 + 2x_1 R_1 \cos E_1 \sin A_1 + 2y_1 R_1 \cos E_1 \cos A_1\right]^{\frac{1}{2}}$$

$$A_o = \tan^{-1}\left[\frac{x_1 + R_1 \cos E_1 \sin A_1}{y_1 + R_1 \cos E_1 \cos A_1}\right]$$

$$E_o = \sin^{-1}\left[\frac{R_1 \sin E_1}{R_o}\right]$$

The G_k - Matrix:

(a) xyz - System

$\hat{x}, \hat{y}, \hat{z}$ - from the predicted state vector $\hat{\underline{x}}_{k/k-1}$

$$\hat{r}_1 = \left[(x_1 - \hat{x})^2 + (y_1 - \hat{y})^2\right]^{\frac{1}{2}}$$

$$\hat{r}_o = \left[\hat{x}^2 + \hat{y}^2\right]^{\frac{1}{2}}$$

$$\hat{R}_1 = \left[\hat{r}_1^2 + \hat{z}^2\right]^{\frac{1}{2}}$$

$$\hat{R}_o = \left[\hat{r}_o^2 + \hat{z}^2\right]^{\frac{1}{2}}$$

$$\hat{c}_1 = x_1(\hat{x}-x_1) + y_1(\hat{y}-y_1)$$

$$\hat{c}_2 = x_1(\hat{y}-y_1) - y_1(\hat{x}-x_1)$$

we have

$$g_{11} = [\hat{R}_1^2+\hat{c}_1]/\hat{R}_o\hat{R}_1$$

$$g_{12} = \hat{c}_2/\hat{R}_o$$

$$g_{13} = -\hat{z}\hat{c}_1/\hat{R}_o\hat{r}_1$$

$$g_{21} = -\hat{c}_2/\hat{R}_1\hat{r}_o^2$$

$$g_{22} = [\hat{r}_1^2+\hat{c}_1]/\hat{r}_o^2$$

$$g_{23} = \hat{z}\hat{c}_2/\hat{r}_1\hat{r}_o^2$$

$$g_{31} = \left[\frac{\hat{z}}{R_1} - \frac{\hat{z}}{R_o} g_{11}\right]/\hat{r}_o$$

$$g_{32} = -\frac{\hat{z}}{\hat{R}_o} g_{12}/\hat{r}_o$$

$$g_{33} = \left[\hat{r}_1 - \frac{\hat{z}}{\hat{R}_o} g_{13}\right]/\hat{r}_o$$

(b) RAE - System

$\hat{R}_o, \hat{A}_o, \hat{E}_o$ - from the predicted state vector $\hat{\underline{x}}_{k/k-1}$

$$\hat{r}_o = \hat{R}_o\cos\hat{E}_o$$

$$\hat{z} = \hat{R}_o\sin\hat{E}_o$$

$$\hat{x} = \hat{r}_o\sin\hat{A}_o$$

$$\hat{y} = \hat{r}_o\cos\hat{A}_o$$

then apply the same formulas in (a) to obtain $\underline{G}_k$.

(II) Transformation II: From (R_1, R_2, R_3) to (R_1, A_1, E_1)

Given (see Figure A.2):

Radar I at (0,0,0) measures R_1 or (R_1, A_1, E_1)

Radar II at $(x_2, y_2, 0)$ measures R_2

Radar III at $(x_3, y_3, 0)$ measures R_3

Transformation Formula:

$$R_1 = R_1$$

$$A_1 = \tan^{-1}\left(\frac{x}{y}\right)$$

$$E_1 = \sin^{-1}\left(\frac{z}{R_1}\right)$$

$$x = a_1R_1^2 + a_2R_2^2 + a_3R_3^2 + a_o$$

$$y = b_1R_1^2 + b_2R_2^2 + b_3R_3^2 + b_o$$

$$z = \left[R_1^2 - (x^2+y^2)\right]^{1/2}$$

$$a_1 = c_o\left[y_3 - y_2\right],\ a_2 = -c_oy_3,\ a_3 = c_oy_2,\ a_o = c_o(y_3r_2^2 - y_2r_3^2),$$

$$b_1 = c_o\left[x_2 - x_3\right],\ b_2 = c_ox_3,\ b_3 = -c_ox_2,\ b_o = c_o(x_2r_3^2 - x_3r_2^2)$$

$$c_o = 1/\left[2(x_2y_3 - x_3y_2)\right],\ r_2^2 = x_2^2 + y_2^2,\ r_3^2 = x_3^2 + y_3^2$$

The $\underline{G}_k$ Matrix:

(a) xyz - system

$\hat{x}, \hat{y}, \hat{z}$ from the predicted state vector $\hat{x}_{k/k-1}$

$$\hat{r}_o = \left[\hat{x}^2 + \hat{y}^2\right]^{1/2}$$

$$\hat{R}_1 = \left[\hat{r}_o^2 + \hat{z}^2\right]^{1/2}$$

$$\hat{R}_2 = \left[(\hat{x}-x_2)^2 + (\hat{y}-y_2)^2 + \hat{z}^2\right]^{1/2}$$

$$\hat{R}_3 = \left[(\hat{x}-x_3)^2 + (\hat{y}-y_3)^2 + \hat{z}^2\right]^{1/2}$$

we have

$$g_{11} = 1,\ g_{12} = 0,\ g_{13} = 0$$

$$g_{21} = 2\hat{R}_1(a_1\hat{y} - b_1\hat{x})/\hat{r}_o^2$$

$$g_{22} = 2\hat{R}_2(a_2\hat{y} - b_2\hat{x})/\hat{r}_o^2$$

$$g_{23} = 2\hat{R}_3(a_3\hat{y} - b_3\hat{x})/\hat{r}_o^2$$

$$g_{31} = \frac{\hat{R}_1}{\hat{r}_o\hat{z}}\ (1-2a_1\hat{x}-2b_1y) - \frac{\hat{z}}{\hat{R}_1\hat{r}_o}$$

$$g_{32} = -\frac{2\hat{R}_2}{\hat{r}_o\hat{z}}\left[a_2\hat{x} + b_2\hat{y}\right]$$

$$g_{33} = -\frac{2\hat{R}_3}{\hat{r}_o\hat{z}}\left[a_3\hat{x} + b_3\hat{y}\right]$$

b) RAE - System

$\hat{R}_1, \hat{A}_1, \hat{E}_1$ - from the predicted vector $\hat{\underline{x}}_{k/k-1}$

$$\hat{r}_o = \hat{R}_1\cos\hat{E}_1$$

$$\hat{z} = \hat{R}_1\sin\hat{E}_1$$

$$\hat{x} = \hat{r}_o\sin\hat{A}_1$$

$$\hat{y} = \hat{r}_o \cos\hat{A}_1$$

then apply the same formulas in (a) to obtain $\underline{G}_k$.

(III) <u>Transformation III: From (R_1, R_2, R_3) to (R_o, A_o, E_o)</u>

<u>Given</u> (see Figure A.3):

Radar 0 at (0,0,0) measures (R_o, A_o, E_o)

Radar I at $(x_1, y_1, 0)$ measures R_1

Radar II at $(x_2, y_2, 0)$ measures R_2

Radar III at $(x_3, y_3, 0)$ measures R_3

<u>Transformation Formula</u>:

$$a_1 = c_o\,[y_3 - y_2]\,,\quad a_2 = -c_o\,[y_3 - y_1]\,,\quad a_3 = c_o\,[y_2 - y_1]\,,$$

$$b_1 = c_o\,[x_2 - x_3]\,,\quad b_2 = c_o\,[x_3 - x_1]\,,\quad b_3 = -c_o\,[x_2 - x_1]\,,$$

$$a_o = c_o\,\left[(y_3 - y_1)r_2^2 - (y_2 - y_1)r_3^2\right],$$

$$b_o = c_o\,\left[(x_2 - x_1)r_3^2 - (x_3 - x_1)r_2^2\right],$$

$$c_o = \frac{1}{2}\left[(x_2 - x_1)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1)\right]^{-1}$$

$$r_2^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2,\quad r_3^2 = (x_3 - x_1)^2 + (y_3 - y_1)^2$$

$$R_o = \left[R_1^2 - x_1^2 - y_1^2 + 2xx_1 + 2yy_1\right]^{\frac{1}{2}}$$

$$A_o = \tan^{-1}\frac{x}{y}$$

$$E_o = \sin^{-1}\frac{z}{R_o}$$

$$x = a_1R_1^2 + a_2R_2^2 + a_3R_3^2 + a_o + x_1$$

$$y = b_1R_1^2+b_2R_2^2+b_3R_3^2+b_o+y_1$$

$$z = \left[R_o^2-x^2-y^2 \right]^{\frac{1}{2}}$$

The $\underline{G}_k$ Matrix:

(a) xyz - system

$\hat{x},\hat{y},\hat{z}$ - from the predicted state vector $\hat{x}_{k/k-1}$.

$$\hat{r}_o = \left[\hat{x}^2+\hat{y}^2 \right]^{\frac{1}{2}}$$

$$\hat{R}_o = \left[\hat{r}_o^2+\hat{z}^2 \right]^{\frac{1}{2}}$$

$$\hat{R}_1 = \left[(\hat{x}-x_1)^2 + (\hat{y}-y_1)^2 + \hat{z}^2 \right]^{\frac{1}{2}}$$

$$\hat{R}_2 = \left[(\hat{x}-x_2)^2 + (\hat{y}-y_2)^2 + \hat{z}^2 \right]^{\frac{1}{2}}$$

$$\hat{R}_3 = \left[(\hat{x}-x_3)^2 + (\hat{y}-y_3)^2 + \hat{z}^2 \right]^{\frac{1}{2}}$$

we have

$$g_{11} = \hat{R}_1 \left[1+2a_1x_1+2b_1y_1 \right] /\hat{R}_o$$

$$g_{12} = 2\hat{R}_2 \left[a_2x_1+b_2y_1 \right] /\hat{R}_o$$

$$g_{13} = 2\hat{R}_3 \left[a_3x_1+b_3y_1 \right] /\hat{R}_o$$

$$g_{21} = 2\hat{R}_1 \left[a_1\hat{y}-b_1\hat{x} \right] /\hat{r}_o^2$$

$$g_{22} = 2\hat{R}_2 \left[a_2\hat{y}-b_2\hat{x} \right] /\hat{r}_o^2$$

$$g_{23} = 2\hat{R}_3 \left[a_3\hat{y}-b_3\hat{x} \right] /\hat{r}_o^2$$

$$g_{31} = \frac{\hat{R}_1}{\hat{r}_o\hat{z}}\left[1+2a_1(x_1-\hat{x})+2b_1(y_1-\hat{y})\right] - \frac{\hat{z}}{\hat{r}_o\hat{R}_o}\ g_{11}$$

$$g_{32} = \frac{2\hat{R}_2}{\hat{r}_o\hat{z}}\left[a_2(x_1-\hat{x})+b_2(y_1-\hat{y})\right] - \frac{\hat{z}}{\hat{r}_o\hat{R}_o}\ g_{12}$$

$$g_{33} = \frac{2\hat{R}_3}{\hat{r}_o\hat{z}}\left[a_3(x_1-\hat{x})+b_3(y_1-\hat{y})\right] - \frac{\hat{z}}{\hat{r}_o\hat{R}_o}\ g_{13}$$

(b) RAE - System

$\hat{R}_o, \hat{A}_o, \hat{E}_o$ - from the predicted state vector $\hat{\underline{x}}_{k/k-1}$

$$\hat{r}_o = \hat{R}_o\cos\hat{E}_o$$

$$\hat{z} = \hat{R}_o\sin\hat{E}_o$$

$$\hat{x} = \hat{r}_o\sin\hat{A}_o$$

$$\hat{y} = \hat{r}_o\cos\hat{A}_o$$

then apply the same formulas in (a) to obtain $\underline{G}_k$.

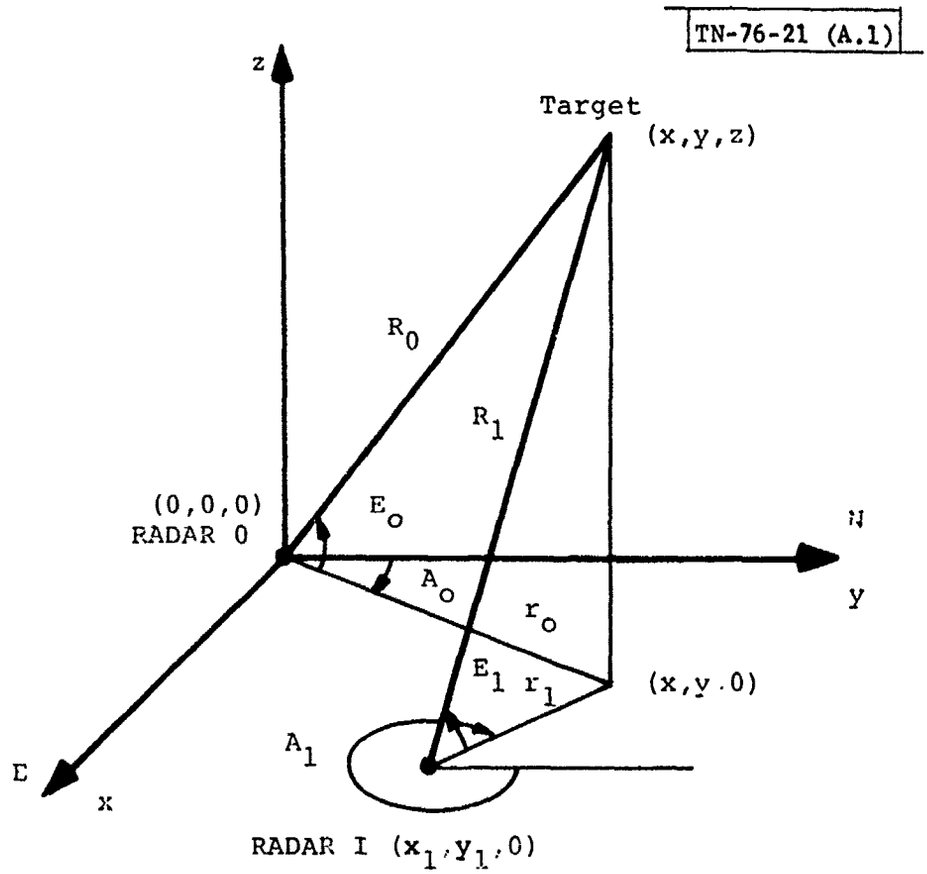


Figure A.1 Measurement transformation I.

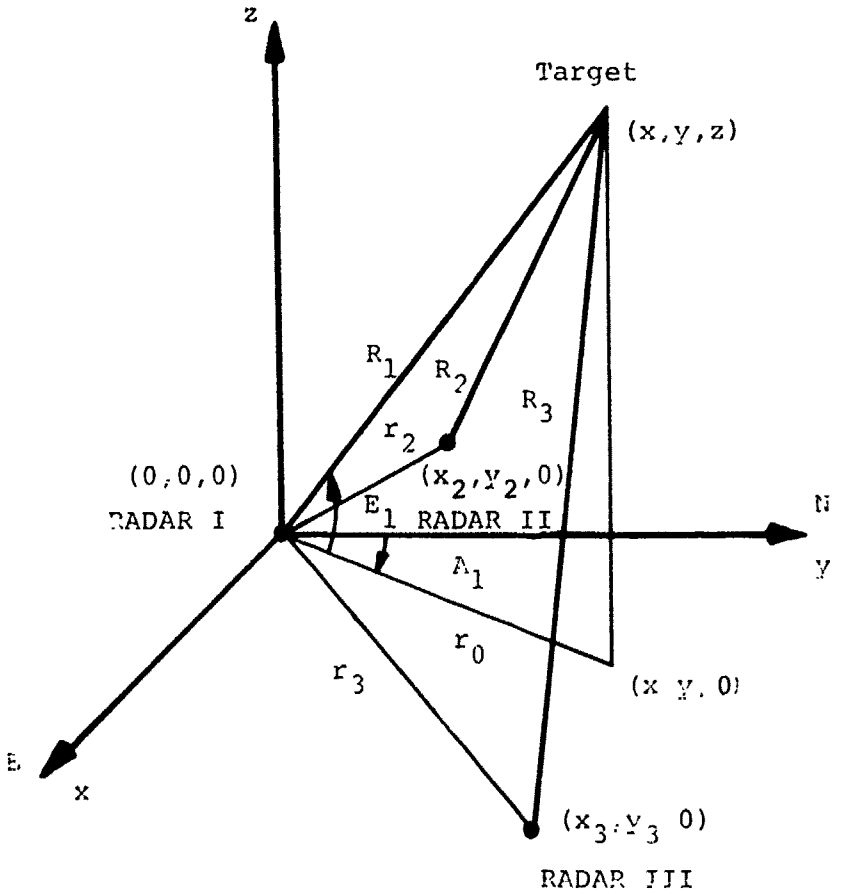


Figure A.2 Measurement transformation II.

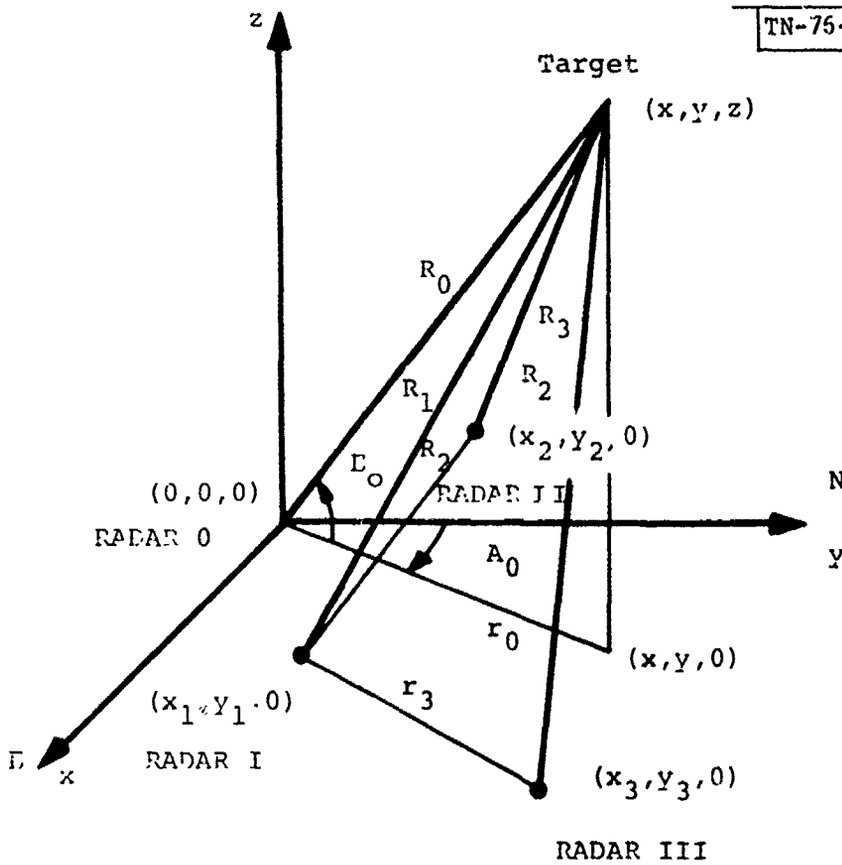


Figure A.3 Measurement transformation III.

APPENDIX B

Derivation of Data and Estimate Compression Equations

Let $\tilde{\underline{x}}_i$ denote the measurement (or estimate) of $\underline{x}$ from the i-th sensor (estimator) with mean $\underline{x}$ and covariance P_{ii}. In addition, it is assumed that the correlation between $\tilde{\underline{x}}_i$ and $\tilde{\underline{x}}_j$, P_{ij}, is known and $\underline{x}_i$, i=1,...,N and $\underline{x}$ are all expressed in the same coordinate, then

$$\tilde{\underline{x}}_i = \underline{x} + \underline{n}_i \tag{B.1}$$

A weighted least square estimate of $\underline{x}$ from $\tilde{\underline{x}}_i$,i=1,...,N is the $\underline{x}$ which minimizes

$$J = (\tilde{\underline{x}} - H\underline{x})^T P^{-1} (\tilde{\underline{x}} - H\underline{x}) \tag{B.2}$$

where $P = \begin{bmatrix} P_{11} & P_{12} & --- & P_{1N} \\ P_{21} & P_{22} & --- & P_{2N} \\ \vdots & & & \\ P_{N1} & P_{N2} & --- & P_{NN} \end{bmatrix}$

$$\underline{\tilde{x}} = \begin{bmatrix} \underline{\tilde{x}}_1 \\ \underline{\tilde{x}}_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \underline{\tilde{x}}_N \end{bmatrix}$$

$$H = \begin{bmatrix} \underline{I} \\ \underline{I} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ I \end{bmatrix}.$$

I= an identity matrix with the same order as $\underline{x}$. The solution is

$$\underline{\hat{x}} = (H^T P^{-1} H)^{-1} H^T P^{-1} \underline{\tilde{x}} \tag{B.3}$$

$$(H^T P^{-1} H)^{-1} = \text{covariance of } \underline{\hat{x}} \tag{B.4}$$

If the order of $\underline{x}$ is n, then the dimension of P is (nNxnN). The above equations require the inverse of a large size matrix. For the case when $P_{ij} = 0 \quad \forall\ i \neq j$, the above equations may be simplified to

$$\hat{\underline{x}} = \left(\sum_{i=1}^{N} P_{ii}^{-1} \right)^{-1} \sum_{i=1}^{N} P_{ii}^{-1} \underline{x}_i \tag{B.5}$$

$$\left(\sum_{i=1}^{N} P_{ii}^{-1} \right)^{-1} = \text{covariance of } \hat{\underline{x}} \tag{B.6}$$

Notice that in this case the matrices to be inverted have dimension (nxn).

In the data compression case, the measurements are uncorrelated. Equations (B.5) and (B.6) are used for this purpose. For the estimate compression case, the estimates are correlated. In order to optimally use estimate compression, one has to

(a) Compute all correlations, P_{ij} $\forall i=j$

(b) Invert a large matrix, P.

Both are optimum, however the estimate compression is computationally extremely inefficient.

APPENDIX C

Proof of Filter Equivalence

In this Appendix, the equivalence of sequential filter, parallel filter, and the filter using compressed data are shown. It should be noted that they are equivalent only in the linear case. The results for nonlinear systems such as RV tracking should still be close to optimal. The prediction equations of the three filters are the same. Only the equality of the update equations need to be proven. The equations of the parallel filter will be used as the reference. All the other filters will be shown to be the same as the parallel filter. For convenience, the update equations of the parallel filter are restated below.

(State) $$\hat{\underline{x}}_{k+1/k+1} = \hat{\underline{x}}_{k+1/k} + \sum_{i=1}^{I} K_{k+1,i} \left(z_{k+1,i} - H_{k+1,i} \hat{\underline{x}}_{k+1/k}\right) \tag{C.1}$$

(Gain) $$K_{k+1,i} = P_{k+1/k+1} H^T_{k+1,i} R^{-1}_{k+1,i} \tag{C.2}$$

(Covariance) $$P^{-1}_{k+1/k+1} = P^{-1}_{k+1/k} + \sum_{i=1}^{I} H^T_{k+1,i} R^{-1}_{k+1,i} H_{k+1,i} \tag{C.3}$$

The proofs are stated individually.

(a) The equivalence of the sequential filter.

The covariance matrices can be easily shown to be the same. Iterating (5.11a) I times yields

$$P^{-1}_{k+1/k+1,I} = P^{-1}_{k+1/k} + \sum_{i+1}^{I} H^{T}_{k+1,i} R^{-1}_{k+1,i} H_{k+1,i} \qquad (C.4)$$

and

$$P^{-1}_{k+1/k+1,I} = P^{-1}_{k+1/k+1}$$

This is the same as (C.3). Next we show the state estimate equation. Substituting (5.8) and (5.9) to (5.7) and after a few manipulations, we obtain

$$\hat{\underline{x}}_{k+1/k+1} = \hat{\underline{x}}_{k+1/k} + \sum_{i=1}^{I-1} \prod_{j=i+1}^{I} (\underline{I} - K_{k+1,j}H_{k+1,j}) K_{k+1,i}$$

$$(\underline{z}_{k+1,i} - H_{k+1,i} \hat{\underline{x}}_{k+1/k})$$

$$+ K_{k+1,I}(\underline{z}_{k+1,I} - H_{k+1,I} \hat{\underline{x}}_{k+1/k}) \qquad (C.5)$$

where $\underline{I}$ = an identity matrix, and $K_{k+1,i}$ is the gain defined by (5.10), not (C.2). The are equal when i=I. Using the following relation of the sequential filter,

$$P_{k+1/k+1,i+1} = (I - K_{k+1,i+1} H_{k+1,i+1}) P_{k+1/k+1,i} \qquad (C.6)$$

then,

$$\prod_{j=i+1}^{I} (\underline{I} - K_{k+1,j} \quad H_{k+1,j}) \quad K_{k+1,i}$$

$$= \prod_{j=i+2}^{I} (\underline{I} - K_{k+1,j} \; H_{k+1,j}) \quad (\underline{I} - K_{k+1,i+1} \; H_{k+1,i+1})$$

$$P_{k+1/k+1,i} \; H^T_{k+1,i} \; R^{-1}_{k+1,i}$$

$$= \prod_{j=i+2}^{I} (\underline{I} - K_{k+1,j} \; H_{k+1,j}) \quad P_{k+1/k+1,i+1} \; H^T_{k+1,i} \; R^{-1}_{k+1,i}$$

.

.

.

$$= P_{k+1/k+1,I} \; H^T_{k+1,i} \; R^{-1}_{k+1,i}$$

$$= P_{k+1/k+1} \; H^T_{k+1,i} \; R^{-1}_{k+1,i}$$

$$= K_{k+1,i} \text{ of (2.2)}$$

This completes the proof.

(b) The equivalence of the filter using compressed data.

In order to use the data compression method, all measurements must be first transformed to a common coordinate, i.e., they must have the same measurement matrix. Let the measurement

of the i-th sensor be denoted by $\underline{z}_{k+1,i}$, the transformation by T_i , and the transformed measurement by $\underline{z}_{k+1,i}$, then

$$\underline{z}'_{k+1,i} = T_i \underline{z}_{k+1,i}$$

$$= T_i \, H_{k+1,i} \, x_{k+1} + T_i n_{k+1,i} \qquad \text{(C.7)}$$

and

$$T_i \, H_{k+1,i} = H_{k+1} \text{ for all } i=1, \ldots, I.$$

It should be noted that the transformation T_i may not exist for all i. They do exist however for the multistatic radar application discussed in this report. The covariance of $n_{k+1,i}$ is $R_{k+1,i}$ and that of $T_i \, n_{k+1,i}$ is $R'_{k+1,i} = T_i \, R_{k+1,i} \, T_i^T$. The compressed covariance is denoted by

$$\tilde{R}^{-1}_{k+1} = \sum_{i=1}^{I} R^{-1}_{k+1,i}$$

$$= \sum_{i=1}^{I} T_i^{-T} \, R^{-1}_{k+1,i} \, T_i^{-1} \qquad \text{(C.8)}$$

Applying the above results to the filter covariance equations yields

$$P^{-1}_{k+1/k+1} = P^{-1}_{k+1/k} + H^T_{k+1} \tilde{R}^{-1}_{k+1} H_{k+1}$$

$$= P^{-1}_{k+1/k} + \sum_{i=1}^{I} H^T_{k+1} T_i^{-T} R^{-1}_{k+1,i} T_i^{-1} H_{k+1}$$

$$= P^{-1}_{k+1/k} + \sum_{i=1}^{I} H^T_{k+1,i} R^{-1}_{k+1,i} H_{k+1,i}$$

This proves that the covariance propagates the same way. Next we show the state estimate. Let $\underline{z}_{k+1}$ denote the compressed data, then

$$\hat{\underline{x}}_{k+1/k+1} = \hat{\underline{x}}_{k+1/k} + P_{k+1/k+1} H^T_{k+1} \tilde{R}^{-1}_{k+1} (\underline{z}_{k+1} - H_{k+1} \hat{\underline{x}}_{k+1/k})$$

$$= \hat{\underline{x}}_{k+1/k} + P_{k+1,k+1} H^T_{k+1} \tilde{R}^{-1}_{k+1} (\tilde{R}_{k+1} \sum_{i=1}^{I} R'^{-1}_{k+1,i} z'_{k+1,i} - H_{k+1} \hat{\underline{x}}_{k+1/k})$$

$$= \hat{\underline{x}}_{k+1/k} + P_{k+1/k+1} H^T_{k+1} (\sum_{i=1}^{I} R'^{-1}_{k+1,i} \underline{z}'_{k+1,i} - \sum_{i=1}^{I} R'^{-1}_{k+1,i} H_{k+1} \hat{\underline{x}}_{k+1/k})$$

$$= \hat{\underline{x}}_{k+1/k} + \sum_{i=1}^{I} P_{k+1/k+1} H^T_{k+1} R'^{-1}_{k+1,i} (z'_{k+1,i} - H_{k+1} \hat{\underline{x}}_{k+1/k})$$

$$= \hat{\underline{x}}_{k+1/k} \sum_{i=1}^{1} P_{k+1/k+1} H^T_{k+1,i} T_i^T T_i^{-T} R^{-1}_{k+1,i} T_i^{-1}$$

$$(T_i \underline{z}_{k+1,i} - T_i H_{k+1,i} \hat{\underline{x}}_{k+1/k})$$

$$= \hat{\underline{x}}_{k+1/k} + \sum_{i=1}^{I} P_{k+1/k+1} H^T_{k+1,i} R^{-1}_{k+1,i} (\underline{z}_{k+1,i}$$

$$- H_{k+1,i} \hat{\underline{x}}_{k+1/k})$$

This completes the proof.

APPENDIX D

Computational Efficiency of Various Kalman Filter Configurations

The computational requirements of an algorithm can be estimated by the number of multiplications per cycle. One cycle of a Kalman filter can be divided into predict part (subscript p) and update part (subscript u). Using the standard formula the number of multiplications M is computed, assuming that the components of the measurement are independent and taking advantage of the symmetric matrices i.e., only the upper triangular matrix has to be computed.

D.1 Predict Part

$\tilde{P} = APA^T + Q$

A, P nxn

	Product	# of multiplications
	PA^T	n^3
$\hat{x} = A\hat{x}$	$A(PA^T)$	$\frac{1}{2} n^2 (n+1)$
$\hat{x}_{nx1}$	$A\hat{x}$	n^2

$$M_p = \frac{3}{2} n^2 (n+1)$$

D.2 Update Part

$$K = \tilde{P}H^T(H\tilde{P}H^T + R)^{-1}$$

$$H_{mxn},\ \tilde{P}_{nxn},\ R_{mxm}$$

	Product	# of mult.
	$\tilde{P}H^T$	n^2m
	$H(\tilde{P}H^T)$	$\frac{1}{2}(nm^2 + nm)$
	$(\quad)^{-1}_{mxm}$	$\frac{2}{3}(m^3 - m)$
	$(\tilde{P}H^T)_{nxm}(\quad)^{-1}_{mxm}$	nm^2
$P = \tilde{P} - K(H\tilde{P})$	$K(H\tilde{P})$	$\frac{1}{2}(n^2m + nm)$
$\hat{z} = H\hat{x}$	$H\hat{x}$	nm
$K(z - \hat{z})$	$K(z - \hat{z})$	nm

$$M_u = nm\ \frac{3}{2}(n+m)+3\ + \frac{2}{3}(m^3-m)$$

D.2.1 Update via Inverse

$$P = (\tilde{P}^{-1} + H^T R^{-1} H)^{-1}$$

$$P_{nxn} \quad H_{mxn} \quad R_m = \text{Diag.}\{r_i\}$$

Product	# of multiplications
$H^T R^{-1} H$	$\frac{1}{2} n^2 m + \frac{3}{2} nm$
$\tilde{P}^{-1}$	$\frac{3}{2} (n^3 - n)$
$(P^{-1})^{-1}$	$\frac{3}{2} (n^3 - n)$
$K = P_{nxn} (H^T R^{-1})_{nxm}$	$n^2 m$
$\tilde{z} = Hx$, $k(z - z)$	$2nm$

$$M_{u,I} = nm \left(\frac{3}{2} n + \frac{7}{2}\right) + \frac{4}{3} (n^3 - n)$$

D.3 Data Compression

given: k m-dim. data sets z_i $i=1, \ldots, k$

1) transform z_i to z_o - coordinate system

2) transform diagnoal covariance matrices R_i

3) compress

Product	# of mult.
1) $z_i = G_{ii}\, z_i$	$k\, m^2$
2) $\bar{R}_i = G_i\, R_i\, G_i^T$	$k\, \frac{1}{2}\, (m^3 + 3_m{}^2)$
3) $(\bar{R}_i)^{-1}$	$(k+1)\, \frac{2}{3}\, (m^3 - m)$
4) $(\Sigma\, \bar{R}_i^{-1})^{-1}\ \ (\Sigma \bar{R}_i^{-1}\, \bar{z}_i)$	$(k+1) m^2$

$$M_D = m^3 (\tfrac{7}{6}\, k + \tfrac{2}{3}) + m^2\, (\tfrac{7}{2}\, k + 1) - m\tfrac{2}{3}(k+1)$$

D.4 Estimate Compression

given: k n - dim. estimates x_i, $i=1,\ldots,k$ using only the P_{ii} matrices (the P_{ij} $j \neq i$ matrices are not available) the # of multiplications for the compressed estimate is computed.

Product	# of results
P_{ii}^{-1}	$(k+1)\, \frac{2}{3}(n^3 - n)$
$(\Sigma P_{ii}^{-1})^{-1} (\Sigma P_{ii}^{-1}\, \hat{x}_i)$	$(k+1) n^2$

$$M_E = (k+1) n \quad \tfrac{2}{3}(n^2 - 1) + n$$

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The purpose of this report is to examine several Kalman filter algorithms that can be used for state estimation with a multiple sensor system. These algorithms are described in detail and their results are compared with a suboptimum tracking algorithm which processes only multiple range measurements. A state estimate compression algorithm is also described. Various radar measurement transformation formulas are listed. Algorithms for a nonsynchronous data collection system are not examined in detail but possible approaches are suggested.

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