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AFRPL-TR-76-14

PLUME ATTENUATED RADAR CROSS SECTION CODE:

USER'S MANUAL

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JUNE 1976

Approved for Public Release
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Special Technical Report

Prepared for

Air Force Rocket Propulsion Laboratory
Director of Science and Technology
Air Force Systems Command
Edwards AFB, CA 93523

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FOREWORD

This is the Computer Programming Manual for the Plume Attenuated Radar Cross Section Code (PARCS), developed under Contract F04611-75-C-0021, covering the period 2 December 1974 to 31 May 1977.

The authors would like to acknowledge the considerable help provided by Dr. R. Fante of AFGL who served as Technical Monitor for this work and the support of Capt. W. Rothschild and Lt. R. Sperlein, the AFRPL Project Engineers.

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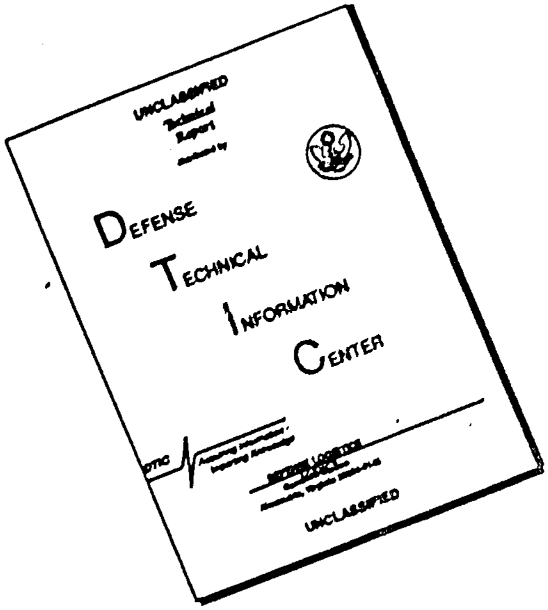

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REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: AFRPL-TR-76-14	2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle): PLUME ATTENUATED RADAR CROSS SECTION CODE: USER'S MANUAL	5. TYPE OF REPORT & PERIOD COVERED: Special Technical Rep[redacted]t
	6. PERFORMING ORG. REPORT NUMBER: ARI-RR-68
7. AUTHOR(s): J. Rickman, K. Tait, D. Mann	8. CONTRACT OR GRANT NUMBER(s): F04611-75-C-0021
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Aerodyne Research, Inc., Bedford Research Park, Bedford, MA 01730	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS: Air Force Rocket Propulsion Laboratory, Lt. Sperlein AFRPL/DYSP, Edwards AFB, CA 93523	12. REPORT DATE: Jun[redacted]76
	13. NUMBER OF PAGES: 83
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report): Unclassified
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for Public Release
Distribution Unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

RADAR CROSS SECTION
PLUME
PLASMA

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The PARCS CODE calculates the coherent, incoherent, and overdense surface radar cross sections of a rocket plume. The modified Born approximation calculation includes attenuation, local index of refraction, Doppler shift and range cell truncation. The program accepts plume data directly from the AeroChem LAPP code, but may be interfaced with other sources of plume definition.

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1. INTRODUCTION

This report describes the structure and use of the program PARCS (Plume Attenuated Radar Cross Section). This is a CDC FORTRAN IV code that calculates the coherent, incoherent, and overdense surface radar cross sections of a rocket plume. The calculations include attenuation, local index of refraction, doppler shift, and range cell. The program accepts plume data directly from the AeroChem LAPP code [1], but may be interfaced with other sources of plume definition.

The program was developed for the Air Force Rocket Propulsion Laboratory under contract No. F04611-75-C-0021, monitored by Capt. W. Rothschild and Lt. R. Sperlein. The Technical Monitor was Dr. R. Fante (AFGL).

The RCS model was developed at Aerodyne by Drs. D. Mann and J. Draper.[2, 3] This report summarizes the model equations and gives an outline of the program.

1. R. R. Mikatarian, C. J. Kau and H. S. Pergament, "A Fast Computer Program for Nonequilibrium Rocket Plume Predictions," AeroChem Research Laboratories, Inc., August 1972, AFRPL-TR-72-94.

2. D. Mann, J. Draper, and J. Rickman, "Rocket Plume Radar Cross Sections: Theory and Data," JANNAF 9th PLUME TECHNOLOGY MEETING.

3. JANNAF Rocket Exhaust Plume Technology Handbook," Plume RCS," J.S. Draper and D. Mann, soon to be published.

2. DESCRIPTION OF THE MODEL

2.1 Table of Symbols

Text	Program	
a	ATT	Attenuation coefficient (m^{-1})
c	C	Velocity of light (2.998×10^8 m/sec)
f	RFREQ	Radar frequency (Hz)
Δf		Doppler frequency shift (Hz)
Δf_D	DOPFRQ	Radar frequency resolution (Hz)
$\vec{K}$		Radar wave vector (m^{-1})
L	FL	Maximum plume length intersected by radar beam (RJET)
$\hat{N}$		Normal to overdense surface
N		Complex index of refraction
$\bar{N}$	XIND	Real part of index of refraction
$\tilde{N}$	YIND	Negative of imaginary part of index of refraction
$Ne_{i,k}$	PLE (I, K)	Plume electron density (cm^{-3})
N_e	ZNE	Interpolated electron density (cm^{-3})
P	PHASE	Coherent cross section phase (radians)
r_j	RJET	Jet radius at matched pressure (m)
r_{ij}	PLR (I, J)	Radial location of plume data point (m/jet radius)
r_{od_i}	PLROVD(I, J)	Radius of OVERDENSE surface at z_i (m/jet radius)
R	RANGE	Radar source to plume range (m)
s	S	Ray S coordinate
S_{min}, S_{max}	SMIN, SMAX	Ray integration limits
$\vec{v}$	PLVC (I, J)	Plume gas axial velocity (m/sec)
v_D	DOPVEL	Radar velocity resolution
W	B WIDTH	Radar beam width (m)
x_0, z_0	XO, ZO	Ray intersection with x, z - plane
x_{Lim}	XLIM	x coordinate integration limit
z_{00} z_{Lim}	ZOO, ZLIM	z coordinate integration limit

Text	Program	
β_u		Forward scattering angle (radians)
γ	GAM	Mean fluctuation factor
Δs	STEP	Step size along s-axis (jet radii)
Δx_0	DXO	Step size along x-axis (jet radii)
Δz_0	DZO	Step size along z-axis (jet radii)
θ	THETAD	Radar aspect angle (DEG) (0° nose on viewing)
$\tilde{\mu}$	FMUOD(1), FMUOD(2)	Complex overdense surface return amplitude (m^{-2})
ν_{ij}	PLFC(I, J)	Plume collision frequency
π	PI	3.141592654
ρ		Overdense surface reflection coefficient amplitude
σ	SIG	Total radar cross section (RCS) (m^2)
σ_{coh}	SCOH	Coherent RCS (m^2)
σ_{inc}	SINC	Incoherent RCS (m^2)
σ_{od}	SOD	Overdense RCS (m^2)
$\tilde{\tau}$	TTILDA(1), TTILDA(2)	Coherent RCS complex amplitude (m^{-2})
ψ		Angle between overdense surface x, z plane projection and the z-axis
ω	OMEG	Radar frequency (radians)

2.2 Coordinate Systems

The rocket plume coordinate system is shown in Figure 1. The plume is assumed to be symmetric about the z-axis. The x, y plane is parallel to the nozzle exit plane, but displaced by an amount dependent on the interval over which the plume data is given. The coordinate values are in units of jet radius at matched pressure.

The plume data is given at discrete points r_{ij} by the AeroChem LAPP code.[1] The plume properties given are electron density N_e, electron-neutral collision frequency ν and axial gas velocity v.

Volume and surface integrations are performed in the skew coordinate system obtained by rotating the plume system y-axis so that it lies parallel to the radar line-of-sight. The rotated axis is called the s-axis, and it can be seen in Figure 1 that $y = -s \sin\theta$, where θ is the radar aspect angle.

Volume integrations are performed over a matrix of rays parallel to the s-axis, and a volume $dv = dx\, dy\, dz = dx\, dz\, (-ds \sin\theta)$ is associated with each integration step.

2.3 Incoherent RCS

The underdense incoherent cross section is calculated from the formulas:

$$\sigma_{inc} = 4\pi \int_{z_{00}}^{z_{Lim}} 2 \int_{0}^{x_{Lim}} \tilde{\sigma}(x_0, z_0)\, r_j^2\, dx_0\, dz_0 \tag{1}$$

with

$$\tilde{\sigma} = \int_{s_{min}}^{s_{max}} \left(\frac{d\sigma}{d\Omega}\right) \exp\left(-4\int_{s_{min}}^{s} a r_j ds'\right) r_j \sin\theta\, ds\ , \tag{2}$$

where the limits are chosen to include the entire volume of the plume.

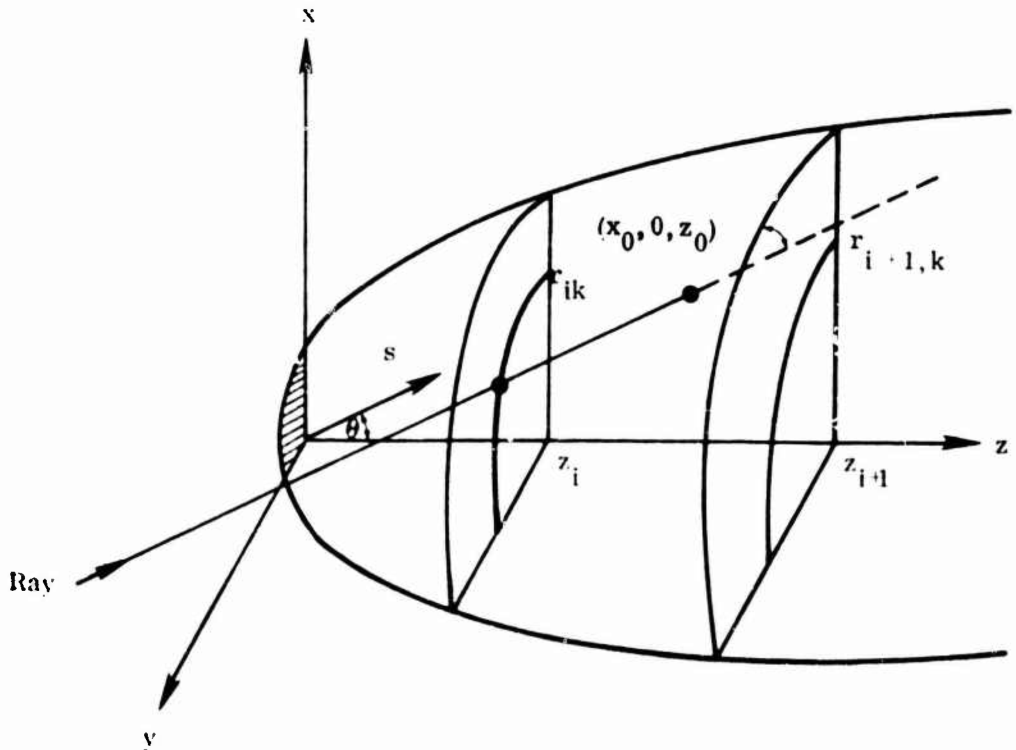

Figure 1 - Coordinate System

The integration limits are given by:

$$x_{Lim} = PLRMAX \quad (3)$$

$$z_{00} = -PLRMAX/\tan\theta \quad (4)$$

$$z_{Lim} = PLRMAX/\tan\theta + L \quad , \quad (5)$$

where PLRMAX is the maximum plume radius, L is the length of the plume, and θ is the aspect angle.

Other definitions required are:

$$\frac{d\sigma}{d\Omega} = \frac{\left(r_e n_e \gamma\right)^2}{1+\left(\frac{\nu}{\omega}\right)^2}\,\Phi \quad ; \quad \Phi = 15.6\ \Lambda^3\left[1 + \left(2\bar{N}\frac{\omega}{c}\Lambda\right)^2\right]^{-11/6} , \tag{6}$$

$$a = \frac{\omega}{c}\tilde{N} + \pi\left(1 + \cos\beta_m\right)\left(\frac{d\sigma}{d\Omega}\right) , \tag{7}$$

$$\bar{N} = R_e\sqrt{N^2} \quad ; \quad \tilde{N} = -\mathrm{Im}\sqrt{N^2} , \tag{8}$$

and

$$N^2 = \left(1 - \frac{\frac{N_e}{N_c}}{1+\left(\frac{\nu}{\omega}\right)^2}\right) - i\,\frac{\nu}{\omega}\,\frac{\frac{N_e}{N_c}}{1+\left(\frac{\nu}{\omega}\right)^2} . \tag{9}$$

The quantity $\tilde{\sigma}$ is the (nondimensional) radar cross section per unit area along the ray that passes through $(x_0, 0, z_0)$. The limits of integration are obtained by intersecting the ray with a plume defined as a set of cylindrical slabs (see Figure 2). The integrand in the $\tilde{\sigma}$ calculation is the product of a nondimensional attenuation factor and the cross section per unit volume $d\sigma / d\Omega$ $(m^{-1}\ sr^{-1})$. The function $\Phi(m^3)$ is the Kolmogorov correlation function, and a (m^{-1}) is the attenuation coefficient per unit length.

Other definitions required are:

r_e	=	electron radius = 2.8178×10^{-15} (m)
γ	=	mean fluctuation factor
Λ	=	turbulence scale = r_j (m) (jet radius at matched pressure)
c	=	velocity of light = 2.998×10^8 (m/sec)
ω	=	signal frequency = $2\pi f$ (sec^{-1})
N^2	=	square of index of refraction
N_c	=	critical density = $3.14 \times 10^{-10}\ \omega^2$ (cm^{-3})
β_m	=	forward scatter 1/2 angle = π

The mean fluctuation factor, γ, is the ratio of the fluctuating to the mean electron density. Typical values are between 0.5 and 1.0

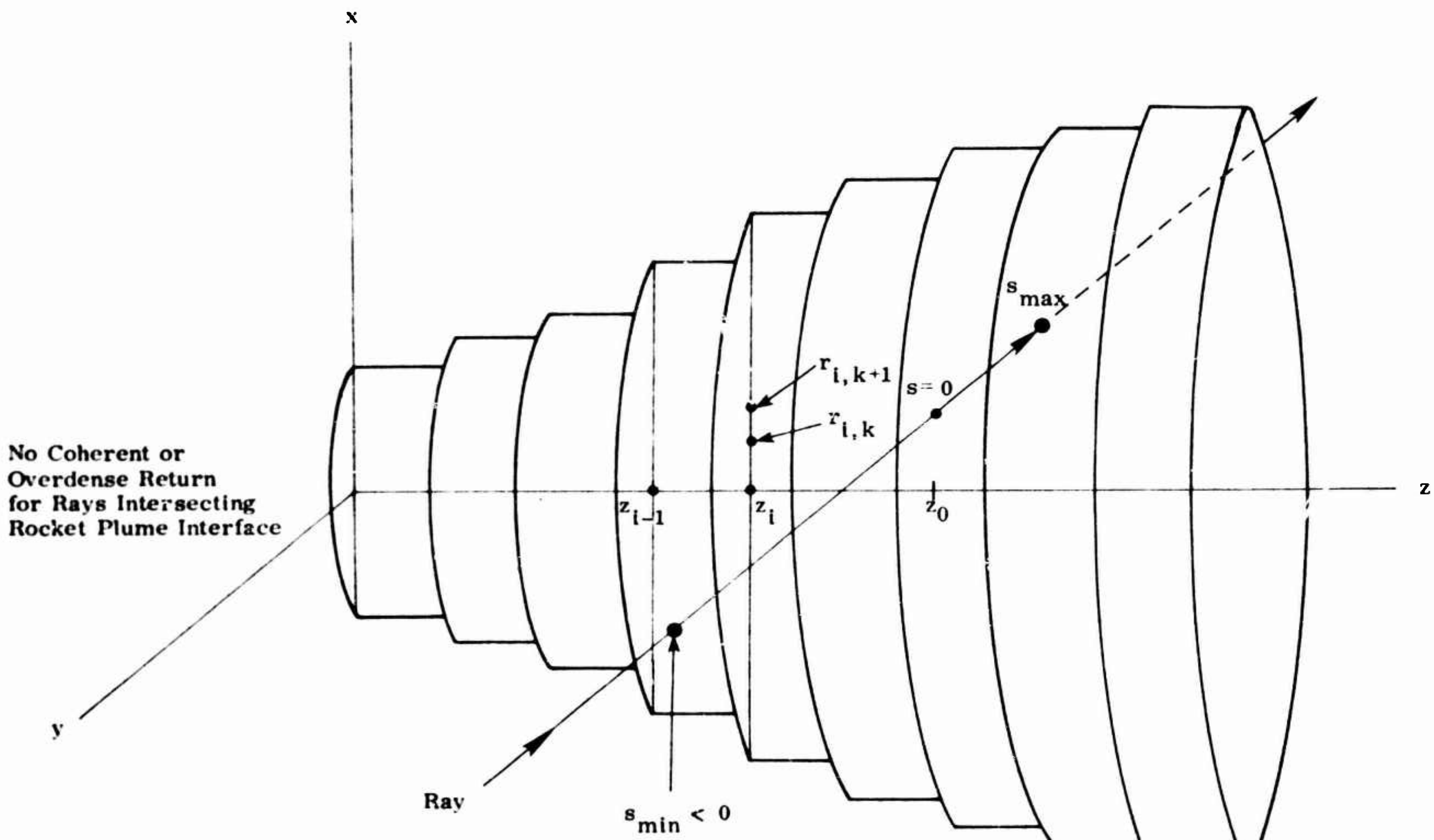

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Figure 2 - Plume Boundary

2.4 Coherent RCS

The underdense coherent cross section is calculated from the formulas:

$$\sigma_{coh} = 4\pi \left| \int_0^{z_{Lim}} \int_0^{x_{Lim}} \tilde{\tau}(x_0, z_0)\, r_j^2 \, dx_0 \, dz_0 \right|^2 , \qquad (10)$$

where

$$\tilde{\tau} = \int_{s_{min}}^{s_{max}} \frac{r_e N_e}{\sqrt{1 + \left(\frac{\nu}{\omega}\right)^2}} \exp\left(-2 \int_{s_{min}}^{s} a r_j ds' \right) (\cos P - i \sin P) \cdot r_j \sin\theta \, ds \qquad (11)$$

and

$$P = \int_{s_{min}}^{s} 2\bar{N} \frac{\omega}{c} r_j ds' + 2 \frac{\omega}{c} r_j (s_{min} + z_0 \cos\theta) . \qquad (12)$$

This calculation is complex to include both amplitude and phase. The quantity $|\tilde{\tau}|$ has the units of m^{-1}. All other symbols have been defined in Subsection 2.2. The quantity $(s + z_0 \cos\theta)$ is the distance from the edge of the plume (s_{min} in Figure 2) to a fixed plane passing through the origin normal to the integration ray. Those rays that pass through the plume rocket interface (Figure 2) are made to give a zero coherent and overdense return because the plume face is obscured by the vehicle.

2.5 Overdense RCS

The integrations in Subsections 2.3 and 2.4 with respect to s are terminated when an overdense surface is reached. This is defined to be the locus of points where,

$$N_e = N_{od} = N_c \left[1 + \left(\frac{\nu}{\omega}\right)^2 \right] \left[1 + \left(\frac{\nu}{2\omega}\right) \right]^{-1} \sin^2\theta . \qquad (13)$$

The radar cross section contribution from reflection by the overdense surface is given by:

$$\sigma_{od} = \frac{1}{\pi} \left| \int_0^{z_{Lim}} \int_0^{x_{Lim}} 2\mu \left(x_0, z_0\right) r_j^2 \sin dx_0 dz_0 \right|^2 , \tag{14}$$

with

$$\tilde{\mu} = \left(\frac{1}{\sqrt{\pi}} \frac{\omega}{c} \rho\right) \frac{|\hat{N} \cdot \hat{K}|}{|\hat{N} \cdot \hat{y}|} \xi^{1/2} \exp\left(-2 \int_{s_{min}}^{s} a r_j ds\right) (\cos P - i \sin P) , \tag{15}$$

$$\rho = \frac{1 - N}{1 + N} \qquad (N \equiv \text{complex index of refraction}) , \tag{16}$$

and

$$\xi = \left[1 + 4 \left(\frac{r_j \beta}{r_{od}}\right)^2\right] \exp\left(-\beta^2\right) \quad \begin{cases} \beta = 0.8 \frac{\omega}{c} r_j \sin \theta \\ r_{od} = \text{radius of overdense surface} \end{cases} , \tag{17}$$

where $\hat{K}$ is the unit wave vector, $\hat{N}$ is the normal to the overdense surface, ρ is the reflection coefficient amplitude, and ξ is a surface roughness correction. The surface roughness becomes important when the mean roughness (h) is on the order of a radar wavelength. In the above formulation, $h = 0.4 r_j$ and surface roughness should be included for $kh > 0.5$.

The radii of the overdense surface r_{od_i} at each axial station z_i are determined from the overdense condition

$$N_e \geq N_c \left[1 + \left(\frac{\nu}{\omega}\right)^2\right] \left(1 + \frac{\nu}{2\omega}\right)^{-1} \sin^2 \theta . \tag{18}$$

The intermediate values $r_{od}(z)$ are linearly interpolated from the plume axial station values r_{od_i} and $r_{od_{i-1}}$, where,

$$z_{i-1} < z < z_i \quad .$$

The normal to the overdense surface in the x z-plane is

$$N(z) = \cos \psi \ , \quad 0, \ -\sin \psi \ , \tag{19}$$

where

$$\tan \psi = \frac{r_{od_{i+1}} - r_{od_i}}{z_{i+1} - z_i} \ , \text{ with } \quad z_i < z < z_i + 1 \quad . \tag{20}$$

The three dimensional normal at an arbitrary point (x, y, z) is obtained by rotation about the z-axis.

$$\hat{N}(x, y, z) = (\cos \psi \cos \beta, \ \cos \psi \sin \beta, \ - \sin \psi) \quad , \tag{21}$$

for

$$\cos \beta = \frac{x}{\sqrt{x^2 + y^2}} \quad ; \quad \sin \beta = \frac{y}{\sqrt{x^2 + y^2}} \quad . \tag{22}$$

The radar wave vector dot product with the surface normal is given by:

$$\hat{N} \cdot \hat{K} = -\sin \theta \cos \psi \sin \beta - \cos \theta \sin \psi \quad , \tag{23}$$

where

$$K = (0, \ -\sin \theta, \ \cos \theta) \quad . \tag{24}$$

2.6 Range Cell

The radar resolution is limited by the range cell length (ℓ) and beam width (w) as shown in Figure 3. For small aspect angles (θ) the resolution is limited by the range cell length, and the maximum plume length intersected by the radar beam is

$$L = \frac{w}{\sin\theta} \quad , \quad 0 \leq \theta \leq \tan^{-1}\left(\frac{w}{\ell}\right) \quad . \tag{25}$$

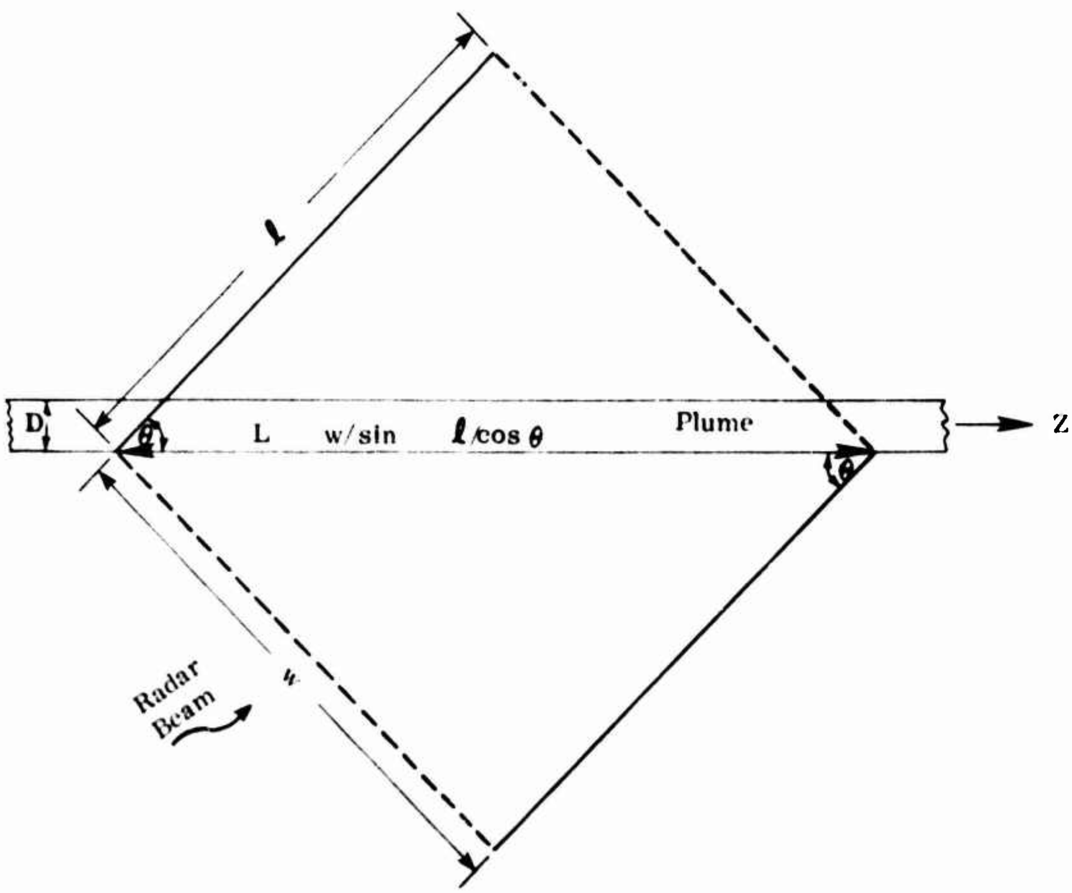

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Figure 3 - Finite Radar Beam Incident on Cylindrical Plume (for $w = \ell = 10D$, $\theta = \tan^{-1}(w/\ell)$)

For large aspect angles the maximum plume length intersected is determined by the range cell length.

$$L = \frac{\ell}{\cos\theta} \quad , \quad \tan^{-1}\left(\frac{w}{\ell}\right) \leq \theta \leq \frac{\pi}{2} \quad . \tag{26}$$

The large and small aspect angle equations are equal for $\theta = \tan^{-1}(a/\ell)$ as shown in Figure 3. The beam width is determined from the range (R) and beam divergence (δ) according to

$$w = R\,\delta \quad . \tag{27}$$

The cross sections are summed and printed for each range cell when the integration intervals in the axial direction (z in Figure 3) are multiples of the plume intersection length L. The low aspect angle case ($\theta \leq \theta_0$) is not range cell resolved. but appears in a single range cell (see Subsection 2.8).

2.7 Doppler Shift

The incoherent cross section is determined as a function of doppler frequency shift. The shift is due to the local gas velocity ($\vec{v}$) relative to the missile, and is given by:

$$\Delta f(\text{Hz}) = 2\pi\vec{K} \cdot \vec{V} \quad , \tag{28}$$

where $\vec{K}$ is the radar wave vector. The contributions to the incoherent cross section at each point along the integration ray are put into a Doppler bin determined by the local gas velocity.

The velocity resolution of the radar,

$$V_D = \frac{\Delta f_D}{2\pi K} \quad , \quad \Delta f_D \equiv \text{Radar frequency resolution} \quad . \tag{29}$$

and the maximum gas velocity are used to define the velocity width of the Doppler bins (V_D) and the number of bins. As the integration along the ray proceeds the local gas velocity V is projected along the radar line-of-sight $\hat{K}$ and a Doppler bin N is selected such that

$$(N-1)\, V_D < \vec{V} \cdot \hat{K} < N\, V_D \quad . \tag{30}$$

The local incoherent cross section contribution is stored in Doppler bin N and is taken to be the integral along the ray from the preceding integration point.

2.8 Low Aspect Angle

The cross sections for aspect angles in the range

$$|\theta| \le \theta_0 \qquad (\theta_0 \equiv \text{low aspect angle definition from input data}) \quad , \tag{31}$$

are determined by integrating over a matrix of rays defined in the x, y-plane. (The rays are defined in the x, z-plane for higher aspect angles.) The coherent and incoherent cross sections are determined from the equations of Subsections 2.3 and 2.4 with the substitutions

$$z_0 \rightarrow y_0$$

and

$$\sin\theta\, ds \rightarrow (\hat{t} \cdot \hat{z})\, ds \quad , \tag{32}$$

where $\hat{t}$ is a unit vector tangent to the ray and s is the path length.

The integration ray bends as it steps through the plume according to

$$\hat{t}(p') = \hat{t}(p) + \left.\frac{d\hat{t}}{ds}\right|_p ds \quad , \tag{33}$$

where

$$\frac{d\hat{t}}{ds} = \left(\frac{\vec{\nabla}N \cdot \hat{U}}{N}\right)\hat{U} = \hat{r} \; \frac{1}{N}\left(\frac{\partial N}{\partial r} U_r + \frac{\partial N}{\partial z} U_z\right) U_r$$

$$+ \hat{z} \; \frac{1}{N}\left(\frac{\partial N}{\partial r} U_r + \frac{\partial N}{\partial z} U_z\right) U_z \tag{34}$$

and

$$\hat{U} = \left(\frac{\hat{t} \times \vec{\nabla}N}{|\vec{\nabla}N|}\right) \times \hat{t} = \frac{1}{\nabla N} \left\{ \left[\frac{\partial N}{\partial r} - \left(t_r \frac{\partial N}{\partial r} + t_z \frac{\partial N}{\partial z}\right) t_r\right] \hat{r} \right.$$

$$\left. + \left[\frac{\partial N}{\partial z} - \left(t_r \frac{\partial N}{\partial r} + t_z \frac{\partial N}{\partial z}\right) t_z\right] \hat{z} - \left(t_r \frac{\partial N}{\partial r} + t_z \frac{\partial N}{\partial z}\right) t\phi \; \hat{\phi} \right\} , \tag{35}$$

where $\hat{r}$ is a unit vector along the cylindrical radius from the plume axis, $\hat{\phi}$ is perpendicular to $\hat{r}$ such that $\hat{\phi} \times \hat{r} = \hat{z}$ and $\vec{\nabla}N$ is the real part of the gradient of the index of refraction.

The gradient of the index of refraction is determined from:

$$\vec{\nabla}N = \frac{1}{4N}\left(\frac{\vec{\nabla}x \left\{ x\left[1 + \left(\frac{\nu}{\omega}\right)^2\right] - \left(\frac{\nu}{\omega}\right)^2 \right\} + (1-x)^2 \frac{\nu\vec{\nabla}\nu}{\omega^2}}{\sqrt{x^2 + (1-x)^2 \left(\frac{\nu}{\omega}\right)^2}} + \vec{\nabla}x\right) \tag{36}$$

and

$$\vec{\nabla}x = \frac{1}{2 \times N_c \left(1 + \frac{\nu^2}{\omega^2}\right)} \left[\frac{N_e \, 2\nu\vec{\nabla}\nu}{\omega^2 \left(1 + \frac{\nu^2}{\omega^2}\right)} - \vec{\nabla}N_e\right] \tag{37}$$

with

$$x = 1 - \frac{\frac{N_e}{N_c}}{1 + \left(\frac{\nu}{\omega}\right)^2} \quad . \tag{38}$$

The initial value of $\hat{t}$ is determined from

$$d\vec{R} = t_r \hat{r} + t_\phi \hat{\phi} + \hat{t}_z \hat{z} \quad ds = (-\sin\theta \; \hat{y} + \cos\theta \; \hat{z}) \; ds \tag{39}$$

to be

$$\hat{t}_0 = -\sin\theta \cos\alpha \; \hat{r} + \sin\theta \sin\alpha \; \hat{\phi} + \cos\theta \; \hat{z} \quad , \tag{40}$$

where

$$\cos\alpha = \frac{y_0}{x_0^2 + y_0^2} \quad , \tag{41}$$

$$\left(x_0, y_0\right) \equiv \text{Ray intersection with x, y-plane} \quad ,$$

and

$$\theta = \text{aspect angle.}$$

The low aspect angle rays are terminated when the ray exits from the plume or the value of the exponential damping term is in the range

$$10 \; \text{Log}_{10} \left| \exp\left(-4 \int_{s_{min}}^{s} ar_j \; ds' \right) \right| \leq -20 \; \text{db} \quad . \tag{42}$$

There is no overdense surface ray termination of cross section return for the low aspect angle case. The cross sections are not range gate resolved for low aspect angle, but they are Doppler resolved.

3. MAIN PROGRAM

A flow chart of the main program is shown in Figure 4. The main program performs the x and z integrations of the cross sections using the Trapezoidal rule and calls subroutine RAY to perform the integration along the ray (in s). For low aspect angles ($\theta \leq \theta_0$), the integration in z is transformed into an integration over y (see Figure 1) by a suitable modification of the step sizes integration limits.

The radar properties are read in by the main program, and LAPPIN is called to read and store the plume properties from the AeroChem LAPP code.[1] If the radar aspect angle is greater than 90° the plume is reflected about a bisecting x, y-plane (Figure 1) and the aspect angle (θ) is replaced by its complement ($180^\circ - \theta$).

The integration step sizes are set equal to a fraction of the larger of the radar free space wavelength or the jet radius by an input data card (C in Figure 6).

For low aspect angles, the s and z integration step sizes are interchanged. If the input parameter INCOH is set equal to 1, the step sizes are all set equal to the step size in z regardless of aspect angle.

The limits of the z integration are taken to be

$$ZOO = -\frac{PLRMAX}{\tan\theta} \tag{43}$$

and

$$ZLIM = \frac{PLRMAX}{\tan\theta} + PLZ(NZ) \quad . \tag{44}$$

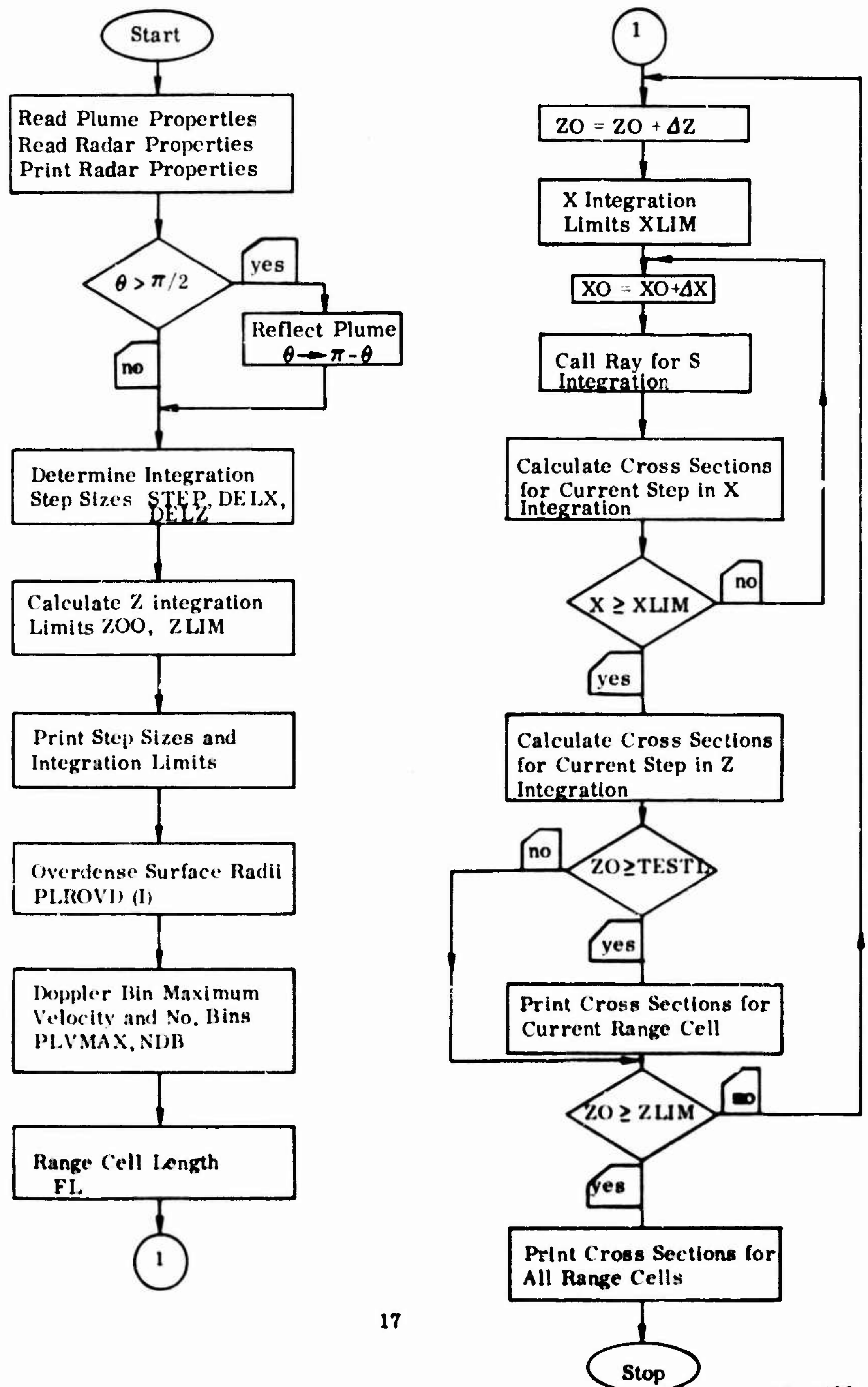

Figure 4 - Main Program

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where PLRMAX is the maximum plume radius and PLZ(NZ) is the length of the plume. The values above are multiplied by $\tan\theta$ for low aspect angle cases to give the corresponding limits for the y integration.

The plume data points at a given axial station are tested for the overdense election density condition (Subsection 2.5) starting with the maximum radius and stepping inwards. The radius of the overdense surface at the axial station is taken to be the linear interpolation between the first overdense point encountered and the preceding point.

The number of Doppler bins required for storage of the Doppler resolved incoherent cross section is determined by searching for the maximum gas velocity (PLVMAX) projected along the radar line-of-sight and dividing the maximum velocity by the radar velocity resolution.

$$NDB = \frac{PLVMAX}{DOFVEL} + 1 \quad . \tag{45}$$

The velocity resolution (V_D) may be given directly by the radar properties read in, or, if the velocity resolution is read as 0, it will be calculated from the frequency resolution (Δf_D)

$$V_D = \frac{\pi c}{\omega} \Delta f_D \quad . \tag{46}$$

The x and z integrations are performed using the Trapezoidal Rule,

$$\int_0^1 f(x)\, dx = \frac{(x_1 - x_0)}{2} \left[f(x_1) - f(x_0) \right] \quad . \tag{47}$$

The cross sections are printed when z is equal to a multiple of the range cell plume intersection length (FL). The incoherent cross sections is presented in

Doppler bins with a maximum projected gas velocity and corresponding frequency shift printed for each bin. The total cross sections for the current range cell are printed in units of square meters (sm), and the value of 10 $\text{Log}_{10} \left| \sigma/1\,\text{m}^2 \right|$ (dbsm) is also presented. The total cross sections and the Doppler resolved incoherent cross section are summed over all range cells and printed at the conclusion of the integration.

Subroutine Ray is called to perform the integration along the s-axis (see Figure 1). The arguments include the ray intersection with the x, z-plane (x_0, z_0). The intersection points form the grid over which the volume integrations are carried out. The rays are formed by incrementing the intersection points according to

$$x_0 = \left(k_x - 1\right) \Delta x \quad , \quad k_x = 1, 2, \ldots, \frac{\text{XLIM}}{\Delta x} \tag{48}$$

and

$$z_0 = \left(k_z - 1\right) \Delta z + z_{00} \quad , \quad k_z = 1, 2, \ldots, \frac{\text{ZLIM}}{\Delta z} \quad , \tag{49}$$

as the integration proceeds.

For low aspect angle cases the integration over z is carried out with limits and step sizes appropriate to an integration over y, and the point (x_0, z_0) is the intersection of the ray with the x, y-plane.

4. SUBROUTINES

4.1 Subroutines LAPPIN and CYLDER

Subroutine LAPPIN reads in the plume properties from the AeroChem LAPP code.[1] The LAPP output format with LAPP FORTRAN variables are shown on the next page. To be used as input to the present program, the deck must be preceded by a title card in 80 column alphanumeric format, and followed by an end card which has -1. E30 punched in columns 75-80.

The plume properties are given in the x, z-plane (see Figure 1) at the points

$$\left(x_i, z_i\right) = (\text{PLR(I)},\ \text{PLZ(J)}) \quad . \tag{50}$$

The plume properties stored are:

PPLE(I, J) = electron density at (x_i, z_i) (cm^{-3})

PPLF(I, J) = electron – neutral collision frequency (sec^{-1})

PPLV(I, J) = axial gas velocity (m/sec)

PPLR(I, J) = plume radius

NZ = number of axial stations along z-axis

NR(I) = number of radial points at z_i .

All distances are in units of jet radii and the axial gas velocity is converted from the LAPP units of ft/sec to Aerodyne LAPPIN m/sec.

Various consistency checks are made on the input data. If any input errors are encountered, the variable IER is set equal to 1, and a message is printed.

The values of electron density, collision frequency and gas velocity are set equal to 1. if their AeroChem LAPP value is less than 1.

CYLDER can be called in place of LAPPIN by changing the parameter ITEST on a data card in the main program. The test plume defined by CYLDER is a cylinder of constant electron density. CYLDER supplies a plume data point every tenth of a jet radius in the radial direction and integer multiple of a jet radius in the axial direction. The plume properties are set by data cards in CYLDER to:

Length	=	2 Jet radii
Radius	=	1 Jet radius
N_e	=	$10^8/cm^3$ (electron density)
ν	=	10^{11} Hz (collision frequency)
v	=	1. (axial gas velocity)

TABLE 1. LAPP CARD OUTPUT

Card Type 1 (1. E10. 3, 60X, E10. 3)

XORJ	z_i	=	axial station scaled by r_j
BIG	1. E30	=	card type 1 identifier

Card Type 2 (1P8E10. 3)

YOUT	r_{ik}	=	radial location scaled by r_j
ECC	Ne_{ik}	=	electron density (cm^{-3})
XNEU	ν_{ik}	=	electron-neutral collision frequency (sec^{-1})
T	T_{ik}	=	exhaust temperature (^oK)
PPCO	P_{CO}	=	partial pressure of CO (atm)
$PPCO_2$	P_{CO_2}	=	partial pressure of CO_2 (atm)
PPH2O	P_{H_2O}	=	partial pressure of H_2O (atm)
U	u_{ik}	=	exhaust axial velocity (ft/sec)

Molecular species partial pressures are part of the normal LAPP output but are not used in the RCS calculation.

Note: Each type 1 card is followed by a variable number of type 2 cards but not more than 25.

4.2 Subroutine RAY

Ray performs all of the calculations necessary for a single integration ray. The ray is parallel to the radar line-of-sight (s-axis in Figure 1) and passes through the x, z-plane at $(x_0, 0, z_0)$ or the x, y-plane at $(x_0, z_0, 0)$ for low aspect angle cases. The plume properties are passed to RAY through COMMON/PLUME/ and the output is passed through COMMON/RAY/. The subroutines called are INTER, SUND, and INDEX. A flow chart of RAY is shown in Figure 5.

The minimum and maximum ray-plume intersection points are determined by dividing the plume into cylindrical slabs as shown in Figure 2. A test is made at each axial station for a ray-cylinder edge intersection. The y coordinate of an intersection point is (from Figure 2)

$$y = -S \sin \theta \quad . \tag{51}$$

The ray is parallel to the y, z-plane so that the radial position vector $\vec{R}$ from the z-axis to the intersection point is given by:

$$\vec{R} = \vec{x}_0 + \vec{y} \quad . \tag{52}$$

Therefore, the S coordinate of the intersection point is

$$S = \pm \frac{\sqrt{R^2 - x_0^2}}{\sin \theta} \quad , \tag{53}$$

where the sign is negative for SMIN and positive for SMAX.

SMIN or SMAX may occur at a cylindrical slab face (see Figure 2) rather than a slab edge, therefore, each axial station is tested for ray intersection with the slab face. The S coordinate of the face intersection at z_i is

$$S = \frac{\left(z_i - z_0\right)}{\cos \theta} \quad . \tag{54}$$

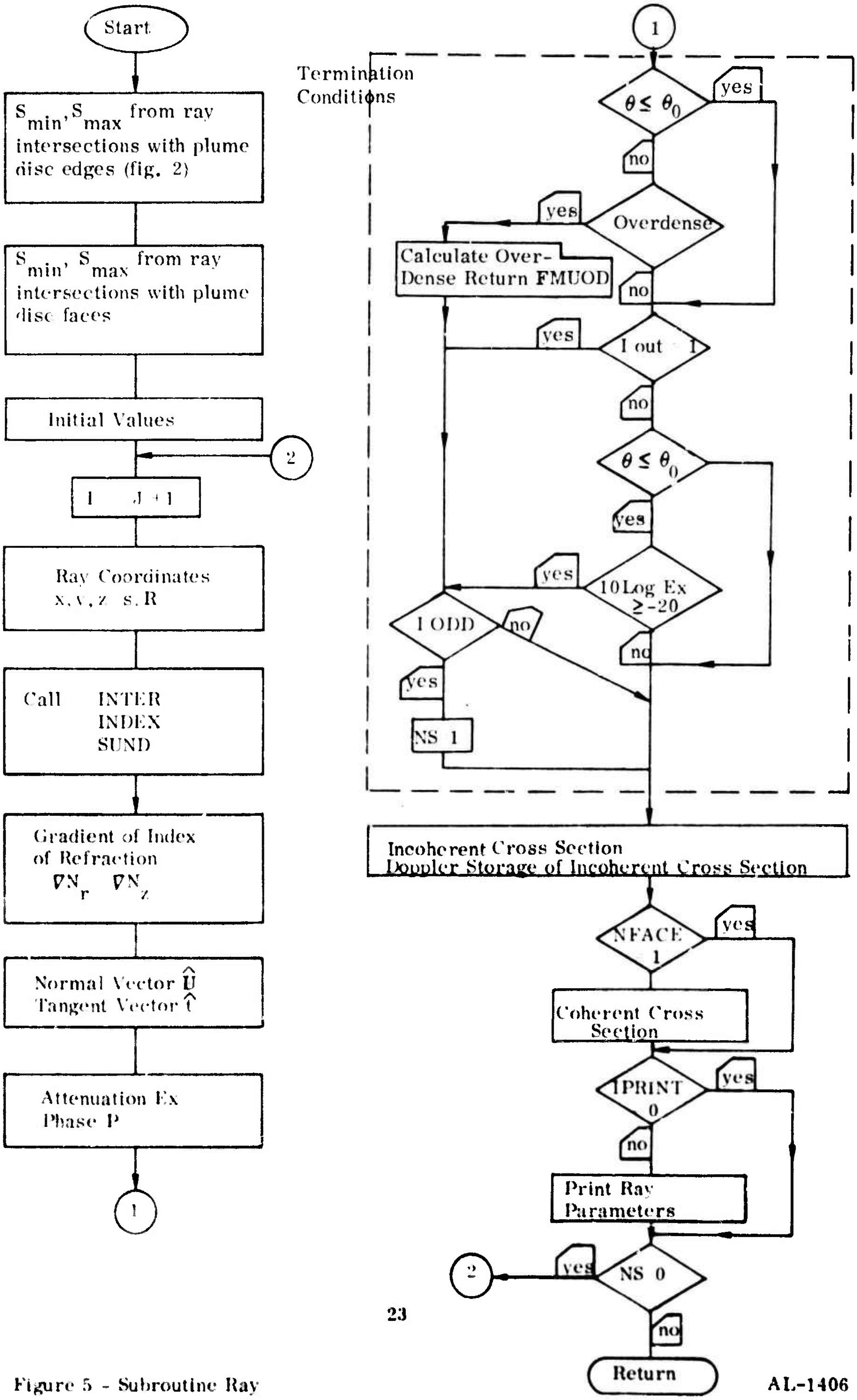

Figure 5 - Subroutine Ray

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Each intersection S value is compared with previous maximum and minimum values to determine the new SMIN and SMAX.

The ray coordinates for high aspect angle ($\theta > \theta_0$) are given by:

$$S = SMIN + (I - 1) STEP \quad (55)$$

$$X = XO \quad (56)$$

$$Y = -S \, SIN (\theta) \quad (57)$$

$$R = \sqrt{x^2 + y^2} \; . \quad (58)$$

For low aspect angles z and R are given by:

$$z = ZOLD + TZ \, STEP \quad (59)$$

and

$$R = ROLD + TR \, STEP \; , \quad (60)$$

where TZ and TR are components of the unit tangent to the ray, and STEP is the integration step size.

INTER is called to determine the plume properties at the point (z, R) by interpolation in the AeroChem LAPP plume data. INDEX is called to calculate the index of refraction using the interpolated plume properties. SUND calculates the integrands.

The tangent vector ($\hat{t}$) calculated during the current integration step is the tangent of the ray at the next point. Therefore, the currently calculated value determines the position of the integration point following the next point.

Portions of the plume are excluded from the volume integrations for the cross sections by terminating the ray according to the conditions shown in

Figure 5. For low aspect angles ($\theta \leq \theta_0$, Subsection 2.8) the ray is terminated when the attenuation term

$$\exp\left(-2 \int_{SMIN}^{S} ar_j dS'\right) \tag{61}$$

is less than -20 db or when the value of IOUT is equal to 1. Subroutine INTER returns a value of 1 for IOUT when an attempt is made to extrapolate the plume data to a point outside the plume.

High aspect angle rays are terminated by IOUT = 1 or when the ray encounters the overdense surface defined in Subsection 2.5.

The Simpson's rule integration scheme for the ray integration of the cross sections,

$$\int_0^2 f(x)\,dx = \frac{1}{3}\Delta x\left(f_2 + 4f_1 + f_0\right) \quad , \tag{62}$$

requires that the ray terminate on an odd number of integration steps. Furthermore, the attenuation and phase portions of the integrands require the evaluation of integrals at Simpson's rule midpoints (even numbers of integration steps). This is accomplished by using the "3/8 rule,"

$$\int_0^3 f(x)\,dx = \frac{3}{8}\Delta x\left(f_0 + 3f_1 + 3f_2 + f_3\right) \quad , \tag{63}$$

to evaluate the integrals in the attenuation and phase (P in Subsection 2.4) for even number integration steps and Simpson's "1/3 rule" for odd number steps. This ensures that the combined integration has a truncation error of Δx^5.

The incoherent cross section contribution for each integration step is stored in a "Doppler bin" dependent on the local axial gas velocity. The Doppler resolved contributions from the incoherent cross section are determined by a Trapezoidal integration over each integration step.

The coherent and overdense cross sections for the current ray are set equal to zero if NFACE equals 1. The first section of RAY that determines SMIN and SMAX will set NFACE equal to 1 if the ray intersects the plume-rocket interface at z = 0.

4.3 Subroutines INTER and PROF

Subroutines INTER and PROF perform interpolation of the plume data to obtain the values of N_e, ν, and v at the point R, Z. A linear interpolation in Z is performed by INTER and a quadratic interpolation in R is performed by PROF.

INTER finds a pair of axial stations such that

$$Z_i \leq Z < Z_{i+1} \quad .$$

PROF is called for each axial station to perform a radial interpolation obtaining the plume parameter values at (R, Z_i) and (R, Z_{i+1}). INTER performs a linear interpolation in Z between the axial station values to give the plume values at (R, Z). Points outside the plume generate exponentially decaying plume values. The attenuation factor is the inverse of the free space wavelength.

4.4 Subroutines INDEX and SUND

INDEX determines the complex index of refraction (N = XIND - i YIND) from,

$$\mathrm{Re}\sqrt{N^2} = \sqrt{\frac{1}{2}\left(\sqrt{xx^2 + yy^2} + xx\right)} \tag{64}$$

and

$$\mathrm{Im}\sqrt{N^2} = -\sqrt{\frac{1}{2}\left(\sqrt{xx^2 + yy^2} - xx\right)} \quad . \tag{65}$$

where

$$xx = 1 - \frac{\frac{N_c}{N_c}}{1 + \left(\frac{\nu}{\omega}\right)^2} \qquad (66)$$

and

$$yy = \frac{\nu}{\omega} \frac{\frac{N_e}{N_c}}{1 + \left(\frac{\nu}{\omega}\right)^2} \quad . \qquad (67)$$

If INDEX = 0 the index of refraction is set to the free space value N = 1.

SUND calculates the integrands from the interpolated values of N_e, ν and the index of refraction. The parameters $\nu^2 = 1/2\,(1 + \bar{r}^2)$ and cos β_m are set to 1 and -1 respectively. The output from SUND is:

$$DSIG = \frac{d\sigma}{d\Omega} \text{ (Subsection 2.3)} \qquad (68)$$

$$ATT = a \text{ (attenuation coefficient, Subsection 2.3)} \qquad (69)$$

$$DTAU = d\tau = r_e N_c \left[1 + \left(\frac{\nu}{\omega}\right)^2\right]^{-1/2} \qquad (70)$$

$$Q = 2\bar{N}\,\omega/c \text{ (Phase Integrand, Subsection 2.4)} \quad . \qquad (71)$$

5. INPUT/OUTPUT FORMATS

5.1 Input Format

The card deck for a run using a FORTRAN source deck on a CDC 6600 with a SCOPE operating system is shown in Figure 6. Card B gives the radar properties, Card C gives the integration step sizes and Card D sets the option switches. The AeroChem LAPP data defining the plume is described in Section F. The required plume data card format is shown in Subsection 4.1. Card G signals the end of the plume data cards. Cards A-D may be repeated in Section H for additional runs using the same plume data.

CARD A: RADAR Properties Title Card (20A4)

CARD B: RADAR Properties (Free Format)

θ (deg)	Aspect angle (0° for nose on viewing)
f (Hz)	Radar frequency
r_j (m)	Jet radius at matched pressure
Range (m)	Missile range
RESOLUTION (m)	Radar range cell resolution
Divergence (deg)	Radar beam divergence
Doppler Velocity (m/sec)	Radar doppler velocity resolution
Doppler Frequency (Hz)	Radar doppler frequency Resolution (set to 0. if velocity resolution is given)
Low Aspect Angle (deg)	Angles less than or equal to this angle are low aspect angles treated as described in Subsection 2.8
Gamma	Mean fluctuation factor

CARD C: Integration Step Sizes (Free Format)

FRACTS	Fraction of free space radar wavelength chosen for integration step size along the ray
FRACTX	Fraction of free space wavelength chosen for integration step size along x-axis
FRACTZ	Wavelength fraction used for integration step size along z-axis for high aspect angle cases and y-axis for low aspect angle cases

CARD D: OPTION Switches (Free Format)

ITEST =	0	Plume data from card deck in Section 6, Figure 6
	1	Constant electron density, cylindrical plume data from subroutine CYLDER
IPRINT =	0	No printout of integrands every Nth step in ray integration
	N	Printout of integrands every Nth step in ray integration
IATT =	1	Integrand attenuation factor attenuates cross sections $\left[\exp\left(-2\int_{s_{min}}^{s} ar_j ds'\right)\right]$
	0	No attenuation, attenuation factor set equal to 1
IDEX =	1	Index of refraction calculated from local electron density and collision frequency
	0	Index of refraction set equal to 1
IOVD =	1	Ray integration terminated when overdense surface encountered for high ($\theta \geq 15$) aspect angle
	0	No overdense surface ray termination

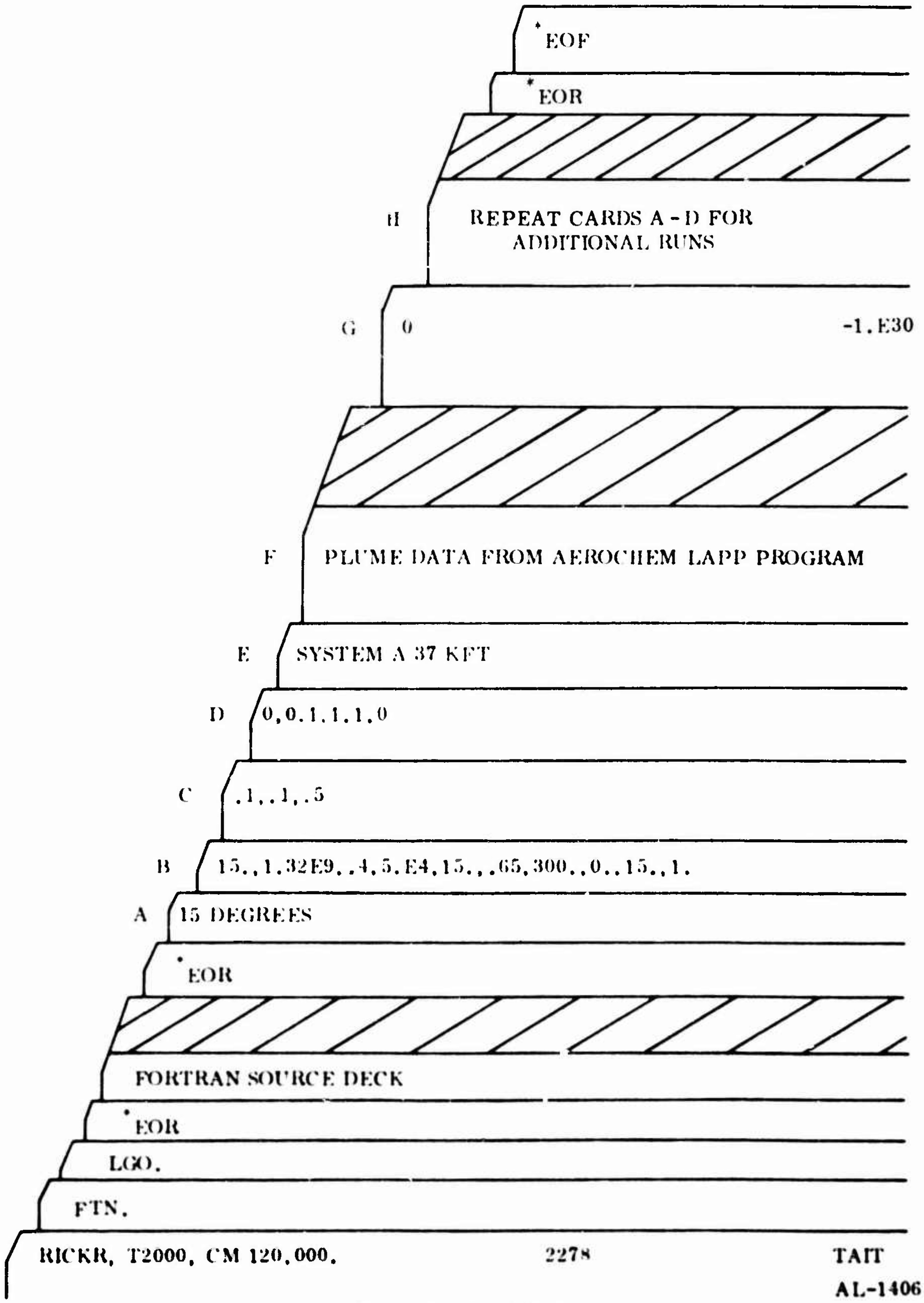

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Figure 6 - Card Deck

IRUFF = 1 Surface roughness factor and applied to overdense cross section return (Subsection 2. 5)

0 Surface roughness factor set equal to 1.

CARD E: PLUME DATA TITLE CARD (20A4)

SECTION F: Refer to Subsection 4. 1 – Table 1. For a constant cylinder test case with ITEST = 1 on Card D, Section F is omitted.

CARD G: END OF PLUME DATA 0. in columns 1 and 2, -1. E3 in columns 75-80

SECTION H: Repeat Cards A-D for additional runs using same plume data in Section F.

5. 2 Output Format

Appendix B shows the PARCS output for System A using the input data of Figure 6, Subsection 5. 1. The first page of the output shows the plume data used in the output subheading ''SYSTEM A 37 KFT'' taken from plume title card E (Figure 6). The data from input data cards A through D is printed below the plume title card ''SYSTEM A 37 KFT'' in the same units as those shown in Subsection 5. 1. The last line of the first output page shows the lower and upper limits of the y integration ZOO and ZLIM in units of jet radii (RJET). The low aspect angle definition θ_0 and the radar aspect angle θ are both equal to 15 deg so that this is a low aspect angle case (Subsection 2. 8). Therefore, the integration in z (along the plume axis, Figure 1) is transformed into an integration along the y-axis. The number of integration steps taken along the y-axis is given by NZM and DZO is the y integration step size in units of jet radii (RJET). If this had been a high aspect angle case ($\theta > \theta_0$), ZOO, ZLIM, NZM, and DZO would all have referred to the z-axis and there would have been no integration along the y-axis. DXO is the step size along the x-axis and STEP is the integration step size along the ray (Figure 1). Both DXO and STEP are in units of jet radii.

The second and third pages of the sample output show the cross sections for that portion of the plume in the first range gate (Subsection 2.6). The range cell number and length are shown on the first line of output page B-2 as 1 and 10^{10} jet radii. The plume is contained entirely within the first range cell, but if additional range cells were required to cover the entire plume their printouts would follow one another in order after the first range cell. The incoherent cross section of each range cell is "Doppler resolved" (Subsection 2.7). The first "Doppler bin" (page B-2) gives the total incoherent cross section for those portions of the plume where the plume axial gas velocity projected along the radar line-of-sight (ray, Figure 1) is less than or equal to the "DOPPLER VELOCITY" of 300 m/sec. The second Doppler bin contains contributions from regions of the plume where the projected gas velocity is in the range of 300 to 600 m/sec. The "FREQUENCY SHIFT" is the corresponding Doppler frequency shift of the radar wave in hertz.

The cross sections are shown in two systems of units; "sm" or square meters and "dbsm" where

$$\sigma\,(\mathrm{dbsm}) = 10\ \mathrm{Log}_{10}\left|\frac{\sigma\,(\mathrm{sm})}{1\ \mathrm{m}^2}\right| \quad . \tag{72}$$

The total cross sections for the first range cell are shown on page B-3. The total incoherent cross section (SINC) is the sum of the contribution from each Doppler bin, SCOH is the coherent cross section, SOD is the contribution from the overdense surface, and RCS is the total radar cross section given by the sum (in units of m^2) of the incoherent, coherent, and overdense cross sections. Output pages similar to pages B-2 and B-3 will appear for each range cell.

Pages B-4 and B-5 show the total cross sections for all range cells. The Doppler bin incoherent cross sections are equal to the sum (in m^2) of the cross sections from the corresponding Doppler bins in each range cell. The total cross sections for all range cells and Doppler bins are shown on page B-5.

The "Day File" on the last page shows a CP (central processor) time of "1850.48 sec" for the CDC 6600. This run would take

3.4 x 1850 = 6,290 sec

on a CDC 6400 computer.

5.3 Program Use

In order to ascertain the accuracy of the PARCS calculations, a number of test cases (for which analytical solutions are available) are employed. These test cases also permit a rapid assessment of the effects of varying step sizes employed in integrations along the rays, Δs and changing the total number of rays, i. e., changing Δx and Δz. The general conclusion of these tests cases is that the incoherent, underdense RCS is least sensitive to step size changes. The incoherent RCS requires five steps per wavelength to give a 2 db agreement with the analytical solution. At least 10 steps per wavelength are needed for a 2 db agreement with the closed solution for the coherent overdense and underdense returns.

Figure 7 shows the calculated L-band underdense, incoherent RCS as a function of aspect angle for a uniform cylinder 0.8m long, 0.4m radius (R_c) having an electron density of $10^{10}/cm^3$ and collision frequency of 10^{11}/sec. There is exact agreement at 90° with the equation for the collision corrected first order Born approximation to the incoherent cross section[4]

$$\sigma_{inc} = \frac{4\pi r_e^2}{1 + \left(\frac{\nu}{\omega}\right)^2} \gamma N_e^2 \Phi V \quad , \tag{73}$$

where V is the volume and the other terms are given in Section 2. At lower aspect angles there is a decrease in the RCS due to increasing path lengths and resultant attenuation in the plasma.

4. J. Jarem, "Radar Reflectivity of Turbulent Rocket Exhaust Plumes — Derivation of Equations," AeroChem Research Laboratories, TP-167 Supplement, November 1969, p. 20.

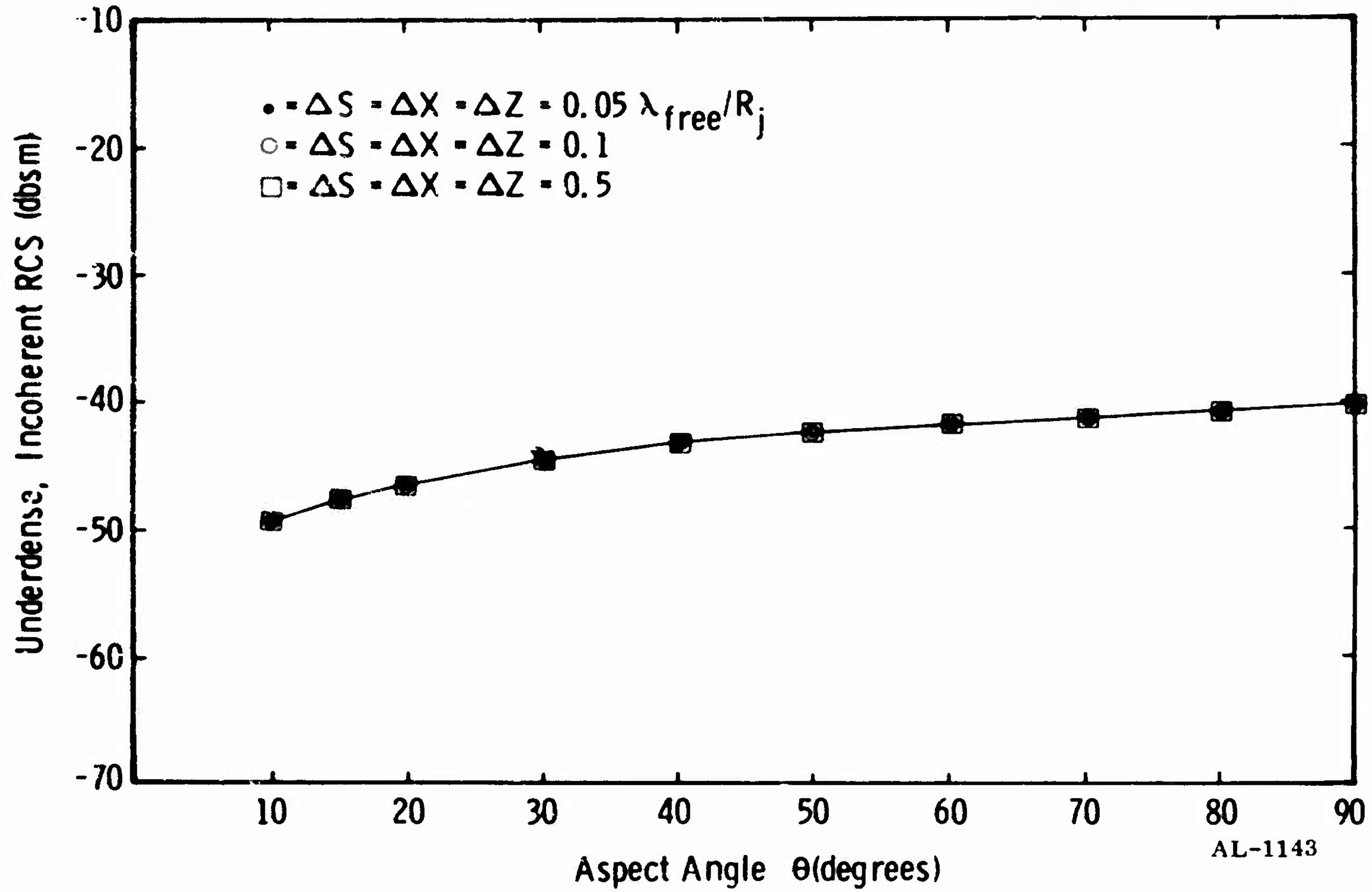

Figure 7 - Underdense, Incoherent RCS for Uniform Cylinder ($n_e = 10^{10}$ cm^{-3}, $\nu = 10^{11}$ sec^{-1} L = 0.8m, R = 0.4m)

The underdense, coherent return for the same cylinder is shown in Figure 8. The closed form solution is obtained from[5]

$$\sigma = \frac{32\pi^2 r_e^2 L^2 R_c^4 N_e^2}{1 + \left(\frac{\nu}{\omega}\right)^2} \left[\frac{\sin (kL \cos \theta)}{kL \cos \theta}\right]^2 \left[\frac{\sin \left(2kR_c \sin \theta\right) - \frac{\pi}{4}}{\left(2kR_c \sin \theta\right)^{3/2}}\right]^2 . \tag{74}$$

At 90° there is again good agreement with the exact solution for all steps sizes. However, at aspect angles less than about 70°, it becomes necessary to take at least ten steps per wavelength to attain less than 5 db error.

The PARCS calculated return from an overdense smooth cylinder of finite conductivity is shown in Figure 9. The closed form solution (solid line) is calculated from[6]

$$\sigma = \frac{\rho^2 k^2}{\pi} \left| \int_S (\hat{N} \cdot \hat{k})\, e^{2i\vec{k} \cdot \vec{r}}\, dS \right|^2 \tag{75}$$

with

$$\rho^2 = \frac{\sqrt{N^2} - N^2}{\sqrt{N^2} + N^2} , \tag{76}$$

where ρ^2 is the reflectivity (Subsection 2.5) and the surface integral is evaluated on the surface of the cylinder. It is necessary to employ greater than 10 steps per wavelength for the overdense cross section in order to get good agreement with the closed solution.

5. J. Jarem, "Radar Reflectivity of Turbulent Rocket Exhaust Plumes - Derivation of Equations," AeroChem Research Laboratories, TP-167 Supplement, November 1969, p. 50.

6. Methods of Radar Cross Section Analysis, ed. J.W. Crispin and K.M. Siegel; p. 105, Academic Press, 1968.

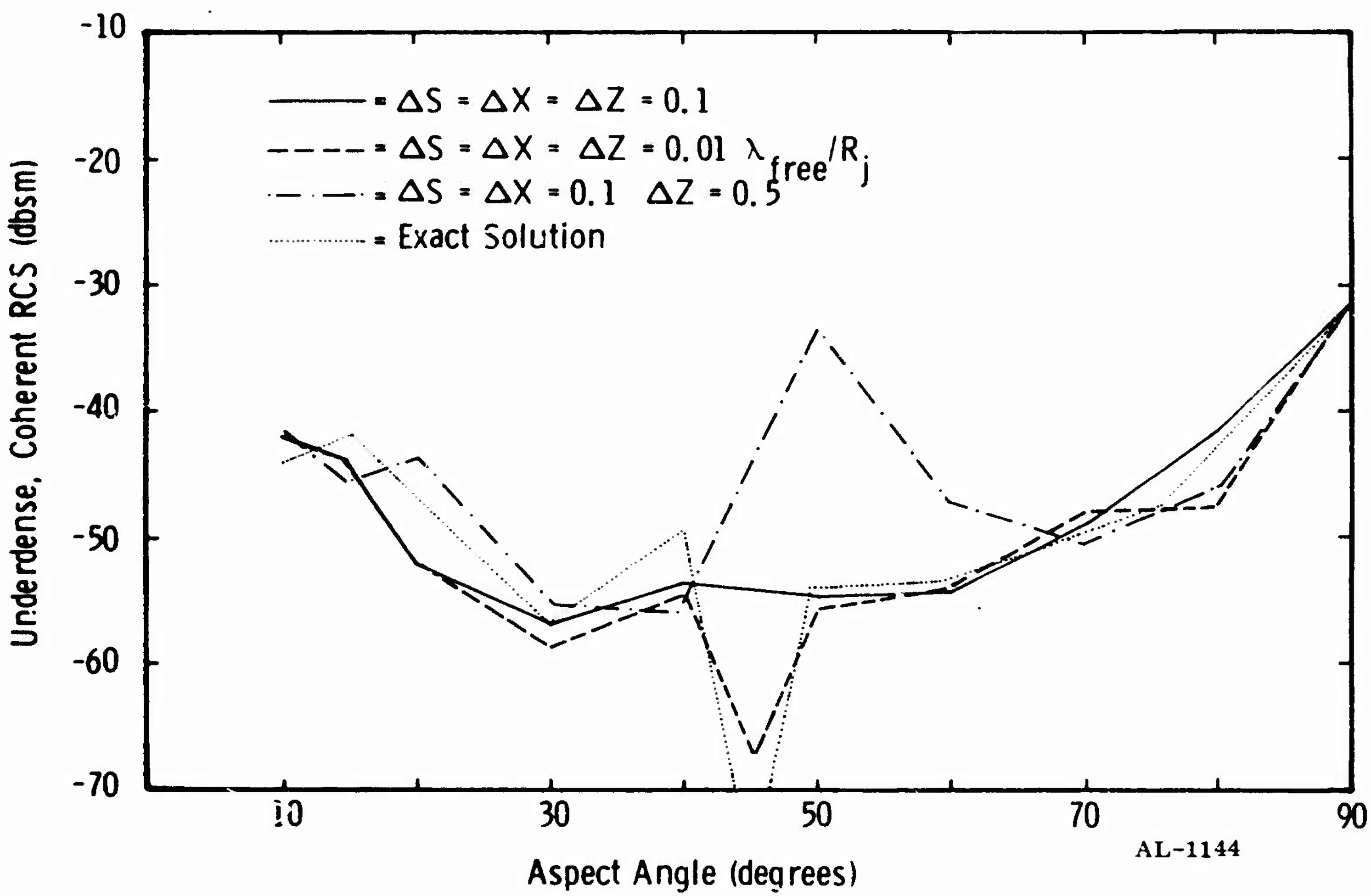

Figure 8 - Underdense, Coherent RCS for Uniform Cylinder ($n_e = 10^{10}$ cm^{-3}, $\nu = 10^{11}$ sec^{-1}, L = 0.8m, R = 0.4m)

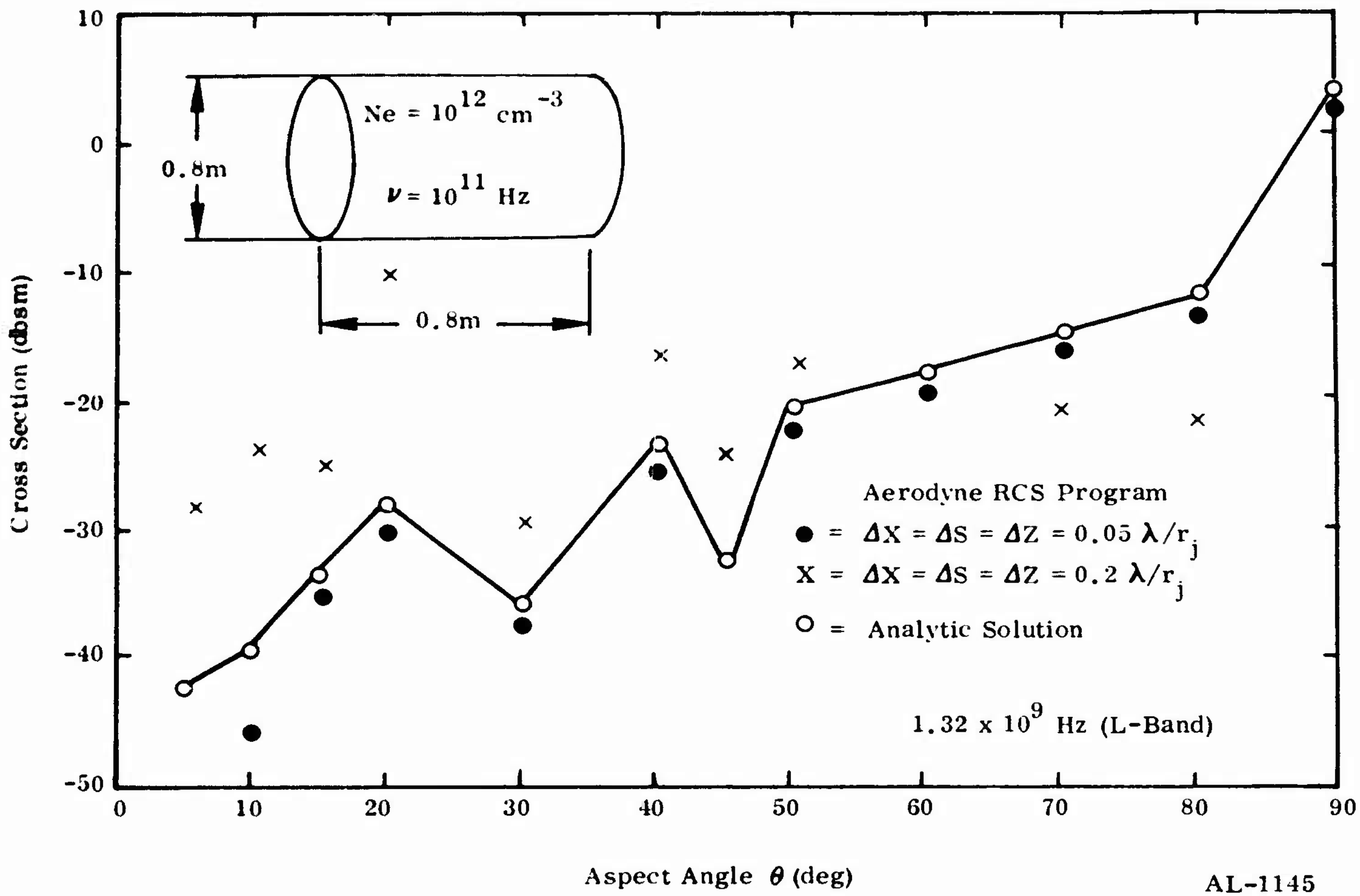

Figure 9 - Test Case for AFRPL Overdense RCS Routine

The total coherent and incoherent cross sections for SYSTEM D at 23.9 km was calculated for various step sizes using the PARCS code and the input data of Appendix C. The results are shown below.

Step Size FRACTX,FRACTS, FRACTZ			Total Incoherent Cross Section (dbsm)	Total Coherent Cross Section (dbsm)	CPU Time (sec) CDC 6600
0.1	0.1	0.5	-45.5	-59.7	100
0.2	0.2	0.2	-45.9	-69.9	84
0.1	0.1	0.1	-45.9	-67.7	490
0.05	0.05	0.05	-45.9	-67.7	3,212

All step sizes must be taken as one tenth of a radar wavelength to get convergence for the coherent cross section, but the incoherent cross section is close to convergence with FRACTZ = FRACTX = FRACTS = 0. 2.

The CPU time is approximately a linear function of the step size in each direction. For example, decreasing the step size in the z direction by a factor of five in going from the first to the third case in the table above increased the run time by a factor of 4. 9. The CPU time on a CDC 6400 computer would be a factor of 3.4 times longer than for the CDC 6600 (see Appendix C, System D).

APPENDIX A

PROGRAM LISTING

```
C
C AERODYNE RCS PROGRAM   CY=201
C
      PROGRAM PARCS(OUTPUT,TAPE6=OUTPUT,TAPE1,TAPE5)
      COMMON/PLUME/NZ,PLZ(100),NP(100),PLR(100,25),PLE(100,25)
     *,PLF(100,25),IATT,IDEX,IOVD,PLV(100,25),NNR(100),IRUFF,
     2 PPLE(100,25),PPLF(100,25),PPLV(100,25),PPLP(100,25)
      COMMON/RAY/STILDA,TTILDA(2),FMUOD(2),DOPSTR(50),PLROVD(100),GAMMA
      COMMON/ANGLE/THETA,THETAD,THETA0,THETCD,COSTHE,SINTHE,TTHETA
      DIMENSION TITLE(20),TITLE1(20)
      DIMENSION TCOH(2),SMUOD(2),XCOH(2),XOD(2),
     1 OLDCHX(2),OLDODX(2),OLDCHZ(2),OLDODZ(2),
     2 DOPINC(50),DOPX(50),OLDDOF(50),OLDOPZ(50),DOSDOP(50),
     3 TDOP(50),DBTDOP(50)
      REAL LAMBDA
      DATA DEG/.0174532925/,PI/3.141592654/,C/2.998E8/
C
C IPPINT = N FOR INTEGRANDS PRINTED EVERY NTH STEP IN S
C IATT = 1 FOR CALCULATED ATTENUATION
C        0 FOR NO ATTENUATION
C IDEX = 1 FOR CALCULATED INDEX OF REFRACTION
C        0 FOR INDEX = 1
C IOVD = 1 FOR OVERDENSE SURFACE RAY TERMINATION
C        0 NO OVERDENSE SURFACE RAY TERMINATION
C IRUFF = 1 OVERDENSE SURFACE ROUGHNESS CALCULATION
C         0 NO SURFACE ROUGHNESS CALCULATION
C ITEST = 1 CYLINDRICAL PLUME OF CONSTANT DENSITY FROM CYLDEP
C         0 PLUME DATA FROM LAPPIN
C FRACTS,FRACTX,FRACTZ = FRACTION OF FREE SPACE WAVELENGTH CHOSEN
C       FOR INTEGRATION STEP SIZE
C THETAAD = LARGEST ASPECT ANGLE USING LOW ASPECT ANGLE SCHEME (DEGREES)
C
C FUNCTION STATEMENTS
C
      FOD(XX,YY,ZZ,OM) = XX - ZZ*(1. + YY*YY/(OM*OM))/(1. + .5*YY/OM)
      DBFTN(X)=10.*ALOG10(AMAX1(X,1.E-10))
C
      NTIMES = 0
  100 CONTINUE
C
C READ AND PRINT RADAR DATA
C
      READ (5,200) TITLE
      IF (EOF(5).NE.0.) STOP
      READ(5,*) THETAD,RFREQ,RJET,RANGE,RESOLU,DIVERG,
     1 DOPVEL,DOPFRQ,THETCD,GAMMA
      READ(5,*) FRACTS,FRACTX,FRACTZ
      READ(5,*) ITEST,IPPINT,IATT,IDEX,IOVD,IRUFF
```

```
C
C READ IN PLUME DATA
C
      IF(NTIMES.NE.0) GO TO 30
      IF(ITEST) 10,20,10
   10 CALL CYLDER(IER,TITLE1)
      GO TO 30
   20 CALL LAPPIN(IER,TITLE1)
   30 IF(IER.NE.0) STOP
      NTIMES = 1
      WRITE(6,205)
  205 FORMAT(*1AERODYNE RCS PROGRAM*/)
      WRITE(6,200) TITLE1,TITLE
  200 FORMAT(20A4)
      WRITE(6,220) THETAD,RFREQ,RJET,RANGE,RESOLU,DIVERG,
     1DOPVEL,DOPFRQ,THETCD,GAMMA
  220 FORMAT(*0*,T5,*THETAD*,T15,*RFREQ*,T25,*RJET*,T35,*RANGE*,T45,
     1*RESOLUTION*,T57,*DIVERGENCE*/5X,1P6G10.3/*0*,T5,*DOPVEL*,T15,
     2*DOPFRQ*,T25,*THETCD*,T35,*GAMMA*/5X,1P4G10.3)
      WRITE(6,222) FRACTS,FRACTX,FRACTZ
  222 FORMAT(*0*,T5,*FRACTS*,T15,*FRACTX*,T25,*FRACTZ*/5X,1P3G10.3)
      WRITE(6,224) ITEST,IPRINT,IATT,IDEX,IOVD,IRUFF
  224 FORMAT(*0*,5X,*ITEST*,T15,*IPRINT*,T25,*IATT*,T35,*IDEX*,T45,
     1 *IOVD*,T55,*IRUFF*/6I10)
C
C PLUME REFLECTED ABOUT BISECTING XY PLANE FOR THETA GT 90 DEGREES
C
      DO 60 I=1,NZ
      NMAP = I
      IF(THETAD.GT.90.) NMAP = NZ - I + 1
      NRI = NNR(I)
      DO 50 J=1,NRI
      PLE(NMAP,J) = PPLE(I,J)
      PLF(NMAP,J) = PPLF(I,J)
      PLV(NMAP,J) = PPLV(I,J)
      PLR(NMAP,J) = PPLR(I,J)
      NR(NMAP) = NNR(I)
   50 CONTINUE
   60 CONTINUE
      IF(THETAD.GT.90.) THETAD = 180.-THETAD
C
C MAXIMUM PLUME RADIUS PLRMAX
C
      PLRMAX = 0.
      DO 65 I=1,NZ
      PLRMAX = AMAX1(PLRMAX,PLR(I,NR(I)))
   65 CONTINUE
```

```
C
C     AREA = PROJECTED AREA
C     OMEG = RADAR FREQUENCY IN RADIANS/SEC
C     THETA = ASPECT ANGLE IN RADIANS
C     DXO & STEP ARE INTEGRATION STEP SIZES IN X AND S IN UNITS OF RJET
C     DZO IS THE STEP SIZE IN THE AXIAL DIRECTION
C     FRACS = FRACTS
      FRACZ = FRACTZ
      FRACX = FRACTX
      IF(THETAD.GT.THETCD) GO TO 70
      FRACS = FRACTZ
      FRACZ = FRACTX
   70 OMEG = RFREQ*2.*PI
      LAMBDA = 2.*PI*C/OMEG/RJET
      STEP = LAMBDA*FRACS
      DXO = LAMBDA*FRACX
      DZO = LAMBDA*FRACZ
      IF(LAMBDA-1.) 90,80,80
   80 STEP = FRACS*RJET
      DXO = FRACX*RJET
      DZO = FRACZ*RJET
   90 THETA = THETAD*DEG
      THETAC = THETCD*DEG
      IF(THETAD.LE.89.) GO TO 225
      TTHETA = 1.0E+10
      GO TO 228
  225 TTHETA = TAN(THETA)
  228 COSTHE = COS(THETA)
      SINTHE = SQRT(1.-COSTHE*COSTHE)
      AREAOD = 2.*DXO*DZO*RJET**2
      AREA = AREAOD*SINTHE
      IF(THETA.LE.THETAO) AREA = AREAOD
      FFOD = 7.14E-10*OMEG*OMEG*SINTHE*SINTHE
      RANGE = RANGE/RJET
      RESOLU = RESOLU/RJET
      DIVERG = DIVERG*DEG
C
C ZLIM = UPPER LIMIT OF Z INTEGRATION CHOSEN TO INCLUDE END SECTION
C NZM = NUMBER OF POINTS IN AXIAL DIRECTION
C
      ZOO = 0.
      DELZ = 0.
      ZLIM = PLZ(NZ)
      IF(THETAD.GE.89.) GO TO 240
      IF(THETAD.LE.1.) GO TO 230
      DELZ = PLRMAX/TTHETA
      ZOO = -1.*DELZ
      GO TO 240
```

```
  230 ZLIM = PLRMAX
      Z00 = -1.*PLRMAX
      GO TO 245
  240 ZLIM = ZLIM + DELZ
      IF(THETA.GT.THETA0) GO TO 245
      ZLIM = ZLIM*TTHETA
      Z00 = Z00*TTHETA
  245 NZM = INT(1.0001*(ZLIM-Z00)/DZ0) + 1
      WRITE(6,250) Z00,ZLIM,NZM,DX0,DZ0,STEP
  250 FORMAT(*0*,T5,*Z00*,T15,*ZLIM*,T25,*NZM*,T35,
     1 *DX0*,T45,*DZ0*,T55,*STEP*/* *,1P2G16.3,I5,4X,3G16.3//)
      IF(IPRINT.GT.0) WRITE(6,255)
  255 FORMAT(*0*,T10,*RAY MATRIX*/*0*,T5,*S/RJ*,T20,*EX*,
     1 T35,*STILDA*,T50,*TTILDA(1)*,T65,*TTILDA(2)*,T80,
     2 *FMUOD(1)*,T95,*FMUOD(2)*,T110,*DOPSTR(ND3)*//)
C
C THIS LOOP DETERMINES THE RADIUS OF THE OVERDENSE SURFACE(PLROVD(I))
C AT EACH AXIAL STATION(Z = PLZ(I))
C
C I = AXIAL STATION      J = RADIAL STATION
C
      DO 300 I=1,NZ
      NRI = NR(I)
      DO 290 JJ=1,NRI
      J = NRI - JJ + 1
C
C TEST FOR RADIAL STATION INCREMENTED BEYOND OVERDENSE SURFACE
C
      IF(J.EQ.1) GO TO 280
      IF(FOD(PLE(I,J),PLF(I,J),FFOD,OMEG)) 290,270,270
  270 CONTINUE
C
C LINEAR INTERPOLATION BETWEEN CURRENT AND PREVIOUS RADIAL STATION
C DETERMINES PLROVD(I)
C
      FODTST = FOD(PLE(I,J),PLF(I,J),FFOD,OMEG) -
     1 FOD(PLE(I,J-1),PLF(I,J-1),FFOD,OMEG)
      IF(FODTST.LT.0.001) GO TO 280
      PLROVD(I) = FOD(PLE(I,1),PLF(I,1),FFOD,OMEG)*
     1 (PLR(I,J-1) - PLR(I,J))/
     2 (FOD(PLE(I,J),PLF(I,J),FFOD,OMEG) -
     3 FOD(PLE(I,J-1),PLF(I,J-1),FFOD,OMEG))
      GO TO 300
  280 PLROVD(I) = PLR(I,J)
      GO TO 300
  290 CONTINUE
  300 CONTINUE
```

```
C
C DOPPLER VELOCITY RESOLUTION & NO. DOPPLER BINS
C
      NDB = 1
      IF(THETAD.GE.89.) GO TO 314
      IF(DOPVEL.EQ.0.) DOPVEL = DOPFRQ*PI*C/OMEG
      PLVMAX = 0.
      DO 313 I=1,NZ
      NRI = NR(I)
      DO 312 J=1,NRI
      PLVMAX = AMAX1(PLVMAX,PLV(I,J))
  312 CONTINUE
  313 CONTINUE
      PLVMAX = PLVMAX*COSTHE
      NDB = INT(1.0001*PLVMAX/DOPVEL) + 1
      IF(NDB.LE.50) GO TO 314
      WRITE(6,315)
  315 FORMAT(*0TOO MANY DOPPLER BINS FOR ARRAY STORAGE ALLOCATED*)
      STOP
C
C CALCULATE THE RANGE CELL LENGTH FL
C
  314 BWIDTH = RANGE*DIVERG
      TANGLE = ATAN(BWIDTH/RESOLU)
      IF(TTHETA.LE.TANGLE) GO TO 320
      FL = BWIDTH/SINTHE
      GO TO 330
  320 FL = RESOLU/COSTHE
  330 IF(THETA.LE.THETAC) FL = 1.E+10
C
C THE DO LOOPS RANGE OVER NZM Z-VALUES(AXIAL STATIONS) AND
C NXM X-VALUES(RADIAL STATIONS). THE TRAPEZOIDAL RULE IS USED
C FOR THE X AND Z INTEGRATIONS.
C
      SING = 0.
      TSCOH = 0.
      TSOD = 0.
      TSING = 0.
      TSIG = 0.
      TCOH(1) = 0.
      TCOH(2) = 0.
      SMUOD(1) = 0.
      SMUOD(2) = 0.
      DO 332 I=1,NDB
      DOPINC(I) = 0.
      TDOP(I) = 0.
```

```
  332 CONTINUE
      KK = 1
      KKK = 1
      KCOUNT = 0
      TESTL = FL + Z00
C ****
      DO 430 KZ=1,NZM
C ****
      KCOUNT = KCOUNT + 1
      XING = 0.
      XCOH(1) = 0.
      XCOH(2) = 0.
      XOD(1) = 0.
      XOD(2) = 0.
      DO 334 I=1,NDB
      DOPX(I) = 0.
  334 CONTINUE
C
      ZC=ZCC+(KZ-1)*DZ0
C
C CALCULATE THE VALUES OF XLIM AND NXM
C
  340 IF(KK.GT.NZ) GO TO 350
      IF(ZC.LT.PLZ(KK)) GO TO 350
      XLIM = PLR(KK,NR(KK))
      NXM = INT(1.0001*XLIM/DX0) + 1
      KK = KK + 1
      IF(KK.GT.NZ) GO TO 350
      GO TO 340
  350 IF(ZC.LT.PLZ(1)) NXM = INT(1.0001*PLZ(1)/DX0) + 1
      IF(THETA.LE.THETAD) NXM=INT(1.0001*PLRMAX/DXC) + 1
C ****
      DO 380 KX=1,NXM
C ****
      X0 = (KX-1)*DX0
C
C RAY CALCULATES THE INTEGRALS IN THE S DIRECTION ALONG THE RAY
C
      CALL RAY(X0,ZC,OMEG,RJET,STEP,IPRINT,DOPVEL,NDB,LAMBDA)
C
      IF(KX.GT.2) GO TO 370
      OLDIGX = 0.
      OLDCHX(1) = 0.
      OLDCHX(2) = 0.
      OLDODX(1) = 0.
      OLDODX(2) = 0.
      DO 362 I=1,NDB
      OLDDOP(I) = 0.
```

```
  362 CONTINUE
  370 XINC = XINC + .5*(STILDA + OLDICX)
      XCOH(1) = XCOH(1) + .5*(TTILDA(1) + OLDCHX(1))
      XCOH(2) = XCOH(2) + .5*(TTILDA(2) + OLDCHX(2))
      XOD(1) = XOD(1) + .5*(FMUOD(1) + OLDODX(1))
      XOD(2) = XOD(2) + .5*(FMUOD(2) + OLDODX(2))
      DO 372 I=1,NDB
      DOPX(I) = DOPX(I) + .5*(DOPSTR(I) + OLDDOP(I))
      OLDDOP(I) = DOPSTR(I)
  372 CONTINUE
      OLDICX = STILDA
      OLDCHX(1) = TTILDA(1)
      OLDCHX(2) = TTILDA(2)
      OLDODX(1) = FMUOD(1)
      OLDODX(2) = FMUOD(2)
C
  380 CONTINUE
C
      IF(KCOUNT.GT.2) GO TO 390
      OLDICZ = 0.
      OLDCHZ(1) = 0.
      OLDCHZ(2) = 0.
      OLDODZ(1) = 0.
      OLDODZ(2) = 0.
      DO 382 I=1,NDB
      OLDOPZ(I) = 0.
  382 CONTINUE
  390 SINC = SINC + .5*(XINC + OLDICZ)
      TCOH(1) = TCOH(1) + .5*(XCOH(1) + OLDCHZ(1))
      TCOH(2) = TCOH(2) + .5*(XCOH(2) + OLDCHZ(2))
      SMUOD(1) = SMUOD(1) + .5*(XOD(1) + OLDODZ(1))
      SMUOD(2) = SMUOD(2) + .5*(XOD(2) + OLDODZ(2))
      DO 392 I=1,NDB
      DOPINC(I) = DOPINC(I) + .5*(DOPX(I) + OLDOPZ(I))
      OLDOPZ(I) = DOPX(I)
  392 CONTINUE
      OLDICZ = XINC
      OLDCHZ(1) = XCOH(1)
      OLDCHZ(2) = XCOH(2)
      OLDODZ(1) = XOD(1)
      OLDODZ(2) = XOD(2)
```

```
C
      IF(KZ.GE.NZM) GO TO 400
      IF(Z0.LT.TESTL) GO TO 430
  400 WRITE(6,410) KKK,FL
  410 FORMAT(*1*,*RANGE CELL NUMBER *,I2,5X,*CELL LENGTH*,1PG9.2,
     1 2X,*JET RADII*//)
      DO 404 I=1,NDB
      DOPINC(I) = DOPINC(I)*AREA*4.*FI
      DBSDOP(I) = DBFTN(DOPINC(I))
      TDOP(I) = TDOP(I) + DOPINC(I)
      FI = I
      VELDOP = FI*DOPVEL
      FRDDOP = VELDOP*OMEG/PI/C
      WRITE(6,402) I,VELDOP,FRDDOP,DOPINC(I),DBSDOP(I)
  402 FORMAT(*0DOPPLER BIN  *,I2,5X,*DOPPLER *,
     1 *VELOCITY (M/SEC) = *,1PG10.3,5X,*FREQUENCY SHIFT*,
     2 *(HZ) =*,G10.3/T15,*SINC*/1X,*SM*,T10,1PE10.3/
     3 1X,*DBSM*,T10,1PE10.3/)
  404 CONTINUE
C
      SINC = SINC*4.*PI*AREA
      DBSI = DBFTN(SINC)
      SCOH=4.*PI*AREA**2*(TCOH(1)**2+TCOH(2)**2)
      DBSC=DBFTN(SCOH)
      SOD = AREAOD*AREAOD*(SMUOD(1)*SMUOD(1) + SMUOD(2)*SMUOD(2))/PI
      DBSOD = DBFTN(SOD)
      SIG=SINC + SCOH + SOD
      DBSM=DBFTN(SIG)
C
C SUM CROSS SECTIONS FROM EACH RANGE CELL
C
      TSCOH = TSCOH + SCOH
      TSOD = TSOD + SOD
      TSINC = TSINC + SINC
      TSIG = TSIG + SIG
C
C PRINT TOTAL CROSS SECTION FOR RANGE CELL
C
      WRITE(6,415) KKK
  415 FORMAT(*0TOTAL CROSS SECTIONS FOR RANGE CELL *,I2)
      WRITE (6,420) SINC,SCOH,SOD,SIG,DBSI,DBSC,DBSOD,DBSM
  420 FORMAT(T15,*SINC*,T25,*SCOH*,T35,*SOD*,T45,*RCS*/
     *1X,*SM*,T10,1P4E10.3/1X,*DBSM*,T10,1P4E10.3/)
```

```
C
      KKK = KKK + 1
      TESTL = TESTL + FL
      SING = 0.
      TCOH(1) = 0.
      TCOH(2) = 0.
      SMUOD(1) = 0.
      SMUOD(2) = 0.
      DO 425 JJ=1,NDR
      DOPING(JJ) = 0.
  425 CONTINUE
      KCOUNT = 0
C
  430 CONTINUE
C
C PRINT TOTAL CROSS SECTIONS FOR ALL RANGE CELLS
C
      WRITE(6,432)
  432 FORMAT(*1TOTAL DOPPLER CROSS SECTIONS FOR ALL RANGE CELLS*/)
      DO 435 I=1,NDR
      DBTDOP(I) = DBFIN(TDOP(I))
      FI = I
      VELDOP = FI*DOPVEL
      FRQDOP = VELDOP*OMEG/PI/2.998E+08
      WRITE(6,438) I,VELDOP,FRQDOP,TDOP(I),DBTDOP(I)
  438 FORMAT(*0DOPPLER BIN *,I2,5X,*DOPPLER VELOCITY (M/SEC) = *,
     1 1PG10.3,5X,*FREQUENCY SHIFT (HZ) =*,G10.3/
     2T15,*SING*/1X,*SM*,T10,1PE10.3/1X,*DBSM*,T10,1PE10.3/)
  435 CONTINUE
      DBTCOH = DBFIN(TSCOH)
      DBTOD = DBFIN(TSOD)
      DBTINC = DBFIN(TSINC)
      DBTSIG = DBFIN(TSIG)
      WRITE(6,440)
  440 FORMAT(*0TOTAL CROSS SECTIONS FOR ALL RANGE CELLS*)
      WRITE(6,450) TSINC,TSCOH,TSOD,TSIG,DBTINC,DBTCOH,DBTOD,DBTSIG
  450 FORMAT(T15,*TSING*,T25,*TSCOH*,T35,*TSOD*,T45,*TRCS*/
     11X,*SM*,T10,1P4E10.3/1X,*DBSM*,T10,1P4E10.3/)
C
      GO TO 100
      END
```

```
C ********************************************
C
C
      SUBROUTINE RAY (X0,Z0,OMEG,RJET,STEP,IPRINT,DOPVEL,NDB,LAMBDA)
      COMMON/PLUME/NZ,PLZ(100),NR(100),PLR(100,25),PLE(100,25)
     *,PLF(100,25),IATT,IDEX,IOVD,PLV(100,25),NNR(100),IRUFF,
     2 PPLE(100,25),PPLF(100,25),PPLV(100,25),PP_R(100,25)
      COMMON/RAY/STILDA,TTILDA(2),FMUOD(2),DOPSTR(50),PLROVD(100),GAMMA
      COMMON/ANGLE/THETA,THETAD,THETA0,THETGD,COSTHE,SINTHE,TTHETA
      DIMENSION AT(3),ATTN(3),PH(3),PHSE(3),RS(100)
      REAL NEWSTL,NEWTT1,NEWTT2,NEWATT,NEWPH,NDOTK,NDOTV,NORMEX,NSTL,
     1 LAMBDA
      DATA C/2.998E+08/, ENC/3.14E-10/, PI/3.14159265/
      DATA CUTOFF/-20./
C
C DETERMINE SMIN AND SMAX INTEGRATION LIMITS
C PLUME DIVIDED INTO NZM1 DISCS.
C LOOP IN I INCREMENTS DISCS.
C RAY TESTED FOR INTERSECTION WITH DISC
C
      ZZ0 = Z0
      IF(THETA.GT.THETAD) GO TO 152
      ZZ0 = 0.
      IF(THETAD.GE.1.) ZZ0 = Z0/TTHETA
  152 NFACE = 0
      SMIN=1.E6
      SMAX=-1.E6
      NZM1 = NZ - 1
      DO 110 I=1,NZM1
C
C DISC RADIUS = MAXIMUM PLUME RADIUS AT AXIAL STATION I OR I+1
C
      R=AMAX1(PLR(I,NR(I)),PLR(I+1,NR(I+1)))
      RS(I)=R
C
C TEST: RAY MISSES DISC IF X0 GREATER THAN R
C
      IF (X0.GT.R) GO TO 110
      IF ( ABS(SINTHE).LT..01) GO TO 110
C
C S = DISTANCE FROM RAY INTERSECTION WITH X,Z-PLANE AT (X0,ZZ0)
C TO INTERSECTIONS WITH  INFINITELY LONG CYLINDAR WITH RADIUS EQUAL TO
C DISC RADIUS R
C
      S=SQRT(R**2-X0**2)/SINTHE
      DO 100 J=1,2
      IF (J.EQ.2) S=-S
```

```
C
C J-LOOP DETERMINES Z VALUES OF BOTH INTERSECTIONS OF RAY WITH
C INFINITELY LONG CYLINDAR. A TEST IS MADE TO DETERMINE WHETHER OR
C NOT THE INTERSECTION Z VALUE IS WITHIN THE DISC Z VALUE LIMITS
C
      Z=ZZ0+S*COSTHE
      IF ((PLZ(I+1)-Z)*(Z-PLZ(I)).LT.0.) GO TO 110
      SMAX=AMAX1(SMAX,S)
      SMIN=AMIN1(SMIN,S)
  100 CONTINUE
  110 CONTINUE
C
C THE RAY MAY INTERSECT A DISC X,Y PLANE FACE AT SMIN OR SMAX.
C THE I LOOP DETERMINES RAY INTERSECTIONS WITH THE X,Y PLANE AT EACH
C AXIAL STATION. THE INTERSECTION POINT (AT RADIUS R) IS TESTED TO
C WHETHER OR NOT IT LIES WITHIN THE DISC (OF RADIUS RS(I))
C
      IF(ABS(COSTHE).LT..01) GO TO 150
      RS(NZ)=RS(NZM1)
      DO 140 I=1,NZ
      S=(PLZ(I)-ZZ0)/COSTHE
      R=SQRT(X0*X0+S*S*SINTHE*SINTHE)
      IF (R.GT.RS(I)) GO TO 140
      IF(I.LT.1) NFACE = 1
      SMAX=AMAX1(SMAX,S)
      SMIN=AMIN1(SMIN,S)
  140 CONTINUE
  150 CONTINUE
C
C INTEGRATION LOOP
C
      IIOVD = IOVD
      IF(THETA.LE.THETA0) IIOVD = 0
      FOO= FNC*OMEG*OMEG*SINTHE*SINTHE
      PHASEC = 2.*OMEG*RJET*(SMIN+ZZ)*COSTHE)/C
      IOUT = 0
      PILAM = PI/LAMBDA
      TR = 0.
      TZ = 0.
      ROLD = SQRT(X0*X0+Z0*Z0)
      COSALP = 1.
      IF(ROLD.GE..001) COSALP = Z0/ROLD
      TRS = -1.0*SINTHE*COSALP
      TPHIS = SINTHE*SQRT(1.-COSALP*COSALP)
      TZS = COSTHE
      PHASES = PHASEC
      DO 155 I=1,NOR
      DOPSTR(I) = 0.
```

```
  155 CONTINUE
      ATTENS = 0.
      FMUOD(1) = 0.
      FMUOD(2) = 0.
      TTILDA(1) = 0.
      TTILDA(2) =0.
      STILDA = 0.
      DS = STEP*RJET
      KK = 4
      J = 0
      NS = 0
      IOD = 0
      I = 0
      IF(SMIN.GT.SMAX) RETURN
C
  160 I = I+1
C
C RAY COORDINATES
C
      FIM1 = I-1
      S = SMIN + FIM1*STEP
      X = X0
      Y = -1.*S*SINTHE
      Z = ZZ0 + S*COSTHE
      R = SQRT(X*X+Y*Y)
      IF(THETA.GT.THETA0) GO TO 166
      R = ROLD
      IF(I.LE.1) GO TO 166
      DS = STEP*RJET*TZ
      Z = ZOLD +TZ*STEP
      R = ROLD + TR*STEP
      IF(R.GT.0.) GO TO 166
      R = ABS(R)
      TRS = ABS(TRS)
      R = ABS(R)
  166 ZOLD = Z
      ROLD = R
C
C INTER INTERPOLATES LAPP INPUT TO DETERMINE THE ELECTRON DENSITY
C AND COLLISSION FREQUENCY AT THE POINT (X,Y,Z)
C
      CALL INTER(R,Z,ES,FS,VEL,LAMDDA,GRADEZ,GRADFZ,GRADER,GRADFR,ICUT)
C
C
C INDEX DETERMINES THE INDEX OF REFRACTION  N = XIND - I YIND
C
      CALL INDEX(ES,FS,OMEG,XIND,YIND,IDEX)
```

```
C
C SUND CALCULATES THE INTEGRANDS
C
      CALL SUND(ES,FS,GAMMA,-1.,OMEG,RJET,
     1         DSIG,AIT,DTAU,0,XIND,YIND)
C
C RAY TRACING FOR LOW ASPECT ANGLE
C
      IF(THETA.GT.THETA0) GO TO 168
C
C GRADIENT OF INDEX OF REFRACTION
C
      BRAKET = OMEG*OMEG + FS*FS
      ARG = 1.-ES/FNC/BRAKET
      IF(ARG.LT.0.) ARG = 1.E
      FN = SQRT(ARG)
      GRADNF = 0.5/FNC/BRAKET/FN
      GRADFF = 2.0*ES*FS/BRAKET
      GRADXR = GRADNF*(GRADFF*GRADFR-GRADER)
      GRADXZ = GRADNF*(GRADFF*GRADFZ-GRADEZ)
      F1 = FS*FS/OMEG/OMEG
      F2 = (1.-ARG)*(1.-ARG)
      F3 = SQRT(ARG*ARG + F2*F1)
      GRADNR = F2*F1*GRADFR/FS
      GRADNR = GRADNR + GRADXR*(ARG*(1.+F1) - F1)
      GRADNR = .25*(GRADNR/F3 + GRADXR)/XIND
      GRADNZ = F2*F1*GRADFZ/FS
      GRADNZ = GRADNZ + GRADXZ*(ARG*(1.+F1) - F1)
      GRADNZ = .25*(GRADNZ/F3 + GRADXZ)/XIND
      GRADNM = SQRT(GRADNR*GRADNR+GRADNZ*GRADNZ)
C
C UNIT NORMAL VECTOR U
C
      TR = TRS
      TZ = TZS
      TPHI = TPHIS
      UFACTR = TR*GRADNR + TZ*GRADNZ
      UR = GRADNR - UFACTR*TR
      UR = UR/GRADNM
      UZ = GRADNZ - UFACTR*TZ
      UZ = UZ/GRADNM
      UPHI = -1.*UFACTR*TPHI
```

```
C
C UNIT TANGENT VECTOR T
C
      FDTDS = GRADNR*UR + GRADNZ*UZ
      FDTDS = FDTDS/FN
      DTDSR = FDTDS*UR
      DTDSZ = FDTDS*UZ
      DTDSP = FDTDS*UPHI
      TRS = TR + DTDSR*STEP
      TZS = TZ + DTDSZ*STEP
      TPHIS = TPHI + DTDSP*STEP
      TMAG = SQRT(TRS*TRS + TZS*TZS + TPHIS*TPHIS)
      TRS = TRS/TMAG
      TZS = TZS/TMAG
      TPHIS = TPHIS/TMAG
C
C EVEN I
C
  168 IF(MOD(I,2).NE.0) GO TO 170
      SIMP = 4.
      GO TO 180
C
C ODD I
C
  170 SIMP = 1.
C
C PREVIOUS INTEGRAND ADDED EVERY OTHER POINT FOR SIMPSON'S RULE
C
  180 IF(I.EQ.KK) GO TO 190
      OLDSTL = 0.
      OLDTT1 = 0.
      OLDTT2 = 0.
      OLDATT = 0.
      OLDPH = 0.
      GO TO 200
  190 KK = KK + 2
C
C SIMPSON'S RULE
C
  200 NEWATT = ATT*DS*SIMP/3.
      ATTENS = ATTENS + NEWATT + OLDATT
      OLDATT = NEWATT
      NEWPH = Q*DS*SIMP/3.
      PHASES = PHASES + NEWPH + OLDPH
```

```
      OLDPH = NEWPH
      ATTEN = ATTENS
      PHASE = PHASES
C
C SIMPSON 2 RULE
C
      IF(I.LT.4) GO TO 210
      IF(MOD(I,2).NE.0) GO TO 210
      AT4PTS = (AT(3) + 3.*AT(2) + 3.*AT(1) + ATT)*.375*DS
      ATTEN = AT4PTS + ATTN(3)
      PH4PTS = (PH(3) + 3.*PH(2) + 3.*PH(1) + Q)*.375*DS
      PHASE = PH4PTS + PHSE(3)
C
C THE FIRST AND SECOND POINTS ARE SPECIAL
C
  210 IF(I.EQ.1) ATTEN = 0.
      IF(I.EQ.1) PHASE = PHASEC
      IF(I.EQ.2) ATTEN = DS*(AT(1) + ATT)/2.
      IF(I.EQ.2) PHASE = DS*(PH(1) + Q)/2. + PHASEC
C
C STORE LAST THREE POINTS
C
      IF(I.EQ.1) GO TO 230
      IF(I.EQ.2) GO TO 220
      AT(3) = AT(2)
      ATTN(3) = ATTN(2)
      PH(3) = PH(2)
      PHSE(3) = PHSE(2)
  220 AT(2) = AT(1)
      ATTN(2) = ATTN(1)
      PH(2) = PH(1)
      PHSE(2) = PHSE(1)
  230 AT(1) = ATT
      ATTN(1) = ATTEN
      PH(1) = Q
      PHSE(1) = PHASE
C
C ATTENUATION
C
      FEXP = 2.*ATTEN
      EX = 1./EXP(FEXP)
      IF(IATT.EQ.0) EX=1.
C
C TERMINATION CONDITIONS & OVERDENSE SURFACE CROSS SECTION
```

```
C
      IF(IOD.EQ.1) GO TO 231
      IF(IIOVD.EQ.0) GO TO 240
      RAT = FS/OMEG
      ZNOD = FOD*(1. + RAT*RAT)/(1. + .5*RAT)
      IF(FS.LT.ZNOD) GO TO 240
      IOD = 1
  231 IF(NFACE.EQ.1) GO TO 250
      DO 232 N=2,NZ
      IF(Z.LE.PLZ(N)) GO TO 234
  232 CONTINUE
  234 IF(N.GT.NZ) N=NZ
      TANPSI = PLROVD(N) - PLROVD(N-1)
      TANPSI = TANPSI/(PLZ(N) - PLZ(N-1))
      COSPSI = 1./SQRT(1.+TANPSI*TANPSI)
      SINPSI = SQRT(1.-COSPSI*COSPSI)
      XI = 1.
      IF(IRUFF.EQ.0) GO TO 235
      ROD = TANPSI*(Z-PLZ(N)) + PLROVD(N)
      H = .4*RJET
      IF(ROD.LE.H) GO TO 235
      XII = .8*OMEG*RJET*SINTHE/C
      XII = XII*XII
      XI = (1. + .16*XII/ROD/ROD)*EXP(-1.*XII)
      XI = SQRT(XI)
  235 IF(Y.LT.STEP) Y=STEP/2.
      SINBTA = Y/SQRT(X*X+Y*Y)
      NDOTK = -1.*SINTHE*COSPSI*SINBTA - COSTHE*SINPSI
      NDOTK = ABS(NDOTK)
      NDOTY = COSPSI*SINBTA
      RATIO = NDOTK/NDOTY
      DENOM = (1.+XIND)*(1.+XIND) + YIND*YIND
      DEXMAG = 1. - XIND*XIND - YIND*YIND
      NORMEX = OMEG*RATIO*EX*XI/C
      EMUOD(1) = NORMEX*(DEXMAG*COS(PHASE) + 2.*YIND*SIN(PHASE))/DENOM
      EMUOD(2) = NORMEX*(2.*YIND*COS(PHASE) - DEXMAG*SIN(PHASE))/DENOM
      GO TO 250
C
C LAST I MUST BE ODD FOR SIMPSON'S RULE INTEGRATION
C
  240 IF(IOUT.EQ.1) GO TO 255
      IF(THETA.GT.THETA0) GO TO 255
      DBEX = 10.*ALOG10(AMAX1(EX,1.E-10))
      IF(DBEX.LE.CUTOFF) GO TO 250
      DTDSM = 0.
      IF(THETA.LE.THETA0) DTDSM = SQRT(DTDSR*DTDSR+DTDSZ*DTDSZ+
     1 DTDSP*DTDSP)
      GO TO 255
  250 IF(MOD(I,2).NE.0) NS = I
  255 CONTINUE
```

```
C
C INCOHERENT RADAR CROSS SECTION STILDA
C
      NSTL = DSIG*EX*EX*DS
      NEWSTL = NSTL*SIMP/3.
      STILDA = STILDA + NEWSTL + OLDSTL
      OLDSTL = NEWSTL
C
C DOPPLER CELL STORAGE OF INCOHERENT CROSS SECTION
C
      IF(I.LE.1) GO TO 265
      SIGMA = .5*(STLOLD + NSTL)
      BINVEL = 0.
      VEL = VEL*COSTHE
      DO 264 J=1,NDB
      BINVEL = BINVEL + DOPVEL
      IF(VEL.GT.BINVEL) GO TO 264
      DOPSTR(J) = DOPSTR(J) + SIGMA
      GO TO 265
  264 CONTINUE
  265 STLOLD = NSTL
C
C COHERENT RADAR CROSS SECTION TTILDA(1) + I TTILDA(2)
C
      IF(NFAGE.EQ.1) GO TO 268
      NEWTT1 = DTAU*EX*COS(PHASE)*DS*SIMP/3.
      NEWTT2 = DTAU*EX*SIN(PHASE)*DS*SIMP/3.
      TTILDA(1) = TTILDA(1) + NEWTT1 + OLDTT1
      TTILDA(2) = TTILDA(2) + NEWTT2 + OLDTT2
      OLDTT1 = NEWTT1
      OLDTT2 = NEWTT2
C
C PRINT INTEGRALS EVERY 'IPRINT' TIME THROUGH LOOP
C
  268 IF(IPRINT.EQ.0) GO TO 300
      IF(I.EQ.1) GO TO 280
      IF(MOD(I,IPRINT).NE.0) GO TO 300
      IF(IOD.NE.0) WRITE(6,270)
  270 FORMAT(50X,*OVERDENSE*)
  280 WRITE(6,290) S,EX,STILDA,TTILDA(1),TTILDA(2),
     1 EMUOD(1),EMUOD(2),DOPSTR(NDB)
  290 FORMAT(1H ,1P8G15.3)
C
  300 IF(NS.EQ.0) GO TO 160
C
C END OF INTEGRATION LOOP
```

```
C
      IF(I.GT.1) GO TO 310
      STILDA = 0.
      TTILDA(1) = 0.
      TTILDA(2) = 0.
  310 CONTINUE
      RETURN
      END
C ******************************************
      SUBROUTINE PROF(I,R,E,F,VR,LAMBDA,GRADER,GRADFR,IOUT)
C
C PROF IS CALLED BY INTER TO PERFORM INTERPOLATION IN R.
C THE INTERPOLATION IS LINEAR FOR THE COLLISSION FREQUENCY AND
C AXIAL VELOCITY, BUT QUADRATIC IN THE ELECTRON DENSITY
C
      COMMON/PLUME/NZ,PLZ(100),NR(100),PLR(100,25),PLE(100,25)
     *,PLF(100,25),IATT,IDEX,IOVD,PLV(100,25),NNR(100),IRUFF,
     2 PPLE(100,25),PPLF(100,25),PPLV(100,25),PPLR(100,25)
      REAL LAMBDA
      IF(R.GT.PLR(I,1)) GO TO 10
      E = PLE(I,1)
      F = PLF(I,1)
      VR = PLV(I,1)
      GRADER = 0.
      GRADFR = 0.
      RETURN
   10 JMAX = NR(I)
C
C THIS LOOP DETERMINES PLR(I,J) SUCH THAT
C PLR(I,J-1).LT.R.LT.PLR(I,J)
C
      DO 100 J=2,JMAX
      IF (R.LE.PLR(I,J)) GO TO 110
  100 CONTINUE
      IF(R.LT.PLR(I,JMAX)+.1) IOUT = 1
      DELR2 = R - PLR(I,JMAX)
      A = 1./LAMBDA
      AD = A*DELR2
      EM = PLE(I,JMAX)
      RATIOF = 1.
      RATIOV = 1.
      IF(EM.GE.1.) RATIOF = PLF(I,JMAX)/EM
      IF(EM.GE.1.) RATIOV = PLV(I,JMAX)/EM
      E = EM*EXP(-1.*AD)
      IF(E.LT.1.) E = 1.
      F = RATIOF*E
      VR = RATIOV*E
      GRADER = -1.*E*A
      GRADFR = GRADER*RATIOF
      RETURN
```

```
  110 DELR = (PLR(I,J) - PLR(I,J-1))
      DELTA = (PLR(I,J) - R)/DELR
      F = PLF(I,J)*(1.-DELTA) + PLF(I,J-1)*DELTA
      VR = PLV(I,J)*(1.-DELTA) + PLV(I,J-1)*DELTA
      GRADFR = (PLF(I,J)-PLF(I,J-1))/DELR
      IF(F.LT.1.) F=1.
      IF(VR.LT.1.) VR=1.
      IF(J-2) 120,120,130
  120 E = PLE(I,J)*(1.-DELTA) + PLE(I,J-1)*DELTA
      GRADER = (PLE(I,J)-PLE(I,J-1))/DELR
      IF(E.LT.1.) E=1.
      RETURN
  130 CONTINUE
C
C QUADRATIC INTERPOLATION FOR THE ELECTRON DENSITY
C WHERE  E = C0 + C1 R + C2 R**2
C
      C0 = PLE(I,J)*PLR(I,J-2)*(PLR(I,J-2)+DELR)/2./DELR/DELR
      C0 = C0 - PLE(I,J-1)*PLR(I,J-2)*(PLR(I,J-2)+2.*DELR)/DELR/DELR
      C0 = C0 + PLE(I,J-2)*(PLR(I,J-2)**2 + 3.*DELR*PLR(I,J-2) +
     1 2.*DELR*DELR)/2./DELR/DELR
C
      C1 = -1.*PLE(I,J)*(2.*PLR(I,J-2)+DELR)
      C1 = C1 + 4.*PLE(I,J-1)*(PLR(I,J-2)+DELR)
      C1 = C1 - 1.*(2.*PLR(I,J-2) + 3.*DELR)*PLE(I,J-2)
      C1 = C1/2./DELR/DELR
C
      C2 = PLE(I,J) - 2.*PLE(I,J-1) + PLE(I,J-2)
      C2 = C2/2./DELR/DELR
C
      E = C0 + C1*R + C2*R*R
      GRADER = C1 + 2.*C2*R
      IF(E.LT.1.) E=1.
      RETURN
      END
C
C ******************************************
      SUBROUTINE INTER(R,Z,E,F,V,LAMBDA,GRADEZ,GRADFZ,GRADER,
     1 GRADFR,ICUT)
C
C INTER INTERPOLATES THE ELECTRON DENSITY,
C COLLISSION FREQUENCY, AND AXIAL VELOCITY.
C THE INTERPOLATION IN R IS DONE BY CALLING PROF.
C INTER PERFORMS LINEAR INTERPOLATION IN Z.
```

```
C
      COMMON/PLUME/NZ,PLZ(100),NR(100),PLR(100,25),PLE(100,25)
     *,PLF(100,25),IATT,IDEX,IOVD,PLV(100,25),NNR(100),IRUFF,
     2 PPLE(100,25),PPLF(100,25),PPLV(100,25),PPLR(100,25)
      REAL LAMBDA
      DO 100 I=1,NZ
      IF(PLZ(I)-Z) 100,110,110
  100 CONTINUE
      IOUT = 1
      I2 = NZ
      I1 = NZ-1
      CALL PROF(I2,R,E,F,V,LAMBDA,GRADER,GRADFR,IOUT)
      RATIOF = 1.
      RATIOV = 1.
      IF(E.GE.1.) RATIOF = F/E
      IF(E.GE.1.) RATIOV = V/E
      IF(I2.LE.1) GO TO 115
      CALL PROF(I1,R,E1,F1,VR1,LAMBDA,GRDE1R,GRDF1R,IOUT)
      DELZ2 = Z - PLZ(I2)
      A = 1./LAMBDA
      AD = A*DELZ2
      E = E*EXP(-1.*AD)
      IF(E.LT.1.) E = 1.
      F = RATIOF*E
      V = RATIOV*E
      GRADEZ = -1.*E*A
      GRADFZ = GRADEZ*RATIOF
      RETURN
  110 I2=I
      I1=I-1
      CALL PROF(I2,R,E,F,V,LAMBDA,GRADER,GRADFR,IOUT)
      RATIOF = 1.
      RATIOV = 1.
      IF(E.GE.1.) RATIOF=F/E
      IF(E.GE.1.) RATIOV = V/E
      IF(I.GT.1) GO TO 120
  115 DECAY = EXP(Z/LAMBDA)
      E = E*DECAY
      F = RATIOF*E
      V = RATIOV*E
      GRADEZ = E/LAMBDA
      GRADFZ = RATIOF*GRADEZ
      RETURN
  120 CALL PROF(I1,R,E1,F1,VR1,LAMBDA,GRDE1R,GRDF1R,IOUT)
      DELZ = PLZ(I2) - PLZ(I1)
      DEL1 = (PLZ(I2)-Z)/DELZ
      E = (E1*DEL1 + E*(1.-DEL1))
      F = (F1*DEL1 + F*(1.-DEL1))
```

```
      V = (VR1*DEL1 + V*(1.-DEL1))
      GRADFR = GRDF1R*DEL1 + GRADFR*(1.-DEL1)
      GRADER = GRDE1R*DEL1 + GRADER*(1.-DEL1)
      GRADEZ = (E-E1)/DELZ
      GRADFZ = (F-F1)/DELZ
      RETURN
      END
C
C ******************************************
      SUBROUTINE LAPPIN(IER,TITLE)
      COMMON/PLUME/NZ,PLZ(100),NR(100),PLR(100,25),PLE(100,25)
     *,PLF(100,25),IATT,IDFX,IOVD,PLV(100,25),NNR(100),IBUFF,
     2 PPLE(100,25),PPLF(100,25),PPLV(100,25),PPLR(100,25)
      DIMENSION BUFF(8),TITLE(20)
      READ (1,100) TITLE
  100 FORMAT (20A4)
      OLDZ = -1.
      READ (1,110) BUFF
  110 FORMAT(8F10.0)
      IF (BUFF(8).NE.1.E30) GO TO 190
      DO 170 I=1,100
      J = 1
      IF((BUFF(1).LE.OLDZ).OR.(BUFF(1).GT.200.)) GO TO 190
      OLDZ = BUFF(1)
      PLZ(I)=BUFF(1)
      DO 150 J=1,26
      READ (1,110) BUFF
      IF (EOF(1).NE.0.) GO TO 190
      IF (ABS(BUFF(8)).GE.1.E30) GO TO 160
      IF (J-1) 120,120,130
  120 IF (BUFF(1).NE.0.) GO TO 190
      GO TO 140
  130 IF((BUFF(1).LE.OLD).OR.(BUFF(1).GT.10.)) GO TO 190
  140 OLD=BUFF(1)
      PPLR(I,J)=OLD
      IF(BUFF(2).LT.1.0) BUFF(2) = 1.0
      IF(BUFF(3).LT.1.0) BUFF(3) = 1.0
      IF(ABS(BUFF(8)).LT.1.0) BUFF(8) = 1.0
      PPLV(I,J) = BUFF(8)*.3048
      PPLE(I,J)=BUFF(2)
  150 PPLF(I,J)=BUFF(3)
      J=27
      GO TO 190
  160 NNR(I)=J-1
      IF (BUFF(8).LT.0.) GO TO 180
  170 CONTINUE
      I=5
      GO TO 190
```

```
  180 NZ = I
      IER=0
      RETURN
  190 IER=1
      WRITE(6,200) I,J
  200 FORMAT(/1H0,*INPUT DATA ERROR*,2I5/)
      RETURN
      END
C
C ************************************************
      SUBROUTINE INDEX(ZNE,ZNU,OMEG,X,Y,IDEX)
      IF(IDEX) 20,10,20
   10 X = 1.
      Y = 0.
      RETURN
   20 RAT = ZNU/OMEG
      YY=ZNE/OMEG/3.14E-10/OMEG
      YY=YY/(1.+RAT**2)
      XX=1.-YY
      YY=RAT*YY
      X=(SQRT(XX**2+YY**2)+XX)/2.
      X=SQRT(X)
      Y = .5*(SQRT(XX*XX+YY*YY) - XX)
      Y = SQRT(ABS(Y))
      RETURN
      END
C
C ************************************************
      SUBROUTINE SUND(ZNE,ZNU,GAMMA,CBETA,OMEG,CLAM,DSIG,ATT,
     *DTAU,ETA,XIND,YIND)
      DATA PI/3.14159265/
      ZK=OMEG/2.998E8
      GAMSQ = GAMMA*GAMMA
      FKOLM=15.6*CLAM**3/(1.+(2.*XIND*ZK*CLAM)**2)**1.833
      REZNE=2.8178E-9*ZNE
      DTAU=REZNE/SQRT(1.+(ZNU/OMEG)**2)
      DSIG=DTAU**2*FKOLM*GAMSQ
      ETA=2.*XIND*ZK
      ATT=ZK*YIND+PI*DSIG*(1.+CBETA)
      RETURN
      END
```

```
C
C ****************************************************
      SUBROUTINE CYLDER(IER,TITLE1)
      COMMON/PLUME/NZ,PLZ(100),NR(100),PLR(100,25),PLE(100,25),
     1 PLF(100,25),IATT,IDEX,IOVD,PLV(100,25),NNR(100),IRUFF,
     2 PPLF(100,25),PPLF(100,25),PPLV(100,25),PPLP(100,25)
      DIMENSION TITLE1(20)
C
C CONSTANT CYLINDER TEST CASE:
C  LENGTH = NZ-1 JET RADII
C  RADIUS = .1*(NRR-1) JET RADII
C  ELECTRON DENSITY = D (CM-3)
C  COLLISSION FREQUENCY = F (HZ)
C  AXIAL GAS VELOCITY = V (M/SEC)
C
      DATA NZ,NRR/3,11/
      DATA D,F,V/1.E+08,1.E+11,1./
C
      READ(1,10) TITLE1
   10 FORMAT(20A4)
      FL = NZ-1
      FR = .1*(NRR-1)
      WRITE(6,15) FL,FR,D,F,V
   15 FORMAT(*1CONSTANT CYLINDER PROPERTIES*/
     1* LENGTH =*,1PG10.3,2X,*JET RADII*/
     2* RADIUS =*,1PG10.3,2X,*JET RADII*/
     3* ELECTRON DENSITY =*,1PG10.3,2X,*CM-3*/
     4* COLLISSION FREQUENCY =*,1PG10.3,2X,*HZ*/
     4* AXIAL GAS VELOCITY =*,1PG10.3,2X,*M/SEC*//)
      IER = 0
C
      DO 30 I=1,NZ
      FIM1 = I-1
      PLZ(I) = FIM1
      NNR(I) = NRR
      NRI = NNR(I)
C
      DO 20 J=1,NRI
      FJM1 = J-1
      PPLP(I,J) = FJM1*.1
      PPLV(I,J) = V
      PPLE(I,J) = D
      PPLF(I,J) = F
   20 CONTINUE
   30 CONTINUE
      RETURN
      END
```

APPENDIX B

SAMPLE OUTPUT

AERODYNE RCS PROGRAM

SYSTEM A 37KFT (3/75)

15 DEGREES

THETAD	PERFC	RJET	RANGE	RESOLUTION	DIVERGENCE
15.0	1.3205+09	.400	5.000E+04	15.0	.650

DOP VEL	DOPFRQ	THETOP	GAMMA
700.	0.	15.0	1.00

FRACTS	FRACTX	FRACTZ
.100	.100	.500

ITEST	IPRINT	IATT	IDEV	IOVR	TPURE
0	0	1	1	1	0

Z00	ZLTM	NZM	DX0	DZ0	STEP
-5.22	18.8	423	5.678E-02	5.6785-02	.284

```
RANGE CELL NUMBER  1        CELL LENGTH 1.00E+10   JET RADII

DOPPLER BIN     1       DOPPLER VELOCITY (M/SEC) =      300.        FREQUENCY SHIFT(HZ) = 2.642E+03
                SINC
SM        5.147E-11
D*SM     -1.000E+02

DOPPLER BIN     2       DOPPLER VELOCITY (M/SEC) =      600.        FREQUENCY SHIFT(HZ) = 5.284E+03
                SINC
SM        6.516E-09
D*SM     -8.196E+01

DOPPLER BIN     3       DOPPLER VELOCITY (M/SEC) =      900.        FREQUENCY SHIFT(HZ) = 7.925E+03
                SINC
SM        5.654E-06
D*SM     -5.248E+01

DOPPLER BIN     4       DOPPLER VELOCITY (M/SEC) =  1.200E+03       FREQUENCY SHIFT(HZ) = 1.057E+04
                SINC
SM        4.707E-05
D*SM     -4.327E+01

DOPPLER BIN     5       DOPPLER VELOCITY (M/SEC) =  1.500E+03       FREQUENCY SHIFT(HZ) = 1.321E+04
                SINC
SM        1.212E-04
D*SM     -3.917E+01

DOPPLER BIN     6       DOPPLER VELOCITY (M/SEC) =  1.800E+03       FREQUENCY SHIFT(HZ) = 1.585E+04
                SINC
SM        6.785E-05
D*SM     -4.195E+01
```

```
DOPPLER BIN    7        DOPPLER VELOCITY (M/SEC) =  2.100E+03      FREQUENCY SHIFT(HZ) = 1.849E+04
                SINC
SM         5.991E-05
DPSM      -4.223E+01

DOPPLER BIN    8        DOPPLER VELOCITY (M/SEC) =  2.400E+03      FREQUENCY SHIFT(HZ) = 2.113E+04
                SINC
SM         5.594E-05
DPSM      -4.253E+01

DOPPLER BIN    9        DOPPLER VELOCITY (M/SEC) =  2.700E+03      FREQUENCY SHIFT(HZ) = 2.378E+04
                SINC
SM         7.217E-05
DPSM      -4.142E+01

DOPPLER BIN   10        DOPPLER VELOCITY (M/SEC) =  3.000E+03      FREQUENCY SHIFT(HZ) = 2.642E+04
                SINC
SM         4.292E-05
DPSM      -4.367E+01

TOTAL CROSS SECTIONS FOR RANGE CELL   1
                SINC      SCOH      SOD       RCS
SM         4.655E-04 7.355E-03 0.        7.821E-03
DPSM      -3.332E+01-2.133E+01-1.000E+02-2.107E+01
```

TOTAL DOPPLER CROSS SECTIONS FOR ALL RANGE CELLS

Doppler RTN	Doppler Velocity (M/SEC)	Frequency Shift (HZ)	SM (SINC)	DBSM (SINC)
1	300.	2.642E+03	5.147E-11	-1.000E+02
2	600.	5.284E+03	6.516E-09	-8.196E+01
3	900.	7.925E+03	5.654E-06	-5.248E+01
4	1.200E+03	1.057E+04	4.707E-05	-4.327E+01
5	1.500E+03	1.321E+04	1.212E-04	-3.917E+01
6	1.800E+03	1.585E+04	6.385E-05	-4.195E+01

DOPPLER BIN 7 DOPPLER VELOCITY (M/SEC) = 2.100E+03 FREQUENCY SHIFT (HZ) = 1.849E+04

	SINC
SM	5.981E-05
DBSM	-4.223E+01

DOPPLER BIN 8 DOPPLER VELOCITY (M/SEC) = 2.400E+03 FREQUENCY SHIFT (HZ) = 2.113E+04

	SINC
SM	5.584E-05
DBSM	-4.253E+01

DOPPLER BIN 9 DOPPLER VELOCITY (M/SEC) = 2.700E+03 FREQUENCY SHIFT (HZ) = 2.378E+04

	SINC
SM	7.217E-05
DBSM	-4.142E+01

DOPPLER BIN 10 DOPPLER VELOCITY (M/SEC) = 3.000E+03 FREQUENCY SHIFT (HZ) = 2.642E+04

	SINC
SM	4.292E-05
DBSM	-4.367E+01

TOTAL CROSS SECTIONS FOR ALL RANGE CELLS

	TSINC	TSCOH	TSCP	TRCS
SM	4.655E-04	7.355E-03	0.	7.821E-03
DBSM	-3.332E+01	-2.133E+01	-1.000E+02	-2.107E+01

```
04/16/76   SCOPE 3.4.3        * CDC7740  A.F.G.L.
19.08.41.PICKR2P  FROM CATR
19.08.41.IP  00013312 WORDS - FILE INPUT   , DC 00
19.08.41.PICKP,T2000,CM120000.
19.08.41.          2270    TAIT
19.08.43.FTN(T,SL,R=3)
19.08.43.                                       000110
19.09.19.     5.625 CP SECONDS COMPILATION TIME
19.09.19.LGO.
19.09.19.                                       000140
04.30.53.LOCKTN.
04.47.54.     STOP
04.47.54.  1850.408 CP SECONDS EXECUTION TIME
04.47.54.OP  00020352 WORDS - FILE OUTPUT , DC 40
04.47.54.MS   21504   WORDS  (   35840 MAX USED)
04.47.54.CPA    1858.247 SEC.     1858.247 ADJ.
04.47.54.IO        5.281 SEC.        5.281 ADJ.
04.47.54.CM   59273.505 KWS.     3517.767 ADJ.
04.47.54.SS                      5481.295
04.47.54.PP       30.399 SEC.     DATE 04/16/76
04.47.54.EJ  END OF JOB, **

***********          PICKR2P  //// END OF LIST ////
***********          PICKR2P  //// END OF LIST ////
```

APPENDIX C

INPUT DATA SHEETS FOR TYPICAL RUNS

APPENDIX C

INPUT DATA FOR TYPICAL RUNS

SYSTEM A 37 KFT

CARD A: 15 DEGREES (20A4)

CARD B: 15., 1.32E9, .4, 5.E4, 15., .65, 300., 0., 15., 1. (free format)

θ (deg), f (Hz), JET RADIUS (m), RANGE (m), RESOLUTION (m), DIVERGENCE (deg), DOPPLER VELOCITY (m/sec), DOPPLER FREQUENCY (Hz), LOW ASPECT ANGLE (deg), GAMMA

CARD C: .1, .1, .5 (free format)

FRACTS, FRACTX, FRACTZ

CARD D: 0, 0, 1, 1, 1, 0 (free format)

ITEST, IPRINT, IATT, IDEX, IOVD, IRUFF

CPU TIME ON CDC 6600: 1850 sec

SYSTEM A 5 KFT

CARD A: 30 DEGREES

CARD B: 30., 1.32E+09, .4, 1.E+04, 5., .65, 150., 0., 15., 1.

CARD C: .1, .1, .5

CARD D: 0, 0, 1, 1, 1, 0

CPU TIME ON CDC 6600: 235 sec

SYSTEM D 23.9 KM

CARD A: 138 DEGREES

CARD B: 138., 1.32E+09, .621792, 3.05E+04, 15., .9, 340., 0., 15., 1

CARD C: .1, .1, .5

CARD D: 0, 0, 1, 1, 1, 0

CPU TIME ON CDC 6600: 108 sec

CPU TIME ON CDC 6400: 344.8 sec