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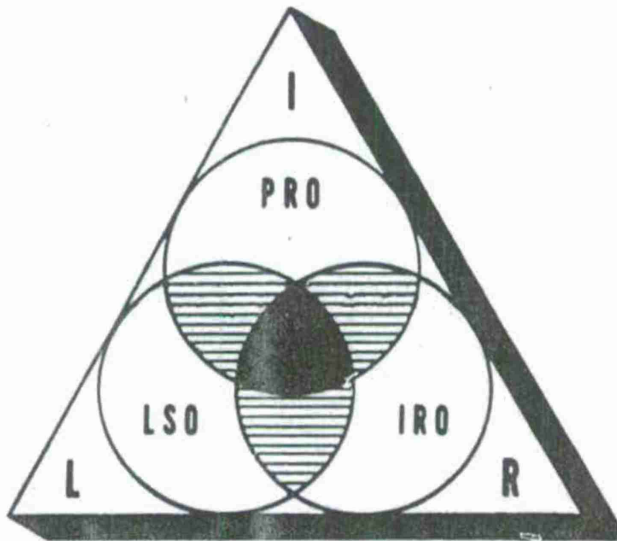## Defense Supply Agency

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DEMAND FORECASTS USING PROCESS MODELS & ITEM CLASS PARAMETERS: APPLICATION OF ANCILLARY VARIABLES

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DEMAND FORECASTS USING PROCESS MODELS & ITEM

CLASS PARAMETERS: APPLICATION OF ANCILLARY

VARIABLES

FINAL REPORT

BY

DONALD ORR

APRIL 1976

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19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Demand Forecasting   Error Measures
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Theoretical and statistical results are presented on forecasting a time series (D_t) in conjunction with a correlated series (H_t). In the particular problem D_t is demand for a part which is on an aircraft which flies H_t hours in period t. Reasonable recursive models of the underlying demand process are postulated and it is shown that theoretically rigorous or heuristically satisfactory forecast algorithms can be obtained by applying Kalman filter or weighted moving average techniques to 4 time series: D_t, D_t/H_t, log D_t, log D_t/H_t. An important forecasting

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ABSTRACT (CONT)

parameter - denoted k - is developed for the structural models; k is the ratio of the noise variance of a process to the variance of random changes in the process mean. Included are stratified results on error measure values of the algorithms. Their performances are also tested in a simulation of the supply system.

It is found that forecasts utilizing flying hours do give improved performance; the "best" algorithm is a Kalman filter with a varying weighting parameter which depends upon the flying hours in a period and k, which is determined by the item's demand frequency class. When the ancillary variable (program variable) is end item density rather than flying hours, the algorithm is identical but with different k-values.

Projected savings, over the current Army method of forecasting demands on the wholesale supply system, were 1.8 million dollars annually on the 10,000 parts in the data base.

ACKNOWLEDGEMENTS

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SUMMARY

1. Background

In early 1974, after some preliminary investigative work by IRO, study was assigned to that same office by the AMC (now DARCOM) Directorate of Supply, to formulate and test methodologies with potential for improving demand forecasts for Army managed secondary items. Moving average and exponential smoothing forecasting schemes had been investigated extensively in the past and have specific data retention rules. Other more structured forecasting models had not been tested; the catalog approach for other than insurance items, Bayesian estimators which combine item history and demand distributions over a whole catalog of items, and techniques (e.g. Kalman filters) which handle changing demand rates should be added to a list of potential techniques.

At the time, Martin Cohen was studying techniques for forecasting that utilized program data (flying hours, end item densities). The study reported here did not duplicate the effort with those techniques, but it was found that the procedures (algorithms) developed herein could be applied to demands or to demand per unit program rates.

2. Purpose & Objectives

P1   Investigate untried but theoretically rigorous forecast techniques including methods applicable to items for which a program factor is not feasible.

P2   Develop implementation procedures and specifications for the retention and upkeep of item past history.

O1   Use the available 28 quarters of AVSCOM demand and program data for forecast model building and comparative evaluation.

O2   Determine how much item history should be retained, how often should the retained data be updated, and what importance or weight should be attached to various demand estimators (e.g. how the age of the data should be weighted).

O3   Determine the merit of various performance measures for comparing forecast techniques.

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3. Scope

The study is limited to developing procedures for the forecasting of world-wide recurring demands for Army managed Class IX secondary items (repair parts and spares) including Stock Fund and PEMA items. The procedures are to be applied in the Commodity Command Standard System (CCSS) inventory management function implemented at the Army's National Inventory Control Points (NICP).

The focus of this report in terms of a "best" forecast technique is on an algorithm which uses an ancillary variable (FH program or density program). A further study and subsequent report will concentrate on the techniques described herein which may be applied to items for which program data is not meaningful, and on the results utilizing a data base of such items.

This current study was not intentionally limited to aircraft items but the only adequate program data available over 7 years was for such items. Methodology is developed which extends the scope to items from other Commands for which end item density is the ancillary variable.

4. Methodology

a. Postulate models of the demand process.

b. Develop algorithms which theoretically would best forecast this process. At this point is also determined the time series on which to apply the algorithms - demand D, demand per flying hour D/H, logarithm of demand log D, logarithm of demand per flying hour log D/H.

c. Determine algorithm parameter values for various stratifications of items, e.g. classify items by annual requisition frequency.

c. Screen the many algorithms by their performance with several statistical error measures (e.g. mean square error by item class).

d. Evaluate the remaining candidate algorithms by their cost-performance averaged over individual items in the simulation model of the Army supply management system.

The final selection was made on the basis of smallest aggregate simulated inventory cost for constant time-weighted requisitions backordered.

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5. Results

    a. "Best" algorithm: Kalman filter (see Section 5.3) to estimate demand per unit program (DP).

        (1) It is akin to exponential smoothing with a varying smoothing or weighting parameter which depends upon the program in a period (quarter) and a "k-factor".

        (2) The "k-factor" is updated yearly from the items demand frequency class.

        (3) Older data is given less weight.

        (4) Periods with high program given more weight.

        (5) Forecast = DP x program in future period.

    b. Cost-Performance Comparison: "Best" versus Present Army Program Factor (1794), based on simulation projections for 10,000 items.

        (1) For same average days wait, $1,800,000 annual savings are realized.

        (2) At constant average yearly cost, wait is reduced $\sim 12\%$.

    c. Tables of parameters are presented for forecasting by item class, as are extensive tables of error measure values by item class for the various algorithms.

    d. Several candidates (non-program related) algorithms for forecasting common items have been found.

6. Conclusions

    a. This study has reinforced Cohen's findings - that forecast algorithms utilizing flying hours perform better on the AVSCOM data base than strictly demand dependent algorithms.

    b. Recommended technique yields substantial improvement in terms of cost savings.

    c. The technique can be applied across Commodity Commands with only a change in algorithm parameter values.

d. Methodology has been developed which can be applied to a broad spectrum of common items.

CHAPTER I

INTRODUCTION

In many approaches to demand forecasting, several general forecast techniques (moving averages; single and double exponential smoothing; linear regressions on time or another independent variable) are applied to groups of items. Average performances with respect to an error measure(s) are compared; optimal forecast parameters (smoothing parameters, moving average base) need to be determined by search or enumeration, observing the error measure values. Such approaches are somewhat haphazard; the forecast parameters would be justified a posteriori, no consistent theory on the structure of the underlying processes would have been developed, and extensions of empirical results and the techniques would be made more difficult.

In this study, several models of a demand process or a demand – flying hour (FH) process are postulated. Basically the models consider a process mean corrupted by some noise in the observed values; in addition the mean itself of the process is changing randomly and/or non-randomly. For the most part, nothing is assumed about the probability distributions of the random variates. Models for how the demand series over time D_t is changing and for how the demand per FH series D_t/H_t is changing are described. For example one might expect that if a demand – FH relation exists, the mean of the rate D/H would be relatively stable.

The advantage of this modeling is that available and newly developed (/3) theory dictates what are the optimal and suboptimal forecast algorithms associated with each model. For example, since in general the models assume the process mean to be changing, a sample mean of all past history is not the best forecast technique. Kalman filters (akin to exponential smoothing with changing smoothing constants) and moving averages with variable base lengths are indicated. Also parameters of the forecast algorithm are related to noise variances in the process. Analysis of the time series of the process for groups of items can determine these process noise parameters

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on an average basis. If patterns in these average values develop over groups of items, this is one indication that the model in question is appropriate. Finally, these process parameters lead to the forecast parameters without the need of a search; and the performance of the algorithm relative to others indicates which model best describes the process.

Before proceeding with models of the processes and associated forecast algorithms (Chapter II), we briefly describe in Chapter II the data base of items used in the analysis. Chapter IV describes the computer program for evaluating the forecasts via statistical error measures. Stratified empirical results comparing about 25 algorithms are presented, as are 10 average values of an important forecasting parameter - the k factor - for items grouped by requisition frequency. Trends and relative values in the tables are analyzed. In Chapter V, the most promising candidate algorithms are used as forecast routines in the simulator of the Army wholesale supply operation. Based on cost performance in the simulator runs, three final algorithms for forecasting using FH are compared to the current AVSCOM program factor technique. Projected savings are discussed in Chapter VI, as are implementation considerations and modifications of the best algorithm to utilize end item density as the program factor rather than a usage variable (FH).

A short chapter on conclusions, recommendations for forecasting and further work on common items, and some caveats or aids to future researchers ends this report.

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CHAPTER II

DATA BASE

The conclusions in this report are based on studies made using chronological demand data from AVSCOM. Flying hours were obtained from DCSLOG. Details of the data organization and editing are found in an IRO report by Cohen (4). The final data base contained over 10,000 "peculiar" parts – those that are on only one type of aircraft and hence can be associated with specific flying hour values. A larger data base of common items ($\sim$ 30000) has been retained to study forecast algorithms developed here which do not depend on usage (FH); this will be a future task (see Chapter VII).

All data has been summarized by quarter. For each quarter we have worldwide totals of the number of requisitions $\{R_t\}$, the quantity demanded $\{D_t\}$, and the flying hours $\{H_t\}$. The flying hour totals are broken out by aircraft type/model/series (TMS). The data spans the 28 quarters from Jan 1967 thru Dec 1973. The scope of this work is limited to recurring demand; requisitions for initial issue, mobilization, and rebuild are not included.

Table 2.1 shows the distribution of items in the final data base by classes (definitions follow).

TABLE 2.1 DISTRIBUTION BY ITEM CHARACTERISTICS

ITEM CLASS	TOTAL COUNT	PEMA	ASF	NON-REP	REP	INS
LDV Non-Dynamics	10350	29	10321	9877	448	25
LDV Dynamic	1008	24	984	957	51	
LDV Total	11358	53	11305	10834	499	25
HDV Non-Dynamic	174	30	144	66	108	
HDV Dynamic	99	52	47	26	73	
HDV Total	273	82	191	92	181	
Total Non-Dynamics	10524	59	10465	9943	556	25
Total Dynamic	1107	76	1031	983	124	
Total	11631	135	11496	10926	680	25

The last columns give breakouts by funding (PEMA, ASF) and segment (non-reparable, reparable or insurance items). Usually PEMA are expensive, reparable items. HDV items have in at least one year an average yearly dollar demand of at least $50,000 or average yearly frequency (requisitions) of at least 100; the LDV class is comprised of the other items (low and medium dollar value). Dynamic components are defined based on a description of the items FSC, and are those experiencing high rotation rates (rotor blades, transmissions, engine components) - the demand for which may be quite dependent on FH. Non-dynamic components are more structural in nature. The LDV non-dynamic class is intended to contain relatively cheap non-reparables.

Again refer to Cohen (4) for a fuller description.

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CHAPTER III

MODELS OF PROCESSES AND FORECAST ALGORITHMS

3.1 Models of Processes

3.1.1 Structural Forms of Underlying Processes

Dynamic Mean

$$y_t = x_t + \gamma_t$$

$$x_t = x_{t-1} + v_t \tag{31}$$

$$E(\gamma_t) = E(v_t) = o$$

$$\text{Var } \gamma_t = r_t^2$$

$$\text{Var } v_t = q_t^2$$

y_t = observed value of process at time (qtr) t

x_t = mean of process at time t

γ_t = additive noise random variable with variance r_t^2

v_t = additive random change in mean x from time t-1 to time t. Variance is q_t^2

This model is sufficiently complex to explain short term trends in a time series. Its mean is non-stationary in that it changes from period to period. Moving averages and single exponential smoothing work well on this process.

Linear Growth

$$y_t = x_t + \eta_t$$

$$x_t = x_{t-1} + \beta_t + v_t \tag{311}$$

$$\beta_t = \beta_{t-1} + \delta_t$$

$$E(\delta_t) = o \quad \text{Var } \delta_t = p_t^2$$

β_t = incremental growth in mean of process at time t

δ_t = random change in growth term

Other definitions as above.

This model allows for linear growth over time of the process mean. Its forecast algorithm is a general version of double exponential smoothing. Linear regression over time would do well.

Dynamic Proportion

$$z_t = u_t \cdot \rho_t \ , \ \rho_t \geq o$$

$$u_t = u_{t-1} \, \omega_t \ , \ \omega_t \geq o \tag{3111}$$

$$E(\rho_t) = E(\omega_t) = 1$$

$$\text{Var } \rho_t = \exp (r_t^2) - 1$$

$$\text{Var } \omega_t = \exp (q_t^2) - 1$$

z_t = observed value of process at time t

u_t = mean of process at time t

ρ_t = multiplicative noise random variable

ω_t = multiplicative random change or "percentage" change in mean u from t-1 to t.

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This model is useful for avoiding theoretically possible negative values as in (3i). Random changes can be regarded as percentages. With the variances expressed as in (3iii) we may make the transformations[1],

$$x_t = \log(u_t) - 1/2 \; r_t^2$$

$$y_t = \log(z_t) \tag{3iv}$$

and thereby use system (3ii) with

$$\beta_t = -1/2 \; q_t^2 \quad , \quad \text{Var } \delta_t = 0$$

3.1.2 Processes Utilized in Structural Forms

The time series $\{D_t/H_t\}$ and $\{D_t\}$ are natural candidates for investigation in the three structural forms. The former will yield forecast algorithms for a demand per FH rate which in turn can be used to predict future demand based on projected FH. Algorithms for the latter process will also be developed here and for comparison purposes be applied to the current peculiar item data base, but their real potential will be realized when forecasting common items or where the use of a program (FH) factor is not feasible.

In (3i)   let $x_t = E(D_t)$,   then $y_t = D_t$

In (3i)   let $x_t = E(D_t/H_t)$ then $y_t = D_t/H_t$

In (3iii) let $u_t = E(D_t)$ then $y_t = \log D_t$

In (3iii) let $u_t = E(D_t/H_t)$ then $y_t = \log(D_t/H_t)$

It is now apparent that past history of four time series $\{D_t\}$, $\{D_t/H_t\}$, $\{\log D_t\}$, $\{\log D_t/H_t\}$ may be used in algorithms to forecast their upcoming values. Appropriate transformations will then yield forecasts for demand.

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[1] See Reference [3]. u, ρ, ω are assumed log-normally distributed.

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There are many combinations as we shall see and the investigation becomes quite comprehensive.

3.2 Forecast Algorithms

The following sections describe each algorithm tested, relating it to a model. As applied to the four time series in Section 3.1.2, the algorithms forecast a value y over a given lead time where y represents D, D/H, log D, log D/H. Initialization procedures are described in Chapter IV. The theory for the development of these procedures is given in Orr [13].

Each section has designated abbreviations for referring to tabulated results in Chapter IV. The underlying model is also noted.

For each of the three model structures in Section 3.1, there is an optimal algorithm, which minimizes mean square error of forecast of a future period. These algorithms are Kalman filters and are designated as such in the following sub-sections. Sub-optimal[*] algorithms are also described; exponential smoothing is seen to be a special case of Kalman filtering; moving average algorithms are in a separate class but are particularly suited for dynamic mean models, with the base period parameter related to the Kalman filter parameters.

3.2.1 Kalman Filter - 1st Order

(i) Designator - <u>KAL-1</u>   Model - Dynamic Mean

$$\hat{y}_n(\ell) = \hat{x}_n \tag{1}$$

$$\hat{\dot{x}}_n = \hat{x}_{n-1} + G_n \cdot (y_n - \hat{x}_{n-1}) \tag{2}$$

$$G_{n+1} = \frac{q_n^2 + r_n^2 \, G_n}{q_n^2 + r_n^2 \, G_n + r_{n+1}^2} \tag{3}$$

where

y_n = observed value in period (QTR) n

---

[*] In our context, sub-optimal refers to methods which also can "fit" the model generated data and use the process parameters.

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$$\hat{x}_n = \text{estimate of mean of process at end of period } n$$

$$\hat{y}_n(\ell) = \text{forecast at end of period } n \text{ of the process value } \ell$$
$$\text{periods later}$$

$$G_n = \text{variable weight, "smoothing parameter"}$$

$$q_n^2, r_n^2 \quad : \text{as defined in (31)}$$

(ii) Exponential Smoothing
        Designator – EXPSM – a sub-optimal algorithm for
            Dynamic Mean

Let G_n be a constant G. Then (2) is exponential
smoothing relation.

It is seen in Section (3.3) that an appropriate G
for a corresponding moving average of base B is

$$G = \frac{\sqrt{1 + 4c} - 1}{2c} \tag{4}$$

$$\text{with } c = (2B^2 - 1)/6 \tag{5}$$

(iii) Moving Averages with Fixed or Variable Base Lengths B
        Designator – MA4, MA8, MA12 Model – Dynamic Mean
                    (Denotes # Qtrs)

$$\hat{y}_n(\ell) = \hat{x}_n \tag{6}$$

$$\hat{x}_n = \frac{1}{B} \sum_{j=1}^{B} y_{n-j+1}, \; B = 4, 8, \text{ or } 12 \tag{7}$$

Designator – MAKB

as in (6) and (7) with

$$B = \sqrt{(1+6k)/2} \tag{8}$$

where k is forecast parameter discussed in Section 3.3.
With this B, a moving average is suboptimal.

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3.2.2 Modified KAL-1 Based on Cohen's Results

(1) Designator - $\underline{KAL1-H^2}$ Model - Dynamic Mean for

$$y = D/H$$

In Cohen's investigation [4], the best algorithm tested was basically REG8 described below. We can postulate a model for which this REG8 is a suboptimal algorithm.

$$D_t = a_t \, H_t + \gamma_t' \tag{9'}$$

$$a_t = a_{t-1} + \gamma_t \tag{10}$$

Dividing (9') by H_t we obtain

$$D_t/H_t = a_t + \gamma_t \tag{9}$$

where equations (9) and (10) are in Dynamic Mean form (31) and where
$$\text{Var } \gamma_t = 1/H_t^2 \, \text{Var } \gamma_t' \tag{11}$$

We assume a constant k (see Section 3.3.1) defined by

$$\frac{\text{Var } \gamma_t}{\text{Var } \gamma_t} = r_t^2/q_t^2 \equiv k \tag{12}$$

Assuming Var γ_t' not dependent on H_t (homoscedasticity),

$$r_t^2 \, H_t^2 = \text{constant} = r_{t+1}^2 \, H_{t+1}^2 \tag{13}$$

Therefore equation (3) becomes (using subscript n now to denote algorithm iterations)

$$G_{n+1} = \frac{q_n^2 + r_n^2 \, G_n}{q_n^2 + r_n^2 \, G_n + r_n^2 \, H_n^2/H_{n+1}^2}$$

$$= \frac{1 + k \, G_n}{1 + k \, G_n + k \, H_n^2/H_{n+1}^2} \tag{14}$$

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and as before in Section 3.2.1

$$\hat{y}_n(\ell) = \hat{x}_n \qquad (15)$$

$$\hat{x}_n = \hat{x}_{n-1} + G_n \cdot (y_n - \hat{x}_{n-1}) \qquad (16)$$

Note that, as with exponential smoothing, (16) gives less and less
weight to older quarters. On the other hand (14) indicates that quarters
with relatively high FH are given more weight; if $H_{n+1} \gg H_n$ then
$G_{n+1} \to 1$ and $\hat{x}_{n+1} \sim y_{n+1}$

(ii) Regression Technique from Cohen [4]

Designator – <u>REG8</u>   Model – Dynamic Mean (9),(10)

Use only with y = D/H

$$\hat{y}_n(\ell) = \hat{x}_n \qquad (17)$$

$$\hat{x}_n = \sum_{j=1}^{8} y_{n-j+1} \omega_{n-j+1} \qquad (18)$$

where weight ω_{n-j} is given,

$$\omega_{n-j} = H_{n-j}^2 / \sum_{i=1}^{8} H_{n-i+1}^2 \qquad (19)$$

Designator – <u>REGKB</u>

Use B given by (8) in place of 8 Qtrs

Equation (18) is in the form of a weighted moving average. Cohen's
algorithm written in this way demonstrates its appropriateness for forecast
model (9),(10). According to theory, the weights should be inversely
proportional to the variance of the process variables y_i when forming
minimum variance estimators of the type (18). REGKB is an obvious
modification, to allow the base to vary.

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3.2.3 Kalman Filter – 2nd Order

Designator – <u>KAL 2 – GEN</u>   Model – Linear Growth

$$\hat{y}_n(\ell) = \hat{x}_n + \ell\beta_n \tag{20}$$

$$\hat{x}_n = \hat{x}_{n-1} + \hat{\beta}_{n-1} + G_n \cdot (y_n - \hat{y}_{n-1}(1)) \tag{21}$$

$$\hat{\beta}_n = \hat{\beta}_{n-1} + H_n \cdot (y_n - \hat{y}_{n-1}(1)) \tag{22}$$

G_n changes as in (3)

$$H_n \approx \sqrt{(1-G_n)/(100\ r_n^2/q_n^2)} \tag{23}$$

H_n is an approximation assuming p_n^2 in system (3ii) is small.

(21) and (22) are again the basic filter where variable weights G_n, H_n are applied to the one-step-ahead error to obtain adjustments to the previous estimates of the process level and growth means, x and β

3.2.4 Modified KAL 2 for Log Series

Designator – <u>KAL 2 – BQ</u>   Model – Dynamic Proportion

Use (20), (21), (22), (23) with

$$H_n = o \tag{24}$$

$$\hat{\beta}_n = -1/2\ q_n^2 \tag{25}$$

This algorithm is used with log D or log D/H series of values, by which (3iii) is transformed to (3ii) thru (3iv).

3.3 k-Factors

An important parameter of both the models and the algorithm is the k-factor.

$$k = \frac{\text{Variance of noise in process}}{\text{Variance of random change in process mean}}$$

$$= r_n^2/q_n^2 \tag{26}$$

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q_n^2 is associated with short term correlations in changes in the mean. k is assumed constant in all of the models.

If in addition r_n^2 does not vary by period, i.e.

$$r_n^2 = r^2 \tag{27}$$

then (3) becomes

$$G_{n+1} = \frac{1 + k\, G_n}{1 + (G_n + 1)k} \tag{28}$$

G_n in (28) approaches a limit $G = \dfrac{\sqrt{1 + 4k} - 1}{2k}$

Orr [/3] shows that given a dynamic mean model with parameter k, the "best" (in the sense of minimizing mean square error of ℓ-period ahead forecasts) moving average algorithm should use B periods (quarters) as its base with B given by (8).

k is an indicator of how stationary the process is; high k values imply relatively small changes in the process mean and more reliance should be put on past history for forecasting; low k indicates changes in the mean, short term trends, and relatively low observation noise, and more weight should be put on recent observations (note (28)).

There are several ways of obtaining estimates of k. Orr [/3] obtains formulas for k using mean square errors of moving average forecasts. Average values of k for items falling in cells of various stratifications were obtained in this study. A stratification by yearly requisitions showed the most definite patterns and the average values of k are tabulated in Chapter IV for the four time series. These tabulated results are used to update k every year in the above algorithm in cases where an item migrates from one requisition class to another. In a sense the algorithms have now become dependent on a catalog parameter (average k) derived for groups of items which is updated yearly.

k is quite valuable in determining the parameters for suboptimal algorithms. If one is constrained to use a MA or exponential smoothing algorithm rather than a Kalman filter, equations (4), (5), and (8) give all the necessary relations among G, B, k and are a rigorous alternative to Brown's [3] $G \sim \dfrac{2}{B+1}$

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CHAPTER IV

ERROR ANALYSIS

4.1 Computer Program to Gather Error Statistics

The program, though long, is conceptually quite simple. Only the forecasting subroutine is changed in a given run. Different error measures (Section 4.2) are averaged over a time horizon by item and stored on a tape by item for subsequent stratifying procedures. A preliminary output gives the error measures averaged over items in two strats:

AYD x UP   :   0 - $5000, $5000 - 50000, $50,000 & up

AYF (req) :   0 - 3, 4-12, 13 & up

Significant Logic:

1. 735 items deleted for zero FH in last quarter.
2. 9 items deleted with an absolute error larger than 2000 in any quarter, using moving average of 8 quarters.
3. Estimates of AYD, AYF are averages over the time horizon of the 8 quarter moving averages.
4. Time horizon goes from quarter ID to qtr 28 where ID is the first non-zero FH qtr for the item.
5. Forecasts start in 8th qtr after ID (i.e. 2 year warmup)
6. Error statistics accumulate in 12th qtr after ID.

4.2 Error Measures for One Item

Described in the following subsections are all the error statistics accumulated on demand forecast error. Note

$$\hat{D}_{\tau}(\ell) = \text{forecast at quarter } \tau \text{ of demand in quarter } \tau + \ell$$

So if process forecast $\hat{y}_{\tau}(\ell)$ is for the observed variable D/H then $\hat{D}_{\tau}(\ell) = \hat{y}_{\tau}(\ell) \cdot H_{\tau,\ell}$ where $H_{\tau,\ell}$ is the projected[2] FH in period $\tau + \ell$

These measures are averaged over items in particular stratification schemes.

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[2] Study used actual FH for "future" periods, but these did not differ significantly from projections, which are really target programs.

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Mean Absolute Deviation

This error measure and the following are computed for one quarter MAD_1, and four quarters MAD. All un-subscripted measures refer to yearly values.

$$MAD_1 = \frac{1}{T} \sum_{\tau}^{\tau+T} \left| D_{\tau+1} - \hat{D}_\tau(1) \right| \qquad (1)$$

$$MAD = \frac{1}{T} \sum_{\tau}^{\tau+T} \left| \sum_{\ell=1}^{4} (D_{\tau+\ell} - \hat{D}_\tau(\ell)) \right| \qquad (2)$$

Mean Square Error

MSE_1 : replace $|\cdot|$ by $(\cdot)^2$ in (1)

MSE : replace $|\cdot|$ by $(\cdot)^2$ in (2)

Error Bias

$Error_1$: replace $|\cdot|$ by $(\cdot)$ in (1)

$Error$: replace $\cdot$ by $(\cdot)$ in (2)

Absolute Error Over Forecast F

$|\varepsilon_1|/F_1$: replace $|\cdot|$ by $|\cdot|/\hat{D}_\tau(1)$ in (1)

$|\varepsilon|/F$: replace $|\cdot|$ by $|\cdot|/\sum_{\ell=1}^{4} \hat{D}_\tau(\ell)$ in (2)

Absolute Error Over Actual Demand A

$|\varepsilon_1|/A_1$: replace $|\cdot|$ by $|\cdot|/\hat{D}_{\tau+1}$ in (1)

$|\varepsilon|/A$: replace $|\cdot|$ by $|\cdot|/\sum_{\ell=1}^{4} D_{\tau+\ell}$ in (2)

Absolute Error Over Average of Demand and Forecast

$\left. \begin{array}{l} |\varepsilon_1|/\; 1/2\; (A_1+F_1) \\ \\ |\varepsilon|/\; 1/2\; (A+F) \end{array} \right\}$ obvious

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Relative MAD

$$\text{MAD}/\text{AYD}$$

Relative MSE

$$\text{MSE}/(\text{AYD})^2$$

In the final tabulation of the algorithms, we looked at error
measures of one year forecasts since this corresponds to representative
lead times. $|\varepsilon|/F$ and $|\varepsilon|/A$ have built-in disadvantages when F or A
equal 0 or F,A are large. MSE is more sensitive to large error than MAD.
Final four measures selected are:

$$\text{MAD}, \ \text{MAD}/\text{AYD}, \ \text{MSE}/(\text{AYD})^2, \ |\bar{\varepsilon}|/1/2(\text{A+F})$$

The last three are relative measures, necessary when combining items with
different demands. The MAD is useful for low demand items.

Before presenting the tabulated results (Section 4.5), tabulated
values on the k-factor are presented.

4.3 k-factor Tables

Methodology is presented in Orr $(/3)$. For log D and log D/H, both k
and q are obtained since $\hat{\beta} = -1/2 \ q^2$ is needed (equation (25) Chapter III).

Stratifications using the following variables were investigated –

requisitions per year	req
average yearly demand	AYD
1/ Unit Price	1/UP
dollar demand	AYD · UP
average order size	AYD/req

req and AYD · UP give similar patterns by strat cell which are explainable.
Other stratifications did not yield strong patterns. Table 4.1 gives the
k-values by item requisition for the four processes, D, log D, D/H,
log D/H.

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TABLE 4.1　k-FACTORS BY REQUISITION CLASS

Cell	Upper Bound	# Items	Avg Reg	D		log D			D/H		log D/H		
				k	MA Qtrs	q	k	MA Qtrs	k	MA Qtrs	q	k	MA Qtrs
1	1	1489	.59	0	1	.16	3.69	3	0	1	.34	14.56	6
2	2	1354	1.52	3.164	3	.20	6.77	4	7.34	5	.38	31.59	8
3	3	1176	2.50	4.251	4	.22	8.12	5	14.18	7	.36	48.13	9
4	4	873	3.49	4.399	4	.26	8.18	5	20.79	8	.38	52.71	9
5	5	636	4.49	4.71	4	.26	9.75	5	31.19	10	.38	54.71	10
6	6	513	5.51	3.464	3	.30	7.67	4	28.31	10	.46	54.7	10
7	8	768	6.95	3.864	3	.34	6.87	4	75.9	15	.40	54.7	10
8	12	943	9.74	3.674	3	.36	7.23	4	∞	∞	.42	50.16	10
9	18	747	14.68	3.120	3	.42	5.43	3	∞	∞	.38	58.88	10
10	∞	1204	35.99	2.022	3	.44	3.96	3	∞	∞	.34	77.0	12

For D, D/H:　MA Qtrs found from $B = \sqrt{\dfrac{1+6k}{2}}$

For log D, log D/H:　MA Qtrs found by search to minimize (A.7)

$V_{q,\beta}$ error in forecasting process value

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Analysis

Remember r^2 is process variance around a mean and q^2 is associated with short term correlation in changes in the process mean. High k reflects stationary processes; low k is associated with short term trends.

k values for D/H and log D/H processes increase with increasing activity (req). This is because demand becomes more correlated with FH as activity rises. The mean of the D/H rate becomes more stationary (higher k). Also the k values are higher than those for the corresponding D and log D series since these processes are more volatile and reflect trends in FH.

Note that for D and log D series that k increases, then decreases with req. For low # of requisitions, q^2 is high (relative to r^2) indicating that demands tend to come in correlated "bunches". For high requisition activity, these demand series show trends due to changes in FH. Hence k is somewhat lower at top and bottom of these columns than in the middle.

The behavior of q under log transformations (see columns) is not fully understood.

4.4 Forecast Algorithms - Initialization and k-Updating in Computer Program

Equations for KAL-1 in (2) and (3) Chapter III are started up using

$$\hat{x}_o = \mu + G_o \ (y_o - \mu) \tag{3}$$

$$G_o = \frac{\tau^2}{\tau^2 + r_o^2} \tag{4}$$

where

y_o = initial observed value of process

μ = mean of a prior distribution on X

τ^2 = variance of a prior distribution on X

r_o^2 = variance of estimate y_o

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In lieu of a catalog approach, using statistics on groups of items to get μ, τ^2, which was not used (see Chapter VII), the 8 qtr warmup period was used. μ was obtained from an 8 qtr average and y_0 from the last 4 qtrs. Assuming some constant process variance h^2 in warmup, $\tau^2 \propto h^2/8$ and $r_0^2 \propto h^2/4$. Therefore G_0 was assigned a weight of 1/3.

For KAL-2 , $\hat{\beta}_0 = 0$

For moving average algorithms, the initial $\hat{x}_0$ is obtained from the previous B quarters.

Every 4 qtrs, starting with 8th qtr, k or B is updated by a table lookup for the appropriate process based on the current 8 qtr moving average estimate of the yearly requisitions.

4.5 <u>Tabulated Results on Error Measures</u>

The following 16 tables present the average values of the four error measures - MAD/AYD, MSE/(AYD)^2, MAD, $|\mathcal{E}|/1/2$ (A+F) - for forecast techniques applied to the four time series, $\{D\}$, $\{D/H\}$, $\{\log D\}$, $\{\log D/H\}$. The results were obtained using the same stratification (on average requisitions which gave the final k-values. This natural consistency in performing stratifications allows one to observe how error measure values vary as k-factors change, and indicates how implemented forecast procedures which may vary by requisition class would perform.

Refer to Section 3.2 for a description of the designators. Not all designators - time series - error measure combinations are included. Some forecast techniques were eliminated due to preliminary runs with unpromising results; as the experimental design evolved some branches of the combinatorial tree were not climbed to a great extent; not all of the modified algorithms based on Cohen's results (see Section 3.2.2) were run, since all of these are tested in the simulator.

ALGORITHM

Cell	# Items	Avg. Reqs/yr	MA4	MAKB	KAL1	KAL2-GEN	EXPSM G: MA4		
1	1489	.59	1.402	1.291	1.757	4.832	2.764		
2	1354	1.52	1.337	1.418	1.422	1.608	1.375		
3	1176	2.50	1.148	1.185	1.177	1.262	1.136		
4	873	3.49	1.064	1.071	1.067	1.141	1.052		
5	636	4.49	.976	.978	.960	1.017	.944		
6	513	5.51	.965	.950	.929	.994	.944		
7	768	6.95	.913	.898	.869	.918	.885		
8	943	9.74	.850	.838	.802	.839	.825		
9	747	14.68	.774	.755	.717	.742	.753		
10	1204	35.99	.669	.641	.610	.623	.653		
	9703	TOTAL	1.045	1.036	1.089	1.599	1.235		

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ALGORITHM

Cell	# Items	Avg. Reqs/yr	MA4	MA8	MA12	MAKB	KAL1	REG KB	REG8
1	1489	.59	1.134	3.764	10.265	3.529	3.682	2.705	3.293
2	1354	1.52	1.088	1.382	1.946	1.629	1.614	1.386	1.217
3	1176	2.50	.892	.960	1.201	1.082	1.093	1.005	.934
4	873	3.49	.875	.988	1.174	1.102	1.091	.955	.886
5	636	4.49	.782	.806	.912	.912	.873	.845	.789
6	513	5.51	.768	.836	.931	.928	.880	.850	.786
7	768	6.95	.723	.745	.831	.879	.792	.781	.727
8	943	9.74	.658	.671	.734	.776	.687	.720	.663
9	747	14.68	.593	.616	.663	.701	.621	.648	.610
10	1204	35.99	.479	.491	.524	.563	.493	.515	.482
	9703	TOTAL	.829	1.285	2.392	1.351	1.341	1.149	1.175

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ERROR MEASURE: MAD/AVD

ALGORITHM

Cell	# Items	Avg. Reqs/yr	MA4	MAKB	KAL1	KAL2-BQ	KAL2-GEN		
1	1489	.59	1.219	1.428	8.659	7.991	7.918		
2	1354	1.52	1.242	1.325	1.808	1.605	1.733		
3	1176	2.50	1.041	1.073	1.144	.999	1.135		
4	873	3.49	.889	.891	.899	.778	.905		
5	636	4.49	.793	.789	.717	.672*	.788		
6	513	5.51	.758	.749	.717	.627*	.742		
7	768	6.95	.706	.702	.660	.588*	.690		
8	943	9.74	.667	.669	.622	.569*	.660		
9	747	14.68	.662	.630	.581	.533*	.623		
10	1204	35.99	.550	.551	.515	.451*	.570		
	9703	TOTAL	.892	.939	2.076	1.886	1.975		

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ALGORITHM

Cell	# Items	Avg. Reqs/yr	MA4	MA8	MAKB	KAL1	KAL2-BQ		
1	1489	.59	.551	.748	.773	.770	.547		
2	1354	1.52	.736	.708	.710	.694	.694*		
3	1176	2.50	.747	.731	.725	.714*	.749		
4	873	3.49	.744	.724	.721	.709*	.754		
5	636	4.49	.728	.722	.720	.710	.774		
6	513	5.51	.707	.698	.690	.674	.743		
7	768	6.95	.676	.674	.678	.661	.746		
8	943	9.74	.643	.647	.656	.642	.738		
9	747	14.68	.575	.588	.607	.587	.698		
10	1204	35.99	.461	.474	.499	.482	.642		
	9703	TOTAL	.646	.671	.679	.666	.693		

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ERROR MEASURE: MAD/AYD

ALGORITHM

Cell	# Items	Avg. Reqs/yr	MA4	MAKB	KAL1	KAL2-GEN	EXPSM G: MA4		
1	1489	.59	12.967	12.940	33.872	184.346	81.278		
2	1354	1.52	4.127	5.081	4.917	6.093	4.078		
3	1176	2.50	2.661	2.997	2.896	3.266	2.432		
4	873	3.49	2.288	2.498	2.362	2.653	2.072		
5	636	4.49	1.889	2.089	1.886	2.092	1.640		
6	513	5.51	1.888	1.904	1.764	1.970	1.718		
7	768	6.95	1.680	1.732	1.545	1.713	1.478		
8	943	9.74	1.402	1.436	1.269	1.417	1.246		
9	747	14.68	1.142	1.130	.997	1.107	1.011		
10	1204	35.99	.874	.841	.728	.802	.761		
	9703	TOTAL	3.818	4.034	6.878	28.427	13.295		

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$MSE/(AYD)^2$

ALGORITHM

Cell	# Items	Avg. Reqs/yr	MA4	MA8	MA12	MAKB	KAL1	REG KB	REG8
1	1489	.59	10.309	519.026	3093.089	481.246	437.013	155.219	365.904
2	1354	1.52	4.983	29.499	44.812	31.073	19.281	7.051	5.446
3	1176	2.50	1.711	3.126	7.192	5.230	3.223	3.172	2.580
4	873	3.49	3.152	7.404	8.955	5.724	5.784	2.331	1.880
5	636	4.49	1.278	1.760	2.691	2.873	1.897	1.886	1.438
6	513	5.51	1.592	2.685	3.261	3.241	2.335	2.239	1.641
7	768	6.95	1.170	1.397	2.489	2.801	1.60.	1.461	1.174
8	943	9.74	.935	1.112	1.579	1.899	1.144	1.270	1.001
9	747	14.68	.757	.864	1.167	1.322	.864	.925	.818
10	1204	35.99	.523	.563	.784	1.015	.590	.624	.532
	9703	TOTAL	3.295	78.465	443.859	76.630	65.714	24.423	53.634

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SERIES: D/H·   ERROR MEASURE: $MSE/(AYD)^2$

ALGORITHM

Cell	# Items	Avg. Reqs/yr	MA4	MAKB	KAL1	KAL2-BQ	KAL2-GEN		
1	1489	.59	13.776	23.377	269.750	231.188	228.724		
2	1354	1.52	3.440	3.684	4.572	3.567	4.327		
3	1176	2.50	1.983	2.004	1.916	1.465	1.969		
4	873	3.49	1.466	1.447	1.296	1.006*	1.384		
5	636	4.49	1.133	1.120	.969	.776*	1.068		
6	513	5.51	1.085	1.072	.896	.716*	1.016		
7	768	6.95	.990	1.006	.834	.706*	.960		
8	943	9.74	.881	.917	.757	.660*	.881		
9	747	14.68	.747	.790	.655	.561*	.793		
10	1204	35.99	.548	.608	.504	.398*	.662		
	9703	TOTAL	3.367	4.899	41.915	35.842	35.801		

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ALGORITHM

Cell	# Items	Avg. Reqs/yr	MA4	MA8	MAKB	KAL1	KAL2 - BQ		
1	1489	.59	3.400	5.893	7.273	4.244	3.196*		
2	1354	1.52	1.538	1.422	1.435	1.351*	1.416		
3	1176	2.50	1.319	1.278	1.262	1.220*	1.321		
4	873	3.49	1.279	1.205	1.197	1.145	1.247		
5	636	4.49	1.070	1.064	1.053	1.027	1.162		
6	513	5.51	1.099	1.038	1.002	.943	1.078		
7	768	6.95	.963	.952	.980	.928	1.086		
8	943	9.74	.845	.858	.893	.844	1.020		
9	747	14.68	.670	.708	.746	.693	.878		
10	1204	35.99	.445	.465	.514	.471	.700		
	9703	TOTAL	1.387	1.719	1.926	1.447	1.408		

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SERIES: log D/H   ERROR MEASURE: $MSE/(AYD)^2$

ALGORITHM

Cell	# Items	Avg. Reqs/yr	MA4	MAKB	KAL1	KAL2-GEN	EXPSM G: MA4		
1	1489	.59	3.267	3.007	3.328	5.436	4.102		
2	1354	1.52	7.588	7.844	7.873	8.711	7.690		
3	1176	2.50	11.105	11.058	10.942	11.627	10.845		
4	873	3.49	16.020	16.508	16.362	17.344	15.864		
5	636	4.49	20.784	20.725	19.860	21.176	19.761		
6	513	5.51	19.384	19.065	18.869	20.457	19.206		
7	768	6.95	30.241	29.976	29.130	31.050	29.295		
8	943	9.74	42.792	42.287	40.320	42.356	41.225		
9	747	14.68	64.763	63.639	60.641	63.696	63.102		
10	1204	35.99	156.807	150.811	143.636	147.556	152.131		
	9703	TOTAL	38.668	37.795	36.326	38.208	37.674		

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SERIES   ERROR MEASURE: MAD

ALGORITHM

Cell	# Items	Avg. Reqs/yr	MA4	MA8	MA12	MAKB	KAL1	REG KB	REG8
1	1489	.59	2.548	4.693	9.003	5.068	5.001	4.071	4.184
2	1354	1.52	5.960	6.909	8.819	7.773	7.916	7.145	6.530
3	1176	2.50	8.849	9.299	10.733	9.666	9.834	9.189	9.252
4	873	3.49	13.117	13.949	15.509	15.163	15.122	13.872	13.137
5	636	4.49	16.983	16.814	18.026	18.211	17.921	17.351	16.542
6	513	5.51	15.634	17.281	19.029	18.771	18.192	17.580	16.283
7	768	6.95	23.936	24.525	26.510	27.717	26.233	25.036	24.046
8	943	9.74	33.320	33.050	34.758	35.716	33.162	33.942	32.708
9	747	14.68	49.848	51.325	54.903	57.288	51.926	53.562	51.334
10	1204	35.99	112.445	112.545	117.403	122.116	112.024	114.163	109.144
	9703	TOTAL	29.218	29.899	32.414	32.511	30.558	30.338	29.093

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SERIES: D/H   ERROR MEASURE: MAD

ALGORITHM

Cell	# Items	Avg. Reqs/yr	MA4	MAKB	KAL1	KAL2-BQ	KAL2-GEN		
1	1489	.59	2.072	2.163	4.960	4.560	4.608		
2	1354	1.52	5.400	5.599	6.509	5.919	6.401		
3	1176	2.50	8.211	8.299	8.399	7.693	8.491		
4	873	3.49	11.559	11.575	11.388	10.555	11.837		
5	636	.4.49	15.440	15.353	14.629	13.739*	15.282		
6	513	5.51	14.522	14.410	13.850	12.821*	14.298		
7	768	6.95	21.672	21.627	20.183	18.863*	21.448		
8	943	9.74	32.032	32.424	30.039	28.652*	32.108		
9	747	14.68	51.838	52.502	48.739	46.155*	53.226		
10	1204	35.99	129.535	129.478	120.449	106.202*	138.863		
	9703	TOTAL	30.635	30.772	29.384	26.766	32.816		

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SERIES: log D   ERROR MEASURE: MAD

ALGORITHM

Cell	# Items	Avg. Reqs/yr	MA4	MA8	MAKB	KAL1	KAL2-BQ		
1	1489	.59	1.180	1.231	1.239	1.199	1.059*		
2	1354	1.52	3.881	3.775	3.785	3.720	3.692*		
3	1176	2.50	6.896	6.795	6.745	6.671*	6.827		
4	873	3.49	10.564	10.606	10.618	10.454*	10.819		
5	636	4.49	15.156	14.686	14.503	14.515	15.175		
6	513	5.51	14.266	14.370	14.313	14.033	14.948		
7	768	6.95	21.088	20.882	20.982	20.585	22.243		
8	943	9.74	30.912	31.116	31.367	30.925	34.257		
9	747	14.68	48.513	50.029	51.481	50.269	57.534		
10	1204	35.99	108.579	110.350	116.950	113.580	153.826		
	9703	TOTAL	26.990	27.261	23.308	27.665	34.075		

SERIES: log D/H   ERROR MEASURE: MAD

ALGORITHM

Cell	# Items	Avg. Reqs/yr	MA4	MAKB	KAL1	EXPSM G: MA4			
1	1489	.59	.539	.463	.591	.893			
2	1354	1.52	.966	.964	1.051	1.101			
3	1176	2.50	1.013	1.025	1.060	1.064			
4	873	3.49	.995	.996	1.017	1.025			
5	636	4.49	.963	.962	.958	.957			
6	513	5.51	.956	.953	.944	.956			
7	768	6.95	.913	.911	897	.910			
8	943	9.74	.876	.869	.849	.869			
9	747	14.68	.816	.802	.783	.814			
10	1204	35.99	.719	.697	.684	.725			
	9703	TOTAL	.853	.838	.869	.933			

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SERIES: D   ERROR MEASURE: $|\varepsilon|/1/2(A+F)$

ALGORITHM

Cell	# Items	Avg. Reqs/yr	MA4	MA8	MA12	MAKB	KAL1	REG KB	REG8
1	1489	.59	.498	.726	.966	.577	.633	.577	.728
2	1354	1.52	.906	1.013	1.073	.994	1.030	1.000	1.030
3	1176	2.50	.956*	.989	1.015	.992	1.012	.998	1.004
4	873	3.49	.959*	.983	.998	.989	.993	.995	1.003
5	636	4.49	.927	.928	.952	.932	.922*	.946	.951
6	513	5.51	.903	.903	.920	.908	.894*	.922	.921
7	768	6.95	.869	.864	.872	.885	.852*	.895	.885
8	943	9.74	.821	.818	.836	.838	.791*	.851	.838
9	747	14.68	.745	.751	.766	.782	.731*	.789	.774
10	1204	35.99	.612	.618	.639	.657	.608*	.658	.630
	9703	TOTAL	.796	.852	.910	.840	.837	.846	.868

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SERIES: D/H   ERROR MEASURE: $|\varepsilon|_{1/2}$ (A+F)

ALGORITHM

Cell	# Items	Avg. Reqs/yr	MA4	MAKB	KAL1				
1	1489	.59	.406	.429	1.538				
2	1354	1.52	.890	.944	1.342				
3	1176	2.50	1.018	1.058	1.186				
4	873	3.49	1.017	1.023	1.090				
5	636	4.49	.973	.972	.990				
6	513	5.51	.954	.951	.964				
7	768	6.95	.708	.905	.894				
8	943	9.74	.860	.857	.845				
9	747	14.68	.803	.807	.783				
10	1204	35.99	.695	.690	.677				
	9703	TOTAL	.820	.836	1.074				

SERIES: loz D   ERROR MEASURE: $|\varepsilon|/1/2(A+F)$

ALGORITHM

Cell	# Items	Avg. Reqs/yr	MA4	MA8	MAKB	KAL1			
1	1489	.59	.388*	.485	.475	.556			
2	1354	1.52	.872*	.920	.910	.917			
3	1176	2.50	1.047	1.088	1.078	1.069			
4	873	3.49	1.121	1.136	1.131	1.119			
5	636	4.49	1.140	1.163	1.165	1.151			
6	513	5.51	1.103	1.117	1.114	1.101			
7	768	6.95	1.071	1.091	1.095	1.073			
8	943	9.74	1.006	1.031	1.049	1.031			
9	747	14.68	.881	.913	.951	.926			
10	1204	35.99	.688	.720	.776	.753			
	9703	TOTAL	.880	.922	.930	.931			

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SERIES: log D/H   ERROR MEASURE: |E(1)/2(ALE)|

4.6 Analyses of Relative Performances and Trends in Tables

Trends in a Column

For the MAD measure, values increase as activity (reqs) increases, since the demand is growing larger and hence errors increase. For the other three relative error measures, the general tendency is for column values to decrease with activity; the denominator tends to increase faster than the error function in the numerator. In some cases (especially note $|\varepsilon|/1/2 \ (A+F))$ there is an initial increase before this general tendency takes over; for the first few cells with very low requisition activity, there are aberrations - frequent zero error with actual and forecast being zero, giving very low average values of the measure - after which increased occurrence of demand spikes amongst the zero periods raises the error measure value for a time.

Relative Performance of Algorithms by Series

D series: Expected results hold across the four measures. For majority of cells, KAL1 is best. Good performance of EXPSM indicates that part of KAL1 performance is due to the iterative formula of exponential smoothing. MAKB does a little better than MA4.

D/H series: Longer base periods did not do well (MA4 best, MA12 worst). The data base has strong impact here since rates based on very low FH in the initial warmup period can give quite inaccurate estimates of the D/H rate for the remainder of the horizon (shorter base periods do not pick up these low FH or do not keep them a. long). KAL1 does better than most of the MA's and is the best for most cells under measure $|\varepsilon|/1/2 \ (A+F)$. REGKB and REG8 do better than KAL1 for three measures and are quite close in performance between themselves.

log D series: MAKB outperforms MA4 in the middle cells but not overall; this is contra theory. KAL1 is generally better than these as expected and KAL2-BQ, which accounts for the theoretical bias due to log transformations, does the best in the majority of cells.

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log D/H series: Explanation is basically same as D/H series.
Log transform has tempered impact of low FH and cell 1 results are not
as horrendous. KAL1 is outperforming the MA's in most of the cells.
KAL2-BQ's performance is disappointing.

Comparative Performance Between Series

D vs D/H: Values for comparable columns indicate smaller values
for D/H series, especially for the higher activity. This is theoretically
expected; more active items are correlated with FH, D/H rates are relatively
stable, k-values are higher, hence forecast errors are smaller.

D vs log D: Except for measure $|\epsilon|/1/2$ (A+F) (the significance
of which shall be seen in Section 4.7 and Chapter V), the error measure
values for log D are smaller than those for D. This is apparently due to
log D forecasts having smaller variance; hence the overall variance of
forecast error may be smaller, although the bias may be significant.

D/H vs log D/H: Same comments as above.

log D vs log D/H: The general observation is that values for
comparable columns are smaller for log D/H for less active items. This
is counter-intuitive; a theoretical explanation is given in Orr [/3].

4.7 Candidates for Further Study

Section 4.6 indicated some general patterns and some particular
comparisons. When analyzing all 16 tables simultaneously to select several
promising candidates, one must be circumspect. The "totals" must not be
too influential, because one or two cells might have had large impact. Cell
1 acts strangely in many cases due to the occurrence of many zero-demand
quarters; anyway the final forecast algorithm would not be utilized for
very inactive items.

The "best" algorithm (I shan't give reasons. One may check tables)
which is not too complex is a hybrid:

$$\begin{array}{lll}
\text{Cells } 1,2,3 & \text{Use KAL2-BQ} - \log \text{D/H} & (4.1)\\
\text{Cells } 4\text{-}10 & \text{Use KAL2-BQ} - \log \text{D} &
\end{array}$$

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Of those involving FH which does not duplicate (4.i) and is not a hybrid

$$KAL1 - \log D/H \qquad\qquad (4.ii)$$

Based on Cohen's [4] results, REG8, and its modification REGKB and the
theoretically modified $KAL-H^2$ will be studied further

$$REG8 \quad - \quad D/H \qquad\qquad (4.iii)$$

$$REGKB \quad - \quad D/H \qquad\qquad (4.iv)$$

$$KAL-H^2 \quad - \quad D/H \qquad\qquad (4.v)$$

Two algorithms not involving FH are chosen. KAL2-BQ-log D is already part
of 4.i. MAKB - log D was chosen over KAL1 - log D since it was a bit more
consistent and not much worse than MA4 - log D, but more interesting.
Finally KAL1 was chosen as the algorithm on the pure $\{D\}$ series

$$MAKB - \log D \qquad\qquad (4.vi)$$

$$KAL1 - D \qquad\qquad (4.vii)$$

The performance of these algorithms in terms of cost-effectiveness
curves based on simulation results are presented in Chapter V.

4.8 Relative Merits of Statistical Error Measures

We wish to have some indication of how well the 4 statistical error
measures rank the algorithms in Section 4.7 as compared to their rankings
by cost performance in the simulator. We use results from the forthcoming
chapter on simulation runs for some 60 active items (HDV-Dynamic group).
To be comparative we obtained average rankings for the last 5 cells in the
appropriate columns of the tables. The results are shown in Table 4.3.

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TABLE 4.3: RANKINGS OF 5 ALGORITHMS BY PERFORMANCE MEASURE

MEASURE	ALGORITHMS				
	REGKB D/H	REG8 D/H	KAL1 D	MAKB log D	KAL1 log D/H
SIMULATOR COST PERFORMANCE	1	2	3	4	5
MAD/AYD	4	2	5	3	1
MSE/AYD^2	4	2	5	3	1
MAD	4	2	5	3	1
$\lvert \bar{c} \rvert /1/2$ (A+F)	2	1	3	4	5

For the 3 measures MAD/AYD, MSE/AYD^2, MAD, we had seen previously that the algorithms on log series had generally lower error measure values than those of D, D/H methods. These 3 measures are sensitive to variance of the forecast, which is lower for "log" algorithms. Since

$$\text{Variance of forecast error} = \text{variance of forecast} + (\text{bias})^2$$

$$+ \text{process variance,}$$

(5)

these "log" types perform well despite bias term. However, simulator cost performance apparently is sensitive to a bias (consistent over or under forecasting).

The algorithms involving D, D/H have higher variance of error (and hence higher MAD, MSE) but lower bias and this is reflected in their simulator performance.

Note that the $\lvert \bar{c} \rvert /1/2$ (A+F) measure "tracks" the simulator rankings fairly well. This is explainable; briefly, the measure is related to a mean square error on the logarithm of forecasts and in calculating variance of the log of the forecast, little inherent advantage is accrued by log series algorithms. However, this measure can obtain values only between 0 and 2; it is less discriminate than the other 3 as can be seen from tables.

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The following summary table summarizes the merits of the error measures.

TABLE 4.4   MERITS OF MEASURES

PERFORMANCE MEASURE	TYPE	USEFULNESS
MAD/AYD $MSE/(AYD)^2$	Forecast Error Statistic	Initial Screen: gives good discrimination of log algorithms amongst themselve or non-log models amongst themselves
MAD	"	As above. Especially useful for inactiv items (cells 1,2)
$\lvert E \rvert/1/2$ (A+F)	"	Initial Screen - qualitative rankings of algorithms correlates with simulator rankings. Suffers from "sameness" of values.
SIM	Cost Performance	Final Screen - useful for several candidates. Tends to be costly.

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CHAPTER V

SIMULATION ANALYSIS

5.1 Simulator Overview & Special Features

The final selections were made by observing the algorithms' performance in Army NICP environment using the DoDI 4140.39 simulator (See Cohen [17]). Subsequent changes to the simulator and a description of its operation is found in Cohen [4].

The results of the simulation runs appear in the next sections in the form of cost-performance curves.
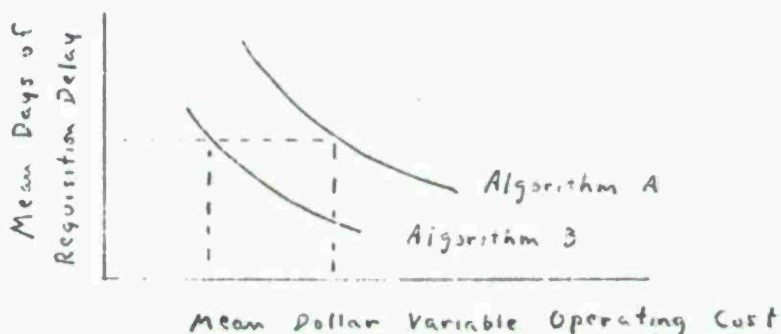
The curves are traced thru several "λ" points for each algorithm. The lambda (λ) values reflect an operating policy, in terms of a marginal cost for time weighted requisitions short, based on budgetary constraints. See Deemer, Kruse [6] for a complete description.

Features of Simulator Operation Differing from Cohen [4] Runs

The algorithms have the same starting conditions prior to accumulating performance statistics. To do this, during the warmup period (2 years) all algorithms utilize the MA8-D algorithm. Of course, also during warmup, the algorithms obtain their various forecast parameter starting values.

At the end of the simulation, excess cost is charged to assets over and above RO assuming projected annual demand obtained from the highest

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forecast among MA4-D, MA8-D, MA12-D predictions. Projected assets re-
maining after this maximal forecast is a conservative estimate of excess.

Forecasting Between Quarters

In many cases policy dictates forecasts at times other than on
the quarter. The algorithms were derived for forecasting periodically
(quarterly) and updating parameters every quarter. The modification for
forecasting between quarters is to use the algorithms basic relation for
x on a "moving" quarter of observations on the process variable y. The
new values are not retained after the forecast, nor are any parameters
updated.

Example: KAL1 at time $n+\triangle$ where $\triangle$ is a fraction of qtr.

$$z = \hat{x}_n + G_n \ (\mathcal{y}_{n+\triangle} - \hat{x}_n)$$

where $\mathcal{y}_{n+\triangle}$ is a "moving" qtr up to time $n+\triangle$. Then forecast
$\hat{y}_{n+\triangle} (\Sigma) = z$. These values are not retained after $n+\triangle$.

5.2 Initial Runs

The seven algorithms of Section 4.6 were programmed in subroutines
and simulation runs were made for various λ values on 60 HDV - dynamic
items. These runs are not too costly because of the few number of items;
but these costly active items give an immediate indication of the impact
of using program factors. The resulting curves are presented in Figure 5.1

One can see that the log type algorithms do not do well on a cost
performance measure. The three D/H algorithms, REG8, REGKB, KAL-H^2 are
clearly superior, indicating that use of FH in forecasting is warranted.
(This agrees with Cohen's results). Hereafter we shall investigate only
the FH algorithms and modifications; the results for D and log D series
will be studied in a future project for forecasting common items.

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HDV - D4r.

HYBRID 4 ?.

KALL - log D/H

MAKB - log D
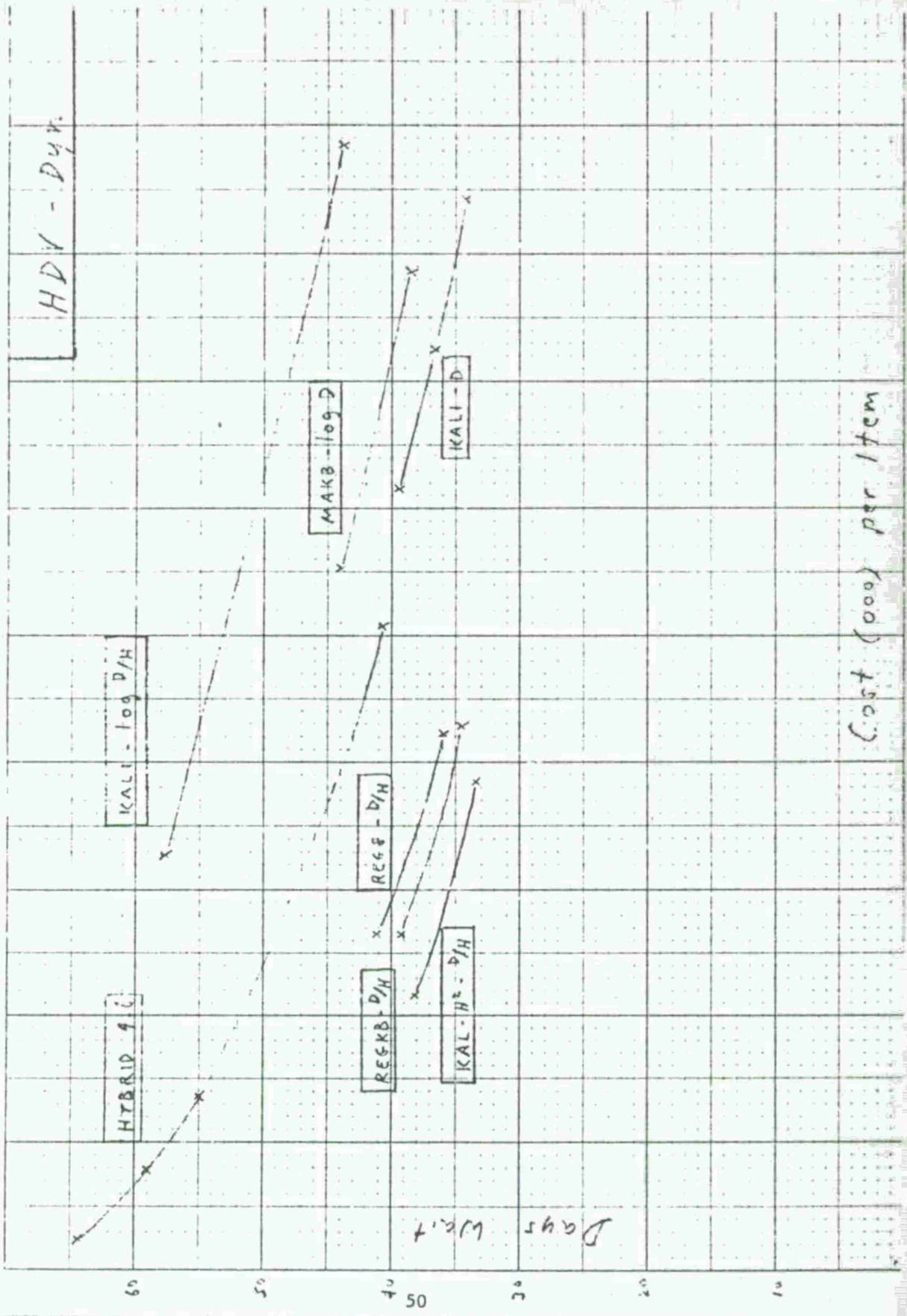
KALL - D

REGG - D/H

REGKB D/H

KAL - H² - D/H

Cost (000) per Item

Days W2.t

5.3 Final Four Algorithms and Resulting Curves

The final program factor algorithms are summarized below. 1794 is the designator for the current Army program factor technique and is the base case for obtaining relative performances and cost savings (Chapter VI). All algorithms operate on the variable $y = D/H$, three of which are weighted moving averages.

REG8:

$$\hat{x}_n = \sum_{j=1}^{8} w_{n-j+1}\, y_{n-j+1} \tag{1}$$

$$w_{n-j+1} = H_{n-j+1}^2 / \sum_{i=1}^{8} H_{n-i+1}^2 \tag{2}$$

REGKB:

As above with KB qtrs for 8 qtrs

KB updated yearly based on requisition class.

1794:

$$\hat{x}_n = \sum_{j=1}^{8} w_{n-j+1} y_{n-j+1}$$

$$w_{n-j+1} = H_{n-j+1} / \sum_{i=1}^{8} H_{n-i+1} \tag{3}$$

Note difference in weights (2), (3)

KAL-H^2:

$$\hat{x}_n = \hat{x}_{n-1} + G_n \,(y_n - \hat{x}_{n-1}) \tag{4}$$

$$G_{n+1} = \frac{1 + k\, G_n}{1 + k\, G_n + k\, H_n^2/H_{n+1}^2} \tag{5}$$

With k updated yearly based on requisition class.

Yearly forecasts are obtained from $\hat{x}_n$ and the projected FH for the next 4 quarters.

$$\hat{D}(year) = \hat{x}_n \cdot (H_{n+1} + H_{n+2} + H_{n+3} + H_{n+4}) \tag{6}$$

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Simulation Groups: The simulations were run for 4 groups of items separately (see Chapter II).

HDV - Dynamic   60 items

HDV - non-Dynamic   151 items

LDV - Dynamic   736 items (722 PEMA items)

LDV - non-Dynamic $\sim$ 8500 items (the few PEMA items were not included in the results). Due to cost of runs, 1/4 of these items were randomly selected for simulation in obtaining average costs per item in group.

The final combined curves for all items were obtained from the other 4 graphs by weighting costs by # items in the group and by weighting days by # requisitions and # items.

Overall rankings were (best to worst):

KAL-H^2, REGKB, REG8, 1794

This was theoretically expected. Details of the cost savings are discussed in the next chapter.

LDV - NonDyn
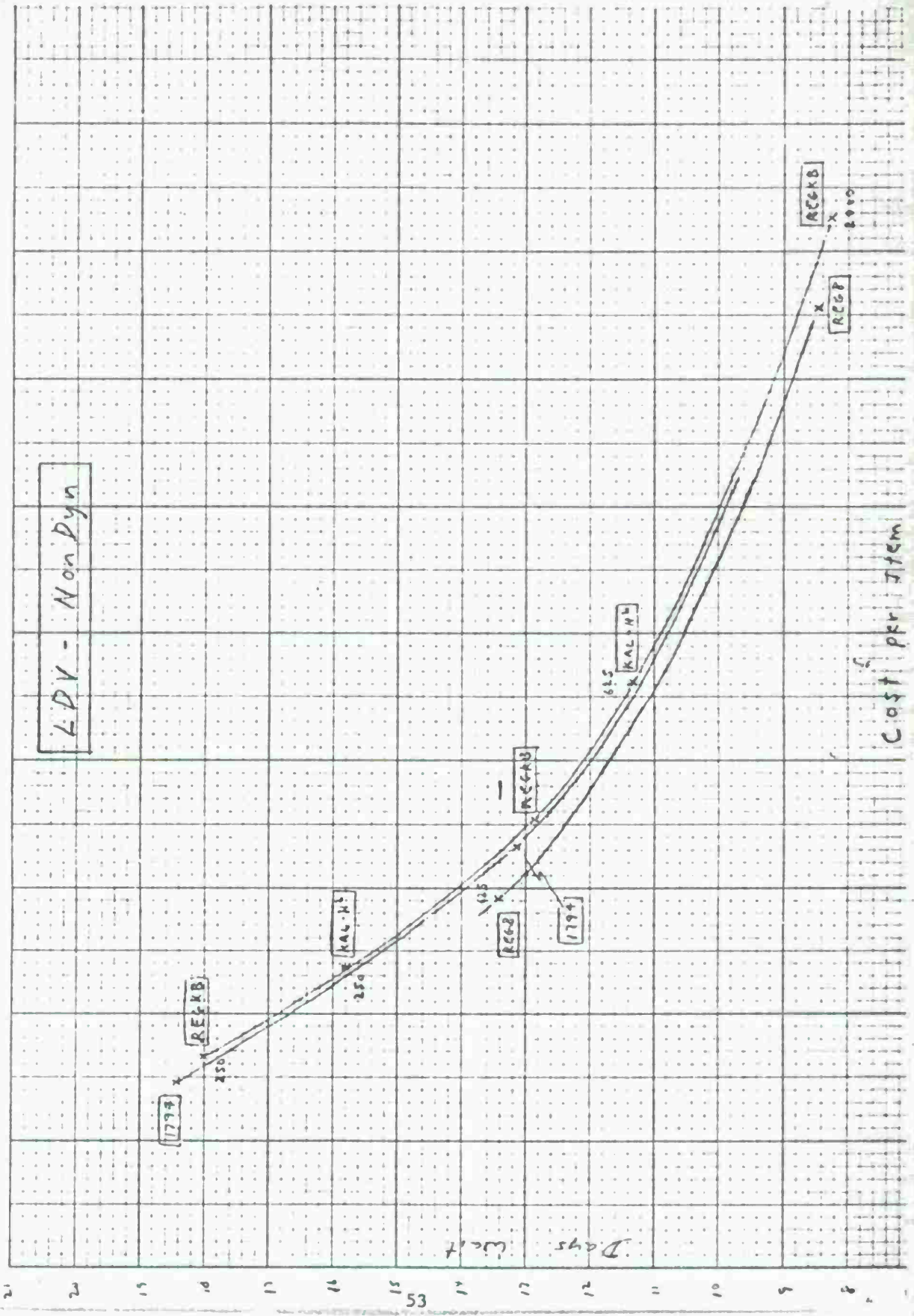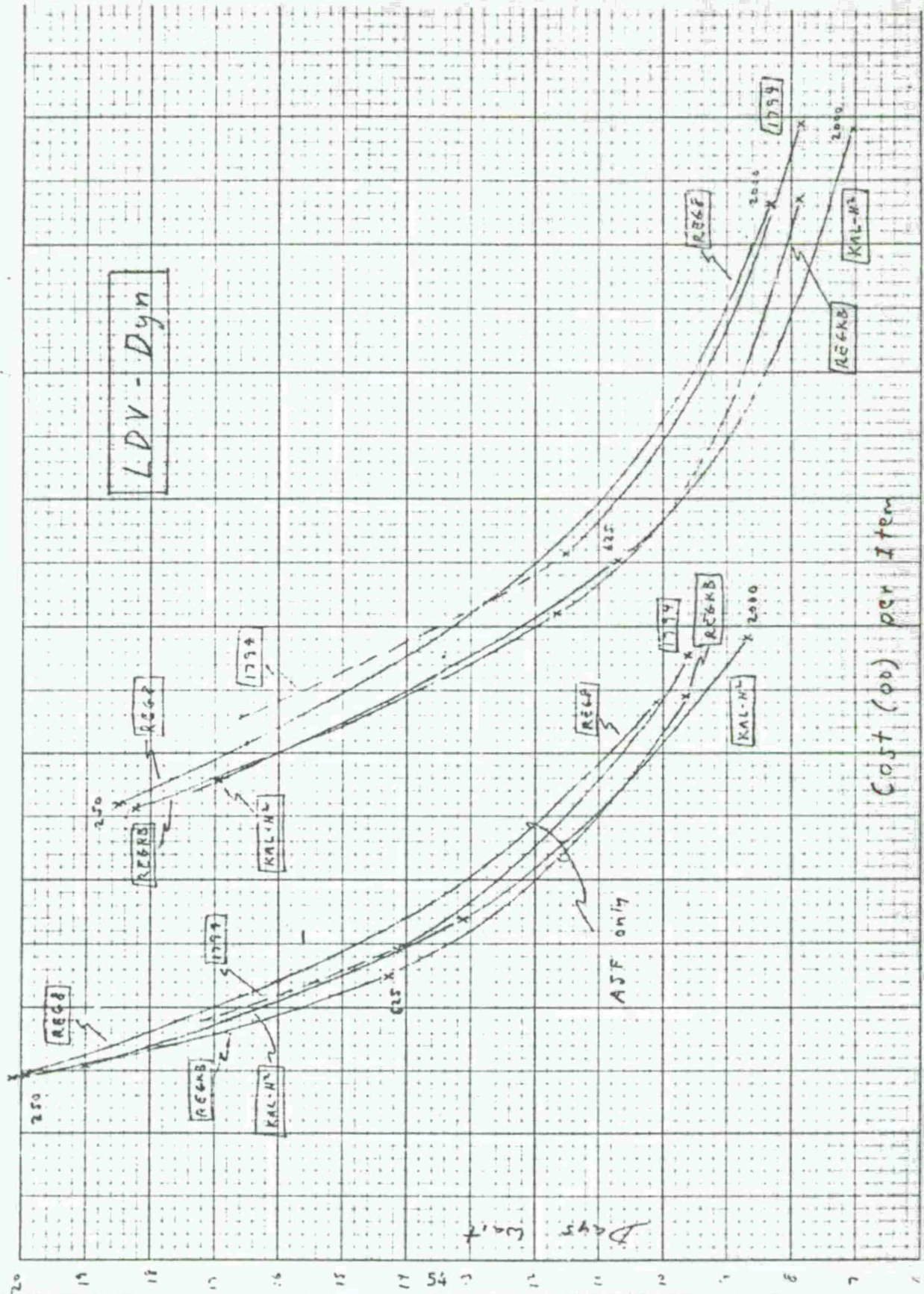
LDV - Dyn

HDV - Von Dyn

Days Wait

Cost (000) Per Item

1754

REGE

KAL·M

RC6KB

55

H.D.V - Dyn

Days Unit

Cost (000) per Item

AF-GF

KC-GFAB

KAL-N'

179t

All Items

1794

REG8

RE4KB

KAL-H²
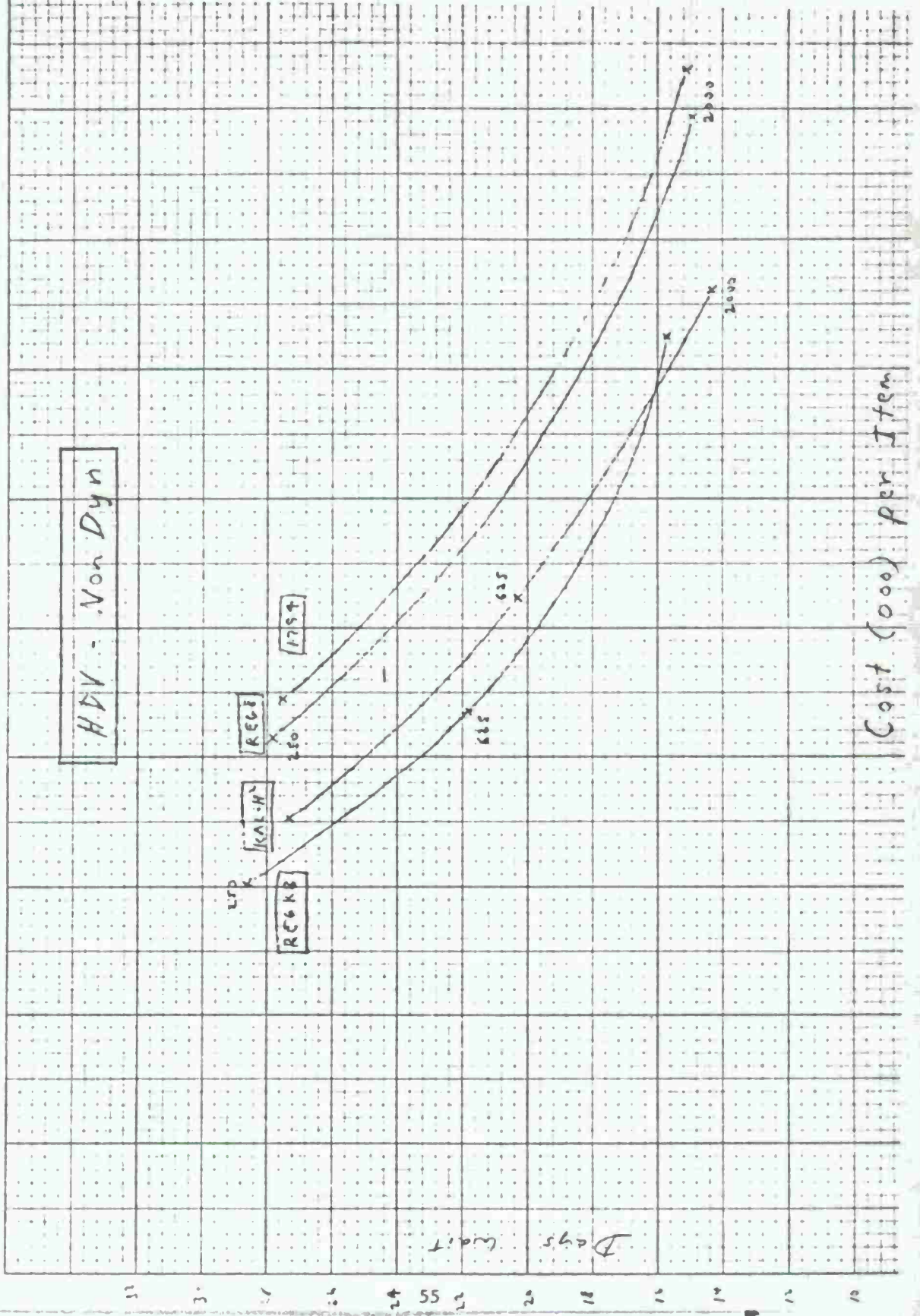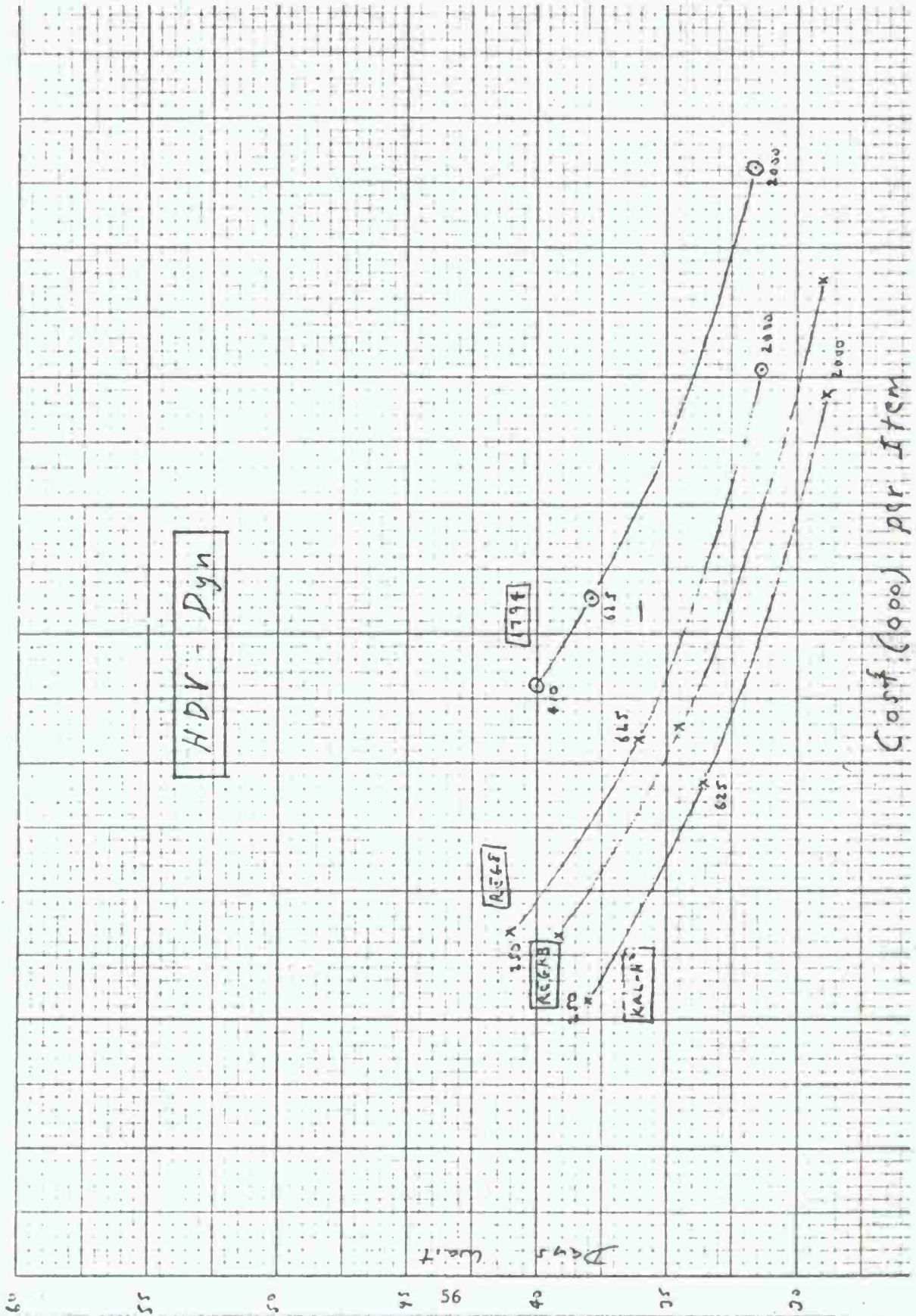
Cost per Item
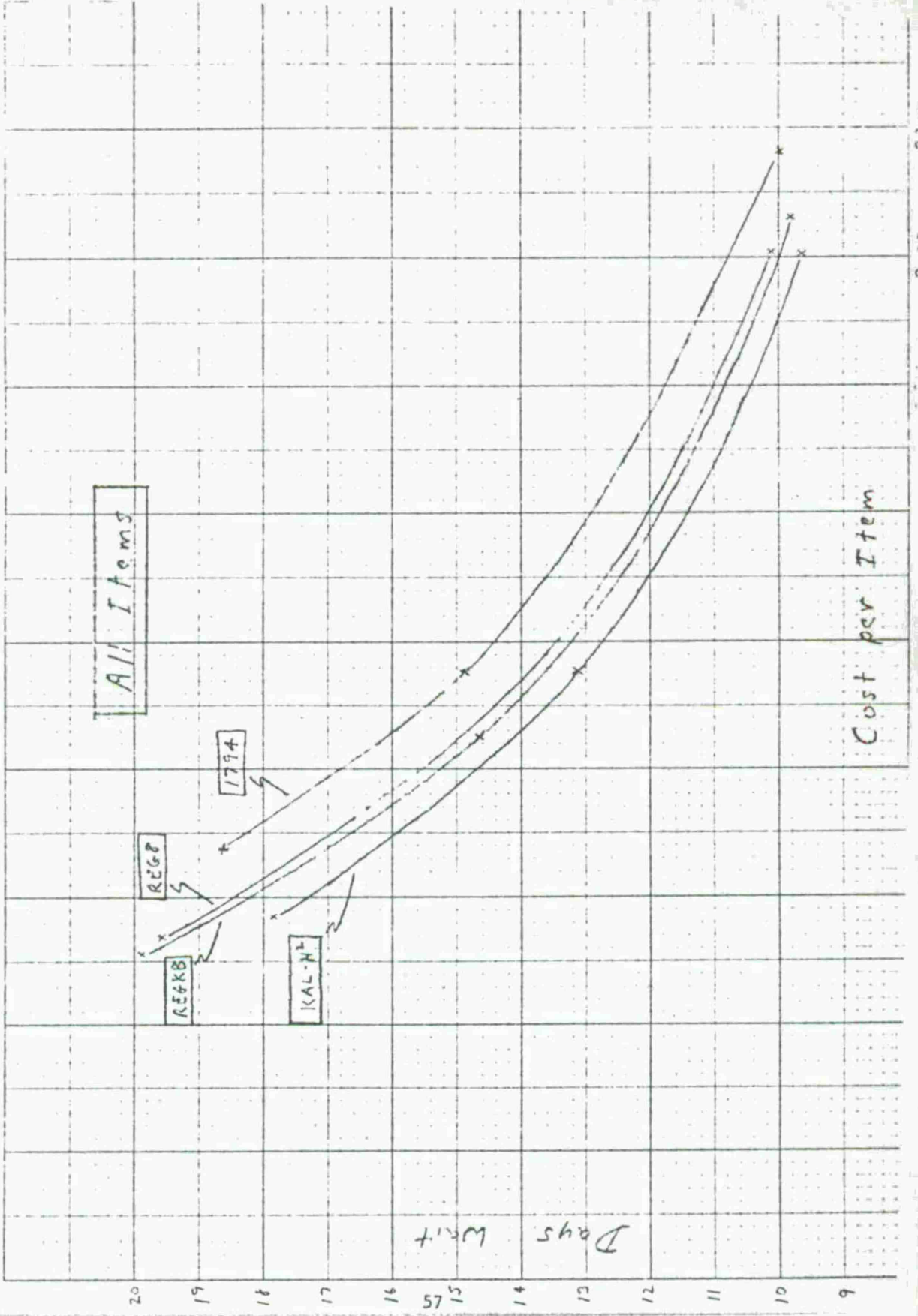
Days Wait

CHAPTER VI

SAVINGS & IMPLEMENTATIONS

6.1 Cost Savings Over 1794 Policy

From the curves in Figure 5.6, we can obtain the annual cost savings per item at a given days requisition delay. Seventeen days wait is an average using the 1794 policy operating at current λ values.

TABLE 6.1   PER ITEM SAVINGS AT 17 DAYS WAIT

REG8	vs	1794	$100
REGKB	vs	1794	$120
KAL-H^2	vs	1794	$180

These averages were obtained from a data base of 9433 items[3] (see Section 5.3). However, it is reasonable for exposition and mnemonics to present for total savings a standard AVSCOM base of 10,000 aircraft-peculiar items (and which constitute the bulk of safety level investment at AVSCOM). Most of this savings is in the HDV-dynamic group (a small group of active, costly items), as can be seen in Figure 5.2. Table 6.2 summarizes the pertinent cost-savings for this 10,000 item base.

6.2 Implementation Considerations

The $1 million savings for REG8, over the 10,000 items, on annual investment costs to obtain the same performance (in terms of mean days requisition delay) is a significant improvement - the result of Cohen's investigations. The algorithm is easily implementable since it is a moving average type, differing from the current 1794 algorithm only in its weights.

The REGKB algorithm yields marginal improvement over REG8 and presents

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[3]Further editing reduced data base of Chapter IV, e.g., two aircraft types with extremely low initial FH were removed.

TABLE 6.2   SAVINGS, PERFORMANCE ON 10,000 ITEMS

COMPARISON	COST SAVINGS AT 17 DAYS WAIT	PERCENT SAVINGS DUE TO HDV-DYN.	REDUCED DAYS WAIT (FROM 17) AT CONSTANT COST
REG8 VS 1794	$1,000,000	78.3%	15.7 DAYS
REGKB VS 1794	$1,200,000	75.5%	15.4 DAYS
KAL-H^2 VS 1794	$1,800,000	86.4%	14.8 DAYS

On average, excess cost savings $\sim 44\%$ of total savings

implementation problems; some of the requisition classes (see Table 4.1) require 15 qtrs or more of past history on demand and FH to be retained. (~ 4 years or more).

The additional savings for KAL-H^2 is dramatic and implementation is feasible but presents new considerations. The formulae in equations (4) and (5) of Chapter V look forbidding, but are not really complex. We now summarize what is involved.

Data Retention (above current requirements):

Table of k-values by requisition class

Current value of k-factor

Current value of estimate $\hat{x}$ of mean of rate D/H

Current value of "smoothing constant" G

Updating : Qtrly

2 year files of $\{D_t\}$ and $\{H_t\}$ and $\{R_t\}$

$$G_n = \frac{1 + k\, G_{n-1}}{1 + k\, G_{n-1} + k\, (H_{n-1}^2/H_n^2)} \tag{1}$$

$$\hat{x}_n = \hat{x}_{n-1} + G_n \cdot (D_n/H_n - \hat{x}_{n-1}) \tag{2}$$

Updating : Yearly

Compute average yearly requisitions from last 8 qtrs

Table look up to find new k

Requisitions	0-1	1-2	2-3	3-4	4-5	5-6	6-8	> 8
k	0	7.34	14.18	20.79	31.19	28.31	75.9	999

Forecasting : At nth Qtr

$$\hat{D} = (\hat{x}_n) \cdot (H_{n+1} + H_{n+2} + H_{n+3} + H_{n+4}) \tag{3}$$

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<u>Forecasting</u> : Between Qtrs

$$z = \hat{x}_n + G_n (\gamma - \hat{x}_n) \tag{4}$$

where γ found from interpolation of DRD and FH

files to yield a current quarter rate

$D = z \cdot$ (years projection of FH at current time)

There are also necessary special procedures, due to the exponential smoothing structure, to handle adjustments of forecasts due to backorder cancellations and to breakout forecasts by area (if an item migrates to an HDV class). ALMSA can modify the necessary CCSS routines, with IRO assistance, to implement these procedures.

6.3 Modifications of Algorithms to Use End Item Density as a Program Factor

IRO did some additional analysis which broadens the scope of application of the $KAL\text{-}H^2$ algorithm. Defining ρ as an end item density variable and $\{\rho_t\}$ as the corresponding time series, the algorithm of Section 6.2 may be used with ρ substituted for H and a different table of k-values as seen below. Commands may use ρ as the program factor; there must be some justification in that end item density $\sim$ should be correlated with some usage variable (e.g. flying hours, miles, rounds fired). The estimate $\hat{x}$ is related to the rate D/ρ in this case and the forecast

$$D = (x_n) \cdot (\rho_{n+1} + \rho_{r+2} + \rho_{n+3} + \rho_{n+4})$$

Orr [/3] does some theoretical analysis which relates the k factor for D/ρ to the k factors for D/H by the residual variance of a regression fit of H by ρ and by the variance of the time series $\{\rho_t\}$ itself. It was found for AVSCOM data that 72% of the variance in H could be explained by variance in ρ. Therefore the demand/end item rate is less stationary than the demand/FH rate; hence smaller k's and shorter base periods B (for MA algorithms) are called for; i.e. less weight is given to observation on D/ρ far in the past. The following table is instructive.

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TABLE 6.3   K-FACTOR FOR 3 TIME SERIES

D		D/ρ		D/H	
k	B	k	B	k	B
0	1	0	1	0	1
3.164	3	4.02	4	7.34	5
4.251	4	5.765	4	14.18	7
4.399	4	6.25	4	20.79	8
4.71	4	6.91	5	31.19	10
3.464	3	5.16	4	28.31	10
3.564	3	5.55	4	75.9	15
3.674	3	5.88	4	∞	∞
3.120	3	4.99	4	∞	∞
2.022	3	3.235	3	∞	∞

Note that forecasting using only D utilizes short base periods in MA
algorithms and small k's (less stationary process), whereas k and B
increase for D/ρ and D/H accordingly.

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CHAPTER VII

CONCLUSIONS, RECOMMENDATIONS & FUTURE WORK

7.1 Conclusions

This study has reinforced Cohen's findings - that forecast algorithms utilizing flying hours perform better on the AVSCOM data base than strictly demand dependent algorithms. REG8 is clearly superior to the current Army method of utilizing FH as a program factor. KAL-H^2 yields additional substantial improvement in terms of cost savings. We have also developed a rationale and presented empirical tables for forecasting by item class (requisition frequency). A modified algorithm and tables for forecasting using end item density as a program factor have been presented.

Other Valuable Conclusions and Results

Moving average base periods should be short (~ 4 Qtrs) for $\{D\}$ series. MA base periods should vary by item class for $\{D/H\}$ series. Several candidates (non-program factor) for forecasting common items have been found. Three such algorithms are KAL1-D, MAKB-D, KAL2-BQ-logD.

7.2 Recommendations

a. KAL1-H^2 be implemented as the FH-based algorithm for AVSCOM. ALMSA, with IRO assistance, to determine the best way of modifying CCSS routines.

b. Analogous algorithms be used at other major subordinate Commands. If end item density is the program factor, parameter values need to be adjusted.

c. IRO be tasked to obtain the best non-program factor forecast procedure for common items (or where program factor not feasible). Common item data base from AVSCOM be used to further screen candidates found in this study.

7.3 Future Research - Aids & Caveats

Any future studies on demand forecasting at the wholesale level for Army secondary items which use this report and Cohen as starting points

should be aware of other aspects not fully discussed in this report. Forecasting at other Services and/or other support levels should find investigation of the models - algorithms useful but not necessarily with the same parameter values.

k-values - These were obtained from time series 7 years in length. The most basic assumption in this forecasting is that the past describes the future. However, as more history is accumulated, these k values should be redetermined on some periodic basis and over some "moving" base length (say 5 or 7 years).

Outlier Analysis - Little work was done on determining which observations in a time series are erroneous or outliers in some sense. Some items (see Section 4.1) were deleted because of very large forecast errors. Also it was found that 1171 items had over 50% of their total demand over the 28 Qtr horizon in 1 Qtr. Mostly these are inactive items and are quite unforecastable by any technique.

Other Stratifications - Section 4.3 listed some of the item stratifications. Reparable - non-reparable breakouts and weapon system breakouts were appended to these strats; some weapon systems had distinctive k values for D series, but this was not pursued due to performance of D/H based algorithms. Stratifications by IMPC or FSC were not attempted.

Error Measures - There is interest in finding statistical error measures that correlate better with the simulator cost performance. Weighting quantitative error functions (such as MAD) by unit price or order size may prove fruitful. Future work which could quantify a relation between cost performance and forecast error properties would reduce considerably the number of simulator runs.

Prior Distributions - Catalog Approach

For the Kalman algorithms where prior parameters (μ, σ^2 ; see Section 4.4) are required for startup, some time was spent on devising techniques to use statistics on a catalog of items during initial experience or warmup. The catalog aspect was finally scrapped because:

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a. The items in the data base are more heterogeneous than insurance items, where a catalog approach is more sensible. This is the case even using a sub-catalog based on an item stratification. Due to heterogeneity, catalog technique can give quite bad estimates for μ, τ^2 on a particular item.

b. We were unable to develop a reasonable procedure for scaling or normalizing a catalog.

c. After 2 years of warmup, for most items, much weight should be given to data experience on that item.

d. Such a catalog procedure would not accurately reflect real world where maintenance factors (engineering estimates of consumption rates) and prior distributions based on some other classification of similar items may be used.

Other Forecast Techniques Excluded from Investigation

a. Subjective - not practical

b. Segmentation - individual forecasting methods or use of particular information for classes of items - not included per se.

c. Monitoring methods - problems with untrained personnel, clerical effort, undesirable response to transients.

d. Adaptive filters, stochastic approximation - too many data points required; item dependent.

e. Box - Jenkins models - Orr [13] shows the relationship to techniques in this study. However, it has same disadvantages as in d.

f. Econometric - Regression methods - only done for FH. KAL2 is actually a general form of a time dependent linear regression model.

g. Triple Exponential Smoothing - performed quite badly in preliminary investigation by Orr and Cohen.

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BIBLIOGRAPHY

1   Aoki, _Optimization of Stochastic Systems_, Academic Press, 1968.

2   Box & Jenkins, _Time Series Analysis: Forecasting & Control_,
    Holden-Day, 1970.

3   Brown, R, _Smoothing, Forecasting & Prediction_, Prentice-Hall, 1963.

4   Cohen, Martin, "Demand Forecasting with Program Factors," USAMC
    Inventory Research Office, IRO #182, Sept 1975, AD #A017858.

5   Croston, J.D., "Forecasting & Stock Control for Intermittent Demands,"
    Operations Research Quarterly (UK), Sept 1972.

6   Deemer, R & W. Kruse, "Evaluation of Several VSL/EOQ Models" USAMC
    Inventory Research Office, May 1974, AD #781948.

7   Efron & Morris, "Limiting the Risk of Bayes & Empirical Bayes
    Estimators, Part I," _J. Amer Stat Assoc_, Dec 1970.

8   Goodman, M.L., "A New Look at Higher Order Exponential Smoothing for
    Forecasting," _Operations Research_, July-Aug 1974.

9   Harrison, P.J., "Exponential Smoothing & Short Term Sales Forecasting,"
    Management Science, July 1967.

10 Harrison, P.J. & Stevens, "A Bayesian Approach to Short-Term
    Forecasting," _Operations Research Quarterly (UK)_, Dec 1971.

11 Hayes, R.E., "A Comparison of Short-Term Forecasting Models," Naval
    Post Graduate School, Sept 1971, AD #734865.

12 Jewell, W.J., "The Credible Distribution," OR Center, Report #77-13,
    University of California, Berkeley, Aug 1973.

13 Orr, D.A., "Kalman & Moving Average Filters for Forecasting,"
    DRC Inventory Research Office (To be published).

14 Rhodes, I.B., "A Tutorial Introduction to Estimation & Filtering,"
    IEEE trans on Automatic Control, AC-16, Dec 1971.

66

15   Sage, A.P., *Optimum Systems Control*, Prentice Hall, 1968.

16   Theil & Wage, "Some Observations on Adaptive Forecasting," *Management Science* 10, 1964.

17   Cohen, Martin, "ALPHA 4140.39 Simulator," DRC Inventory Research Office, May 1973, AD #762-348.

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