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ANALYSIS OF MULTIVARIATE SENSOR RESPONSES  
IN A SURVEILLANCE ENVIRONMENT

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The purpose of this paper is to develop a methodology for analyzing surveillance sensor signals and to translate this analysis into decision rules. The use of Bayesian statistical techniques provides a powerful tool for a quantitative approach to decision making, especially when the analyst has the opportunity to observe or sample prior to making any judgements. The increased reliance placed on using a variety of sensors as an early warning system on the modern battlefield dictates the use of more sophisticated mathematical methods to digest the information sent by these devices. The most recent example of this has occurred in the Middle East where the presence of a U.S. surveillance team is an integral facet of an interim Egyptian-Israeli settlement.

The September 1975 settlement engineered by Dr. Kissinger's diplomatic shuttle provided for a U.S. Sinai Field Mission near the Mitla and Giddi Passes to monitor military movements of Israel and Egypt. This force has the potential for providing surveillance of not only the key passes but also of the entire length of the U.N. Buffer Zone, from the Mediterranean Sea to the Gulf of Suez. While it can be assumed that these sensor responses will be subject to scrutiny at the highest policy making levels, the initial analysis of the output will rest with the operators. The surveillance within the demilitarized area can be expected to consist of signals from unattended ground sensors (UGS) and aerial tracking. The constraint on manpower to support U.S. participation probably will not allow for the luxury of even rudimentary manual analysis of information on where to concentrate the surveillance effort. Hence, the observer force will have to avail itself to state-of-the-art computer systems, their

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display capabilities, and operations research techniques to assist in arraying even basic decision rules. A similar problem faced U.S. forces in Southeast Asia in the uses of sensor systems. The plethora of information electronically transmitted from UGS and aerial systems such as SLAR (side-looking airborne radar) and infrared surveillance required automated analysis to assist decision makers. In order to meet this challenge and prepare for those that might be expected on a European battlefield, the Army exerted a considerable effort to update its command and control procedures by integrating automated data processing (ADP) into command and staff organizations. The expertise gained from these research and development efforts, coupled with that received through the extensive employment of sensors in Vietnam, could serve as the basis for the establishment of a sophisticated surveillance system not only in the Sinai Peninsula but also in a European environment.

In this situation, sensor responses can be aggregated into two categories: Those received from tracking aircraft and those generated by ground forces. The signals from each system are feed electronically to a centralized observation post where they are recorded and routed into a computer for correlation and storage. Through a predetermined program, the signals are analyzed for intensity, type, frequency, and proximity to other signals. Hence, the set of systems can have a single random variable (not necessarily integer) resulting from a constant updating of the computer's master file. The development of decision parameters, coupled with operational and empirical data from these signals, would allow for the use of a computer subroutine to fit a probability distribution to the random variate associated with each set of sensor responses. At anytime during a previously delineated observation period, a decision maker can receive information about both systems. However, at sometime a decision will have to be made regarding the status of the surveillance plan---if surveillance will be increased in a particular area, continued, or terminated. (This assumption negates any sequential sampling and it will be this point in time that will be addressed in this study.)

DECISION ELEMENTS

The signal analyst can define the elements of a decision matrix as an outcome space (W), a decision space (D), and a loss function,  $L(W,D)$ . For the purposes of a simplistic environment,  $W = \{w_1, w_2\}$ , where  $w_1$  is the state of nature indicating unusually high military activity in the area of surveillance and  $w_2$  is normal activity for the particular area in question. The decision space,  $D = \{d_1, d_2, d_3\}$ , contains the alternatives available for the observers:

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$d_1$ : Increase surveillance of an area by concentrating resources on that locale.

$d_2$ : Continue current observation posture.

$d_3$ : Decrease surveillance posture in the area in favor of another location.

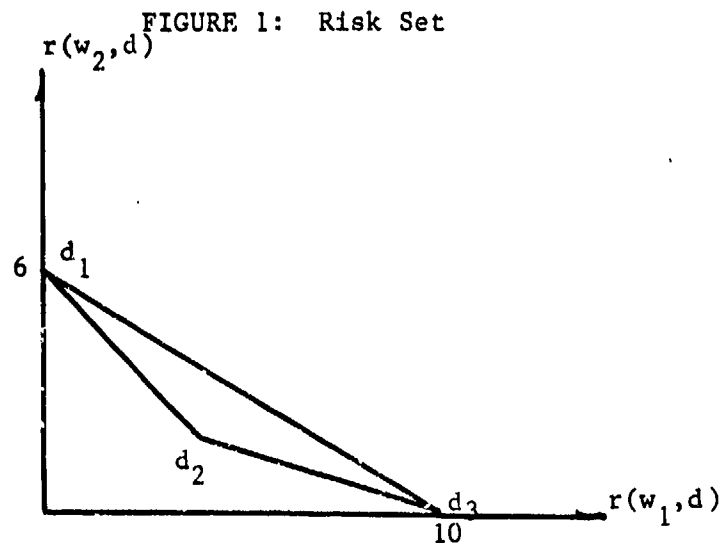
The expected/estimated losses that result from these decisions are discrete and, for the purposes of this formulation, could range from 0, the best situation, to 10, the worst situation.

TABLE 1: Loss Function for D and W

	$d_1$	$d_2$	$d_3$
$w_1$	0	4	10
$w_2$	6	2	0

(The numbers in Table 1 are, in fact, risks  $(r(w_1, d_j))$  ..... or expected losses.)

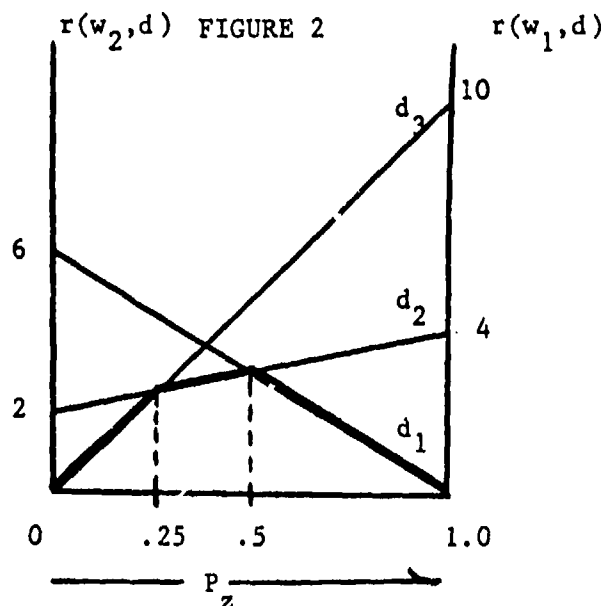
Graphically, these risks are shown in Figure 1.



(The line segments,  $d_1 - d_2$  and  $d_2 - d_3$ , define the admissible and Bayes boundaries.)

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Since D and W are both finite, a minimax decision approach is appealing because it minimizes the worst possible loss.



If the analyst could not make any observations or samples before arriving at a decision,  $P_z$  shown in Figure 2 would determine the admissible decisions. Hence, for a  $P_z < .25$ , the analyst would choose  $d_3$  (decrease the surveillance posture in that area in favor of concentration in another area). If  $P_z > .5$ ,  $d_1$  would be implemented (increase surveillance of the area).

However, as previously stated, the key element in this formulation is the ability to associate a probability distribution, with a known mean ( $m$ ) and variance ( $s^2$ ), to each type of sensor response. Given the current status of technology and the vast number of tests and simulations conducted with these systems, the assumption of a conditional probability distribution for each value of  $w \in W$  is not presumptuous. If  $X$  is the random variable associated with UGS responses and  $Y$  is the random variable from the aerial surveillance, then each would approximate some distribution  $\phi$ .

$$X | w_i \sim \phi(m_i, s_i^2) \quad Y | w_j \sim \phi(m_j, s_j^2)$$

#### METHODOLOGY

Although  $X$  and  $Y$  are independent, the analyst must make a decision ( $d_i$ ) based on all information available. This would

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require sampling outputs from both ground and aerial surveillance systems and incorporating them into a posterior probability function. This is accomplished by using the prior distribution (e.g.,  $z(w)$ , the analyst's feeling that the state of nature,  $w_1$ , does exist ( $P(W=w_1)$ )) and the likelihood,  $f(x,y|w)$ . The posterior distribution takes the form

$$z(w | x,y) = \frac{f(x,y|w) z(w)}{\int_W f(x,y|w) z(w) dw} \quad (\text{Equation 1})$$

or in other notation

$$z(w | x,y) = \frac{f_1(x,y)}{f_1(x,y) + f_2(x,y)} = \frac{1}{1 + \frac{f_2(x,y)}{f_1(x,y)}} \quad (\text{Equation 2})$$

Equation 1 expresses Bayes' Theorem which states that the posterior probability equals the prior probability multiplied by the likelihood function. Mathematically, the posterior probability is the probability that some hypothesis is true given certain evidence, the prior probability is the probability that the hypothesis was true before the evidence was collected, and the likelihood is the probability of obtaining the observed evidence given the hypothesis is true. (If the observations come from a continuous distribution, the likelihood is defined as the joint density function).

$P_2$  is the probability that minimizes the expected losses since the Bayes solution for the problem with no observations is the desired Bayes action when observations of X and Y are considered (see Figure 2). Therefore, by equating  $P_2$  to  $z(w|x,y)$ , Equation 2 can be solved for those values of x and y which will allow the analyst to choose  $d_1$ ,  $d_2$ , or  $d_3$  with minimal concomitant risk. As the prior probability ranges from 0.0 to 1.0, a decision envelope is established for the specified  $\phi$  and  $L(D,W)$ .

## DECISION RULES

For the purposes of this analysis, the random variables, X and Y, approximate the following distributions:

$$\begin{array}{ll} X | W = w_1 \sim N(7,1) & X | W = w_2 \sim N(2,1) \\ Y | W = w_1 \sim N(5,1) & Y | W = w_2 \sim N(1,1) \end{array}$$

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(While the condition of normality enhances mathematical tractability, the evaluation of multiple inputs is in no way limited to this probability distribution.)

Since X and Y are independent and the normal distribution is continuous, the resulting likelihood is the joint probability density function which takes the form

$$f(x,y | w) = \frac{1}{2\pi s_x s_y} \exp \left\{ -\frac{1}{2} \left[ \left( \frac{x - m_x}{s_x} \right)^2 + \left( \frac{y - m_y}{s_y} \right)^2 \right] \right\}$$

(Equation 3)

Using the prior distribution as  $z = z(w)$ , the posterior distribution can be found.

$$z(w | x,y) = \frac{z/2\pi \exp \left\{ -\frac{1}{2} \left[ (x-7)^2 + (y-5)^2 \right] \right\}}{z/2\pi \exp \left\{ -\frac{1}{2} \left[ (x-7)^2 + (y-5)^2 \right] \right\} + (1-z)/2\pi \exp \left\{ -\frac{1}{2} \left[ (x-2)^2 + (y-1)^2 \right] \right\}}$$

(Equation 4)

If  $z(w | x,y) = P_z$ , then using Equation 2 and the values in Figure 2

$$P_z = \begin{cases} .25 \\ \text{or} \\ .50 \end{cases} = \frac{1}{1 + \frac{f_2(x,y)}{f_1(x,y)}}$$

Therefore,  $\frac{f_2(x,y)}{f_1(x,y)} = 3$  when  $P_z = .25$  and 1 when  $P_z = .5$ ; also,

$$\frac{f_2(x,y)}{f_1(x,y)} = \frac{1-z}{z} \exp \left\{ -\frac{1}{2} \left[ (x-2)^2 + (y-1)^2 - (x-7)^2 - (y-5)^2 \right] \right\}$$

Expanding this expression

$$\begin{aligned} \frac{1-z}{z} \exp \left\{ (-5x - 4y + 34.5) \right\} &= 3 && \text{(when } P_z = .25) \\ \text{or} &&& \\ \frac{1-z}{3z} \exp \left\{ (-5x - 4y + 34.5) \right\} &= 1 && \text{(when } P_z = .50). \end{aligned}$$

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Taking the natural log of both sides of the equation

$$\log \left( \frac{1-z}{3z} \right) + 34.5 - 5x - 4y = 0 \quad \text{or}$$

$$x + .8y = 6.9 + .2 \log \left[ \frac{1-z}{3z} \right] \quad (\text{Equation 5}).$$

A similar operation when  $P_z = .5$  yields

$$x + .8y = 6.9 + .2 \log \left[ \frac{1-z}{z} \right] \quad (\text{Equation 6}).$$

Referring back to Figure 2, the new decision rules are:

Rule A. For  $x + .8y < 6.9 + .2 \log \left( \frac{1-z}{3z} \right)$  choose  $d_3$  (decrease surveillance posture).

Rule B. For  $x + .8y > 6.9 + .2 \log \left( \frac{1-z}{z} \right)$  choose  $d_1$  (increase surveillance posture).

Rule C. For  $x + .8y \geq 6.9 + .2 \log \left( \frac{1-z}{3z} \right)$  and for  $x + .8y \leq 6.9 + .2 \log \left( \frac{1-z}{z} \right)$  choose  $d_2$  (continue current posture).

By varying the prior probability from 0.0 to 1.0, the analyst can determine values of X and Y which would indicate the best decision.

TABLE 2: Decision Envelope for Multiple Inputs

$z = P(W=w_1)$	Choose $d_3$ when $x + .8y <$	Choose $d_1$ when $x + .8y >$
.1	7.119	7.339
.2	6.957	7.117
.3	6.849	7.069
.4	6.761	6.981
.5	6.680	6.900
.6	6.599	6.819
.7	6.510	6.731
.8	6.403	6.622
.9	6.240	6.461

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FIGURE 3

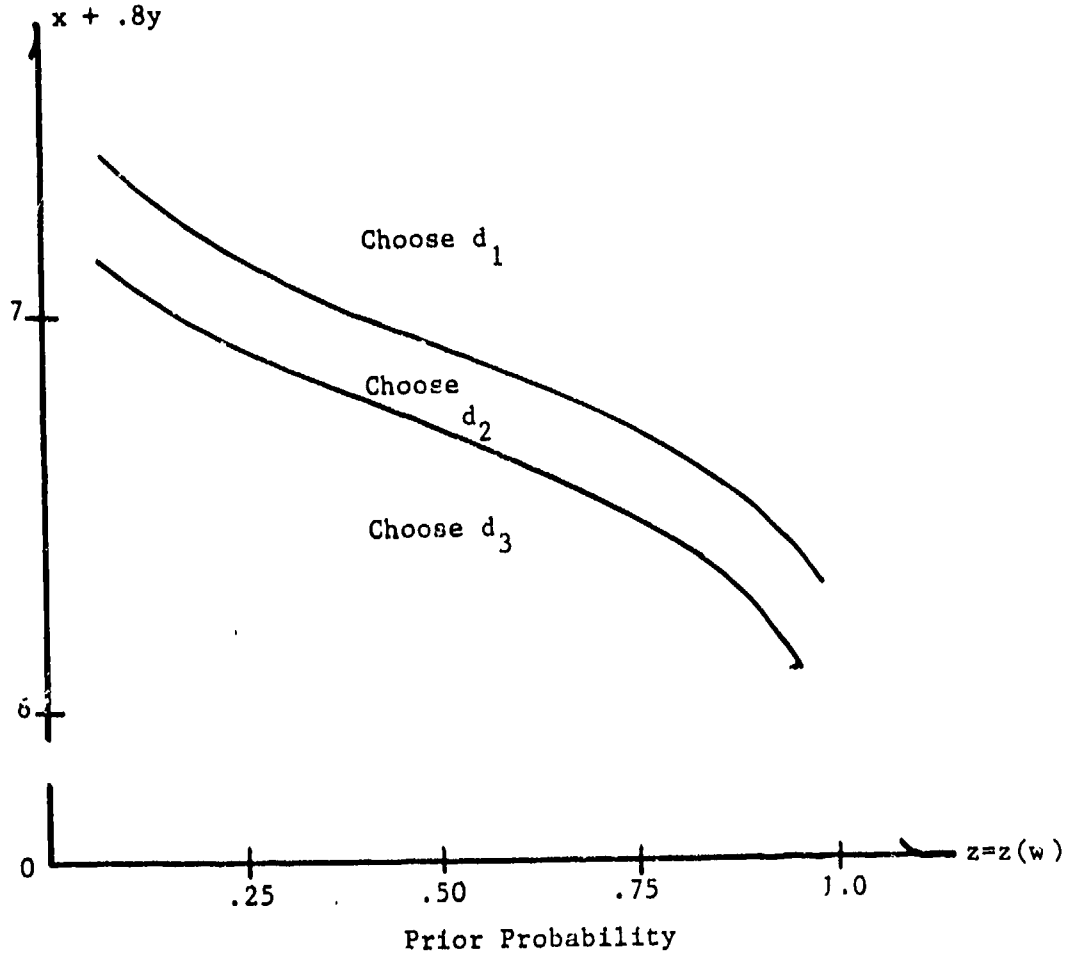


Table 2 indicates that the decision space  $d_2 \in D$  is very small compared to the other alternatives. The region is so constrained that the analyst is essentially deciding whether to increase or decrease his surveillance posture.

Using the same method of calculation (except the likelihood in the form  $f(x|w)$  or  $f(y|w)$ ), decision rules can be obtained when the analyst receives input from a single source.

Sensor Input Only: Choose  $d_3$  if  $x < 4.5 + .21 \log(1-z/3z)$  and  $d_1$  if  $x \geq 4.5 + .21 \log(1-z/z)$ . Choose  $d_2$  otherwise.

Aerial Input Only: Choose  $d_3$  if  $y < 3.0 + .125 \log(1-z/3z)$  and  $d_1$  if  $y \geq 3.0 + .125 \log(1-z/z)$ . Choose  $d_2$  otherwise.



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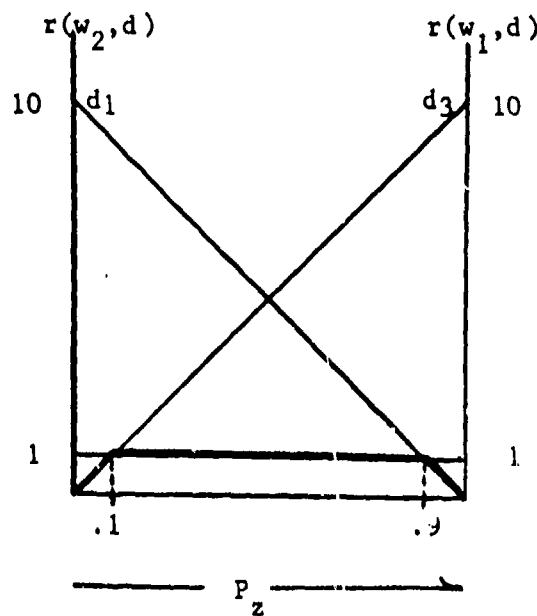
While Figure 3 shows the sensitivity of  $x$  and  $y$  to the variance in the prior probability, the analyst would also be expected to change his loss function periodically to correspond to a particular surveillance area. This subjective determination would alter the original decision rules and the decision envelope. If equal "loss" were attached to making the "wrong" decisions ( $d_1$  when  $w_2$  exists and  $d_3$  when  $w_1$  exists), the new  $L(D,W)$  might look like this:

TABLE 3: Loss Function for D and W

	$d_1$	$d_2$	$d_3$
$w_1$	0	1	10
$w_2$	10	1	0

Similarly, the  $L(D,W)$  can be depicted graphically and minimizing probabilities extracted.

FIGURE 4



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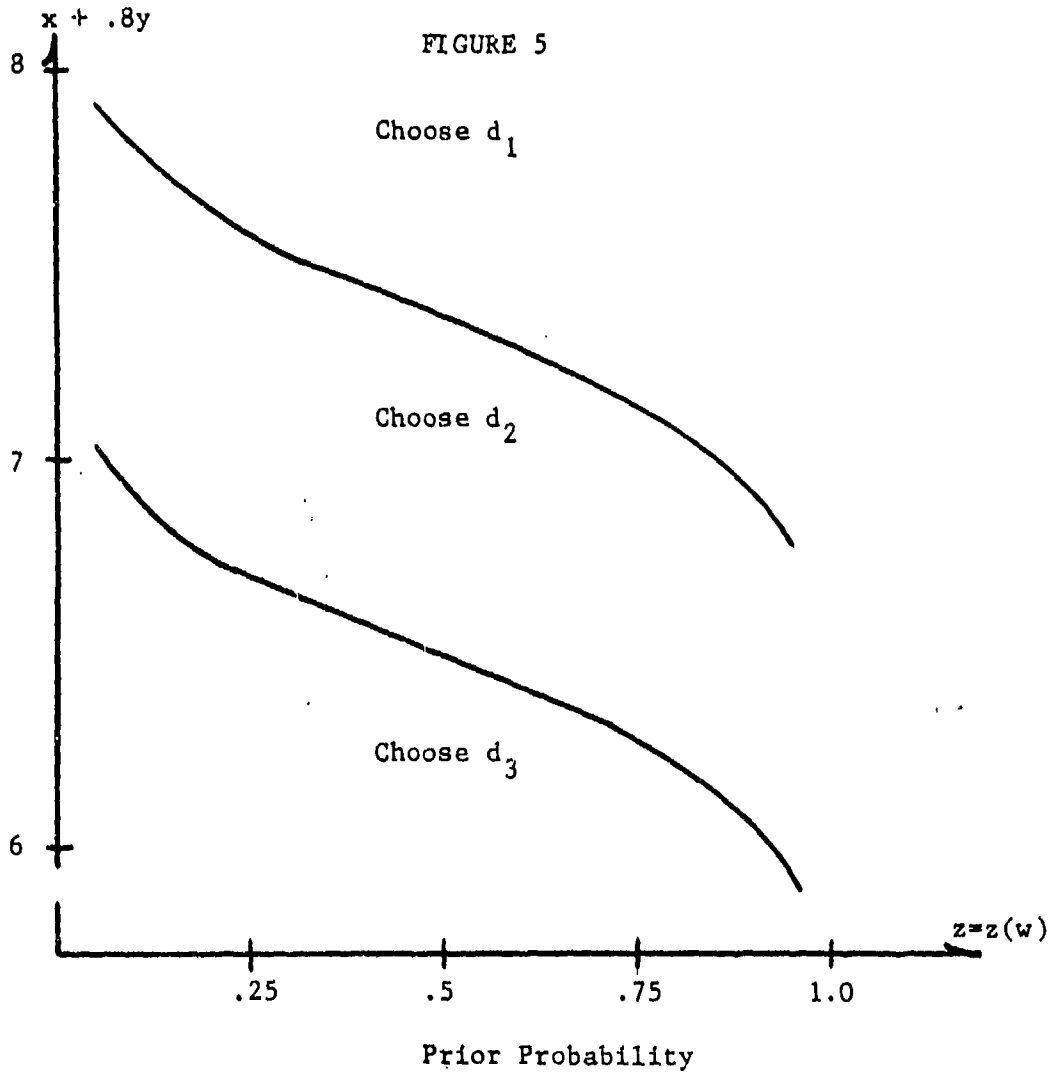
The associated decision rules would be:

Rule A: If  $x + .8y < 6.9 + .2 \log (1 - z/9z)$ , choose  $d_3$ .

Rule B: If  $x + .8y > 6.9 + .2 \log (9(1 - z)/z)$ , choose  $d_1$ .

Rule C: If  $x + .8y \geq 6.9 + .2 \log (1 - z/9z)$  and  $x + .8y \leq 6.9 + .2 \log (9(1 - z)/z)$ , choose  $d_2$ .

By varying the prior from 0.0 to 1.0, values of X and Y that indicate the best decision can be determined.



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### CONCLUSIONS

The Bayesian methodology presents a viable technique for the formulation of decision rules in a surveillance environment. Yet, there is some controversy about this body of statistical theory since it revolves around just what one is willing to assume concerning the problem. However, the idea behind using such a procedure is to guard against a catastrophic loss that might be encountered in a military situation. Its utility in this instance is dependent upon the capability of modern ADP hardware to aggregate groups of signals and translate them into a usable form, while maintaining the flexibility to perform the subsequent numerical analysis.

With the advent of more machinery to assist the operations on the battlefield, sophisticated analytical tools must be employed to handle the increased volume of information. Data which might have previously been lost is now readily available through computerized storage and recall. However, this reliance on machinery has a tendency to divorce the individual from the action and negates much of the "seat of the pants" decision making that has been so common in the past. In this atmosphere, Bayesian theory can be of great assistance to the decision maker who probably feels alienated in the antiseptic world of computer hardware. There will be critics who say that major decisions are made based on a few mathematical formulas and mechanical calculations. However, even from this simple problem, it is evident that the mathematics and the hardware do not eliminate the human being. The analyst's judgment about the prior probability and the formulation of the loss function are the most important aspects of the whole operation, and hence, all results are dependent on them. In the final analysis, the analyst or the commander will still have to make his decision based on all information, experience, and assistance that is available to him.

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