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Buoyancy Cross-Flow Effects on the Boundary Layer of a Heated Horizontal Cylinder

L. S. Yao and Ivan Catton

A Report prepared for

DEFENSE ADVANCED RESEARCH PROJECTS AGENCY

Rand
SANTA MONICA, CA. 90406

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PREFACE

Under the sponsorship of the Tactical Technology Office of the Defense Advanced Research Projects Agency, The Rand Corporation has been engaged in analysis for and development of hydrodynamic design criteria employing concepts of boundary layer control, including shaping, suction, and heating.

For a body moving at lower speeds, laminar flow phenomena never before considered may arise. These phenomena are associated with the interaction between the buoyancy force induced by small temperature differences and a basic boundary layer flow symmetrical about the axis of the body. Using analytic methods, this report considers the combined effects of buoyancy and forced flow over a heated cylinder, as a preliminary step in the delineation of cross-flow effects on a flow configuration that is otherwise symmetric about the line of the body.

This report should be useful to hydrodynamicists, to designers of submersibles, and to others interested in applying fluid mechanics to the prediction and improvement of the performance of underwater vehicles.

Other, related Rand publications include:

- R-1752-ARPA/ONR, *Low-Speed Boundary-Layer Transition Workshop*, W. S. King, June 1975.
- R-1789-ARPA, *Controlling the Separation of Laminar Boundary Layers in Water: Heating and Suction*, J. Aroesty and S. A. Berger, September 1975.
- R-1863-ARPA, *The Effects of Wall Temperature and Suction on Laminar Boundary-Layer Stability*, W. S. King (forthcoming).
- R-1898-ARPA, *"e^9": Stability Theory and Boundary-Layer Transition*, S. A. Berger and J. Aroesty (forthcoming).
- R-1966-ARPA, *The Effect of Variable Viscosity on Buoyancy Cross Flow on a Heated Horizontal Cylinder*, L. S. Yao (forthcoming).

SUMMARY

Heat transfer and shear stress may be significantly affected by buoyancy forces and associated free-convection motions in many forced-convection flows. A cross flow is induced when a uniform, horizontal stream passes along a heated, semi-infinite horizontal cylinder. The cross-flow effects on heat transfer and shear stress grow as the fluid flows downstream, and eventually become one of the dominant mechanisms even for moderate-speed forced-convection flow. It is important to know the development of the buoyancy-force effect, the conditions under which the effects of free convection may be ignored in heat-transfer and shear-stress calculations, and the regions beyond which the effects of free convection may be as important as the forced-convection flow.

In this report, a similarity solution is found for the flow described above. An asymptotic expansion based on the ratio of the Grashof number to the square of the Reynolds number indicates the region where buoyancy forces can be treated as second order. The ordinary differential equations, reduced by a similarity transformation from the three-dimensional boundary-layer equations, are solved by numerical integration over the Prandtl number range from 0.01 to 10. The results supply a quantitative criterion for distinguishing the conditions under which the effects of free convection may not be neglected.

The criterion, when applied to the axial motion in water of a 100°F heated cylinder with 1-ft radius, indicates that free convection effects should be considered for axial distances greater than a diameter or two when the speed is low (~ 4 ft/sec) and at distances greater than five diameters at higher speeds (~ 20 ft/sec). Furthermore, the free convection effect increases as the radius of the cylinder increases.

While the secondary flow profile is required to be small for the present exact analysis to apply, it always exhibits an inflection point. The effect of this on boundary-layer stability and transition is potentially significant and should be investigated further.

ACKNOWLEDGMENT

The authors wish to express their sincere appreciation to G. M. Harpole for the numerical analysis necessary to complete this work.

CONTENTS

SYMBOLS

a = radius of the cylinder

$\bar{g}$ = gravitational acceleration, Eq. (3)

g = similarity nondimensional temperature, Eq. (7)

G = similarity nondimensional temperature independent of ϕ, Eqs. (9) and (10)

Gr = Grashof number, Eq. (3)

f = nondimensional stream function, Eqs. (5), (6), and (7)

F = similarity nondimensional stream function, Eqs. (9) and (10)

Nu = Nusselt number, Eq. (13) and (14)

Pr = Prandtl number, Eq. (3)

r = radial coordinate

Re = Reynolds number, Eq. (3)

T = temperature

u = axial velocity

v = circumferential velocity

w = radial velocity

x = axial coordinate

α = thermal diffusivity

β = thermal expansion coefficient

ε = Gr/Re^2, Eq. (3)

η = $r/\sqrt{2x}$, Blasius similarity variable, Eq. (5)

θ = nondimensional temperature, Eq. (3)

ν = kinematic viscosity

ϕ = circumferential coordinate

τ = shear stress

Subscripts

0 = zeroth order solution

1 = first order solution

w = surface

∞ = free stream

Superscripts

— = dimensional quantities

′ = derivative with respect to η

x = local quantities at location x

I. INTRODUCTION

Free-forced convection has been a subject of great importance in such engineering devices as solar collectors and nuclear reactors and in such applications as boundary-layer stabilization. The effect of free convection on two-dimensional boundary layers has been extensively studied because of its simple nature--for example, the heated (or cooled) vertical flat plate with upflow or downflow. The relative importance of free convection in forced convection depends on the ratio $Gr_{\bar{x}}/Re_{\bar{x}}^{2} = \beta g \bar{x} \Delta T / u_{\infty}^{2}$, which can be small for a short plate and becomes one of the dominant effects for a large plate. Acrivos [1] established the ranges of different regions using an integral approach. Similar results were obtained by Sparrow and Gregg [2]. A more complex two-dimensional problem of co-flow or counter-flow was investigated by Gersten and Körner [3]. They looked at stagnation flow with suction or blowing under mixed free-forced convection conditions. The buoyancy forces were shown to act as an adverse pressure gradient to counter-flow and to lead to an earlier separation.

Cross-flow buoyancy forces may be generated by a heated horizontal surface. Under conditions which do not lead to thermal instabilities, analyses have been reported by Mori [4], Sparrow and Minkowycz [5], Hauptmann [6], and more recently by Redekopp and Charwat [7]. It was found that the buoyancy effect had to be considered if $Gr_{\bar{x}}/Re_{\bar{x}}^{3/2}$ was greater than 0.01. The heated-from-below situation causes a more dramatic effect than the heated-from-above situation. This is because the heated-from-below case results in an inflection point in the velocity profile and magnifies separation as well as initiating thermal instabilities.

A hot horizontal cylinder in cross-flow has been the subject of several investigations. The most recent work of Oosthuizen and Hart [8] presents a criterion for consideration of buoyancy forces which depends on the direction of the flow relative to the vertical. A vertical heated cylinder in upflow or downflow has also been investigated [9].

An exact solution of the three-dimensional laminar boundary layer over a heated surface when gravity-driven buoyancy effects introduce significant cross flow into an otherwise axially symmetric flow geometry is presented in this report. The effect of surface heating on laminar boundary-layer stability and transition has been analyzed by Wazzan, Okamura and Smith [10], and the effect on separation by Aroesty and Berger [11]. The buoyancy cross-flow effects have not, however, been included in such studies.

The physical model chosen for study is a semi-infinite cylinder of radius a, which is aligned with its axis parallel to a uniform flow and normal to the direction of gravity. The uniform flow is assumed to have a velocity u_∞ and temperature T_∞. The surface of the cylinder is heated to a constant temperature T_w $(T_w > T_\infty)$. For most external flows, the buoyancy force can be neglected in a small pure forced convection region of size $0\ (u_\infty \nu^{1/3}/[\beta g(T_w - T_\infty)]^{2/3})$ downstream of the leading edge. Beyond that region the effect of buoyancy cross-flow increases as the fluid flows downstream. There is, however, a region where it is still small and can be treated as second order. Further downstream, a distance of order $aRe/Gr^{1/2}$, the initially small buoyancy effect becomes as important as the forced convection effects and can no longer be treated as a second order effect. The solution of the intermediate region where the buoyancy effect is second order is presented in this paper. In this region, the fluid which is entrained by the basic forced laminar boundary-layer flow is given a cross-flow component by the buoyancy effect of the heated walls. Because of this entrainment, the secondary flow velocity grows linearly in the downstream direction. This is a different situation from the usual pure free convection flow, where all boundary-layer entrainment is through buoyancy, and where the axial entrainment is zero.

II. ANALYSIS

The nondimensional Boussinesq boundary-layer equations in cylindrical coordinates, as shown in Fig. 1, are

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial r} + \frac{\partial v}{\partial \phi} = 0 , \tag{1-a}$$

$$u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial r} + v \frac{\partial u}{\partial \phi} = \frac{\partial^2 u}{\partial r^2} , \tag{1-b}$$

$$u \frac{\partial v}{\partial x} + w \frac{\partial v}{\partial r} + v \frac{\partial v}{\partial \phi} = \frac{\partial^2 v}{\partial r^2} + \varepsilon \cdot \sin \phi \cdot \theta , \tag{1-c}$$

$$u \frac{\partial \theta}{\partial x} + w \frac{\partial \theta}{\partial r} + w \frac{\partial \theta}{\partial \phi} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial r^2} , \tag{1-d}$$

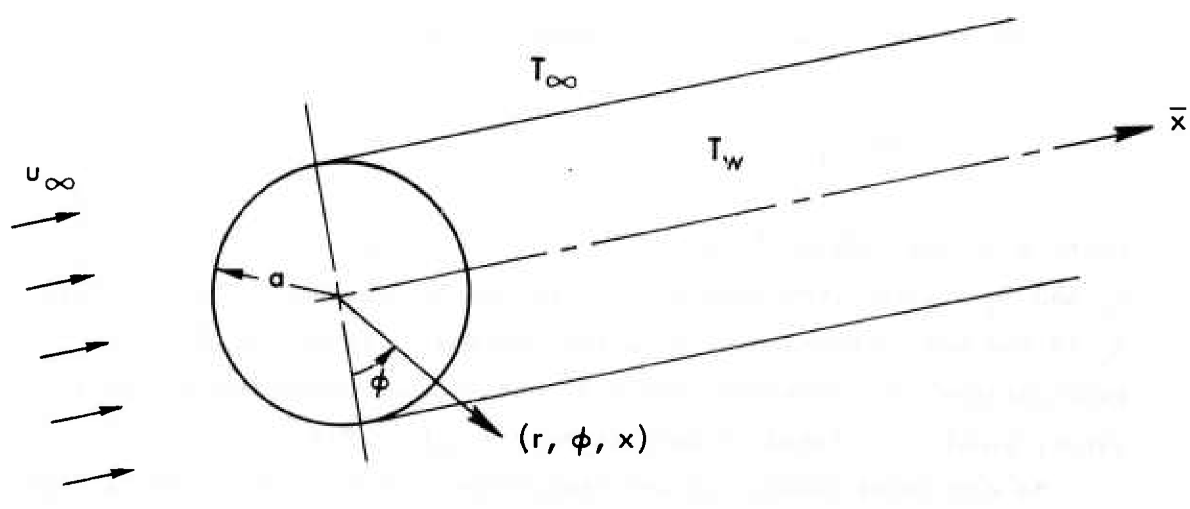

Fig. 1 — Physical model and coordinates

after neglecting smaller order terms under the condition that

$$Gr/Re^{3/2} > 1 \ . \qquad (2)$$

The nondimensional variables used in scaling Eqs. (1) are

$$\left.\begin{aligned}
&u = \frac{\bar{u}}{u_\infty} \ , \ v = \frac{\bar{v}}{u_\infty} \ , \ w = \frac{\bar{w}\sqrt{Re}}{u_\infty} \qquad \text{(the velocities) ;} \\
&\theta = \frac{T - T_\infty}{T_w - T_\infty} \qquad \text{(the temperature) ;} \\
&x = \frac{\bar{x}}{a} \ , \ r = \frac{(\bar{r} - a)\sqrt{Re}}{a} \ , \qquad \text{(the coordinates) ;} \\
&Re = \frac{u_\infty a}{\nu} \qquad \text{(the Reynolds number) ;} \\
&Gr = \frac{\beta g a^3 (T_w - T_\infty)}{\nu^2} \qquad \text{(the Grashof number) ;} \\
&Pr = \nu/\alpha \qquad \text{(the Prandtl number) , and} \\
&\varepsilon = Gr/Re^2 \ ,
\end{aligned}\right\} \qquad (3)$$

where a is the radius of the cylinder, ν is the kinematic viscosity, u_∞ and T_∞ are the free stream velocity and temperature, respectively, T_w is the wall temperature, α is the thermal diffusivity, g is the gravitational acceleration, and β is the thermal expansional coefficient, which is related to density by $\rho = \rho_\infty[1 - \beta(T - T_\infty)]$.

As mentioned above, for the region very close to the leading edge of the cylinder, the buoyancy effect is negligible. The magnitude of this region, defined by Eq. (2), is of the order

$$\bar{x} \sim u_\infty \nu^{1/3}/[\beta g(T_w - T_\infty)]^{2/3} \ .$$

Downstream of the leading-edge region, the solution of equations in (1) can be expanded into a series of ε, if ε is small, so that

$$u = u_0 + \varepsilon u_1 + \dots \qquad (4\text{-a})$$

$$v = \varepsilon v_1 + \dots \qquad (4\text{-b})$$

$$w = w_0 + \varepsilon w_1 + \dots \qquad (4\text{-c})$$

$$\theta = \theta_0 + \varepsilon \theta_1 + \dots \qquad (4\text{-d})$$

Substitution of the expansion given by Eqs. (4) into Eqs. (1), and the collection of terms of equal order will result in the perturbation equations. The perturbation equations of lowest order are

$$(\varepsilon^\circ): \quad f_0''' + f_0 f_0'' = 0 \ , \qquad (5\text{-a})$$

$$\theta_0'' + \mathrm{Pr} f_0 \theta_0' = 0 \ , \qquad (5\text{-b})$$

where the prime denotes a derivative with respect to η, with $\eta = r/\sqrt{2x}$ being the Blasius similarity variable. The stream function f_0 is defined by

$$u_0 = f_0'(\eta) \ , \qquad (5\text{-c})$$

$$w_0 = \frac{1}{\sqrt{2x}} (\eta f_0' - f_0) \ . \qquad (5\text{-d})$$

The solution of Eq. (5-a) is the Blasius solution. Equation (5-b) is the forced-convection energy equation, whose solution was first given by Pohlhausen in 1921.

The second order perturbation equations are

$$(\epsilon^1): \quad f_1''' + f_0 f_1'' - 4f_0' f_1' + 5f_0'' f_1 + f_0'' \frac{\partial f_2}{\partial \phi} = 0 , \tag{6-a}$$

$$f_2''' + f_0 f_2'' - 2f_0' f_2' + \sin\phi \cdot \theta_0 = 0 , \tag{6-b}$$

$$g'' + Pr\left(f_0 g' - 4f_0' g + 5\theta_0' f_1 + \theta_0' \frac{\partial f_2}{\partial \phi}\right) = 0 . \tag{6-c}$$

The stream functions f_1 and f_2, and the temperature g are defined by

$$\left.\begin{aligned} u_1 &= (2x)^2 f_1'(\eta,\phi) , \\ v_1 &= (2x) f_2'(\eta,\phi) , \\ w_1 &= (2x)^{3/2}\left(\eta f_1' - 5f_1 - \frac{\partial f_2}{\partial \phi}\right) , \\ \theta_1 &= (2x)^2 g(\eta,\phi) . \end{aligned}\right\} \tag{7}$$

The boundary conditions for Eqs. (6) are

$$\left.\begin{aligned} & f_1(0,\phi) = f_1'(0,\phi) = f_1'(\infty,\phi) = 0 , \\ & f_2'(0,\phi) = \frac{\partial f_2}{\partial \phi}(0,\phi) = f_2'(\infty,\phi) = f_2'(\eta,0) = 0 , \\ & g(0) = g(\infty) = 0 . \end{aligned}\right\} \tag{8}$$

Equations (6) are separable in terms of η and ϕ with dependent variables of the form

$$\left.\begin{aligned} f_1(\eta,\phi) &= F_1(\eta) \cos\phi , \\ f_2(\eta,\phi) &= F_2(\eta) \sin\phi , \\ g(\eta,\phi) &= G(\eta) \cos\phi . \end{aligned}\right\} \tag{9}$$

Equations (6) can be further reduced to the following ordinary differential equations:

$$F_1''' + f_0F_1'' - 4f_0' F_1' + 5f_0'' F_1 = -f_0'' F_2 , \qquad (10\text{-a})$$

$$F_2''' + f_0F_2'' - 2f_0' F_2' = -\theta_0 , \qquad (10\text{-b})$$

$$\frac{1}{Pr} G'' + (f_0G' - 4f_0' G) = -\theta_0' (5F_1 + F_2) . \qquad (10\text{-c})$$

The associated boundary conditions for Eqs. (10) are

$$\left.\begin{aligned} &F_1(0) = F_1'(0) = F_1'(\infty) = 0 , \\ &F_2(0) = F_2'(0) = F_2'(\infty) = 0 , \\ &G(0) = G(\infty) = 0 . \end{aligned}\right\} \qquad (11)$$

Solutions of Eqs. (10) are integrated numerically for Pr = 0.01, 1, 8. These results will be utilized below in heat transfer and shear-stress calculations. It is of interest to point out that Eqs. (7) indicate that the effect of cross-flow becomes stronger as the fluid moves downstream. Combining Eqs. (4), (5-c), (5-d), and (7) gives

$$\left.\begin{aligned} &u = F_0' + \varepsilon(2x)^2 F_1' \cos\phi + \ldots , \\ &v = \varepsilon(2x) F_2' \sin\phi + \ldots , \\ &w = \frac{1}{\sqrt{2x}} (\eta F_0' - F_0) + \varepsilon(2x)^{3/2} (\eta F_1' - 5F_1 + F_2) \cos\phi + \ldots ; \\ &\theta = \theta_0 + \varepsilon(2x)^2 G \cos\phi + \ldots . \end{aligned}\right\} \qquad (12)$$

The above expansion breaks down at a distance of $O(a \cdot Re/Gr^{\frac{1}{2}})$, corresponding physically to the point beyond which the buoyancy cross-flow effect, initially small, becomes as important as the forced-convection effects. Solutions for the region beyond $O(a \cdot Re/Gr^{\frac{1}{2}})$ will not be considered in this report.

III. RESULTS AND DISCUSSION

Numerical values of the functions F_1, F_1', F_2, F_2', and G are presented in Figs. 2, 3, and 4 for Pr = 0.01, 1, 10. Equations (12) show that the buoyancy force stabilizes the flow and accelerates its speed over the lower half of the cylinder ($-\pi/2 < \phi < \pi/2$). Over the upper half of the cylinder ($\pi/2 < \phi < 3\pi/2$), the axial flow is decelerated and may be destabilized. The cross flow is accelerated by the buoyancy force from the lower stagnation point ($\phi = 0$) to attain its maximum value at $\phi = \pi/2$, and then decelerated to its upper stagnation point ($\phi = \pi$). It is interesting to note that the solution (12) represents the case of a vertical flat plate when $\phi = \pi/2$ for the region near the leading edge where the cross flow is small. For a vertical flat plate, the cross flow is decoupled from the axial flow; in other

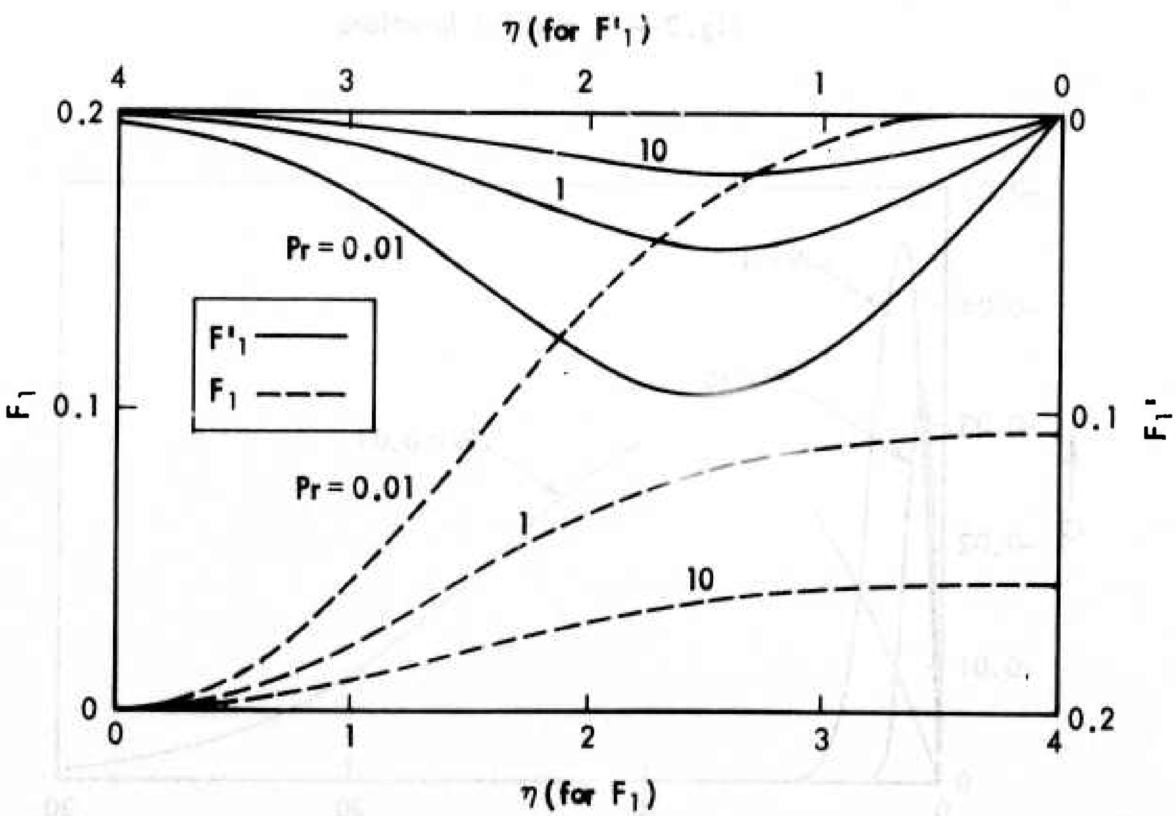

Fig. 2 — F_1 and F_1' functions

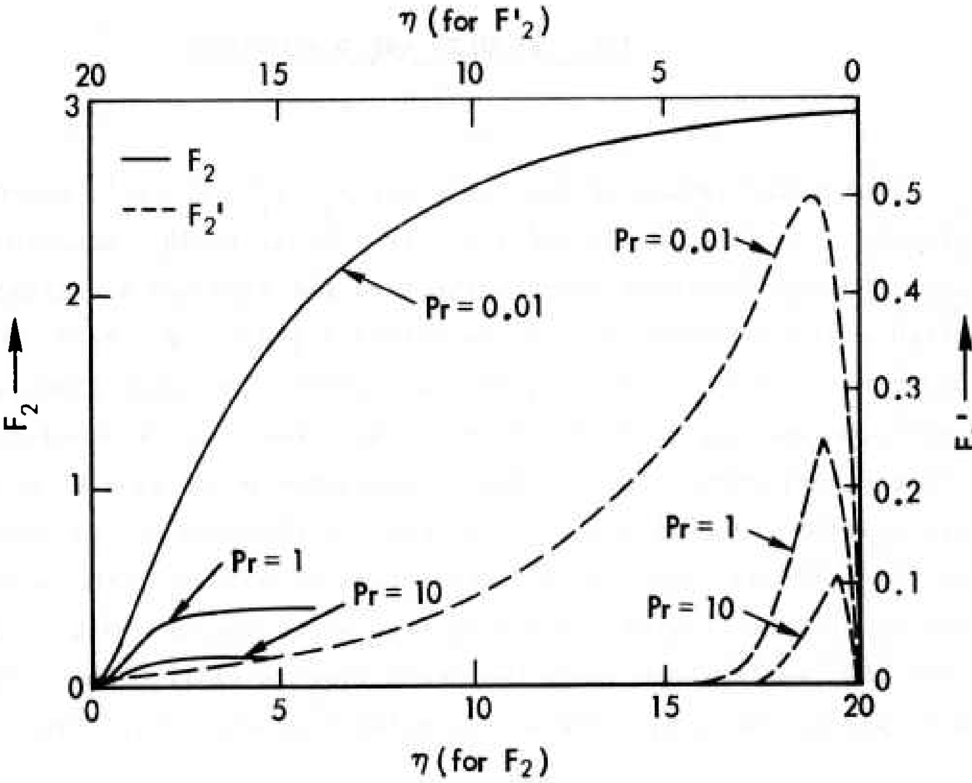

Fig. 3 — F_2 and F_2' functions

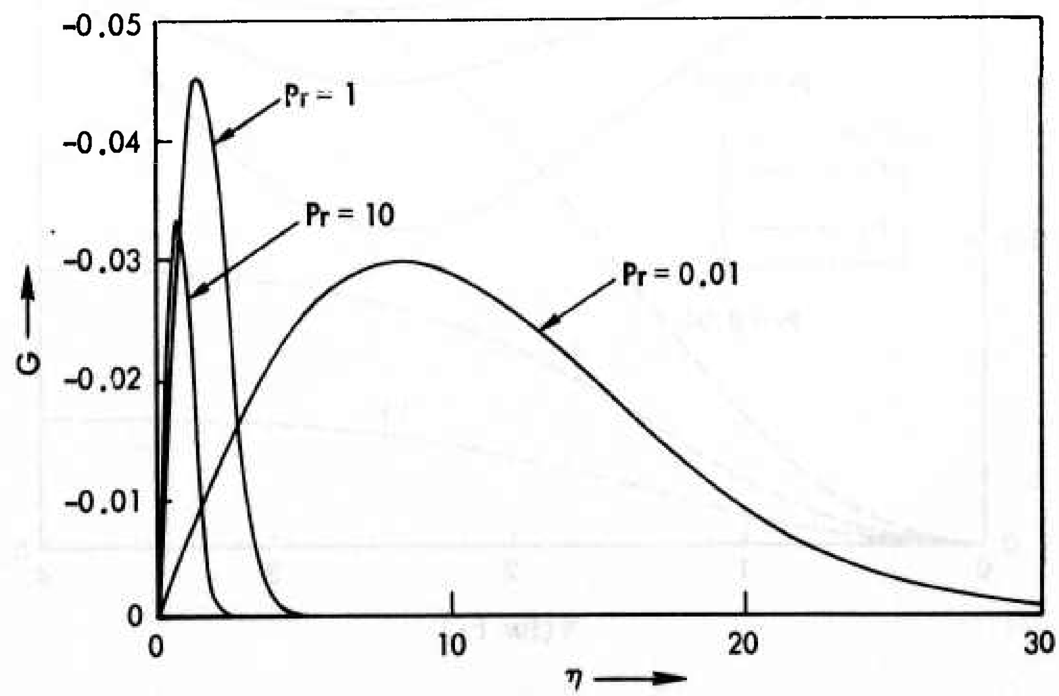

Fig. 4 — G function

words, the cross flow does not affect the axial flow and the temperature distribution. The magnitude of the circumferential velocity, v, is being increased as a linear function of the axial coordinate, and eventually becomes one of the dominant velocity components when the fluid flows far enough downstream.

The local Nusselt number with respect to a and $(T_w - T_\infty)$ can be derived from Eq. (12), which is

$$Nu_x = -\sqrt{\frac{Re}{x}}\left[\theta_0'(0) + \varepsilon(2x)^2\, G'(0)\cdot\cos\phi + \ldots\right]. \qquad (13)$$

The first term of Eq. (13) represents the Nusselt number for a pure forced convection. The relative importance of heat transfer due to cross flow can be indicated by

$$Nu_x/(Nu_x)_{fc} = 1 + \varepsilon(2x)^2\, G_1'(0)/\theta_0' \cdot \cos\phi + \ldots, \qquad (14)$$

where fc denotes the forced convection. Equation (14) shows that the cross flow enhances the heat transfer on the lower half of the cylinder and degrades it on the upper half of the cylinder. Values of $G'(0)/\theta_0'(0)$ are given in the following table for Pr = 0.01, 1, 10. Equation (14)

Pr	$G'(0)/\theta_0'(0)$	$F_1''(0)/f_0''(0)$	$F_2''(0)$
0.01	0.08638	0.19509	0.92627
1	0.09959	0.09959	0.61904
10	0.06576	0.04458	0.39761

has been plotted on Fig. 5 for $\phi = 0$ and π. The curves clearly show the developing cross-flow effect along the x-axis and show where the originally small cross-flow effect grows and will eventually overwhelm the pure forced-convection one, beyond which the expansions (12) are no longer valid.

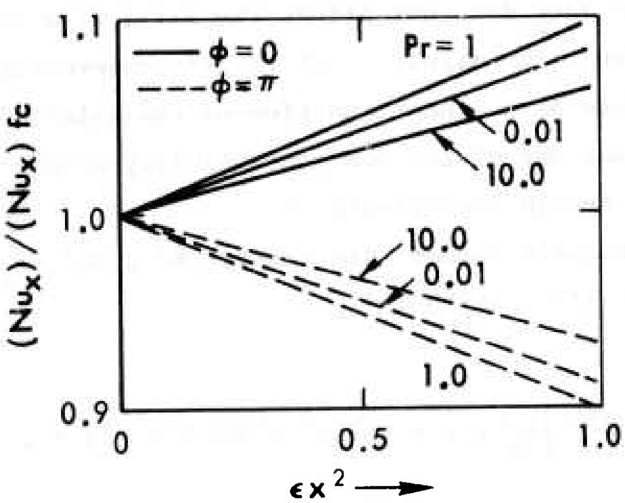

Fig. 5— Heat flux distribution

The local shear stress at the cylinder surface can be computed from the equation

$$\tau_{rx} = \mu \left(\frac{\partial u}{\partial r}\right)_{r=0} \quad \text{and} \quad \tau_{r\phi} = \mu \left(\frac{\partial v}{\partial r}\right)_{r=0} .$$

Introducing the series expansions (12), the relative importance of the cross-flow effect on the axial shear stress can be found by

$$\tau_{rx}/(\tau_{rx})_{fc} = 1 + \varepsilon(2x)^2 \; F_1''(0)/f_0''(0) \cdot \cos\phi + \ldots \; . \qquad (15)$$

The circumferential shear stress can be shown to be proportional to

$$\tau_{r\phi} \sim \varepsilon\sqrt{2x}\; F_2''(0) \cdot \sin\phi \; . \qquad (16)$$

Values of $F_1''(0)/f_0''(0)$ and F_2'' also are given in the table above. The ratio given in Eq. (15) has been plotted in Fig. 6 for $\phi = 0$ and $\phi = \pi$. As can be seen, for the $\phi = \pi$ curves the ratio tends to approach zero, for large enough εx^2, which indicates that separation is imminent. For moderate free-stream velocities, this can occur within several radii of

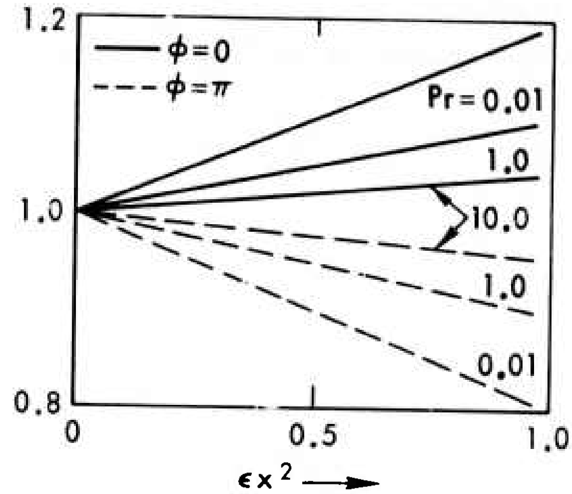

Fig. 6— Shear stress distribution

the leading edge. Further, the unevenly distributed axial shear stress, Eq. (15), can induce a pitch moment and could cause an oscillation of the moving cylinder.

Equations (14) and (15) indicate that the cross-flow effect on heat transfer and shear stress for a heated horizontal cylinder grows rapidly when the fluid flows downstream, proportionally to x^2. This means that an initially small cross-flow effect, which may be neglected in the region close to the leading edge of the cylinder, cannot be ignored for a heated, long slender body.

The analysis is strictly applicable to the flow over the outer surface of a heated hollow cylinder. It is also relevant to the flow over a slender body of revolution, provided that the pressure gradient associated with nose shape is negligible. The effect of pressure gradient and nose shape will be reported later.

The parameter ε has been evaluated for water, subject to a 30° wall overheat and a cylinder radius of 1 ft. When u_∞ is 3 ft/sec > $\varepsilon \sim .01$ and when u_∞ is 30 ft/sec, $\varepsilon \sim .0001$. The magnitude of the cross flow is proportional to εx^2; this suggests that the present analysis is valid when $x < 10$ ft, at low speeds. Within this region, the effects on laminar heat transfer and shear stress are not overwhelming; however, the existence of the inflection point in the cross-flow velocity profile can have profound impact on the hydrodynamic stability of the laminar flow.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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Using analytical methods, this report considers the effect of forces acting counter to laminar flow around heated bodies moving through water at low speeds. A cross flow is induced when a uniform horizontal stream passes along a heated semi-infinite horizontal cylinder. The cross-flow effects on heat transfer and shear stress grow as the fluid flows downstream and eventually becomes one of the dominant flow mechanisms. When the analysis applied to specifically a cylinder of one foot radius, heated to 100 degrees F, at speeds of about four feet per second, free convection effects should be considered for distances as great as four feet from the leading edge; at about 20 feet per second, out to ten feet from the leading edge. (BK)