

U.S. DEPARTMENT OF COMMERCE
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AD-A024 969

CUMULATIVE DEFLECTION AND RIGID PAVEMENT SERVICEABILITY

CLARKSON COLLEGE OF TECHNOLOGY

PREPARED FOR
AIR FORCE CIVIL ENGINEERING CENTER

NOVEMBER 1975

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AFCEC-TR-75-20

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AD A 024969

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CLARKSON COLLEGE OF TECHNOLOGY
POTSDAM, NEW YORK

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DISTRIBUTION

FINAL REPORT: JULY 1974 - SEPTEMBER 1975

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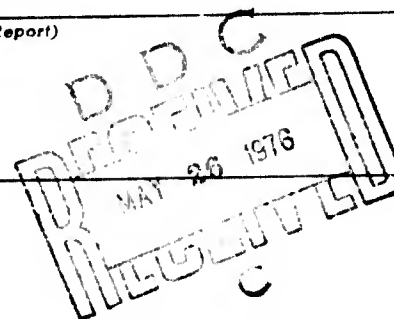
AIR FORCE CIVIL ENGINEERING CENTER
(AIR FORCE SYSTEMS COMMAND)

TYNDALL AIR FORCE BASE
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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM								
1. REPORT NUMBER AFCEC-TR-75-20	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER								
4. TITLE (and Subtitle) CUMULATIVE DEFLECTION AND RIGID PAVEMENT SERVICEABILITY		5. TYPE OF REPORT & PERIOD COVERED Final Report July 1974 - September 1975								
		6. PERFORMING ORG. REPORT NUMBER								
7. AUTHOR(s) William H. Highter		8. CONTRACT OR GRANT NUMBER(s) F29601-75-C-0002 <i>nu</i>								
9. PERFORMING ORGANIZATION NAME AND ADDRESS Clarkson College of Technology Potsdam, New York 13676		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Program Element 63723F Project 21041A17								
11. CONTROLLING OFFICE NAME AND ADDRESS Air Force Civil Engineering Center Air Force Systems Command Tyndall Air Force Base, Florida 32401		12. REPORT DATE November 1975								
		13. NUMBER OF PAGES 62								
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified								
		15a. DECLASSIFICATION DOWNGRADING SCHEDULE								
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.										
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)										
18. SUPPLEMENTARY NOTES Available in DDC.										
19. KEY WORDS (Continue on reverse side if necessary and identify by block number)										
<table border="0"> <tr> <td>Pavement performance</td> <td>Airfield pavement performance</td> </tr> <tr> <td>Pavement energy</td> <td>Airfield pavement deflections</td> </tr> <tr> <td>Airfield pavement evaluation</td> <td>Civil engineering</td> </tr> <tr> <td>Energy concepts</td> <td></td> </tr> </table>			Pavement performance	Airfield pavement performance	Pavement energy	Airfield pavement deflections	Airfield pavement evaluation	Civil engineering	Energy concepts	
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Airfield pavement evaluation	Civil engineering									
Energy concepts										
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This report describes efforts to relate the energy imparted to a rigid pavement system (as measured by cumulative deflections) to the condition of that system. Analysis of raw data from the Americal Association of State Highway Officials Road Test showed no results that could be used for predictive purposes. Use of averaged data points indicated there exists a threshold cumulative deflection beyond which pavement serviceability changes rapidly.										



SUMMARY

Section II reviews three methods by which rigid pavement deflection can be predicted. The Winkler hypothesis and solid elastic solutions require that the engineer assume a model that includes parameters that characterize the pavement system and these parameters must be known. Transfer function theory, on the other hand, does not require the engineer to assume a model. The transfer function itself contains the descriptors needed to characterize the pavement.

Also discussed in Section II are various pavement performance criteria. The concept of the Present Serviceability Index (PSI) was described in some detail. The results of previous laboratory research on the fatigue of Portland cement concrete are reviewed and the significance of rigid pavement deflection is discussed.

Section III presents the working hypothesis of the present research and its theoretical basis. The strain energy of a loaded continuous beam supported by a Winkler foundation and the strain energy of a deformed circular slab supported by a Winkler foundation are both shown to be related to the load-induced deflection. The procedure whereby the cumulative pavement deflections can be obtained from existing American Association of State Highway Officials (AASHO) Road Test data is also described in Section III.

The analysis of AASHO Road Test data and the results of the analysis are described in Section IV. Regression analysis was used to determine a functional relationship between the serviceability (PSI) of rigid pavements and the cumulative deflection it had experienced. However, because of the wide range in data and the wide variation in behavior or replicate pavement test sections, the regression analysis did not yield promising results. The data were then classified according to slab thickness and averaged. The results indicate that the PSI-cumulative deflection relationship for each slab thickness consists of two straight lines when cumulative deflection is plotted to a logarithmic scale. It was found that, initially, the PSI decreases very little as cumulative deflection increases. A sharp break in the curve then occurs after which large changes in PSI result from increases in cumulative deflection. The cumulative deflection corresponding to the break in the curve is referred to as the threshold cumulative deflection. The threshold cumulative deflection increases as slab thickness increases. The relationship between the averaged cumulative deflection and PSI lends strong support to the validity of the working hypothesis. For illustrative purposes, the usefulness of PSI - cumulative deflection relationships was shown by predicting the service life of a pavement when traffic forecasts are available.

FOREWORD

This report was prepared by Clarkson College of Technology under contract No. F29601-75-C-0002, job order 21041A17, with the Air Force Civil Engineering Center, Tyndall Air Force Base FL.

This report summarizes work accomplished between July 1974 and September 1975. 1st Lt R. A. Bourquard was the former Center project officer. Major George Ballentine is the current project officer.

This report has been reviewed by the Information Officer (IO) and is releasable to the National Technical Information Service (NTIS). At NTIS, it will be available to the general public, including foreign nations.

This technical report has been reviewed and is approved for publication.

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SECTION I

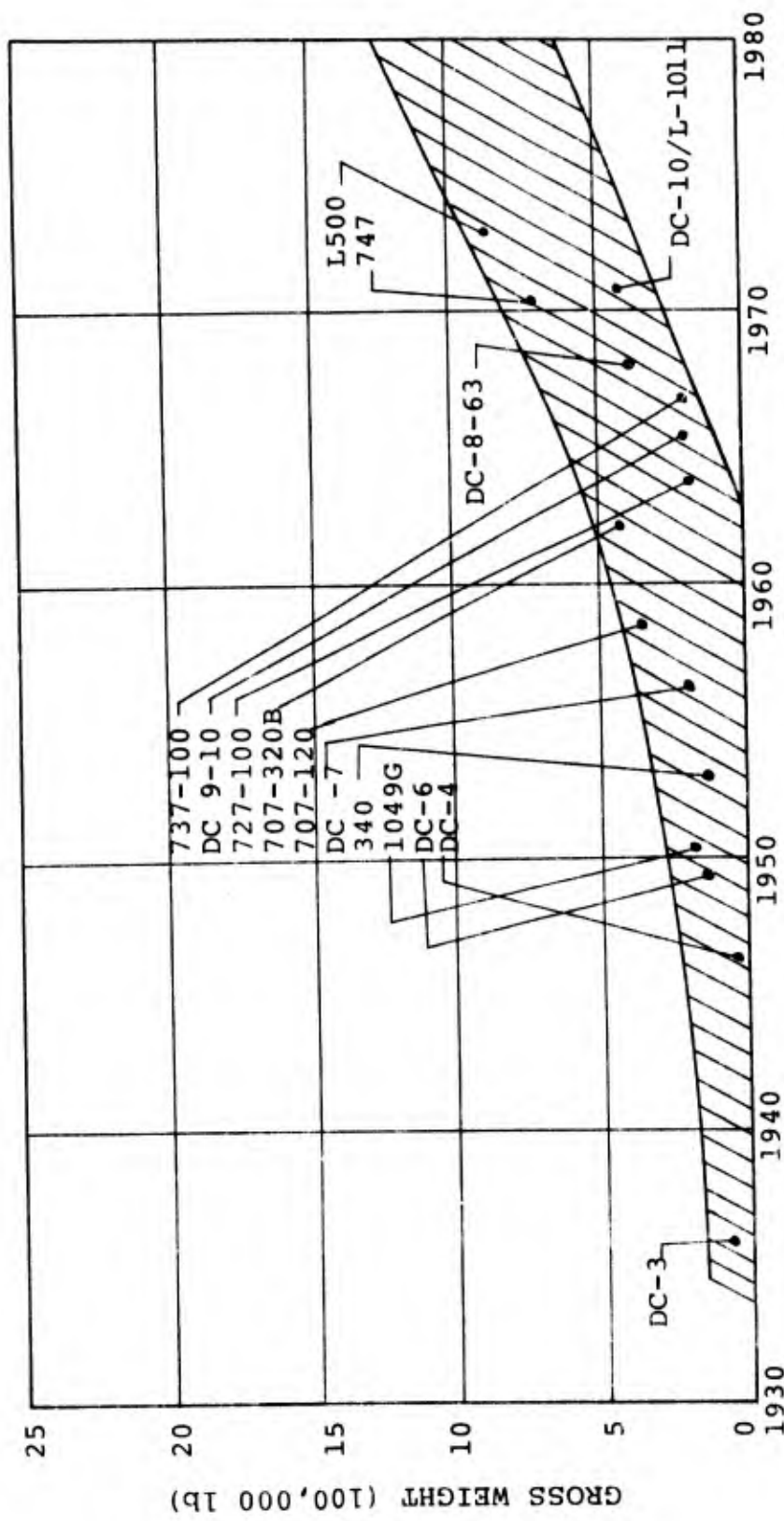
INTRODUCTION

Most methods of rigid pavement design for airfields and highways are based on considerations of load-induced stresses in elastic slabs. Airfield pavements are subjected to much heavier loads than highway pavements but the number of load repetitions and the frequency of loading is smaller for airfield pavements than for highway pavements. Despite these differences, it is recognized that factors which must be considered in the design of both airfield and highway pavements include the slab thickness, reinforcing, and characteristics of the base course and subgrade material.

Yoder (Reference 1) observed that while excessive deformation of the foundation, and shear of one or more of the pavement components are major causes of pavement distress, distress due to fatigue of the concrete under repeated loads may be equally significant. Pavement distress due to fatigue may become more important in the future as aircraft wheel loads and highway legal loads increase and exceed those contemplated by designers when the pavements were constructed because the number of load repetitions which produce fatigue distress decreases as the load-induced stress increases.

The trend in increasing gross weight of aircraft entering service is illustrated in Figure 1. Individual wheel loads are also increasing: the Boeing 747 has a wheel load of 41,600 pounds; a DC-10 model scheduled to enter service in a few years will have an individual wheel load of 52,600 pounds.

This research effort uses cumulative deflections to correlate the performance of rigid pavements with the volume of traffic applied. The approach allows pavement performance as influenced by foundation deformation, shear, and fatigue to be taken into account. Deflections rather than stresses are used because in the field it is much easier to measure dynamic deflections than dynamic stresses, and dynamic deflection measuring devices are presently available. American Association of State Highway Officials (AASHO) Road Test data are used to establish a cumulative deflection-performance relationship for rigid pavements.



YEAR ENTER SERVICE

After Robinson (Ref. 2)

Figure 1. Aircraft Gross Weight Trend

SECTION II

REVIEW OF THE LITERATURE

INTRODUCTION

It has been known for some time that the repeated application of loads which induce stresses well below the modulus of rupture of a given material can cause the material to fail. This phenomenon is called fatigue failure and it is attributed to the fact that materials are not ideal homogeneous solids (Reference 3). Portland cement concrete exhibits this behavior. Curves depicting the fatigue phenomenon usually have stress or strain on the ordinate versus cycles of load on a logarithmic abscissa. Such relationships are difficult for the pavement engineer to apply to in-service rigid pavements because it is difficult and time consuming to measure in-situ stresses or strains. Deflection measurements, on the other hand, are made much more easily; the Air Force currently has a vehicle-mounted laser deflection measuring system under development which will be able to measure and compile deflections accurately with little or no interruption to traffic. Thus a correlation of deflections with a rigid pavement performance index, which includes fatigue effects, would provide a valuable tool to the pavement engineer.

As a first step toward a solution of the problem of preventing fatigue failures in pavements, methods of predicting and measuring deflections are valuable and a summary of important contributions in this area is presented here. The Winkler and solid elastic theories offer models to predict the behavior of the pavement; transfer function theory provides a relationship between a response of a pavement system and an operational input to that system. Previous work on fatigue in Portland cement concrete is also discussed in this section as well as the PSI, a measure of the relative ability of pavements to serve traffic which was developed for use in the AASHO Road Test.

THE WINKLER HYPOTHESIS

Winkler (Reference 4) was among the first to consider the transfer of load between surface material and an underlying supporting material having different characteristics. The subgrade was modeled as a dense fluid which is equivalent to a system of independent, linear springs. This is a discontinuous type of foundation because the load is transferred through the surface material only and springs (supporting material) located outside the perimeter of the surface material do not deflect when the load is applied. Winkler hypothesized that the reaction of the foundation at any point beneath the loaded area

was

$$P(x) = k W(x) \quad (1)$$

where

- P(x) = the reaction of the foundation at point x
- W(x) = the deflection of the subgrade surface at point x
- k = a proportionality constant frequently called the foundation modulus or modulus of subgrade reaction

Winkler did not discuss the physical significance of k, its range of magnitude, nor what characteristics it would depend upon. Zimmerman (Reference 5) applied the Winkler hypothesis when analyzing railroad tracks and noted that k depended on the type of subgrade material.

Hayashi (Reference 6) modified the Winkler hypothesis to account for the shear which exists between a beam and underlying soil when the beam is deformed. Florin (Reference 7) noted that the inclusion of the shear force did not significantly affect the resulting vertical pressure (P) exerted on the foundation.

Westergaard (Reference 8) modeled a rigid highway pavement as a solid elastic plate or slab supported by a bed of springs. The analysis was later extended to include airfield rigid pavements (Reference 9). It was recognized that the value of k for a given foundation depended not only on the characteristics of the foundation but also on the dimensions of the loaded area. Yet, because a fourfold increase in k, from 50 to 200 lb/in³, resulted in only minor changes in the computed pavement stresses, it was assumed that k was constant at all points under consideration. The recommendation was made that a standard test procedure be adopted to determine k values.

Yoder (Reference 1) pointed out that small errors in measuring the foundation modulus do not affect the design of a rigid pavement significantly: for typical highway wheel loads the reduction in required thickness resulting from increasing k from 100 to 500 lb/in³ was only 1 inch, all other factors being the same.

Timoshenko (Reference 10) analyzed the bending of a railroad track by modeling the support as a Winkler foundation. Since the foundation modulus appeared as a fourth root ($\sqrt[4]{k}$) in the solutions for rail stresses and foundation pressure, it was concluded that errors in k would produce much smaller errors in the computed stresses and pressures.

Terzaghi (Reference 11) pointed out that the foundation modulus depended on the dimensions of the loaded area as well as elastic properties of the foundation material. It was noted

that the Winkler hypothesis had met with success in predicting stresses and bending moments but was not reliable as a predictor of deflections. Vesic and Saxena (Reference 12) reached a similar conclusion as a result of work with AASHO Road Test rigid pavement data. It was found that no single value of k yielded perfect agreement between statically induced bending moments, contact pressures and deflections. The suggestion was made that the foundation modulus, in addition to being a function of foundation soil type and condition, is inversely proportional to the slab thickness. Sebastyan (Reference 13) indicated that the foundation modulus depended on the load intensity to which the soil was subjected. This would help explain some of the discrepancy between observed deflections and those predicted using the Winkler hypothesis.

In summary, the literature indicates that the foundation modulus k is a function of the type of soil and its condition, the dimensions of the loaded area, and the level of load intensity. A single value of k cannot provide agreement between bending moments, contact pressures, and deflections. Theories using the Winkler hypothesis can be used to determine values of bending moments and stresses that agree reasonably well with observations, but such theories generally have not met with much success in predicting deflections.

SOLID ELASTIC MODELS

As previously mentioned, Westergaard (Reference 8) was the first to apply elastic theory to rigid pavements by assuming that the concrete pavement slab was homogeneous and isotropic, and that the subgrade reaction was vertical and proportional to the deflections, i.e., a Winkler foundation.

Because of the lack of agreement between all statical influences such as bending moments, shearing forces, contact pressures, and deflections that were obtained when the Winkler hypothesis was applied, other researchers sought models that would yield such agreement. Biot (Reference 14) solved for the contact pressures between an infinite elastic beam and its foundation which was modeled as a semi-infinite elastic solid. Once the contact pressures were known, Biot did a back-calculation which could determine the equivalent foundation modulus, k . The equivalent k was found to be a function of the rigidity of the beam as well as the elastic properties of the subgrade. From this analysis the conclusion could be that a given foundation material does not have a unique foundation modulus.

Hogg (Reference 15) solved the problem of a thin plate infinite in two dimensions supported by a semi-infinite elastic solid. Neglecting axial stresses perpendicular to the plate and shearing stresses parallel to the plate, solutions for the deflection and curvature of the plate for concentrated and circular uniformly distributed loads were obtained.

Burmister (References 16 and 17) solved the problem of a multilayered solid elastic system subjected to a uniformly distributed circular load. It was assumed that each layer was homogeneous and isotropic, infinite in two dimensions, and continuously supported; furthermore, the interface between layers was either perfectly rough (no slip) or perfectly smooth (no shear), the deformations of each layer were small, and the mass inertia of the system was negligible. Each layer, then, had two parameters - Young's modulus (E) and Poisson's ratio (ν). These assumptions are usually applied to elastic layer theory and simplify the mathematics.

Vesic and Saxena (Reference 12) found that the load response of AASHO Road Test subgrades could be described by a homogeneous, isotropic, elastic solid with two characteristics, a deformation modulus and Poisson's ratio. Burmister (Reference 18) applied elastic layer theory to evaluate the strength properties of various layers of the pavements used in the Western Association of State Highway Officials road test using data from in-service observations.

The advantage of models using solid elastic theory with respect to those using the Winkler hypothesis is that individual layers can be easily characterized. Lateral deformations can also be taken into effect and various interface conditions can be described mathematically.

TRANSFER FUNCTION THEORY

A transfer function is the ratio of an output, expressed as the Laplace transfer of the output, to an operational input, also expressed as the Laplace transform, of a time dependent system. This definition can be expressed mathematically as

$$\bar{G}(s) = \frac{\bar{O}(s)}{\bar{I}(s)} \quad (2)$$

where

- $\bar{I}(s)$ is the Laplace transfer of a time-dependent input function, $I(t)$
- $\bar{O}(s)$ is the Laplace transfer of the resulting time-dependent output function, $O(t)$
- $\bar{G}(s)$ is a transfer function

Although to date the application of transfer function theory to pavements has been limited to asphaltic concrete pavements, its potential applications to rigid pavements are important enough so that a brief description of the published work in the area is included here.

Swami, Goetz, and Harr (Reference 19) were the first to use transfer functions to characterize the time-dependent behavior of asphaltic concrete. It was found that the transfer function represented material properties that were independent of the type of load input and, as would be expected, the temperature of the test specimen had the greatest single effect on the transfer function. The fact that the transfer function was found to be independent of the type of loading increases the efficacy of the method because, referring to Equation (2), once the transfer function $G(s)$ is known the operation output can be computed for any time-dependent input whose Laplace transform can be found, without having to run additional tests.

Ali (Reference 20) developed a technique for generating a reproducible impulse loading in the laboratory. This was an important contribution to the development of the transfer function approach because the Laplace transform of an impulse loading is unity, and therefore the response to the impulse loading in the Laplace plane is the transfer function itself (see Equation (2)). Ali tested the procedure by predicting the pavement response to a step load by applying the transfer function obtained from an impulse load.

Boyer and Harr (References 21 and 22) used data from full scale field tests (three different aircraft were used on three different flexible pavements), and found that the characteristics of a flexible pavement could be represented by transfer functions obtained from dynamic tests. It was determined that the time-dependent transfer functions contained some descriptors that were related to the aircraft gear configuration, and some additional descriptors that were related only to the behavior of the pavement.

The advantage of the transfer function approach is that it is capable of accommodating more material descriptors than are normally included in solid elastic theory. Furthermore, it precludes the necessity of assuming a mathematical model to characterize the behavior of the pavement - a model that may or may not be representative of the pavement system.

PAVEMENT PERFORMANCE CRITERIA

There is no universal agreement on any one performance criterion for either rigid or flexible pavements. Two approaches are discussed here; (1) vehicle vertical acceleration which has been applied to airfield pavements, and (2) a subjective performance rating system that was used in the AASHO Road Test.

Acceleration response of aircraft is felt to be an excellent roughness criterion because aircraft response motion continues for some time after a particular input, i.e., the aircraft response to one section of a runway is dependent on preceding sections of the runway. Acceleration response takes this into effect.

Houbolt (Reference 23) recommended that when the acceleration experienced in an aircraft exceeds 0.3g remedial measures should be taken. The Port of New York Authority (Reference 24) found the maximum level of aircraft vibration attained before passenger discomfort was noted was 0.12g in normal operation areas and 0.3g in infrequently trafficked areas. Hall and Kopelson (Reference 25) indicated that the response of an aircraft to a runway was undesirable when the acceleration in either the cockpit or at the aircraft center of gravity exceeded 0.5g.

Other criteria have been applied to highway pavements. A serviceability-performance concept was developed for use in the AASHO Road Test (Reference 26). Since Road Test data are used in the present research, this unique functional failure criterion is described in some detail here. The concept was developed on the basis that "highways are for the comfort and convenience of the traveling public". A Present Serviceability Rating (PSR) on a scale of 0 to 5 (very poor serviceability to very good serviceability) was assigned to a section of pavement by a panel consisting of representatives from a wide spectrum of highway users. A given rating was a subjective assessment of the serviceability (ability of the pavement to serve a large number of cars and trucks operating at high speeds) of a highway section at the time at which the rating was made. A single PSR value, then, does not indicate past serviceability characteristics of the pavement. It was found that panels could replicate or reproduce their ratings such that replication differences were in the range of 0.0 to 0.5 with a mean of 0.2 and a standard deviation of 0.5. However, these data were determined when replications were performed on the same day as the original PSR was made; it is possible that the data would show greater differences between original and replicated ratings if the time interval between ratings was larger.

Because the PSR is subjective and also required a section of highway longer than the test sections constructed for the AASHO Road Test, a PSI which is a mathematical combination of values of physical characteristics which could easily be measured on highway pavements was developed to predict the PSR of the pavement. With this development, rating panels were no longer required and an objective index could be obtained. By measuring certain physical characteristics of the pavement, a PSI could be determined which was related to the PSR value a panel would have obtained had a rating PSR been made. Mathematically then,

$$PSR = PSI + \epsilon \quad (3)$$

where ϵ is an error term or residual not explained by the

function relating PSI to physical characteristics of the pavement.

The relationship between PSI and relevant pavement characteristics was obtained in the following form:

$$PSI = A_0 + A_1 \log(1 + \overline{SV}) + A_2 \sqrt{C+P} \quad (4)$$

where

A_0 , A_1 , A_2 are constants determined by regression analysis when Equation (4) is substituted into Equation (3),
 \overline{SV} is the mean wheelpath slope variance,
 $C+P$ is a measure of cracking and patching in a highway section.

Equation (4) indicates that any phenomenon which changes the level of \overline{SV} or $(C+P)$ will change the PSI and the nature of the phenomenon need not be identified. Thus, for example, cracking caused by either overloading and/or fatigue will change the PSI.

Both PSR and PSI are measures of the serviceability of a pavement section at the time the particular value is determined. If it is accepted that performance is related to the ability of the pavement to serve traffic over a period of time, then a measure of performance can be obtained from the serviceability-time history of a given pavement section. This concept is accepted in the present research insofar as analysis of AASHTO Road Test data is concerned - the objective being to show that a serviceability-cumulative deflection history can also serve as a basis of performance. This would enable future performance trends to be predicted if deflections could be predicted.

FATIGUE OF PORTLAND CEMENT CONCRETE

Mellinger (Reference 27) discussed two primary causes of rigid pavement failure. It was noted that the number of failures resulting from construction and material defects has decreased significantly as knowledge of Portland cement concrete and slab joints has increased. The most significant cause of failure of rigid airfield pavements is overloading. Mellinger states that rigid pavement overloading can occur when loads are so excessive that the pavement fails almost immediately, or pavement overload can occur over a period of time, the duration of which is related to the fatigue strength of the concrete and the foundation. Yoder (Reference 1) observed that failure of a rigid pavement under repeated loads can result from fatigue of the concrete and that while excessive deformation of the subgrade and shear of one of the pavement components are major causes of pavement failure, fatigue failure may be equally significant.

Packard (Reference 28) discussed two methods of investigating the effects of mixed aircraft repeated traffic loads on rigid pavements. It was shown that there exists a close correlation between the coverage method (Corps of Engineers) and the fatigue method (Portland Cement Association) and recommended a procedure for designing and evaluating rigid pavements that operate at, or near, full operational capacity.

The fatigue phenomenon can be illustrated by plotting typical laboratory data as shown in Figure 2 taken from Reference 29. Fatigue curves for two materials are shown, one having a fatigue limit and the other having no fatigue limit, i.e., no stress level below which the material can withstand an infinite number of load cycles without fatigue failure occurring. Materials which do not have a fatigue limit will fail in fatigue after a finite number of load applications no matter how small the load induced stress. It will be seen later in this section that Portland cement concrete has no fatigue limit at least up to 10^7 load cycles.

Fatigue of concrete has been investigated in terms of stress by several investigators (e.g., Nordby (Reference 30); McCall (Reference 31); Lloyd, Lott, and Kesler (Reference 32)). Nordby (Reference 30) reviewed research findings involving the fatigue of Portland cement concrete and noted that most of the research performed on both plain concrete specimens and those with reinforcement similar to that of highway pavements was motivated by the fact that many failures of concrete pavements by cracking were due to repeated applications of stress. Fatigue research on plain concrete beams indicate that (Reference 30):

1. It is doubtful that plain concrete possesses a fatigue limit at least within 10 million cycles of load;
2. Inadequately aged and cured concrete is less resistant to fatigue than well-aged and cured concrete; and,
3. As the induced stress is decreased the fatigue strength is increased substantially.

There is substantial agreement among fatigue investigators that for reinforced concrete specimens (Reference 30):

1. Most failures of reinforced beams were due to failure of the reinforcing steel which was accompanied by severe cracking in the concrete and stress concentrations associated with these cracks; and,
2. Beams accumulated residual deflections over many cycles of load but recovered somewhat during rest periods. This indicates that the fatigue of reinforced concrete beams is a function of the frequency of loading. This rate of loading phenomenon is not taken into account in current rigid pavement design procedures that attempt to consider fatigue (Reference 1).

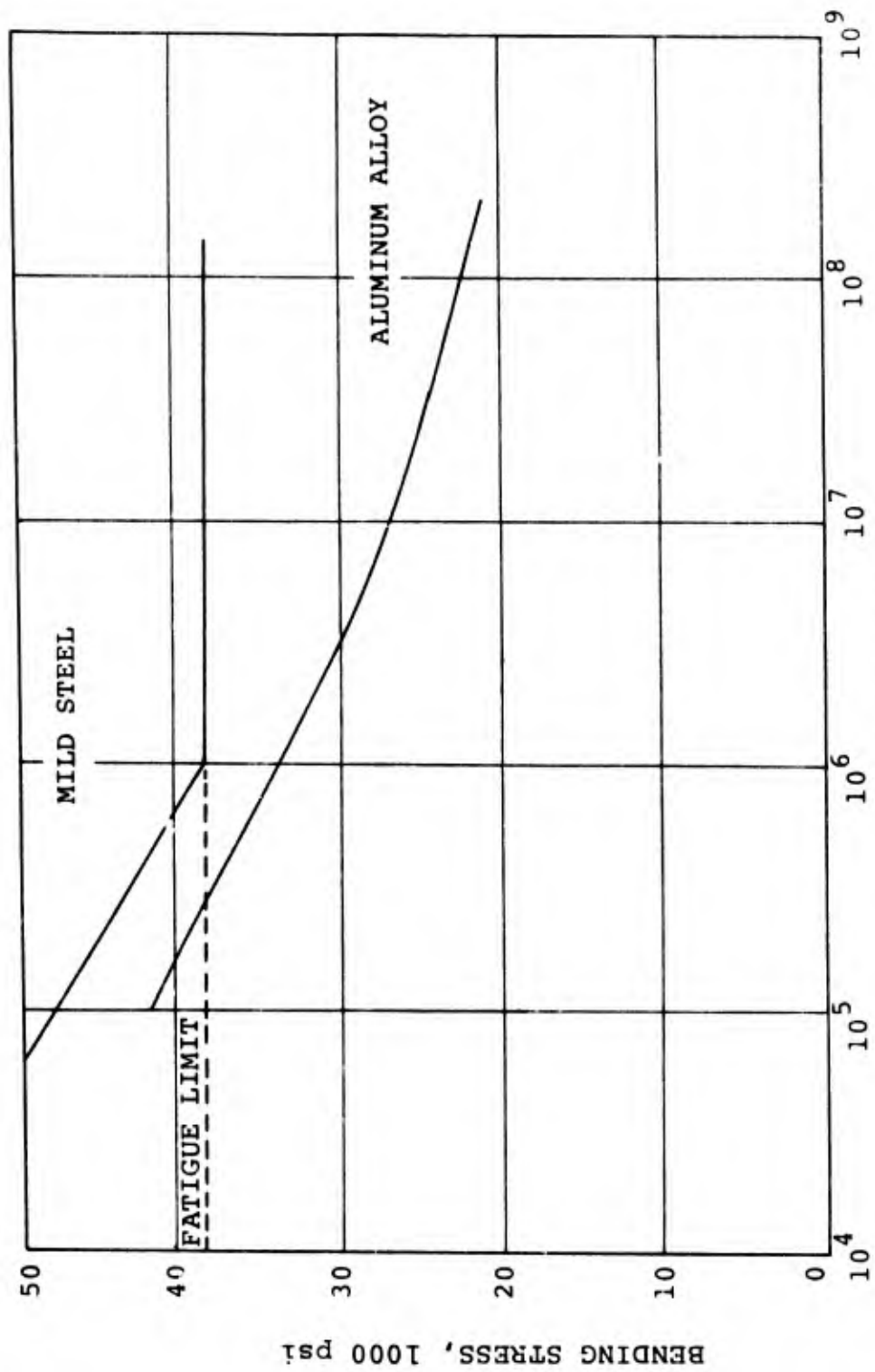


Figure 2. Typical Fatigue Curves for Ferrous and Non-Ferrous Metals

Lloyd, Lott, and Kesler (Reference 32) found that for repeated loading the strength of a specimen was reduced and the strength at failure was usually much less than the static flexural strength. Portland cement concrete was found to behave similarly to other materials under cyclic loading in that the strength reduction was found to be proportional to the logarithm number of load cycles to failure. This relationship is illustrated in Figure 3. Also of importance is the fact that Portland cement concrete does not seem to have a fatigue limit, i.e., there is no stress level below which concrete can be stressed an indefinite number of times without failure occurring. This is also illustrated in Figure 3.

Concrete does not have a fatigue limit, therefore reference is frequently made to the fatigue strength of concrete. The fatigue strength is the strength expressed as a percentage of the static ultimate strength corresponding to a given number (frequently 10 million) of cycles of load (see for example Kesler, Reference 33).

Linger and Gillespie (Reference 34) reported a comprehensive evaluation of results of previous investigations of the fatigue characteristics of Portland cement concrete and determined that the cumulated deformation was an indication of the fatigue damage.

Awad and Hilsdorf (Reference 35) found that damage to plain concrete caused by large repeated loads depends on the number of stress cycles and the total time the concrete has to sustain the stress. This seems to indicate that fatigue of a rigid pavement could depend on the speed of a vehicle and the length of its wheelbase - both would affect the frequency of loading. Traffic spacing, then, could also be a factor.

It is important to point out that all of the experimental research on concrete beam fatigue that was found in the literature was done on plain or reinforced sections that were simply supported. The support conditions remained constant for the duration of each test. This is in contrast to in-service conditions for rigid pavements. Special studies undertaken as a part of the AASHO Road Test (Reference 36) revealed that during periods of changing air temperature, points on the surface of slabs were in continuous vertical motion which would cause a continuous change in the geometry of slab support. At times, the slab would only be partially supported.

SIGNIFICANCE OF RIGID PAVEMENT DEFLECTIONS

When Burmister (Reference 16) developed the theory of the structural behavior of rigid pavements, based on a layered solid elastic model in 1943, it was suggested that the design be based on a criterion of limited deformation under load. Although Hveem (Reference 37) tentatively suggested that the maximum

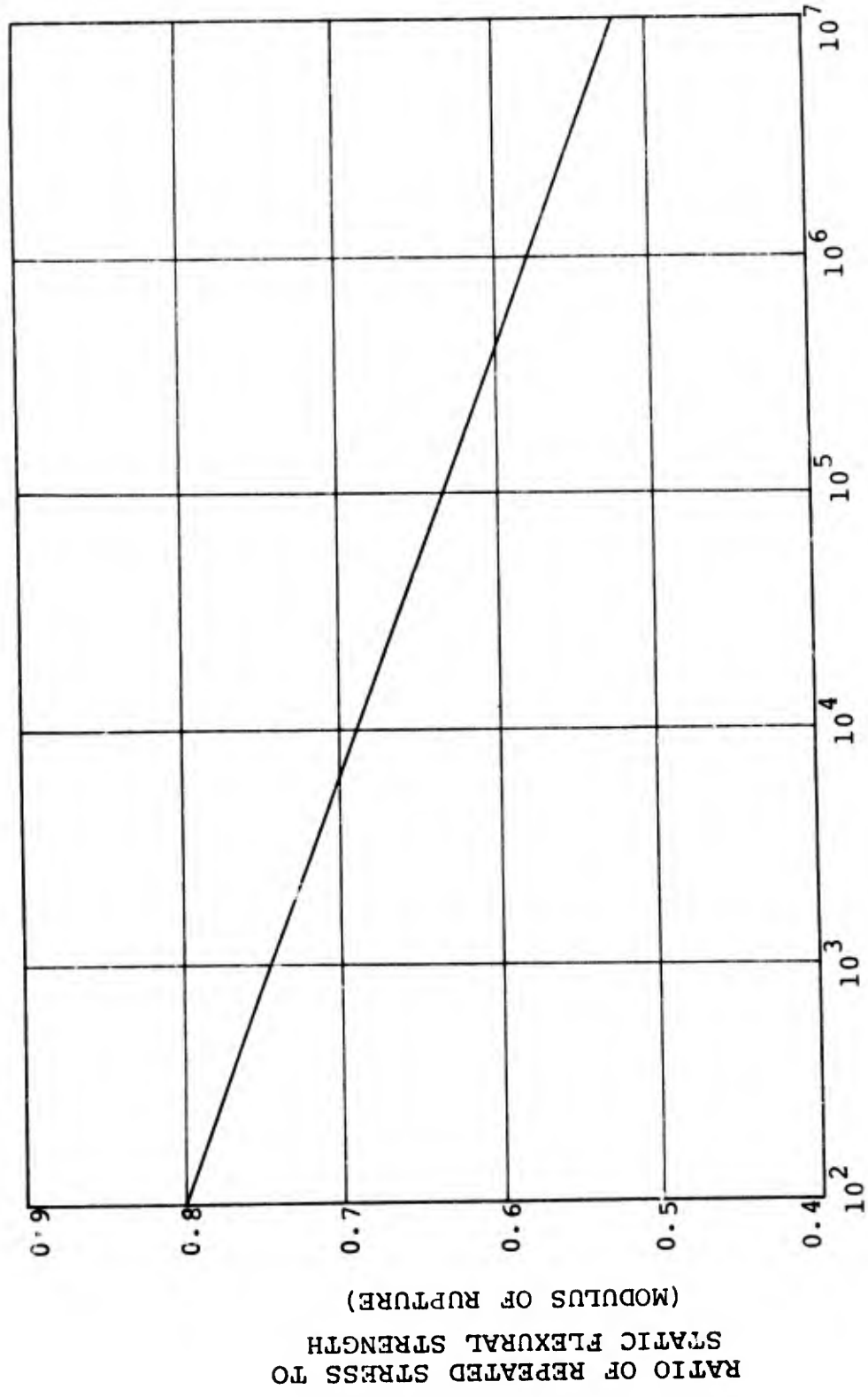


Figure 3. Typical Fatigue Curve for Plain Concrete

allowable deflection of an 8-inch thick Portland cement concrete pavement should be about 0.012 inch to prevent fatigue failure, most design procedures used today are based on stress because a relationship between deflection and performance has not yet been developed for rigid pavements (e.g., Reference 38).

Treybig (Reference 39) found that pavement type and thickness, subbase type and subgrade affected the load induced deflections of continuously reinforced concrete pavements. He also found that pavement deflection or response to load is an indicator of the relative performance of continuously reinforced concrete pavements of different designs. Thus, important pavement parameters have been found to affect the deflections of rigid pavements, i.e., rigid pavement deflection is a function of the temperature gradient across the slab thickness, load, subbase, subgrade, slab thickness, etc.

Ahlvin et al. (Reference 40) recommended that consideration be given to the determination of maximum allowable deflections which can be tolerated in a rigid pavement structure. Furthermore, they questioned whether designing for a given number of loadings by the largest load expected to use the pavement is still realistic and suggested "that designs which incorporate strain and/or deflection histories should be investigated" to analyze random loading (mixed traffic with periodic loading frequency) characteristics.

SECTION III

THEORY

GENERAL

Energy methods have seen wide application in mechanical engineering and form the basis of the well known Lagrange and Hamilton equations of motion. Although civil engineers have long used energy principles in structural analysis and in the derivation of dynamic pile driving formulas, it was not until recently that Hightler and Harr (References 41 and 42), on the basis of energy considerations, showed that there exists a relationship between the cumulative deflection that a flexible pavement experiences in its service life and the corresponding performance history of the system. Their approach incorporates mixed traffic and, because deflections are used, it provides a valuable application for transfer function theory as applied to flexible pavements by Boyer and Harr (References 21 and 22). The purpose of this section is to provide for rigid pavements a theoretical basis for the application of the approach developed previously (References 41 and 42) for flexible pavements.

PRESENTATION AND THEORETICAL BASIS OF THE WORKING HYPOTHESIS

It is hypothesized that:

There is a functional relationship between the total energy imparted to a given pavement system as measured by cumulative deflections and the condition of that system.

Pavements can be conveniently characterized as either highway or airport pavements and, although there are basic differences, both serve to provide a smooth wearing course and to distribute the applied vehicular loadings. Table 1 points out some of the basic differences between the load characteristics of airfield and highway pavements. The combined effects of heavier loads and higher speeds inherent to airfields indicate that dynamic effects and pavement roughness may be much more important for airfield pavements than for highway pavements. This would also suggest that the determination of stresses induced in pavement system may be more difficult to determine for airfields than for highways.

The magnitude of dynamic loads imposed by an aircraft in its ground operations is governed by a complex interaction between the aircraft and the pavement system. The aircraft weight, location of center of gravity, aerodynamic lift, landing gear configuration, tire air pressure, and tire size and spacing have all been shown to enter this interaction (Reference 43). It is not unreasonable to assume that, all other factors being equal, the training and experience of the pilot may also be an important factor. It is known that the variability and degree

of pavement roughness also influences the dynamic loading because aircraft response motion continues for some time after its initiation (Reference 25), so that the response of the aircraft to a particular section of pavement could depend on the direction in which the aircraft traverses that particular section of pavement. It is not surprising then that the actual loads imposed on pavements by aircraft defy anything more than qualitative description.

TABLE 1. COMPARISON OF HIGHWAY AND AIRPORT LOAD CHARACTERISTICS

LOAD CHARACTERISTIC	HIGHWAY	AIRPORT
Magnitude of load	normal load \approx 10,000 lb, tire pressure \leq 70 psi	normal load \approx 40,000 lb, tire pressure \leq 200 psi
Traffic volume	high	low
Distribution of loading	more nearly uniform than airport pavements	many parts of runway areas are seldom loaded
Speed of vehicle	maximum 55 MPH	up to 240 MPH
Shear (horizontal) loads due to braking and accelerating	moderate or negligible	can be very severe
Load pattern	single or dual wheels	many types of gear configuration

Furthermore, the measurement of load-induced stresses is difficult and time consuming. It would thus seem that methods of pavement analysis which require more or less precise knowledge of induced loads or stresses have a small probability of success in predicting performance. Unfortunately, in spite of this, the existence of a unique and reproducible load-deflection or stress-strain relationship is central to most methods of rigid pavement design and analysis in use today. Therefore, a method which does not rely on the determination of dynamic load-induced stresses would provide a valuable step in the development of a rational rigid pavement design procedure.

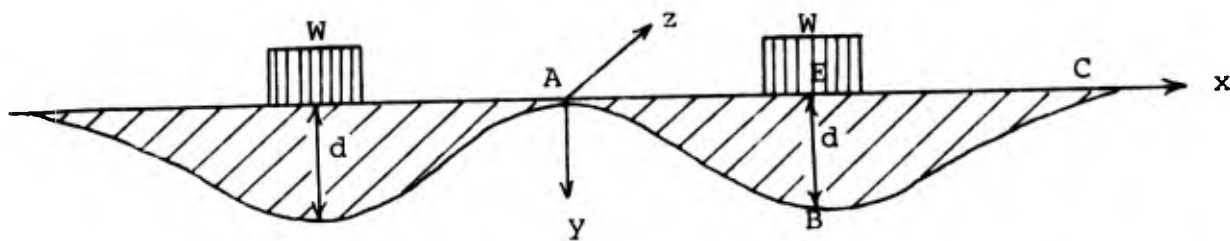
Methods of pavement design and analysis currently in use that are based on elastic theory were developed from the consideration of a single point in the material and are therefore local in nature. Thus, even if the loads which are required as input data to such analyses were known, the development would

lead to a quantitative evaluation of only isolated points in the pavement system. Even if rigid pavements were assumed to be grossly homogeneous there still would be variations from point to point because of changing foundation conditions, and the overall behavior of the pavement may deviate substantially from predictions based on single points within the system. In other words, such methods might provide knowledge of the condition of individual statistical points in a runway, but would not indicate the variation between these points - information which is vital in determining the condition of the whole runway.

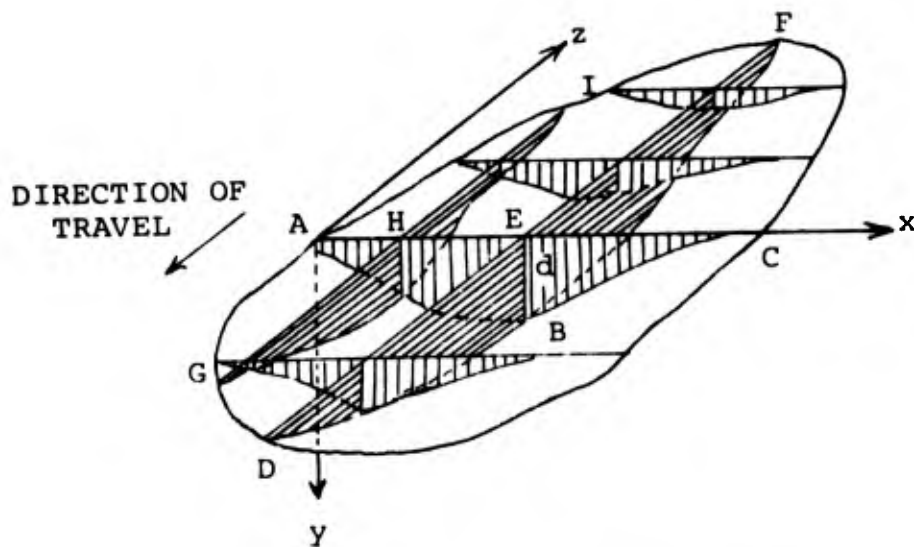
Except for the method developed by Boyer and Harr (References 21 and 22) discussed previously, methods which seek to characterize the dynamic response of pavements to loads assume that pavements can be modeled mathematically; assume a given mathematical model is representative; and, seek to determine the values of parameters that are required for the selected model.

With this background, a need for an index which characterizes the dynamic response of the pavement and which can easily be measured becomes apparent. Hightner and Harr (References 41 and 42) showed that for flexible pavements there is reason to believe that cumulative deflections may provide such an index. The following developments indicate that it is not unreasonable to assume that cumulative deflections may also provide such an index for rigid pavements also. The first development, taken from Hightner and Harr (Reference 41), considers a beam on an elastic (Winkler hypothesis) foundation; the second development is based on a plate (slab) on a Winkler foundation. The latter development was used by Westergaard (References 8 and 9).

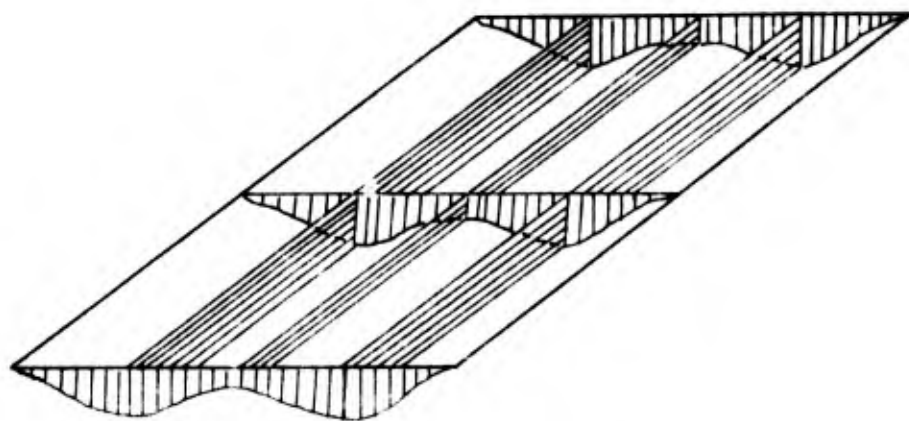
Figure 4 is a stop-motion schematic of a cross-section of the deflection pattern (or deflection basin) created by a moving load. The size and shape of the deflection basin reflects the weight, speed, and wheel configuration of the vehicle as well as properties of the pavement and its foundation. In this context the weight of the vehicle is the actual load applied to the pavement and, depending on the type of vehicle and the roughness of the pavement, the load may be greater than, equal to, or less than the static weight of the vehicle. The maximum deflection, d , occurs directly under the load and the pavement distributes the load so that unloaded portions of the pavement deflect and the deflection basin is wider than it would have been had the pavement been less rigid or absent altogether. Conceptually, then, the pavement serves to distribute and attenuate the deflection (or load) away from the wheels and in doing so creates a three-dimensional deflection basin which extends in all directions around the load. Figure 4 (b) is a three-dimensional schematic of half of the symmetrical deflection basin. Point D in this figure is on the periphery of the deflection basin and is in the wheel path of



(a) - TWO-DIMENSIONAL VIEW OF THE PLANE CONTAINING THE PEAK DEFLECTIONS



(b) - THREE-DIMENSIONAL ILLUSTRATION OF HALF OF THE SYMMETRICAL DEFLECTION BASIN



(c) - THREE-DIMENSIONAL DEFLECTION TROUGH

NOT TO SCALE

Figure 4. Schematic Representations of a Deflection Basin and a Deflection Trough

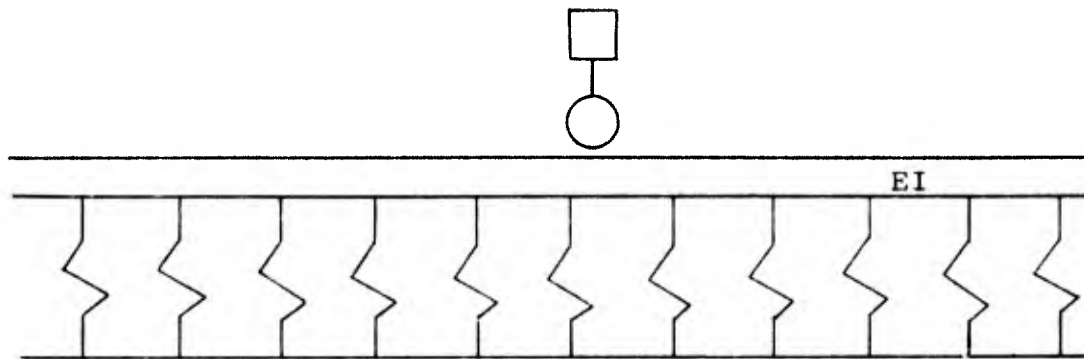
the load located at point E in this representation moving toward D. As the load moves in the negative Z direction (out of the page) point D is deflected, eventually attaining its peak deflection (at a time slightly after the passage of the vehicle) designated on the figure as d. As the vehicle moves farther and farther away, the deflection decreases and the pavement returns to its original position, point F.

Conceptually the plane containing points D, B, and F, which is parallel to the Z axis in Figure 4 (b), can be thought of as the deflection-time response of a single point lying in the wheel path of the moving vehicle. Similarly, the plane containing points G, H, and I can be thought of as the deflection-time response of a point adjacent to the wheel path. The vehicle can be thought of as creating a trough of deflections represented by the plane containing the points A, B, and C as it traverses the pavement. Figure 4 (c) is a three-dimensional illustration of the deflection trough created by a vehicle as it traverses the pavement. If it is assumed that the response of the pavement is essentially the same at different points along given longitudinal axes, then the problem can be reduced from three dimensions (the deflection basin) to a two-dimensional problem in a plane. With this assumption the system can be modeled as a beam of unit width whose length represents the width of the pavement. In this regard, the beam can be thought of as being infinitely long because the width of the deflection trough, i.e., edge effects can be ignored.

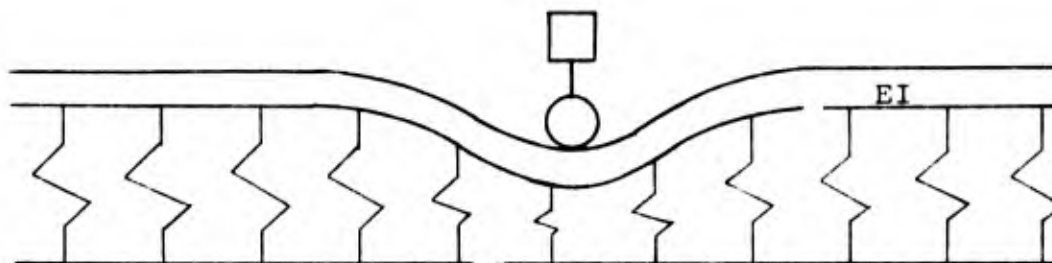
Insight for the theoretical basis of the working hypothesis can most easily be gained by considering two important models: (1) a beam on an elastic foundation and (2) a slab (or plate) on an elastic foundation. An elastic system is not an unreasonable model, for illustrative purposes, because well-designed rigid pavement systems do behave nearly elastically for single coverages. It is only after many vehicle coverages that non-elastic characteristics become apparent. The beam on an elastic foundation assumes that longitudinal variations in the pavement response to load are minimal. This model is the simpler of the two and will be considered first. The development follows that given in Reference 41.

Figure 5 (a) illustrates an airport runway modeled as an infinitely long solid elastic beam supported by a series of independently acting elastic springs (Winkler foundation), where the length of the beam represents the width of the runway. The change in energy in the system when it is loaded statically is the sum of the strain energy stored in the elastic springs and the strain energy stored in the beam itself. The strain energy in the deflected elastic beam, Figure 5 (b), is the sum of four components:

1. Strain energy due to axial load - The model selected does not offer resistance in the lateral direction and thus this



(a) - NO LOAD CONDITION



(b) - STATICALLY LOADED CONDITION

Figure 5. Theoretical Model of Elastic Beam on a Winkler Foundation

strain energy component is zero. In real pavement systems most of the induced load is normal to the surface of the pavement. There is little longitudinal displacement, elongation or contraction, of the pavement when compared to the bending mode and the axial strain component is negligible. Thus the elastic beam models the prototype pavement system well in this respect.

2. Strain energy due to shear force - For beams with length to depth ratios of 10 or more the contribution of the shear forces to the total strain energy is negligible (Reference 44). For a rigid airfield runway this ratio would be 50 or more and therefore the strain energy component due to shear forces can be neglected.
3. Strain energy due to twisting - As discussed previously, the deflection trough reduces the problem to a two-dimensional or "plane deflection" consideration. Therefore, for this model there is no tendency for the beams to twist and this component of the strain energy is zero.
4. Strain energy due to bending - The strain energy due to bending is a function of the curvature or change in slope of the deflected beam. Expressed symbolically:

$$U_{\text{bending}} = \int_{-\infty}^{\infty} \frac{EI}{2} \frac{d^2 W(x)}{dx^2}^2 dx \quad (5)$$

where

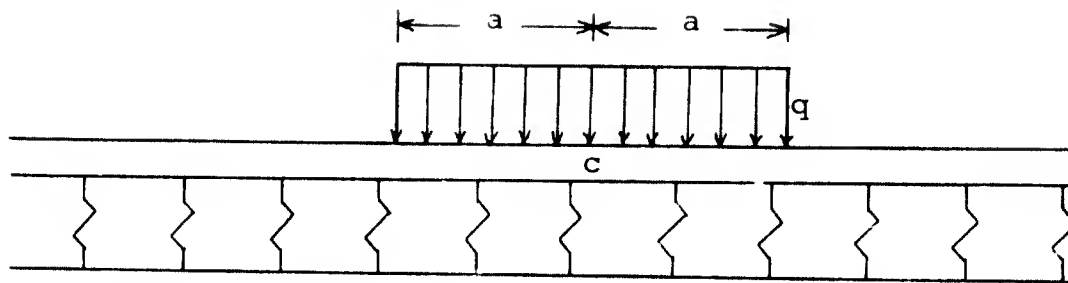
U_{bending} = the component of strain energy due to bending
 E = the modulus of elasticity
 I = the moment of inertia of the beam
 $W(x)$ = the deflection at point x

The results of field tests carried out by Highter and Harr (Reference 41) indicate that for flexible pavements the curvature of the loaded pavement surface was small except within the close proximity of the aircraft wheels. Within this region, it was found to be related in a general way to the magnitude of the deflection directly under the aircraft wheels. It is not unreasonable to assume that for rigid pavements the curvature would be even smaller than that for comparable flexible pavements due to the superior "beam action" characteristics of rigid pavements. Figure 6 shows the relationship between maximum deflection and curvature for an elastic beam on a Winkler foundation.

The strain energy in the Winkler foundation is a function of the spring stiffness and the beam deflection (Reference 45):

$$U_{\text{foundation}} = \int_{-\infty}^{\infty} \frac{k}{2} (W(x))^2 dx \quad (6)$$

where k is the foundation modulus (spring stiffness) and $W(x)$ is



ELASTIC BEAM SUPPORTED BY A WINKLER FOUNDATION

THE MAXIMUM DEFLECTION OCCURS AT POINT C.

$$D_c = \frac{q}{k} (1 - e^{-\lambda a} \cos \lambda a) \quad \text{where } \lambda = \sqrt[4]{\frac{k}{4EI}}$$

THE MOMENT AT C IS GIVEN BY

$$EI \frac{d^2 D}{dy^2} = \frac{q}{2\lambda^2} e^{-\lambda a} \sin \lambda a$$

SUBSTITUTING

$$D_c = \frac{q}{k} \left(1 - \frac{2\lambda^2 EI}{q} \frac{d^2 D}{dx^2} \cot \lambda a \right)$$

Figure 6. Expressions for the Deflection and Moment at the Midpoint of a Symmetrically Loaded Elastic Beam Supported by a Winkler Foundation

as previously defined. The total strain energy of the deflected beam then is:

$$U_{\text{total}} = U_{\text{bending}} + U_{\text{foundation}} \quad (7a)$$

or

$$U_{\text{total}} = \frac{EI}{2} \int_{-\infty}^{\infty} \frac{d^2W(x)}{dx^2}^2 dx + \frac{k}{2} \int_{-\infty}^{\infty} (W(x))^2 dx \quad (7b)$$

Both terms in Equation (7b) are related to the load induced deflections of the beam and are independent of the rate and the path by which these deflections are obtained.

Rigid pavements are commonly modeled as plates supported by elastic (Winkler) foundations. Such a model has more utility than the previously investigated beam on an elastic foundation because it can accommodate variations in k along the length as well as the width of a runway. This means that the trough illustrated in Figure 4 (c) need not have the same deflections in each cross-section along the runway, and variations in the pavement rigidity and support can be considered. Therefore, a deflection trough illustrated schematically in Figure 7 in which the deflections vary due to variations in rigidity and foundation support can be predicted. The advantage of this model is that three-dimensional variations in pavement and pavement support parameters can be considered, whereas in the previous model (beam supported by a Winkler foundation) only in-plane variations can be considered.

Timoshenko and Woinowsky-Krieger (Reference 46) presented an approximate expression for a circular plate supported by an elastic foundation (Figure 8). Assuming as an approximation that the deflection can be expressed as

$$W(r) = A + Br^2 \quad (8)$$

where A and B are constants they found that the total strain energy of the system for a load P applied at the center is

$$U_{\text{total}} = U_{\text{plate}} + U_{\text{foundation}} \quad (9a)$$

$$= 4B^2 D \pi a^2 (1+\nu) + \int_0^{2\pi} \int_0^a \frac{k(x) (W(r))^2}{2} r dr d\theta \quad (9b)$$

For the case where k is a constant, the strain energy is

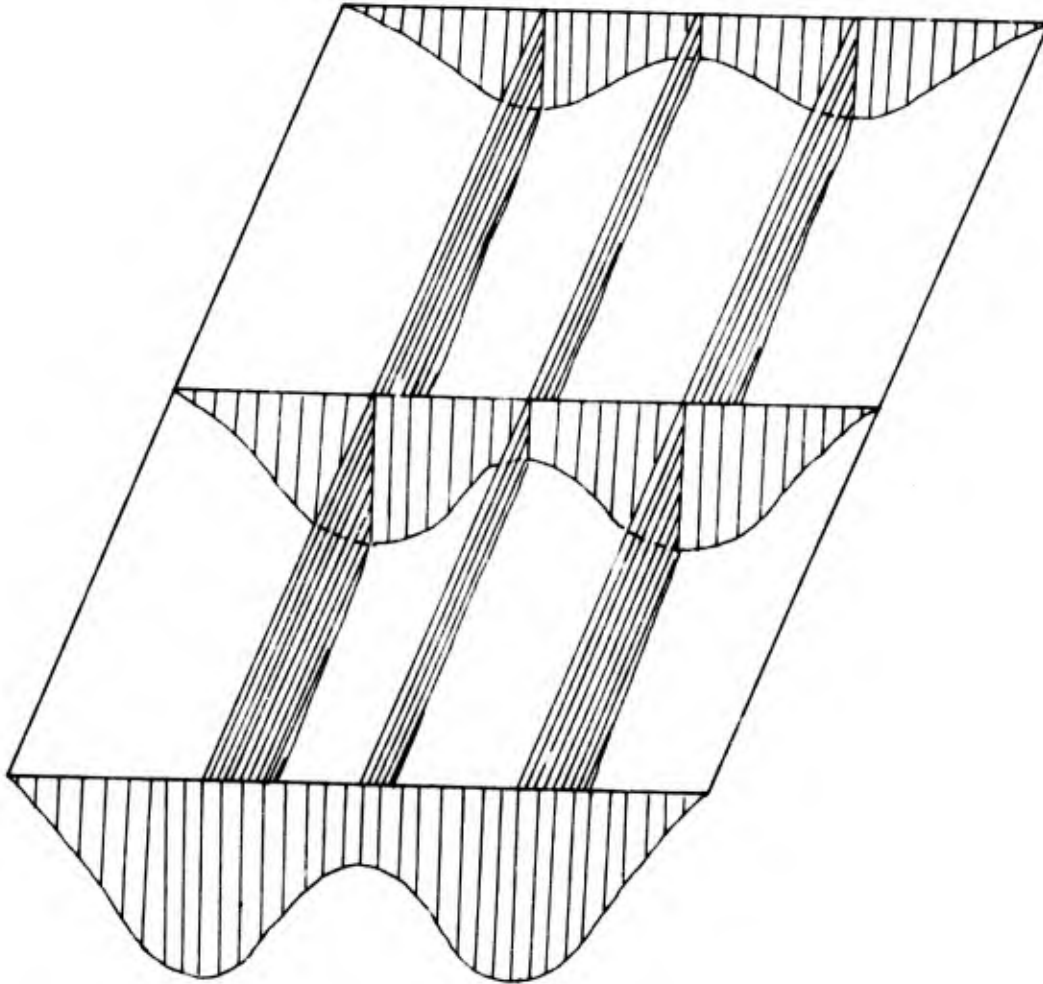


Figure 7. Schematic of Deflection Trough Resulting from Variations in Pavement Support

$$\text{Flexural Rigidity} = D = \frac{Eh^3}{12(1-\nu^2)}$$

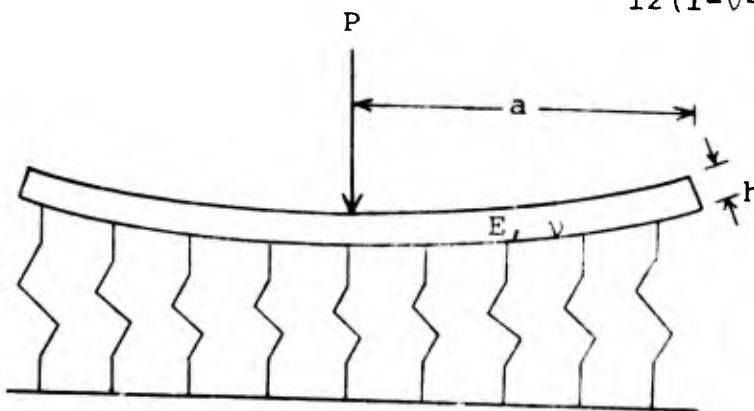


Figure 8. Deflection of a Circular Plate Supported by a Winkler Foundation Due to a Concentrated Load at the Center

$$U_{\text{total}} = 4B^2 D \pi a^2 (1+\nu) + \pi k \left(\frac{A^2 a^2}{2} + \frac{Aba^4}{2} + \frac{B^2 a^6}{6} \right) - PA \quad (10)$$

where

$$D = \frac{Eh^3}{12(1-\nu^2)} \quad \text{the flexural rigidity of the plate}$$

h = the thickness of the plate
 E = the modulus of elasticity
 a = the plate radius
 ν = Poisson's ratio of the plate
 k = the foundation modulus
 W = the deflection of the plate

A and B are constants that are determined by imposing the condition that the total strain energy of the system is minimum when stable equilibrium is achieved. Since the maximum deflection occurs directly under the load ($r=0$), Equation (8) indicates that A is the maximum deflection, W_{max} . Therefore, examination of Equation (10) indicates that the strain energy is a function of the plate and foundation parameters, and the deflection. This was also the case for the model of a beam on an elastic foundation.

Equation (10) could have been presented in cartesian coordinates with direct application to a rectangular plate or a pavement slab. The expression for strain energy would be more complex, but the salient point that strain energy is related to deflection would remain.

Equations (7), (9) and (10) indicate that the strain energy in a beam or plate supported by an elastic (Winkler) foundation is a function of the system parameters, and is related in at least a general way to the load induced deflections. Within this context, the deflection becomes a measure of the load induced energy available to do work to a given pavement system, or the maximum deflected shape of a given pavement provides a measure of the net energy introduced into a pavement by a load (vehicle). Of considerable importance in this regard is the recent application of transfer function theory to pavements by Boyer and Harr (References 21 and 22). The approach allows the deflection response of a pavement to any vehicle to be determined quickly and accurately. Thus, in the future, if the working hypothesis of this research effort is proved to be valid, pavement engineers will be able to quantify and compile the cumulative deflection data for rigid pavements needed to predict the performance trends of airfields and highways by a nondestructive method with a minimum of interruption to traffic. The following paragraphs indicate how the validity of the working hypothesis was tested.

PROCEDURE USED TO TEST THE WORKING HYPOTHESIS

Highter and Harr (Reference 41) found that for flexible pavements a complete record of deflection data over the service

life of a pavement system existed in the extensive data records collected at the AASHO Road Test. They were able to develop a performance-cumulative deflection relationship from these data and then extended the concept to include flexible airfield pavements. Data by which the existence of a performance-cumulative deflection relationship could be determined for rigid pavements are also available from the AASHO Road Test. The establishment of a rigid pavement performance-cumulative deflection relationship for highways will enable ongoing Air Force Civil Engineering Center research, using the transfer approach, to be used to compile deflection data required to determine such a relationship for airfield pavements.

The rigid pavement study phase of the AASHO Road Test consisted of trafficking test sections arranged continuously in a loop until failure occurred. The test sections varied in design by having different slab and subbase thicknesses. In addition, some test sections were reinforced while others were constructed of plain Portland cement concrete. The data collected included periodic Benkelman beam deflection measurements, periodic surveys that assessed the performance of the pavement in terms of the Present Serviceability Index (PSI) as well as the number of load applications corresponding to a given PSI.

Figure 9 is a flow chart which shows schematically the approach which was used to find the cumulative deflection corresponding to a given performance level. Benkelman beam deflection data were taken at periodic intervals during the Road Test. Because the deflection data are for static tests, and it is known that deflections are dependent upon the speed of the load vehicle, the data were adjusted to account for the 35 mph speed of the Road Test trafficking vehicles. This was done by applying the speed-deflection relationship found at the Road Test for rigid pavements (Reference 36). In effect, this allowed "performing" (on paper) dynamic deflection tests. These dynamic deflections were then scaled linearly to account for the differences in load between the vehicles used in making the Benkelman beam deflection measurements and those used in trafficking the test sections.

The next step (Figure 9) in finding cumulative deflections involved the Road Test traffic record. By knowing the number of vehicles trafficking the test sections between contiguous Benkelman beam deflection measurements, and knowing the deflections produced by the vehicles, the deflections were summed to determine the cumulative deflections. The cumulative deflections determined for each period that deflection measurements were made had a corresponding performance rating. Regression analysis was then applied to the rigid pavement-performance data to determine a relationship between the cumulative deflection and the corresponding serviceability level.

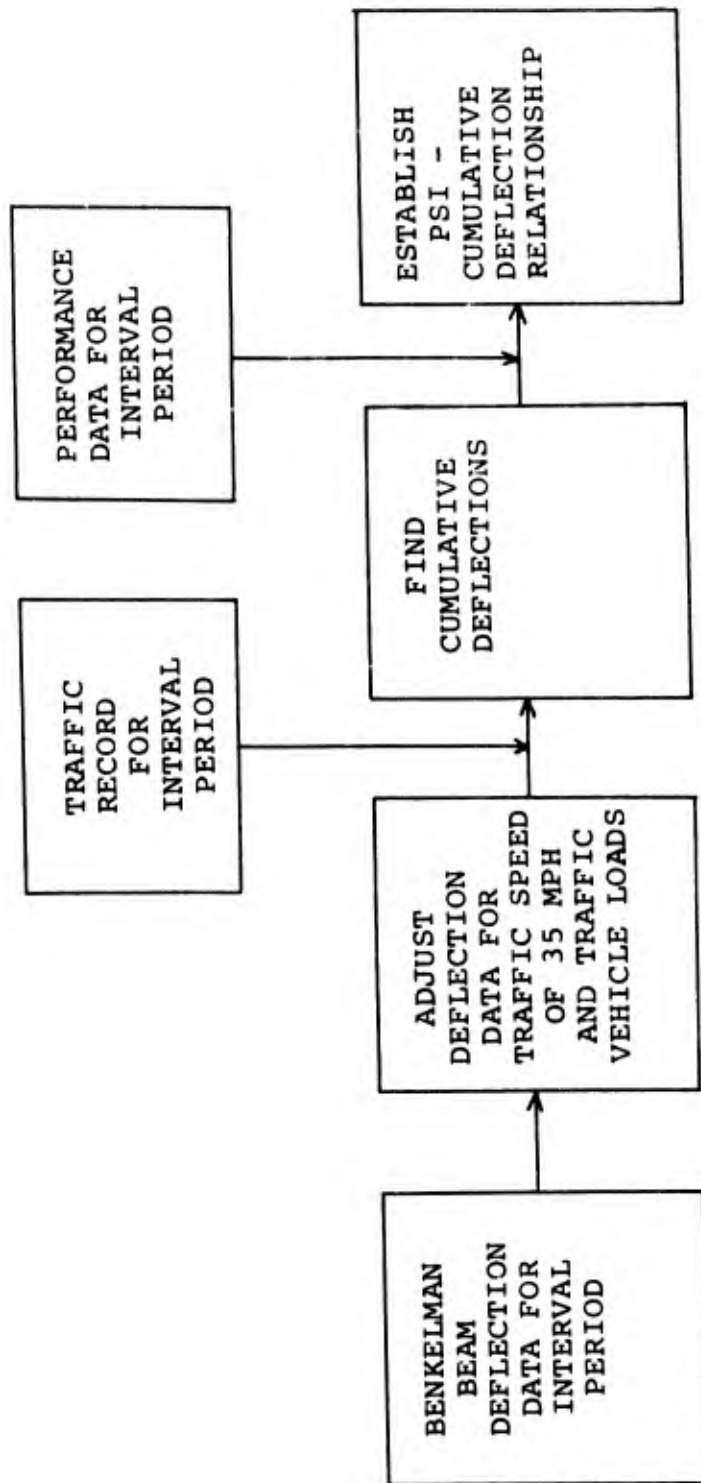


Figure 9. Flow Chart for Determining PSI - Cumulative Deflection Relationship for Rigid Pavements

The determination of a performance-cumulative deflection relationship would establish the validity of the working hypothesis. The relationship could then be used in conjunction with future traffic volume forecasts and estimates of the accompanying deflection to estimate future PSI levels.

SECTION IV

ANALYSIS OF AASHO ROAD TEST DATA

INTRODUCTION

To test the working hypothesis, rigid pavement deflection measurements and corresponding assessments of the serviceability of the pavement over a sufficient period of time are needed. The time interval over which these data are required must be long enough so that the serviceability of the pavement changes markedly within the time interval. A sufficient time interval then depends on the design of the pavement, ambient conditions and the nature and frequency of the traffic. All other variables being equal, the required time interval (related to the volume of traffic) is proportional to the thickness of the rigid pavement, i.e., it can be expected that a relatively thick rigid pavement would take longer to deteriorate to a given low serviceability level than a relatively thin pavement.

Data required to test the working hypothesis are available in the histories of some of the test sections trafficked in the AASHO Road Test. The PSI of some test sections did not change appreciably over the duration of the Road Test. Thus data from these test sections, which can be considered to be overdesigned for purposes of the present study, could not be used. Typical PSI history curves of "overdesigned" test sections are shown in Figure 10. Typical PSI history curves of test sections meeting the criteria of the present study as outlined above are shown in Figure 11.

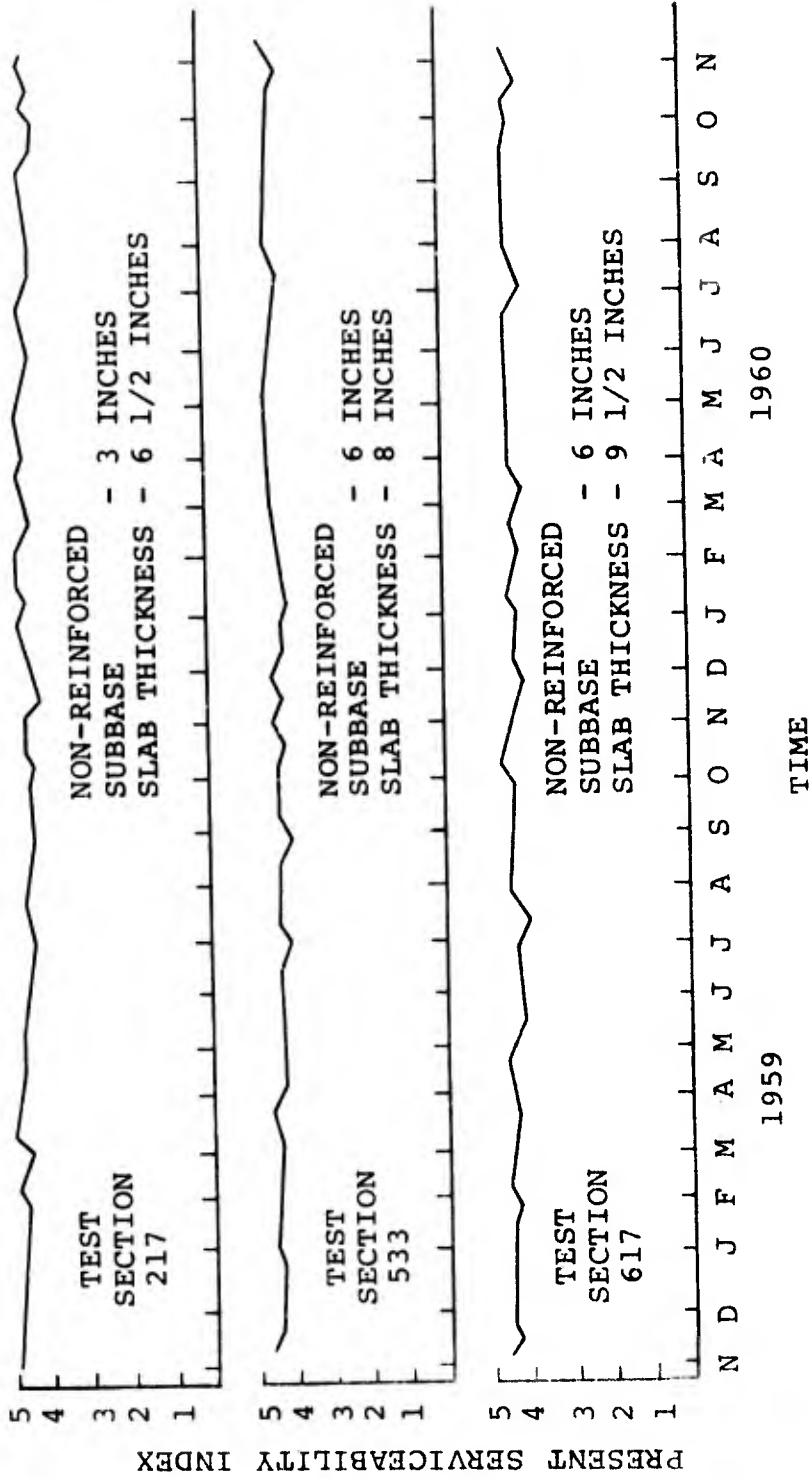
ANALYSIS

When assembling AASHO Road Test data, five variables were selected as likely to be related to the PSI. These were: (1) cumulative deflection, (2) load (L), (3) reinforcing (R), (4) subbase thickness (SB), and (5) slab thickness (S). However, it was found that in performance analysis carried out on Road Test data by others (Reference 36), subbase thickness and reinforcing were not statistically significant and these variables were not further considered in this research.

A function was sought that would predict PSI as a function of cumulative deflection, load, and pavement slab thickness:

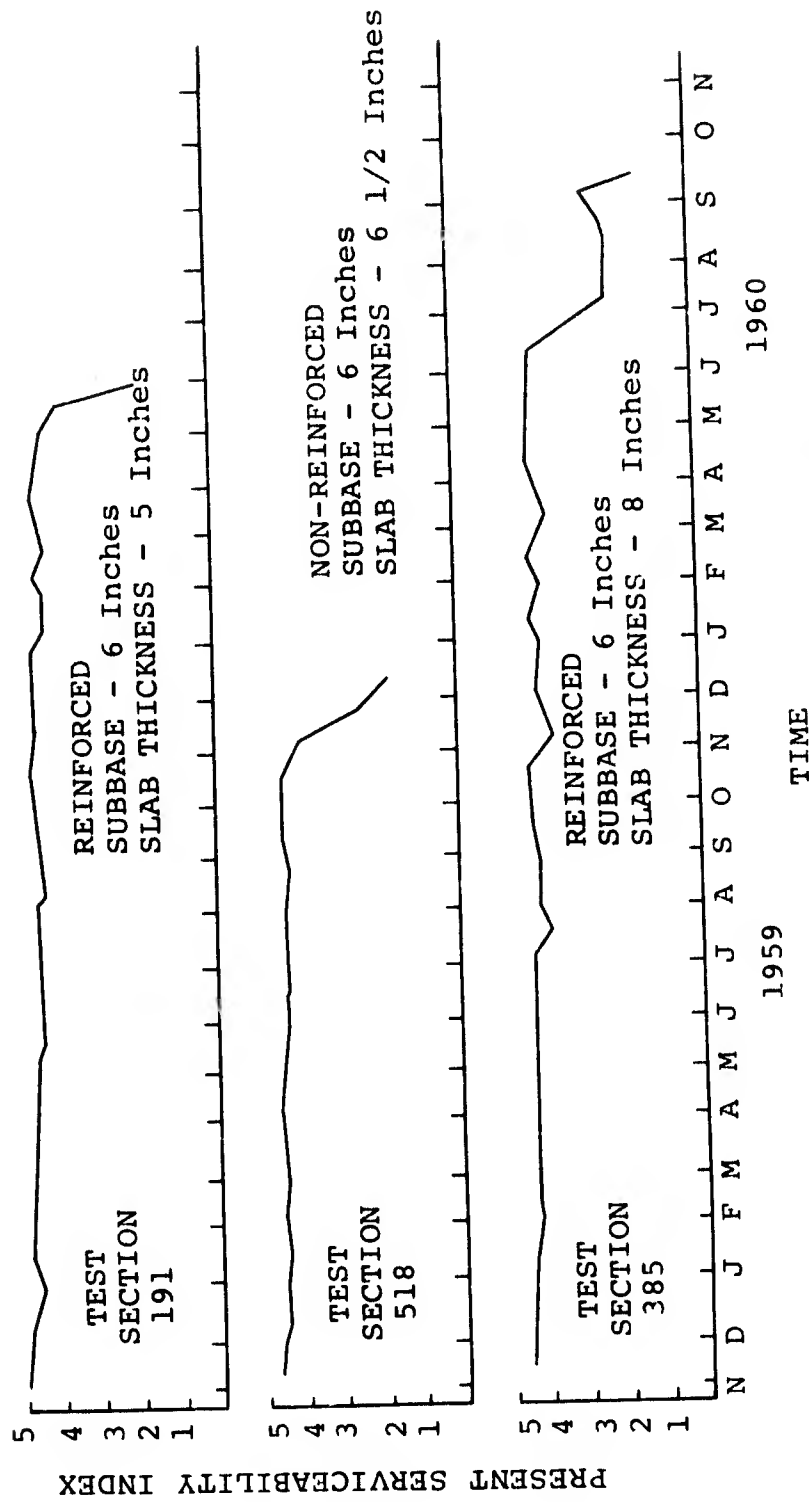
$$PSI = f_1(\text{Cumulative deflection, Load, Slab Thickness}) \quad (11)$$

With this function known, it would be possible to predict the serviceability level of a rigid pavement if its deflection history were compiled.



(DATA FROM AASHO ROAD TEST DATA SYSTEM DS-4292-R)

Figure 10. Typical Serviceability History of Test Sections Not Used in Analysis



(DATA FROM AASHO ROAD TEST DATA SYSTEM DS-4292-R)

Figure 11. Typical Serviceability History of Test Sections Used in Analysis

A fundamental assumption in regression analysis is that independent variables are independent of one another. Therefore, before regression analysis could be applied to Equation (11) it was necessary that the assumption of independence of variables be validated. A linear correlation analysis was carried out to determine the linear correlation between variables. The results are given in the correlation matrix in Table 2.

The correlation matrix indicates a very slight linear correlation between slab thickness, load and all other variables. The cumulative edge deflection and the cumulative corner deflection showed the strongest correlation with PSI. The high

TABLE 2. LINEAR CORRELATION MATRIX
(CUMULATIVE DEFLECTIONS)

VARIABLE	VARIABLE				
	PSI	CUMULATIVE EDGE DEFLECTION	CUMULATIVE CORNER DEFLECTION	SLAB THICKNESS	LOAD
PSI	1.000	-0.552	-0.616	0.000	-0.027
Cumulative Edge Deflection		1.000	0.984	0.329	0.382
Cumulative Corner Deflection			1.000	0.334	0.335
Slab Thickness				1.000	0.476
Load					1.000

correlation between cumulative edge deflection and cumulative corner deflection indicates these variables are not independent. The correlation matrix suggests that load and slab thickness be eliminated from Equation (11) and regression analysis be carried out on the following functions:

$$PSI = f_2(\text{cumulative edge deflection}) \quad (12a)$$

$$PSI = f_3(\text{cumulative corner deflection}) \quad (12b)$$

However, to include the effect of load, the cumulative deflections were normalized, i.e., divided by load, to create a

new parameter. The correlation matrix with normalized deflections is shown in Table 3. The results are similar to the correlation matrix shown in Table 2 except the linear correlations between PSI and normalized cumulative deflections are slightly greater than those between PSI and cumulative deflections. Regression analyses on normalized cumulative deflection data were carried out as indicated by the following functions:

$$\text{PSI} = f_4 \text{ (normalized cumulative edge deflection)} \quad (13a)$$

and

$$\text{PSI} = f_5 \text{ (normalized cumulative corner deflection)} \quad (13b)$$

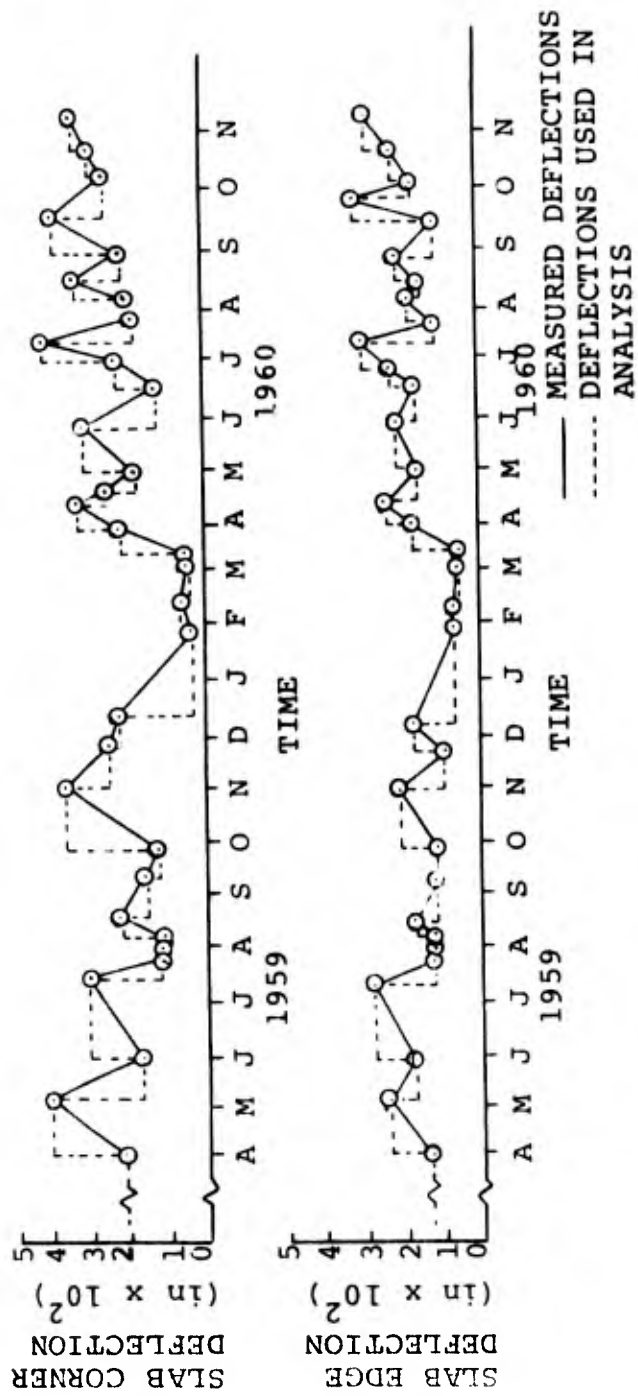
Because of the strong linear correlation between edge and corner deflections shown in Table 3, it can be expected that Equations (13a) and (13b) will have similar forms.

Throughout the duration of the AASHO Road Test, edge and

TABLE 3. LINEAR CORRELATION MATRIX
(NORMALIZED CUMULATIVE DEFLECTIONS)

VARIABLE	VARIABLE			
	PSI	NORMALIZED CUMULATIVE EDGE DEFLECTION	NORMALIZED CUMULATIVE CORNER DEFLECTION	SLAB THICKNESS
PSI	1.000	-0.622	-0.637	0.000
Normalized Cumulative Edge Deflection		1.000	0.982	0.073
Normalized Cumulative Corner Deflection			1.000	0.038
Slab Thickness				1.000

corner Benkelman beam deflection measurements were recorded periodically. A typical deflection history for one test section is shown in Figure 12. Such deflection histories served as the basis in determining cumulative deflections to be used in Equations (13a) and (13b). Because deflection measurements were taken periodically and not continuously, it was necessary to estimate the deflection that occurred in the interval between contiguous deflection measurements. It was assumed that the deflection measurement taken at the end of an interval was representative of the deflections throughout the interval.



(DATA FROM AASHO ROAD TEST DATA SYSTEM DS-4292-R)

Figure 12. Typical Deflections History of Test Sections

Individual deflections used in computing cumulative deflections are indicated by broken lines in Figure 12.

The wheel loads of the vehicles used in measuring the Benkelman beam deflection were different from the wheel loads of the vehicles used for trafficking. To account for the change in load, it was necessary to scale the Benkelman beam deflection data so that the deflections were representative of those caused by the vehicles used for trafficking. It was assumed that the load-deflection relationship was linear. This assumption seems to be justified by the findings (References 21 and 36).

The traffic history of each test section used in the present study was then applied to the scaled deflection measurements to obtain cumulative edge deflections and cumulative corner deflections.

The procedure whereby cumulative deflections were estimated for each test section can be summarized in an equation:

$$\text{Cumulative Deflection} = \sum_{i=1}^n \frac{L}{B_L} MD_i N_i \quad (14)$$

when

- L = Load used in trafficking
- B_L = Load used in Benkelman beam deflection
- MD_i^L = The ith Benkelman beam deflection measurement
- N_i = Number of trafficking loads applied between the ith and i-1 Benkelman beam deflection measurement

To investigate the nature of the functions in Equations (13a) and (13b), PSI versus normalized cumulative deflection per unit load data were plotted before regression analysis was carried out. The PSI-normalized cumulative corner deflection curves are shown for seven test sections in Figure 13. Two conclusions are suggested from observation of the figure: (1) the data extend over a wide range; (2) most of the curves are nearly horizontal for PSI values above 4.0 and then a break occurs after which a relatively small change in normalized cumulative deflection results in relatively large changes in the level of serviceability.

The wide range in data is illustrated further in Figure 14 which shows the PSI history of two identical test sections. The PSI of one section did not change appreciably throughout the duration of the Road Test whereas the other section exhibits a break in the curve after which the level of PSI decreases quickly and substantially. The inconsistency of the behavior of identical test sections indicates that regression analysis may not be successful.

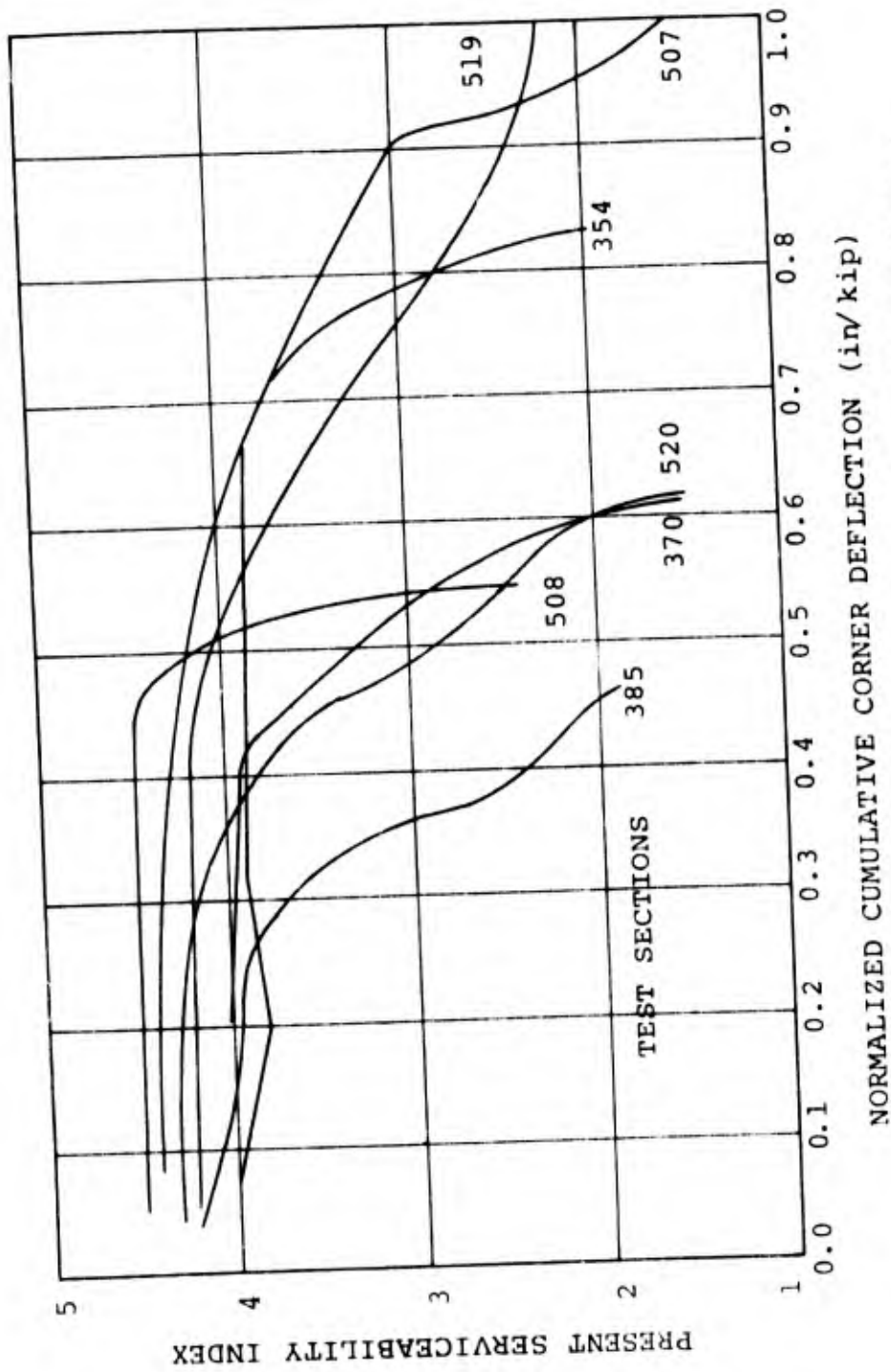
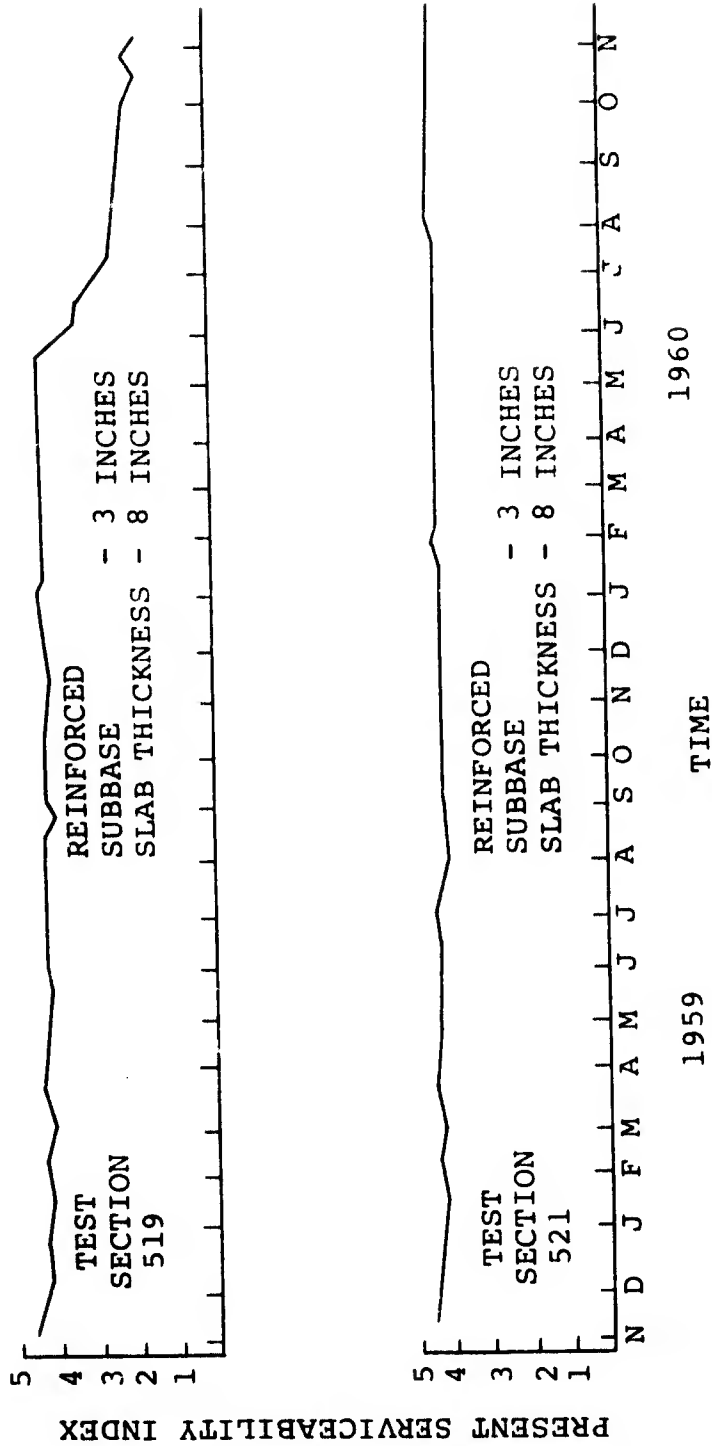


Figure 13. Serviceability versus Normalized Cumulative Corner Deflection Data for 8 Inch Slab



(Data from AASHO Road Test Data System DS-4292-R)

Figure 14. Serviceability Histories of Duplicate Test Sections

Before regression analysis was carried out, attempts were made to linearize the data. The shapes of the curves in Figure 13 suggest that linearization might be achieved by transforming the data by taking the natural logarithm of both the PSI and the normalized cumulative deflection. Thus Equations (13a) and (13b) take the form

$$\ln(5-PSI) = a + b \ln(\text{normalized cumulative deflection}) \quad (15)$$

where a and b are constants to be determined by regression analysis. With a and b known, Equation 15 becomes:

$$PSI = 5 - e^a (\text{normalized cumulative deflection})^b \quad (16)$$

Using data from 18 test sections, functional relationships for edge and corner deflections were found to be:

$$PSI_E = 5 - e^{0.645} (\text{normalized cumulative edge deflection})^{0.548} \quad (17a)$$

and

$$PSI_C = 5 - e^{0.524} (\text{normalized cumulative corner deflection})^{0.534} \quad (17b)$$

where the subscripts E and C refer to slab edge and slab corner, respectively. The statistical results of the regression analysis are tabulated in Table 4. The R² value of about 0.5 for both normalized cumulative edge and corner deflection functions indicate that only about one-half the variation in the PSI level is explained by the terms in Equations (17a) and (17b). The regression analysis then indicates that there is at least a general relationship between PSI and normalized cumulative deflections but Equations (17a) and (17b) should not be used for predictive purposes.

The regression analysis did not differentiate between the various slab thicknesses because of the linear correlation between cumulative edge deflection and slab thickness and between cumulative corner deflection and slab thickness shown in Table 2. However, it is possible to take slab thickness into account if the data from test sections having the same slab thickness were separated and then analyzed.

In addition to separating the data according to slab thickness, the attempt was made to establish a relationship between PSI and cumulative deflection rather than between PSI and normalized cumulative deflection. This approach was used previously for asphaltic concrete pavements and met with success (References 41 and 42).

TABLE 4. RESULTS OF REGRESSION ANALYSIS

$$PSI = 5-e^a (\text{normalized cumulative deflection})^b$$

PARAMETER	SLAB EDGE DEFLECTION	SLAB CORNER DEFLECTION
a	0.645	0.524
b	0.548	0.534
R ²	0.478	0.499
Standard Error of Estimate	0.76	0.74
Percent Error	19.5	19.0
F-Test	137.1	149.6

Data from seven test sections having a slab thickness of 8 inches are shown in Figure 15. Comparing Figure 15 with Figure 13 indicates that the range in the data for the non-normalized curves is less than when the deflection data are normalized with respect to load. The same effect was noted for the test sections having slab thicknesses of 5 and 6.5 inches.

The data for each slab thickness were then averaged so a single curve represented the test sections having different slab thickness. When these averaged curves were plotted to a semi-logarithm scale it was apparent that curves could be represented by two straight lines. One line represented the PSI - cumulative deflection relationship for the initial part of the curve when there were small changes in PSI, and the second line approximated the curve when small changes in cumulative deflection resulted in large change in the level of PSI.

Simple linear regression analysis was then carried out on the averaged data to obtain the parameters of the following regression equation:

$$PSI = a_1 + b_1 \log_{10}(\text{cumulative edge deflection}) \quad (18a)$$

and

$$PSI = a_2 + b_2 \log_{10}(\text{cumulative corner deflection}) \quad (18b)$$

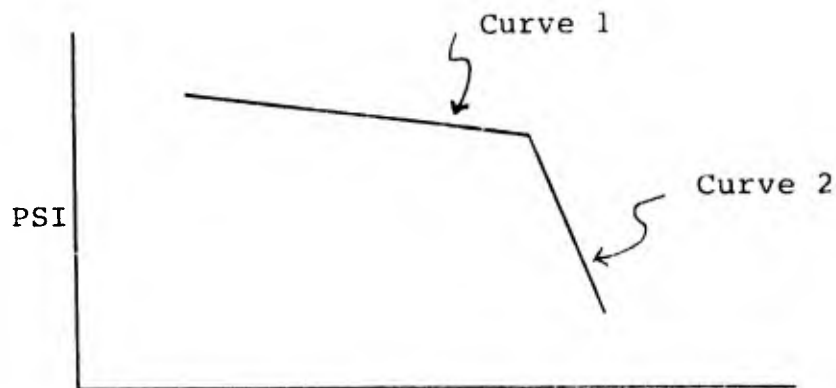
The results of the analysis are tabulated in Tables 5 and 6 and

TABLE 5. LEAST SQUARES COEFFICIENTS FOR
PSI-AVERAGED CUMULATIVE CORNER DEFLECTION DATA

$$PSI = a_2 + b_2 \log_{10}(\text{CUMULATIVE CORNER DEFLECTION})$$

SLAB THICKNESS (IN)	CUMULATIVE CORNER DEFLECTION					
	CURVE 1*			CURVE 2*		
	a ₂	b ₂	CUMULATIVE DEFLECTION RANGE	a ₂	b ₂	CUMULATIVE DEFLECTION RANGE
5.0	6.77	-0.63	<24,000	60.86	-13.04	>24,000
6.5	5.80	-0.35	<30,000	84.97	-18.05	>30,000
8.0	6.03	-0.45	<41,000	53.36	-10.74	>41,000

*See sketch below for explanation



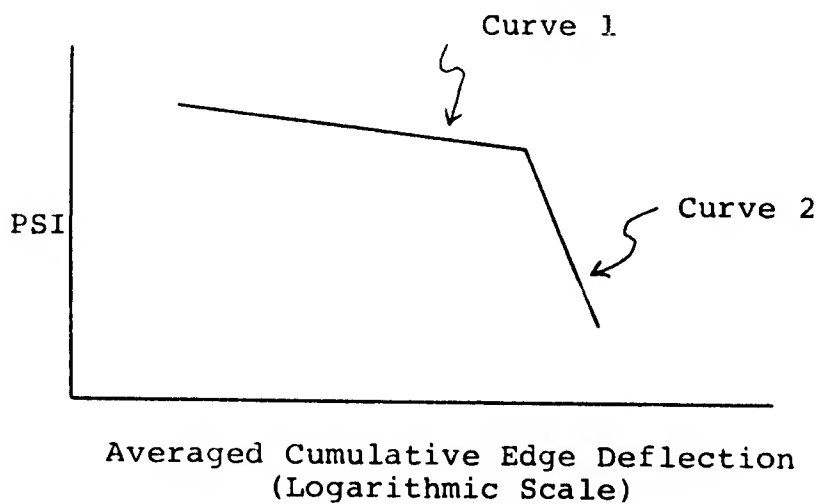
Averaged Cumulative Corner Deflection
(Logarithmic Scale)

TABLE 6. LEAST SQUARES COEFFICIENTS FOR
PSI-AVERAGED CUMULATIVE EDGE DEFLECTION DATA

$$PSI = a_1 + b_1 \log_{10} (\text{CUMULATIVE EDGE DEFLECTION})$$

SLAB THICKNESS (IN)	CUMULATIVE EDGE DEFLECTION					
	CURVE 1*			CURVE 2*		
	a_1	b_1	CUMULATIVE DEFLECTION RANGE	a_1	b_1	CUMULATIVE DEFLECTION RANGE
5.0	7.57	-0.85	<16,500	52.25	-11.44	>16,500
6.5	4.98	-0.14	<23,000	92.92	-20.27	>23,000
8.0	5.36	-0.30	<37,000	112.87	-23.81	>37,000

*See sketch below for explanation



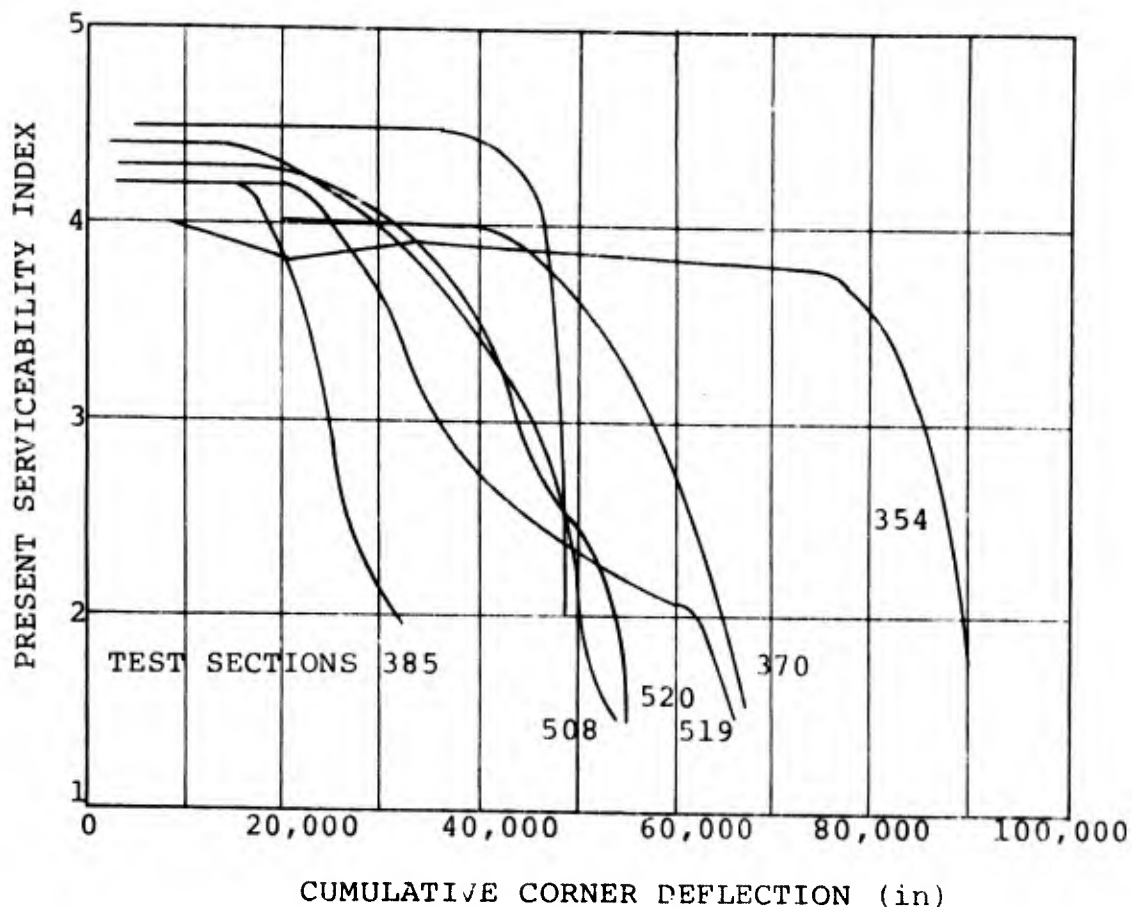


Figure 15. Serviceability versus Cumulative Corner Deflection Data for 8 Inch Slab

are plotted in Figures 16 and 17 for corner and edge cumulative deflections, respectively, along with the averaged data points used in the analysis. Inspection of these figures indicates that:

1. The relationship between PSI and cumulative deflections is well defined for the averaged data.
2. The cumulative deflection at the intersection of the two straight lines comprising each curve, referred to as the threshold cumulative deflection, increases as the pavement slab thickness increases. This indicates that a thicker slab tolerates a greater cumulative deflection before its serviceability decreases to a low value.
3. The position and slope of the initial part of each curve are nearly the same for each slab thickness. It is not unreasonable to assume the positions of the initial portions of the curves are related to the as-built condition of the pavement rather than slab thickness.

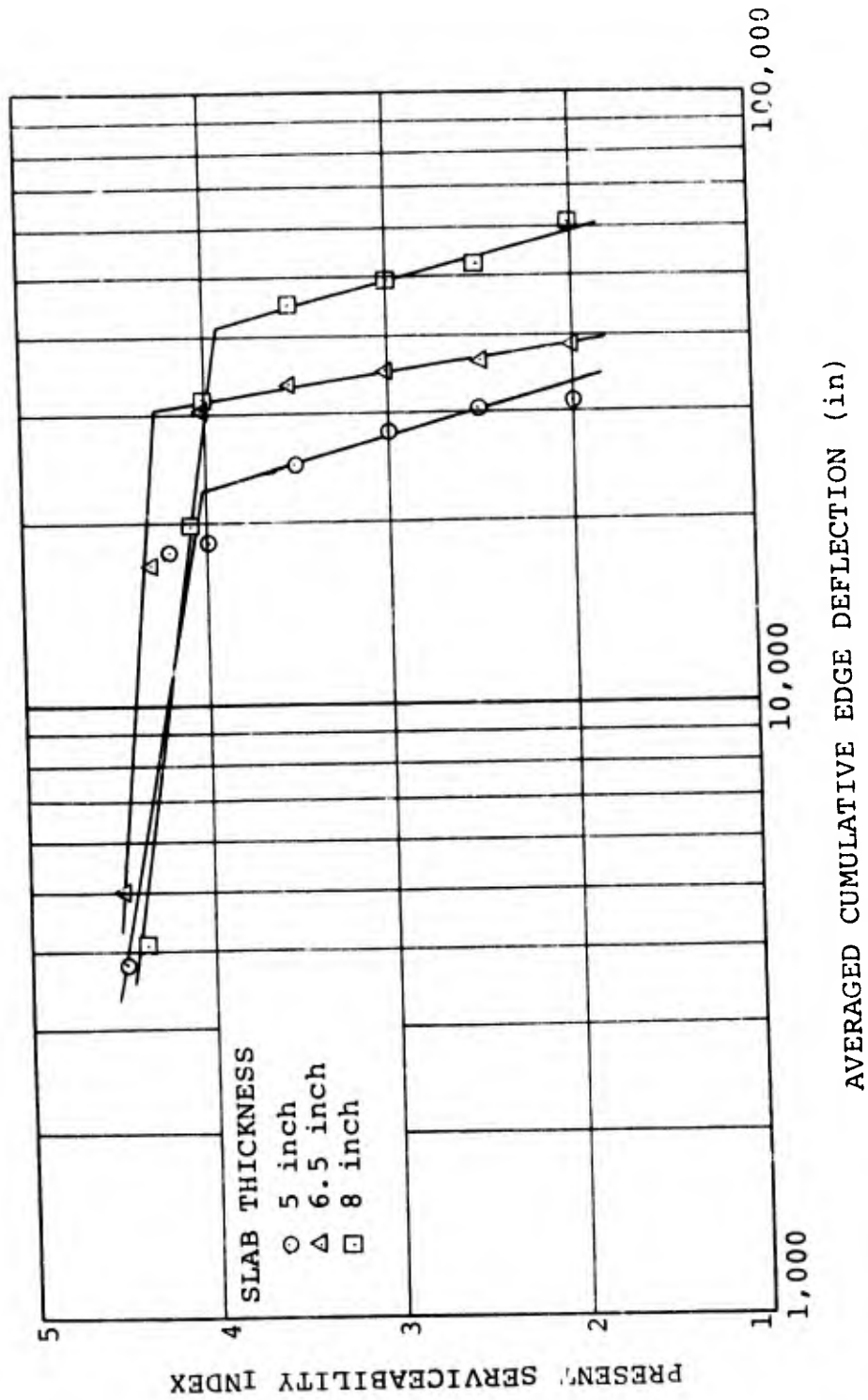


Figure 16. Serviceability versus Averaged Cumulative Corner Deflections for Three Slab Thicknesses

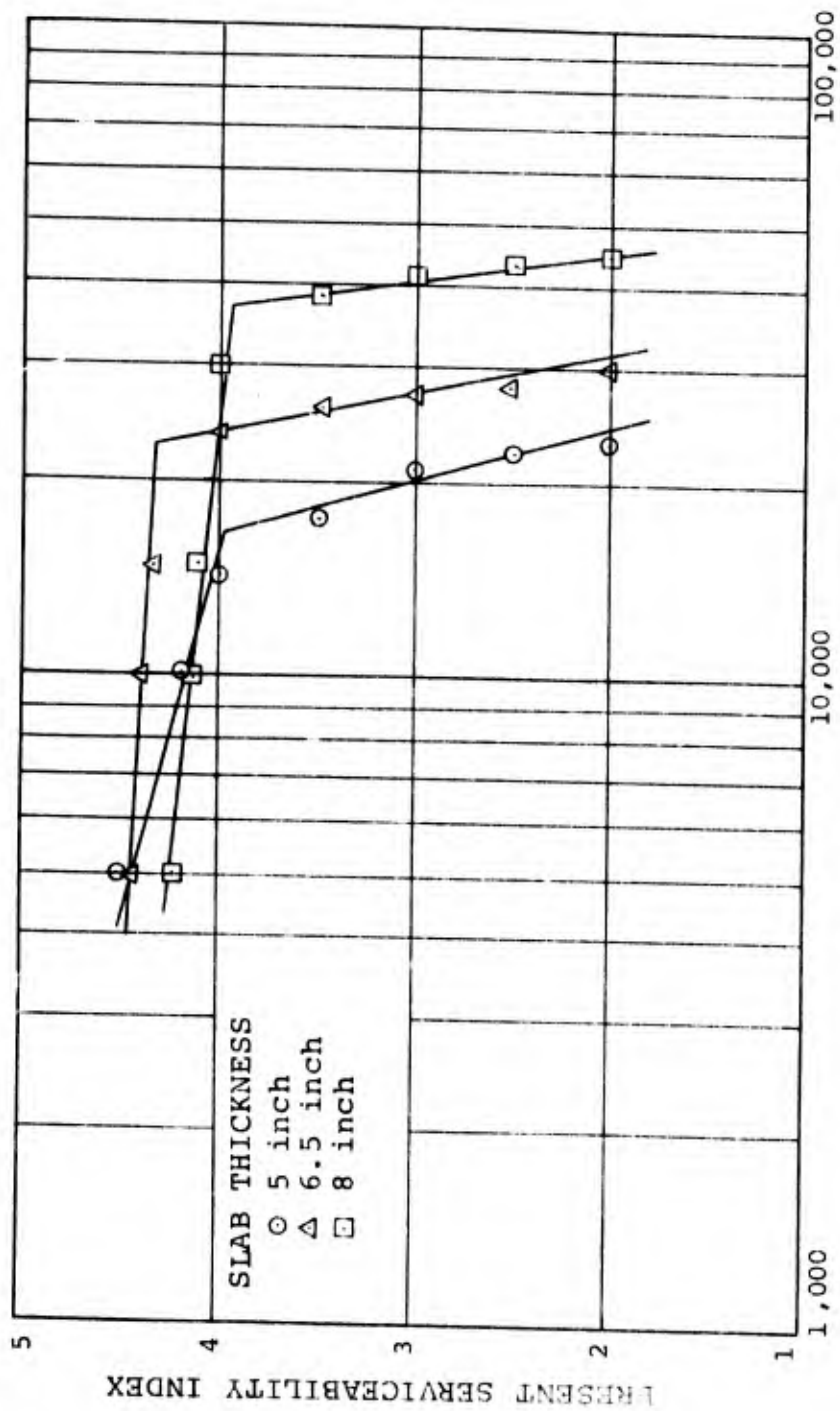


Figure 17. Serviceability versus Averaged Cumulative Edge Deflections for Three Slab Thicknesses

4. Although the slopes of the second part of each curve appear to be about the same when plotted to a logarithmic scale, the cumulative deflection associated with reducing the PSI of the 8-inch thick slab from 3 to 2 is nearly 3 times as great as that corresponding to a similar change in PSI for the 5 inch slab.
5. The observations noted above are applied to both the PSI - cumulative corner deflection relationship (Figure 16) and the PSI - cumulative edge deflection relationships (Figure 17).

The averaged data lend strong support to the working hypothesis, i.e., there does exist a relationship between cumulative deflection and pavement condition or serviceability. The behavior of the average test section trafficked in the AASHO Road Test is consistent with the working hypothesis.

APPLICATION

Although Figures 16 and 17 are not presented as design curves, the utility and usefulness of the information contained in such curves can be illustrated by an example. The purpose of presenting an application is to show why similar curves (cumulative deflection versus pavement condition) should be developed for airfield pavements. Of course, the method of measuring the serviceability of airfield pavements would be different than the PSI concept used in the AASHO Road Test.

Assume that a highway is constructed by similar methods, with similar materials, and in a climate similar to that of Ottawa, Illinois which was the location of the AASHO Road Test. Traffic forecasts indicate that 5000 cars and 500 trucks can be expected to use the highway daily. The average weight of the cars is 4 kips and that of the trucks is 50 kips. A designer wishes to predict the service life of a reinforced pavement having a slab thickness of 8 inches before the PSI decreases to 2.0.

Previous analyses of Road Test data (Reference 36) indicate that for a 20°F temperature gradient across the slab thickness an 8-inch pavement corner deflects approximately 0.0005 in/kip of load. Assuming these conditions apply to the slab under consideration, and noting that the cumulative corner deflection for an 8 inch slab is 62,000 inches for PSI = 2 (Figure 16), a designer could predict the service life of the pavement as shown in Table 7.

It should be pointed out that since averaged data were used to construct Figure 16, the estimate of service life and calculated in Table 7 should be interpreted as that of an average 8-inch thick slab.

TABLE 7. SERVICE LIFE CALCULATION EXAMPLE

Data Used in Calculation

Daily Traffic: 5000 cars, 500 trucks

Vehicle Weight: cars - 4 kips, trucks - 50 kips

Average Corner Deflection: $0.0005 \frac{\text{inch}}{\text{kips}}$

Cumulative Corner Deflection for PSI = 2 - 62,000 inches (See Figure 16)

$$\text{Service Life (Years)} = \frac{62,000 \text{ inches}}{\left[\left(5000 \frac{\text{cars}}{\text{day}} \right) \left(4 \frac{\text{kips}}{\text{car}} \right) \left(0.0005 \frac{\text{inch}}{\text{kips}} \right) + \left(500 \frac{\text{trucks}}{\text{day}} \right) \left(50 \frac{\text{kips}}{\text{truck}} \right) \left(0.0005 \frac{\text{inch}}{\text{kips}} \right) \right] \left(365 \frac{\text{days}}{\text{year}} \right)}$$

Service Life = 7.6 years

SECTION V

CONCLUSIONS

The objective of this research was to verify the following hypothesis for rigid highway pavements:

There is a functional relationship between the total energy imparted to a pavement system as measured by cumulative deflections and the condition of that system.

Analyses of AASHO Road Test data indicate that the hypothesis is valid for rigid pavements. In addition as a result of this research the following conclusions seem to be warranted:

1. The determination of a well-defined serviceability-cumulative deflection relationship will allow an engineer to estimate the service life of a rigid pavement provided traffic volume data can be forecast and the accompanying pavement deflections are measured or can be estimated.
2. The Present Serviceability Index-cumulative deflection relationship determined from averaged AASHO Road test data indicates that early in the service life of a pavement, increases in cumulative deflection result in small changes in PSI until a threshold cumulative deflection is reached. For cumulative deflections greater than the threshold value, the pavement serviceability decreases more rapidly as the cumulative deflection increases. The phenomenon was noted for both slab edge and slab corner cumulative deflections.
3. The threshold cumulative deflection at which a sharp break occurs in the PSI - cumulative deflection curve increases as the slab thickness of a rigid pavement increases.
4. All other factors being equal, the effects of increasing rigid pavement slab thickness are twofold:
 - a. the deflections caused by a given load decrease as the slab thickness increases
 - b. the cumulative deflection corresponding to a given level of pavement serviceability increases as the slab thickness increases.
5. A well defined pavement serviceability - cumulative deflection relationship for airfield pavements would enable the engineer to apply the working hypothesis, predict future serviceability levels, and plan and budget programs of pavement maintenance.

SECTION VI

RECOMMENDATIONS FOR FUTURE RESEARCH

A serviceability - cumulative deflection relationship should be determined for both flexible and rigid airfield pavements. Such relationships can be determined by implementing the following:

1. Records of landings and takeoffs categorized according to type of aircraft should be maintained at all airfields.
2. Periodic deflection measurements should be made at airfields. The successful development of a laser deflection measuring device would allow periodic deflection measurements to be made with a minimum of traffic interruption.
3. Develop a performance or serviceability rating system for airfield pavements. The system should be cognizant of the mission of the airfield, i.e., to provide a surface upon which landing, taxiing and takeoff operations can be carried out safely and efficiently. The system should be objective, applicable to all airfields, and should recognize that the degree of pavement distress that can be tolerated at a given airfield will be a function of the types of aircraft using the field, the volume and frequency of traffic, etc. In addition, the criteria upon which the rating system is based should involve pavement characteristics that can be determined with a minimum of traffic interruption or airfield downtime and should be a non-destructive method.

As a first step in implementing the preceding at airfields having rigid pavements where traffic records are available, the original pavement design and subsequent pavement maintenance records should be analyzed. These data would enable an estimate of cumulative deflection to be determined and the cumulative deflection could be correlated with the remedial measures that were carried out at the airfield. Thus, for example, it could be determined that at XYZ Air Force Base, a "t" inch thick overlay was applied when the cumulative deflection reached "R" inches. This would enable an estimate of the threshold cumulative deflection to be determined. This type of analysis was carried out by Hightner and Harr (Reference 41) for flexible airfield pavement and the results indicated there was a threshold cumulative deflection at which cracking developed. It is recommended that a similar analysis be carried out on rigid pavements.

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ABBREVIATIONS AND SYMBOLS

A_0, A_1, A_2	Constants
AASHO	American Association of State Highway Officials
B	Constant
B_L	Load used for Benkelman beam deflections, kips
C+P	Parameter related to cracking and patching of test sections of AASHO Road Test
D	Flexural rigidity of plate, lb-in
E	Young's modulus (lb/in ²)
$\bar{G}(s)$	Transfer function
I	Moment of inertia, in ⁴
$\bar{I}(s)$	Laplace transform of time dependent input function, I(t)
L	Load, kips
MD	Benkelman beam deflection, inches
N	Number of traffic loads between contiguous Benkelman beam deflection measurements
$\bar{O}(s)$	Laplace transform of time dependent output function, O(t)
P	Load, kips
P(x)	Reaction of foundation at point x, lb/ft ²
PSI	Present Serviceability Index
PSR	Present Serviceability Rating
R	Reinforcing
S	Slab thickness, inches
SB	Subbase thickness, inches
U	Strain energy, in-lb
W(x)	Deflection of subgrade surface at point x, inches
a, b	Constants

a	Radius of circular plane, feet
h	Thickness of plate, inches
k	Modulus of subgrade reaction
ΣD	Cumulative deflection, inches
ϵ	Error term in regression analysis
ν	Poisson's ratio