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A COMPARISON OF TWO METHODS FOR PREDICTING LOSS OF LEARNING DUE TO A BREAK IN PRODUCTION

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A COMPARISON OF TWO METHODS FOR PREDICTING LOSS OF LEARNING DUE TO A BREAK IN PRODUCTION

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67

FOREWORD

The research discussed in this report was accomplished as part of the Maintenance Effectiveness Engineering Graduate Program conducted jointly by the DARCOM Intern Training Center and Texas A&M University. As such, the ideas, concepts and results herein presented are those of the author and do not necessarily reflect approval or acceptance by the Department of the Army.

This report has been reviewed and is approved for release. For further information on this project contact Dr. Ronald C. Higgins, Intern Training Center, Red River Army Depot, Texarkana, Texas 75501.

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Learning curves have received increased emphasis from private industry and the government in recent years. A reality that is associated with the learning curve but has received little formal attention is a break in production and the effect it has on follow-on first unit cost. Since a major activity of the government is the procurement of spare parts after initial production of a system is completed, reliable prediction techniques are needed for estimating first unit costs following a break in production.

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This report compares two such methods that have been developed by two separate and independent sources that predict the direct labor hours for the first item to be produced after a break in production has occurred.

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TABLE OF CONTENTS

FIGURES

CHAPTER 1

INTRODUCTION

The learning phenomenon has been studied by philosophers and psychologists for centuries. In fact, Aristotle was the first to set forth laws in an attempt to explain the basis of learning. (17)*

In Mednick's book, Learning, learning has been defined in terms of four characteristics. (14) These are:

1. Learning results in a behavioral change. This characteristic is the basic goal of any efforts at learning.

2. Learning is a result of practice. This eliminates behavioral changes due to illness, maturation, or motivation. Although performance may be greatly altered by these variables, learning is not.

3. Learning is a relatively permanent change. A task which was learned sometime in the past can be easily resumed after a little practice.

4. Learning is not directly observable. Performance is affected by variables other than learning. Therefore, a record of successive performance is just that, and cannot be considered an exact representation of the learning process.

As a result of studying the usefulness of learning, various methods have been developed for measuring the amount of learning acquired and projecting its effects on things to happen at some future time. The theories behind these methods of learning applications have become very useful tools in the field of industrial

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*Numbers in parentheses refer to the list of references following this report.

forecasting. One concept, which will be the foundation of this research paper, has been developed utilizing the relations between the number of units produced and the labor hours required to produce those units.

The statistical measure of this relationship between the number of units produced and their respective labor hours has become known as a learning curve. This curve also may be referred to as a progress, improvement or experience curve because variables, other than learning, contribute to determining the slope of the curve. (16) These variables are: (12)

1. Improved Production Methods
2. Direct Labor Learning
3. Management Learning
4. More Effective Procurement
5. Eliminating Engineering Problems
6. Simplification of Design

The first publication leading to the industrial application of the learning curve has been credited to T. P. Wright. His article was published in the February, 1936 issue of the <u>Journal of Aeronautical Science</u>. (11) He showed that as the number of aircraft produced increases, the cumulative average per unit cost to produce an aircraft decreases at a constant rate. This has since become known as the cumulative average theory of the learning curve. (12)

Since this first publication, the learning curve theory has been extended into many areas ranging from the setting of contract prices to production planning and control. (1) In situations where the learning curve principles can be applied, the government is also using it in evaluating contract proposals.

In the book, Purchasing and Materials Management, Lee and Dobler state that: (13)

> The basic point revealed by the learning curve is that a specific and constant percentage reduction in the average direct labor hours required per unit results each time the number of units produced is doubled.

This means that the direct labor hours required to produce a second unit will be a certain percentage less than the labor required for the first unit; the direct labor required for the fourth unit will be the same percent less than that of the second unit; the direct labor for the eighth unit will be the same percent less than the fourth unit; and this constant percentage of reduction will continue as long as uninterrupted production of the same item continues. (12)

In using the learning curve, there are at least two pieces of data required. First, the hours required to produce the first unit must be known or determinable. This data point is arrived at by the use of production standards or historical data which has been made available from previous builds. The second piece of data, the slope of the learning curve to be used, is arrived at by fitting a curve to the historical data, if available, or using a standard curve accepted by a particular industry.

If theoretical data is used, the accuracy of the calculations would be only as good as the assumptions used in generating them. For this reason, the best results are obtained when actual or historical data is used with the learning curve.

Very often the government has access to historical or actual data which its contractors have accumulated during past contracts of like or similar items. This is especially true where follow-on contracts for spares or replacement parts are concerned. A phenomenon associated with these types of contracts which has received very little formal attention is the effect a break in production might have on a follow-on contract's first unit direct labor hours. More and more concern is being directed to this area of learning curve application because the problem of pricing the first item to be produced following a break in production exists in numerous contract negotiations.

Two methods, one by Allen A. Pichon and Charles L. Richardson of the USAF and another by Robert Blair Ilderton of the Defense Contracts

Audit Agency, have been developed for dealing with follow-on contracts which have experienced production breaks. The objective of this research paper will be to examine these two methods to see how they compare in predicting the first unit cost following a break in production.

CHAPTER 2

LEARNING CURVE BACKGROUND AND RESEARCH JUSTIFICATION

In the previous chapter mention was made of the learning curve as a tool for estimating labor hours for production contracts. This chapter will provide an introduction to the learning curve for the benefit of those who are unfamiliar with its principles and a more detailed justification for this research paper.

The Learning Curve

The kind of curve that expresses the learning phenomenon is called an inverse variation. The formula used to express this relationship is $Y=KX^c$, where

Y = the number of direct labor man hours required to produce the X^{th} unit.
K = the number of direct labor man hours required to produce the first unit.
X = the unit number
c = log B/log 2 where B equals the learning curve factor (.90, .85, .77, etc.)

Also, since the inverse variation, in general, means that the dependent variable (Y) gets smaller as the independent variable (X) gets larger, this relationship is also referred to as an exponential (log-linear) equation. For a given learning curve, K and c are constants where K can assume any positive value and c is a negative constant between 0 and -1. (15)

It is important to understand K and c since they control the vertical position and rate of decrease of Y. Due to the fact that $1^n = 1$, it is evident that $Y = K$ for $X = 1$. Thus, the magnitude of the vertical height of Y, the dependent variable, is determined by K. For increases in X the rate of decrease in Y is controlled by the size of c which means that as c approaches 0, Y approaches a horizontal line K units high and tends to decrease very little for increases in X. Yet, the rate of decrease of Y grows larger as c approaches -1. (15)

As can be seen from Figure 2-1, the greatest absolute decreases in Y values occur at the lowest range of X values for the curve described by $Y = KX^c$. Upon closer observation of the curve, it becomes apparent that, for a unit change of X, the absolute decrease in Y gets smaller as X increases. In fact, each time the value of X is doubled, the value of Y will decrease by a constant proportion. The amount of decrease will depend upon the value of the constant c. For example, on a 90 percent curve the cost of the 100th item of a production run will be 90 percent of the cost of the 50th item, the cost of the 50th item will be 90 percent of the cost of the 25th item, etc.

By taking the logarithm of the learning curve equation and remembering that $\log (AB) = \log A + \log B$ and $\log Z^y = y \log Z$, then

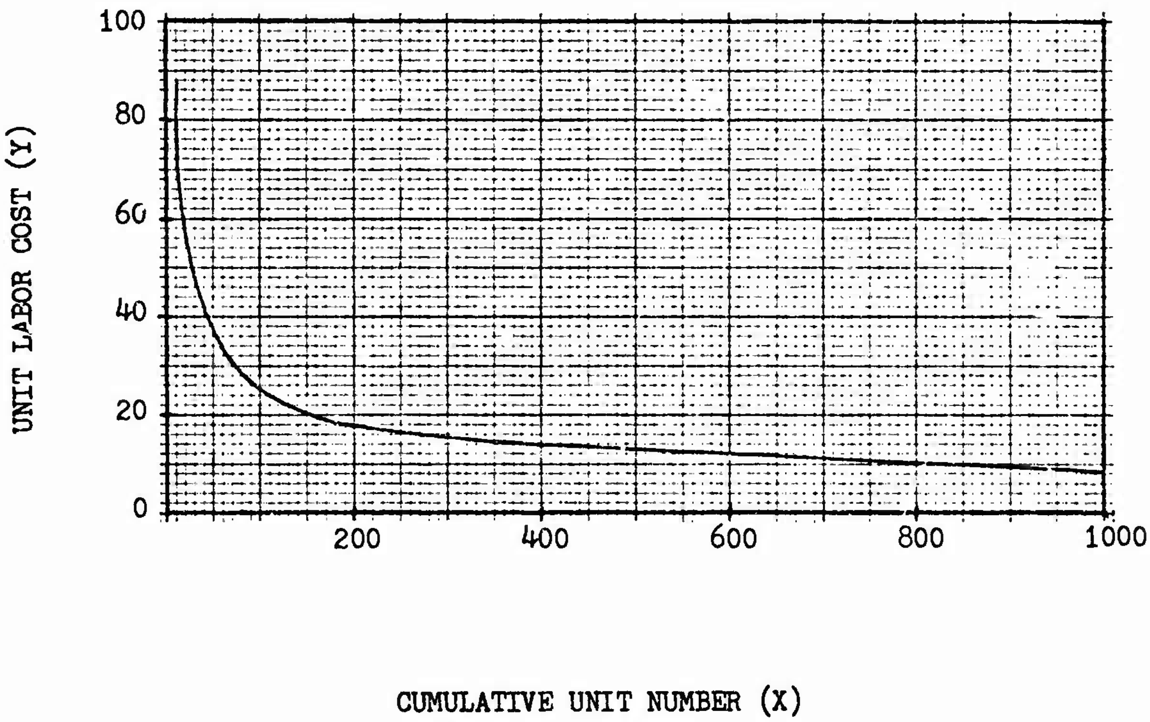

Figure 2-1. 90 Percent Learning Curve On Arithmetic Grid For $Y = KX^c$ Where $c = \text{Log } .9/\text{Log } 2$ (15)

$Y = KX^c$ becomes

$$\log Y = \log K + c \log X \qquad \text{(Eq. 2-1)}$$

If Equation 2-1 is compared to the standard linear equation, $Y = A + BX$, the similarity becomes readily apparent. Thus, for logarithmic values of Y, K, and X, Equation 2-1 can be plotted as a straight line having a negative slope.

Due to the tediousness of transferring data to logarithmic values, log-log graph paper should be used to graph Equation 2-1. Figure 2-2 represents the 90 percent learning curve plotted on logarithmic grid with both the X and Y axes subdivided logarithmically. Thus, an arithmetic value plotted on log-log paper corresponds to logarithmic values plotted on arithmetic graph paper. (15)

The slope of the learning curve is another interesting characteristic to observe. The mathematical definition of slope is given as a logarithmic function of c. Since Y is customarily expressed as a function of X where X_N differs from X_{N-1} by a factor of 2, the equation $Y = KX^c$ can be used to express the slope as follows:

$$\text{Slope} = \frac{Y_{2X} = K\ (2X)^c}{Y_X \ = KX^c}$$

or

$$\text{Slope} = S = 2^c$$

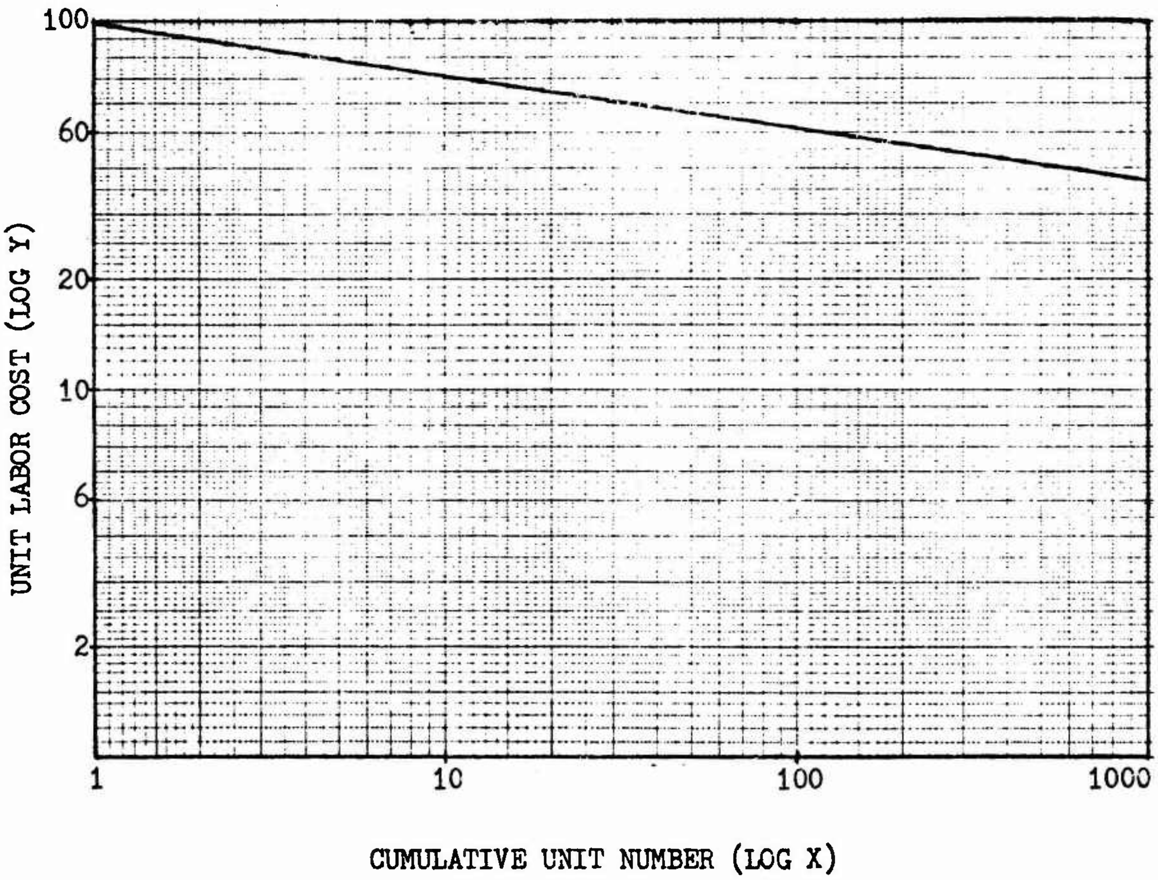

Figure 2-2. 90 Percent Learning Curve On Logarithmic Grid For Log Y = Log K + c Log X, Where c = Log .9/Log 2 (15)

Thus, c can be expressed as a function of S by taking the log of both sides of the above equation. (15)

$$\log (\text{slope}) = \log S = c \log 2 \qquad \text{(Eq. 2-2)}$$

Therefore,

$$c = \frac{\log S}{\log 2} \qquad \text{(Eq. 2-3)}$$

For the 90 percent learning curve, c can be expressed as

$$c = \log .9/\log 2$$

Therefore, by knowing the type of learning curve involved, $Y = KX^c$ can be used as a model for predicting various values of Y.

Justification for Research

One of the assumptions that is important in the application of the learning curve is that production runs will be stable and will not encounter a break in production. In real world situations however, production runs may not parallel the above theoretical assumption.

In fact, a problem currently facing contract negotiators who use the learning curve as an evaluating tool is that of relating the effects of a break in production to the learning curve. In his unpublished study, "Production Break and Related Learning Loss," George Anderlohr, a former employee of Defense Contracts Administration Services (DCAS), discussed an example of a break in production. (3)

The following quote from his study is pertinent to the problems faced by government negotiators:

> The production break is the time lapse between the completion of a contractual requirement for the manufacturing of certain units of equipment and the commencement of a follow-on order for identical units of equipment. This time lapse disrupts the continuous flow of products. This could, in smaller shops, include a condition where the follow-on order was received prior to the delivery of the last units of the first order. An example of this would be the completion of circuit board assemblies, and all personnel had been moved into the final assembly area. Thus, the circuit board assembly line would have to be reestablished to accommodate the new order. (3)

In establishing a new assembly line, lost learning would be implied.

It is the above learning loss that negotiators are concerned with when negotiating a follow-on contract which has experienced a break in production. Mr. Anderlohr further emphasized this problem in the above mentioned study as follows:

> A major problem with the application of the improvement (learning) curve has always been that it addresses itself to a perfect environment which rarely exists. A major condition for this perfect environment is an uninterrupted production cycle (one lot of identical units following another). When plotting actual labor hours on a curve, it has been long noted that any interruption in the orderly and continuous flow of work from one work station to another is accompanied by an increase of labor hours when production is resumed. This has been commonly referred to as start up costs which relates directly to loss of improvement.
>
> In the real world of government procurement there is, almost always, a break in the production cycle. There has been no established reliable method of compensating for the loss of improvement resulting from a break in production. General Electric Cost Accounting Service Bulletin No. PC-5 recommends a fifty percent loss of learning for a three to six month break and a seventy-five percent loss for a twelve month break. This is such a general approach that

> it would be extremely difficult to support in cost negotiations. Because of the lack of guidance, most cost analysts take almost arbitrary positions ranging from the use of unexpected percentages, as mentioned above, to the position that no learning was retained after a production break. The total loss of learning is usually based on a common misconception that learning or improvement is directly related to personnel know-how only.
>
> Negotiators and Cost Analysts facing their counterparts across a negotiation table are frequently plagued with the recurring problem of estimating loss of improvement (learning). (3)

The problem facing contract negotiators is the determination of how much learning has been lost and what point should be used upon reentering the learning curve to determine unit cost. This problem of determining the effects of a break in production on a follow-on contract has not been researched extensively as of yet. In performing the literature search on this topic, two models were found which have been developed for the purpose of predicting lost learning. The purpose of this research paper will be to make a comparison of the results of these two methods when predicting the cost of the first unit produced following a break in production.

CHAPTER 3

DESCRIPTION OF MODELS

This chapter presents the results of a literary search that this author has made on the subject of determining effects of lost learning due to a break in production. An attempt will be made to briefly describe two mathematical models, both designed for use on the computer, that have been developed by two separate sources.

Model 1:

The United States Air Force (USAF), which relies a great deal on the learning curve as an estimating tool in many of its contract negotiations, prior to 1974 had no formal method for handling loss of learning due to a break in production. Thus, the USAF supported a study by Allen A. Pichon, Jr. and Charles L. Richardson which resulted in a mathematical model for predicting the first unit costs following a break in production by use of step-wise regression techniques. (15)

In developing their method, Pichon and Richardson used the Newtonian approximation method, simple calculus and the computer to transform collected data into meaningful data, provided the number of units in a lot, the cumulative direct labor hours involved, and the break in production are known. Thus, a means of approximating the first unit direct labor hours, the last unit direct labor hours and the learning curve for a lot was formulated.

The above authors formulated a mathematical model that was developed by using step-wise regression techniques. These techniques were used based on the objectives of regression analysis listed:

> 1. The first purpose of regression analysis is to provide estimates of values of the dependent variable from values of the independent variable(s)...
>
> 2. A second goal of regression analysis is to obtain a measure of the error involved in using the regression line as a basis for estimating. For this purpose the "standard error of estimate" or its square, the "error variance around the regression line" are calculated...
>
> 3. The third objective, which...(is)...classified as correlation analysis, is to obtain a measure of the degree of association or correlation that exists between... two variables. The coefficient of determination, calculated for this purpose measures the strength of the relationship that exists between...two variables (15)...

The model resulting from the above study was of the form:

$$Y = A_0 + A_1X_1 + A_2X_2 + A_3X_3 \qquad \text{(Eq. 3-1)}$$

where

Y = the calculated independent variable (first unit cost after a break in production)

A_0 = the regression constant

A_1 = regression coefficient for X_1

A_2 = regression coefficient for X_2

A_3 = regression coefficient for X_3

X_1 = learning curve factor

X_2 = last unit direct labor hours for the lot(s)

X_3 = the length of a break in production

In approximating values for the variables, one must determine the learning curve for the production process and the number of hours to produce the first unit. If the recorded data reflects actual hours for the unit, then it is easy to determine both the learning curve and the first unit total hours. On the other hand, if the data recorded only reflects the cumulative hours for the units produced, and the number of units in the lot, then the learning curve and the direct labor for the first unit must be calculated by the use of some approximation technique. One such technique used by Pichon and Richardson in developing this model is described here for information to the reader.

It is generally accepted that the area under a curve "f" is represented by the integral of the function "f" bounded by an interval (a, b). This area, in terms of the learning curve, is represented by the cost of a particular lot or the total direct labor hours of a lot. Thus, the learning curve can be determined by integrating $Y = KX^c$; therefore,

$$D_j = \int_{.5}^{t + .5} KX^c dx \qquad \text{(Eq. 3-2)}$$

where D_j equals the direct labor hours of a particular lot of size "t".

This integration then is actually an approximation of a step function since direct labor hours is a discrete variable. After expanding

$$D_i = \frac{K}{c+1} \left[(t + .5)^{c+1} - (.5)^{c+1} \right] \qquad \text{(Eq. 3-3)}$$

for a lot of size L, D can be expressed as:

$$D_L = \frac{K}{c+1} \left[(L + .5)^{c+1} - (.5)^{c+1} \right] \qquad \text{(Eq. 3-4)}$$

and the direct labor hours for both lots, ie. the sum of lot 1 and lot 2 can be expressed as:

$$D_M = \frac{K}{c+1} \left[(M + .5)^{c+1} - (.5)^{c+1} \right] \qquad \text{(Eq. 3-5)}$$

Expressing Equations 3-4 and 3-5 in another form:

$$\frac{c+1}{K} = \frac{1}{D_L} \left[(L + .5)^{c+1} - (.5)^{c+1} \right] \qquad \text{(Eq. 3-6)}$$

and

$$\frac{c+1}{K} = \frac{1}{D_M} \left[(M + .5)^{c+1} - (.5)^{c+1} \right] \qquad \text{(Eq. 3-7)}$$

Solving Equations 3-6 and 3-7 simultaneously:

$$D_L \left[(M + .5)^{c+1} - (.5)^{c+1} \right] - D_M \left[(L + .5)^{c+1} - (.5)^{c+1} \right] = 0 \qquad \text{(Eq. 3-8)}$$

For Equation 3-8, the following information is determinable from actual data collected: (15)

D_L = Direct labor hours for the first lot

D_M = Direct labor hours for two or more consecutive lots

L = Number of units produced for the first lot

M = Number of units produced for two or more consecutive lots

We see that the only variable that is not defined is c and Equation 3-8 can be used to solve for this unknown. Since c equals log B/log 2, the learning curve factor, B, can also be determined. However, calculating the value for c is a long and tedious process whereby a value for c must be selected and plugged into Equation 3-8 to see if it equals zero. Chances are very slim that this first selection will be the exact value that satisfies the equation. Therefore, other values (one higher and one lower than the first) must be selected in efforts to find an interval in which c must lie. Once this interval is determined, the selection process must be continued until a value for c is found for which Equation 3-8 is satisfied. Since such a process of elimination could be quite inefficient because of the selections made and the accuracy desired, other methods for approximating the learning curve can be used.

One such method is to make use of learning curve tables which have been generated to assist in applying the learning curve as an estimating tool. These tables contain factors for various learning curves which can be used to establish the amount of time required to produce any unit in a production lot. In order to use such tables, one needs to know or be able to determine the lot size, the cumulative or unit costs from previous productions and the learning curve for the items produced. A portion of such tables for an 85% learning curve is shown in Figures 3-1A and 3-1B. (9)

To illustrate how learning curve tables can be used, suppose that you have built a lot of 25 units and have gotten a follow-on order for two option quantities of 25 units each and you need to estimate the cost of these options. You have recorded actual hours for the 25 units at 8500 hours and an 85% curve was experienced.

From the example,

$$\text{Hours for Unit 1} = \frac{\text{Total Cumulative Hours}}{\text{Cumulative Factor from Learning Curve Table for N = 25}} = \frac{8500}{14.800727} \text{ (Ref. Fig. 3-1A)}$$

$$= \underline{\underline{574.2961 \text{ Hours}}}$$

one can determine the total cost for the two follow-on options.

<u>Option 1:</u>

Total hours for 25 additional units = Total hours for 50 units- Total hours for 25 units

Total hours for 50 units = Hours for Unit 1 x Cumulative Factor for 50 units

$$= 574.2961 \times 25.51311 \text{ (Ref. Fig. 3-1A)}$$

$$= \underline{\underline{14,652 \text{ Hours}}}$$

	85.0			86.0			87.0		
N	CUM TOTAL	C.A.	UNIT	CUM TOTAL	C.A.	UNIT	CUM TOTAL	C.A.	UNIT
1	1.00000000	.00000000	.00000000	1.00000000	.00000000	.00000000	1.00000000	.00000000	.00000000
2	1.84999999	.92500000	.84999999	1.86000000	.93000000	.86000000	1.87000000	.93499999	.87000000
3	2.62291484	.87430494	.77291483	2.64737659	.88245886	.78737659	2.67193706	.89064568	.80193706
4	3.34541483	.83635370	.72250000	3.38697659	.84674414	.73960000	3.42883705	.85720926	.75690000
5	4.03108589	.80621718	.68567106	4.09152383	.81830476	.70454723	4.15255285	.83051056	.72371580
6	4.68806350	.78134391	.65697761	4.76864770	.79477794	.67714387	4.85023809	.80837301	.69768523
7	5.32171997	.76024571	.63365646	5.42347559	.77478223	.65480789	5.52664663	.78952094	.67640853
8	5.93584497	.74198062	.61412500	6.05953159	.75744145	.63605600	6.18514962	.77314370	.65850300
9	6.53324232	.72591581	.59739734	6.67949349	.74216594	.61996190	6.82825267	.75869474	.64310304
10	7.11606272	.71160627	.58282040	7.28540412	.72854041	.60591061	7.45788541	.74578854	.62963274
11	7.68600333	.69872757	.56994060	7.87887835	.71626166	.59347423	8.07557601	.73414327	.61769059
12	8.24443430	.68703619	.55843096	8.46122208	.70510184	.58234373	8.68256217	.72354685	.60698616
13	8.79248279	.67634483	.54804849	9.03351117	.69488547	.57228909	9.27986509	.71383578	.59730292
14	9.33109079	.66650648	.53860799	9.59664596	.68547471	.56313478	9.86834052	.70488146	.58847542
15	9.86105613	.65740374	.52996533	10.15138996	.67675932	.55474400	10.44871504	.69658100	.58037452
16	10.38306238	.64894140	.52200624	10.69839812	.66864988	.54700816	11.02161265	.68885079	.57289761
17	10.89770113	.64104124	.51463874	11.23823787	.66107281	.53983974	11.58757454	.68162203	.56596189
18	11.40548887	.63363827	.50778774	11.77140510	.65396695	.53316724	12.14707419	.67483746	.55949964
19	11.90688007	.62667789	.50139120	12.29833661	.64728087	.52693150	12.70052901	.66844890	.55345481
20	12.40227742	.62011386	.49539734	12.81941974	.64097098	.52108313	13.24830949	.66241547	.54778048
21	12.89203990	.61390666	.48976249	13.33500015	.63500001	.51558041	13.79074656	.65670221	.54243706
22	13.37648942	.60802224	.48444951	13.84538800	.62933581	.51038784	14.32813738	.65127897	.53739081
23	13.85591603	.60243113	.47942661	14.35086300	.62395056	.50547500	14.86075018	.64611957	.53261279
24	14.33058236	.59710760	.47466632	14.85167861	.61881994	.50081561	15.38882814	.64120117	.52807795
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26	15.26656839	.58717570	.46584122	15.84023403	.60923976	.49216861	16.43224623	.63200947	.51965354
27	15.72830565	.58252984	.46173727	16.32837752	.60475472	.48814349	16.94797440	.62770275	.51572816
28	16.18612245	.57807580	.45781679	16.81267344	.60045262	.48429591	17.45994802	.62356957	.51197361
29	16.64018793	.57379958	.45406547	17.29328556	.59632019	.48061212	17.96832477	.61959741	.50837675
30	17.09065846	.56968861	.45047053	17.77036540	.59234551	.47707984	18.47325060	.61577502	.50492582
31	17.53767903	.56573158	.44702056	18.24405350	.58851785	.47368809	18.97486097	.61209229	.50161036
32	17.98138434	.56191826	.44370531	18.71448051	.58482751	.47042701	19.47328188	.60854006	.49842092
33	18.42189989	.55823939	.44051555	19.18176824	.58126570	.46728772	19.96863087	.60511002	.49534896
34	18.85934283	.55468655	.43744293	19.64603042	.57782442	.46426217	20.46101771	.60179464	.49238685
35	19.29382274	.55125207	.43447990	20.10737351	.57449638	.46134309	20.95054525	.59858701	.48952754
36	19.72544231	.54792895	.43161958	20.56589733	.57127492	.45852382	21.43730995	.59548083	.48676469
37	20.15429801	.54471076	.42885569	21.02169567	.56815393	.45579834	21.92140246	.59247033	.48409251
38	20.58048054	.54159159	.42618252	21.47485676	.56512780	.45316109	22.40290815	.58955021	.48150568
39	21.00407535	.53856603	.42359481	21.92546380	.56219137	.45060703	22.88190750	.58671557	.47899935
40	21.42516309	.53562907	.42108773	22.37359530	.55933988	.44813149	23.35847653	.58396191	.47656902
41	21.84381996	.53277609	.41865687	22.81932548	.55656891	.44573018	23.83258712	.58128504	.47421059
42	22.26011807	.53000281	.41629811	23.26272463	.55387440	.44339915	24.30460737	.57868113	.47192024
43	22.67412576	.52730525	.41400769	23.70385937	.55125254	.44113473	24.77430184	.57614655	.46969447
44	23.08590785	.52467972	.41178209	24.14279291	.54869983	.43893354	25.24183185	.57367799	.46753001
45	23.49552593	.52212279	.40961806	24.57958536	.54621300	.43679244	25.70725569	.57127234	.46542383
46	23.90303855	.51963127	.40751262	25.01429386	.54378899	.43470850	26.17062882	.56892671	.46337312
47	24.30850147	.51720215	.40546292	25.44697287	.54142495	.43267900	26.63200409	.56663838	.46137527
48	24.71196785	.51483266	.40346637	25.87767430	.53911821	.43070142	27.09143192	.56440483	.45942782
49	25.11348837	.51252016	.40152052	26.30644767	.53686628	.42877337	27.54896042	.56222368	.45752849
50	25.51311145	.51026223	.39962308	26.73334032	.53466681	.42689265	28.00463558	.56009271	.45567516

FIGURE 3-1A
85% LEARNING CURVE TABLE

	85.0			86.0			87.0		PAGE 502
N	CUM TOTAL	C.A.	UNIT	CUM TOTAL	C.A.	UNIT	CUM TOTAL	C.A.	UNIT
51	25.91088337	.50805654	.39777192	27.15839750	.53251760	.42505717	28.45850140	.55800983	.45386581
52	26.30684841	.50590093	.39596503	27.58166251	.53041659	.42326501	28.91059998	.55597308	.45209858
53	26.70104896	.50379337	.39420054	28.00317684	.52836183	.42151432	29.36097168	.55398060	.45037169
54	27.09352564	.50173195	.39247668	28.42298024	.52635148	.41980340	29.80965518	.55203065	.44868350
55	27.48431742	.49971486	.39079177	28.84111087	.52438383	.41813063	30.25668763	.55012159	.44703244
56	27.87346170	.49774038	.38914428	29.25760536	.52245723	.41649448	30.70210467	.54825187	.44541705
57	28.26099440	.49580692	.38753270	29.67249890	.52057015	.41489353	31.14594060	.54642001	.44383592
58	28.64695006	.49391293	.38595565	30.08582532	.51872113	.41332642	31.58822837	.54462463	.44228777
59	29.03136188	.49205698	.38441182	30.49761719	.51690876	.41179187	32.02899972	.54286440	.44077134
60	29.41426183	.49023769	.38289995	30.90790585	.51513176	.41028866	32.46828519	.54113808	.43928547
61	29.79568072	.48845378	.38141888	31.31672151	.51338887	.40881565	32.90611424	.53944449	.43782705
62	30.17564819	.48670400	.37996747	31.72409327	.51167892	.40737176	33.34251526	.53778250	.43640102
63	30.55419289	.48498718	.37854469	32.13004922	.51000078	.40595594	33.77751564	.53615104	.43500338
64	30.93134240	.48330222	.37714951	32.53461646	.50835337	.40456723	34.21114185	.53454909	.43362620
65	31.30712339	.48164805	.37578099	32.93782115	.50673570	.40320469	34.64341941	.53297568	.43227756
66	31.68156161	.48002366	.37443821	33.33968859	.50514679	.40186744	35.07437302	.53142989	.43095361
67	32.05468194	.47842809	.37312032	33.74024323	.50358572	.40055463	35.50402655	.52991084	.42965353
68	32.42650843	.47686041	.37182649	34.13950870	.50205159	.39926547	35.93240312	.52841769	.42837656
69	32.79706438	.47531977	.37055594	34.53750789	.50054359	.39799918	36.35952505	.52694964	.42712193
70	33.16637229	.47380532	.36930791	34.93426295	.49906090	.39675505	36.78541401	.52550571	.42588896
71	33.53445401	.47231625	.36808171	35.32979533	.49760275	.39553237	37.21009097	.52408578	.42467695
72	33.90133065	.47085181	.36687664	35.72412582	.49616841	.39433049	37.63357625	.52268855	.42348528
73	34.26702271	.46941127	.36569206	36.11727457	.49475718	.39314875	38.05588958	.52131356	.42231332
74	34.63155006	.46799392	.36452734	36.50926114	.49336839	.39198656	38.47705007	.51996014	.42116048
75	34.99493195	.46653909	.36338189	36.90010449	.49200139	.39084335	38.89707628	.51862763	.42002621
76	35.35718710	.46522614	.36225514	37.28982303	.49065556	.38971854	39.31598622	.51731560	.41890994
77	35.71833365	.46387446	.36114655	37.67843465	.48933031	.38861161	39.73379742	.51602334	.41781119
78	36.07838924	.46254345	.36005559	38.06595670	.48802508	.38752205	40.15052684	.51475034	.41672943
79	36.43737100	.46123254	.35898176	38.45240606	.48673932	.38644936	40.56619105	.51349609	.41566420
80	36.79529558	.45994119	.35792457	38.83779915	.48547247	.38539308	40.98080610	.51226007	.41461505
81	37.15217917	.45866887	.35688359	39.22215191	.48422410	.38435276	41.39438763	.51104182	.41358153
82	37.50803751	.45741509	.35585834	39.60547987	.48299365	.38332795	41.80695085	.50984086	.41256321
83	37.86288592	.45617934	.35484841	39.98779813	.48178070	.38231826	42.21851056	.50865675	.41155970
84	38.21673932	.45496118	.35385339	40.36912140	.48058478	.38132326	42.62908117	.50748906	.41057061
85	38.56961222	.45376014	.35287289	40.74946401	.47940546	.38034260	43.03867674	.50633737	.40959556
86	38.92151875	.45257580	.35190653	41.12883987	.47824232	.37937587	43.44731093	.50520129	.40863419
87	39.27247269	.45140773	.35095394	41.50726261	.47709497	.37842273	43.85499708	.50408042	.40768615
88	39.62248747	.45025554	.35001477	41.88474546	.47596301	.37748285	44.26174819	.50297441	.40675111
89	39.97157616	.44911883	.34908868	42.26130134	.47484608	.37655587	44.66757694	.50188283	.40582874
90	40.31975152	.44799723	.34817535	42.63694284	.47374380	.37564149	45.07249568	.50080550	.40491873
91	40.66702600	.44689039	.34727447	43.01168225	.47265585	.37473941	45.47651647	.49974193	.40402079
92	41.01341172	.44579795	.34638572	43.38553157	.47158186	.37384931	45.87965109	.49869186	.40313461
93	41.35892055	.44471958	.34550882	43.75850248	.47052153	.37297092	46.28191104	.49765495	.40225994
94	41.70356404	.44365493	.34464348	44.13060644	.46947453	.37210394	46.68330752	.49663093	.40139649
95	42.04735348	.44260371	.34378944	44.50185457	.46844057	.37124813	47.08385152	.49561948	.40054399
96	42.39029989	.44156562	.34294642	44.87225779	.46741935	.37040322	47.48355372	.49462035	.39970221
97	42.73241407	.44054035	.34211417	45.24182675	.46641058	.36956896	47.88242461	.49363324	.39887089
98	43.07370651	.43952762	.34129244	45.61057185	.46541400	.36874510	48.28047440	.49265790	.39804979
99	43.41418751	.43852714	.34048100	45.97850327	.46442932	.36793141	48.67771311	.49169407	.39723870
100	43.75386713	.43753867	.33967962	46.34563095	.46345630	.36712768	49.07415050	.49074150	.39643739

FIGURE 3-1B
85% LEARNING CURVE TABLE

Thus,

Total hours for 25 additional units = 14,652 - 8500 = $\underline{\underline{6,152 \text{ Hours}}}$

or

$$\frac{6152 \text{ Hours}}{25 \text{ additional units}} = \underline{\underline{246.08 \text{ Hours/Unit}}}$$

Option 2:

Total hours for second 25 additional units = Total hours for 75 units - Total hours for 50 units

Total hours for 75 units = Hours for Unit 1 x Cumulative factor for 75 units

= 574.2961 x 34.9949 (Ref. Fig. 3-1B)

= $\underline{\underline{20,097 \text{ Hours}}}$

Thus,

Total hours for second 25 additional units = 20,097 - 14,652 (from Option 1)

= $\underline{\underline{5,445 \text{ Hours}}}$

or

$$\frac{5,447 \text{ Hours}}{25 \text{ additional units}} = \underline{\underline{217.81 \text{ Hours/Unit}}}$$

Since the learning curve is an exponential function, Pichon and Richardson decided to look at transformations of the linear model in efforts to improve its predictive capability. Their transformed model came as a result of developing twenty-five different models, one of which was chosen to replace the original model. The following model was chosen: (15)

$$Y = e^{A_0 + A_1 X_1 + A_2 (\ln X_2)} \qquad \text{(Eq. 3-9)}$$

or

$$\ln Y = A_0 + A_1 X_1 + A_2 (\ln X_2) \qquad \text{(Eq. 3-10)}$$

where

Y = the calculated dependent variable (first unit cost after a break in production)

A_0 = the regression constant

A_1 = regression coefficient for X_1

A_2 = regression coefficient for X_2

X_1 = last unit direct labor hours for the lot(s) preceding the break in production

X_2 = learning curve factor

This transformed model is the one selected by Pichon and Richardson in making their analysis because it yielded a standard error of estimate of only 1.5696 at a .05 level of significance compared to a standard error of 25.6335 for the basic model. This model was also selected because a break of as much as 23 months was shown to be statistically insignificant in estimating the cost of a production lot following a break in production. For these reasons, the natural logarithm version (Eq. 3-10) of this transformed model will be used in making the comparisons in Chapter 4.

In order to determine values for the unknowns in the above equations, stepwise regression and the learning curve tables can be used. Pichon and Richardson calculated values for the regression constant and coefficients (A_0, A_1, A_2) in reference 15. They were calculated by applying stepwise regression to data that came from small cost items, items which took less than ten hours to produce. Those constants produce reasonably good results for items which require less than 50 direct labor hours (DLH). However, as the DLH for the last unit of a previous lot, X_1, get larger, the estimate for Y increases to the point that the estimate for the next unit to be produced takes more time than was required to produce the first unit of lot one. At this point new constants should be calculated so that the equation will better represent direct labor data for the item being evaluated.

If the unit DLH are available, then the above constants can be calculated by applying stepwise regression using actual data. The reader is referred to Wiley's <u>Applied Regression Analysis</u> by Draper and Smith for a procedure in applying stepwise regression. If, however, only the total DLH are available, then the DLH for each unit must be estimated to provide data for calculating the regression constants and coefficients.

Since the learning curve can be determined as stated previously, the unit DLH for each unit produced can be estimated as follows.

$$\text{First Unit DLH} = \frac{\text{Total DLH}}{\text{Cum. F.}}$$

(Eq. 3-11)

$$\text{Unit I DLH} = \text{First Unit DLH X U. F. (I)}$$

where

Cum. F. = Cumulative Factor for the total number of units produced for the appropriate learning curve.

U. F. (I) = Unit Factor for Unit I for the appropriate learning curve.

These unit DLH can be generated by hand or by use of a computer and then used to calculate values for A_0, A_1, and A_2. One such computer technique is described in reference 15. Once the unit DLH for each unit is determined, the DLH for the last unit produced will be used for X_1 and the learning curve for the item produced will be used for X_2 (.76, .80, .83, etc.).

When values are found for all the unknowns in the above equations, an approximation of the DLH for the first unit of a follow-on lot can be made. The total DLH for that lot can then be estimated by using the learning curve tables.

This method treats each production lot as if it were the first lot produced. Past learning is not taken into account when approximating a follow-on lot's first unit DLH. Thus, the above prediction technique provides good approximations for items where all learning is assumed lost due to a break in production. Also, if the last unit DLH is less than

50 hours then this model yields good results when using the regression constants calculated by Pichon and Richardson (see Eq. 4-3).

Model 2:

Another government agency, the Defense Contract Audit Agency (DCAA), also uses the learning curve as an estimating tool. Although DCAA is primarily an auditing agency, the organization also furnishes support to other government agencies in evaluating and negotiating government contracts. In performing this support activity, DCAA realized that a need existed for objectively measuring the learning lost due to a break in production. In efforts to satisfy this need, a project was initiated in 1971 to determine if such a task could be accomplished. As a result of this project, a study, directed by Mr. Robert B. Ilderton, was performed. As a result of this study, a method was developed whereby a weighted least-squares line is fitted, under the unit curve theory, to direct labor data before and after a production break in efforts to determine how many units are lost due to a break in production (11).

In developing this model, Mr. Ilderton also used simple calculus, linear regression analysis and the computer to approximate lot midpoints from cumulative data in order that an analysis of regression could be performed. Although the same tools were used in developing this model as were used to develop Model One, the end results were different.

Model Two is a modification of the basic learning curve and takes the form $Y = K(X - AZ)^c$ where A equals the number of units of learning lost because of the break, Z equals zero before the break and 1 afterwards and the other variables are the same as the learning curve given in Chapter 2.

In using Mr. Ilderton's method an initial least-squares fit to the equation $Y = KX^c$ is accomplished in essentially the same way, whether labor hours are available for individual units or lots. Depending upon the amount of accuracy placed on the estimate, the data can be fitted in various ways. One way is to visually fit a curve through data points positioned on logarithmic graph paper using a straight edge to approximate a least-squares fit to these points. If a more accurate fit is desired, the data can be fitted to the equation:

$$Y_X = \log K + c \log X$$

where Y_X represents the logarithm of the average hours required to make units 1 through X.

Another alternative is that of fitting a curve to only two points: (i) cumulative average hours through the last completed unit and (ii) cumulative average hours through the completion of half that number of units. The slope is equal to the first average divided by the second.

For better accuracy and faster estimating, a computer can be used to fit curves to historical data by applying simple linear regression formulae to the logarithms of the average hours and the number of units.

Once the learning curve has been determined, the parameter A in this model is set equal to 1 and a least-squares fit to the data is obtained after deducting one unit from all the units numbered after the break. The values obtained for the index of determination, (r^2), from the two calculations are compared where

$$r^2 = \frac{\left[N\sum(\text{Log } X \cdot \text{Log } Y) - \sum \text{Log } X \cdot \sum \text{Log } Y\right]^2}{\left[N\sum(\text{Log } X)^2 - (\sum \text{Log } X)^2\right] \cdot \left[N\sum(\text{Log } Y)^2 - (\sum \text{Log } Y)^2\right]}$$

$$= \frac{\text{Regression Sum of Squares}}{\text{Total Sum of Squares}} \qquad \text{(Eq. 3-12)}$$

where

N = Number of units produced
X = Unit number
Y = DLH required to produce unit X

If the first index is greater, then no better fit is obtained from the model and no further calculations are required. If the second index is greater, the process is continued to provide fits to repositioned data with A = 2, 3, ..., 29, 30, 32, ..., 98, 100, 105, ..., 195, 200, 210, ..., 490, 500, 525, ..., 975, 1000, 1050, ..., 1950, 2000, 2100, ..., 4900, 5000, 5250, ..., 9750, 10,000, 10,500, ... until the values obtained for r^2 stop increasing and start decreasing. Thus, using the results from the least-squares fit which produced the highest value of r^2, the hours for the first unit following a break in production can be approximated. (11)

The above method treats each production lot as if it were a continuation of a past build. For this reason, past learning is used as a factor in predicting the DLH for the first unit of a follow-on production. When this method is applied, it is in essence shifting the learning curve up. Such a shift causes the unit cost of all units in a follow-on production to be larger when compared to the unit DLH without a break, as seen in Figure 3-2.

In viewing the results in a different light, the DLH for the first unit after a break would be equivalent to the cost of some previous unit. Thus, the break moves the starting position for the follow-on lot backward along the curve. Should all units be assumed lost, this method would yield approximately the same results as Method 1.

This method provides good approximations for the first unit of follow-on lots when learning is assumed to be retained after a break in production occurs.

The above methods for determining the direct labor hours after a break in production may be used for several purposes, some of which are listed below: (11)

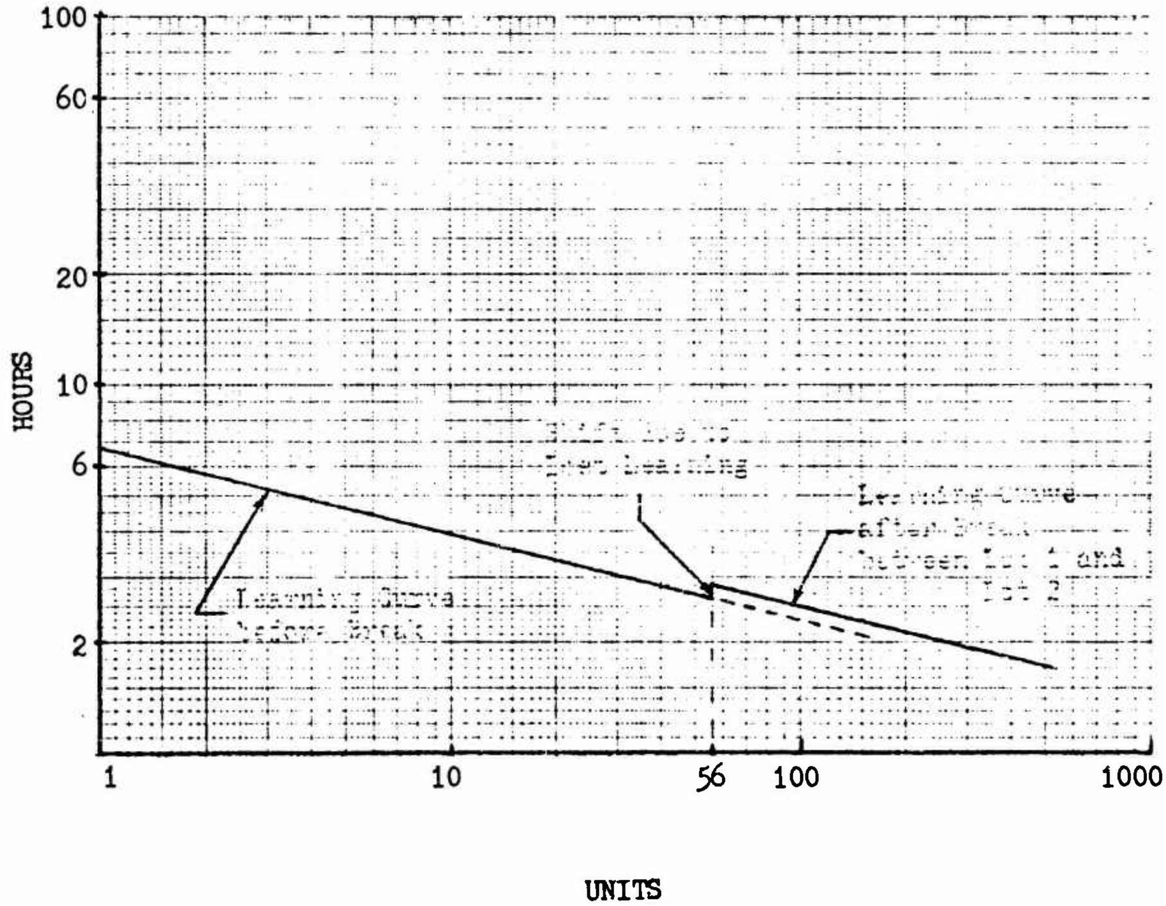

Figure 3-2. 85 Percent Learning Curve Before and After a Break in Production.

a. A contractor has submitted a proposal for a follow-on procurement of an item for which there has been a break in production in the past. These methods can be used to determine the units of learning lost as a result of the break, reposition the units after the break and project the direct labor required for the proposed contract.

b. A contractor has submitted a proposal for the follow-on procurement of an item which is not currently in production or for which a break in production will occur prior to the start of production under the proposed contract. It may be possible to estimate the loss of learning which will occur because of the break by using the above methods to determine the learning lost which occurred as a result of similar breaks in the production of similar items in the past.

c. A contractor claims damages due to an interruption in production caused by Government error. These methods can be applied to normal data before and after the interruption to estimate the labor costs which the contractor would have incurred had there been no interruption.

It should be pointed out that computer programs have been developed for performing the calculations for both Method One and Method Two. These programs can be found in reference 15 and 11 respectively.

CHAPTER 4

APPLICATION OF MODELS

In efforts to show how the two methods presented in Chapter 3 can be applied, the following situation will be considered.

Suppose that a contract is to be let for 25 units of item X which has been previously built by the ABC Company. The first lot consisted of 25 units of item X which required 503.5 direct labor hours (DLH) to produce. Should the contract be awarded to the ABC Company, a break of 6 months will be realized by the time production could be resumed. With this in mind, the ABC Company submitted a proposal for 530 hours to build the follow-on 25 units. A local evaluation engineer has been asked to evaluate the contractor's proposal to determine if it is reasonable.

Since a six month production break will be realized, the evaluator should use an evaluation technique which considers the effects of a break in production upon future builds. Two such techniques will now be applied to the above situation.

Method 1:

This approximation method applies when all learning is assumed to be lost due to a break in production. That is, no learning is retained from previous productions. Since the break is only six months, and the contractor still employs 98% of the employees that worked on the first contract for item X, it should be reasonable to assume that some learning is retained. Thus, this method will not be applicable to our particular situation. However, the method will be illustrated to give the reader a procedure that can be followed when this method can be applied.

Recalling the model for the first method:

$$Y = e^{A_0 + A_1 X_1 + A_2 (\ln X_2)} \qquad \text{(Eq. 4-1)}$$

or

$$\ln Y = A_0 + A_1 X_1 + A_2 (\ln X_2) \qquad \text{(Eq. 4-2)}$$

where

Y = First unit DLH after a break in production
X_1 = Last unit DLH for the lot preceding the break in production
X_2 = Learning Curve (.83)

In order to apply this model, the following procedure is suggested.

Step 1.

The slope of the learning curve which applies to the item to be produced, X_2, must be determined. Since the prediction is for a follow-on production, actual data is available and appears in Appendix A. From this data, the learning curve which describes the production of item X is approximately an 83% curve, as shown in Appendix A. If, however, such data were not available, then the learning curve for similar items which the contractor has built should be used. If there are no similar items, then the standard curve for the industry should be used for estimation purposes.

Step 2.

Determine values for the constants A_0 (the regression constant), A_1, and A_2 (regression coefficients). Values for these constants have been calculated by Pichon and Richardson and their values are: (15)

$$A_0 = 1.09948$$
$$A_1 = 0.06020$$
$$A_2 = -7.95450$$

Since the last unit of item X produced requires less than 50 DLH, these constants will be used in this illustration (see Chapter 3). Thus, Eq. 4-2 becomes

$$\ln Y = 1.09948 + 0.06020\ X_1 - 7.95450\ (\ln X_2) \qquad \text{(Eq. 4-3)}$$

Had the unit DLH been greater than 50 hours, then values for the above constants would be calculated using the method described in Chapter 3.

Step 3.

The last piece of information needed to apply the model above is the direct labor hours for the last unit of the preceding lot, X_1. Since actual unit DLH are available, the DLH recorded for the last unit produced, 15.5 hours, will be used for X_1. If, however, the unit DLH were not available, then the learning curve tables could be used to estimate the DLH for the last unit produced. Since the learning curve has been determined and the total DLH are known, the DLH for the last unit produced could be estimated as follows:

$$\text{Last Unit DLH} = \frac{\text{Total DLH}}{\text{Cum. F.}} \times \text{U. F.}$$

where

Cum. F. = Cumulative Factor for the total number of units produced for the 83% learning curve

U. F. = Unit Factor for the last unit produced for the 83% learning curve

Step 4.

Determine the first unit DLH for Lot 2.

$$\begin{aligned} \ln Y &= 1.09948 + 0.06020\ X_1 - 7.95450\ (\ln X_2) \\ &= 1.09948 + 0.06020\ (15.5) - 7.95450\ (\ln .83) \\ &= \underline{\underline{3.51474\ \text{Hours}}} \end{aligned}$$

Thus,

$$Y = \underline{\underline{33.607\ \text{Hours}}} \text{ for Unit 1 of Lot 2}$$

Step 5.

Now that the DLH for the first unit of the follow-on lot is known, the total DLH for the follow-on lot can be estimated. The estimated total DLH for the 25 units to be produced is

$$33.607 \times 13.756^{*} = \underline{\underline{462.30\ \text{Total Hours}}}$$

* Cumulative Factor for 25 units using an 83% learning curve.

Since there is no set rule for determining reasonableness, the discretion lies with the evaluator. Thus, the contractor's proposal would be considered reasonable by this evaluator had all learning been lost due to the break in production. The reasonableness would be justified since the proposal and estimate differ only by 67.7 total hours, or 2.71 hours per unit.

<u>Method 2:</u>

This approximation method applies when learning is assumed to be retained from previous productions even though a break in production occurs. Since the ABC Company has retained 98% of the employees that worked on the first build of item X, learning is assumed to be retained. Thus, this method will be used to determine the reasonableness of the contractor's proposal.

Recalling the model for this method:

$$Y = K(X - AZ)^{c} \qquad \text{(Eq. 4-4)}$$

where

X = Number of hours to produce the first unit

X = Unit number to be produced

A = Number of units of learning lost because of the break

Z = 0 before the break and 1 after the break

$c = \log B/\log 2 = \log .83/\log 2 = -.26882$

In order to apply this model, the following procedure is suggested.

Step 1.

The slope of the learning curve which applies to item X is determined as in Step 1 of Method 1. Thus, the 83% learning curve will be used.

Step 2.

Once the slope of the learning curve is determined, a value for c can be calculated by Eq. 2-3 from Chapter 2.

$$c = \frac{\text{Log } S}{\text{Log } 2}$$

Where S is the slope of the learning curve, .83. Thus, c takes on a value of -0.26882 for this evaluation.

Step 3.

A value for K, the DLH for the first unit produced, can be calculated by using Eq. 3-11.

$$\text{First Unit DLH} = \frac{\text{Total DLH}}{\text{Cum. F.}}$$

In situations where actual data is available, as in our case, the actual DLH can be used. However, when the actual DLH for the first unit built differs from the value of K as calculated by Eq. A-2 in Appendix A, then the calculated value should be used. The calculated value would yield an estimate which conforms to the learning curve and reflects a more realistic value since it is calculated from DLH for all units produced. Also, the actual total hours for the first unit could include time spent correcting unforeseen problems that developed as well as production time. For these reasons, the value for K to be used is 36.328 (see Appendix A).

Step 4.

The next unknown, Z, will be equal to one in our evaluation since a break in production is expected.

Step 5.

The unknown yet to be determined is A, the number of units of learning lost due to a break in production. In efforts to determine the value for A to be used, r^2 was calculated, using Eq. 3-12, from the least-squares fit to the equation $Y = KX^c$. In calculating the index of determination, r^2, $N = 25$, $A = 0$, $Z = 1$, X takes on values from 26 through 50, and Y equals the DLH for unit X. The resulting value obtained for the index was .9998 (see Appendix A). Then a least-squares fit was obtained for $Y = K(X - AZ)^c$ setting $A = 1$, $Z = 1$, $N = 25$, X and Y are the same as above. The resulting r^2 for this fit was .9918 which is less than the previous index (see Appendix A). Therefore, the value for A to be used in approximating the first unit of Lot 2 is zero. Thus, unit one of Lot 2, unit 26 to be produced, should require approximately 16.15 hours of production time as shown below.

$$Y = K(X - AZ)^{c}$$
$$= 36.328\ (26 - 0 \cdot 1)^{-0.26882}$$
$$= \underline{\underline{16.150 \text{ Hours}}} \text{ to produce the 26th unit of item X}$$

Had the first index been smaller than the second, then the procedure would be continued to provide least-squares fits to the repositioned data incrementing A by one from 2 through 30, by two from 30 through 100, by five from 100 through 200, etc., until such time as the value obtained for r^2 stops increasing and begins to decrease. The value for A then, is that value which produces the highest value for r^2. Since the task of determining the largest value for r^2 could be quite time-consuming, a computer should be used to perform the calculations. A copy of a program developed by Robert B. Ilderton of the Defense Contract Audit Agency which handles situations where breaks occur is included in Appendix B.

Step 6.

Once the DLH for the first unit of the follow-on lot are calculated, the total cost for that lot can be approximated. The total DLH for the follow-on lot will be the sum of the hours for the units from X - A + 1 to X - A + N inclusive, where X is the last unit produced, A is the number of units lost and N is the number of units to be produced. The total DLH can be approximated using the following equation.

(Cum. F. L. U. - Cum. F. F. U.) X N = Total DLH (Eq. 4-5)

where

Cum. F. L. U. = Cumulative Factor for last unit (Unit X - A + N)

Cum. F. F. U. = Cumulative Factor for first unit (Unit X - A + 1)

N = Number of units in follow-on lot

Using the learning curve tables for an 83% curve and Eq. 4-5, the total DLH should be approximately 226 hours as shown here.

(23.2198 - 14.1727) X 25 = 226.178 Hours

In comparing the contractor's proposed DLH to the estimated DLH above, the 530 hours proposed by the ABC Company are 2.34 times the estimated hours. Thus, the contractor's proposed hours are too high and it is recommended that the contractor's proposal be rejected.

Comparison:

In comparing the two methods presented in this chapter, there are some things that should be considered before applying either of the two models. For Method 1 to be applicable, there should exist a situation in which it can be assumed that no learning is retained whenever a break in production occurs. Also, one model does not work for all production items. For instance, when using the model developed by Pichon and Richardson, the accuracy deteriorates as the DLH of the items being evaluated increase. Their model appears to yield reasonable estimates for items requiring up to approximately 50 hours of production but begins to worsen as the costs increase beyond that point. Thus, for higher cost items, a new equation would have to be generated.

For the second method, it is assumed that learning is passed on from lot to lot. That is, learning obtained from previous productions is retained for follow-on productions. Thus, when learning is assumed to be passed from one lot to another, this prediction method will yield good approximations for direct labor hours. If, however, all learning is assumed to be lost for one reason or another, then this method will produce approximately the same results as Method 1.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

In concluding this report, it is the opinion of this author that both prediction methods described and illustrated earlier in this report provide relatively close approximations for first units following a break in production, provided the proper assumptions are made. These again are: Method 1 presented an equation in which it was assumed that all learning is lost when a break is encountered. Thus, the estimated direct labor hours for the first unit following a break in production is made as though the follow-on production is the first. Put another way, the first unit of the follow-on production is treated as though it were the first unit to be produced. Method 2 presented an equation which predicts DLH assuming that learning can be retained from previous production runs. Even when a break in production occurs, some, but generally not all, learning may be retained.

The two models analyzed in this research paper are the only two models found by this author that attempt to incorporate a break in production into the approximation of follow-on productions. Thus, since breaks so often occur in product production, there is a great need for research in the area of production breaks and the effects they have on follow-on productions. Therefore, additional approximation techniques are needed that can be used to estimate the amount of learning lost due to production breaks. One approach might be to set up experiments whereby data is collected on various production products in which breaks of varying lengths are experienced. This data could be used to formulate other models which would possibly yield more realistic and more accurate results.

It is the hope of this author that additional research will be conducted in the area of breaks in production and new and better approximation techniques will be developed.

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APPENDIX A

DATA FOR APPLYING BOTH PREDICTION MODELS DESCRIBED IN CHAPTER 3

Lot #1:

Item No. X_i	Direct Labor Hours Y_i
1	40.0
2	32.0
3	26.0
4	23.0
5	22.0
6	21.0
7	22.0
8	21.0
9	20.0
10	19.0
11	19.5
12	19.0
13	18.0
14	18.5
15	17.5
16	17.0
17	17.5
18	16.5
19	17.0
20	16.5
21	16.0
22	16.5
23	16.5
24	16.0
25	15.5

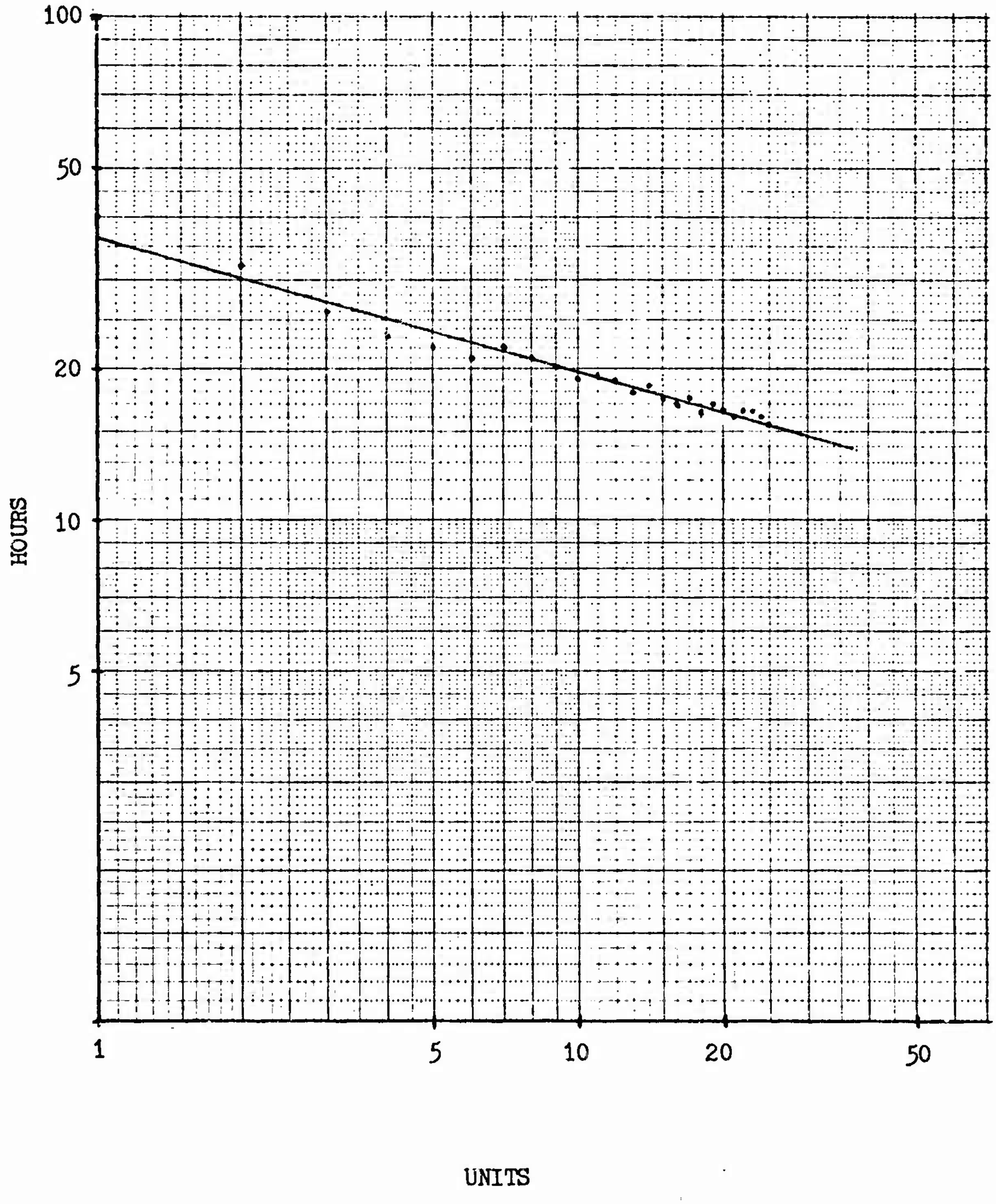

Figure A-1. Line fitted to the actual data from Lot 1 using logarithmic graph paper.

$$N = 25$$

$$c^* = \frac{N \cdot \Sigma(\text{Log } X \cdot \text{Log } Y) - \Sigma \text{Log } X \cdot \Sigma \text{Log } Y}{N \cdot \Sigma(\text{Log } X)^2 - [\Sigma \text{Log } X]^2} \qquad \text{(Eq. A-1)}$$

$$c = \underline{\underline{-0.26623}}$$

$$\text{Log } K^* = \frac{\Sigma \text{Log } Y - B \cdot \Sigma \text{Log } X}{N} \qquad \text{(Eq. A-2)}$$

$$\text{Log } K = \underline{\underline{1.56025}}$$

Thus,

$$K = \underline{\underline{36.32853 \text{ Labor Hours}}}$$

% Slope* = 2^c X 100% = 83.149% rounded to 83% so the same factor as used in the learning curve tables will be used.

Line fitted to the above data takes the form of

$$\text{Log } Y^* = \text{Log } K + c \cdot \text{Log } X \qquad \text{(Eq. A-3)}$$

where

$K = 36.328$; $c = -0.26882$ (Factor for 83% Learning Curve)

and $X = X_i$

The DLH for the first unit of Lot #1 is

$$\text{Log } Y = \text{Log } 36.328 + (-0.26882) \text{ Log } 1$$

$$= \underline{\underline{1.56025}}$$

Thus,

$Y = \underline{\underline{36.328 \text{ Hours}}}$ for the first unit produced in Lot #1

* See Reference (8)

The DLH for the last unit for Lot #1 is

$$\text{Log } Y = \text{Log } 36.328 + (-0.26882) \text{ Log } 25$$

$$= \underline{\underline{1.18908}}$$

Thus,

$$Y = \underline{\underline{15.45526 \text{ Hours}}} \text{ for the last unit produced in Lot \#1}$$

Calculation of Index of Determination (r^2):

$$r^2 = \frac{\left[N \sum \log X \cdot \log Y - \sum \log X \cdot \sum \log Y\right]^2}{\left[N \sum (\log X)^2 - (\sum \log X)^2\right] \cdot \left[N \sum (\log Y)^2 - (\sum \log Y)^2\right]}$$

where

$N = 25$

$Y = 36.328\ (X - A \cdot 1)^{-.26882}$

X = Unit number (26 through 50)

For $A = 0$:

$$\sum \log X \cdot \log Y = 44.656390$$
$$\sum \log X = 39.292430$$
$$\sum (\log X)^2 = 61.934727$$
$$\sum \log Y = 28.443447$$
$$\sum (\log Y)^2 = 32.374117$$

Thus

$$r^2 = .99989$$

For $A = 1$:

$$\sum \log X \cdot \log Y = 44.782180$$
$$\sum \log X = 39.292430$$
$$\sum (\log X)^2 = 61.934727$$
$$\sum \log Y = 28.524310$$
$$\sum (\log Y)^2 = 32.559201$$

Thus

$$r^2 = .99181$$

APPENDIX B

This appendix containes a computer program developed by Robert B. Ilderton that fits weighted least-squares equations to the models $Y = AX^B$ and $Y = A(X-CZ)^B$

```
?00 REM R. ILDERTON
701GOSUB90010
702DIMF(200),L(200),Y(200),M(200)
704DATA9E9,9E9
706READD,C
708IFC=9E9THEN 1006
710LETC=D+C
712IFC>200THEN1002
714IFC>=3THEN720
716PRINT"THERE MUST BE AT LEAST THREE POINTS OF DATA."
718STOP
720LETL=0
722LETT=0
724LETX1=0
726LETX2=0
728LETY1=0
730LETY2=0
732LETZ=0
734FORI=1TOC
736REA@F(I),L(I),Y(I)
737IFF(I)>LTHEN740
738PRINT"FIRST UNIT IN LOT";I;"IS";F(I);".  THIS DOES NOT EXCEED"
739PRINTL;"WHICH IS THE LAST UNIT IN THE PRIOR LOT.  CHECK YOUR DATA"
40LETL=L(I)
,41IFF-INT(F)>0THEN1002
?42IFL>INT(L)THEN1002
743IFL<F(I)THEN1002
744LETN=L-F(I)+1
745LETT=T+N
746LETX=LOG((F(I)+L)/2)
748LETX1=X1+N*X
750LEPX2=X2+N*X*X
752LETY=LOG(Y(I))
754LETY1=Y1+N*Y
756LETY2=Y2+N*Y*Y
758LETZ=Z+N*X*Y
760NEXTI
762LETY3=Y1/T
764LETY4=Y2-Y1*Y3
766READA
768IFA<>9E9THEN1002
770LETB=(Z-X1*Y1/T)/(X2-X1*X1/T)
772LETS=0
774FORH=0TO9
776GOSUB930
778IFABS(B1-B)<.00001THEN784
780LETB=B1
782NEXTH
?84IFB1<0THEN788
786PRINT"DATA YIELDS SLOPE OF MORE THAN 100 PC"
787STOP
788PRINT
790PRINT "LEAST-SQUARES FIT TO Y=AX↑B"
792PRINT
```

```
794GOSUB1048
796LETE1=E
797LETS1=1
798LETS2=0
799IFL(C)>99THEN801
800LETS1=.01
801FORS=S1TO29*S1STEPS1
802GOSUB836
804NEXT S
806FORS=30*S1TO98*S1STEP2*S1
808GOSUB836
810NEXTS
812LETS1=S1*5
814FORS=20*S1TO39*S1STEPS1
816GOSUB836
818NEXTS
820FORS=40*S1TO98*S1STEP2*S1
822GOSUB836
824NEXTS
826FORS=100*S1TO195*S1STEP5*S1
828GOSUB836
830NEXTS
832LETS1=S1*10
834GOTO814
836IFS<F(D+1)-1THEN840
838LETS=F(D+1)-1
840GOSUB930
842IFS=F(D+1)-1THEN862
844IFE/E1<.9999THEN856
846IFE<E1THEN854
848LETE1=E
850LETB=B1
852LETS2=S
854RETURN
856LETS=INT(100*S2+.5)/100
858IFS=0THEN874
860GOSUB930
862PRINT"LEAST-SQUARES FIT TO Y=A(X-CZ)↑B, WHERE Z=0 BEFORE"
864PRINT"BREAK IN PRODUCTION AND 1 AFTER"
866PRINT
868PRINT"C=",S
870GOSUB1048
872GOTO878
874PRINT"NO BETTER FIT IS OBTAINED FROM THE MODEL"
876PRINT"Y=A(X-CZ)↑B"
878 LET C5=C
880 LET C=1
882LETI=1
884PRINT
886PRINT"WHEN A QUESTION MARK APPEARS, TYPE THE FIRST (F) AND LAST (L)"

888PRINT"UNITS OF A LOT FOR WHICH CALCULATION OF PROJECTED HOURS OR"
890PRINT"COST IS DESIRED.  WHEN NO MORE CALCULATIONS ARE NEEDED,"
892PRINT"ENTER '0,0'."
894PRINT
```

```
895PRINT"F,L=";
896INPUTF1,L(1)
897IFF1+L(1)=0THEN1084
898IFF1-INT(F1)>0THEN1002
899IFL(1)-INT(L(1))>0THEN1002
900IFF1<1THEN1002
901IFL(1)<F1THEN1002
902LETF(1)=F1
904IFF1<LTHEN912
906F(1)=F(1)-S
908L(1)=L(1)-S
912GOSUB930
914PRINT"MIDPOINT=","  ",M(1)
916LETU=A*M(1)↑B1
918IFF1<=L(C5)THEN924
920PRINT "REPOSITIONED VALUE OF F=",F(1)
922PRINT "REPOSITIONED VALUE OF L=",L(1)
924PRINT"PROJECTED UNIT VALUE=",U
926PRINT"PROJECTED TOTAL VALUE=",U*N
928GOTO894
930LETX1=0
932LETX2=0
934LETZ=0
936LETW=0
938FORI=1TOC
940LETF=F(I)
942LETL=L(I)
944IFI<=DTHEN950
946LETF=F-S
948LETL=L-S
950LETN=L-F+1
952LETW=0
954FORK=FTOL
956IFK>50THEN962
958LETW=W+K↑B
960NEXTK
961GOTO964
962LETW=W+((L+.5)↑(1+B)-(K-.5)↑(1+B))/(1+B)
964LETM(I)=(W/N)↑(1/B)
966IFC=1THEN984
968LETX=LOG(M(I))
970LETX1=X1+N*X
972LETX2=X2+N*X*X
974LETZ=Z+N*X*LOG(Y(I))
976NEXTI
978LETZ1=Z-X1*Y3
980LETB1=Z1/(X2-X1*X1/T)
982LETE=B1*Z1
984RETURN
1002PRINT "DATA DOES NOT CONFORM TO PRESCRIBED FORMAT.  PLEASE CHECK
1004IFC=1THEN894
1006PRINT"THIS PROGRAM FITS WEIGHTED LEAST-SQUARES EQUATIONS TO THE"
1008PRINT"MODELS Y=AX↑B AND Y=A(X-CZ)↑B, WHERE"
1010PRINT"  Y=DIRECT LABOR HOURS OR COST PER UNIT"
1012PRINT"  X=UNIT NUMBER OR LOT MIDPOINT"
1014PRINT"  A=THEORETICAL UNIT 1 HOURS OR COST"
1016PRINT"  B=IMPROVEMENT CURVE SLOPE COEFFICIENT"
```

```
1018PRINT"  C=POSITIVE INTEGER REPRESENTING UNITS OF LEARNING"
1020PRINT"     LOST AS A RESULT OF A PRODUCTION BREAK"
1022PRINT"LINES 1 TO 699 ARE AVAILABLE FOR USE AS DATA STATEMENTS"
1024PRINT"ENTER FIRST THE NUMBER OF LOTS PRIOR TO THE PRODUCTION BR
1026PRINT"THEN THE NUMBER AFTER THE BREAK, AND THEN THE FIRST UNIT,"
1028PRINT"LAST UNIT AND AVERAGE HOURS OR COST PER UNIT FOR EACH LOT
1030PRINT"SEQUENCE.  TYPE 'RUN' ON THE NEXT LINE.  FOR EXAMPLE:"
1032PRINT
1034PRINT"    1 DATA 3,2"
1036PRINT"    11 DATA 1,1,1102"
1038PRINT"    12 DATA 2,3,825"
1040PRINT"    13 DATA 7,10,551.4"
1042PRINT"    21 DATA 11,11,616"
1044PRINT"    22 DATA 12,16,517"
1046STOP
1048LETA=EXP(Y3-B1*X1/T)
1050PRINT "A=",A
1052PRINT "B=",B1
1054PRINT"PCT.=",100*2^B1
1056PRINT "INDEX=",E/Y4
1058PRINT
1060PRINT"MIDPOINT","CALCULATED Y","ACTUAL Y","PCT. DIFF."
1062PRINT
1064FORI=1TOC
1066LETK=A*M(I)^B1
1068PRINTM(I),K,Y(I),
1070LETP=INT(1000*(Y(I)/K-1)+.5)/10
1072IFP<0THEN1076
1074PRINT"       ";
1076PRINTP
1078NEXTI
1080PRINT
1082RETURN
1084PRINT
90000GOTO90110
90010DISABLE ALL
90020A$=UID
90030B$="*"
90040B$(2,6)=A$
90050FILE APPEND#1=B$
90060PRINT#1USING90070,DAT,UID,PID,TIM
90070:*BREAK     ######## ###### ###### ########
90080FILES
90090ENABLE
90100RETURN
90110PRINT 'END OF JOB
READY
```

INSTRUCTIONS FOR BREAK PROGRAM

```
*BREAK    05:09    07/26/73

THIS PROGRAM FITS WEIGHTED LEAST-SQUARES EQUATIONS TO THE
MODELS Y=AX↑B AND Y=A(X-CZ)↑B, WHERE
  Y=DIRECT LABOR HOURS OR COST PER UNIT
  X=UNIT NUMBER OR LOT MIDPOINT
  A=THEORETICAL UNIT 1 HOURS OR COST
  B=IMPROVEMENT CURVE SLOPE COEFFICIENT
  C=POSITIVE INTEGER REPRESENTING UNITS OF LEARNING
    LOST AS A RESULT OF A PRODUCTION BREAK
ENTER FIRST THE NUMBER OF LOTS PRIOR TO THE PRODUCTION BREAK,
THEN THE NUMBER AFTER THE BREAK, AND THEN THE FIRST UNIT,
LAST UNIT AND AVERAGE HOURS OR COST PER UNIT FOR EACH LOT IN
SEQUENCE.  TYPE 'RUN' ON THE NEXT LINE.  FOR EXAMPLE:

   1 DATA 3,2
   11 DATA 1,1,1102
   12 DATA 2,3,825
   13 DATA 7,10,5514
   21 DATA 11,11,616
   22 DATA 12,16,317
NOW AT 1770
SRU'S:0.3
READY
```

```
TAPE
READ PAPER TAPE
TP ON
1 DATA5,11
2 DATA1,48867,.73831
3 DATA48868,150566,.42494
4 DATA150567,215367,.53933
5 DATA215368,355406,.38395
6 DATA355407,545124,.31550
10 DATA940276,1086362,.29361
11 DATA1086363,1324810,.23832
12 DATA1324811,1568678,.22629
13 DATA1568679,1816422,.19670
14 DATA1816423,2135068,.18107
15 DATA2135069,2412150,.16989
16 DATA2412151,2763654,.15854
17 DATA2763655,3047023,.13867
18 DATA3047024,3336169,.14188
19 DATA3336170,3670508,.13856
20 DATA3670509,3954108,.13061

END OF PAPER TAPE INPUT
READY
RUN

*BREAK    05:16    07/26/73

LEAST-SQUARES FIT TO Y=AX↑B

A=              33.0905
B=             -.360149
PCT.=           77.9084
INDEX=          .924082
```

MIDPOINT	CALCULATED Y	ACTUAL Y	PCT. DIFF.	
14169.5	1.05827	.73831	-30.2	
93412.1	.536556	.42494	-20.8	
181656.	.422266	.53933		27.7
281434.	.360672	.38395		6.5
445685.	.305639	.3155		3.2
1.01213E+6	.227468	.29361		29.1
1.20291E+6	.213752	.23832		11.5
1.44441E+6	.200121	.22629		13.1
1.6905E+6	.189098	.1967		4
1.97282E+6	.178867	.18107		1.2
2.27169E+6	.170007	.16989	-.1	
2.5852E+6	.162273	.15854	-2.3	
2.90378E+6	.155622	.13867	-10.9	
3.19009E+6	.150439	.14188	-5.7	
3.50153E+6	.145476	.13856	-4.8	
3.81112E+6	.141104	.13061	-7.4	

LEAST-SQUARES FIT TO Y=A(X-CZ)↑B, WHERE Z=0 BEFORE BREAK IN PRODUCTION AND 1 AFTER

C=	550000
A=	37.5533
B=	-.376124
PCT.=	77.0505
INDEX=	.956135

MIDPOINT	CALCULATED Y	ACTUAL Y	PCT. DIFF.
13960.7	1.03647	.73831	-28.8
93336.3	.507332	.42494	-16.2
181641.	.39494	.53933	36.6
281388.	.334991	.38395	14.6
445631.	.281794	.3155	12
460662.	.2783	.29361	5.5
650573.	.244415	.23832	-2.5
892924.	.216972	.22629	4.3
1.13946E+6	.19796	.1967	-.6
1.42165E+6	.182152	.18107	-.6
1.72105E+6	.169518	.16989	.2
2.03442E+6	.159161	.15854	-.4
2.35338E+6	.150696	.13867	-8
2.63978E+6	.144325	.14188	-1.7
2.95117E+6	.138397	.13856	.1
3.26088E+6	.133299	.13061	-2

WHEN A QUESTION MARK APPEARS, TYPE THE FIRST (F) AND LAST (L) UNITS OF A LOT FOR WHICH CALCULATION OF PROJECTED HOURS OR COST IS DESIRED. WHEN NO MORE CALCULATIONS ARE NEEDED, ENTER '0,0'.

F,L=?545125,2080000

MIDPOINT=	1.19926E+6
PROJECTED UNIT VALUE=	.194188
PROJECTED TOTAL VALUE=	298055.

F,L=?5080001,7580000

MIDPOINT=	5.71727E+6
PROJECTED UNIT VALUE=	.107921
PROJECTED TOTAL VALUE=	269803.

F,L=?0,0