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Intercept control is the process of guiding an interceptor toward a target. This process requires the transmission of appropriate altitude, speed and heading commands to the interceptor in proper sequence. A procedure for determining interceptor commands in computer controlled systems was developed approximately two decades ago. This procedure with minor variations is currently being used in operational systems throughout the world.

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20. Abstract (Continued)

The specifications for the various operational systems are available. However, these documents provide neither rationale nor adequate derivations for the intercept control procedure and are consequently difficult to comprehend. In spite of the difficulties in understanding the specifications, the salient features in determining interceptor guidance commands are relatively straightforward and can be presented in a comprehensible manner. This paper represents such an attempt.

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SECTION I

INTRODUCTION

INTERCEPT CONTROL

Intercept control or guidance is the process of guiding an interceptor toward a target. This process requires the transmission of appropriate altitude, speed and heading commands to the interceptor in proper sequence.

PURPOSE

A procedure for determining interceptor commands in computer controlled systems was developed approximately two decades ago. This procedure, with minor variations, is currently being used in several systems throughout the world.

Specifications for the various operational systems are available. However, these documents provide neither rationale nor adequate derivations for the intercept control procedure and are consequently difficult to comprehend. In spite of the difficulties in understanding the specifications, the salient features in determining interceptor guidance commands are relatively straightforward and can be presented in a comprehensible manner. This paper represents such an attempt.

SCOPE

Section II presents an operationally oriented description of an intercept mission. Subsequent sections present procedures for obtaining and periodically verifying the acceptability of the guidance solution for several classes of intercept missions.

Sections III, IV and V present procedures with accompanying rationale to obtain the minimum time guidance solution for intercept missions with differing assumptions and geometries.

Section III considers the constant speed, straight line intercept mission wherein the interceptor is assumed to be airborne and capable of performing an instananeous turn from its initial heading to the desired command heading. A procedure is shown for assessing whether interception is possible and if so, for determining the number of guidance solutions.

Section IV augments Section III by considering the variable speed, three dimensional intercept mission wherein the interceptor is required to approach the target from a particular direction. The interceptor is assumed to be capable of performing an instantaneous turn from the present to the command heading and from the command to the terminal heading.

Section V further augments Sections III and IV by modifying the procedure developed therein to consider the time and distance in the initial and final interceptor turns. The various interceptor turn combinations, the interceptor flight path during turns, and the displacement of the interceptor position to account for the motion of both the interceptor and target during turns are discussed.

Sections III through V present procedures for obtaining guidance solutions to various classes of intercept missions. Section VI presents procedures for periodically verifying the acceptability of the current guidance solution using more recent interceptor and target information.

SECTION II

INTERCEPT MISSION OPERATIONAL DESCRIPTION

INTRODUCTION

There are five mission phases associated with an interceptor-target pairing. Superimposed upon the mission phases is the geometry of the intercept. Intercept profile and tactic are descriptors which determine the interceptor speed, altitude, geometry and time in the combat phase of the mission. The mission phases, intercept geometry, intercept profile and tactic are described under their respective titles.

MISSION PHASES

An intercept mission may be described in terms of five phases namely scramble, climb, cruise, transition, and combat. Figure 1 shows the phases of an intercept mission.

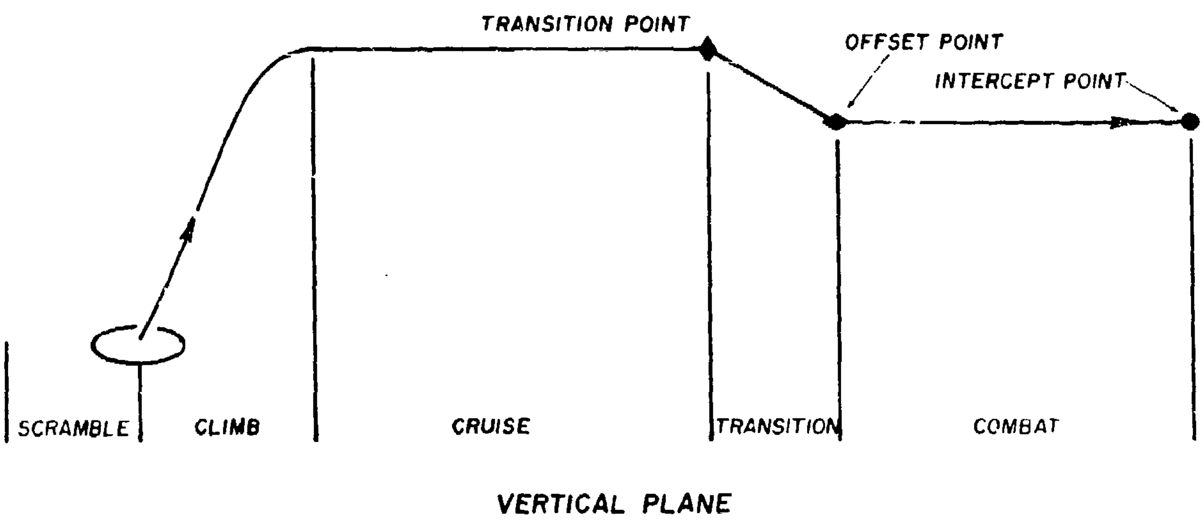

Figure 1 PHASES OF AN INTERCEPT MISSION

Scramble Phase

An interceptor enters the Scramble Phase when it is located at an airbase and is paired with a target by an operator. When pairing occurs, a guidance solution is generated which assumes a nominal time delay for the interceptor to become airborne.

Climb Phase

An interceptor enters the Climb Phase upon recognition by the computer program that it is airborne. During this phase, the interceptor attains and proceeds at the command heading and climbs to the cruise altitude. The acceptability of the current guidance solution is examined periodically both in this and in subsequent mission phases. If the guidance solution is deemed unacceptable, a new solution is generated.

Cruise Phase

An interceptor enters the Cruise Phase when it reaches cruise altitude. During the Cruise Phase, the interceptor attains the cruise speed and flies at its command heading until the transition point.

Transition Phase

An interceptor enters the Transition Phase when the transition point is attained. The transition point is defined as the position at which the acceleration or deceleration to combat speed and the climb or descent to combat altitude begin. It is located so that any transition maneuvers are completed by the offset point.

Combat Phase

An interceptor enters the Combat Phase when the offset point is reached. The offset point has either of two meanings depending upon the intercept geometry. For single turn intercept geometries, the offset point is defined as the expected interceptor position a designated time interval before interception will occur. For double turn intercept geometries, the interceptor begins the final turn from the command to the terminal heading at the offset point.

INTERCEPT GEOMETRY

There are three types of geometry used for intercept missions. These are the single turn, double turn and pursuit geometries. Pursuit geometry, which will not be addressed further herein, results in the generation of periodic commands that direct the interceptor toward the current target position. The interceptor's heading is the target bearing or the angle from the interceptor's location to the target's position relative to true north. The interceptor's command heading generally changes with each computation due to the motion of the interceptor and the target.

Single Turn Geometry

Single turn geometry provides for an initial turn from the interceptor's initial or current heading to the desired command heading, followed by a straight line path to the intercept point. The interceptor's terminal heading to the intercept point is not constrained. Figure 2 shows the single turn intercept geometry.

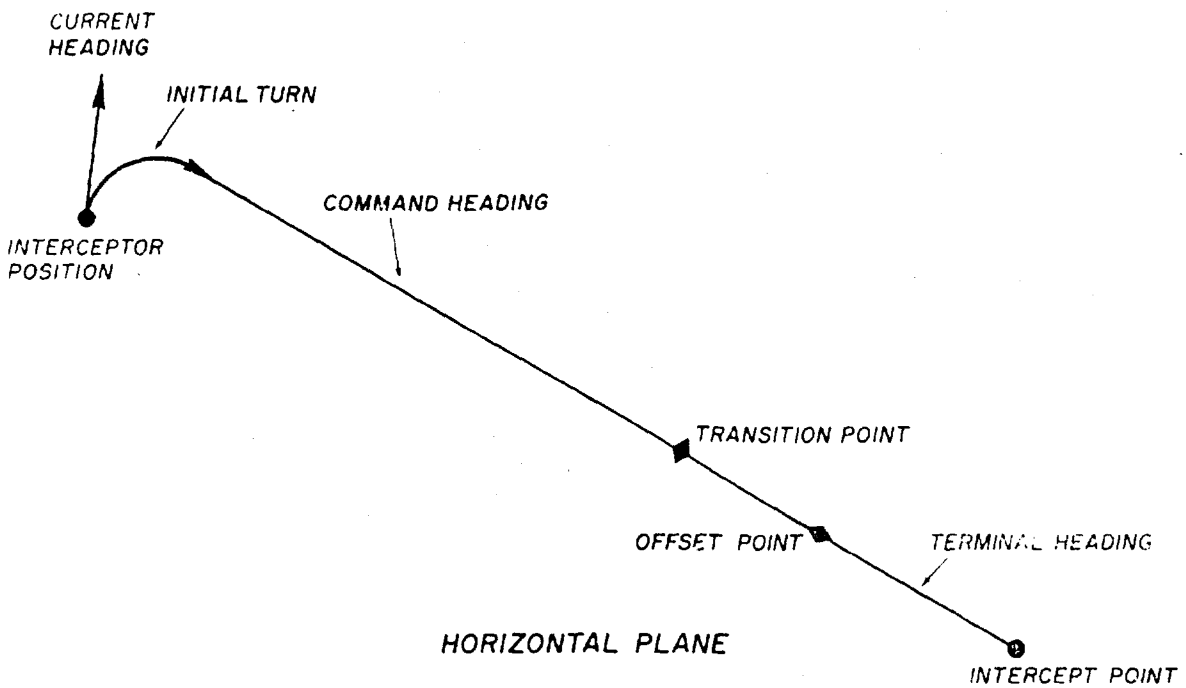

Figure 2 SINGLE TURN INTERCEPT GEOMETRY

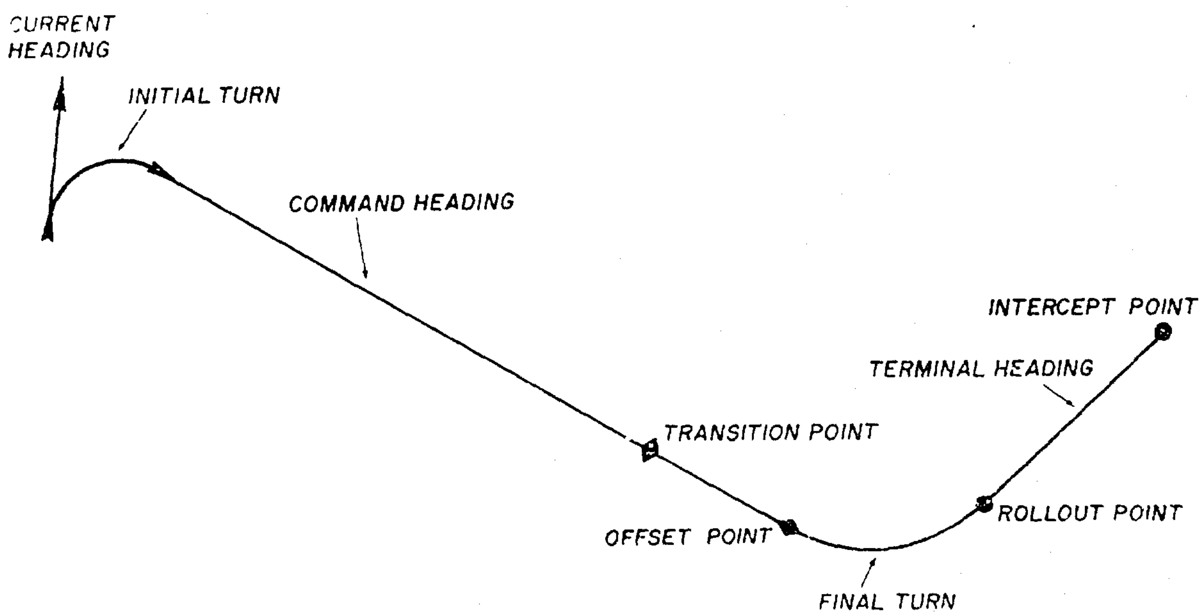

Figure 3 DOUBLE TURN INTERCEPT GEOMETRY

Double Turn Geometry

Double turn geometry results in the interceptor approaching the target from a particular direction with respect to the target heading. This geometry provides for an initial turn from the interceptor's current heading to the command heading, a straight line path to bring the interceptor into proximity with the target,and a final turn from the command heading to the terminal heading. Figure 3 shows the double turn intercept geometry.

PROFILE AND TACTIC PARAMETERS

Selection of an intercept profile and tactic determines the interceptor climb speed, cruise speed, cruise altitude, combat speed, combat altitude, duration of the combat phase and the type of intercept geometry. Figure 4 shows the phases of a double turn intercept mission that are affected by selection of an intercept profile and tactic.

Intercept Profile

A profile defines the phases of an intercept mission from take-off until the transition point. Choice of a profile determines the interceptor climb speed as well as the cruise speed and altitude that are used during the cruise phase of the mission.

Profile parameters are selected based upon consideration of whether fuel conservation or minimization of target penetration is of primary importance. If minimization of target penetration is the primary consideration, Profile I parameters, which sacrifice interceptor range by requiring a shorter time-to-interception at increased

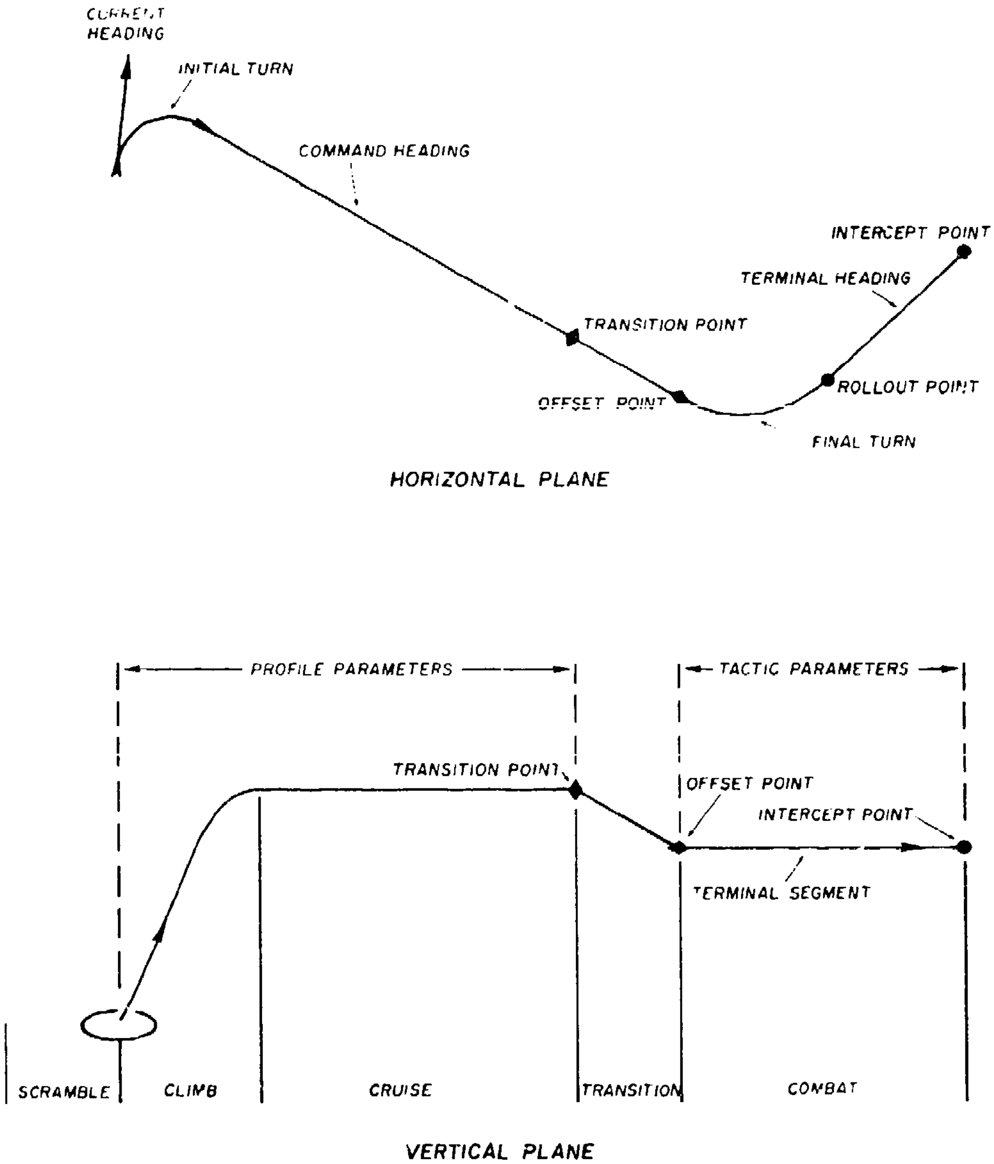

Figure 4 PROFILE AND TACTIC PARAMETERS

fuel consumption, are used. If fuel conservation is significant and time-to-interception is less critical, Profile III parameters, which allow the interceptor to achieve maximum range, are used. Profile II parameters provide a compromise between the other profile parameters by combining the climb characteristics of Profile III with a cruise speed approximately that of the Profile I parameters. The Profile II parameters provide extended interceptor range with some sacrifice of time-to-interception and target penetration over the Profile I parameters.

Interceptor Tactic

A tactic defines those portions of an intercept mission from offset point to intercept point. Choice of a tactic determines the combat speed, combat altitude, duration of the terminal segment or combat phase and whether single or double turn intercept geometry is to be used. If double turn geometry is to be used because of the requirements of the interceptor's armament and avionics system, the direction from which the interceptor should approach the target is determined. The tactic and its associated parameters are obtained based upon the target altitude, target speed, interceptor type and armament.

SECTION III

CONSTANT SPEED INTERCEPT MISSION

INTRODUCTION

This section considers the constant speed, straight line intercept mission. The interceptor is assumed to be co-altitude with the target and to fly at a constant speed and heading from its initial location to the intercept point. The interceptor is also assumed capable of performing an instantaneous turn from its initial heading to the desired command heading.

As a prelude to consideration of the constant speed, straight line intercept mission, the coordinate system to be used is presented. Thereafter, a procedure is shown for assessing whether interception is possible and if so for determining the number of guidance solutions. Finally, a procedure to generate the guidance solution that would result in a minimum time-to-interception is presented.

COORDINATE SYSTEM

It is assumed that the position and velocity of both the interceptor and the target are maintained in a rectangular coordinate system. In obtaining the guidance solution, the interceptor position is expressed in a coordinate system wherein the target is stationary and located at the origin and the target velocity vector is collinear with the Y axis.

To obtain the interceptor position in the desired coordinate system, two transformations are performed. The initial transformation expresses the interceptor position in a coordinate system parallel to the original coordinate system but centered at the target location. The second transformation rotates the axes of the translated coordinate system until the Y axis becomes collinear with the target velocity vector.

To express the interceptor position (X_1, Y_1) in a coordinate system centered at the target location, the following equations are used:

$$X_1 = X_0 - X_T \qquad (1)$$
$$Y_1 = Y_0 - Y_T$$

where:

X_0, Y_0 are the interceptor coordinates

X_T, Y_T are the target coordinates.

The second transformation rotates the axes so that the Y axis is superimposed upon the target velocity vector. The following equations express the interceptor position (X_2, Y_2) in the target coordinate system that is depicted via dashed lines in Figure 5.

$$\begin{bmatrix} X_I, Y_I \end{bmatrix} = \begin{bmatrix} X_2, Y_2 \end{bmatrix} = \begin{bmatrix} X_1, Y_1 \end{bmatrix} \begin{bmatrix} \text{Cos } \psi_T & \text{Sin } \psi_T \\ -\text{Sin } \psi_T & \text{Cos } \psi_T \end{bmatrix} \qquad (2)$$

where:

$$\psi_T = \text{Tan}^{-1} \left[\frac{\dot{X}_T}{\dot{Y}_T} \right]$$

$\dot{X}_T, \dot{Y}_T$ are the target velocity components

The angle ψ_T is measured in a clockwise direction from the positive Y_1 axis as shown in Figure 5.

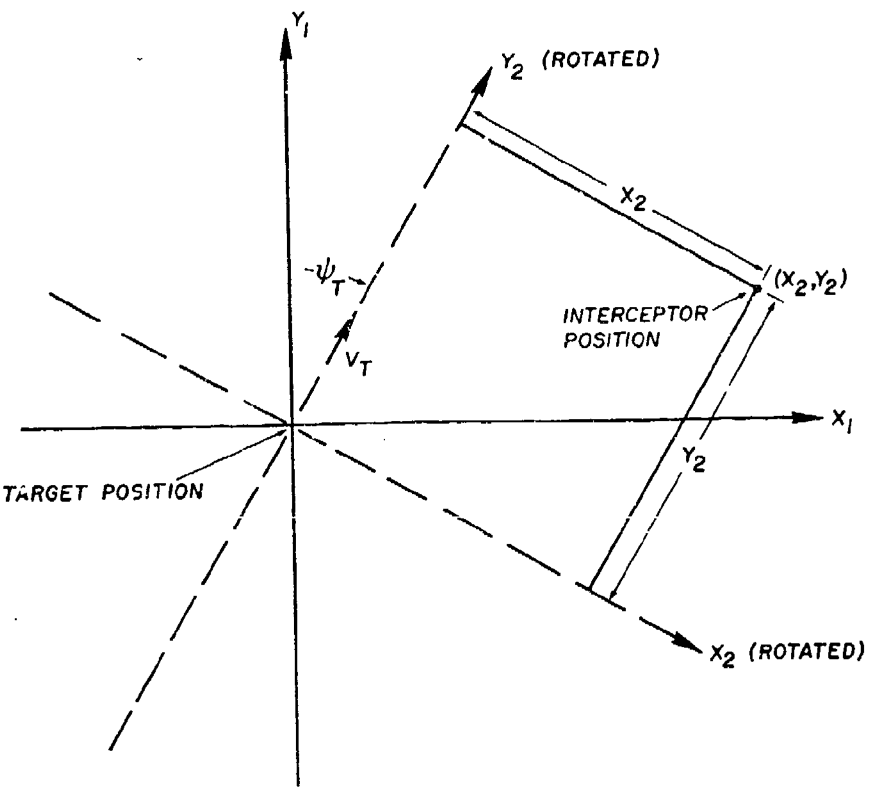

Figure 5 TARGET COORDINATE SYSTEM

The coordinates (X_2,Y_2) represent the instantaneous position of the interceptor in the target coordinate system that is centered at the target position with the positive Y_2 axis superimposed on the target velocity vector. In the target coordinate system, positive heading angles are measured in a clockwise direction from the Y_2 axis.

As previously indicated, the interceptor position will be expressed in the target coordinate system with the stipulation that the target remain stationary at the origin. The target is constrained to the origin by attributing the target motion to the interceptor.

The interceptor extrapolated position (X_I', Y_I') is expressed in the target coordinate system wherein the target remains stationary at the origin as follows:

$$X_I' = X_I + (|V_I| \text{Sin } \theta)t$$
$$Y_I' = Y_I + (|V_I| \text{Cos } \theta)t - |V_T|t \qquad (3)$$

where:

X_I, Y_I is the interceptor position namely (X_2, Y_2)

$|V_I|$ is the interceptor speed

$|V_T|$ is the target speed

θ is the interceptor heading in the target coordinate system

t is the extrapolation interval.

The concept of attributing target motion to the interceptor will be discussed further in the following paragraph.

NUMBER OF GUIDANCE SOLUTIONS

The number of guidance solutions for the constant speed, straight line intercept mission depends upon the initial geometry as well as the target and interceptor speeds. If the interceptor is flying faster than the target, a single guidance solution exists. If the interceptor is flying slower than the target, two, one or no guidance solutions exist, depending upon the initial geometry.

To verify these assertions, we note that a constant speed, straight line interception will occur in time t if:

$$T_0 + V_T t = I_0 + V_I t \qquad (4)$$

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where:

T_0 and I_0 are the initial positions of the target and interceptor

V_T and V_I are the velocity vectors of the target and interceptor

t is the time until interception.

Equation (4) may alternately be expressed as:

$$T_0 = I_0 + (V_I - V_T)t \tag{5}$$

Equation (5) is the vector equivalent of equations (3) assuming interception will occur. Equation (5) expresses the intercept problem in a coordinate system wherein the virtual target remains stationary at the origin and its motion is attributed to the interceptor. In this coordinate system, the virtual interceptor traverses the relative velocity $(V_I - V_T)$ as shown in Figure 6.

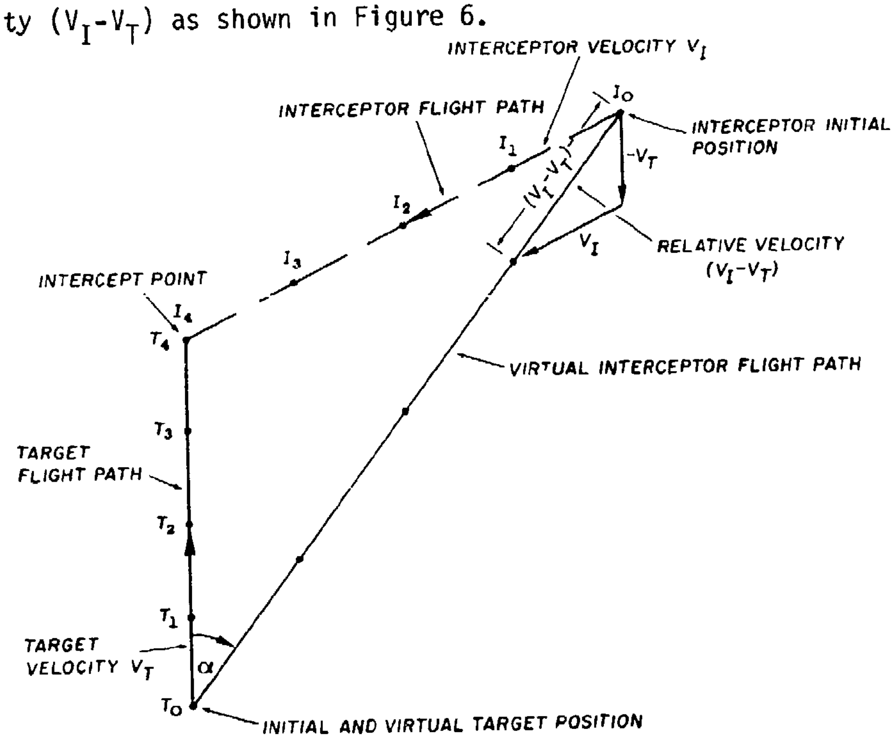

Figure 6 RELATIVE VELOCITY VECTOR

In Figure 6, the interceptor initially located at I_0 proceeds toward the intercept point along the dashed line corresponding to the interceptor velocity vector V_I. The target initially located at T_0 proceeds toward the intercept point along the velocity vector V_T. Successive interceptor and target positions are designated by subscripts. In the coordinate system wherein the target remains static and its motion is attributed to the interceptor, the virtual interceptor proceeds toward the static virtual target located at T_0 along the relative velocity (V_I-V_T). For interception to occur, the resultant velocity (V_I-V_T) must lie along the line of sight between T_0 and I_0. If the velocity vector V_I is chosen so that (V_I-V_T) lies along the line of sight and is oriented toward T_0, interception will occur.

From Figure 6, we note that varying the velocity vector V_I to cause (V_I-V_T) to lie along the line of sight is analogous to varying both the interceptor speed and heading. The minimum interceptor speed required for interception is dependent upon the target speed $|V_T|$ and the angle α.

To verify the assertion regarding the number of guidance solutions, we will consider the three distinct geometries shown in Figures 7, 8, and 9. In the initial geometry, the absolute value of the angle between the target heading and the line of sight designated $|\alpha|$ is 90^0. The other geometries considered are $|\alpha|$ greater than 90^0 and $|\alpha|$ less than 90^0.

$|\alpha|$ Equals 90 Degrees

From Figure 7, $|V_I|$ must exceed $|V_T|$ in order for the relative velocity vector $(V_I - V_T)$ to lie along the line of sight between P_T and P_I. Otherwise, no guidance solution is possible. For $|V_I|$ chosen such that $|V_I| > |V_T|$ and so that $(V_I - V_T)$ lies along the line of sight between P_I and P_T a single guidance solution exists.

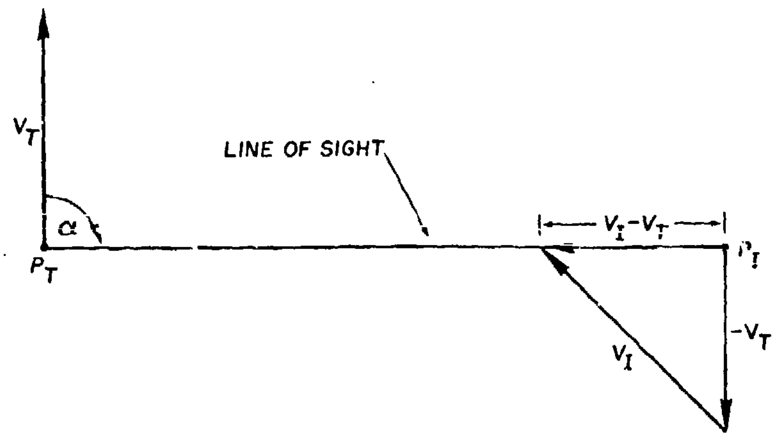

Figure 7 INTERSECTION ANGLE $|\alpha| = 90°$

$|\alpha|$ Greater than 90 Degrees

From Figure 8, $|V_I|$ must exceed $|V_T|$ in order for $(V_I - V_T)$ to lie along the line of sight between P_T and P_I. Otherwise, no guidance solution is possible. For $|V_I|$ chosen so that $|V_I| > |V_T|$ and so that $(V_I - V_T)$ lies along the line of sight between P_I and P_T, a single guidance solution exists.

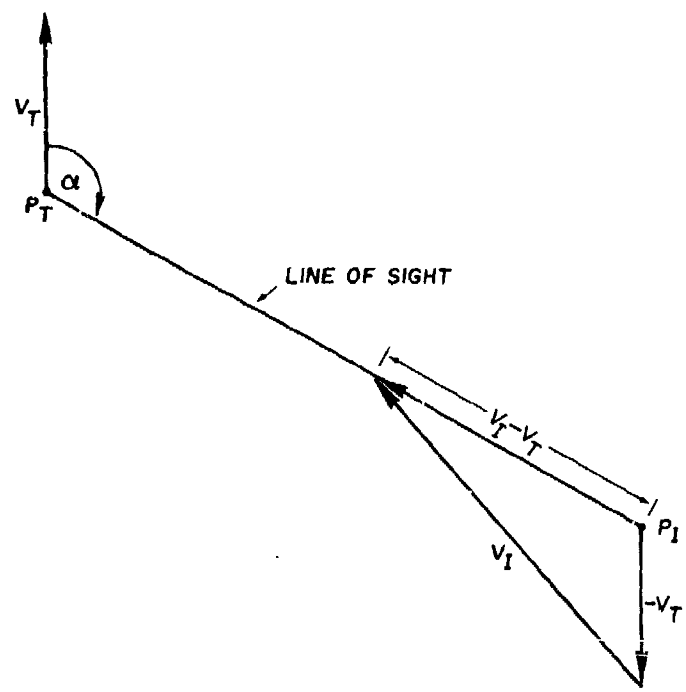

Figure 8 INTERSECTION ANGLE $|\alpha| > 90°$

$|\alpha|$ Less Than 90 Degrees

Figure 9 shows, via a dashed line, the minimum interceptor speed that will result in a straight line intercept. This corresponds to the interceptor speed being chosen as $|V_T|$ Sin$|\alpha|$ and the interceptor heading being chosen perpendicular to the line of sight. With the interceptor velocity so chosen, a single guidance solution exists.

If $|V_I|$ is chosen smaller than $|V_T|$ Sin$|\alpha|$, the relative velocity vector $(V_I - V_T)$ will not lie along the line of sight. Therefore, when $|V_I|$ is smaller than $|V_T|$ Sin$|\alpha|$, there are no guidance solutions.

For an interceptor speed larger than $|V_T|$ Sin$|\alpha|$ but smaller than $|V_T|$, two interceptor headings could result in interception. These headings, designated as H_1 and H_2 in Figure 9, would both

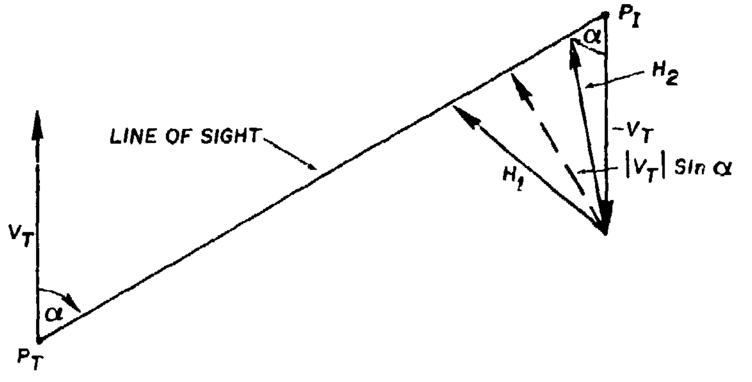

Figure 9 INTERSECTION ANGLE $|\alpha| < 90°$

result in a constant speed, straight line intercept. However, if the interceptor traversed the path designated H_1, interception would be accomplished more rapidly than if the interceptor traversed the path designated by H_2. Thus, in the target coordinate system wherein the target velocity vector is collinear with the Y axis, the interceptor heading closer to 180 degrees always results in an earlier intercept assuming two different headings will effect interception.

Finally, for an interceptor speed which equals or exceeds the target speed chosen so that $(V_I - V_T)$ lies along the line of sight and is oriented toward P_T, one guidance solution exists.

Solution Versus Intersection Angle

Table I shows the number of guidance solutions depending upon the intersection angle and the relationship of interceptor to target speed.

Table I

Number of Solutions Versus Intersection Angle

Intersection Angle	Speed Relationship	Number of Solutions
$\lvert\alpha\rvert \geq 90^0$	$\lvert V_I\rvert \leq \lvert V_T\rvert$	0
	$\lvert V_I\rvert > \lvert V_T\rvert$	1
$\lvert\alpha\rvert < 90^0$	$\lvert V_I\rvert < \lvert V_T\rvert \text{ Sin } \lvert\alpha\rvert$	0
	$\lvert V_I\rvert = \lvert V_T\rvert \text{ Sin } \lvert\alpha\rvert$	1
	$\lvert V_T\rvert \text{ Sin } \lvert\alpha\rvert < \lvert V_I\rvert < \lvert V_T\rvert$	2
	$\lvert V_I\rvert \geq \lvert V_T\rvert$	1

Solutions Versus Speed Relationship

Table II shows the number of guidance solutions with the interceptor to target speed relationship as the primary variable.

Table II

Number of Guidance Solutions Versus Speed Relationship

Speed Relationship	Intersection Angle	Number of Solutions
$\lvert V_I \rvert > \lvert V_T \rvert$	All	1
$\lvert V_T \rvert = \lvert V_T \rvert$	$\lvert\alpha\rvert \geq 90^{\circ}$	0
	$\lvert\alpha\rvert < 90^{\circ}$	1
$\lvert V_I \rvert < \lvert V_T \rvert$	$\lvert\alpha\rvert < 90^{\circ}$ and $\lvert V_I \rvert < \lvert V_T \rvert \operatorname{Sin} \lvert\alpha\rvert$	0
	$\lvert\alpha\rvert < 90^{\circ}$ and $\lvert V_I \rvert = \lvert V_T \rvert \operatorname{Sin} \lvert\alpha\rvert$	1
	$\lvert\alpha\rvert < 90^{\circ}$ and $\lvert V_I \rvert > \lvert V_T \rvert \operatorname{Sin} \lvert\alpha\rvert$	2
	$\lvert\alpha\rvert \geq 90^{\circ}$	0

DETERMINATION OF GUIDANCE SOLUTION

Having determined the number of guidance solutions for the constant speed, straight line intercept mission, the solution that will result in a minimum time-to-interception will be determined. The elements of the guidance solution are the interceptor command heading as well as the time and path length for the interceptor to reach the intercept point.

Figure 10 shows the geometry for the straight line intercept mission. In determining the guidance solution, the interceptor is assumed to be co-altitude with the target and capable of executing an instantaneous turn to the command heading. Interceptor speed is assumed to be constant and that associated with the selected profile.

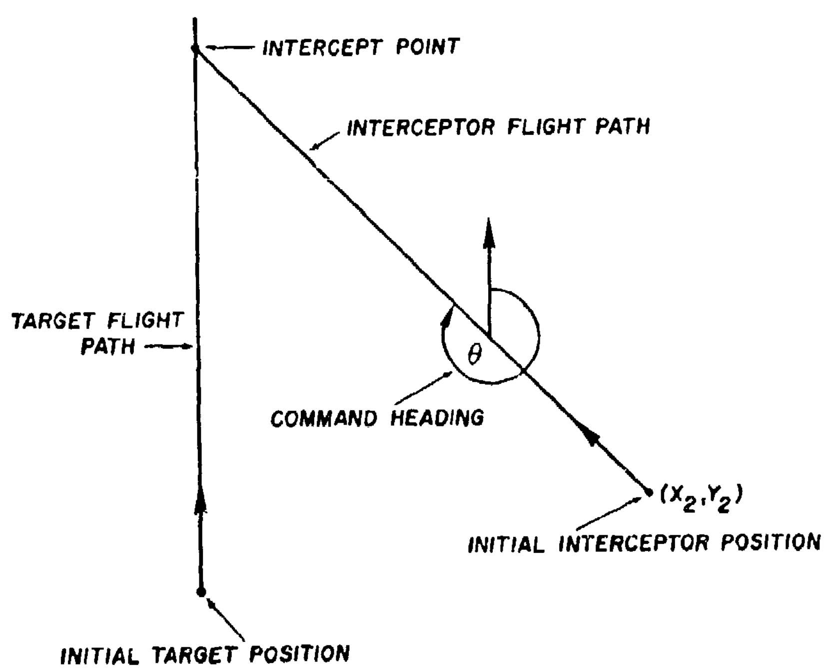

Figure 10 STRAIGHT LINE INTERCEPT GEOMETRY

In obtaining the guidance solution, some preliminary computations are required. These calculations are measures of separation between the target and interceptor as well as measures of the time and path length for the interceptor to reach the point of closest approach to the target. Upon obtaining these measures, the guidance solution command heading, time and path length to point of closest approach will be obtained for a stationary target and thereafter for a moving target.

Interceptor and Target Separation

Equation (3) expressed the interceptor position in a coordinate system wherein the target remains stationary at the origin. Solving equation (3) for the time t_y at which the interceptor crosses the Y axis, which is equivalent to setting X_I' to zero and solving for t_y, we obtain:

$$t_y = \frac{-X_I}{|V_I| \sin \theta} \qquad (6)$$

The distance D_y, which the interceptor travels from its initial position until it crosses the Y axis, is:

$$D_y = |V_I| t_y = \frac{-X_I}{\sin \theta} \qquad (7)$$

Substituting t_y from equation (6) in the Y component equation from (3), we obtain the Y separation, designated ΔY, between the interceptor and the target at the time the interceptor crosses the Y axis. This separation is:

$$\Delta Y = Y_I - \frac{X_I \cos\theta}{\sin\theta} + \frac{\rho X_I}{\sin\theta} \qquad (8)$$

where:

$$\rho = |V_T|/|V_I|$$

Figure 11 shows the interceptor position (X_I, X_I), the path length D_y from the interceptor position until it crosses the Y axis and the Y separation between the target and the interceptor at the instant the interceptor crosses the Y axis.

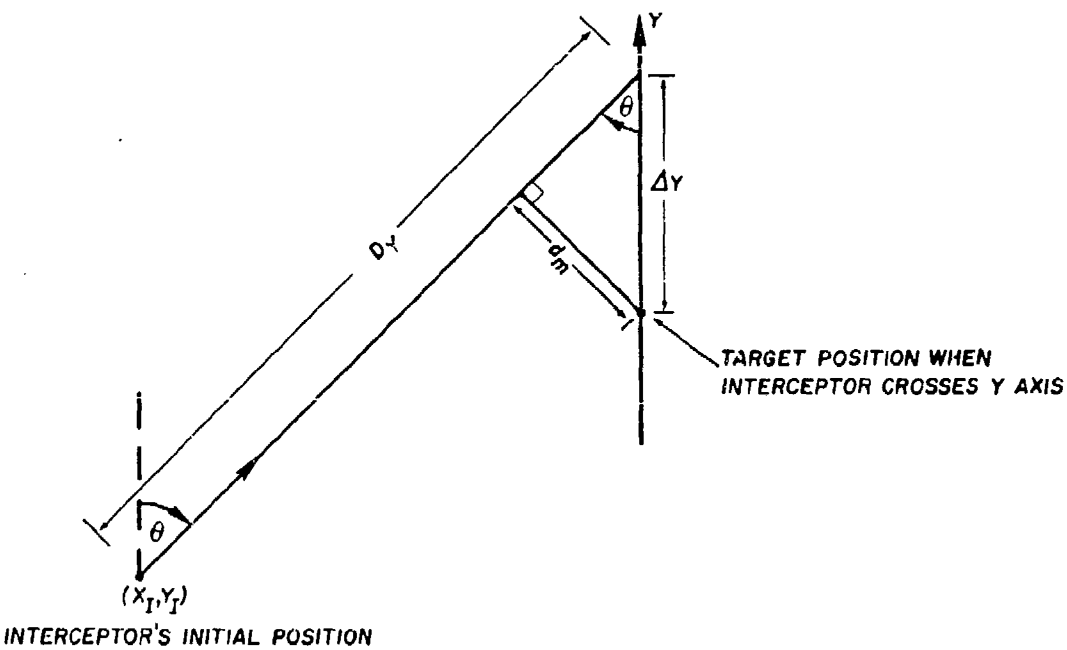

Figure 11 PATH LENGTH AND Y SEPARATION

In addition to showing the path length D_y and the separation ΔY, Figure 11 also shows the miss distance d_m. The miss distance is defined as the perpendicular distance from the target position to the interceptor flight path measured at the instant the interceptor crosses the Y axis or target flight path. From Figure 11, the miss distance d_m is expressed by:

$$d_m = \Delta Y \text{ Sin } \theta = Y_I \text{ Sin } \theta - X_I \text{ Cos } \theta + \rho X_I \qquad (9)$$

Examination of Figure 11 shows that either ΔY or d_m provides an excellent measure of the suitability of a particular heading to effect an intercept. A command heading θ, which results in both ΔY and d_m equaling zero, will result in a successful intercept. Furthermore, when ΔY is zero, so also is d_m and vice versa.

Point of Closest Approach

Figure 12 shows the path length L_m from the initial interceptor position to the interceptor's point of closest approach to the target for a geometry where the target is heading northward and the interceptor is headed toward the east. Successive target positions are designated by T_0, T_1, T_2 and T_3 whereas corresponding interceptor positions are denoted by I_0, I_1, I_2 and I_3. The miss distance d_m, the ΔY separation and the separation at the point of closest approach d_{min} are also shown.

From equation (A-2) of Appendix A, the time t_m at which the separation between the interceptor and the target is a minimum is:

$$t_m = \frac{-X_I \text{ Sin } \theta - Y_I \text{ Cos } \theta + \rho Y_I}{|V_I|(1 - 2\rho \text{Cos } \theta + \rho^2)} \qquad (10)$$

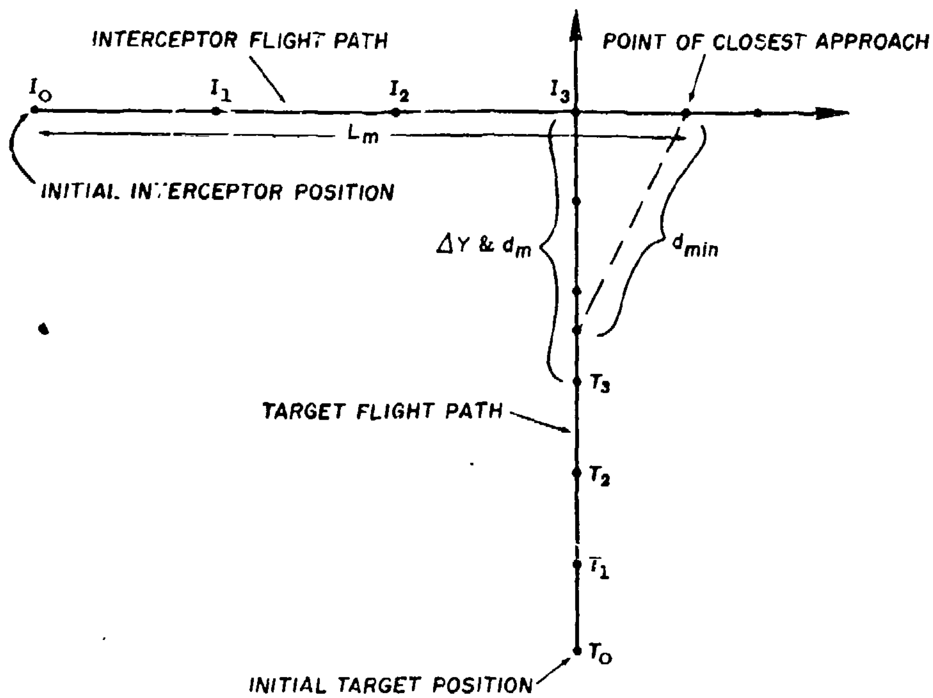

Figure 12 MISS DISTANCE AND PATH LENGTH TO POINT OF CLOSEST APPROACH

The interceptor path length L_m to the point of closest approach to the target is obtained from the expression

$$L_m = |V_I| \, t_m = \frac{1}{\gamma^2}\left[-X_I \text{ Sin } \theta - Y_I \text{ Cos } \theta + \rho \, Y_I\right] \qquad (11)$$

where:

$$\gamma^2 = 1 - 2 \, \rho \text{ Cos } \theta + \rho^2 \qquad (12)$$

Equations (8) and (9) express the separation between the interceptor and the target in terms of the command heading θ. Similarly, equations (10) and (11) represent the time and distance respectively

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for the interceptor to proceed from its initial position to the point of closest approach to the target in terms of the command heading θ.

It remains to determine the heading θ that would result in a satisfactory interception. Upon determining the command heading, the time and distance to the point of closest approach can be determined from equations (10) and (11).

<u>Guidance Solution for a Stationary Target</u>

To delineate the fundamentals involved, the guidance solution will initially be obtained for a stationary target. For this situation, the ratio of target to interceptor speed ρ is zero, resulting in equations (8) and (9) being rewritten as:

$$\Delta Y = Y_I - \frac{X_I \text{ Cos } \theta}{\text{Sin } \theta} \tag{13}$$

$$d_m = Y_I \text{ Sin } \theta - X_I \text{ Cos } \theta \tag{14}$$

Since X_I and Y_I are the known coordinates of the interceptor, the separations ΔY and d_m that result for any choice of θ may trivially be calculated. A heading θ, which results in a separation of zero, is a guidance solution.

Setting either ΔY in equation (13) or d_m in equation (14) to zero and solving for the interception heading θ, we obtain:

$$\theta = \text{Tan}^{-1}\left(\frac{-X_I}{-Y_I}\right) = \left[\text{Tan}^{-1}\left(\frac{X_I}{Y_I}\right) + 180\right] \text{ Mod } 360 \tag{15}$$

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Figure 13 shows the separations ΔY and d_m versus the heading θ for a stationary target when the interceptor coordinates are $X_I = 100$ and $Y_I = 200$.

Examination of Figure 13 in conjunction with equations (13) and (14) reveals that:

(a) The curves for both d_m and ΔY pass through the line $Y = 0$ at the same heading θ.

(b) The miss distance curve d_m has a smaller range of values than the ΔY curve.

(c) The value of d_m for a particular value of θ is the negative of d_m for $(\theta + 180)$ degrees; therefore, redundant information is provided.

(d) The ΔY curve repeats identically from 180 to 360 degrees what was shown from 0 to 180 degrees; therefore, redundant information is provided.

(e) Both the d_m and the ΔY curves pass through the line $Y = 0$ at two headings, which are separated from each other by 180 degrees.

Figure 13 appears to indicate that an interceptor located at coordinates $X_I = 100$, $Y_I = 200$ will intercept a stationary target by assuming a heading of either 26.6 or 206.6 degrees. Since the interceptor is initially located northeast of the target, assuming a heading of 26.6 degrees would cause the interceptor to be going away from the target. To explain this apparent anomaly, it is necessary to briefly examine equation (10), which is restated below

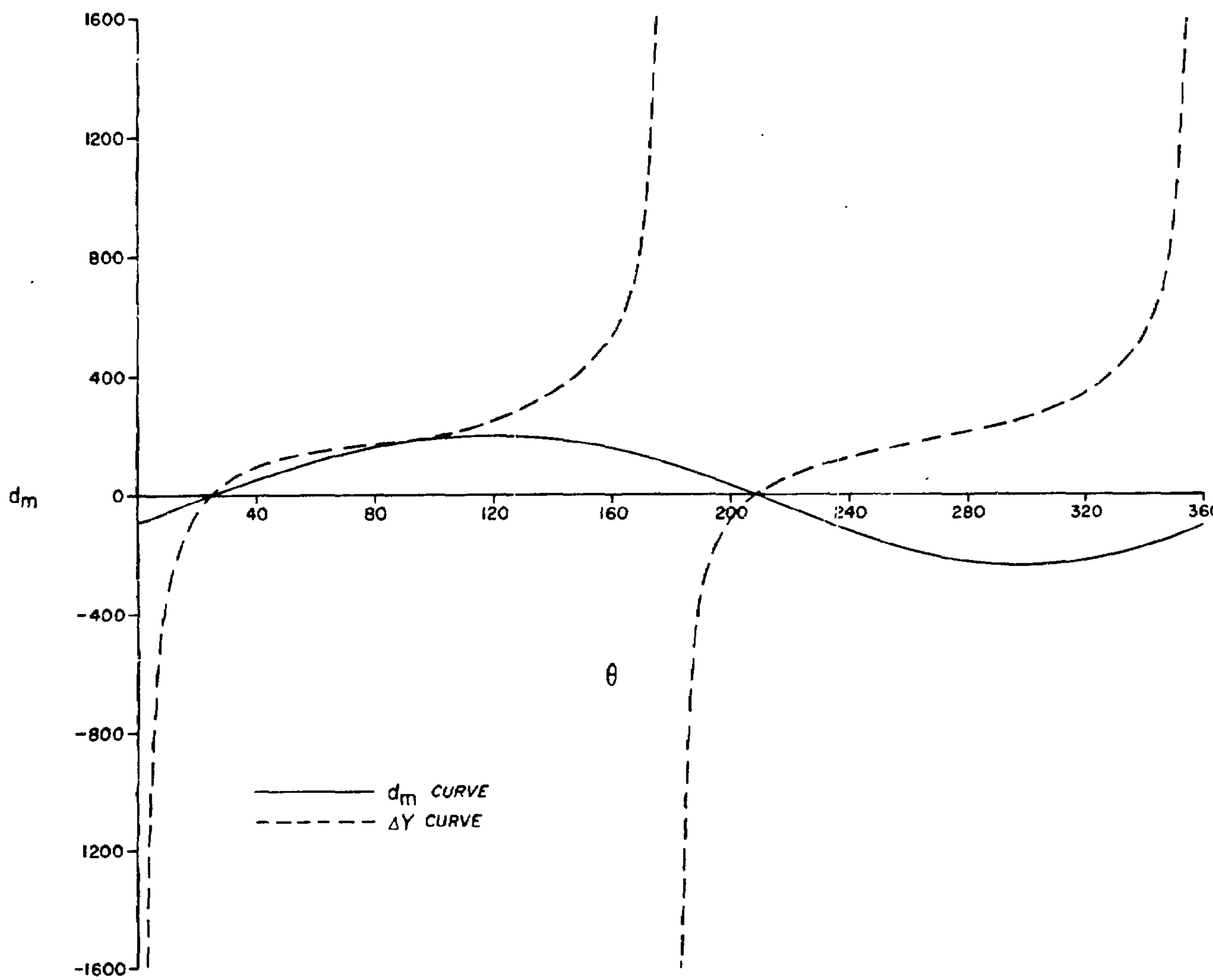

Figure 13 MISS DISTANCE VERSUS HEADING FOR STATIONARY TARGET

for the case when the target is stationary. In this case, the time to the point of closest approach is:

$$t_m = \frac{-(X_I \text{ Sin } \theta + Y_I \text{ Cos } \theta)}{|V_I|} \tag{16}$$
$$= \frac{X_I \text{ Sin } (180 + \theta) + Y_I \text{ Cos } (180 + \theta)}{|V_I|}$$

The time t_m until the point of closest approach is positive for a heading of 206.6 degrees and negative for a heading of 26.6 degrees. What Figure 13 in conjunction with equation (16) implies, is that interception can be accomplished either by proceeding along a heading of 206.6 degrees for a positive time t_m or by flying a heading of 26.6 for a negative time t_m. The negative time solution, which corresponds physically to the interceptor flying backwards, must be eliminated from consideration.

Eliminating the negative time solution, the interceptor heading that would result in a minimum time guidance solution is 206.6 degrees. The associated time to the point of closest approach is obtained from equation (16) when $\theta = 206.6$. The path length L_m to the point of closest approach is obtained from equation (11) when ρ is zero and when $\theta = 206.6$ degrees.

From the previous discussion, the command heading which results in a minimum time-to-interception is obtained by solving the miss distance equation (9) for θ when d_m is zero. The geometry of the problem constrains the heading θ to the appropriate half of the 360 degree interval. Therefore, if the interceptor is located in the first or second quadrants of the target coordinate system as shown in Figure 14, the only acceptable choices of heading are values of θ between 180 and 360 degrees. If the interceptor is located in the third or fourth quadrants, the heading θ is chosen from the interval between 0 and 180 degrees. Upon solving equation (9) for the command heading, the time and distance to the point of closest approach are obtained from equations (10) and (11), respectively.

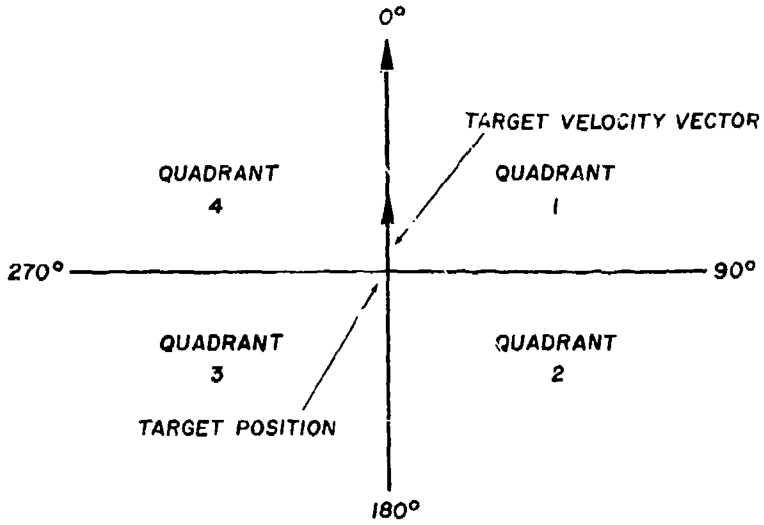

Figure 14 RANGE OF POSSIBLE HEADINGS

Guidance Solution for a Moving Target

The procedure for obtaining the minimum time guidance solution for a moving target is similar to that for the stationary target. The miss distance equation is solved for the command heading which yields an acceptable separation. Then, the time and distance to the point of closest approach are obtained.

In determining the command heading, several situations that result in differing numbers of guidance solutions will be considered. Table I has shown that the number of guidance solutions depends upon the interceptor/target geometry and speeds. In an effort to independently verify the results of Table I, we will choose interceptor/target positions and speeds which correspond to the last four conditions of Table I.

Table III presents four sets of interceptor positions (X_I, Y_I), interceptor speeds $|V_I|$ and target speeds $|V_T|$ that will be considered. For each of the four sets of parameters, the appropriate command heading will be determined. Thereafter, the time and path length to the point of closest approach will be obtained.

Table III

Interceptor/Target Parameters

X_I	Y_I	$\|V_I\|$	$\|V_T\|$	Condition
100	100	200.0	400	$\|V_I\| < \|V_T\| \sin \alpha$
100	100	282.8	400	$\|V_I\| = \|V_T\| \sin \alpha$
100	100	340.0	400	$\|V_T\| \sin \alpha < \|V_I\| < \|V_T\|$
100	100	600.0	400	$\|V_I\| > \|V_T\|$

Figure 15 shows a graphical presentation of the miss distance d_m versus the interceptor heading for values of θ that result in positive

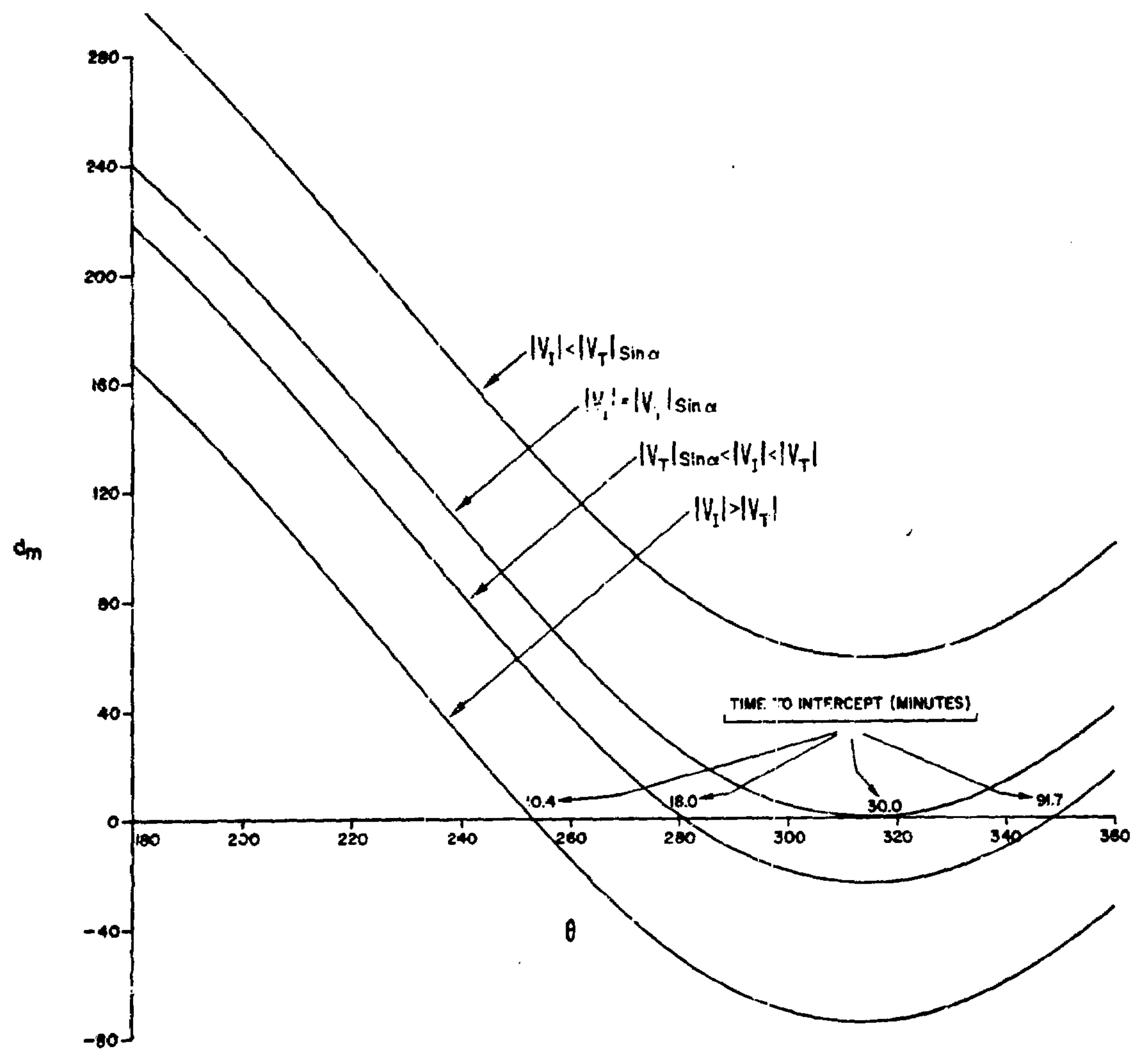

Figure 15 MISS DISTANCE VERSUS HEADING FOR MOVING TARGET

time solutions for each of the four sets of parameters. Examination of the figure shows that the number of guidance solutions is in agreement with the data presented in Tables I and II. For the interceptor and target parameters shown in Table III, when $|V_I| < |V_T| \sin \alpha$ there is no solution. When $|V_I| = |V_T| \sin \alpha$, only the heading of 315.0 degrees which is perpendicular to the line of sight would result in interception. When $|V_T| \sin \alpha < |V_I| < |V_T|$, an interceptor heading of either 281.3 or 349.0 degrees would result in interception. Finally, when $|V_I| > |V_T|$, only a heading of 252 degrees would result in interception.

Figure 15 also shows the time to the point of closest approach as obtained from equation (10) for the value of θ at which d_m is zero. For the interceptor and target parameters shown in Table III, when $|V_I| > |V_T|$, the time-to-interception is 10.4 minutes. When $|V_T| \sin \alpha < |V_I| < |V_T|$, two headings would result in interception. As has previously been stated, selecting the value of θ closer to 180 degrees in the target coordinate system minimizes the time to interception. For the situation shown in Figure 15, the heading closer to 180 degrees yields a time-to-interception of 18.0 minutes whereas the heading farther from 180 degrees but at the same interceptor speed requires 91.7 minutes of flight time.

With knowledge of both the interceptor position (X_I, Y_I) and the ratio of target to interceptor speed ρ, equation (9) determines d_m for any value of θ. This technique was used to obtain the data presented in Figure 15. What is desired however is a rapid approach for determining the roots of equation (9); i.e., for d_m approximately zero, what values of θ satisfy the equation? Unfortunately, values of θ that satisfy the equation cannot be obtained directly. The iterative procedure described hereafter yields the command heading to the desired degree of accuracy.

General Approach

To determine the interceptor command heading using equation (9), the range of headings is initially constrained to the appropriate half of the 360 degree interval, thereby eliminating negative time solutions. Thereafter, the remaining heading interval is split in such a way that each sub-interval contains no more than one root. This may be accomplished by splitting the interval at the extremes of the miss distance equation. However, since the extremes of d_m are difficult to locate, those of (d_m/Sin θ) are used instead. This is acceptable because the roots of both d_m and (d_m/Sin θ) are identical except when θ is either 0 or 180 degrees. Because 0 and 180 degrees will be defined as boundaries of the interval to be examined, splitting the interval at the extremes of (d_m/Sin θ) is sufficient to ensure that each sub-interval contains at most one interceptor heading.

Returning to Figure 15, we note that the initial range of interceptor headings is constrained between 180 and 360 degrees. The next step in determining the command heading is to split the interval from 180 to 360 degrees at the extremes of (d_m/Sin θ). Having divided the interval from 180 to 360 degrees into sub-intervals, the existence of a solution within a sub-interval is detected by examining the sign of d_m at the two endpoints of the sub-interval. If the two signs of d_m are identical, no root exists within the sub-interval. If the two signs of d_m differ, further examination is required to determine the root within the sub-interval. When the two signs of d_m differ, the sub-interval is divided in half and d_m is evaluated at the midpoint. A comparison of the sign of d_m at the midpoint with the sign of d_m at the two ends of the sub-interval indicates which sub-interval contains the root. This technique of interval splitting is repeated until the desired degree of heading accuracy is obtained.

Derivative of the Miss Distance Function

To determine the value of θ at which the heading interval should be subdivided in order to ensure that each interval contains at most one root, the derivative of $(d_m/\sin\theta)$ with respect to θ will be obtained. From equation (9), $(d_m/\sin\theta)$ is defined by:

$$(d_m/\text{Sin } \theta) = Y_I - X_I \text{ Cot } \theta + \rho X_I/\text{Sin } \theta \qquad (17)$$

The derivative of $(d_m/\sin\theta)$ with respect to θ is:

$$\frac{d}{d\theta}\left(\frac{d_m}{\text{Sin } \theta}\right) = \frac{X_I}{\text{Sin}^2 \theta} - \frac{\rho X_I \text{ Cos } \theta}{\text{Sin}^2 \theta}$$

Multiplying by $\sin^2\theta$ and setting the result to zero:

$$\rho X_I \text{ Cos } \theta = X_I$$

$$\theta = \text{Cos}^{-1}(1/\rho) \qquad (18)$$

Equation (18) indicates that extremes of the function (d_m/Sin θ) exist only if the interceptor is at a speed disadvantage ($\rho > 1$). When the interceptor is at a speed disadvantage, the two extremes of (d_m/Sin θ) occur at:

$$\theta_m = \text{Cos}^{-1}\,(1/\rho) = \text{Cos}^{-1}\,(|V_I|/|V_T|)$$

$$\text{and } \theta_n = 360^0 - \theta_m \qquad (19)$$

Equations (19) represent the values of θ at which the range of interceptor headings should be subdivided. The equations are consistent with the data in Table II, which indicates that two guidance solutions are possible only when the interceptor has a speed disadvantage.

Guidance Solution

Figure 16 illustrates a procedure for obtaining the minimum time guidance solution. The solution includes the command heading, the path length and time-to-interception. Inputs to the procedure are the interceptor speed $|V_I|$ and the target speed $|V_T|$.

To obtain the intercept solution which yields the minimum time-to-interception, θ_m and θ_n are calculated via equation (19) if $|V_I|$ is less than $|V_T|$. The interceptor coordinates (X_I, Y_I) are calculated from equation (2). If X_I is in the first or second quadrant, the heading interval is constrained between 180 and 360 degrees. If X_I is in the third or fourth quadrant, the heading interval is constrained from 0 to 180 degrees. If the interceptor is at a speed disadvantage, the heading interval is split at either θ_m or θ_n.

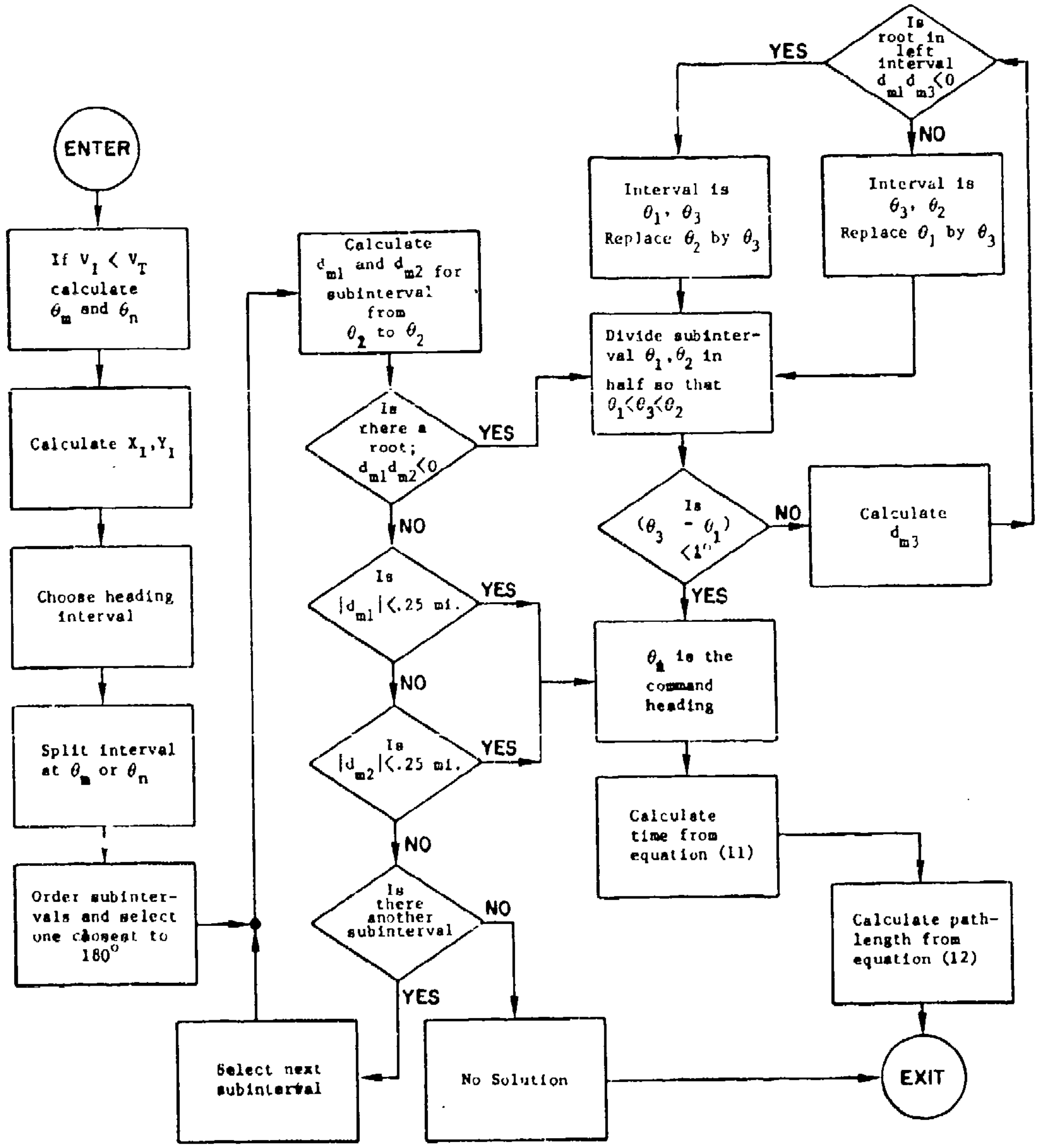

Figure 16 INTERCEPT SOLUTION FOR STRAIGHT LINE GEOMETRY

Basic to the procedure of Figure 16 is an ordering of the sub-intervals so that a sub-interval with a range of headings closer to 180 degrees is examined first. For example, if the sub-intervals are 180 to 315 degrees and 315 to 360 degrees and if a root is found within the interval 180 to 315 degrees, the other interval is not examined. Clearly, in the target coordinate system, a command heading within the interval 315 to 360 degrees would require a longer path length and time-to-interception than a heading within the other interval.

The values of the miss distance corresponding to headings of θ_1 and θ_2 at the endpoints of a sub-interval are evaluated from equation (9). There is a solution between θ_1 and θ_2 only if:

$$d_{m1}\, d_{m2} < 0 \qquad (20)$$

Here and in Figure 16, the second subscript indicates to which value of θ the quantity d_m corresponds. If condition (20) does not hold and neither $|d_{m1}|$ nor $|d_{m2}|$ is approximately zero, the next sub-interval is examined.

If condition (20) is satisfied, the value of the angle midway between the bounds is found.

$$\theta_3 = .5\ (\theta_1 + \theta_2) \qquad (21)$$

If the interval is sufficiently small, the appropriate command heading has been determined. With the value specified in Figure 16, θ_3 is the correct solution to within 1 degree. If the interval is not sufficiently small, d_{m3} is calculated, the interval is split in half again and the process is repeated. When the appropriate command heading is finally ascertained, the path length and time to the point of closest approach are obtained from equations (10) and (11) respectively.

SECTION IV

VARIABLE SPEED INTERCEPT MISSION

INTRODUCTION

Section III considered the constant speed, two dimensional, straight line intercept mission. However, the material presented therein provides insight as well as the framework for the solution to the more general class of intercept problem. This section augments Section III by considering the variable speed, three dimensional intercept mission that may require the interceptor to approach the target from a particular direction.

VARIABLE SPEED THREE DIMENSIONAL FLIGHT PATH

Figure 17 illustrates the interceptor flight path for the variable speed, three dimensional intercept mission. The interceptor climbs to cruise altitude. Upon attaining cruise altitude, the interceptor accelerates to cruise speed and proceeds at the cruise speed and command heading to the transition point. At the transition point, the acceleration or deceleration to combat speed and the climb or descent to combat altitude begin. The transition point is located so that the transition maneuvers are completed before the interceptor reaches the offset point. The combat phase commences when the offset point is attained. At the offset point the final turn, if required, from the command heading to the terminal heading is begun. Upon attaining the terminal heading, the interceptor proceeds at combat speed and altitude to the intercept point.

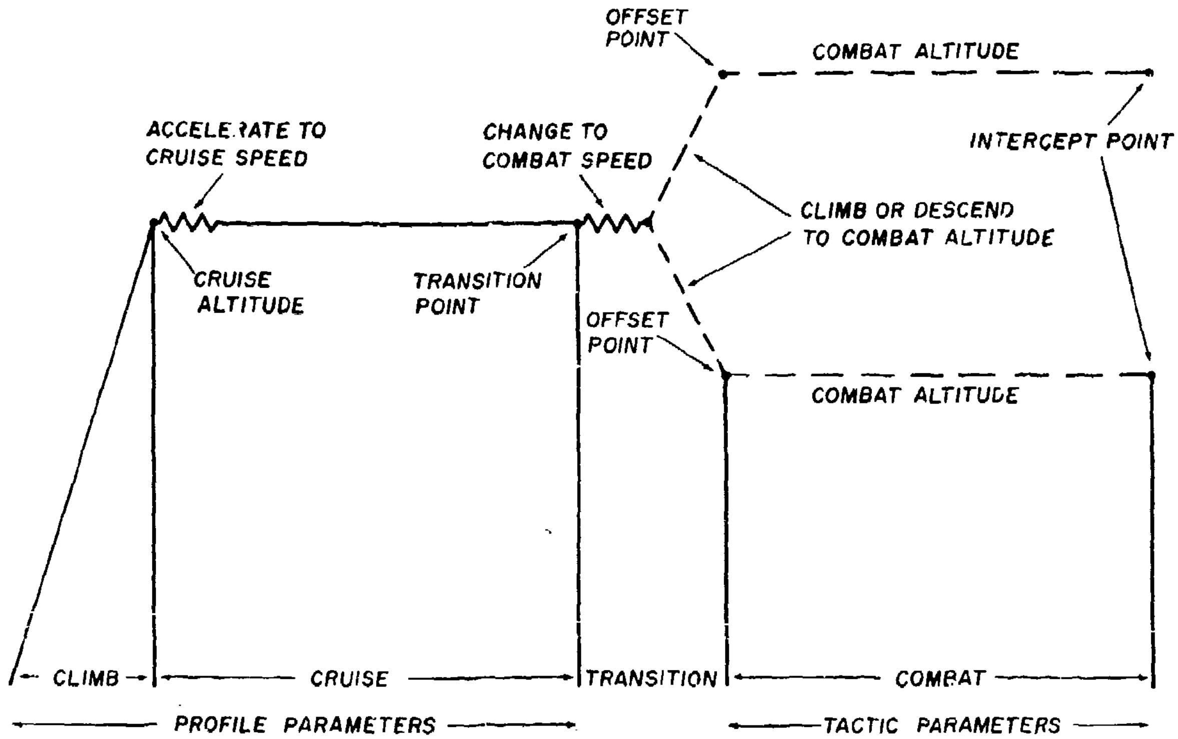

Figure 17 INTERCEPTOR FLIGHT PATH

The variable speed, three dimensional interceptor flight path may be synopsized as follows:

(a) Climb to cruise altitude,

(b) Accelerate to cruise speed,

(c) Proceed at cruise speed to the transition point,

(d) Accelerate or decelerate to combat speed,

(e) Climb or descend to combat altitude,

(f) Perform turn, if required, to terminal heading, and

(g) Proceed at combat speed and altitude to the intercept point.

VARIABLE SPEED THREE DIMENSIONAL INTERCEPT MISSION

Having described the variable speed, three dimensional intercept flight path, a procedure for determining the guidance solution resulting in a minimum time-to-interception will be presented. The elements of the guidance solution are the climb speed, cruise speed, cruise altitude, command heading, time and path length to intercept point or offset point, combat speed, combat altitude, duration of the combat phase and the terminal heading.

The procedure for obtaining a guidance solution is analogous to that of Section III wherein the interceptor was assumed to be co-altitude with the target and to complete the entire mission at a constant speed and altitude. The objective herein will be to modify the guidance equations from Section III to account for a variable speed, three dimensional flight path that may require the interceptor to approach the target from a particular direction.

The guidance solution will initially be obtained for an interceptor flight path that does not require a particular approach heading with respect to the target. Thereafter, the guidance solution will be developed for an intercept that requires a particular approach heading. In obtaining the guidance solution, the interceptor is assumed to be capable of performing an instantaneous turn from its present heading to the command heading and from the command heading to the terminal heading.

Guidance Sclution for Mission With No Terminal Segment

Profile selection results in the determination of the interceptor climb speed, cruise speed and cruise altitude. Tactic selection determines combat speed, combat altitude and duration of the combat phase. Thus, the remaining elements of the guidance solution to be obtained are the command heading and the time and path length to intercept point.

The fundamental distinction between the variable speed, three dimensional intercept mission and the constant speed, straight line mission described in Section III is that the interceptor performs accelerations, decelerations, climbs and descents. Thus, the guidance equations from Section III must be modified to compensate for these maneuvers.

Interceptor performance data are used to estimate the duration of the climb and transition phases. Tactic selection determines the duration of the combat phase. Thus, the cruise phase of a mission is unique because its duration is unknown until the command heading is determined.

Figure 18 shows the ground distance an interceptor travels versus time for an interception mission. The slope of the curve at any point is the interceptor speed at that time. The solid curve depicts an initial climb corresponding to OA, an instantaneous acceleration to the cruise speed AB and an instantaneous acceleration to the combat speed BC. The dashed curve shows the same distance being traversed in less time at a constant cruise speed.

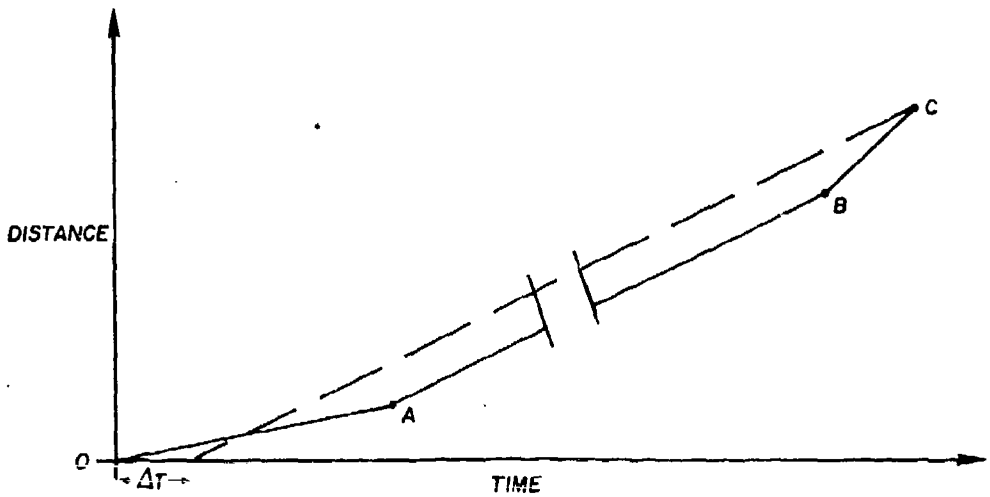

Figure 18 DISTANCE VERSUS TIME DIAGRAM

Examination of Figure 18 shows that, independent of the duration of the cruise phase AB, the same intercept point C may be realized from either the complex solid schedule OABC or via the simpler dashed schedule. Therefore, if the interceptor is assumed to remain stationary for a time ΔT and then to fly the entire mission at the cruise speed, as shown in Figure 18, the same intercept point is realized.

The time interval ΔT_i is the difference between the actual time for a particular phase of the mission and the time to traverse the same distance assuming it was flown at the cruise speed. The time interval ΔT as shown in Figure 18 is the sum of the ΔT_i for all phases of the mission. The interval ΔT is used to displace the interceptor position to account for interceptor climbs, descents, accelerations and decelerations. This displacement is required because the guidance equations from Section III implicitly assume that the interceptor flies the entire mission at a constant speed and altitude.

To calculate ΔT and in turn compensate for the variable speed, three dimensional flight path, knowledge of the actual interceptor speed for each phase of the mission is required. Likewise, the duration of each phase of the intercept mission other than the cruise phase must be known. The interceptor speed during each phase of the mission is obtained from the profile and tactic selection processes. The duration of climbs, descents, accelerations, and decelerations is obtained from interceptor performance data. With knowledge of these parameters, the time displacement ΔT may be calculated.

The ground distance GD traversed by an interceptor for a particular phase of a mission is:

$$GD = t_A|V_A| \qquad (22)$$

where t_A and $|V_A|$ are the actual time duration and the actual interceptor speed for that phase of the mission. The ground distance covered by an interceptor for a particular phase of the mission may also be expressed in terms of the cruise speed $|V_I|$ and an unknown time T required to traverse the same distance at the cruise speed by:

$$GD = T|V_I| \qquad (23)$$

Since ΔT_i is the difference between the actual time spent in a particular phase of the mission and the time it would take to cover the same distance at the cruise speed, ΔT_i is defined by:

$$\Delta T_i = t_A - T = t_A \left[1 - \frac{|V_A|}{|V_I|} \right] \quad (24)$$

The time displacement ΔT_i due to an interceptor acceleration or deceleration is obtained from:

$$\Delta T_i = t_A \left[1 - \frac{(V_{IN} + V_{OUT})}{2|V_I|} \right] \quad (25)$$

where V_{IN} and V_{OUT} are the interceptor speed at the start and completion of the acceleration or deceleration.

The time displacement ΔT_i due to an interceptor climb or dive is obtained from:

$$\Delta T_i = t_A - D_A/|V_I| \quad (26)$$

where t_A and D_A are the time spent and the distance traversed during the climb or dive, respectively. The parameters t_A and D_A are obtained from interceptor performance data.

For example, Figure 19 shows the ground distance an interceptor travels versus time for a mission with three phases. The interceptor is assumed to perform an initial climb corresponding to OA. During this phase, the interceptor travels at 5 miles a minute for two minutes. During the cruise phase which extends from A to B, the interceptor is assumed to travel at 10 miles a minute for three minutes. Finally, during the combat phase which extends from B to C, the interceptor travels at 20 miles a minute for one half of a minute. This information is presented in Table IV.

Table IV

Speed and Duration Per Phase

Phase	Speed	Duration
Climb (OA)	5 mi/min	2 min's
Cruise (AB)	10 mi/min	3 min's
Combat (BC)	20 mi/min	.5 min's

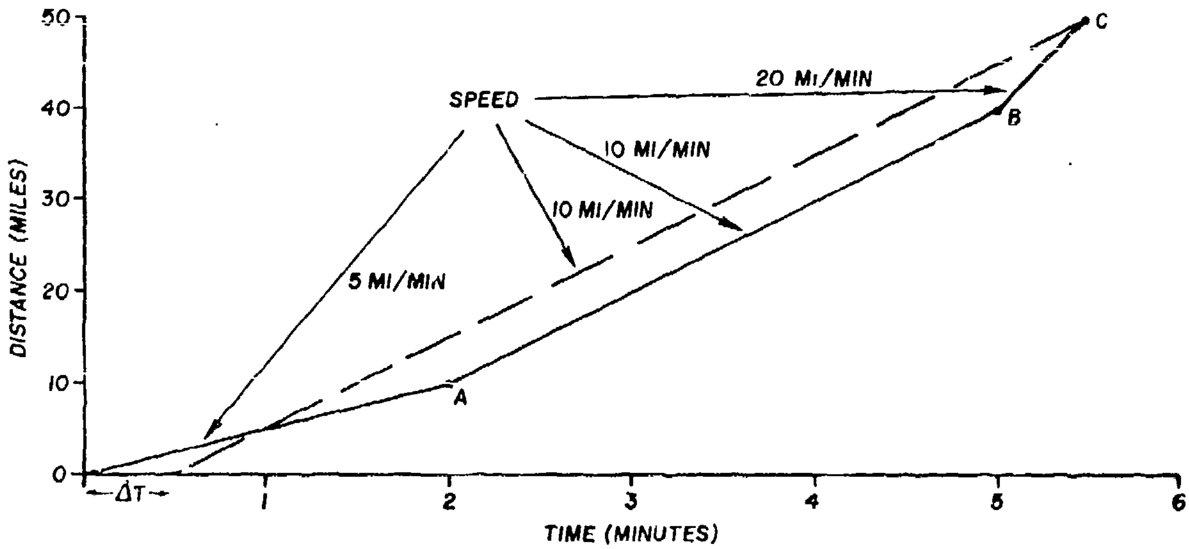

Figure 19 DISTANCE VERSUS TIME DIAGRAM

Assuming a capability to perform an instantaneous change of speed, the time displacement ΔT is obtained from:

$$\Delta T = \sum t_A \left[1 - \frac{|V_A|}{|V_I|} \right] = 2\ (1-5/10) + 3\ (1-10/10) + .5(1-20/10) \quad (27)$$

or $\Delta T = .5$

Thus, holding the interceptor stationary for 0.5 minutes and then assuming the interceptor travels at the cruise speed of 10 miles per minute as shown by the dashed curve in Figure 19 results in the same intercept point that would occur from using the more complicated geometry OABC with its three different speeds.

To modify the guidance equations from Section III in accordance with this concept, the target is extrapolated along its velocity vector for a time ΔT during which time the interceptor remains motionless. In the coordinate system wherein the target remains static at the origin with its velocity vector collinear with the Y axis, the forward motion of the target is attributed to the interceptor in a negative direction. Thus, to account for the target motion during the time interval ΔT, the interceptor position (X_2,Y_2) in the target coordinate system is displaced to the location (X_I,Y_I) as follows:

$$X_I = X_2$$
$$Y_I = Y_2 - \Delta T \, |V_T| \tag{28}$$

To obtain the command heading which results in a minimum time-to-interception, the procedure shown in Figure 16 is employed where (X_I,Y_I) are defined by equations (28). Then, having obtained the command heading via the procedure indicated in Figure 16, the time and path length to the point of closest approach are obtained from equations (10) and (11) respectively.

Terminal Segment Geometry

For interceptors with certain armaments and avionics systems, the tactic selection process prescribes a particular heading crossing angle (HCA) which determines the interceptor's terminal heading. The HCA is formed by the interceptor and target flights paths with a HCA of 180 degrees corresponding to a head-on approach. Examination of Figure 20 reveals it is identical to Figure 10 except that a terminal segment of length D with a HCA of Ø has been appended to the interceptor flight path. As indicated in the assumptions, the transition from the command heading to the terminal heading θ_A is performed instantaneously.

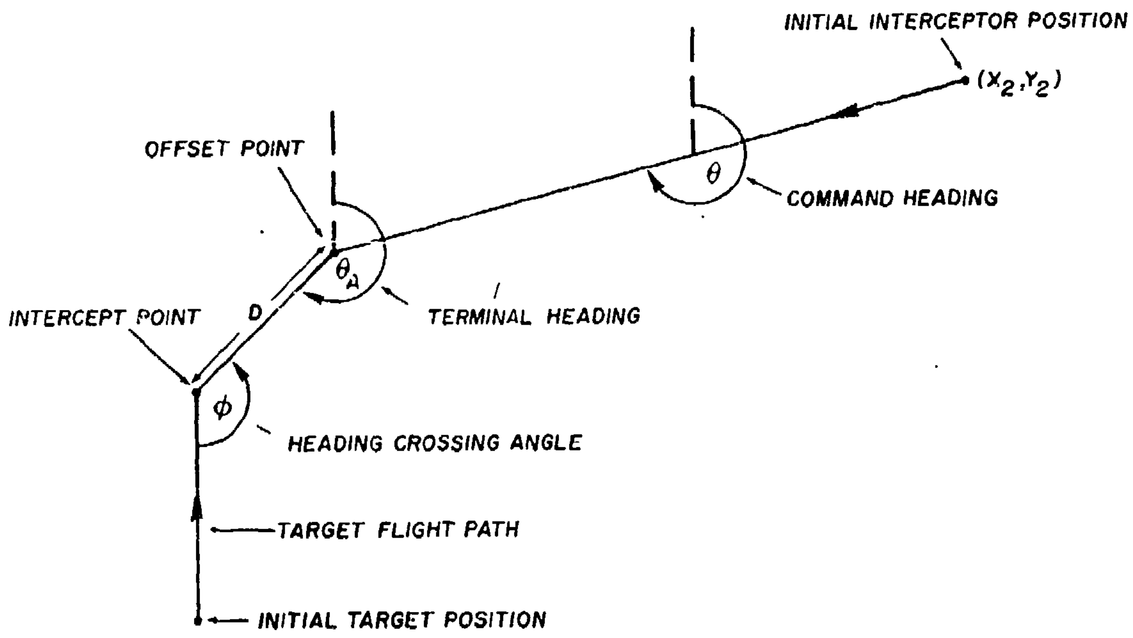

Figure 20 STRAIGHT LINE GEOMETRY WITH TERMINAL SEGMENT

Given the prescribed HCA, the terminal heading θ_A is calculated. Thereafter, using D and θ_A, the position of the interceptor is displaced to account for the motion of both the interceptor and the target from the offset to the intercept point. The result is a displaced interceptor position which essentially reduces the intercept mission with a terminal segment to that which was discussed in Section III.

The terminal heading θ_A in the target coordinate system is obtained from the HCA $\emptyset$ and the side of attack parameter σ according to:

$$\theta_A = \left[- \sigma \emptyset\right] \text{Mod } 360 \qquad (29)$$

where σ is +1 if X_2 is positive and -1 if X_2 is negative. Equation (29) implies that the interceptor should not cross in front of the target flight path.

The interceptor position (X_2, Y_2) is displaced to account for the motion of both the interceptor and the target from the offset point to the intercept point as follows:

$$\begin{aligned} X_I &= X_2 + D \text{ Sin } \theta_A \\ Y_I &= Y_2 + D \text{ Cos } \theta_A - D\rho \end{aligned} \qquad (30)$$

Equations (30) implicitly assume a constant speed, two dimensional interceptor flight path wherein the interceptor has traversed the segment of length D at a terminal heading θ_A. The equations further assume that while the interceptor has been traversing the segment of length D for a time $(D/|V_I|)$, that the target will have flown a distance $|V_T|\ (D/|V_I|)$ or $D\rho$. Since the target will have progressed a distance D_P, this motion is attributed to the interceptor position in a negative direction as indicated in equation (30). Figure 21 depicts the situation. Equations (30) will be modified in the following paragraph so that they are appropriate to a variable speed, three dimensional interceptor flight path with a prescribed approach heading relative to the target.

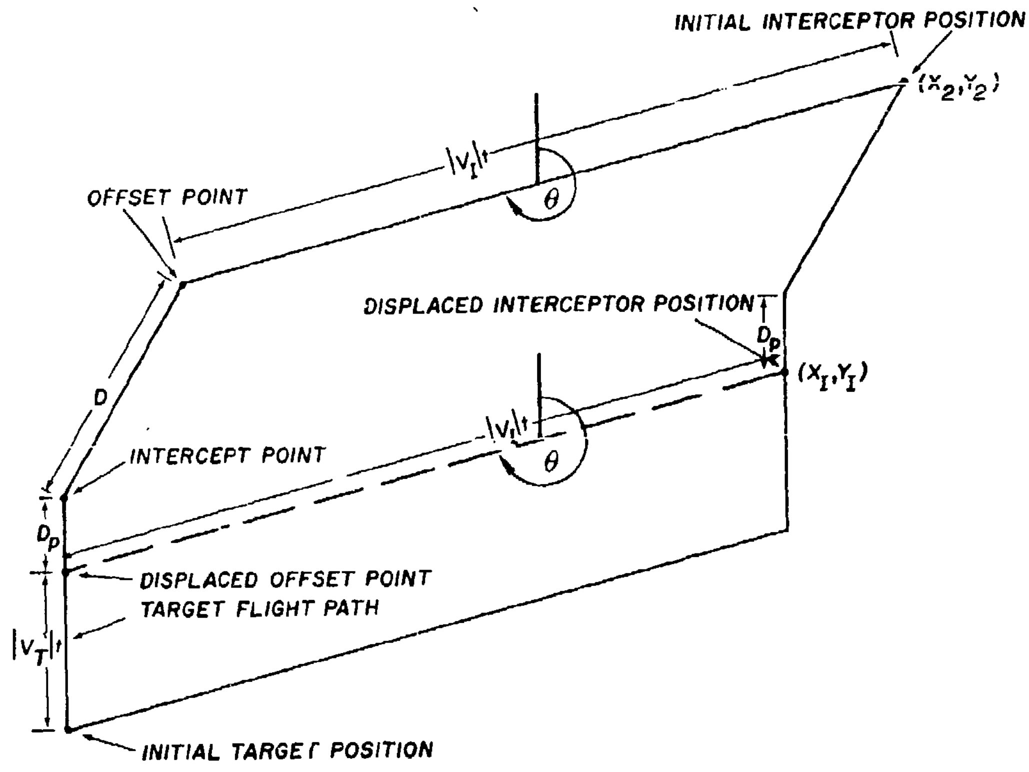

Figure 21 INTERCEPTOR DISPLACED FOR TERMINAL SEGMENT

The command heading θ in Figure 21 has been chosen so that while the interceptor travels the distance $|V_I|t$, the target will have traveled the distance $|V_T|t$. Examination of the figure reveals that when the actual interceptor initially located at (X_2,Y_2) has proceeded to the offset point, the displaced interceptor initially located at (X_I,Y_I) will have proceeded to the displaced offset point. When the actual and displaced interceptors have attained their respective offset points, the target will also have arrived at the displaced offset point. Assuming an instantaneous interceptor turn at the offset point, the actual interceptor will be separated from the intercept point by a distance D whereas the target will be displaced from the intercept point by a distance Dρ. Consequently, a successful intercept will result. Thus, displacing the interceptor position from (X_2,Y_2) to (X_I,Y_I) to account for the relative motion of both the interceptor and the target from offset point to intercept point has reduced the straight line intercept mission with a terminal segment to the straight line intercept geometry described in Section III.

Guidance Solution for Mission with Terminal Segment

To account for a variable speed, three dimensional intercept that includes a prescribed approach heading relative to the target, the interceptor position (X_2,Y_2) is displaced to the location (X_I,Y_I) as follows:

$$X_I = X_3 = X_2 + D \sin \theta_A$$
$$Y_I = Y_3 = Y_2 + D \cos \theta_A - (t_D + \Delta T)\,|V_T| \qquad (31)$$

where t_D is $(D/|V_I|)$.

(X_3,Y_3) is a simplified expression for the right side of equation (31) that will be useful in the following section of the report. It is noted that for a constant speed, two dimensional intercept mission with no prescribed HCA, D, t_D and ΔT are zero. Therefore, (X_I,Y_I) reduces to (X_2,Y_2) as in Section III.

To obtain the command heading that yields a minimum time-to-interception, the procedure of Figure 16 is used. For a variable speed, three dimensional flight path with no prescribed HCA, (X_I,Y_I) are defined by equation (28). If a particular HCA is required, (X_I,Y_I) are defined by equation (31). Then, having obtained the command heading by the procedure indicated in Figure 16, the time and path length to the point of closest approach are obtained from equations (10) and (11), respectively.

SECTION V

DOUBLE TURN INTERCEPT MISSION

INTRODUCTION

The term "double turn" does not imply that there are only two turns in an entire mission. Rather, it conveys the concept that the time and distance required to complete two turns are considered each time a guidance solution is obtained. During the course of an intercept mission, there may be several distinct guidance solutions each of which requires a different command heading.

The two turns that are considered in obtaining a guidance solution are the initial and the final turn. The initial turn is executed so that the interceptor can proceed from its current heading to the command heading. The final turn, which is only required when the interceptor is constrained to approach the target in accordance with a particular HCA, is executed so that the interceptor can proceed from the command heading to the terminal heading. Figure 22 illustrates an interceptor flight path which includes both an initial and a final turn.

The guidance solutions that have been presented in Sections III and IV are based upon the assumption that the interceptor can perform instantaneous turns. Obviously neither the initial nor the final turn can be performed instantaneously. Consideration of the time and distance during the initial and final turns requires modifications to the previously developed guidance equations, which in turn introduce additional guidance solutions. Mathematically, an infinite number of

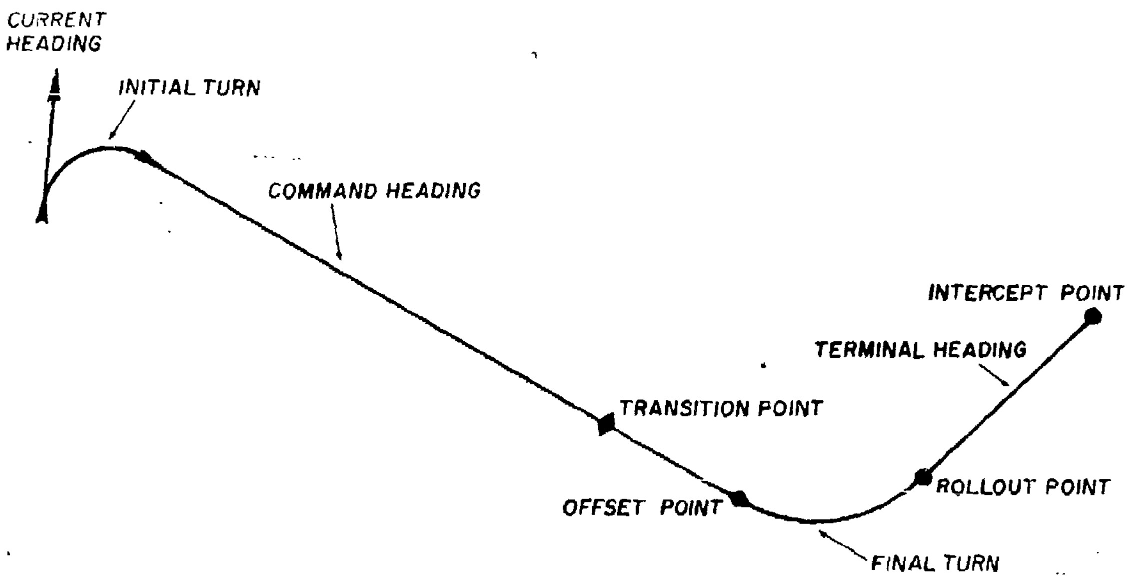

Figure 22 INITIAL AND FINAL TURNS

guidance solutions exist for an interceptor with a speed advantage since the interceptor could turn in a circle any number of times and then effect an intercept. Fortunately, reasonable solutions require minimal times-to-interception; therefore, turns larger than 360 degrees may be eliminated from consideration.

This section augments Sections III and IV by modifying the procedure developed therein to account for the time and distance during initial and final interceptor turns. As an introduction, the various turn combinations, the interceptor flight path during turns, and the displacement of interceptor position to account for the motion of both the interceptor and the target during turns are discussed. Then, the guidance solution that would result in a minimum time-to-interception for the double turn intercept mission is presented.

TURN COMBINATIONS

There are two directions in which an interceptor may turn from its present heading to the command heading in order to intercept a target. These directions are right and left. When constrained to approach a target in accordance with a prescribed HCA, there are four ways of describing the path to the intercept point when turns of less than 360 degrees are considered. These paths have right right, right left, left right and left left initial and final turns, respectively. Figure 23 shows the four interceptor flight paths to a stationary target when the interceptor's required terminal heading is θ_A.

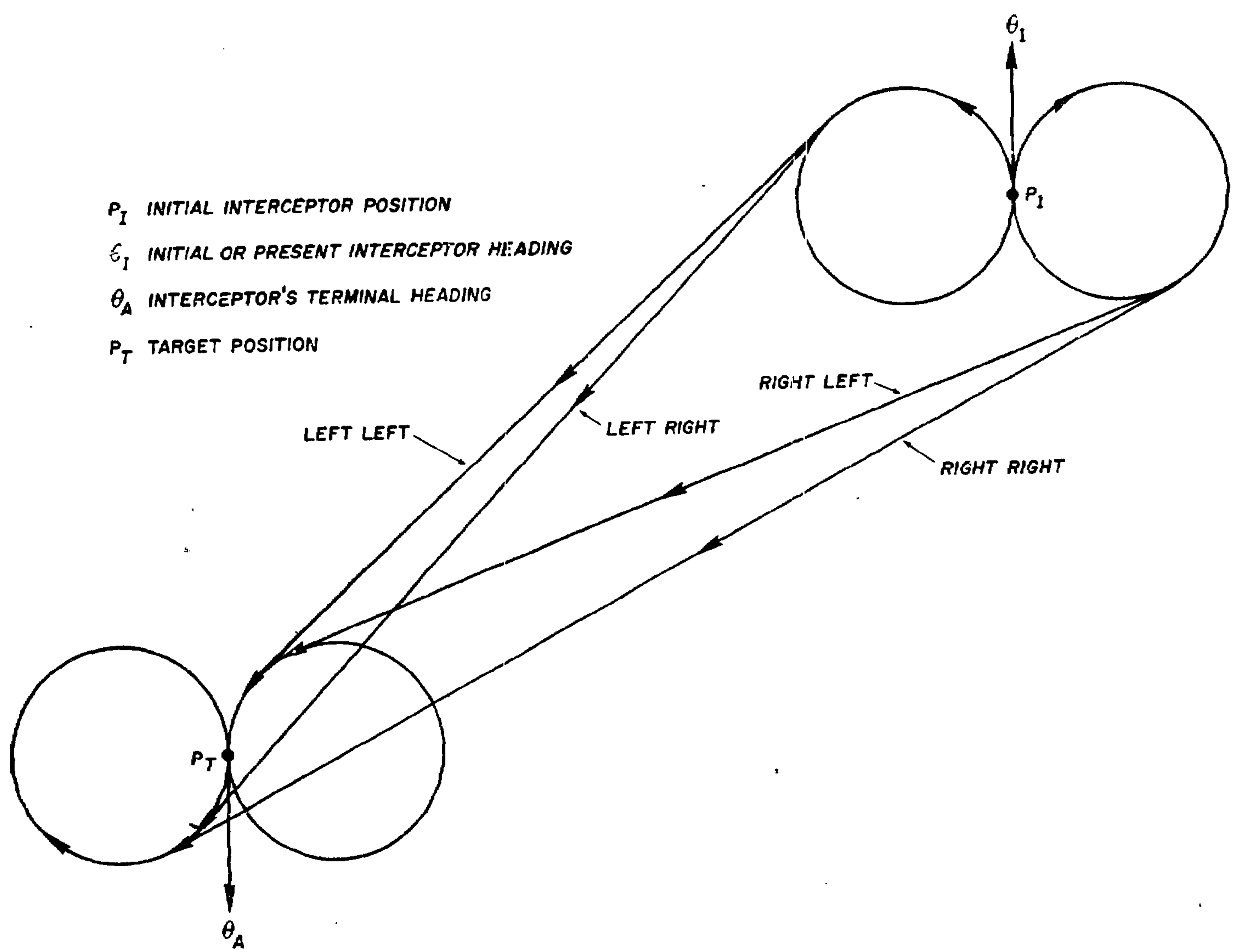

Figure 23 DOUBLE TURN INTERCEPTOR PATH

INTERCEPTOR FLIGHT PATH DURING TURNS

To consider the effect of turns on the guidance solutions, it is desirable to obtain the components of the interceptor's positional displacement during a turn. Figure 24 shows the flight path for an interceptor that is performing a right or clockwise turn when the turn radius is R.

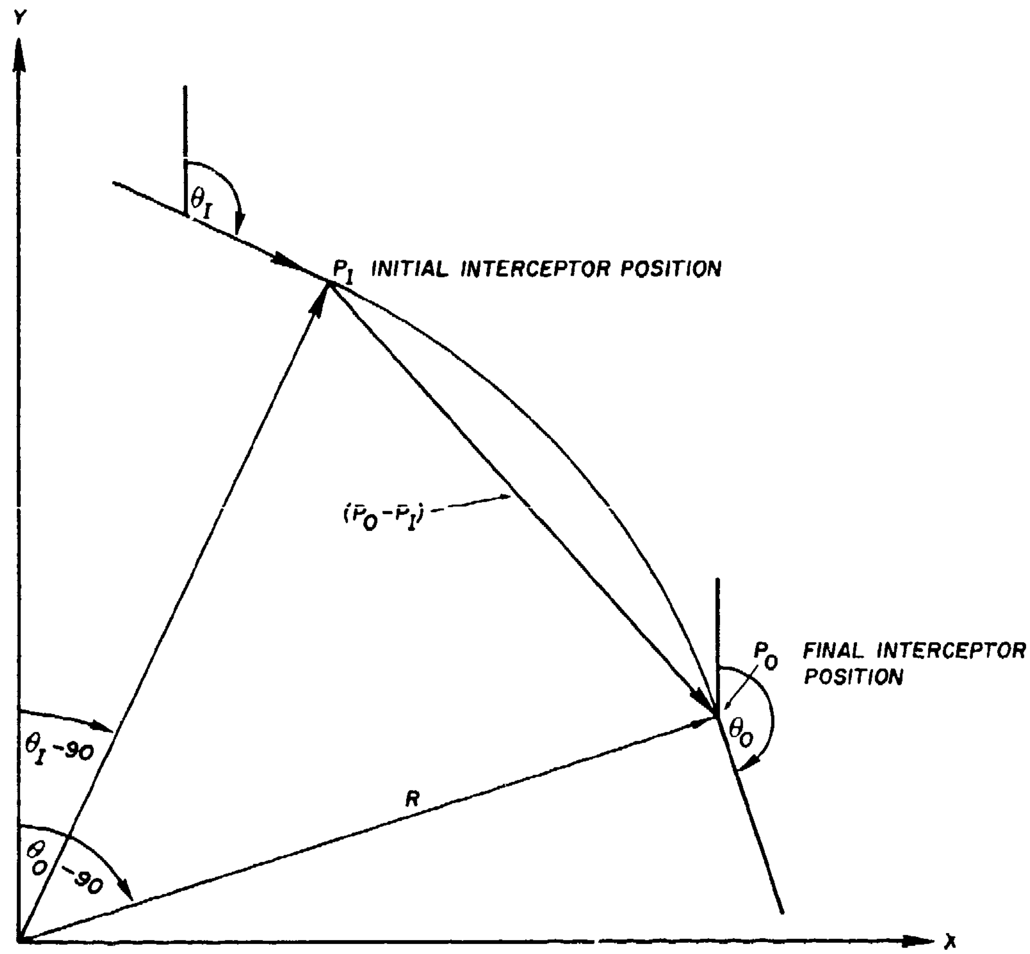

Figure 24 CLOCKWISE TURN

The interceptor position P_I entering the right turn is:

$$P_I = \begin{bmatrix} R \, Sin \, (\theta_I - 90) \\ R \, Cos \, (\theta_I - 90) \end{bmatrix} = \begin{bmatrix} -R \, Cos \, \theta_I \\ R \, Sin \, \theta_I \end{bmatrix} \quad (32)$$

The interceptor position P_0 exiting the right turn is:

$$P_0 = \begin{bmatrix} R \, Sin \, (\theta_0 - 90) \\ R \, Cos \, (\theta_0 - 90) \end{bmatrix} = \begin{bmatrix} -R \, Cos \, \theta_0 \\ R \, Sin \, \theta_0 \end{bmatrix} \quad (33)$$

Thus, the interceptor displacement during a clockwise turn is obtained from:

$$P_0 - P_I = \begin{bmatrix} -R \, Cos \, \theta_0 + R \, Cos \, \theta_I \\ R \, Sin \, \theta_0 - R \, Sin \, \theta_I \end{bmatrix} = \begin{bmatrix} \varepsilon \, R \, (Cos \, \theta_I - Cos \, \theta_0) \\ -\varepsilon \, R \, (Sin \, \theta_I - Sin \, \theta_0) \end{bmatrix} \quad (34)$$

where ε is + 1 for a clockwise turn.

To facilitate calculating the components of the interceptor's positional displacement during a counterclockwise turn, Figure 25 shows the path of an interceptor while performing a left turn.

The interceptor position P_I entering the left turn is:

$$P_I = \begin{bmatrix} R \, Sin \, (\theta_I + 90) \\ R \, Cos \, (\theta_I + 90) \end{bmatrix} = \begin{bmatrix} R \, Cos \, \theta_I \\ -R \, Sin \, \theta_I \end{bmatrix} \quad (35)$$

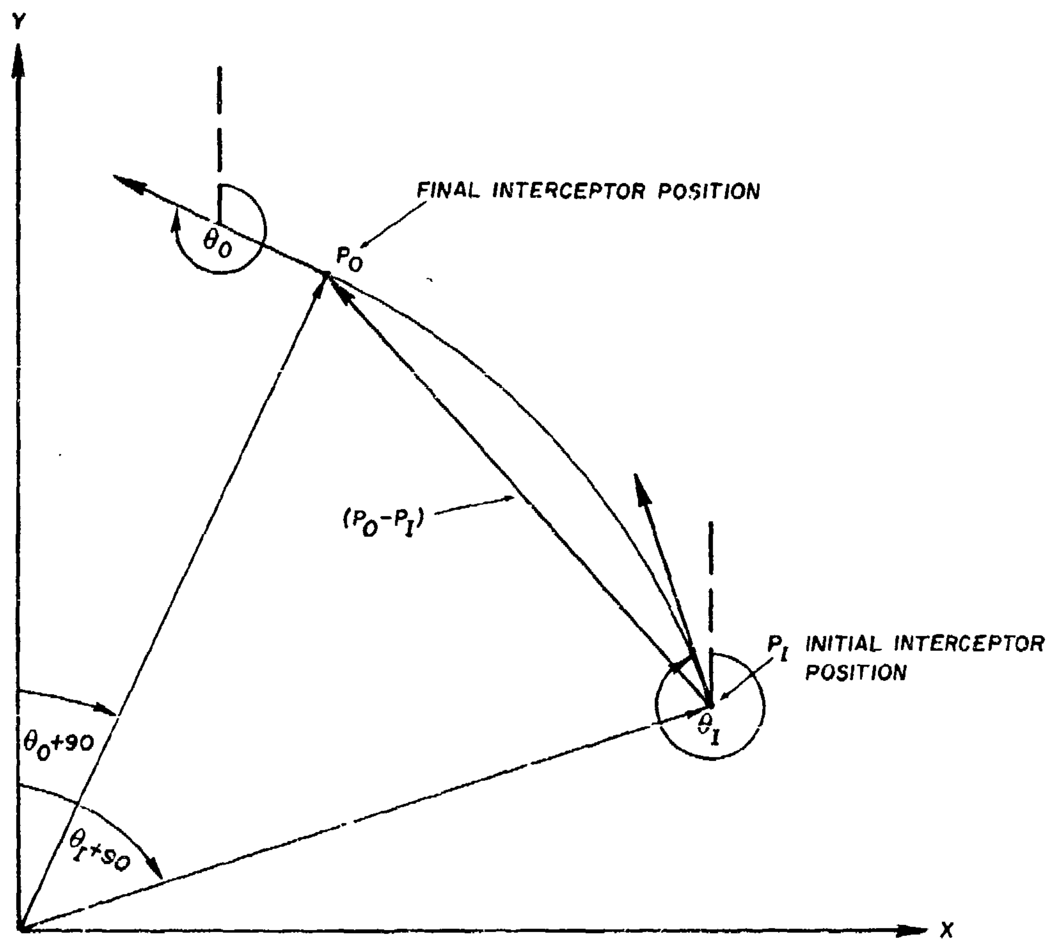

Figure 25 COUNTERCLOCKWISE TURN

The interceptor position P_0 exiting the left turn is:

$$P_0 = \begin{bmatrix} R \sin (\theta_0 + 90) \\ R \cos (\theta_0 + 90) \end{bmatrix} = \begin{bmatrix} R \cos \theta_0 \\ -R \sin \theta_0 \end{bmatrix} \qquad (36)$$

The interceptor displacement during a counterclockwise turn is obtained from:

$$P_0 - P_I = \begin{bmatrix} R \cos \theta_0 - R \cos \theta_I \\ -R \sin \theta_0 + R \sin \theta_I \end{bmatrix} = \begin{bmatrix} \varepsilon R(\cos \theta_I - \cos \theta_0) \\ -\varepsilon R(\sin \theta_I - \sin \theta_0) \end{bmatrix} \qquad (37)$$

where ε is -1 for a counterclockwise turn.

Examination of equations (34) and (37) reveals that they are identical. As a consequence, these equations may be used to calculate the positional displacement of an interceptor due to any turn. The value of ϵ is determined by the direction of turn.

INTERCEPTOR DISPLACEMENT DUE TO TURNS

Given an expression for the interceptor displacement during a turn, the interceptor position is displaced to account for the motion of both the interceptor and the target during the interceptor's initial and final turns. The result is a displaced interceptor position which reduces the relatively complicated double turn intercept problem to the simpler one discussed in Sections III and IV.

It is assumed that the interceptor position in the target coordinate system (X_2,Y_2) has been displaced to (X_3,Y_3) according to equation (31) to account for a variable speed, three dimensional flight path with a terminal segment of length D. If the particular mission does not require these complexities, (X_3,Y_3) will reduce to (X_2,Y_2).

To account for the motion of both the interceptor and the target during the interceptor's initial turn from the present heading θ_I to the command heading θ, the interceptor position (X_3,Y_3) is displaced as follows:

$$
\begin{aligned}
X_I &= X_3 + \varepsilon_1 R_1 \text{ Cos } \theta_I - \varepsilon_1 R_1 \text{ Cos } \theta \\
Y_I &= Y_3 - \varepsilon_1 R_1 \text{ Sin } \theta_1 + \varepsilon_1 R_1 \text{ Sin } \theta - 1_{ti}\rho
\end{aligned}
\tag{38}
$$

where:

X_I, X_I is the interceptor position displaced to account for the initial turn

ε_1 is the direction of the initial turn

R_1 is the initial turn radius

1_{ti} is the length of the initial turn

To compensate for the motion of the interceptor and the target during both the initial turn and the final turn from the command heading θ to the terminal heading θ_A, the interceptor position (X_3,Y_3) is displaced as follows:

$$
\begin{aligned}
X_I &= X_3 + \varepsilon_1 R_1 \text{ Cos } \theta_I - \varepsilon_1 R_1 \text{ Cos } \theta + \varepsilon_2 R_2 \text{ Cos } \theta - \varepsilon_2 R_2 \text{ Cos } \theta_A \\
Y_I &= Y_3 - \varepsilon_1 R_1 \text{ Sin } \theta_I + \varepsilon_1 R_1 \text{ Sin } \theta - \varepsilon_2 R_2 \text{ Sin } \theta + \varepsilon_2 R_2 \text{ Sin } \theta_A - 1_t\rho
\end{aligned}
\tag{39}
$$

where: X_I, Y_I is the interceptor position displaced to account for the initial and final turns

ε_2 is the direction of final turn

R_2 is the final turn radius

l_t is the length of both the initial and final turns

Equations (39) are based upon the assumption that the interceptor travels at the cruise speed $|V_I|$ during the initial and final turns.

Equations (39) may be rewritten in terms of the auxiliary variables X_4, Y_4 and d as:

$$\begin{aligned} X_I &= X_4 - d \cos \theta \\ Y_I &= Y_4 + d \sin \theta - l_t \rho \end{aligned} \tag{40}$$

where:

$$\begin{aligned} X_4 &= X_3 + \varepsilon_1 R_1 \cos \theta_I - \varepsilon_2 R_2 \cos \theta_A \\ Y_4 &= Y_3 - \varepsilon_1 R_1 \sin \theta_I + \varepsilon_2 R_2 \sin \theta_A \\ d &= \varepsilon_1 R_1 - \varepsilon_2 R_2 \end{aligned} \tag{41}$$

$$\varepsilon_1 = \begin{cases} +1 & \text{for a right initial turn} \\ -1 & \text{for a left initial turn} \end{cases}$$

$$\varepsilon_2 = \begin{cases} +1 & \text{for a right final turn} \\ -1 & \text{for a left final turn} \end{cases}$$

The length of the initial and final turns l_t in equation (40) is calculated from:

$$l_t = \frac{2\pi}{360} \left\{ \left[\epsilon_1 R_1 (\theta-\theta_I) \right] \text{Mod } 360 + \left[\epsilon_2 R_2(\theta_A-\theta) \right] \text{Mod } 360 \right\} \quad (42)$$

GUIDANCE SOLUTION FOR DOUBLE TURN MISSION

Having presented the requisite preliminary material, a procedure to determine the guidance solution for the double turn intercept mission will be described. The elements of the guidance solution required beyond those determined from the profile and tactic selection processes are the command heading and the time and path length to the point of closest approach.

For a particular choice of initial and final turns (ϵ_1, ϵ_2) and with knowledge of the interceptor turn radii (R_1, R_2) as well as interceptor position (X_3, Y_3), present heading (θ_I) and terminal heading (θ_A), the variables X_4, Y_4 and d are calculated by equation (41). However, for the double turn intercept mission (X_I, Y_I) as defined in equation (40) cannot be calculated directly since these parameters are dependent upon the command heading θ. Thus, obtaining a guidance solution by the technique of Figure 16, which required the computation of X_I, Y_I by equation (31), is inappropriate for the double turn intercept mission.

Path Length and Time to Interception

To obtain the path length to the point of closest approach for the double turn intercept mission, the definition of the path length L_m from Section III will be considered. For convenience, L_m as defined in equation (11) is restated below:

$$L_m = \frac{1}{\gamma^2} \left[- X_I \sin\theta - Y_I \cos\theta + \rho Y_I \right] \quad (43)$$

For the double turn intercept mission with provision for a variable speed, three dimensional flight path that may include a terminal segment, (X_I, Y_I) as defined from equation (40) may be rewritten:

$$X_I = X_4 - d \cos \theta$$

$$Y_I = d_1 + d \sin \theta \qquad (44)$$

where:

$$d_1 = Y_4 - \rho d$$

Before proceeding, it is desirable to restate that equations (44) implicitly assume that the interceptor has traversed the initial and final turns as well as the terminal segment and that the interceptor has remained motionless for the time period ΔT. Equations (44) are based upon the further assumption that the target has continued its flight path for the time required by the interceptor to accomplish these maneuvers. In essence, then, the interceptor has been displaced from the coordinates (X_2, Y_2) to (X_I, Y_I) by equation (44) to account for the motion of both the interceptor and the target for any combination of desired interceptor maneuvers.

Substituting (X_I, Y_I) from equation (44) into equation (43) we obtain:

$$L_m = \frac{1}{\gamma^2} \left[(-X_4 + \rho d) \sin \theta + d_1 (\rho - \cos \theta) \right] \qquad (45)$$

Equation (45), which yields the path length to the point of closest approach is simply a more complicated form of equation (43). Equation (45) is applicable to intercept problems that include any combination of an initial turn, a final turn, climbs, descents, variable interceptor speeds and a terminal segment. Inasmuch as the interceptor position has been displaced to account for the motion of both the interceptor and the target during any combination of these interceptor maneuvers, equation (45) represents the path length from the completion of the initial turn to the beginning of the final turn. In other words L_m, as defined by equation (45), is the distance which the interceptor travels along its command heading.

The time required by the interceptor to traverse the distance between the completion of the initial turn and the beginning of the final turn is:

$$t_m = \frac{L_m}{|V_I|} \qquad (46)$$

Assuming the interceptor traverses the final turn and the terminal segment at a speed $|V_I|$, the time in turns and the time required by the interceptor to attain the rollout point are:

$$t_t = \frac{l_t}{|V_I|}; \; t_{RO} = \frac{L_m + l_t}{|V_I|} \qquad (47)$$

The time-to-interception designated t_I is the sum of the time-to-rollout t_{RO}, the time in the terminal segment and the time ΔT that the interceptor is assumed to have remained motionless. Therefore, the time-to-interception is:

$$t_I = \frac{L_m + l_t + D}{|V_I|} + \Delta T \tag{48}$$

Determination of Command Heading

The procedure for obtaining the minimum time guidance solution for the double turn intercept mission with the provision for interceptor climbs, descents, accelerations, decelerations and a terminal segment is similar to that of previous sections. First, the miss distance equation is solved for the command heading which yields an acceptable separation. Then, the distance to the point of closest approach and the time-to-rollout are obtained from equations (45) and (47) respectively.

Miss Distance Versus Command Heading

To obtain the miss distance equation that is appropriate to the double turn intercept mission with the provision for interceptor climbs, descents, accelerations and decelerations, the miss distance equation for the constant speed, straight line intercept mission as defined by equation (9) will initially be considered. For convenience this equation is restated:

$$d_m = Y_I \operatorname{Sin} \theta - X_I \operatorname{Cos} \theta + \rho X_I \tag{49}$$

To obtain the miss distance equation that is appropriate to the double turn intercept geometry described herein, (X_I, Y_I) from equation (44) are substituted into equation (49). The result is:

$$d_m = d_1 \sin\theta + d_2 \cos\theta + d_3 \qquad (50)$$

where:

$$d_1 = Y_4 - \rho l_t$$

$$d_2 = -X_4 - \rho d$$

$$d_3 = \rho X_4 + d$$

In solving the miss distance equation, each of the two possible turn combinations are considered for intercepts with no prescribed HCA. If a particular HCA is required, each of the four possible turn combinations are considered.

Figure 26 illustrates the miss distance d_m versus interceptor heading θ for values of θ which result in positive times until interception. Values for the parameters used in the calculation of the miss distance d_m from equation (50) are shown in Table V.

Table V

Input Parameters to Miss Distance Calculation

X_2 = 50 mi.	$\lvert V_I \rvert$ = 400 knots	R_2 = 10 mi.
Y_2 = 100 mi.	$\lvert V_T \rvert$ = 500 knots	θ_I = 260 deg.
D = 0 mi.	R_1 = 10 mi.	θ_A = 200 deg.

(X_2, Y_2) is the interceptor position in the target coordinate system. D is the length of the terminal segment. With D equal to zero, interception should occur as the interceptor attains the terminal heading

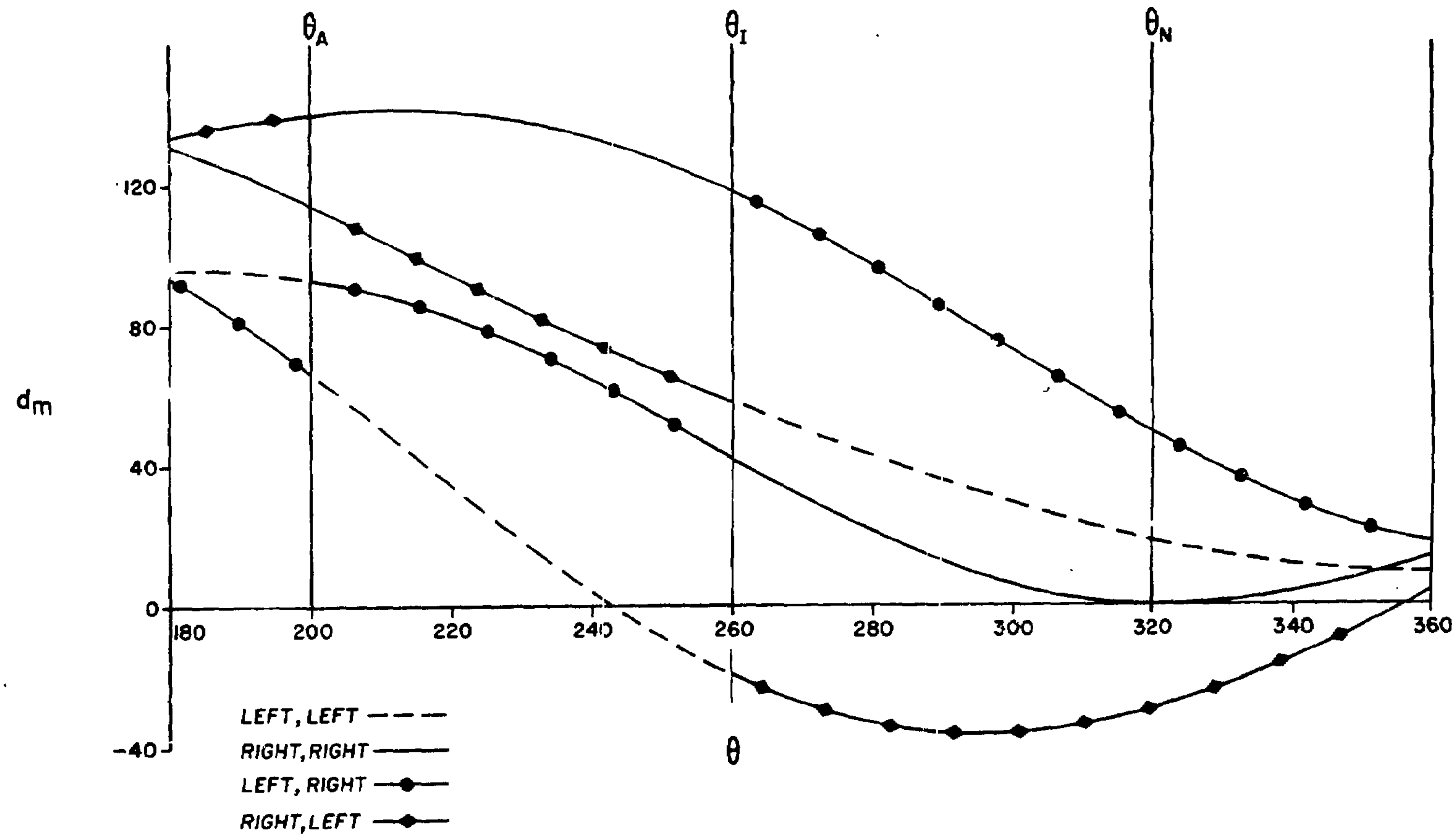

Figure 26 MISS DISTANCE VERSUS HEADING WITH TURNS CONSIDERED

θ_A. $|V_I|$ and $|V_T|$ are the interceptor and target speeds respectively. R_1 and R_2 are the interceptor turn radii for the initial and final turns. θ_I is the current interceptor heading and θ_A is the interceptor's terminal heading.

Examination of Figure 26 shows:

a) Distinct miss distance curves for each of the four turn combinations.

b) Discontinuities in each miss distance curve at the current heading θ_I and the terminal heading θ_A.

c) The left, left turn combination curve results in a zero miss distance at a command heading of approximately 242 degrees.

d) The right, right turn combination curve results in a miss distance of approximately zero at a command heading of 325 degrees.

e) The right, left turn combination curve results in a miss distance of approximately zero at a command heading of 355 degrees.

Table VI shows for each turn combination, the solution to the miss distance equation to the nearest degree, the miss distance, the path length L_m and the distance l_t which the interceptor travels during the initial and final turns.

Table VI

Solutions To Miss Distance Equation

Turn Combination	Command Heading	Miss Distance	Path Length	Distance in Turn
Left, Left	242^0	0	47 mi.	10 mi.
Right, Right	325^0	1.0 mi.	100 mi.	52 mi.
Right, Left	355^0	-.2 mi.	220 mi.	44 mi.
Left, Right	None	N/A	N/A	N/A

With knowledge of the parameters listed in Table V, equation (50) may be solved for the miss distance d_m for any desired value of θ. This technique was used to obtain the data that is graphically portrayed in Figure 26. What is desired however is a rapid approach for obtaining the roots of equation (50); i.e., for d_m approximately zero, what values of the command heading θ satisfy the equation? Unfortunately, values of θ which satisfy equation (50) cannot be obtained directly. The iterative technique described hereafter yields the command heading to the desired degree of accuracy.

Discontinuities and Extremes of the Miss Distance

To facilitate the determination of solutions to equation (50), the heading interval is divided in such a manner that each sub-interval contains no more than one root. This is accomplished by splitting the interval at the discontinuities and extremes of the miss distance function. As stated previously since the extremes of the miss distance function are difficult to locate, those of (d_m/Sin θ) are used instead.

From Figure 26, it is apparent that the miss distance curve has discontinuities at both the interceptor's current heading θ_I and the terminal heading θ_A. This phenomenon is caused by the modular nature of the arithmetic. For example, if a right final turn is being considered and the present interceptor heading is 5 degrees left of the desired terminal heading, a five degree right final turn is required. However, if the present interceptor heading is 5 degrees right of the desired terminal heading, a 355 degree right final turn is required. Consequently, the discontinuities in the miss distance function are induced by the modular nature of the arithmetic.

To determine the additional values of θ at which the heading interval should be split to ensure that each sub-interval contains no more than one root, the extremes of (d_m/Sin θ) will be obtained. From consideration of equation (50), (d_m/Sin θ) is defined by:

$$\left[{}^{d_m}/_{\text{Sin }\theta}\right] = \left[(Y_4 - \rho l_t) - (X_4 + \rho d) \text{ Cot } \theta + \frac{(d + \rho X_4)}{\text{Sin } \theta}\right] \quad (51)$$

The derivative of (d_m/Sin θ) with respect to θ is:

$$\frac{d}{d\theta}\left[\frac{d_m}{\text{Sin } \theta}\right] = -\rho d + \frac{(X_4 + \rho d)}{\text{Sin}^2\theta} - \frac{(\rho X_4 + d) \text{ Cos } \theta}{\text{Sin}^2\theta} \quad (52)$$

since the derivative of l_t is:

$$\frac{dl_t}{d\theta} = \frac{d}{d\theta}\left[\varepsilon_1 R_1 (\theta - \theta_I) + \varepsilon_2 R_2 (\theta_A - \theta)\right] = d \quad (53)$$

Multiplying the right side of equation (52) by $\sin^2\theta$ and setting the left side to zero we obtain:

$$\rho d \cos^2\theta - (\rho X_4 + d)\cos\theta + X_4 = 0 \tag{54}$$

Solving for the two roots of this quadratic equation, we obtain:

$$\theta_1 = \cos^{-1}\left[\frac{1}{\rho}\right] \text{ and } \theta_2 = \cos^{-1}\left[\frac{X_4}{d}\right] \tag{55}$$

The first root of equation (55) implies that two extremes of the miss distance function exist within the heading interval if the interceptor is at a speed disadvantage ($\rho > 1$). When the interceptor is at a speed disadvantage, the extremes of ($d_m/\sin\theta$) due to this condition occur at:

$$\theta_m = \cos^{-1}\left(\frac{1}{\rho}\right) = \cos^{-1}\left(\frac{|V_I|}{|V_T|}\right) \tag{56}$$

and

$$\theta_n = 360^0 - \theta_m$$

The second root of equations (55) implies that two additional extremes of the miss distance function exist within the heading interval if $|X_4|$ is less than $|d|$. These two extremes of the function ($d_m/\sin\theta$) are:

$$\theta_p = \cos^{-1}\left[\frac{X_4}{d}\right] \text{ where } 0 \leq \theta_p \leq 180$$

and (57)

$$\theta_q = 360 - \theta_p$$

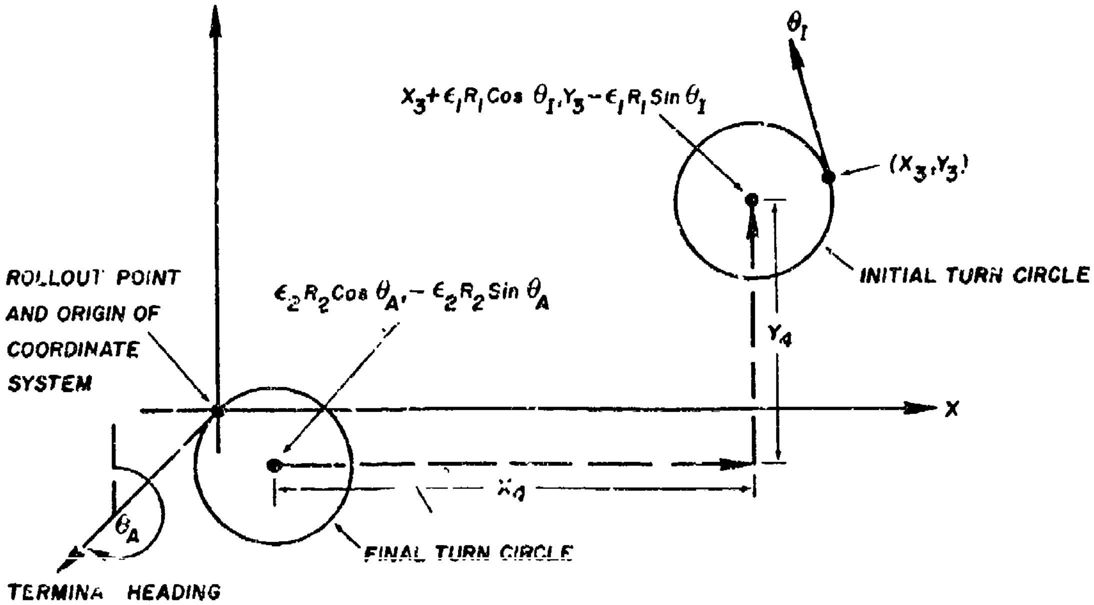

Figure 27 X_4, Y_4 COORDINATES

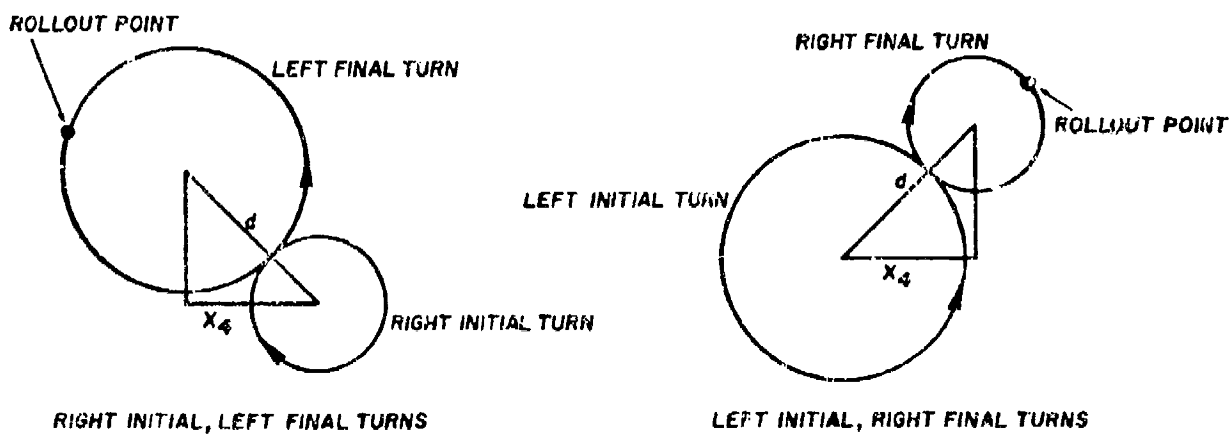

Figure 28 RIGHT LEFT AND LEFT RIGHT TURN COMBINATIONS

Equations (57) are used in the following section and in Appendix B to locate the range of interceptor headings within which a physically realizable solution to the guidance equations might exist.

Range of Headings

To determine the range of headings that should be examined to find a root of the miss distance function, an understanding of the implications of equation (57) is required. The two terms therein are X_4 and d; thus, a physical interpretation of each of these parameters as well as the Y_4 parameter will be presented.

X_4, Y_4 and d are defined from equation (41) as:

$$
\begin{aligned}
X_4 &= X_3 + \epsilon_1 R_1 \cos \theta_I - \epsilon_2 R_2 \cos \theta_A \\
Y_4 &= Y_3 - \epsilon_1 R_1 \sin \theta_I + \epsilon_2 R_2 \sin \theta_A \qquad (58) \\
d &= \epsilon_1 R_1 - \epsilon_2 R_2
\end{aligned}
$$

If the final turn circle is located in such a manner that the interceptor's roll-out point occurs at the origin of the coordinate system and at the proper terminal heading, the coordinates (X_4,Y_4) represent the position of the center of the initial turn circle with respect to the center of the final turn circle. Figure 27 depicts the situation.

For a left right or for a right left turn combination, d may be interpreted as the distance from the center of one to the center of the other turn circle when the circles are externally tangent to one another. Figure 28 shows d and X_4 for right left and for left right turns.

For a left left or for a right right turn combination, d may be interpreted as the distance from the center of one to the center of the other turn circle when the circles are internally tangent to one another. Figure 29 shows d and X_4 for left left and for right right turns.

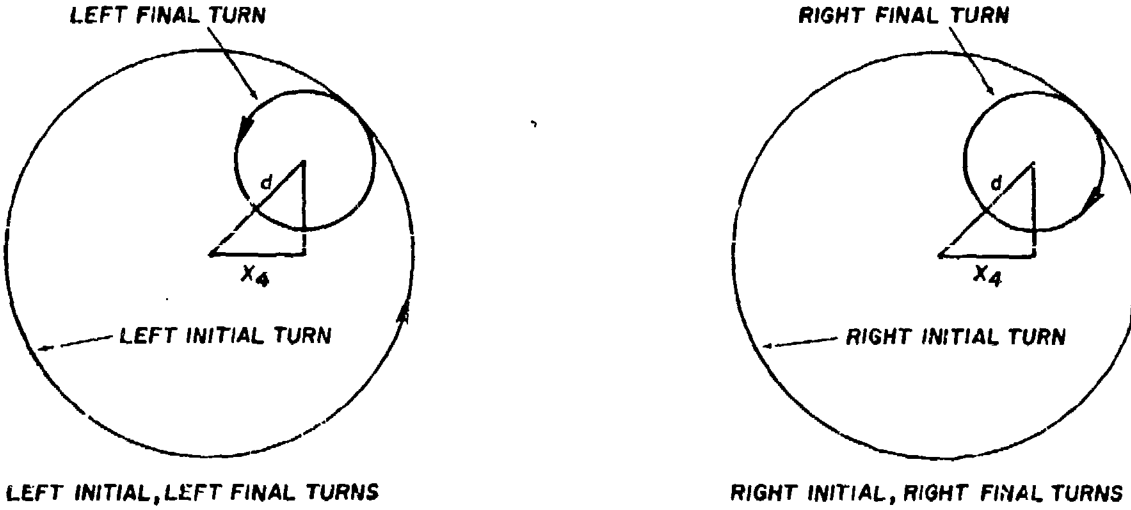

Figure 29 LEFT LEFT AND RIGHT RIGHT TURN COMBINATIONS

Having briefly discussed the physical interpretation of the parameters X_4,Y_4 and d, we now present the range of headings to be examined to determine the roots of the miss distance function for varying interceptor and target geometries. The geometrical conditions and the associated heading interval to be examined in seeking the command heading that will result in interception are presented in Table VII.

Table VII

Heading Interval

Geometrical Conditions	Interval
a) $\|X_4\| \geq \|d\|$ and $X_4 \leq 0$	0 to 180
b) $\|X_4\| \geq \|d\|$ and $X_4 > 0$	180 to 360
c) $\|X_4\| < \|d\|$ and $d > 0$	0 to θ_p and 180 to $(360-\theta_p)$
d) $\|X_4\| < \|d\|$ and $d < 0$	θ_p to 180 and $(360-\theta_p)$ to 360

Conditions a) and b) of Table VII imply that when the interceptor turn circles are adequately separated, the command heading that will result in interception must be within the appropriate half of the 360 degree interval. Otherwise the interceptor would be flying away from the target.

Figure 30 shows a situation where the interceptor's initial and final turn circles are sufficiently separated and where X_4 and Y_4 are positive. From Figure 30, it is apparent that to transition from the initial onto the final turn circle, the interceptor must assume a command heading θ between 180 and 360 degrees as shown in Table VII. The heading θ which results in a miss distance of zero from within this interval is shown in Figure 30.

Conditions c) and d) of Table VII occur when the interceptor's initial and final turn circles are sufficiently proximate so that the

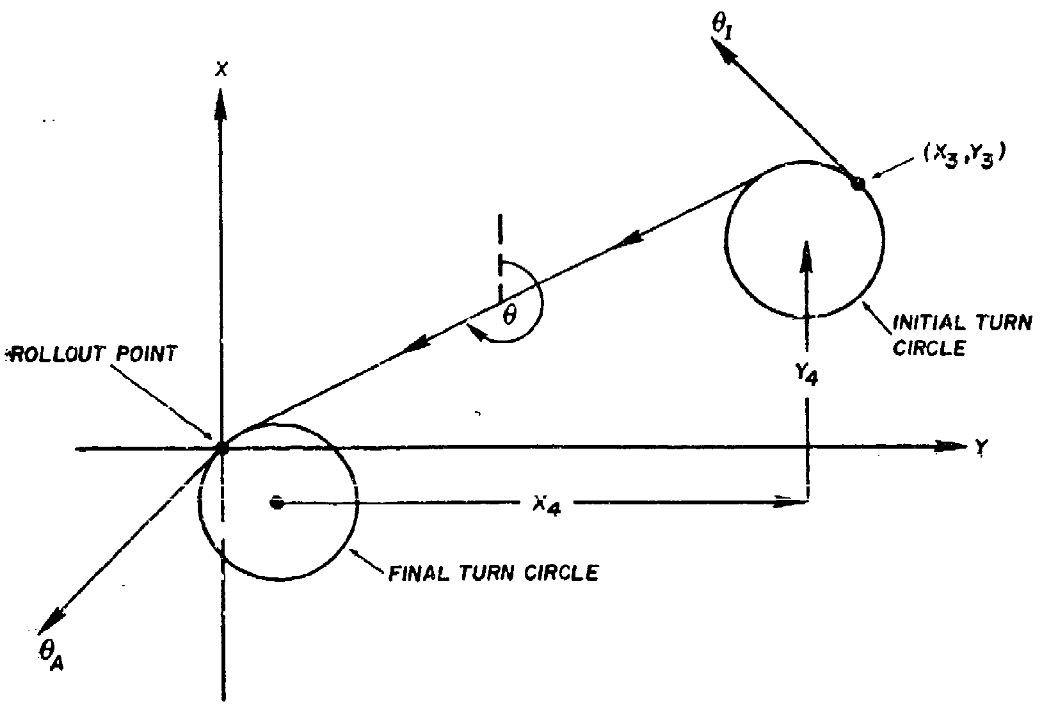

Figure 30 TURN CIRCLES NON PROXIMATE

interceptor must temporarily proceed away from the target flight path in order to proceed from the initial onto the final turn circle. Figure 31 illustrates a geometry wherein the interceptor's initial and final turn circles are proximate and the directions of the initial and final turns are right, left respectively ($d > 0$). From examination of Figure 31, it is apparent that $|X_4|$ is less than $|d|$ both for the initial and for the displaced initial turn circle that is shown via dashed lines. When the interceptor's actual initial turn circle is the dashed circle, the proper command heading to transition from the initial onto the final turn circle is θ_p. If the actual initial turn

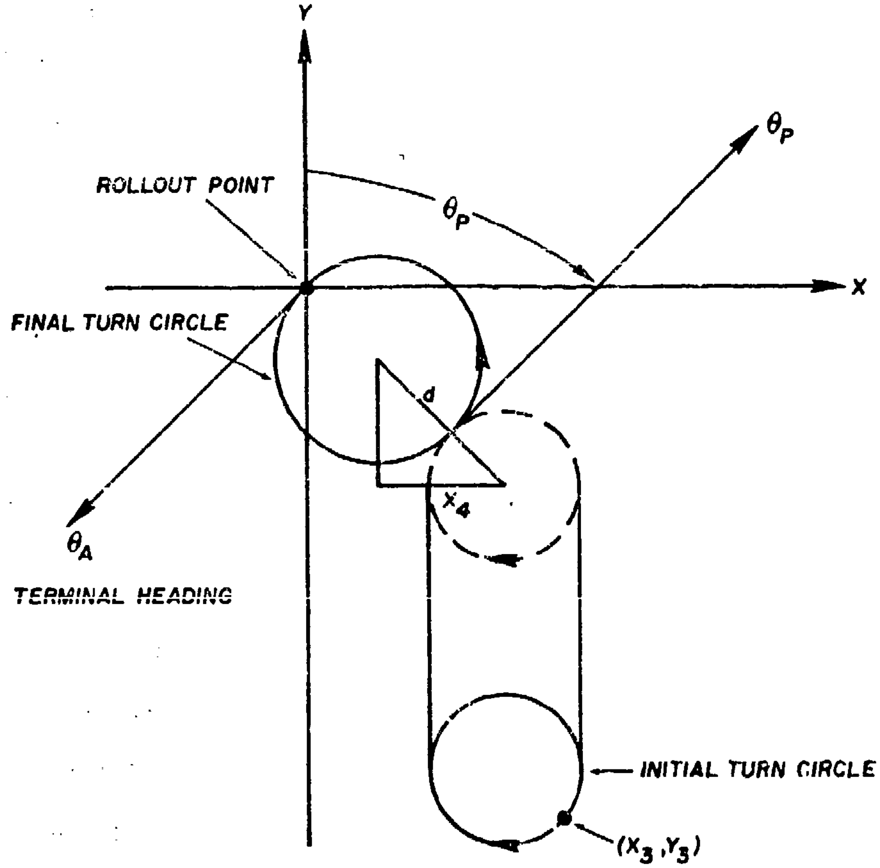

Figure 3i TURN CIRCLES PROXIMATE

circle is located farther from the dashed circle but parallel to the Y axis in a southerly direction, the appropriate command heading to transition from the initial onto the final turn circle approaches zero degrees. Thus, for $|X_4| < |d|$ and $d > 0$, the appropriate heading interval is 0 to θ_p, as shown in Table VII.

Reasoning similar to that used in conjunction with Figures 30 and 31, is employed in Appendix B to obtain the heading interval to be examined for each geometry and turn combination. The resultant heading intervals are consistent with Table VII.

Command Heading Solution

Having determined the discontinuities and extremes of the miss distance function as well as the heading interval to be examined, a procedure to obtain the command heading for missions which include any combination of initial and final turns, climbs, descents, variable interceptor speeds and a prescribed HCA may be presented.

Figure 32 illustrates a rapid procedure for obtaining the minimum time guidance solution. This procedure determines not only the command heading but also the direction of the initial turn, the time and path length to the rollout point and for intercepts with a prescribed HCA the direction of the final turn.

Referring to Figure 32, θ_m and θ_n are calculated from equation (56). The directions of the initial and final turns are initially chosen as clockwise. X_4, Y_4, d_2 and d_3 are obtained from equations (41) and (50). The appropriate interval to be examined in order to determine the command heading is obtained from Table VII. If the interceptor is

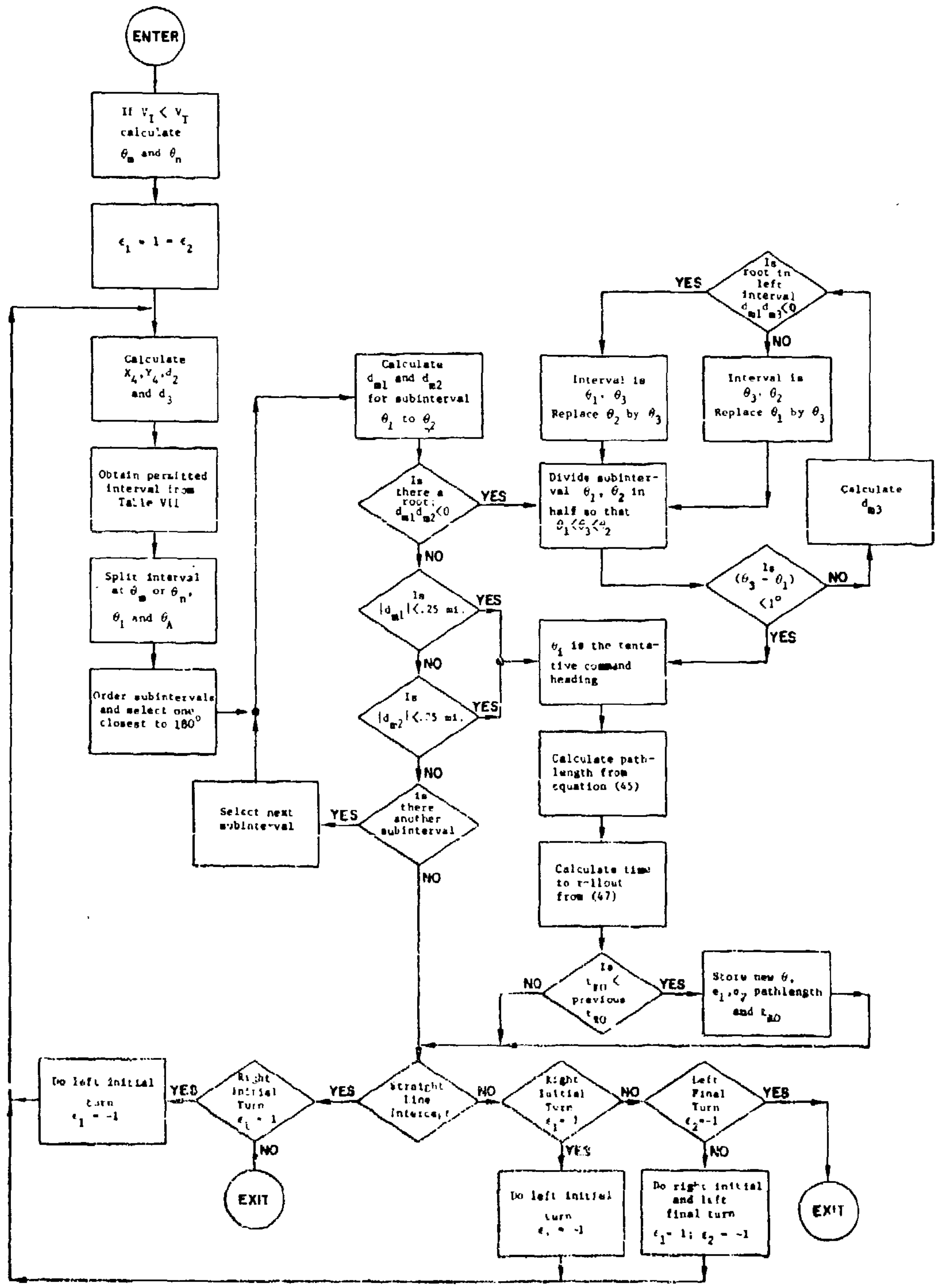

Figure 32 GENERAL ROOT DETECTION TECHNIQUE

at a speed disadvantage so that $\rho > 1$, the heading interval containing either θ_m or θ_n is subdivided in two at that point. Note that both θ_m and θ_n cannot lie within a heading interval since they are separated by 180 degrees. If the interval contains points of discontinuity at the current heading θ_I or at the terminal heading θ_A for a double turn geometry, the interval is further subdivided at these headings.

For a particular turn combination, there are up to four sub-intervals within the heading interval. These are defined by the minimum heading for the interval, θ_m or θ_n when the interceptor is at a speed disadvantage, the present heading θ_I, the terminal heading θ_A, and the maximum heading for the interval.

Basic to the procedure of Figure 32 is an ordering of sub-intervals for a particular turn combination so that sub-intervals with headings closer to 180 degrees are examined sequentially. For example, if the heading interval is 0 to 90 degrees, sub-intervals should be examined starting at 90 degrees and proceeding toward the sub-interval which contains 0 degrees as an endpoint. For, in the target coordinate system, any solution to the miss distance equation with a command heading closer to 90 degrees would result in a shorter time-to-interception than one proximate to 0 degrees.

Having obtained the heading interval, having subdivided the interval at its discontinuities and extremes, having calculated X_4, Y_4, d_2 and d_3 and having ordered the sub-intervals, the values of the miss distance at the endpoints θ_1 and θ_2 for the initial sub-interval are evaluated from equation (50). There is a root between θ_1 and θ_2 if $d_{m1}\ d_{m2} < 0$ where the second subscript indicates to which value of θ the quantity d_m corresponds. If $d_{m1}\ \ d_{m2}$ is not negative and if neither $|d_{m1}|$ or $|d_{m2}|$ is close to zero, the next sub-interval for the selected turn combination is examined.

If $d_{m1}\ d_{m2} < 0$, the sub-interval from θ_1 to θ_2 is subdivided as described in Section III and ultimately a tentative command heading is calculated. Similarly, if $d_{m1}\ d_{m2} > 0$ but either $|d_{m1}|$ or $|d_{m2}|$ is approximately zero, a tentative command heading is obtained.

The procedure described in Figure 32 yields at most one tentative command heading for each turn combination. If the interceptor's avionics system does not require a particular HCA, only right and left initial turns are considered. If the intercept requires a particular HCA, all four turn combinations are considered as shown in Figure 32.

Upon obtaining a tentative command heading for a particular turn combination, the path length and time-to-rollout associated with the command heading are calculated from equations (45) and (47) respectively. The tentative command heading θ, the applicable turn directions ε_1 and ε_2, the path length and the time-to-rollout t_{RO} are saved. Upon obtaining a command heading associated with a different turn combination, the time-to-rollout associated with the new solution is compared with the time-to-rollout for the previous solution. If the new solution results in a lesser time-to-rollout, the new solution parameters replace those that were previously saved. When each turn combination has been examined, the directions of the initial and final turns, the path length, the time-to-rollout and the command heading which results in a minimum time-to-interception will have been obtained.

SECTION VI

ACCEPTABILITY OF GUIDANCE SOLUTION

INTRODUCTION

Sections III through V have presented procedures for obtaining guidance solutions to various classes of intercept missions. The resultant guidance solution is based upon the instantaneous position and velocity of the interceptor and target at the time of the guidance calculation.

If the target were to maintain its velocity, and if the interceptor responded to its guidance commands, it would be unnecessary to generate further guidance solutions. Inasmuch as these assumptions are unrealistic, the acceptability of the current guidance solution must be verified periodically using more recent interceptor and target positional and velocity data to ascertain whether a successful intercept will occur with the current guidance parameters.

HEADING ACCEPTABILITY

The acceptability of the current command heading may be tested by either post or pre-computational filtering. With either technique it is undesirable to alter the command heading unless the current heading is significantly in error.

Post-Computational Filtering

Post-computational filtering requires the periodic calculation of the command heading. Upon obtaining a new heading, it is compared with the previously computed command heading. If the difference between the two headings exceeds a threshold, the command heading is accepted. Otherwise, the more recently computed heading is rejected.

Obtaining the command heading generally requires the examination of all appropriate turn combinations. Single turn intercept missions have no HCA requirement; hence, only two turn combinations are examined. For missions with a designated HCA, all four possible turn combinations are generally examined to determine a new command heading. The turn combination and command heading that provide the minimum time-to-intercept constitute the new solution. It is apparent that savings in processing time can be made by computing a new command heading only when it is required rather than at periodic intervals.

Pre-Computational Filtering

Pre-computational filtering requires the periodic determination of the acceptability of the current command heading. The expected separation between the interceptor and the target at their point of closest approach is calculated using the most recent estimates of the interceptor and target positions and velocities and the current command heading. If the expected separation is acceptable, the current command heading is maintained. Otherwise, a new command heading is obtained via the procedure indicated in Figure 16 or 32.

The acceptability of the current command heading is determined by comparing the expected separation with a threshold that is proportional to the length of the interceptor's flight path to intercept point. At large distances from the intercept point, the expected separation between the interceptor and the target at their point of closest approach is permitted to be larger than would be tolerated later in the mission.

From equation (A-4) of Appendix A, the expected separation d_{min} between the interceptor and the target at their point of closest approach is:

$$d_{min} = \frac{d_m}{\gamma} = \frac{d_1 \text{ Sin } \theta + d_2 \text{ Cos } \theta + d_3}{(1 - 2\rho \text{ Cos } \theta + \rho^2)^{\frac{1}{2}}} \tag{59}$$

Values for d_1, d_2, d_3 and ρ are calculated based upon the most recent estimates of the interceptor and target parameters. Then, the expected separation d_{min} is computed using the current command heading θ. The acceptability of the current command heading is tested by the inequality:

$$d_{min} = \frac{d_m}{\gamma} < \text{Max } (A, BL_m) \tag{60}$$

where:

A is approximately 0.75 NM

B is approximately 0.1

Equation (60) compares the expected separation between the interceptor and the target at their point of closest approach with a fraction of the path length L_m. If the separation is less than the maximum of A or a fraction of the path length L_m, the current command heading is maintained. Otherwise, a new command heading is calculated. The parameter A is included in equation (60) to prevent the threshold from becoming unnecessarily small and thereby resulting in needless changes in command heading.

APPENDIX A

RELATIONSHIP BETWEEN MISS DISTANCE AND MINIMUM SEPARATION

The miss distance d_m is defined as the perpendicular distance from the target position to the interceptor flight path measured at the instant the interceptor crosses the target flight path. The minimum separation distance d_{min} is defined as the separation between the interceptor and the target at their point of closest approach. Figure A-1 shows d_m and d_{min} for a geometry where the target is heading northward and the interceptor is heading toward the east. Successive target positions are denoted by T_0, T_1, T_2 and T_3 whereas corresponding interceptor positions are denoted by I_0, I_1, I_2 and I_3.

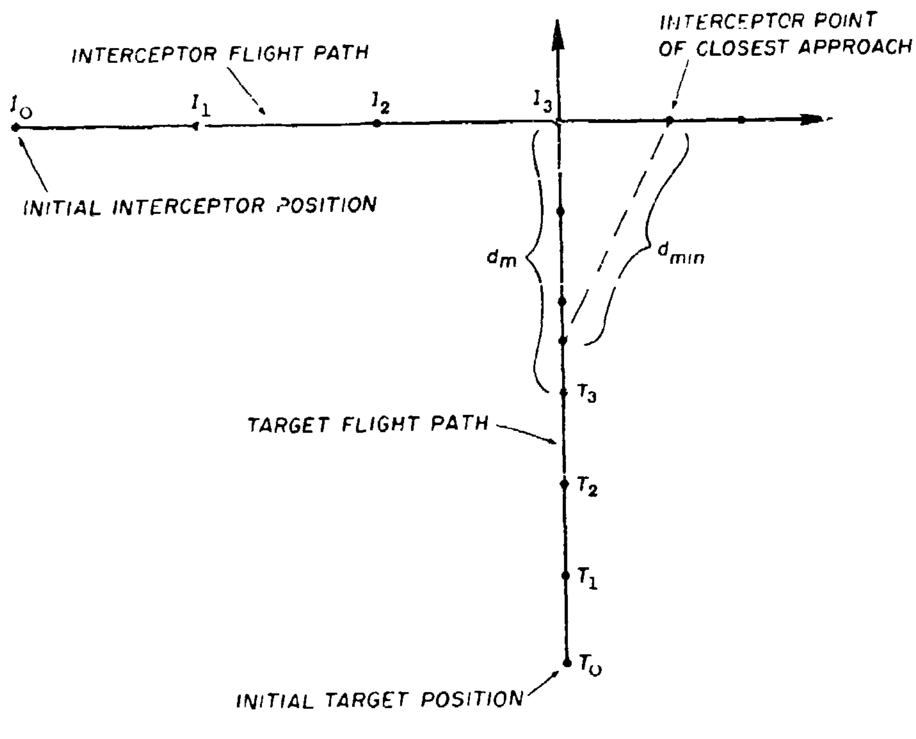

Figure A-1 MISS DISTANCE d_m AND MINIMUM SEPARATION d_{min}

To relate d_{min} and the miss distance d_m, the separation d between the interceptor and the target will be expressed in a coordinate system wherein the target remains stationary at the origin and its motion is attributed to the interceptor. The coordinate system is centered at the target position with the positive Y axis superimposed on the target velocity vector. In obtaining the relationship between d_{min} and d_m, the X and Y subscripts denoting interceptor coordinates will not be used. Also, the terms V_T, V_I and V_I^2 are assumed to be scalars.

$$d^2 = (X + \dot{X}t)^2 + (Y + \dot{Y}t - V_T\ t)^2$$

$$d^2 = (X^2 + Y^2) + 2t\ (X\dot{X} + Y\dot{Y} - Y\ V_T) + t^2\ V_I^2\ \gamma^2 \qquad \text{(A-1)}$$

where:

$$\gamma^2 = (1 - 2\rho \cos\theta + \rho^2) \text{ and } \rho = (V_T/V_I)$$

To obtain the time at which the separation between the interceptor and the target is a minimum, we equate the derivative of d^2 to zero as follows:

$$\frac{\partial d^2}{\partial t} = 2(X + \dot{X}t)\ \dot{X} + 2(Y + \dot{Y}t - V_T t)\ (\dot{Y} - V_T) = 0$$

The time t_m of minimum separation is:

$$t_m = \frac{Y_I\ V_T - X_I\ \dot{X}_I - Y_I\ \dot{Y}_I}{V_I^2\ \gamma^2} \qquad \text{(A-2)}$$

Substituting from equation (A-2) into equation (A-1), we obtain:

$$d_{min}^{\;2} = (X^2 + Y^2) - \frac{2(X\dot{X} + Y\dot{Y} - Y\,V_T)^2}{V_I^{\,2}\gamma^2} + t_m^2\,V_I^{\,2}\gamma^2$$

$$d_{min}^{\;2} = (X^2+Y^2) + \frac{(-X^2\dot{X}^2 - 2XY\dot{X}\dot{Y} + 2XY\dot{X}V_T - Y^2\dot{Y}^2 + 2Y^2\dot{Y}V_T - Y^2V_T^{\,2})}{V_I^{\,2}\gamma^2}$$

$$= (X^2+Y^2) + L$$

$$\text{Define } K = \frac{X^2(\dot{X}^2+\dot{Y}^2) + Y^2(\dot{X}^2+\dot{Y}^2) - 2\dot{Y}V_T(X^2+Y^2) + V_T^{\,2}(X^2+Y^2)}{V_I^{\,2}\gamma^2}$$

$$d_{min}^{\;2} = (X^2+Y^2) + (L+K) - K$$

$$d_{min}^{\;2} = (X^2+Y^2) + \frac{(X^2\dot{Y}^2 - 2XY\dot{X}\dot{Y} + 2XY\dot{X}V_T + Y^2\dot{X}^2 - 2\dot{Y}V_TX^2+V_T^{\,2}X^2)}{V_I^{\,2}\gamma^2} - (X^2+Y^2)$$

$$d_{min} = \frac{Y\dot{X} - X\dot{Y} + XV_T}{V_I\gamma} = \frac{Y_I\ \mathrm{Sin}\ \theta - X_I\ \mathrm{Cos}\ \theta + X\rho}{\gamma} \qquad \text{(A-3)}$$

Considering equations (A-3) and (9), we obtain the relationship:

$$d_{min} = \frac{d_m}{\gamma} \qquad \text{(A-4)}$$

Thus, intercept parameters that result in a zero miss distance correspondingly achieve a collision interception.

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APPENDIX B

COMMAND HEADING INTERVAL

INTRODUCTION

For any intercept geometry, Table B-1 presents the interval to be examined in seeking the command heading that will result in a minimum time-to-interception.

TABLE B-1

HEADING INTERVAL

	GEOMETRY	RANGE OF HEADINGS
(a)	$\lvert X_4 \rvert \geq \lvert d \rvert$ and $X_4 \leq 0$	0 to 180
(b)	$\lvert X_4 \rvert \geq \lvert d \rvert$ and $X_4 > 0$	180 to 360
(c)	$\lvert X_4 \rvert < \lvert d \rvert$ and $d > 0$	0 to θ_p and 180 to $(360 - \theta_p)$
(d)	$\lvert X_4 \rvert < \lvert d \rvert$ and $d < 0$	θ_p to 180 and $(360 - \theta_p)$ to 360

Conditions (a) and (b) of Table B-1 indicate that when the interceptor initial and final turn circles are adequately separated, the command heading which will result in interception must lie within half of the 360 degree heading interval. Otherwise, the heading selected would cause the interceptor to proceed away from the target. The more sophisticated conditions (c) and (d) occur when the interceptor initial and final turn circles are sufficiently proximate so that the interceptor must temporarily point away from the target flight path during a portion of its flight to proceed from the initial onto the final turn circle.

INTERPRETATION OF PARAMETERS

Table B-1 expresses the proximity between the interceptor initial and final turn circles in terms of the parameters X_4 and d. To facilitate comprehension of the table, a brief explanation of the parameters X_4, Y_4 and d is presented.

If the final turn circle is located so that the interceptor's rollout point occurs at the origin of the coordinate system and at the desired terminal heading, the coordinates X_4, Y_4 represent the position of the center of the initial turn circle with respect to the center of the final turn circle. Figure B-1 depicts the situation.

From equation (41), d is defined by $(\varepsilon_1 R_1 - \varepsilon_2 R_2)$. Thus, for the right left turn combination shown in Figure B-1, d equals $(R_1 + R_2)$. For a left right turn combination d equals $-(R_1 + R_2)$. For either si uation, d may be interpreted as the distance from the center of the initial turn circle to the center of the final turn circle when the two circles are externally tangent to one another.

Figure B-2 shows d and X_4 for both right left and left right turn combinations. The right left and left right turns are sufficiently proximate so that the interceptor must point away from the target flight path to proceed from the initial onto the final turn circle.

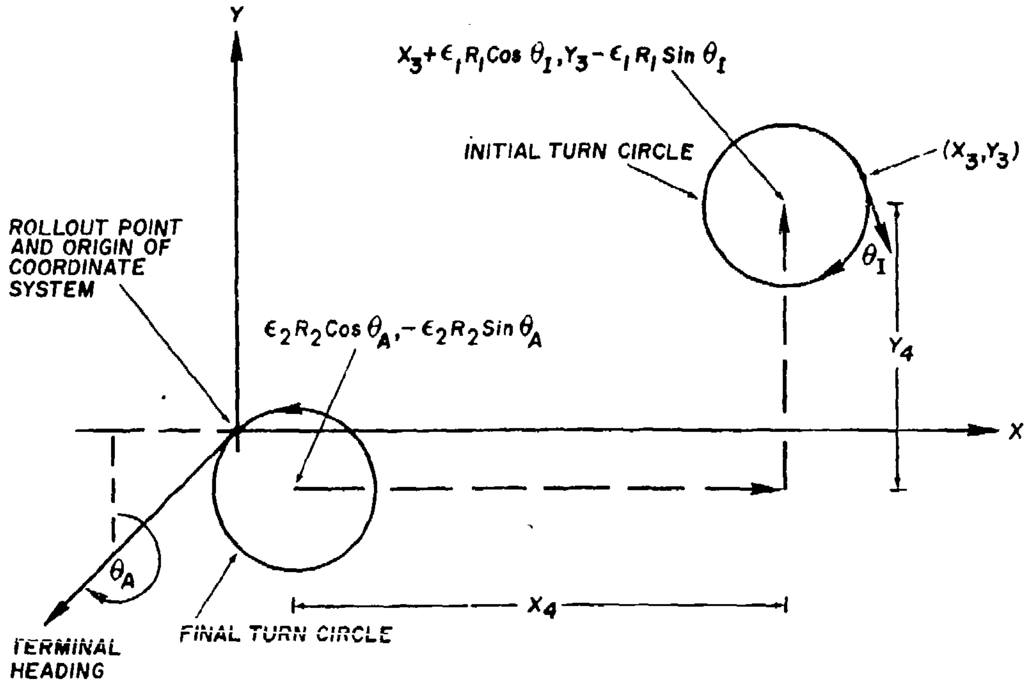


Figure B-1 X_4,Y_4 COORDINATES

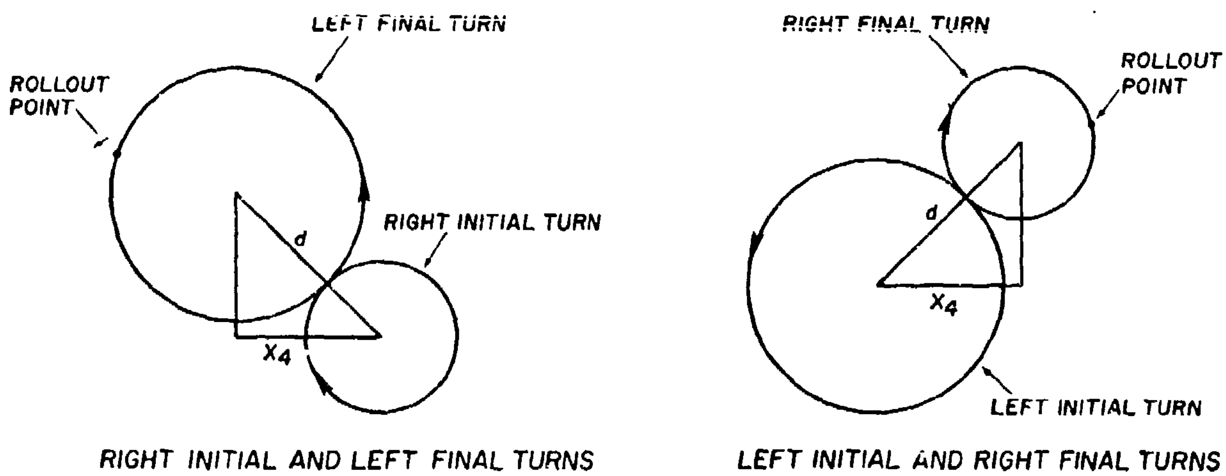


Figure B-2 RIGHT LEFT AND LEFT RIGHT TURNS

For a left left turn combination, d equals $(R_2 - R_1)$ whereas for a right right turn combination d equals $(R_1 - R_2)$. For either situation d may be interpreted as the distance from the center of one turn circle to the center of the other turn circle when one of the turn circles is internally tangent to the other. Figure B-3 shows d and X_4 for left left and for right right turns.

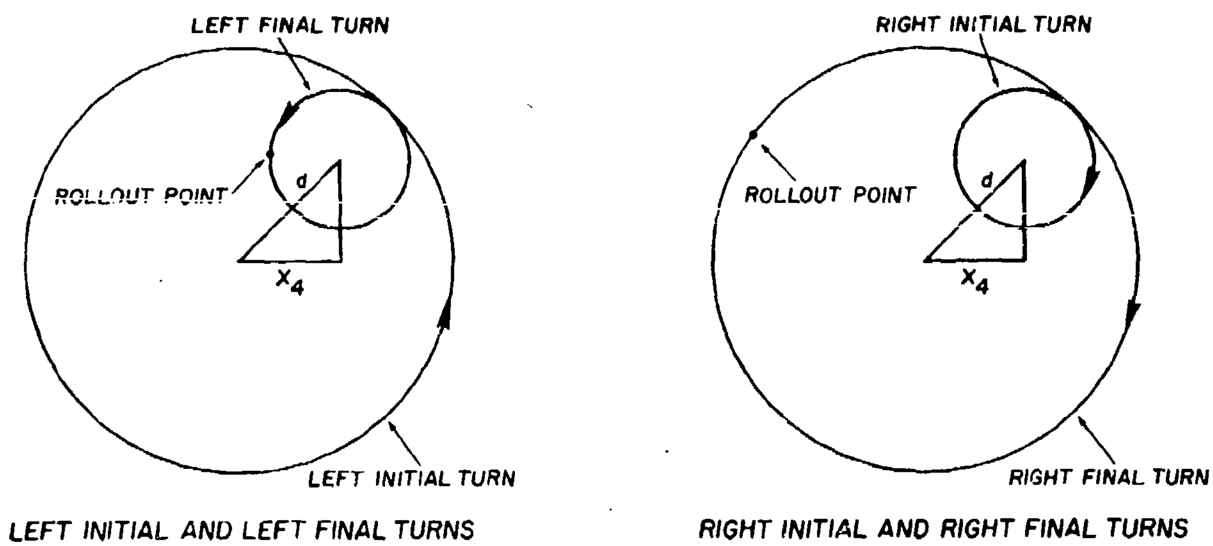


Figure B-3 LEFT LEFT AND RIGHT RIGHT TURN COMBINATIONS

Having defined the parameters X_4, Y_4 and d, the interval to be examined in order to obtain the command heading will be determined for varying intercept geometries when $|X_4| < |d|$. Initially, right left and left right geometries are considered. Thereafter, left left and right right geometries are addressed.

RIGHT LEFT AND LEFT RIGHT TURNS

Considering a right initial followed by a left final turn there are four variations of the parameters X_4 and Y_4. These variations correspond to the initial turn circle being located either to the right or to the left and either above or below the final turn circle.

For a left initial followed by a right final turn, there are four further cases to be considered. The four cases corresponding to right left turns and the four additional cases corresponding to left right turns are presented in Table B-2. The signs of the parameters X_4, d and Y_4 for each of the eight cases are shown.

TABLE B-2

RIGHT LEFT AND LEFT RIGHT TURNS

Case	Turn Directions	Sign of X_4	Sign of d	Sign of Y_4
1	RL[1]	+	+	-
2	RL	+	+	+
3	RL	-	+	-
4	RL	-	+	+
5	LR[1]	+	-	-
6	LR	+	-	+
7	LR	-	-	-
8	LR	-	-	+

(1) RL denotes a right left and LR denotes a left right turn combination

As a prelude to consideration of the intercept geometries of Table B-2, we recall the definition of θ_p from equation (57).

$$\theta_p = \text{Cos}^{-1}\frac{X_4}{d}; \quad 0 \le \theta_p \le 180 \tag{B-1}$$

Right Left Turns

The right left turn combinations corresponding to cases 1 through 4 of Table B-2 will be considered. The interval to be examined in order to determine the command heading that will result in interception will be obtained.

Initial Circle Right and Below Final Circle

Figure B-4 shows a right left turn combination when the initial clockwise turn is to the right and below the final counterclockwise turn. As observed from Figure B-4, if the initial turn circle is displaced parallel to the Y axis until it is externally tangent to the final turn circle, the angle of tangency between the two circles is θ_p. From examination of Figure B-4, it is apparent the $|X_4|$ is less than $|d|$ both for the initial turn circle and for the displaced initial turn circle that is portrayed via dashed lines. If the interceptor's actual initial turn circle is the dashed circle, the appropriate command heading to transition from the initial onto the final turn circle is θ_p. As the actual initial turn circle is located farther away from the dashed circle but parallel and southerly to the Y axis, the appropriate command heading becomes more proximate to zero degrees. Consequently, for the geometry shown in Figure B-4, the heading interval to be examined in order to obtain the command heading to transition from the initial turn circle onto the final turn circle is from 0 to θ_p degrees.

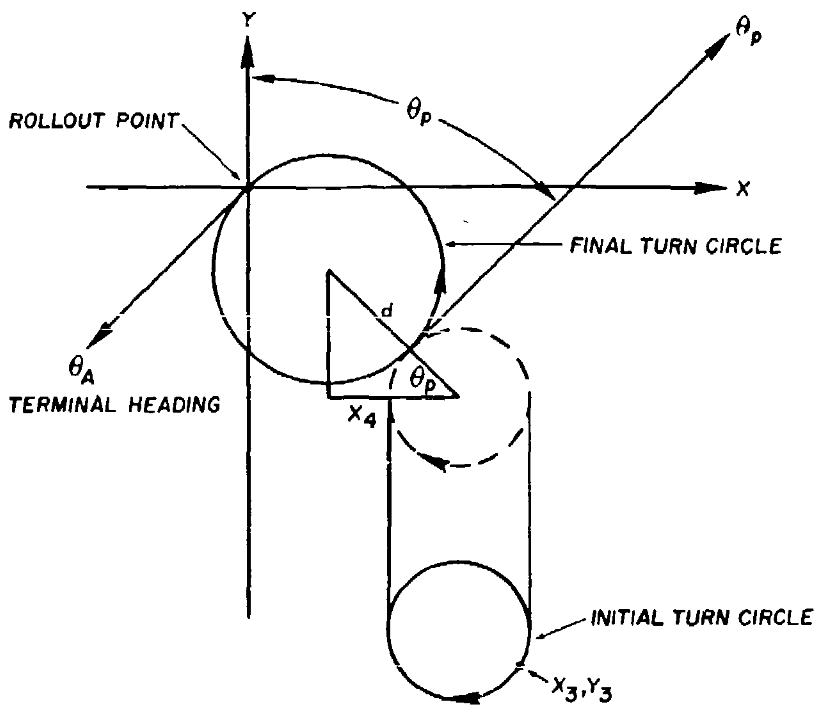


Figure B-4 INITIAL CIRCLE RIGHT AND BELOW FINAL CIRCLE (CASE 1)

Initial Circle Right and Above Final Circle

Figure B-5 shows a right left turn combination when the initial clockwise turn is to the right and above the final counterclockwise turn. In Figure B-5 as well as in the remaining figures within this Appendix, the coordinate system, the rollout point, and the terminal heading will no longer be shown. Consideration of Figure B-5 in a manner analogous to that of Figure B-4 reveals that the appropriate range of interceptor headings that should be examined to obtain the command heading to transition from the initial onto the final turn circle is 180 to $(360-\theta_p)$ degrees.

Initial Circle Left and Below Final Circle

Figure B-6 shows a right left turn combination when the initial clockwise turn is to the left and below the final counterclockwise turn. It is noted that X_4 is a negative quantity; thus, the following definition is introduced:

$$\theta_p' = \cos^{-1}\left[\frac{|X_4|}{|d|}\right] = \begin{cases} \theta_p \text{ when } (X_4/d) \geq 0 \\ (180-\theta_p) \text{ when } (X_4/d) < 0 \end{cases} \tag{B-2}$$

Consideration of Figure B-6 reveals that the appropriate range of headings to be examined is 0 to θ_p degrees.

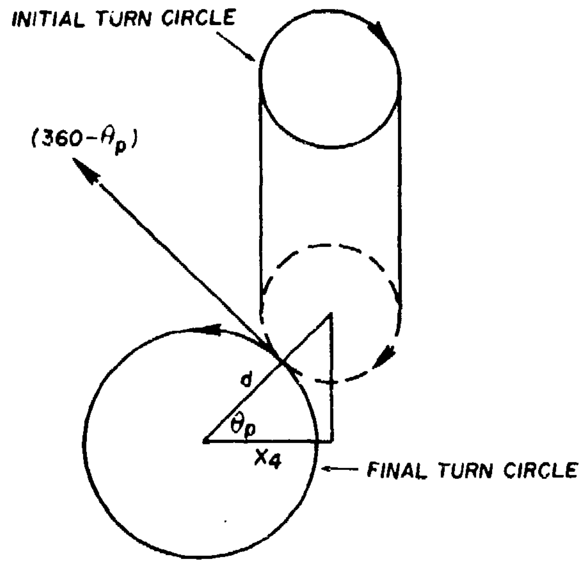


Figure B-5 INITIAL CIRCLE RIGHT AND ABOVE FINAL CIRCLE (CASE 2)

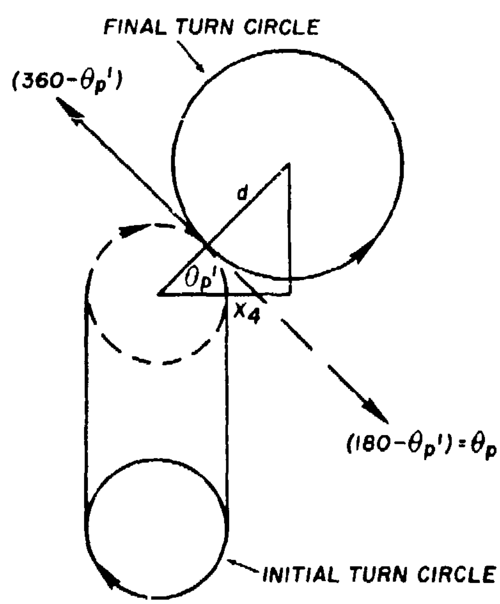


Figure B-6 INITIAL CIRCLE LEFT AND BELOW FINAL CIRCLE (CASE 3)

Initial Circle Left and Above Final Circle

Figure B-7 shows a right left turn combination when the initial clockwise turn is to the left and above the final counterclockwise turn. Examination of Figure B-7 reveals that the appropriate heading interval is 180 to $(360-\theta_p)$ degrees.

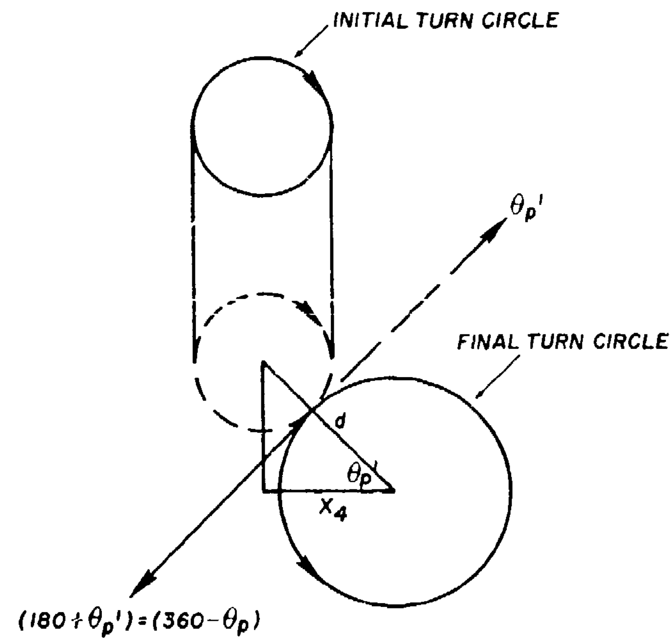
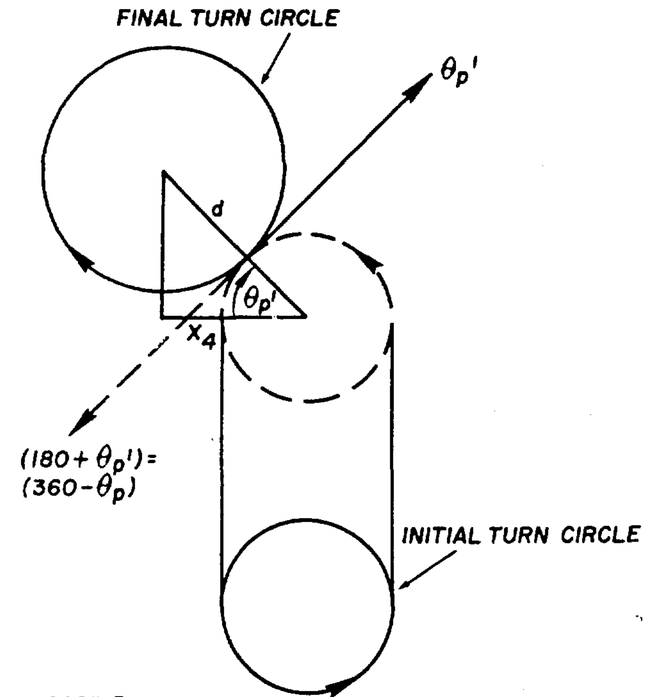


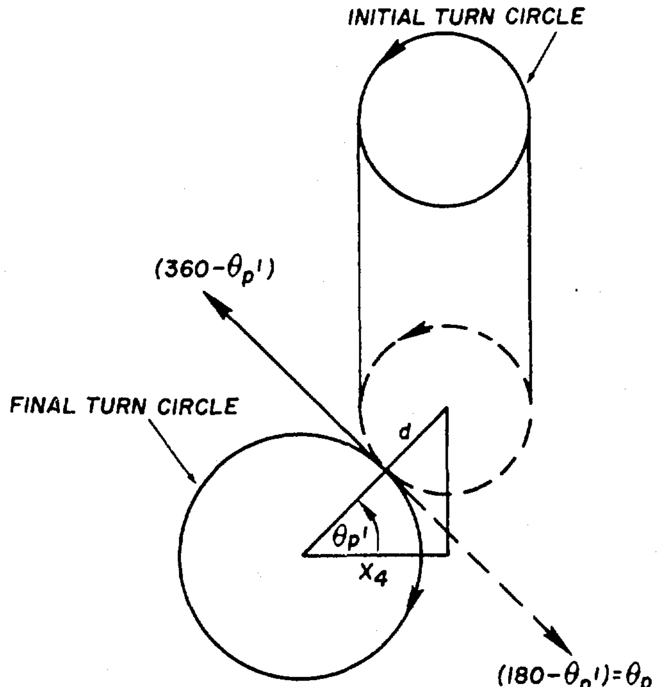
Figure B-7 INITIAL CIRCLE LEFT AND ABOVE FINAL CIRCLE (CASE 4)

Left Right Turns

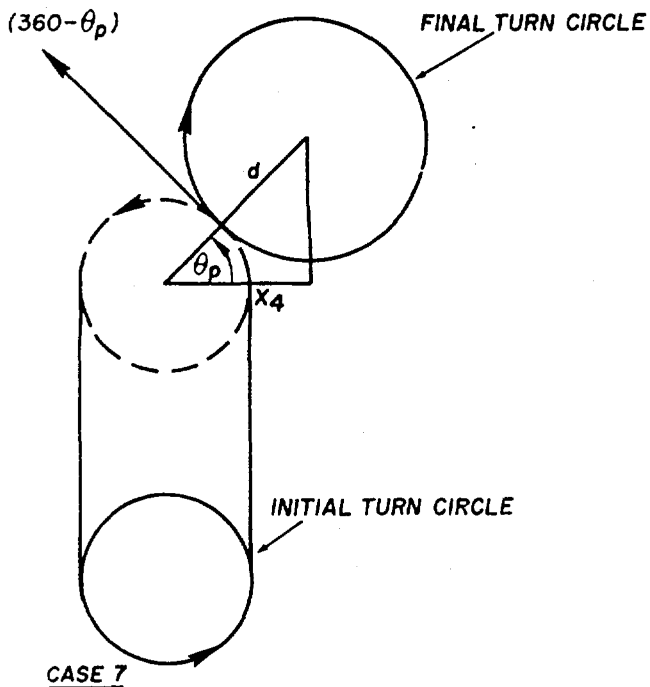
For a left initial turn followed by a right final turn there are four distinct cases to be considered. Figure B-8 shows each of these cases as well as the heading interval to be examined in order to determine the command heading. The rationale for the choice of the heading interval to be examined is apparent from consideration of Figure B-8 in a manner analogous to that used for the right left turns.



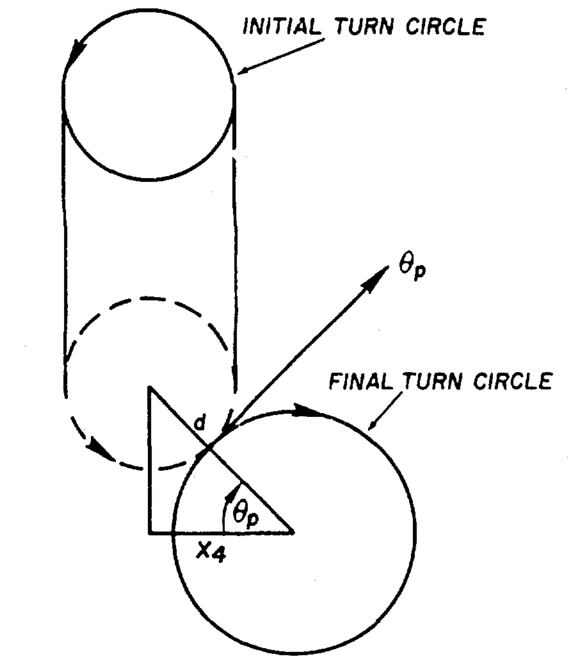
CASE 5
INITIAL CIRCLE RIGHT AND BELOW FINAL CIRCLE
PERMITTED INTERVAL $(360-\theta_p)$ TO 360



CASE 6
INITIAL CIRCLE RIGHT AND ABOVE FINAL CIRCLE
PERMITTED INTERVAL θ_p TO 180



CASE 7
INITIAL CIRCLE LEFT AND BELOW FINAL CIRCLE
PERMITTED INTERVAL $(360-\theta_p)$ TO 360



CASE 8
INITIAL CIRCLE LEFT AND ABOVE FINAL CIRCLE
PERMITTED INTERVAL θ_p TO 180

Figure B-8 LEFT RIGHT TURNS

Heading Interval for Right Left and Left Right Turns

Table B-3 presents the heading interval versus the sign of the parameters X_4, d and Y_4 based upon consideration of Figures B-4 through B-8 and the accompanying discussion.

TABLE B-3

HEADING INTERVAL FOR RIGHT LEFT AND LEFT RIGHT TURNS

Case	Sign of X_4	Sign of d	Sign of Y_4	Heading Interval
1	+	+	-	0 to θ_p
2	+	+	+	180 to $(360-\theta_p)$
3	-	+	-	0 to θ_p
4	-	+	+	180 to $(360-\theta_p)$
5	+	-	.	$(360-\theta_p)$ to 360
6	+	-	+	θ_p to 180
7	-	-	-	$(360-\theta_p)$ to 360
8	-	-	+	θ_p to 180

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LEFT LEFT AND RIGHT RIGHT TURNS

Considering a left initial turn followed by a left final turn, there are eight distinct variations of the parameters X_4, d and Y_4. This assertion may be verified by noting that the initial turn circle may be either to the right or to the left, either above or below, and either smaller or larger than the final turn circle.

Considering a right initial turn followed by a right final turn, there are eight additional cases to be examined. The eight cases corresponding to left left turns and the eight additional cases corresponding to right right turns are shown in Table B-4. The signs of the parameters X_4, Y_4 and d are shown for each of the 16 cases.

Figure B-9 illustrates each of the 16 cases presented in Table B-4. The first four illustrations, designated cases 1 through 4, are left left turns where d is defined by $(R_2 - R_1)$. Since the initial turn circle is smaller than the final turn circle, d is positive. Cases 5 through 8 are left left turn combinations with the initial turn circle larger than the final turn circle so that d is negative. For each of these eight cases, the range of interceptor headings to be examined is stated below the appropriate figure.

Cases 9 through 12 are right right turn combinations with d defined as $(R_1 - R_2)$. The initial turn circle is larger than the final turn circle and d is positive. Cases 13 through 16 are right right turn combinations with the initial turn circle smaller than the final turn circle so d is negative. For each of the eight right right turn combinations, the range of interceptor headings to be examined is stated below the appropriate figure.

TABLE B-4

LEFT LEFT AND RIGHT RIGHT TURNS

Case	Sign of X_4	Turn Directions	Sign of d	Sign of Y_4
1	+	LL[1]	+	-
2	+	"	+	+
3	-	"	+	-
4	-	"	+	+
5	+	"	-	-
6	+	"	-	+
7	-	"	-	-
8	-	"	-	+
9	+	RR[1]	+	-
10	+	"	+	+
11	-	"	+	-
12	-	"	+	+
13	+	"	-	-
14	+	"	-	+
15	-	"	-	-
16	-	"	-	+

[1] LL denotes a left left and RR denotes a right right turn combination.

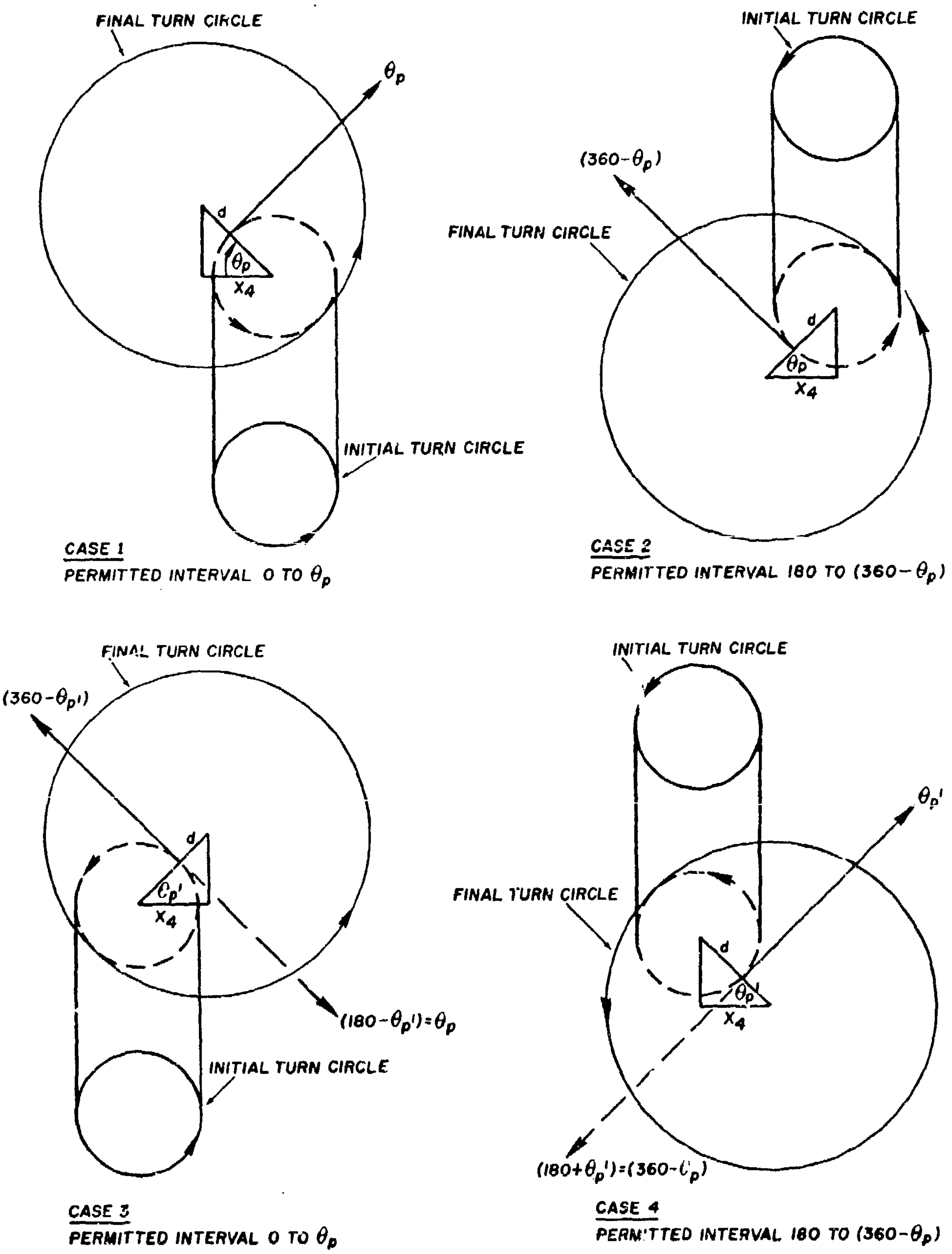
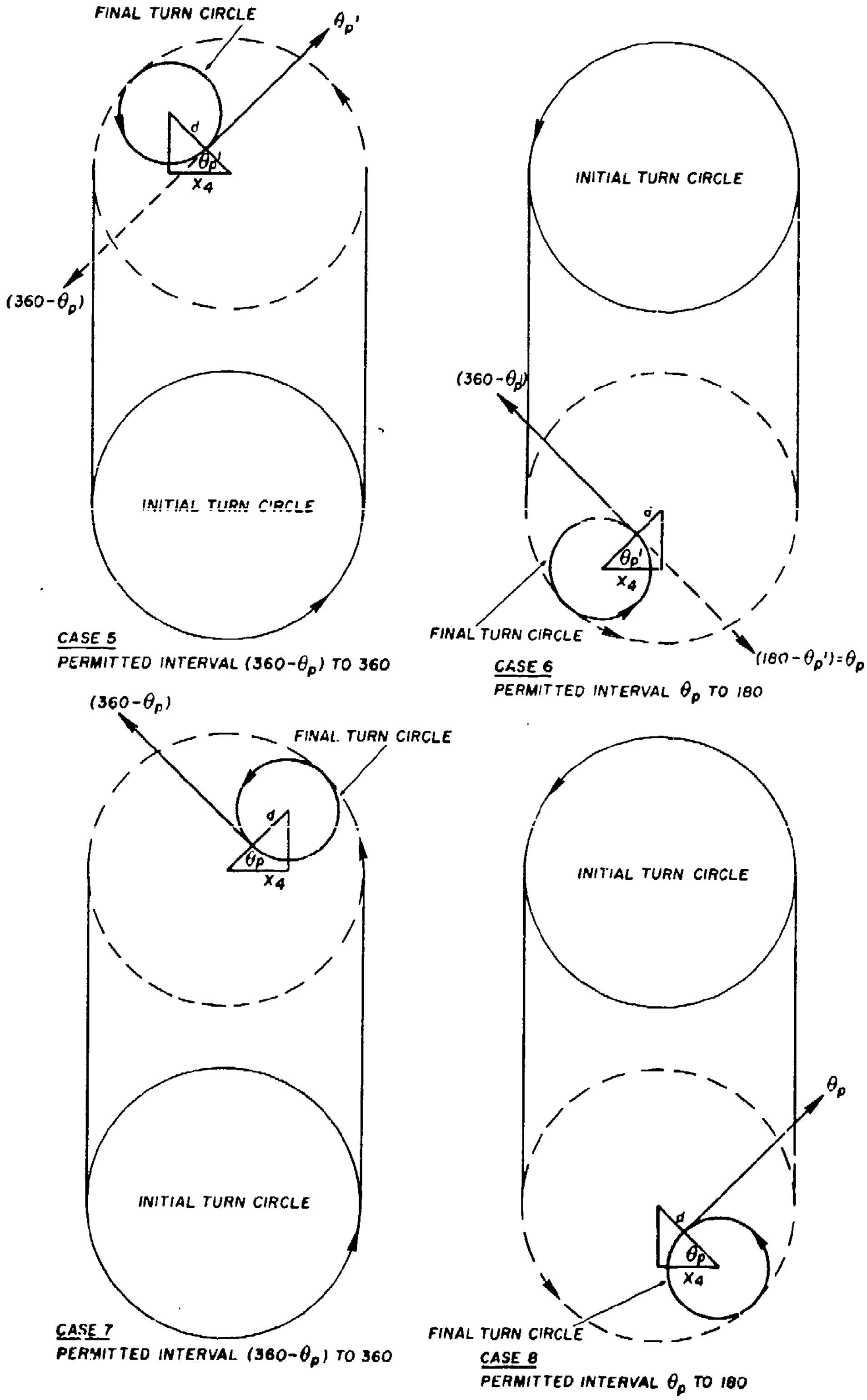


Figure B-9 LEFT LEFT AND RIGHT RIGHT TURNS

Figure B-9 cont'd



CASE 5
PERMITTED INTERVAL $(360-\theta_p)$ TO 360

CASE 6
PERMITTED INTERVAL θ_p TO 180

CASE 7
PERMITTED INTERVAL $(360-\theta_p)$ TO 360

CASE 8
PERMITTED INTERVAL θ_p TO 180

Figure B-9 cont'd

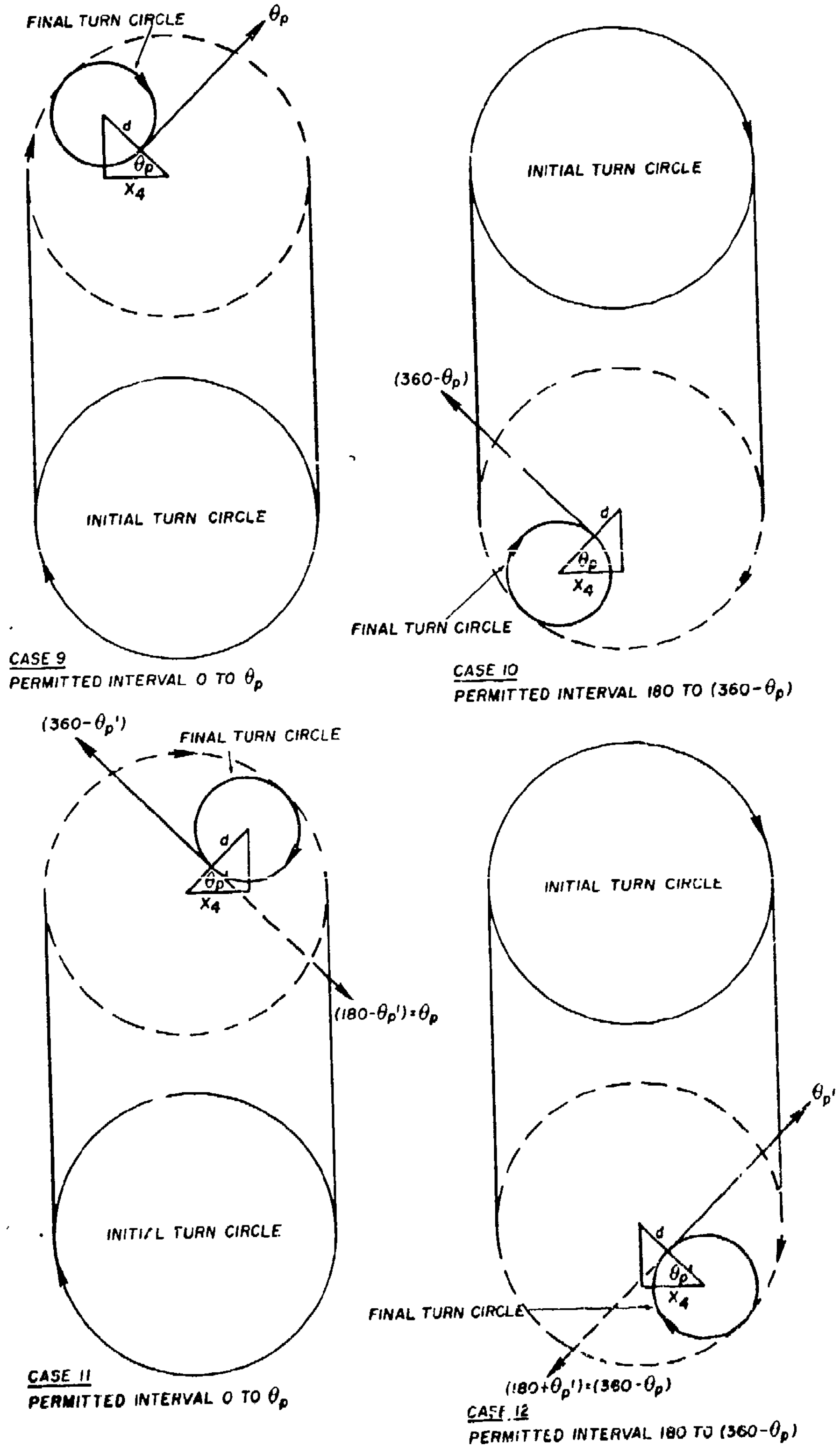
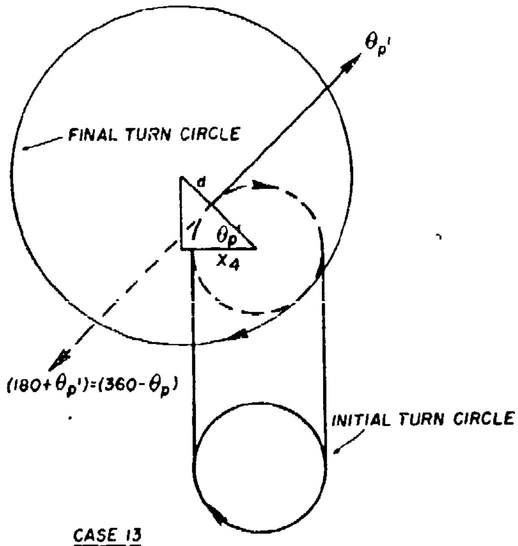
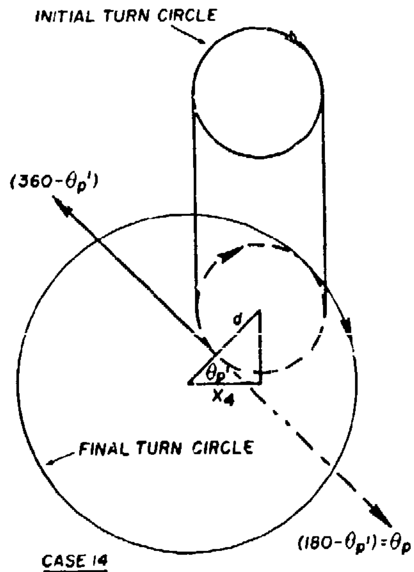


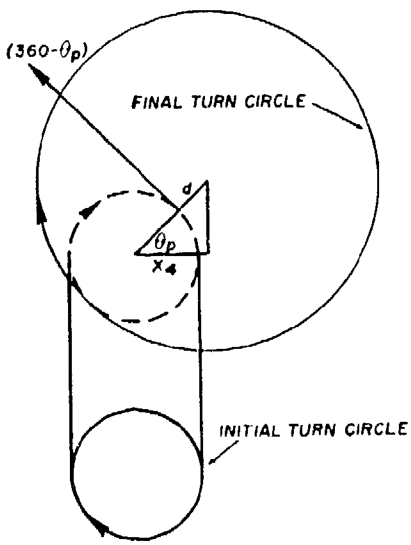
Figure B-9 cont'd



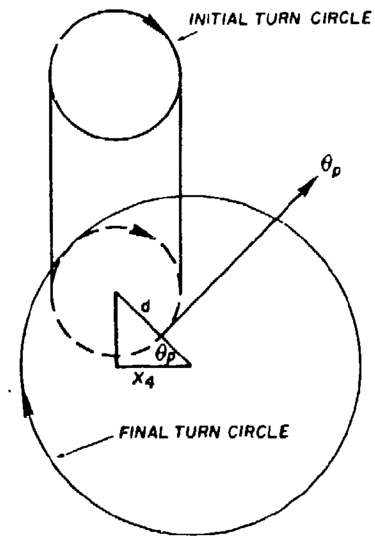
CASE 13
PERMITTED INTERVAL $(360-\theta_p)$ TO 360



CASE 14
PERMITTED INTERVAL θ_p TO 180



CASE 15
PERMITTED INTERVAL $(360-\theta_p)$ TO 360



CASE 16
PERMITTED INTERVAL θ_p TO 180

Heading Interval for Left Left and Right Right Turns

Table B-5 presents the range of interceptor headings to be examined versus the sign of the parameters X_4, d and Y_4 for each of the 16 cases depicted in Figure B-9.

TABLE B-5

HEADING INTERVAL FOR LEFT LEFT AND RIGHT RIGHT TURNS

Case	Sign of X_4	Sign of d	Sign of Y_4	Heading Interval
1	+	+	-	0 to θ_p
2	+	+	+	180 to $(360-\theta_p)$
3	-	+	-	0 to θ_p
4	-	+	+	180 to $(360-\theta_p)$
5	+	-	-	$(360-\theta_p)$ to 360
6	+	-	+	θ_p to 180
7	-	-	-	$(360-\theta_p)$ to 360
8	-	-	+	θ_p to 180
9	+	+	-	0 to θ_p
10	+	+	+	180 to $(360-\theta_p)$
11	-	+	-	0 to θ_p
12	-	+	+	180 to $(360-\theta_p)$
13	+	-	-	$(360-\theta_p)$ to 360
14	+	-	+	θ_p to 180
15	-	-	-	$(360-\theta_p)$ to 360
16	-	-	+	θ_p to 180

RANGE OF INTERCEPTOR HEADINGS

Table B-3 presented the heading intervals to be examined in seeking the command heading that will result in an interception for right left and left right turns when $|X_4| < |d|$. Table B-5 presented the same information for left left and for right right turns. Table B-6 summarizes the information from both tables and presents the range of interceptor headings to be examined in seeking the command heading that will result in a minimum time-to-interception when $|X_4| < |d|$.

TABLE B-6

HEADING INTERVAL FOR $|X_4| < |d|$

Geometrical Condition	Range of Headings
$d > 0$	0 to θ_p and 180 to $(360-\theta_p)$
$d < 0$	θ_p to 180 and $(360-\theta_p)$ to 360

The information presented in Table B-6 corresponds to the last two entries in Table B-1.

REFERENCES

1. System Development Corporation, "SAGE Computer Program Operational Specifications, Volume II"; September 1969.

2. System Development Corporation, "BUIC III Air Defense Computer Program, Volume 3"; May 1971.

3. COMCO Electronics Corporation, "Computer Program Development Specification, Volume II"; December 1974.