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COMPRESSIVE STRENGTH OF WELDED STEEL SHIP GRILLAGES

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## COMPRESSIVE STRENGTH OF WELDED STEEL SHIP GRILLAGES

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Naval Construction Research Establishment St Leonard's Hill Dunfermline Fife

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REPORT NO NCRE/R611 (Item N6A2)

COMPRESSIVE STRENGTH OF WELDED STEEL SHIP GRILLAGES

#### ABSTRACT

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Results are presented of a series of tests on full scale welded steel grillages representing typical warship deck and single bottom structures under compressive load combined in some cases with lateral pressure. Experimental results are discussed in relation to currently available analysis methods. Design recommendations are made together with suggestions on the need for further research.

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Superintendent

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## COMPRESSIVE STRENGTH OF WELDED STEEL SHIP GRILLAGES

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#### C S Smith

#### INTRODUCTION

A vital aim of ship structural design, of particular importance in the case of slender warship hulls, is provision of deck and bottom-shell structures having sufficient compressive strength to withstand vertical bending of the ship's hull. The importance and complexity of this requirement have resulted over the years in extensive research, including major experimental projects such as the trials on PRESTON, BRUCE, ALBUERRA and OCEAN VULCAN (1)\*. Introduction of welding in place of riveting and adoption of longitudinal framing in most large ship hulls have led during the last 25 years to a focussing of attention on the strength of welded plate grillages having closely spaced longitudinal girders and relatively widely spaced transverse frames under longitudinal compression combined with lateral pressure. Research relating to ship grillages has been paralleled by research on stiffened shells in the aerospace field, together recently with a wide-ranging programme of research on box-girder bridges prompted largely by the 1970-71 bridge disasters in Europe and Australia.

2. The important influence of plating behaviour on compressive strength of stiffened panels has been reflected in theoretical and experimental studies of elastic and inelastic buckling, post-buckling and collapse behaviour of individual rectangular plates (1 to 24) including careful examination of the effects of initial distortions and residual stresses caused by welding. Extensive theoretical and experimental studies have also been carried out referring to

- (i) elastic buckling and post-buckling behaviour of panels with singledirection stiffening (25 to 27);
- (ii) inelastic buckling and collapse of panels with single-direction stiffening (28 to 40);

\*() = References on page 37

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- (iii) elastic buckling and beam-column behaviour of orthogonally stiffened grillages (41 to 44);
- (iv) inelastic, post-buckling and collapse behaviour of orthogonally stiffened grillages and box girders (45 to 53).

Despite this substantial body of research, present understanding of collapse behaviour in welded steel grillages is far from complete. Accurate predictions of compressive strength for ship type grillages referring to all possible modes of failure, allowing for the complex interactions which occur between plating and stiffeners and for the influence of initial distortions and residual stresses, cannot yet be made.

3. As part of a programme of research aimed at providing an improved understanding of grillage behaviour and hence more accurate design methods, a series of tests has been carried out at NCRE on large-scale grillages representing warship single-bottom and deck structures under compressive load combined in some cases with lateral pressure. The object of the present report is to present the results of these experiments, to discuss test results in relation to theoretical analysis methods, to make design recommendations where appropriate and to identify problem areas in which further research is necessary.

## DETAILS OF TEST STRUCTURES

<sup>4.</sup> Except in the case of light superstructure decks, where overall grillage buckling is a possible collapse mode, compressive failure of longitudinally stiffened deck and single-bottom structures in warship hulls is always likely to take the form of local, inelastic buckling of longitudinal girders and attached plating. The parameters most strongly influencing compressive strength are, in addition to the material yield strength  $\sigma_v$  and Young's modulus E, the

plate slenderness ratio b/t (or  $\beta = \frac{b}{t} \sqrt{\frac{b}{E}}$ ) and the slenderness ratio a/k

(or  $\lambda = \frac{a}{k\pi} \sqrt{\frac{y}{E}}$ ) of longitudinal girders where

- 2 -

- a = spacing of transverse frames
- b = spacing of longitudinals
- t = plating thickness
- k = radius of gyration of longitudinals acting with assumed effective breadth of plating.

A survey of deck and single-bottom designs in existing British warships indicated values of  $\beta$  in the range 1.0 to 4.5 and  $\lambda$  in the range 0.15 to 0.9. Test grillages were designed in mild steel with various combinations of  $\beta$  and  $\lambda$ falling within these ranges. Broad representation of current design practice was intended, no attempt being made to duplicate particular ship designs or to achieve optimum design of test structures.

5. The test grillages included four pairs of nominally identical structures (1a, 1b, 2a, 2b, 3a, 3b, 4a, 4b) representing possible ship-bottom configurations, together with two grillages (5 and 7) representing frigate strength decks and one grillage (No 6) corresponding to a light superstructure deck. One additional grillage (No 8) was constructed specifically for evaluation of weld-induced residual stresses. The overall imensions of each grillage were 20 ft long by 10 ft 6 in wide. Plate thicknesses and stiffener dimensions and spacings are listed in Table 1. With the exception of fabricated girders in Grillages 4a and 4b, all stiffeners were standard Admiralty tee bars.

6. Grillages were constructed in the NCRE workshop, following as far as possible normal shipyard fabrication and welding procedures. The plating of each structure was formed by two strakes joined by a longitudinal butt weld at (or close to) the grillage centreline. All stiffeners were continuously welded to the plating. During fabrication, plates were initially tack-welded together and stiffeners tacked to the plating, typically by 3 in runs of weld at 18 in spacing. Welding was then completed manually, normally by a single welder operating first on one side of each stiffener and then the other, giving in effect a two-pass welded attachment. In Grillages 2a and 3b, however, singlepass welds were simulated by employing two welders simultaneously on each

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stiffener; as indicated in Table 5, weld-induced residual stresses were noticeably higher in these structures. An attempt was made to achieve reasonable uniformity of plate thickness, stiffener dimensions and yield strengths by ordering twice the required quantity of steel and selecting plates and stiffeners for grillage construction on the basis of measured dimensions and material properties. The mean total area of weld metal deposited at each longitudinal and transverse stiffener attachment is indicated for each grillage in Table 1. Yield strengths obtained from standard tensile tests with a strain rate of approximately 0.0005/minute, are summarized in Table 2. Compression tests were also carried out on a total of 170 specimens from selected plates and tee-bars; compressive yield stresses were on average 6% higher than corresponding tensile values.

7. Non-dimensional grillage parameters derived from as-fitted dimensions and measured material properties are listed in Table 3.

## DETAILS OF TEST RIG

8. Tests were carried out in the Large Testing Frame (LTF) at NCRE in a specially designed rig as shown in Figure 1. Grillages were supported horizontally on a 20 ft x 10 ft steel platform and were held down at their ends against lateral pressure by light flexure plates and along their sides by tiebars with 6 in spacing. Tie-bars incorporated bottle screws to allow vertical adjustment. This form of support was intended to impose conditions of zero vertical displacement and zero rotational restraint (simple support) at the ends and sides of test structures, with the ends and sides free to move longitudinally in the plane of the grillage and the sides also free to move transversely in-plane.

<sup>9</sup>. Longitudinal compression was provided by six (or in some cases seven) hydraulic jacks evenly spaced across the ends of the grillages. Compression was applied by activating the jacks at one end of the rig from a common pressure source and reacting loads through a passive set of jacks, also connected to a common (closed) hydraulic line, at the other end. At the active end loads were

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applied through calibrated hydraulic load transducers giving an indication of each applied jack force with better than 2% accuracy. Differences between jack loads, attributable to friction in the jacks, were always less than 10% and in most cases less than 5%. Differences between total loads at active and passive ends, attributable to friction between grillages and the support platform and to horizontal components of force in tie-bars and flexure plates, were also normally less than 5%. Jacks and load-transducers were mounted on cradles incorporating vertical screw adjustment to provide accurate control over the line of action of compressive forces. As shown in Figure 1, jack loads were applied through ball-bearings in order to minimize rotational restraint at the ends of test structures. Jack forces were reacted through an arrangement of load beams into the walls of the LTF.

10. In order to distribute jack loads into the test structure in a reasonably uniform manner, avoiding the risk of premature failure close to the ends, each grillage was reinforced by fitting doubler plates to the webs and tables of longitudinal girders and to plate vanels between girders over 2/3 of the span of end bays. Because of the conditions of simple support at the ends, reinforcement in these regions had negligible influence on overall behaviour of the grillages and proved a successful means of ensuring that interframe collapse occurred in a representative region of each structure away from the ends. 11. Lateral load was applied by means of a water-filled rubber bag contained between test panel and support platform as shown in Figure 1. Provision was made for lateral pressures of up to 30 psi. Lateral loads were monitored by a calibrated pressure gauge and checked by a water manometer.

#### TEST PROCEDURE

#### Measurement of Initial Deformation

12. Before testing, the initial deformations of each grillage were thoroughly surveyed. The overall vertical deformation of each grillage relative to its ends was measured at stiffener intersection positions by means of stretched wires. Local, interframe vertical deformations of longitudinal girders were

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also measured, together in some cases with sideways distortion of stiffener tables. Extensive measurements were made of plating distortion along the centrelines of plate panels between girders.

#### Measurement of Weld-Induced Residual Stress

13. During construction of the grillages (with the exception of la and lb) weldinduced residual strains were measured by taking extensometer readings on both sides of the plating and on the stiffeners before and after welding using a Demec mechanical extensometer. Measurements on the plating of Grillages 2 to 6 were generally confined to longitudinal strains at the centrelines of plate panels. 14. In view of the unexpectedly high level of residual strains recorded in Grillages 2 to 6, two additional structures, Grillages 7 and 8, were constructed with the object of obtaining a more thorough evaluation of residual stress in typical structures having high and low plate slenderness. Grillage 7 was subsequently tested to collapse; Grillage 8 was instead used for further assessment of residual stresses by means of destructive testing, including trepanning-out of strain-gauged specimens. Longitudinal extensometer readings were taken at closely spaced intervals across selected pirte panels in Grillages 7 and 8 in order to evaluate distributions of residual stress; transverse plating strains were also measured, together with strains in the webs and tables of longitudinal and transverse stiffeners. Strain readings were corrected where necessary for temperature effects. Each set of extensometer readings was taken at least twice, normally by different personnel; a high standard of repeatability was achieved, suggesting an accuracy well within ± 5% in measurement of maximum strains.

#### Load Application

15. Grillages 1a, 2b, 3b, 4a, 5, 6 and 7 were tested to collapse under compressive load alone. Grillages 1b, 2a, 3a and 4b were tested to collapse under compressive load combined with lateral pressure. The applied lateral load, maintained during collapse tests at a constant level with adjustment where necessary to compensate for the effects of structural deformation, was taken in

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each case as the pressure just sufficient, when acting alone, to cause yield in the outer fibres of longitudinal or transverse stiffeners. Tests on Grillages 1 to 4 thus provided a direct indication of the influence of lateral pressure on compressive collapse. Before commencement of the collapse test, each grillage was subjected within the elastic range to various combinations of in-plane and lateral loads. During collapse tests compressive loads were applied incrementally with frequent returns to zero to allow evaluation of permanent set of deflections and strains.

16. In adjusting the position of jacks for application of compressive load an attempt was made to anticipate loss of plating effectiveness (caused by buckling of slender panels and yielding associated with weld-induced residual stress), with consequent upward shift of the plate-stiffener neutral axis, by introducing a slight initial upward eccentricity of jack loads. It was found, however, that while this initial eccentricity caused downward bending of stiffeners in reinforced end bays, the effect of jack eccentricity had largely disappeared (and even in some cases caused slight upward bending) in adjacent, unreinforced bays as a result of "grillage action" by transverse frames. Jack loads were therefore finally applied at positions corresponding approximately to the initial neutral axis of each plate-stiffener cross-section. In multi-bay grillages with stiff transverse frames and reinforced end-bays, the effect of load eccentricity on interframe collapse is likely to be small; tests on orthogonally stiffened grillages should in this respect give a satisfactory representation of actual ship bottom and deck structures in which the effective line of action of compressive load will to some extent follow any shift of neutral axis caused by change of plate effectiveness. In experiments on single-bay panels having longitudinal stiffeners only, fixed alignment of axial load in the presence of a shifting neutral axis may in some cases cause serious experimental error.

## Deflection Measurements

17. During tests, extensive measurements of stiffener and plating deflections were carried out using dial gauges. Overall vertical deflections of each

grillage were measured at stiffener intersection positions by gauges mounted on an independent datum frame. Vertical interframe deflections of longitudinal girders relative to transverse frames were recorded in selected regions, together in some cases with horizontal deflections of girder tables. Detailed measurements of vertical deflection were also made along the centrelines of selected plate panels relative to transverse frames.

#### Strain Measurement

18. Extensive measurements of strain in stiffeners and plating were carried out during the tests. Single gauges were attached to the tables of longitudinal and transverse stiffeners together with back-to-back pairs at mid-depth positions on longitudinal webs; these were used to evaluate bending stresses under lateral load and to estimate mean and bending components of stress in longitudinals under compressive or combined loads. On Grillages la and lb, extensive use of bidirectional pairs of back-to-back gauges was made to examine plating strains. Each of these structures was fitted with over 270 strain gauges. Considerable difficulty was however experienced in interpreting plating strains, which were very irregular and clearly influenced by yielding at an early stage in load application; stress changes could thus normally only be evaluated by reference to unloading strains, removal of load being assumed to occur elastically. Less priority was, therefore, given to measurement of plating strains in subsequent tests, although detailed strain distributions were recorded across at least one transverse cross-section of each grillage tested.

#### DISCUSSION OF RESULTS

#### Initial Deformations

19. A summary of initial deformations is contained in Table 4, including maximum interframe displacements at the centrelines of plate panels and central interframe displacements of longitudinal girders, in each case measured relative to the bounding transverse frames. Mean values and coefficients of variation (cov = standard deviation/mean value) of plate deformations were estimated treating both upward and downward deflections as positive. Initial vertical deformation of

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longitudinal girders was usually upwards but was in some cases downward; percentages of upward and downward displacements, together with maximum and mean values and cov estimated separately for upward and downward deflections, are indicated in Table 4.

#### Weld-Induced Residual Stresses

20. Average longitudinal compressive heart-of-plate residual stresses  $\sigma_{nc}$  at the centres of plate panels in grillages 2 to 6, estimated from longitudinal strains on the assumption that transverse stresses were negligible, are listed in Table 5 together with outer-fibre stresses measured in the girders of Grillages 2a and 2b. Mean plating and outer-fibre stiffener residual stresses for Grillages 7 and 8 are also included in Table 5. Plating stresses in these structures were estimated from both longitudinal and transverse strains; distributions of longitudinal stress at sections mid-way between transverse frames are shown in Figure 2. Plating stresses measured in Grillage 7 were slightly lower than those in Grillages 5 and 6; results for Grillage 8 confirmed the generally high level of compressive residual stress recorded in the plating of Grillages 2, 3 and 4. Transverse residual stresses were in all cases small. Plating residual stresses were found to be irregular, both in intensity and in distribution, probably as a result of irregularities, eg stops and starts, in the welding procedure. The non-uniform, almost parabolic distribution of compressive residual stress  $\sigma_{nc}$  shown in Figure 2(b) should be noted; this differs from the idealized distribution of residual stress, shown in Figure 3, which is commonly assumed in theoretical analysis (7, 8, 15, 70). Published data (9, 29, 39, 50, 51, 53) referring to weld-induced residual stresses in stiffened plating are varied and somewhat contradictory but do include recorded instances of  $\sigma_{nc}$ as high as  $\frac{2}{3}\sigma_v$  and of non-uniform distributions of  $\sigma_{rc}$  similar to those shown in Figure 2(b). (53).

21. Residual stresses measured in the outer fibres of stiffeners were very variable, probably because of irregularities in the welding procedure; these

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stresses were in most cases compressive, though much lower than  $\sigma_{\rm rc}$  in the plating, but were tensile in some cases. Weld-induced residual stresses in the webs of deep fabricated tee girders in Grillage 8 are shown in Figure 2(b); strains recorded in the tables of fabricated tee-stiffeners were inelastic and corresponding stresses therefore could not be deduced.

## Elastic Behaviour of Grillages under Lateral Pressure

22. Under lateral pressures up to the levels indicated in Table 7, deflections and stresses in grillage stiffeners behaved virtually linearly and generally agreed well with theoretical deflections and stresses estimated by beam grillage analysis, ie assuming each stiffener with an attached effective breadth of plating to behave in accordance with beam theory. It was however found, as illustrated in Figure 4 and observed previously by Clarkson (55), that local, interframe deflections and bending stresses in the smaller stiffeners were generally higher than theoretical values even where all the applied lateral load was assumed to act on the longitudinal girders. This effect, which is not at present fully understood, is not accounted for by the non-uniform, approximately parabolic distribution of normal force along plate edges or by the fact that distributed edge forces exceed the applied lateral load, being balanced by concentrated forces close to the plate corners; the discrepancy may be caused partly by the destabilizing action of transverse compressive stresses in the plating associated with bending of transverse frames.

23. Grillage calculations carried out assuming various different effective breadths suggested that in analysing elastic behaviour under lateral pressure plating effective breadths  $b_{eb}$  in the range 0.75b to 1.0b may safely be assumed for closely spaced stiffeners (b/t < 45) while for more widely spaced stiffeners (b/t > 60) the recommendation by Clarkson that an effective breadth of 0.5b or 50t be adopted (whichever is less) is reasonable if usually pessimistic. The sensitivity of grillage response to effective breadth assumptions is illustrated in Figure 4 which refers to the following cases:

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Solution 1 : b<sub>eb</sub>/b = 1.0 for longitudinal girders,

 $b_{eb}/a = 1.0$  for transverse frames Solution 2 :  $b_{eb}/b = b_{eb}/a = 0.75$ 

Solution 3 :  $b_{eb}/b = b_{eb}/a = 0.5$ 

Solution 4 : b /b = 0.5, b = 50t for transverse frames.

In Solutions 1 to 3 lateral pressure was assumed to be uniformly distributed along both longitudinal and transverse members; in Solution 4 lateral load was assumed to act uniformly on longitudinals only.

#### Elastic Buckling and Beam-Column Behaviour

24. Theoretical elastic buckling characteristics of Grillages 1 to 7 are summarized in Table 6. These include initial buckling stresses corresponding to:

(i) buckling of plate panels between stiffeners, estimated using the Bryan formula for long or square simply supported plates

$$\sigma_{\rm cr} = \frac{\pi^2 {\rm Et}^2}{3(1-\mu^2){\rm b}^2}.$$
 (1)

and checked in some cases by rigorous folded-plate analysis (25);(ii) interframe flexural buckling of longitudinal girders estimated using the Euler formula (corrected for shear effects)

$$\sigma_{\rm cr} = \frac{\pi^2 EI}{Aa^2} \left/ \left( 1 + \frac{\pi^2 EI}{a^2 GA_{\rm s}} \right) \right.$$
(2)

where A is the total cross-sectional area,  $A_s$  the effective shear (web) area and I the effective moment of inertia of a stiffener with attached plating, checked in some cases by folded-plate analysis;

- (iii) interframe tripping (lateral-torsional instability) of longitudinal girders, estimated for selected grillages using folded-plate analysis;
  - (iv) overall grillage buckling estimated using the formula for a simply supported orthotropic plate under uniaxial compression

$$\sigma = \frac{n^2 \pi^2 D_x}{n_y B^2} \left[ \frac{D_y B^2}{D_x L^2} + \frac{2m^2 D_x y}{n^2 D_x} + \frac{m^4 L^2}{n^4 B^2} \right]$$
(3)

in which L and B are overall length and breadth, m and n are the numbers of buckling half-waves in transverse and longitudinal directions respectively,  $h_y$  is the average cross-sectional area per unit width of plating and longitudinal girders,  $D_x$  and  $D_y$  are effective flexural rigidities per unit width of stiffeners with attached plating in transverse (x) and longitudinal (y) directions and  $D_{xy}$  is the twisting rigidity per unit width, negligible in the case of open section stiffeners; results obtained using equation (3) were checked selectively by finite element analysis (42).

25. Elastic buckling data of the type contained in Table 6 give no direct indication of ultimate grillage strength but do in some cases provide a pointer to the likely failure mode. The torsional collapse modes of stiffeners in Grillages 1 and 4 (see Figures 8, 15 and 16) were consistent with computed elastic buckling modes, as were the interframe flexural buckling failures of Grillages 2, 3 and 5 (see Figures 10, 12 and 19). The collapse mode of Grillage 6 (see Figure 21) was consistent with computed n = 2 and n = 3 overall elastic buckling modes. Elastic buckling data are also of use as a basis for approximate evaluation of beam-column effects by judicious application of the magnification factor  $1/(1-\frac{\sigma}{\sigma_{cr}})$  to deflections and stresses caused by lateral load (42).

## Collapse Behaviour

26. A summary of Grillage collapse loads is contained in Tabl 7; the collapse behaviour of each structure is described briefly below.

### Grillages la and lb

27. Failure of Grillages la and lb was preceded by buckling of plate panels accompanied by a substantial loss of plating stiffness. Loss of plating

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effectiveness in Grillage la is illustrated in Figure 6, which shows distributions of mean longitudinal stress estimated from unlcading strains. (Unlike initial load application, during which early permanent set of plating strains was recorded, removal of load occurred virtually elastically allowing corresponding stress changes to be found directly from strains). Progressive buckling deformations and permanent set of selected plate panels are shown in Figure 7. 28. Collapse of Grillage la occurred finally by interframe tripping of longitudinal girders primarily between Frames T2 and T3, at an average compressive stress of 12.4 tsi. Progressive development of vertical and horizontal interframe deflection of stiffener tables is illustrated in Figure 7, which shows that tripping deformation of the girders was more marked than flexural deformation. A photograph taken at an early stage in the collapse of Grillage la, showing plate buckling and incipient stiffener failure, is contained in Figure 8(a); photographs showing fully developed collapse are contained in Figures 9(b) to 8(e). 29. Collapse of Grillage lb occurred under a constant lateral pressure of 15 psi at an average compressive stress of 12.1 tsi; although lateral pressure had a marked effect on plate deformations, as shown in Figure 9, its influence on collapse foad was slight. The form of stiffener failure was almost identical with that in Grillage la, collapse occurring primarily between Frames T4 and T5.

## Grillages 2a and 2b

30. Collapse of Grillage 2a occurred by interframe buckling of longitudinal girders associated with inelastic buckling of plate panels, as shown in Figure 10(a), at an average compressive stress of 15.9 tsi combined with a lateral pressure of 7 psi. In the collapse bay stiffeners buckled upwards in a flexural mode with negligible sideways displacement (except for the edge girders which were restrained vertically and therefore buckled torsionally). In adjacent bays some sideways tripping of girders occurred close to the transverse frames. The lower side of the grillage following collapse is shown in Figure 10(b) illustrating the extent of plate buckling.

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31. Collapse of Grillage 2b occurred at an average compressive stress of 14.7 tsi. The form of collapse was virtually identical with that of Grillage 2a. 32. In both grillages substantial permanent set of longitudinal strains in stiffeners and plating, attributable to high  $\sigma_{\rm rc}$  and associated with reduced plating stiffness, commenced at an average compressive stress of about 8.5 tsi. Buckling deformations of plate panels were not observed until collapse occurred and were then confined primarily to the collapse bay in each grillage. Significant permanent set of vertical interframe deflections in longitudinal girders, indicating stiffener yield, commenced in the collapse bay at an average compressive stress of 12.5 tsi in Grillage 2a and 13 tsi in Grillage 2b.

#### Grillages 3a and 3b

33. Collapse of Grillage 3a occurred by interframe buckling of longitudinal girders associated with inelastic plate buckling, as shown in Figure 11, at an average compressive stress of 11.1 tsi combined with a lateral pressure of 3 psi. Upward flexural buckling of girders occurred in one of the central bays together with downward buckling in an adjacent bay, inhibited by the action of lateral pressure and involving some tripping of stiffeners close to the central transverse frame.

34. Grillage 3b collapsed at an average compressive stress of 9.8 tsi. As shown in Figure 12, failure involved upward flexural buckling of longitudinal girders between Frames T3 and T4, accompanied by downward buckling in the adjacent bay. Downward deflection was associated with some local stiffener tripping at the centre of the interframe span.

35. In both grillages substantial permanent set of long tudinal strains, attributable mainly to the influence of residual stresses, commenced at an average compressive stress of about 7.5 tsi. As illustrated in Figures 13 and 14, which show progressive deformation of typical girders and plate panels, significant yielding and permanent set of the stiffeners commenced at about  $\sigma_{ave} = 8$  tsi. The marked downward buckling and permanent set of longitudinal girders between Frames T2 and T3 in Grillage 3b should be noted.

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## Grillages 4a and 4b

36. Grillage 4a collapsed at a mean compressive stress of 13.3 tsi. Failure occurred by flexural buckling of small longitudinal girders and sideways tripping of deep fabricated girders between Frames T2 and T3, accompanied by local inelastic buckling of the plating and of the webs of deep girders. The form of failure is indicated in Figure 15(a), which shows the initial stages of collapse, and Figure 15(b) which shows fully developed collapse.

37. Collapse of Grillage 4b occurred at an average compressive stress of 13.5 tsi combined with a lateral pressure of 8 psi. As shown in Figures 16(a), (b) and (c), failure was similar in form to that of Grillage 4a except that collapse occurred in the central bay and deep girders all buckled in one direction. Elastic buckling analysis, as illustrated in Figure 5(b), indicated that buckling in which deep girders deflect in the same direction would occur at virtually the same stress as buckling in which alternate girders deflect in opposite directions.

38. Extensive permanent set of strains commenced in both grillages at an average compressive stress of about 8.5 tsi. Development of deflections and permanent set in typical longitudinal girders and plate panels is shown in Figures 17 and 18. The relatively small amplitude of tripping deformations in deep girders of Grillage 4b is probably attributable to tensile stresses induced by lateral load. Grillage 5

39. Grillage 5 collapsed at an average compressive stress of 11.4 tsi. Failure, which was preceded by buckling of plate panels, occurred by interframe buckling of longitudinal girders as illustrated in Figure 19, upwards and essentially flexural in one of the central bays, downwards and involving some tripping in the adjacent bay. Development of deflections and permanent set in a typical longitudinal girder and plate panels is shown in Figure 20.

#### Grillage 6

40. Collapse of Grillage 6 occurred at a mean compressive stress of 8.1 tsi by overall instability involving upward and downward bending of transverse frames as

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well as longitudinal girders, as illustrated in Figure 21. Failure was preceded by buckling of plate panels. Development of plating and stiffener deflections is illustrated in Figure 22.

#### Grillage 7

41. Collapse occurred at a mean compressive stress of 12.0 tsi. The form of failure was virtually identical with that of Grillage 5.

#### THEORETICAL EVALUATION OF GRILLAGE STRENGTH

42. The collapse behaviour of orthogonally stiffened welded steel grillages under compressive load can be assumed to fall into the following four categories:

(i) Plate failure;

- (ii) Interframe flexural buckling of stiffeners and plating;
- (iii) Interframe tripping of stiffeners and plating;
- (iv) Overall grillage instability.

The main features of these failure modes are discussed below.

#### Plate Failure

43. In this case the ultimate strength of plate panels is exceeded before extensive yield occurs in the stiffeners; as end-shortening is increased beyond this point, reduction of load in the plating proceeds more rapidly than increase of load in the stiffeners so that the ultimate load for the stiffened panel is reached before stiffener failure occurs.

44. Reliable estimates of ultimate strength for rectangular plates under uniaxial compression can now be made by reference to published theoretical and experimental collapse data (2, 3, 4, 7, 8, 9, 11, 16, 70). Most existing data refer to plates in which unloaded edges are free to deflect in-plane; it has been shown (52) that the difference between this boundary condition and the condition (more representative of plated grillages) in which unloaded edges of plate panels remain straight ( $\frac{\partial u}{\partial y} = 0$ ) is negligible for steel plates of low and medium slenderness ( $\frac{b}{t} < 60$ ); in plates with  $\frac{b}{t}$  of 80 the difference between collapse

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loads for edges-free and edges-straight conditions was found to be no more than 5%. Less information is available on plate strength under combined compression and lateral pressure but reference to theoretical analysis (10) and test data (11, 13) suggests a linear interaction between compressive and lateral loads and indicates that lateral pressure is unlikely to cause significant loss of plating compressive strength in normal ship designs.

45. For the purpose of analysing grillage failure, a knowledge of plating behaviour throughout the load range is necessary; evaluation of ultimate plate strength alone is insufficient. A particularly effective means of describing plating behaviour during grillage collapse is provided by load-shortening or "stress-strain" curves for rectangular plates, which have been established theoretically (8, 15, 16) and experimentally (9) for a wide range of plate geometries, initial distortions and residual stress distributions. Typical plate stress-strain curves, based on Cambridge University and TRRL data (8, 9, 15), are shown in Figure 23. It is evident that in plates with slenderness in the range  $\frac{b}{+}$  > 50 having little or no residual stress and only slight initial deformation, compressive failure is followed by a rapid loss of load-carrying capacity; in grillages containing such plating (or in hybrid structures where the yield strength of the stiffeners is substantially higher than that of the plating) collapse would be likely to occur by "plate failure". In most practical grillages, however, which contain substantial weld-induced distortions and residual stresses, and in grillages incorporating very stocky plating  $(\frac{b}{t} < 45)$ , little if any load-reduction occurs in the plating; the ultimate strength of the plating will not normally be reached until the average compressive strain is well in excess o. the yield strain  $\varepsilon_{u}$ ; at this stage extensive yielding and hence elasto-plastic buckling of the stiffeners will usually have occurred. Interframe Flexural Buckling of Longitudinal Stiffeners

46. Failure in this case occurs by column-like flexural buckling of stiffeners and plating between transverse frames. Initial deformation of the stiffeners, together in some cases with lateral load, will often ensure that buckling occurs towards the stiffener outstand. In some cases, however, as indicated in Table 4, initial distortion may be directed towards the plating, inducing buckling in this direction; bending of the stiffeners away from the plating in one interframe bay may also induce buckling towards the plating in adjacent bays. Where buckling occurs towards the plating, as illustrated in Figures 11, 12, 19 and 21, flexure may be coupled with sideways tripping of the stiffeners.

47. Provided that correct account is taken of reduced plating stiffness and that buckling is assumed to be purely flexural in form, interframe collapse of a stiffened panel may be investigated using inelastic column analysis based either

- (i) on an incremental finite element method (56, 57, 58), or
- (ii) on numerical solution of the beam-column equilibrium equations using elasto-plastic moment-thrust-curvature relationships computed for the appropriate section geometry with allowance for initial residual stresses (35, 37, 59, 60, 61).

Data curves developed using analysis of this type (59, 62, 63) provide an approximate means of estimating collapse strength under combined compression and lateral pressure.

48. In order to examine the collapse behaviour of test grillages described in the present report, a computer program developed for analysis of inelastic buckling of plane frames (58) has been adapted to deal with beam-column failure of stiffened-panels. The analysis method involves subdivision of a longitudinal girder with attached strip of plating along its length into uniform beam elements with subdivision of girder cross-sections into elemental areas, or "fibres", as illustrated in Figure 25. Any dist. Bution of residual stress over an element cross-section may be represented as a set of initial stresses at fibre centroidal positions. The analysis follows a step-by-step procedure in which loads are applied incrementally, a linear solution being obtained for each incremental load application by the usual matrix Displacement method, ie by solving the incremental matrix equation

$$(K + K_{c})\delta = R$$

where K is a conventional stiffness matrix,  $K_{\rm G}$  is a geometric stiffness matrix representing the destabilizing influence of axial forces in beam elements,  $\delta$  is a column matrix of incremental nodal displacements and R is a column matrix representing applied loads and initial deformations. Following each incremental solution, cumulative values of nodal displacements, element axial forces and fibre stresses and strains are updated; the state of stress in each fibre of each element is examined and where the total stress (including initial residual stress) exceeds yield, the fibre is assumed to make no contribution to element stiffness in the following incremental solution, ie an elastic-perfectly plastic stress-strain curve is assumed, strain-hardening effects and the influence of shear on yield being ignored. Allowance is made for recovery of elastic stiffness where strain-reversal occurs in yielded fibres. In each incremental solution the contribution of the plating to element sectional properties is estimated

from the plating stiffness bt  $\frac{d\sigma_{ave}}{d\epsilon}$ , derived directly from the slope of loadshortening curves of the type shown in Figure 23 which are represented numerically in the computer. A direct indication of collapse load is obtained by evaluating the determinant of the stiffness matrix (K + K<sub>G</sub>) in each incremental solution; collapse is assumed to occur when the determinant changes sign. The accuracy of the analysis method is illustrated in Figure 24, which shows

- (i) a computed load-deflection curve for a uniform, simply-supported beam (corresponding to a longitudinal girder with attached plating in Grillage 3a) under central concentrated load: the computed collapse load corresponds closely with the exact fully-plastic collapse load for the beam, suggesting that inelastic behaviour is satisfactorily represented in the analysis;
- (ii) a plot of stiffness matrix determinant against applied axial load for the same structure, with plasticity avoided by assuming a high yield

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stress; the finite element buckling load corresponds closely with the exact Euler load, suggesting that destabilizing effects are accurately treated in the analysis.

49. Collapse loads computed for Grillages 3a and 5, in which flexural buckling of stiffeners occurred, are summarized in Table 8 for various assumed conditions of loading, initial deformation and residual stress. Calculations 1 to 17, detailed in Table 8, referred to a single interframe span of each grillage with a plane of symmetry assumed at mid-span. Each half-span was divided into ten equal length beam elements. Cross-sections of Grillage 3a were subdivided into fibres as shown in Figure 25; stiffener cross-sections in Grillage 5 were similarly subdivided while the plating was represented by a single fibre (except in calculations 1, 2 and 15, where the plating was subdivided in the same way as that of Grillage 3a). A condition of simple support at the ends of the column (ie at transverse frames) was assumed in calculations 1 to 15; a clamped-end condition was examined in calculations 16 and 17. Compressive yield stresses for plating and stiffeners were taken to be 6% higher than the tensile values listed in Table 2.

50. Standard initial vertical deformation of the stiffener and plating was assumed to have the form of uniform curvature towards the stiffener with central amplitude equal to the mean value indicated in Table 4; downward initial deformations were also examined. Zero, standard and double-standard deformations are denoted 0, 1 and 2 in Table 8; downward deformations are indicated by a negative sign. The influence of lateral load on compressive strength was examined using a standard pressure for each grillage equal to the experimental value listed in Table 7; zero, standard and double-standard lateral pressures are denoted 0, 1 and 2 in Table 8. As a standard condition throughout the calculations the line of action of applied compressive load was assumed to follow any shift of neutral axis at the end cross-section; this assumption (denoted 1 in Table 8) is believed to be more realistic both for test grillages and for actual ship structures than the alternative assumption that the line of action of axial

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load remains fixed. In one case (denoted 0 in Table 8) the line of action of load was kept fixed in order to examine the significance of the assumption. 51. Compressive residual stress  $\sigma_{rc}$  in the plating of Grillage 3a was assumed to have the idealized distribution shown in Figure 3; the assumed width 2nt of the tension block adjacent to the stiffener attachment was adjusted so that the total initial force on the cross-section was zero in each case. The average value of  $\sigma_{rc}$  in Grillage 3a was taken to be 2/3 of the measured central value indicated in Table 5 on the assumption, suggested by the results shown in Figure 2(b), that  $\sigma_{rc}$  was distributed parabolically across each plate panel; it was assumed what plate buckling effects were negligible and that loss of plating stiffness in Grillage 5, assumed to be influenced by residual stresses. Plating stiffness, was estimated (except in calculations 1, 2 and 15) from the loadshortening curves shown in Figure 23(b). Five conditions of residual stress, labelled 0 to 4 in Table 8, were examined for each grillage:-

Condition 0 - zero residual stress;

Condition 1 - residual stress confined to plating;

Condition 2 - residual stress in plating combined with residual stress in stiffener varying linearly from  $\sigma_{nc}$  (com-

pressive) at the base to zero at the outer fibre; Condition 3 - same as 2 except that stiffener stress varied from  $\sigma_{rc}$  at the base to  $\sigma_{rof}$  (compressive) at the outer fibre;

Condition 4 - same as 3 except that stress at the outer-fibre of the stiffener was  $-\sigma_{rof}$  (tensile).

The stiffener outer-fibre compressive residual stress  $\sigma_{rof}$  was taken to be 2.5 tsi in both Grillages 3a and 5, consistent with measured values in similarly proportioned structures as shown in Figure 2;  $\sigma_{rc}$  for Grillage 5 was taken

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equal to the mean measured value indicated in Table 5.

52. In addition to the single-span analysis described above, calculations 18 to 23 were carried out for two continuous spans of each structure assuming a condition of simple support at the central transverse frame and a plane of symmetry at the centre of each interframe span. Each half-span of stiffener and plating was divided into ten equal-length elements and each cross-section was subdivided into fibres as in the single-span calculations. Collapse loads were computed assuming a standard upward initial deformation in the right-hand span combined with various upward and downward deformations of the left-hand span as indicated in Table 8. Progressive development of plasticity in Grillage 3a prior to collapse is illustrated in Figure 26 for two cases corresponding to calculations 19 and 23; in both cases the right hand span buckled upwards and the left-hand span downwards. In calculation 19 (assuming residual stress condition 3) yield was found to commence at the base of the stiffener, where the compressive residual stress was highest, and to spread along the length and upwards over the full height of the stiffener at the centre of the left-hand span, the plating remaining elastic; this process was influenced by the fact that the yield strength of the plating was 10% higher than that of the stiffener. In calculation 23 (assuming that only the plating carried residual stress) stiffener yield was found to start in the outer-fibre at the centre of the left-hand span and to spread downwards through the stiffener accompanied by partial yield of the plating confined to the right-hand span in way of initial compressive residual stress.

53. From the results listed in Table 8 the following observations may be made.

(i) Calculations 1 and 15 indicate that where buckling occurs downwards (towards the plating) the collapse load may be substantially less than where collapse occurs upwards; this difference is more marked in Grillage 3a than in the less slender Grillage 5. Calculations 1 and 15 refer to "basic" columns, uncomplicated by residual stress and plate buckling effects. Similar though less marked differences

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between upward and downward collapse occur where plating stiffness is reduced by residual stress action and buckling as indicated by calculations 5 and 12.

- (ii) Calculations 1 and 2 indicate that plating residual stresses can have a significant influence on collapse of a stiffened panel. Calculations 4, 5, 6, 19 and 23 show that collapse loads may also be influenced quite strongly by the distribution of residual stress in the stiffeners; as might be expected, compressive residual stress in stiffener tables causes a reduction of compressive strength while tensile residual stress in stiffener tables can have a beneficial effect.
  (iii) Calculations 5 and 7 indicate that significant differences may occur between the case in which the line of action of compressive load remains fixed at the end of a single-span structure and the case in
  - (iv) Calculations 5, 10 and 11 suggest that upward collapse of a singlespan panel is insensitive to upward initial deformation; calculations indicate a greater sensitivity of downward collapse to the amplitude of downward initial deformation.

which the line of action of load follows the shift of neutral axis.

- (v) Calculations 5, 8 and 9 show that upward lateral pressure reduces the collapse load of a single-span structure where buckling occurs upwards; calculations 12 and 14 indicate that upward lateral pressure has a beneficial influence on downward buckling.
- (vi) Comparison of calculations 5, 12, 16 and 17 shows that clamping at the end of a single-span structure has little effect on upward buckling but may have a stronger effect on downward buckling.
- (vii) Comparison of calculations 1 to 15 with calculations 18 to 25 indicates that significant coupling may occur between adjacent interframe spans of an orthogonally stiffened grillage, particularly in a structure with high interframe slenderness such as Grillage 3a. Analysis of a two-span model, which accounts correctly for the restraining or

destabilizing influence of each span upon the other, is thus preferable to analysis of a single span. Calculations 20, 21 and 22 show that even where all initial deformations are towards the stiffener, upward bending in the span with largest deformation may cause downward bending and hence collapse in an adjacent span at a load substantially less than that for upward buckling failure. Calculations 20 and 22 suggest that where initial deformations are entirely upward, lateral pressure may reduce the collapse load of a two-span system, but calculations 24 and 25 demonstrate that if downward initial deformations are present, lateral pressure may increase the collapse load.

(viii) Calculations for Grillages 3a and 5 were based on nominal conditions of initial deformation and residual stress, including rather arbitrary assumed distributions of stiffener residual stress; no account was taken of possible interaction between parallel stiffeners having different initial distortions and residual stresses. Comparison of theoretical and experimental results therefore does not provide a conclusive check on the accuracy of the theoretical method. Theoretical collapse loads are nevertheless reasonably consistent with experimental results. On the evidence of results listed in Table 8, the difference between experimental collapse loads for Grillages 3a and 3b could be attributed to differences in stiffener initial deformations and residual stresses and to a beneficial action by lateral load. Theoretical collapse loads for Grillage 5 are slightly lower than the experimental value, possibly because of excessively pessimistic assumptions regarding stiffener residual stresses.

#### Interframe Tripping (Lateral-Torsional Buckling) of Longitudinal Girders

54. This form of failure, illustrated in Figures 8, 15 and 16, is particularly likely to occur in short, flexurally stiff girders and in stiffeners with low lateral-torsional rigidity, eg flat-bars and bulb-plates. As shown in Figures 11 and 12, tripping may take place in association with flexure of the stiffeners

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where buckling occurs towards the plating.

55. No satisfactory method appears to exist at present for analysis of inelastic tripping of stiffeners welded to continuous plating. Elastic tripping stresses can be estimated using approximate formulas (64) or, more accurately, by foldedplate (25, 26) or finite element analysis. Evaluation of initial elastic buckling behaviour may provide a pointer to the likely failure mode but gives no direct indication of inelastic collapse strength. As in the case of inelastic flexural buckling, use of tangent moduli in conjunction with elastic buckling analysis cannot provide a reliable estimate of collapse load unless "structural" tangent moduli are established referring to the appropriate stiffener geometry and allowing correctly for initial deformations and residual stresses; such tangent moduli can only be obtained from rigorous analysis of inelastic tripping behaviour or, empirically, from tripping test data.

56. Incremental finite element analysis, allowing for progressive development of plasticity and hence reduction of torsional rigidity, transverse flexural rigidity and warping rigidity, including the effects of initial distortion and residual stress, has proved an effective method of analysing the inelastic lateral-torsional buckling behaviour of I-beams and other prismatic sections of the type used in civil engineering structures (66, 67, 68). It seems clear that this type of analysis could be extended to deal with coupled tripping and bending of stiffeners welded to continuous plating, including approximate representation of reduced in-plane stiffness of the plating as in the purely flexural finite element analysis described above, together with representation of transverse rotational restraint exerted by the plating on the base of the stiffener (folded-plate analysis (25) has shown that such restraint may have a dominant influence on stiffener tripping; it is also possible for buckled plating to exert a destabilizing influence on stiffener tripping). Such analysis, based on beam-theory representation of the stiffener, might be expected to give satisfactory results for panels stiffened by tee or inverted angle-bars in which lateral-torsional stiffness is dominated by transverse flexural rigidity of the

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stiffener table; where stiffeners are formed by flat-bars or bulb-plates a different approach, involving finite element treatment of plate-bending behaviour in stiffener webs, may be necessary.

## Overall Grillage Br. kling

57. This form of failure, illustrated in Figure 21, involves buckling of a grillage over its entire length into one or more half-waves with bending of transverse as well as longitudinal stiffeners. As in the case of interframe stiffener buckling, collapse may be strongly influenced by reduced plating stiffness; failure may also be affected by local stiffener tripping where bending occurs towards the plating, as shown in Figure 21.

58. Elastic overall buckling loads and modes can be estimated accurately for uniform rectangular grillages by use of an orthotropic plate formula and may be calculated for non-uniform grillages by finite element analysis (42). Such buckling data may be useful as a basis for estimating elastic beam-column magnification of deformations and bending stresses caused by lateral load, but have little direct relevance to prediction of collapse loads which in all practical cases are likely to be influenced by plasticity. As in the case of interframe collapse, use of "structural" tangent moduli in conjunction with elastic buckling analysis is only likely to yield reliable results if such moduli are based empirically on relevant test data or on rigorous elasto-plastic analysis. In a very slender grillage for which the elastic buckling stress is well below yield point, a significant post-buckling reserve of strength may exist. Scope clearly exists for development of incremental elasto-plastic analysis methods for grillages under combined loads, allowing for largedeflection membrane effects in plating and stiffeners with approximate representation of plate stiffness as in the analysis of interframe collapse. An analysis method of this type has been developed by Parsanejad and Ostapenko (48); application of this analysis to Grillage 6 yielded a theoretical collapse load within 5% of the value indicated in Table 7. Further comparisons with test data and with alternative theoretical solutions are however needed before the

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reliability of this and other analysis methods (49) can be judged conclusively. DESIGN CONSIDERATIONS AND THE NEED FOR FURTHER RESEARCH

## Plate Failure Versus Stiffener Failure

59. Tests described in the present report have demonstrated various forms of grillage failure including flexural, torsional and coupled flexural/torsional interframe buckling together with overall grillage instability. Test results, supported by theoretical analysis, suggest that yielding and hence inelastic buckling of longitudinal girders will usually play a dominant part in grillage collapse; the inelastic "column strength" of stiffened panels must therefore be examined carefully in design. Most ship grillages contain heavy continuous welding and hence are affected by substantial residual stresses and plate distortions; in such structures "plate failure" as defined in the present paper is unlikely to occur (except where the yield strength of stiffeners is higher than that of the plating); loss of plating stiffness, caused in stocky plating by high residual stresses and in slender plating by buckling, will however in many cases strongly influence grillage strength.

#### Plate Stiffness

60. For design purposes the stiffness of plate panels under uniaxial compression can probably be estimated satisfactorily from load-shortening curves of the type shown in Figure 23. Plate panels may in some cases also carry transverse compression caused by hydrostatic pressure on the ship's sides and by bending of transverse frames under lateral pressure. Test data referring to unstiffened, welded plates under biaxial compression (11) suggest that where the transverse compressive stress is less than  $0.25 \sigma_y$ , reduction of longitudinal plate strength and stiffness may be estimated from a parabolic interaction formula (70). This approach should normally be conservative since, particularly in thin plates, transverse compression may, depending on the loading sequence, cause a substantial increase in longitudinal compressive strength (11). A need clearly exists however for further test data and theoretical analysis referring to plates under biaxial compression. Existing finite element (8, 15) and finite difference (23, 24) programs appear to offer an effective means of generating design data and are currently being applied by NCRE for this purpose.

## "Column Failure" of Stiffened Panels

61. Inelastic flexural buckling of stiffened panels between transverse frames can be examined fairly accurately, allowing for residual stresses, initial deformation and reduced plating stiffness, using incremental finite element analysis of the type described in the present paper. For design purposes analysis of this type appears to provide the best basis presently available for evaluation of grillage compressive strength and also offers a means of computing design columncurves for a range of standard plate-stiffener sections. In carrying out such calculations it is advisable to adopt a two-span idealization of the structure which allows for interaction between adjacent spans.

62. The most serious deficiency of flexural column analysis is that it takes no account of stiffener tripping, either where this is a dominant cause of failure as illustrated in Figures 8, 15 and 16, or where tripping occurs in association with flexural buckling as shown in Figures 10, 11, 12 and 21. (It cannot unfortunately be established with certainty from the experimental data or by theoretical analysis that the tripping deformations shown in Figures 10, 11, 12 and 21 influenced collapse significantly; it is possible that tripping did not occur until after the maximum load was reached. Equally however the possibility cannot be excluded that failure in these grillages was influenced by tripping). Buckling of unsymmetrical sections such as inverted angle-bars and bulb-plates will of course always involve coupled flexure and tripping. An urgent need exists for development of analysis methods capable of dealing accurately with coupled torsional-flexural buckling of stiffeners welded to continuous plating. A need also clearly exists for further test data referring to flexural-torsional failure of stiffened panels. The present lack of adequate test data and analysis methods is a compelling reason for cautious design and for avoidance where possible of stiffeners with low lateral-torsional rigidity (eg flat-bars) and unsymmetrical stiffeners such as inverted angle-bars and bulb-plates in grillages designed to

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carry high compressive. load.

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63. A further objection to column analysis is that it takes no account of interaction between parallel members of a stiffened, interframe panel which, as illustrated by Tables 4 and 5 and Figure 2, may have widely different initial deformations and residual stresses and might therefore, as indicated by Table 8, have widely different failure loads if treated as a system of independent columns. Clearly, failure of a stiffened panel must involve an "averaging" process depending on the post-ultimate load-carrying capacity and stiffness of individual members and on a linking action between parallel members provided by membrane and flexural stiffness of the plating. Rigorous analysis of this process probably is not possible at present, although theoretical analysis based on approximate representation of plate behaviour might be worthwhile. The best immediate approach to assessment of parallel interaction between the members of a stiffened panel may lie in correlation of grillage test data with individual column calculations aimed at establishing reliable assumptions regarding effective initial deformation and residual stress, perhaps as a function of mean and maximum values, for use in column analysis. The statistical aspects of this problem have been discussed by Grundy (69).

64. Deep girders susceptible to tripping failure are sometimes provided with lateral support in the form of tripping brackets which impose rotational continuity between plating and stiffeners at one or more positions along their span. Tripping brackets have the obvious disadvantages of increasing fabrication cost and of introducing hard spots at the toes of brackets which may cause fatigue problems and may also, in warships, weaken the structure under explosive loading. It seems unlikely that tripping brackets usually contribute significantly to compressive strength since the rotational stiffness provided by the plating in a buckled or near-buckled condition is likely to be slight and may even be negative; inspection of Figures 8, 15 and 16 shows however that inclusion of midspan brackets would at least have forced changes in the collapse modes of Grillages 1 and 4. It is suggested that in developing computer programs for
analysis of torsional-flexural failure, provision be made for simulation of tripping brackets and that evaluation of tripping brackets should be included in future experiments on stiffened panels.

## Overall Grillage Buckling

65. No proven theoretical analysis method and very little test data exist at present for this form of failure. A need clearly exists for provision of further test data, for use of such data to evaluate existing theory (48, 49) and probably for further development of theory.

66. Overall grillage buckling is only likely to precede interframe buckling in unusually slender grillages, eg warship superstructure decks. Where such decks are designed to contribute to the hull-girder section modulus, loss of compressive stiffness must obviously be minimized; in the absence of a more accurate design method it is suggested that the initial elastic overall buckling stress, which is readily calculated, should be kept above yield point and well above the expected maximum service stress level. It should usually be possible to suppress overall instability by introduction of pillars or by judicious use of support from secondary structural components such as minor bulkheads and engine casings; it is important however that this requirement be recognised at an early stage and that pillars, minor bulkheads etc should be carefully positioned and designed with sufficient strength and stiffness and sufficiently strong attachments to provide the necessary support.

## Effects of Lateral Load

67. Test results and beam-column calculations described in the present paper together with previous theoretical (10) and experimental data.(11, 13) referring to square and rectangular plates suggest that lateral pressure is unlikely to cause significant reduction of longitudinal compressive strength in normally proportioned single bottom and deck structures. Lateral pressure may have a more marked effect on plate stiffness and hence on grillage strength where transverse compression is also present (11); Faulkner (70) has tentatively proposed a linear interaction formula to deal with this case. Experimental

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evidence is however very scanty and a need clearly exists for further data referring to plate stiffness under uniaxial and biaxial compression combined with lateral pressure. Existing finite element and finite difference programs (8, 15, 23, 24) appear to offer an effective means of generating design data and are currently being employed by NCRE for this purpose.

68. The influence of lateral load on inelastic column strength of stiffened panels may be examined using finite element analysis of the type described in the present paper; two-span theoretical models should clearly be adopted in such calculations. Scope exists for parametric application of this type of analysis in developing data curves for ship-type sections under combined later:1 and compressive loads. Such curves have already been derived for single-span panels (62, 63) but these should be used with caution because of the possibility of interaction between adjacent panels in a continuous grillage.

## Effects of Residual Stress and Initial Deformation

69. Recent theoretical and experimental studies (8, 9, 15, 16, 23) have led to a much improved understanding of the influence of residual stress and initial deformation on strength and stiffness of square and rectangular plates under uniaxial compression. Further investigation is however needed into the effects of initial deformation and bidirectional residual stress on plate behaviour under biaxial compression.

70. The results summarized in Table 8 indicate that high compressive plating residual stresses can cause substantial reductions in column strength of stiffened panels. Results also indicate that strength of stiffened panels may be sensitive to the distribution of residual stress in longitudinal girders. The apparent importance of residual stress effects suggests that some priority should be given to

(i) obtaining an improved statistical knowledge of residual stress distributions in ships' plating and stiffeners by carrying out measurements during hull construction;

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- (ii) seeking an improved understanding of the influence of welding procedure on residual stress;
- (iii) carrying out further theoretical and experimental investigation into the influence of residual stresses on compressive strength.

Consideration should be given to the behaviour of fabricated tee-stiffeners in which high tensile residual stresses, induced in the table by welding and flamecutting, seem likely to have a beneficial effect: consideration should perhaps also be given to methods of inducing favourable residual stress distributions by heat-treatment of plating and rolled stiffeners during construction. Attention should be paid to the influence on compressive strength of cold-bending of stiffeners which may occur during the fabrication process: recent analysis (58) has shown that residual stresses induced in tee-bars by fully distic coldbending with elastic springback, followed by welding the tee-bars to a plate, can result in losses of up to 35% in column strength of the stiffened plate. The amount of curvature required to induce fully plastic bending of rolled stiffeners is small and could readily occur as a result of straightening of distorted stiffeners during construction of a flat stiffened panel.

71. It should be borne in mind that residual stresses induced in a ship's hull by welding and other manufacturing operations will to some extent be "shaken out" by alternating tensile and compressive loads associated with wave-induced bending of the ship's hull. This process has been discussed by Faulkner (70), who suggests that for design purposes effective residual stresses should be assumed to be about 75% of initial, as-fabricated values. Further investigation of the shake-out process is needed involving statistical evaluation of the extent to which residual stresses reduce before extreme compressive loads occur, perhaps based partly on direct measurement of residual stress changes on ships in service.

72. Initial deformation of stiffened panels generally causes reduction of compressive strength, although in some cases the effect of initial distortion of stiffeners can be beneficial (see Table 8) by causing tensile bending stresses

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which delay compressive yield in stiffener outer-fibres; the influence of initial stiffener deformations is complicated by interaction between adjoining interframe spans of a continuous grillage. A need clearly exists for thorough statistical evaluation of plating and stiffener distortions in ships' hulls. It is suggested that the best approach to this problem is to employ a "ripplescanning" device of the type developed at Cambridge University (71) to record continuous profiles of plating and stiffener distortion and to analyse statistically the modal (Fourier) components of deflection profiles.

## Note on Experimental Procedure

73. A need has been indicated for further collapse tests on welded steel grillages, both as a basis for evaluating new theory and as a source of empirical design data. In carrying out such experiments it is suggested that the following experimental procedures should be followed where possible:-

- (i) Design of Test Structures: in order to avoid experimental error caused by non-uniformity and eccentricity of compressive load and by unrealistic support conditions, orthogonally stiffened grillages containing at least three and preferably four or more interframe spans should be tested rather than single-span stiffened panels; in such test structures realistic allowance will be made for interaction between adjoining spans. End-bays should be reinforced (or reduced in length or manufactured in higher strength material) in order to avoid premature failure in these regions.
- (ii) Haterial Properties: Compressive as well as tensile yield strength should be measured using standard test methods with reasonably low strain rates (eg in accordance with BS 18); test specimens should be sufficient in number to provide a statistical definition of material variability.
- (iii) Initial Deformations: plating and stiffener distortions should be thoroughly surveyed, including horizontal deformation of stiffener tables; continuous profiles of plating distortion should be recorded

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using a scanning device in order to allow evaluation of modal deflection components.

- (iv) Residual Stresses: high priority should be given to measurement of residual stresses in stiffeners as well as plating, ideally at various stages during the welding procedure. Weld-induced strains are readily measured using a small portable extensometer (eg a Demec extensometer); readings should be checked for repeatability and corrected for temperature effects. Particulars of the welding procedure should be recorded, including the amount of weld material deposited and/or the welding voltage and current employed.
- (v) Deflection Measurements: a thorough record of plating and stiffener deformations should be obtained, including horizontal deflection of stiffeners wherever tripping is likely to occur and including permanent sets.
- (vi) Strain Measurement: measurement of strains at carefully selected positions is clearly desirable as a means of monitoring local plating and stiffener deformations and of checking the uniformity of compression across test grillages; it should however be borne in mind that, particularly during the latter stages of load application when parts of the structure have yielded extensively causing permanent set of strains, evaluation of stresses may be impossible.
- (vii) Load Application: ideally test structures should be subjected to controlled end-shortening up to and beyond collapse rather than to controlled loads; in this way deformations and strains may be correlated accurately with applied load in the sensitive range close to collapse where stiffness is low and may be negative, and a record may be obtained of post-collapse load-carrying capacity and stiffness (which would be of interest, for example, in evaluating the strength of a multi-deck ship where collapse of one deck might not precipitate hull failure). Controlled application of end-shortening is likely to be achieved more

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effectively using a mechanical, screw-jack type of testing machine than using hydraulic load application unless a sensitive, travelrelated servo-control is incorporated in the hydraulic system. Loading procedures should include frequent returns to zero load

to provide a record of permanent set of deflections and strains. Statistical Evaluation of Grillage Strength

74. The compressive strength of a welded grillage depends on several quantities, including yield strength, plate thickness, stiffener dimensions, initial deformations and residual stresses, which are inherently irregular, whose exac values cannot be specified for design purposes and which should therefore be defined statistically. A need clearly exists (70, 72) for a statistical approach to evaluation of grillage strength; several obstacles remain to be overcome, however, before reliable statistical design methods can be established. In pursuing such design methods it is suggested that there is a need for:-

- (i) Evaluation of basic grillage statistics, referring to manufacturing imperfections and irregularity of geometry and material properties, by direct measurements carried out on ships during construction and in service (including assessment of corrosion effects); most existing data refer to small-scale models constructed in laboratory workshops in which weld-induced deformations and residual stresses and variability of yield strength, plate thicknesses etc are urrepresentative of actual ship structures.
- (ii) Further development of deterministic analysis methods and design data (including statistical assessment of accuracy) as a basis for converting statistically defined dimensions, material properties and manufacturing imperfections into a statistical definition of grillage strength.
- (iii) Investigation of various complications in the statistics of grillage behaviour including dependence on sequence of loading, interdependence of variables such as residual stress and initial deformation and

interactions between different components of a grillage and different modes of failure.

Further experimental evaluation of grillage strength also has a key part to play but cannot be expected to provide direct statistical descriptions of grillage strength; large-scale tests of the type described in the present report are too expensive to carry out in sufficient numbers and small-scale tests are statistically unrepresentative for the reasons mentioned above. It is suggested that the main object of further grillage tests should therefore be to guide the development of improved analysis methods and to check the accuracy of analysis methods and design data with provision of empirical corrections where necessary. ACKNOWLEDGMENT

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Statistic Statistics and Statistics

# DINGNSIONS OF TEST STRUCTURES (INCHES)

	Plating Thickness	1 iokme ss			Longitudina	idinal Girders	8				Trant	Transverse Franes		
Grillage No	Mean	800	Spacing	Mean Overall Depth	Mean Table Width	Nominal Table Thickness	Mean Web Thickness	Mean Weld Area (in <sup>2</sup> )	Spacing	Mean Overall Depth	Mean Table Width	Nominal Table Thickness	Mean Web Thickness	Mean Weld Area (in <sup>2</sup> )
7	0.315	0.02	24	6.05	3.11	0.56	0.284	0•067	84	10.14	4004	0.72	0•369	0.108
đĩ	0.310	£0•0	24	•0•9	3.00	95*0	0.28	1	4,8	10.0*	5.0*	0.72	0.36*	ł
28	0.304	0.02	ส	4.55	1.81	0.375	0.214	9+10=0	60	8.07	4.04	0.64	0.327	0.053
2b	0.290	0.02	ส	4.50	1.76	0.375	0.212	0*037	60	8.02	<b>4</b> •04	0.64	0.328	0. (ULB
Ŕ	0.251	0.01	77	3.06	1.02	0•25	0.178	0*01*0	60	6.15	3.11	0.56	0.268	0.044
ጽ	0.252	0.02	ជ	3.04	1.10	0.25	0,183	0.033	60	6•06	3.12	0.56	0.271	0.057
4	0.253	0•02	10+	3.02	1.09	0,25	0.191	0*036	8 <del>1</del>	7.95	4•03	0.64	0•340	0*049
<del>१</del> १	0.252	0.02	10++	<b>3.</b> 03	1.03	0,25	0*1%	450.0	4,8	8.05	<b>4•0</b> 4	0.64	0.329	0•045
5	0.253	0.02	24	4.57	1.82	0.375	0.210	0.027	60	6.07	3.04	0.56	0.266	0.032
9	0.249	0.01	24	3.00	1,08	0.25	0.179	0°034	4,8	4.51	1.82	0.375	0.211	0.035
7	0.248	0.01	24	4.53	78	<u>525</u> 0	0.203	6£0°0	60	6.06	3.10	0.56	0.262	0.042
œ	0. 353	0.03	12**	4.5.	•c/	0.375	0.20*	1	48	8.0*	4.0*	0.64	0.32*	1
	and the factor of the													

Nominal values
 interspersed by 12.1 x 0.253x 3.07 x 0.375 fabricated tee-bars at 40 in spacing
 interspersed by 12.1 x 0.252x 2.99 x 0.375 fabricated tee-bars at 40 in spacing
 interspersed by 12.0 x 0.355 x 3.0 x 0.375 fabricated tee-bars at 60 in spacing

And States and States and American

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## NATKRIAL YIELD STRENGTH

	Plating	âni	Longitudinals	ina l.s	Fabricated Tee-Bars	Tee-Bars	Transverses	erses
Grillage No	Mean Yisld tsi	AOD	Mean Yield tsi	COV	Mean Yield tsi	OOV	Mean Yield . tsi	400
18	16.4	<b>6</b> •03	16.7	0.04			18.5	0.05
41	16.6	0.05	16.5	<b>•</b> *0	1		18.4	0.07
28	17.2	<b>10</b> •0	17.7	60 • 0		I I I	16.1	0.11
2b	17.1	0.02	18.1	0.10		1	15.5	70°07
3a	16.5	0.04	15.0	0.02	-	1	17.5	0.12
<b>3</b> b	16.6	0.03	14.7	0.02	-		18.0	0.07
4a	17.1	0• 06	15.4	0.03	17.7	0.07	16 <b>•</b> 9	0.10
<b>द</b> †	17.4	0.05	15.0	0.02	17.7	0.07.	16.1	0.07
5	16.3	40.04	15.2	<b>60.03</b>	1		17.8	<b>60</b> •0
6	16.9	0.02	15.9	10.0	1		17.5	0.04
7	19.1	0*06	20.1	90.06	1		17.4	0.05
8	18.0	0•07	19.8	0.08	19.3	0.08	15.3	0.13

## DERIVED GRILLAGE PARAMETERS

	Pl	ating	Lo	ngitudinal	Ls
Grillage No	b/t	$p = \frac{b}{c} \frac{\sigma_y}{E}$	a/k	$\lambda = \frac{a}{k\pi} \int_{E}^{\sigma} \frac{y}{E}$	$\frac{A_{st}}{bt}$
la	76.2	2.67	21	0.24	0.42
lb	77.4	2.72	21	0.23	0.43
2a	39.5	1.42	36.5	0.42	0.40
2b	41.4	1.48	36	ി.42	0.42
3a	47.8	1.68	66	0.70	0.24
3b	47.6	1.68	66	0.70	0.24
4a	39.5	1.41	50 <b>*</b>	0.54*	0.28*
4Ъ	39.7	1.43	50 <b>*</b>	0.53*	0.28*
5	94.9	3.31	42	0.45	0.24
6	96.4	3.42	68	0.75	0.12
7	96.8	3.65	42	0.52	0.24
8	34.0	1.25	30*	0.37*	0.35*

• refers to small longitudinals

 $A_{st} = stiffener area$ 

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## <u>TABLE 4</u> SUMMART OF DRITTAL DEPORMATIONS

idth         Vertion         Derivation         Vertion         Vertion           idth         cov         Upmind         Max         Maan         ov         Demined           mann         cov         Upmind         Max         Maan         ov         Demined           mann         cov         Upmind         Max         Maan         ov         Domined           0.0050         0.446         67         0.0015         0.0011         0.55         0         0           0.0077         0.446         100         0.0015         0.0011         0.55         0         0         0           0.0050         0.446         78         0.0012         0.0101         0.55         11           0.0050         0.22         78         0.0028         0.408         22           0.0051         0.402         0.0028         0.408         22         33           0.0051         0.402         0.0028          33         33           0.0051         0.402         0.0028          33         33           0.0051         0.4017         0.0028         0.418         34         33           0.0051 <th>Nax P</th> <th>Max Plate Deflection</th> <th>tion</th> <th>Centre</th> <th>I Interfi</th> <th>The Defle</th> <th>otion e</th> <th>Central Interframe Deflection of Longitudinal Girders (as fraction of frame spacing</th> <th>nal Girde</th> <th>bra (aa fa</th> <th>raction</th> <th>of frame</th> <th>specing)</th> <th></th>	Nax P	Max Plate Deflection	tion	Centre	I Interfi	The Defle	otion e	Central Interframe Deflection of Longitudinal Girders (as fraction of frame spacing	nal Girde	bra (aa fa	raction	of frame	specing)	
Mat.         Mat.         Ove         Upmerial         Mat.         Mat.         Ove         Demmerial           0.0105         0.0050         0.44         67         0.0015         0.0077         0.77         25           0.0105         0.0050         0.44         67         0.0015         0.0071         0.55         25           0.0156         0.0044         0.52         89         0.0045         0.0011         0.55         11           0.0156         0.0044         0.52         89         0.0045         0.0012         0.55         11           0.0105         0.0044         0.52         89         0.0045         0.0026         11         22           0.0150         0.22         3094         0.52         78         0.0029         22         11           0.0215         0.0150         0.44         67         0.0029         0.010         56         25           0.0224         0.0150         0.017         0.0029         0.017         20048         35           0.0224         0.017         0.017         0.0029         0.017         20048         35           0.0224         0.017         0.017         0.017		te sidth	;	Vert loa	1 Deflect	iten (Upm	(pu	Vert loal	Deflectio	n (Downwa	(PJ)	Horison	Horisontal Deflection	otion
0.0105         0.0050         0.44         67         0.0015         0.077         25           0.0145         0.0077         0.46         100         0.0015         0.435         0           0.0145         0.0044         0.42         89         0.0015         0.46         10         25         11           0.0145         0.0044         0.42         89         0.0045         0.46         11         22           0.0145         0.0044         0.42         89         0.0045         0.46         11           0.0165         0.0044         0.42         78         0.0010         0.48         22           0.0145         0.0044         0.42         78         0.0010         0.48         22           0.0202         0.0059         0.44         67         0.0028          55           0.0215         0.41         0.0017         0.017         0.017         56         56           0.0244         0.5016         0.5016         0.017         0.017         56         56           0.0244         0.5016         0.5016         0.5017         0.017         56         56           0.0244         0.5016<	H	-	004	Upmind	Kax	5	800	Dommerd	Ma X	Ken	400	NaX NaX	ar Juni Mean 00	100
0.0136         0.0077         0.46         100         0.0015         0.0011         0.535         0           0.0135         0.0044         0.32         89         0.0043         0.055         11           0.0145         0.0044         0.32         89         0.0043         0.0525         13         1           0.0145         0.0044         0.23         78         0.0010         0.48         22           0.0202         0.0093         0.44         67         0.0029         0.0028          33           0.0202         0.0150         0.51         44         0.0024         0.019         0.87         56           0.0249         0.0150         0.51         44         0.0024         0.019         56           0.0249         0.0150         0.51         0.012         0.012         0.97         56           0.0244         0.024         4.5         0.0023         0.012         0.012         56           0.0244         0.012         0.012         0.0023         0.012         0.012         56           0.0244         0.012         0.012         0.012         0.012         0.01         56			°	67	0.0015	0,0007	0.77	25	0.0015	0.0013	0	ł	ł	:
0.0015         0.0044         0.32         89         0.0045         0.055         11           0.0015         0.0046         0.25         78         0.0010         0.46         22           0.00165         0.0005         0.044         67         0.0029         0.0010         0.48         22           0.00202         0.0015         0.44         67         0.0029         0.0019         26         33           0.00215         0.51         44         0.0011         0.4013         0.67         33           0.0224         0.0012         0.51         44         0.0012         0.67         33           0.0244         0.501         0.5024         0.5024         0.012         0.67         33           0.0244         0.5011         0.5024         0.017         0.017         0.67         33           0.0244         0.5012         0.5012         0.0023         0.57         33         33           0.0244         0.5012         0.5012         0.5013         0.56         33         33           0.0244         0.5012         0.5013         0.5013         0.56         33         33           0.0240         0.5013 <td>-</td> <td></td> <td></td> <td>100</td> <td>0.0015</td> <td>0,0011</td> <td>0.53</td> <td>o</td> <td></td> <td>-</td> <td>1</td> <td>0*0010</td> <td>0.0004</td> <td>0, 76</td>	-			100	0.0015	0,0011	0.53	o		-	1	0*0010	0.0004	0, 76
0.0085         ~.0060         0.25         78         0.0010         0.46         22           0.0202         0.0095         0.44         67         0.0029         0.067         55           0.0203         0.0150         0.51         44         0.0029         0.0619         67         56           0.0204         0.0150         0.51         44         0.0024         0.67         56           0.0244         0.0150         0.51         44         0.0024         0.67         56           0.0244         0.012         0.012         0.012         0.012         56         56           0.0244         0.012         0.0024         0.017         56         56         56           0.0244         0.0245         0.0024         0.017         56         56         56           0.0245         0.025         0.026         1.00         7         56         56           0.0245         0.017         0.0028         1.04         56         57         56           0.0245         0.0125         0.52         58         0.0028         1.04         56           0.0125         0.0125         0.22         58		-		68	0.0043	0.0025	0.35	n	0,0011	0,0011	•	:	:	
0.0202         0.0095         0.44         67         0.0029         0.0028          53           0.0215         0.0150         0.51         44         0.0029         0.667         56           0.0215         0.0150         0.51         44         0.0044         0.0019         0.87         56           0.02445         0.0011         0.54         67         0.0026         0.637         56           0.1017         0.1017         0.0021         0.0323         0.37         56           0.1017         0.017         0.0020         0.023         0.37         56           0.0173         0.0212         0.0017         0.0023         0.34         56           0.0125         0.023         0.012         0.0033         0.34         56           0.0125         0.223         0.023         0.34         56         57           0.0125         0.223         0.023         0.023         0.43         42				78	0.0019	0.0010	0.48	22	0,0012	0.0006	0.86	i	1	;
0.0255         0.0150         0.57         44         0.0041         0.67         56           0.0244         0.0081         0.51         44         0.0046         0.637         55           0.0244         0.0081         0.51         67         0.0046         0.0025         (.57         55           0.144         0.6017         0.0026         1.04         56         55         55         55           0.145         0.655         44         0.0017         0.0028         1.04         56           0.145         0.0125         0.55         58         0.0031         0.029         57           0.0175         0.0125         0.22         58         0.0031         0.029         57				67	0,0029	0.0028		33	0,0006	0• 0006	•	1	;	:
0.02445         0.0081         0.34         67         0.0036         (.37         33           0. 4045         -0.053         0.655         44         0.0017         0.0028         1.04         55           0. 4045         -0.653         44         0.0017         0.0028         1.04         56           0. 4045         -0.653         65         7         7.0012         0.0268         1.04         56           0. 0253         0.025         0.025         0.0263         1.04         26         57           0.0173         0.0253         0.22         58         0.0020         0.43         42           0.0173         0.0212         0.22         58         0.0021         0.020         0.43         42				***	0.0041	0,0019	0.87	X	0.0030	0.0017	0+59	•	-	
0. Lord        0755         0.65         44         0.60017         6.0008         1.04         56           0. Lord        0755         0.655         64         0.60010         1.042         56           0. 2.22         58         0.0051         0.0020         0.45         42           0.0175         0.022         0.25         58         0.0051         1.27				67	0.00 %	0.0023	(.37	55	0.0030	0*0018	0•80	0.0161*	0*00#7*	1.18*
• • • • • • • • • • • • • • • • • • •		_	5	73	0.6017	6.0008	1.04	38	0*00*0	0, 0012	0,86	0.0073	0.0024	•66*0
0.0125 0.22 58 0.0051 0.0020 0.43 42	5 <sup>1</sup> . 2		3	\$9	0100°s	8-00 <b>-</b> 8	0,29	57	0 <b>-0</b> 0U3	0,0002	05•0		-	-
	6 0.01		ŏ	R	0.0051	0*0050	64.0	3	0,0025	0,0012	0.72	1	1	1
	1 0.0208	96 0°000	0.58	58	0.0015	2000*0	92 °0	24	0,0003	0, 0002	0.62	ł	•	1

· in deep fabricated girders

\*\* insufficient readings to provide neunineful cov

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# VELD-DIDUCED RESIDUAL STRESSES

## (Stresses in tai) .

	Conpress	ive Stree	Compressive Stress ( $\sigma_{ m rc}$ ) at centres of plate panels	res of pla	ite panels	Longitudinal Girder Outer-Fibre Stress	al Girder • Stress
Grillege		Longitudinal	dinal	Transverse	1756	(compressive)	ssive)
0	<b>ne</b> an	COV	Bst imated J	nean	400	Bean	<b>A</b> 00
la I		-			ł		1
91	8	-	8			1	
28	12.4	0.13	• <b>†</b> •9	-	ļ	2.8	0• 70
2b	8.5	0.38	5.1*	1	-	0.8	0.73
Ŗ	4.6	0 <b>، بد</b>	6.6*	-	•	889	
R	10.7	0.52	7.2*	•		9 9 8	8
484	9-8	0.12	5.5*	8	-	8	8
<b>व</b> †	10•7	0.35	5.8*			8	8
5	4.0	0.16	<b>**</b> 6	•	8	8	
و	5.2	U.24	11.4			5	8
7	2.4	0.41	5.4	1.9	0• 75	2.4	0.48
80	7.4	0.28	3.7	3.1	0.43	0.72	2.44

16

• assuming average  $r_{c} = 2/3$  x values at centres of plate panels

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THEORETICAL ELASTIC BUCKLING STRESSES (131)

	Plate	Plate Buckling	Interfra <b>me</b> Flexur Buckling (n = 1)	Flexural (n = 1)	Interframe Tripping (n = 1)	Overall Grillage Buckling (m = 1)	rillage (m = l)
	Bryan Formula	Folded Plate Analysis	Buler Formula	Folded Plate Analysis	Folded Plate Analysis	Orthotropic Plate Formula	Finite Blement Analysis
		17.40 (n = 1)			•6•£1	136 (n = 2)	131 (n = 2)
la	8.34 (n = 2)	$11.1^{\circ} (n = 2)$	212	219*	95.5*	121 (n = 3)	119 $(n = 3)$
		11.2 (n = 3)				152 $(n = 4)$	149 (n = 4)
28	31.1 (n = 5)	8	88.4		8		-
						34•3 (n = 2)	
Ř	21.2 (n = 5)	t 8	28.9	ł		28.3 (n = 3)	
						31.4 (n = 4)	
		30 <b>.</b> 3 (n = 4)			29°0*		
a.	22.4 (n = 4)	$29.7^{\circ} (n = 5)$	49•5	51° 3*	29 <b>.</b> 0*	1	1
	·	31.0 (n = 6)					
						112 (n = 1)	110 $(n = 1)$
5	5.38 (n = 2)	i	65.5	ł	1	43.2 (n = 2)	(2 = u) {.th
						48.4 (n = 3)	51.7 (n = 3)
						42.2 (n = 1)	(I = u) †*t†
9	5.21 (n = 2)	ł	26.6		1	14.5 (n = 2)	14.3 (n = 2)
						14.0 (n = 3)	13.9 (n = 3)

TABLE 6 (Contd)

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	• • •			
rilla <i>g</i> e (m = l)	Finite Element Analysis		1	
Overall Grillage Buckling (m = l)	Orthotropic Plate Formula	(1 = 1) 7TI	43.2 (n = 2)	48.4 (n = 3)
Interframe Tripping (n = 1)	Folded Plate Ânalysis		1	
Flexural n = 1)	Folded Plate Analysis			
<pre>Interframe Flexural Buckling ( n = 1)</pre>	Euler Formula		65.5	
lok1 ing	Folded Plate Analysis			
Plate Buckling	Bryan Formula		5.17 (n = 2)	
	ON		2	

\* see figure 5

## SUMMARY OF COLLAPSE LOADS

Grillage No	Lateral Pressure psi	Average Applied Compressive Stress $\sigma_u$ (tsi)	Strength Factor ¢ *
19	0	12.4	0.76
lb	15	12.1	0.73
2a	7	15.9	0 <b>.9</b> 1
2ъ	0	14.7	0.83
<u>3</u> a	3	11.1	0.69
3b	0	9.8	0.61
4a	0	13.3	0.82
4Ъ	8	13.5	0.83
5	0	11.4	0.72
6	0	8.1	0.49
7	0	12.0	0.65

•  $\phi = \sigma_u / \sigma_{yave}$  (Ref 45)

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INELASTIC FLEXURAL BUCKLING: COMPUTED COLLAPSE LOADS

Г																	
;	Collapse (tsi)	Grillage 5	15•5*	34°4*	10.6	10.4	10.0	10.4	9•5	9•6	9•2	10.0	10.0	9•6	6*6	6•6	13.5*
	Computed Stress	Grillage Ja	14.7	12.8	12,8	12.2	11.4	13.0	12.2	10.9	9•6	11.4	11.4	8•0	9*6	0*6	9*6
	Lateral	DBOM	0	0	0	0	0	0	0	Ъ	2	0	0	0	0	1	0
	Neutral Axis	Shift	г.	1	1	Ъ	1	1	0	ı	1.	1	J	7	г	l	1
	Residual	<b>UT10</b> 88	0	л	J	2	3	47	3	. 3	Ś	3	3	3	3	3	0
	Initial	Jer ormanion	г	1	1	-1	Ч	1	1	г	l	0	2	-1		-1	-1
	Boundary	Condition	Simple Support	F	E	F	T	E	E	E	E	E	E	E	E	t	E
	Calculation	O N	Ч	7	3	4	Ŋ	9	2	æ	6	10	11	21	13	<b>†</b> т	15

TABLE 8 (Contd)

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<b>Calculation</b>	Boundary	Initial	Residual	Neutral	Lateral	Computed Collapse Stress (tsi)	Collapse (tsi)
No	Condition	Deformation	Stress	AX15 Shift	Load	Grilla <i>g</i> e Ja	Grillage 5
16	Clamped	1	3	1	0	7.11	10.2
17	E	-1	3	ı	0	11.4	6*6
18	T <b>wo-s</b> pan	<del>-</del> 1 -2	3	ı	0	9•3	6•7
19	E	0 1	3	I	0	9.8	9•8
20	E	<del>4</del> 1	3	1	0	10.4	6•6
21	E	<u>1</u> 2	3	ı	0	10.6	6*6
22	F	4 1	3	1	Т	9.8	6•7
23	<b>4</b>	0 1	1	I	0	11.8	, <b>7.</b> 01
24	E	-L <u>-</u>	3	1	0	9•0	9.7
25	t	-1 2	3	1	1	9•6	6*6

\* plate buckling effects excluded



ARRANGEMENT OF TEST RIG

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## GRILLAGE 7: WELD - INDUCED RESIDUAL STRESSES MEAN VALUES IN PLATING, OUTER FIBRE VALUES IN GIRDERS



## IDEALIZED DISTRIBUTION OF PLATING RESIDUAL STRESS







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and the second second second second second

FIGURE 7 NCRE/R611





GRILLAGE IG FULLY DEVELOPED COLLAPSE



FIGURE B (a)

	GRILLAGE	10
FULLY	DEVELOPED	COLLAPSE

FIGURE 8 (b)

GRILLAGE 10 FULLY DEVELOPED COLLAPSE





FIGURE B(c)

FIGURE 8 (d)

GRILLAGE 10 FULLY DEVELOPED COLLAPSE



FIGURE 8 (e)

## (DEFLECTION DATA FOR LONGITUDINAL GIRDERS AFFECTED BY DISTURBANCE OF DATUM FRAME)

## GRILLAGE ID



GRILLAGE 20 COLLAPSE VIEWED FROM STIFFENER SIDE

<u>GRILLAGE 20</u> Collapse viewed from plating side



FIGURE IO (a)



FIGURE IO (b)

GRILLAGE 30 - COLLAPSE



FIGURE II

GRILLAGE 35 - COLLAPSE





- Standard





and a second state of the date from both the second state of the

GRILLAGE 40 INITIAL STAGES OF COLLAPSE



FIGURE 15 (a)

GRILLAGE 46 INITIAL STAGES OF COLLAPSE GRILLAGE 4a FULLY DEVELOPED COLLAPSE



FIGURE 15 (b)

GRILLAGE 45 FULLY DEVELOPED COLLAPSE





FIGURE 16(a)

FIGURE 16 (b)

GRILLAGE 45 COLLAPSE VIEWED FROM PLATING SIDE



FIGURE 16(c)

GRILLAGE 4a



GRILLAGE 45





and a specific the second billion and a second





## GRILLAGE 5 - COLLAPSE

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GRILLAGE 6 - COLLAPSE



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GRILLAGE 6

the state of the



FIGURE 22









## SUBDIVISION OF CROSS - SECTION INTO "FIBRES"

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