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INFLUENCE OF END RESTRAINT IN CYCLIC TRIAXIAL TESTS

by

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Theoretical considerations concerning the effect of end restraint on the strength of soil in triaxial tests are briefly discussed herein. The theoretical aspects are supplemented by a review of all laboratory test data available to the writer. Most of the data pertain to static drained tests on sands. There are also data from static undrained tests on saturated clays. A lesser amount of previously unpublished data from the writer's files on static and
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cyclic loading undrained tests on saturated sands are also included. The theoretical considerations and the available data are all consistent within themselves in demonstrating and explaining the significant conclusions which follow directly from basic considerations of the behavior of soil under various loading conditions. End restraints, such as are found in most ordinary triaxial test equipment, prevent lateral strains at the ends of the specimen and thereby lead to nonuniformities and concentrations in the stresses and strains throughout the specimen. As a result, volume changes in soils which dilate strongly when sheared will be affected. If tested undrained, this will be reflected by variations in pore pressure from that associated with uniform stress and strain distributions throughout. Since most granular soils tend to dilate, especially at low effective confining pressure, this volume change tendency leads to lower pore pressures and hence stronger saturated specimens with free ends than with restrained ends, whether tested statically or under cyclic loading. For the soils from which data are available, this strength increase with frictionless versus regular end platens is about 15-20 percent for static tests and up to 25-40 percent for cyclic tests. It appears that because clay soils show less dilatant tendency when sheared than sands, the effect of end restraint is less with clays than with sands and is almost negligible for normally consolidated clay from which data are available. Although there are considerable data from static tests, the data from cyclic tests are quite limited, involving only one sand and no clays. Therefore, the conclusions above must be considered as preliminary, pending further laboratory test results with both regular and frictionless end specimens.

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PREFACE

This report was prepared by Professor K. L. Lee under Contract DACW 39-75-M-1674 as part of ongoing work at the U. S. Army Engineer Waterways Experiment Station (WES) under CWIS 31145 work unit entitled "The Liquefaction Potential of Earth Dams and Foundations."

The work was directed by Dr. W. F. Marcuson III, Research Civil Engineer, Earthquake Engineering and Vibrations Division (EE&VD), Soils and Pavements Laboratory (S&PL). General guidance was provided by the following S&PL personnel: Messrs. J. P. Sale, Chief; S. J. Johnson, Special Assistant; and W. C. Sherman and Dr. F. G. McLean, former Chief and Chief, EE&VD, respectively.

Director of WES during this study and the preparation and publication of this report was COL G. H. Hilt, CE. Technical Director was Mr. F. R. Brown.

CONTENTS

CONVERSION FACTORS, U. S. CUSTOMARY TO METRIC (SI) UNITS OF MEASUREMENT AND METRIC (SI) TO U. S. CUSTOMARY UNITS

Units of measurement used in this report can be converted as follows:

Multiply	By	To Obtain
U. S. Customary to Metric (SI)		
inches	2.54	centimetres
pounds (force)	4.448222	newtons
pounds (force) per square inch	6.894757	kilopascals
pounds (mass) per cubic foot	16.01846	kilograms per cubic metre
degrees (angle)	0.01745329	radians
Metric (SI) to U. S. Customary		
millimetres	0.0393701	inches
centimetres	0.393701	inches
kilograms	2.20462	pounds
kilograms per square centimetre	14.2233	pounds per square inch

INFLUENCE OF END RESTRAINT IN CYCLIC TRIAXIAL TESTS

PART I: INTRODUCTION

Background

1. During the past 10 yr, thousands of cyclic triaxial tests have been conducted on a research and a commercial basis related to the problems of soil liquefaction potential, foundation and slope stability under earthquake, and other cyclic loading conditions. The early tests of this type reported in the literature[1-6] used 1.4-in.-diam* by 3- to 3-1/2-in.-long specimens with regular (fixed, frictional) end platens. The subsequent development of stability analyses methods for field problems has been based strongly on results of tests conducted in this manner, coupled with semiempirical factors to correlate the laboratory test results with the observed field behavior in case history studies.[7-11] These semiempirical correlation factors are required to allow for the acknowledged deficiency between actual field conditions and current laboratory and analytical capabilities.

2. Some of these deficiencies and corresponding correlation factors include the following:

a. Truly nonlinear, three-dimensional seismic stress analyses are currently not feasible. As an approximation, simplified plane-strain, equivalent linear calculation methods are used which incorporate strain-dependent linear elastic and damping factors.[12]

b. Cyclic testing with actual irregular load shapes is not currently feasible on a design basis. Therefore, as an approximation, uniform cyclic loads are applied and the data used with an equivalent number of uniform stress cycles from an equivalent linear seismic response analysis.[1]

c. Laboratory equipment which produces the correct three-dimensional total cyclic stresses or strains on the boundaries

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* A table of factors for converting U. S. customary units of measurement to metric (SI) units and metric (SI) units to U. S. customary units is given on page 4.

of a test specimen is not currently available. As a compromise approximation, cyclic triaxial and, in a few cases,[7,11] cyclic simple-shear equipment are used to produce design data.

3. Recognition of these many deficiencies has led to the abovementioned pragmatic approximations, so that the considerable knowledge developed to date can be used in practical designs rather than delaying until the total clear picture is developed. Confidence in using such approximations has developed as a result of several thorough case history studies.[4,9-11] In consideration of the many approximations involved, it is clear that much research is required before a thoroughly accurate method of seismic-stability analysis can be developed. Fortunately, this research is actively proceeding along many lines, at many research centers. However, for the present, the process of improvement is a long and difficult task. Each subtopic must be considered separately and thoroughly examined. Then, before the results can be applied in design, each topic must also be considered in connection with the semiempirical correlation factors which are also part of the overall design procedure. For example, an improved method of seismic-stress analysis may require simultaneously incorporating a somewhat different correlation factor (than that currently used) into the overall seismic-stability calculation so that the end results remain consistent with actual field case history observations. Similarly, use of cyclic strengths obtained from an improved laboratory testing procedure could only be justified, provided they were incorporated into an analysis method which recognized in the inherent correlation factors the improved testing techniques.

4. The introductory comments above were made so that the reader might keep the results of the study reported herein in perspective with respect to the overall use of laboratory test data in seismic-stability analyses. The studies described herein have concentrated on only one subaspect of the many ideal deficiencies in the present seismic-stability analysis procedures--the effect of end restraint in the cyclic triaxial tests. This aspect represents one small step in the enormous program of improving actual knowledge so as to be able to reduce the need for semiempirical correlation factors. However, it is emphasized that the

results from this study, or any single study of this nature, cannot be applied directly in seismic-stability analyses without simultaneously altering the appropriate correlation factors.

Past Experience on the Significance of End Restraint in Triaxial Testing

5. The literature abounds with results of previous studies conducted on the influence of end conditions in triaxial testing of materials. For example, tensile-test specimens of most nonsoil materials are made with long rods, often tapered gently in the midsection to avoid stress concentrations over the zone of interest. Unfortunately, long, thin compression-test specimens tend to buckle, so short specimens must be used. Thus the nonuniform conditions which may exist at the ends of the specimen may significantly affect the entire short specimen.

6. Part of the end-effect problems can be overcome by using smooth plane and parallel end platens to avoid obvious stress concentrations from rough or nonparallel ends. However, even with smooth ends, stress concentrations may develop due to the Poisson effect which exists in most materials; axial compression is accompanied by lateral expansion. If this lateral-strain tendency is prevented at the ends by friction between the specimen and the platen, stress concentrations will result. The amount of stress concentration will depend on Poisson's ratio. The influence of this stress concentration on the overall behavior of the specimen will depend on the length-to-diameter (ℓ/d) ratio of the specimen and on the material of which the specimen is composed. According to Timoshenko,[13] this end effect was recognized early in the development of laboratory compression testing techniques, and various steps were tried to minimize it. In 1900, A. Foppal reported using paraffin on the ends to eliminate the friction.[13] He found that with the treatment short cubic specimens ($\ell/d = 1$) failed by splitting vertically. He also found that if test specimens with regular ends were made longer than two to three times their diameter ($\ell/d = 2$ to 3), there was an apparent approximately uniform stress distribution in the middle portion, and the

end effect was practically eliminated. Timoshenko also reports that in 1927 Siebel and Pomp tried using conical ends with cone angles equal to the angle of friction, to eliminate the end friction effects. Similar and identical techniques have also been used in compression testing of soil specimens. These various experimental methods have been sufficiently described in recent literature;[14-26] therefore, repetition here is unnecessary.

7. Experimental work reported by Taylor[27] in the early 1940's, and summarized by Rutledge[28] in 1947, on the development of the triaxial test for soils led to the conclusion that reliable results could be obtained from triaxial tests on specimens with regular ends and with ℓ/d in the range about 1.5 to 3.0. On the basis of these early studies, the triaxial test specimens have been more or less standardized at $\ell/d = 2.0$ to 2.5, and end restraint has not been considered. However, it must be borne in mind that this conclusion was based largely on results of drained tests on sand and undrained tests on clay, without particular care for accurate measurements of stress-strain, volume change, or pore pressure. The adequacy of the test was based on terms of its ability to measure the overall strength of the specimen.

8. Several analytical studies have also been conducted to calculate the nature and extent of stress concentrations near the end of test specimens with frictional restraint at the platens. The earliest major work of this nature appears to have been by Filon[29] who in 1902 (long before the advent of computers or rapid hand calculators) presented results of many intricate calculations evaluating series functions to define stress distributions in compression test cylinders. Additional analytical results have also been more recently presented by Balla[30] and others.[31-34] The modern computer and the finite element method today make analytical solutions of this nature rather straightforward, including investigations of various elastic parameters and end conditions.

9. Finally, experimental measurements by means of load and deformation transducers have also been reported.[35-38] Most of the experimental data have been limited to measurements of vertical stress or strain. To

the extent of available experimental data, the results are consistently similar to the analytical results in showing that frictional platens lead to stress concentrations near ends. The stress concentrations decrease with distance from the platen, and with increasing ℓ/d.

10. To the writer's knowledge, no direct experimental measurements have been made of the shear stress distribution in a triaxial test specimen. However, analytical results have been obtained which should be considered credible because of the good agreement between analytical and experimental vertical stresses. For example, the theoretical shear stress τ_{xr} distribution throughout one quadrant of a triaxial test specimen, using numerical tabulations from Balla[30], is shown in Figure 1. The ideal τ_{xr} shear stresses should be zero throughout, but as indicated in Figure 1, a large portion of the specimen near the platens has significant shear stresses.

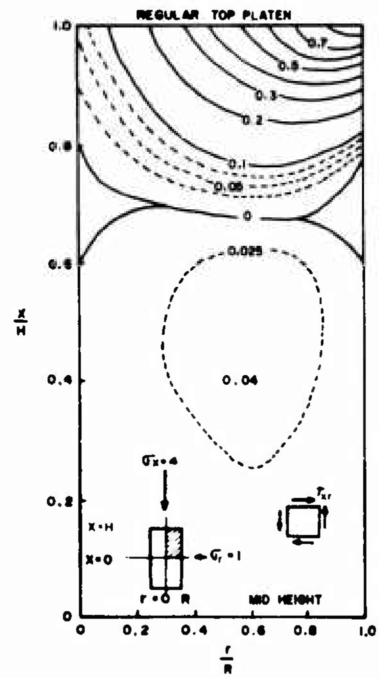


Figure 1. Distribution of shear stress throughout a triaxial test specimen with end friction (after Balla[30])

11. Recognition of stress concentrations at the ends of regular triaxial test specimens has led serious investigators in soil mechanics to question the commonly accepted principle that with ℓ/d greater than 2.0, the effect of end restraint may be ignored. This view has been particularly expressed with regard to the ever increasing demands of the triaxial test beyond a simple measurement of the soil strength. When triaxial tests are used to obtain accurate stress-strain, volume change, or pore pressure data, the concept that a long specimen will eliminate the need for special end-effect considerations needs further verification. Thus, Casagrande[39] suggests that the case for development of frictionless end platens, which was closed as a result of Taylor's[27] early work, should perhaps be reopened. In the

ensuing 15 yr since that comment, a number of additional investigations were made with frictionless (or rather, nearly frictionless) end platens on triaxial test specimens.

12. The earliest of these second generation studies was a detailed stress-strain-volume change investigation on granular material, conducted by Rowe and reported in the Proceedings of the Royal Society of London.[24] This publication, which was not generally available to soil engineers and which did not concentrate on the problem of frictionless platens, was followed shortly after by a comprehensive treatment of the subject published expressly for soil engineers.[25]

13. The method of developing frictionless platens used by Rowe in these studies was very simple. It involved only the use of hard and smooth plane ends, lubricated by inert silicone grease, which was in turn covered by a thin rubber sheet to separate the grease from the soil. By this means the angle of sliding friction between the soil specimen and the platens was reported generally less than 1 deg in the range of confining pressures used for many tests (10 psi or greater). Prior to this, Roscoe[40] had used greased rubber to eliminate friction between the soil specimen and the sidewalls of his simple shear apparatus, and Leonards and Girault[41] used lubricated sleeves to reduce sidewall friction in a consolidation test; however, Rowe seems to have been the first to use this technique for reducing end friction with triaxial test specimens.

14. Publication of these articles inspired two discussions on the frictionless aspects[22] and many subsequent studies which have continued up to the present time. Virtually all have used the silicone grease-rubber sheet concept developed by Rowe, some with one layer and some with more.[22] Some studies have used frictionless platens, as did Rowe,[24] simply as a means of obtaining results better than those obtained with regular platens.[42] Others have concentrated more on the relative benefits to be gained by frictionless ends as opposed to regular ends, and have compared test data from both types of equipment.[20,26,35,36] Unique among these comparative studies are the results of vertical stress measurements made at the frictionless platen of an

8-in.-diam by 8-in.-high triaxial compression test specimen, reported by Kirkpatrick, Seals, and Newman.[35] Within the range of experimental scatter, the test data indicate completely ideal uniform vertical stress across the entire end platens, equal to the average applied stress. As mentioned earlier, another series of tests using regular platens were reported in the same paper[35] which showed over a 200 percent variation in the normal stress on the platen from the center to the edge of the specimen.

15. Among the many studies of the effect of greased rubber as a means of reducing soil-platen friction are some previously unpublished data from Lee[43] although referred to briefly by Lee and Seed.[22] Sliding tests were conducted using 3- by 3-in. square blocks of platens and sands using high-vacuum silicone grease and rubber sheet to reduce friction.

16. Silicone grease was selected after several other compounds were tried and found to be greatly inferior. These other compounds included:

a. Teflon plastic with and without greased rubber
b. Powdered molybdenum disulfide
c. Powdered molybdenum disulfide mixed with oil
d. Surgical lubricating compound
e. S.T.P., a mineral oil compound used in the automotive industry to reduce bearing friction

17. In addition, sliding tests were conducted with sand over smooth blocks of several types of solid materials without lubricants. A well-defined linear relation was found in all cases between sliding resistance and normal loads that could be conveniently expressed in terms of an angle of sliding friction ϕ_μ. The normal stresses for these tests ranged from about 10 to 100 psi. The angle of sliding resistance for the materials tested was as follows:

Material Without Lubricant	ϕ_μ, deg
Sand on sand	43
Sand on Lucite	25
Sand on Teflon-coated steel	22
Sand on aluminum	20
Sand on steel	13

18. Following these tests, several sliding friction tests were conducted with greased rubber between sand and the smooth block. Monterey No. 20 sand was used; it is very uniform with subangular grains and a mean diameter $D_{50} = 0.6$ mm . Two different rubber membranes 0.002 and 0.015 in. thick, respectively, were used in the tests. The sliding tests were conducted with one layer of greased rubber on smooth 3-in.-square blocks of steel or Lucite. Identical results were obtained with each type of block, with sliding force directly proportional to normal force up to the limit of tests at 100-psi normal stress.

a. 0.002-in.-thick rubber: $\phi_\mu = 12$ deg

b. 0.015-in.-thick rubber: $\phi_\mu = 2.5$ deg

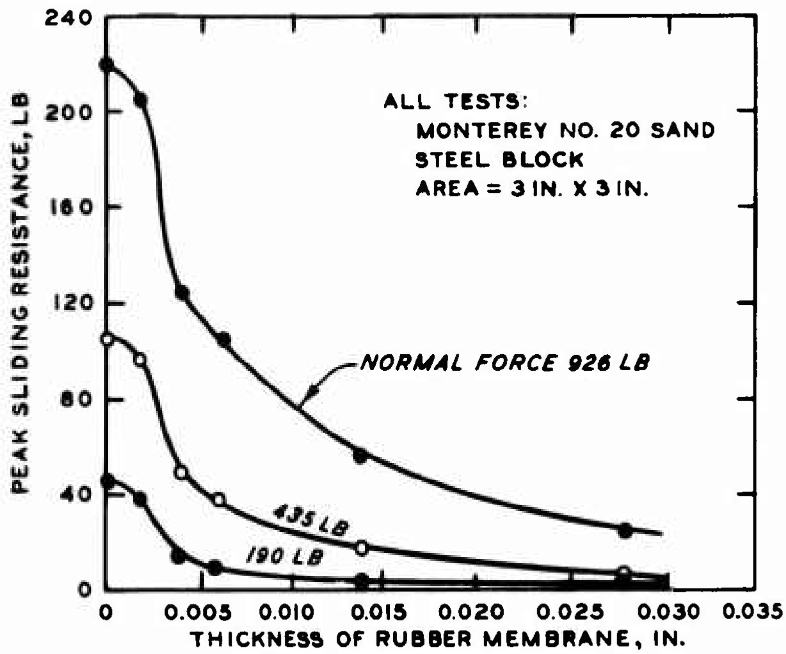
19. These sliding friction angles were somewhat larger than reported by Rowe,[24,25] and it was believed that they were probably due in part to the large, angular particles which tended to penetrate the grease and rubber and touch the solid block. Therefore, some additional tests were conducted using various thicknesses of rubber made up by using different numbers of layers of rubber sheets. Each sheet was separated by a layer of silicone grease; the results of these tests are shown in Figure 2a. Note that the frictional sliding resistance decreases rapidly with increasing thickness of greased rubber.

20. To further determine the effect of soil penetrating the greased rubber, tests were conducted with various sizes of sand ranging from silt to Ottawa Standard 20-30 sand; these data are shown in Figure 2b. Note that for a constant thickness of greased rubber, the sliding resistance greatly reduces with decreasing grain size.

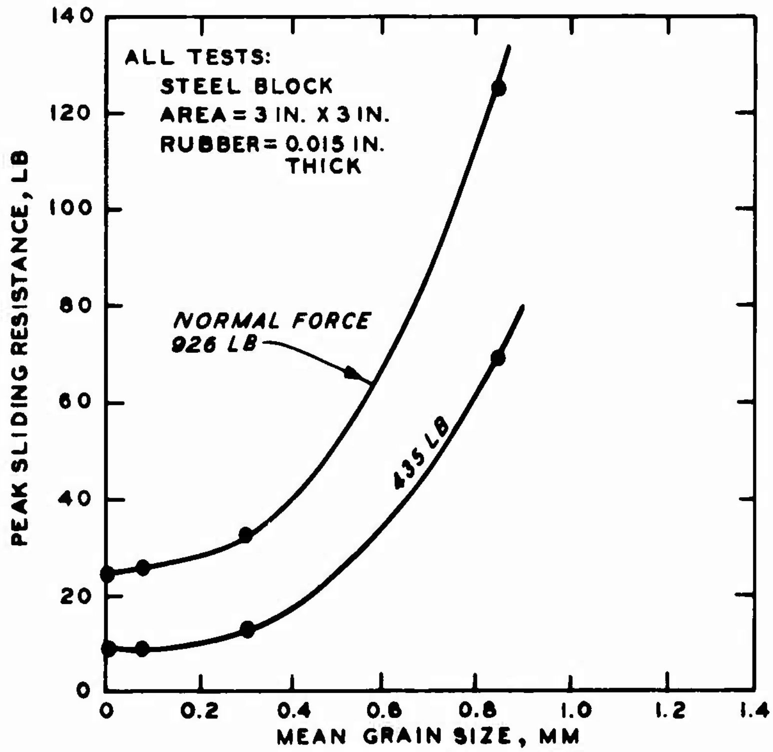
21. Finally, to determine the effect of prolonged consolidation, one test was conducted after 15 hr rest. This test was followed by a second test (at a higher normal load) on the same greased rubber without changing the grease or unloading the normal force. The results were as follows, as compared with the previously described tests.

a. Reference test after 5-min rest — $\phi_\mu = 2.5$ deg

b. Peak frictional resistance after 15-hr rest — $\phi_\mu = 25$ deg



a. EFFECT OF RUBBER THICKNESS



b. EFFECT OF GRAIN SIZE

Figure 2. Effect of rubber membrane thickness and solid grain size in sliding resistance tests (from Lee[43])

c. Peak frictional resistance after test but with increased normal load — $\phi_\mu = 12$ deg

22. As indicated by this last series of tests, a potential disadvantage of using silicone-grease lubricant is that if it is fluid enough to reduce friction, it may also squeeze out from between the rubber and the platens during a prolonged loading period. To overcome this possibility, the more viscous, high-vacuum silicone grease is recommended.[20,25]

23. The rate at which the grease is squeezed out from between the rubber and the platen and the subsequent effect on the sliding friction were investigated by Duncan and Dunlop.[20] Their results are summarized in Figure 3; the results of sliding block tests on frictionless platens are shown in Figure 3a. Note that both the low-viscosity stopcock grease and the high-viscosity, high-vacuum grease are equally effective in reducing friction, providing they are of adequate thickness. A film of grease only 0.1 mm thick provides almost no measurable frictional resistance. This film is readily obtained immediately after the test is set up. In time, the thickness of the grease film reduces according to the following equation:[20]

$$h^2 = \frac{h_i}{1 + \frac{4}{3\pi} \frac{h_i^2 Ft}{\nu r^4}} \tag{1}$$

where

h = theoretical thickness of the grease at any time t

h_i = initial thickness of the grease at time $t = 0$

F = normal force pushing down on the greased platen

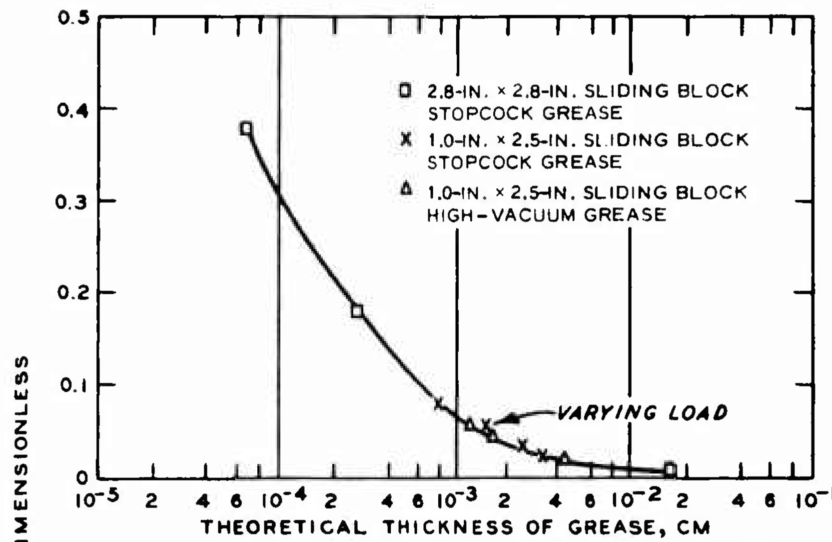
ν = viscosity of the grease

= 0.1 kg-sec/cm^2 for Dow's Corning silicone stopcock grease

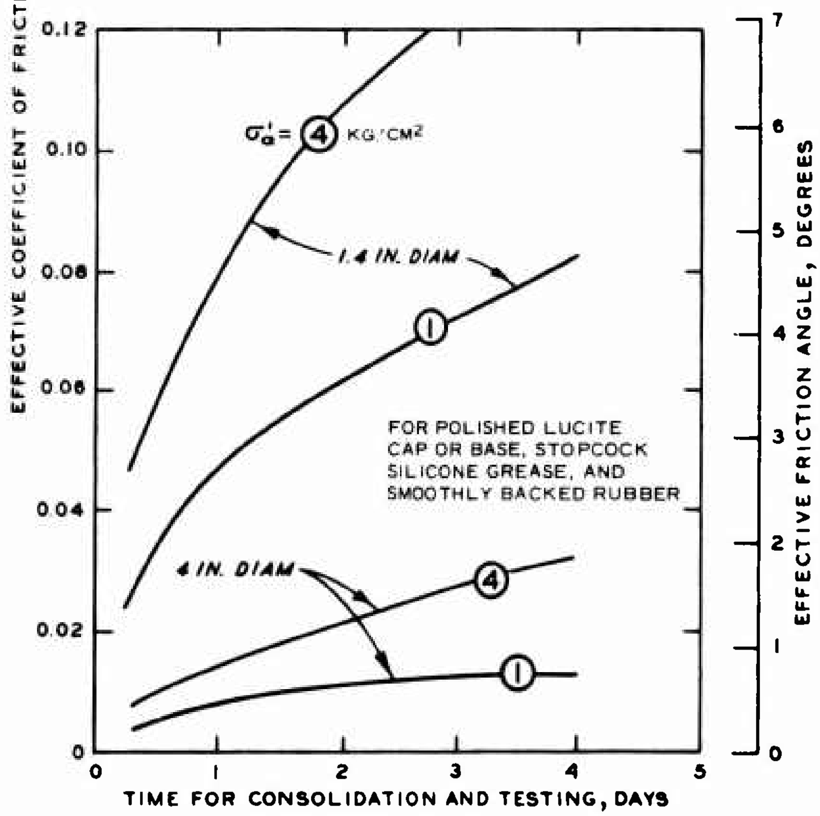
= 0.23 kg-sec/cm^2 for high-vacuum silicone grease

r = radius of the circular platen

24. The effect of prolonged loading at one confining pressure on the frictional resistance of the frictionless platen is shown directly in Figure 3b. Note that for 4-in.-diam samples, and confining pressures



a. RELATIONSHIP BETWEEN THE EFFECTIVE COEFFICIENT OF FRICTION AND THE THEORETICAL THICKNESS OF GREASE BETWEEN POLISHED LUCITE AND RUBBER



b. VARIATION OF FRICTION WITH TIME FOR LUBRICATED CAPS AND BASES ON TRIAXIAL SPECIMENS

Figure 3. Sliding resistance for lubricated end platens (from Duncan and Dunlop[20])

up to 4 kg/cm^2, the effective angle of sliding friction remains less than 1 deg for almost 2 days. If the effective confining pressure is 1 kg/cm^2, the angle of sliding resistance appears to remain less than 1 deg for an almost indefinite length of time. Other investigators[14,24,25] also found angles of sliding resistance on the order of 1 deg or less for low effective confining pressures. However, for higher confining pressures and smaller diameter specimens, the grease squeezes out much more rapidly and the angle of sliding resistance increases to 1 deg and beyond in only a few hours.

25. The results of frictional resistance studies cited above were the peak values obtained by controlled-load measurement after specimens were rested for periods of time. It was noted that after the initial movement (following a static rest period) the sliding resistance was greatly reduced in that sliding movement proceeded at an accelerated rate.[20] Quantitative data on the post-peak sliding friction of silicone grease platens were presented by Barden and McDermott.[14] Their data indicate that displacements on the order of only about 0.025 to 0.05 in. are sufficient to reduce the sliding friction angle to 1 deg, even though the peak value may be considerably lower.

26. Most of the reported investigations on sands with frictionless platens have been made using relatively low effective confining pressures and short loading times so that squeezing out of the grease would not be expected to be a serious problem. However, Lee and Seed[22] conducted tests on 1.4-in.-diam specimens with confining pressures up to 280 psi and observed good frictionless behavior with silicone grease-rubber ends; in that study, the specimens showed no barreling effect at high strains. A particularly noteworthy study in this regard was a series of drained triaxial tests on sand using 2- and 4-in.-diam specimens at confining pressures up to 800 psi reported by Roy and Lo.[26] The specimens were all prepared with $\ell/d = 2$. Only one greased rubber layer was used at the ends. Photographs of deformed specimens and other data presented convincing evidence that frictionless platen behavior was obtained.

27. One curious aspect investigated by Roy and Lo was the effect

of end restraint on the distribution of particle crushing throughout the specimen. Following comparative tests at 800-psi confining pressure, the deformed specimens were divided into five horizontal layers, and grain size distributions were determined on each layer. Specimens tested with lubricated ends showed an almost equal amount of crushing along the entire vertical height. In contrast, specimens tested with regular ends showed somewhat more particle crushing in the middle zones than at the end zones. Since particle crushing is strongly augmented by increasing amounts of shearing strains,[44,45] the tendency for rough platens to suppress particle crushing in the end zones is apparently due to the restraint offered by the rough platens in reducing the localized shear deformation in those zones.

28. The observations cited previously, that very small sliding movements on lubricated platens will greatly reduce the sliding resistance, may explain why greased platens used with tests at high confining pressures lead to apparently good frictionless behavior. The small lateral strains which tend to develop in the early stages of the test (when frictionless and regular platens show little difference) may be sufficient to break the bond which develops at the greased ends. As the lateral strains continue, the frictional resistance in the grease rapidly reduces so that, near failure when strains are large, the lubricating effect of the greased ends has been almost completely restored. Thus, the high frictional resistance deduced from Equation 1 and Figure 2, representing peak sliding resistance after various normal loading conditions, may be overly pessimistic.

29. The combined results of the many studies which have been made using the Rowe silicon grease-rubber sheet arrangement leave little doubt that this technique is extremely effective in reducing end frictional restraint. As a result, several studies have been conducted using this type of lubricated platens. The stated benefits are as follows:

- a. Frictionless platens lead to uniform strains throughout the specimen; therefore, there is no barreling effect in the center at large strains, and area calculations can be made with confidence.

b. Uniform strains throughout a specimen lead to more accurate volume-change measurements than possible with regular platens.

c. Regular platens which produce nonuniform strains and stress will result in a tendency for nonuniform distribution of pore pressures. Equalization of pore pressures requires movement of water, which in clay soil is a time-dependent process. Since pore pressures are conventionally measured at the ends of a specimen, accurate measurements can only be made provided there is sufficient time for the pore pressures to equalize throughout the specimen.[46,47] Frictionless ends which remove these nonuniformities lead to much faster acceptable times for testing clay soils with pore pressure measurements.

d. Long specimens ($\ell/d \geq 2$) are more capable of containing nonuniform initial conditions than short specimens, because short specimens have less material. Nonuniformities may cause the development of a single shear plane at failure. When a shear plane is developed, the strains, volume changes, or pore pressure changes are highly localized and nonuniform throughout the specimen. Therefore, accurate specimen response data require strains without the development of a failure plane; this condition can be avoided by using short specimens. If frictionless platens are also used, short specimens can be tested to avoid premature failure plane development without the adverse effect of severe stress concentrations at the ends.

30. Thus, in summary, the advantages of using frictionless platens in static triaxial tests are that specimens can be tested more rapidly and more accurately than with regular platens.

31. The main disadvantages are that specimens with frictionless platens are difficult to assemble. The grease is messy and extra effort is required to properly assemble the test equipment. Drainage must be limited to a small central zone where radial strains will alweys be small; this increases the time required for consolidation. As an alternative, complicated filter strip assemblies leading to a stone behind the greased platen can be arranged[16] but this is awkward. Conventional practice is difficult to change. Although there are studies which espouse the benefits of frictionless platens, engineers appear to remain unconvinced. Therefore, most commercial and research laboratories ignore the advantages of frictionless platens, and continue to use regular ends for their triaxial testing.

32. The reluctance to accept frictionless platens seems to be induced by a combination of fear and the lack of motivating evidence of definite advantage: fear of changing a proven technique to an unknown test; fear that the test results may not be acceptable to the client or user; fear that the testing might prove too difficult to outweigh the advantages; and the lack of available data which indicate that use of $\ell/d = 2$ with regular ends leads to unsatisfactory results (it is useful to examine the data obtained concerning frictionless versus regular platens).

33. Available triaxial test data on the drained strength behavior of sand with frictionless as compared with regular platens with $\ell/d = 2$ indicate the following:

- a. Slightly flatter initial slopes of stress-strain curves (initial tangent modulus 60 percent lower).
- b. Slightly less strength (10 percent or less).
- c. Somewhat less compressive and greater expansion volume change tendencies, the difference increasing with increasing axial strains.

34. Available data concerning undrained tests on clay using frictionless as compared with regular platens with $\ell/d = 2$ and appropriate testing times indicate the following:

- a. Slightly lower undrained strengths (about 5 percent or less); the difference is of a similar order of magnitude as the scatter in the test results in any one series.
- b. Similar pore pressure and effective stress parameters.

35. Thus, for routine testing, there is little evidence to promote a strong motivation for changing from regular to frictionless platens. The disadvantage of extra difficulties and of uncertainties in the effective drainage arrangements, especially for clays, seems to offset the advantage gained by reduced testing times. Even among researchers in the field there is no unanimous appeal. Barden and McDermott[14] call for the use of frictionless platens in research and routine testing, while Duncan and Dunlop[20] conclude that regular platens are satisfactory except under very special circumstances.

Use of Frictionless Platens with Cyclic Loading Tests

36. To date (1976) no published data are available concerning the possible effect of end conditions on the cyclic triaxial strength of soil. Judging from the available data from static tests, the greatest effect would be associated with sands rather than clays, since the use of frictionless platens in static tests on clays showed almost no effect on the strength at the developed pore pressure, whereas for drained tests a significant volume change influence has been reported. Volume changes in drained tests translate into pore pressure changes in undrained tests, and the pore pressure change tendencies in cyclic tests define the cyclic strength. It is therefore of interest to consider how the use of frictionless platens might affect the cyclic strength of sand, especially for liquefaction tests.

37. It is recognized that many cyclic tests on sands, especially for research purposes, are conducted quickly. The specimens are prepared, consolidated, and tested within a period of 1 to 2 hr. Therefore, as indicated in Figure 3 from Duncan and Dunlop,[20] little grease is expected to squeeze out from the platens, and the sliding friction angle is likely to be less than 1 deg. Of course, if longer consolidation times are used, more grease will be extruded, and higher sliding friction angles will result.

38. The sliding friction angle referred to in the above paragraph corresponds to measured sliding resistances under static loads. It seems clear that for very slow rates of loading associated with most normal testing, the viscosity of the grease will not adversely affect the nearly frictionless behavior of lubricated platens.

39. However, cyclic loading tests for earthquake analyses are conventionally conducted at a frequency of about 1 to 2 Hz.[3,7,9] At these high loading frequencies, the concept of a sliding friction angle loses significance. It is then more appropriate to consider the sliding resistance as a viscous rather than frictional force. Using the basic definition of viscosity, it is possible to estimate the lateral shear resistance τ_r , at the lubricated ends of a cyclic triaxial test,

$$\tau_r = \nu \frac{dr}{dt\ h} \qquad (2)$$

where

ν = viscosity of the grease

dr = radial movement of an element at the base of the specimen in time dt . These various quantities can be estimated from published data

h = thickness of the grease

40. Lee and Fitton[2] have shown that prior to liquefaction, double amplitude axial strains in sands are generally less than 3 to 5 percent. Since Poisson's ratio of saturated sand is about 0.5, this corresponds to a single amplitude radial strain of about 2 percent, and an absolute radial movement at the perimeter of a 1.4-in. specimen of about 0.014 in. There will always be zero movement at the center and, therefore, a linear distribution of radial movement 0 to 0.014 in. from center to edge of a 1.4-in.-diam specimen prior to liquefaction.

41. The probable thickness of the silicone grease may be estimated from the measurements made by Duncan and Dunlop,[20] as indicated in Figure 3 or by Equation 1. For tests conducted soon after the specimen is formed, the thickness of the grease may be as much as 0.02 cm. Greater thicknesses may be obtained by using more than 1 layer of rubber and grease. For tests conducted at a cyclic frequency of 1 Hz, the time for a compression cycle is 0.5 sec; the viscosity of high-vacuum silicone grease is given by Duncan and Dunlop[20] as 0.23 kg-sec/cm^2. Combining these assumed values in Equation 2 leads to

$$\tau_r = \frac{0.23 \times 0.035}{0.5 \times 0.02} = 0.8 \text{ kg/cm}^2$$

42. Thus the radial shear stress at the ends of a cyclic triaxial test with frictionless platens may range from zero at the center to as much as about 0.8 kg/cm^2 at the circumference where, in fact, the frictionless platen should produce zero shear stress at the boundary.

43. To put this in perspective, it is noted that a typical peak cyclic shear stress on sands consolidated to 1.0-kg/cm^2 effective pressure

will be on the order of only about 0.2 to 0.45 kg/cm^2, depending on the soil density and number of cycles to cause failure. This cyclic strength is less than half the peak cyclic stress produced at the ends of the specimen due to friction between the soil and the rough platen. Thus comparatively large radial shear stresses can develop at the ends of triaxia test specimens with greased platens, if tests are conducted at the normal frequency of about 1 Hz.

44. Lower radial shear stresses at the ends can be obtained by using more than one grease-rubber layer and/or longer cyclic frequencies. Fortunately, for cyclic tests on sands, the cyclic frequency does not appear to affect the cyclic strength.[48,49] However, for clays, there may be cyclic frequency effects[50] so that positive evaluation of the effect of frictionless platens with clay soils may be more difficult than with sands. As an example, for increasing the lubrication effect, doubling the grease thickness would halve the radial cyclic shear stress at the ends. Reducing the cyclic frequency to 0.1 Hz would reduce the viscous shear resistance by an order of magnitude to an acceptably low value.

45. The shape of the loading pattern may also affect the cyclic shear strength of saturated sands with greased ends. Fortunately, for sands with regular ends, the shape of the load pattern does not significantly affect the cyclic strength, but for clays this factor may be important.[50] However, even for sands, if tested with greased ends, the very rapid rise time for a square load wave will lead to very large viscous shear resistance at the ends, thereby nullifying the lubricating effect of the grease when the load is first applied. Since clean sand responds immediately to load changes, any stress concentration effect caused by the rapid rise in load would probably not be voided by a subsequent long rest period during which the viscous greased ends may deform. Therefore, in order to investigate the effect of frictionless platens on the cyclic strength of sand, it is necessary to conduct tests with more than one layer of greased rubber, use slow cyclic frequency, and avoid use of square-shaped load patterns.

PART II: RESULTS OF CYCLIC TESTING USING END RESTRAINTS

Influence of End Restraint on the Static Undrained Strength of Sand

46. As an introduction to the cyclic strength considerations, it is of interest to investigate the effect of end restraint on the undrained strength of saturated sands. Furthermore, as an introduction to the undrained static strength problem, it is useful to consider the relative effect of end restraint and lubrication on the drained strength of the same sand.

47. The soil tested in these studies was the Sacramento River sand used by Seed and Lee in several of their static and cyclic strength studies;[5,6,22,45,51] this was a uniformly graded fine silica sand with $D_{50} \approx 0.2$ mm . All static tests reported herein were conducted at a void ratio of 0.71, corresponding to about 78 percent relative density. The tests were conducted in a standard triaxial apparatus using 1.4-in.-diam samples 3.4 in. high ($\ell/d = 2.4$). Polished stainless steel platens were used in both types of test. The frictionless or free ends were produced using two layers of 0.012-in.-thick rubber separated by a thin layer of high-vacuum silicone grease, as originally developed by Rowe.[24] In the first few tests, the specimens with free ends tended to slide off one side of the platens. This sliding was later prevented by using a 1/8-in.-diam, 1/2-in.-long dowel extending into the axis of the specimen from each platen. The dowel at the base was hollow and was connected to the drain line. The arrangement and materials are illustrated in Figures 4 and 5. All specimens were saturated by boiling the sand in water and then raining it underwater into the water-filled forming jacket. Drainage was provided only at the base. Because of the high permeability of this clean material, no special precautions were required to ensure equal pore pressure distribution throughout the specimen during the test. The specimens were each isotropically consolidated to the desired effective stress, and then axially loaded to beyond failure at a strain rate of about 0.35 percent per minute.

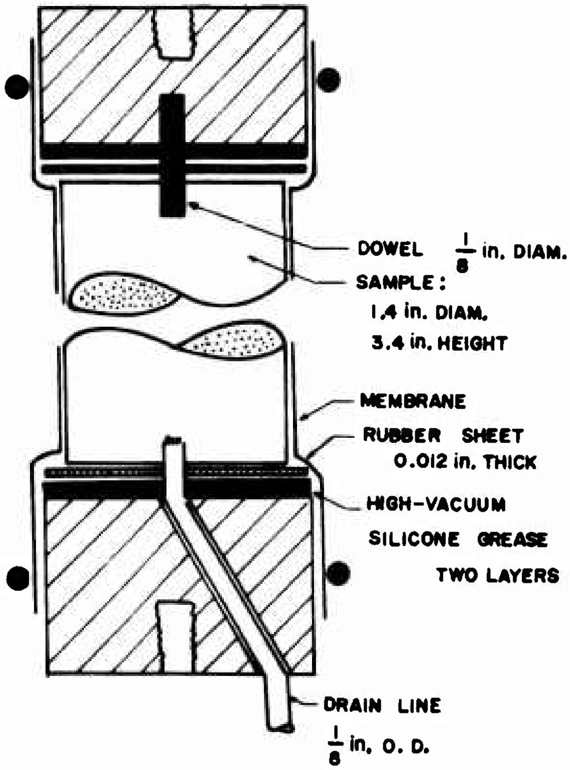


Figure 4. Frictionless cap and base

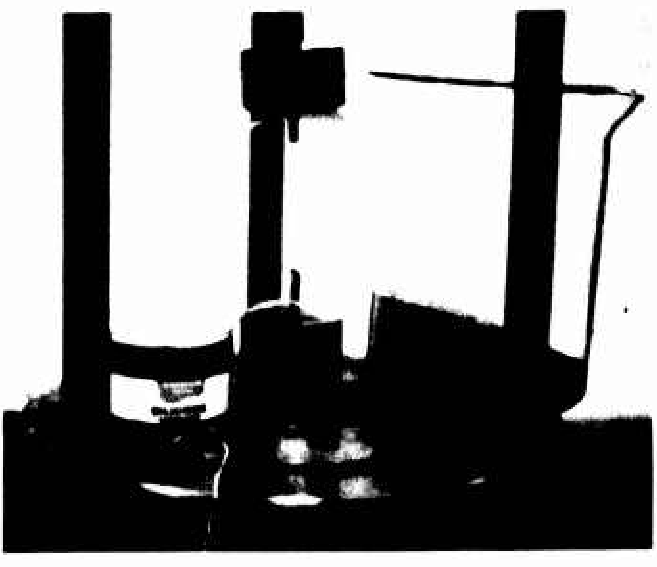


Figure 5. Equipment for frictionless end platens. Note that two layers of rubber sheets were used, each separated by a thin layer of high-vacuum silicone grease

48. A nominal back pressure of 15 psi was used in all drained tests. The undrained tests were consolidated under a back pressure sufficiently high to prevent cavitation in the pore water during the undrained shearing stage. For the low effective consolidation stresses, this procedure required a back pressure in excess of up to 200 psi.

Results of Drained and Undrained Static Loading Tests

49. Axial stress-strain and volume change data for two typical sets of tests are shown in Figure 6, one set at a low pressure and one set at a high pressure. As a first approximation, the behavior is similar to that observed for other soils. At low confining pressures, these medium dense specimens dilated at failure, whereas at high pressures they contracted.

50. Direct comparison of results from the two tests at the same pressure also indicates behavior typical of that reported by others for the effect of end restraint on the drained strength behavior of sand. At

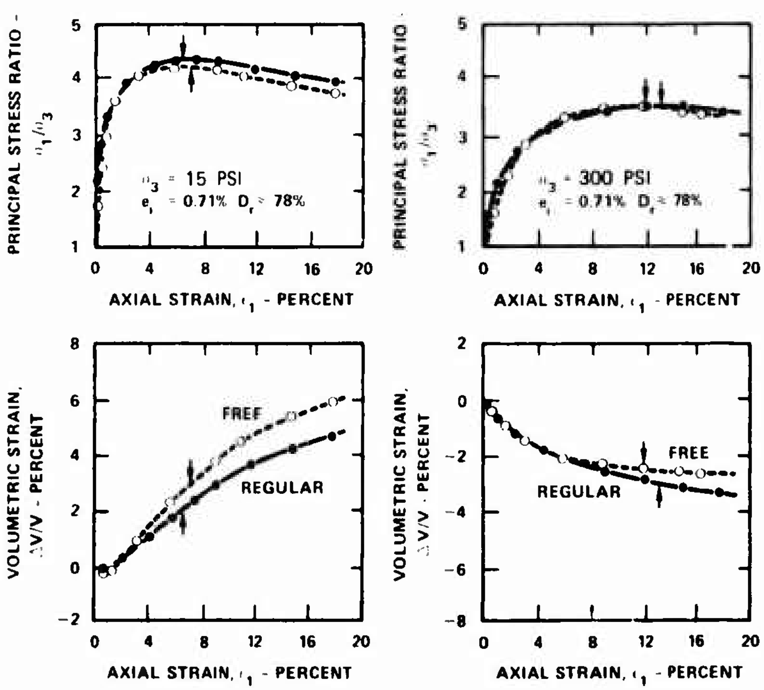


Figure 6. Drained triaxial tests using regular and free ends. Arrows indicate the point of maximum stress

the low pressure, the regular-ended specimen was slightly stronger. At the higher pressure, the strengths were equal for both end conditions. At both low and high pressures, the lubricated or free-ended specimens showed a greater tendency to dilate than the regular-ended specimens.

51. Axial stress-strain and pore pressure change data for two typical sets of undrained tests are shown in Figure 7. Again, the general nature of these data is similar to that of data reported elsewhere for undrained tests on saturated sands. At the low consolidation pressure, the tendency to dilate is reflected by a pore pressure decrease, whereas the high-pressure tests produced an increase in pore pressure.

52. As might be expected from the volume change data of the drained tests, in each set of undrained tests the free-ended specimen, which had a greater tendency to dilate, produced lower pore pressures than the corresponding specimen tested with regular ends. Since the undrained strength is primarily a function of effective stress, the relatively lower pore pressure produced relatively higher strengths for the free-ended undrained tests.

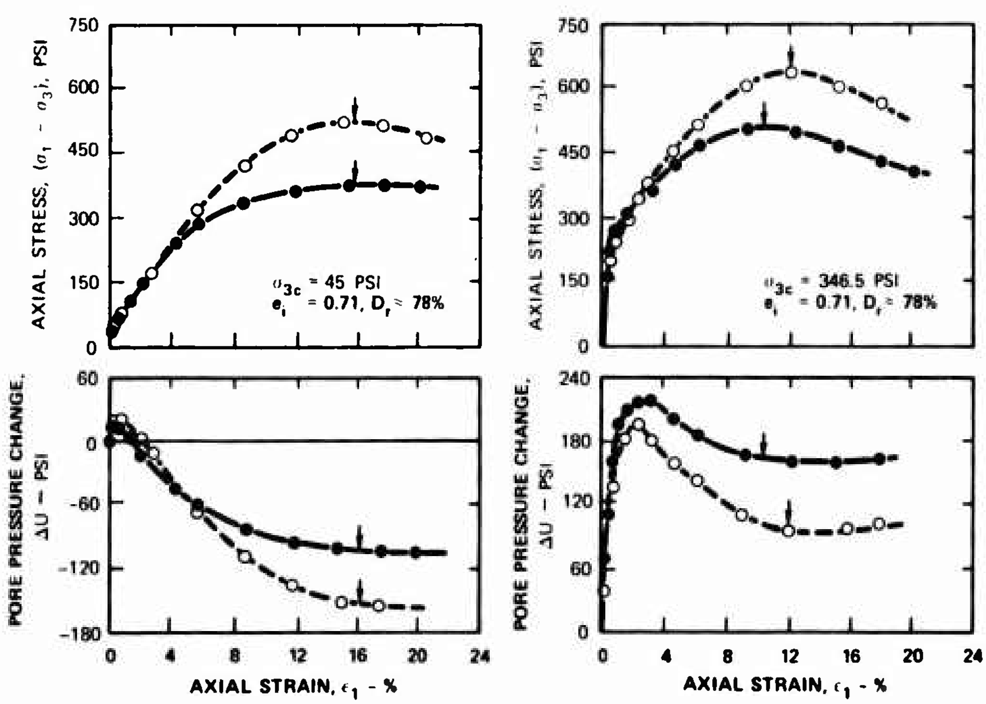


Figure 7. Undrained triaxial tests using regular and free ends. Arrows indicate the point of maximum stress

Strength Components

Undrained tests on sand

53. The relative significance of the several parameters making up the drained and undrained strengths of granular soil has been discussed previously. Expressed in terms of deviator or axial stress at failure, the drained and undrained strengths of sand after isotropic consolidation and no pore water cavitation are given by

$$\text{Drained:} \quad \left(\sigma_1 - \sigma_3\right)_f = \sigma_{3c}(K_f - 1) \tag{3}$$

$$\text{Undrained:} \quad \left(\sigma_1 - \sigma_3\right)_f = \sigma_{3crit}(K'_f - 1) \tag{4}$$

where

$K_f = \left(\sigma_1/\sigma_3\right)_f = \tan^2 (45 + \phi_d/2)$

σ_{3crit} = effective isotropic consolidation pressure

$K'_f = \left(\sigma_1/\sigma_3\right)_f = \tan^2 (45 + \phi'/2)$

The term K_f for drained tests is somewhat dependent on the confining pressure and the dilation tendency. Modification factors have been suggested to aid in quantifying the dilation effect,[24,45] but for most practical purposes it is usually sufficient to work directly with the unmodified K_f in Equation 3.

54. The effect of end restraint on the drained strength of this sand is manifested entirely by its effect on the value of K_f or drained angle of internal friction ϕ_d. Values of K_f or ϕ_d from drained tests at various confining pressures are shown in Figure 8. At very low consolidation pressures, the value of ϕ_d is up to 3 deg greater for regular ends than for free ends; however, at higher consolidation pressures this difference vanishes.

55. In undrained tests the strength is a function of two soil properties, K'_f and σ_{3crit}. Therefore, the effect of end restraint must be evaluated in terms of the effect on these two parameters. The term K'_f for undrained tests is essentially independent of the consolidation pressure. It is approximately equal to the drained K_f after the

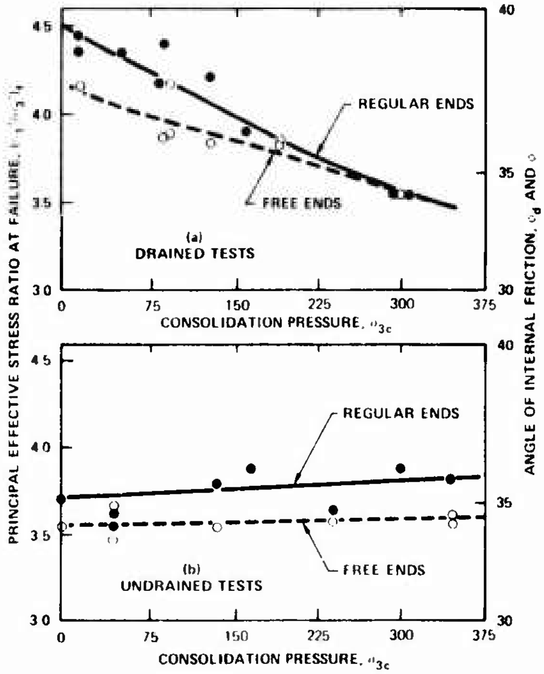


Figure 8. Effect of end restraint on ϕ_d and ϕ'

dilation effect has been eliminated either experimentally by undrained testing or by drained testing at a high σ_{3c}, or modified by an energy absorption term.[45] The effect of end restraint on the undrained effective K_f' is illustrated in Figure 8. Over the full range of consolidation pressures, the values of ϕ' are essentially constant for each type of apparatus, with the value being about 1 deg higher for regular ends than for free ends.

56. As discussed by Seed and Lee,[51] in undrained tests at any consolidation pressure σ_{3c}, the pore pressure tends to change until it reaches an equilibrium condition which defines an effective stress equal to σ_{3crit}, provided that the absolute pore pressures are high enough to prevent cavitation. Thus, the effective minor principal stress at failure is always σ_{3crit}, and may be defined by

$$\sigma_{3crit} = \sigma_{3c} - \Delta u_f \tag{5}$$

57. Alternatively, σ_{3crit} may be defined from an interpolation

of data from a number of drained or undrained tests as illustrated in Figure 9, which summarizes data from all the tests conducted for this study. In agreement with previous investigations, the values of σ_{3crit} from drained and undrained tests are of similar magnitude; however, both drained and undrained tests show that the value of σ_{3crit} is about 45 psi greater for free ends than for regular ends.

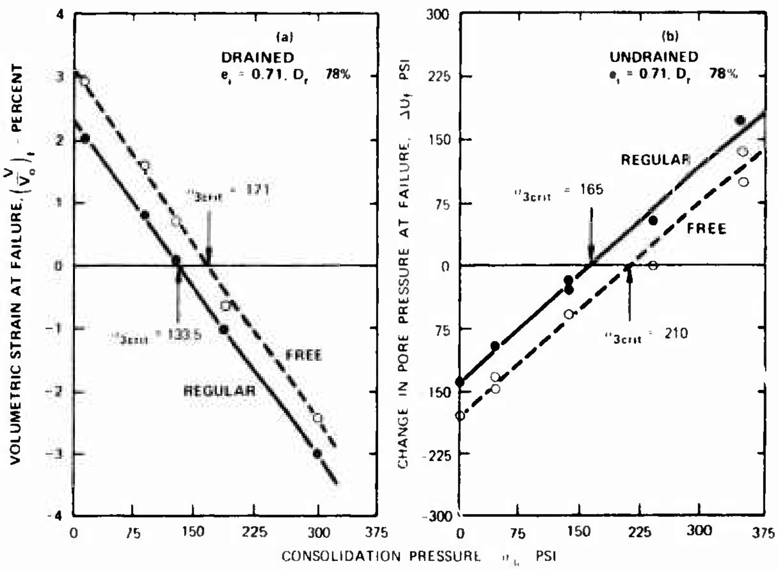


Figure 9. Determination of σ_{3crit} from drained and undrained tests

58. Thus the effect of end restraint on the two soil properties which make up the undrained strength of this sand is as follows:

K_f' or ϕ' - free ends less than regular ends by about 1 deg

σ_{3crit} - free ends greater than regular by about 45 psi

Substitution of this range of values into Equation 3 shows that although the two effects are in opposite directions, the effect of σ_{3crit} is significantly the most important, suggesting that for undrained tests on sands, frictionless platens should lead to strengths higher than those associated with regular platens.

59. A modified Mohr diagram summarizing the undrained strengths plotted on a total stress basis is shown in Figure 10. As expected from the pattern of the two typical tests shown in Figure 7, and from the above considerations, the specimens tested with free ends are consistently some 15 percent stronger than specimens tested with regular ends. This variation contrasts the results from drained tests, in which the strength is a function of only ϕ_d . Although end restraint does indicate a difference in volume change behavior in drained tests, this difference is not very important in governing the drained soil strength. In undrained tests, however, the difference in volume change tendency produced by end restraint, as reflected by the pore pressure changes during loading, significantly influences the effective stress and hence the soil strength.

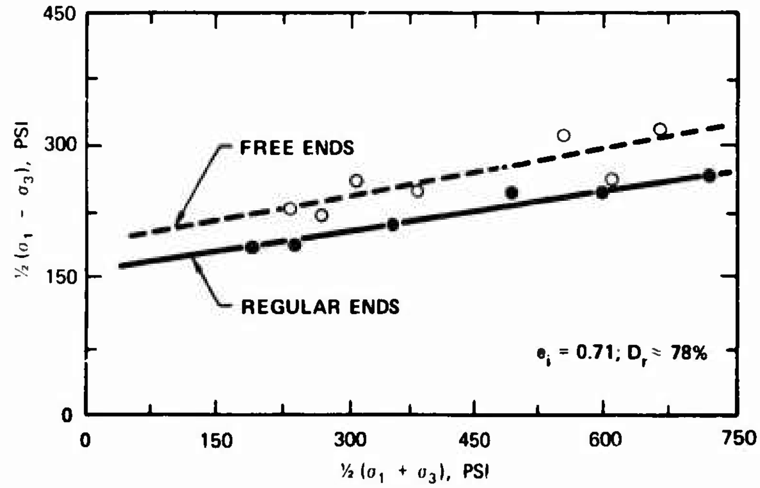


Figure 10. Modified diagrams for undrained tests, total stress basis

Application to other soils

60. Although this test series was limited to one sand at one initial density, it seems logical that the same trend would be valid for other granular soils and consolidation pressures, since the strength of all cohesionless soils is governed by Equation 4. Since the numerical values of K_f' and σ_{3crit} differ with soil type and density, the absolute effect of end restraint should also differ for different soils.

The stronger the dilatant tendency of the soil, the more influence end restraint would be expected to have on the undrained strength.

61. Conceptually, Equation 4 should also apply to clays with low effective cohesion intercept. Clays are generally more compressible than sands; therefore, the value of σ_{3crit} would be expected to be lower and more sensitive to changes in consolidation pressures than for sands. Stress-strain-pore pressure change data for undrained tests on clay[14,16,20] show a trend that is similar to but less pronounced than that observed here for sands. Free platens apparently cause a small reduction in ϕ' and a slightly lower excess pore pressure at failure (higher σ_{3crit}) than for regular platens. For high overconsolidation ratios, Blight[16] found that free ends lead to higher pore pressures than rough ends, but there was no significant difference in undrained strength if compared at the same effective stress. The effects of end restraint on the strength of clays appear to be small and generally in compensating directions. The net result is that end restraint should cause no significant change in the undrained strength of clays, which in fact is borne out by experimental data.[20]

Conclusions from Static Tests with Frictionless Platens

62. The results of the data from static loading tests on soils reviewed and discussed herein lead to the following conclusions.

a. Use of rough regular platens leads to nonuniform distribution of normal and shear stresses and strains, with more strain occurring in the middle of the specimen than near the ends. To minimize this effect on the measured strength, regular specimens are conventionally made with $\ell/d \approx 2$.

b. In drained tests with $\ell/d = 2$, regular platens overestimate the angle of internal friction of dense sand by up to about 3 deg at low confining pressures, but this effect is reduced to an insignificant amount at high confining pressures greater than σ_{3crit}. Therefore, for static strength purposes, the continual use of regular ends for drained tests on sand using specimen $\ell/d \approx 2$ would seem to be justified although the resulting strength data may be slightly unsafe.

c. Use of regular platens with $\ell/d = 2$ tends to accentuate nonuniformities in the specimen with the result that one or a few major shear planes may develop prematurely. The concept of an homogeneous, isotropic specimen vanishes and accurate stress-strain data are unobtainable when single shear planes develop. Therefore, since frictionless platens lead to uniform stress and strain distributions, tests may be conducted with them to obtain accurate results using short ($\ell/d = 1$) specimens in which localized shear planes are unlikely to develop.

d. Volume changes in soils are strongly dependent on shear strains; therefore, the nonuniformities caused by regular rough ends lead to a nonuniform distribution of shear strains and shear-dependent volumetric strains. Dead zones near the rough ends do not undergo large shear strains and hence do not experience dilatancy tendencies as strong as those in zones of major shear strains in the middle portion of the specimens. As a result, the overall dilatant volumetric strain tendency in specimens with regular ends is less than in specimens with lubricated ends. This reduction leads to lower critical confining pressures σ_{3crit} for specimens tested with regular ends than for those tested with frictionless ends.

e. The behavior of soil in undrained tests is directly analogous to the behavior described above for drained tests, substituting pore pressure changes for volume changes, and realizing that pore pressures tend to equalize throughout the specimen whether the volumetric strains are uniform throughout or not.

f. In undrained tests, use of regular ends overestimates ϕ' by about 1 deg.

g. In undrained tests, use of regular ends underestimates σ_{3crit} by about 45 psi (for medium dense Sacramento River sand).

h. The combined effect of ϕ' and σ_{3crit} leads to lower undrained strength for regular ends than for frictionless ends by about 15 percent (for medium dense Sacramento River sand).

i. The method outlined herein for interpreting the results of undrained tests on sands should also apply to clay soils. Available data suggest that both c' and Δu_f or σ_{3crit} for normally consolidated clay are only slightly influenced by the type of end restraint, with the net result that end restraint shows no significant influence on the measured undrained strength of clay.

<u>Prediction of the Effect of End Restraint
in Cyclic Triaxial Test Results</u>

63. From the foregoing review of the effects of end restraint on static loading tests, some predictions may be made as to the probable behavior under cyclic loading. However, a few preliminary comments on the behavior of soil in a cyclic triaxial test are of interest as background reference.

64. The numerous cyclic loading studies conducted to date demonstrate that, for sand soils at least, the cyclic strength or liquefaction potential is directly dependent on the rate of pore pressure buildup during cyclic loading. A typical cyclic load test on an isotropically consolidated specimen of loose saturated sand is shown in Figure 11. Note that the specimen remains stable and almost completely undeformed during the first stage of a cyclic loading test. With each cycle of load the pore pressure increases and decreases in phase with the direction of the load. Because the cyclic loads are generally small in relation to the static strength, the soil remains in the compression range of the volume change or pore pressure change even for dense sands. Because soil is not elastic, there is always a residual portion of the pore pressure buildup remaining at the end of each stress cycle so that the net pore pressure has increased a small amount. Therefore, each successive cycle acts on a specimen having slightly lower effective confining pressure than in the previous cycle. The same cyclic stress always extends slightly into the virgin range of the stress-strain curve. This process continues as the cyclic loading proceeds until such time as the net pore pressure reaches such a high value that the soil no longer has strength to carry the next cycle of stress. That is, on an effective stresses basis, the soil is near the point of imminent failure, such that the full effective angle of internal friction is mobilized when the next peak cyclic stress is applied.

65. At this point, a small strain occurs during the next cycle. This strain disturbs the soil grain structure, leading to a large transient increase in pore pressure beyond the regular pattern. When the

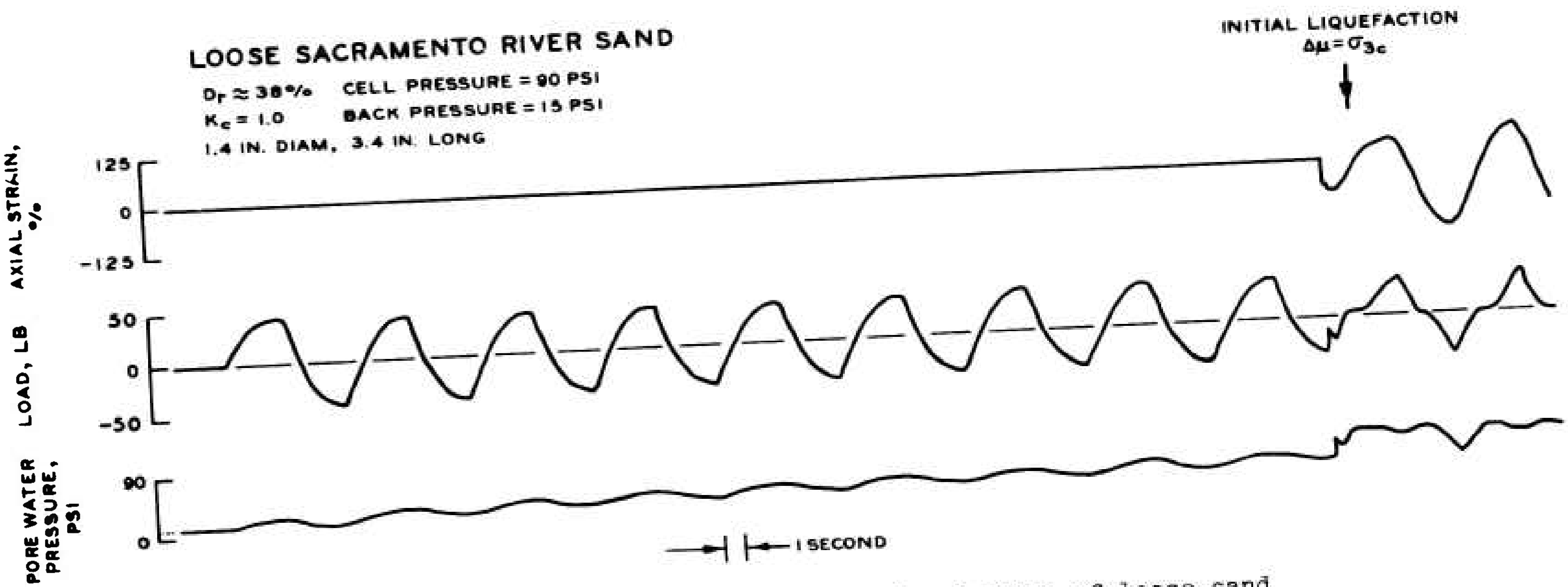


Figure 11. Detailed record showing liquefaction of loose sand

cyclic load reduces to zero, the pore pressure then jumps to the value of the confining pressure. This sudden increase in pore pressure to the level of the confining pressure has been termed "initial liquefaction." Beyond the initial liquefaction condition the strains begin to increase fairly rapidly.

66. The number of cycles required to reach this critical initial liquefaction condition depends on the cyclic stress level and the response of the soil in terms of pore pressure increase per cycle. If the net pore pressure increases by a large amount in each cycle, then the residual pore pressure will also increase rapidly and vice versa. From this it follows that specimens which have strong dilatancy tendencies, such as dense sand, take longer to reach initial liquefaction than specimens with a weak tendency to dilate when loaded. In the previous sections it was shown that specimens with frictionless platens tended to dilate more than specimens with regular platens. Therefore, it should be expected that initial liquefaction with frictionless platens would require more cycles than with regular end platens.

67. Following the initial liquefaction condition the pore pressure response changes. The highest pore pressures occur in the part of the cycle when the axial stress is zero. This high value is generally close to or equal to the total confining pressure, so that the effective stress at this stage is about zero. Then, as the specimen is strained by application of the axial load (compression or extension), it tends to dilate, the pore pressure decreases, and the effective stress increases sufficiently to carry the applied load on that cycle. The specimen develops the maximum effective angle of internal friction with every load cycle. In so doing, strains or plastic yielding occur, with the pore pressure decreasing to compensate for the increasing shear stress as the load is applied.

68. The rate at which the specimen builds up effective stress with strain in each load cycle depends on the tendency for the sand to dilate. Unless at an extremely low density, close to or before the conventionally defined minimum density, all sands will dilate at zero effective stress provided the strains are large enough. The test data

in Figure 12 show the time rate of pore pressure decrease and axial load increase in a static constant rate of strain test on a loose and a dense specimen following liquefaction and cyclic deformation up to ±10 percent axial strain. Note that the dense specimen dilates and gains strength at a much faster rate than the loose specimen. The same tendency occurs in each load pulse of a cyclic test following liquefaction. The specimen strains and tends to dilate until there is a sufficient reduction in pore pressure to develop the strength needed to carry the applied load, then the movement stops. Therefore, strongly dilatant specimens, e.g. dense sands, tend to strain less per cycle than weakly dilatant specimens, e.g. loose sands. Based on the observations from the previous section, it follows that since frictionless ends tend to increase dilatancy, they should also serve to reduce the strain per load pulse in a cyclic loading test. From this, it may be expected that the use of frictionless end platens in cyclic tests on sands should lead to to a greater number of cycles to cause initial liquefaction. Following liquefaction, specimens with frictionless ends should require more cycles to reach the strain reached using specimens with regular ends.

Results of Some Comparative Cyclic Triaxial Tests

69. In order to investigate the effect of end restraint, a number of comparative cyclic triaxial tests were conducted. The specimens were prepared from the Sacramento River sand, as in the previous tests described herein, the same sand used for the cyclic triaxial tests described by Seed and Lee.[3,5] The 1.4-in.-diam by 3.4-in.-long specimens and the same frictionless cap and base arrangement were also used.

70. The frictionless platens consisted of two layers of rubber each 0.012 in. thick, and each separated by a generous smear of high-vacuum silicone grease. The lubricated end specimens were all tested within at least 30 min of the time that the specimens were set up. The effective confining pressure was 15 psi in all tests. Two densities of specimens were tested: loose, $D_r \approx 38$ percent; and dense, $D_r \approx 100$ percent.

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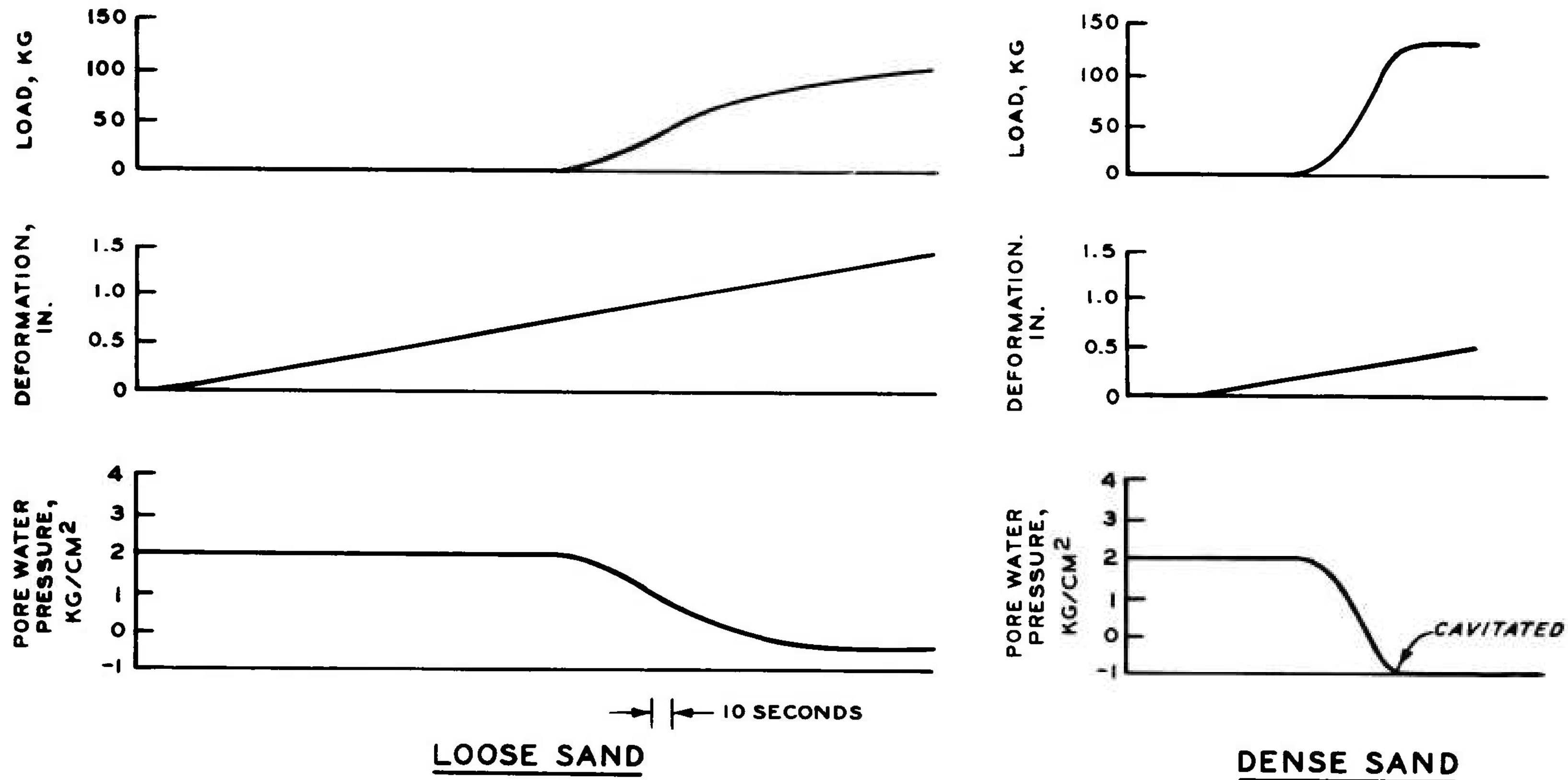


Figure 12. Record of a static test following liquefaction in a pulsating load test. (From Seed and Lee5)

71. Because the same type of sand would be used, the comparative regular platen tests were those originally presented by Seed and Lee.[3,5] However, some additional check tests were conducted to ensure that the old data could be reproduced.

72. Two series of frictionless end tests were conducted. The first series used a near-square load pattern at 1 Hz cyclic frequency. Some question arose as to the amount of lubrication which could be developed in this type of loading. It was feared that the rapid rise time and the short frequency would not be sufficient to allow the viscous grease to truly act as a lubricant. Therefore, a few additional check tests were conducted using a near-sine wave load form and cyclic frequency of only 0.05 Hz.

Loose sand

73. A record of an early cyclic loading test on loose sand for the previous study[5] is shown in Figure 13. These tests were conducted at 2 Hz using a square wave form. Typical of all tests on loose sands, initial liquefaction and very large strains occurred almost simultaneously. A record of a typical test on loose sand with the first series of frictionless platens (near-square wave and 1 Hz) is shown in Figure 14. For comparison, a record of a typical test on loose sand with the second series of frictionless platens is shown in Figure 15.

74. Following the conventional practice used in the University of California, Los Angeles (UCLA) laboratory, the axial strain and pore pressure test data were plotted to semilog scale. Data from typical tests of each type are shown in Figure 16. Note that there is no distinction in the shape of the curves for the different types of test. In each case the specimen undergoes almost no strain until initial liquefaction, whereupon it suddenly strains to very large amounts in the next cycle or two. The amount of this post-liquefaction strain is limited by the travel of the loading piston, and not by the sand gaining strength due to dilation. Presumably, if the specimen could have strained far enough, the dilation tendency would have begun to become operative as shown in Figure 12. However, within the limits of movement of the cyclic loading equipment, the sand particles were apparently

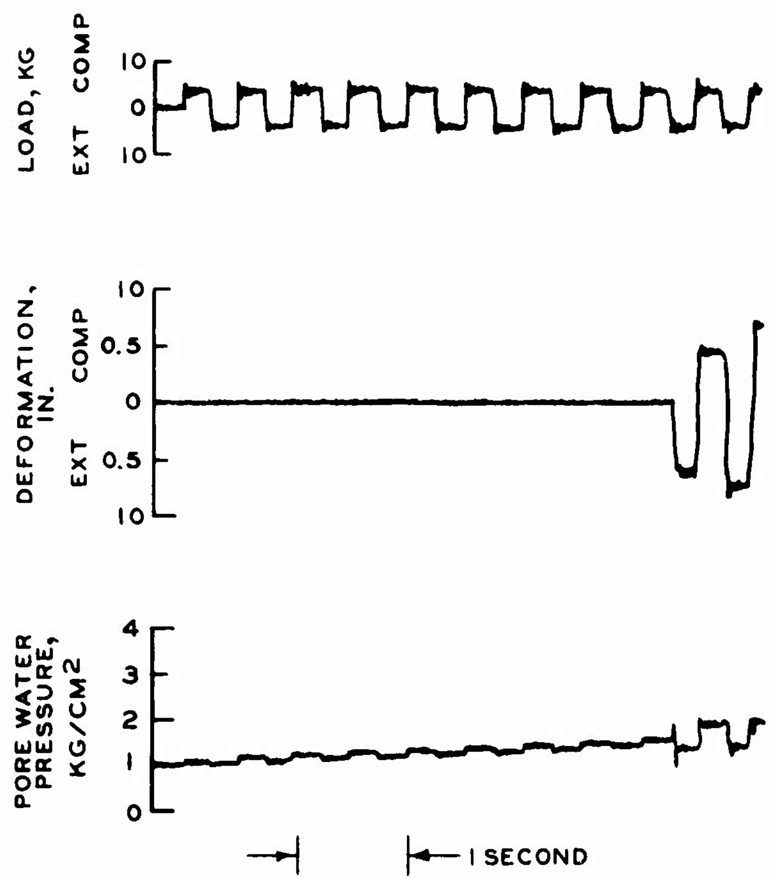


Figure 13. Record of a typical cyclic load triaxial test on loose Sacramento River sand; regular ends, 2 Hz. (From Seed and Lee[2])

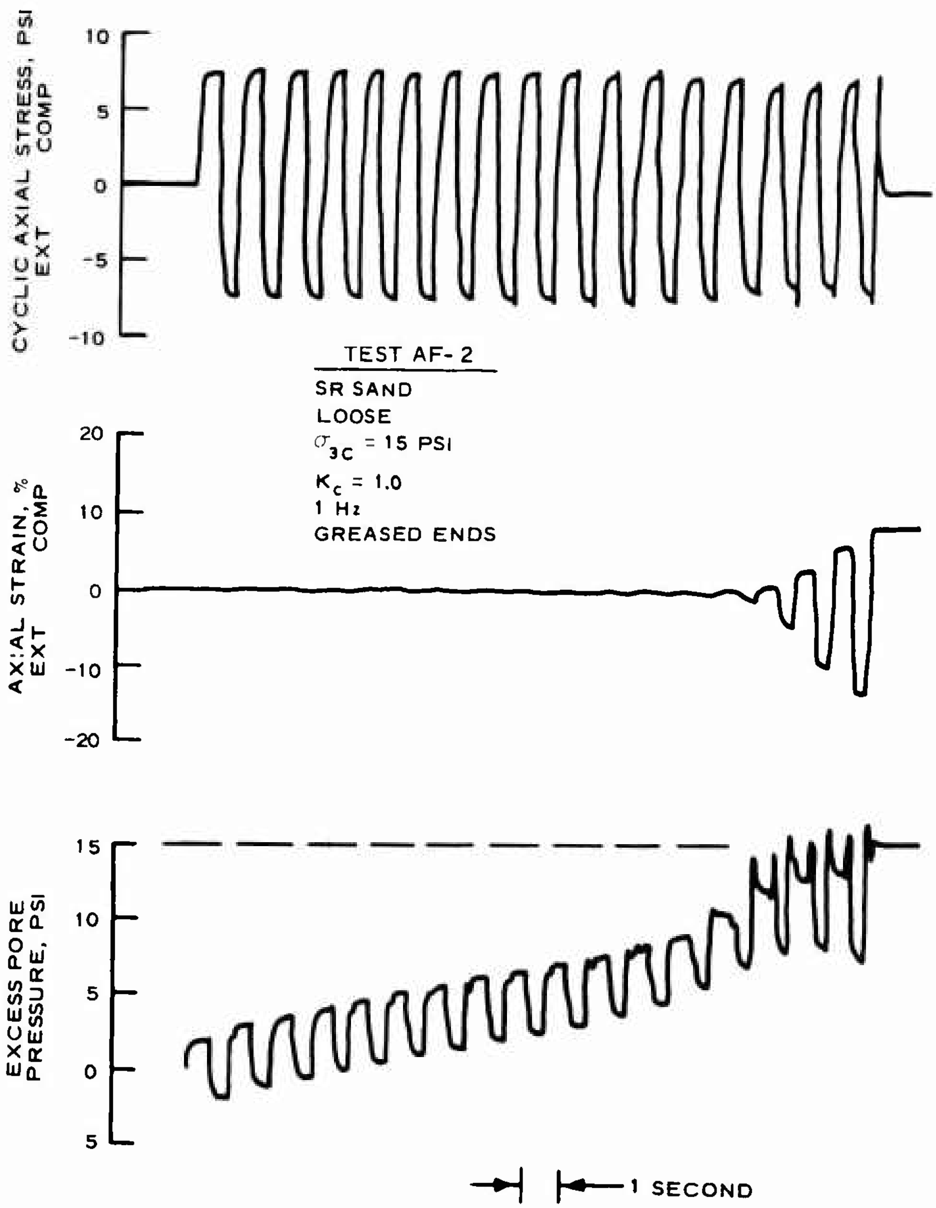


Figure 14. Record of typical cyclic triaxial test on loose Sacramento River sand, lubricated ends, square load forms, 1 Hz

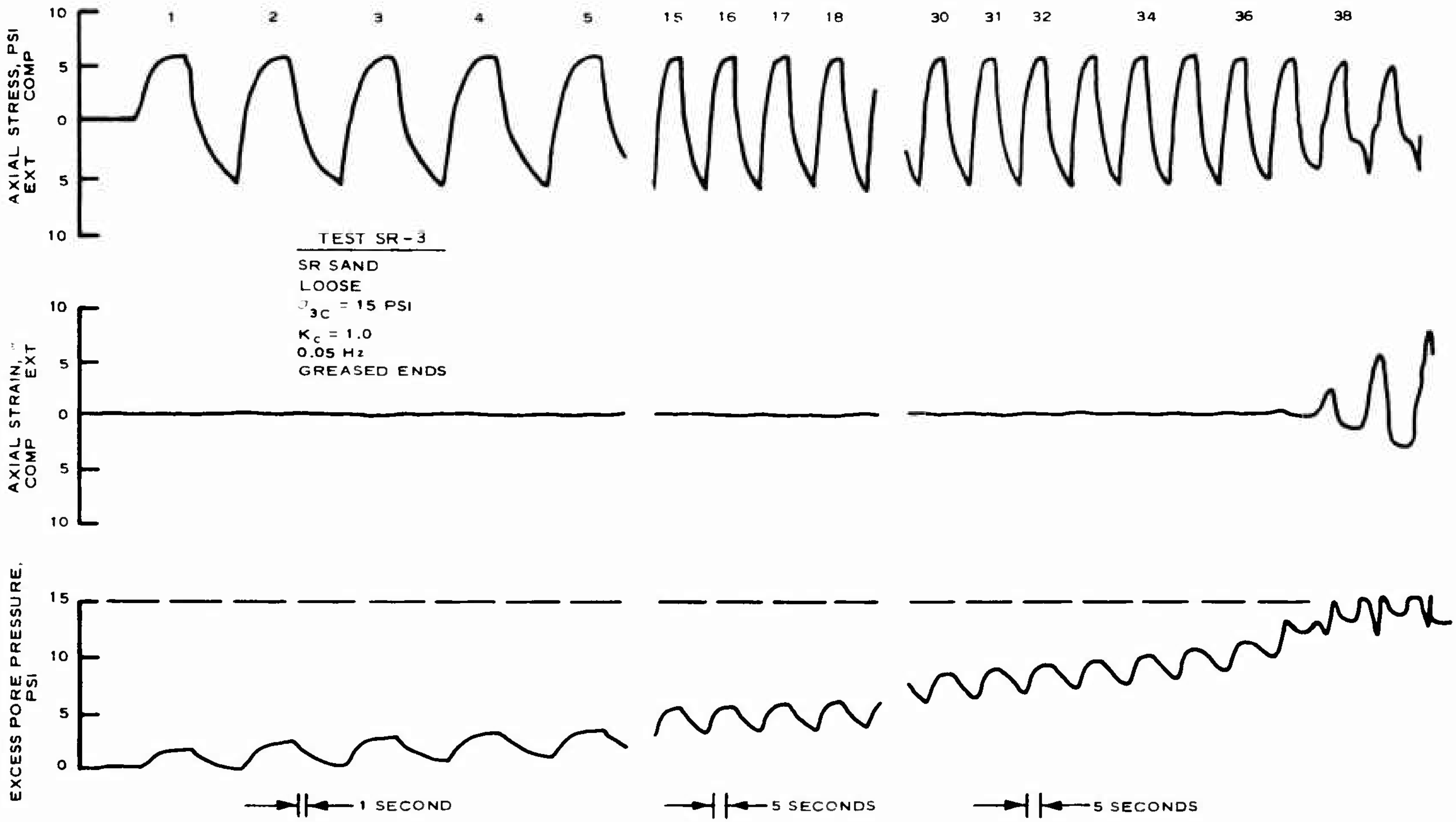


Figure 15. Record of a typical cyclic load triaxial test on loose Sacramento River sand, lubricated ends, peak load trace, 0.05 Hz

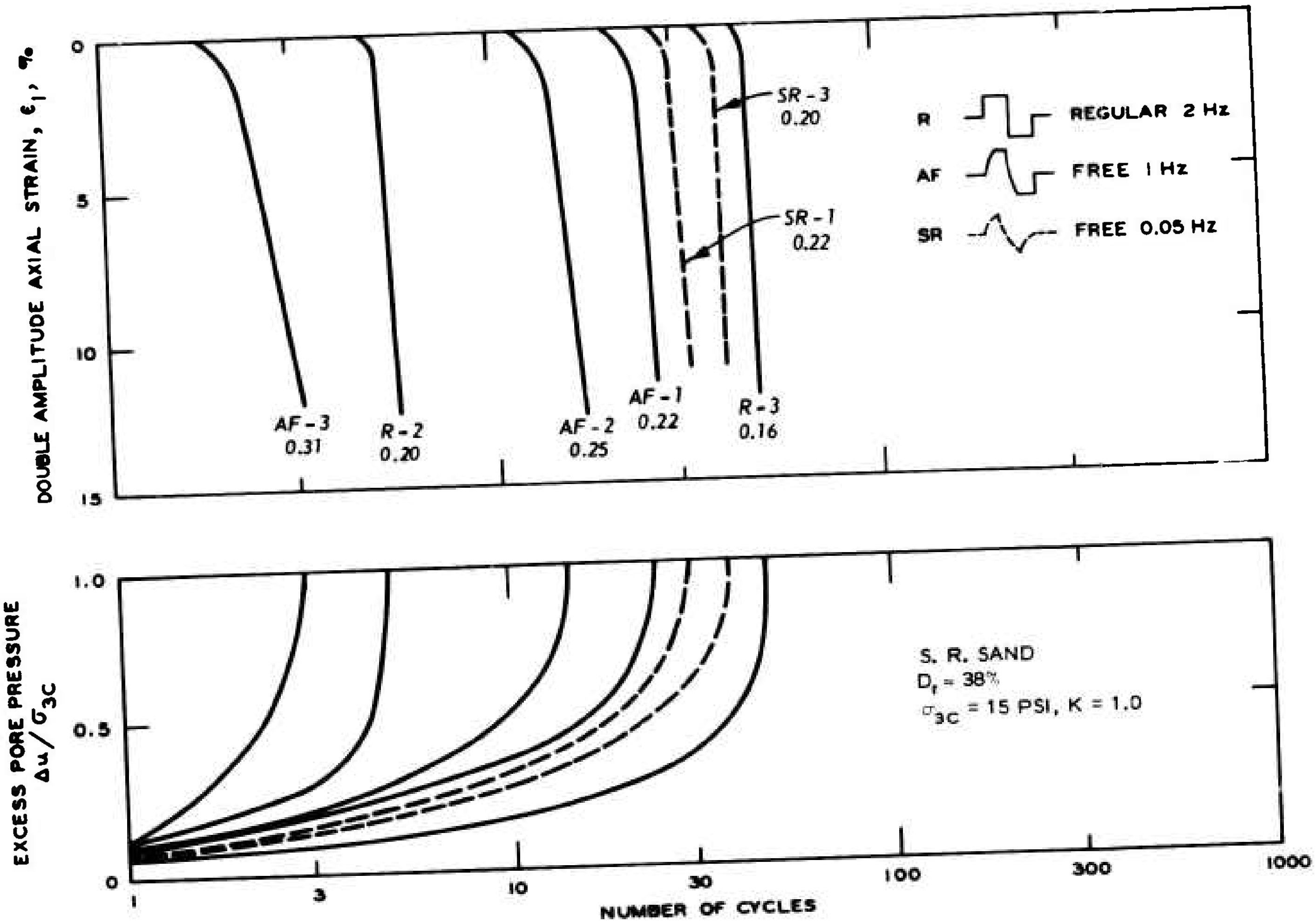


Figure 16. Typical data from cyclic triaxial tests on loose Sacramento River sand

completely separated from each other and suspended in the pore water in a truly liquefied state. The concept of frictionless platen effect for these fully liquefied conditions loses its significance. Cyclic strength comparisons are therefore only valid for the initial liquefaction data.

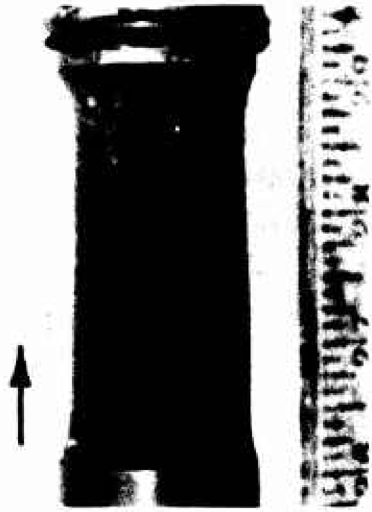
75. One of the shortcomings of a triaxial test is that the specimen changes area as it strains. With regular ends, the specimen will bulge more in the middle than at the frictional ends for compressive loads, leading to a barrel-shaped specimen. In extension, the specimen area decreases and, if carried to large strains, the specimen may neck at one particular zone of weakness. If necking occurs, it usually develops near the end of a cyclic triaxial test after the specimen has already reached initial liquefaction. When necking occurs, the test must be stopped as data obtained after necking are not meaningful.

76. One of the advantages of lubricated end platens is to reduce the lateral strain variations over the specimen which lead to barreling or necking. This beneficial effect is shown in Figure 17 for a loose specimen with lubricated ends as it deforms under cyclic loading. Note that the cross-sectional area seems to be fairly constant over the entire length of the specimen, even at large strains. There is no obvious barreling or necking.

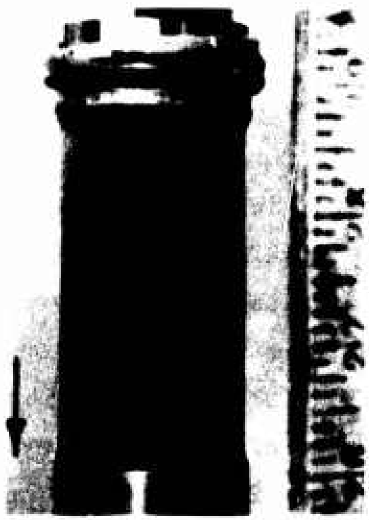
77. When very loose triaxial specimens liquefy, the soil begins to settle to the bottom under its own buoyant weight, leaving excess water trapped below the cap at the top. The rate of this internal particle readjustment depends on the grain size distribution and on the frequency of cyclic loading. Coarse sands and/or slow loading frequency provide time for the soil particles to settle before the next cycle stirs up the sand within the membrane. Fine-grained sands and/or rapid cyclic loading tend to keep the liquefied soil particles in suspension longer.

78. The wrinkles on the membrane near the top of the specimen in Figure 17d are the result of this particle settlement inside the membrane after liquefaction. The slow cyclic loading for this test (0.05 Hz) allowed time for a considerable amount of particle settlement

a. Before cyclic loading (zero strain)



b. First extension cycle after liquefaction (-5% strain)



c. First compression cycle after liquefaction (-5% strain)

d. Compression cycle several cycles after liquefaction (-5% strain)

Figure 17. Appearance of a specimen of loose Sacramento River sand before, at, and following liquefaction. Frictionless ends, 0.05 Hz

between cyclic peaks so that after a few cycles an excess amount of water had collected, replacing the sand which had settled to the bottom and densified. This phenomenon prevented a meaningful test from being conducted for large numbers of cycles after liquefaction. The tests conducted at 2 Hz did not show this tendency because the loading frequency was fast enough to keep the liquefied sand agitated, and prevented it from settling to the bottom.

79. The cyclic stress ratios causing initial liquefaction in the loose Sacramento River sand are shown in Figure 18. The data are fairly consistent in showing that the lubricated end platens lead to cyclic strengths about 25 percent higher than those obtained with regular platens. Thus these data are consistent with the hypothesis developed from the review of static tests, which is that frictionless ends should lead to higher strengths for saturated soil.

80. An exception is for the two data points at 0.05 Hz which both failed in one cycle. It may be that one cycle is not sufficient to free the grease, or it may be that at these relatively high cyclic stresses, the soil is too sensitive to small changes in specimens for accurate comparative results.

Dense sand

81. A record of a typical test on dense ($D_r \approx 100$ percent) Sacramento River sand with regular ends is shown in Figure 19. For comparison, typical records of tests on 100 percent relative density sand using lubricated ends are shown in Figures 20 and 21. The record shown in Figure 20 was for the first series (near-square wave and 1 Hz), whereas the data in Figure 21 were obtained from a second series test (near-sine wave and 0.05 Hz).

82. Semilog plots of peak strains and peak pore pressures for each of these three series of tests are shown in Figures 22-24. Note that the axial strain scale for these dense specimen tests is expanded considerably from that used for the loose specimen data in Figures 13-15.

83. A quick comparison of the shape of the number of cycles-strain curves for the three series of tests suggests that the strain after liquefaction proceeds slower with lubricated ends than with regular

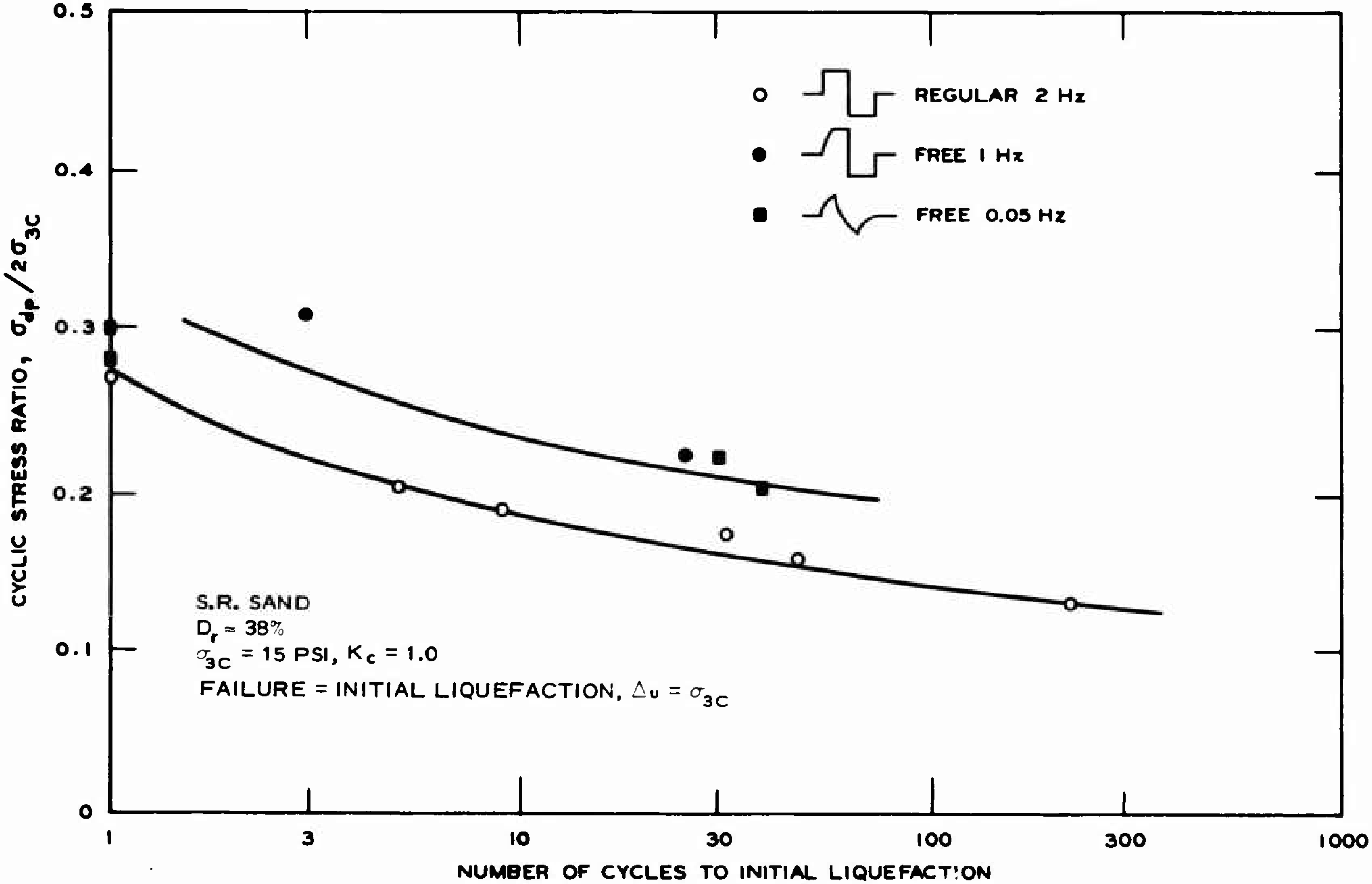


Figure 18. Strength data from cyclic triaxial tests on loose Sacramento River sand

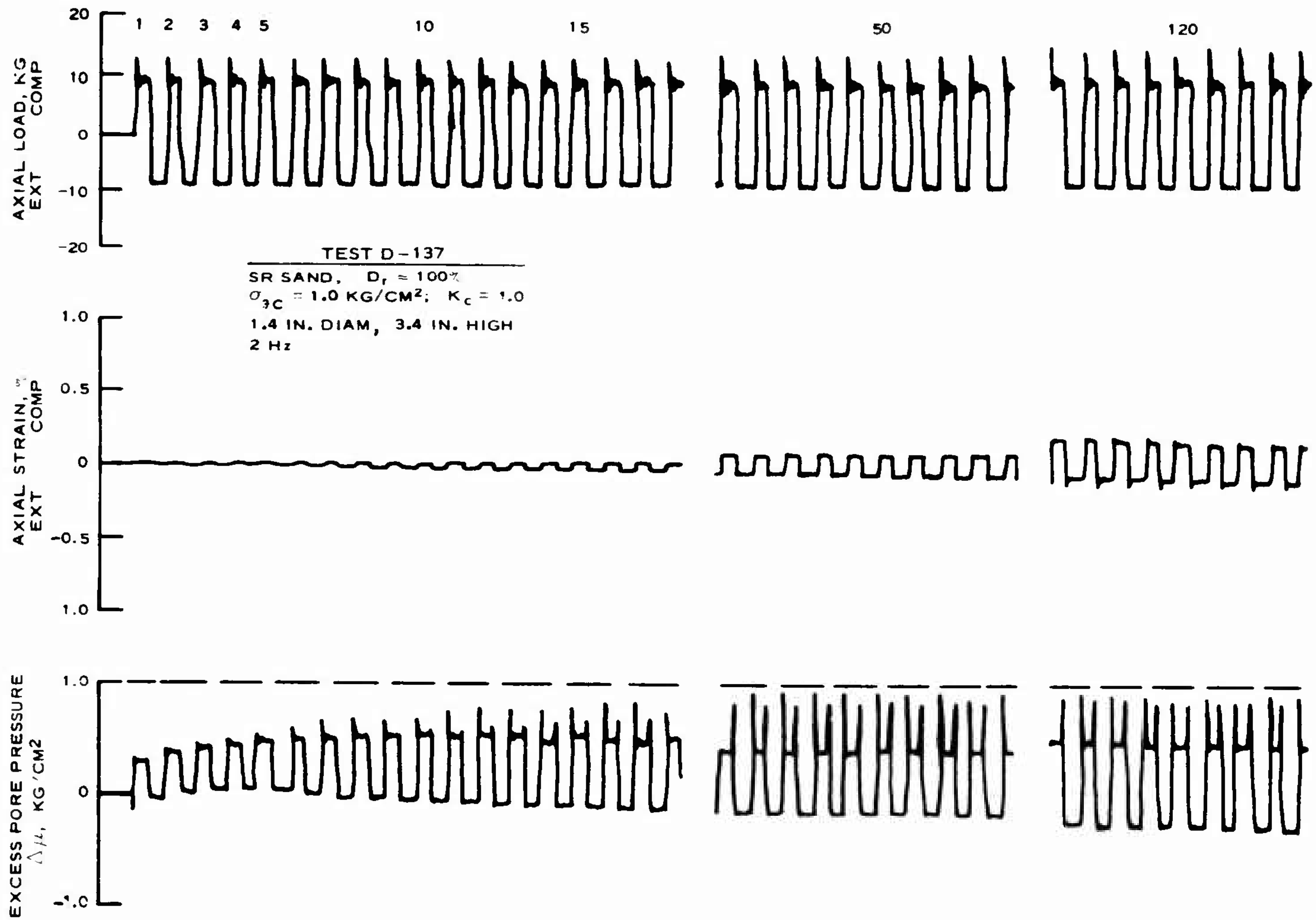


Figure 19. Record of a typical cyclic load test on dense Sacramento River sand; regular ends, 2 Hz. (From Seed and Lee[51])

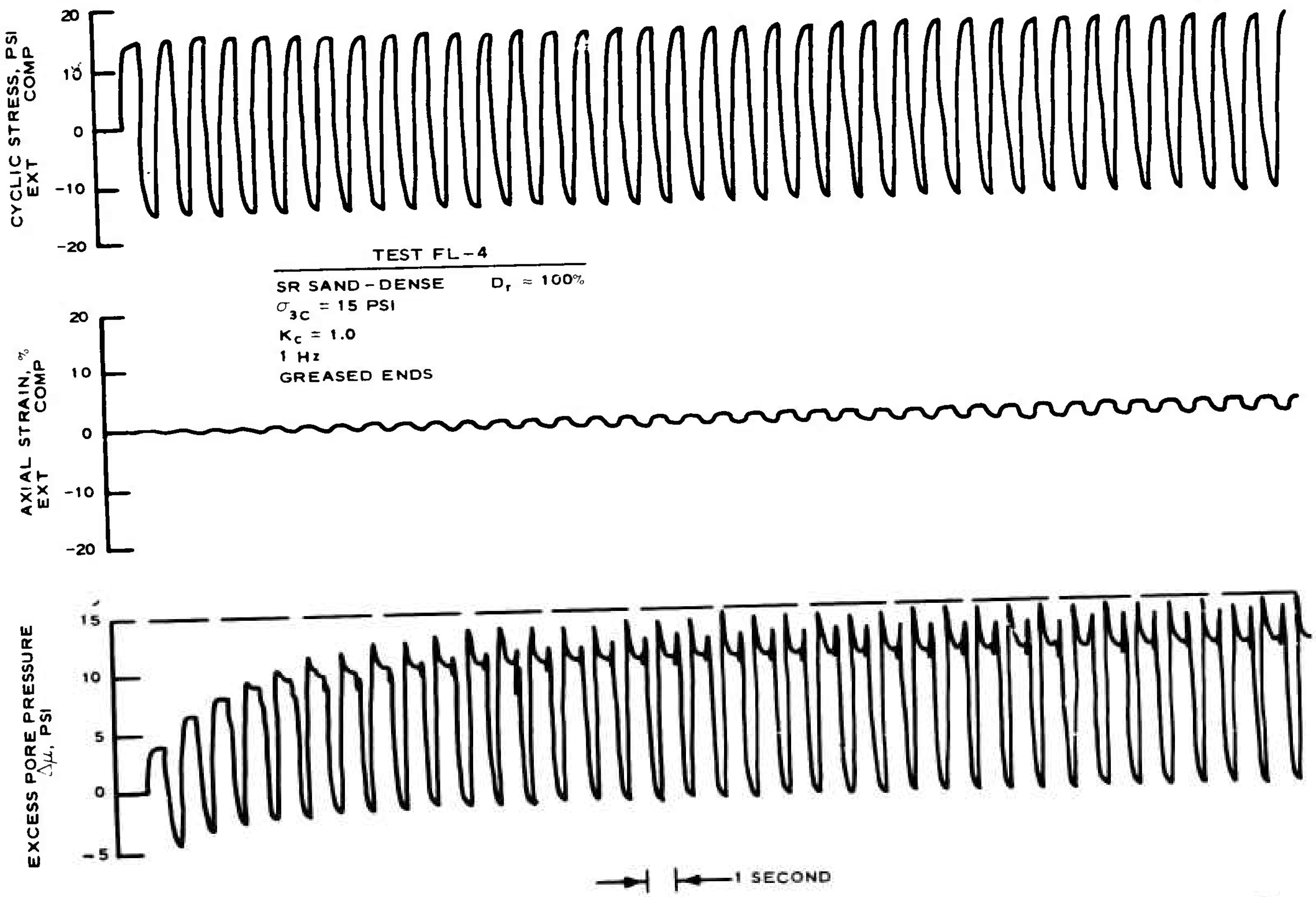


Figure 20. Record of a typical cyclic load triaxial test on dense Sacramento River sand; lubricated ends, 1 Hz

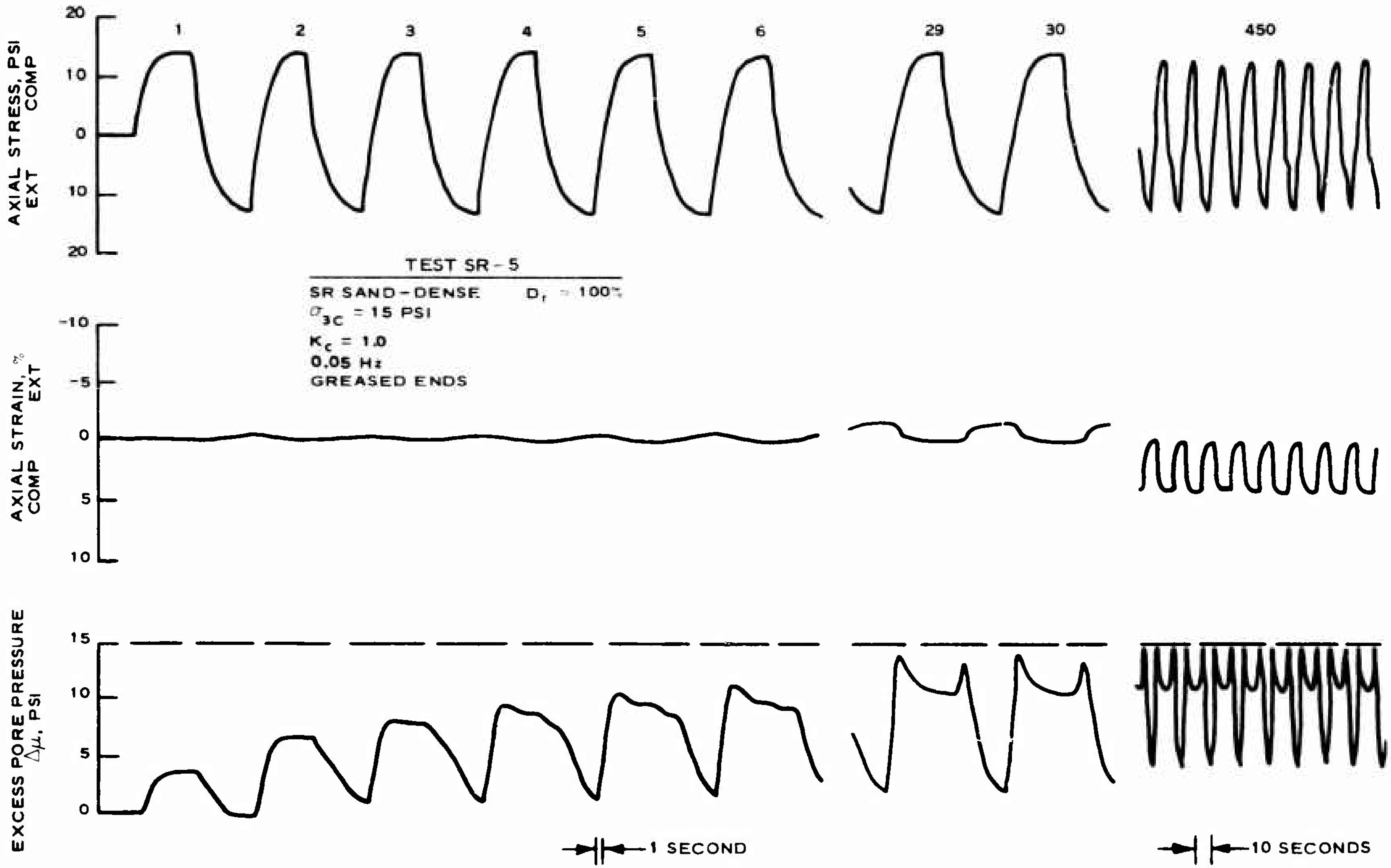


Figure 21. Record of a typical cyclic load triaxial test on dense Sacramento River sand; lubricated ends, peak load trace, 0.05 Hz

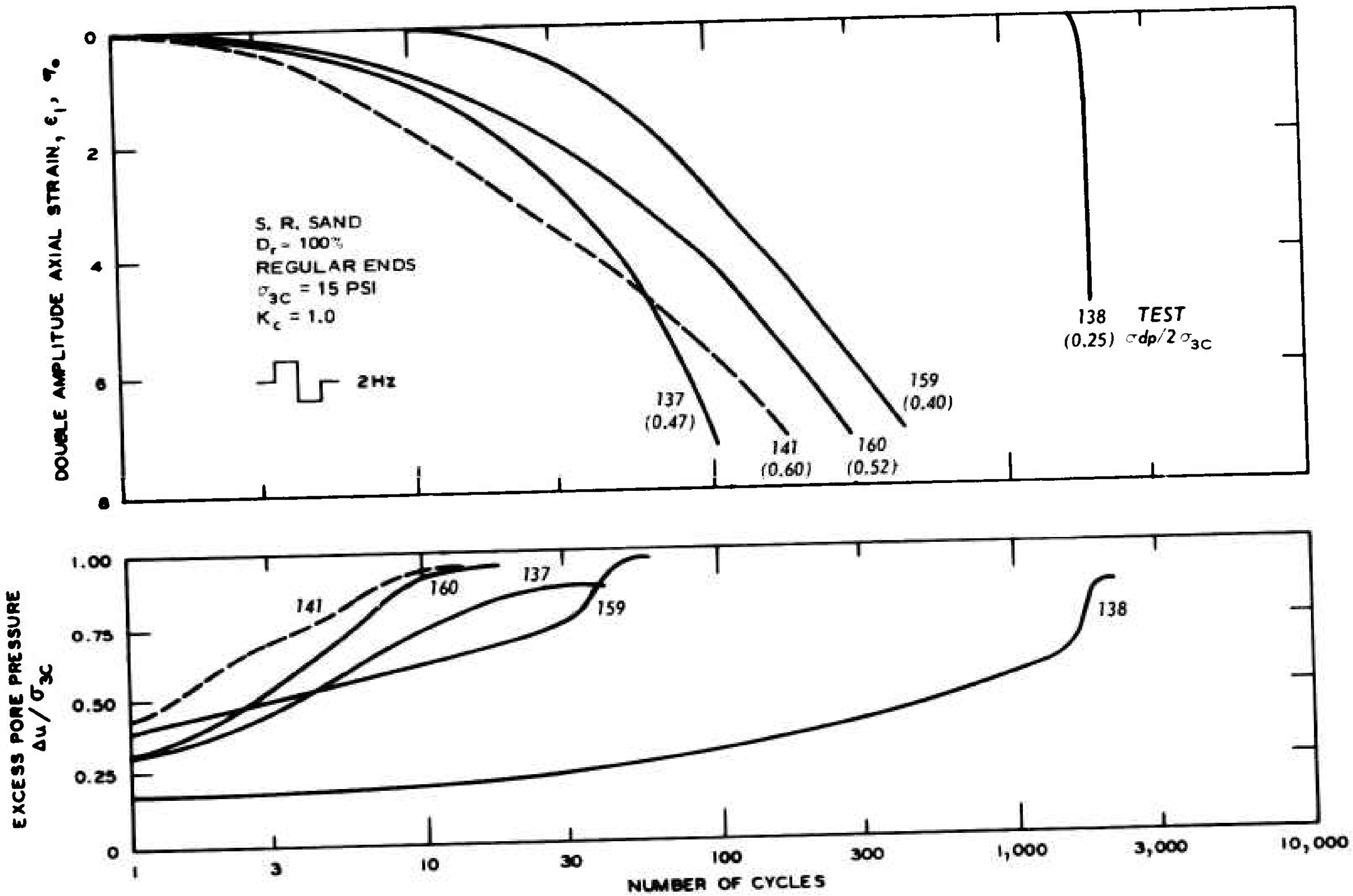


Figure 22. Summary of typical data from cyclic triaxial tests on dense sand using regular ends

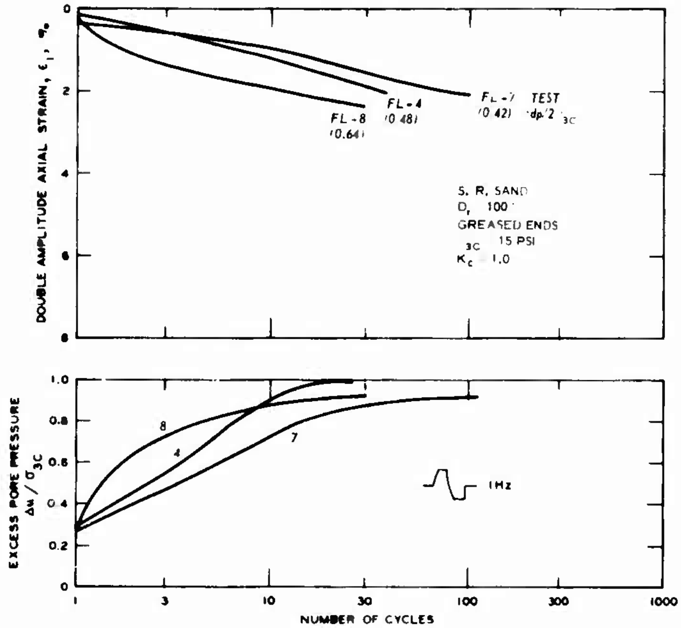


Figure 23. Summary of data from cyclic triaxial tests on dense sand using greased ends, 1 Hz

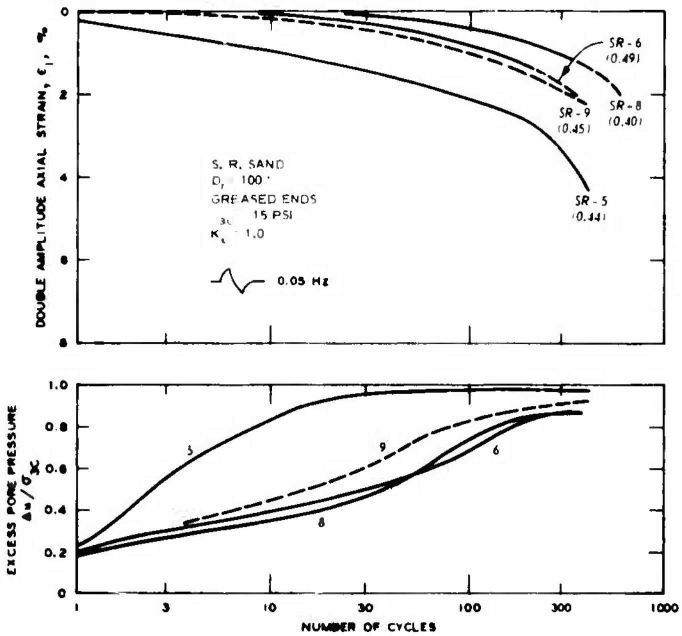


Figure 24. Summary of data from cyclic triaxial tests on dense sand using greased ends, 0.05 Hz

ends. However, the semilog scale can distort the shape of the curves so that a direct comparison is difficult.

84. The test records in Figures 19-21 do not show the well-defined transition to initial liquefaction that is evident with tests on loose sands. In fact, in some tests the peak excess pore pressure never does quite reach the confining pressure. This is typical of tests on dense sands.[2]

85. In early studies, an initial liquefaction state for dense soil was sometimes defined by the last cycle in which the pore pressure showed a significant increase, or by the first cycle in which a sharp spike was evident in the pore pressure record when the load passed through zero. However, these definitions were rather arbitrary and not necessarily unique. A more consistent definition describes failure in terms of a certain specified axial strain.[6] For example, 5 percent single amplitude strain has been used as a failure criterion for earth dam analysis.[4,10] The test data on dense sand produced in this study show that none of the tests with lubricated ends were continued to axial strains approaching 5 percent single amplitude (10 percent double amplitude); to have done so would have required in excess of 1000 cycles for most tests. Therefore, for purposes of this comparison, a failure criterion was arbitrarily selected as 2 percent double amplitude (1 percent single amplitude axial cyclic strain). The cyclic strength data by this definition for the dense specimens are presented in the usual semilog form in Figure 25. The data from the specimens with regular ends are fairly consistent in defining a smooth curve typical of many obtained on a large variety of soils. The strength data from the first series of tests (1 Hz) on specimens with greased ends are also fairly consistent, although there were only three tests. On this failure criterion basis, there was little difference between the two series of test data.

86. Only four tests were conducted in the third series (frictionless ends, near-sine form, 0.05 Hz). Three of the four tests show a very large increase in number of cycles to failure as compared with the regular end tests. However, the results of the first test in this

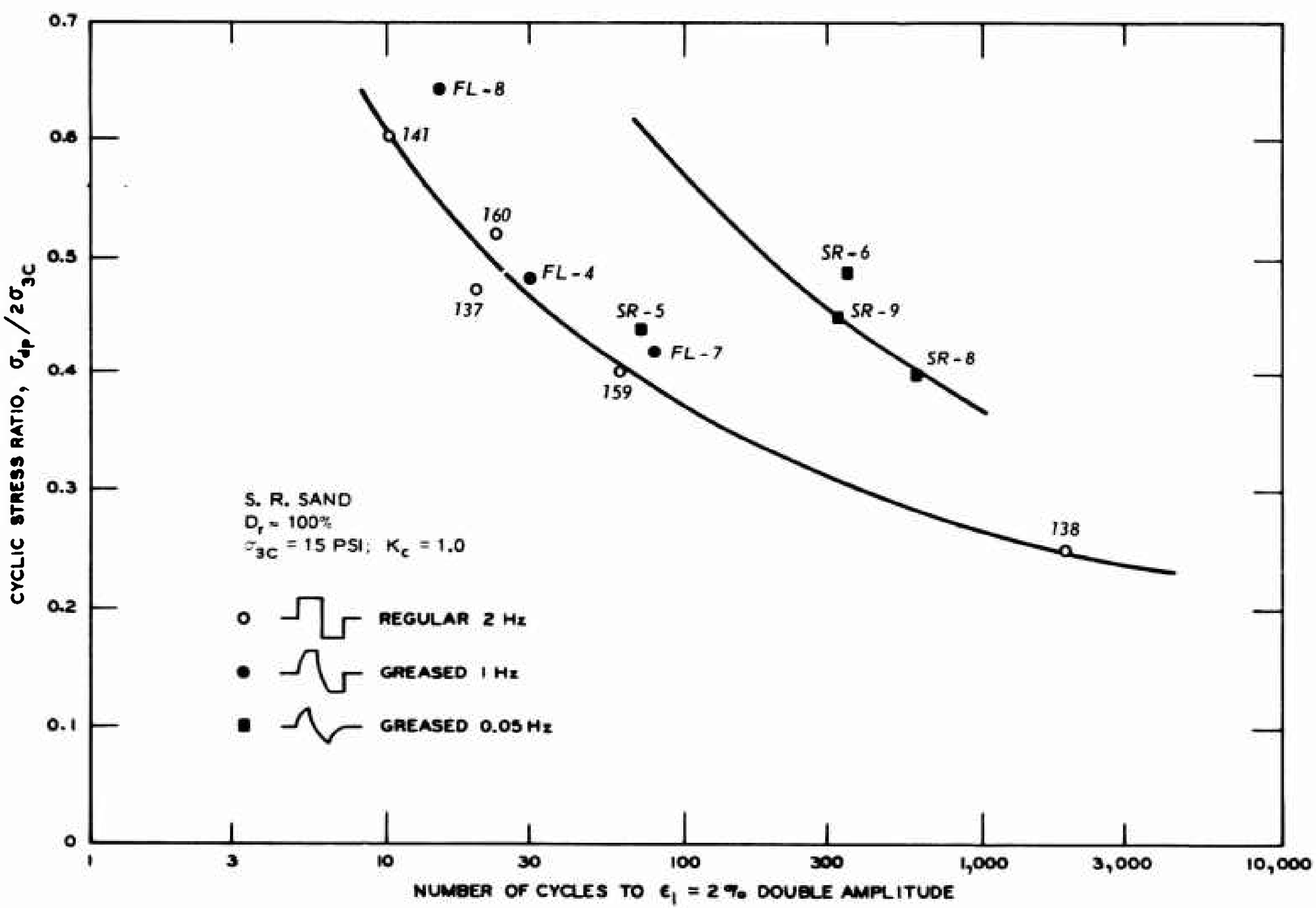


Figure 25. Summary of strength data from cyclic triaxial tests on dense sand with regular and lubricated ends

series (SR-5) fell within the range of data for regular ends.

87. Strength comparisons were not made for other failure criteria. However, the basic data curves in Figures 22-24 indicate that if failure were to be defined by say 5 percent double amplitude strain, the test specimens with lubricated ends would very likely be much stronger. Although all tests with frictionless ends were conducted with cyclic stress ratios greater than 0.4, none of these tests were carried to 5 percent strain, but extrapolating the data suggests that at least 1000 or 2000 cycles of stress would be required, whereas for regular ends, 5 percent strain was achieved in less than 300 cycles for all tests with cyclic stress ratio greater than 0.4.

88. Unfortunately, the data from the third series of cyclic tests at 0.05 Hz were scattered, and only 4 tests were conducted. The one test which plotted near the regular end data was not obviously wrong; however, it was the first test conducted in this series. Some difficulties were experienced in obtaining 100 percent relative density specimens with frictionless ends, and it may have been that this specimen was not prepared exactly like the three which followed. It may also have been that the lubricating grease was extra thin so that it did not provide as free a boundary as with the other three tests. However, three of the four "frictionless" end tests were consistent with each other in defining cyclic strengths that were significantly greater than the strengths for any of the regular end tests.

89. Thus, the limited results from cyclic loading tests on dense sands also agree with the proposed hypothesis that frictionless platens lead to a slower accumulation of axial strains than for regular ends.

Possible effects of load wave forms

90. In order to assure good frictionless behavior with greased end specimens, it was necessary to conduct tests using slow cyclic loading and near-sine wave form. The comparative regular end tests were conducted using a square-wave load pattern at 2 Hz. Therefore, an alternative explanation for the observed difference in cyclic strength might be ascribed to the form or the frequency of the cyclic wave

loading. Fortunately, considerable data exist from other UCLA laboratory studies and elsewhere to show that for sands, such as used herein, the shape and the cyclic frequency of the wave loading have no apparent affect on the cyclic strength.

91. Early data obtained by the writer from tests on medium dense Sacramento River sand are shown in Figure 26. The observed variation in strength with tests conducted using different wave forms was within the range of scatter of the data. Similar early data were obtained by Peacock and Seed[7] from cyclic simple shear tests on sand. A graduate student, M.-S. Wang, working under the direction of Peacock at the University of Western Ontario, also produced data[48] to show that wave form and frequency have no effect on the cyclic strength of Ottawa Standard Sand at D_r = 50 percent. Every factor was kept constant except the cyclic loading frequency and the wave form. At low frequencies, the wave form was square; however, at high frequencies, the loading equipment could not keep up and the attempted square wave degenerated into a curved sawtooth or near-sine form. The results of the seven tests are shown below:

Cyclic Frequency Hz	Number of Cycles for Initial Liquefaction
2	300 square load wave
4	300
6	290
8	300
12	350
16	250
28	250 near-sine load wave

92. On a semilog plot, the usual format for presenting this type of data, these results would suggest a remarkably consistent pattern, indicating that high frequencies did not affect the cyclic strength.

93. Recently Lee and Focht[49] presented data from cyclic loading

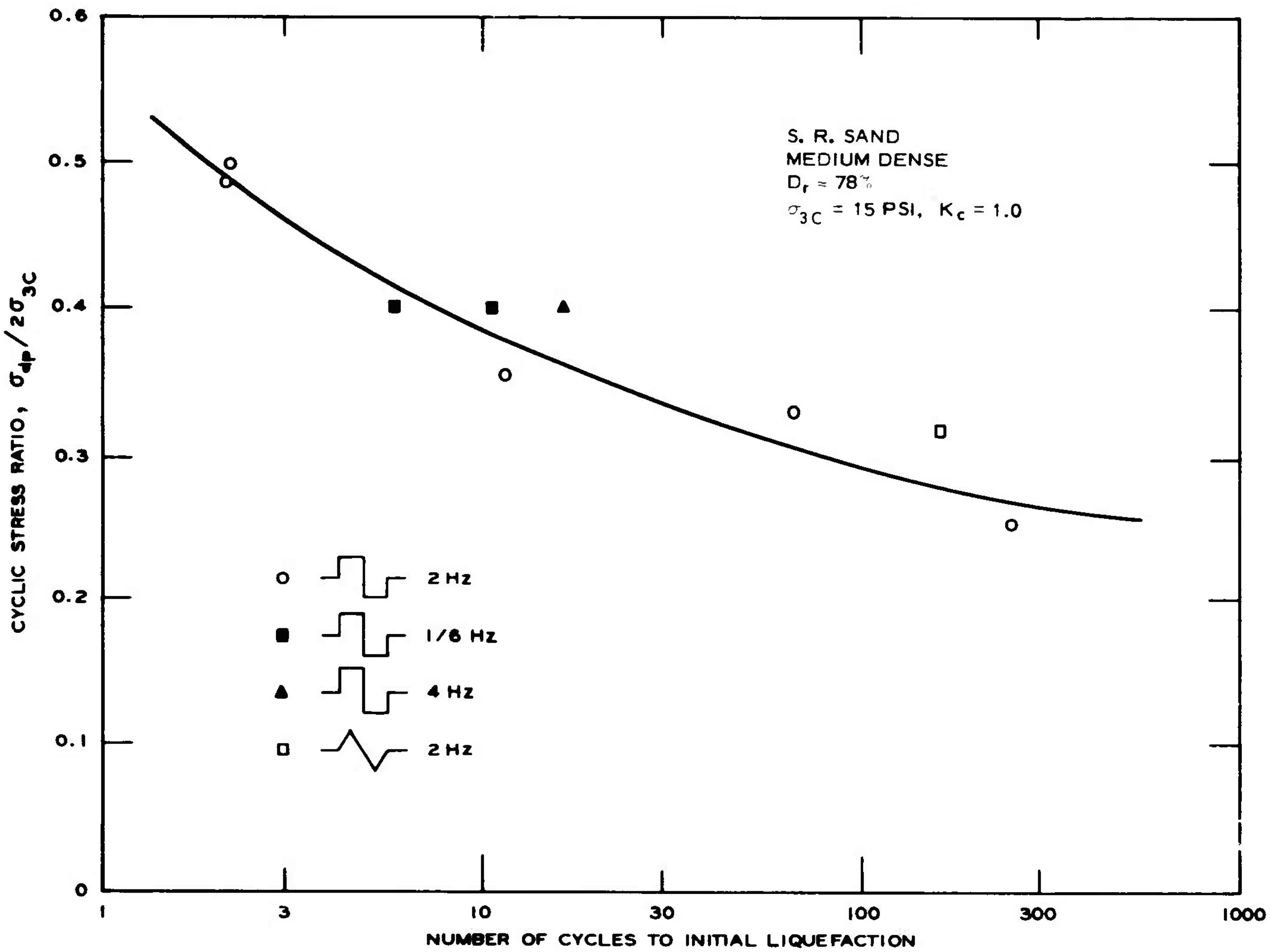


Figure 26. Effect of load shape and frequency on the cyclic triaxial strength of medium dense Sacramento River sand

tests at 1 and 0.08 Hz. These data, reproduced in Figure 27, also show a consistent pattern that low cyclic frequencies do not affect the cyclic strength.

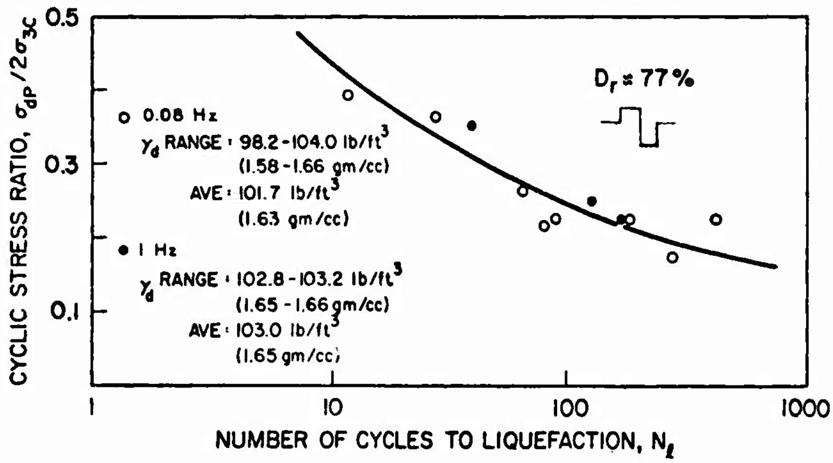


Figure 27. Effect of loading frequencies of 0.08 and 1 Hz on the cyclic triaxial strength of sand. (From Lee and Focht[49])

94. Finally, triaxial test data on Monterey No. 20 sand at 50 percent relative density are summarized in Figure 28. An extraordinary number of data points are shown for the square wave loading. The reason

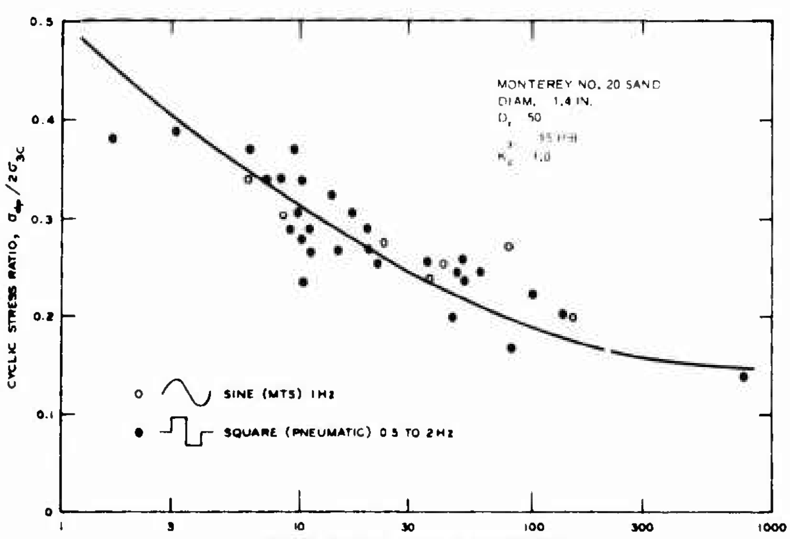


Figure 28. Effect of shape of load trace on the liquefaction potential in a cyclic triaxial test on sand

is that this sand has been used as a calibration standard for the UCLA laboratory. The data include results of tests conducted by Peacock and Seed[7] at Berkeley in 1968, and data obtained by six UCLA graduate students at various times after they had conducted enough preliminary tests to feel confident in this area of testing. The scatter in the results apparently represents operation error in making the specimen, setting the cyclic stress gages, etc., and may indicate the scatter that can be expected in this type of testing. Note that the data from the sine wave loading are completely consistent with the data from the square wave loading.

Effect of
height-to-diameter ratio

95. As described earlier in this report, conventional compression testing of materials, including soils, is usually conducted on cylindrical specimens using regular end platens and with length-to-diameter ratios of about 2 to 2.5. One of the advantages of frictionless ends is that they relax the requirement for this specified ℓ/d ratio. In fact, short cylindrical specimens using $\ell/d = 1$ with frictionless ends have given good results in static tests. Therefore, if frictionless ends were to be adopted as a standard for cyclic triaxial tests, one might also consider using them with short ($\ell/d = 1$) test specimens. This would produce the advantage that the specimens might be more homogeneous since localized inhomogeneous layers or pockets could be eliminated and still retain enough of a specimen for testing. Also, quality undisturbed samples could be divided in the middle, yielding two test specimens each; this practice should lead to more consistent results than currently obtainable testing only one specimen per undisturbed sample tube.

96. Concerning length-to-diameter ratio, a series of cyclic triaxial tests on loose Ottawa Standard sand with regular ends were conducted by Wang,[48] under the direction of Peacock (see Figure 29). Note that the short specimens ($\ell/d = 1$) were approximately 20-50 percent stronger than the regular specimens ($\ell/d = 2.3$). The writer knows of no cyclic triaxial tests on short specimens with frictionless end platens.

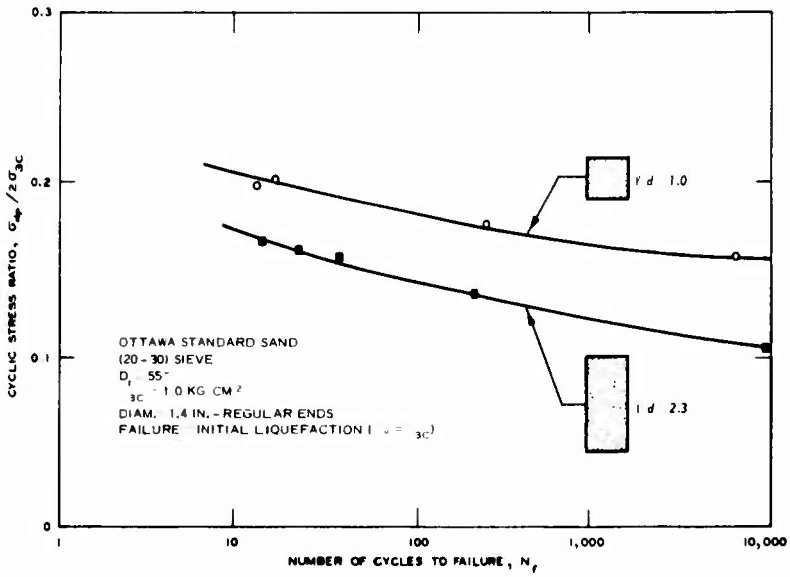


Figure 29. Effect of length-to-diameter ratio on the cyclic triaxial strength of loose sand with regular ends. (After Wang[48])

PART III: SUMMARY AND CONCLUSIONS

97. This report has surveyed the literature relative to the effect of lubricated or "frictionless" versus regular end platens in static triaxial testing of soils in general. In addition, original data have been presented concerning the effect of end restraint on static undrained tests and cyclic triaxial tests on saturated sand.

98. Most of the available data from the literature and from original sources are consistent in the overall conclusions. Regular end platens lead to nonuniform stress and strain distribution throughout the specimen; this has little effect on the static drained strength of sand or the undrained strength of clay. However, end restraint significantly influences volume change tendencies in triaxial test specimens, reducing the amount of dilatant volume change which would otherwise develop with frictionless ends. Soils which owe much of their strength to this dilatant volume change tendency will show larger strength when tested with frictionless platens than with regular platens. Thus, the use of frictionless platens increases the static undrained strength and the cyclic strength of saturated sand.

99. For the soil tested in this study, Sacramento River sand, the effect of frictionless ends led to an increase in static undrained strength of about 15-20 percent as compared with results of tests with regular ends. For cyclic triaxial tests, the strength with frictionless ends was 25-40 percent greater than when testing with regular ends. Expressed alternatively, at the same cyclic stress ratio, specimens with frictionless ends required about one order of magnitude more cycles than did specimens with regular ends to cause the same type of failure (initial liquefaction or specified cyclic strain). These effects of frictionless platens, as compared with those associated with regular platens, were of the same general order of magnitude whether the specimens were tested at low or high confining pressures (for static tests) or loose or dense (for cyclic tests).

100. These strength influences apply only to the Sacramento River sand for which test data are available. Other soils may be expected to

show different absolute and relative strengths, although the same general trend should be found for all granular soils.

101. The static undrained strength of saturated clays does not appear to be significantly affected by the type of end conditions used in triaxial testing. Perhaps this results from the absence of strong dilation tendencies of the undrained clays which have been studied. Although no data are available, it is believed that the type of end restraint will not significantly affect the cyclic strength of clays, since it does not affect the static strength. However, use of frictionless ends with cyclic testing of clays might allow more accurate cyclic pore pressure measurement, provided the grease viscosity and the rate of loading effect are properly accounted for.

102. The use of grease to produce frictionless ends in cyclic tests theoretically requires sine wave loading at fairly low frequencies to ensure that the viscosity of the grease does not lead to an effective restraint at the end of the specimen. The limited experimental data suggest that this reasoning is sound, at least for dense soil. Fortunately, for sands, the form and frequency of the loading wave have no measurable influence on the cyclic strength.

103. If frictionless platens were to be adopted for routine design testing, then the other semiempirical factors which enter into the overall design must also be changed, since these factors were developed from data from tests with regular ends.

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Prepared for Office, Chief of Engineers, U. S. Army, Washington, D. C., under Contract DACW 75-M-1674 and CWIS 31145.
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