Stratospheric Turbulence and Vertical Effective Diffusion Coefficients

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STRATOSPHERIC TURBULENCE AND VERTICAL EFFECTIVE DIFFUSION COEFFICIENTS

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ABSTRACT: We have obtained an estimate of residence times in the stratosphere in terms of an effective diffusion coefficient, K_e. Our approach is based on the hypotheses that (1) CAT (clear-air turbulence) is the major source of vertical transport, and that (2) almost all CAT is caused by the Kelvin-Helmholtz shear instability. According to the best current evidence, turbulent instability occurs whenever the Richardson number (Ri) is less than approximately 0.25.

Our calculations used velocity data from a NASA report of 200 rocket smoke-trail wind profiles at 25 m resolution. Our analysis of stratospheric shears as a function of altitude from this sample (30,000 data points) revealed that about 2% of the altitude consists of thin sporadic layers of high shear, separated by large regions of low shear. Richardson numbers and turbulence frequencies were computed from these shears on the basis of a standard temperature profile.

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INTRODUCTION

In this paper we report computations of the observed frequency distribution of the magnitudes of vertical shears of horizontal winds between 5 and 20 km altitude. We then derive the probability distribution of turbulent layers for various values of vertical thickness, on the basis of accepted relationships between shear and turbulence. An effective vertical-diffusion coefficient, K_e, is estimated from a simple model using this empirical turbulence probability distribution.

Specifically, we calculate the effect of clear-air turbulence (CAT) upon vertical transport, using a model consisting of a vertical column of thin, randomly-spaced mixing (turbulent) layers separated by thick non-mixing atmospheric layers. The turbulent layers correspond to the intermittent sporadic CAT "blini" described in the literature (Bretherton, 1969). We also assume that essentially all CAT is due to shear instability of the Kelvin-Helmholtz type, which can occur in stratified fluids. This assumption is shown to have wide experimental support in the current literature. Our results lead to an estimated diffusion coefficient of approximately 0.3 m^2/s in the stratosphere. This is roughly consistent with vertical diffusivities estimated from radioactive fallout, which have values ranging from 0.1 m^2/s to 1.0 m^2/s for tropical and polar stratospheres respectively (Junge, 1963).

CLEAR-AIR TURBULENCE AND KELVIN-HELMHOLTZ BILLOWS

In order to clarify our model, it seems appropriate to briefly review the main background information concerning the Kelvin-Helmholtz (K-H) instability, as well as the evidence that CAT, at the altitude of interest, is almost always due directly to the K-H phenomenon.

The K-H Instability

Kelvin's original paper (Kelvin, 1910) on this phenomenon treated the influence of wind on water waves. Helmholtz was the first, however, to discuss the instability of surfaces separating

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fluids which have different velocities. Subsequent literature assigns their names to the more general instability which occurs when there are vertical shears of horizontal velocities across finitely thick layers of vertically stratified fluids. This instability frequently occurs in both the ocean and the atmosphere. Whenever a horizontal layer is buoyantly stable but has a sufficiently high velocity-shear across it, a small perturbation will result in a growing wave which eventually breaks and generates a patch of turbulence. Such breaking "gravity waves" are usually organized in clusters and result in horizontally wide but vertically thin turbulent layers. The criterion for instability is given in terms of Richardson number, Ri, which is defined as

$$\mathrm{Ri} = -g(\partial\rho/\partial z)/[\rho(\partial u/\partial z)^2]$$

$$= (+g/T)\,((\partial T/\partial z) + \Gamma)/(\partial u/\partial z)^2 \quad (1)$$

The first form on the right is often used in oceanography, and the second form in atmospheric physics. g is the acceleration gravity, ρ is the fluid density, z the vertical coordinate, u the horizontal velocity, T the temperature in °K, and Γ the adiabatic lapse rate (dry air assumed) (Monin and Yaglom, 1971).

The criterion for stability is that Ri be >0.25 everywhere in the flow (Taylor, 1931; Goldstein, 1931; Miles, 1961; Hazel, 1972; Turner, 1973). This is a general result which has been accepted (within certain restrictions). This criterion does not mean, however, that instability and turbulent breakdown occur whenever and wherever Ri < 0.25; it simply means that Ri $<$ 0.25 is a *necessary* condition for turbulence to occur. The physical reason for the value 0.25 is that, for this value of Ri (Ludlam, 1967; Businger, 1969a), the available kinetic energy due to the velocity difference across the layer is equal to the work which must be done against buoyancy forces in order to exchange fluid parcels across the layer. Once the energy is available, a perturbation may result in a growing nonlinear oscillation of the layer, in the form of a wave. When turbulence has started (after the wave breaks), mixing can occur *within* the layer. Businger (1969) has shown that in this case, Ri = 1 represents equality between potential energy and available kinetic energy (thus explaining Richardson's original Ri = 1 criterion for the instability threshold). Once a layer has become turbulent, then, one would expect it to continue to be "fed" energy until Ri has increased to 1. This occurs when the layer thickens enough to sufficiently lower the shear across it. After such a point, turbulence would be damped by the forces of stable buoyancy.

Internal K-H billows below the ocean surface have been investigated by oceanographers. One simple model for such observations has been given by Woods (1969). He assumed that Ri = 0.25 is also a *sufficient* condition for turbulence. Experimental evidence to date seems to indicate that a value of Ri around 0.25 is indeed a sufficient condition in the free ocean and atmosphere (away from boundaries and in the absence of obliquely-shearing oblique winds, etc. (see Hines, 1971). Woods also assumed that Ri = 1 was the "cutoff" condition for turbulence. He thus postulated a "hysteresis effect" that would give a stable layer until Ri <0.25, and would then become turbulent until the layer thickened enough to make Ri = 1.

Figure 1 (based on his report) shows the sequence of events he observed by means of dye tracers. The initially stable layer goes into oscillation. A wave builds up until nonlinear effects predominate, causing a characteristic "roll-up." Finally there is turbulent breakdown and layer spread until Ri = 1. He assumed that the density (ρ) and velocity (V) gradients were zero outside the layer, but that ρ and V at the top and bottom layer surfaces remained constant. Under these assumptions, the transition from Ri = 0.25 to Ri = 1 causes the layer to thicken by a factor of 4. At Ri = 1 the turbulence subsides and the layer becomes stable.

The original (and now famous) paper which first described this "roll-up" effect was by Rosenhead (1931); it showed a numerically-generated billow effect. This effect has also been beautifully reproduced in the laboratory by Thorpe (1973), who generated K-H billows by inducing a shear between two initially stable liquids of different densities. His measurements indicated that the actual "cutoff" for turbulence is more like 0.4 ± 0.1 rather than the overly simple theoretical value mentioned above, so Ri = 0.4 is known in the literature as the Thorpe number (Garrett and Munk, 1972).

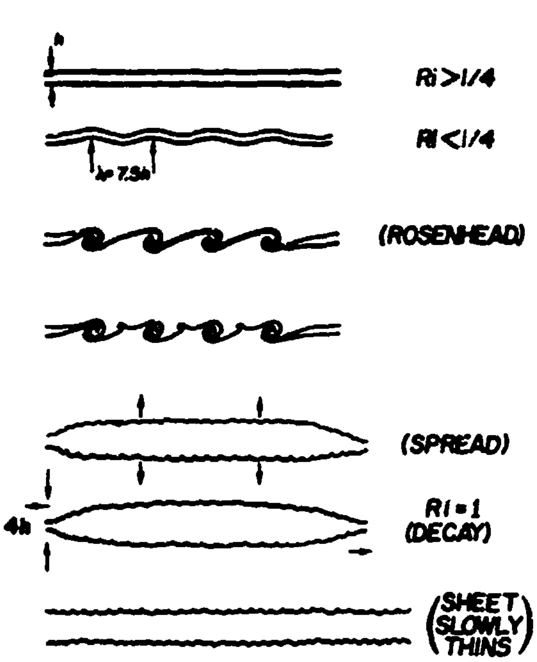

Figure 1. Development of an underwater Kelvin-Helmholtz-type instability and subsequent roll-up, breakdown, spread and decay. (After Woods, 1969.)

According to Woods and Wiley (1972) such K-H mixing and spreading events constitute the principal mechanism for vertical mixing throughout the World Ocean. Their work has been enlarged upon by others (e.g., Garrett and Munk, 1972). In view of the hypothesis that CAT is due to K-H billow events, it seems eminently reasonable to hypothesize that it also plays the dominant role in vertical mixing of the upper atmosphere.

CAT and K-H Billow Events

Radar observations of CAT by Richter and Gossard (see Battan, 1973) clearly show the K-H billow shape. In fact, the entire sequence observed by Woods has also been seen in CAT; it is illustrated by Browning and Watkins (1970) with the schematic shown in Figure 2 (see also Atlas, 1970). The spreading shown appears to be negligible in contrast to what is seen in the ocean. Earlier work of Ludlam (1967) also showed similar structures in clouds. When Browning (1971) analyzed seventeen K-H events by means of simultaneous radar and balloon soundings, he found that almost all of these events were preceded by a period of time in which Ri was around 0.25. The events themselves lasted approximately 500 sec. on the average, and the condition Ri <0.25 was often maintained for over one-half hour prior to the K-H events in the figures he gave. This and other evidence reviewed by Dutton (1971, 1973) make a convincing case that CAT is most likely due to the K-H phenomenon. Thorpe himself refers (1973) to underwater K-H as "underwater CAT."

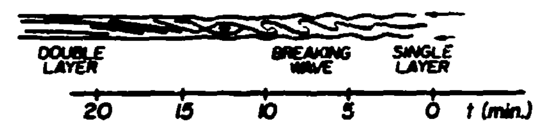

Figure 2. Schematic of a K-H billow event in the atmosphere as detected by means of radar. (After Browning and Watkins, 1970.)

WIND-PROFILE DATA AND THE STATISTICAL SPATIAL STRUCTURE OF CAT

The experimental data base for our calculation was a series of reports containing 200 vertical profiles of horizontal winds measured from smoke trails by Miller, Henry, and Rowe* (1959-1962, 1965, 1968). They gave wind velocity vectors as a function of height at 25-m intervals with a precision of 0.1 m/s. Figure 3 shows a typical smoke trail. This montage shows the progressive distortion due to shears. In all there were 90,000 data points: 50,000 below 12 km, 30,000 between 12 and 16 km, and 10,000 above 16 km. From these, shears were calculated as $[(\Delta V_x/\Delta_z)^2 + (\Delta V_y/\Delta Z)^2]^{1/2}$, where V_x and V_y represent the horizontal wind components.

Figure 4 shows the cumulative frequency distribution for component shears measured at 25-m spacing, where the ordinate is given in units of standard deviations. The linear portions of these curves (the central 90%) correspond to a Gaussian distribution with a standard deviation of 0.014 s^{-1} in both components at both altitudes. However, high shears occur more

* All profiles were obtained in beautiful weather, and this selection aspect of the method of data collection must be kept in mind.

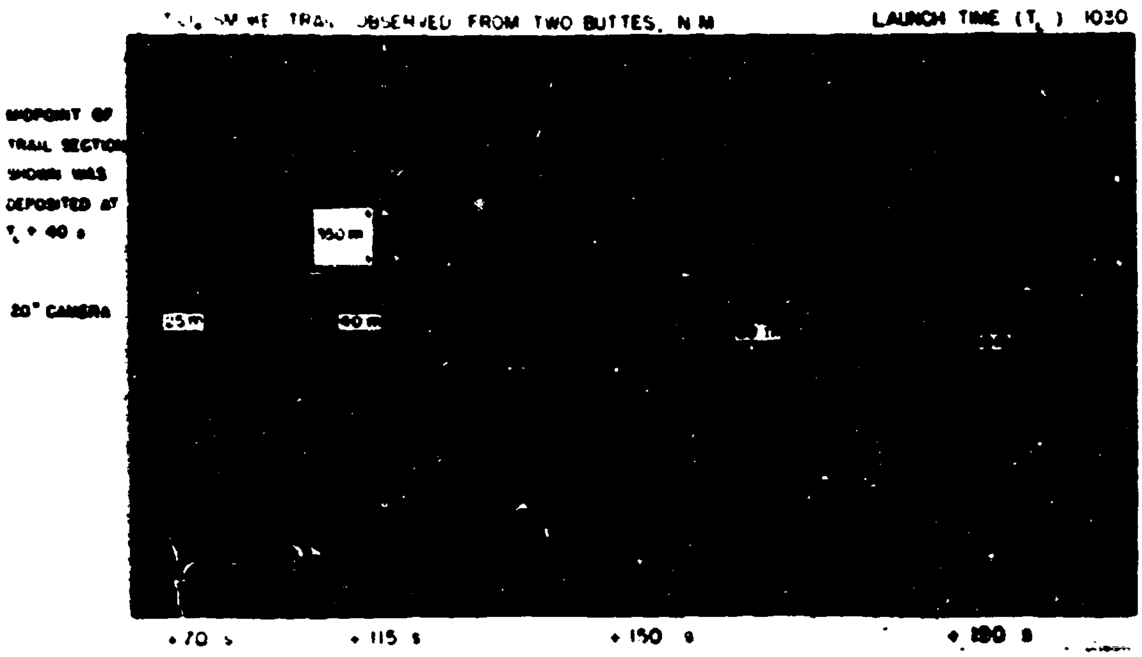

Figure 3. Typical smoke-trail profile showing the distortion due to wind shears.

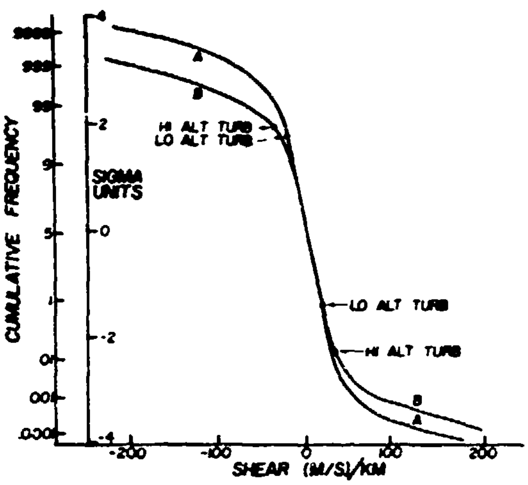

Figure 4. Cumulative frequency distributions for component shears. A is based on tropospheric winds while B represents stratospheric shears. The threshold shears for turbulence are indicated by dots. Note that distributions become non-Gaussian (actually log-normal) for supercritical shears. Observe also the asymmetry about the vertical axis.

predict. When the non-Gaussian portions of these curves were plotted with a log-normal scale for the ordinate, they became linear. According to Gibson, Stegen, and Williams (1970), who referred to predictions of Kolmogoroff, Obukhoff, and Yaglom, there is now good evidence that the probability distributions of velocity derivatives are log-normal, provided that these velocities are part of an inertial-range turbulent velocity field. This raises a very interesting question: Why should the high shears which would *bring about* turbulence have the log-normal statistics which would be expected from the turbulent process itself? One might speculate that the high shears (of horizontal and presumably *laminar* winds) have their origin in a much larger scale of turbulence. We will not discuss this phenomenon further in this paper, but simply note that it seems to be of interest.

The critical shear was computed from $Ri = 0.25$, but since temperature measurements made simultaneously with the wind data were not available, we used U.S. Standard Atmosphere mean temperature gradients of -6°/km in the troposphere and 0°/km in the stratosphere, with a -9.8°/km adiabatic lapse rate. This led to a frequently than a Gaussian distribution would critical shear, S_c, of 0.025 s^{-1} in the 5-12 km region (troposphere), while for the 12 km - 19 km region (stratosphere), S_c was 0.045 s^{-1}. The latter higher value, of course, reflects the higher stability of the stratosphere. Figure 4 shows that many of the above-threshold shears are in the log-normal portion of the curves, especially for the stratosphere.

Using the above values for S_c, we obtained the cumulative frequency distribution, $P_I(L)$, for finding turbulent layers of thickness L or greater. Note that only 2% of shears at 25 m resolution exceed threshold in Figure 4. $P_I(L)$ is related to the probability $P(L)$ of finding a layer having a thickness between L and L + dL by

$$P_I(L) = 1 - \int_0^L P(L')dL' \tag{2}$$

We can therefore derive $P(L)$ from our data by calculating the difference between $P_I(L)$ for neighboring values of L.

$$(dP_I(L))/dL = P(L) \tag{3}$$

Figure 5 shows a plot of $P_I(L)$ vs. L. We used the empirical results of Miller et al. (1965) directly in our calculations. An extrapolation was made for L = 0 by a least-squares fit of the form $P_I(L) \propto \sqrt{L}$ on available points.

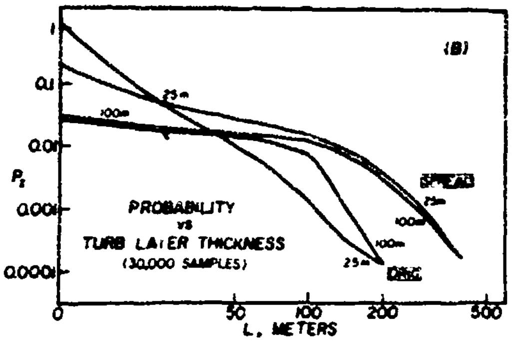

Figure 5. Cumulative frequency distributions ($P_I(L)$) for various lengths L of unstable layers.

THE VERTICAL-STACK DIFFUSION MODEL

A one-dimensional model relating effective vertical diffusivity K_e to $P_I(L)$ is derived below. In other words, the vertical structure of CAT layers is determined from wind-shear data, using the criterion $Ri < 0.25$, and this structure is then used to determine the vertical transport by means of the model.

Assumptions

First we shall assume that there is no vertical transport between turbulent layers: in other words, all such transport is assumed to take place within CAT mixing layers. Second, we shall assume that the horizontal rearrangements of the layers and material being transported will have no effect on the vertical transport. This type of assumption is not unusual in oceanography and allows the use of a simple one-dimensional model (Garrett and Munk, 1972).

Model

The fundamental definition of the coefficient of diffusivity provides the basis for the derivation of our model. Figure 6 shows a horizontal slab through which material (or heat, in the general case) diffuses vertically. Suppose that the concentration C of the material is held to a constant at the top of the slab by means of an infinite reservoir, and assume that the downward diffusion is steady-state. Assume that all material reaching the bottom of the slab drops into an infinite sink at zero concentration. The profile of C would then be a straight line as indicated.

The definition of the coefficient of diffusivity, K, is

$$K = (dn/dt)/(dc/dz) \tag{4}$$

where dn/dt is the number of moles of material flowing out through a unit of surface area at the bottom of the slab per unit of time, and dc/dz is the constant gradient of the concentration with respect to the altitude, z. When vertical motion is not due to molecular transport effects, but is instead due to turbulence of some sort, this motion can be expressed in terms of effective diffusivity, K_e.

To simulate stratospheric vertical motion, we imagine a series of thin horizontal mixing

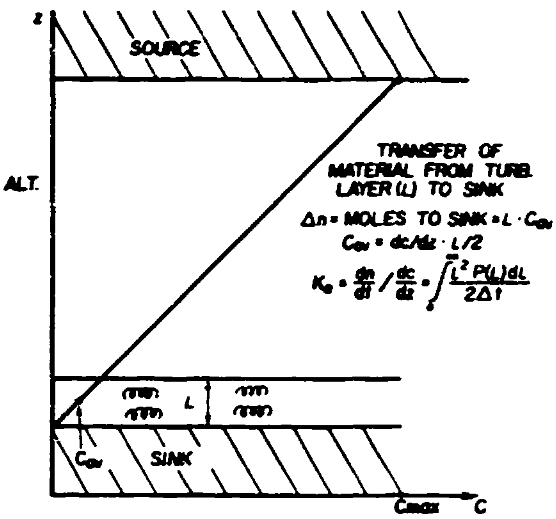

Figure 6. Diagram for K_e model relating wind-profile statistics (P(L)) to vertical transport.

layers which are randomly arranged vertically and vary randomly in their thickness. We suppose that within each layer there is complete mixing and that after a time interval t, the entire profile is replaced by a different one from an ensemble of statistically similar profiles. In reality the CAT layers will appear sporadically and intermittently, but we shall assume that our "time step" approach will not affect the results. Whenever a mixing layer occurs at the bottom of the stack, material will flow out to the sink. This would happen only occasionally, after a large number of time steps, each of duration Δt. Due to the assumed steady-state random nature of the process, the *average* concentration gradient will remain constant. In Figure 6, when a mixing layer of thickness L occurs at the bottom of the slab, the volume of material involved is L times the unit of area, and the average concentration within this volume is (L/2) (dc/dz) leading to Δn (moles transferred) = L(L/2) (dc/dz). If we know the probability P(L) that a layer of thickness L will occur at the bottom in the interval Δt, we can obtain

$$Ke = (\Delta n/\Delta t)/(dc/dz) = \int_0^\infty P(L)L^2 dL/(2\Delta t) \tag{5}$$

which takes into acount all possible layer thicknesses.

Since we assume that the random nature of our turbulent layers will ensure that vertical transport will be, on the average, diffusive in nature, we can assume that K_e also satisfies the diffusion equation:

$$K_e \nabla^2 C = \partial C/\partial t \tag{6}$$

From this we can calculate the residence time for a layer of pollution. Assuming an initial Gaussian distribution, Eq. (6) leads to

$$C(z,t) = \frac{1}{\sqrt{t}} \exp(-z^2/4\,K_e t) \tag{7}$$

(Korn and Korn, 1968). The residence time t_R, i.e., the time needed for the Gaussian radius (one-dimensional) to reach a distance (z) = H, is therefore

$$t_R = H^2/4\,K_e \tag{8}$$

In order to calculate K_e from Eq. (5) and t_R from Eq. (8) we need to have an estimate of Δt, and this will be discussed in the next section.

GROWTH AND DECAY TIME, Δt, FOR A K-H BILLOW EVENT

The Δt in our model does not correspond to the duration of turbulence, but rather to the time needed for a K-H event to develop after Ri has descended below 0.25. Thus Δt, or the time between profiles in the model, corresponds to the interval between Ri < 0.25 and the time when turbulent breakdown makes Ri > 0.25.

We shall estimate Δt directly from some observations by Browning (1971) of 17 K-H events. He made measurements simultaneously by radar and balloon soundings. Figure 2, taken from Browning and Watkins (1970), shows that in a typical event, the build-up and breakdown can take place in a period of approximately 1000 seconds. In Browning (1971), information on Δt times was available for 6 of the 17 K-H events. The time elapsed between Ri < 0.25 and the end of a billow event (Ri > 0.25) varied between approximately 1000 seconds and 5000 seconds, averaging about 3000 seconds, and the average duration of the 17 billow events themselves is approximately 500 seconds. (The exceptional case of a 4-hour billow event was omitted in the calculation of this average.) In view of the above, we chose $\Delta t \simeq 3{,}000$ sec for the 5-12 km altitude region which was studied by Browning.

The value of Δt for the stratosphere would differ from the above values. From Rosenhead (1931) it can be seen that Δt is proportional to λ/U, where λ is the most unstable wavelength $\approx 7h$ and where h is the layer thickness (Turner, 1973), and where U is half the difference between the velocities on each side of the layer. Thus

$$\Delta t \propto 7h(du/dz)^{-1}(h/2)^{-1} \propto du/dz^{-1} \qquad (9)$$

where we take du/dz to be S_c. Since the stratospheric S_c is approximately twice the size of the tropospheric S_c, Δt in the stratosphere would be 1,500 sec, if we accept 3,000 seconds as the tropospheric Δt.

The turbulence should start to decay when Ri exceeds 0.4, the Thorpe number mentioned above. As we have seen from the data of Browning, the duration of a billow is about 500 sec on the average, which presumably is the time needed to raise the Ri above the threshold once turbulence has commenced.

RESULTS

Figure 7 summarizes our main findings. The relation

$$P(L) = 1 - \int_0^L P(L')dL'$$

was explained earlier, and illustrated in Figure 5. We have also explained how the effective diffusion coefficient, K_e, can be obtained from

$$K_e = \frac{1}{2\Delta t} \int_0^L P(L')(L')^2 dL' \qquad (10)$$

Figure 7 shows K_e as a function of the upper limit of integration L. Among the 30,000 atmospheric shears, 60 were found with a thickness greater than 200 m ($P_l(L) = 0.002$), but none with a thickness greater than 300 m ($P_l(L) = 0$). A comparison of Figures 5 and 7 shows how $P_l(L)$ affects the shape of $K_e(L)$.

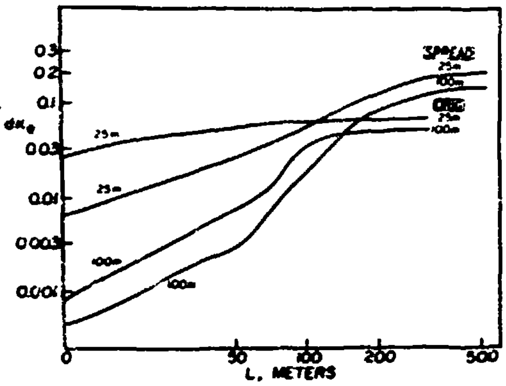

Figure 7. Diffusion coefficient K_e vs. turbulent-layer thickness, showing results at 25 m and 100 m resolution, and the effects of layer spreading on them.

Figure 7 indicates that $K_e = 0.068$ m^2/s for data at 25 m resolution. We were also interested in the effect of data resolution on K_e. A four-point moving average was used to smooth component velocity profiles to simulate 100-m resolution. Figure 7 shows that this results in no significant change in the final value of K_e, although the dependences of P_l and K_e on L have been markedly altered ($K_e \approx 0.054$).

Next we consider the effect of vertical spreading of turbulence upon our estimate of K_e. As was discussed earlier, once turbulence has been initiated in regions of high shear, it spreads vertically until the mean shear decreases so much that the increasing Richardson number reaches the "extinction" value of about 0.4.

Figure 8 shows "original" 25-m and 100-m resolution shear profiles. It also shows what happens if we allow turbulence spreading to bring supercritical shears down to their critical values (Ri = 0.25).

Figure 9 demonstrates how the spreading was computed. The left side shows a jagged profile of shear vs. altitude, and the right side a hodograph (showing the velocity profile as seen from above). When a supercritical shear is encountered at an altitude z (e.g., altitude 16.50 km in Figure 9, between point 7 and 8 in the hodograph), a search is made to find the maximum height separation, centered at z, which is still supercritical (Ri < 0.25). It is assumed that the profile will take on a constant shear between those two altitudes, with the excess energy going into turbulence. Thus we joined heights 16.30 km and 16.65 km with a constant shear, and the

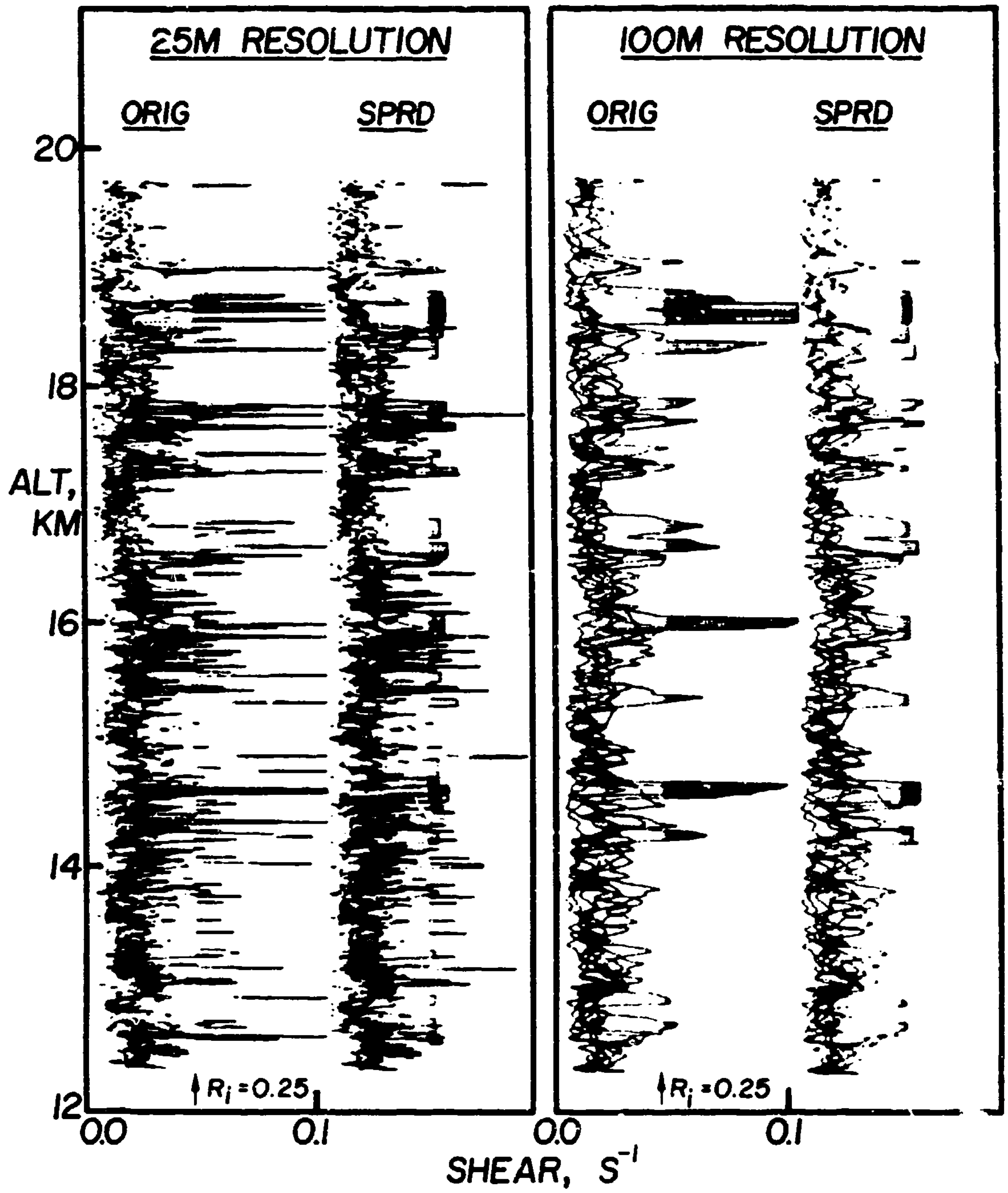

Figure 8. Effects of resolution and layer spreading of supercritical shear layers. The curve on the right side of each box shows the profile after spreading. These profiles consist of a superposition of 8 trails.

hodograph between these points with a straight-line vector.

Figures 5 and 7 show the effects of spreading on $P_l(L)$ and $K_e(L)$ at 25 m resolution. The K_e estimate has been raised to 0.21 m^2/s because of the thickened layers. A decrease in resolution to 100 m is seen to decrease estimated K_e to 0.15 m^2/s. From this we would expect that, if the resolution were improved beyond 25 m, one might find a larger value for K_e. Trails

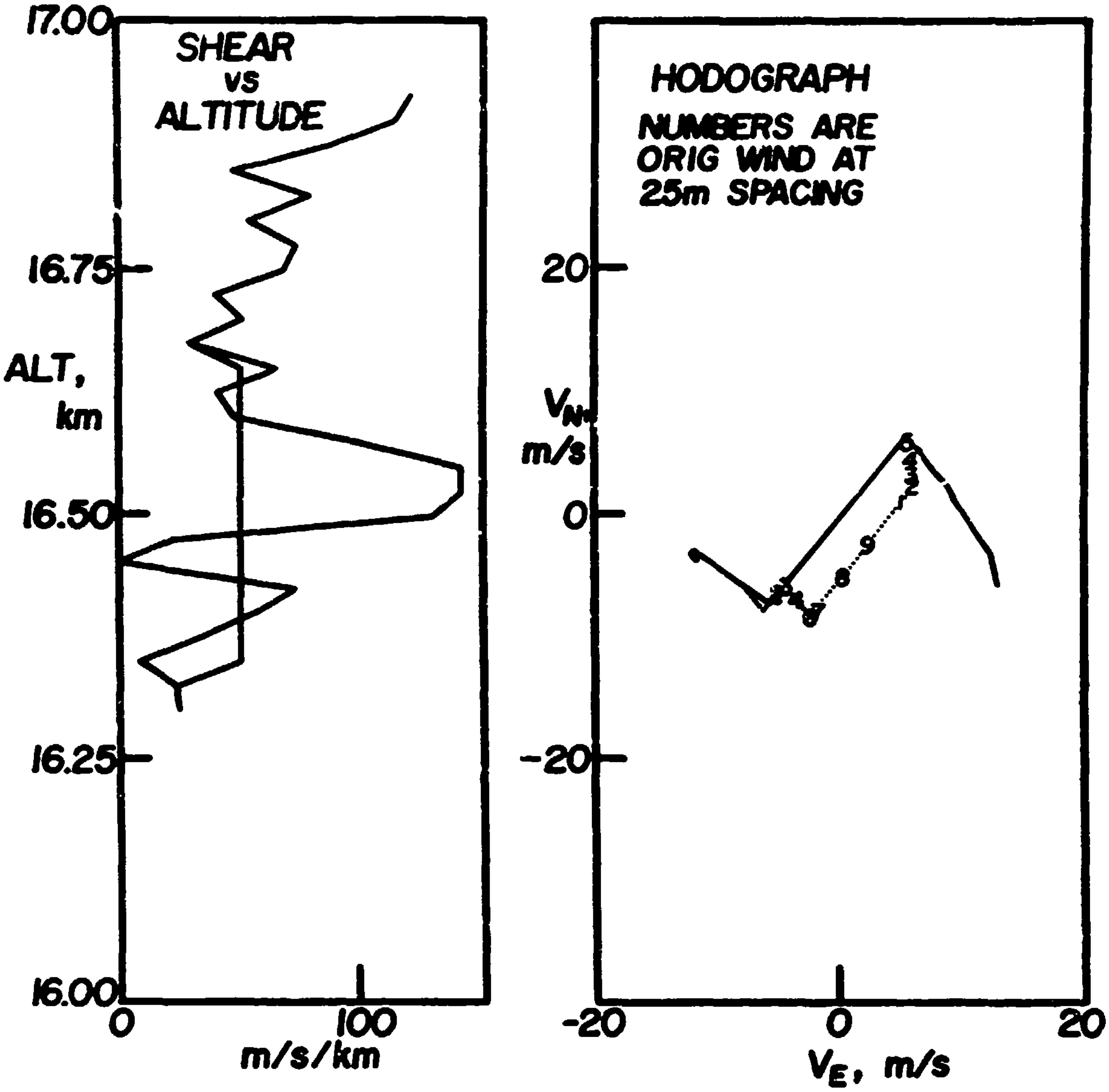

Figure 9. Shear profile and velocity hodograph before and after spreading.

with such higher resolution (10 m) are currently being analyzed in our laboratory. If the spreading were allowed to continue until Ri = 0.4, K_e would be increased. The spread which would account for a change in Ri from 0.25 to 0.4 is found from

$$Ri = -g\left(\frac{\theta'}{\overline{\theta}}\right)\frac{1}{(u')^2} = -\frac{g\theta' h^2}{\overline{\theta}(\Delta u)^2} \tag{11}$$

where $\overline{\theta}$ is the average potential temperature in a layer, θ' is the potential-temperature gradient, h is the layer thickness, and Δu is the difference in horizontal velocity across the layer. θ' and Δu can be presumed to remain approximately constant as the layer expands; thus Ri is proportional to h^2. From this we see that h^2 would grow by a factor of $(0.4/0.25) = 1.6$. From Eq. (10) we see that an increase of all values of L (or h) in this way amounts to multiplying K_e by a factor of 1.6. A spread 25-m profile would then result in

$$K_e = 0.21 \times 1.6 \approx 0.3\ m^2/s.$$

In order to see whether the extremely high shears (in excess of 2 S_c) were an important factor for the value of K_e, we edited out these high shears (amounting to 0.1% of the sample) and repeated the calculations. We obtained essentially identical results, and so ignored the very high shears thereafter.

Using Eq. (8) with H = 10 km (corresponding to the growth of a Gaussian radius located at 20 km down to the tropopause at 10 km), we obtain a residence time $t_R \approx 3$ years for $K_e = 0.3\ m^2/s$. Is this an overestimate or an underestimate? It is difficult to answer this question without further information on the reliability of our estimate of Δt.

CONCLUDING REMARKS

We used a statistical analysis of 200 wind profiles in conjunction with a vertical-stack diffusion model to calculate the effective vertical diffusivities to be expected from CAT in the stratosphere. (It was also necessary to use the radar and balloon soundings of Browning for these calculations.) Our results indicate that it is likely that K_e in the stratosphere is in the 0.3 m^2/s range if the spreading of turbulent layers is taken into account. Our results seem to agree with measurements of radioactive fallout (see Junge, 1963) as well as measurements of CH_4 loss (see Wofsy and McElroy, 1973). The results are also consistent with the findings of Lilly, Waco, and Adelfang (1973), who derived vertical diffusivities from turbulence spectra observed by aircraft-borne instrumentation, and with the studies discussed by Justus (1973) for the altitude range of interest.

In this way we see that one need not resort to such mechanisms as stratospheric penetration by thunderstorms, "dumping" by global circulation to the poles, aerosol "precipitation", etc. in order to explain observed stratospheric residence times. In other words, it now appears that CAT plays the same prominent role in vertical transport in the stratosphere that "underwater CAT" plays in the World Ocean.

The next steps in this research should involve (a) a careful study of the high shears and possible instrumental effects, (b) better estimates of Δt, and (c) analysis of higher-resolution velocity profiles.

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DISCUSSION

REITER: You've obtained diffusion coefficients on a relatively small scale. If you include synoptic disturbances, the residence times in the lower stratosphere become shorter.

ROSENBERG: Yes, this is only the vertical diffusivity; if other processes contribute, each will have to be weighted accordingly.