AD-A017 180

A SIMPLE DESCRIPTION OF AN ASCENDING NUCLEAR FIREBALL AND A FORTRAN SOLUTION

Denzel D. Waltman, Jr.

Air Force Institute of Technology
Wright-Patterson Air Force Base, Ohio

March 1975

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER GNE/PH/75-14	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) A SIMPLE DESCRIPTION OF AN ASCENDING NUCLEAR FIREBALL AND A FORTRAN SOLUTION		5. TYPE OF REPORT & PERIOD COVERED MS Thesis 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Denzel D. Waltman, Jr. Capt, USAF		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Air Force Institute of Technology (AFIT-EN) Wright-Patterson AFB, Ohio 45433		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Analysis Division (SAA) Air Force Weapons Laboratory Kirtland AFB, New Mexico 87117		12. REPORT DATE March, 1975 13. NUMBER OF PAGES 76
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report) Approved for public release; IAW AFR 190-17 JERRY C. HIX, Captain, USAF Director of Information		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Nuclear Fireball Nuclear Effects Fireball		

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A set of four differential equations describing a rising, homogeneous, spherical, atomic fireball were developed from basic laws of physics. These four equations were: a time rate of change of temperature derived from energy conservation; a time rate of change of altitude derived from the definition of velocity; a time rate of change of velocity derived from momentum conservation; and a time rate of change of radius derived from the ideal gas law.

A computer program was written to solve the equations. This program assumed that the values for the four parameters were known at some time after the second

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

thermal maximum, and the program uses a Runge-Kutta method to determine the solutions. The value for the entrainment coefficient had to be determined by parametric evaluation. The value was chosen to be 0.00055.

Data for a 1-Kt, sea level nuclear detonation was used, the equations solved, and the solutions analyzed. The peak altitude was higher than expected and the velocity and radius initially behaved as expected.

A SIMPLE DESCRIPTION OF AN ASCENDING

NUCLEAR FIREBALL AND A FORTRAN SOLUTION

THESIS

GNE/PH/75-14 Denzel D. Waltman, Jr.
Captain USAF

A SIMPLE DESCRIPTION OF AN ASCENDING NUCLEAR FIREBALL AND A FORTRAN SOLUTION

THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology

Air University

in Partial Fulfillment of the

Requirements for the Degree of

Master of Science

by

Denzel D. Wal[illegible]man, Jr.
Captain USAF

Graduate Nuclear Engineering

March 1975

Approved for public release, distribution unlimited.

// /

Preface

This thesis is a result of my investigating the possibility of developing and solving a set of differential equations which describe an ascending nuclear fireball. Although complex models have been developed, a relatively simple model is needed to aid in the initial study of fireballs. Whenever possible I have made assumptions which keep the model simple. Included within this report is the FORTRAN program that I wrote to solve the equations.

My sincerest thanks are given to Dr. Charles J. Bridgman, my thesis advisor. Not only did he willingly answer my questions, but he asked pertinent questions which served to stimulate my thinking. Also, he introduced me to various persons who were knowledgeable in certain related areas.

I would also like to thank Dr. Donn G. Shanklin, who provided the subroutine which I used to solve the equations; Prof. Harold C. Larsen, who answered several of my questions involving fluid dynamics and mechanics; and Captain Daniel A. Matuska of the Air Force Weapons Laboratory, who furnished the data which I used as input into my equations and to which I compared my solutions. To other members of the faculty at the Air Force Institute of Technology who gave some guidance or encouragement, I say thank you.

Finally, I would like to thank Ms. Howdyshell for typing this thesis.

Contents

Contents

List of Figures

List of Tables

List of Symbols

Symbol	Definition
A	Area of sphere projected on plane normal to direction of motion (m^2)
c	2.9979×10^8 $m \cdot sec^{-1}$, speed of light
C_D	Drag coefficient (dimensionless)
c_v	Specific heat at constant volume ($J \cdot kgm^{-1} \cdot {}^oK^{-1}$)
d	Diameter of sphere (m)
$\frac{dc_v}{dT}$	Change in specific heat at constant volume WRT temperature ($J \cdot kgm^{-1} \cdot {}^oK^{-2}$)
$\frac{dE}{dt}$	Change in energy of fireball WRT time ($J \cdot sec^{-1}$)
$\frac{dM_f}{dt}$	Change in mass of fireball WRT time (kgm·sec-1)
$\frac{dp_a}{dz}$	Change in ambient air pressure WRT altitude ($Nt \cdot m^{-3}$)
$\frac{dp_f}{dt}$	Change in pressure in fireball WRT time ($Nt \cdot m^{-2} \cdot sec^{-1}$)
$\frac{dp_f}{dz}$	Change in pressure in fireball WRT altitude ($Nt \cdot m^{-3}$)
$\frac{dr_f}{dt}$	Change in radius of fireball WRT time ($m \cdot sec^{-1}$)
$\frac{dT_f}{dt}$	Change in temperature of fireball WRT time (${}^oK \cdot sec^{-1}$)
$\frac{dV_f}{dt}$	Change in volume of fireball WRT time ($m^3 \cdot sec^{-1}$)
$\frac{dz}{dt}$	Change in altitude WRT time ($m \cdot sec^{-1}$)
E	Energy of fireball (J)
F_B	Buoyant force (Nt)
F_D	Drag force (Nt)
F_T	Total force acting upon fireball (Nt)

List of Symbols

Symbol	Definition
g	9.80671 $m \cdot sec^{-2}$, earth's gravitational acceleration constant
m	Molecular weight of dry air ($kgm \cdot kmole^{-1}$)
M	Mass of gas (kgm)
M_T	Total mass of fireball system (kgm)
M_a	Mass of ambient air (kgm)
M_f	Mass of fireball (kgm)
p	Pressure of gas ($Nt \cdot m^{-2}$)
p_a	Pressure of ambient air ($Nt \cdot m^{-2}$)
p_f	Pressure in fireball ($Nt \cdot m^{-2}$)
R	8.3143×10^3 $J \cdot kmole^{-1} \cdot {}^oK^{-1}$, universal gas constant
R_d	287.05 $J \cdot kgm^{-1} \cdot {}^oK^{-1}$, gas constant for dry air
Re	Reynold's number (dimensionless)
r_f	Radius of fireball (m)
R'	Specific gas constant ($J \cdot kgm^{-1} \cdot {}^oK^{-1}$)
t	Time (sec)
T	Temperature of gas (${}^oK$)
T_a	Temperature of ambient air (${}^oK$)
T_f	Temperature of fireball (${}^oK$)
V	Volume of gas (m^3)
V_a	Volume of ambient air (m^3)
V_f	Volume of spherical fireball (m^3)
v_{rel}	Velocity of moving sphere relative to air ($m \cdot sec^{-1}$)
v_f	Velocity of fireball ($m \cdot sec^{-1}$)
z	Altitude (m)
z_{max}	Maximum altitude (m)

List of Symbols

Symbol	Definition
ι	Specific volume of gas ($m^3 \cdot kgm^{-1}$)
ε	Entrainment coefficient (dimensionless)
ν	Kinematic viscosity of air ($m^2 \cdot sec^{-1}$)
π	3.14159, the constant pi
ρ	Density of fluid ($kgm \cdot m^{-1}$)
ρ_a	Density of ambient air ($kgm \cdot m^{-3}$)
ρ_f	Density of fireball ($kgm \cdot m^{-3}$)
σ	5.6696×10^{-8} $J \cdot m^{-2} \cdot sec^{-1} \cdot {}^oK^{-1}$, Stephan-Boltzmann constant

Abstract

A set of four differential equations describing a rising, homogeneous, spherical, atomic fireball were developed from basic laws of physics. These four equations were: a time rate of change of temperature derived from energy conservation; a time rate of change of altitude derived from the definition of velocity; a time rate of change of velocity derived from momentum conservation; and a time rate of change of radius derived from the ideal gas law.

A computer program was written to solve the equations. This program assumed that the values for the four parameters were known at some time after the second thermal maximum, and the program uses a Runge-Kutta method to determine the solutions. The value for the entrainment coefficient had to be determined by parametric evaluation. The value was chosen to be 0.00055.

Data for a 1-Kt, sea level nuclear detonation was used, the equations solved, and the solutions analyzed. The peak altitude was higher than expected and the velocity and radius initially behaved as expected.

A SIMPLE DESCRIPTION OF AN ASCENDING NUCLEAR FIREBALL AND A FORTRAN SOLUTION

I. Introduction

Although computer programs have been written which describe a rising fireball from a nuclear explosion, the codes are unusable at the Air Force Institute of Technology (AFIT). In many of the codes the fireball has been described by some empirical relationship based on findings from experimental data and not based on physical concepts. Generally, these relationships have been used as a subroutine within a complex program designed primarily to study some other nuclear effect. The programs written specifically to describe a fireball are complex. In fact, the computer time required to process one of these programs is prohibitive at AFIT. Therefore, the need for a simple computer program which could be used to describe a nuclear fireball has led to this thesis.

Statement of Problem

The objective of this thesis was twofold. First, a set of basic equations which could be used to describe a late-time fireball had to be developed. The term 'late-time' has been used here to indicate time after the second thermal maximum which occurs from a nuclear explosion. The set of equations had to be developed with the knowledge that they would be of little practical use unless they could be solved by some numerical technique using a computer. This led to the second part of the thesis problem.

The second segment of the problem was to develop a computer program that would solve the set of equations previously mentioned. The program was to be written in FORTRAN code since AFIT has a terminal to a CDC 6600 computer. The actual solutions would be obtained by using an available subroutine that incorporated a Runge-Kutta technique.

Analysis of Problem

Equations. The equations that describe a nuclear fireball had to consider the conservation of energy, mass, and momentum. If one began with a set of equations that described the energy, mass, and the momentum of a fireball, then the only requirement remaining was to insure that the properties were conserved.

Programming. The computer program to solve the equations had to be written satisfying two criteria. First, the monetary and time costs to the user of the code had to be within the limits of AFIT. The second criterion was that the code had to be written so that variable input data could be used.

Assumptions

To solve the thesis problem several assumptions had to be made. The main assumptions were:

1. The nuclear detonation was an air burst (i.e., fireball did not touch the ground).

2. The fireball could be treated as a homogeneous sphere

3. The drag force acting upon the fireball could be treated as the form drag which acts upon a solid sphere as it moves through an ambient atmosphere of air

4. The pressure within the fireball at late-times is approximately the same as the ambient pressure

5. The initial input data would be available from some external source.

Organization

The remainder of the thesis has been organized just as the problem was solved. The derivation of the set of equations describing the nuclear fireball is contained within the next chapter, and a method of solving the equations is discussed in the following chapter. Next is Chapter IV which has the discussion on the results obtained from the computer program. The final two chapters contain the conclusions and the recommendations, respectively.

A bibliography and a number of appendices are located immediately following the last chapter. The bibliography may aid anyone who has a desire to do more investigation into the problem, and the appendices may help refresh the reader's mind on certain aspects of physics as they apply to the problem of late-time fireballs.

II. Derivation of Equations

To describe an ascending nuclear fireball one can begin with the equations for the fireball energy, the time rate of change in that energy, the simple definition of velocity, the momentum equation, and the equation of state of an ideal gas. With the proper applications of physical equations as they apply to the fireball and the atmosphere, a set of four basic equations can be developed that describe the fireball temperature, size, and motion as functions of time.

The fireball as used in this model is based on several simplifying assumptions. First, the fireball, even at late-times, is assumed to be a homogeneous sphere of radius r_f. Even though a torus is formed by late-times (Ref: 18,8), when the objective is to develop a simple model to describe the fireball, then the fireball can be assumed to act as a sphere (Ref: 7,3).

The second assumption is that the temperature of the fireball is the same throughout the interior. In reality, this is not true (Ref: 3,168); however, for the purpose of simplicity a constant temperature is assumed. The fireball temperature, T_f, might best be thought of as an average temperature within the fireball.

The third, and final, major assumption concerning the fireball involves the treatment of entraining the atmospheric air into the fireball. This model assumes that the ambient air is taken into the fireball at the ambient density and is instantaneously accelerated to the velocity of the fireball. Also, this model assumes that the entrained air is instantaneously mixed with the gases within the fireball to maintain the homogeneity of the sphere. A better and more complicated description of the interior of a fireball is being developed by considering the fireball as consisting of three fluids: a hot, unmixed gas; a cold, unmixed gas which is the entrained air; and a mixture of the two (Ref: 12,19). But since the simple model assumes a homogeneous sphere with a temperature T_f, the instantaneous mixing is assumed. The three-fluid model is complicated and is impossible to describe using simple equations; therefore, it will not be incorporated into this model.

The method of approach used in developing this simple model is to start by considering the energy equations and arriving at an equation describing the temperature of the fireball. Then, the basic velocity equation is considered. Following this, the forces acting upon the fireball system are considered to insure conservation of momentum. This leads to an equation describing the acceleration of the fireball. The final expression developed gives the time rate of change in the fireball radius. Each of the final equations is a function of the fireball temperature, radius, velocity, and/or altitude.

Before proceeding into the actual development of the set of equations, the reader is referred to page vii which contains a listing of the symbols used in this model. For the purpose of clarity, the first time that a symbol is used, it will be defined. The dimensions of the variable quantities are given in parentheses following the definition. If the symbol represents a constant, the value and units are given followed by the name of the constant.

Energy

The energy within a nuclear fireball is the sum of the radiant energy and the internal energy which, in equation form, is

$$E = \frac{16\pi\sigma r_f^3 T_f^4}{3c} + M_f c_v T_f \tag{1}$$

where

E = energy of fireball (J)

π = 3.1416, constant pi

σ = $5.6696\mathrm{x}10^{-8}$ nt/(m-sec-$^{o}K^{4}$), Stephan-Boltzmann constant

c = $2.9979\mathrm{x}10^{8}$ $\mathrm{m \cdot sec^{-1}}$, speed of light

r_f = radius of fireball (m)

T_f = temperature of fireball (^{o}K)

M_f = mass of fireball (kgm)

c_v = specific heat at constant volume ($\mathrm{J \cdot kgm^{-1} \cdot {}^{o}K^{-1}}$)

At late-times, any time after the second thermal maximum, the temperature of the fireball is on the order of a few thousand degrees or less, the radius is on the order of a few hundred meters, and the specific heat is on the order of a thousand Joules per kilogram per degree. Therefore, eq (1) can be approximated by

$$E \approx M_f c_v T_f \tag{2}$$

Differentiating this approximation with respect to time (WRT) gives, when the chain rule is applied to dc_v/dt, the following equation:

$$\frac{dE}{dt} \approx M_f c_v \frac{dT_f}{dt} + M_f T_f \frac{dc_v}{dT_f} \frac{dT_f}{dt} + c_v T_f \frac{dM_f}{dt} \tag{3}$$

where $\frac{dE}{dt}$ = time rate of change in fireball energy ($J \cdot sec^{-1}$)

$\frac{dT_f}{dt}$ = time rate of change in fireball temperature ($^oK \cdot sec^{-1}$)

$\frac{dc_v}{dT_f}$ = rate of change in c_v WRT T_f ($J \cdot kgm^{-1} \cdot {^oK}^{-2}$)

$\frac{dM_f}{dt}$ = time rate of change in fireball mass ($kgm \cdot sec^{-1}$).

The rate of change in mass is due to entrainment and can be expressed in terms of the ambient density, the surface area of the fireball, and the velocity of the fireball relative to the atmosphere through which the fireball is moving (Ref: 21, 1644). This expression is valid since the fireball is assumed to be spherical. The rate at which the fireball mass changes is

$$\frac{dM_f}{dt} = 4\pi\varepsilon \rho_a r_f^2 v_{rel} \tag{4}$$

where ε = entrainment coefficient (dimensionless)

ρ_a = ambient air density ($kgm \cdot m^{-3}$)

v_{rel} = $|v_f|$, fireball velocity relative to ambient atmosphere ($m \cdot sec^{-1}$)

The entrainment coefficient is assumed to be constant. From nuclear tests, a range of values from 0.13 to 0.26 for ε has been found (Ref: 7, 8). The only method available to determine the actual value for the entrainment coefficient is to apply the descriptive equations being developed herein to some known data from a fireball.

An expression for the mass and density of the fireball is derived from the equation of state for dry air since the fireball is composed mainly of heated air in one form or another. The equation of state for dry air is developed from the equation of state for an ideal gas. This is actually done in Appendix A. The equation of state for dry air is

$$p_a V_a = M_a R_d T_a \tag{5}$$

where p_a = pressure of ambient air ($Nt \cdot m^{-2}$)

V_a = volume of ambient air (m^3)

M_a = mass of ambient air (kgm)

R_d = 287.05 $J \cdot kgm^{-1} \cdot {}^oK^{-1}$, gas constant for dry air

T_a = temperature of ambient air (^oK)

Since the density is, by definition, mass per unit volume, eq (5) can be solved for M_a/V_a which is the density of air; therefore,

$$\rho_a = \frac{p_a}{R_d T_a} \tag{6}$$

where ρ_a is the density of ambient air ($kgm \cdot m^{-3}$). This is the form for the air density that will be used in eq (4).

An expression for the mass of the fireball is obtained by applying eq (5) to the fireball and by using

$$V_f = \frac{4\pi r_f^3}{3} \tag{7}$$

where V_f (m^3) is the volume of a fireball when taken to be a sphere with radius r_f. The only change is eq (5) when it is applied to a fireball is the changing of the subscript 'a' to 'f'. This change yields the equation of state for a fireball. By using this and eq (7), one can write the mass of the fireball as

$$M_f = \frac{4\pi p_f r_f^3}{3R_d T_f} \tag{8}$$

where p_f is the pressure within the fireball ($Nt \cdot m^{-2}$).

The specific heat of air at constant volume is a function of temperature (Ref: 17). However, for the purpose of simplicity a constant value of $5R_d/2$ is assumed for this model. The reasons for selecting this particular value for c_v are discussed in Appendix B. Substituting in the numerical value for R_d leads to

$$c_v = 717.63\ J \cdot kgm^{-1} \cdot {}^oK^{-1} \tag{9}$$

This implies that for this simple model,

$$\frac{dc_v}{dT} = 0 \tag{10}$$

Eq (3) can now be written in terms of fewer variables by using eqs (4), (6), (8), and (10). When these equations are substituted into eq (3) the common terms are factored, the dE/dt can be approximated by

$$\frac{dE}{dt} \approx \left[\frac{4\pi r_f^2 c_v}{R_d} \frac{p_f r_f}{3} \frac{dT_f}{dt} + \frac{\epsilon p_a T_f v_{rel}}{T_a}\right]$$

But by using the first law of thermodynamics, the time rate of change in the fireball energy can be written as the sum of the internal energy lost by radiation and the energy given up in the form of work done by the expanding fireball. In this model, the energy gained by the fireball through the release of latent heat is not considered. One knows from basic physics that as phase changes take place either energy is gained or lost by the body undergoing the changes (Ref: 8, 482). In this model the quantity of latent heat is assumed small when compared to the amount of energy being radiated or lost due to expansion.

Writing dE/dt as the sum of the radiant energy lost and the work energy lost gives

$$\frac{dE}{dt} = -4\pi\sigma r_f^2 T_f^4 - p_f \frac{dV_f}{dt} \tag{12}$$

where dV_f/dt is the time rate of change in the volume of the fireball, $(m^3 \cdot sec^{-1})$.

Differentiating the equation of state for a fireball, $p_f V_f = M_f R_d T_f$, with respect to time and solving for $p_f dV_f/dt$ gives

$$p_f \frac{dV_f}{dt} = M_f R_d \frac{dT_f}{dt} + R_d T_f \frac{dM_f}{dt} - V_f \frac{dp_f}{dt} \tag{13}$$

where dp_f/dt is the time rate of change in the fireball pressure, $(Nt \cdot m^{-2} \cdot sec^{-1})$.

When the chain rule is applied to dp_f/dt, the following equation can be obtained:

$$\frac{dp_f}{dt} = \frac{dp_f}{dz}\frac{dz}{dt} \tag{14}$$

where dp_f/dz is the change in the pressure within the fireball with respect to altitude, $(\mathrm{Nt}\cdot\mathrm{m}^{-3})$, and dz/dt is, by definition, the velocity of the fireball $(\mathrm{m}\cdot\mathrm{sec}^{-1})$. Since the equation representing the velocity is an extremely important equation, as will be evident later in this development, let it be written as

$$\frac{dz}{dt} = v_f \tag{15}$$

Using eqs (4), (6), (7), (8), (14), and (15) in eq (11) yields the following expression for the work energy:

$$p_f\frac{dV_f}{dt} = 4\pi r_f^2\left(\frac{p_f r_f}{3T_f}\frac{dT_f}{dt} + \frac{\epsilon p_a T_f v_{rel}}{T_a} - \frac{r_f v_f}{3}\frac{dp_f}{dz}\right) \tag{16}$$

Using the above form for the work energy in eq (12) and collecting the common terms give

$$\frac{dE}{dt} = -4\pi r_f^2\left(\sigma T_f^4 + \frac{p_f r_f}{3T_f}\frac{dT_f}{dt} + \frac{\epsilon p_a T_f v_{rel}}{T_a} - \frac{r_f v_f}{3}\frac{dp_f}{dz}\right) \tag{17}$$

Temperature

Now, since eqs (11) and (17) are both approximately equal to the time rate of change in the energy of a fireball, the right hand sides (RHS) of the equations must be approximately equal. Therefore:

$$\frac{4\pi r_f^2}{R_d}\left(\frac{p_f r_f c_v}{3T_f}\frac{dT_f}{dt} + \frac{c_v \varepsilon p_a T_f v_{rel}}{T_a}\right) \approx$$

$$-4\pi r_f^2\left(\sigma T_f^4 + \frac{p_f r_f}{3T_f}\frac{dT_f}{dt} + \frac{\varepsilon p_a T_f v_{rel}}{T_a} - \frac{r_f v_f}{3}\frac{dp_f}{dz}\right) \qquad (18)$$

The last equation can be solved for dT_f/dt. Since the pressure is a function of altitude, and the specific heat value has been chosen to be constant, the resulting equation for dT_f/dt is a function of the altitude, radius, temperature, and velocity of the fireball. After applying some basic algebra, eq (18) yields the following approximation for the time rate of change in the fireball temperature:

$$\frac{dT_f}{dt} \approx -\left[\frac{3R_d T_f}{\left(c_v + R_d\right) \quad p_f r_f}\right] \mathrm{X}$$

$$\left[\sigma T_f^4 + \frac{\varepsilon p_a T_f v_{rel}}{T_a}\left(1 + \frac{c_v}{R_d}\right) - \frac{r_f v_f}{3}\frac{dp_f}{dz}\right] \qquad (19)$$

Before proceeding into the development of the remaining equations, a brief discussion of the pressure and the specific heat are appropriate. This allows eq (19) to be written in the final form.

The pressure within the fireball can be assumed to be equal to the ambient air pressure, p_a, for the times being considered in this model. The shock front has moved from the vicinity of the fireball, and the pressure within the fireball has had a relatively long time in which to return to the ambient pressure (Ref: 14, 7). For 100-kt yields or less, the time required to return to ambient pressure is approximately one second. The model being developed herein will be based on this assumption; that is,

$$p_f = p_a \tag{20}$$

This model will use the standard atmosphere as adopted by the National Advisory Committee for Aeronautics (NACA). The NACA has accepted 10,769m as the height of the tropopause; therefore, two sets of equations describing a fireball must eventually be considered. The only differences in the sets will be the equations that will be used to describe the properties of the atmosphere. The atmospheric temperature and pressure as functions of altitude are given by the following equations (Ref: 9, 52):

$$T_a = 288 - .0065z \qquad z \leq 10{,}769\text{m} \tag{21a}$$

$$T_a = 218 \qquad z > 10{,}769\text{m} \tag{21b}$$

$$p_a = \left(3.8029\text{x}10^{-20}\right)\left(44{,}308 - z\right)^{\frac{1}{.19023}} \qquad z \leq 10{,}769\text{m} \tag{22a}$$

$$p_a = 23452 \exp\left[-(z - 10{,}769)/6381.6\right] \qquad z > 10{,}769\text{m} \tag{22b}$$

where the temperature is in ^{o}K and the pressure is in $\text{Nt}\cdot\text{m}^{-2}$.

The change in pressure found in eq (17) can be found by differentiating eqs (22a) and (22b) with respect to altitude. The simplified forms of dp_a/dz are

$$\frac{dp_a}{dz} = \left(-1.9991\text{x}10^{-19}\right) \quad (44{,}308 - z)^{\frac{.80977}{.19023}}, z \leq 10{,}769\text{m} \quad (23a)$$

$$\frac{dp_a}{dz} = -3.6749 \exp\left[-(z - 10{,}769)/6381.6\right] \quad , z > 10{,}769\text{m} \quad (23b)$$

Now eq (19) can be written in the final form. Naturally, since the atmosphere has been divided into two regions, there are actually two expressions for dT_f/dt. The numerical values for R_d and c_v have been inserted into eq (19) as have been eqs (20) through (23) for the purpose of simplifying the final forms.

For $z \leq 10{,}769$m:

$$\frac{dT_f}{dt} \approx -\left[\frac{2.2539\text{x}10^{19}T_f}{r_f\ (44{,}308 - z)^{\frac{1}{.19023}}}\right] X$$

$$\left[\sigma T_f^4 + \frac{\left(1.3310\text{x}10^{-19}\right)(44{,}308 - z)^{\frac{1}{.19023}}\epsilon T_f v_{rel}}{288 - .0065z}\right.$$

$$\left. + 6.6637\text{x}10^{-20} r_f v_f (44{,}308-z)^{\frac{.80977}{.19023}}\right] \quad (24a)$$

For $z > 10{,}769$m:

$$\frac{dT_f}{dt} \approx -\left\{\frac{3.6549\times10^{-5}T_f}{r_f \exp\left[-(z-10{,}769)/6381.6\right]}\right\} X \left\{\sigma T_f^4 + 3.7652\times10^{2}\varepsilon T_f v_{rel} \exp\left[-(z-10{,}769)/6381.6\right] + 1.2250 r_f v_f \exp\left[-(z - 10{,}769)/6381.6\right]\right\} \quad (24b)$$

So far the equations giving the time rate of change in the temperature and the altitude of the fireball are available. They are given by eqs (24a) and (24b) as well as eq (15). Since ε is assumed to be constant, the equations are in terms of the fireball temperature, altitude, radius, and velocity.

Velocity

As previously mentioned as eq (15), the time rate of change in the altitude of the fireball is nothing more than the velocity of the fireball. Now, an equation describing the time rate of change in the velocity, commonly called the acceleration, is needed. The desired equation must be in terms of the same parameters as mentioned in the preceeding paragraph.

Acceleration

By considering the conservation of momentum one can determine an expression for dv_f/dt. The time rate of change in the momentum must be equal to the sum of the forces acting upon the fireball. At late-times there are only two forces acting upon the fireball. They are the buoyant force (F_B) and the drag force (F_D). A more detailed discussion on these forces will be given later, but first recall that the basic equation for momentum can be written as

$$\frac{d}{dt}(M_T v_f) = F_T \tag{25}$$

where M_T is the total mass of the fireball system (kgm) and F_T is the total force acting upon the fireball (Nt).

When momentum is being considered, the mass of the fireball system is greater than just the mass of the fireball. This is a result of the presence of the ambient air around the fireball. As the fireball moves through the atmosphere, the apparent mass of the fireball is actually equal to the mass of the fireball plus half of the mass of the fireball but consisting of air (Ref: 13, 124). Thus, when the mass is written as the product of the volume and the density,

$$M_T = \frac{4\pi r_f^3}{3} (\rho_f + \rho_a/2) \tag{26}$$

where ρ_f is the density of the fireball ($\mathrm{kgm \cdot m^{-3}}$). The apparent mass effect is indicated by the $\rho_a/2$-term.

Performing the differentiation in eq (25) and making use of eqs (4) and (26) gives

$$\frac{4\pi r_f^3}{3} (\rho_f + \rho_a/2) \frac{dv_f}{dt} + 4\pi\varepsilon \rho_a r_f^2 v_{rel} v_f = F_T \tag{27}$$

Before this equation can be simplified, a brief discussion on the forces acting upon the fireball is needed. F_B is discussed first, and then F_D is discussed.

Buoyant Force. The density of the fireball initially is much less than the density of the ambient air surrounding the fireball because of the extremely high temperature within the fireball. The result is an initial upward force acting upon the fireball. This is an example of Archimedes' principle (Ref: 8, 363). The buoyant force can be written as

$$F_B = g(M_a - M_f) \tag{28}$$

where $g = 9.80671\ \mathrm{m \cdot sec^{-2}}$, the earth's gravitational acceleration constant.

Writing the mass as the product of the volume and the density gives the following equation for the buoyant force:

$$F_B = g(V_a \rho_a - V_f \rho_f) \tag{29}$$

Since, by necessity, the volume of the air displaced by the fireball must be equal to the volume of the fireball; therefore,

$$V_a = V_f \tag{30}$$

By substituting eq (30) into eq (29) for V_a, factoring out the common term V_f, and using the equivalent form of V_f as given by eq (7), one can write F_B as

$$F_B = \frac{4\pi g r_f^3}{3} (\rho_a - \rho_f) \tag{31}$$

This is the form of F_B that will be used later in developing the remainder of the equations.

Drag Force. Any body moving through a fluid, even if that fluid is air, experiences a retarding force which opposes the motion. This force is commonly referred to as the drag force. The drag force is dependent upon the shape, size, and velocity of the moving body as well as upon the properties of the fluid through which the object is moving (Ref: 19, 91).

Any text on fluid dynamics gives an equation for the drag force experienced by a sphere moving through a fluid. One such equation is (Ref: 16, 5-10)

$$F_D = 1/2 C_D \rho v_{rel}^2 A \tag{32}$$

where

F_D = drag force on sphere (Nt)

C_D = drag coefficient (dimensionless)

ρ = density of fluid ($kgm \cdot m^{-3}$)

v_{rel} = relative velocity between sphere and fluid ($m \cdot sec^{-1}$)

A = area of sphere projected on plane normal to direction of motion (m^2).

When eq (32) is applied to a fireball, it can be written as

$$F_D = \frac{\pi C_D}{2} \rho_a r_f^2 v_{rel}^2 \tag{33}$$

where the projected area has been replaced by πr_f^2.

The value of C_D is usually obtained from a graph of the drag coefficient versus a quantity called the Reynolds' number, Re. To obtain a value for C_D, one must first compute the value of Re, and then obtain the value of C_D by referring to a C_D vs Re graph for a sphere. The values for the Re for a spherical nuclear fireball have an order of magnitude of 10^7 to 10^9, and the values of C_D are from approximately 0.2 to 0.1 in this Re-range (Ref: 11, 3-8). Appendix C gives a more detailed discussion of C_D and Re as it applies to the fireball problem. Since the upper limit occurs more often in the study of fireball rise, $C_D = 0.1$ is used within this model.

Eq (33) assumes that the direction of motion of the fireball is unchanging. Since the fireball may oscillate around an altitude of stabilization, eq (33) must be modified to indicate the change in the drag force. The drag force must always oppose the buoyant force. When the fireball is above the stabilization altitude ($\rho_f = \rho_a$), then F_B <0; therefore, the effect of F_D must be positive. To insure that F_D always opposes F_B, eq (33) should be written as

$$F_D = \frac{\pi}{2} C_D \rho_a r_f^2 v_f v_{rel} \tag{34}$$

Using $F_T = F_B - F_D$ and the RHS of eq (27) replaced with eqs (31) and (34), one can describe the forces acting on the fireball as

$$\frac{4\pi r_f^3}{3} (\rho_f + 1/2\ \rho_a) \frac{dv_f}{dt} + 4\ \pi\ \varepsilon\ \rho_a r_f^2 v_f v_{rel} =$$

$$\frac{4\pi g r_f^3}{3} (\rho_a - \rho_f) - \pi C_D \rho_a r_f^2 v_f v_{rel} \tag{35}$$

By applying some rules of algebra, one can solve the above equation for the acceleration of the fireball, dv_f/dt. The equation becomes

$$\frac{dv_f}{dt} = \frac{2g(\rho_a - \rho_f) - \frac{3\rho_a v_f v_{rel}}{4r_f}(C_D + 8\varepsilon)}{2\rho_f + \rho_a} \tag{36}$$

From this equation one can see that the effect of entrainment is like the aerodynamic drag. Using the equation of state for dry air and the fireball plus eqs (20) and (30), one finds that

$$\frac{\rho_a}{\rho_f} = \frac{T_f}{T_a} \tag{37}$$

When both the numerator and denominator of eq (36) are divided by ρ_f and eq (37) is substituted in the resulting expression, one gets

$$\frac{dv_f}{dt} = \frac{2g\left(\frac{T_f}{T_a} - 1\right) - \frac{.75T_f v_f v_{rel}}{r_f T_a}(C_D + 8\varepsilon)}{2 + \frac{T_f}{T_a}} \tag{38}$$

This is one of the equations describing a nuclear fireball which must eventually be solved. From this equation, the initial acceleration must be approximately 2g since $T_f >> T_a$ and v_f is zero. Only when T_f and T_a become the same order of magnitude will the drag force appreciably effect the acceleration.

In order for this equation to be solved, T_a must be written in terms of the fireball altitude. This is done by making use of eqs (21a) and (22b). The results are given on the following page.

For $z \leq 10{,}769$m:

$$\frac{dv_f}{dt} = \frac{2g\left(\dfrac{T_f}{288 - .0065z} - 1\right) - \dfrac{.75T_f v_f v_{rel}\,(C_D + 8\varepsilon)}{r_f(288 - .0065z)}}{2 + \dfrac{T_f}{288 - .0065z}} \tag{39a}$$

For $z > 10{,}769$m:

$$\frac{dv_f}{dt} = \frac{2g\left(\dfrac{T_f}{218} - 1\right) - \dfrac{.75T_f v_f v_{rel}\,(C_D + 8\varepsilon)}{218 r_f}}{2 + \dfrac{T_f}{218}} \tag{39b}$$

Radius

Since the equations for dT_f/dt and dv_f/dt involve r_f, an expression for the time rate of change in the radius, dr_f/dt, is needed. When this equation is obtained, then the set of equations which describe a rising nuclear fireball will be complete.

The equation of state for the fireball is the beginning point in developing the desired equation. Using eq (7) to express V_f in terms of r_f and the assumption that $p_f = p_a$ yields the following form of the equation of state:

$$\frac{4\pi p_a r_f^3}{3} = M_f R_d T_f \tag{40}$$

Taking the time derivative of this equation will give a time derivative of r_f, which is the item of interest. When this is done and eqs (4) and (6) are properly substituted into the result, the equation found on the next page is obtained.

$$\frac{dr_f}{dt} = \frac{r_f}{3T_f}\frac{dT_f}{dt} + \frac{\varepsilon T_f v_{rel}}{T_a} - \frac{r_f v_f}{3p_a}\frac{dp_a}{dz} \qquad (41)$$

The parameters of T_a, p_a, and dp_a/dz are given as functions of altitude by eqs (21a) through (23b). One can see that dr_f/dt must be expressed by two equations since the parameters each have two forms. Substitution of eqs (21a), (22a), and (23a) into eq (41) and solving for dr_f/dt gives the following expression which is valid for $z \leq 10,769$m:

$$\frac{dr_f}{dt} = \frac{r_f}{3T_f}\frac{dT_f}{dt} + \frac{\varepsilon T_f v_{rel}}{288 - .0065z} + \frac{1.7523 r_f v_f}{44,308 - z} \qquad (42a)$$

Substitution of eqs (21b), (22b), and (23b) gives the following equation for dr_f/dt which is valid for $z > 10,769$m:

$$\frac{dr_f}{dt} = \frac{r_f}{3T_f}\frac{dT_f}{dt} + \frac{\varepsilon T_f v_{rel}}{218} + 5.2232\mathrm{x}10^{-5} r_f v_f \qquad (42b)$$

Now that the time rate of change in the radius of the fireball is written in terms of T_f, r_f, and v_f, all of the necessary equations to describe a rising fireball at late-times are available for solution.

Summary

The four equations that are necessary to describe a rising nuclear fireball have just been developed. The equations form a simple descriptive model since several assumptions were made in the developing process. Since the NACA standard atmosphere was used and since it is based upon the tropopause being at 10,769-meters, a set of equations had to be written in the final form that was valid for each region of the atmosphere. Thus, two separate sets were actually derived.

The equations are written in terms of four properties of the fireball. They are: temperature, T_f; altitude, z; radius, r_f; and the velocity, v_f. The set of equations involves the time derivatives of these four parameters.

The two sets of descriptive equations are repeated here since they do not appear anywhere as a consolidated list.

For $z \leq 10{,}769$m:

$$\frac{dT_f}{dt} = -\left[\frac{2.2539\text{x}10^{19}\ T_f}{r_f\ (44{,}308 - z)^{\frac{1}{.19023}}}\right] \text{X}$$

$$\left[\sigma T_f^4 + \frac{1.3310\text{x}10^{-19}(44{,}308 - z)^{\frac{1}{.19023}}\varepsilon T_f v_{rel}}{288 - .0065z} + 6.6637\text{x}10^{-20} r_f v_f (44{,}308 - z)^{\frac{.80977}{.19023}}\right] \qquad (24a)$$

$$\frac{dz}{dt} = v_f \qquad (15)$$

$$\frac{dv_f}{dt} = \frac{2g\left(\dfrac{T_f}{288 - .0065z} - 1\right) - \dfrac{.75T_f v_f v_{rel} (C_D + 8\,\varepsilon)}{r_f(288 - .0065z)}}{2 + \dfrac{T_f}{288 - .0065z}} \qquad (39a)$$

$$\frac{dr_f}{dt} = \frac{r_f}{3T_f}\frac{dT_f}{dt} + \frac{\varepsilon T_f v_{rel}}{288 - .0065z} + \frac{1.7523 r_f v_f}{44{,}308 - z} \qquad (42a)$$

For $z > 10{,}769$m:

$$\frac{dT_f}{dt} \approx -\left\{\frac{3.6549\text{x}10^{-5} T_f}{r_f \exp\left[-(z - 10{,}769)/6381.6\right]}\right\} \text{ X } \left\{\sigma T_f^4 + \varepsilon 3.7652\text{x}10^2 T_f v_{rel} \exp\left[-(z - 10{,}769)/6381.6\right] + 1.2250 r_f v_f \exp\left[-(z - 10{,}769)/6381.6\right]\right\} \qquad (24b)$$

$$\frac{dz}{dt} = v_f \qquad (15)$$

$$\frac{dv_f}{dt} = \frac{2g\left(\dfrac{T_f}{218} - 1\right) - \dfrac{.75T_f v_f v_{rel} (C_D + 8\,\varepsilon)}{218 r_f}}{2 + \dfrac{T_f}{218}} \qquad (39b)$$

$$\frac{dr_f}{dt} = \frac{r_f}{3T_f}\frac{dT_f}{dt} + \frac{\varepsilon T_f v_{rel}}{218} + 5.2232\text{x}10^{-5} r_f v_f \qquad (42b)$$

III. Solving the Equations

Numerical Techniques Used

Modified Euler-Cauchy Method. The four equations developed in the previous chapter describe a rising nuclear fireball at late-times. The equations were initially solved by using the Modified Euler-Cauchy Method (Ref: 15, 259) and a Hewlett-Packard, model 35, hand calculator. However, the purpose of using this method was to study the effect of each term in order to see if any of the terms were insignificant. For this reason, only three iterations were actually performed.

Also, the modified Euler-Cauchy Method is only an improved version of the point-slope technique. This method uses the slope at the mid-point between two values instead of using the slopes at an end-point. Thus, the solutions are more accurate for slowly varying functions than for rapidly varying functions. Around the points of inflection the Modified Euler-Cauchy Method can yield erroneous answers since the actual slopes can be greater than the slopes generated by using the mid-point.

Runge-Kutta Technique. After checking the order of magnitude of each term, the Runge-Kutta Technique was selected. An existing computer code called BLCKDQ has been developed by Donn L. Shanklin of AFIT (Ref: 22) which uses an eighth-order Runge-Kutta Technique.

BLCKDQ

The subroutine BLCKDQ solves a set of first-order differential equations by an eighth order Runge-Kutta method. The initial values of the independent variable, time, and the corresponding dependent variables, temperature, altitude, velocity, and radius, must be furnished as input data to BLCKDQ. Also, the desired degree of accuracy in each variable and the desired final value of the independent variable must be given as input data. The exact form of the input data is discussed below.

Input. The input variables necessary to access BLCKDQ are FCN, NEQ, ND, XIN, XOUT, YS, X, Y, F, ALWNC, ALLOW, HEST, SIZE, and IBLCK. Each of these terms are now discussed to illustrate the necessary input data required for BLCKDQ.

1. FCN is an external function subroutine called EXTERNAL FCN (X, Y, F) which contains the actual differential equations to be solved, here equations 24, 15, 39, and 42. X is the independent variable which is time, Y is a matrix of the current values of the dependent variables, and F is a matrix of the values of the derivatives of the dependent variables at X.

2. NEQ is the number of equations to be solved. In this case NEQ = 4.

3. ND is the dimension limit for the arrays Y and F, and in this case ND = 4.

4. XIN is the initial value of the independent variable which in this case is time. In this study the initial value is set equal to zero and then incremented by the value of XOUT.

5. XOUT is the final desired value of the independent variable. In this case, XOUT is the time increment for establishing the time between the desired solutions.

6. YS is a matrix which initially contains the values of the dependent variables at time = XIN and which upon returning from BLCKDQ contains the solutions at time = XOUT.

7. X is an array of length 12 which contains a table of values of the independent variable around XOUT upon returning from BLCKDQ. This array is used internally.

8. Y is a scratch-pad matrix which at return contains the values of the dependent variables for each value in X. The matrix Y is dimensioned (4, 12) in this model. Therefore, if Y (1) corresponds to the dependent variable of temperature, then Y (1, J) corresponds to the temperature values at X (J).

9. F is a scratch-pad matrix which at return contains the values of the derivatives of the dependent variables for each time value in X. In this model F is dimensioned (4, 12); therefore, the values in F have the same relationship to X as does Y.

10. ALWNC is the array, dimensioned ALWNC (4), which contains the relative error tolerances. The actual values of the tolerances are read into the matrix by the main program, and the values are discussed later in this report.

11. ALLOW is a scratch-pad array with a length of four in this model.

12. HEST is the estimated step size and is automatically adjusted by the subroutine BLCKDQ. At return, HEST contains the last step size used y BLCKDQ. An initial, extremely bad guess will waste computer time, but an initial good guess is not critical for BLCKDQ to work. In this model the initial values of HEST are 10^{-6}.

13. SIZE is an array which contains the guesses for the solutions' sizes. In this model the dimensioning of the array is (4). The subroutine is written so that no initial values need to be given; therefore, since no size estimates are known for the solutions, no initial values are given for SIZE.

14. IBLCK is an indicator to BLCKDQ. If no estimates are given in SIZE, then IBLCK must be negative. But, if estimates are given in SIZE, then IBLCK must be zero. Since no values are given in this model for SIZE, IBLCK = -1.

The above list can be reduced to the following input data: the initial time and the desired time increment between solutions; the initial values of the dependent variables; and the relative error tolerance for each dependent variable.

A short main program was written to process this input, to call BLCKDQ, to produce the solutions, and to process the output. This program, plus some subroutines, are contained in Appendix D. Only the essential input data and the output options are now discussed.

Input Data. The main computer program is written to read five data cards. The cards can be thought of as containing the time information on the first card, the initial values of the dependent variables on the second card, the tolerances on the third card, the number of iterations minus one on the fourth card, and the last card contains an indicator to select the type of output desired. Each of these are discussed in greater detail in the following paragraphs.

1. Time. On the first data card, two numbers appear. The first number is the initial time of the data which is actually the time after the explosion. This time is given in seconds. The second number on the card is the time increment in seconds, and this corresponds to the first XOUT-value.

2. Dependent variables. The second data card contains four numbers corresponding to the dependent variables. The values are read into the program as YS(I), I = 1,4. YS(1) is the temperature of the fireball, YS(2) is the altitude, YS(3) is the velocity, and YS(4) is the radius. The actual values on the card are determined by the size of the nuclear device and the initial time selected. The actual data that is used in studying the set of equations is given here as Table I, which is located on the following page.

3. Tolerances. The third card also contains four numbers, but these four numbers are the relative error tolerances. The values for this model are not all equal since the magnitudes of the expected range for the various dependent variables are different. The tolerance for the velocity is chosen to be equal to one since the expected change in the velocity for a time increment is small. The three remaining tolerances are each chosen to be equal to ten.

Table I

Initial Input Data

Data at One
Second After Burst of
of 1-Kt, Sea-level Detonation

Variable	Value
$T(^{o}K)$	6000
z(m)	0
v(m/sec)	0
r(m)	100

(Ref: 23)

4. Number of iterations. The fourth data card contains a single integer which represents the number of solutions to be found by BLCKDQ minus one. The number is used as an upper limit on a DO-loop in the main program. The number chosen for this study is 599 since the total number of iterations is then 600. By this judicious selection, the times values, although originally in seconds, can easily be converted to minutes.

5. Output indicator. The fifth, and final, data card also contains a single number. But this number is used within the main program only as a means of selecting the form in which the solutions are to be displayed; therefore, this final number does not effect the solutions. There are two possible forms of output in which the solutions may be obtained. The first form corresponding to the number one is a series of plotted solutions and a complete listing of the numerical answers. And the second form which corresponds to the number two is the plotted solutions only.

So far the required input data, the subroutine BLCKDQ, and the output options have been discussed; but, before th equations can be solved for any nuclear fireball, test solutions must be done to determine the value of the entrainment coefficient.

Determining Entrainment Coefficient

As mentioned on page 7, the only method to determine the numerical value of the entrainment coefficient is to vary the coefficient with a set of test solutions. When the computer solutions match the data from experiments, then the value of the entrainment coefficient which produced the solutions is accepted as the correct value. The input data are taken from Table I which is on the preceeding page. According to Captain D. Matuska (Ref: 23), the peak or maximum altitude reached by the fireball from this initial data is approximately 4-km, and the time from burst to that altitude is about 180-seconds. This value will be used as a benchmark to determine ε.

Varying the values for the coefficient are expected to indicate the one value which will yield the most accurate solutions. This value is then inserted as a constant into the external function FCN. Once the value is determined, then the computer program is complete. By changing the input data to match the values of the variables for different sizes of nuclear devices, the computer program can then be used to describe the changing late-time fireball of different devices. The data must be obtained initially from some external source and placed on data cards.

Summary

This chapter contains the discussions on the computer program that is written to solve the set of differential equations that were developed in the previous chapter. The program reads the initial data from five cards, calls the subroutine BLCKDQ, and then produces the solutions as either a complete list plus a series of plots or just as a series of plots. BLCKDQ uses a high-order iterative technique to solve the equations which are actually written as an external function subroutine. But before the program can be applied to any size of device, a test must be used to empirically determine the correct value of the entrainment coefficient. The results of using the computer program are the topic of the following chapter.

IV. Results

The solutions of the four differential equations are discussed in detail within this chapter. The first part of the discussion concerns the results of using various values for the entrainment coefficient. After one value has been finally chosen to be used, the solutions for a one-kiloton nuclear device are discussed in detail. In this portion of the discussion is included a description of the dominating terms in each of the four equations. To conclude this chapter, a summary of the results is given.

Entrainment Coefficient

To determine the value of the entrainment coefficient that will be accepted as the value to use in the set of equations, the following method was used with the indicated results. A particular value for the coefficient was chosen and placed into the equations. The program was then run on the computer, and the solutions were determined. Next, the computer maximum altitude and the corresponding time were compared with the peak altitude and time values furnished by Captain Matuska.

As previously mentioned, the expected peak altitude was 4-kilometers which was expected to occur approximately 180-seconds after detonation. If the computed value of the peak altitude was too high, then the value for ε was increased; however, if the computed value was too low, then ε was decreased. Then the entire process was repeated. Although several values for ε were used, ranging from 0.0 to 0.1, only the results for three test programs are presented in this discussion since they illustrate all of the main results.

Table II

Effect of ε on Maximum Altitude

Based on Input Data from Table I

ε	z_{max}(km)	t(min)
0.00000	28.0	5.9
0.00055	19.0	4.5
0.00600	4.3	1.3

Table II contains the computed maximum altitude and the time of occurrence for each of the three values of ε. As is evident from the listed maximum altitudes, all values of ε smaller than 0.006 yield a maximum altitude greater than the desired value of 4-kilometers. Also, the time to the desired maximum altitude, or approximately three minutes,must correspond to a computer maximum altitude of higher than 5-kilometers.

The data entered into Table II were taken from Figures 1, 2, and 3. Figure 1 shows a peak altitude of 28-kilometers could be achieved by a rising fireball from a 1-kiloton air burst detonated at sea-level. And Figure 3 shows that if ε=0.006, then the same type of fireball will reach a maximum altitude of approximately 4-kilometers, but the fireball is more dense than the ambient air so the fireball 'falls' to earth. Actually, the fireball accelerates downward and reaches sea-level before the drag forces can stop its descent. Since the fireball does entrain some of the ambient air but does not accelerate to earth in reality, the actual value to use for ε must be between 0.0 and 0.006. This is two orders of magnitude less than other studies have indicated (see page 7).

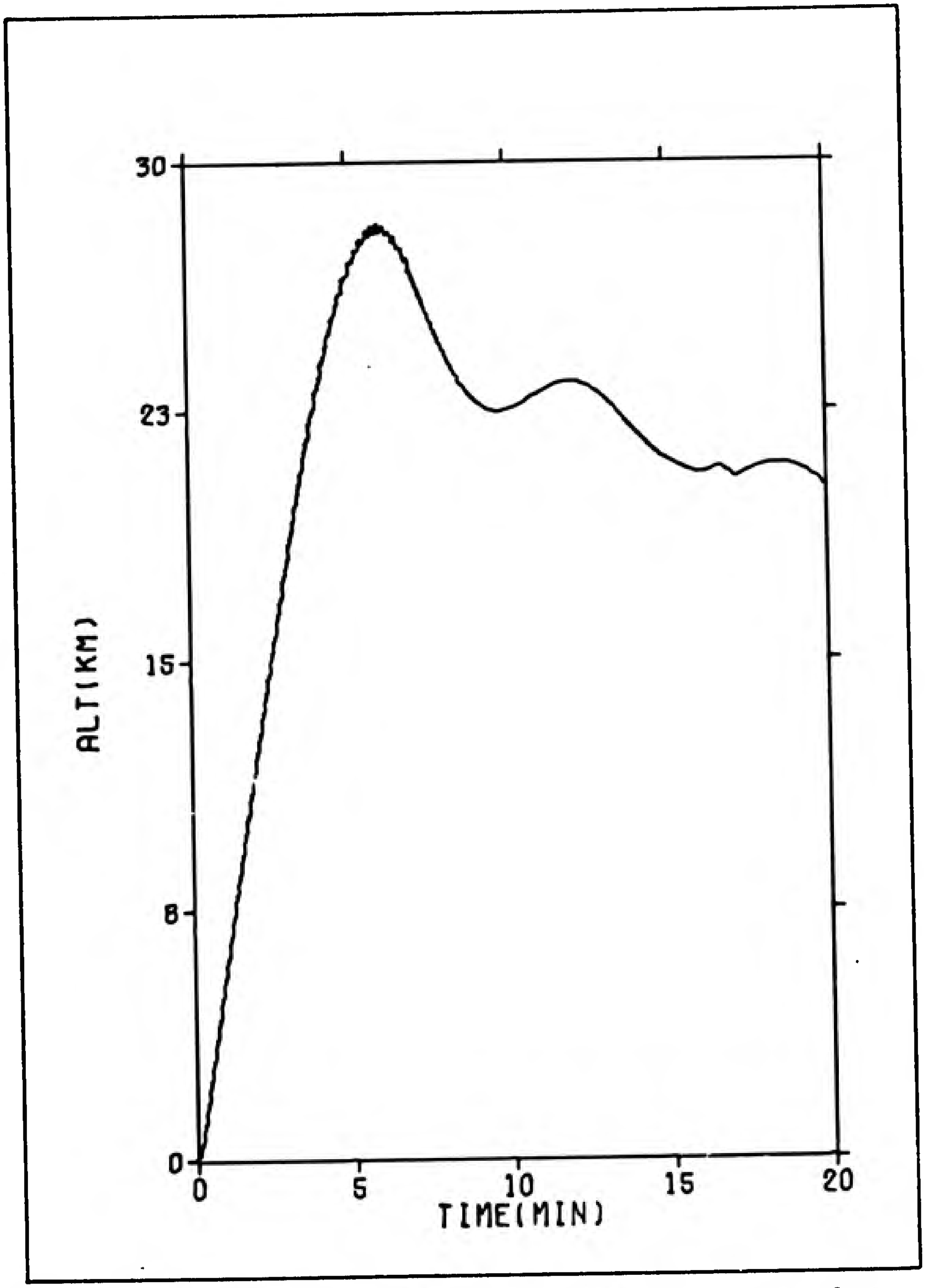

Fig. 1. Altitude vs Time, ε = 0.00000. Based on input data from Table I.

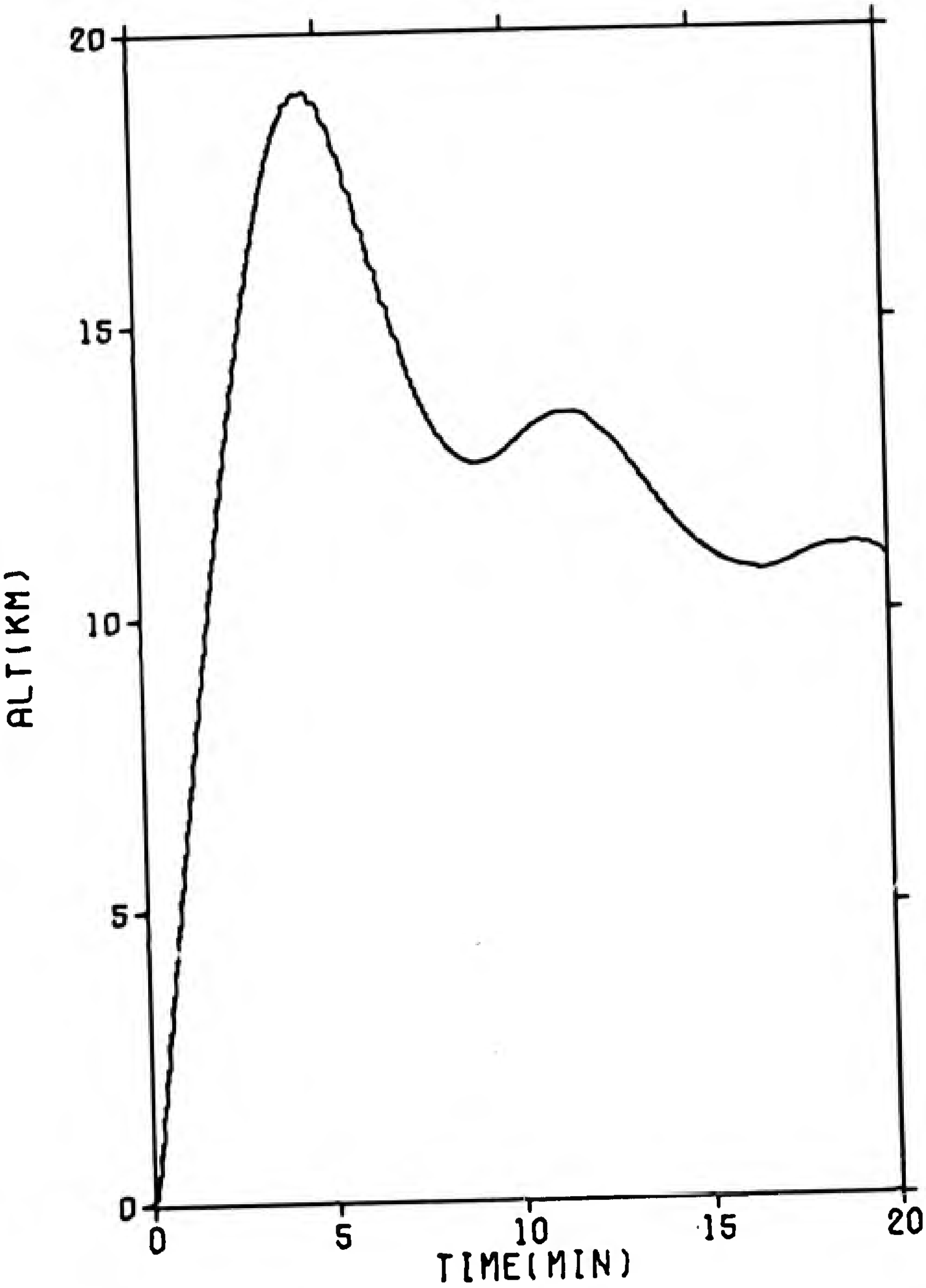

Fig. 2. Altitude vs Time, ε = 0.00055. Based on input data from Table 1.

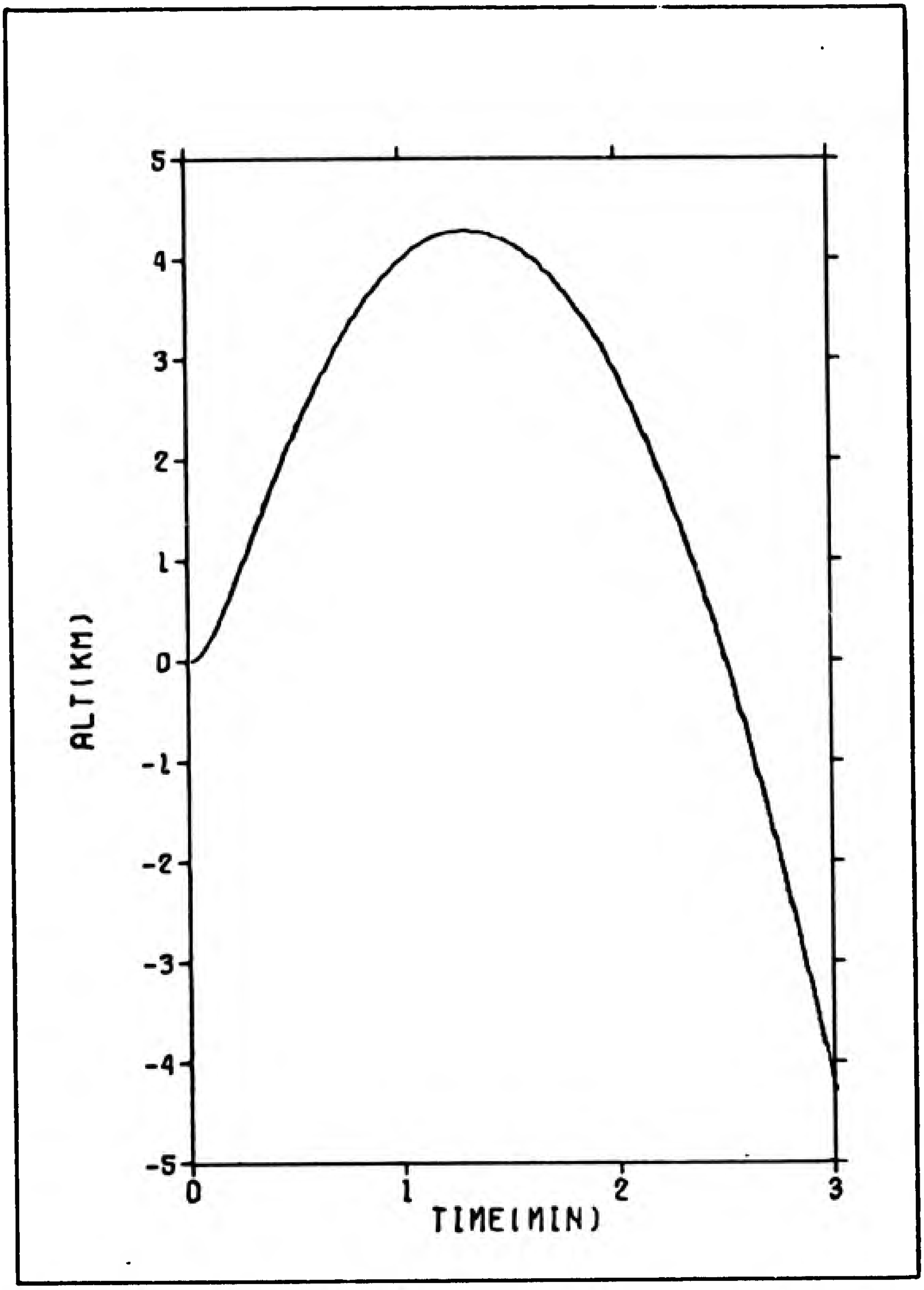

Fig. 3. Altitude vs Time, ε = 0.00600. Based on input data from Table I.

The value of $\varepsilon = 0.00055$ was finally accepted for use in completion of this study. This value was finally accepted because it is the largest value of ε studied which does not cause the fireball to accelerate earthward so fast as to reach sea level. This value was accepted even though the maximum altitude achieved by the fireball was approximately five times greater than the desired altitude.

Solutions for 1-Kt Fireball

Using the above value for ε and the initial data for a 1-kt device given in Table 1, the set of equations was solved for a time period of approximately 20-minutes. The 20-minute time period was chosen because it illustrates the effects of the various terms in the equations. The graphical forms of the solutions were obtained as output and included in this report as Figures 2, 4, 5, and 6. Figure 2 is located on page 36, and the remainder of the figures are located on the following three pages. Each resulting figure furnishes some insight into what terms are the dominating terms, and these are discussed in the following paragraphs.

Altitude. The initial altitude for this fireball is sea-level, but the maximum altitude reached is approximately 19-kilometers as shown in Figure 2. And as time passes the change in the altitude decreases. Thus, although not shown in the figure, the fireball seems to be stabilizing at an altitude of approximately 11-kilometers. The expected altitude for stabilization for this fireball is about 3-kilometers (Ref: 23). Therefore, closer agreement in the stabilization altitudes is achieved than in the maximum altitudes.

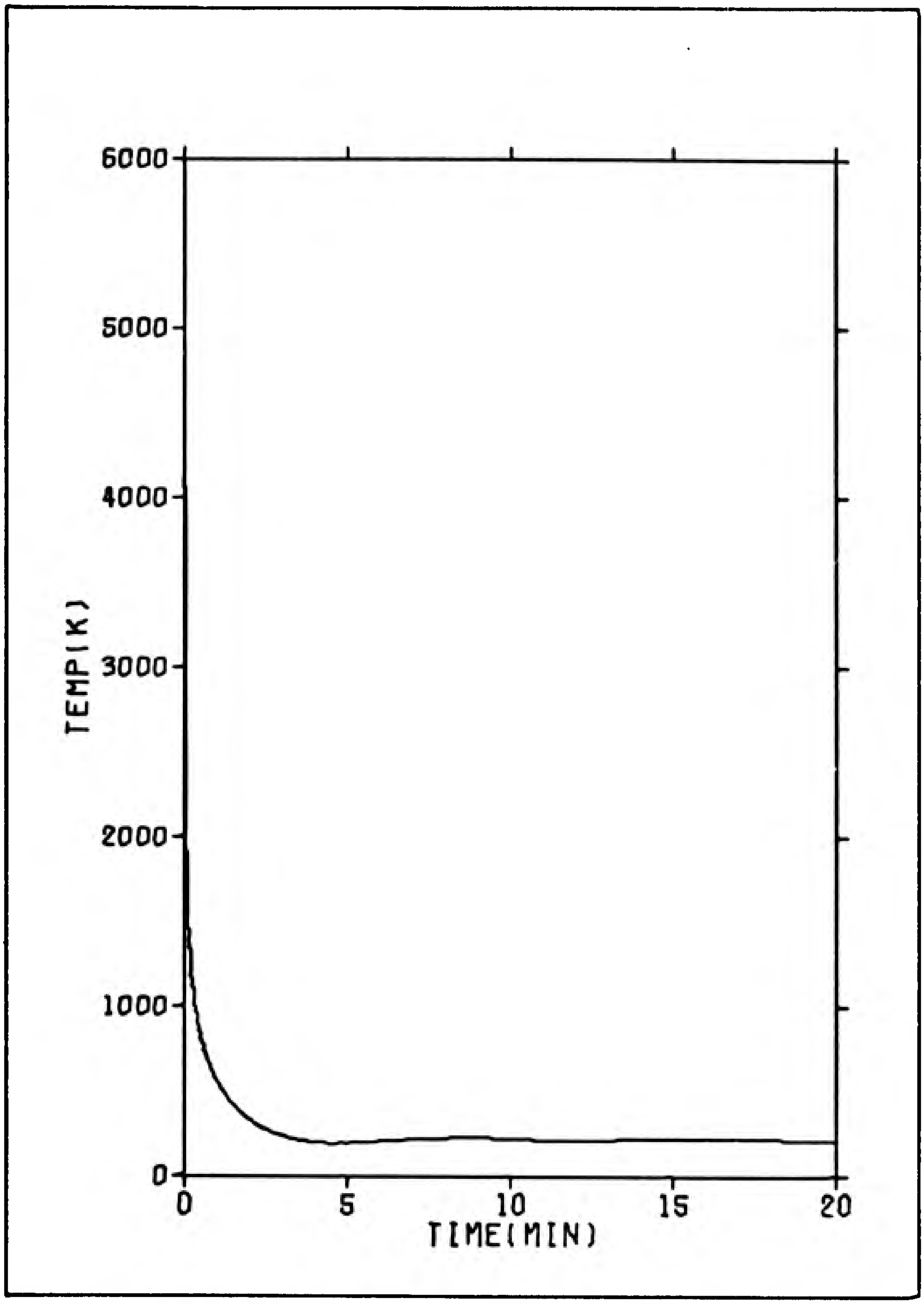

Fig. 4. Temperature vs Time, $\varepsilon = 0.00055$. Based on input data from Table I.

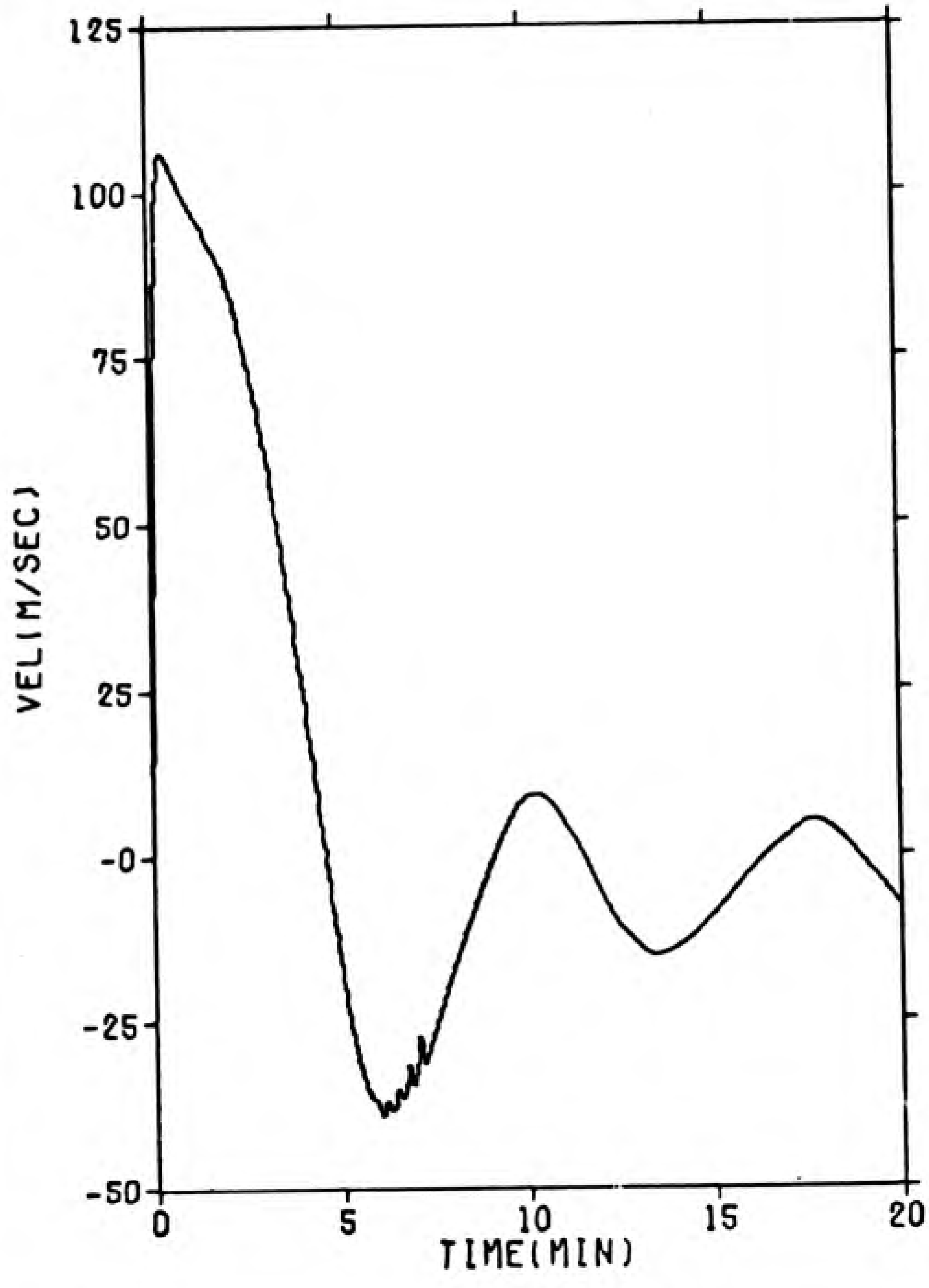

Fig. 5. Velocity vs Time, ε = 0.00055. Based on input data from Table I.

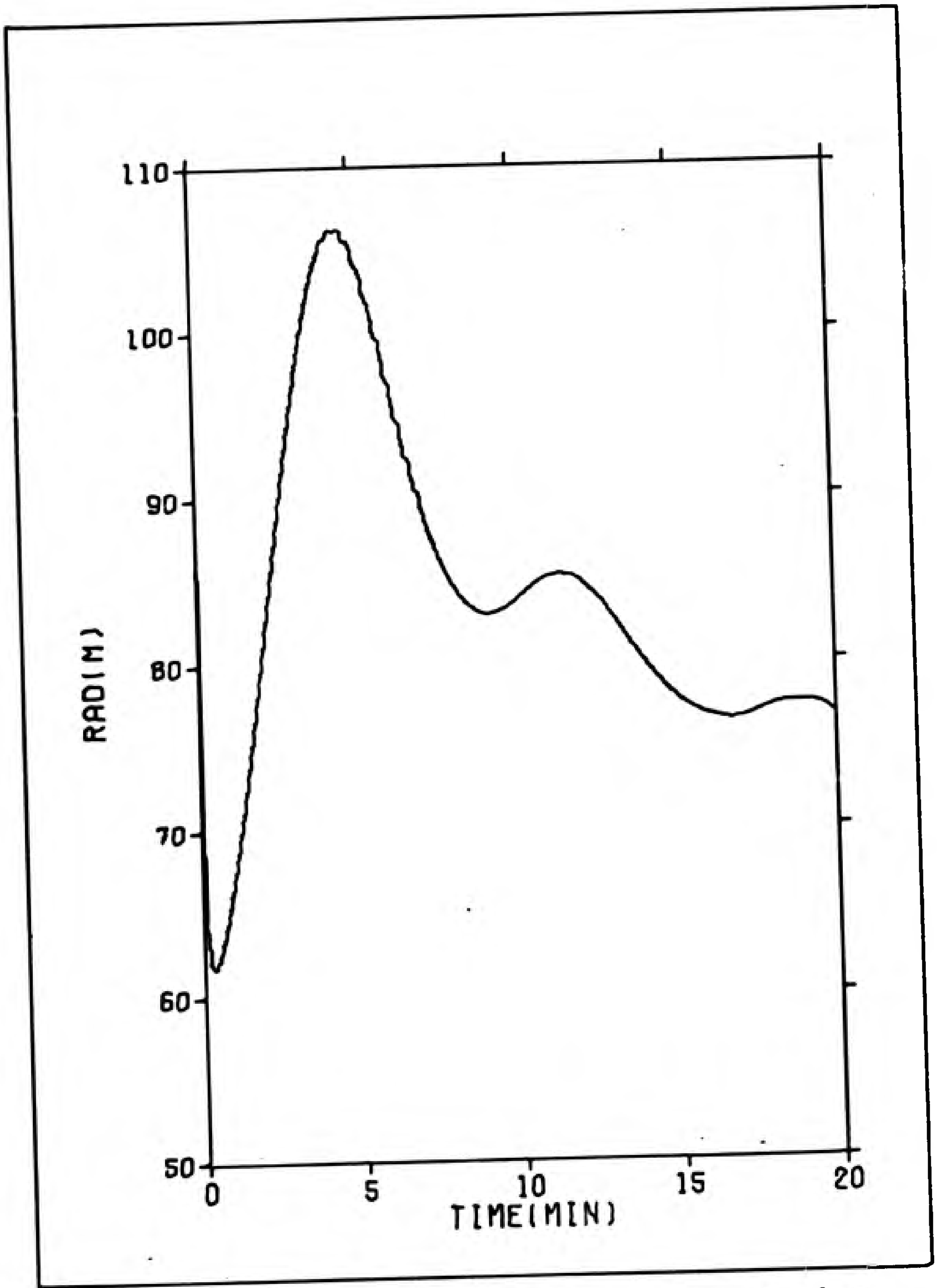

Fig. 6. Radius vs Time, ε = 0.00055. Based on input data from Table I.

Temperature. Figure 4, found on page 39, shows that the temperature of this fireball decreases rapidly, and within five-minutes after the burst the temperature of the fireball is the same order of magnitude as the ambient air. Initially, the cooling is primarily by radiation; therefore, the dominant term is σT_f^4. But, when the temperature is only a few hundred degrees Kelvin, then all of the terms in eq (24) contribute to the change in the temperature of the fireball.

At times, the temperature in the fireball is actually cooler than the ambient air due to the cooling by expansion, radiation, and entrainment. This is the reason that the stabilization altitude is less than the maximum altitude. The momentum of the fireball causes it to go beyond the altitude of stabilization. This will be discussed later in more detail.

When the temperature in the fireball is cooler than the ambient air, the entrainment of the relatively warm air increases the temperature of the fireball. This is especially true when the relative velocity is large. This effect is more evident by studying the numerical solutions instead of Figure 4; however, a careful study of the figure will detect this phenomena.

Velocity. The vertical velocity of the fireball considered in this study is shown in Figure 5, page 40. From this figure one can see that when the temperature of the fireball is much higher than the ambient temperature, then a large velocity is achieved. This fact is as expected since initially the air is much more dense than the fireball; therefore, the buoyancy effect is large. As was mentioned in the derivation, the initial acceleration is approximately 2g.

The effect of the temperature can best be studied by using eq (38). As the temperature of the fireball approaches that of the surrounding air, the acceleration decreases. When the buoyancy is equal to the drag induced by the moving sphere and the entrainment, then the acceleration is zero. Even when this happens, the fireball has a non-zero velocity. The momentum of the fireball will cause it to continue in motion until the fireball is decelerated to the point where $v_f = 0$. When $v_f = 0$ and $T_f = T_a$, then the fireball must either accelerate upward or downward. The determining factor is whether T_f is greater than or less than T_a.

The velocity is zero only when either the fireball is at the altitude of stabilization or the momentum of the fireball is zero. Of course the initial velocity is not being considered at this time. Since, as is evident in Figure 2 on page 36, the fireball has not stabilized within the 20-minute time period considered in this study, the times when $v_f = 0$ in Figure 5, page 40, correspond to inflection points on the altitude curve, Figure 2. The same points on the velocity curve correspond to the maxima and minima points on the radius plot, Figure 6, page 41.

Radius. From the available literature (Ref: 4, 77) the expected maximum radius was approximately 67-meters. This is extremely interesting since the initial radius (Ref: 23) was 100-meters. From this, one is forced to analyze the plot of the solutions on their own merits.

Initially, the radius of the fireball decreases very suddenly. In fact, the radius decreases by approximately one-third of the initial value within less than one minute. This is a result of the initial large change in the fireball temperature.

As the rate of cooling decreases and the velocity becomes large, the effect of the velocity becomes more important. The importance of the velocity is vividly seen by comparing Figures 5 and 6. Whenever the sign of the velocity changes, the trend of the radius also changes. Thus, if the radius is increasing and the sign of the velocity changes, then the radius begins to decrease.

Since ε was accepted to be a very small value, the effect of the entrainment on dr_f/dt is negligible. This allows eq (42) to be simplified by omitting the middle term on the RHS of the equation. However, since the term was needed to determine the accepted value of ε and since that value is questionable, no simplification of eq (42) has been made in the computer program.

Summary

An accepted value for the entrainment coefficient was determined by comparing the computed solutions to the solution furnished by the Air Force Weapons Laboratory. Then, plots of the solutions were obtained as output from the computer program using the accepted value for ε. These plots were used to study the set of descriptive equations that were developed in this report. Some terms were shown to dominate the solutions during the first few minutes and then become insignificant while the contribution due to other terms increases as time passes.

V. Conclusions

As a result of this investigation, four conclusions can be drawn. The first three conclusions involve the equations, and the final conclusion involves the validity of the results.

The equations needed to describe a rising fireball resulting from a nuclear explosion can be developed from basic laws of physics and solved using some numerical technique. Four equations are necessary to describe the rising fireball. They involve the time rate of change in the altitude, radius, temperature, and velocity of the fireball. Most of these equations involve the four parameters just mentioned.

The solutions of the four equations can be obtained by using an iterative numerical technique within a computer program. The cost of running the computer program on the author's computer was less than six cents, and this includes the cost of obtaining the output in the form of four plots.

The initial time changes in the parameters can be approximated by very simple relations. The temperature changes as a radiating blackbody, the radius changes by approximately one-third of the initial radius, and the acceleration initially is approximately 2g.

The final conclusion involves the results. The peak altitude is too high, and the radius decreases after reaching a maximum. This is not in agreement with the data furnished by the Air Force Weapons Laboratory. The input data may not have been compatible with the model as developed within this report since the value of ε was forced to be extremely small.

VI. Recommendations

Three recommendations are made within this chapter. They are made with the hope that they might improve the validity of the solutions to the set of equations.

First, more data is needed. Other initial data is needed from various sources so that a comparison between the initial values used in this study and the values for the same parameters from other sources can be made. Also, a time history of a fireball would aid greatly in comparing the solutions obtained from this model and those obtained from a more complicated model.

Secondly, the formation of a torus should be studied to investigate its effect on the equations developed within this study. The assumption that the fireball can be treated as a homogeneous sphere has been the accepted method of treatment in other studies; however, the fact that the nuclear debris will be trapped within the torus will be critical if this model is to be used in any radiation studies. Also, the values for the coefficients associated with the drag and entrainment might be radically changed.

Finally, the interior of the fireball should receive more investigation. If the three-fluid approach to describing the interior could be incorporated into the set of equations, then the solutions should be more accurate. The question that would have to be answered is "Is the increase in accuracy worth the additional cost of the computer time?"

Bibliography

1. Besancon, Robert M., ed. Encyclopedia of Physics, 2nd ed. New York: Van Nostrand Reinhold Company, 1974.

2. Bridgmann, Charles J. "The Physics of Nuclear Explosives." Wright-Patterson Air Force Base: Air Force Institute of Technology, 1972.

3. Brode, Harold L. "Review of Nuclear Weapons Effects." Annual Review of Nuclear Science 18 (1968): 153-202.

4. Glasstone, Samuel, ed. The Effects of Nuclear Weapons, revised ed. Washington D.C.: United States Atomic Energy Commission, 1964.

5. Goldstein, S., ed. Modern Developments in Fluid Dynamics, Vol II. New York: Dover Publications, Inc., 1965.

6. Gray, Dwight E., ed. American Institute of Physics Handbook, 3rd ed. New York: McGraw-Hill Book Company, 1972.

7. Ha gh, Wayne W., and David D. Mantram. Fireball Entrainment Study. Semiannual Report - 15 January to 31 August 1972. Redondo Beach, California: TRW Systems Group, August 1972.

8. Halliday, David, and Robert Resnick. Physics for Students of Science and Engineering, Combined ed. New York: John Wiley & Sons, Inc., 1965.

9. Haltiner, George J., and Frank L. Martin. Dynamical and Physical Meteorology. New York: McGraw-Hill Book Company, 1957.

10. Hess, Seymour L. Introduction to Theoretical Meteorology. New York: Holt, Rinehart and Winston, 1959.

11. Hoerner, Sighard F. Fluid-Dynamic Drag. Midland Park, New Jersey: By the Author, 148 Busteed Drive, 1958.

12. King, H. H., and A. Q. Eschenroeder. Bulk Turbulence Equations for Fireball Modeling. Report - 1 April 1973 to 1 February 1974. Santa Barbara, California: General Research Corporation, February 1974.

13. Lamb, Sir Horace. Hydrodynamics, 6th ed. New York: Dover Publications, 1932.

14. Mantrom, David D., and Wayne W. Haigh. Fireball Entrainment Study. Final Report - 18 August to 30September 1973. Redondo Beach, California: TRW Systems Group, September 1973.

15. Moursund, David G., and Charles S. Duris. Elementary Theory and Applications of Numerical Analysis. New York: McGraw-Hill Book Company, 1967.

16. Perry, Robert H., ed. Chemical Engineer's Handbook, 4th ed. New York: McGraw-Hill Book Company, 1963.

17. Predvoditelev, A. S., ed. Charts and Tables of Thermodynamic Functions of Air, 3 vols. Translated by Associated Technical Services, Inc. New Jersey: Associated Technical Services, Inc., 1962.

18. Rohringer, Gerhard. Entrainment and Expansion Controlled Fireball Rise. Report RM 63TMP-25. Santa Barbara, California: General Electric Company, July 1963.

19. Tietjens, O. G. Applied Hydro-and Aeromechanics. Translated by J. P. Den Hartog. New York: Dover Publications, Inc., 1957.

20. U.S. Committee on Extension to the Standard Atmosphere. U.S. Standard Atmosphere Supplements, 1966. 1967.

21. Wang, C. P., "Motion of an Isolated Buoyant Thermal," Physics of Fluids 14, 8 (August 1971): 1643-1647.

22. Private Conversation: Bridgman, Charles J., and Donn G. Shanklin. Air Force Institute of Technology, 9 August 1974.

23. Private Conversation: Matuska, Daniel A., Capt. Air Force Weapons Laboratory, 5 December 1974.

APPENDIX A

Equation of State for Dry Air

The equation of state for dry air can be developed from the equation of state for an ideal gas (Ref: 9, 9). For an ideal gas,

$$p\alpha = R'T \tag{43}$$

where

p = pressure of gas ($Nt \cdot m^{-2}$)

α = specific volume of gas ($m^3 \cdot kgm^{-1}$)

R' = specific gas constant ($J \cdot kgm^{-1} \cdot {}^{o}K^{-1}$)

T = temperature of gas (${}^{o}K$)

Now, the specific volume is, by definition, the volume per unit mass, or

$$\alpha = \frac{V}{M} \tag{44}$$

where V (m^3) is the volume of the gas and M (kgm) is the mass of the gas.

Substituting this equation into eq (43) and multiplying both sides by M/T leads to

$$\frac{pV}{T} = MR' \tag{45}$$

According to Avogadro's Law (Ref: 10, 16), one mole of any two gases at the same pressure and temperature will occupy the same volume. Stated another way, this says that

$$\frac{pV}{T} = \text{constant} \equiv R \tag{46}$$

where R is the universal gas constant with a numerical value of $8.3143 \times 10^3\ \mathrm{J \cdot kmole^{-1} \cdot {}^oK^{-1}}$.

Obviously, the RHS of eq (45) must be equal to R by eq (46). If eq (45) is written so that it stands for the equation of state for dry air, and R_d, the specific gas constant for dry air, is used in place of R', then

$$mR_d = R \tag{47}$$

where m is the molecular weight of dry air.

The apparent molecular weight of dry air, based on the carbon-12 isotope scale, is 28.9644 $\mathrm{kgm \cdot kmole^{-1}}$ (Ref: 6, 2-134). When m in eq (47) is replaced by its numerical value and the equation is solved for the dry air constant, then one finds that

$$R_d = 287.05\ \mathrm{J \cdot kgm^{-1} \cdot {}^oK^{-1}} \tag{48}$$

As a result, the equation of state for dry air can be written as

$$p_a V_a = M_a R_d T_a \tag{5}$$

where p_a = pressure of ambient air ($\mathrm{Nt \cdot m^{-2}}$)

V_a = volume of ambient air ($\mathrm{m^3}$)

M_a = mass of ambient air (kgm)

R_d = 287.05 J $kgm^{-1} \cdot {}^{o}K^{-1}$

T_a = temperature of ambient air (^{o}K)

APPENDIX B

Specific Heat at Constant Volume for Dry Air

By classical statistical mechanics, diatomic molecules have a predicted value for the specific heat at constant volume, c_v, of 7R/2 where R is the universal gas constant; however, most diatomic molecules at room temperatures have a value of 5R/2 for c_v (Ref: 1, 414).

The key word to determining the value of c_v is "temperature". From equipartion theory, each coordinate or momentum component appearing as a squared term in the internal energy expression for a gas contributes a value of R/2 to c_v. For a diatomic gas, seven contributions are possible: three by translational motion; two by rotational motion; and two by vibrational motion. But, vibrational motion contributes only when $T \geq h\nu/k$, where $h = 6.6256\text{x}10^{-34}$ J·sec, $k = 1.38054\text{x}10^{-23}$ J·$^oK^{-1}$, and ν is the natural frequency of oscillation (Ref: 1, 414).

Nitrogen, N_2, is used to study the vibrational contribution in air. The vibrational frequency of N_2 is approximately $7\text{x}10^{13}$ sec^{-1} (Ref: 6, 7-178); therefore, the minimum temperature at which vibrational motion contributes to the c_v-value is approximately 3000°K. So for temperatures of a few thousand degrees Kelvin or less, there is no contribution due to vibrational motion.

Tables and graphs of the value of c_v for air as a function of temperature have been published (Ref: 17). This data shows how c_v varies with temperature.

However, for incorporation into a simple model, the value of c_v is approximated. The value chosen is 5R/2. This value has been chosen for two basic reasons. First, even if the initial temperature of the fireball is a few thousand degrees Kelvin, the fireball cools rapidly by radiating as a blackbody. Therefore, the temperature is quickly in the applicable range for justifying the use of 5R/2. The second reason for selecting this value for c_v is the fact that the fireball is entraining ambient air which does have $c_v = 5R/2$ (Ref: 6, 2-134).

Thus, this model uses

$$c_v = \frac{5R}{2} \tag{49}$$

By using eq (47), found in Appendix A, the value of the specific heat of dry air at constant volume is numerically given by

$$c_v = 717.63 \text{ J}\cdot\text{kgm}^{-1}\cdot{}^{o}\text{K}^{-1} \tag{50}$$

APPENDIX C

Drag Coefficient

The drag force acting upon a sperical fireball is assumed to be proportional to the ambient air density, the area of the sphere normal to the direction of motion, and the square of the relative velocity of the fireball with respect to the ambient atmosphere. This is stated as eq (32) in the main text.

In addition to the previously mentioned quantities, a drag coefficient, C_D, appears in the equation for the drag force. The numerical value of C_D for a sphere is found by a two-step procedure. First, a quantity called the Reynold's number, Re, is calculated. Then the value of C_D is obtained from of graph of C_D vs Re for a sphere. These graphs are contained in almost any text on fluids and aeromechanics (Ref: 11, 3-8; 5, II-495) as well as in various science handbooks (Ref: 6, 2-268; 16, 5-60).

To calculate Re, the following definition for Re is used:

$$Re = \frac{vd}{\nu} \tag{51}$$

where v is the velocity of the moving sphere relative to the air ($m \cdot sec^{-1}$), d is the diameter of the sphere (m), and ν is the kinematic viscosity of air ($m^2 \cdot sec^{-1}$).

For a nuclear fireball, Re has a magnitude of approximately 10^7 to 10^9 except when the velocity is zero. The magnitude of the diameter for small yields is on the order of 10 to 10^2 meters, and the magnitude of the kinematic viscosity of the ambient atmosphere is approximately

10^{-5} $m^2 \cdot sec^{-1}$ (Ref: 20). The magnitude of the velocity is normally 10 to 10^2 meters per second; therefore, by using these magnitudes in eq (50), the magnitude of Re is 10^7 to 10^9.

Although most data on the Reynold's number have 10^6 as the upper limit, at least one graph of C_D vs Re is available in the open literature which has a greater upper limit (Ref: 11, 3-8). The upper value of Re in this graph is 10^8. This graph is shown as Figure 7 below.

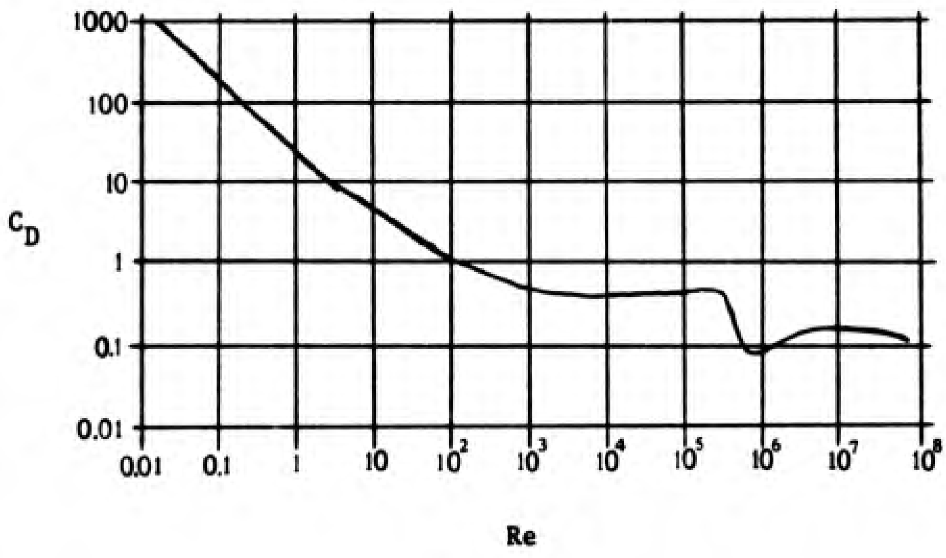

Fig. 7. C_D vs Re for a Sphere. (Ref: 11, 3-8)

The value of C_D used within this model is based upon this plot. The figure shows that C_D varies between 0.1 and 0.2 when the range of Re is from 10^6 to 10^8. Although no data is available for Re > 10^8, an approximate value can be determined by projecting the graph. When the projection is made and the value of C_D is obtained, one finds that for Re = 10^9, C_D .09. Therefore, considering the range of Re for a rising fireball, the value accepted to be used within this model for C_D is

$$C_D = 0.1 \tag{52}$$

APPENDIX D

The Computer Program Used To Solve the Equations

The computer program contained within this appendix is the program which was used to solve the set of equations that describe the rising fireball. The equations are contained in the external subroutine FCN. Since numerous comment cards have been included in the program and since the subroutine BLCKDQ has already been discussed in Chapter III, no further explanations are necessary to use this program.

```
      PROGRAM FRBALL (INPUT,OUTPUT,PLOT)
      DIMENSION SIZE(4),ALWNC(4),YS(4),X(12),Y(4,12),F(4,12),ALLOW(4),Z(
     1602),T(602),H(602),V(602),R(602)
C
      EXTERNAL FCN
C     ******************************************************************
C     *  THIS PROGRAM SOLVES THE FOUR FIRST-ORDER DIFFERENTIAL         *
C     *  EQUATIONS THAT DESCRIBE THE TIME CHANGES IN THE FIREBALL'S    *
C     *  TEMPERATURE, ALTITUDE, VELOCITY, AND RADIUS.  THIS PROGRAM    *
C     *  USES A SUBROUTINE BLCKDQ TO PERFORM THE ACTUAL NUMERICAL      *
C     *  ANALYSIS.  INCORPORATED WITHIN BLCKDQ IS AN EIGTH-ORDER RUNGE *
C     *  KUTTA METHOD.  THE SET OF EQUATIONS TO BE SOLVED IS GIVEN IN  *
C     *  EXTERNAL FCN.  THE SOLUTIONS AT THE SPECIFIED DT ARE RETURNED *
C     *  BY BLCKDQ AS NEW YS(I) VALUES.                                *
C     ******************************************************************
C
C        THE FOLLOWING IS A LISTING OF THE SYMBOLS USED WITHIN FRBALL:
C              SYMBOL     MEANING
C                CD       DRAG COEFFICIENT, DIMENSIONLESS
C                DT       TIME INCREMENT, SECONDS
C                EPS      ENTRAINMENT COEFFICIENT, DIMENSIONLESS
C                K        NUMBER OF ITERATIONS TO BE DONE MINUS ONE
C                N        DETERMINES IF ANSWERS WILL BE PRINTED N=1 YES
C                                                               N=2 NO
C                TI       INITIAL TIME OF DATA, SECONDS
C              YS(1)      TEMPERATURE IN BLCKDQ, DEGREES KELVIN
C              YS(2)      ALTITUDE IN BLCKDQ, METERS
C              YS(3)      VELOCITY IN BLCKDQ, METERS PER SECOND
C              YS(4)      RADIUS IN BLCKDQ, METERS
```

```
C     ALWNC(I)  TOLERANCES TO BE MET IN BLCKDQ:                         FR 30
C                    I=1   TEMPERATURE                                  FR 31
C                    I=2   ALTITUDE                                     FR 32
C                    I=3   VELOCITY                                     FR 33
C                    I=4   RADIUS                                       FR 34
C       Z       STORAGE MATRIX FOR TIME                                 FR 35
C       T       STORAGE MATRIX FOR TEMPERATURE                          FR 36
C       H       STORAGE MATRIX FOR ALTITUDE                             FR 37
C       V       STORAGE MATRIX FOR VELOCITY                             FR 38
C       R       STORAGE MATRIX FOR RADIUS                               FR 39
C                                                                       FR 40
C  THE REQUIRED FORMAT FOR THE INPUT DATA IS:                           FR 41
C     CARD #     VARIABLE    FORMAT    COLUMNS                          FR 42
C       1           TI        F5.2       1-5                            FR 43
C                   DT        F5.2       6-10                           FR 44
C       2         YS(1)       E10.1      1-10                           FR 45
C                 YS(2)       E10.1     11-20                           FR 46
C                 YS(3)       E10.1     21-30                           FR 47
C                 YS(4)       E10.1     31-40                           FR 48
C       3       ALWNC(1)      E10.1      1-10                           FR 49
C               ALWNC(2)      E10.1     11-20                           FR 50
C               ALWNC(3)      E10.1     21-30                           FR 51
C               ALWNC(4)      E10.1     31-40                           FR 52
C       4           K         I5         1-5                            FR 53
C       5           N         I2         1-2                            FR 54
C  IF MORE THAN 600 ITERATIONS ARE DESIRED THEN THE MATRICES Z,T,       FR 55
C  H, AND R MUST BE RE-DIMENSIONED TO THE SIZE OF (K+3)                 FR 56
C                                                                       FR 57
```

```
C
C         THE FOLLOWING VALUES ARE USED IN THE SUBROUTINE BLCKDQ.
C
      ND=4
      NEQ=4
      HEST=1.E-6
      IBLCK=-1
      XOUT=0.
C
C         READ IN THE INITIAL DATA.
C
      READ 1,TI,DT
    1 FORMAT(2F5.2)
      READ 2,(YS(I),I=1,NEQ)
    2 FORMAT(4E10.1)
      READ 3,(ALWNC(I),I=1,NEQ)
    3 FORMAT(4E10.1)
      READ 4,K
    4 FORMAT(I5)
      READ 5,N
    5 FORMAT(I2)
C
      PRINT 6,TI
C         PRINT THE INITIAL DATA
    6 FORMAT(////1X,16HTIME AFTER BURST,/1X,16HOF INITIAL DATA:,F6.2,8HS
     1ECONDS.)
      PRINT 7,DT,YS
    7 FORMAT(//5X,15HINITIAL VALUES:,/10X,16HTIME INCREMENT =,2X,F7.2,/1
     13X,13HTEMPERATURE =,F9.2,/16X,10HALTITUDE =,F9.2,/16X,10HVELOCITY
     2=,3X,F6.2,/18X,8HRADIUS =,1X,F8.2)
```

```
C         STORE THE INCREMENTAL VALUES RETURNED FROM BLCKDQ IN:        FR 88
      DO 10 I=1,K                                                       FR 89
      XIN=XOUT                                                          FR 90
      XOUT=XIN+DT                                                       FR 91
C     CONVERT TIME TO MINUTES.                                          FR 92
      Z(I)=(TI+XIN)/60.                                                 FR 93
      T(I)=YS(1)                                                        FR 94
      H(I)=YS(2)                                                        FR 95
      V(I)=YS(3)                                                        FR 96
      R(I)=YS(4)                                                        FR 97
C                                                                       FR 98
C         THE SUBROUTINE BLCKDQ IS USED TO SOLVE THE FOUR DIFFERENTIAL  FR 99
C         EQUATIONS GIVEN IN FCN                                        FR100
C                                                                       FR101
      CALL BLCKDQ(FCN,NEQ,ND,XIN,XOUT,YS,X,Y,F,ALWNC,ALLOW,HEST,SIZE,IBL FR102
     1CK)                                                               FR103
   10 CONTINUE                                                          FR104
      I=K                                                               FR105
      K=I+1                                                             FR106
C     CONVERT TIME TO MINUTES.                                          FR107
      Z(K)=(TI+XOUT)/60.                                                FR108
      T(K)=YS(1)                                                        FR109
      H(K)=YS(2)                                                        FR110
      V(K)=YS(3)                                                        FR111
      R(K)=YS(4)                                                        FR112
      IF(N.EQ.2) GO TO 30                                               FR113
      PRINT 8                                                           FR114
    8 FORMAT(///5X,*TIME*,12X,*TEMP*,10X,*ALT*,12X,*VEL*,9X,*RADIUS*)   FR115
      DO 20 I=1,K                                                       FR116
      PRINT 9, Z(I),T(I),H(I),V(I),R(I)                                 FR117
    9 FORMAT(1X,1P5E14.4)                                               FR118
   20 CONTINUE                                                          FR119
```

```
C                                                                       FR120
C        PLOT THE RESULTS USING ONE OR MORE OF THE PLOT SUBROUTINES.    FR121
C        THE SUBROUTINE OPTIONS ARE:                                    FR122
C              1.  ALTITUDE VS TIME.                                    FR123
C              2.  TEMPERATURE VS TIME.                                 FR124
C              3.  RADIUS VS TIME.                                      FR125
C              4.  VELOCITY VS TIME.                                    FR126
C        K MUST BE INCREASED BY 2 FOR PLOT SUBROUTINES.                 FR127
C                                                                       FR128
   30 K=K+2                                                             FR129
      CALL ALT(Z,H,K)                                                   FR130
      CALL TEMP(Z,T,K)                                                  FR131
      CALL RAD(Z,R,K)                                                   FR132
      CALL VEL(Z,V,K)                                                   FR133
      STOP                                                              FR134
      END                                                               FR135
```

```
      SUBROUTINE FCN(X,Y,F)
      DIMENSION Y(4),F(4)
      DATA  C1,C2,C4,C5,C6,C7,C9,C10,C11,C12,C13,C14,C15,C16,G,SIGMA/-2.
     12539E+19,44308.,1.331E-19,288.,.0065,6.6637E-20,1.7523,-3.6549E-5,
     210769.,6381.6,376.52,1.225,5.2232E-5,218.,9.80665,5.67E-8/
C
C        THIS SUBROUTINE CONTAINS THE DIFFERENTIAL EQUATIONS WHICH
C             DESCRIBE A RISING NUCLEAR FIREBALL
C        EQUATION SET 51 DESCRIBE THE FIREBALL WHEN ITS HEIGHT IS LESS
C             THAN OR EQUAL TO 10,769 METERS.  EQUATION SET 52 DESCRIBE
C             THE FIREBALL WHENEVER IT IS ABOVE 10,769 METERS
C        THE CONSTANTS C1 THROUGH C14 ARE CONSTANTS CONTAINED WITHIN
C             THE DIFFERENTIAL EQUATIONS
C        G IS THE GRAVITATIONAL ACCELERATION CONSTANT.  THIS IS ASSUMED
C             TO BE CONSTANT
C        SIGMA IS THE STEPHAN-BOLTZMANN CONSTANT
C
      C3=1./.19023
      C8=.80977/.19023
      EPS=.00655
      CD=0.1
      D=CD+8.*EPS
C        TEST TO SEE IF THE FIREBALL IS ABOVE THE TROPOPAUSE.
      IF(Y(2).GT.C11) GO TO 52
      F(1)=(C1*Y(1)/(Y(4)*((C2-Y(2))**C3)))*((SIGMA*(Y(1)**4))+(C4*((C2-
     1Y(2))**C3)*EPS*Y(1)*ABS(Y(3))/(C5-C6*Y(2)))+(C7*Y(4)*Y(3)*((C2-Y(2
     2))**C8)))
      F(2)=Y(3)
      F(3)=(2.*G*(Y(1)/(C5-C6*Y(2))-1.)-.75*Y(1)*Y(3)*ABS(Y(3))*D/(Y(4)*
     1(C5-C6*Y(2))))/(2.+(Y(1)/(C5-C6*Y(2))))
      F(4)=(Y(4)*F(1)/(3.*Y(1)))+EPS*Y(1)*ABS(Y(3))/(C5-C6*Y(2))+(C9*Y(4
     1)*Y(3)/(C2-Y(2)))
      GO TO 10
```

```
52    F(1)=(C10*Y(1)*EXP((Y(2)-C11)/C12)/Y(4))*((SIGMA*(Y(1)**4))+(C13*E
     1PS*Y(1)*ABS(Y(3))/EXP((Y(2)-C11)/C12))+(C14*Y(3)*Y(4)/EXP((Y(2)-C1
     21)/C12)))
      F(2)=Y(3)
      F(3)=((2.*G*((Y(1)/C16)-1.))-.75*Y(1)*Y(3)*ABS(Y(3))*D/(C16*Y(4)))
     1/(2.+(Y(1)/C16))
      F(4)=(Y(4)*F(1)/(3.*Y(1)))+(EPS*Y(1)*ABS(Y(3))/C16)+C15*Y(4)*Y(3)
1C    END
```

```
      SUBROUTINE ALT(X,Y,N)                                        ALT   1
      DIMENSION X(N),Y(N)                                          ALT   2
C                                                                  ALT   3
C     THIS SUBROUTINE PLOTS ALTITUDE AS A FUNCTION OF TIME.        ALT   4
C                                                                  ALT   5
      CALL PLOT(0.,-9.,-3)                                         ALT   6
      CALL PLOT(0.,2.,-3)                                          ALT   7
      CALL SCALE(X,5.5,N-2,1)                                      ALT   8
      CALL SCALE(Y,7.5,N-2,1)                                      ALT   9
      CALL AXIS(0.,0.,9HTIME(MIN),-9,5.5,0.,X(N-1),X(N))           ALT  10
      CALL AXIS(0.,0.,6HALT(M),6,7.5,90.,Y(N-1),Y(N))              ALT  11
      CALL LINE(X,Y,N-2,1,10,4)                                    ALT  12
      CALL PLOTE                                                   ALT  13
      RETURN                                                       ALT  14
      END                                                          ALT  15
```

```
      SUBROUTINE TEMP(X,Y,N)                                       TEMP  1
      DIMENSION X(N),Y(N)                                          TEMP  2
C                                                                  TEMP  3
C     THIS SUBROUTINE PLOTS TEMPERATURE AS A FUNCTION OF TIME.     TEMP  4
C                                                                  TEMP  5
      CALL PLOT(0.,-9.,-3)                                         TEMP  6
      CALL PLOT(0.,2.,-3)                                          TEMP  7
      CALL SCALE(X,5.5,N-2,1)                                      TEMP  8
      CALL SCALE(Y,7.5,N-2,1)                                      TEMP  9
      CALL AXIS(0.,0.,9HTIME(MIN),-9,5.5,0.,X(N-1),X(N))           TEMP 10
      CALL AXIS(0.,0.,7HTEMP(K),7,7.5,90.,Y(N-1),Y(N))             TEMP 11
      CALL LINE(X,Y,N-2,1,10,4)                                    TEMP 12
      CALL PLOTE                                                   TEMP 13
      RETURN                                                       TEMP 14
      END                                                          TEMP 15
```

```
      SUBROUTINE RAD(X,Y,N)
      DIMENSION X(N),Y(N)
C
C     THIS SUBROUTINE PLOTS RADIUS AS A FUNCTION OF TIME.
C
      CALL PLOT(0.,-9.,-3)
      CALL PLOT(0.,2.,-3)
      CALL SCALE(X,5.5,N-2,1)
      CALL SCALE(Y,7.5,N-2,1)
      CALL AXIS(0.,0.,9HTIME(MIN),-9,5.5,0.,X(N-1),X(N))
      CALL AXIS(0.,0.,6HRAD(M),6,7.5,90.,Y(N-1),Y(N))
      CALL LINE(X,Y,N-2,1,10,4)
      CALL PLOTE
      RETURN
      END
```

```
      SUBROUTINE VEL(X,Y,N)
      DIMENSION X(N),Y(N)
C
C     THIS SUBROUTINE PLOTS VELOCITY AS A FUNCTION OF TIME.
C
      CALL PLOT(0.,-9.,-3)
      CALL PLOT(0.,2.,-3)
      CALL SCALE(X,5.5,N-2,1)
      CALL SCALE(Y,7.5,N-2,1)
      CALL AXIS(0.,0.,9HTIME(MIN),-9,5.5,0.,X(N-1),X(N))
      CALL AXIS(0.,0.,10HVEL(M/SEC),10,7.5,90.,Y(N-1),Y(N))
      CALL LINE(X,Y,N-2,1,10,4)
      CALL PLOTE
      RETURN
      END
```

VITA

Denzel D. Waltman, Jr. [redacted] He attended Butler University where he was awarded a Bachelor of Science degree in education in 1966. At the same time he was commissioned into the United States Air Force. Upon entering active duty he was sent to the University of Utah where he received a Bachelor of Science degree in meteorology in 1967. He then spent the next six years as an Air Force Weather Officer. In 1973 he entered the Air Force Institute of Technology Graduate Engineering Physics program.

[redacted]

This thesis was typed by D. Howdyshell.