AD-A014 359

RELIABILITY ASSESSMENT OF AIRCRAFT STRUCTURES BASED ON PROBABILISTIC INTERPRETATION OF THE SCATTER FACTOR

Alfred M. Freudenthal

Prepared for:

Air Force Materials Laboratory

April 1975

DISTRIBUTED BY:

NTIS
National Technical Information Service
U. S. DEPARTMENT OF COMMERCE

258071

AFML-TR-74-198

AD A014359

RELIABILITY ASSESSMENT OF AIRCRAFT STRUCTURES BASED ON PROBABILISTIC INTERPRETATION OF THE SCATTER FACTOR

APRIL 1975

TECHNICAL REPORT AFML-TR-74-198

Approved for public release; distribution unlimited

AIR FORCE SYSTEMS COMMAND
AIR FORCE MATERIALS LABORATORY
Wright-Patterson Air Force Base, Ohio

AIR FORCE SYSTEMS COMMAND
United States Air Force

Reproduced by
NATIONAL TECHNICAL
INFORMATION SERVICE
US Department of Commerce
Springfield, VA 22151

NOTICE

When Government drawings, specifications, or other data are used for any purpose other than in connection with a definitely related Government procurement operation, the United States Government thereby incurs no responsibility nor any obligation whatsoever; and the fact that the government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data, is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use, or sell any patented invention that may in any way be related thereto.

This technical report has been reviewed and is approved for publication.

Robert C. Donat

Robert C. Donat
Project Engineer

FOR THE COMMANDER

Vincent J. Russo

VINCENT J. RUSSO
Metals Behavior Branch
Metals and Ceramics Division
Air Force Materials Laboratory

Copies of this report should not be returned unless return is required by security considerations, contractual obligations, or notice on a specific document.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFML-TR-74-198	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Reliability Assessment of Aircraft Structures Based on Probabilistic Interpretation of the Scatter Factor		5. TYPE OF REPORT & PERIOD COVERED Final Report Sept 1973-March 1975
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Alfred M. Freudenthal		8. CONTRACT OR GRANT NUMBER(s) F33615-74-C-5003
9. PERFORMING ORGANIZATION NAME AND ADDRESS Dr. Alfred M. Freudenthal 4515 Willard Avenue Chevy Chase, Maryland 20015		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Proj. 7351 Metallic Materials Task 735106 Behavior of Metals
11. CONTROLLING OFFICE NAME AND ADDRESS Air Force Materials Laboratory (AFML/LLN) Air Force Systems Command Wright-Patterson Air Force Base, Ohio 45433		12. REPORT DATE April 1975
		13. NUMBER OF PAGES 57
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Reliability	Shape parameter (distribution)
Fatigue Scatter	Scale parameter (distribution)
Mean life	Maximum likelihood estimate
Characteristic life	Fatigue variability
	Weibull distribution

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The "scatter factor" S as used in fatigue design of aircraft is defined as the ratio between the location parameter (estimate) of the "population" of all aircraft, obtained from n full-scale tests, and the first failure in a fleet of m aircraft. Introducing the Third Asymptotic distribution of smallest values for the fatigue life of the population, this definition produces a Pareto-type distribution of the scatter-factor, on the basis of which S can be related to the numbers n and m and the reliability level R. Tables of S for different combinations of n, m, R and the "minimum fatigue life" are evaluated. Useful

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

AIR FORCE - [illegible]

values of the scatter factor for different materials and purposes are suggested.

FOREWORD

The research work reported herein was conducted by Dr. Alfred M. Fruedenthal, 4515 Willard Avenue, Chevy Chase, Maryland 20015, for the Metals and Ceramics Division, Air Force Materials Laboratory, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio, under USAF Contract No. F33615-74-C-5003. The contract was initiated under Project No. 7351 "Metallic Materials", Task No. 735106 "Behavior of Metals" with Mr. Robert C. Donat (AFML/LLN) Acting as Project Engineer.

This report covers work conducted during the period from September 1, 1973 to March 1, 1975. The manuscript was submitted by the author in March 1975.

TABLE OF CONTENTS

Reliability Assessment of Aircraft Structures
Based on Probabilistic Interpretation
of the Scatter Factor

1. Introduction.

The planning, design and development of advanced aircraft structures of superior structural integrity and reliability requires the establishment of rational procedures for reliability assessment, assurance and demonstration. Such procedures must necessarily extend from the planning phase of materials evaluation to the final phase of acceptance of completed structures through reliability demonstration testing and the setting-up of optimal inspection and maintenance programs. It is the lack of recognition of the fact that the service performance of structural metals depends on the complex interactions, in the structure, of inherent material properties, applied design criteria, operational conditions and manufacturing processes, that has been responsible for the faulty materials evaluation on the basis of which some of the most costly errors in materials selection have been committed in recent years. Relevant materials evaluation for the purpose of assurance of structural integrity and reliability is therefore not identical with the conventional evaluation of inherent material properties, but involves comparative study of alternative systems and process technologies in which all interactions are carefully considered and from which basic parameters for subsequent reliability analysis can be deduced.

Conventional procedures of reliability estimation are based on the assumption of arbitrary scatter factors of between 2 and 4 which, applied to an "estimated" life determined from development tests, are supposed to produce the "safe" or "certifiable" life. The probability of occurrence of individual lives shorter than the "safe" life in the fleet of airplanes remains thus completely undefined but is presumably considered acceptable (or zero). No statement concerning the reliability level of the design can be made, since no confidence level can be attached to the selected scatter factor, nor can any statistical significance be attached to the "estimated" life, simply because in conventional development testing systematic attempts are hardly ever made to use loading sequences that would be fully representative of expected operational conditions. However, in view of the unreliability of available conventional analytical methods of damage prediction, testing under loading sequences representing operational loads provides the single means of arriving at realistic life estimates.

Attempts to improve conventional procedures of structural reliability estimation by the application of procedures of reliability analysis for electronic and other complex systems developed in connection with missile and spacecraft design and testing are obviously futile since for such systems "failure" is a contingency to be guarded against mainly by multiple stand-by redundancies on the basis of reliable evaluation of the "mean-time-to-failure" (MTF) of the single elements making-up the system by sufficiently large test replication (which is possible because of the low cost of both the elements and the testing procedure). The mean-time-to-failure of the system can then be computed on the basis of simple assumptions concerning element assemblies in series or in parallel, or in combinations of both. The effect of scatter is provided for by assurance of a design life (MTF of the system) that is such a large multiple of the (usually rather short) expected operational life that even under the most adverse condition of pure chance failure the computed probability of failure during the usually one-shot operational life is extremely small. It is probably not sufficiently realized that in spite of the fact that the operational times of such systems are only of the order of days, weeks or months the obvious success of this procedure in our space-effort had to be paid for by an improvement of the level of qualify control in the production process of elements as well as of complete systems the economic consequences of which would be unbearable in any industrial production process that is still governed, at least to a certain extent, by considerations of economy and by limitations in the allocation of resources.

A basic improvement of the conventional procedures based on arbitrary scatter factors applied to an ill-defined "expected life" can be achieved through the application of the concept of the "expected time to first failure in a fleet" (A. M. Freudenthal, Tech. Rep.AFML-TR-66-37, AFML-TR-66-241 and AFML-TR-67-149). This concept recognizes the facts that in the design and development of complex large structural systems such as airplanes

(a) the concept of the mean (or median) time to failure is useless since a reasonable value cannot be determined by multiple tests because of excessive cost and time expenditure, nor can it be computed because of complex interactions and sequential redundancies; it is, moreover, operationally irrelevant since it implies that at that time about one-half of the fleet of airplanes has already failed, and

(b) the application of a more or less arbitrary scatter factor based on the implicit assumption of a quasi-symmetric distribution function of operational lives serves no purpose since, in the absence of any knowledge concerning the character of the distribution, it provides no information whatsoever concerning the probability of a member of the fleet not to attain the certified or "safe" life. It disregards, moreover, the crucial fact that, if the conventionally specified "safe" life is assumed, by implication, to represent the life up to which no failure is anticipated this would, in fact, make it identical with a lower bound for the "time to first failure"; it becomes, therefore, a function of the anticipated fleet size and depends significantly on the type of structural damage that is expected to produce the failures, since it is the damage rate that determines the shape of the distribution function of structural lives and thus the ratio between the "expected" and the shortest life in the population. Certification of an operational safe life disregarding the above facts is not more than a numbers game that has no relation to the engineering reality. The result of the introduction of the "time to first failure" as the central parameter of structural reliability analysis involves the replacement, in this analysis, of the methods of conventional statistical theory based on averages and deviations from them, by the modern theory of order statistics and of extreme value distributions. Moreover, the identification of the "time to first failure" with the "certifiable life" of the fleet establishes immediately the missing relation between this life and the probability of its exceedance in the fleet, which is the fleet reliability, since identification as an extremal statistic uniquely determines the shape of the associated distribution function. Finally, it provides the basis for a simple and direct procedure of reliability demonstration in acceptance tests that can become an easily enforceable part of the procurement acceptance procedures instead of being, as at present, a source of ambiguous or arbitrary decisions usually arrived at by a compromise between conflicting "expert" opinions.

To establish the feasibility of the developed method and to verify its usefulness in order to reduce it to a procedure that would be introduced into the actual practice of aircraft design, life prediction, reliability assessment and demonstration, as well as maintenance planning and certification, research studies have been and are being undertaken under AFML sponsorship mainly by the Boeing Company (AFML-TR-69-65 and subsequent reports), with complementary research by the McDonnell Douglas Corp. and the Lockheed-Georgia Company, which evaluate and utilize the large groups of available in-service operational data on full scale aircraft structures as well as multiple structural part and material specimen test data. Consideration of some of the results of those studies seems to lead to the following tentative conclusions:

(a) the development of practical procedures of life prediction and reliability assessment and demonstration based on the concept of the "time to the first failure in a fleet" constitutes a significant improvement over current empirical or semi-empirical methods;

(b) the resulting shift of concern from the (unknown) distribution of lives in the whole fleet, to the distribution of lives of the weakest members of this fleet ("shortest-lives") justifies the selection of the Third Asymptotic Distribution of extremal (smallest) values, also known as the Weibull distribution, as being the physically germane function for the purposes of reliability analysis; this justification, which is far more important than the statistically demonstrated fitting of available multiple-test results, thus provides the basis for extrapolation of these results beyond the practical testing probability range, without which life predictions and reliability assessments are pure fictious,

(c) estimates of the shape parameters (α) of these distributions on the basis of the collected data, under the assumption of zero minimum life (two-parameter distribution) suggest the following (preliminary) values for the different types of structural aircraft metals used for long life service:

aluminum alloys $\alpha = 4.0$

titanium alloys $\alpha = 3.0$

steel alloys 100-200 ksi $\alpha = 3.5$

steel alloys 200-300 ksi $\alpha = 2.5$

(d) the estimates of the shape parameters show significant statistical variation, the values specified under (c) representing values in the quantile range $0.7 < q < 0.8$ and thus close to the most probable (characteristic) value if a two-parameter Weibull distribution were also used to represent the variation of the shape factors; the probability of values smaller than those specified under (c), implying larger scatter, is therefore not more than $0.2 < (1-q) < 0.3$.

(e) scatter of fatigue lives in structures for short-life service (fighter aircraft) is significantly lower than that in structures for long service lives (long range transport), with preliminary results suggesting that this difference could be reflected by values of the shape parameters about 50 percent higher than those specified under (c) for different materials.

While these conclusions reflect a considerable advance towards the goal of a rational practical procedure of reliability assessment, they also point towards several unresolved questions which, however, could not even be formulated before the present stage in the research effort was reached.

Springfield, VA 22151

The first question concerns the fact that the "scatter factor", though clearly a fiction if used, as in the past, as an empirically specified multiplier, is nevertheless an extremely practical concept in the context of a workable simple procedure of life-prediction which, like the "safety factor" in civil engineering design practice, has been widely accepted by aircraft designers and operators because of its comforting simplicity. On the other hand, the time to the first failure in a fleet, while obviously a clearly and rationally defined theoretically concept, confronts both the structural designer and the testing engineer with two basic questions the previously simple answer to which is disappearing together with the removal of the scatter factor: for what "life" is the structure to be designed and what is the meaning of the results of the necessarily small number of fatigue tests of critical structural parts and, in particular, of the (usually single) full-scale fatigue test of the structure?

The second question concerns the use, for the sake of simplicity and convenience, of the two-parameter extreme value distribution, with the tacit implication that the third parameter, the minimum life, can always be assumed to be small enough to justify its replacement by zero. However, even for small specimens interpretation of multiple-replication fatigue tests (A. M. Freudenthal and E. J. Gumbel, Proc. Ray. Soc. A 216, 1953, 309; J. Am. Stat. Assoc., 49, 1954, 575) do not support this assumption; for large specimens or structural or machine parts the assumption of zero minimum life contradicts the basic definition of fatigue as progressive damage.

The disproportionately large ratios between the expected life of the fleet and the time to the first failure computed on the basis of the two-parameter distribution at high reliability levels (R>0.8), which might be significantly reduced if even a small "minimum life" were introduced, suggest the necessity of a careful study of the relation between the shape parameter and the minimum life in the three-parameter extreme value distribution. Obviously, the use of the three-parameter distribution in parameter estimation magnifies the computational effort by at least one order of magnitude, while substantially reducing the confidence level. This is the basic reason for the use of the two-parameter form. Nevertheless, at this stage of the research effort, a study of the implications of the introduction of the third parameter on the reliability analysis can no longer be postponed since the results obtained so far make such study not only possible but unavoidable.

The following section of this report is concerned with the first question, the answer to which lies in the probabilistic interpretation of the "scatter factor" by introducing this factor itself as a new statistical variable, retaining however the two-parameter form of the Weibull distribution. The second question which concerns the effect of the introduction of the third parameter on the redefined scatter factor is dealt with in the subsequent section.

2. The Scatter Factor.

The only operationally useful definition of the scatter-factor that satisfies the requirements of the designer as well as of the operator of a fleet of aircraft, and establishes its relation to the results of the very small number of development tests of structural parts, or to the usually single test of the full-scale structure is as the ratio between the point estimator of the location parameter of the distribution of the population of fatigue lives and the time to the first failure. While the distribution of the total population is unknown, the fact that the interest in this distribution centers on its location parameter (central region), makes the estimation of the parameter rather insensitive to the selection of a form of this distribution, the Logarithmic Normal, the Gamma and the two-parameter Weibull distributions in the central region being practically indistinguishable from each other. Selecting therefore for the sake of the simplicity of the analysis the two-parameter Weibull distribution with location (scale) parameter (characteristic value) β and shape parameter α as the distribution function of the population y of fatigue lives in the fleet, the definition of the scatter factor S becomes

$$S = \hat{\beta}/y_1 \tag{1}$$

where $\hat{\beta}$ is the maximum likelihood point estimator of the scale parameter β for a sample of size n and y_1 is the (statistically variable) time to the first failure in a fleet of size m.

The point estimator $\hat{\beta}$ for n observations y_1 (Gumbel, Statistics of Extremes, p. 297)

$$\hat{\beta} = [\frac{1}{n}\sum_1^n y_1^{\alpha}]^{1/\alpha} \tag{2}$$

provided α is known. The distribution function $f(\hat{\beta})$ can be obtained by introducing the new variable $z_i=(y_i/\beta)^{\alpha}$ and utilizing the known fact (Feller, Vol.2) that the sum

$$2nW = 2\sum_1^n z_i = 2\sum(y_i/\beta)^{\alpha} = 2n(\frac{\hat{\beta}}{\beta})^{\alpha} \tag{3}$$

has a chi-square distribution with 2n degrees of freedom

$$f(W)dW = [2^n\Gamma(n)]^{-1}W^{n-1}e^{-n/2}dW \tag{4}$$

and therefore

$$f(\hat{\beta})d\hat{\beta} = \frac{n^n}{\Gamma(n)}\frac{\alpha}{\beta}(\frac{\hat{\beta}}{\beta})^{\alpha n-1}\exp[-n(\frac{\hat{\beta}}{\beta})^{\alpha}]d\hat{\beta} \tag{5}$$

The distribution of the smallest value y_1 of the Weibull variate with parameters (β,α) in a sample size m

$$f_1(y_1) = \frac{\alpha}{\beta_1}(\frac{y_1}{\beta_1})^{\alpha-1} e^{-(y_1/\beta_1)^{\alpha}} \tag{6}$$

where the scale parameter of $f(y_1)$ $\beta_1=\beta_m{}^{1/\alpha}$ while the shape parameter is the same as for $f(y)$. Hence

$$f_1(y_1) = \frac{\alpha m}{\beta}\left(\frac{y_1}{\beta}\right)^{\alpha-1}\exp\left[-m\left(\frac{y_1}{\beta}\right)^{\alpha}\right] \tag{7}$$

The distribution of the quotient $S=\hat{\beta}/y_1$ is therefore

$$\begin{aligned} f(S) &= \int_0^\infty f(\hat{\beta})\cdot f_1(y_1)\,y_1\,dy = \\ &= \int_0^\infty f(\hat{\beta})\cdot f_1(\hat{\beta}/S)(\hat{\beta}/S)\,d(\hat{\beta}/S) = \\ &= \int_0^\infty (\hat{\beta}/S^2)\,f(\hat{\beta})\,f_1(\hat{\beta}/S)\,d\hat{\beta} = \\ &= \frac{n^n m\alpha^2}{\Gamma(n)\,\beta\cdot S^{\alpha+1}}\int_0^\infty \left(\frac{\hat{\beta}}{\beta}\right)^{\alpha(n+1)-1}\exp\left[-\left(n+\frac{m}{S^\alpha}\right)\left(\frac{\hat{\beta}}{\beta}\right)^{\alpha}\right]d\hat{\beta} \end{aligned} \tag{8}$$

With the abbreviations

$$A = n + mS^{-\alpha}, \quad y = A\left(\frac{\hat{\beta}}{\beta}\right)^{\alpha}$$

and therefore

$$\left(\frac{\hat{\beta}}{\beta}\right)^{\alpha} = \frac{u}{A}, \quad \frac{\alpha}{\beta}\left(\frac{\hat{\beta}}{\beta}\right)^{\alpha-1}d\hat{\beta} = \frac{du}{A}$$

the integral, Eq.(8), is transformed into

$$f(S) = \frac{n^n m\alpha^2}{\Gamma(n)\,S^{\alpha+1}}\int_0^\infty u^n e^{-u}\,du = \frac{\alpha m n^{n+1} S^{\alpha n-1}}{(m+nS^\alpha)^{n+1}} \tag{9}$$

since $\int_0^\infty u^n e^{-u}\,du = n!\,(n)$.

By integration of the density function Eq.(9)

$$\int_0^\infty f(S)\,dS = \left[\frac{nS^\alpha}{m+nS^\alpha}\right]^n = F(S) \tag{10}$$

the distribution function F(S) is obtained. Since the reliability R is the probability of values $\leq S$ it follows that

$$R = F(S) = \left[\frac{S^\alpha}{(m/n)+S^\alpha}\right]^n = [F'(S)]^n \tag{11}$$

where

$$F'(S) = \frac{S^\alpha}{m/n+S^\alpha} \tag{12}$$

F(S) is therefore the distribution of the largest value in a sample size n of the population F'(S).

A degenerate form of Eqs.(9) and (10) is obtained when instead of a fleet of size m a single individual of the population (m=1) is considered. This form has been obtained by L. F. Impellizzeri and coworkers (McDonnell Douglas Interim Report MDC A 1870, No. 1, Aug. 15, 1972) and erroneously designated as the distribution of the "scatter factor." However, being in fact the distribution of the ratio between the point estimator of the scale parameter $\hat{\beta}$ for a sample of size n and any member of the Weibull population, it is not a "scatter factor" that can be meaningfully associated with a structurally significant reliability level. The small values of S obtained in the reported investigation are therefore misleading: it is not the (small) sample size n used to estimate the scale parameter $\hat{\beta}$, but the size m of the fleet that determines the fleet reliability by relating it to its weakest (shortest lived) member.

It can, in fact, be shown that the effect of the sample size n on the actual scatter factor S is surprisingly small. This is a conclusion of considerable practical significance, particularly with respect to the results of full-scale tests or of tests of large parts: if the improvement in the estimator $\hat{\beta}$ that results from an increase in the sample size from n=1 to n>1 is not very significant, the usually considerable cost of replication (n>1) of full-scale tests can hardly be justified and the estimate of the scatter factor and associated reliability can be safely based on the results of a single test for the estimator $\hat{\beta}$. The results of the comparative computations presented in Tables 1 to 3 show that with the single exception of the exponential distribution (α=1) at the reliability level R=0.5, the error in the estimation of the scatter factor S by using n=1 instead of n=2 or 3 at the significant reliability levels $R \geq 0.75$ does not exceed two percent and that even this small gain from test replication practically vanishes beyond n=3.

With n=1 Eqs. (9) and (11) take the form

$$f(S) = \frac{\alpha m S^{\alpha-1}}{(m+S^{\alpha})^2} \tag{9a}$$

and

$$F(S) = \frac{S^{\alpha}}{m+S^{\alpha}} = R \tag{11a}$$

With the substitution $S^{\alpha}=z$ and $S^{\alpha-1}=dz/dS$, Eq. (9a) is transformed into

$$f(S)\,dS = \frac{m\,dz}{(m+z)^2} \tag{13}$$

which is the Type-2 Inverted Beta Function

$$f(z)\,dz = \frac{1}{B(p,q)} \frac{z^{p-1} b^{q}}{(z+b)^{p+q}}\,dz \tag{14}$$

with b=m, p=q=1 since the complete beta function

$$B(p,q) = \frac{(p-1)!\,(q-1)!}{(p+q-1)!} = B(1,1) = 1;$$

this Pareto-type function has no moments for q<1 (Raiffa, et al Appl. Stat. Decision Theory, Harvard U.P. 1961, p. 221; also Cramer, p. 242) and therefore no "expectation."

However either the mode or median can be specified as its location parameter. Differentiating Eq. (9) with respect to S, the mode $\tilde{S}$ is obtained from $df(S)/dS = 0$

$$\tilde{S} = (m)^{1/\alpha} \left(\frac{\alpha-1/n}{\alpha+1}\right)^{1/\alpha} \tag{15}$$

while the median

$$\breve{S} = (m)^{1/\alpha} \left[\frac{1}{n(\sqrt[n]{2}-1)}\right]^{1/\alpha} \tag{16}$$

For n=1 therefore

$$\tilde{S} = (m)^{1/\alpha}(\frac{\alpha-1}{\alpha+1})^{1/\alpha} \qquad (15a)$$

and

$$\breve{S} = (m)^{1/\alpha} > \tilde{S} \qquad (16a)$$

with the quantiles $R(\tilde{S})=\frac{\alpha-1}{2\alpha}$ and $R(\breve{S})=0.5$; with increasing value of α the location of the mode of the distribution f(S) for n=1, which is at R=0.25 for α=2 approaches rather rapidly that of the median (R=0.5). Expressing Eq. (11a) in terms of the reduced variable $(S/\breve{S})=s$ the simple equation

$$F(s) = \frac{s^{\alpha}}{1+s^{\alpha}} = R \qquad (11b)$$

is obtained.

The relation between scatter factor and reliability is obtained by solving Eq.(11) for S:

$$S = (\frac{m}{n})^{1/\alpha}[\frac{R^{1/n}}{1-R^{1/n}}]^{1/\alpha} \qquad (17)$$

and for n=1

$$S = S_1 = m^{1/\alpha}(\frac{R}{1-R})^{1/\alpha} \qquad (18)$$

or

$$S_1/\breve{S}_1 = s = (\frac{R}{1-R})^{1/\alpha} \qquad (18a)$$

The ratio (S/S_1) illustrates the effect of the sample size n used in estimating $\hat{\beta}$

$$S/S_1 = (\frac{1}{n})^{1/\alpha}[R^{(1/n-1)}\ \frac{1-R}{1-R^{1/n}}]^{1/\alpha} \qquad (19)$$

Eqs.(18) and (19) have been evaluated for different combinations of m, n, α and R. The results are presented in Tables 1 to 3 in order to support the conclusion that the scatter factor S_1 can be applied to the results of a single full-size test (n = 1).

3. Effect of Minimum Fatigue Life on the Scatter Factor

The principal, well known difficulty in the use of the three-parameter distribution function

$$- \ln[1-P(y)] = \left[\frac{y-\omega}{\beta-\omega}\right]^{\alpha} \tag{20}$$

valid for $y>\omega>0$, the minimum life, is the interrelation between the three parameters α, β and ω which precludes their independent estimation. A first estimate of the minimum life can be based on a visual inspection of the "linearity" of the plot of Eq. (20) on extreme value probability paper with the aid of its transformation into the straight-line relation

$$\ln(y-\omega) = \ln(\beta-\omega) + z/\alpha \tag{21}$$

where

$$z = \ln\{-\ln[1-P(y)]\}$$

The transformation into this straight line relation between $\ln(y-\omega)$ and z of the curvilinear relation between $\ln\ y$ and z reflects an adequate a priori estimate of the parameter ω. However, the effectiveness of this procedure is limited to small and moderate values of the parameter α since for large values of α the existence of a value $\omega>0$ is not always clearly reflected.

Presenting Eq. (21) in the normalized form

$$\eta = \varepsilon + (1-\varepsilon)e^{z/\alpha} \tag{22}$$

where $\eta = y/\beta$ and $\varepsilon = \omega/\beta$ so that the two parameters ε and $1/\alpha$ are both bounded between zero and one, it seems that for large values of α the normalized variate η is a nearly linear function of z although ε is not zero. When both α and ε are large the scatter of η is narrow; when both α and ε are small the scatter is wide. Since the effect of the two parameters has the same direction they can compensate each other, a fact that obviously produces problems in the parameter estimation. However, within the range of values of $\alpha<5$, which is the range of this parameter that is significant for fatigue performance of high-strength aircraft structural metal alloys, the increasing nonlinearity of Eq. (22) with increasing ε is quite pronounced and can be used for a first estimation of this parameter.

The distribution function $P(\eta_1)$ of the smallest value η_1 of the normalized variate η in a sample of size m is obtained from

$$1 - P(\eta_1) = [1-P(\eta)]^m = \exp\left[-m\left(\frac{\eta_1-\varepsilon}{1-\varepsilon}\right)^\alpha\right] \qquad (23)$$

which is of the same shape as Eq. (20) but with the characteristic value reduced by the factor $m^{-1/\alpha}$. At the reliability level $[1-P(\eta_1)] = R$ therefore

$$\eta_1(R) = \varepsilon + (1-\varepsilon)m^{-1/\alpha}\left[\ln\left(\frac{1}{R}\right)\right]^{1/\alpha} \qquad (24)$$

while the expectation of η according to the theory of extreme values

$$E(\eta) = \varepsilon + (1-\varepsilon)\ \Gamma(1+1/\alpha) \qquad (25)$$

The scatter factor at the reliability level R with respect to the expectation is therefore

$$S(R) = S_R = \frac{E(\eta)}{\eta_1(R)} = \frac{\varepsilon + (1-\varepsilon)\ \Gamma(1+1/\alpha)}{\varepsilon + (1-\varepsilon)m^{-1/\alpha}[\ln(1/R)]^{1/\alpha}}$$

or

$$S_R = S_{RO} \cdot \frac{1 + \varepsilon[\Gamma(1+1/\alpha)^{-1} - 1]}{1 + \varepsilon\left\{m^{1/\alpha}[\ln(1/R)]^{-1/\alpha} - 1\right\}} \tag{26}$$

where

$$S_{RO} = m^{1/\alpha}\ \Gamma(1+1/\alpha)[\ln(1/R)]^{-1/\alpha} \tag{27}$$

denotes the scatter factor for $\varepsilon=0$. The existence of $\varepsilon>0$ thus reduces this latter by a factor that depends on ε, α, m and R

$$f(\varepsilon,\alpha,m,R) = \frac{1 + \varepsilon[\Gamma(1+1/\alpha)^{-1} - 1]}{1 + \varepsilon\left\{m^{1/\alpha}[\ln(1/R)]^{-1/\alpha} - 1\right\}} \tag{28}$$

For the characteristic reliability $R = e^{-1}$ this expression reduces to

$$f(\varepsilon,\alpha,m) = \frac{1 + \varepsilon[\Gamma(1+1/\alpha)^{-1} - 1]}{1 + \varepsilon(m^{1/\alpha} - 1)} \tag{28a}$$

The effect of ε on the scatter factor increases with increasing sample size m and decreasing α; thus, for instance, for m=250 and $\alpha=3$ a value of $\varepsilon=0.1$ reduces S_{RO} by roughly one-third, a value of $\varepsilon=0.05$ by one-fifth.

It is at high reliability levels that the effect of ε becomes much more pronounced. For R=0.95, for instance, the reduction in the above example for $\varepsilon=0.1$ would be to roughly 40 percent for R=0.99 to roughly one-quarter of S_{RO} at these reliability levels. At the R=0.99 level even as small a value as $\varepsilon=0.05$ would reduce the scatter factor to less than one-half of its value for $\varepsilon=0$, a fact which illustrates the significance of the parameter ε.

The definition of the scatter factor with respect to the expectation of η according to Eq. (26), which replaces the estimate of the scale parameter on the basis of a (small) sample of size n by the expectation of the normalized variable, can be substantially improved by introducing the concept of the scatter factor as a statistical variable in accordance with Eq. (1), considering, however, the existence of a positive value ω in the three-parameter distribution function Eq. (20).

Introducing the auxiliary variable

$$h(S) = S' = \frac{\hat{\beta}-\omega}{Y_1-\omega} = \frac{\hat{\beta}(1-\varepsilon)}{Y_1\left(1 - \frac{\hat{\beta}}{Y_1}\varepsilon\right)} = S\,\frac{1-\varepsilon}{1-S\varepsilon} \geq 0 \qquad (29)$$

the inverse relation

$$g(S') = S = \frac{S'}{1 + \varepsilon(S'-1)} = \frac{\hat{\beta}}{y_1} > 1 \tag{30}$$

is obtained. Since the distribution of the variable $x = y-\omega$ has the form of a two-parameter extremal distribution

$$-\ln[1-P(x)] = \left[\frac{x}{v}\right]^{\alpha} \tag{31}$$

with location parameter $v=\beta-\omega$, the distribution of the smallest value $x_1=y_1-\omega$ in a sample of size m has the same form

$$-\ln[1-P(x_1)] = \left[\frac{x_1}{v_1}\right]^{\alpha} \tag{32}$$

with $v_1=vm^{-1/\alpha}=(\beta-\omega)m^{-1/\alpha}$. Since the density function $f(S')$ of $S' = \frac{\hat{\beta}-\omega}{y_1-\omega} = \frac{v}{x_1}$ is given by Eq. (9), the density function of $S = g(S')$ is obtained by substituting $h(S) = S'$ for S' in the form

$$f(S) = f[h(S) \cdot h'(S)] \tag{33}$$

from which the distribution function $F(S)$ follows by integration between the limits $S'=0$ or $S=0$ and $S' = S\,\frac{1-\varepsilon}{1-S\varepsilon}$ with upper limit $S'=\infty$ or $S=\varepsilon^{-1}$. Hence

$$F(S) = \left\{\frac{1}{1 + \frac{m}{n}\left[\frac{1-S\varepsilon}{S(1-\varepsilon)}\right]^{\alpha}}\right\}^{n} \tag{34}$$

between the limits $F(S) = 0$ at $S=0$ and $F(S)=1$ at $S=\varepsilon^{-1}$. The median

$$\check{S} = m^{1/\alpha} \frac{1}{[n(\sqrt[n]{2}-1)]^{1/\alpha}\left\{1 + \varepsilon\left[\frac{m}{n(\sqrt[n]{2}-1}\right]^{1/\alpha} - \varepsilon\right\}^{1/\alpha}} \qquad [35]$$

which, for $\varepsilon=0$, degenerates into Eq. (16). Since the reliability R is the probability of values $\leq$ S it follows that $R=F(S)=[F'(S)]^n$ where $F'(S)$ is the expression inside the brackets of Eq. (34), which with n=1 represents the distribution of S for a single test.

Eq. (34) has been solved for S for different sets of values of the parameters n, m, α and ε at different levels of R = F(S) and the results are presented in Tables 4 to 21. Comparison of the values of S for different assumptions of the minimum life at different values of α and reliability levels R shows the significant effect of this assumption and illustrates the necessity of much more elaborate experimental studies of the magnitude of the minimum fatigue life in different structural metal alloys as well as in different designs for the same metal. It is in fact the normalized minimum fatigue life ε of a whole design or of a design detail which reflects the quality of design and fabrication more than that of the material, and thus provides a quantitative measure of the quality of fatigue design which has, so far, been missing, as well as a means of considering such quality in the reliability assessment of a structural detail or a whole structure.

Comparing the results at the selected reliability levels R = 0.5 to R = 0.999 for the three sample sizes n = 1, 2 and 3 the practical independence of the figures of the sample size n is most striking. However, without actually evaluating Eq. (34) for these sample sizes at different reliability levels $R = F(S)$, the conclusion that in order to assess the time to first failure in a fleet at a reliability R, the computed scatter factors can be applied to the result of a single full-scale test, could not be justified for distributions with a non-zero minimum life, although it has been proved above for distributions with zero minimum life. The practical significance of this conclusion is quite obvious since it provides an answer to the difficult question that is always being asked with respect to single full-scale tests: what statistical significance can be associated with their results? The insignificant differences of the computed scatter factors for n = 1, 2 and 3 establish the result of a single full-scale prototype fatigue test as an adequate basis for the estimation of the probability of survival of all members of a fleet of size m at a specified fraction of the result.

4. Practical Applications.

In order to draw conclusion of practical significance from the results of this report presented in Tables 1 to 21 the physical relevance of the parameters α and ε must be considered.

The general interrelation between the shape parameter α of the external distribution and the structural material has been referred to in Section 1. Considering the effect, on the shape parameter, of the design stress level and the order of magnitude of the expected life, as well as of the strength - and toughness -- level of the structural alloys as reflected in the results of the investigation by the Boeing Company under AFML sponsorship (AFML Techn. Rep. TR-72-236), the following values can be considered to represent reasonable estimates for the purpose of estimation of scatter factors to be used in design at reliability levels not exceeding $R \sim 0.90$:

Material	Short Life	Long Life
Aluminun	$\alpha = 4.5$	$\alpha = 3.5$
Titanium	$\alpha = 3.0$	$\alpha = 2.5$
Steel (100-200 ksi)	$\alpha = 3.5$	$\alpha = 3.0$
Steel (200-300 ksi)	$\alpha = 2.5$	$\alpha = 2.0$

In view of the significant scatter of the shape parameter it is quite doubtful whether in design a higher reliability level than $R \sim 0.90$ can be aimed for even if the existence of a minimum life as a characteristic feature of fatigue is postulated. There is sufficient evidence to justify this assumption, which would counteract the large scatter to be expected particularly in the fatigue life of high-strength structural metals, such as titanium and steel alloys, while for aluminum it has been customary to assume zero minimum life. However, although the minimum life for aluminum alloys is certainly lower than for either titanium or steel, its existence is implied in the definition of fatigue. It appears, therefore, safe and reasonable to assume a nominal value of $\varepsilon = 0.01$ for aluminum alloys and values of $\varepsilon = 0.05$ and $\varepsilon = 0.10$ for titanium and steel alloys, respectively.

Concerning fleet size a distinction should be made between the scatter factor to be used for small ($m = 3$) "lead-the-fleet" groups, medium-size fleets not expected to exceed $m = 250$ and large fleets intended for many years of service. In all three groups, however, distinctions should be made between structures designed for a relatively small number of flight-hours and short missions at high-intensity load exposure and those designed for very large number of flight hours and long missions with a very small ratio of high-intensity load exposures. In constructing tables of scatter factors for practical use along the lines indicated above, differences of a few tenths in the scatter factor are neglected.

Tables 22 to 25 represent an attempt to abstract the information contained in Tables 1 to 21 in a simple form that would encourage practical use (a) by designers of new systems who, at best, can aim at attaining reliability levels $R \sim 0.9$ and (b) by reliability specialists called upon to evaluate the performance record of operating fleets and who, therefore, must attempt to assess the chances of premature failures at reliability levels significantly exceeding the level $R = 0.9$. It would be a fiction to assume that a level of $R = 0.99$ could actually be assured particularly in view of the scatter of the parameter α, but the figures in part (b) of the tables are intended to reflect reliability levels between 0.95 and 0.99. Part (c) of the tables reflects reliability levels of $R = 0.5$ and its purpose is to illustrate the inadequacy of the conventional, currently used scatter factors the magnitude of which, at the relevant values of the parameters of α and m, is well represented by this part of the tables.

Table 1. Scatter Factor S_1 for Zero Minimum Life.

(a) Reliability Level R = 0.5.

n = 1, α =		1	2	3	4	5	6
m =	3	3.0	1.7	1.4	1.3	1.25	1.2
	25	25	5.0	2.9	2.2	1.9	1.7
	100	100	10.0	4.6	3.2	2.5	2.15
	250	250	15.8	6.3	4.0	3.0	2.5
	1000	1000	31.6	10.0	5.6	4.0	3.2

Ratio S/S_1 for numbers $2 \leq n \leq 6$

n = 2	1.21	1.10	1.07	1.05	1.04	1.03
3	1.28	1.13	1.09	1.06	1.05	1.04
6	1.36	1.17	1.11	1.08	1.06	1.05

(b) Reliability Level R = 0.75.

n = 1, α =		1	2	3	4	5	6
m =	3	9.0	3.0	2.1	1.7	1.55	1.4
	25	75	8.7	4.2	2.9	2.4	2.05
	100	300	17.3	6.7	4.2	3.1	2.6
	250	750	27.4	9.1	5.2	3.8	3.0
	1000	3000	54.8	14.4	7.4	5.0	3.8

Ratio S/S_1 for numbers $2 \leq n \leq 6$

n = 2	1.08	1.04	1.03	1.02	1.02	1.01
3	1.10	1.05	1.03	1.03	1.02	1.02
6	1.13	1.06	1.04	1.03	1.03	1.02

Table 2. Scatter Factor S_1 for Zero Minimum Life

(a) Reliability Level R = 0.9.

n = 1, α =		1	2	3	4	5	6
m =	3	27	5.2	3.0	2.3	1.9	1.7
	25	225	15.0	6.1	3.9	2.95	2.5
	100	900	30.0	9.7	5.5	3.9	3.1
	250	2250	47.4	13.1	6.9	4.7	3.6
	1000	9000	94.9	20.8	9.7	6.2	4.6

Ratio S/S_1 for numbers $2 \leq n \leq 6$

n = 2	1.03	1.01	1.01	1.01	1.01	1.01
3	1.04	1.02	1.01	1.01	1.01	1.01
6	1.05	1.02	1.02	1.01	1.01	1.01

(b) Reliability Level R = 0.95.

n = 1, α =		1	2	3	4	5	6
m =	3	5.7×10	7.6	3.9	2.8	2.3	2.0
	25	4.8×10^2	21.8	7.8	4.7	3.4	2.8
	100	1.9×10^3	43.6	12.4	6.6	4.5	3.5
	250	4.8×10^3	68.9	16.8	8.3	5.4	4.1
	1000	1.9×10^4	137.	26.7	11.7	7.2	5.2

Ratio S/S_1 for numbers $2 \leq n \leq 6$

n = 2	1.01	1.00	1.00	1.00	1.00	1.00
3	1.02	1.01	1.01	1.00	1.00	1.00
6	1.02	1.01	1.01	1.01	1.00	1.00

Table 3. Scatter Factor S_1 for Zero Minimum Life.

(a) Reliability Level R = 0.99.

n = 1, α =		1	2	3	4	5	6
m =	3	3.0×10^2	1.7×10	6.7	4.2	3.1	2.6
	25	2.5×10^3	5.0×10	13.5	7.1	4.8	3.7
	100	9.9×10^3	9.9×10	21.5	10.0	6.3	4.6
	250	2.5×10^4	1.6×10^2	29.1	12.5	7.6	5.4
	1000	9.9×10^4	3.1×10^2	46.3	17.7	10.0	6.8

Ratio S/S_1 for numbers $2 \leq n \leq 6$

n = 2-6	1.0	1.0	1.0	1.0	1.0	1.0

(b) Reliability Level R = 0.999.

n = 1, α =		1	2	3	4	5	6
m =	3	3.0×10^3	5.5×10	14.4	7.4	4.9	3.8
	25	2.5×10^4	1.6×10^2	29.2	12.6	7.6	5.4
	100	10.0×10^4	3.2×10^2	46.4	17.8	10.0	6.8
	250	2.5×10^5	5.0×10^2	63.0	22.4	12.0	7.9
	1000	1.0×10^6	1.0×10^3	100.0	31.6	15.9	10.0

Ratio S/S_1 for numbers $2 \leq n \leq 6$

n = 2-6	1.0	1.0	1.0	1.0	1.0	1.0

Table 4. Scatter Factors S_n for Minimum Life Ratios $0 \leq \varepsilon \leq 0.10$ and Fleet Sizes $3 \leq m \leq 1000$.

Reliability Level R = 0.5, n = 1.

		α = 2	3	4	5
m = 3, ε =	0.00	1.7	1.4	1.3	1.3
	0.01	1.7	1.4	1.3	1.2
	0.05	1.7	1.4	1.3	1.2
	0.10	1.6	1.4	1.3	1.2
m = 25, ε =	0.00	5.0	2.9	2.2	1.9
	0.01	4.8	2.9	2.2	1.9
	0.05	4.2	2.7	2.1	1.8
	0.10	3.6	2.5	2.0	1.8
m = 100, ε =	0.00	10.0	4.6	3.2	2.5
	0.01	9.2	4.5	3.1	2.5
	0.05	6.9	3.9	2.9	2.3
	0.10	5.3	3.4	2.6	2.2
m = 250, ε =	0.00	15.8	6.3	4.0	3.0
	0.01	13.8	6.0	3.9	3.0
	0.05	9.1	5.0	3.5	2.7
	0.10	6.4	4.1	3.1	2.5
m = 1000, ε =	0.00	31.6	10.0	5.6	4.0
	0.01	24.2	9.2	5.4	3.9
	0.05	12.5	6.9	4.6	3.5
	0.10	7.8	5.3	3.9	3.1

Table 5. Scatter F[illegible]tors S_n for Minimum Life Ratios $0 \leq \varepsilon \leq 0.10$ and Fleet Sizes $3 \leq m \leq 1000$.

Reliability Level R = 0.5, n = 2.

		α = 2	3	4	5
m = 3, ε =	0.00	1.9	1.6	1.4	1.3
	0.01	1.9	1.6	1.4	1.3
	0.05	1.8	1.5	1.4	1.3
	0.10	1.7	1.5	1.3	1.3
m = 25, ε =	0.00	5.5	3.2	2.3	2.0
	0.01	5.3	3.0	2.3	2.0
	0.05	4.5	2.8	2.2	1.9
	0.10	3.8	2.6	2.1	1.8
m = 100, ε =	0.00	11.0	5.0	3.3	2.6
	0.01	10.0	4.8	3.2	2.5
	0.05	7.3	4.1	2.9	2.4
	0.10	5.5	3.5	2.7	2.2
m = 250, ε =	0.00	17.4	6.7	4.2	3.2
	0.01	14.9	6.4	4.0	3.1
	0.05	9.6	6.2	3.6	2.8
	0.10	6.6	4.3	3.2	2.6
m = 1000, ε =	0.00	34.8	10.7	6.2	4.2
	0.01	26.0	9.7	5.6	4.0
	0.05	13.0	7.1	4.7	3.6
	0.10	7.9	5.4	4.0	3.1

Table 6. Scatter Factors S_n for Minimum Life Ratios $0 \leq \varepsilon \leq 0.1$ and Fleet Sizes $3 \leq m \leq 1000$.

Reliability Level R = 0.5, n = 3.

m	ε \ α =	2	3	4	5
m = 3	0.00	1.9	1.5	1.4	1.3
	0.01	1.9	1.5	1.4	1.3
	0.05	1.9	1.5	1.4	1.3
	0.10	1.8	1.5	1.3	1.3
m = 25	0.00	5.6	3.2	2.3	2.0
	0.01	5.4	2.1	2.3	2.0
	0.05	4.6	2.9	2.2	1.9
	0.10	3.9	2.6	2.1	1.8
m = 100	0.00	11.3	5.0	3.4	2.6
	0.01	10.3	4.8	3.3	2.6
	0.05	7.5	4.2	3.0	2.4
	0.10	5.6	3.6	2.7	2.3
m = 250	0.00	18.0	6.9	4.2	3.2
	0.01	15.3	6.5	4.1	3.1
	0.05	9.7	5.3	3.6	2.9
	0.10	6.7	4.3	3.2	2.6
m = 1000	0.00	36.8	10.9	5.9	4.2
	0.01	26.6	10.0	5.7	4.1
	0.05	13.1	7.3	4.8	3.6
	0.10	8.0	5.5	4.0	3.2

Table 7. Scatter Factors S_n for Minimim Life Ratios $0 \leq \varepsilon \leq 0.1$ and Fleet Sizes $3 \leq m \leq 1000$.

Reliabiliby Level R = 0.75, n = 1.

	α =	2	3	4	5
m = 3, ε =	0.00	3.0	2.1	1.7	1.6
	0.01	2.9	2.0	1.7	1.5
	0.05	2.7	2.0	1.7	1.5
	0.10	2.5	1.9	1.6	1.5
m = 25, ε =	0.00	8.7	4.2	2.9	2.3
	0.01	8.0	4.1	2.9	2.3
	0.05	6.3	3.6	2.7	2.2
	0.10	4.9	3.2	2.5	2.1
m = 100, ε =	0.00	17.3	6.7	4.2	3.1
	0.01	14.9	6.3	4.0	3.0
	0.05	9.5	5.2	3.6	2.8
	0.10	6.6	4.3	3.2	2.6
m = 250, ε =	0.00	27.4	9.1	5.2	3.8
	0.01	21.7	8.4	5.0	3.7
	0.05	11.8	6.5	4.3	3.3
	0.10	7.5	5.0	3.7	3.0
m = 1000, ε =	0.00	54.8	14.4	7.4	5.0
	0.01	35.6	12.7	7.0	4.8
	0.05	14.8	8.6	5.6	4.1
	0.10	8.6	6.2	4.5	3.6

Table 8. Scatter Factors S_{L} for Minimum Life Ratios $0 \leq \varepsilon \leq 0.1$ and Fleet Sizes $3 \leq m \leq 1000$.

Reliability Level R = 0.75, n = 2.

m = 3, α =		2	3	4	5
ε =	0.00	3.1	2.2	1.8	1.6
	0.01	3.1	2.1	1.7	1.6
	0.05	2.8	2.0	1.7	1.5
	0.10	2.6	1.9	1.6	1.5
m = 25, ε =	0.00	9.1	4.4	3.0	2.5
	0.01	8.3	4.2	2.9	2.4
	0.05	6.4	3.7	2.7	2.2
	0.10	5.0	3.2	2.5	2.1
m = 100, ε =	0.00	17.9	6.9	4.3	3.2
	0.01	15.4	6.5	4.1	3.1
	0.05	9.7	5.3	3.6	2.9
	0.10	6.7	4.3	3.2	2.6
m = 250, ε =	0.00	28.5	9.5	5.4	3.9
	0.01	22.3	8.6	5.1	3.7
	0.05	12.0	6.6	4.4	3.3
	0.10	7.6	5.1	3.7	3.0
m = 1000, ε =	0.00	57.4	14.9	7.6	5.1
	0.01	36.5	13.0	7.1	4.8
	0.05	15.0	8.8	5.7	4.2
	0.10	8.6	6.2	4.6	3.6

Table 9. Scatter Factors S_n for Minimum Life Ratios $0 \leq \varepsilon \leq 0.1$ and Fleet Sizes $3 \leq m \leq 1000$.

Reliability Level R = 0.75, n = 3.

m = 3, α =		2	3	4	5
ε =	0.00	3.2	2.2	1.8	1.6
	0.01	2.1	2.1	1.8	1.6
	0.05	2.9	2.0	1.7	1.5
	0.10	2.6	1.9	1.6	1.5
m = 25, ε =	0.00	9.3	4.4	3.0	2.5
	0.01	8.4	4.2	3.0	2.4
	0.05	6.5	3.7	2.7	2.3
	0.10	5.0	3.3	2.5	2.1
m = 100, ε =	0.00	18.3	7.0	4.3	3.2
	0.01	15.5	6.5	4.1	3.1
	0.05	9.8	5.3	3.7	2.9
	0.10	6.7	4.3	3.2	2.6
m = 250, ε =	0.00	28.8	9.5	5.4	3.9
	0.01	22.5	8.7	5.1	3.7
	0.05	12.0	6.6	4.4	3.4
	0.10	7.6	5.1	3.7	3.0
m = 1000, ε =	0.00	57.5	14.8	7.7	5.1
	0.01	36.8	13.1	7.1	4.9
	0.05	15.0	8.8	5.7	4.2
	0.10	8.6	6.2	4.6	3.6

Table 10. Scatter Factors S_n for Minimum Life Ratios $0 \le \varepsilon \le 0.1$ and Fleet Sizes $3 \le m \le 1000$.

Reliability Level R = 0.90, n = 1.

m = 3, α =		2	3	4	5
ε =	0.00	5.2	3.0	2.3	1.9
	0.01	5.0	2.9	2.3	1.9
	0.05	4.3	2.7	2.1	1.8
	0.10	3.7	2.5	2.0	1.8
m = 25, ε =	0.00	15.0	6.1	3.9	3.0
	0.01	13.1	5.8	3.8	2.9
	0.05	8.8	4.9	3.4	2.7
	0.10	6.3	4.0	3.0	2.5
m = 100, ε =	0.00	30.0	9.7	5.5	3.9
	0.01	23.2	8.9	5.2	3.8
	0.05	12.2	6.7	4.5	3.4
	0.10	7.7	5.2	3.8	3.0
m = 250, ε =	0.00	47.4	13.1	6.9	4.7
	0.01	32.4	11.7	6.5	4.5
	0.05	14.3	8.2	5.3	4.0
	0.10	8.4	5.9	4.3	3.4
m = 1000, ε =	0.00	94.9	20.8	9.7	6.2
	0.01	48.9	17.3	9.0	5.9
	0.05	16.7	10.5	6.8	4.9
	0.10	9.1	7.0	5.2	4.1

Table 11. Scatter Factors S_n for Minimum Life Ratios $0 \leq \varepsilon \leq 0.1$ and Fleet Sizes $3 \leq m \leq 1000$.

Reliability Level R = 0.9, n = 2.

m = 3, α =		2	3	4	5
ε =	0.00	5.3	3.0	2.3	2.0
	0.01	5.1	3.0	2.3	1.9
	0.05	4.3	2.7	2.2	1.9
	0.10	3.7	2.5	2.0	1.8
m = 25, ε =	0.00	15.2	6.2	4.0	3.0
	0.01	13.3	5.8	3.8	2.9
	0.05	8.9	4.9	3.4	2.7
	0.10	6.3	4.1	3.0	2.5
m = 100, ε =	0.00	30.3	9.7	5.6	3.9
	0.01	23.5	8.9	5.3	3.8
	0.05	12.3	6.8	4.5	3.4
	0.10	7.7	5.2	3.8	3.0
m = 250, ε =	0.00	47.9	13.2	7.0	4.8
	0.01	14.3	11.8	6.5	4.5
	0.05	8.4	8.2	5.3	4.0
	0.10	4.1	6.0	4.4	3.4
m = 1000, ε =	0.00	96.0		9.8	6.3
	0.01	49.3	17.5	9.0	5.9
	0.05	16.7	10.5	6.8	4.9
	0.10	9.1	7.0	5.2	4.1

Table 12. Scatter Factors S_n for Minimum Life Ratios $0 \leq \varepsilon \leq 0.1$ and Fleet Sizes $3 \leq m \leq 1000$.

Reliability Level R = 0.90, n = 3.

m = 3, α =		2	3	4	5
ε =	0.00	5.3	3.0	2.3	1.9
	0.01	5.1	3.0	2.3	1.9
	0.05	4.4	2.8	2.2	1.9
	0.10	3.7	2.5	2.0	1.8
m = 25, ε =	0.00	15.3	6.2	3.9	3.0
	0.01	13.4	5.9	3.8	2.9
	0.05	8.9	4.9	3.4	2.7
	0.10	6.3	4.1	3.0	2.5
m = 100, ε =	0.00	30.6	9.8	5.6	3.9
	0.01	23.6	9.0	5.3	3.8
	0.05	12.3	7.0	4.5	3.4
	0.10	7.7	5.2	3.8	3.0
m = 250, ε =	0.00	48.3	13.2	7.0	4.8
	0.01	32.8	11.8	6.6	4.6
	0.05	14.4	8.2	5.4	4.0
	0.10	8.4	6.0	4.4	3.4
m = 1000, ε =	0.00	97.1	21.0	9.8	6.3
	0.01	49.3	17.5	9.0	5.9
	0.05	16.7	10.5	6.8	4.9
	0.10	9.1	7.0	5.2	4.1

Table 13. Scatter Factors S_n for Minimum Life Ratios $0 \leq \varepsilon \leq 0.1$ and Fleet Sizes $3 \leq m \leq 1000$.

Reliability Level R = 0.95, n = 1.

m = 3, α =		2	3	4	5
ε =	0.00	7.6	3.9	2.8	2.3
	0.01	7.1	3.7	2.7	2.2
	0.05	5.7	3.4	2.5	2.1
	0.10	4.6	3.0	2.3	2.0
m = 25, ε =	0.00	21.8	7.8	4.7	3.4
	0.01	18.0	7.3	4.5	3.4
	0.05	10.7	5.8	3.9	3.1
	0.10	7.1	4.6	3.4	2.8
m = 100, ε =	0.00	43.6	12.4	6.6	4.5
	0.01	30.6	11.1	6.3	4.4
	0.05	13.9	7.9	5.2	3.9
	0.10	8.3	5.8	4.2	3.4
m = 250, ε =	0.00	68.9	16.8	8.3	5.4
	0.01	41.0	14.5	7.7	5.2
	0.05	15.7	9.4	6.1	4.5
	0.10	8.9	6.5	4.8	3.8
m = 1000, ε =	0.00	137	26.7	11.7	7.2
	0.01	58.2	21.2	10.6	6.8
	0.05	17.6	11.7	7.6	5.5
	0.10	9.4	7.5	5.7	4.4

Table 14. Scatter Factors S_n for Minimum Life

Ratios $0 \leq \varepsilon \leq 0.1$ and Fleet Sizes $3 \leq m \leq 1000$.

Reliability Level R = 0.95, n = 2.

		α = 2	3	4	5
m = 3, ε =	0.00	7.7	3.9	2.8	2.3
	0.01	7.1	3.8	2.7	2.2
	0.05	5.7	3.4	2.5	2.1
	0.10	4.6	2.9	2.3	2.0
m = 25, ε =	0.00	22.0	7.8	4.7	3.4
	0.01	18.1	7.3	4.5	3.4
	0.05	10.7	5.9	4.0	2.1
	0.10	7.1	4.7	3.4	2.8
m = 100, ε =	0.00	44.0	12.4	6.6	4.5
	0.01	30.7	11.2	6.3	4.4
	0.05	14.0	7.9	5.2	3.9
	0.10	8.3	5.8	4.2	3.4
m = 250, ε =	0.00	70.0	16.8	8.3	5.4
	0.01	41.2	14.6	7.8	5.2
	0.05	15.7	9.4	6.1	4.5
	0.10	8.9	6.5	4.8	3.8
m = 1000, ε =	0.00	138	26.7	11.7	7.2
	0.01	58.4	21.3	10.6	6.8
	0.05	17.5	11.7	7.7	5.5
	0.10	9.4	7.5	5.7	4.4

Table 15. Scatter Factors S_n for Minimum Life Ratios $0 \leq \varepsilon \leq 0.1$ and Fleet Sizes $3 \leq m \leq 1000$.

Reliability Level R = 0.95, n = 3.

m	ε	α = 2	3	4	5
m = 3	0.00	7.7	3.9	2.8	2.3
	0.01	7.1	3.8	2.7	2.2
	0.05	5.7	3.4	2.5	2.1
	0.10	4.6	3.0	2.3	2.0
m = 25	0.00	22.0	7.9	4.7	3.4
	0.01	18.2	7.3	4.5	3.3
	0.05	10.7	5.8	4.0	3.1
	0.10	7.1	4.7	3.4	2.8
m = 100	0.00	44.0	12.5	6.6	4.5
	0.01	30.7	11.2	6.3	4.4
	0.05	14.0	7.9	5.2	3.9
	0.10	8.3	5.8	4.2	3.4
m = 250	0.00	69.5	17.0	8.3	5.4
	0.01	41.3	14.6	7.8	5.2
	0.05	15.7	9.4	6.1	4.5
	0.10	8.9	6.5	4.8	3.8
m = 1000	0.00	138	27.0	11.7	7.2
	0.01	58.4	21.3	10.6	6.8
	0.05	17.6	11.7	7.7	5.5
	0.10	9.4	7.5	5.7	4.4

Table 16. Scatter Factors S_n for Minimum Life Ratios $0 \leq \varepsilon \leq 0.1$ and Fleet Sizes $3 \leq m \leq 1000$.

Reliability Level R = 0.99, n = 1.

		α = 2	3	4	5
m = 3, ε =	0.00	17.2	6.7	4.2	3.1
	0.01	14.8	6.3	4.0	3.1
	0.05	9.5	5.2	3.6	2.8
	0.10	6.6	4.3	3.2	2.6
m = 25, ε =	0.00	49.8	13.5	7.1	4.8
	0.01	33.4	12.0	6.7	4.6
	0.05	14.5	8.3	5.4	4.0
	0.10	8.5	6.0	4.4	3.5
m = 100, ε =	0.00	99.5	21.5	10.0	6.3
	0.01	50.1	17.8	9.2	6.0
	0.05	16.8	10.6	6.9	5.0
	0.10	9.2	7.0	5.3	4.1
m = 250, ε =	0.00	157	29.1	12.5	7.6
	0.01	61.4	22.7	11.2	7.1
	0.05	17.8	12.1	8.0	5.7
	0.10	9.5	7.6	5.8	4.6
m = 1000, ε =	0.00	314	46.3	17.7	10.0
	0.01	76.1	31.8	15.2	9.2
	0.05	18.9	14.1	9.7	6.9
	0.10	9.7	8.4	6.6	5.3

Table 17. Scatter Factors S_n for Minimum Life Ratios $0 \leq \varepsilon \leq 0.1$ and Fleet Sizes $3 \leq m \leq 1000$.

Reliability Level R = 0.99, n = 2.

m = 3, α =		2	3	4	5
ε =	0.00	17.2	6.7	4.2	3.1
	0.01	14.8	6.3	4.0	3.1
	0.05	9.5	5.2	3.6	2.8
	0.10	6.6	4.3	3.2	2.6
m = 25, ε =	0.00	49.8	13.5	7.1	4.8
	0.01	33.5	12.0	6.7	4.6
	0.05	14.5	8.3	5.4	4.0
	0.10	8.5	6.0	4.4	3.5
m = 100, ε =	0.00	99.5	21.5	10.0	6.3
	0.01	50.2	17.8	9.2	6.0
	0.05	16.8	10.6	6.9	5.0
	0.10	9.2	7.1	5.3	4.1
m = 250, ε =	0.00	157	29.1	12.5	7.6
	0.01	61.4	22.8	11.3	7.1
	0.05	17.9	12.1	8.0	5.7
	0.10	9.5	7.6	5.8	4.6
m = 1000, ε =	0.00	314	46.3	17.7	10.0
	0.01	76.7	31.9	15.2	9.2
	0.05	18.9	14.2	9.7	6.9
	0.10	9.7	8.4	6.6	5.3

Table 18. Scatter Factors S_n for Minimum Life Ratios $0 \leq \varepsilon \leq 0.1$ and Fleet Sizes $3 \leq m \leq 1000$.

Reliability Level R = 0.99, n = 3.

m = 3, α =		2	3	4	5
ε =	0.00	17.0	6.7	4.2	3.1
	0.01	14.9	6.3	4.0	3.1
	0.05	9.5	5.2	3.6	2.8
	0.10	6.6	4.3	3.2	2.6
m = 25, ε =	0.00	50.0	13.5	7.1	4.8
	0.01	33.5	12.0	6.7	4.5
	0.05	14.5	8.3	5.4	4.1
	0.10	8.5	6.0	4.4	3.5
m = 100, ε =	0.00	99.0	21.5	10.0	6.3
	0.01	50.2	17.8	9.2	6.0
	0.05	16.8	10.6	6.9	5.0
	0.10	9.2	7.0	5.3	4.1
m = 250, ε =	0.00	160	29.1	12.5	7.6
	0.01	61.4	22.8	11.3	7.1
	0.05	17.8	12.1	8.0	5.7
	0.10	9.5	7.6	5.8	4.6
m = 1000, ε =	0.00	310	46.3	17.7	10.0
	0.01	76.1	31.9	15.2	9.1
	0.05	18.9	14.2	9.7	6.9
	0.10	9.7	8.4	6.6	5.3

Table 19. Scatter Factors S_n for Minimum Life Ratios $0 \leq \varepsilon \leq 0.1$ and Fleet Sizes $3 \leq m \leq 1000$.

Reliability Level R = 0.999, n = 1.

m = 3, α =		2	3	4	5
ε =	0.00	54.7	14.4	7.4	5.0
	0.01	35.6	12.7	7.0	4.8
	0.05	14.8	8.6	5.6	4.1
	0.10	8.6	6.2	4.5	3.6
m = 25, ε =	0.00	158	29.2	12.6	7.6
	0.01	61.5	22.8	11.2	7.1
	0.05	17.9	12.1	8.0	5.7
	0.10	9.4	7.6	5.8	4.6
m = 100, ε =	0.00	316	46.4	17.8	10.0
	0.01	76.2	31.9	15.2	9.2
	0.05	18.9	14.2	9.7	6.9
	0.10	9.7	8.4	6.6	5.3
m = 250, ε =	0.00	500	63.0	22.4	12.0
	0.01	83.4	38.9	18.4	10.8
	0.05	19.3	15.4	10.8	7.7
	0.10	9.8	8.7	7.1	5.7
m = 1000, ε =	0.00	999	100	31.6	15.9
	0.01	91.0	50.2	24.2	13.8
	0.05	19.6	16.8	12.5	9.1
	0.10	9.9	9.2	7.8	6.4

Table 20. Scatter Factors S_n for Minimum Life Ratios $0 \le \varepsilon \le 0.10$ and Fleet Sizes $3 \le m \le 1000$.

Reliability Level R = 0.999, n = 2.

	α =	2	3	4	5
m = 3, ε =	0.00	54.7	14.4	7.4	5.0
	0.01	35.6	12.7	7.0	4.8
	0.05	14.9	8.6	5.6	4.1
	0.10	8.6	6.2	4.5	3.6
m = 25, ε =	0.00	158	29.2	12.6	7.6
	0.01	61.5	22.8	11.3	7.1
	0.05	17.9	12.1	8.0	5.7
	0.10	9.5	7.6	5.8	4.6
m = 100, ε =	0.00	316	46.4	17.8	10.0
	0.01	76.1	31.9	15.2	9.2
	0.05	18.9	14.2	9.7	6.9
	0.10	9.7	8.4	6.6	5.3
m = 250, ε =	0.00	500	63.0	22.4	12.0
	0.01	83.5	33.9	18.5	10.8
	0.05	19.2	15.4	10.8	7.7
	0.10	9.8	8.8	7.1	5.7
m = 1000, ε =	0.00	999	100	31.6	15.9
	0.01	90.1	50.3	24.2	13.8
	0.05	19.6	16.8	12.5	9.1
	0.10	9.9	9.2	7.[illegible]	6.4

Table 21. Scatter Factors S_n for Minimum Life Ratios $0 \leq \varepsilon \leq 0.1$ and Fleet Sizes $3 \leq m \leq 1000$.

Reliability Level R = 0.999, n = 3.

	ε	α = 2	3	4	5
m = 3, ε =	0.00	55.0	14.4	7.4	4.9
	0.01	35.6	12.7	7.0	4.8
	0.05	14.8	8.6	5.6	4.1
	0.10	8.6	6.2	4.5	3.6
m = 25, ε =	0.00	160	29.2	12.6	7.6
	0.01	61.5	22.8	11.2	7.1
	0.05	17.9	12.1	8.0	5.7
	0.10	9.5	7.7	5.8	4.6
m = 100, ε =	0.00	320	46.4	17.8	10.0
	0.01	76.1	31.9	15.2	9.2
	0.05	18.9	14.1	9.7	6.9
	0.10	9.7	8.4	6.6	5.3
m = 250, ε =	0.00	500	63.0	22.4	12.0
	0.01	83.5	38.9	18.4	10.8
	0.05	19.3	15.4	10.8	7.8
	0.10	9.8	8.8	7.7	5.7
m = 1000, ε =	0.00	1000	100	31.6	15.9
	0.01	91.0	50.2	24.2	13.8
	0.05	19.6	16.8	12.5	9.1
	0.10	9.9	9.2	7.8	6.4

Table 22. Use Values of Scatter Factors for Various Purposes and Associated Reliability Ranges. Aluminum Alloys.

(a) For Design ($0.75 < R < 0.90$)

	Short Life	Long Life
Prototype Group ($m = 3$) ("Lead-the Fleet")	2.0	2.5
Medium Size Fleet ($100 < m < 250$)	4.5	7.0
Large Fleet ($250 < m < 1000$)	6.0	9.0

(b) For Reliability and Maintainability Assessment ($0.95 < R < 0.99$)

	Short Life	Long Life
Prototype Group ($m = 3$) ("Lead-the-Fleet")	3.0	4.0
Medium Size Fleet ($100 < m < 250$)	7.0	10.0
Large Fleet ($250 < m < 1000$)	9.0	12.0

(c) Expected (Median) Values ($R = 0.5$)

	Short Life	Long Life
Prototype Group ($m = 3$) ("Lead-the-Fleet")	1.3	1.5
Medium Size Fleet ($100 < m < 250$)	3.0	4.5
Large Fleet ($250 < m < 1000$)	4.0	6.0

Table 23. Use Values of Scatter Factors for Various Purposes and Associated Reliability Ranges. Titanium Alloys.

(a) For Design (0.75 < R < 0.90)

	Short Life	Long Life
Prototype Group (m = 3) ("Lead-the-Fleet")	2.5	3.0
Medium Size Fleet (100 < m < 250)	6.5	8.5
Large Fleet (250 < m < 1000)	8.5	11.0

(b) For Reliability and Maintainability Assessment (0.95 < R < 0.99)

	Short Life	Long Life
Prototype Group (m = 3) ("Lead-the-Fleet")	4.5	6.0
Medium Size Fleet (100 < m < 250)	10.0	13.0
Large Fleet (250 < m < 1000)	12.0	16.0

(c) Expected (Median) Values (R = 0.5)

	Short Life	Long Life
Prototype Group (m = 3) ("Lead-the-Fleet")	1.5	2.0
Medium Size Fleet (100 < m < 250)	4.5	6.0
Large Fleet (250 < m < 1000)	6.0	8.0

Table 24. Use Values of Scatter Factors for Various Purposes and Associated Reliability Ranges. Steel Alloys (Strength 100-200 ksi).

(a) For Design ($0.75 < R < 0.90$)

	Short Life	Long Life
Prototype Group ($m = 3$) ("Lead-the-Fleet")	2.0	2.5
Medium Size Fleet ($100 < m < 250$)	5.0	6.0
Large Fleet ($250 < m < 1000$)	6.0	8.0

(b) For Reliability and Maintainability Assessment ($0.95 < R$ 0.99)

	Short Life	Long Life
Prototype Group ($m = 3$) ("Lead-the-Fleet")	3.0	3.5
Medium Size Fleet ($100 < m < 250$)	6.0	7.0
Large Fleet ($250 < m < 1000$)	7.0	9.0

(c) Expected (Median) Values ($R = 0.5$)

	Short Life	Long Life
Prototype Group ($m = 3$) ("Lead-the-Fleet")	1.5	1.5
Medium Size Fleet ($100 < m < 250$)	3.0	4.0
Large Fleet ($250 < m < 1000$)	4.0	5.0

Table 25. Use Values of Scatter Factors for Various Purposes and Associated Reliability Ranges. Steel Alloys (Strength 200-300 ksi).

(a) For Design (0.75 < R < 0.90)

	Short Life	Long Life
Prototype Group (m = 3) ("Lead-the-Fleet")	2.5	3.0
Medium Size Fleet (100 < m < 250)	6.5	8.0
Large Fleet (250 < m < 1000)	8.0	11.0

(b) For Reliability and Maintainability Assessment (0.95 < R < 0.99)

	Short Life	Long Life
Prototype Group (m = 3) ("Lead-the-Fleet")	4.5	6.0
Medium Size Fleet (100 < m < 250)	8.0	10.0
Large Fleet (250 < m < 1000)	10.0	14.0

(c) Expected (Median) Values (R = 0.5)

	Short Life	Long Life
Prototype Group (m = 3) ("Lead-the-Fleet")	2.0	2.0
Medium Size Fleet (100 < m < 250)	4.5	6.0
Large Fleet (250 < m < 1000)	6.0	8.0

☆ U S GOVERNMENT PRINTING OFFICE 1975—657 630/35