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M. L. Burrows

Capacitive Profile Tapering for Towed ELF Loop Antennas

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY

LEXINGTON, MASSACHUSETTS

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FOR THE COMMANDER

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY

LINCOLN LABORATORY

CAPACITIVE PROFILE TAPERING FOR TOWED ELF LOOP ANTENNAS

M. L. BURROWS Group 61

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ABSTRACT

A method of capacitive connection to the signal winding of a towed ELF loop antenna is proposed which achieves a smoothly tapered sensitivity profile using a series of strictly uniform and standard fabrication steps. Thus the expense, risk and waste involved in the non-uniform methods of core permeability tapering, core cross section tapering or turns density tapering can be avoided.

Capacitive Profile Tapering for Towed ELF Loop Antennas

I. Introduction

It has already been established that vibration noises in a submarinetowed ELF loop antenna can be substantially reduced by making the antenna long and by tapering its sensitivity profile gradually to zero at each end [1-7]. Direct methods of tapering are varying the turns density [8], the core permeability [9] or the core cross section [9] or any combination of these as a function of position. All of these require non-uniform production procedures, however, and are therefore expensive and difficult to control. The two core modification methods also have the troublesome feature that the precise piece of core to be used as the antenna core is fixed from the start. If some unacceptable damage or faulty construction should subsequently occur anywhere along this length, the whole length has to be scrapped.

This note describes a method of achieving the profile tapering which allows uniform production processes to be used in the antenna construction. It also allows one to complete essentially all the stages of antenna fabrication on an arbitrarily long antenna core before selecting the separate lengths that are to become the separate antennas. Damaged or faulty portions can therefore be avoided.

The method, in essence, consists of making connection to the two ends of the signal winding capacitively instead of conductively. By distributing this capacitive connection over a substantial length at each end of the signal winding, one achieves a signal connection that essentially bypasses the very

end turns of the winding, thereby weighting them very low in their contribution to the received voltage, and gradually weights successive turns more and more. Figure 1 shows schematically the uniformly wound signal winding and the two connection electrodes which are also applied uniformly but only to the end sections. Also shown in Fig. 1 is a sketch of what the current distribution in the signal winding would be if a current source were applied to the antenna terminals. By the principle of reciprocity, the antenna sensitivity profile is directly proportional to this current distribution [2].

For the sensitivity to go to zero at each end of the antenna, it is necessary to ensure that no inductive coupling exists between the ferromagnetic core (if any) and each connection electrode. If the core consists of a ferromagnetic tape helix, the simple device of winding the electrode as a conductive helix of the same pitch and sense as the core helix should bring about the desired lack of coupling [2].

Thus, in practice, the antenna would consist of a uniformly wound ferromagnetic tape helix as the core, a thin layer of polymeric insulation on top of that followed by the uniformly wound signal winding, and then another thin layer of insulation on top of which the uniformly wound electrode helices would be applied over each end section. The sense and pitch of the electrode helix would be the same as that of the core helix. Since the signal winding is now uniformly wound, it can be used also to demagnetize and to bias the antenna core. Two more connection leads would then be required, conductively connected to opposite ends of the signal winding.



Fig. 1. The capacitive tapering method shown schematically.

In the following sections, the capacitively tapered end sections are analyzed using a transmission line model with a given distributed series impedance and shunt admittance. Of concern are the effective length and resistance of the end section compared with the same quantities for an end section using a tapered turns density, and also the discontinuity in the slope of the profile at its end point and at the junction with the antenna's mid-section. (These discontinuities are a source of motion-induced noise [2].)

The properties of a complete antenna using the capacitive tapering method are treated next, followed by a practical example using actual numerical magnitudes of the important quantities.

It is assumed throughout that the connection electrode has negligible series impedance per unit length. That no mutual inductive coupling exists between it and the signal winding is required for the full effectiveness of the tapering method, as mentioned above. It is also assumed that the distributed capacitance per unit length between the signal winding and the return conductor is negligible. Thus the only distributed capacitance to be taken into account is that between the connection electrode and the signal winding.

II. The Capacitively Tapered End

By virtue of the assumptions, the equivalent circuit of the antenna consists of three impedances and three signal voltage sources in series. They are the two identical impedances Z_t of the capacitively coupled end sections, together with their equal signal-induced voltages, and the impedance Z_m of the mid section, together with its signal-induced voltage. Thus the total

antenna impedance is the sum of the separate impedances and the total effective length is the sum of the separate effective lengths. For the mid section, the current is distributed uniformly, Z_m is given by the product of the impedance per unit length of the signal winding and the mid-section length, and the formula for the effective length is equally simple [2]. The corresponding characteristics of the end sections are not so simple. They are examined below.

If the capacitance per unit length between connection electrode and signal winding is C, if the impedance per unit length of the signal winding is $Z = R + j\omega L$, where R and L are the resistance and inductance per unit length, and if the physical length over which the connection electrode is applied is b (see Fig. 2), then the current distribution I in the signal winding is given by

$$I = I_{o} \frac{\sin kz}{\sin kb} .$$
 (1)

Here I_o is current flowing into the connection electrode, z is the distance from the start of the signal winding and $k^2 = -j\omega CZ = \omega^2 LC[1-jR/(\omega L)]$. The formula follows directly from standard transmission line theory. The voltage V of the electrode with respect to the signal winding is therefore given by

$$V(z) = \frac{1}{j\omega C} \frac{dI(z)}{dz} = \frac{I_o}{j\omega C} \frac{k \cos kz}{\sin kb}$$
(2)

The impedance Z_t of the tapered end section is the ratio V_t/I_o , where V_t is the voltage developed between the points A and B in Fig. 2 at which the current I_o enters and leaves the tapered section. Since the connection electrode has been assumed to have negligible series impedance per unit length,



Fig. 2. Notation for the tapered end analysis.

this voltage is V(z) evaluated at z = b. Thus Z_{+} is given by

$$Z_{t} = \frac{V(b)}{I_{0}} = \frac{1}{j\omega C} \frac{k}{\tan kb} = -\frac{Z}{k \tan kb} .$$
(3)

Here the definition of k^2 has been used to cast the result in the equivalent alternative forms.

As C approaches zero, the current distribution and impedance, given generally by (1) and (3), simplify to I = $I_0 z/b$ and $Z_t \approx (j\omega Cb)^{-1} + Zb/3$. Since the sensitivity profile u(z) is proportional [2] to the product IN, it is therefore given by u(z) = $\gamma N_0 z/b$ where γ is the constant of proportionality and N_0 is the turns denisty of the (uniformly wound) signal winding. The resistance R_t of the tapered end section is the real part of Z_t . That is, for this small C case, $R_t \approx R_b/3$.

If the end tapering is achieved by varying the turns density of the winding instead of using the capacitive method, the corresponding expressions for u(z) and R_t are u(z) = $\gamma N_o z/b$ and R_t = Rb/2. These expressions were obtained by assuming the turns density N is given by N = $N_o z/b$ and that the resistance per unit length is proportional to the turns density.

These results are summarized in Table I.

Table I

Cap. Taper*Turns TaperSens. Profile u(z) $\gamma N_o z/b$ $\gamma N_o z/b$ Resistance RRb/3Rb/2

Capacitive Versus Turns-Density Tapering

* Small C approximation

Table I shows that if C is small enough, the sensitivity profile of the capacitively tapered end section is identical to that of a tapered end section in which the tapering is effected by a linear variation of turns density. The taper achieved would therefore be equally effective in reducing vibration noise.

Comparing the expressions for R_t in Table I, one sees that the resistance of the capacitively tapered end section, in the small C approximation, is only 2/3 of that of the end section tapered by a linear variation in turns density.

These results are promising. They suggest that the capacitive tapering method can be as effective in suppressing vibration noise, and can generate less thermal noise, than the linearly varying turns density method of tapering. However, this conclusion depends upon the distributed capacitance C being small enough. It may transpire that when C is small enough to achieve a satisfactory tapered profile, the reactive impedance it thereby places in series with the antenna impedance will be too large, causing an unacceptable degradation in pre-amplifier noise figure. Thus it is necessary to examine the behavior of the sensitivity profile, of the resistance and of the reactance of the tapered end section as the capacitance C increases from zero.

These effects are most conveniently handled in normalized form. That is, the quantity in question is divided by the value it takes when the capacitance is zero. Then, the effect of varying the capacitance is seen as a departure of the quantity from unity. Whenever possible, this is the way the results will be presented.

The shape of the profile is important in two ways. First, the area

under the profile is proportional to the effective length of the end section and to its signal sensitivity. By integrating I, given by (1), from 0 to b and then dividing the result by I $_{0}^{b/2}$, its low-C limit, one obtains the following expression for the normalized effective length l_{t} of the tapered end section:

$$\frac{\chi_{t}}{\chi_{t}}(o) = \frac{2(1-\cos kb)}{kb \sin kb} .$$
(4)

Here $l_t^{(0)}$ is the notation adopted for the value of l_t when C approaches zero. Figures 3 and 4 show $l_t/l_t^{(0)}$ plotted in magnitude and phase respectively, as a function of $\omega\sqrt{LC}$ b with R/(ω L) as a parameter. The curves show that if R/(ω L) is 10 or less, then C can lie anywhere between 0 and 0.4/($\omega^2 b^2 L$) without practical effect on the signal sensitivity. This range of R/(ω L) is a practical one; for the Lincoln experimental antenna, for example, R/(ω L) = 5.75 at 45 Hz.

The second important attribute of the profile is the nature of its discontinuities, for these are a source of motion-induced noise. Since the current in the signal winding is constant at zero for z < 0 and constant at I_o for z > b, there are two discontinuities associated with the tapered end section, one at z = 0 and the other at z = b. The profile itself is continuous at these points. The discontinuities are exhibited by the first derivative of the profile. Thus the normalized discontinuities in profile slope at z = 0 and z = b are given by normalizing dI(z)/dz with respect to its value at C = 0 and evaluating it at z = 0 and z = b. The result is



Fig. 3. The amplitude of the normalized effective length of the tapered end section as a function of $\omega\sqrt{LC}$ b with $R/(\omega L)$ as a parameter.



Fig. 4. The phase of the normalized effective length of the tapered end section as a function of $\omega\sqrt{LC}$ b with R/(ω L) as a parameter.

$$\frac{\Delta(du/dz)}{\Delta(du^{(o)}/dz)} = \begin{cases} kb/\sin kb & , z = 0\\ kb/\tan kb & , z = b \end{cases}$$

The magnitude of these two normalized profile discontinuities is plotted in Figs. 5 and 6 as a function of $\omega\sqrt{LC}$ b with R/(ω L) as a parameter. The curves are plotted in dB (i.e., as 10 times \log_{10} of the squared magnitude of the discontinuities) because they then give directly the increase, attributable to a non vanishing C, in the discontinuity-generated vibration noise.

The resistance R_t of the tapered end section is simply the real part of its impedance Z_t given by (3). The normalized resistance is therefore $R_t/R_t^{(o)}$, where $R_t^{(o)}$ is the low-C approximation for R_t and is given in Table I as Rb/3. The curves of $R_t/R_t^{(o)}$ versus $\omega\sqrt{LC}$ b with $R/(\omega L)$ as a parameter are plotted in Fig. 7. They show that, for example, if $R(\omega L) \leq 10$ and $\omega\sqrt{LC}$ b ≤ 0.4 , then R_t is within about 4% of $R_t^{(o)}$.

Finally, Fig. 8 shows the variation of the Q-factor Q_t of the impedance of the tapered end section. Here Q_t is defined as $Im\{Z_t\}/Re\{Z_t\}$. (It is mostly negative because for low C, Z_t is mainly capacitive.) The figure shows that Q_t stays within reasonable bounds for all $R/(\omega L)$ provided $\omega\sqrt{LC}$ b ≥ 0.4 . Since the earlier figures have shown that the other performance measures do not deteriorate too badly provided $\omega\sqrt{LC}$ b ≤ 0.4 , one concludes that there exists a range of values of C, centered on the value defined by $\omega\sqrt{LC}$ b ≈ 0.4 , for which the capacitive tapering concept is viable.

The distributed resistance and inductance of the Lincoln experimental antenna are numerically [2] equal to $1.69\Omega/m$ and 1.04×10^{-3} H/m. At 45 Hz



Fig. 5. The magnitude in dB, of the normalized discontinuity in profile slope occuring at z = 0 as a function of $\omega\sqrt{LC}$ b with R/(ω L) as a parameter.



Fig. 6. The magnitude, in dB, of the normalized discontinuity in profile slope occuring at z = b as a function of $\omega\sqrt{LC} b$ with R/(ωL) as a parameter.



Fig. 7. The normalized resistance of the tapered end section as a function of $\omega\sqrt{LC}$ b with R/(ωL) as a parameter.



Fig. 8. The Q-factor of the tapered end section as a function of $\omega\sqrt{LC}\,\,b$ with R/($\omega L)$ as a parameter.

therefore, $R/(\omega L) = 5.75$ and the value of distributed capacitance C required to make $\omega \sqrt{LC}$ b = 0.4 at 45 Hz is 1.92×10^{-7} F/m if b = 100 m. If the outer diameter of the signal winding is 0.3 inches, for example, this could be achieved with a dielectric film about 0.14 mil thick and having a dielectric constant of 3.16. Thus, for the parameters of the Lincoln antenna, a value of C of the right magnitude could be obtained by using a thin tape of polyethylene terephthalate (Mylar) between the signal winding and the connection electrode.

III. The Complete Antenna

If two capacitively tapered end sections of length b are used in conjunction with a center section of length l, the total length of the antenna is l + 2b and its properties are obtained by combining the expressions derived in the last section with the corresponding (and much simpler) expressions for the properties of the center section.

Thus, the sensitivity profile of the center section is $\gamma N_0^{(0)}$, and its impedance is ZL. The normalized effective length $\ell_e/\ell_e^{(0)}$ for the whole ant-enna is therefore given by

$$\frac{\frac{\ell_e}{\ell_e}}{\ell_e} = (\ell + 2\frac{1-\cos kb}{k\sin kb}) \frac{1}{\ell+b} , \qquad (5)$$

and its impedance Z by

$$Z_{o} = Z\ell - \frac{2Z}{k \tan kb}.$$
 (6)

The normalized antenna resistance $R_0/R_0^{(o)}$ is therefore given by

$$\frac{R_{o}}{R_{o}} = \frac{R\ell - Re \left\{\frac{2Z}{k \tan kb}\right\}}{R\ell + 2bR/3} .$$
(7)

Other quantities of interest are the Q-factor Q_o for the whole antenna, given by $Im\{Z_o\}/R_o$, and the normalized signal to thermal noise power ratio $(\ell_e^2/R_o)/(\ell_e^{(o)2}/R_o^{(o)})$ given by combining (5) and (7).

For the particular example for which b/l = 0.75 (i.e., when the tapered sections each occupy 3/10 of the antenna's total length), the behavior of $l_e/l_e^{(0)}$, $R_o/R^{(0)}$, $(l_e^2/R_o)/(l_e^{(0)2}/R_o^{(0)})$ and Q_o as $\omega\sqrt{LC}$ b varies is plotted in Figs. 9 through 12. The results show no surprises. They confirm that if C is such that $\omega\sqrt{LC}$ b ≈ 0.4 and if $R/(\omega L) \geq 4$, then the method of capacitive tapering is effective and practicable.

IV. Conclusions

The capacitive tapering technique allows one to fabricate an antenna in a series of uniform manufacturing operations. The non-uniform operations required by turns density tapering, core permeability tapering and core cross section tapering are eliminated. It also allows one to defer, until late in the manufacturing sequence, the decision of precisely how the division of the arbitrarily long antenna assembly into separate antennas is to be made. In this way, points of damage or improper fabrication, which could spoil a complete antenna of the core tapered type, can be avoided.

Expressions for the various performance parameters for both a tapered end section and for a complete antenna have been derived and numerical evaluations of them presented graphically. They show that a range of values exists for the distributed capacitance per unit length of the connection



Fig. 9. The normalized effective length of a complete, capacitively tapered, antenna as a function of $\omega\sqrt{LC}$ b with R/(ωL) as a parameter.



Fig. 10. The normalized input resistance of a complete, capacitively tapered, antenna as a function of $\omega\sqrt{LC}$ b with R/(ωL) as a parameter.



Fig. 11. The normalized signal to thermal noise power ratio of a complete, capacitively tapered, antenna as a function of $\omega\sqrt{LC}$ b with R/(ωL) as a parameter.



Fig. 12. The Q-factor of a complete, capacitively tapered, antenna as a function of $\omega\sqrt{LC}\,^{1}b$ with R/(ωL) as a parameter.

electrode for which both good performance is achieved and the impedance is not too highly reactive. In addition, this range of values is readily achievable in practice by the use of conventional materials and standard fabrication techniques.

References

- M. L. Burrows, "Motion-Induced Noise in Towed Flexible Sensors," Project Report NAC-15 (Navy Communications), Lincoln Laboratory, M.I.T. (24 (24 March 1969), not generally available.
- 2. M. L. Burrows, "On the Design of a Towed ELF H-Field Antenna," Technical Note 1972-34, Lincoln Laboratory, M.I.T. (28 December 1972), DDC AD-754949.
- M. L. Burrows, "Other Sources of Motion-Induced Noise in a Towed ELF H-Field Antenna," Technical Note 1973-21, Lincoln Laboratory, M.I.T. (25 May 1973), DDC AD-762936.
- R. L. Crane, "Motion-Induced Noise in Flexible Electrode Pair and Loop Antennas," Technical Memorandum No. 1 RCA David Sarnoff Research Center, Princeton, New Jersey (July 1972).
- A. Pelios, "Motion-Induced Noise in Trailed Antennas," Technical Memorandum, No. 2 RCA David Sarnoff Research Center, Princeton, New Jersey (October 1972).
- 6. M. L. Burrows, "The Lincoln Submarine-Towed ELF Loop Antenna," Technical Note 1975-24, Lincoln Laboratory, M.I.T. (27 May 1975).
- M. L. Burrows, "Performance of the ELF Antenna Water-Flow Tunnel," Technical Note 1975-19, Lincoln Laboratory, M.I.T. (27 May 1975).
- M. L. Burrows and S. H. Prince, "Winding a Long Coil with a Pre-programmed Turns Density Variation," Technical Note 1975-30, Lincoln Laboratory, M.I.T. (27 May 1975).
- 9. M. L. Burrows <u>et al.</u>, "Fabrication of Flexible Loop Antenna," Technical Note 1970-31, Lincoln Laboratory, M.I.T. (5 October 1970) DDC AD-717718.

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