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This report deals with the utilization of bulk cushioning materials in shock mitigation systems. The current techniques used in designing bulk cushioning systems are discussed, and an improved technique is presented through the development of a mathematical model of impact response which is based on the viscoelastic properties of bulk cushioning materials. A General Model of impact response which is applicable to all types of bulk cushioning materials and predicts g-level response in terms of drop height, static stress,

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Block 20 Abstract continued

thickness of cushion, and temperature is developed. A technique for determining the optimal cushioning system design is developed, and examples of the use of the technique are presented for a cross-linked polyethylene foam cushioning material.

CONTENTS

Chapter I

INTRODUCTION

A cushioning system is an interface device which isolates a protected item from the shock loads which develop in its external environment. A cushioning system is required when the protected item cannot survive the shock loads imposed by the external environment, unless these loads are attenuated by some form of cushioning system. The design of a cushioning system, therefore, involves stipulations concerning both the item to be protected and the external environment. These two stipulations of how much shock an item can survive and how much shock it is expected to experience in its environment are questions that must be answered in the design of any cushioning system, commercial or military. It is the concern for extreme external environments that is emphasized most in the military designs.

The Military Environment

A cushioning system designed for military use, such as a shock isolating component of a missile or missile system, is required to perform its basic functions throughout its operating life. Current military policy, based on the need for worldwide deployment, dictates that military materiel be capable of withstanding the rigors of deployment on a worldwide basis. Consequently, military materiel, including cushioning systems, must withstand all the environments to which they are exposed and continue to operate satisfactorily if the materiel is to be considered acceptable.

The environment in which military materiel operates is frequently quite severe and may produce substantial damage. Military operations encompass all the geographic regions and the peculiar aspects of their environments. Each environment introduces hazards peculiar to geographic zones which materiel will encounter when deployed.

The tropics present a hot, humid atmosphere, subjecting materiel to what is commonly referred to as "jungle-rot". Excessive rainfall, high humidity, heat, and fungus, in addition to the prevalence of vermin, insects, and reptiles present problems relating to materiel protection and to the safety of personnel.

The arctic regions subject materiel to a very cold atmosphere. Temperatures of -65°F are common and temperatures of -85°F have been recorded in underground ammunition storage "igloos". The physical characteristics of components, in particular resilient cushioning systems, are radically altered under these conditions and their operational integrity is jeopardized.

Also, high-speed, fast-climbing aircraft introduce rapid and severe variations in temperature and pressure, and provisions must be made to withstand the effects of these changes, when required.

Cushioning Systems in the Military

In evaluating the environmental factors, the military has established certain extreme limits on the particular conditions that have become accepted as a basis for worldwide environments. Temperature extremes, which produce dramatic changes in the resiliency of cushioning materials, are important in cushion system design. The worldwide temperature extremes are generally defined as -65°F and 160°F; cushioning systems in military applications are expected to perform their shock interface function under these adverse environments.

Many of the military cushioning systems use some form of bulk cushioning material (e.g. plastic foam) as the cushioning system. This provides a low-cost, lightweight cushioning system that is easily incorporated into the design; however, designers have some reservations concerning the use of bulk cushioning systems in the military. One limiting factor is the difficulty the designer has in predicting the response of bulk cushioning materials when subjected to the extreme environmental conditions encountered under military deployment. The temperature sensitivity of these cushioning materials, which causes variations in the impact response, is also a primary concern. It was shown in a recent study of thermoplastic foam cushioning systems that temperature had a significant effect on cushion system response [1]. Therefore, temperature effects must be factored into the design of cushion systems using bulk cushioning materials.

To fully utilize bulk cushioning systems, designers require sufficient information to accurately predict response variations. The current practice is to provide the designer with cushioning data for each type and thickness of cushioning material. These data are provided in the form of dynamic cushioning curves as prescribed in the current theory of cushioning design (Chapter II). For any particular shock isolation system design program, the designer is generally given a maximum allowable fragility level which the protected item is permitted to experience. Also the particular organization involved will have an established testing policy defining appropriate impact tests. These parameters provide the basis for the design of the shock mitigation system. Given a satisfactory prediction of impact response for various candidate cushioning materials, a designer can use this prediction to select the appropriate cushioning scheme to meet the particular design requirements.

Research Objective

The research objective of this study is to develop a reliable impact response model that accurately predicts the dynamic cushioning performance of bulk cushioning systems. A secondary objective is to develop an optimization technique that utilizes the model in determining an optimal cushioning system design.

In answering the research objective a systematic study of background material was conducted. The current theory of cushioning design is discussed in Chapter II. The ingredients of the General Model of impact response are identified in Chapter III and the basis of the underlying structure of the developed model is based upon viscoelastic theory.

The modeling process and the analysis techniques used to formulate the General Model of impact response are also discussed in Chapter III. Chapter IV presents the two finalized models, a General Model of impact response and the Minicel Model, the model of impact response of a particular cushioning material (Minicel is a 2 lb/ft^3 cross-linked polyethylene foam manufactured by Hercules, Inc).

The validity of the models are demonstrated in Chapter V through a systematic series of tests and analysis. Finally, in answer to the secondary objective of the research, an optimization technique is presented in Chapter VI that generates the optimal bulk cushion design. The optimization technique utilizes the predictive capability of the General Model of impact response to determine optimality and provide, as output, a set of dynamic cushioning curves at the optimal conditions.

Chapter II

CUSHIONING DESIGN THEORY

Anything that is subject to movement is subject to mechanical damage due to shock. Shock may be defined as a sudden change in direction or velocity of the motion of a body. The magnitude or intensity of shock is expressed in G's, which is defined in terms of the time rate of change of the velocity (acceleration), and is measured in feet/second/second. Mathematically, G's = a/g where a is the acceleration experienced by the body and g is the acceleration due to gravity (32.2 ft/sec/sec).

A given body in a static condition has one gravity unit of G acting on it and, therefore, exerts one G upon its support. If this body is raised and allowed to fall freely, it will accelerate in its fall, due to the force of gravity, until it collides with its support or the earth. The stop causes the body to experience a sudden deceleration that can be expressed in terms of G's. If the body experiences, upon impact, a deceleration of 20 times that of its static condition, it is said to have experienced 20 G's of deceleration. A jet pilot experiences such a condition when he pulls out of a dive. He must not exceed his G limit or he will black out, and if the aircraft is not designed and stressed to withstand high G's during such maneuvers, severe damage will occur to the aircraft and it may crash. This same situation exists in regard to fragile objects. The fragility level of an object, measured in G's, is its ability to withstand deceleration. The purpose of a cushioning system is to reduce the shock transferred to the protected object to a degree below its fragility level, thus protecting it against physical damage.

Cushioning System Design

To understand how a cushioning material functions during shock transfer, one can consider what happens when a body is dropped onto a rigid surface such as a concrete floor. At impact the body is falling at a velocity ($V = \sqrt{2\ gh}$), where V is the velocity at impact in feet/second, g is the acceleration due to gravity of 32.2 feet/second/second, and h is the height of drop measured in feet. In a very short time after impact, this velocity is reduced to zero in a very small distance. Thus, a rapid decrease in velocity occurs, due to impact, and the body is subjected to a very high deceleration.

If the same situation is considered except that the body is dropped onto a resilient cushioning material which rests on the same rigid surface, the body has the same velocity at impact. However, the time required for the velocity to be reduced to zero is much greater than in the previous situation; the rate at which the velocity decreases is considerably less; and the distance traveled afte initial contact with the cushion until the time the velocity is reduced to zero is considerably greater. Compared to the first situation, the body is subjected to lower G's. The resilient cushioning material has, therefore, attenuated the shock pulse by dissipating the kinetic energy present in the body at the time of impact over a longer time period. This has been expressed mathematically [2] as follows:

$$G = \frac{72}{t}\sqrt{h} \qquad \text{(II-1)}$$

where

G = acceleration G - level

t = shock pulse rise time in milliseconds

h = drop height in inches.

Equation (II-1) shows that as the shock pulse rise time is increased, the G's experienced by the body are proportionally decreased.

Increases in shock pulse rise time can usually be accomplished by increasing the thickness of the resilient cushioning material. For example, the G's experienced by a body falling on a tangentially elastic cushioning material can be predicted as follows [2]

$$G = \frac{3.9\ h}{T} \qquad \text{(II-2)}$$

where T = thickness of cushion in inches.

Equation (II-2) shows that the predicted G levels are reduced proportionally with increases in the thickness of the cushioning material. If the cushion thickness is increased, then, during impact, the excursion envelope of the body is increased proportionally and the body moves through an additional amount of cushioning material before it comes to rest. This increase in excursion directly increases the shock pulse rise time with an accompanying reduction in G levels. This reduction is consistent with the change in G levels that would be predicted by Equation (II-1) with an increase in shock pulse rise time.

History of Cushion System Design

Equation (II-2) is based on Mindlin's work in 1945 [3] that marked the beginning of the scientific approach to cushion design. Mendlin's paper was followed by a number of discussions [4-8] by author's who adopted his procedures. The next substantial step forward was the development of optimum efficiency design points by Janssen [9]. Janssen utilized material properties determined by quasistatic means, in particular, stress-strain curves determined on a conventional compression tester with the speed of compression quite slow (not more than 2 inches/minute). He derived a cushion factor, "J", for several cushioning materials that was the ratio of optimum stress to optimum strain. Then G-level can be predicted on the basis of

$$G = J\,\frac{h}{T} \qquad \text{(II-3)}$$

where J = Janssen's cushion factor.

This was a significant improvement over Equation (II-2) in that the J values allowed for different performance factors for different materials. However, it soon became apparent that single curves based on the static stress-strain characteristics of a material did not describe the dynamic behavior. Several methods of presentation, all involving families of curves, were attempted. Gradually, Kerstner's approach [10] became the most popular. In this approach, a family of curves is drawn for each material thickness at each drop height, plotting peak dynamic stress (or acceleration) against the original static stress. A typical set of such curves, taken from Humbert and Hanlon [11], is shown in Figure 1.

These curves, referred to as dynamic cushioning curves, are generated for a particular type and thickness of cushion by performing drop tests using standard weight specimens that are dropped onto the cushion.

The static stress (σ_s) is determined by:

$$\sigma_s = \frac{W}{A} \quad , \qquad \text{(II-4)}$$

where σ_s is the static stress (psi), W is the specimen weight, and A is

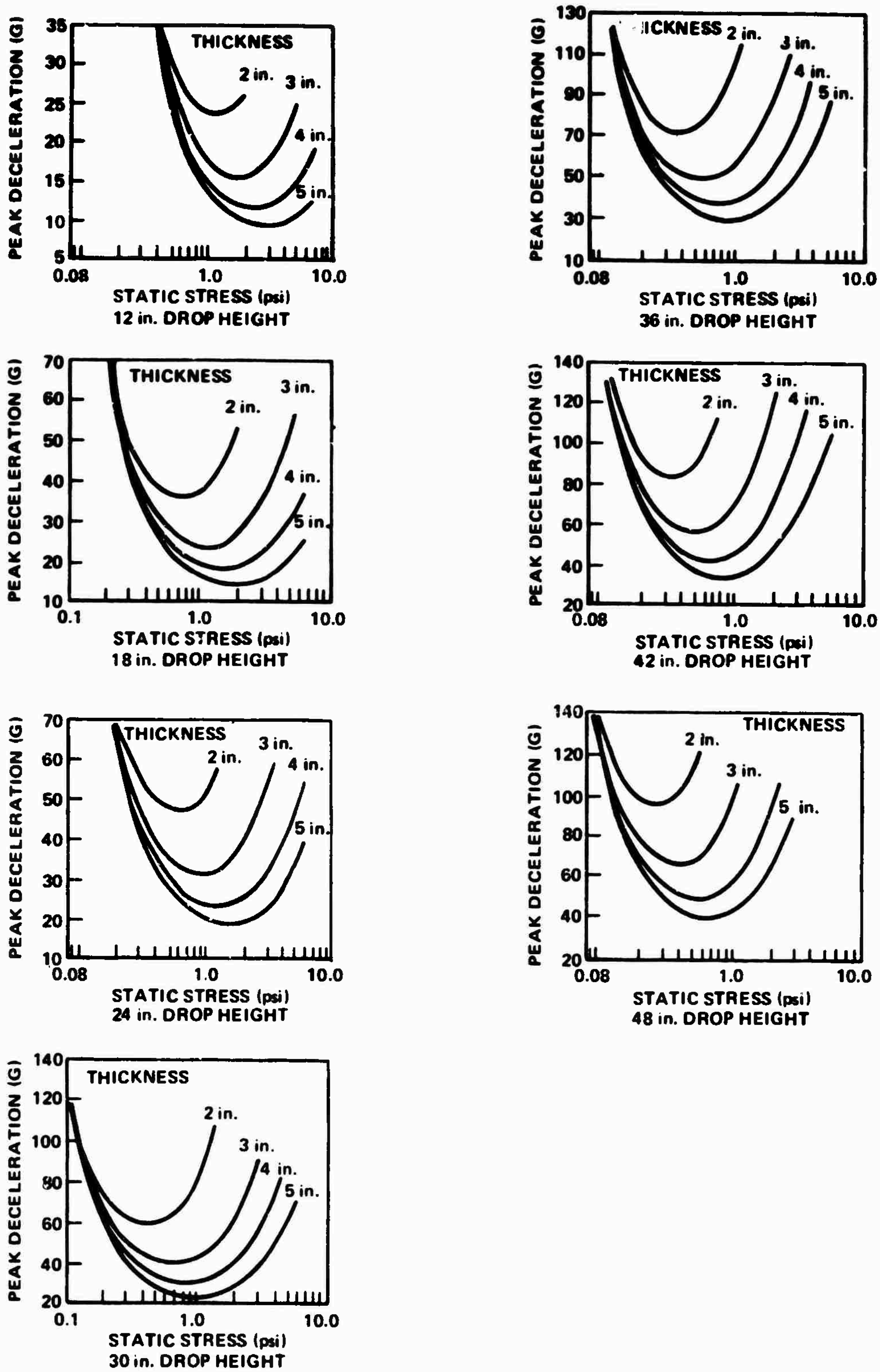


Figure 1. Peak-acceleration versus static-stress curves for polyethylene foam [11].

the footprint of the specimen in the cushion (in.^{2}). A different curve is required for each drop height, thickness, and type of material. The curves are a good indication of the protection to be expected for a particular cushion scheme.

The current practice utilizes the Kerstner procedure in that manufacturers of cushioning products demonstrate the cushioning performance capabilities of their particular products by generating a series of dynamic cushioning curves at various drop heights and thicknesses of cushions (Figure 1). Also, Military Handbook -304, "Cushioning Design Handbook", was published in 1964 which provides families of dynamic cushioning curves for a large number of frequently utilized cushioning materials. Additional work was done by Mustin [12] who used a single value for correlating each dynamic cushioning curve in a family of curves such as Figure 1. This proved to be an over simplification in developing a general model of impact response, as will be seen in Chapter IV.

Reservations Concerning Current Cushioning System Design

In recent years, equipment designers have become increasingly aware of the detrimental effects of extreme temperature upon equipment performance. Consequently, there are now included in the qualification tests of equipment, some tests conducted at temperatures that are representative of the temperature extremes that the equipment is likely to encounter.

This extreme temperature testing has received substantial attention in the military, where the range of temperatures encountered is quite extreme, and the failures can produce catastrophic results. The transportation of military equipment is one area of concern and a study was made of several military containers that used bulk cushioning systems. It was found that temperature appeared to have a significant effect on impact response [1]. The cushioning systems in the containers did not perform properly due to changes in the performance of the bulk cushioning materials that were induced by extreme temperatures. Consequently, the items packaged in the containers (guided missile systems and system components) did not receive adequate protection, and the missile system reliability was compromised. This type of failure is a potential problem that can occur in very expensive equipment and produce a malfunction in weapon systems that compromises the combat power of a military organization. Also, if proper failsafe provisions are not incorporated into the protection of ordinance items, the safety of any of the personnel that handle the equipment within the logistics system is jeopardized.

Rather than incorporate additional protection into equipment, it is much more cost effective to improve the reliability of cushioning systems. This can be done if a reliable method of predicting cushioning performance can be developed.

A technique has been suggested for modifying the conventional dynamic cushioning curves in such a manner as to address the effect of temperature on the impact response of bulk cushioning systems [1]. The technique utilizes superimposed dynamic cushioning curves that are a super-positioning of the dynamic cushioning curves at temperature extremes upon the ambient dynamic cushioning curve. One such curve is presented in Figure 2. This type of curve demonstrates the effect of temperature for a selected set of conditions and provides the cushioning system designer with the capability of designing cushioning systems with a reduced likelihood of design failure under extreme temperature conditions.

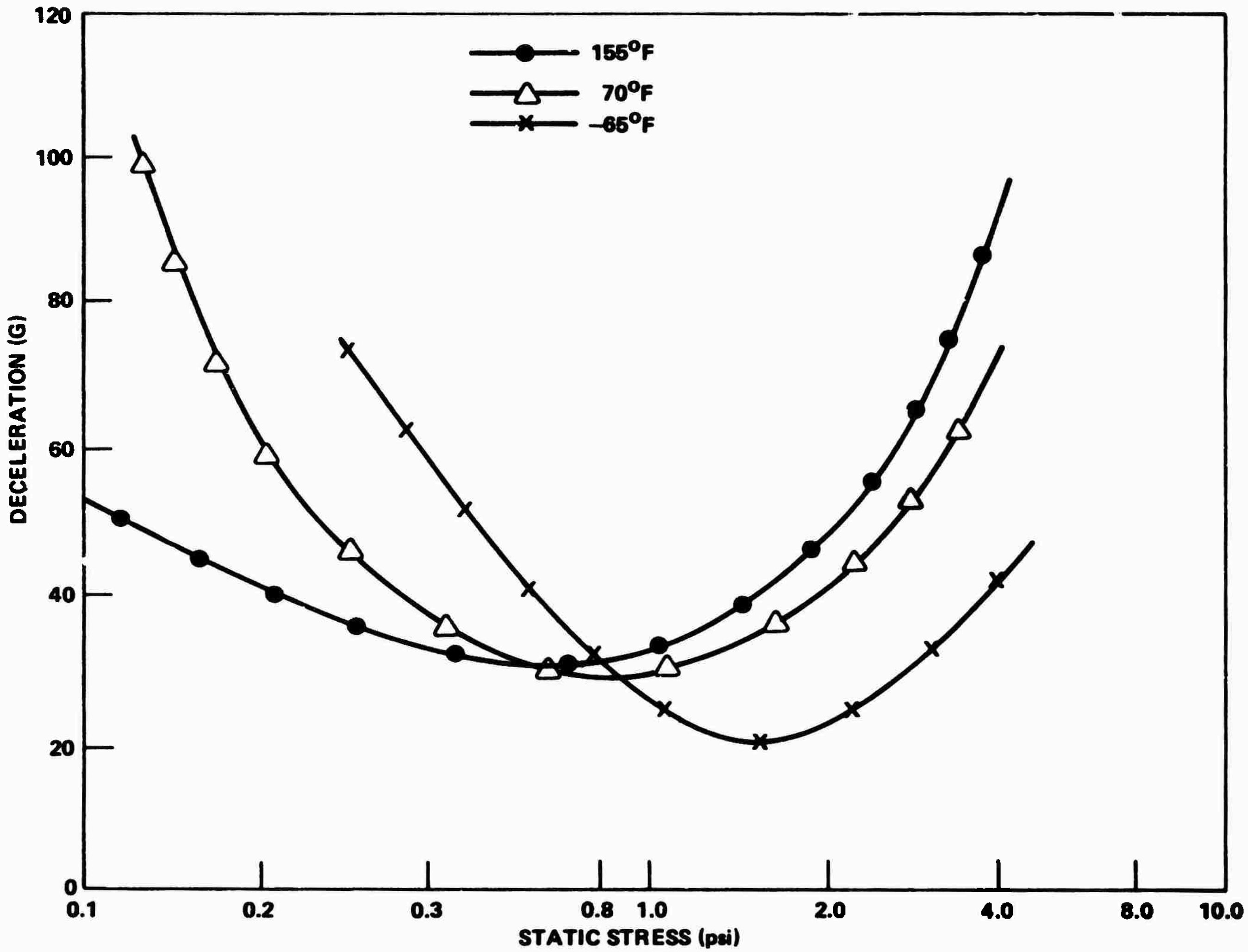


Figure 2. Superimposed dynamic cushioning curve for polyethylene foam (2 lb/ft^3 density, -65°, 70°, and 155°F, 30-in. drop height, 4-in. thickness).

Chapter III

BASIS FOR THE IMPACT RESPONSE MODEL

A valid model of impact response must incorporate all parameters that are expected to have a significant effect on impact response. Temperature has a significant effect on impact response [1], and it is postulated that viscoelastic theory can be utilized to formulate a model of impact response that incorporates temperature effects. The current design practice for predicting impact response is predicated on dynamic cushioning curves. Dynamic cushioning curves do not account for temperature effects on impact response. To improve the predictability of cushioning systems, a model of impact response must account for temperature effects. The temperature effects would be expected to be the most dramatic as the temperature tends towards the extremes; therefore, the temperature extremes encountered by a cushioning system are of particular concern in modeling impact response.

Temperature Extremes in the Model

Army Regulation 70-38, "Research, Development, Test and Evaluation of Materiel for Extreme Climatic Conditions," requires that the extreme external environments that are likely to be encountered, be considered in the design of Army materiel. The temperature extremes to be used in the design are defined in the AR according to the intended deployment of the materiel being designed. Figures 3 and 4 (from AR 70-38) present maps of the extreme temperature conditions to be used in the

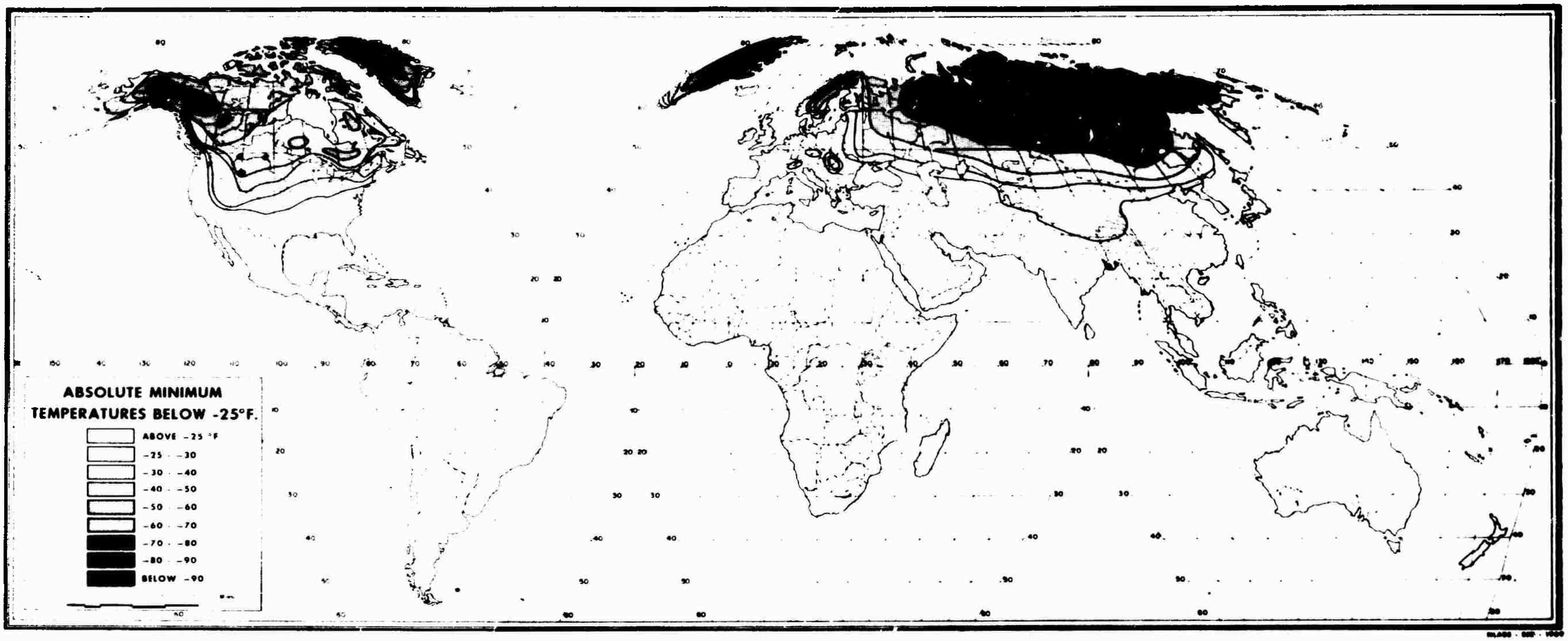


Figure 3. Distribution of absolute minimum temperatures.

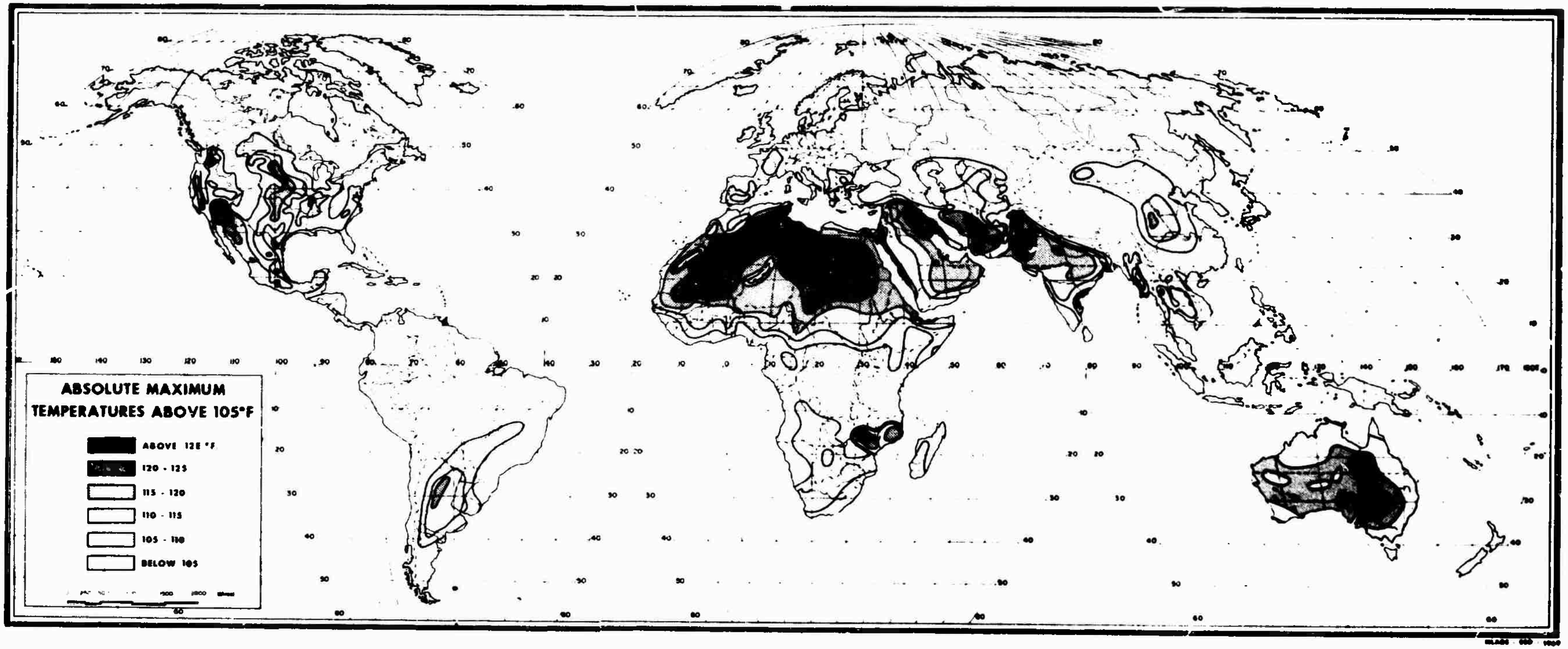


Figure 4. Distribution of absolute maximum temperatures.

design of Army materiel. For example, for worldwide deployment, the operational temperature range is given as -65° to 160°F. However, for continental United States deployment only, the range is from -30° to 145°F. Army materiel is expected to function properly within these ranges and consequently the qualification testing of Army materiel is conducted at these extremes.

A research effort was initiated at the Army Missile Command (MICOM) in 1973 to develop superimposed dynamic cushioning curves that address the effect of temperature on the impact response of cushioning materials. The drop tests were conducted at MICOM and the results analyzed by the University of Alabama in Huntsville (UAH) under a supporting research contract [13]. The initial experimentation was conducted on Minicel material, a cross-linked polyethylene foam material with a 2 pound/foot^3 density manufactured by Hercules, Inc.

The drop test program that was conducted on the Minicel material used drop heights of 12, 18, 24, and 30 inches; temperatures of -65°, 70°, and 160°F; cushion thicknesses of 1, 2, and 3 inches; and static stress levels that varied from 0.04 to 5.0 psi. The G-level response and shock pulse duration were recorded for each of 2736 drop tests. An automated data handling system that included outlier tests and other statistical analyses was developed and used to analyze the Minicel data.

This experimental effort resulted in the generation of a data base of 2409 statistically valid data points. A family of second order polynomial equations was found to be the best predictor of impact response of the Minicel cushioning material. The data are given in Appendix A together with the families of regression polynomials for the 12 inch and 24 inch drop heights, and the correlation coefficients.

Theoretical Basis of the Model

The construction of an impact response model for cushioning materials at varying temperatures requires the development of a functional relationship of the variables. The required relationship can be expressed mathematically as follows:

$$G = F(\sigma_s, T, \theta, h) \qquad \text{(III-1)}$$

where

G = acceleration G-level

σ_s = static stress in psi

T = thickness of cushion inches

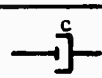
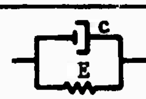
θ = cushion temperature in °F

h = drop height in inches.

After considerable research, it was determined that the theory of viscoelasticity could be utilized to provide a theoretical basis for the model. Viscoelastic theory recognizes cushioning materials as belonging to a class of materials which have mechanical properties that are common to perfect solids and perfect liquids. Various theories have been developed over the past century for describing the behavior of perfect solids and perfect liquids. Among these, the oldest theories are the classical theory of elasticity and the theory of hydrodynamics. The classical theory of elasticity deals with the behavior of solids for which the stress is directly proportional to the strain but is independent of rate of strain. This type of solid is known as a Hookian solid (perfect elastic solid). The theory of hydrodynamics describes the behavior of perfect viscous liquids for which, in accordance with Newton's viscosity law, the stress is directly proportional to the rate of strain, rather than the strain itself. In certain instances, solids and liquids may have their stress related to strain, rate of strain, and higher time derivatives of strain. Behavior of such materials is termed viscoelastic when the stress is linearly proportional to strain. Materials whose behavior is viscoelastic display solid-like and liquid-like characteristics [14].

Mathematical models have been formulated for viscoelastic materials that have validity over both short and long time periods; for example, Mustin [12] gives the creep and relaxation functions (long term behavior) for a number of simple mathematical models made up of simple spring elements that are assumed linear and massless, and of dashpots in which a piston is moving through a liquid that obeys Newton's law of viscosity (velocity is proportional to strain). The short term stress law and the long term creep and relaxation functions are given in Table I. Creep

TABLE I. SOME CREEP AND RELAXATION FUNCTIONS

Function	Linear	Dashpot	Voigt Solid	Maxwell Solid
Pictorial representation				
Stress law	$\sigma = E\epsilon$	$\sigma = c\,\frac{d\epsilon}{dt}$	$\sigma = E\epsilon + c\,\frac{d\epsilon}{dt}$	$\frac{\sigma}{c} + \frac{1}{E}\frac{d\sigma}{dt} = \frac{d\epsilon}{dt}$
Creep function	$\frac{1}{E} \cdot H(t)$	$\frac{t}{c} \cdot H(t)$	$\frac{1}{E}(1 - e^{-Et/c}) \cdot H(t)$	$\left(\frac{1}{E} + \frac{t}{c}\right) \cdot H(t)$
Relaxation function, $R(t)$	$E \cdot H(t)$	$c \cdot \delta(t)$	$E \cdot H(t) + c\,\delta(t)$	$(Ee^{-Et/c}) \cdot H(t)$

Notes:
E = spring constant
σ = stress
ϵ = strain
c = damping coefficient
$H(t)$ = unit step function
$\delta(t)$ = Dirac delta function

functions are shown in Figure 5 while Figure 6 illustrates the relaxation behavior. The appearance of the relaxation functions indicates that the stress is infinite at the instant that strain occurs. This is due to the dashpot element which, unlike a spring, cannot give a finite instantaneous strain response to a finite instantaneous force change. Therefore, an infinite force is required to produce a finite instantaneous strain.

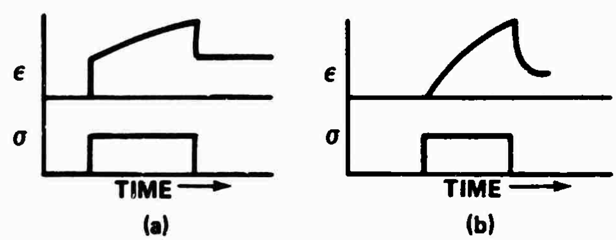


Figure 5. Behavior of some simple creep functions: (a) Maxwell solid, and (b) Voigt solid.

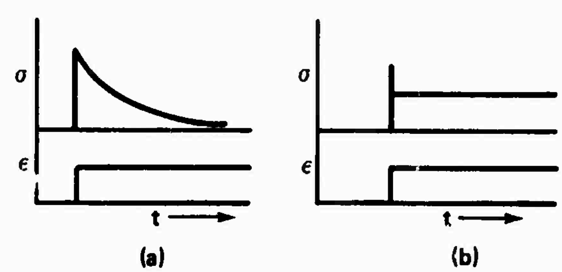


Figure 6. Behavior of some simple relaxation functions (R(t)): (a) Maxwell solid, and (b) Voigt solid.

These relationships are expressed compositely in constitutive equations which, in viscoelastic theory, describe the response of materials to mechanical excitation. The constitutive equation in conjunction with the energy, momentum, and continuity equations permits the prediction of complicated responses to complicated excitations, like the flow of liquids in irregularly shaped channels or the deflections of beams of varying cross sections under complicated loading conditions.

There are two approaches to obtaining the constitutive equation of a material: it can be obtained either experimentally with the help of some simple, well-defined tests (like the stress-strain curve) or the response of the material to some strain or stress history can be calculated with the help of a model describing its structure. Hooke's law is an example for the first group, the phenomenological equations. It is based solely on experimental observation. The best known example in the latter category of structural theories is the kinetic theory of rubber elasticity predicting the elastic response of vulcanized elastomers from their structure.

While the phenomenological constitutive equations describe the results of experimentally obtained data and are usually applied to predict more complicated behavior, the structural ones offer an insight into structure-property relations [15].

Since temperature effects are required in the constitutive equation that models impact response, one must consider that portion of viscoelastic theory which accounts for the behavior of materials at varying temperatures. Most of this theory arises from the consideration of the behavior of the molecule. The molecule may be visualized as a long curled elastic chain which may have cross-linking with other molecular chains. Since molecular activity is a function of temperature, strain response (the summation of individual molecule responses) is also a function of temperature. On this basis, in the glassy zone (the zone below the temperature at which the polymer structure starts to become brittle), molecular "curling and uncurling" cannot occur rapidly enough to follow the stress. In the rubbery zone (the zone above the temperature at which the polymer structure becomes rubbery), curling and uncurling are in phase with the stress which is not conducive to energy dissipation.

Let the long term elasticity of a material at a given temperature, θ_1 , be plotted against the natural logarithm of time. Suppose, now, that the curve shifts horizontally to the right as the temperature is lowered. This situation is illustrated in Figure 7 for three temperatures.

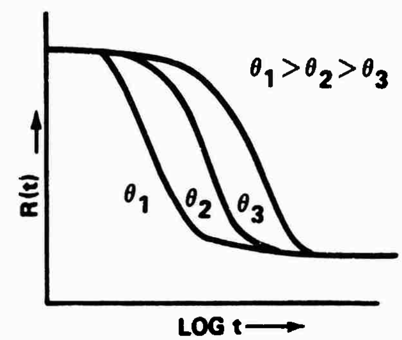


Figure 7. Shift in elasticity characteristics as a function of time and temperature.

The behavior sketched in Figure 7 is typical of many materials utilized as cushions. Materials which behave in this manner are called thermorheologically simple. Amorphous polymers behave in this manner while polymers with crystalline structures do not. Most cushioning materials may be considered amorphous. Teflon and plasticized polyvinyl chloride are not amorphous, but they are rarely used as cushions [16].

Viscoelastic Considerations

A constitutive equation that relates the maximum G-level experienced by a body when subjected to a deceleration impact into a cushioning material is required for the impact response model stated in Equation (III-1). A partial basis is found in a paper on the viscoelastic properties of thermorheologically simple cushioning materials by Cost [17]. Cost considers a viscoelastic Kelvin body which is identical to the Voight model shown in Table I. The stress laws are identical with Figure 7 except for notation, in that Cost uses K and η for E and c, respectively.

Consider a body of mass M falling under the influence of gravity from a height h and impacting a cushion material of area A_c and thickness T. The static stress σ_s is defined as the weight of the body divided by the area of the cushion A_c.

Assume the cushion material behaves as a viscoelastic Kelvin body whose stress strain relation can be expressed as

$$\sigma = K\epsilon + \eta\dot{\epsilon} \qquad \text{(III-2)}$$

where η and K are material properties. When Equation (III-2) is applied to the problem under consideration, σ becomes the static stress in a bulk cushion, σ_s, and ϵ, the unit strain, can be expressed in terms of the displacement of the upper surface of the cushion as $\epsilon = x/T$. If the principles of Newtonian mechanics are applied to the falling body after time of impact where force equals mass times acceleration, then the equation of motion for the body can be expressed as

$$M\ddot{x} = -\sigma_s A_c \qquad \text{(III-3)}$$

Upon substitution of Equation (III-2) into Equation (III-3)

$$M\ddot{x} = -\frac{A_c K}{T}\, x - \frac{A_c \eta}{T}\, \dot{x} \qquad \text{(III-4)}$$

where x is measured relative to the initial location of the upper surface of the cushion and is considered positive downward. Rearranging the equation of motion gives

$$\ddot{x} + \frac{A_c \eta}{MT} \dot{x} + \frac{A_c K}{MT} x = 0 \quad . \qquad \text{(II -5)}$$

Equation (III-5) can be cast in the following canonical form:

$$\ddot{x} + 2\beta_c \omega_n \dot{x} + \omega_n^2 x = 0 \qquad \text{(III-6)}$$

provided β_c and ω_n are defined as follows:

$$\beta_c = \frac{\eta g}{2\sigma_s T \omega_n} = \frac{\eta A_c}{2MT \omega_n}$$

$$\omega_n = \sqrt{\frac{gK}{\sigma_s T}} = \sqrt{\frac{A_c K}{MT}} \quad . \qquad \text{(III-7)}$$

The parameters β_c and ω_n have physical significance as being the damping factor and natural frequency of the undamped system, respectively. For the case of "underdamped" motion, the solution of Equation (III-6) can be expressed as

$$x(t) = e^{-\beta_c \omega_n t} [B_1 \cos \mu t + B_2 \sin \mu t] \qquad \text{(III-8)}$$

where

$$\mu = \omega_n \sqrt{1 - \beta_c^2} \qquad \text{(III-9)}$$

and B_1 and B_2 are arbitrary constants which can be determined from the initial conditions. Applying the initial conditions

$$x(0) = 0 \quad ,$$

$$\dot{x}(0) = \sqrt{2gh} \qquad \text{(III-10)}$$

to evaluate the two constants B_1 and B_2 gives

$$B_1 = 0 \quad ,$$

$$B_2 = \frac{\sqrt{2gh}}{\omega_n \sqrt{1 - \beta_c^2}} \quad . \qquad \text{(III-11)}$$

Therefore, the acceleration history can be expressed as

$$\ddot{x}(t) = \frac{\omega_n \sqrt{2gh}}{\sqrt{1 - \beta_c^2}} e^{-\beta_c \omega_n t} \left[(2\beta_c^2 - 1) \sin \omega t - 2\beta_c \sqrt{1 - \beta_c^2} \cos \omega t \right] . \qquad \text{(III-12)}$$

In this research, only the maximum value of the acceleration is of interest. Consequently, it is necessary to separate the solution into oscillatory and decaying components. To accomplish this, a trigonometric identity must be introduced into Equation (III-12) to obtain the following expression:

$$\ddot{x}(t) = -\frac{\omega_n \sqrt{2gh}}{\sqrt{1 - \beta_c^2}} e^{-\beta_c \omega_n t} \cos \left[\omega_n \sqrt{1 - \beta_c^2}\, t + \gamma \right] \qquad \text{(III-13)}$$

where

$$\gamma = \tan^{-1} \left[\frac{2\beta_c^2 - 1}{2\beta_c \sqrt{1 - \beta_c^2}} \right] . \qquad \text{(III-14)}$$

Equation (III-13) expresses the acceleration of the falling mass as a function of time. It is observed that the character of the response is that of a damped sinusoid with the exponential and trigonometric components of the solution governing the transient behavior. Since the magnitude of the exponential or the trigonometric term can achieve a maximum value of one, the coefficient of these terms governs the absolute magnitude of the acceleration. Thus, in regard to the form of the equation, it can be seen that

$$\ddot{x}_{max} \approx \frac{\omega_n \sqrt{2gh}}{\sqrt{1 - \beta_c^2}} . \qquad \text{(III-15)}$$

Recalling the expansion

$$(1 - Z^2)^{-1/2} = 1 + \frac{1}{2} Z^2 + \frac{3}{8} Z^4 + \ldots \quad , \qquad \text{(III-16)}$$

the peak acceleration can be expressed as

$$\ddot{x}_{max} \approx \omega_n \sqrt{2gh} \left(1 + \frac{1}{2} \beta_c^2\right) \qquad \text{(III-17)}$$

where only terms up through the second order in β_c have been retained. Returning to the primary variables σ_s, T, h, and θ, and Equation (III-7) gives

$$\ddot{x}_{max} \approx g \left(\frac{hK}{\sigma_s T}\right)^{1/2} \left[1 + \frac{1}{8} \frac{\eta^2 g}{\sigma_s T K}\right] \quad . \qquad \text{(III-18)}$$

Inserting arbitrary constants instead of the specific numerical coefficients gives

$$\ddot{x}_{max} \approx C_1 \frac{h^{1/2}}{\sigma_s^{1/2} T^{1/2}} + C_2 \frac{h^{1/2}}{\sigma_s^{3/2} T^{3/2}} \eta^2 \qquad \text{(III-19)}$$

where η is a material property dependent on temperature.

Assuming thermorheologically simple behavior for the viscoelastic material, η can be expressed as

$$\eta(\theta) = \eta_0 \, a(\theta) \qquad \text{(III-20)}$$

where $a(\theta)$ is the "shift factor". Furthermore, $a(\theta)$ has been shown by experience [16] to be expressible as

$$a(\theta) = C_1' \, e^{-C_2' (\theta - \theta_0)} = C_1'' \, e^{-C_2' \theta} \quad . \qquad \text{(III-21)}$$

Substituting into Equation (III-20) gives

$$\eta(\theta) = \eta_0 \, C_1'' \, e^{-C_2' \theta} \qquad \text{(III-22)}$$

and

$$\eta^2(\theta) = C_3 \, e^{-C_4 \theta} \qquad \text{(III-23)}$$

which can be expanded in a Taylor series to give

$$\eta^2(\theta) = C_3 \left[1 - C_4\theta + C_5\theta^2\right] \qquad \text{(III-24)}$$

where only terms up through the second order have been retained. Upon substitution of this expression into Equation (III-19), we get

$$\ddot{x}_{max} \approx K_1 \frac{h^{1/2}}{\sigma_s^{1/2}\, T^{1/2}} + K_2 \frac{h^{1/2}}{\sigma_s^{3/2}\, T^{3/2}} + K_3 \frac{h^{1/2}}{\sigma_s^{3/2}\, T^{3/2}}\, \theta$$

$$+ K_4 \frac{h^{1/2}}{\sigma_s^{3/2}\, T^{3/2}}\, \theta^2 \qquad \text{(III-25)}$$

as an expression relating the variables σ_s, T, h, and θ to the peak acceleration.

If additional terms are retained, a more general expression will result. Such a general expression is

$$\ddot{x}_{max} \approx C_0 \left(\frac{h}{\sigma_s\, T}\right)^{1/2} + \sum_{i=1}^{N} \frac{C_i h^{1/2}}{(\sigma_s\, T)^{i+1/2}} \left[\sum_{j=1}^{M} K_{ij}\ \theta^j\right] \text{(III-26)}$$

where the C_i (i = 0, 1, 2, ..., N) and K_{ij} (j = 1, 2, ..., M) are constants to be determined by curve fitting procedures.

Expressions of the form given in Equations (III-25) and (III-26) appear likely candidates for empirically curve-fitting cushioning system experimental data. This concludes Cost's derivation.

Verification

Once a candidate relationship was selected, it was verified. A statistical comparison of the G-level predicted by the function to the data base listed in Appendix A was used to verify the model.

This was accomplished by modifying the Stepwise Regression Procedure given by Draper and Smith [18]. In the Draper and Smith version, variables are entered and removed from the regression at each stage on the basis of F-tests performed on the coefficients of the independent variables. This procedure was modified in this research in that the variables were entered into the regression equation and remain there as long as they make a contribution to the overall correlation. This modification was considered most appropriate during the preliminary phases of the modeling. It precludes the removal of a variable from the regression equation at one stage because other variables can explain most of the variation, and then find in a later stage that the variable was needed but was not available. In the Draper and Smith version, the establishment of critical F values can inadvertently cause the removal of a desirable variable.

It is considered most important in the early stages of model development that all variables that can possibly be retained remain in solution. Once they are discarded as irrelevant, it is difficult to reincorporate them and there is a substantial risk that they will be lost. However, the modification can cause the stages in the solution to carry superfluous variables through several iterations and ultimately retain them in the final solution. This does require some small amount of additional computer time. However, these unnecessary variables are easily identified and can be discarded during the fine tuning of the final solution with little risk to the validity of the model. If variables are discarded prematurely, which could happen in the Draper and Smith version of the regression analysis, the validity of the model may be compromised.

A program listing of the Stepwise Regression Procedure that was utilized, called MLRD, is given in Appendix B. One form of output from the program is printer plots in the form of dynamic cushioning curves. These are a nesting of various thicknesses of a particular cushioning at a particular drop height and temperature. The curves are plotted using numbers corresponding to the thickness of cushion. The dynamic cushioning curves that were selected from the UAH study are displayed in Appendix C using the MLRD format. The initial validation exercise for the modeling effort, then, consisted primarily of displaying the curve shape and nesting of the model being validated to determine if the model generated dynamic cushioning curves that compared favorably with those in the UAH study.

Theoretical Validity

The required form of the model as given in Equation (III-1) can be identified in viscoelastic theory as a phenomenological constitutive equation. The model proposed by Cost was selected as the most viable candidate, where $\ddot{x}_{max}$ is the maximum G-level experienced by a body during impact into a cushion, and C_0, C_1, K_1, K_2, etc. are regression constants. This equation was used as the basic underlying structure of the model because it provides a functional relationship of all the variables required in Equation (III-1) and has a theoretical basis in viscoelastic theory. There was one transformation of variables made prior to using the equation. It can be reasoned that when θ is a significant aspect of the model, then for θ^j, where an exponent j is added, the polarity of θ^j will oscillate. To avoid this difficulty, θ was transformed to °R, so the extreme temperatures encountered in the data are always positive.

Validity of Initial Trials

A program code of Equation (III-26) was run on MLRD and the MLRD plot (Figure 8) shows that $\ddot{x}$ was a progressively decreasing value as σ_s increased. This was anticipated to an extent by Cost during his finite element analysis [17]. He generated a plot of peak acceleration versus static stress in the region where stress is proportional to strain (Figure 9).

This deficiency in a constitutive equation is not at all uncommon as Meinecke [15] suggests that even though the behavior of actual materials is usually different from that predicted by various classical theories, for engineering purposes it may be worthwhile to approximate the actual behavior to the idealized behavior. But for cases where it is not possible to approximate the actual behavior of the material to the idealized behavior without sacrificing the accuracy in prediction, it is very essential to consider the anomalies from the idealized character. The departure of actual material from its idealized character may be due to various reasons. For instance, the deviation from the idealized character may be such that the stress, instead of being directly proportional to the strain, is related to it in a complicated manner. Solids display this behavior when they are stressed beyond the so called elastic range or when the deformation becomes so large as to introduce nonlinearity between stress and displacements. Likewise, liquids for which the stress is nonlinearly related to rate of strain are termed non-Newtonian liquids. Equation (III-26) is a good representation of the linearity of impact response but a nonlinearity must be introduced as an initial correction for shape to get a U shaped curve.

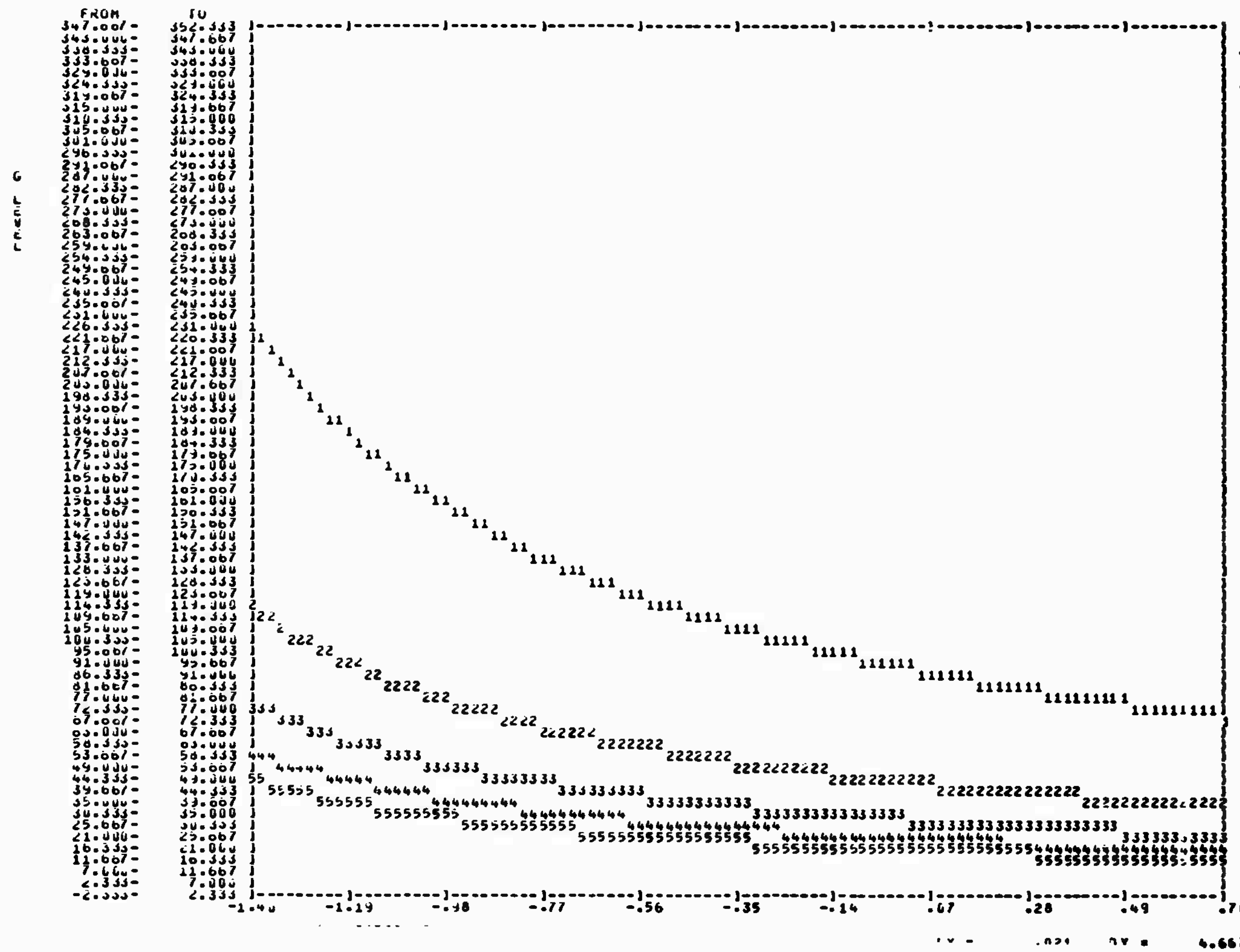


Figure 8. Preliminary MLRD plots of dynamic cushioning curves of the response model at 70°F and 12-inch drop height.

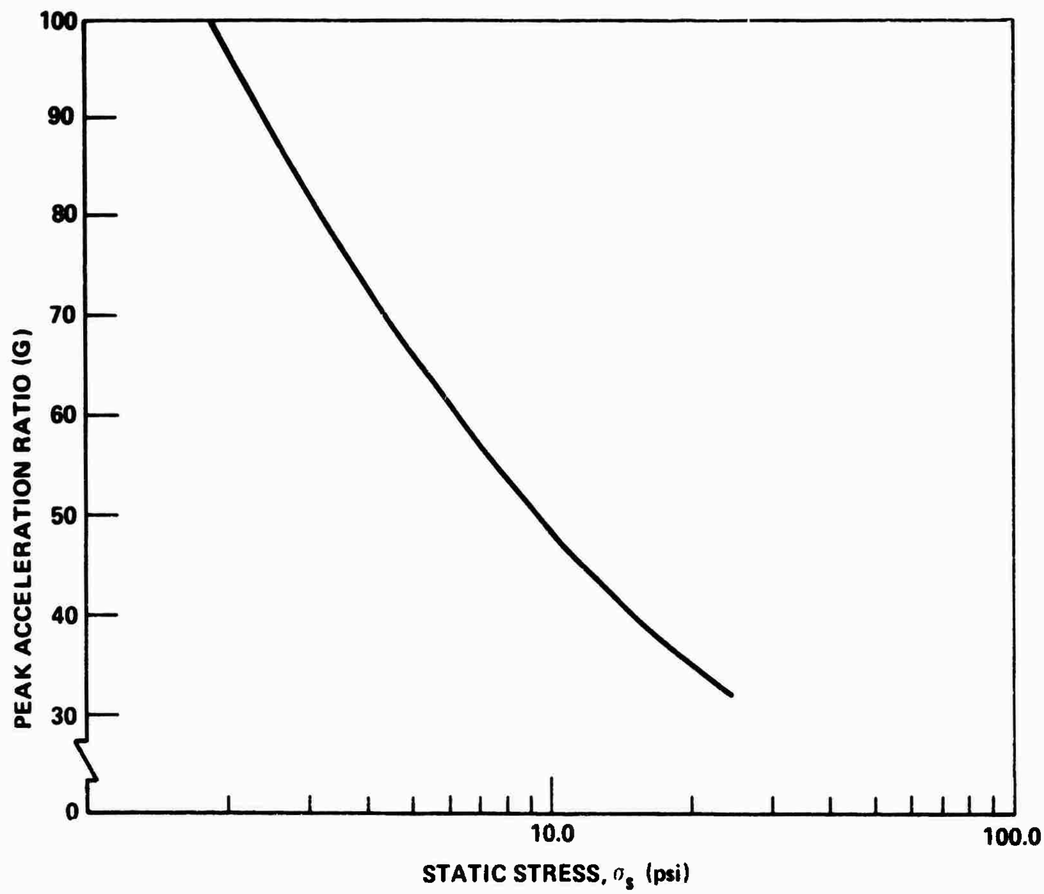


Figure 9. Finite element solution results for peak acceleration.

Initial Correction for Shape

All known dynamic cushioning curves exhibit a characteristic U shape. Consider a compression spring approximately 6 inches in diameter and a free height of 6 inches, similar to those used in the front suspension of an automobile. On this spring a rigid steel plate that serves as an impacting surface is placed. Then, a safety pin, an automobile, and a locomotive are dropped, in turn, onto the spring from approximately 6 inches.

Now the results of the drop test are examined. The compression spring is much too stiff for the small weight of the safety pin, so the spring does not deflect to store the energy of the fall. This rapid deceleration of the pin causes it to experience high G-levels. Because the compression spring was taken from an automobile, the spring deflects with the fall of the automobile. Energy is stored by the spring and the load on the automobile is proportional to the stiffness of the spring and its deflection. For the locomotive, due to the extremely heavy mass, the spring deflects until the coils of the springs have compressed completely. Up to this point the deceleration has progressed slowly with no high G-levels. However, from this point the locomotive will be decelerated to zero velocity almost immediately and will experience high G-levels.

Thus, for a given stiffness and free height of the spring, the appropriate weight dropped from a given height will result in the maximum energy storage in the spring and a minimum G-level experienced by the weight. The U shapes of dynamic cushioning curves, such as those in Figure 1, are the result of this property of bottoming of bulk cushions. The optimal conditions exist at the ogive of the curve as indicated in Figure 10. At this point the maximum energy is stored in the cushion with the accompanying minimum G level.

A simplified model of a foam material such as the one proposed by Gent and Thomas [19] (Figure 11) is helpful in visualizing the transitional aspects of why bottoming occurs. The foam consist of thin threads joined together to form a cubical lattice. Mechlin [20] explains the bottoming effect as the result of the ligaments of the cushioning material structure packing together, one against another.

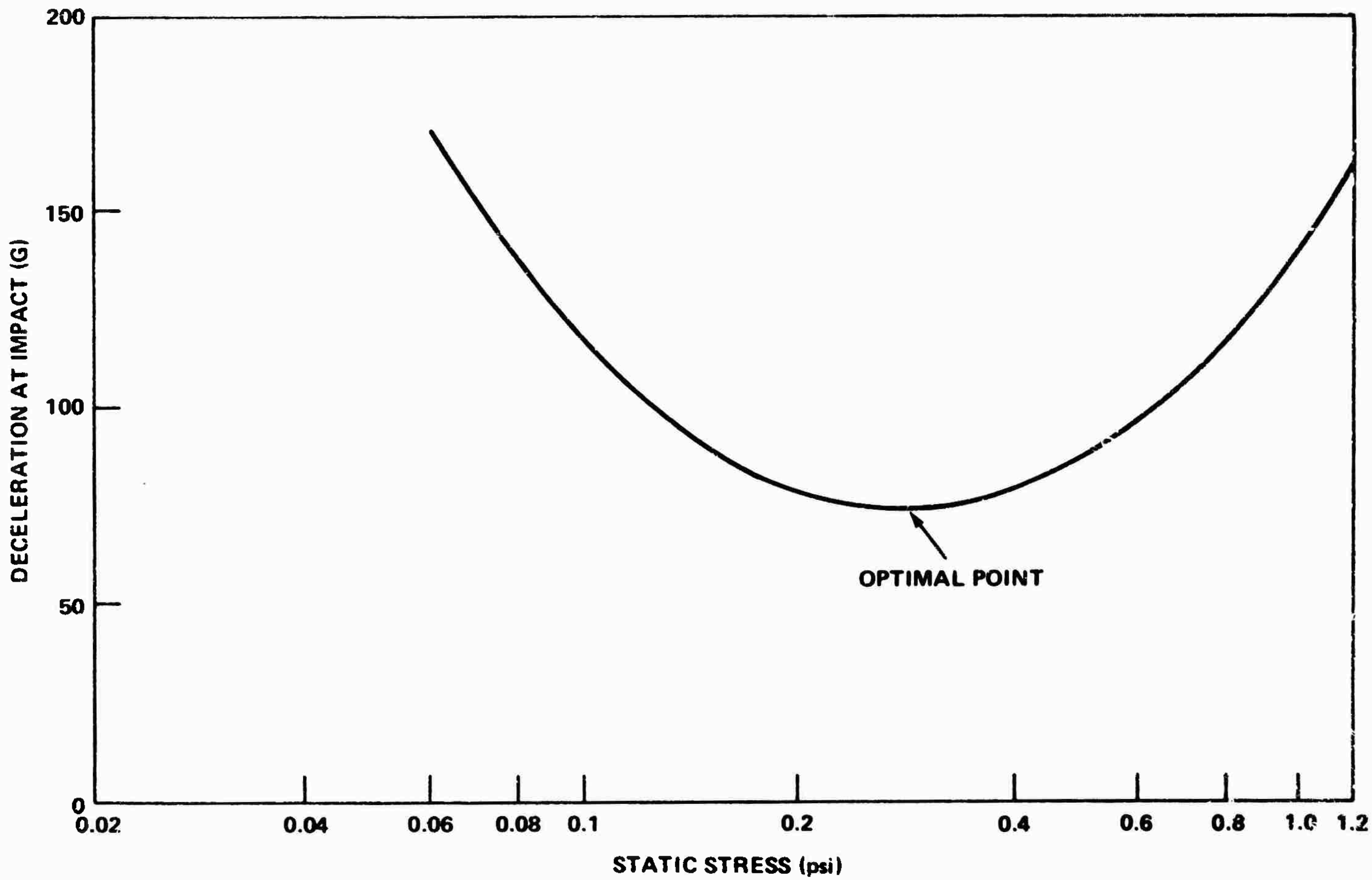


Figure 10. Typical dynamic cushioning curve (Minicel, 30-in. drop height, 70°F, 1 in. thickness).

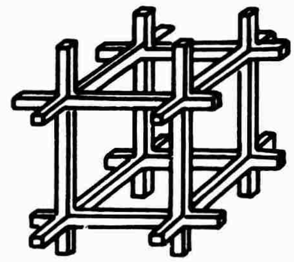


Figure 11. Structural model of a foam material [22].

This situation could be characterized as representing an entirely new material with stress-strain properties substantially different from the original cushioning material. Accordingly, the initial model, Equation (III-26), does not address the non-linearity introduced beyond the bottomed region of the curve, and in itself is not a satisfactory model of cushion response. However, it is reasonable to assume that the functional arrangement of the variables may not be rearranged through the bottomed region of the curve. This possibility was explored using a modular modeling technique and it was found that the basic arrangement of the variables has merit.

Chapter IV

THE IMPACT RESPONSE MODELS

Development of the General Model

The initial model, Equation (III-26), proved deficient in representing the nonlinear characteristics of cushion response. However an extensive literature search showed it to be the only known model that provides a direct relationship of the required variables. Consequently, a modular technique suggested by Shannon [21] was used to modify this model and construct a valid model of impact response. The relationship of each independent variable (σ_s, T, θ, h) and its effect upon the dependent variable (G-level) was studied and the finalized relationship for each independent variable was then incorporated into the model.

Variable 1, Drop Height (h)

The effect of drop height upon G-level that is given in Equation (II-1) is based upon the relationship,

$$V = \sqrt{2gh} \quad . \tag{IV-1}$$

V is the velocity at impact and is related as the square root of drop height (h). Mindlin [3], Janssen [9], and others utilized this relationship of G-level versus drop height, and the derivation by Cost in Equation (III-26) is on this same basis. It was determined that drop height should be incorporated into the model as $h^{1/2}$.

Variable 2, Static Stress (σ_s)

In the UAH study [12], many relationships of G's versus static stress were investigated and it was found that the best agreement was obtained in a second order polynomial of the natural log of stress. Several polynomials of various orders of static stress were tested in this research and a similar conclusion was reached, namely, that a second order polynomial was superior and that the desired U shaped dynamic cushioning curves of G's versus static stress would result.

The initial MLRD Computer runs were made using the following relationship:

$$G = F(\sigma_s, h) \quad . \tag{IV-2}$$

The variables were input with $h^{1/2}$ and a second order polynomial of σ_s. The best fit was obtained using the following functional relationship:

$$G = C_0 + C_1 h^{1/2} + C_2 h^{1/2} \ln \sigma_s + C_3 h^{1/2} (\ln \sigma_s)^2 \quad . \tag{IV-3}$$

These polynomials generated U shaped curves similar to those in the UAH study (compare Figure C-2 and Figure 12).

Variable 3, Thickness of Cushion (T)

Janssen [9], Mindlin [3], and others suggested that G-level was an inverse relationship to thickness as given in Equation (II-2). This seems intuitively correct and many attempts were made using this relationship. T was introduced into these as a negative exponential such as $T^{-1/2}$, $T^{-3/2}$, $T^{-5/2}$, $T^{-7/2}$, etc. Computer runs showed good correlation with thickness input as a negative exponential of the type given. Figure 12 shows the typical nesting effect obtained.

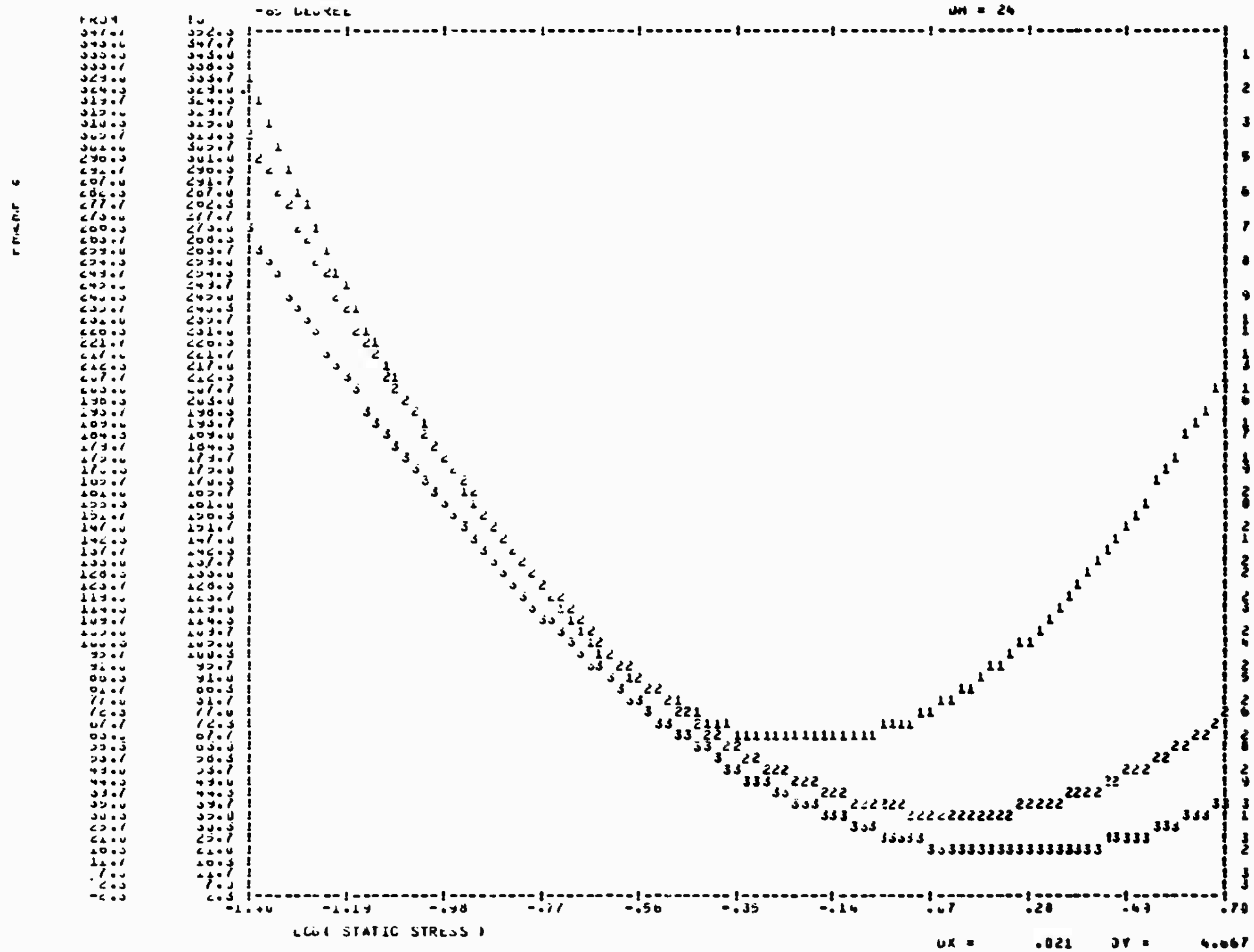


Figure 12. MLRD plots of dynamic cushioning curves of $G = C_0 + C_1\, h^{1/2}\, \ell n\, \sigma_s + C_2\, h^{1/2}\, (\ell n\, \sigma_s)$ at -65°F and 160°F and 24-inch drop height.

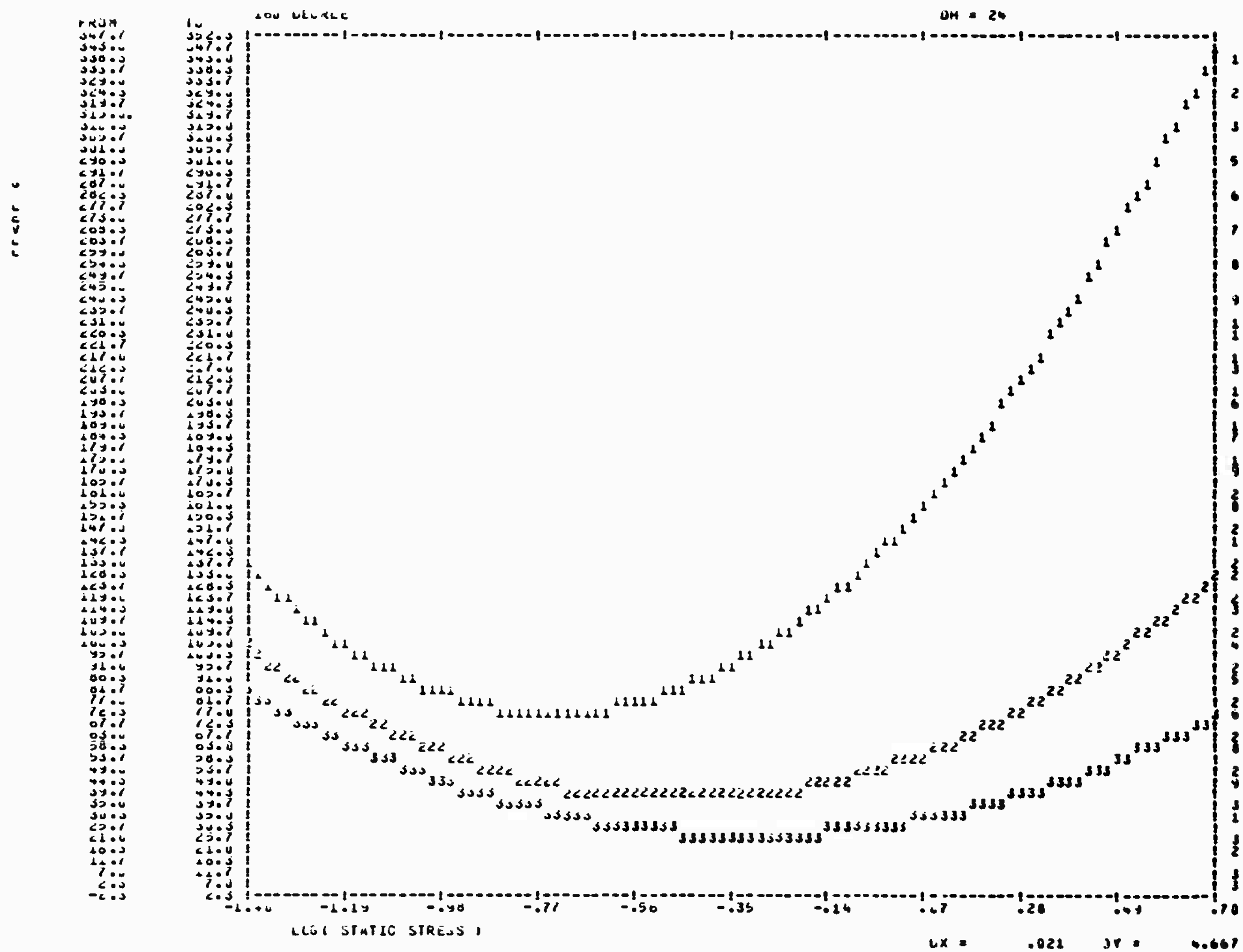


Figure 12. Concluded.

Variable 4. Temperature (θ)

Temperature effects were expected to be the most difficult to incorporate and this turned out to be the case. It can be reasoned that the phase shift effect of temperature discussed in viscoelastic theory for thermorheologically simple materials produces the multiplier effect of the exponentials shown by Cost. Models were tried with several orders of temperature and the polynomial with θ^j, where $j = 1, 2, 3, \ldots, n$ were the most satisfactory. Figure 13 is a typical plot using θ^j (where $j = 1, 2, 3$) that shows the shifting obtained from a cold temperature of -65°F (C) through ambient (A) to hot 160°F (H).

The General Model

Many relationships were tried and rejected. Each time, the basic underlying structure of the variables was rejustified and new formulations were attempted. The process was repeated until a valid General Model of impact response was developed. The General Model is given as follows:

$$G = C_0 + \sum_{\ell=0}^{S} h^{\ell/2} \sum_{k=0}^{R} \frac{1}{T^{(1/2+K)}} \sum_{j=1}^{N} \theta^j \sum_{i=0}^{M} C_{ijk\ell} (\ell n\ \sigma_s)^i \ . \tag{IV-4}$$

This General Model incorporates each of the variables in the manner prescribed in the modular modeling effort. The curves produced by this model are all U shaped and can be displayed using the MLRD plot routine. A series of plots from the General Model for various temperatures and drop heights are given in Appendix D for the 18 and 30 inch drop heights at -65°, 70°, and 160°F. A comparison of these curves

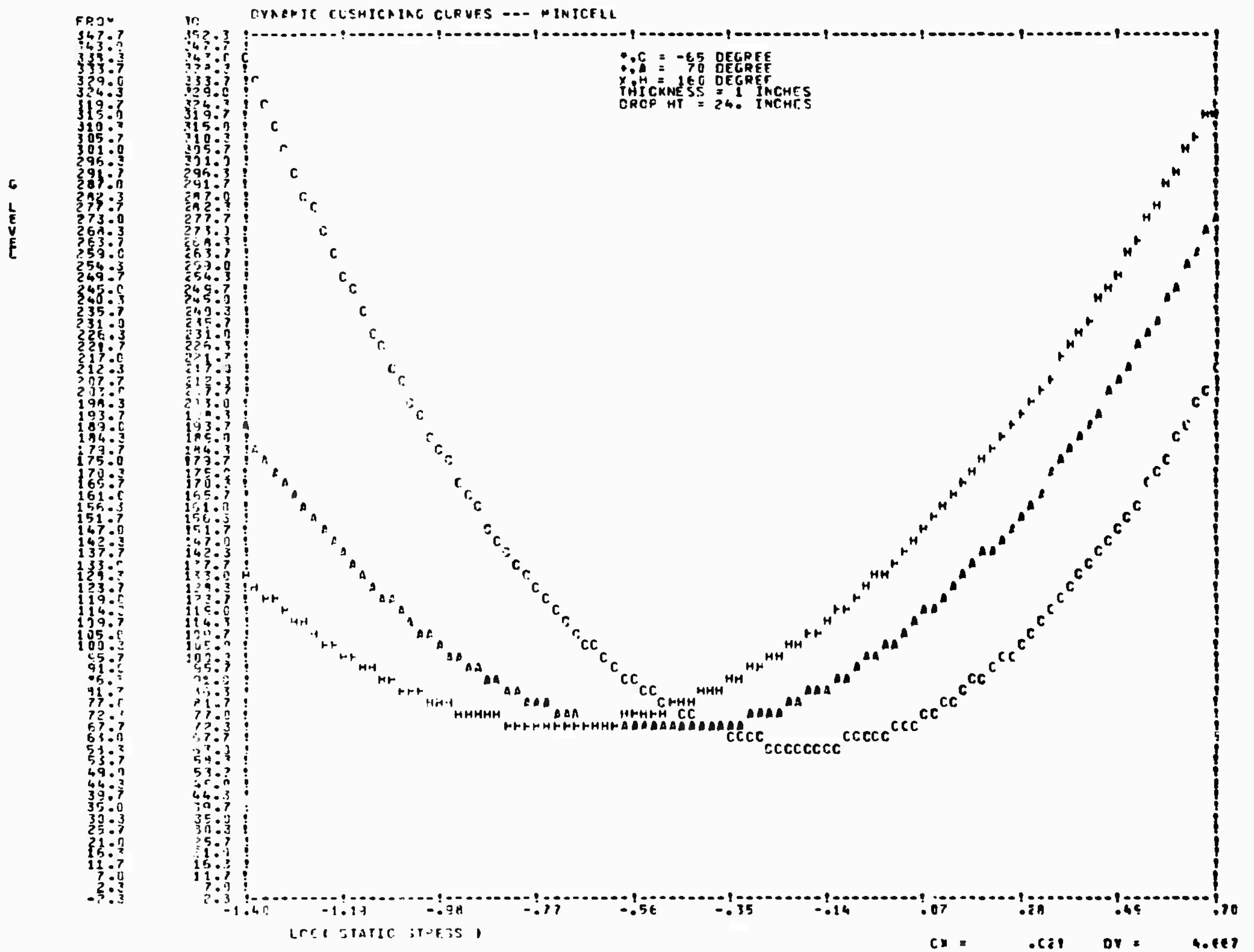


Figure 13. MLRD plots of superimposed dynamic cushioning curves derived from a model using θ^{j}.

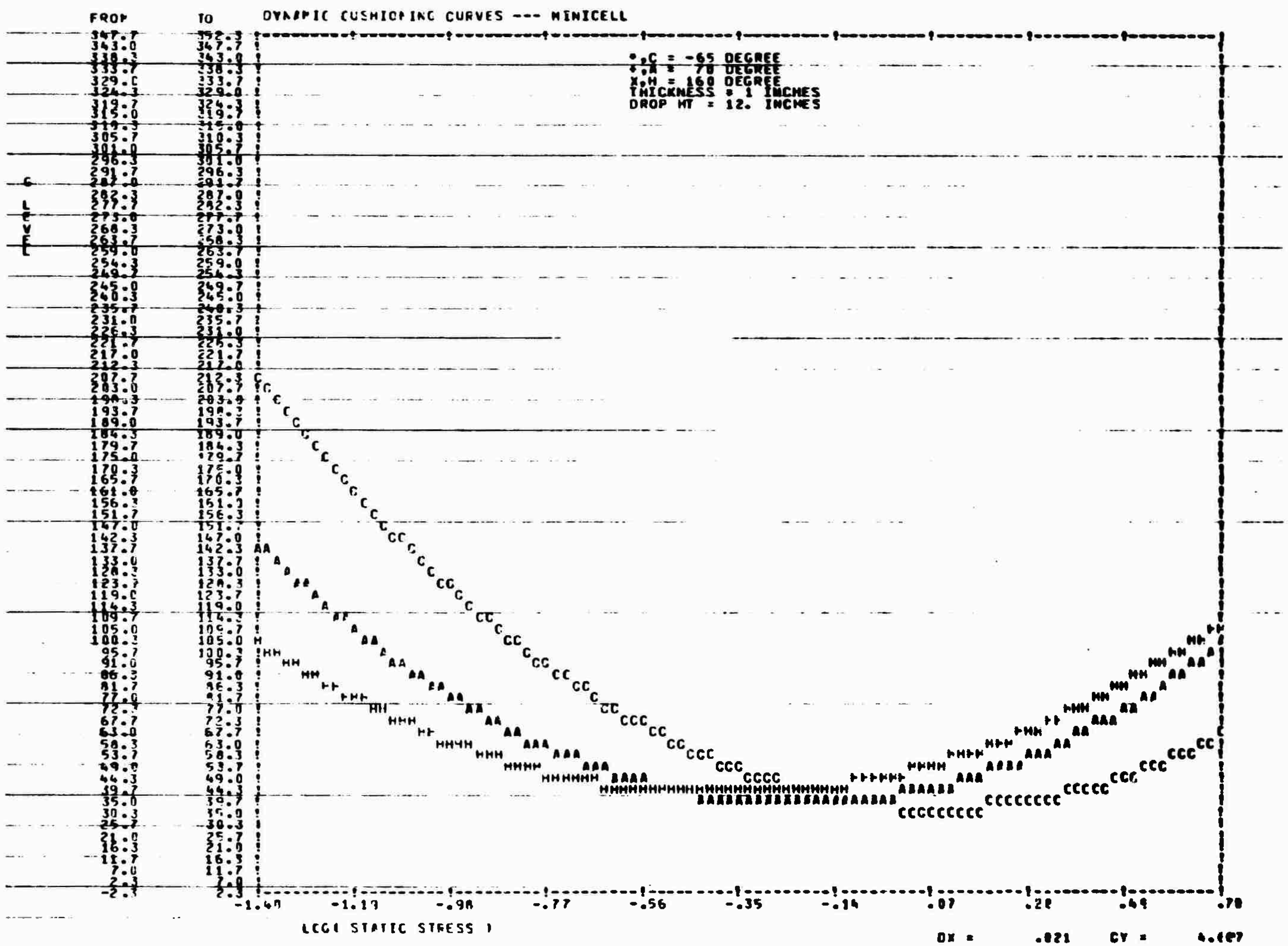


Figure 13. Concluded.

demonstrates the following:

A) The curves of G-level versus static stress are U shaped, consistent with cushioning theory

B) G-level decreases as drop height decreases, consistent with Equation (II-1)

C) The curves are shifted laterally with temperature, consistent with the thermorehologically simple assumption

D) G-level decreases as thickness increases, consistent with Equation (II-2)

E) Thickness curves are nested similar to Humbert and Handlon [11], Figure 1.

<u>Precision of the General Model</u>

The precision of this General Model, Equation (IV-4), has to do with how well it can be made to represent a particular type of cushioning material. This can be determined through comparison with an experimental data base similar to that in Appendix A. Sensitivity analysis can be performed on the model by adjusting the upper limits of the summations, M, N, R, and S to determine if it is possible to obtain the desired precision.

A sensitivity analysis was performed using the Minicel data of Appendix A. S is set at 1, R is set at 1, N is set at 3, M is set at 2, and the Minicel Model takes the following special form:

$$G = C_0 + \sum_{\ell=0}^{1} h^{\ell/2} \sum_{k=0}^{1} \frac{1}{T^{(1/2+K)}} \sum_{j=1}^{3} \theta^j \sum_{i=0}^{2} C_{ijk\ell} (\ell n \sigma_s)^i \quad . \qquad \text{(IV-5)}$$

This is expanded to a 36 term polynomial as follows:

$$
\begin{aligned}
G = C_0 &+ \frac{\theta}{T^{1/2}}\left[C_{0100} + C_{1100}\,\ell n\,\sigma_s + C_{2100}\left(\ell n\,\sigma_s\right)^2\right] \\
&+ \frac{\theta^2}{T^{1/2}}\left[C_{0200} + C_{1200}\,\ell n\,\sigma_s + C_{2200}\left(\ell n\,\sigma_s\right)^2\right] \\
&+ \frac{\theta^3}{T^{1/2}}\left[C_{0300} + C_{1300}\,\ell n\,\sigma_s + C_{2300}\left(\ell n\,\sigma_s\right)^2\right] \\
&+ \frac{h^{1/2}\,\theta}{T^{3/2}}\left[C_{0111} + C_{1111}\,\ell n\,\sigma_s + C_{2111}\left(\ell n\,\sigma_s\right)^2\right] \\
&+ \frac{h^{1/2}\,\theta^2}{T^{3/2}}\left[C_{0211} + C_{1211}\,\ell n\,\sigma_s + C_{2211}\left(\ell n\,\sigma_s\right)^2\right] \\
&+ \frac{h^{1/2}\,\theta^3}{T^{3/2}}\left[C_{0311} + C_{1311}\,\ell n\,\sigma_s + C_{2311}\left(\ell n\,\sigma_s\right)^2\right] \\
&+ \frac{h^{1/2}\,\theta}{T^{1/2}}\left[C_{0101} + C_{1101}\,\ell n\,\sigma_s + C_{2101}\left(\ell n\,\sigma_s\right)^2\right] \\
&+ \frac{h^{1/2}\,\theta^2}{T^{1/2}}\left[C_{0201} + C_{1201}\,\ell n\,\sigma_s + C_{2201}\left(\ell n\,\sigma_s\right)^2\right] \\
&+ \frac{h^{1/2}\,\theta^3}{T^{1/2}}\left[C_{0301} + C_{1301}\,\ell n\,\sigma_s + C_{2301}\left(\ell n\,\sigma_s\right)^2\right] \\
&+ \frac{\theta}{T^{3/2}}\left[C_{0110} + C_{1110}\,\ell n\,\sigma_s + C_{2110}\left(\ell n\,\sigma_s\right)^2\right] \\
&+ \frac{\theta^2}{T^{3/2}}\left[C_{0210} + C_{1210}\,\ell n\,\sigma_s + C_{2210}\left(\ell n\,\sigma_s\right)^2\right] \\
&+ \frac{\theta^3}{T^{3/2}}\left[C_{0310} + C_{1310}\,\ell n\,\sigma_s + C_{2310}\left(\ell n\,\sigma_s\right)^2\right] .
\end{aligned}
\tag{IV-5a}
$$

Minicel Model

To finalize the Minicel Model, the MLRD program was used with Equation (IV-5), and successive analysis of variance tables were constructed as each additional variable was entered into solution. These ANOVA tables were used to determine if the incoming variable made a significant contribution to the overall correlation. A Duncan [22] F test is utilized with this format; it was found that the 25th variable that entered did not make a significant contribution to the G-level response prediction. The test hypothesis can be stated as follows:

H_0 : the entering variable has no effect upon the G-level response prediction.

H_1 : the entering variable has a significant effect upon the G-level response prediction.

The test is

$$F_{calc} = \frac{MS_{due\ to}}{MS_{about}} \tag{IV-6}$$

and the null hypothesis H_0 can be rejected when $F_{calc} > F_{table}$. Utilizing an α level of 0.05, the final ANOVA's and F tests (Tables II and III) show that the 24th variable makes a significant contribution but the 25th does not. One further test is made to verify that all the regression coefficients in the final Minicel Model are significant. This test is conducted using a "t" statistic as follows:

$$t_n = \frac{C_n}{S_n} \tag{IV-7}$$

where

t_n is the t statistic for the nth term

TABLE II. ANOVA TABLE AND F TESTS FOR ENTERING THE 24TH VARIABLE INTO THE MINICEL REGRESSION EQUATION

Source	S.S.	d.f.	M.S.	F_{calc}	$F_{.05}$	Decision
Due to all 24 variables	5,026,531.3	24	209,438.8	1830.76	1.52	SIGN
Due to first 23 variables	(5,024,189.8)	(23)	(218,443)	1834.9	1.52	SIGN
Due to addition of 24th variable to first 23 variables	(2341.5)	(1)	(2341.5)	20.6	3.84	SIGN
About regression on all 24 variables (Residual)	172,858.2	1511	114.4			
About regression on the first 23 variables	175,199.7	1512	115.9			
Total	5,199,389.5	1535				

TABLE III. ANOVA TABLE AND F TESTS FOR ENTERING THE 25TH VARIABLE INTO THE MINICEL REGRESSION EQUATION

Source	S.S.	d.f.	M.S.	F_{calc}	$F_{.05}$	Decision
Due to all 25 variables	5,026,771.0	25	201,070.8	1758.9	1.52	SIGN
Due to first 24 variables	(5,026,531.3)	(24)	209,438.8	1830.8	1.52	SIGN
Due to addition of 25th variable	(239.70)	(1)	239.70	2.0	3.84	NOT SIGN
About regression on all 25 variables	172,618.5	1510	114.39	2.0		
About regression on the first 24 variables	172,858.2	1511				
Total	5,199,389.5	1535				

C_n is the coefficient of the nth term

S_n is the standard error of the nth term.

The test hypothesis is as follows:

H_0 : the nth coefficient is the same as zero

H_1 : the nth coefficient is significantly different from zero.

In conducting the test, the null hypothesis can be rejected when $t_n > t_{table}$. With an α level of 0.05, the test of all the coefficients in the Minicel Model, as given in Table IV, are found to be significantly different from zero.

The resultant Minicel Model is a 25 term regression polynomial. The Minicel Model has a 0.983 correlation coefficient which compares favorably with the UAH data in Appendix A.

The reliability of the correlation coefficient can be tested statistically using a t statistic defined as follows [23]:

$$t_n = r \sqrt{\frac{n - 2}{1 - r^2}}$$

where

r = the correlation coefficient

n = the number of samples used to derive the regression equation

t_n = the resulting number of standard errors of r in the interval between the computed r and r = 0.

The test hypothesis is as follows:

$$H_0 : r = 0$$

$$H_1 : r > 0 .$$

TABLE IV. TEST OF THE SIGNIFICANCE OF THE REGRESSION COEFFICIENTS OF THE MINICEL MODEL

Var	Coefficient Subscript Eq IV-5a	Coefficient	Stand. Error	Coef/Se	F	Decision
0	0	-.83931602E+01				
1	2100	.35419457E+01	.437855E+00	8.0893	65.4	SIGN
2	1111	-.15318724E+02	.817803E+00	-18.7316	350.8	SIGN
3	2111	.33401870E+01	.163007E+00	20.4911	419.8	SIGN
4	0101	.20749326E+03	.402348E+01	51.5707	2659.5	SIGN
5	1101	-.50350553E+02	.107843E+01	-46.6972	2180.6	SIGN
6	2101	.14344456E+01	.105041E+00	13.6561	186.4	SIGN
7	1200	-.66958791E+01	.100147E+01	-6.6860	44.7	SIGN
8	0201	-.54656687E+02	.129727E+01	-42.1322	1775.1	SIGN
9	1201	.11621323E+02	.372076E+00	31.2337	975.5	SIGN
10	0300	-.13016393E+01	.237025E+00	-5.4916	30.1	SIGN
11	1300	.20789886E+01	.140395E+00	14.8081	219.2	SIGN
12	2300	-.22664200E+00	.996818E-02	-22.7366	516.9	SIGN
13	0311	-.40141035E+00	.633909E-01	-6.3323	40.1	SIGN
14	1311	.61173036E+00	.303853E-01	20.1325	405.3	SIGN
15	2311	-.95319017E+01	.453820E-02	-21.0037	441.1	SIGN
16	0301	.39422841E+01	.126069E+00	31.2708	977.8	SIGN
17	1301	-.86603770E+00	.360774E-01	-24.0050	576.2	SIGN
18	0110	-.23377506E+03	.141474E+02	-16.5242	273.0	SIGN
19	1110	.28303458E+02	.395093E+01	7.1637	51.3	SIGN
20	0210	.49678750E+02	.386583E+01	12.8507	165.1	SIGN
21	1210	.26323240E+02	.213922E+01	12.3051	151.4	SIGN
22	2210	-.60678372E+01	.310653E+00	-19.5325	381.5	SIGN
23	1310	-.61520847E+01	.286608E+00	-21.4651	460.7	SIGN
24	2310	.10752888E+01	.509818E-01	21.0916	444.8	SIGN

In conducting the test, the null hypothesis can be rejected when $t_n > t_{table}$. This test of the reliability of the correlation coefficient was conducted on the Minicel Model. The null hypothesis can be rejected since $t_n = 291.2 > t_{table}$. The Minicel Model can be written as follows:

$$
\begin{aligned}
G = -8.39 &+ \frac{3.54\,\theta\,(\ell n\ S)^2}{T^{1/2}} - \frac{15.31\,\theta\,h^{1/2}\,\ell n\ S}{T^{3/2}} \\
&+ \frac{3.34\,\partial\,h^{1/2}\,(\ell n\ S)^2}{T^{3/2}} + \frac{207.49\,\theta\,h^{1/2}}{T^{1/2}} - \frac{50.35\,\theta\,h^{1/2}\,\ell n\ S}{T^{1/2}} \\
&+ \frac{1.43\,\theta\,h^{1/2}\,(\ell n\ S)^2}{T^{1/2}} - \frac{6.70\,\theta^2\,\ell n\ S}{T^{1/2}} - \frac{54.66\,\theta^2\,h^{1/2}}{T^{1/2}} \\
&+ \frac{11.62\,\theta^2\,h^{1/2}\,\ell n\ S}{T^{1/2}} - \frac{1.30\,\theta^3}{T^{1/2}} + \frac{2.08\,\theta^3\,\ell n\ S}{T^{1/2}} \\
&- \frac{0.23\,\theta^3\,(\ell n\ S)^2}{T^{1/2}} - \frac{0.40\,\theta^3\,h^{1/2}}{T^{3/2}} + \frac{0.61\,\theta^3\,h^{1/2}\,\ell n\ S}{T^{3/2}} \\
&- \frac{0.09\,\theta^3\,h^{1/2}\,(\ell n\ S)^2}{T^{3/2}} - \frac{0.87\,\theta^3\,h^{1/2}\,\ell n\ S}{T^{1/2}} - \frac{233.77\,\theta}{T^{3/2}} \\
&+ \frac{28.30\,\theta\,\ell n\ S}{T^{3/2}} + \frac{49.68\,\theta^2}{T^{3/2}} + \frac{26.32\,\theta^2\,\ell n\ S}{T^{3/2}} - \frac{6.07\,\theta^2\,(\ell n\ S)^2}{T^{3/2}} \\
&- \frac{6.15\,\theta^3\,\ell n\ S}{T^{3/2}} + \frac{1.07\,\theta^3\,(\ell n\ S)^2}{T^{3/2}} + \frac{3.94\,\theta^3\,h^{1/2}}{T^{1/2}} \qquad \text{(IV-8)}
\end{aligned}
$$

where $\theta = \dfrac{°F + 460}{100}$ and S = static stress in psi × 100.

This model can be used to predict impact response for Minicel cushioning systems. The model is expected to be 95% reliable when used within the ranges of the independent variables which are as follows:

h = drop height from 12 through 30 inches

σ_s = static stress range from 0.03 to 5.0 psi

θ = temperature from -65° to 160°F

T = thickness of cushion from 1 through 3 inches.

The model will predict with good accuracy at all levels of the independent variables within these ranges. Also it was found that the Minicel Model does a reasonable job of predicting results beyond the ranges stipulated for the independent variables as can be seen when the results of tests of 4 and 5 inch thick Minicel samples are compared in Chapter VI.

Adjustments in Precision

The special form of the General Model as given in Equation (IV-5) was used in the validation of the model for the cross-linked polyethylene foam Minicel material and gave a correlation of 0.983. If additional precision had been required the values of S, R, M, and N could have been increased which may be necessary with other materials but gave only a minimal improvement in precision here. It was apparent, however, that increases in M, which incorporates σ_s with exponentials over 2 is in general not worthwhile. Also, increases in N above 3 that incorporate θ at exponentials of θ over 3 are of marginal value in inproved precision. The changes in R and S that affect the exponentials of thickness and drop height should be explored first if additional precision is required in a model of a particular material.

Once a special form of the General Model is found that represents a particular cushioning material, a measure of its validity can be demonstrated utilizing the same statistical procedures as demonstrated for the Minicel Model. Improvements in the precision of the model will be reflected in increases in the "t" statistic associated with the test of the correlation coefficient. Values comparable to those of the Minicel Model are desirable.

Chapter V

VALIDATION

The model building process proceeded through many iterations of development and verification and culminated in the General Model of impact response stated mathematically as

$$G = C_0 + \sum_{\ell=0}^{S} h^{\ell/2} \sum_{k=0}^{R} \frac{1}{T^{(1/2+K)}} \sum_{j=1}^{N} \theta^j \sum_{i=0}^{M} C_{ijk\ell} (\ell n\ \sigma_s)^i \quad . \tag{V-1}$$

This gives the basic underlying structure for a model of impact response for bulk cushioning materials. The question of how good a model has been developed is answered in the validation process. Naylor and Finger [24] suggest three stages of validation:

1) Verification of internal structure
2) Empirical testing
3) Verification as a predictor.

Verification of Internal Structure

The validating process began when the first model of impact response was developed. At that time the ingredients of a model were selected and their relationships postulated on the basis of prior knowledge and existing theory. The basis of the General Model was founded in the theory of viscoelasticity, and the individual parameters (θ, σ_s, T, h, G) were arranged in the model in a manner that is consistent with theory and intuition. The General Model that resulted from the modeling effort incorporates the following characteristics:

1) The dynamic cushioning curves of G-level versus static stress generated by the model are U shaped. This is consistent with cushioning theory.

2) The predicted G-levels increase with increased drop height. The higher drop heights incorporate more energy into the system which would increase the energy levels experienced by the falling body.

3) The predicted G-levels increase with reduced cushion thickness. The G's experienced by a falling body is dependent on shock pulse duration, Equation (II-1), and a thinner cushion would allow less shock pulse time and an accompanying increase in G-levels.

4) Temperature effects induce lateral shifts in the dynamic cushioning curves. This is consistent with the phase shift function concept of viscoelastic theory. It is also intuitively consistent in that reduced temperatures would be expected to stiffen the cushioning material and require an increased stress level for comparable shock attenuation.

5) The dynamic cushioning curves generated as output from the General Model form a series of curves that are nested within progressive values of thickness and drop height for all possible temperature conditions and static stress conditions. It can be concluded that the internal structure of the General Model is intuitively correct and the output of the General Model is consistent with expectation.

Empirical Testing

The General Model of Equation (V-1) is hypothesized as the model of impact response that is applicable as the basic underlying structure of a model for any one of the many cushioning materials. An impact response model for any one particular cushioning material can be constructed by establishing the summation levels M, N, R, and S, and the values of the regression coefficients in the General Model. This can be done through a testing program that generates a data base similar to Appendix A. Then an analysis program is required that provides a least squares fit of the data base. The test program required can be similar to the one conducted in the UAH study. Once the test program is completed and the data base established, an analysis must be performed to develop the model. The stepwise regression analysis given in Appendix B or a similar analysis routine can be utilized.

When this procedure was followed in constructing the Minicel Model, Equation (IV-8), the General Model was used as the basic underlying structure and the statistical tests performed in verifying the Minicel Model serve to validate the General Model. The ANOVA Table for the Minicel Model (Table II), showing an F_{calc} of 1830.76 and a correlation coefficient of 0.983, is indicative of the fit of the Minicel Model to the UAH results. In addition to the high correlation of the model with experimental data, the Minicel Model also demonstrates the characteristic U shaped dynamic cushioning curves which were one of the more important aspects of the impact response model considered in the model development. The dynamic cushioning curves produced from the Minicel Model were plotted using the MLRD printer plot routine for many conditions of thickness, drop height, and temperature. The classic U shape was evident in all the plots, six of which are given in Appendix E.

An additional measure of validity of the model can be seen when the best fitting polynomials in the UAH study are compared to the Minicel Model. The Minicel Model plots in Appendix E have the independent variables at the same levels as the plots of the UAH curves in Appendix C.

Verification as a Predictor

The final test of validity of the General Model is to assess the ability of the model to predict impact response. It was previously demonstrated that the Minicel Model does an excellent job of predicting impact response when compared to the actual data. However, it must be remembered that the Minicel Model is dependent upon these data when formulating its predictions. It is a limited dependency in that it is using all 2709 data points in predicting a particular dynamic cushioning curve, and in fact only a small portion of the data base, approximately 50 data points, apply directly to the particular conditions with the independent variables at the appropriate levels. The UAH best fitting polynomials, however, are derived using only those data points where the independent variables are at the appropriate levels.

To fully verify the model as a predictor, a data base independent of that used to generate the model must be utilized. Three such data bases are available: a data base of 1, 2, and 3 inch thickness Minicel material at -65°, 70°, and 160°F and at 27 inch drop height, and a data base of 4 and 5 inch Minicel material. The 27 inch drop height data are given in Appendix F and the 4 and 5 inch data in Appendix G. The 27 inch data are contained within the range of the independent variables and was not used in formulating the Minicel Model. However, the 4 and

5 inch data are beyond the data extremes of the developed model. Also the 4 and 5 inch samples themselves were not homogeneous. The samples of the 4 inch material were two-piece cushions which were 2 inches thick. The 5 inch material was made using a 2 inch and a 3 inch piece. This stacking is representative of how cushioning is actually used in shock isolation systems requiring thicker sections than the maximum manufactured thickness of 3 inches. Whether the Minicel Model can do an adequate job of predicting the impact performance of these stacked samples is questionable. The model's ability to predict adequately under these circumstances would indicate that the stacking did not significantly perturb the cushioning performance from that encountered in the 1, 2, and 3 inch continuous samples that form the basis of the model.

To determine statistically how well the model fits a set of independent data, Box and Draper [25] suggest it is appropriate to investigate the bias and variance of the predictor. Two statistical tests can be utilized for this purpose. One test, a test of means, determines whether there is a bias in the predicted values of the model when compared to actual data values. The other test, a test of variances, determines if the variations of the predicted values inherent in the model are comparable with the variations in experimental values.

Test of Means

In all the data bases, including the UAH data, there are three replications of the same conditions of the independent variables. These three replications can be considered a cell. Then, the mean of the G-levels of the three data values in a cell (G_i, i = 1, 2, 3) can be compared with the predicted G-level from the model for that cell, and it is reasonable to expect the sum of the differences to vanish. Any difference that cannot be reasonably attributed to sampling error must be considered a bias that is introduced because the model values are not good predictors.

The first step in this test is to formulate a difference between the data values in a cell and the model prediction for that cell. This can be expressed mathematically as follows:

$$\Delta_j = (GM_j - \overline{GD}_j) \tag{V-2}$$

where

j = number of a cell with fixed values of θ, σ_s, T, h

Δ_j = cell difference for cell j

GM_j = the G-level predicted by the model for cell j

$\overline{GD}_j$ = the mean value of G-level for the three data values in cell j

$$\overline{GD}_j = \frac{\sum_{i=1}^{3} G_i}{3} \quad .$$

Then S, the standard deviation of all the cells, can be defined as follows:

$$S = \sqrt{\frac{\sum_{j=1}^{N} (\Delta_j - \bar{\bar{G}})^2}{N - 1}} \tag{V-3}$$

where

N = the number of cells in the data base

$\bar{\bar{G}}$ = the grand mean of all the cell differences, $\bar{\bar{G}} = \frac{\sum_{j=1}^{N} \Delta_j}{N}$.

The hypothesis to be tested is based on the expected value of the differences which can be written $E(\Delta_j)$ and stated as follows:

$$H_0: \; E(\Delta_j) = 0$$

$$H_1: \; E(\Delta_j) \neq 0 \quad .$$

A two tailed "t" test is used where the t statistic is given as

$$t_m = \frac{\bar{\bar{G}} \sqrt{N}}{S} \qquad \text{(V-4)}$$

where t_m = the test statistic for the model.

The test compares t_m with t_{table} and the null hypothesis can be rejected when $t_m > t_{table}$. Rejection of the null hypothesis implies that $E(\Delta_j)$ is sufficiently greater than zero as to be unexplainable as sampling error and therefore the model predictions would appear not to be representative of the data.

<u>Test of Variances</u>

The other test of goodness of fit of the Minicel Model with actual data determines how the variations in the prediction, using the model, compare with the variations in the data. In each data base, the data points in each cell are the replications of the experiment for each set of conditions of stress level, drop height, and temperature. The sum of these variations for each cell can be written as follows:

$$T_j = \sum_{i=1}^{R} (G_i - \overline{GD}_j)^2 \qquad \text{(V-5)}$$

where

i = the number of the sample in the cell being considered

T_j = the sum of squares of the variation for cell j

R = the number of replications in each cell

G_i = G-level value of the data point being considered

$\overline{GD}_j$ = mean value of the data values in the cell being considered from Equation (V-2).

Then σ_d^2 can be defined as the data within-cells variance which is found by summing the variations within all the cells:

$$\sigma_d^2 = \frac{\sum_{j=1}^{N} T_j}{N \times df} \qquad \text{(V-6)}$$

where df = degrees of freedom in each cell.

The hypothesis to be tested is whether the variance of the data base is equal to the variance in the model predictions.

The test hypothesis can be expressed as follows:

$$H_0 : \sigma_d^2 = \sigma_m^2$$

$$H_1 : \sigma_d^2 \neq \sigma_m^2 \quad .$$

where

σ_d^2 = the variance in the data as given in Equation (V-6)

σ_m^2 = the variance in the predicted values from the model.

An "F" value is used to test the ratio of variances and the F statistic is defined as follows:

$$F_{calc} = \frac{\sigma_m^2}{\sigma_d^2} \qquad \text{(V-7)}$$

The model variance is computed during the regression procedure as the Residual Mean Square in the analysis of variance of the model being tested. The test compares F_{calc} with F_{table}. F_{calc} is computed from Equation (V-7) and F_{table} is set using $\alpha = 0.05$ and is tabled according to the degrees of freedom in the numerator (the degrees of freedom in the data base being considered) and the degrees of freedom of the demoninator (the degrees of freedom of the model being tested).

Rejection of the null hypothesis indicates that there is a difference between the model variance, σ_d^2, and the data variance, σ_m^2, that cannot be attributed to sampling error. This implies that there is a significant difference in the variance of the model when compared with the actual data and that the model is not representative of the data.

Prediction Test Results

The test of means and variances were conducted on the Minicel Model, Equation (IV-8), using the 27-inch drop height data in Appendix F and the 4 and 5 inch data samples of Appendix G. The results are given in Table V and show the following:

1) 27-inch drop height data - The 27-inch drop height data are within the range of the independent variables used in the Minicel Model and the null hypothesis cannot be rejected in the test of means or test of variances. This appears to indicate that there is no significant difference between the means and variances of the data and the values predicted by the model. The model appears to be a statistically valid predictor of these data.

2) 4 and 5 inch data - The test results for the 4 and 5 inch data show that the Minicel Model gives good predictions of the

TABLE V. TESTS OF MINICEL MODEL AS A PREDICTOR OF 4 AND 5 INCH THICKNESS AND 27 INCH DROP HEIGHT

	Minicel Material ($\alpha = 0.05$)		
	27 inch Drop Height	4 inch Thickness	5 inch Thickness
<u>Test of Means</u>			
Number of cells in the data base (N)	99	156	156
Standard deviation of the cells (S)	20.14	8.63	8.56
t_m	1.84	-1.45	-2.61
t_{table}	±1.96	±1.96	±1.96
Decision on H_0 : $E(\Delta_j) = 0$	cannot reject	cannot reject	reject
<u>Test of Variances</u>			
Variance of the data, σ_d^2	111.63	26.41	105.35
Variance of the model, σ_m^2 (Residual Mean Square, Table IV)	114.39	114.39	114.39
F_{calc}	1.02	4.33	1.08
F_{table}	$F_{1511,198} = 1.17$	$F_{1511,312} = 1.15$	$F_{1511,312} = 1.15$
Decision on H_0 : $\sigma_d^2 = \sigma_m^2$	cannot reject	reject	cannot reject

data means at the 4 inch thickness. The test of means on the 4 inch showed no significant difference but the test on the 5 inch showed a significant difference which is not surprising because the model is projecting 2 inches beyond its range at the 5 inch thickness. The test of variances shows that the model variance at the 4 inch thickness is significantly larger, but this is not true at the 5 inch level. The model dispersion is comparable to the dispersion of the 5 inch data.

There were reservations as to whether the model could predict the 4 and 5 inch materials since the samples were stacked and not homogenous and since 4 and 5 inches are beyond the ranges of the independent variables used in the Minicel Model. The results of the tests of means and variances show that the model was not completely acceptable at either the 4 inch or 5 inch thickness. However, the tests indicate that model means were comparable with the data at the 4 inch level and the variances were comparable at the 5 inch level.

Chapter VI

OPTIMIZATION

The procedure which was designed in this research to perform cushion system optimization can best be described as a constrained sequential search technique. The technique uses the knowledge that all dynamic cushioning curves are U shaped, and searches for limits of acceptable G-level values along these curves. The procedure is a direct search technique in that the mathematical model of a material such as the Minicel Model, Equation (IV-8), is used directly as the objective function in the optimization procedure.

The first step in the optimization exercise is to identify the parameters in an optimal cushioning system design; then, the objective or goal of the optimization procedure must be identified. The constraints on the procedure and the objective function are formulated and finally the outputs from the optimization routine itself are identified.

Formatting for Optimal Cushion System Design

An optimal cushioning system is a system that provides the necessary shock isolation to the protected item at a minimum cost. Because the cost of a cushioning system is dependent upon the amount of cushioning material, the optimal cushioning system will employ the minimum thickness of cushion needed. The optimal point on a dynamic cushioning curve such as that shown in Figure 10 is the minimum G's possible which provides the maximum protection for that particular thickness of cushioning. Therefore, it is the identification of this optimal point that must be determined in selecting the optimal cushion system.

A different dynamic cushioning curve is required to depict cushion performance for each drop height, thickness, and temperature of the cushion; the optimal point is different for each curve. When one considers the many types of cushioning materials and a different curve for each condition, it can be seen that a very large library of curves is required to present even the most likely conditions of drop height, temperature, and thickness of a few candidate cushioning materials.

A model of impact response for a cushioning material, e.g. the Minicel Model, computes the impact response directly from the values of the independent variables. This precludes the need for a library of dynamic cushioning curves in predicting impact response. It is also possible, using the cushioning models for various materials, such as the Minicel Model, to determine an optimal cushioning design for each material, if such a design exists. It is also important to provide the designers with the maximum amount of useful design information in the output.

Specification of the Cushion System Constraints

The development of a valid mathematical model of impact response for a particular cushioning material provides a vehicle for the application of optimization techniques. This model can be used as an objective function and explored for an optimal cushioning system. This can be done by defining cushioning system design requirements in terms of the external environment and the amount of exposure the protected item can withstand.

The specification of the external environment must contain a definition of those aspects of the environment that have an impact upon cushion system design. This would include a quantitative measure of the magnitude of the maximum shock pulse to which the system will be exposed, which is usually specified in terms of drop height. Also since temperature has a significant effect on impact response, the specification of the external environment should include the range of temperatures to which the system will be exposed (θ_{min}, θ_{max}).

The specification of the survivability of the protected item is given in terms of its ability to withstand shock. The maximum shock pulse, in G's, that the item can withstand is given as the fragility level of the item (GL_{max}).

These considerations concerning the G-levels and temperatures are incorporated into the model as input values that are utilized to constrain the optimal search to feasible solutions.

Construction of the Objective Function

Consider the parameters of the general cushion design model that can be expressed mathematically in functional notation as

$$G = F(\sigma_s, T, \theta, h) \quad . \tag{VI-1}$$

For the purpose of optimal design, GL is identified as the fragility level of the protected item. The temperature parameter, θ, must consider the range of temperatures of the external environment θ_{min} to θ_{max}. The superimposed dynamic cushioning curve format, shown in Figure 2, can be used for this purpose wherein the curves of extremes of the temperature range are superimposed upon the ambient curve.

One parameter in the model, drop height (h), can be determined based on published testing standards. Other parameters, such as the fragility level of the item to be protected (GL_{max}) and the temperature range (θ_{min}, θ_{max}), are usually specified by overall system requirements. For example, if the cushioning system is to be designed for a military container for a missile, the fragility level of the protected item (GL_{max}) will be specified as part of the hardware specifications in the missile system design criteria. The temperature range (θ_{min}, θ_{max}) is defined in the missile system requirements and the drop height is specified in various Military Standards depending on total container weight. These parameters, GL_{max}, θ_{min}, θ_{max}, and h, are the exogenous variables in the optimization model. Equation (VI-1) can now be written as follows:

$$GL_{max} = F[\sigma_s, T, (\theta_{min}, \theta_{max}), h] \qquad \text{(VI-2)}$$

where GL_{max}, (θ_{min}, θ_{max}), and h are inputs to the equation which are determined by the cushion system design requirements. The optimization technique will search out the optimal design from this expression, and the optimal design solution will be expressed in terms of σ_s and T.

The optimal design searches upon the functional relationship expressed in Equation (VI-2) and with the inputs introduced, reduces to the selection of the minimum thickness of cushion that will perform satisfactorily at a static stress condition that is determined in the search. Classical search techniques would establish Equation (VI-2) as the model to use in the search procedure and it would be rearranged with T as the dependent variable and the objective function as follows:

$$T_{min} = F[\sigma_s, GL_{max}, \theta, h] \quad . \qquad \text{(VI-3)}$$

The search would be subject to certain constraints such as $[\theta_{min} < \theta < \theta_{max}]$ and $[GL \leq GL_{max}]$, and a search made for T_{min} using one of many search techniques.

However, it is not apparent how Equation (VI-2) can be rearranged mathematically into the simple form of Equation (VI-3) where T is solved for directly, nor is it necessarily desirable to do so. It is important to recognize that when the exogenous variables are introduced into the objective function, a three-dimensional search on σ_s and T is all that is required. Since the general shape of the curve of G versus σ_s is known, i.e., it is a classical U shaped dynamic cushioning curve, Equation (VI-2) can be searched directly with little difficulty and with reasonable efficiency.

Once Equation (VI-1) is rewritten as Equation (VI-2) with the values of h, $(\theta_{min}\ \theta_{max})$, and GL_{max} introduced, we have an objective function that can be written as follows:

$$GL_{max} = F\left(\sigma_s, T, (\theta_{min}, \theta_{max}), h\right)$$

where GL_{max}, θ_{min}, θ_{max}, and h are input constraints. This objective function is then searched using the direct search routine.

The Direct Search Routine

The direct search routine (CUSHION OPT) is a three stage process. The initial stage involves the selection of a material type. It is anticipated that a data base will eventually be available that contains valid impact response models for many of the different types of bulk cushioning materials. The model of each type of material in the data base can be investigated by the direct search routine to determine the feasibility of meeting the design objective function. The optimal design conditions of minimum thickness and acceptable static stress range will be output for each type of cushion in the data base if an optimal design exists.

The initial step is to select the type of material. Then, a search is initiated on a minimum thickness cushion (1 inch) at the drop height and minimum temperature (θ_{min}) stipulated in the design requirements. These values are input into the objective function and the search is conducted across the static stress spectrum at the temperature extreme, θ_{min} and then θ_{max}, to determine the feasibility of meeting the stipulated GL_{max}. It is also necessary that the acceptable static stress range at GL_{max} be greater than 0.2 psi to permit design flexibility and preclude creep problems within the design. (This 0.2 psi value can be changed at the discretion of the designer.).

If the search of the acceptable static stress range on the minimum cushion thickness is successful, the answer is output as a feasible solution. If it is not successful, the thickness is incremented 1/2 inch and the search repeated. A flow chart of the CUSHION OPT search routine is given in Figure 14 and the program code is given in Appendix H.

Results

The CUSHION OPT Program output is given in the form of superimposed dynamic cushioning curves such as the typical one given in Figure 15. In Figure 15, the design objective function had the temperature range of -65° to 160°F and a 30-inch drop height. The fragility level was

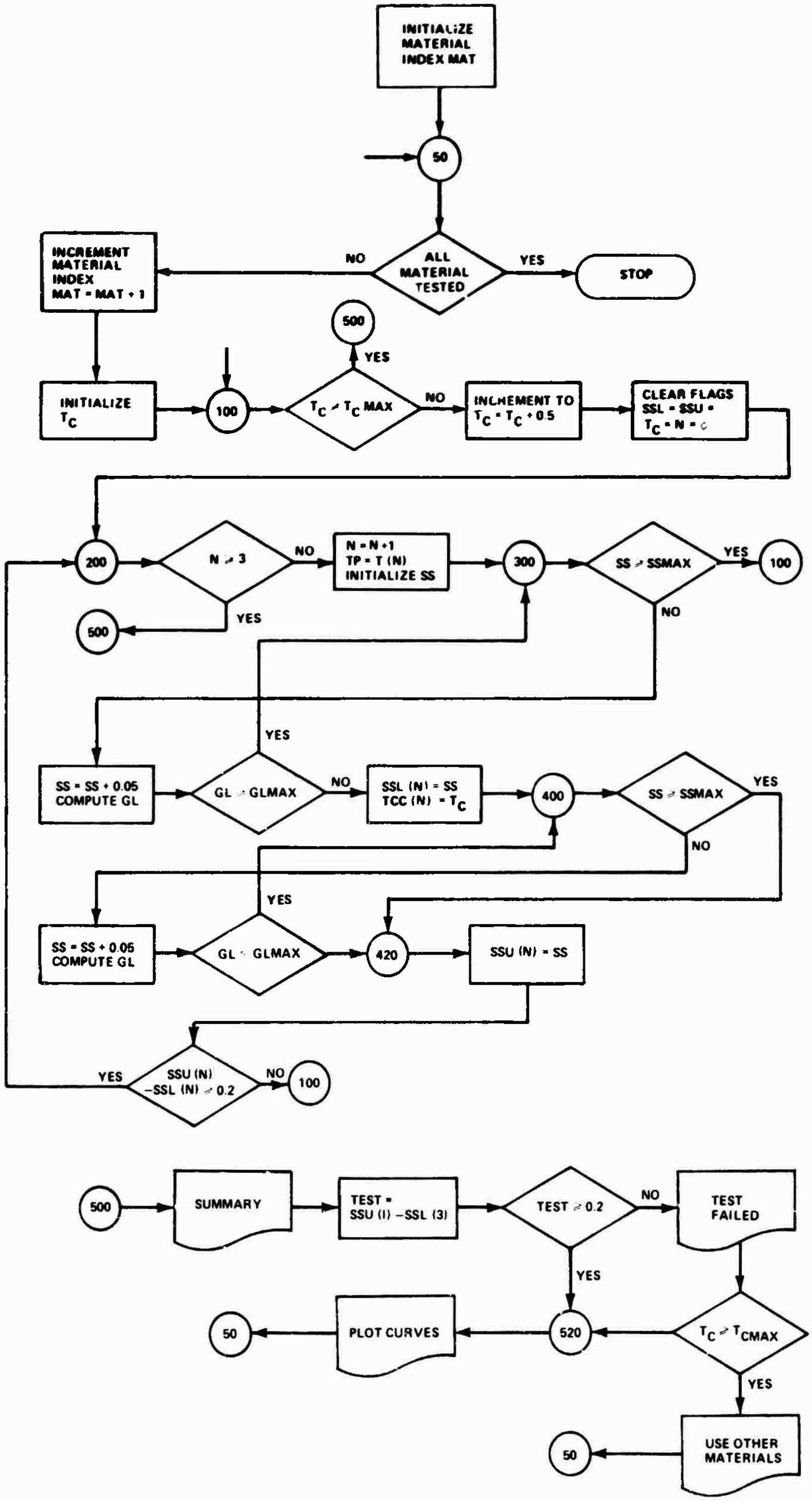


Figure 14. Flow chart of optimization direct search routine (CUSHIONOPT).

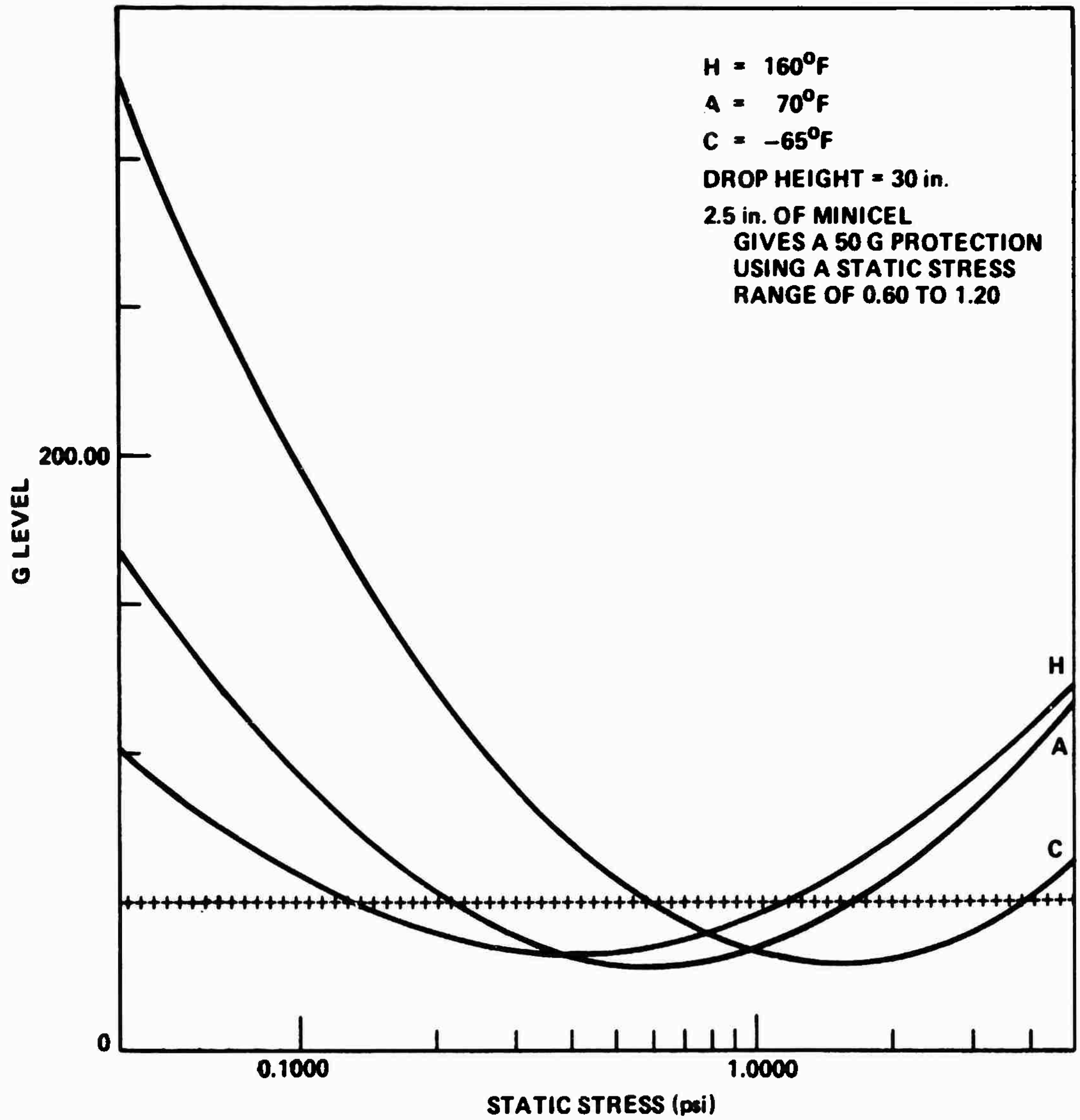


Figure 15. Optimal design output, 2-1/2 in. Minicel.

50 G's which is indicated with a dotted horizontal line. The optimal thickness is determined to be 2.5 inches and the feasible static stress range is found to be 0.60 to 1.20 psi. One of the inherent advantages of using math models is seen here in the capability of determining G-level response at thicknesses other than those in the data base. In this instance a 2.5-inch thickness of cushion was optimal; tests were not conducted at this thickness in the data base. This advantage can also be seen in Figure 16 which is another output of the optimization program where the temperature range has the non-data base values of -20° and 120°F. The optimal thickness is 2.0 inches and the stress range is 0.55 to 0.85 psi to give 50 G protection. Comparison of Figure 15 with Figure 16 demonstrates the effect of temperature on optimal cushion design. When the temperature range was relaxed from the extremes of -65° through 160°F to -20° through 120°F and all other exogenous variables kept the same, the thickness of cushion required for 50 G protection dropped from 2.5 inches to 2 inches. Another comparison can be made between Figure 15 and Figure 17, which demonstrates the increase in thickness of cushion required when the fragility level of the protected item is lowered from 50 G's to 40 G's. One other comparison can be made between Figure 15 and Figure 18, which demonstrates how a reduced drop height requirement reduces the thickness of cushion required.

One additional advantage in using the superimposed dynamic cushioning curve format for the output format is that the designer is presented with a convenient tool for minimizing the creep tendency of the materials by maintaining as low a stress condition as possible and yet be able to ascertain the response of the stress level selected.

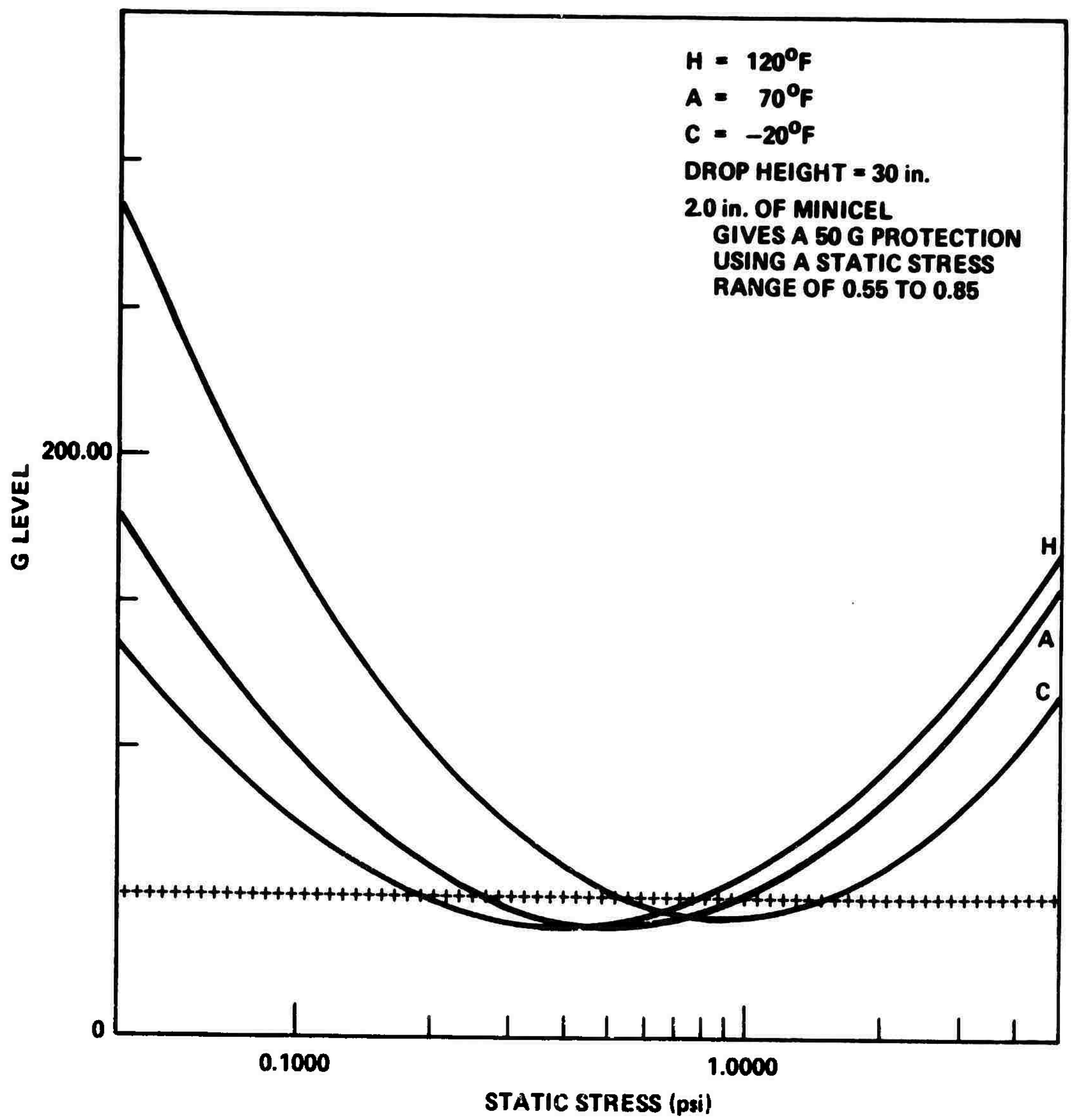


Figure 16. Optimal design output, 2 in. Minicel.

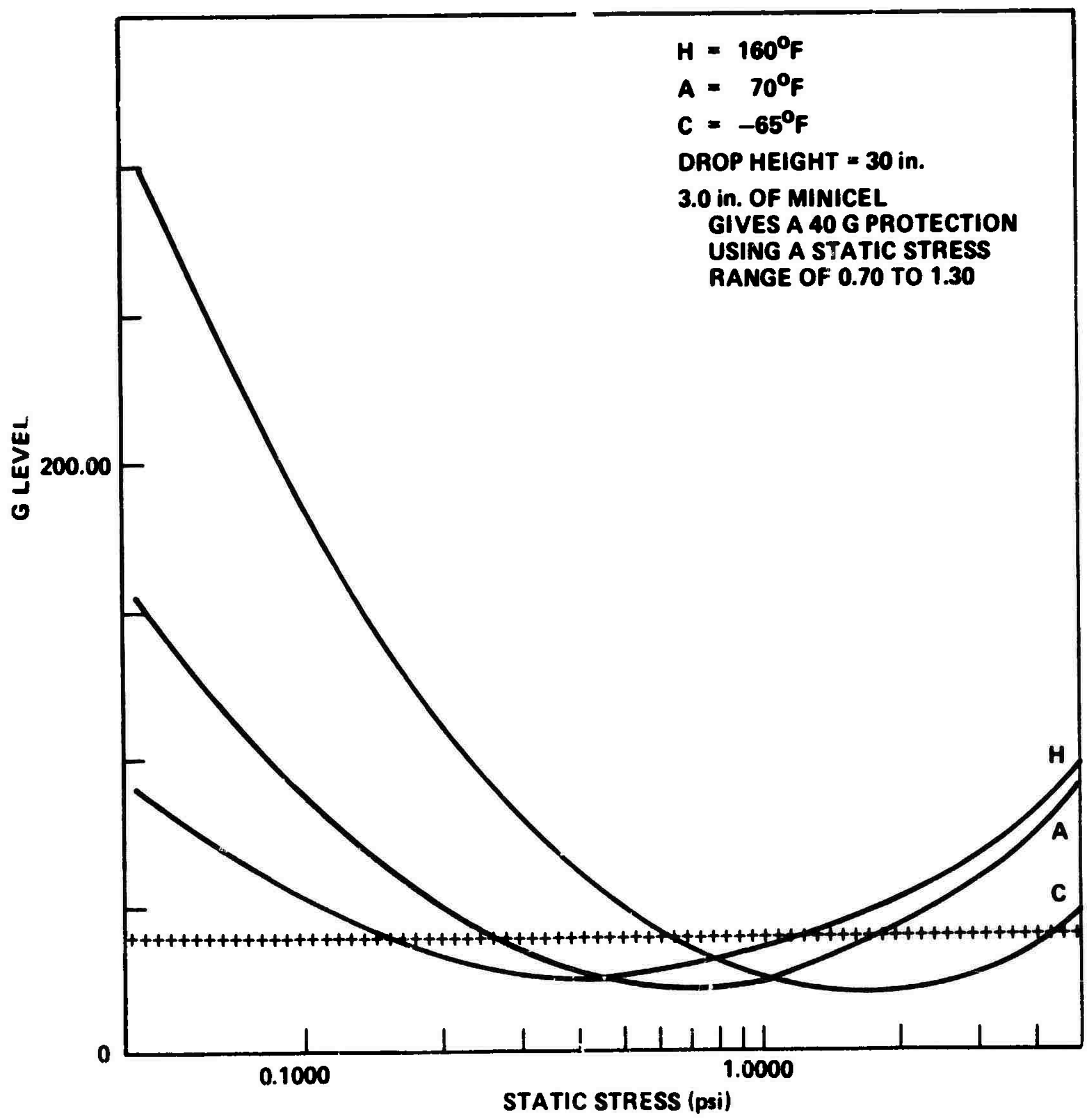


Figure 17. Optimal design output, 3 in. Minicel.

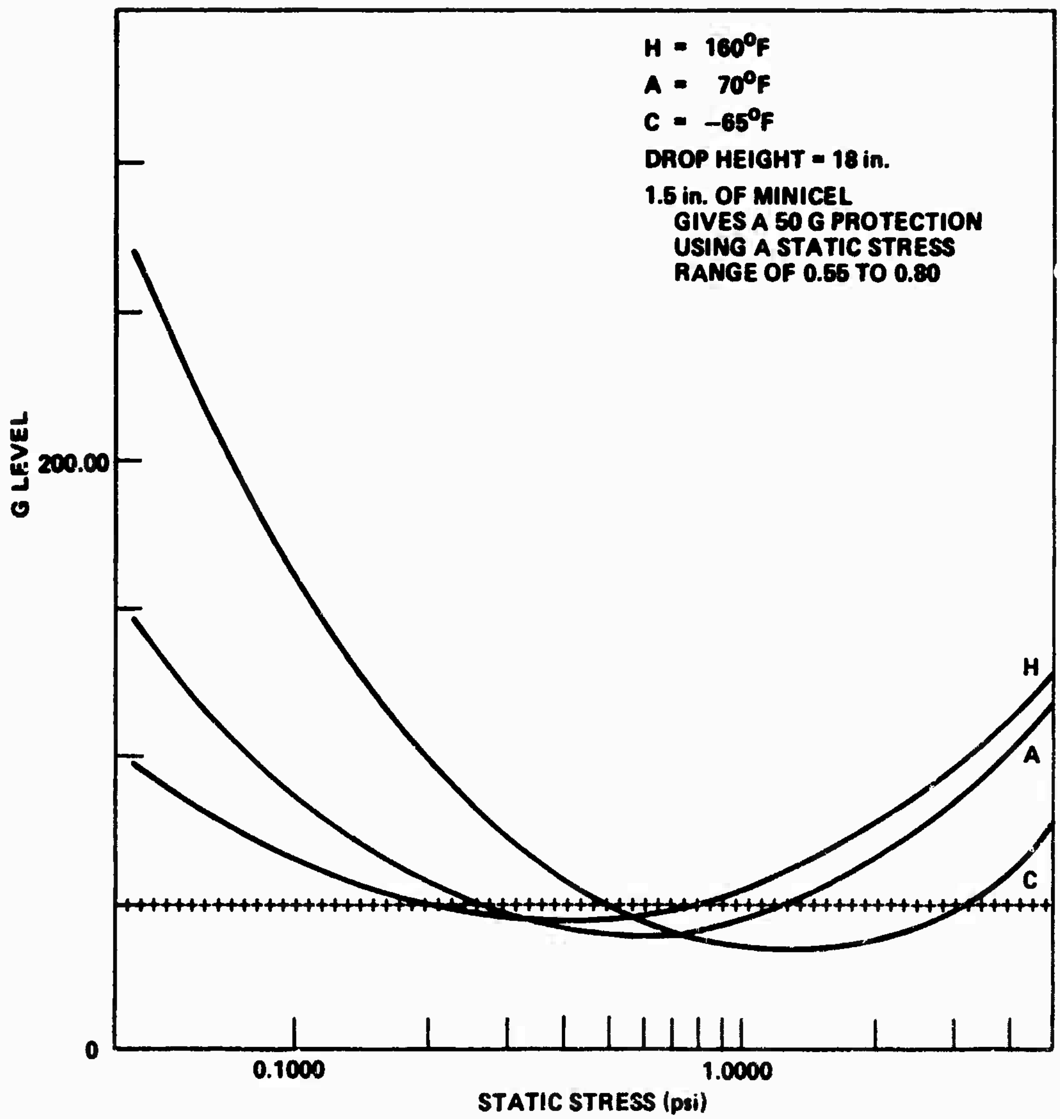


Figure 18. Optimal design output, 1-1/2 in. Minicel.

Chapter VII

CONCLUSIONS AND RECOMMENDATIONS

There has been limited use made of bulk cushioning in shock isolation systems for commercial and military equipment. One reservation that designers have had regarding bulk cushioning is an inability to predict cushioning performance at temperature extremes. Consequently, many designs incorporate mechanical suspension systems such as elastomeric mounts or springs and dash pots and other types of shock isolation systems that are bulky, very heavy, require substantial structural interfaces, and are very expensive.

One result of this research is the development of a valid model of bulk cushioning response that provides a basis for improving the predictability of temperature performance for bulk cushioning materials. Resultant increases in the use of bulk cushioning for shock mitigation systems will generate dollar savings in the accompanying reduced procurement and development costs.

Conclusions

The developed General Model of the impact response of bulk cushioning materials is

$$G = C_0 + \sum_{\ell=0}^{S} h^{\ell/2} \sum_{k=0}^{R} \frac{1}{T^{(1/2+K)}} \sum_{j=1}^{N} \theta^j \sum_{i=1}^{M} C_{ijk\ell}(\ell n\, \sigma_s)^i \quad . \tag{VII-1}$$

The model is predicated on viscoelastic theory and incorporates the effect of temperature, stress, drop height, and thickness of cushion upon the impact response of a cushioning system. This General Model provides the basic underlying structure of impact response of any one of the many bulk cushioning materials used for shock isolation. A sensitivity analysis can be run on the values of S, R, N, and M to obtain the precision desired for an impact model of a particular cushioning material.

Models that are predicated on the basic underlying structure of the General Model are better predictors of impact response than the dynamic cushioning curves currently being utilized, because the effect of temperature has been incorporated into the model. The Minicel Model is one such model that was constructed for the Hercules, Inc. 2 lb/ft^3 Minicel material using the General Model as the basic underlying structure. The Minicel Model is a 25-term polynomial given in Equation (IV-8). The correlation of the Minicel Model with actual data demonstrates the validity of the models and their value as predictors of impact response. The Minicel Model showed high correlation with the actual data within the ranges of the variables and also showed promise as a predictor beyond the variable ranges.

The development of a valid model of impact response of bulk cushioning materials defines the relationships and interrelationships of the variables in response to impact. Using this basis, it was possible to employ a search technique (CUSHION OPT) to determine optimal cushion design and display the findings in terms of superimposed dynamic cushioning curves. The functional form of the model provides the advantage of determining impact response at non-tested levels of the variables with confidence and precludes the need for a library of dynamic cushioning curves for all the different combinations of conditions.

Once a valid model of a particular cushioning material has been developed, it can be incorporated into the CUSHION OPT optimization program. This program accepts the design requirements for a shock isolation system and computes and provides, in the form of superimposed dynamic cushioning curves, the optimal design for each cushioning material in the data base, if one exists. The outputted superimposed dynamic cushioning curves give the pertinent information needed in the optimal design of a shock isolation system.

Recommendations

It is recommended that the model of impact response of bulk cushioning materials, Equation (VII-1), be used as the basic underlying structural design for cushioning systems. This model is considerably better than any previous basis of design. The optimization program used in conjunction with the model can be used to provide accurate predictions of shock mitigation system performance in a time saving manner and in a useful format. It is reasonable to expect that in the design of shock isolation systems, considerable savings can be realized in design time and cost savings by using these more accurate response predictions prior to prototype fabrication and test.

Further, it is recommended that the optimal design program, CUSHION OPT, be used to generate the optimal design of bulk cushioning systems. Cushioning system designers that have a large scale digital computer capability should be encouraged to utilize this procedure. As models of additional cushioning materials become available, CUSHION OPT is provisioned to permit updating to incorporate additional design alternatives using the new materials. To insure maximum utilization of the improved predictive capability of the General Model, it is further recommended that superimposed dynamic cushioning curves of the most likely conditions of the independent variables ([illegible], σ_s, h, T) be published and made available to cushioning system designers who do not have access to a computer facility. It is recognized that one of the major advantages inherent in the CUSHION OPT program, that of obtaining the exact design constraints needed, becomes inoperative and therefore, manual interpolation will be necessary when using published curves. However, the difficulties associated with using and maintaining a library of superimposed dynamic cushioning curves are warranted when the savings accrued through the use of the improved predictions are considered.

Additional drop test programs, similar to the one conducted on Minicel, should be conducted on the bulk cushioning materials. A model of impact response similar to the Minicel Model, Equation (IV-8), using the General Model as the basic underlying structure should be constructed for each new material. This model can then be entered into the CUSHION OPT program to provide an additional optimal design alternative in terms of this new material. It is further recommended that the drop test programs on additional materials be conducted using levels of the independent variables ([illegible], σ_s, h, T) that bracket the values of the variables that might occur in cushion system design. This increase in the range of the variables would insure that the model is predicting values within the range of the model. Particular attention should be given to insuring that the range of thickness is sufficient to obtain G-levels as low as 10 G's, which is not uncommon in some of the more fragile optical and electronic hardware. Also, since only the lower portions of dynamic cushioning curves are used in optimizing cushion design, the test programs can be abbreviated in the understressed and overstressed regions.

It is also recommended that the General Model of impact response be considered as a basis for future research. The mathematical formulation of impact response should provide a vehicle to be utilized for the rigorous analysis of impact response. For example, one particularly lucrative area is that the model be used as a phenomonological constitutive equation in advancing the viscoelastic theory of bulk cushioning materials.

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Appendix A

UAH DATA, 12, 18, 24 AND 30 INCH

MINICEL - 12 in. Drop Height

STRESS LEVELS (PSI)

		0.04			0.10			0.20			0.40			0.80			1.0			1.6			2.0		
		Temperature (°F)																							
Thickness	Replication	-65	70	160	-65	70	160	-65	70	160	-65	70	160	-65	70	160	-65	70	160	-65	70	160	-65	70	160
1"	1	251*	152	103	148	72	58	89	47	45	44	45	47	37	39	49	34	46	52	31	46	49	35	60	70
	2	205	150	108	141	72	56	91	49	49	50	43	43	37	42	45	30	42	48	29	54*	55	34	57	67
	3	175	158*	103	156	75	55	78	49	44	53	42	46	37	43	45	34	41	52	32	45	57	33	61	66
2"	1	208	134*	84	139	78*	43	69	34	28	44	30	25	32*	25	25	24	23	25	21	21	23	18	22	28
	2	206	117	82	148	55	42	81*	34	29	43	29	26	25	23	24	26	22	23	18	22	23	17	24	28
	3	209	119	79	128*	58	40	72	35	29	45	27	25	28	22	25	24	22	24	18	19	22	17	25	25
3"	1	155*	109*	72	94	50	37	63	30	26	40	21	20	25	15	20	22	21	24	15	21	24	16	14	17
	2	217	102	61	108	47	36	64	31	23	42	22	20	24	19	15	20	20	24	14	18	15	15	16	14
	3	188	97	69	108	53	30	67	28	21	38	20	19	23	15	26	21	18	16	14	15	13	12	22*	18

		2.4			3.0			3.6			4.0			4.4			4.6			5.0		
		Temperature (°F)																				
Thickness	Replication	-65	70	160	-65	70	160	-65	70	160	-65	70	160	-65	70	160	-65	70	160	-65	70	160
1"	1	35	59	72	45	75	78	48	89	94	52	81	86*	57	79*	110	61	107*	94	59	106	124*
	2	38	57	70	46	73	84	43	77*	82*	61	81	101	65	90	85*	71	96	106	61	100	114
	3	32	65*	75	50	73	79	48	86	92	56	80	102	68	95	104	70	95	100	69	89*	109
2"	1	16	25	30	12	22	32*	18	28	31	15	29	35	12	24*	36	21	37	39	23	37	45
	2	16	24	25	12	23	25	15	29	31	19	30	33	15	34	41	23	36	41	26	39	40
	3	18	24	27	18	25	28	16	27	35	18	27	37	16	31	39	24	36	41	24	40	43
3"	1	18	14	16	12	15	17	10	18	18	9	15	17	12	15	18	10	20	23	13	20	49*
	2	12	13	15	10	14	17	9	16	12	9	13	20	12	9*	15	11	15	18	9	17	21
	3	16	16	17	8	11	29	10	15	17	7	14	18	9	18	20	10	19	20	11	16	25

MINICEL - 18 in. Drop Height

STRESS LEVELS (PSI)

Thickness	Replication	0.04			0.10			0.20			0.40			0.80			1.0			1.6			2.0		
	Temperatures (°F)	-65	70	160	-65	70	160	-65	70	160	-65	70	160	-65	70	160	-65	70	160	-65	70	160	-65	70	160
1"	1	248	187	113	182	91	77	107	66	65	62	57	70	44	59	81	46	67	84	56	93	107	59	100	115
	2	252	200	120	179	95	72	113	61	66	57	61	66	45	66	79	42	72	85	58	95	117*	68	96	118
	3	250	200	119	183	90	74	103	63	63	56	54	58	45	69	78	45	67	79	54	91	100	68	99	120
2"	1	259	166	86	166	65	52	88	48	37	50	39	35	31	33	35	26	30	56*	25	36	43	28	42	44
	2	250*	216*	105*	166	65	51	87	42	39	46	33	36	31	35	43	27	30	35	24	36	40	26	37	45
	3	260	128	91	157	67	53	73*	43	37	41*	35	40	33	32	37	27	30	36	26	35	44	28	38	48
3"	1	169	112	67	118*	56	41	92*	38	38*	48	29	28	28	29*	27	15	14	24	22	18	23	19	22	24
	2	198	142*	73	153	54	43	76	34	31	44	26	25	27	22	24	22	17	19	20	18	23	22	19	28
	3	242*	125	71	148	49	42	66	40	32	41	27	25	32	20	27	24	21	19	27	21	24	18	23	26

Thickness	Replication	2.4			3.0			3.6			4.0		
	Temperatures (°F)	-65	70	160	-65	70	160	-65	70	160	-65	70	160
1"	1	64*	128	134	100	146	174	104	175	164	131	187	210
	2	78	107*	133	99	143	163	124*	161	173	124	166*	211
	3	73	126	139	98	137	165	99	168	206*	100*	189	202
2"	1	27	44	48	32	52	63	38	64	68	41	73	78
	2	26	45	54	32	57	66	39	59	70	38	61*	84
	3	29	34*	54	31	57	59	37	61	75	35	68	72
3"	1	15	22	19*	18	24	32	18	29	38	25	38	39
	2	12	20	31	15	29	33	15	29	33	21	38	38
	3	15	21	28	12	27	34	19	30	35	21	34	37

MINICEL - 24 in. Drop Height

STRESS LEVELS (PSI)

Thickness	Replication	0.04			0.08			0.10			0.20			0.40			0.60			0.80			1.0			1.4		
	Temperature (°F)	-65	70	160	-65	70	160	-65	70	160	-65	70	160	-65	70	160	-65	70	160	-65	70	160	-65	70	160	-65	70	160
1"	1	231*	180	114	262	122	87	200	106	90	104	77	80	66	71	89	60	83	102	66	96	109	67	98	122	81	123	147
	2	335	215*	121	263	127	104	195	107	87	104	73	81	63	69	86	61	82	97	60	102	108	60	101	122	79	123	150
	3	289	195	113	250*	116	104	198	106	86	97	77	85	63	68	82	61	73*	103	57	88*	120	61	86*	111	62*	124	150
2"	1	277*	163	85	235	112	69	194*	91*	61	102	55	44	48	39	43	49	37	46	35	36	45	34	38	45	31	46	54
	2	254	202*	88	125*	101*	77	213	82	60	88	49	43	50	39	36	41	38	42	36	34	45	35	37	47	28	43	52
	3	245	149	97	270	114	75	205	81	57	93	51	46	71*	41	42	40	34	40	35	38	41	31	36	49	29	39	51
3"	1	220*	111*	94	233	104*	53	110*	68	50	73*	39	39	53	36	27	36	24	25	27	27	24	30	24	27	24	23	27
	2	262	167	86	192*	81	53	155	75*	45	87	40	32	47	33	26	37	23	27	29	25	26	24	32*	25	22	27	30
	3	281	158	68*	232	82	57	169	64	44	89	42	34	46	27*	27	37	25	27	31	24	26	30	23	29	21	24	27

Thickness	Replication	1.8			2.0			2.2			2.4			2.6			3.0			3.4			4.0		
	Temperature (°F)	-65	70	160	-65	70	160	-65	70	160	-65	70	160	-65	70	160	-65	70	160	-65	70	160	-65	70	160
1"	1	112	176*	186	96	148	208				158*	198	235												
	2	108	139	166*	92	164*	205				145	203	254												
	3	90*	137	192	98	136	190				142	198	246												
2"	1	30	49	60	32	16*	64	41	81*	86	47	81	95	52	86	95	45	83	93	57	108	114	68	107	116
	2	32	50	65	32	54	66	46	76	92	44	73	90	48	88	99	45	84	92	58	103	107	75	106	116
	3	33	49	60	30	43	62	42	76	98	49	70	100	102*	87	100	57	78	101	44	99	102	59	100	115
3"	1	21	25	30	23	26	34	24	34	45	22	26	45	21	42	47	21	40	49	22	46	51	30	55	63
	2	21	27	35	22	23	32	28	34	48	18	28	49	24	46	49	20	36	51	20	43	54	31	58	65
	3	20	27	31	19	25	33	23	38	50	24	25	51	27	44	51	31*	41	43	22	43	49	26	56	52*

MINICELL - 30 in. Drop Height

STRESS LEVELS (PSI)

Replication	0.04			0.08			0.1			0.2			0.3			0.4			0.6			Thickness
Temperature (°F)	-65	70	160	-65	70	160	-65	70	160	-65	70	160	-65	70	160	-65	70	160	-65	70	160	
1	365*	208*	143	264	133	118	240	115	100	110	91	98	84	95	102	76	90	120	75	98	144	1"
2	334	195	136	271	129	113	220	100	112	112	88	96	84	93	101	85	91	116	72	106	127	
3	321	187	137	263	134	109	210	100	105	116	92	99	90	93	98	78	90	120	67	103	138	
1	372	173	91	258	99	76	244*	95	65	98	64	54	84	46	49	54	46	53	48	49	54	2"
2	369	169	96	241*	104	76	220	80*	72	106	56	55	82	49	52	57	46	47	47	45	50	
3	369	192*	106*	269	56*	82	215	88	72	88*	58	53	91	54	49	57	49	53	46	42	55	
1	330*	177*	106*	278*	90	60	184	64	56	77	52*	41	71	36	33	50	32	31	38	30	31	3"
2	295	166	89	265	90	57	181	70	58	79	45	39	60*	37	35	52	36	32	43	29	34	
3	301	171	81	257	84	58	195*	80*	57	87*	43	41	76	33	33	46*	35	33	46	30	34	

Replication	0.7			0.8			1.0			1.2			1.3			1.4			1.6			Thickness
Temperature (°F)	-65	70	160	-65	70	160	-65	70	160	-65	70	160	-65	70	160	-65	70	160	-65	70	160	
1	71	108	150	66	113*	170*	79	136	182	105	181	212	99	203*	242	108	201	222	140	217*	287*	1"
2	70	119	147	76	133	145	86	170*	161*	88	173	213	102	178	229	107	206	263*	111	234	264	
3	69	114	145	77	130	154	78	149	191	88	139*	211	97	182	228	125	204	233	123	232	256	
1	41	46	53	41	48	61	38	53	63	37	51	67	34	57	68	32	60	83	35	73	89	2"
2	42	46	56	39	43	58	40	54	62	36	52	72	35	54	73	39	55	77	41	70	41*	
3	47	47	58	40	48	55	40	57	65	37	56	69	35	63	69	40	62	79	37	66	81	
1	37	30	29	36	31	33	35	32	36	27	33	36	25	34	35	27	32	37	23	35	44	3"
2	38	28	32	30*	32	33	31	29	34	27	31	35	24	32	39	25	29	39	24	33	43	
3	36	29	31	35	30	34	32	32	34	26	28	35	27	32	37	26	32	36	26	31	43	

MINICEL - 30 in. Drop Height
(Continued)

STRESS LEVELS (PSI)

Temperature (°F)

Replication	1.8			2.0			2.2			2.4			2.6			3.0			3.4			Thickness
	-65	70	160	-65	70	160	-65	70	160	-65	70	160	-65	70	160	-65	70	160	-65	70	160	
1	140	230	318	184	254	335																1"
2	129	207	315	147	252	334																
3	142	262*	286*	147	229*	322																
1	39	78	90*	44	75	106	56	102	108	59	93	119	51	111	131	76	99*	139	79	135	162	2"
2	36	77	101	49	77	108	54	99	101	51	92	122	59	105	121*	68	128	141	75	125	159	
3	41	72	108	37*	75	105	54	96	110	56	107*	125	51	103	130	68	116	137	89*	105*	157	
1	26	41*	43	25	39	47	22	45	51	30	49	59	28	50	63	26	51	66	32	65	74	3"
2	27	33	45	22	41	51*	23	48	50	30	44	55*	27	29*	56*	37*	54	66	32	64	72	
3	25	34	42	23	34	46	26	46	52	26	44	62	26	48	60	29	48	63	34	50*	60*	

*These values were removed by the outlier procedure during the UAH analysis.

CORRELATION COEFFICIENTS OF THE BEST FITTING POLYNOMIALS IN THE UAH STUDY

Temperature (°F)	Thickness (in.)	Drop Height			
		12 in.	18 in.	24 in.	30 in.
-65	1	0.97	0.98	0.98	0.97
	2	0.99	0.99	0.96	0.99
	3	0.99	0.98	0.99	0.98
70	1	0.98	0.98	0.99	0.98
	2	0.98	0.96	0.97	0.98
	3	0.97	0.98	0.97	0.97
160	1	0.98	0.98	0.97	0.98
	2	0.97	0.97	0.94	0.97
	3	0.93	0.98	0.96	0.97

NOTE: This table gives the sample correlation coefficients from an analysis of regression variance between the data and a polynomial fit of the form

$$y_i = b_0 + b_1 \ell n x_i + b_2 (\ell n x_i)^2.$$

SELECTED BEST FITTING POLYNOMIALS IN THE UAH STUDY

(Hercules Minicel, 2 lb/ft^3 Density)

Thickness (in.)	Temperature (°F)	Design Curve Equation
12 in. Drop Height		
1	-65	$y = 377.74 - 142.48 \ln x + 14.78 (\ln x)^2$
	70	$y = 278.24 - 118.34 \ln x + 14.44 (\ln x)^2$
	160	$y = 197.11 - 84.21 \ln x + 11.27 (\ln x)^2$
2	-65	$y = 367.89 - 131.51 \ln x + 12.22 (\ln x)^2$
	70	$y = 201.97 - 78.34 \ln x + 8.34 (\ln x)^2$
	160	$y = 142.03 - 55.43 \ln x + 6.31 (\ln x)^2$
3	-65	$y = 329.67 - 118.13 \ln x + 10.86 (\ln x)^2$
	70	$y = 159.93 - 58.41 \ln x + 5.77 (\ln x)^2$
	160	$y = 105.43 - 36.66 \ln x + 3.73 (\ln x)^2$
24 in. Drop Height		
1	-65	$y = 691.02 - 301.76 \ln x + 36.16 (\ln x)^2$
	70	$y = 403.76 - 193.48 \ln x + 27.86 (\ln x)^2$
	160	$y = 280.51 - 141.90 \ln x + 23.96 (\ln x)^2$
2	-65	$y = 544.94 - 210.69 \ln x + 21.69 (\ln x)^2$
	70	$y = 333.50 - 150.03 \ln x + 18.71 (\ln x)^2$
	160	$y = 202.97 - 93.58 \ln x + 13.17 (\ln x)^2$
3	-65	$y = 517.16 - 194.42 \ln x + 18.94 (\ln x)^2$
	70	$y = 289.25 - 123.01 \ln x + 13.98 (\ln x)^2$
	160	$y = 170.46 - 73.58 \ln x + 9.21 (\ln x)^2$

Appendix B

STEPWISE REGRESSION PROGRAM LISTING

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      74/74    OPT=1                          FTN 4.2+74278       11/22/74  10.10.21.

C STEPWISE MULTIPLE LINEAR REGRESSION
C
C          WRITTEN BY WAYNE L. JONES,  REDSTONE ARSENAL, ALABAMA
C          BASED UPON PROCEDURES IN DRAPER'S APPLIED REGRESSION ANALYSIS
C          AND SHARE NUMBER 1333
C          TAPES 11  AND 10  ARE USED AS BINARY INPUT TAPES.
C          TAPES  9  AND 10  ARE USED AS WORK TAPES.
C**********                                                           **********
C A. MLR CONTROL CARD 1  FORMAT( 14I5 ) ****************************************
C 01-05 NPROB  = NUMBER TO IDENTIFY PROBLEM.                                   *
C 06-10 NXV    = TOTAL NUMBER OF INDEPENDENT VARIABLES IN INPUT DATA.          *
C 11-15 NYV    = TOTAL NUMBER OF   DEPENDENT VARIABLES IN INPUT DATA.          *
C 16-20 INDEXY = INDEX OF THE DEPENDENT VARIABLE FOR THE PROBLEM.              *
C 21-25 NDATA  = TOTAL NUMBER OF DATA OBSERVATIONS FOR THE PROBLEM.            *
C                  IF UNKNOWN- SET EQUAL MAXIMUM EXPECTED AND SET LAST         *
C                  DATA OBSERVATION EQUAL TO  99999999.                        *
C 26-30 IDEN   = NUMBER OF ALPHABETIC HEADER CARDS ( SEE C ).                  *
C 31-35 INTYPE = 0 FOR REGULAR RUN WITH DATA ON CARDS.                         *
C                1 TO REWIND 10 AND STORE CARD DATA FOR LATER PROBLEM.         *
C                2 TO REWIND 10 AND USE DATA STORED BY A PREVIOUS PROB.        *
C                3 TO STORE CARD DATA ON TAPE 10 WITHOUT REWIND.               *
C                4 TO USE DATA ON TAPE 10 WITHOUT FIRST REWINDING.             *
C                5 TO USE TAPE 11 AS INPUT AFTER REWINDING.                    *
C                6 TO USE TAPE 11 AS INPUT WITHOUT REWINDING.                  *
C                7 REWIND 11, USE AS INPUT, THEN REWIND FOR LATER USE.         *
C 36-40 NREAR  = 0 TO USE DATA WITHOUT REARRANGING IT.                         *
C                1 TO REARRANGE DATA ACCORDING TO CONTROL CARD F.              *
C 41-45 MAXSTP = MAXIMUM NUMBER OF STEPS OR ITERATIONS ALLOWED.                *
C                  TO BYPASS PRINTOUT OF CALCULATIONS PRIOR TO SUMMARY,        *
C                  SET EQUAL TO 999.                                           *
C 46-50 IFBACK = STEP AT WHICH BACK SOLUTION STARTS (ACTUAL VS PRED.).         *
C                  SET EQUAL TO 0 FOR NO BACK SOLUTION.                        *
C                  SET EQUAL TO 999 FOR BACK SOLUTION OF SUMMARY ONLY.         *
C                  NOTE - IF NDATA*(NXV+1) IS GREATER THAN 3000, TAPE 9        *
C                  IS USED TO STORE DATA THEREBY INCREASING RUN TIME.          *
C 51-55 NSTART = NUMBER OF INDEPENDENT VARIABLES THAT YOU WISH TO START        *
C                  THE REGRESSION WITH (SEE D).     NORMAL VALUE IS 0.         *
C                  IF NSTART = -1 THE PROGRAM WILL AUTOMATICALLY PUT           *
C                  ALL NXV VARIABLES IN REGRESSION AT START WITH A             *
C                  TEST OF ONE, WITHOUT CONTROL CARDS IN D. TEST IS ZERO       *
C                  FOR OTHER NEGATIVE VALUES.                                  *
C 56-60 MINSUM = MIN NBR OF IND VAR IN SUMMARY O/P. NORMAL VALUE IS 1.         *
C 61-65 MAXSUM = MAX NBR OF IND VAR IN SUMMARY O/P. NORMAL VALUE IS NXV.       *
C 66-70 MAXREG = MAX NBR OF IND VAR IN REGRESSION.  NORMAL VALUE IS NXV.       *
C**********                                                           **********
C B. MLP CONTROL CARD 2  ***************                                        *
C 01-05 IFWT   = 0 FOR UNWEIGHTED DATA                                         *
C              = 1 IF WEIGHTS ARE READ IN AS INPUT.                            *
C 06-10 TFCNST = 0 IF CONSTANT TERM IS TO BE CALCULATED.                       *
C              = 1 TO DELETE CONSTANT TERM                                     *
C              =-1 IF CONST TERM IS TO BE CONSIDERED AS THE COEFFICIENT        *
C                  OF A NEW INDEPENDENT VARIABLE X0 WHICH ALWAYS HAS THE       *
C                  VALUE 1. THE SIGNIFICANCE OF THE CONSTANT WILL BE           *
C                  INDICATED BY ITS STANDARD ERROR.                            *
C 11-15 IFLIST = 0 TO LIST INPUT DATA, 1 OTHERWISE.                            *
C 16-20 IFSUPS = 0 TO LIST SUM(XI*XJ), 1 OTHERWISE.                            *
C 21-25 IFRES  = 0 TO LIST SUM(XI-XBARI)(XJ-XBARJ), 1 OTHERWISE.               *
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C 26-30 IFCORR = 0 TO LIST SIMPLE CORRELATION COEFFICIENTS, 1 OTHERWISE.        *
C 31-35 NCROSS = 1 IF YOU WISH TO INCLUDE AS ADDITIONAL INDEPENDENT              *
C                    VARIABLES. THE SQUARES AND THE CROSS PRODUCTS OF           *
C                    THE INDEPENDENT VARIABLES.  ZERO OTHERWISE.                *
C                    GENERATED VARIBLES ARE X(NXV+1)=X1*X1                      *
C                    X(NXV+2)=X1*X2, X(NXV+3)=X1*X3,...X(NXV+NXV)=X1*XNXV       *
C                    X(NXV+NXV+1)=X2*X2, X(NXV+NXV+2)=X2*X3 .... ETC.           *
C              = N (GREATER THAN 1) FOR POLYNOMIAL CURVE FIT OF ORDER N.        *
C                NOTE - NSTART SHOULD BE SET TO -2 FOR NORMAL RUN.              *
C 36-40 IFTRA  = ALLOWS FOR TRANSFORMATIONS OF INPUT DATA. SEE G FOR USE.       *
C              = 0 FOR NO TRANSFORMATIONS                                       *
C              = 1 FOR TRANSFORMATIONS.                                         *
C              =-1 TO USE PREVIOUS TRANS WHICH ARE STILL IN CORE.               *
C 41-45 NVOID  = 0 TO PROCESS ALL OBSERVATIONS. = 1 TO READ UP TO 14            *
C                 OBSERVATIONS TO BE LEFT OUT OF REGRESSION. SEE CONTROL        *
C                 CARD E. = -1 TO USE PREVIOUS E CARD .                         *
C 46-50 IFSUB  = ZERO FOR NORMAL RUN. POSITIVE VALUE CALLS IN A                 *
C                 USER SUPPLIED SUBROUTINE CALLED  NEROFX(IFSUB,NXV)            *
C                 TO CHANGE NXV (NOTE- NOROFX MUST BE SUPPLIED EVEN             *
C                 IF IT IS JUST A RETURN). A USER SUPPLIED SUBROUTINE           *
C                 CALLED  EQUAT(IFSUB,DATA) IS USED TO MAKE THE DESIRED         *
C                 CROSS PRODUCT AND TRANSFORMATIONS (NOTE- DEPENDENT            *
C                 VARIABLE SHOULD BE DEFINED AS THE VARIABLE DATA(NXV+1)        *
C                 WHERE DATA IS A SET OF OBSERVATIONS GOING INTO THE            *
C                 S/R AND THE TRANSFORMED SET COMMING OUT ).                    *
C 51-55 NFMT   = 0 FOR REGULAR INPUT FORMAT(7F10.0) .                           *
C              = 1 TO READ INPUT FORMAT( SEE I ).                               *
C              =-1 TO USE FORMAT FROM PREVIOUS RUN.                             *
C 56-60 IFPNCH = 0 TO DELETE PUNCHING OF EQUATION COEFFICIENTS IN SUMMARY.      *
C 61-65 IFDATE = 0 TO PRINT DATE OF COMPUTER RUN                                *
C 66-70 IFNAME = 0 TO READ NAMES OF VARIABLES. SEE H FOR FORMAT                 *
C              =-1 TO USE PREVIOUS H CARD, STILL IN CORE.                       *
C              = 1 TO ASSUME BLANK NAMES                                        *
C**********                                                            **********
C C. ALPHABETIC HEADER  CARDS.      DO NOT USE IF  IDEN=0 .                     *
C     IDEN CARDS WITH FORMAT(16A5)        LAST CARD REPEATED ON EACH PAGE       *
C**********                                                            **********
C D. CARDS FOR VARIABLES IN REGRESSION AT START AND CORRESPONDING TESTS.        *
C      DO NOT USE IF NSTART=0. THERE SHOULD BE "NSTART" FIELDS.7(F8.0,I2)       *
C 01-08 TEST(1) = A TEST CONDITION WHICH DETERMINES WHETHER A VARIABLE          *
C                    WILL BE DELEATED OR ADDED TO THE REGRESSION. ITS           *
C                    VALUE IS 1-R**2. ZERO CORRESPONDS TO A MULTIPLE CORR       *
C                    COEFFICIENT OF 1, WHICH MAKES IT IMPOSSIBLE FOR THE        *
C                    PROGRAM TO DELETE THAT VARIABLE FROM THE SET OF IND        *
C                    VARIABLES. TEST=1 CORRESPONDS TO MULT CORR COEF OF 0.      *
C                    WHICH MAKES SUCH A DELETION CERTAIN.                       *
C 09-10 INDFX(1)= FIRST  VARIABLE TO BE INCLUDED IN REGRESSION AT START.        *
C 11-18 TEST (2)= TEST FOR TWO VARIABLE SET.                                    *
C 19-20 INDFX(2)= SECOND VARIABLE TO BE INCLUDED IN REGRESSION AT START.        *
C 21-28 TEST (3)= TEST FOR THREE VARIABLE SET.                                  *
C 29-30 INDFX(3)= ETC.                                                          *
C**********                                                            **********
C E. OBSERVATIONS TO BE REMOVED FROM REGRESSION. USE ONLY IF  NVOID=1 .         *
C 01-05 NOGOOD(1) = INDEX OF 1ST POINT TO BE REMOVED. ( FORMAT(14I5) )          *
C 06-10 NOGOOD(2) = ETC.                                                        *
C                                                                               *
C**********                                                            **********
```

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              74/74    CPT=1                          FTN 4.2+74278      11/22/74  18.18.21.

C F. CONTROL CARD TO REARRANGE MLR DATA.  DO NOT USE IF NREAR=0.  (14I5)        *
C 01-05 NWORDS = NUMBER OF WORDS IN TAPE OR CARD RECORD.                        *
C 06-10 LOCY   = LOCATION OF DEPENDENT VARIABLE Y.                              *
C 10-15 LOCX(J),J=1,NXV = LOCATIONS OF INDEPENDENT VARIABLES.                   *
C                  IF   IFWT=0 LAST LOCATION IS FOR WEIGHTS.                    *
C***********                                                          **********
C G. TRANSFORMATION CONTROLS. DO NOT USE IF IFTRA=0.  FORMAT(7(F8.0,I2))         *
C     PUT TRANSFORMATIONS AND CORRESPONDING CONSTANTS IN SAME ORDER AS          *
C     X AND Y VARIABLES.                                                        *
C     TRANSFORMATIONS  0=NONE, 1=X+C, 2=X*C, 3=X/C, 4=C/X, 5=X**C,              *
C     6=C**X, 7=LN(X+C), 8=LOG(X+C), 9=E**(C*X), 10=E**(C/X),                    *
C     11=SIN(C*X), 12=COS(C*X), 13=TAN(C*X)                                     *
C 01-08 CONST  = CONSTANT FOR FIRST VARIABLE                                    *
C 09-10 NBRTRA = TRANSFORMATION FOR FIRST VARIABLE                              *
C 11-18        = CONST FOR SECOND VARIABLE ,  ETC.                              *
C 19-20        = TRA FOR SECOND VARIABLE,     ETC.                              *
C***********                                                          **********
C H. INPUT VARIABLE NAMES IN ORDER OF INPUT.  USE ONLY IF IFNAME=C.             *
C 01-10 = NAME OF FIRST INPUT VARIABLE                                          *
C 11-20 = NAME OF SECOND INPUT VARIABLE, ETC.     FORMAT( 7(A6,A4) )             *
C***********                                                          **********
C I. VARIABLE FORMAT FOR INPUT DATA (12A6). USE ONLY IF NFMT=1.                 *
C***********                                                          **********
C J. MLR DATA CARDS SHOULD BE PUNCHED WITH FORMAT(7F10.0), OBSERVATION          *
C     BY OBSERVATION IN THE FOLLOWING ORDER (IF INTYPE=0,1) X1, X2, X3,         *
C     X4, ..., XNXV,  Y1, Y2, Y3, ..., YNYV,  WT(IF IFWT=1 ).                   *
C    DATA CARDS FOR INTYPE=0,1,3 ONLY. BINARY TAPE INPUT FOR OTHER CODES.       *
C    IF NREAR=1 ORDER OF DATA IS DETERMINED BY CONTROL CARD F.                  *
C    OBSERVATIONS WITH BLANK DATA ( -0 ) ARE REMOVED FROM REGRESSION.           *
C***********                                                          **********
C                                                                               *
C TO USE THE PROGRAM FOR AN ORDINARY MULTIPLE REGRESSION (I.E. NO               *
C  ADJOINING OR DELETING), PUT ALL VARIABLES IN THE REGRESSION AT               *
C  THE OUTSET ( NSTART = -NXV ) AND PUT MAXSTP = 1.                             *
C********************************************************************************

C********** CARD OUTPUT  ( IF IFPNCH IS NOT EQUAL ZERO )
C
C          ONE CARD FOR EACH VARIABLE IN EQUATION
C          FORMAT( I5,E20.8,5I5,F20.4,F10.7 )
C      01-05 = I = INDEX OF INDEPENDENT VARIABLES IN EQUATION
C      06-25 = COEFF(I) = COEFFICIENT FOR VARIABLE I
C      26-30 = NPROB  = PROBLEM NUMBER
C      31-35 = NBRNOW = NUMBER OF VARIABLES IN EQUATION
C      36-40 = INDEXY = INDEX OF DEPENDENT VARIABLE
C      41-45 = NOSTEP = STEP NUMBER IN WHICH THE EQUATION WAS COMPLETED
C      46-50 = IFPNCH = INPUT VALUE GREATER THAN ZERO                    MLR 1550
C      51-70 = SIGPCT = STANDARD ERROR OF EQUATION AS A PERCENT OF Y MEAN
C      71-80 = REGRCO = CORRELATION COEFFICIENT OF EQUATION
C
C********** BASIC STATISTICS OUTPUT **********
C
C          XI = X = VALUE OF OBSERVATION FOR VARIABLE I
C          SUM( XI ) = SUMMATION OF VARIABLE I
C          N = NUMBER OF OBSERVATIONS
C          XN = WEIGHTED NUMBER OF OBSERVATIONS
C
```

```
C     MEAN = WEIGHTED AVERAGE = SUM( XI ) / XN
C     STANDARD DEVIATION = SQRT( (SUM(XI**2)-SUM(XI)*MEAN)/(XN-1) )
C
C     SUM CF VARIABLES  = SUM( XI )
C     RAW SUM OF SQUARES AND CROSS PRODUCTS = SUM( XI*XJ )
C     SUM OF SQUARES AND CROSS PRODUCTS ABOUT THE MEAN = CORRECTED SUMS=
C           = SS( I,J ) = SUM(XI*XJ)-SUM(XI)*SUM(XJ)/XN
C     SIMPLE CORRELATION COEFFICIENTS =
C           = R(I,J) = SS(I,J)/SQRT( SS(I,I)*SS(J,J) )
C
C********* RESIDUAL ANALYSIS ( ACTUAL VS PREDICTED ) PRINTOUT *********
C
C     ACTUAL = Y = DEPENDENT VARIABLE
C     PREDICTED = YC = COMPUTED Y USING REGRESSION EQUATION
C                    = A0 + A1*X1 + A2*X2 + ... + AN*XN
C     RESIDUAL = E = YC - Y
C     NORMALIZED DEVIATE = RESIDUAL / STANDARD ERROR
C     PERCENT DEVIATION  = 100 * RESIDUAL / ACTUAL
C     WEIGHT = INPUT WEIGHT OF OBSERVATION
C     SSE = RESIDUAL SUM OF SQUARES
C     CHI SQUARE = SUM( ( RESIDUAL**2 ) / YC )
C
C********* ANALYSIS PRINTOUT *********
C
C     TOTAL(CORRECTED) SUM CF SQUARES = SUM OF SQUARES ABOUT THE MEAN
C           = SUM((Y-YMEAN)**2) = SUM((YC-YMEAN)**2)+SUM((Y-YC)**2) =SS(Y)
C     TOTAL( ORIGIN ) SUM OF SQUARES = SUM OF SQUARES ABOUT THE ORIGIN.
C           USED INSTEAD OF SS(Y) WHEN REGRESSION IS FORCED THRU ORIGIN.
C     REGRESSION SUM OF SQUARES = SUM OF SQUARES DUE TO REGRESSION
C           = EXPLAINED VARIATION = SUM((YC-YMEAN)**2) = SS(R)
C     RESIDUAL SUM OF SQUARES =   SUM OF SQUARES ABOUT THE REGRESSION
C           = UNEXPLAINED VARIATION = SUM((Y-YC)**2) = SS(E)
C     THE MEAN SQUARES COLUMN IS OBTAINED BY DIVIDING THE SUM OF SQUARES
C           ENTRY BY ITS CORRESPONDING DEGREES OF FREEDOM.
C     RESIDUAL MEAN SQUARE = VARIANCE ABOUT THE REGRESSION = S**2 =MS(E)
C     COEFFICIENT OF MULTIPLE DETERMINATION = PCT OF EXPLAINED VARIATION
C           = (SS DUE TO REGRESSION)/(SS ABOUT MEAN) = CORR. FORM SS(R)
C     CORRELATION COEFFICIENT = R = SQRT( CORR. FORM OF SS(R) )
C     STANDARD ERROR OF ESTIMATE = S = SQRT( MS(E) )
C     S.E. AS PCT. OF MEAN = 100 * S / YMEAN
C     F  TEST FOR SIGNIFICANCE OF REGRESSION = MS(R)/MS(E)
C*
C     CONSTANT = A(0) = YMEAN-SUM(A(I)*XMEAN(I)) = CONSTANT TERM
C     COEFFICIENT = A(I) = THE EFFECT ON Y OF A UNIT INCREASE IN XI WHEN
C           THE OTHER VARIABLES ARE HELD CONSTANT.
C     STANDARD ERROR = STANDARD ERROR OF REGRESSION COEFFICIENT.
C           THE 95 PERCENT CONFIDENCE LIMITS FOR A UNIVERSE REGRESSION
C           COEFFICIENT ARE GIVEN BY THE SAMPLE COEFFICIENT PLUS AND MINUS
C           T(0.025) TIMES THE ESTIMATED STANDARD OF THE COEFFICIENT.
C     COEFF/SE = USED IN HYPOTHESIS THAT COEFFICIENT = 0.
C           = COEFFICIENT DIVIDED BY ITS STANDARD ERROR TO GIVE THE NUMBER
C           OF S.E. AWAY FROM HYPOTHEIZED ZERO. SHOULD BE GREATER THAN T
C           VALUE TO REJECT THE HYPOTHESIS THAT THE COEFFICIENT IS NOT
C           SIGNIFICANTLY DIFFERENT FROM ZERO.
C     F    = F VALUE TO REMOVE VARIABLE FROM REGRESSION. **** NOT USED **
C     BETA COEFFICIENT = MEASURE OF THE NET EFFECT OF EACH VARIABLE ON Y.
C     RSQ CHANGE = DECREASE IN RSQ IF THE VARIABLE IS REMOVED FROM REGRESSION
```

```
C*
C     PARTIAL RSQ = THE SQUARE OF THE PARTIAL CORRELATION COEFFICIENT OF
C         VARIABLE K NOT IN THE REGRESSION WITH THE RESPONSE Y.
C         = R(KY.LMN... )**2 WHERE L,M,N,... ARE ALREADY IN REGRESSION.
C         = RELATIVE AMOUNT OF IMPROVEMENT THAT IS BROUGHT ABOUT IF
C         VARIABLE K WERE ADDED TO THE REGRESSION.
C     NORMED SUM/SD = THE NORMALIZED SUM OF SQUARES OF RESIDUALS FOR
C         VARIABLE K  HAD IT TOO BEEN REGRESSED. USEFUL IN DRAWING
C         ATTENTION TO NEAR-LINEAR DEPENDENCIES AMONG THE IND. VARIABLES
C     DELTA RSQ = CHANGE IN RSQ IF VARIABLE K WERE ADDED TO REGRESSION.
C         VARIABLE WITH LARGEST DELTA IS ADDED TO REGRESSION NEXT.
C     F    = F VALUE TO ADD VARIABLE TO REGRESSION. **** NOT USED ****
C
C********* ADDING  AND  DELETING  VARIABLES ***************************
C
C STEP 1 - THE VARIABLE NOT IN THE EQUATION WHICH CAUSES THE GREATEST
C     CHANGE IN RSQ IS ADDED TO THE REGRESSION.
C STEP 2 - THE VARIABLES IN THE EQUATION ARE THEN CHECKED TO SEE IF ONE
C     CAN BE DELETED. THE VARIABLE WHICH CAUSES THE SMALLEST CHANGE IN
C     RSQ IS SELECTED FOR REMOVAL. IF THE EQUATION WITHOUT THIS VARIABLE
C     PRODUCES A SS(R) WHICH IS SMALLER THAN THE PREVIOUS SS(R) FOR THAT
C     NUMBER OF VARIABLES, THE VARIABLE IS REMOVED.
C STEP 3 - IF A VARIABLE WAS REMOVED, REPEAT STEP 2.
C          OTHERWISE REPEAT STEP 1 AND 2.
C
C**********************************************************************
C
C     IT SHOULD BE NOTED THAT THE STATISTICS FOR NON-LINEAR EQUATIONS
C     SHOULD BE USED WITH CARE, AND SHOULD NOT BE COMPARED WITH THOSE
C     FROM LINEAR EQUATIONS, AS THEY HAVE DIFFERENT MEANINGS.
C     FOR EXAMPLE - IF Y IS TRANSFORMED BY TAKING ITS LOGARITHM, THE
C     SUM OF THE SQUARES OF THE ACTUAL RESIDUALS BETWEEN THE CALCULATED
C     AND THE OBSERVED Y VALUES ARE NOT  MINIMIZED, RATHER THE SUM OF
C     SQUARES OF THE LOGARITHMS OF THE  RATIOS  OF THESE VALUES ARE
C     BEING MINIMIZED ( (LOG YC - LOG Y) = ( LOG(YC/Y) ) ).
C     THEREFORE, COMPARISION OF ANY STATISTICS THAT ARE BASED UPON THE
C     SUM OF THE SQUARES OF THE Y RESIDUALS SUCH AS THE F VALUE OR
C     CORRELATION COEFFICIENT MAY BE MISLEADING.
C
C     IT SHOULD ALSO BE NOTED THAT WHEN THE CURVE IS FORCED THROUGH THE
C     ORIGIN OR SOME OTHER SPECIFIED Y INTERCEPT, THE DEGREES OF FREEDOM
C     ARE CHANGED AND THE CURVE NO LONGER GOES THROUGH THE MEANS OF THE
C     VARIABLES, THEREBY, CHANGING THE VALUES OF THE STATISTICS AND
C     MAKING COMPARISIONS OF CURVES WITH UNSPECIFIED Y INTERCEPTS
C     MISLEADING.   ALSO, COMPARISON OF F VALUES WITH THE STANDARD F
C     DISTRIBUTION IS NOT NECESSARILY VALID.
C**********************************************************************
C
C     THE USERS OF THIS PROGRAM ARE URGED TO REVIEW THE STANDARD TEXTS
C     ON REGRESSION ANALYSIS FOR THE USES AND LIMITATIONS OF THIS
C     TECHNIQUE, AND BEAR IN MIND THAT THE STATISTICAL RELATIONSHIPS ARE
C     NO BETTER THAN THE DATA THAT WAS USED TO COMPUTE THEM.
C*********************************************************************

C
C
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```
      PROGRAM  MLR
     * (INPUT=101B,OUTPUT, TAPE5=INPUT, TAPE6=OUTPUT, PUNCH=202B,
     *       TAPE7=PUNCH, TAPE9=1001B, TAPF10=10C1B, TAPE11=1001E )
C
C          WAYNE L. JONES
      COMMCN SIGMA(60),A(52,52),SIMCOR(52,52),AVG(68),TEST(60)              MLR 2490
      COMMON POINT(60),STRING(3000),INDPAC(30,30),INCEXP(61)                MLR 2500
      COMMCN INDEX(60),NCUT(60),KSTEP(60),ALPHA(16),YMEAN,IDEN,IFAVE        MLR 2510
      COMMON MAXSTP,IFPNCH,NSUMRY,NSKIP,NTAPE9,NEW                          MLR 2520
      COMMCN NOVAR,NURNOW,NCSTEP,NDATA,NBRXYN,NERX,LFATM,DEFRM,NVAR         MLR 2530
      COMMCN IFBACK,IFCNST,IFCORR,NPROB,NBRPVR,TCL,NCDEL,JF                 MLR 2540
      COMMCN INDEXY,LBAO,NOGOOO(28)                                         MLR 2550
      COMMCN IFWT,YCONST,NYTRA,V(2,51),YTRA(2)                              MLR 2560
      COMMCN  STDERP(50),CORSQR(58)                                         MLR 2570
      REAL              A,SIMCOR,SIGMA,AVG,TEST                             MLR 2580
C                                                                           MLR 2590
C                                                                           MLR 2600
      DIMENSION  VNAME(2,51),CROSS(8)                                       MLR 2610
      DIMENSICN XDATA(255), LOCX(60), CONST(60), NBRTRA(60)
      DIMENSION  FMT(10)
      DIMENSICN JVOID(14)                                                   MLR 2640
      DIMENSICN TRA(3), ATRA(2,17)                                          MLR 2650
      DATA  ATRA / 34*1H  /
      DATA (ATRA(1,J),J=1,17)/8HX + C   ,8HX * C   ,8HX / C   ,8HC / X  MLR 2660
     1 ,8HX**C    ,7HC**X    ,8HLN(X+C) ,8HLOG(X+C),8HE**(C*X),8HE**(C/X)MLR 2670
     2,8HSIN(C*X),8HCOS(C*X),8HTAN(C*X),9HSINH(C*D),9HCOSH(C*D),9HTANH(CMLR 2680
     3*D),11HCCNST. TERM/                                                   MLR 2690
      DATA (CROSS(J),J=1,8)/4HV(1),4HV(2),4HV(3),4HV(4),4HV(5),4HV(6),
     *                      4HV(7),4HV(8) /
      DATA  FMT / 8H(7E10.0) /
      DATA  ACT,UAL/6HACTUAL,1H /                                           MLR 4730
      DATA NREELS,NEOF/1,0/                                                 MLR 2710
      DATA BLANK/1H /                                                       MLR 2730
      DATA AX/1HX/,AY/1HY/                                                  MLR 2740
      DATA START,VOIDED,SEARCH,TRAN,FORAMT/6HSTART ,6HVOIDED,6HSEARCH,6HMLR 2750
     1 TRA= ,6HJFMT  /                                                      MLR 2760
C                                                                           MLR 2770
      DATA  XDATA(1)/ 60H1SELECTED PERCENTILE VALUES OF THE STUDENT T DIMLR 2780
     1STRIBUTION   /, XDATA(07)/78H           .10        .05        .01
     2   FOR     DIRECTIONAL (ONE-TAILED) TEST  /, XDATA(15)/78H   D.F.
     3    .20        .10        .02      FOR NONDIRECTIONAL (TWO-TAILED) TESMLR 2810
     4T    /                                                                MLR 2820
      DATA ( XDATA(J),J=37,188)/4H   1,3.078,6.313,31.821,4H   2,1.886, MLR 2830
     1 2.920,6.965,4H   3,1.638,2.353,4.541,4H   4,1.533,2.132,3.747,     MLR 2840
     2 4H   5,1.476,2.015,3.365, 4H   6,1.440,1.943,3.143,                 MLR 2850
     3 4H   7,1.415,1.895,2.998, 4H   8,1.397,1.860,2.896,                 MLR 2860
     4 4H   9,1.383,1.833,2.821, 4H  10,1.372,1.812,2.764,                 MLR 2870
     5 4H  11,1.363,1.796,2.718, 4H  12,1.356,1.782,2.681,                 MLR 2880
     6 4H  13,1.350,1.771,2.650, 4H  14,1.345,1.761,2.624,                 MLR 2890
     7 4H  15,1.341,1.753,2.602, 4H  16,1.337,1.746,2.538,                 MLR 2900
     8 4H  17,1.333,1.740,2.567, 4H  18,1.330,1.734,2.552,                 MLR 2910
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PROGRAM MLR        74/74    OPT=1                          FTN 4.2+74278       11/22/74  16.16.21.

          9 4H  19,1.328,1.729,2.539, 4H  20,1.325,1.725,2.520,              MLR 2920
          1 4H  21,1.323,1.721,2.518, 4H  22,1.321,1.717,2.508,              MLR 2930
          2 4H  23,1.319,1.714,2.500, 4H  24,1.318,1.711,2.492,              MLR 2940
          3 4H  25,1.316,1.708,2.485, 4H  26,1.315,1.706,2.479,              MLR 2950
          4 4H  27,1.314,1.703,2.473, 4H  28,1.313,1.701,2.467,              MLR 2960
          5 4H  29,1.311,1.699,2.462, 4H  30,1.310,1.697,2.457,              MLR 2970
          6 4H  40,1.303,1.684,2.423, 4H  50,1.298,1.676,2.403,              MLR 2980
          7 4H  60,1.296,1.671,2.390, 4H  80,1.292,1.664,2.374,              MLR 2990
          8 4H 100,1.290,1.660,2.365, 4H 200,1.286,1.653,2.345,              MLR 3000
          9 4H 500,1.283,1.648,2.334, 4H INF,1.282,1.645,2.326 /             MLR 3010
           WRITE(6,1080) ( XDATA(J), J=1,22 ), (YDATA(J),J=37,188)
C
           TSTART = SECOND(TSTART)
           PSTART = TSTART
C                                                                            MLR 3040
        10 CALL SLITE (0)                                                    MLR 3050
           DO 20 J=1,30                                                      MLR 3060
           DO 20 K=1,30                                                      MLR 3070
        20 INOPAC(J,K)=0                                                     MLR 3080
           JF = 0
C                                                                            MLR 3100
           READ (5,950) NPROB,NXV,NYV,INDEXY,NDATA,IDEN,INTYPE,NREAR,MAXSTF,IMLR 3110
          1FBACK,NSTART,MINSUM,MAXSUM,MAXREG,IFMT,IFCNST,IFLIST,IFSUMS,IFAVE,MLR 3120
          2IFCOPR,NCROSS,IFTRA,NSKIP,IFSUB,NFMT,IFPNCH,IFDATE,IFNAME         MLR 3130
           IF ( EOF(5) .NE. 0.0 )  GO TO 805
           WRITE (6,810)                                                     MLR 3090
C                                                                            MLR 3140
           WRITE (6,1010) NPROB,NXV,NYV,INDEXY,NDATA,IDEN,INTYPE,NREAR,MAXSTFMLR 3150
          1,IFBACK,NSTART,MINSUM,MAXSUM,MAXREG,IFMT,IFCNST,IFLIST,IFSUMS,IFAVMLR 3160
          2E,IFCOPR,NCROSS,IFTRA,NSKIP,IFSUB,NFMT,IFPNCH,IFDATE,IFNAME       MLR 3170
           D1=BLANK                                                          MLR 3180
C               DATE IS MAP S/R TO PICK DATE OFF SEQUENCE CARD              MLR 3220
           IF (IFDATE.EQ.0) CALL DATE (D1 )
           NBRNOW=NSTART                                                     MLR 3230
           IF( NFMT .EQ. 0 )  FMT(1) = RMT
           IF (NDATA.LE.0) NDATA=10000                                       MLR 3280
           NWTT=NXV+NYV                                                      MLR 3290
           IF (IFMT.NE.0) NWTT=NWTT+1                                        MLR 3300
           IF (IFSUB.GT.0) CALL NBROFX (IFSUB,NXV)                           MLR 3310
           NSTEP=0                                                           MLR 3320
           NSUMRY=0                                                          MLR 3330
           TOL=0.00001                                                       MLR 3340
           NBRXY=NXV+1                                                       MLR 3350
C               NBRX  = NUMBER OF INDEPENDENT VARIABLES                      MLR 3360
C               NBRXY = NUMBER OF INDEPENDENT VARIABLES + DEPENDENT VARIABLE MLR 3370
C               NBRXYM= SIZE OF ARRAY = NBRXY + 1                            MLR 3380
C               NBRNOW = NUMBER OF COEFFICIENTS FOR PRESENT  EQUATION        MLR 3390
C               INDEX  = INDEX OF PRESENT  EQUATION                          MLR 3400
           IF (MAXSTP.EQ.0) MAXSTP=998                                       MLR 3410
           IF (MAXREG.LT.5) MAXREG=5                                         MLR 3420
C                                                                            MLR 3430
C            TO STORE DATA ON TAPE FOR USE IN ANOTHER PROBLEM, SET INTYPE=1  MLR 3440
C               FOR REWIND,  INTYPE=3 FOR NO REWIND                          MLR 3450
C               TO USE DATA FROM A PREVIOUS PROBLEM   SET INTYPE = 2 OR 5,   MLR 3460
C               THEREBY CAUSING TAPE TO REWIND.                              MLR 3470
C               INTYPE = 4 OR 6 FOR BIN. TAPE 10 OR 11 TO BE USED AS INPUT   MLR 3480
C                     ALSO PREVENTS TAPE REWIND AT START OF PROBLEM.         MLR 3490
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                NREWB5=1                                                        MLR 3500
                IF (INTYPE.NE.7) GO TO 50                                       MLR 3510
                NREWB5=0                                                        MLR 3520
                INTYPE=5                                                        MLR 3530
             50 NTAPE=10                                                        MLR 3540
                NREAD=0                                                         MLR 3550
                NWRITE=0                                                        MLR 3560
                IF (INTYPE.EQ.0) GO TO 60                                       MLR 3570
                IF (INTYPE.EQ.1.OR.INTYPE.EQ.3) NWRITE=1                        MLR 3580
                IF (INTYPE.EQ.5.OR.INTYPE.EQ.6) NTAPE=11                        MLR 3590
                IF (INTYPE.EQ.1.OR.INTYPE.EQ.2.OR.INTYPE.EQ.5) REWIND NTAPE     MLR 3600
                IF (INTYPE.NE.1.AND.INTYPE.NE.3) NREAD=1                        MLR 3610
          C                                                                     MLR 3620
          C           NUMBER OF INDEPENDENT + DEPENDENT VARIABLES               MLR 3630
             60 NTOTAL=NXV+NYV                                                  MLR 3640
                NEW=NXV+INDEXY                                                  MLR 3650
                IF (NTOTAL.LE.52) GO TO 70                                      MLR 3660
          C           TOO MANY VARIABLES                                        MLR 3670
                NTOTAL=NEW                                                      MLR 3680
                IF (NTOTAL.LE.52) GO TO 70                                      MLR 3690
                WRITE (6,820) NTOTAL                                            MLR 3700
                CALL EXIT                                                       MLR 3710
             70 NOSTEP=0                                                        MLR 3720
          C           CHECK FOR CROSS-PRODUCTS OR POLYNOMIAL
                IF (NCROSS.EQ.0) GO TO 80                                       MLR 3730
                IF( NCROSS .GT. 1 ) GO TO 75
                WRITE(6,1110)
                NOVAR= (NBRXY*(NBRXY+1))/2                                      MLR 3740
                IF (NOVAR.LE.51) GO TO 90                                       MLR 3750
                NCROSS=0                                                        MLR 3760
                WRITE (6,850)                                                   MLR 3770
                GO TO 80
             75 IF( NCROSS .GT. 50 ) NCROSS = 50
                WRITE(6,1120) NCROSS
                IF( NXV .EQ. 1 ) GO TO 76
                NCROSS = 0
                WRITE(6,1130)
                GO TO 80
             76 NBRXY = NCROSS + 1
          C
          C
             80 NOVAR=NBRXY                                                     MLR 3780
             90 IF (IFCNST.LT.0) NOVAR=NOVAR+1                                  MLR 3790
                NBRXYW=NOVAR+1                                                  MLR 3800
                NBRX=NOVAR-1                                                    MLR 3810
                MAXVAR=MIN0(MAXSUM,MAXREG,NBRX)                                 MLR 3820
          C           READ CONTROL CARD C                                       MLR 3830
                IF( IDEN ) 95, 94,110                                           MLR 3840
             94 DO 100 J=1,16                                                   MLR 3850
            100 ALPHA(J)=BLANK                                                  MLR 3860
             95 IDEN = IABS(IDEN)                                               MLR 3870
                GO TO 130                                                       MLR 3880
            110 DO 120 I=1,IDEN                                                 MLR 3890
                READ (5,830) (ALPHA(J),J=1,16)                                  MLR 3900
            120 WRITE (6,840) (ALPHA(J),J=1,16)                                 MLR 3910
            130 CONTINUE                                                        MLR 3920
          C           READ CONTROL CARD D                                       MLR 3930
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PROGRAM MLR        74/74    OPT=1                            FTN 4.2+74278      11/22/74  12.18.21.

              DO 140 J=1,60                                                    MLR 3940
              KSTEP(J)=1                                                       MLR 3950
          140 TEST(J)=+2.0E+30                                                 MLR 3960
              IF (NSTART) 170,200,150                                          MLR 3970
          150 READ (5,930) (TEST(J),INDEX(J),J=1,NBRNCH)                       MLR 3980
C                   PACK INDEX                                                 MLR 3990
              DO 160 J=1,NBRNOW                                                MLR 4000
          160 CALL PACK (NBRNOW,J,INDEX(J),1)                                  MLR 4010
              GO TO 190                                                        MLR 4020
        CC170 NBRNOW=NBRX                                                      MLR 4030
          170 NBRNOW=NXV                                                       MLR 4040
              DO 180 J=1,NBRNOW                                                MLR 4050
              CALL PACK (NBRNOW,J,J,1)                                         MLR 4060
              INDEX(J) = J
              TEST(J) = 1.000                                                  MLR 4070
          180 IF (NSTART.LT.-1) TEST(J)=0.0                                    MLR 4080
          190 WRITE (6,940) START,(TEST(J),INDEX(J),J=1,NBRNCH)                MLR 4090
          200 MINVAR=MAX0(1,MINSUM)                                            MLR 4100
              NTAPE9=1                                                         MLR 4110
              IF (IFBACK.EQ.0) GO TO 210                                       MLR 4120
              IF (NDATA*NBRXYW.LE.3000) GO TO 210                              MLR 4130
              NTAPE9=0                                                         MLR 4140
              REWIND 9                                                         MLR 4150
          210 DO 220 I=1,NBRXYW                                                MLR 4160
              DO 220 J=1,NBRXYW                                                MLR 4170
          220 A(I,J)=0.0                                                       MLR 4180
C                   READ CONTROL CARD E                                        MLR 4190
              IF (NSKIP.LE.0) GO TO 270                                        MLR 4200
              READ (5,950) (JVOID(J),J=1,14)                                   MLR 4210
              JV=0                                                             MLR 4220
              DO 250 L=1,28,2                                                  MLR 4230
              NOGOOD(L)=0                                                      MLR 4240
              NOGOOD(L+1)=0                                                    MLR 4250
          230 IF (JV.EQ.14) GO TO 260                                          MLR 4260
              JV=JV+1                                                          MLR 4270
              IF (JVOID(JV)) 250,230,240                                       MLR 4280
          240 NOGOOD(L)=JVOID(JV)                                              MLR 4290
              NOGOOD(L+1)=JVOID(JV)                                            MLR 4300
              LBAD=L+1                                                         MLR 4310
              IF (JVOID(JV+1)) 230,260,260                                     MLR 4320
          250 NOGOOD(L+1)=IABS(JVOID(JV))                                      MLR 4330
          260 CONTINUE                                                         MLR 4340
          270 NSKIP=IABS(NSKIP)                                                MLR 4350
              IF (NSKIP.NE.0) WRITE (6,960) VOTOED,(NOGOOD(J),J=1,LBAD)        MLR 4360
C                   READ CONTROL CARD F                                        MLR 4370
              LOOK=0                                                           MLR 4380
              IF (NREAR) 290,300,280                                           MLR 4390
C                   IF NREAR=1      READ SET OF SEARCH PARAMETERS.             MLR 4400
          280 NNXJ=NXV                                                         MLR 4410
              IF (IFWT.NE.0) NNXJ=NXV+1                                        MLR 4420
              READ (5,950) NWORDS,LOCY,(LOCX(J),J=1,NNXJ)                      MLR 4430
          290 WRITE (6,960) SEARCH,NWORDS,LOCY,(LOCX(J),J=1,NNXJ)              MLR 4440
              LOOK=1                                                           MLR 4450
          300 CONTINUE                                                         MLR 4460
C                   READ CONTROL CARD G                                        MLR 4470
C                           READ TRANSFORMATIONS                               MLR 4480
              IF (IFTRA.GT.0) READ (5,930) (CONST(I),NBRTRA(I),I=1,NTOTAL)     MLR 4490
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```
      IF (IFLIST.NE.0) GO TO 310                                        MLR 4500
      WRITE (6,990) ALPHA,D1,    (AX,J,J=1,NXV),(AY,J,J=1,NYV)
  310 NWT=NTOTAL                                                        MLR 4520
      IF (IFTRA.NE.0) WRITE (6,940) TRAN,(CONST(I),NBRTRA(I),I=1,NTOTAL)MLR 4530
C         READ CONTROL CARD H      NAME OF VARIABLES                    MLR 4540
      IF( IFNAME ) 315,314,312                                          MLR 4550
  312 DO 313 J=1,2                                                      MLR 4560
      DO 313 L=1,NTOTAL                                                 MLR 4570
  313 VNAME(J,L) = BLANK                                                MLR 4580
      GO TO 315                                                         MLR 4590
  314 READ(5,985) ( (VNAME(J,L), J=1,2), L=1,NTOTAL )                   MLR 4600
  315 IF(IFLIST.EQ.0)WRITE(6,986) ((VNAME(J,L),J=1,2),L=1,NTOTAL)       MLR 4610
C                                                                       MLR 4620
C         READ CONTROL CARD I   ( VARIABLE FORMAT )                     MLR 4630
      IF (NFMT.GT.0) READ (5,970) FMT                                   MLR 4640
      IF (NFMT.NE.0) WRITE (6,970) FORAMT,FMT                           MLR 4650
      IF (IFNT.NE.0) NWT=NTOTAL+1                                       MLR 4660
      NSKIP=IAES(NSKIP)                                                 MLR 4670
      NOAD=0                                                            MLR 4680
      YCONST=CONST(NEW)                                                 MLR 4690
      YTRA(1) = ACT                                                     MLR 4700
      YTRA(2) = CAL                                                     MLR 4790
      J = NBRTRA(NEW)                                                   MLR 4740
      NYTRA=0                                                           MLR 4700
      IF( IFTRA.EQ.0 .OR.  J.EQ.0 )  GO TO 316
      IF (NBRTRA(NEW).EQ.7) NYTRA=-1                                    MLR 4710
      IF (NBRTRA(NEW).EQ.8) NYTRA=+1                                    MLR 4720
      YTRA(1) = ATRA(1,J)                                               MLR 4750
      YTRA(2) = ATRA(2,J)                                               MLR 4760
  316 CONTINUE                                                          MLR 4010
      JDATA = 0
C                                                                       MLR 4020
C.......................................................................
C                                                                       MLR 4030
      DO 520 N=1,NDATA                                                  MLR 4040
      IF (NREAD) 370,370,320                                            MLR 4050
  320 IF (LOCK) 330,330,350                                             MLR 4060
  330 READ(NTAPE)  ( POINT(J),J=1,NWTT )
      IF( EOF(NTAPE) )  800, 420
  350 READ(NTAPE)  ( XDATA(J), J=1,NWORDS )
      IF( EOF(NTAPE) )  800, 400
C                                                                       MLR 5010
  370 IF (LOCK) 380,380,390                                             MLR 5020
  380 READ (5,FMT) (POINT(J),J=1,NWTT)                                  MLR 5030
      IF( EOF(5) )  800,385
  385 IF (NWRITE.NE.0) WRITE (10) (POINT(J),J=1,NWTT)                   MLR 5040
      GO TO 420                                                         MLR 5050
  390 READ (5,FMT) (XDATA(J),J=1,NWORDS)                                MLR 5060
      IF( EOF(5) )  800,395
  395 IF (NWRITE.NE.0) WRITE (10) (XDATA(J),J=1,NWORDS)                 MLR 5070
  400 DO 410 J=1,NNXJ                                                   MLR 5080
      JLOC=LOCX(J)                                                      MLR 5090
  410 POINT(J)=XDATA(JLOC)                                              MLR 5100
      POINT(NWT)=POINT(NNXJ)                                            MLR 5110
      POINT(NEW)=XDATA(LOCY)                                            MLR 5120
C                                                                       MLR 5130
C         CHECK FOR END OF DATA INDICATOR                               MLR 5140
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      420 IF (POINT(1).EQ.99999999.) GO TO 530                         MLR 5150
          IF (IFSUB.GT.0) CALL EQUAT (IFSUB,POINT)                     MLR 5170
          IF (IFLIST.EQ.0) WRITE (6,910) N,(POINT(J),J=1,NWT)          MLR 5180
C             THROW AWAY BLANK DATA  ( BLANK = -0. )
          IFBLK = 0
          IF( IFBLK .NE. 0 ) GO TO 428
          DO 425 J=1,NXV
          IF( POINT(J) .NE. 0. ) GO TO 425
          IF( SIGN(+1., POINT(J) ) )  427,425,425
      425 CONTINUE
          IF( POINT(NEW) .NE. 0. ) GO TO 428
          IF( SIGN(+1., POINT(NEW) )) 427,428,428
      427 WRITE(6,1105)  N
          GO TO 520
C
      428 JDATA = JDATA + 1
          IF (IFTRA.EQ.0) GO TO 430                                    MLR 5190
          CALL CHANGE (POINT,NBRTRA,CONST,NTOTAL)                      MLR 5200
          IF (IFLIST.EQ.0) WRITE (6,920) N,(POINT(J),J=1,NWT)          MLR 5210
C                                                                      MLR 5220
      430 CONTINUE                                                     MLR 5230
                                                                       MLR 5240
          POINT(NOVAR)=POINT(NEW)                                      MLR 5250
          WHT=1.0                                                      MLR 5260
          IF (IFWT.NE.0) WHT=POINT(NWT)                                MLR 5270
          POINT(NBRXYW)=WHT                                            MLR 5280
          IF (NCROSS.EQ.0) GO TO 450                                   MLR 5290
          IF( NCROSS .GT. 1 ) GO TO 445
C                   CROSS PRODUCTS ARE USED AS INDEPENDENT VARIABLES   MLR 5300
          L=NBRXY                                                      MLR 5310
          DO 440 I=2,NBRXY                                             MLR 5320
          DO 440 J=I,NBRXY                                             MLR 5330
          POINT(L)=POINT(I-1)*POINT(J-1)                               MLR 5340
      440 L=L+1                                                        MLR 5350
          GO TO 450
C             GENERATE POWERS FOR POLYNOMIAL
      445 DO 446 J=2,NCROSS
      446 POINT(J) = POINT(J-1) * POINT(1)
      450 IF (IFCNST.LT.0) POINT(NOVAR-1)=1.0                          MLR 5360
          IF (IFBACK.EQ.0) GO TO 480                                   MLR 5370
          IF (KTAPE9.EQ.0) GO TO 470                                   MLR 5380
C             STORE IN STRING IF DATA POINTS * VARIABLES LESS THAN 3000 MLR 5390
          DO 460 J=1,NBRXYW                                            MLR 5400
          JJ=NBRXYW*(N-1)+J                                            MLR 5410
      460 STRING(JJ)=POINT(J)                                          MLR 5420
          GO TO 480                                                    MLR 5430
C         STORE DATA ON TAPE 9 IF DATA POINTS * VARIABLES EXCEED 3000  MLR 5440
      470 WRITE (9) (POINT(K),K=1,NOVAR),WHT                           MLR 5450
C                                                                      MLR 5460
      480 CONTINUE                                                     MLR 5470
          IF (NSKIP.EQ.0) GO TO 500                                    MLR 5480
C             CHECK TO SEE IF POINT IS TO BE DELETED FROM REGRESSION   MLR 5490
          DO 490 J=1,LBAC,2                                            MLR 5500
          IF (N.LT.NOGOOD(J).OR.N.GT.NOGOOD(J+1)) GO TO 490            MLR 5510
          NBAD=NBAD+1                                                  MLR 5520
          GO TO 520                                                    MLR 5530
      490 CONTINUE                                                     MLR 5540
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      C                                                                    NLR 5550
  500 DO 510 I=1,NOVAR                                                     NLR 5560
      C     SUM X(I)                                                       NLR 5570
      A(I,NBRXYN)=A(I,NBRXYN)+POINT(I)*NHT                                 NLR 5580
      DO 510 J=I,NOVAR                                                     NLR 5590
      C     SUM X(I)*X(J)                                                  NLR 5600
  510 A(I,J)=A(I,J)+POINT(I)*POINT(J)*NHT                                  NLR 5610
      A(NBRXYN,NBRXYN)=A(NBRXYN,NBRXYN)+NHT                                NLR 5620
  520 CONTINUE                                                             NLR 5630
      C                                                                    NLR 5640
      C ****************************************************************** NLR 5650
      C                                                                    NLR 5660
  530 NDATA=JDATA-NBAD                                                     NLR 5670
      DEFRM=NDATA                                                          NLR 5680
      DENOM=A(NBRXYN,NBRXYN)-1.0                                           NLR 5690
      IF (IFWT.NE.0) DENOM=DENOM+1.0                                       NLR 5700
      IF (NTAPE9.EQ.0) RENINT 9                                            NLR 5710
      IF (NRENB5.EQ.0) RENINT 11                                           NLR 5720
      C                                                                    NLR 5730
      WRITE (6,1000) NPROB,ALPHA,NDATA,NOVAR,A(NBRXYN,NBRXYN),D1
      K=2                                                                  NLR 5750
      IF (IFTRA.EQ.0) K=1                                                  NLR 5760
      WRITE (6,860) (BLANK,J=1,K)                                          NLR 5770
      WRITE (6,870)                                                        NLR 5780
      IX1=60                                                               NLR 5790
      IX2=0                                                                NLR 5800
      IXP = 1
      C                                                                    NLR 5810
      DO 590 J=1,NOVAR                                                     NLR 5820
      YMEAN=A(J,NBRXYN)/A(NBRXYN,NBRXYN)                                   NLR 5830
      STDEV=SQRT((A(J,J)-A(J,NBRXYN)*YMEAN)/DENOM)                         NLR 5840
      TRA(1)=BLANK                                                         NLR 5850
      TRA(2)=BLANK                                                         NLR 5860
      V(1,J)= VNAME(1,J)                                                   NLR 5870
      V(2,J) = VNAME(2,J)                                                  NLR 5880
      L=J                                                                  NLR 5890
      K=2                                                                  NLR 5900
      IF (J.GT.NXV) GO TO 560                                              NLR 5910
  540 IF (IFTRA.EQ.0) GO TO 580                                            NLR 5920
      I=NBRTPA(L)                                                          NLR 5930
      IF (I.LE.0) GO TO 580                                                NLR 5940
      K=3                                                                  NLR 5950
      TRA(3)=CONST(L)                                                      NLR 5960
  550 TRA(1)=ATRA(1,I)                                                     NLR 5970
      TRA(2)=ATRA(2,I)                                                     NLR 5980
      GO TO 580                                                            NLR 5990
  560 I=17                                                                 NLR 6000
      V(1,J) = 6HCONST.
      V(2,J) = 5H TERM
      IF (IFCNST.LT.0.AND.J.EQ.NOVAR-1) GO TO 550                          NLR 6030
      L=NXV+INDEXY                                                         NLR 6040
      V(1,J) = VNAME(1,L)                                                  NLR 6050
      V(2,J) = VNAME(2,L)                                                  NLR 6060
      IF (J.EQ.NOVAR) GO TO 540                                            NLR 6070
      IF( NCROSS .GE. 2 )  GO TO 575
      C     CROSS PRODUCTS                                                 NLR 6080
      IX1=IX1+1                                                            NLR 6090
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PROGRAM MLR        74/74    OPT=1                          FTN 4.2+74278       11/22/74  18.18.21.

            IF (IX1.LE.NXV) GO TO 570                                          MLR 6100
            IX2=IX2+1                                                          MLR 6110
            IX1=IX2                                                            MLR 6120
        570 WRITE (6,880) J,YMEAN,STDEV,IX2,IX1                                MLR 6130
            V(1,J) = CROSS(IX2)                                                MLR 6140
            V(2,J) = CROSS(IX1)                                                MLR 6150
            GO TO 590                                                          MLR 6160
      C         POLYNOMIALS
        575 V(1,J) = V(1,1)
            IXP = IXP + 1
            ENCODE(6,1140,V(2,J))   IXP
        580 IF(J.EQ.NOVAR) GO TO 590                                           MLR 6170
            WRITE(6,890) V(1,J),V(2,J),J, YMEAN, STDEV, (TRA(L),L=1,K)         MLR 6180
        590 CONTINUE                                                           MLR 6190
            J = NOVAR
            WRITE(6,900) V(1,J),V(2,J),   YMEAN, STDEV, (TRA(L),L=1,K)         MLR 6200
      C                                                                        MLR 6210
            IF (IFSUMS.EQ.0) CALL PRTSUM                                       MLR 6220
            IF (IFCNST.NE.0) GO TO 600                                         MLR 6230
            CALL RESID                                                         MLR 6240
            DEFRM=DEFRM-1.0                                                    MLR 6250
        600 CALL CORREL                                                        MLR 6260
            CALL SLITET(1,LIGHT)                                               MLR 6270
            IF( LIGHT.EQ.1 )  GO TO 10                                         MLR 6280
            NDATA=JDATA                                                        MLR 6290
            NSKIP=NBAD                                                         MLR 6300
            IF(MAXSTP.EQ.999) WRITE(6,1060)                                    MLR 6310
      C     ******************************************************************MLR 6320
      C                                                                        MLR 6330
        605 LPATH = 1                                                          MLR 6340
            KPATH = 1                                                          MLR 6350
            IF (NBRNOW.GT.0) GO TO 710                                         MLR 6360
        610 IF (A(NOVAR,NOVAR).GT.0.0) GO TO 620                               MLR 6370
            WRITE (6,1030) A(NOVAR,NOVAR)                                      MLR 6380
            GO TO 750                                                          MLR 6400
      C                                                                        MLR 6410
        615 JP = 0                                                             MLR 6420
            NBRPVR = NBRNOW                                                    MLR 6430
            NBRNOW = NBRNOW - 1                                                MLR 6440
            DO 617 J=1,NBRPVR                                                  MLR 6450
            CALL PACK( NBRPVR, J, 0, 1 )                                       MLR 6460
            IF( INDEX(J) .EQ. KVAR )  GO TO 617
            JP = JP + 1                                                        MLR 6480
            CALL PACK( NBRNOW, JP, INDEX(J), 1 )                               MLR 6490
        617 CONTINUE                                                           MLR 6500
            DO 618 I=1,NOVAR                                                   MLR 6510
            DO 618 J=1,NOVAR                                                   MLR 6520
        618 A(I,J) = SIMCOR(I,J)                                               MLR 6530
            GO TO 605                                                          MLR 6540
      C                                                                        MLR 6550
        620 CALL ACOTO                                                         MLR 6560
            CALL SLITET (1,LIGHT)                                              MLR 6570
            GO TO (750,630), LIGHT                                             MLR 6580
        630 GO TO (640,720,690), LPATH                                         MLR 6590
      C                                                                        MLR 6600
        640 TEST(NBRNOW)=A(NOVAR,NOVAR)                                        MLR 6610
        650 CALL OUTPUT                                                        MLR 6620
```

```
      CALL SLITET (2,LIGHT)                                             MLR 6630
      IF( LIGHT .EQ. 1 ) GO TO 615                                      MLR 6640
      NSTEP=NSTEP+1                                                     MLR 6650
      IF (NSTEP.GT.MAXSTP) GO TO 750                                    MLR 6660
      IF (NBRNOW.GE.MAXREG) GO TO 750                                   MLR 6670
      CALL SLITET (1,LIGHT)                                             MLR 6680
      GO TO (750,660), LIGHT                                            MLR 6690
  660 GO TO (680,670,10), KPATH                                         MLR 6700
  670 KPATH=1                                                           MLR 6710
C          SEE IF VARIABLE CAN BE REMOVED
  680 IF( NBRNOW.LE.2  .AND.  JF.NE.0 )  GO TO 700
      CALL REMOVE                                                       MLR 6720
      IF( NODEL.EQ.0 )  GO TO 700
C          VARIABLE WAS REMOVED
      LPATH=1                                                           MLR 6750
  690 CALL MATRIX                                                       MLR 6760
      GO TO (650,740,700), LPATH                                        MLR 6770
  700 IF (NBRNOW.LT.NBRX) GO TO 610                                     MLR 6780
      WRITE (6,1020)                                                    MLR 6790
      GO TO 750                                                         MLR 6800
  710 KPATH=2                                                           MLR 6810
  720 L=1                                                               MLR 6820
      LPATH=2                                                           MLR 6830
  730 CONTINUE                                                          MLR 6840
      CALL PACK (NORNOW,L,KVAR,2)
      GO TO 690                                                         MLR 6860
C     ********************* TRY ADJUNCTION *************************** MLR 6870
  740 L=L+1                                                             MLR 6880
      IF (L.LE.NBRNOW) GO TO 730                                        MLR 6890
      GO TO (700,640,780), KPATH                                        MLR 6900
C                                                                       MLR 6910
C          SUMMARY                                                      MLR 6920
  750 WRITE(6,1040) ALPHA,NPROB                                         MLR 6930
      IF (IFBACK.EQ.999) IFBACK=NSTEP                                   MLR 6940
      IF (IFBACK.EQ.999) IFOACK=1                                       MLR 6950
      IF (MAXSTP.EQ.999) MAXSTP=998                                     MLR 6960
      DO 757 J=1,NBRNOW                                                 MLR 6970
      DO 755 L=1,J                                                      MLR 6980
  755 CALL PACK(J,L,INDEX(L),2)                                         MLR 6990
      IF( INDEX(1).LE.0 ) GO TO 757                                     MLR 7000
      WRITE(6,1070)J,KSTEP(J),STOERR(J),CORSCR(J), (INDEX(L),L=1,J)     MLR 7010
  757 CONTINUE                                                          MLR 7020
      KPATH=3                                                           MLR 7030
      NBRNOW=MINVAR                                                     MLR 7040
      NSUMRY=1                                                          MLR 7050
  760 IF( NBRNOW.GT.MAXVAR ) GO TO 795
      CALL PACK (NBRNOW,1,J,2)                                          MLR 7070
      IF (J.LE.0) GO TO 790                                             MLR 7080
      DO 770 I=1,NOVAR                                                  MLR 7090
      DO 770 J=1,NOVAR                                                  MLR 7100
  770 A(I,J)=SIMCOR(I,J)                                                MLR 7110
      GO TO 720                                                         MLR 7120
  780 CALL OUTPUT                                                       MLR 7130
  790 NBRNOW=NBRNOW+1                                                   MLR 7140
      GO TO 760                                                         MLR 7150
C          COMPUTE CP SECONDS FOR PROBLEM
  795 PEND   = SECOND( PEND )
```

```
      PSTART = PEND - PSTART
      WRITE(6,1100) PSTART
      PSTART = PEND
      GO TO 10
  800 WRITE (6,1050) JDATA,(POINT(J),J=1,NWTT)                          MLR 7160
      GO TO 531
  805 WRITE(6,1050)
CCC   STOP
C                                                                       MLR 7190
C                                                                       MLR 7200
C                                                                       MLR 7210
  810 FORMAT (47H1 NONSIMPLE STEPWISE MULTIPLE LINEAR REGRESSION45X23HARMLR 7220
     1MY COMPUTATION CENTER/92X20HARMY MISSILE COMMAND/92X25HREDSTONE ARMLR 7230
     2SENAL, ALABAMA)                                                   MLR 7240
  820 FORMAT (37H0 TOTAL NUMBER OF VARIABLES TOO LARGEI6)               MLR 7250
  830 FORMAT (16A5)                                                     MLR 7260
  840 FORMAT (/3X,16A5)                                                 MLR 7270
  850 FORMAT (47H  CROSS PRODUCTS DELETED, AS THERE ARE TOO MANY//)     MLR 7280
  860 FORMAT (20X35HVAR    WTED AVERAGE     STAND. DEV. ,2A1,3X,25HTRANSMLR 7290
     1FORMATION    CONSTANT)                                            MLR 7300
  870 FORMAT (1X)                                                       MLR 7310
  890 FORMAT (7X,2A6,I3    ,1X,2E16.6,6X,A10,A2,E16.4)
  880 FORMAT (19X    ,I3    ,1X,2E16.6,6X,2HX(I2,6H) *.X(I2,1H) )       MLR 7370
  900 FORMAT (7X,2A6,3H   Y,1X,2E16.6,6X,A10,A2,E16.4)
  910 FORMAT (I9,1X,10F12.3/(10X,10F12.3))                              MLR 7350
  920 FORMAT (5X,1H(I3,1H),10F12.3/(10X,10F12.3))                       MLR 7360
  930 FORMAT (7(F8.0,I2))                                               MLR 7370
  940 FORMAT (1H ,3XA6,10(F10.3,I2)/(10X,10(F10.3,I2)))                 MLR 7380
  950 FORMAT (14I5)                                                     MLR 7390
  960 FORMAT (1HJ,A6,24I5/(7X24I5))                                     MLR 7400
  970 FORMAT( 11A10 )
  985 FORMAT( 7(A6,A4) )                                                MLR 7430
  986 FORMAT( 12X20A6 )                                                 MLR 7440
  990 FORMAT (34H1  * *  I N P U T      D A T A  * *5X16A5,2XA10  //E19X,
     110(7XA1,1H(I2,1H))/))                                             MLR 7460
 1000 FORMAT (37H1STEPWISE REGRESSION PROBLEM NUMBER  I5,10X16A5/23H NUMMLR 7470
     1BER OF OBSERVATIONS14X,I5/20H NUMBER OF VARIABLES17XI5/30H WEIGHTEMLR 7480
     2D DEGREES OF FREEDOM  F12.3,78XA10//)
 1010 FORMAT (//35H  P R O G R A M      C O N T R O L S///10H   NFROB =I MLR 7500
     15,10H   NXV   =I5,10H   MYV   =I5,10H   INDEXY=I5,10H   NDATA =I5,MLR 7510
     210H   IOEM  =I5,10H   IMTYPE=I5//10H   NREAR =I5,10H   MAXSTF=I5,1MLR 7520
     30H   IFBACK=I5,10H   MSTART=I5,10H   MINSUM=I5,10H   MAXSUM=I5,10HMLR 7530
     4   MAXREG=I5//10H   IFWT  =I5,10H   IFCAST=I5,10H   IFLIST=I5,10H MLR 7540
     5   IFSUMS=I5,10H   IFRES =I5,10H   IFCORR=I5,10H   NCROS =I5//10H  MLR 7550
     6 IFTRA =I5,10H   NVOID =I5,10H   IFSUB =I5,10H   NFMT   I5,10H   IMLR 7560
     7FPNCH=I5,10H   IFDATE=I5,10H   IFNAME=,I5)                        MLR 7570
 1020 FORMAT (51H0 ***************** VARIABLES EXHAUSTED ************/)  MLR 7580
 1030 FORMAT (80H0************** PROBLEM TERMINATED, SUM OF SQUARES IS NOMLR 7590
     1N-POSITIVE **************E13.5/)                                  MLR 7600
 1050 FORMAT (//34H END OF FILE REACHED AFTER READINGI5,8H POINTS./20H0LMLR 7610
     1AST OBSERVATION IS//(1X,8E16.8))                                  MLR 7620
 1060 FORMAT(33H1MULTIPLE LINEAR REGRESSION STEPS)                      MLR 7630
 1040 FORMAT(34H1SUMMARY OF BEST SETS OF VARIABLES,4X16A5,9H  PROBLEMI6/MLR 7640
     */25H  M STEP    STD ERR    RSQ / )                                MLR 7650
 1070 FORMAT(1X,I2,I4,F11.3,F9.6,1X,35I3/28X,35I3)                      MLR 7660
 1050 FORMAT( 6A10//8A10/8A10//40( A4,2X,3F10.3/ ) )
 1090 FORMAT( 1H1,50X,10HEND OF JOB )
```

```
PROGRAM MLR        74/74    OPT=1                         FTN 4.2+74278      11/22/74  18.18.21.

        1100 FORMAT(1H1,F10.3/20(/1H-,60(2H* )))
        1105 FORMAT(6H POINT I4,26H DELETED DUE TO BLANK DATA )
        1110 FORMAT(25H0CROSS PRODUCTS GENERATED )
        1120 FORMAT(41H0POWERS GENERATED FOR POLYNOMIAL OF ORDER I3)
        1130 FORMAT(53H0ONLY ONE INDEPENDENT VARIABLE ALLOWED FOR POLYNOMIAL)
        1140 FORMAT(4H ** ,I2)
       C
             END                                                   MLR 7730
```

```
SUBROUTINE OUTPUT      74/74    OPT=1                              FTN 4.2+74278       11/22/74  18.18.37.

      SUBROUTINE OUTPUT                                                 AUTP   0
      COMMON SIGMA(60),A(52,52),SIMCOR(52,52),AVG(60),TEST(60)          AUTP  10
      COMMON PCINT(60),STRING(3000),INDPAC(30,30),INDEXP(61)            AUTP  20
      COMMON INDEX(60),NOUT(60),KSTEP(60),ALPHA(16),YMEAN,IDEN,IFAVE    AUTP  30
      COMMON MAXSTP,IFPNCH,NSUMRY,NSKIP,NTAPE9,NEW                      AUTP  40
      COMMON NCVAR,NBRNOW,NOSTEP,NDATA,NBRXYH,NBRX,LPATH,DEFRM,KVAR     AUTP  50
      COMMON IFBACK,IFCNST,IFCORR,NPROB,NBRPVR,TOL,NCDEL,JF             AUTP  60
      COMMON INDEXY,LBAD,NOGOOD(28)                                     AUTP  70
      COMMON IFWT,YCONST,NYTRA,V(2,51),YTRA(2)                          AUTP  80
      COMMON  STDERR(50),CORSQR(50)                                     AUTP  90
      REAL             A,SINCOR,SIGMA,AVG,TEST                          AUTP 100
C                                                                       AUTP 110
      DIMENSION  COEFF(61), ABC(5)                                      AUTP 120
      REAL             COEFF,CONST                                      AUTP 130
      REAL             SUMSQ,TSS,SIGY2,SIGY                             AUTP 140
      REAL             YPRED,YOBS,DEV,RSO,SQREG,SQREG2                  AUTP 150
      REAL              DEVSQ,CHISO,SUMSCU,CHISQL,DEVU,YO,YC
      DATA BLANK/1H /,VOID/6HVOIDED/,CHECK/6HREVIEW/                    AUTP 170
      DATA  ACTUAL/6HACTUAL/                                            AUTP 180
C         NBRNOW = NUMBER OF COEFFICIENTS FOR PRESENT  ECUATION         AUTP 190
C         INDEX  = INDEX OF PRESENT  EQUATION                           AUTP 200
C         NSUMRY = 0 FOR BUILDING PHASE. = 1 FOR SUMMARY PHASE.         AUTP 210
      IF( NBRNOW.EQ.1 )  JF = 1
      KPATH=1                                                           AUTP 220
      IF (NSKIP.NE.0) KPATH=2                                           AUTP 230
      WMT=1.0                                                           AUTP 240
      TSS = SIGMA(NDVAR)*SIGMA(NDVAR)                                   AUTP 250
      CALL SLITET (1,LIGHT)                                             AUTP 260
      GO TO (10,20), LIGHT                                              AUTP 270
   10 CALL SLITE (1)                                                    AUTP 280
      GO TO 30                                                          AUTP 290
   20 NOSTEP=NOSTEP+1                                                   AUTP 300
      IF (NSUMRY.EQ.1) NOSTEP=KSTEP(NBRNOW)                             AUTP 310
   30 DO 40 J=1,60                                                      AUTP 320
   40 NOUT(J)=0                                                         AUTP 330
      DO 50 J=1,NBRNOW                                                  AUTP 340
      CALL PACK (NBRNOW,J,I,2)                                          AUTP 350
      INDEX(J)=I                                                        AUTP 360
      NOUT(I)=1                                                         AUTP 370
C         BETA = A(I,NOVAR)                                             AUTP 380
   50 COEFF(J)=A(I,NOVAR)*SIGMA(NOVAR)/SIGMA(I)                         AUTP 390
      IF (IFCNST.EQ.0) GO TO 60                                         AUTP 400
      CONST=0.0                                                         AUTP 410
      GO TO 80                                                          AUTP 420
   60 CONST=AVG(NOVAR)                                                  AUTP 430
      DO 70 I=1,NBRNOW                                                  AUTP 440
      J=INDEX(I)                                                        AUTP 450
   70 CONST=CONST-(COEFF(I)*AVG(J))                                     AUTP 460
   80 SUMSQ = A(NOVAR,NOVAR) * TSS                                      AUTP 470
      XVAR=NBRNOW                                                       AUTP 480
      DEFR=DEFRM-XVAR                                                   AUTP 490
      NDEFR=DEFR                                                        AUTP 500
      NDEFRM=DEFRM                                                      AUTP 510
C.........................................................................

      IF (A(NCVAR,NDVAR).LT.0.0) A(NOVAR,NOVAR)=0.0D0                   AUTP1460
CC    SUMSQ=DABS(SUMSQ)                                                 AUTP1470
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      SUMSQ= ABS(SUMSQ)
      IF (NDEFR.GT.0) GO TO 240                                       AUTP1480
      WRITE (6,370) NOSTEP                                            AUTP1490
      SIGY=0.0                                                        AUTP1500
      SIGPCT=0.0                                                      AUTP1510
      REGRCO=0.0                                                      AUTP1520
      CALL SLITE (1)                                                  AUTP1530
      STDERR(NBRNOW) = 0.0                                            AUTP1540
      CORSQR(NBRNOW) = 0.0                                            AUTP1550
      IF (MAXSTP.EQ.999)  GO TO 999
      GO TO 260                                                       AUTP1570
  240 CONTINUE                                                        AUTP1580
      SIGY2=SUMSQ/DEFR                                                AUTP1590
CC    SIGY=DSQRT(SIGY2)                                               AUTP1600
      SIGY = SQRT ( SIGY2 )
      RSQ=1.0D0-A(NOVAR,NOVAR)                                        AUTP1610
      IF( NOSTEP .NE. KSTEP(NBRNOW) )  GO TO 245                      AUTP1620
      STDERR(NBRNOW) = SIGY                                           AUTP1630
      CORSQR(NBRNOW) = RSQ                                            AUTP1640
  245 CONTINUE                                                        AUTP1650
      IF (MAXSTP.EQ.999)  GO TO 999
      AYY=A(NOVAR,NOVAR)                                              AUTP1670
      TR=1.0                                                          AUTP1680
      REGRCO=SQRT(RSQ)                                                AUTP1690
      R2=RSQ/XVAR                                                     AUTP1700
      V2=A(NOVAR,NOVAR)/DEFR                                          AUTP1710
      FTEST=R2/V2                                                     AUTP1720
      SQREG=RSQ*TSS                                                   AUTP1730
      SQREG2=SQREG/XVAR                                               AUTP1740
      DATA CORREC,TED/6HCORREC,3HTED/,ORI,GIN/6H ORIGI,1HN/           AUTP1750
      BASE1=CORREC                                                    AUTP1760
      BASE2=TED                                                       AUTP1770
      IF (IFCNST.EQ.0) GO TO 250                                      AUTP1780
      BASE1=ORI                                                       AUTP1790
      BASE2=GIN                                                       AUTP1800
  250 WRITE (6,440) NOSTEP,ALPHA,NPRCB,BASE1,BASE2,NDEFRM,TSS,TR,NERNCW,AUTP1810
     1SQREG,SQREG2,RSQ,R2,FTEST,NDEFR,SUMSQ,SIGY2,AYY,V2              AUTP1820
      SIGPCT=ABS(SIGY/YMEAN*100.0)                                    AUTP1830
C                                                                     AUTP1840
  260 WRITE(6,380) SIGY,V(1,NOVAR),V(2,NOVAR),SIGPCT,REGRCO           AUTP1850
      DO 290 J=1,NBRX                                                 AUTP1860
CC    IF (A(J,J)-TOL) 270,270,280                                     AUTP1870
CC270 POINT(J)=0.99E22                                                AUTP1880
CC    GO TO 290                                                       AUTP1890
  280 POINT(J)=A(J,NOVAR)*A(NOVAR,J)/A(J,J)                           AUTP1900
  290 CONTINUE                                                        AUTP1910
C                                                                     AUTP1920
      WRITE (6,400)                                                   AUTP1930
      KK=0                                                            AUTP1940
      IF (IFPNCH.NE.0.AND.NSUMRY.EQ.1) WRITE (7,480) KK,CONST,NFRCE,NBRAUTP1950
     1OW,INDEXY,NOSTEP,IFPNCH,SIGPCT,REGRCO                           AUTP1960
      IF (IFCNST.EQ.0) WRITE (6,390) CONST                            AUTP1970
C                                                                     AUTP1980
C     **** LIST COEFFICIENTS                                          AUTP1990
      DO 310 J=1,NBRNOW                                               AUTP2000
      I=INDEX(J)                                                      AUTP2010
      IF (IFPNCN.EQ.0.OR.NSUMRY.EC.0) GO TO 300                       AUTP2020
```

```
      WRITE (7,480) I,COEFF(J),NPROB,NBRNOW,INOEXY,NCSTEP,IFPNCH,SIGFCT,AUTP2030
     1REGRCO                                                             AUTP2040
  300 SEB=SQRT(ABS(A(I,I)))*SIGY/SIGMA(I)                                AUTP2050
      CT=COEFF(J)/SEB                                                    AUTP2060
      IF( SEB .EQ. 0.0 )  CT = 0.0                                       AUTP2070
      F = CT * CT
      WRITE(6,410) V(1,I),V(2,I),I,COEFF(J),SEB,CT,F,A(I,NOVAR),POINT(I)
      IF( POINT(I).NE.0.0 )  GO TO 310                                   AUTP2090
      WRITE(6,510)                                                       AUTP2100
      CALL SLITE (2)                                                     AUTP2110
      KVAR = I                                                           AUTP2120
  310 CONTINUE                                                           AUTP2130
C                                                                        AUTP2140
      IF (NBRNOW.EQ.NBRX) GO TO 360                                      AUTP2150
      IF (NOEFR.LE.0) GO TO 360                                          AUTP2160
      WRITE (6,420)                                                      AUTP2170
      NP=0                                                               AUTP2180
      CT = NOEFR - 1
      DO 350 I=1,NBRX                                                    AUTP2190
      IF( NOUT(I) )  350, 315, 350
  315 F = ( CT * POINT(I) ) / ( AYY - POINT(I) )
      PAR=POINT(I)/AYY                                                   AUTP2200
      IF( A(I,I).LE.TOL )  POINT(I) =3.3333333E33                        AUTP2210
  320 IF (NP) 340,330,340                                                AUTP2230
  330 IH=I                                                               AUTP2240
      FH = F
      RPAR=PAR                                                           AUTP2250
      SSN=A(I,I)                                                         AUTP2260
      DELT=POINT(I)                                                      AUTP2270
      NP=1                                                               AUTP2280
      GO TO 350                                                          AUTP2290
  340 WRITE(6,430) IH,RPAR,SSN,DELT,FH,  I,PAR,A(I,I)-POINT(I),F
      NP=0                                                               AUTP2310
  350 CONTINUE                                                           AUTP2320
      IF( NP.NE.0 )  WRITE(6,430) IH,RPAR,SSN,DELT,FH
  360 CONTINUE
C.........................................................................
C     **** COMPUTE BACK SOLUTION                                         AUTP 550
      IF (MAXSTP.EQ.999) GO TO 999
      IF (IFBACK.EQ.0.OR.IFBACK.GT.NOSTEP) GO TO 999
C                                                                        AUTP 560
      NABC = 1                                                           AUTP 570
      IF( IFWT.NE.0 ) NABC = 2                                           AUTP 580
      IF(NYTRA.NE.0) NABC = 5                                            AUTP 590
      SUMSQ=0.000                                                        AUTP 610
      CHISQ = 0.000                                                      AUTP 620
      SUMSQU = 0.000                                                     AUTP 630
      CHISQU = 0.000                                                     AUTP 640
      NONO = 0                                                           AUTP 450
      NDROP=0                                                            AUTP 660
      LINE=50                                                            AUTP 670
C                                                                        AUTP 680
      DO 220 N=1,NDATA                                                   AUTP 690
      IF (NTAPE9.NE.0) GO TO 90                                          AUTP 700
      READ (9) (POINT(L),L=1,NOVAR),WHT                                  AUTP 710
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          GO TO 110                                                 AUTP 720
   90     JJ=NBRXYH*(N-1)                                           AUTP 730
          DO 100 L=1,NOVAR                                          AUTP 740
          KK=JJ+L                                                   AUTP 750
  100     POINT(L)=STRING(KK)                                       AUTP 760
          JJ=NBRXYH*N                                               AUTP 770
          WHT=STRING(JJ)                                            AUTP 780
C                                                                   AUTP 790
  110     YPRED=CONST                                               AUTP 800
          DO 120 I=1,NBRNOW                                         AUTP 810
          J=INDEX(I)                                                AUTP 820
  120     YPRED=YPRED+COEFF(I)*POINT(J)                             AUTP 830
C                                                                   AUTP 840
          YOBS=POINT(NOVAR)                                         AUTP 850
          DEV=YPRED-YOBS                                            AUTP 860
          DEVN=DEV/SIGY                                             AUTP 870
          IF( NDEFR .LE. 0 )  DEVN = 0.0                            AUTP 880
          PCT = (DEV*100.0)/YOBS
          GOOD=BLANK                                                AUTP 900
          IF (ABS(DEVN).GT.3.5) GOOD=CHECK                          AUTP 910
          GO TO (150,130), KPATH                                    AUTP 920
  130     DO 140 J=1,LBAD,2                                         AUTP 930
          IF (N.LT.NOGOOD(J).OR.N.GT.NOGOOD(J+1)) GO TO 140         AUTP 940
          GOOD=VOID                                                 AUTP 950
          NBAD=NBAD+1                                               AUTP 960
          IF (NBAD.EQ.NSKIP) KPATH=1                                AUTP 970
          GO TO 160                                                 AUTP 980
  140     CONTINUE                                                  AUTP 990
  150     DEVSQ = (DEV*DEV)*WHT                                     AUTP1000
          SUMSQ = SUMSQ + DEVSQ                                     AUTP1010
          CHISQ = CHISQ + DEVSQ/YPRED                               AUTP1020
  160     LINE=LINE+1                                               AUTP1030
          M = NABC                                                  AUTP1040
          ABC(1) = GOOD                                             AUTP1050
          ABC(2) = WHT                                              AUTP1060
          IF (NYTRA) 170,190,180                                    AUTP1070
  170     IF (YOBS.GT.15..OR.YPRED.GT.15.) GO TO 185                AUTP1080
          YO = EXP ( YOBS ) - YCONST
          YC = EXP ( YPRED ) - YCONST
          GO TO 200                                                 AUTP1110
  180     IF (YOBS.GT.8..OR.YPRED.GT.8.) GO TO 185                  AUTP1120
          YO = 10.0D0**YOBS  - YCONST                               AUTP1130
          YC = 10.0D0**YPRED - YCONST                               AUTP1140
  200     CONTINUE                                                  AUTP1150
          DEVU = YC - YO                                            AUTP1160
          ABC(3) = YO                                               AUTP1170
          ABC(4) = YC                                               AUTP1180
          ABC(5) = DEVU                                             AUTP1190
          IF( GOOD.EQ.VOID ) GO TO 190                              AUTP1200
          DEVSQ = (DEVU*DEVU) * WHT                                 AUTP1210
          SUMSQU = SUMSQU + DEVSQ                                   AUTP1220
          CHISQU = CHISQU + DEVSQ/YC                                AUTP1230
          GO TO 190                                                 AUTP1240
  185     NONO = 1                                                  AUTP1250
          M = 2                                                     AUTP1260
  190     CONTINUE                                                  AUTP1270
          IF (LINE.LE.50) GO TO 210                                 AUTP1280
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      WRITE(6,460)NOSTEP,NPROB,(V(J,NOVAR),J=1,2),(YTRA(J),J=1,2),      ALTP1290
     *     (BLANK, J=1,NABC)                                            AUTP1200
      LINE=1                                                            ALTP1210
  210 WRITE(6,470) GOOD,N,YOBS,YPREC,DEV,DEVN,PCT,(ABC(J),J=1,M)        ALTP1320
  220 CONTINUE                                                          ALTP1330

      SIGY = SQRT ( SUMSQ/DEFR )
      SIGYU = SQRT ( SUMSOU/DEFR )
      L=1                                                               AUTP1370
      IF(YTRA(1).NE.ACTUAL) L=2                                         ALTP1380
      WRITE(6,490) SIGY, CHISQ, SUMSQ, (BLANK, J=1,L)                   ALTP1350
      IF( NABC.EC.5 .AND. NCNO.EQ.0 ) WRITE(6,500) SIGYU,CHISCL,SUMSCL  AUTP1400
      IF (NTAPE9.EC.0) REWIND 9                                         ALTP1410
      SUMSC = A(NOVAR,NOVAR) * TSS                                      ALTP1430
C.......................................................................

  999 RETURN
C                                                                       ALTP2350
  370 FORMAT (29H1NO MORE CEGREFS FREEOOH STEPI5/1H0,120(1H*))          ALTP2360
  380 FORMAT (20H0STANDARD ERROR OF Y F16.6,5X2A6/24H S.E. AS PERCENT OFAUTP2370
     * MEAN F12.6/24H CORPELATION COEFFICIENT F12.6 // )                ALTP2380
  390 FORMAT (7X,15HCONST. TERM   0,E16.8)                              AUTP2390
  400 FORMAT(20X,48HVAR    COEFFICIENT          STAND. ERROR   CCEF/SE,8X,ALTP2400
     *1HF, 9X, 4HBETA, 7X,10HRSQ CHANGE )
  410 FORMAT(7X2A6,I3, E16.8,E19.6,F11.4,F11.2, 2F14.8 )
  420 FORMAT(1H-27X58HREGRESSION OF THE VARIAELE K ON THE SET CF VARIABLALTP2430
     1ES ABOVE//   2(5X,44HK    PARTIAL RSD    NORMED SUM/SQ     DELTA RSC,
     2 6X,4HF    ,5X ) )
  430 FORMAT(2(4XI2,F12.7,E17.7,E15.7,1XF8.3, 5X) )
  440 FORMAT (5H1STEPI4,9X,16A5,9H  PROBLEMI6/6H0ANOVA.21X,30H....... ORAUTP2470
     1IGINAL UNITS .......9X,26H.... CORRELATICN FORM ..../8H  SCURCE13XALTP2480
     2,36HC.F.  SUM OF SQUARES    MEAN SQUARES9X,26HSUM SQUARES   MEAN SAUTP2490
     3QUARES11X,9HOVERALL F/7H0TOTAL(A6,A3,1H)I7,E17.8,20X,F15.8/11H REGAUTP2500
     4RESSIONI13.2E17.8,3X,2F15.8,F20.4/11H RESIDUAL  I13,2E17.8,3X,2F15AUTP2510
     5.8/)                                                              AUTP2520
  460 FORMAT (5H1STEPI4,18X,43HRESIDUAL ANALYSIS  ( ACTUAL VS CALCULATEDALTP2530
     1 ).9X,7HPROBLEMI6/1H0,22X,A6,A4/7X,16HOBSERVATION     A1C,A2, I1H
     2CALCULATED,8X,30HRESIDUAL    NOR DFV    PCT DEV.A3,AE,6HWEIGHT,3A1AUTP2550
     3,30H   ACTUAL       CALC.         DFV //)                         ALTP2560
  470 FORMAT (1X,A6,I6,F17.5,2F16.5,F10.3,F11.3,2XA6,F8.4,F12.2,2F11.2) ALTP2570
  480 FORMAT (I5,E20.8,5I5,F20.4,F10.7)                                 AUTP2580
  490 FORMAT (17H0STANDARD ERROR =F10.3,15H , CHI SQUARE =F11.4,2X,     ALTP2590
     *  7H   SSE =E15.8,2A1,34HFOR TRANSFORMED CEPENDENT VARIABLE )     AUTP2600
  500 FORMAT (17H0STANDARD ERROR = F10.3,15H , CHI SQUARE =F11.4,2X,    ALTP2610
     *  7H   SSE =E15.8,2X ,34HFOR RECONVERTED DEPENDENT VARIABLE )     ALTP2620
  510 FORMAT(1X,50(1H*),53H ILL-CCNDITIONED SET --- RESULTS IN DOUBT. DOAUTP2630
     *NOT USE , 19(1H*) )                                               ALTP2640
      END                                                               ALTP2650
```

```
      SUBROUTINE PRTSUM                                                 SOME  10
C     S/R TO PRINT SUMS OF CROSS PRODUCTS                               SOME   8
C       RAW SUMS OF SQUARES AND CROSS PRODUCTS                          SOME  20
      COMMON SIGMA(60),A(52,52),SIMCOR(52,52),AVG(60),TEST(60)          SOME  30
      COMMON PCINT(60),STRING(3000),INOPAC(30,30),INDEXP(61)            SOME  40
      COMMON INDEX(60),NOUT(60),KSTEP(60),ALPHA(16),YMEAN,IOEN,IFAVE    SOME  50
      COMMON MAXSTP,IFPNCH,NSUMRY,NSKIP,NTAPE9,NEW                      SOME  60
      COMMON NOVAR,NBRNOW,NOSTEP,NOATA,NBRXYN,NBRX,LPATH,OEFFM,KVAF     SOME  70
      COMMON IFBACK,IFCNST,IFCORR,NPROB,NBRPVR,TOL,NOOEL,JF             SOME  80
      COMMON INDEXY,LBAO,NOGOOO(28)                                     SOME  90
      COMMON IFWT,YCONST,NYTRA,V(2,51),YTRA(2)                          SOME 100
      COMMON  STOERR(50),CORSQR(50)                                     SOME 110
      REAL            A,SIMCOR,SIGMA,AVG,TEST                           SOME 120
C                                                                       SOME 130
      OATA JB/1H /                                                      SOME 140
      WRITE (6,150)                                                     SOME 150
      WRITE (6,160) (J3,I,A(I,NBRXYH),I=1,NBRX)                         SOME 160
      WRITE (6,170) A(NOVAR,NBRXYH)                                     SOME 170
      WRITE (6,180)                                                     SOME 180
      WRITE (6,190) ((JB,I,J,A(I,J),J=I,NBRX),I=1,NBRX)                 SOME 190
      WRITE (6,200) (J9,I,A(I,NOVAR),I=1,NBRX)                          SOME 200
      WRITE (6,210) A(NOVAR,NOVAR)                                      SOME 210
      RETURN                                                            SOME 220
C     ******************************************************************SOME 230
C     ******************************************************************SOME 240
C     S/R  TO CALCULATE AND PRINT THE RESIOUAL SUM OF SQUARES AND C.F.  SOME 250
      ENTRY RESIO                                                       SOME 260
      IF (A(NBRXYH,NBRXYH)) 10,10,20                                    SOME 270
   10 WRITE (6,220) NPROB                                               SOME 280
      CALL EXIT                                                         SOME 290
   20 OO 40 I=1,NOVAR                                                   SOME 300
      OO 30 J=I,NOVAR                                                   SOME 310
   30 A(I,J)=A(I,J)-(A(I,NBRXYH)*A(J,NBRXYH)/A(NBRXYH,NBRXYH))          SOME 320
   40 AVG(I)=A(I,NBRXYH)/A(NBRXYH,NBRXYH)                               SOME 330
      IF (IFAVE.NE.0) GO TO 50                                          SOME 340
      WRITE (6,230)                                                     SOME 350
      WRITE (6,190) ((JB,I,J,A(I,J),J=I,NBRX),I=1,NBRX)                 SOME 360
      WRITE (6,200) (JB,I,A(I,NOVAR),I=1,NBRX)                          SOME 370
      WRITE (6,210) A(NOVAR,NOVAR)                                      SOME 380
   50 RETURN                                                            SOME 390
C     ******************************************************************SOME 400
C     ******************************************************************SOME 410
C     S/R  TO CALCULATE AND PRINT THE SIMPLE CORRELATION COEFFICIENTS   SOME 420
      ENTRY CORREL                                                      SOME 430
      OO 90 I=1,NOVAR                                                   SOME 440
      IF (A(I,I)) 60,60,80                                              SOME 450
   60 WRITE (6,240) I                                                   SOME 460
      IF( I.EQ.NOVAR )  CALL SLITE(1)                                   SOME 470
      SIGMA(I)=1.0                                                      SOME 480
      OO 70 J=1,NOVAR                                                   SOME 490
      A(I,J)=0.0                                                        SOME 500
   70 A(J,I)=0.0                                                        SOME 510
      GO TO 90                                                          SOME 520
CC 80 SIGMA(I)=OSQRT(A(I,I))                                            SOME 530
   80 SIGMA(I) = SQRT ( A(I,I) )
   90 A(I,I)=1.000                                                      SOME 540
      OO 100 I=1,NBRX                                                   SOME 550
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      SUBROUTINE PRTSUM      74/74    OPT=1                    FTN 4.2+74278      11/22/74  12.10.42.

          IP1=I+1                                                       SOME 560
          DO 100 J=IP1,NOVAR                                            SOME 570
          A(I,J)=A(I,J)/(SIGMA(I)*SIGMA(J))                             SOME 580
      100 A(J,I)=A(I,J)                                                 SOME 590
          DO 110 J=1,NOVAR                                              SOME 600
          DO 110 K=1,NOVAR                                              SOME 610
      110 SIMCOR(J,K)=A(J,K)                                            SOME 620
          IF (IFCNST.NE.0) GO TO 140                                    SOME 630
          IF (IFCORR) 140,120,140                                       SOME 640
      120 WRITE (6,250)                                                 SOME 650
          IF( NBRX.LE.1 )  GO TO 135                                    SOME 660
          NOVM2=NBRX-1                                                  SOME 670
          DO 130 I=1,NOVM2                                              SOME 680
          IP1=I+1                                                       SOME 690
      130 WRITE (6,260) (J3,I,J,A(I,J),J=IP1,NBRX)                      SOME 700
      135 WRITE (6,270) (J8,I,A(I,NOVAR),I=1,NBRX)                      SOME 710
      140 RETURN                                                        SOME 720
C                                                                       SOME 730
C                                                                       SOME 740
C                                                                       SOME 750
      150 FORMAT (1H0,40X,18HSUM  OF  VARIABLES/ )                      SOME 760
      160 FORMAT (4(A1,11H     SUM X(I2,3H) =F14.4))                    SOME 770
      170 FORMAT (6X,11HSUN     Y  =F14.4)                              SOME 780
      180 FORMAT (1H070H                              RAW  SUM  OF  SQUARES  ASOME 790
         1ND  CROSS PRODUCTS/)                                          SOME 800
      190 FORMAT (3(A1,6H    X(I2,7H) VS X(I2,3H) =F17.6))              SOME 810
      200 FORMAT (3(A1,6H    X(I2,12H) VS    Y  =F17.6))                SOME 820
      210 FORMAT (5X,16HY      VS     Y  =F17.6)                        SOME 830
      220 FORMAT (32H0 ZERO NUMBER OF DATA, PROBLEM  I6/)               SOME 840
      230 FORMAT (1H025X56HSUMS  OF  SQUARES  AND  CROSS PRODUCTS  ABOUT  THSOME 850
         1E  MEAN/)                                                     SOME 860
      240 FORMAT (10H0 VARIABLEI5,13H IS CONSTANT //)                   SOME 870
      250 FORMAT (1H0,33X,33HSIMPLE  CORRELATION  COEFFICIENTS/ )       SOME 880
      260 FORMAT (3(A1,6H    X(I2,7H) VS X(I2,3H) =F12.8,5X))           SOME 890
      270 FORMAT (3(A1,6H    X(I2,12H) VS    Y  =F12.8,5X))             SOME 900
          END                                                           SOME 910
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SUBROUTINE ADDTO        74/74    OPT=1                    FTN 4.2+74270      11/22/74  10.10.52.

              SUBROUTINE ADDTO                                              ADJO  10
      C       S/R  TO ADD A VARIABLE                                        ADJO   0
              COMMON SIGMA(60),A(52,52),SIMCOR(52,52),AVG(60),TEST(60)      ADJO  20
              COMMON POINT(60),STRING(3000),INOPAC(30,30),INDEXP(61)        ADJO  30
              COMMON INDEX(60),NOUT(60),KSTEP(60),ALPHA(16),YMEAN,IDEN,IFAVE ADJO  40
              COMMON MAXSTP,IFPNCH,NSUMRY,NSKIP,NTAPE9,NEW                  ADJO  50
              COMMON NOVAR,NBRNOW,NOSTEP,NDATA,NBRXYN,NBRX,LPATH,DEFRM,KVAR ADJO  60
              COMMON IFBACK,IFCNST,IFCORR,NPROB,NBRPVR,TOL,MODEL,JF         ADJO  70
              COMMON INDEXY,LBAD,NOGOOD(20)                                 ADJO  80
              COMMON IFNT,YCONST,NYTRA,V(2,51),YTRA(2)                      ADJO  90
              COMMON  STDERR(50),CORSQR(50)                                 ADJO 100
              EQUIVALENCE (KVAR,K)
              REAL               A,SINCOR,SIGMA,AVG,TEST                    ADJO 110
      C                                                                     ADJO 120
              REAL                 DA,VAR,VMIN,VMAX                         ADJO 130
      C           NBRNOW = NUMBER OF COEFFICIENTS FOR PRESENT  EQUATION     ADJO 140
      C           NBRPRV = NUMBER OF COEFFICIENTS FOR PREVIOUS EQUATION     ADJO 150
      C           INDEX  = INDEX OF PRESENT  EQUATION                       ADJO 160
      C           INDEXP = INDEX OF PREVIOUS EQUATION                       ADJO 170
              DO 10 J=1,NBRNOW                                              ADJO 180
           10 CALL PACK (NBRNOW,J,INDEX(J),2)                               ADJO 190
              DO 20 J=1,NBRX                                                ADJO 200
           20 NOUT(J)=0                                                     ADJO 210
              IF (NBRNOW) 50,50,30                                          ADJO 220
           30 DO 40 J=1,NBRNOW                                              ADJO 230
              NDUMMY=INDEX(J)                                               ADJO 240
           40 NOUT(NDUMMY)=1                                                ADJO 250
           50 VMAX=-1.0                                                     ADJO 260
      C           FIND LARGEST DELTA                           (VAR = DELTA) ADJO 270
              DO 70 I=1,NBRX                                                ADJO 280
      C                                   BYPASS IF ALREADY IN EQUATION     ADJO 290
              IF (NOUT(I).NE.0) GO TO 70                                    ADJO 300
              IF (A(I,I).GE.TOL) GO TO 60                                   ADJO 310
              WRITE (6,510) A(I,I),I,(INDEX(J),J=1,NBRNOW)                  ADJO 320
              GO TO 70                                                      ADJO 330
      C                                                                     ADJO 340
           60 VAR=A(I,NOVAR)*A(NOVAR,I)/A(I,I)                              ADJO 350
              IF (VAR.LE.VMAX) GO TO 70                                     ADJO 360
              VMAX=VAR                                                      ADJO 370
              K=I                                                           ADJO 380
           70 CONTINUE                                                      ADJO 390
      C                HAVE FOUND OPTIMAL VARIABLE                          ADJO 400
              NSTEP=NOSTEP+1                                                ADJO 410
              IF (VMAX) 80,90,90                                            ADJO 420
           80 WRITE (6,520) VMAX                                            ADJO 430
              CALL SLITE (1)                                                ADJO 440
              GO TO 260                                                     ADJO 450
           90 NBRPVR=NBRNOW                                                 ADJO 460
              NBRNOW=NBRNOW+1                                               ADJO 470
              IF (TEST(NBRNOW)-A(NOVAR,NOVAR)+VMAX) 100,100,120             ADJO 480
          100 WRITE (6,530) K,NBRNOW,NSTEP                                  ADJO 490
              DO 110 I=1,NOVAR                                              ADJO 500
              DO 110 J=1,NOVAR                                              ADJO 510
          110 A(I,J)=SIMCOR(I,J)                                            ADJO 520
              LPATH=2                                                       ADJO 530
              GO TO 260                                                     ADJO 540
      C           ADD VARIABLE TO INDEX                                     ADJO 550
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  120 CONTINUE                                                    ADJO 560
      IF (NBRPVR) 230,230,130                                     ADJO 570
  130 DO 140 J=1,NBRPVR                                           ADJO 580
  140 CALL PACK (NBRPVR,J,INDEXP(J),2)                            ADJO 590
      DO 150 J=1,NBRNOW                                           ADJO 600
  150 CALL PACK (NBRNOW,J,INDEX(J),2)                             ADJO 610
      DO 180 J=1,NBRPVR                                           ADJO 620
      IF (INDEXP(J)-K) 180,160,170                                ADJO 630
  160 CALL SLITE (1)                                              ADJO 640
      WRITE (6,480)                                               ADJO 650
      GO TO 260                                                   ADJO 660
  170 JJ=J                                                        ADJO 670
      GO TO 190                                                   ADJO 680
  180 CONTINUE                                                    ADJO 690
      INDEXP(NBRNOW)=K                                            ADJO 700
      GO TO 210                                                   ADJO 710
  190 L=NBRNOW-JJ                                                 ADJO 720
      DO 200 J=1,L                                                ADJO 730
      NR=NBRNOW+1-J                                               ADJO 740
      NS=NBRNOW-J                                                 ADJO 750
  200 INDEXP(NR)=INDEXP(NS)                                       ADJO 760
      INDEXP(JJ)=K                                                ADJO 770
C         CHECK TO SEE IF SET HAS ALREADY BEEN COMPUTED           ADJO 780
  210 DO 220 J=1,NBRNOW                                           ADJO 790
      CALL PACK (NBRNOW,J,I,2)                                    ADJO 800
      IF (INDEXP(J).NE.I) GO TO 240                               ADJO 810
  220 CONTINUE                                                    ADJO 820
      WRITE (6,490) NOSTEP,K,NBRNCH,(INDEX(J),J=1,NBRNCH)         ADJO 830
      LPATH=3                                                     ADJO 840
      GO TO 260                                                   ADJO 850
C                                                                 ADJO 860
  230 INDEXP(1)=K                                                 ADJO 870
  240 TEST(NBRNCH)=A(NOVAR,NOVAR)-VMAX                            ADJO 880
C         NEW SET - PUT INDEXES IN MATRIX                         ADJO 890
      DO 250 J=1,NBRNOW                                           ADJO 900
  250 CALL PACK (NBRNOW,J,INDEXP(J),1)                            ADJO 910
      LPATH=1                                                     ADJO 920
      IF(MAXSTP.EQ.999) GO TO 255                                 ADJO 930
      IF(NSTEP.EQ.1) GO TO 256                                    ADJO 940
  255 WRITE(6,500) NSTEP,K, V(1,K),V(2,K)                         ADJO 950
  256 KSTEP(NBRNCH)=NSTEP                                         ADJO 960
  260 RETURN                                                      ADJO 980
C     ****************************************************************ADJO 990
C     ****************************************************************ADJO1000
C                                                                 ADJO1010
C         ADJUST CORRELATION MATRIX FOR ENTRANCE OF VARIABLE K    ADJO1020
      ENTRY MATRIX                                                ADJO1030
C                                                                 ADJO1040
      DA=1.0/A(K,K)                                               ADJO1050
      DO 330 I=1,NOVAR                                            ADJO1060
      IF (I-K) 270,300,270                                        ADJO1070
  270 DO 290 J=1,NOVAR                                            ADJO1080
      IF (J-K) 280,290,280                                        ADJO1090
  280 A(I,J)=A(I,J)-A(I,K)*A(K,J)*DA                              ADJO1100
  290 CONTINUE                                                    ADJO1110
      A(I,K)=-A(I,K)*DA                                           ADJO1120
  300 CONTINUE                                                    ADJO1130
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      DO 320 J=1,NOVAR                                                  ADJO1140
      IF (J-K) 310,320,310                                              ADJO1150
  310 A(K,J)=A(K,J)*OA                                                  ADJO1160
  320 CONTINUE                                                          ADJO1170
      A(K,K)=OA                                                         ADJO1180
C                                                                       ADJO1190
      IF (NOSTEP.LE.1) NCOMB=MIN1(1000.,2.**(NOVAR-1))                  ADJO1200
      IF (NOSTEP.LE.NCOMB) GO TO 330                                    ADJO1210
      WRITE (6,540) NPROO                                               ADJO1220
      CALL SLITE (1)                                                    ADJO1230
  330 RETURN                                                            ADJO1240
C     ******************************************************************ADJO1260
C     S/R  TO DELETE A VARIABLE                                         ADJO1270
      ENTRY REMOVE                                                      ADJO1280
C                                                                       ADJO1290
      NODEL = 0
      IF( NBRNOW.LE.1 )  GO TO 470
      DO 340 J=1,NBRNOW                                                 ADJO1300
  340 CALL PACK (NBRNOW,J,INDEX(J),2)                                   ADJO1310
      K=0                                                               ADJO1330
      NSTEP=NOSTEP+1                                                    ADJO1340
      VMIN=-2.0E+30                                                     ADJO1350
C          FIND SMALLEST DELTA                        (VAR = DELTA)     ADJO1360
      DO 360 J=1,NBRNOW                                                 ADJO1370
      I=INDEX(J)                                                        ADJO1380
      IF (A(I,I)-TOL) 360,360,350                                       ADJO1390
  350 VAR=A(I,NOVAR)*A(NOVAR,I)/A(I,I)                                  ADJO1400
      IF (VAR.LE.VMIN) GO TO 360                                        ADJO1410
      VMIN=VAR                                                          ADJO1420
      K=I                                                               ADJO1430
  360 CONTINUE                                                          ADJO1440
      IF (TEST(NBRNOW-1)+VMIN-A(NOVAR,NOVAR)) 370,370,380               ADJO1450
  370 CONTINUE                                                          ADJO1460
      GO TO 471                                                         ADJO1470
C          REMOVE VARIABLE TO BE DELETED FROM INDEX                     ADJO1480
  380 JP = 0                                                            ADJO1490
      DO 390 J=1,NBRNOW                                                 ADJO1500
      IF( INDEX(J).EQ.K ) GO TO 390                                     ADJO1510
      JP = JP + 1                                                       ADJO1520
      INDEXP(JP) = INDEX(J)                                             ADJO1530
  390 CONTINUE                                                          ADJO1540
      NBRPVR = NBRNOW - 1                                               ADJO1550
C          CHECK TO SEE IF SET HAS ALREADY BEEN COMPUTED                ADJO1560
      DO 440 J=1,NBRPVR                                                 ADJO1570
      CALL PACK (NBRPVR,J,I,2)                                          ADJO1580
      IF (I.NE.INDEXP(J)) GO TO 450                                     ADJO1590
  440 CONTINUE                                                          ADJO1600
      WRITE (6,550) K                                                   ADJO1610
      GO TO 470                                                         ADJO1620
C          NEW SET - PUT INDEXES IN MATRIX                              ADJO1630
  450 NODEL = 1                                                         ADJO1640
      NBRNOW = NBRNOW - 1                                               ADJO1650
      TEST(NBRNOW)=-VMIN+A(NOVAR,NOVAR)                                 ADJO1660
      DO 460 J=1,NBRNOW                                                 ADJO1670
  460 CALL PACK (NBRNOW,J,INDEXP(J),1)                                  ADJO1680
      WRITE(6,560) NSTEP,K, V(1,K),V(2,K)                               ADJO1690
      KSTEP(NBRNOW)=NSTEP                                               ADJO1700
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SUBROUTINE ADDTO       74/74    OPT=1                          FTN 4.2+74278      11/22/74  18.18.52.

      470 RETURN                                                                    ADJO1710
    C                                                                               ADJO1720
    C                                                                               ADJO1730
    C                                                                               ADJO1740
      500 FORMAT ( 8H0AT STEPI4,20H,  ADJOINED VARIABLEI3 , 3X2A6 )                 ADJO1750
      560 FORMAT ( 8H0AT STEPI4,20H,  DELETED  VARIABLEI3 , 3X2A6 )                 ADJO1760
      490 FORMAT ( 8H0AT STEPI4,20H,  ADJUNCTION OF VARI3,25H PRODUCES THE SADJO1770
         *AME SET OFI3,21H VARIABLES AS BEFORE ,16I3/42I3)                          ADJO1780
      550 FORMAT ( 1H0,11X      ,20H   DELETION   OF VARI3,45H PRODUCES THE SADJO1790
         *AME SET OF VARIABLES AS BEFORE)                                           ADJO1800
      480 FORMAT (42H0ERROR IN ADJOIN S/R... PROBLEM TERMINATED)                    ADJO1810
      530 FORMAT (21H0 ADJOINING VARIABLE I4,30H  PRODUCES NO IMPROVEMENT FOADJO1820
         1R I3,21H  VARIABLES, AT STEP I4)                                          ADJO1830
      510 FORMAT (39H0NORMALIZED RESIDUAL SUM OF SQUARES ISF11.8,17H .......ADJO1840
         1.VARIABLEI3,31H IS NEAR-DEPENDENT ON VARIABLES11I3,/2X40I3)               ADJO1850
      540 FORMAT (49H0MAXIMUM NUMBER OF ITERATIONS EXCEEDED IN PROBLEMI5)           ADJO1860
      520 FORMAT (14H0NEGATIVE VMAX F10.6, 25H ..... PROBLEM TERMINATED)            ADJO1870
          END                                                                       ADJO1880
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      SUBROUTINE PACK (II,JJ,JVALUE,NGO)                                  PACK  20
C         PACK AND EXTRACT INDEX FROM ARRAY NN WHICH HAS 4 VALUES PER WDPACK   0
C                                                                         PACK  10
      COMMON SIGMA(60),A(52,52),SIMCOR(52,52),AVG(60),TEST(60)            PACK  30
      COMMON POINT(60),STRING(3000),INDPAC(30,30),INDEXP(61)              PACK  40
      COMMON INDEX(60),NOUT(60),KSTEP(60),ALPHA(16),YMEAN,IDEN,IFAVE      PACK  50
      COMMON MAXSTP,IFPNCH,NSUMPY,NSKIP,NTAPE9,NEW                        PACK  60
      COMMON NCVAR,NBRNOW,NOSTEP,NDATA,NBRXYW,NBRX,LPATH,DEFRM,KVAR       PACK  70
      COMMON IFBACK,IFCNST,IFCORR,NPROB,NBRPVR,TCL,NCDFL,JF               PACK  80
      COMMON INDEXY,LBAD,NOGOOD(28)                                       PACK  90
      COMMON IFWT,YCCNST,NYTRA,V(2,51),YTRA(2)                            PACK 100
      COMMON  STDERR(53),CORSOR(50)                                       PACK 110
      REAL             A,SIMCOR,SIGMA,AVG,TEST                            PACK 120
      EQUIVALENCE (M,XM)                                                  PACK 130
      DIMENSION MASK(4), MASKC(4), MASKS(4)                               PACK 140
      DATA MASK/77B,7700B,770000B,77000000B/                              PACK 150
      DATA MASKC(1)/77777777777777777700B/
      DATA MASKC(2)/77777777777777770077B/
      DATA MASKC(3)/77777777777777007777B/
      DATA MASKC(4)/77777777777700777777B/
      DATA MASKS/1B,100B,10000B,1000000B/                                 PACK 180
C                                                                         PACK 190
C          INDEX IN NN  ARRAY                                             PACK 200
      I=(II+1)/2                                                          PACK 210
      J=(JJ+1)/2                                                          PACK 220
C         WORD TO BE PACKED                                               PACK 230
      M=INOPAC(I,J)                                                       PACK 240
C         POSITION IN M TO BE PACKED ( 1-4 )                              PACK 250
      NW=2*MOD(II,2)+MOD(JJ,2)+1                                          PACK 260
C                                                                         PACK 270
      GO TO (10,20), NGO                                                  PACK 280
C         PACK INDEX                                                      PACK 290
   10 XM=AND(M,MASKC(NW))                                                 PACK 300
      XM=OR(M,JVALUE*MASKS(NW))                                           PACK 310
      INOPAC(I,J)=M                                                       PACK 320
      RETURN                                                              PACK 330
C         EXTRACT  INDEX                                                  PACK 340
   20 XM=AND(M,MASK(NW))                                                  PACK 350
      JVALUE=M/MASKS(NW)                                                  PACK 360
      RETURN                                                              PACK 370
C                                                                         PACK 380
      END                                                                 PACK 390
```

```
SUBROUTINE CHANGE      74/74    OPT=1                         FTN 4.2+74278      11/22/74  18.28.55.

            SUBROUTINE CHANGE (PCINT,NBRTRA,CONST,NTOTAL)                            CHAN  10
      C     TRANSFORMATION OF DATA                                                   CHAN   0
      C                                                                              CHAN  20
            DIMENSION POINT(60), NBRTRA(60), CONST(60)                               CHAN  30
      C                                                                              CHAN  40
            DO 220 I=1,NTOTAL                                                        CHAN  50
            ITRA=NBRTRA(I)                                                           CHAN  60
            IF (ITRA.LE.0) GO TO 220                                                 CHAN  70
            IF (ITRA.LT.17) GO TO 10                                                 CHAN  80
          5 WRITE (6,230) ITRA                                                       CHAN  90
            NBRTRA(I)=0                                                              CHAN 100
            GO TO 220                                                                CHAN 110
      C                                                                              CHAN 120
         10 D=PCINT(I)                                                               CHAN 130
            C=CONST(I)                                                               CHAN 140
            IF (ITRA.GT.8) GO TO 120                                                 CHAN 150
            GO TO (20,30,40,50,60,80,100,110), ITRA                                  CHAN 160
         20 D=D+C                                                                    CHAN 170
            GO TO 210                                                                CHAN 180
         30 D=D*C                                                                    CHAN 190
            GO TO 210                                                                CHAN 200
         40 D=D/C                                                                    CHAN 210
            GO TO 210                                                                CHAN 220
         50 D=C/C                                                                    CHAN 230
            GO TO 210                                                                CHAN 240
         60 IF (C.LT.0.0) GO TO 70                                                   CHAN 250
            C=D**C                                                                   CHAN 260
            GO TO 210                                                                CHAN 270
         70 C=1./D**(-C)                                                             CHAN 280
            GO TO 210                                                                CHAN 290
         80 IF (D.LT.0.0) GO TO 90                                                   CHAN 300
            C=C**D                                                                   CHAN 310
            GO TO 210                                                                CHAN 320
         90 D=1./C**(-D)                                                             CHAN 330
            GO TO 210                                                                CHAN 340
        100 D=ALOG(D+C)                                                              CHAN 350
            GO TO 210                                                                CHAN 360
        110 D=ALOG10(D+C)                                                            CHAN 370
            GO TO 210                                                                CHAN 380
      C                                                                              CHAN 390
        120 ITRA=ITRA-8                                                              CHAN 400
            IF (C.EC.0.0) C=1.                                                       CHAN 410
            GO TO (130,140,150,160,170,180,190,200), ITRA                            CHAN 420
        130 D=EXP(C*D)                                                               CHAN 430
            GO TO 210                                                                CHAN 440
        140 D=EXP(C/D)                                                               CHAN 450
            GO TO 210                                                                CHAN 460
        150 D=SIN(C*D)                                                               CHAN 470
            GO TO 210                                                                CHAN 480
        160 D=COS(C*D)                                                               CHAN 490
            GO TO 210                                                                CHAN 500
        170 D=SIN(C*D)/COS(C*D)                                                      CHAN 510
            GO TO 210                                                                CHAN 520
      CC180 C=SINH(C*D)                                                              CHAN 530
      CC    GO TO 210                                                                CHAN 540
        180 GO TO 5
      CC190 C=COSH(C*D)                                                              CHAN 550
```

```
SUBROUTINE CHARGE      74/74    OPT=1                          FTN 4.2+74278       11/22/74  10.28.55.

      CC     GO TO 210                                                     CHAN 560
         190 GO TO 5                                                       CHAN 570
         200 D=TANH(C*D)                                                   CHAN 580
         210 POINT(I)=D                                                    CHAN 590
         220 CONTINUE                                                      CHAN 600
             RETURN                                                        CHAN 610
      C                                                                    CHAN 620
      C                                                                    CHAN 630
      C
         230 FORMAT (/1X,14HTRANSFORMATION,I3,18H IS NOT IN TABLES,36H IT WILL BCHAN 640
            1E SET TO ZERO AND IGNORED.//)                                 CHAN 650
             END                                                           CHAN 660
```

```
SUBROUTINE NBROFX      74/74    OPT=1                          FTN 4.2+74270       11/22/74  16.25.01.

          SUBROUTINE  NBROFX ( IFSUB, POINT )
C             DUMMY SUBROUTINE TO MAKE UNUSUAL TYPES OF TRANSFORMATIONS.     SRTR    0
C             THIS SUBROUTINE IS REPLACED AT OBJECT TIME WITH ONE WHICH      SRTR   10
C             PERFORMS THE DESIRED TRANSFORMATIONS.                          SRTR   20
C             POINT= DATA OBSERVATION WHICH WAS READ IN ON CARDS OR TAPE     SRTR   30
C             IFSUB = A NUMBER GREATER THAN ZERO WHICH CALLS S/R EQUAT AND   SRTR   40
C                   NBROFX.  MAY BE USED AS A BRANCH INDICATOR.              SRTR   50
C             NXV = NEW NUMBER OF INDEPENDENT VARIABLES                      SRTR   60
C             MAKE SURE DEPENDENT VARIABLE MATCHES  NXV + INDEXY.            SRTR   70
C                                                                            SRTR   80
C             S/R EQUAT IS USED TO ADD OR CHANGE DATA OBSERVATIONS           SRTR   90
C                                                                            SRTR  100
C             S/R  NBROFX   IS USED TO CHANGE THE VALUE OF NXV.              SRTR  110
C                                                    AND MUST BE USED.       SRTR  120
C     *****************************************************************SRTR  130
C                                                                            SRTR  140
C             EXAMPLE  -  POLYNOMIAL EQUATION      IFSUB = POWER OF X        SRTR  150
C               Y = A0 + A1*X + A2*X**2 + A3*X**3 + ... + A(IFSUB)*X**IFSUB SRTR  160
C                                                                            SRTR  170
          DIMENSION POINT(52)                                                SRTR  250
          EQUIVALENCE ( XV,NXV )
          PRINT 901, IFSUB
      901 FORMAT( "0SUBROUTINE EQUAT WAS CALLED WITH IFSUB =" I3)
          NXV = IFSUB
          POINT(1) = XV

          NY=NXV+1                                                           SRTR  200
          NX=NXV                                                             SRTR  210
          RETURN                                                             SRTR  220
C                                                                            SRTR  230
          ENTRY EQUAT
C             STORE Y                                                        SRTR  260
          K=54-NX                                                            SRTR  270
          L=53                                                               SRTR  280
          DO 10 J=NY,52                                                      SRTR  290
          K=K-1                                                              SRTR  300
          L=L-1                                                              SRTR  310
       10 POINT(L)=POINT(K)                                                  SRTR  320
C                                                                            SRTR  330
          IF( NX.EQ.1 ) RETURN
C             STORE POWERS OF X                                              SRTR  340
          DO 20 J=2,NX                                                       SRTR  350
       20 POINT(J)=POINT(J-1)*POINT(1)                                       SRTR  360
          RETURN                                                             SRTR  370
          END                                                                SRTR  380
```

```
      PROGRAM DIANAL     74/74    OPT=1                          FTN 4.2+74278       11/21/74  10.51.10.

      PROGRAM DIANAL(TAPE1=130,TAPE2=65,TAPE11,
     *    PUNCH=65, INPUT=65, OUTPUT=130, TAPE7=PUNCH  )
C
      DIMENSION  TEMP(3), NC(28), ATEMP(3), V(51), NCARD(8)
      DIMENSION  IND(51),COEFF(51),A(855),X(101),Y(101)
      DIMENSION  HEAD(8), LEFT(8), BOTTOM(8), NSYN(5), DROP(5),DL(5)
      DIMENSION  RIGHT(15)
      DATA  DL/8H DH = 12,8H DH = 18,8H DH = 24, 8H DH = 30,8H DH = 21/
      DATA NSYN/1H1,1H2,1H3,1H4,1H5/,DROP/12.,18.,24.,30.,21./
      DATA  TEMP/-65.,70.,160./, NC/28*0/
      DATA ATEMP/10H-65 DEGREE,10H 70 DEGREE,10H160 DEGREE/
      DATA HEAD/8*1H /, RIGHT/15*1H /
      DATA  TP,DH,TC,SS,GL/5*1./
C
      N = 6
      M = 0
C
      NVR = 18
       NVR = 27
      NVR = 48
       NVR = 45
      NVR = 36
C INSERT NUMBER  OF EQUATION TERMS HERE ......................................
C
      REWIND 7
      READ(7,904)   NTEST
      IF( EOF(7)  .EQ.  0.0 )   GO TO 200
C
C************* PASS 1 **********************************************************
      NRETURN = 1
      CALL DATE( DAY )
      NC(3)=NC(4)=NC(6)=NC(12)=NC(17)=NC(18)=NC(19)=NC(28)  =  1
      NC(5)=2740
      NC(7)=6
      NC(9)=930
      NC(12) = 10
      PRINT 900, N, M, DAY
  900 FORMAT(*1DYNAMIC CUSHIONING ANALYSIS*10X*N =*I2,10X*N =*I2,9X,A10)
      DO  150  NT = 1,1                                                ********
      REWIND 1
      NP = 0
      NC(1)=NC(26)=NT
      NC(2)=NC(13)=NC(14)=    NVR
      TT = TEMP(NT)
C
  100 READ(1,901)  TP, DH, TC, SS, GL
  901 FORMAT( 10X, 4F10.0, I1, F10.3      CYCLE 1
C 901 FORMAT( 7F10.3 )                    CYCLE 2
      IF( EOF(1)  .NE.  0.0 )   GO TO 120
      IF( DH .EQ. 21. )   GO TO 100                                  DH=21
      IF( TC .GE. 4. )   GO TO 100
C INSERT DATA CONDITIONS HERE... ALSO IN AFFECTED DO LOOPS IN PASS 2 ..........
C
      YVAR = GL
C
C         COMPUTE DYNAMIC CUSHIONING MODEL VARIABLES
      GO TO 800
```

```
PROGRAM UTANAL      74/74   OPT=1                        FTN 4.2+74278      11/21/74  10.51.10.

      105 CONTINUE
C
C INSERT YVAR TRANSFORMATION HERE .............................................
          WRITE(11)  ( V(J), J=1, NVR ),  YVAR
          NP = NP + 1
          GO TO 100
C
      120 END FILE 11
          PRINT 902,NP
      902 FORMAT(//*  NP = *I5)
          WRITE(2,904)  NC
          PRINT   904,  NC
      904 FORMAT( 16I5 )
          WRITE(2,906)
          PRINT   906
      906 FORMAT(80X,T2,*DYNAMIC CUSHIONING MODEL* )
      150 CONTINUE
          REWIND 11
          REWIND 2
          GO TO 999
C
C************* PASS 2 ***************************************************************
C
      200 REWIND 7
          NRETURN = 2
          XMIN = ALOG10( .04 )
          XMAX = ALOG10(  5. )
          DX = ( XMAX - XMIN ) / 100.
      210 READ(7,920)  NCARD,  K, CONST, NT, NV,ND, NSTEP, ND, SE, R
      920 FORMAT(8A10,T1, I5 ,E20.4,5I5,F20.4,F10.7)
          IF( EOF(7) .NE. 0.0 )  GO TO 999
C
CC        PRINT 922, ATEMP(NT), NV, R
          PRINT 922,              NV, R
CC922 FORMAT(*1*A4* DEGREES*I5* VARIABLES    CORRELATION COEFFICIENT I3*
      922 FORMAT(*1*              I5* VARIABLES    CORRELATION COEFFICIENT I3*
         *F6.6///* MLR OUTPUT CARDS*//)
          PRINT 910, NCARD
      910 FORMAT(1X,8A10)
          DO 220 J=1,NV
          READ(7,920)  NCARD, IND(J), COEF(J)
      220 PRINT 910, NCARD
          ENCODE(150,911,RIGHT)  ( IND(J), J=1,NV )
      911 FORMAT(100X,50X,T1, 50I3)
C
          DO 260 NT = 1,3                                                   ********
          HEAD(1) = ATEMP(NT)
          TP = TEMP(NT)
C             DROP HEIGHT
          DO 250 ND = 1,5
          DH = DROP(ND)
          IF( DH .EQ. 21. )  GO TO 250                                      D=21
C**
          HEAD(6) = DL(ND)
          CALL SETGRID( A, -76, XMIN, XMAX, 0.0, 350.    )
          CALL LABGRID( A, 1, 20, 20HLOG( STATIC STRESS ) )
          CALL LABGRID( A, 2, 20, 20H               G LEVEL )
```

```
      CALL LABGRID( A, 3, 48, HEAD )
      CALL LABGRID( A, 4, 76, RIGHT )
C
C         THICKNESS
      DO 260 NTC =1,5
      TC = NTC
      IF( TC .GE. 4. )  GO TO 260

      XX = XMIN
      DO 230 JP=1,101
      SS = 10**XX
C         COMPUTE DYNAMIC CUSHIONING MODEL VARIABLES
      GO TO 800
  222 CONTINUE
      Y(JP) = CONST
      DO 225 J=1,NV
      I = IND(J)
  225 Y(JP) = Y(JP) + COEFF(J) * V(I)
      X(JP) = XX
C
C INSERT REVERSE YVAR TRANSFORMATION HERE.    Y(JP) = ----  ......................
C
  230 XX = XX + DX
C
      CALL PLTGRID( A, NSYM(NTC), 101, X, Y )
  240 CONTINUE
C
      CALL PRINTPL( A, 6LOUTPUT )
  250 CONTINUE
  260 CONTINUE                                                     ********
      GO TO 910
C
  800 CONTINUE
C********** DYNAMIC CUSHIONING MODEL *******************************************
      SS = SS * 100.
      AL = ALOG( SS )
      AL2 = AL * AL
      SROM = SQRT( OM )
      TCSH = TC ** (-3.5)
      TR = (TP+460.)/100.
      TR2 = TR * TR
      TR3 = TR * TR2
      TR4 = TR3 * TR
C
      TCOM = TC ** (-0.5)
      TCTH = TC ** (-1.5)
      TCINV= TC ** (-2.5)
C
      V(61) = TR     * TCOM    * 1.0
      V(62) = TR     * TCOM    * 1.0     * AL
      V(63) = TR     * TCOM    * 1.0     * AL2
      V(64) = TR     * TCTH    * SROM
      V(65) = TR     * TCTH    * SROM    * AL
      V(66) = TR     * TCTH    * SROM    * AL2
      V(67) = TR     * TCOM    * SROM
      V(68) = TR     * TCOM    * SROM    * AL
```

```
      PROGRAM UTANAL      74/74    OPT=1                    FTN 4.2+74278      11/21/74  10.51.18.

          V(9) = TR    * TCOM    * SROM    * AL2

          V(10) = TR2  * TCOM    * 1.0
          V(11) = TR2  * TCOM    * 1.0     * AL
          V(12) = TR2  * TCOM    * 1.0     * AL2
          V(13) = TR2  * TCTM    * SROM
          V(14) = TR2  * TCTM    * SROM    * AL
          V(15) = TR2  * TCTM    * SROM    * AL2
          V(16) = TR2  * TCOM    * SROM
          V(17) = TR2  * TCOM    * SROM    * AL
          V(18) = TR2  * TCTM    * SROM    * AL2

          V(19) = TR3  * TCOM    * 1.0
          V(20) = TR3  * TCOM    * 1.0     * AL
          V(21) = TR3  * TCOM    * 1.0     * AL2
          V(22) = TR3  * TCTM    * SROM
          V(23) = TR3  * TCTM    * SROM    * AL
          V(24) = TR3  * TCTM    * SROM    * AL2
          V(25) = TR3  * TCOM    * SROM
          V(26) = TR3  * TCOM    * SROM    * AL
          V(27) = TR3  * TCTM    * SROM    * AL2

          V(28) = TR   * TCTM
          V(29) = TR   * TCTM              * AL
          V(30) = TR   * TCTM              * AL2
          V(31) = TR2  * TCTM
          V(32) = TR2  * TCTM              * AL
          V(33) = TR2  * TCTM              * AL2
          V(34) = TR3  * TCTM
          V(35) = TR3  * TCTM              * AL
          V(36) = TR3  * TCTM              * AL2
C* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
          GO TO ( 185 , 222 ), NRETURN
C
      999 CONTINUE
          END
```

```
      SUBROUTINE  PRINTPL( P, N1, N2, X, Y, Z )
C     SUBROUTINE PRINTPL
C     DESCRIPTION
C         THIS ROUTINE WAS WRITTEN TO ALLOW MULTIPLE PLOTS ON A SINGLE
C         PAGE OF AN ON-LINE PRINTER. THE PRINT CHARACTER MAY BE
C         SELECTED BY THE USER.
C     INPUT
C         THE INPUT TO THIS ROUTINE IS BY THE FOLLOWING ENTRY POINTS.
C       1 CALL SETGRID( P,JL,XMIN,XMAX,YMIN,YMAX )
C         SETGRID MUST BE FIRST CALL. IT SETS UP AND CLEARS ARRAY P.
C         P = ARRAY TO HOLD PLOT. DIMENSION MUST BE AT LEAST  JL*11+18.
C         JL = NUMBER OF LINES IN PLOT. 51 FOR 6 LINES PER INCH OR
C              76 FOR 8 LINES PER INCH IS RECOMMENDED.
C              IF NEGATIVE, INSIDE GRID LINES WILL NOT BE DRAWN.
C         XMIN,XMAX,YMIN,YMAX = MIN AND MAX  OF  X AND Y VALUES.
C       2 CALL LABGRID( P, L, NC, LABEL )
C         LABGRID IS USED TO LABEL PLOT GRID. CALLED FOR EACH LABEL.
C         P = ARRAY THAT HOLDS THE PLOT.
C         L = 1  FOR BOTTOM LABEL
C         L = 2  FOR LEFT   LABEL
C         L = 3  FOR TOP    LABEL
C         L = 4  FOR RIGHT  LABEL
C         NC = NUMBER OF CHARACTERS IN LABEL.
C         LABEL = ARRAY THAT HOLDS THE LABEL.
C                 NOTE- DO NOT CHANGE UNTIL AFTER PLOT HAS BEEN PRINTED.
C       3 CALL PLTGRID( P, NSYM, NP, X, Y )
C         PLTGRID ENTERS DATA INTO PLOT.
C         P = ARRAY THAT HOLDS THE PLOT.
C         NSYM = PLOT SYMBOL ( EX. NSYM=1H* )
C         NP = NUMBER OF POINTS TO BE PLOTTED
C         X = X DATA ARRAY
C         Y = Y DATA ARRAY
C       4 CALL PRINTPL( P, NFILE )
C         PRINTPL PRINTS GRAPH STORED IN ARRAY P. IT DOES NOT CLEAR P.
C         P = ARRAY THAT HOLDS THE PLOT
C         NFILE = OUTPUT FILE THAT THE PLOT IS PRINTED ON.
C     OUTPUT
C         ON-LINE PRINTER PLOTS
C     REMARKS

      DIMENSION   P(1),  X(1),  Y(1),  JWORK(11), WORK(11)
C         P = ARRAY TO HOLD PLOT
C         X =  XMAX, LABEL, X DATA ARRAY
C         Y =  YMIN,         Y DATA ARRAY
C         Z =  YMAX
C         N1 = JL   , L     , NSYM         , NFILE
C         N2 = XMIN, NM    , NP
      EQUIVALENCE ( XJ,JX,XMIN ), ( WORK,JWORK ), ( MASK,PMASK )
      DATA  NFILE/6LOUTPUT/,PMASK/11111111111111111111B/, BLANK/1H /
      GO TO 15
C       SET UP GRID             P, N1, N2  ,X    , Y    ,Z
C************** CALL SET GRID ( P, JL, XMIN,XMAX, YMIN,YMAX ) *****************
C         P = ARRAY TO HOLD PLOT, ITS SIZE MUST BE AT LEAST JL*11+18.
C         JL = NUMBER OF LINES IN PLOT. 51 FOR 6 LINES PER INCH OR
C              76 FOR 8 LINES PER INCH IS RECOMMENDED.
C              IF NEGATIVE, INSIDE GRID LINES WILL NOT BE DRAWN.
      ENTRY  SETGRID
```

```
SUBROUTINE PRINTPL    74/74    OPT=1                    FTN 4.2+ REL        07/25/74  14.19.05.

              JL = IABS( N1 )
              IF( JL .EQ. 0 ) GO TO 90
              GVV = 10HI                                                      PRIN 330
              GVV = 10H]                                                      PRIN 340
              GHH = 10HI---------                                             PRIN 350
              GHH = 10H]---------                                             PRIN 360
              GV = GVV                                                        PRIN 370
              GH = GHH                                                        PRIN 380
              GG = GH                                                         PRIN 390
            1 IF( N1 .GE. 0 )  GO TO 2
              GV = BLANK
              GH = BLANK
              GG = GVV
      C           SET UP CODE THAT SETGRID HAS BEEN CALLED
            2 P(1) = PMASK
      C           NUMBER LINES TO GRAPH
              P(2) = JL
              JJ = 4                                                          PRIN 470
              DO 4 I=1,JL                                                     PRIN 480
              G = GV                                                          PRIN 490
              GB = GVV                                                        PRIN 500
              JJ = JJ + 1                                                     PRIN 510
              IF( JJ.NE.5 ) GO TO 3                                           PRIN 520
              G = GH                                                          PRIN 530
              GB = GG                                                         PRIN 540
              JJ = 0                                                          PRIN 550
            3 DO 4 K = 1,11                                                   PRIN 560
      CCC     P(I,K) = G
              IND = 18 + (I-1)*11 + K                                         ????????
              P(IND) = G
              IF( K.EQ.1 .OR. K.EQ.11 )  P(IND) = GB
              IF( I.EQ.1 .OR. I.EQ.JL )  P(IND) = GHH
            4 CONTINUE
      C           COUNT OF POINTS THAT FELL OUT OF GRID
              P(3) = 0.
      C           XMIN
              JX = N2
              P(4) = XMIN
      C           YMAX
              YMAX = Z
              P(5) = YMAX
              XMAX = X(1)
              YMIN = Y(1)
      C           X AND Y SCALE INCREMENTS
              SX = ( XMAX - XMIN ) / 100.
              SY = ( YMAX - YMIN ) / ( P(2) - 1. )
              OX = 1.5 - XMIN / SX
              OY = 1.5 - YMIN / SY
              P(6) = SX
              P(7) = SY
              P(8) = OX
              P(9) = OY
      C           SET LABLE ADDRESSES = 0
              DO 8 J=10,17
            8 P(J) = 0.
              RETURN
      C                                 P, N1, N2, X
```

SUBROUTINE PRINTPL 74/74 OPT=1 FTN 4.2+ REL 07/25/74 14.19.06.

```
      C************** CALL  LAB GRID ( P, L , NC, LABEL ) ****************************
      C          LABGRID PUTS LABELS ON GRID AXIS
      C          NC     = NUMBER OF CHAR. IN LABEL ARRAY = N2
      C          LABEL = LABEL ARRAY                     = X
            ENTRY  LAB GRID
            XJ = P(1)
            CALL = 7HLABGRID
            IF( JX .NE. MASK )  GO TO 90
            LABELA = LOCF( X(1) ) - LOCF( P(1) )  + 1
            J = N1*2 + 8
      C     LABEL ADDRESS IN REFERENCE TO P
            P(J) = LABELA
      C     NUMBER OF CHARACTERS IN LABEL
            P(J+1) = N2
            RETURN
      C                                       P, N1  , N2, X, Y
      C************** CALL PLT GRID ( P, NSYM, NP, X, Y ) ****************************
      C          PLTGRID ENTERS DATA INTO PLOT GRID P
      C          NSYM = PLOT SYMBOL ( EX. PSYM = 1H* )           = N1
      C          NP   = NUMBER OF POINTS TO PLOT                 = N2
            ENTRY  PLT GRID
            CALL = 7HPLTGRID
            XJ = P(1)
            IF( JX .NE. MASK )  GO TO 90
            NP = N2
            NSYM = N1
            IF( NP .LE. 0 )   RETURN
            SX = P(6)
            SY = P(7)
            DX = P(8)
            DY = P(9)
            DO 30 L = 1, NP
            IF( LEGVAR( X(L) ).NE.0 )  GO TO 20                         PRIN1340
            IF( LEGVAR( Y(L) ).NE.0 )  GO TO 20                         PRIN1350
            JL = P(2)
            J = X(L) / SX + DX
            I = Y(L) / SY + DY
            IF( J.LT.1 .OR. J.GT.101 )   GO TO 20                       PRIN1380
            IF( I.LT.1 .OR. I.GT.JL  )   GO TO 20                       PRIN1390
      C          INSERT PLOT CHARACTER                                  PRIN1400
            JWORD = ( J-1 )/10 + 1                                      PRIN1410
            JPOS  = J - ( JWORD-1 )*10                                  PRIN1420
            IND = 18 + ( I-1 )*11 + JWORD
            DECODE(10,905, P(IND) ) (JWORK(J), J=1,10)
            JWORK(JPOS) = NSYM
            ENCODE(10,905, P(IND) ) (JWORK(J), J=1,10)
        905 FORMAT(10A1)
            GO TO 30
         20 P(3) = P(3) + 1.0
         30 CONTINUE
      C
      C
            RETURN
      C                                       P, N1
      C************** CALL PRINT PL ( P, NFILE ) ************************************
      C          PRINTPL PRINTS GRAPH STORED IN ARRAY P ON OUTPUT FILE NFILE.
      C          IT DOES NOT CLEAR THE GRID.
```

```
SUBROUTINE PRINTPL    74/74    OPT=1                    FTN 4.2+ REL        07/25/74  14.19.06.

   15 XJ = P(1)
      CALL = 7HPRINTPL
      IF( JX .NE. MASK )  GO TO 90
       NFILE = N1
       IF( N1 .EQ. 0 )  NFILE = 6LOUTPUT
       WRITE(NFILE,912)
  912 FORMAT(*T*)                                                        PRIN1600
C            PRINT TOP LABEL
       L1 = P(14)
       IF( L1 .EQ. 0 )  L1 = LOCF( BLANK ) - LOCF( P(1) )  + 1
      L2 = P(14) + P(15)/10.  - .05
       WRITE(NFILE,906)  ( P(L), L=L1,L2 )
  906  FORMAT(1H1,(30X,10A10))
       WRITE(NFILE,907)
  907  FORMAT(8X *FROM        TO*)
       L1 = P(12)
      L2 = P(13)
       L3 = P(16)
      L4 = P(17)
       YMAX = P(5)
       SY = P(7)
C
       Y1 = YMAX - 0.5*SY
       L = -6
       JL = P(2)
       JLP1 = JL + 1
       DO 50 I=1,JL
      L = L + 6
      IF( L .LT. 60 )  GO TO 35
      L1 = L1 + 1
      L3 = L3 + 1
      L = 0
   35 IO2 = 1H
      IO4 = 1H
      IF( L2 .EQ. 0 )  GO TO 40
      IF( I .GT. L2     )  GO TO 40
      IO2 = SHIFT( P(L1), L )
   40 IF( L4 .EQ. 0 )  GO TO 45
      IF( I .GT. L4     )  GO TO 45
      IO4 = SHIFT( P(L3), L )
C
   45 K = JLP1 - I
      Y2 = Y1 + SY
CCC   INO1= 18 +  (K-1)*11
      INO1= 19 +  (K-1)*11
      INO2= INO1+10
      WRITE(NFILE,908)  IO2,Y1,Y2, ( P(J), J=INO1,INO2 ), IO4
  908 FORMAT(1XA1,1XF10.3,1H-,F10.3, 1X,10A10,A1, 1XA1)              PRIN1740
      Y1 = Y1 - SY
   50 CONTINUE
C
      SX = P(6)
      SXX = SX * 10.
      WORK(1) = P(4)
      DO 60 J=2,11
   60 WORK(J) = WORK(J-1) + SXX
      WRITE(NFILE,910)  ( WORK(J), J=1,11 )
```

```
SUBROUTINE PRINTPL    74/74    OPT=1                      FTN 4.2+ REL       07/25/74  14.19.06.

          910 FORMAT(13X,11F10.2/)
        C          BOTTOM LABEL
              L1 = P(10)
              L2 = P(10) + P(11)/10.  - .05
              IF( L1 .EQ  0 )  GO TO 65
              WRITE(NFILE,915)  ( P(L), L=L1,L2 )
          915 FORMAT(30X,10A10)
           65 L  = P(3)
              IF( L .LE. 0 ) GO TO 67
              WRITE(NFILE,902) L
          902 FORMAT(1X,I7, 20H POINTS OUTSIDE GRID )
              GO TO 68
           67 WRITE(NFILE,915) BLANK
           68 WRITE(NFILE,916) SX, SY
          916 FORMAT(*+*,35X,*DX =*F10.3,*     DY =*F10.3)
              WRITE(NFILE,914)
          914 FORMAT(*S*)                                                      PRIN1850
              RETURN
        C
           90 L = LOCF( P(1) )
              WRITE(NFILE,920)  CALL, L
          920 FORMAT(*0*A8*---- S/R SETGRID NOT CALLED FOR P ARRAY AT LOC *O6)
              LERR = LERR + 1
              IF( LERR .GT. 100 )  STOP 111
              RETURN
              END
```

Appendix C

MLRD PLOTS OF DYNAMIC CUSHIONING CURVES OF THE UAH BEST FITTING POLYNOMIALS

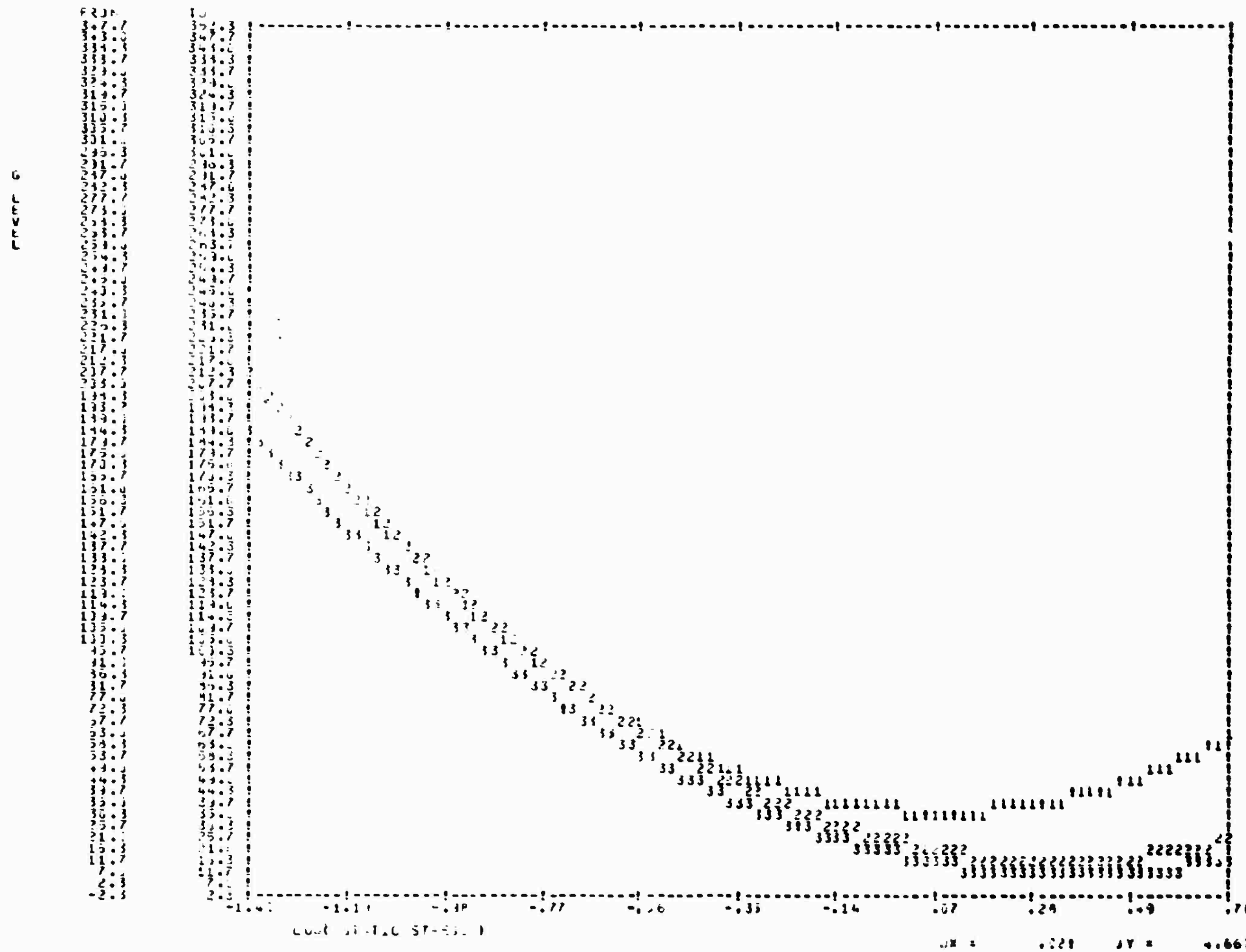

Figure C-1. -65°F, 12-inch drop height.

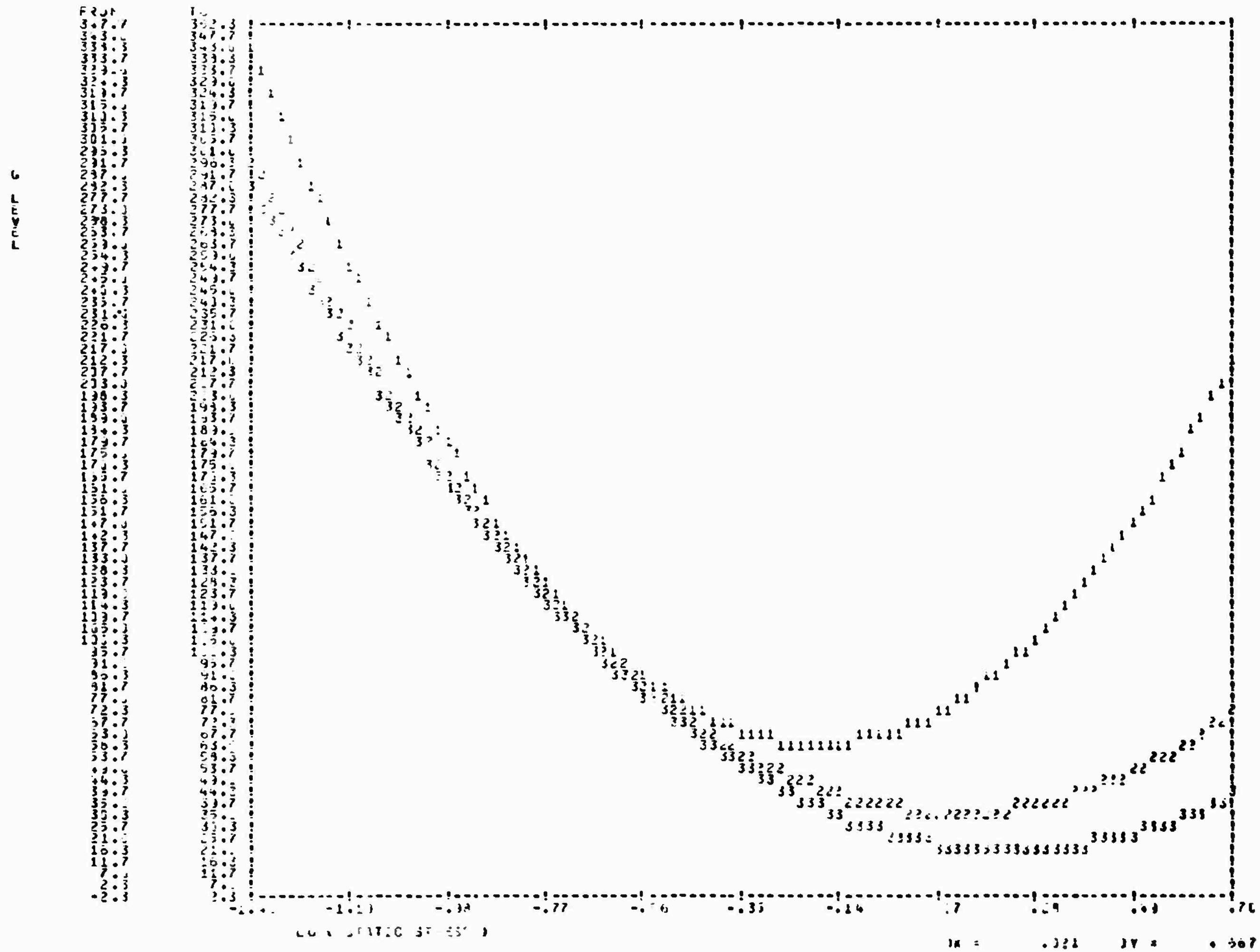

Figure C-2. -65°F, 24-inch drop height.

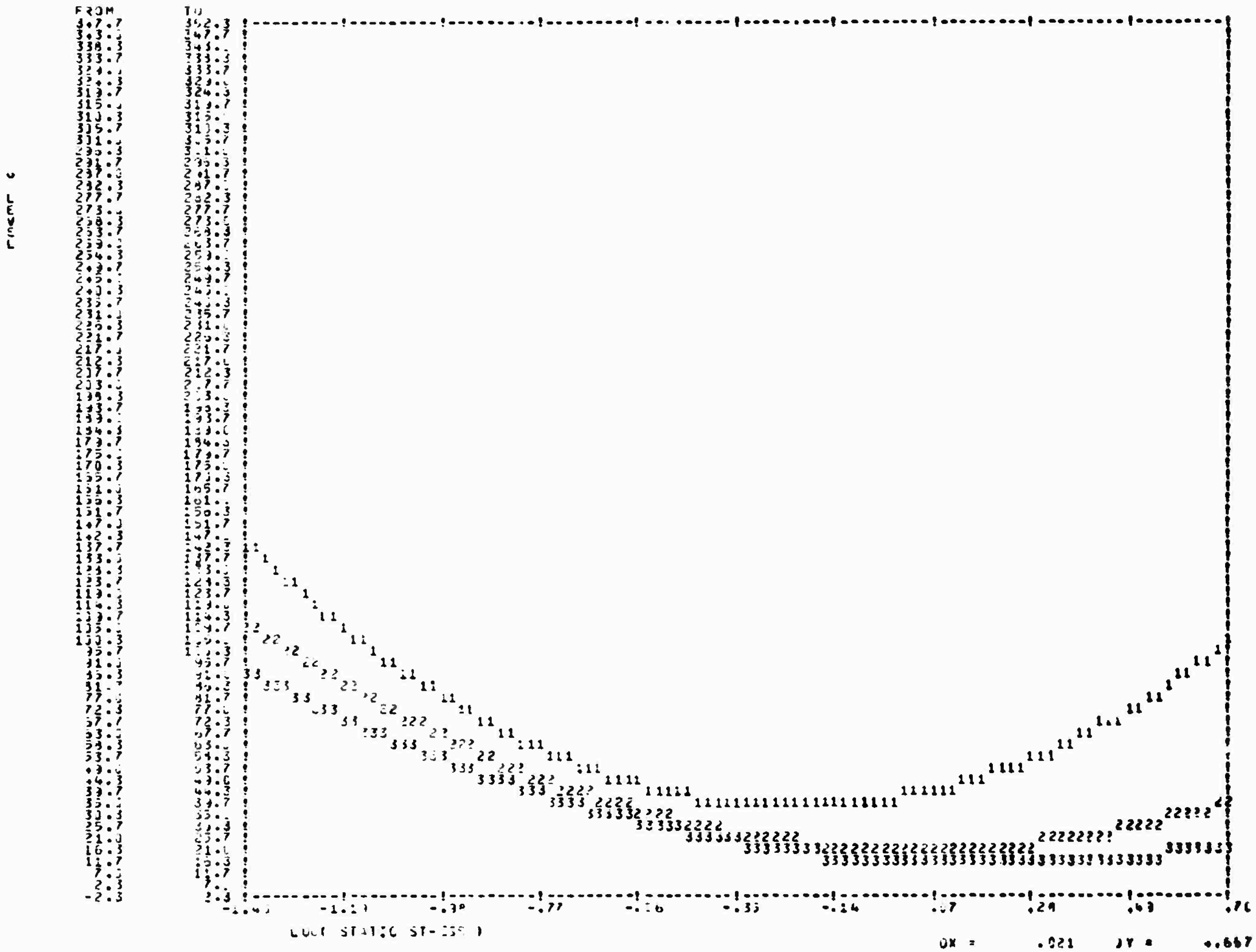

Figure C-3. 70°F, 12-inch drop height.

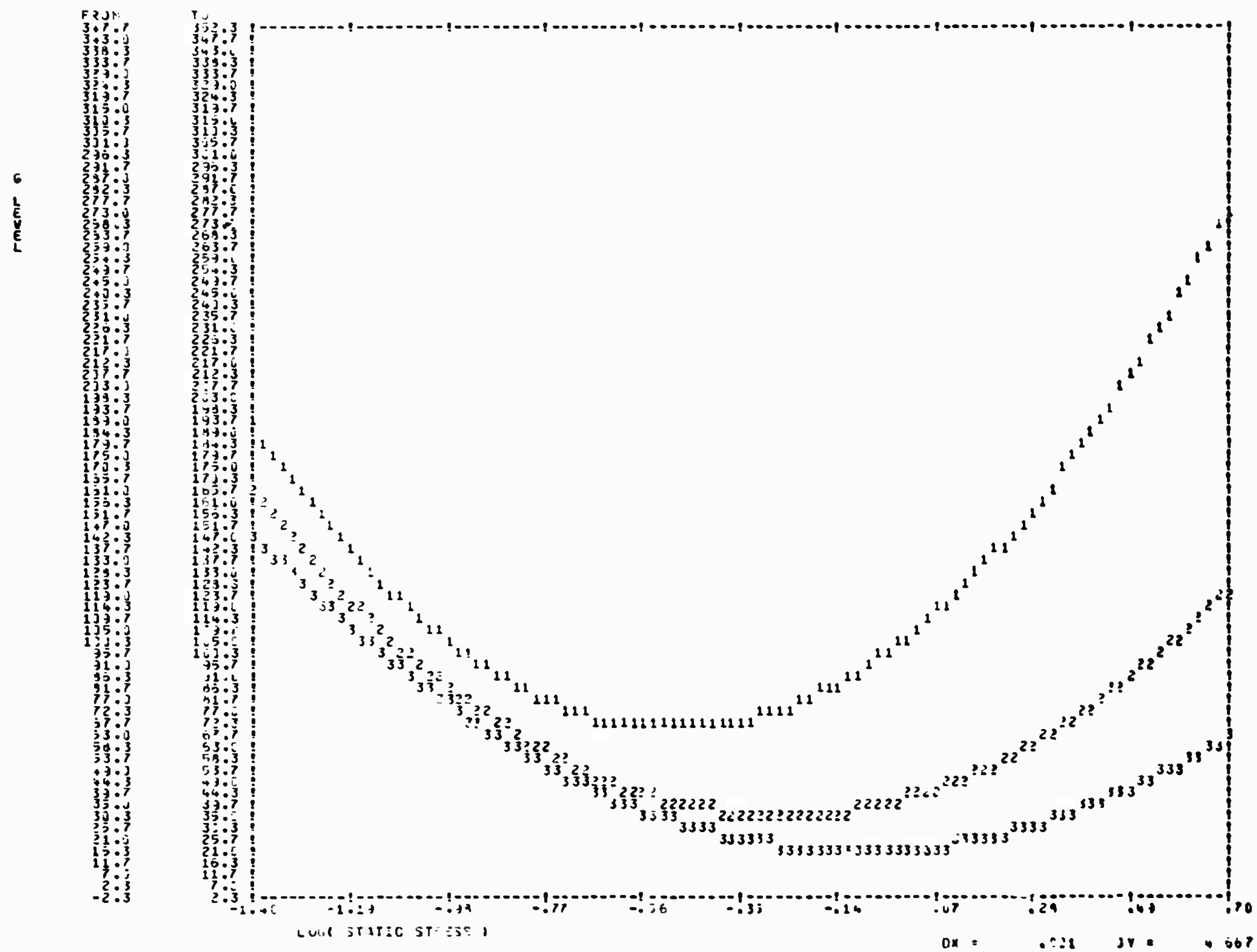

Figure C-4. 70°F, 24-inch drop height.

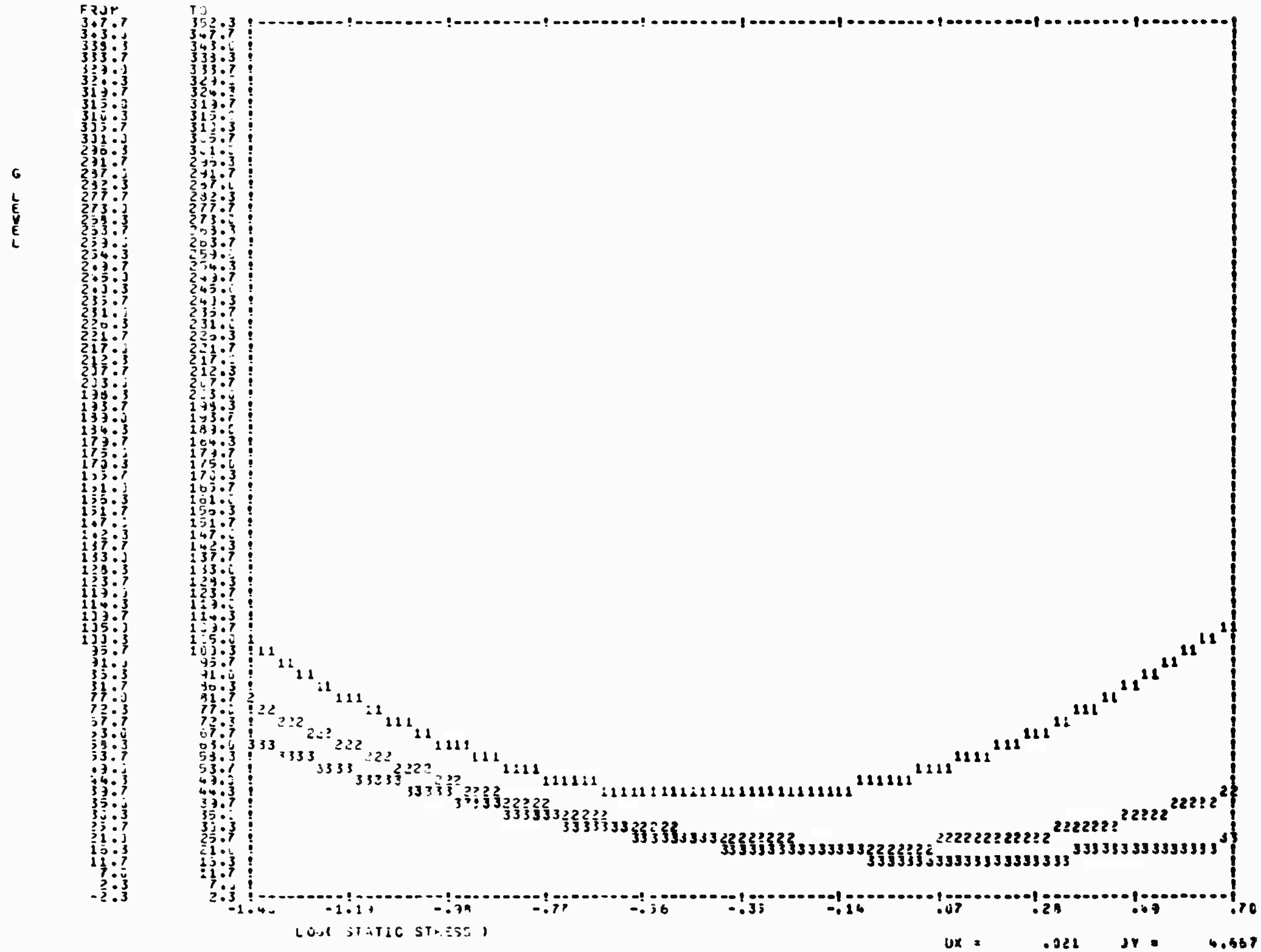

Figure C-5. 160°F, 12-inch drop height.

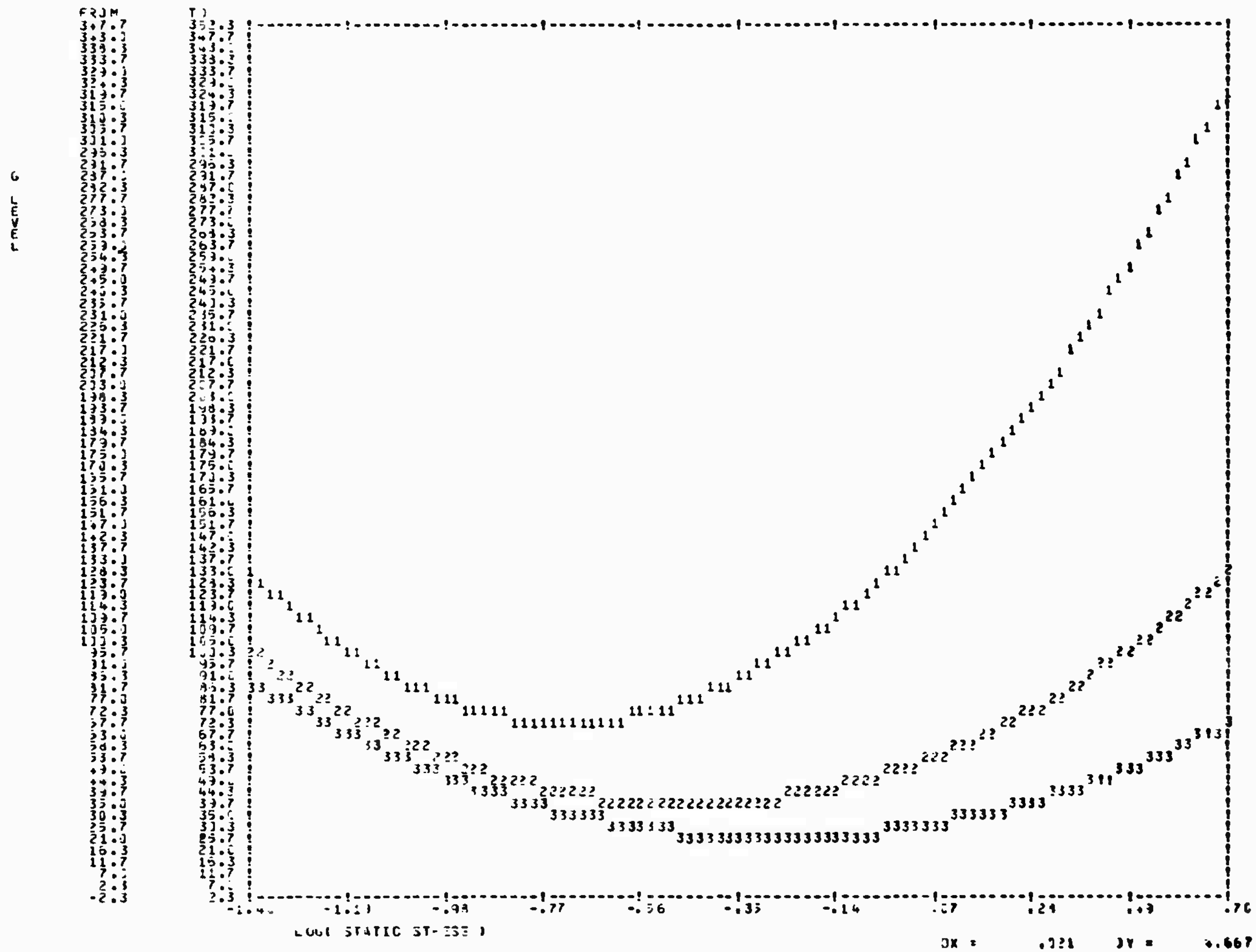

Figure C-6. 160°F, 24-inch drop height.

Appendix D

MLRD PLOTS OF DYNAMIC CUSHIONING CURVES OF THE GENERAL MODEL

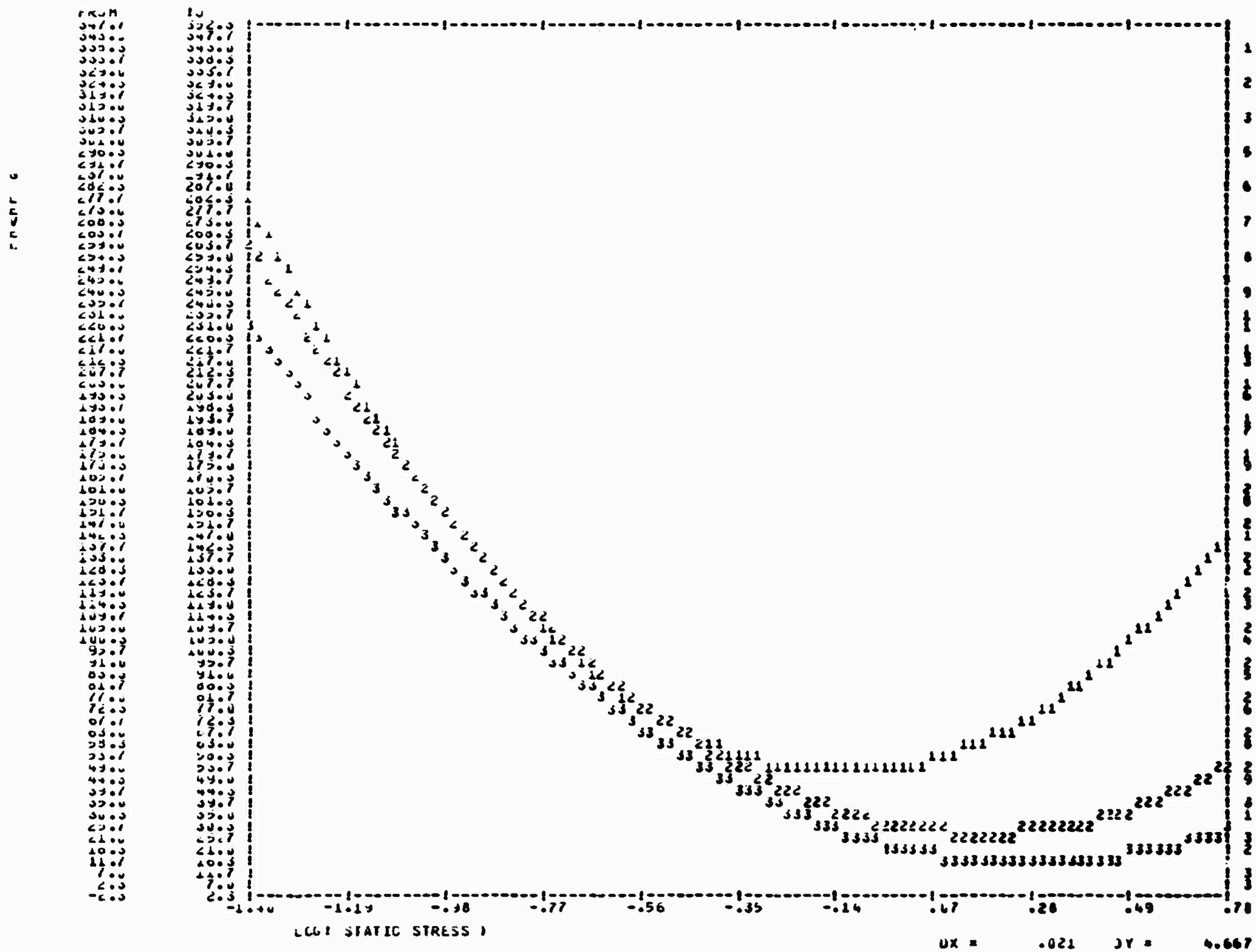

Figure D-1. -65°F, 18-inch drop height.

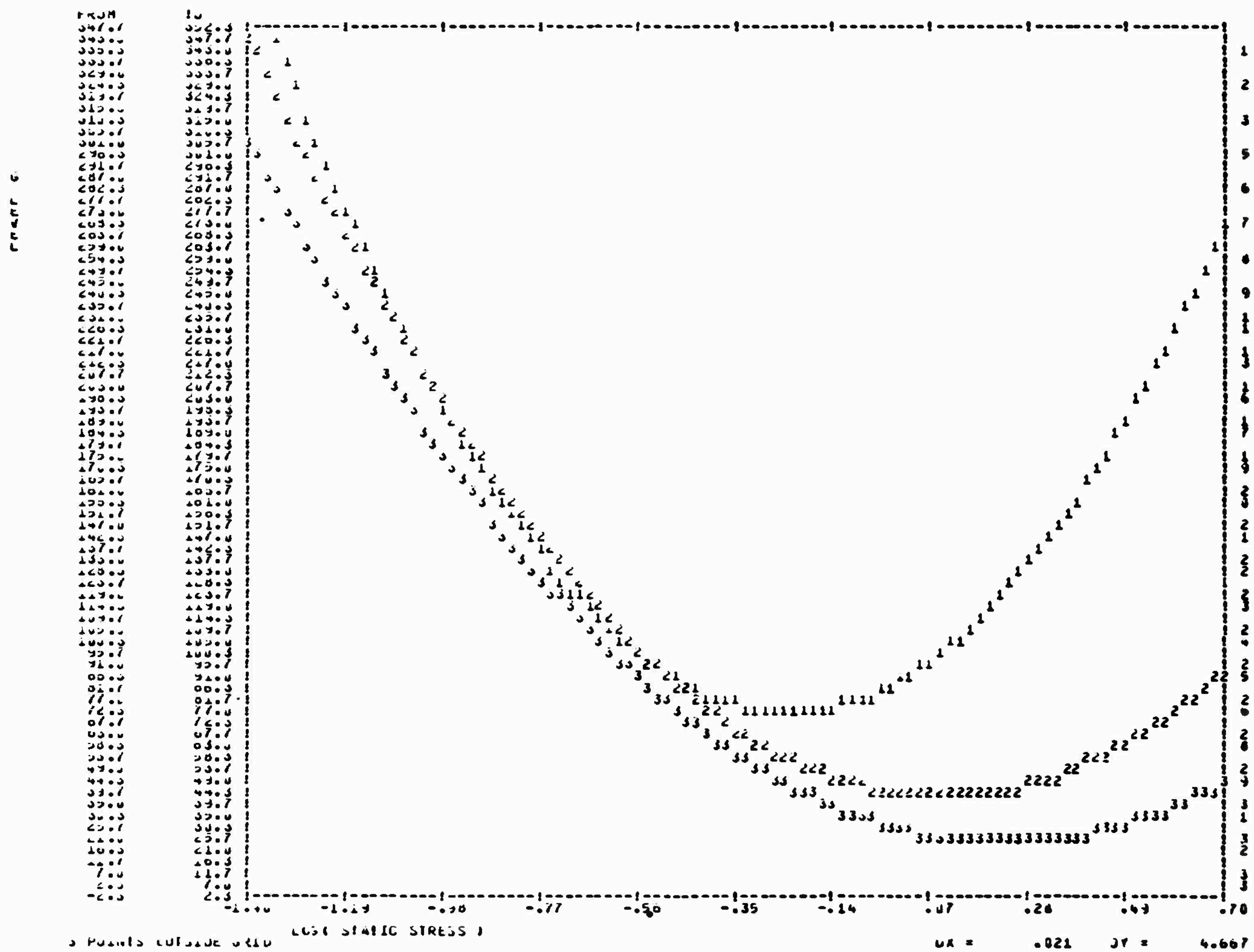

Figure D-2. -65°F, 30-inch drop height.

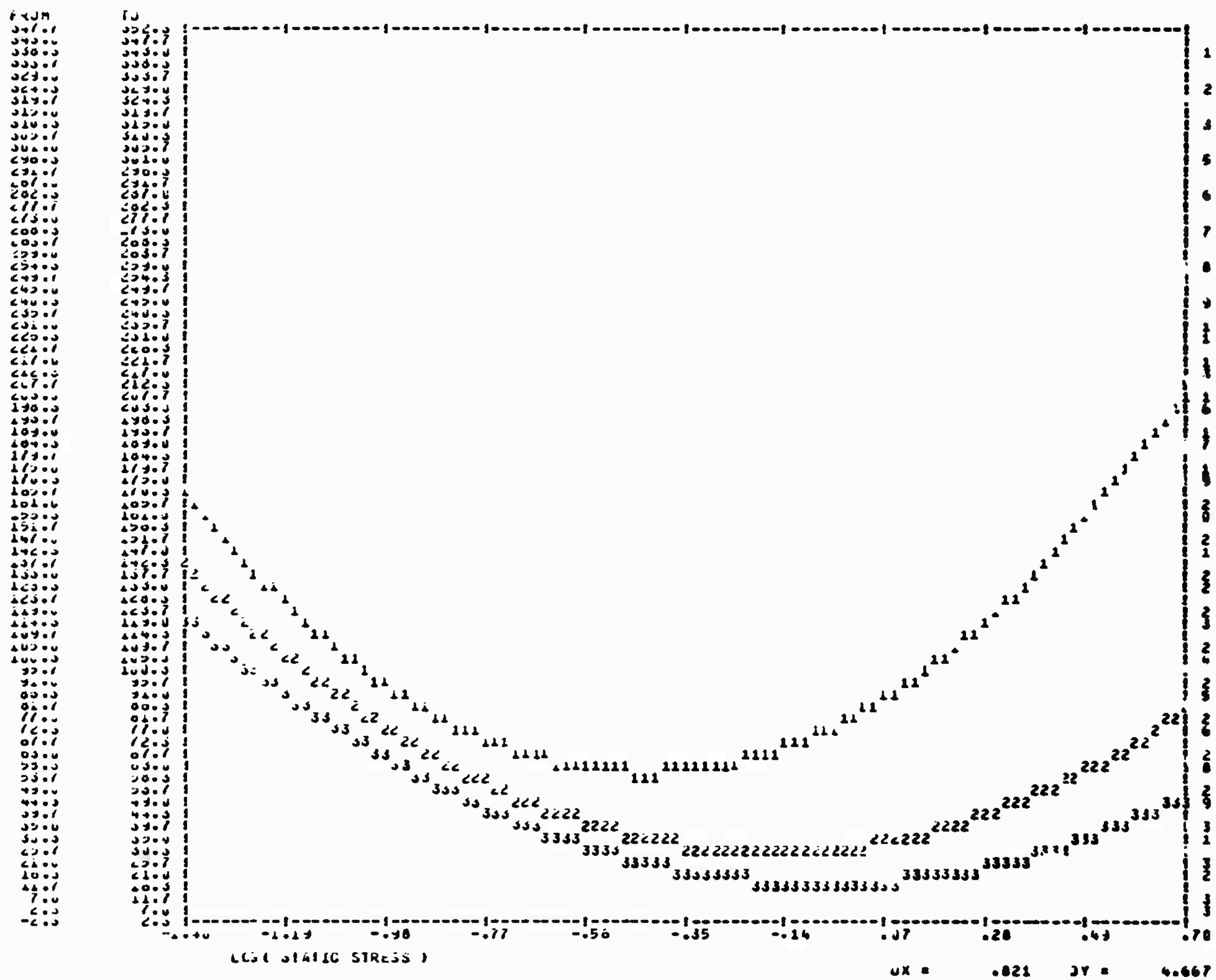

Figure D-3. 70°F, 18-inch drop height.

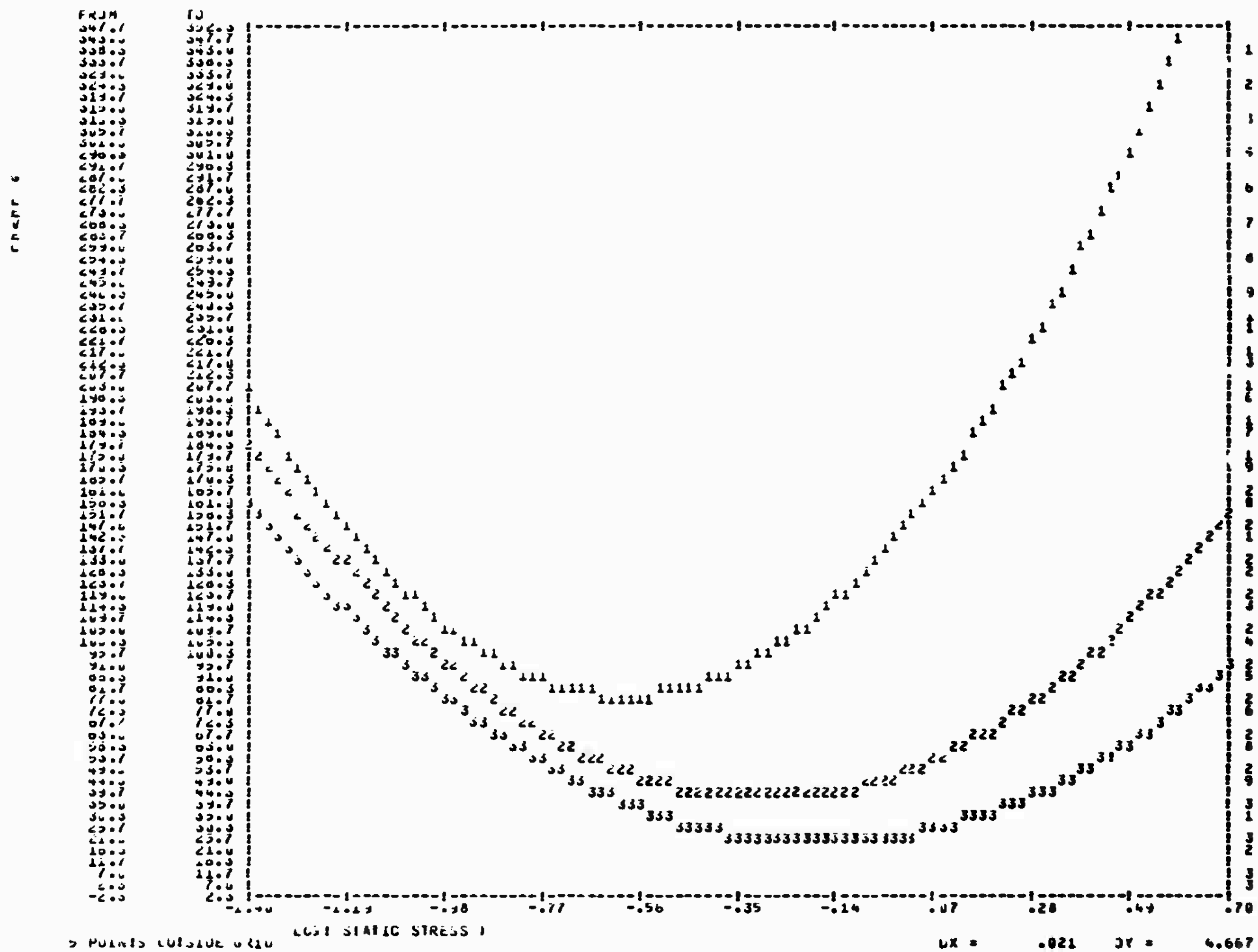

Figure D-4. 70°F, 30-inch drop height.

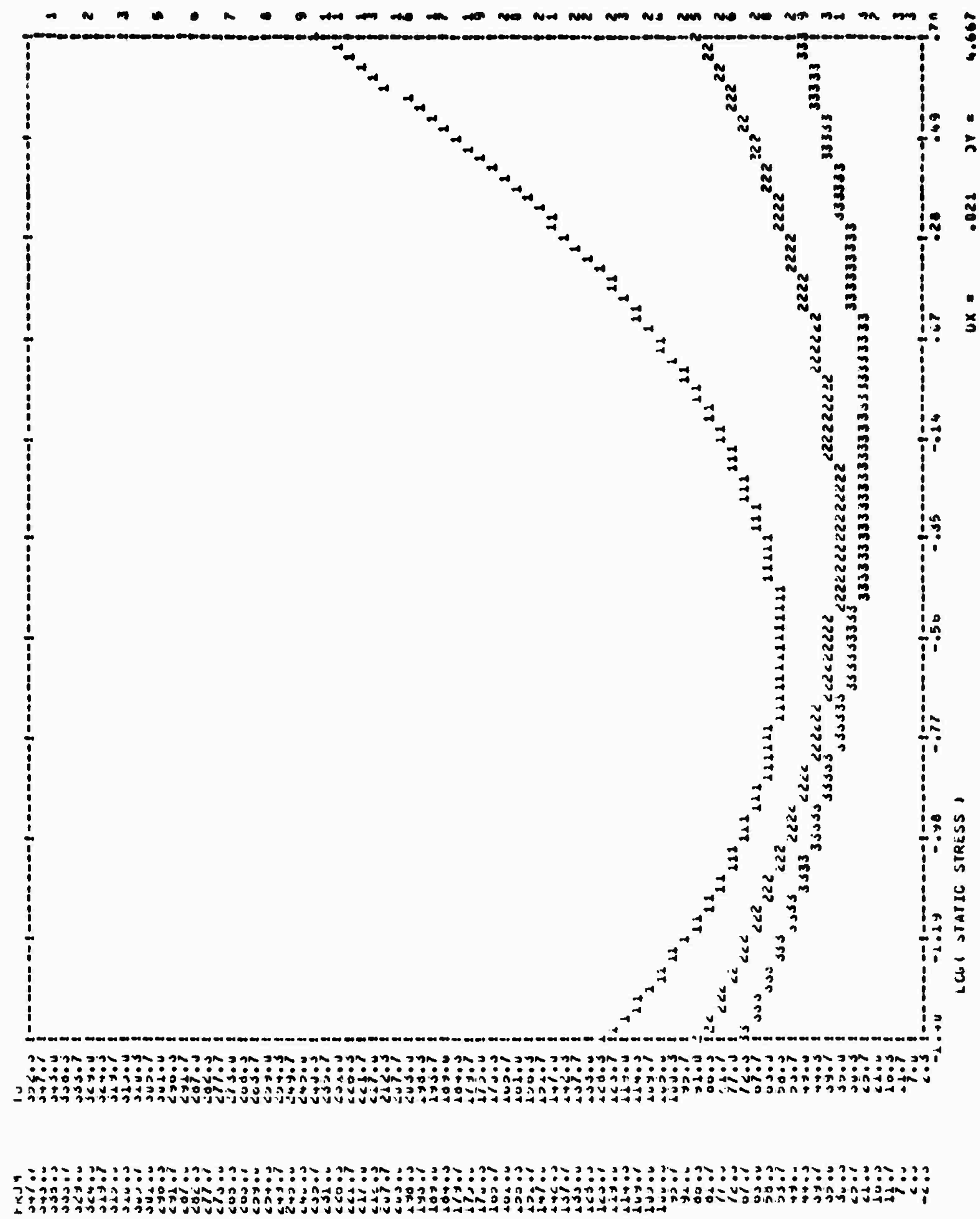

Figure D-5. 160°F, 18-inch drop height.

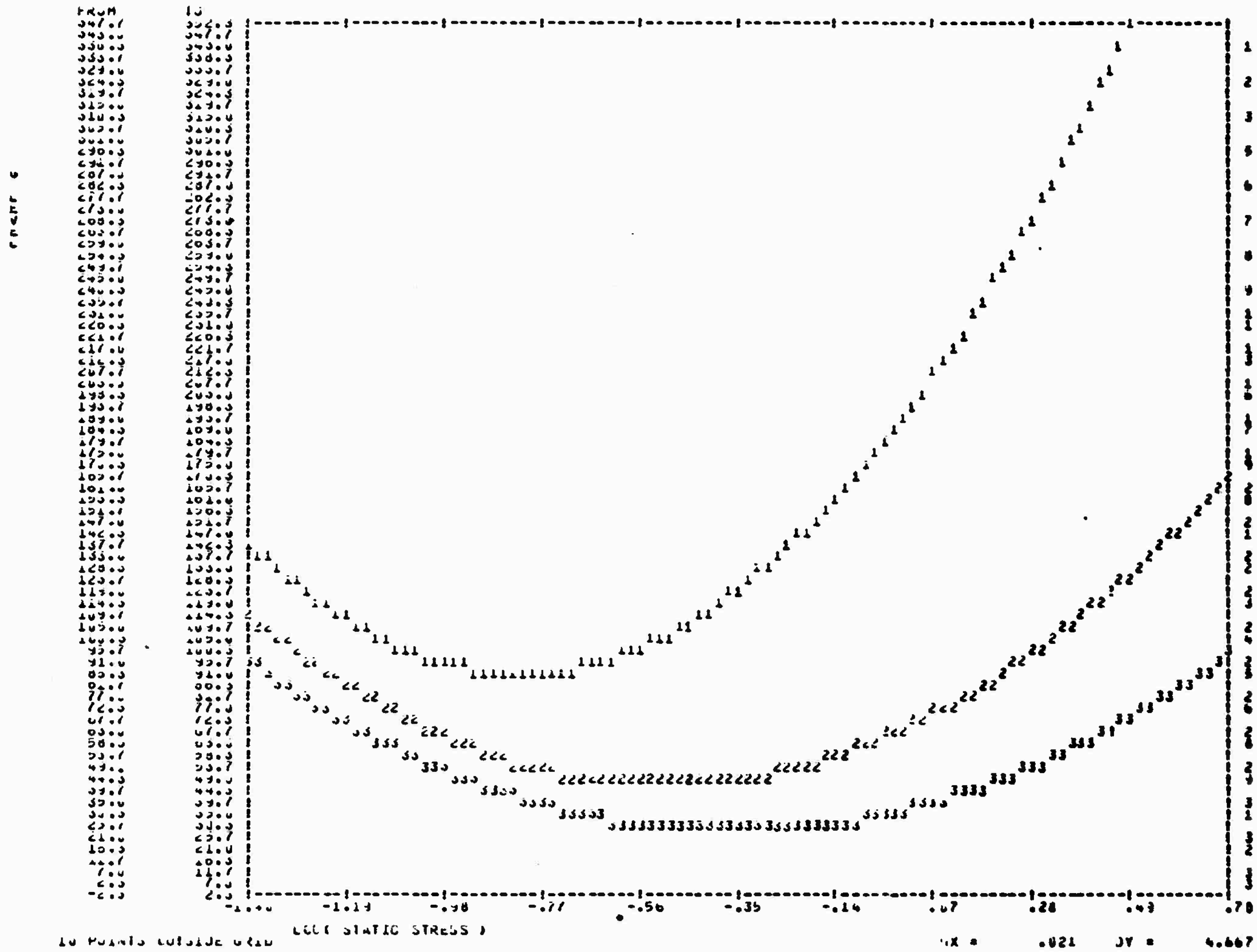

Figure D-6. 160°F, 30-inch drop height.

Appendix E

MLRD PLOTS OF DYNAMIC CUSHIONING CURVES OF THE MINICEL MODEL

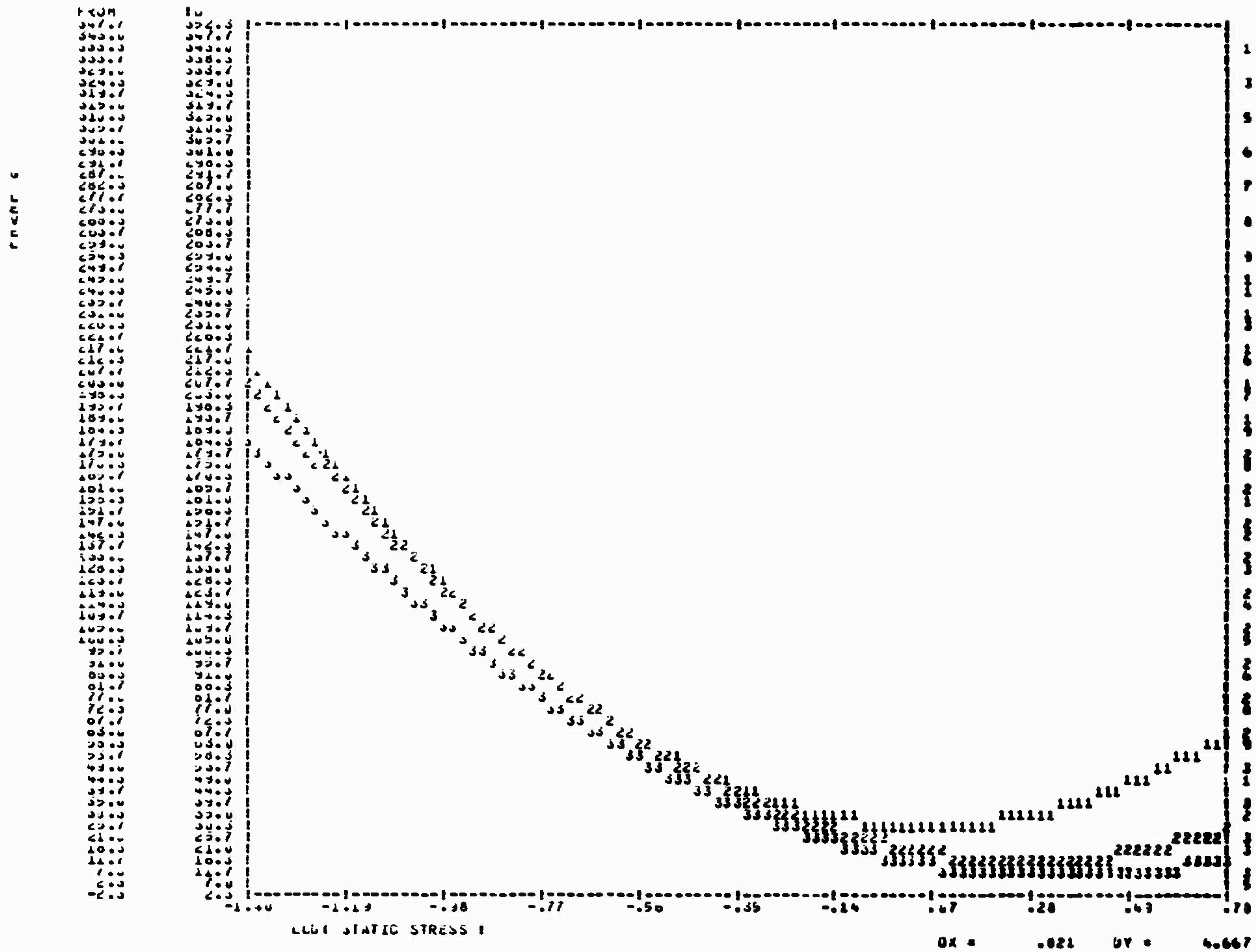

Figure E-1. -65°F, 12-inch drop height.

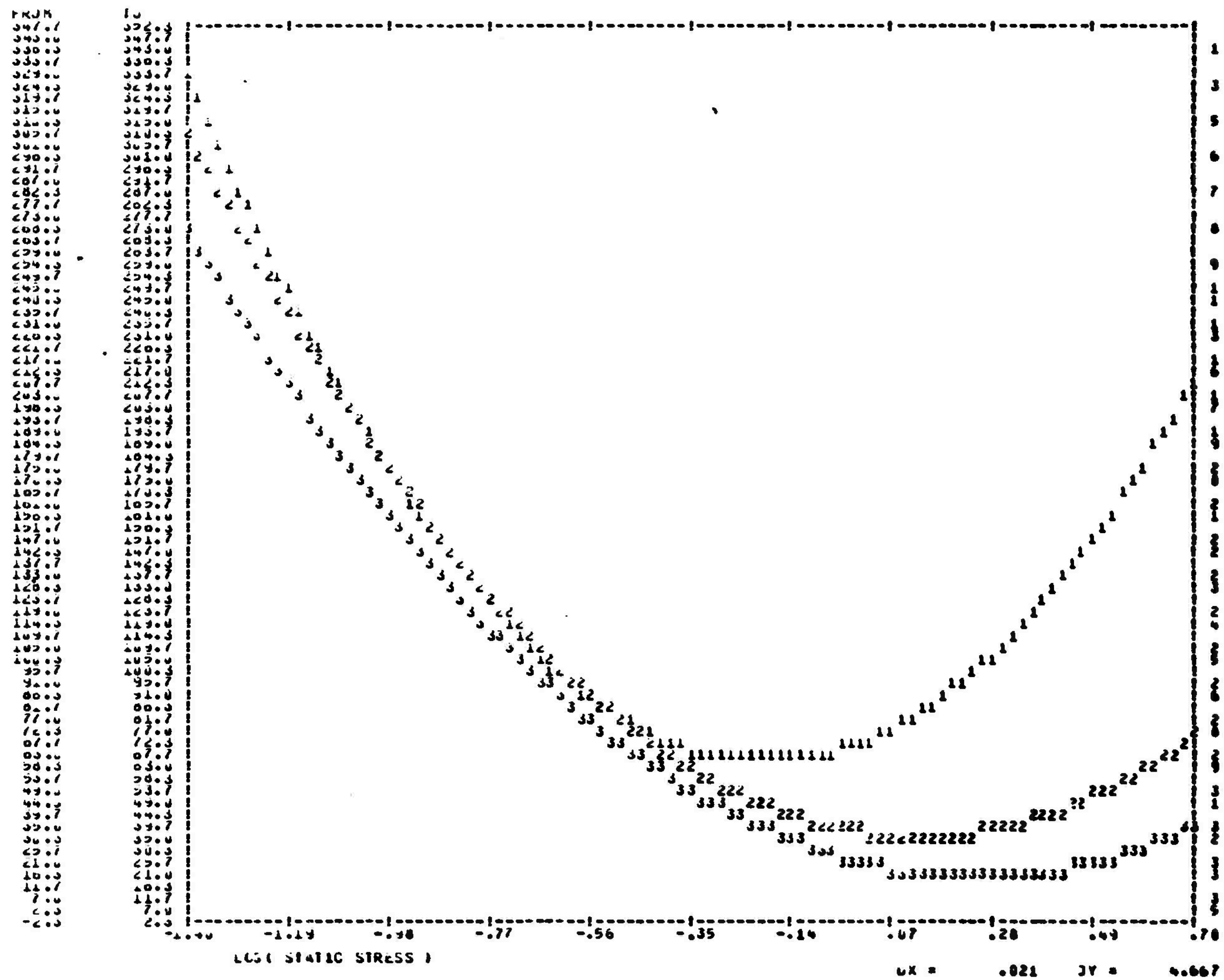

Figure E-2. -65°F, 24-inch drop height.

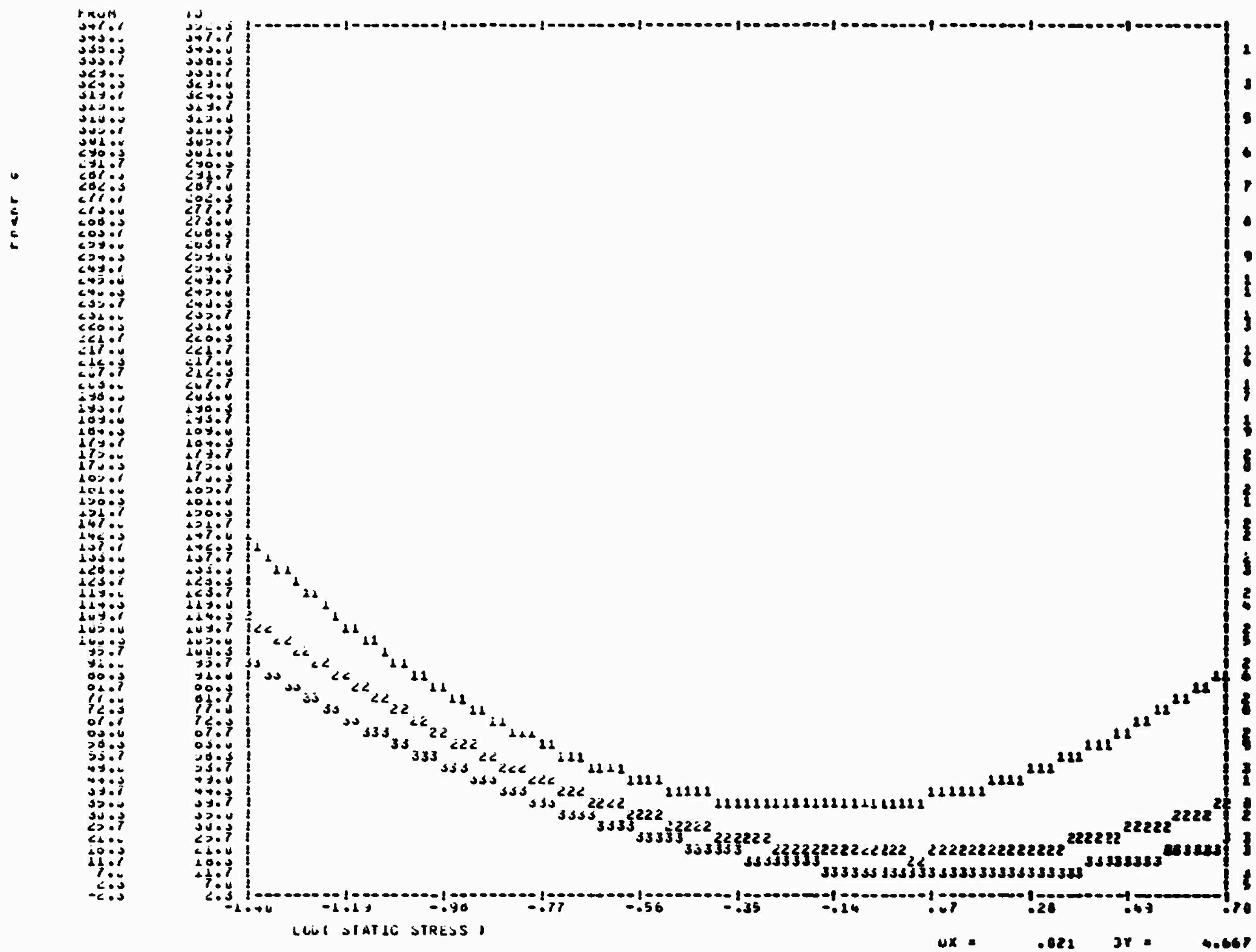

Figure E-3. 70°F, 12-inch drop height.

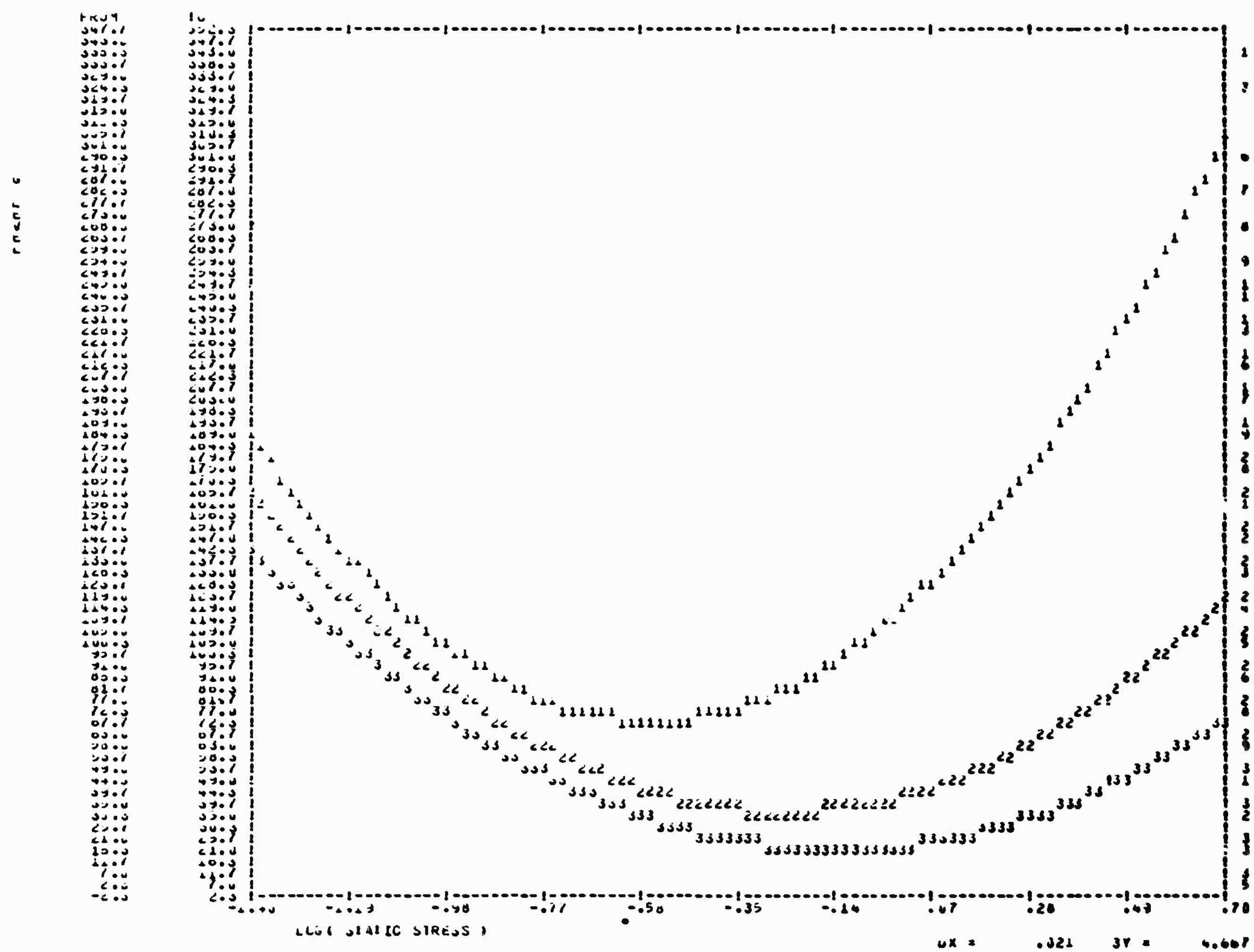

Figure E-4. 70°F, 24-inch drop height.

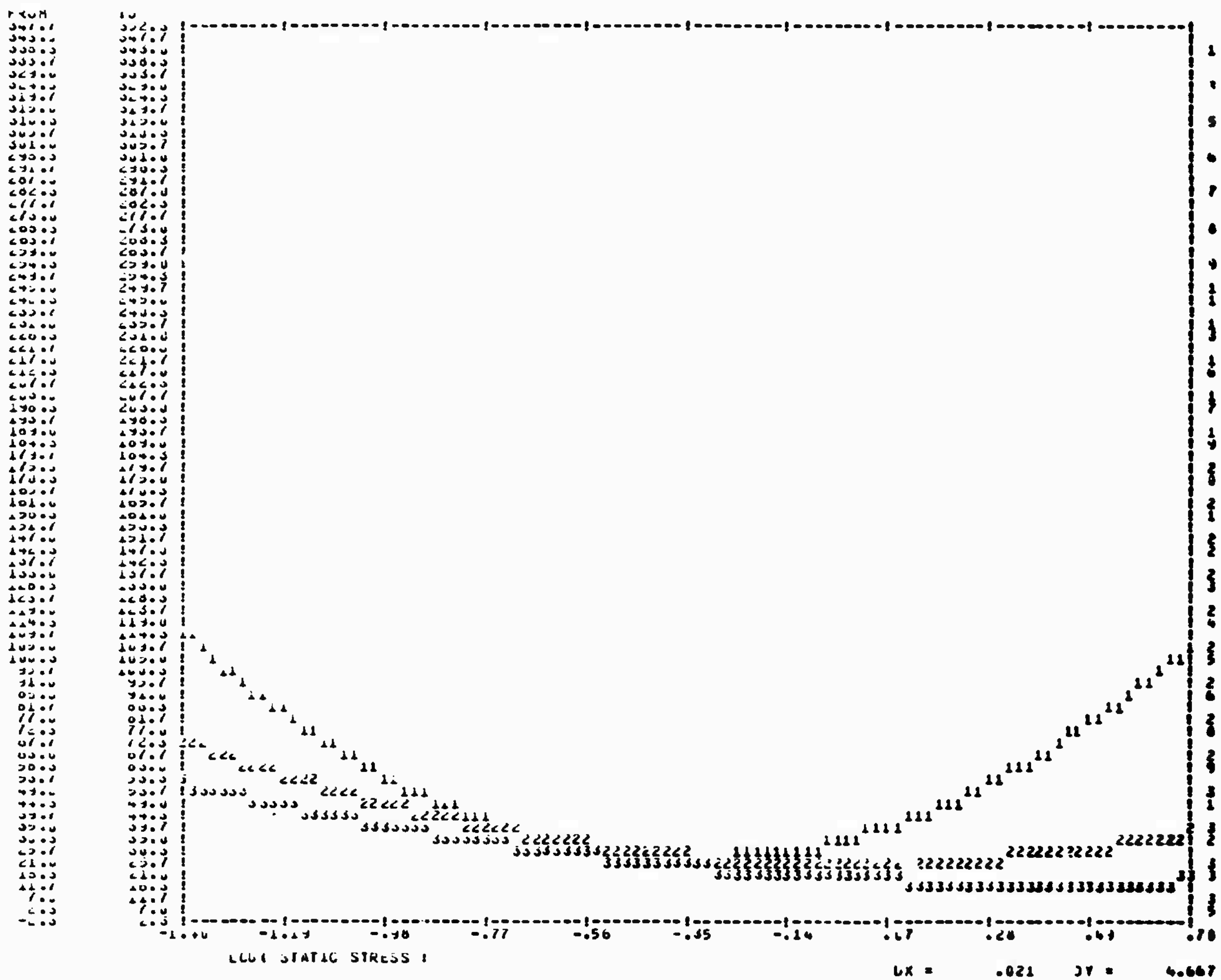

Figure E-5. 160°F, 12-inch drop height.

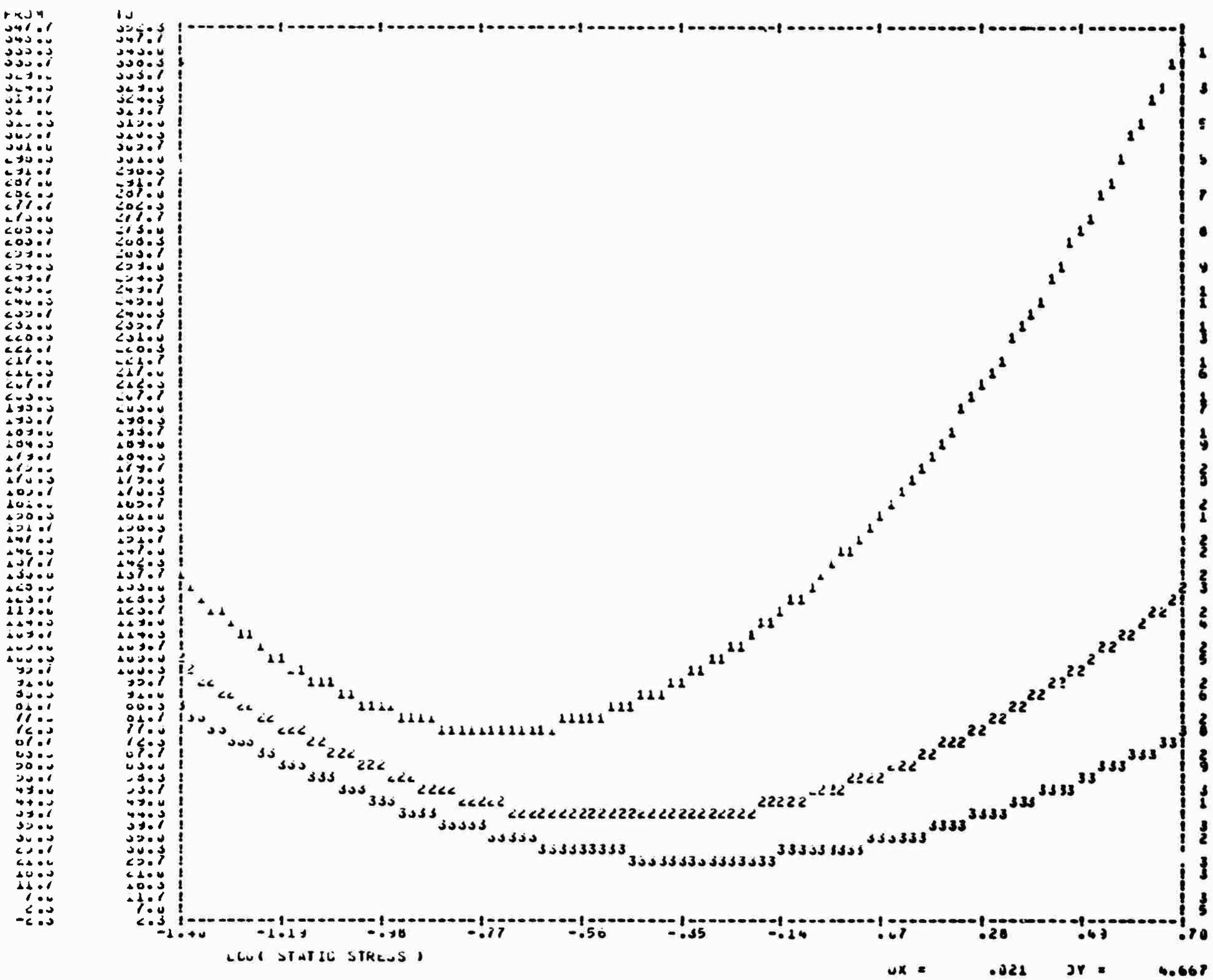

Figure E-6. 160°F, 24-inch drop height.

Appendix F

TWENTY-SEVEN-INCH DROP HEIGHT MINICEL DATA, 1, 2, AND 3 INCH THICK

Thickness	Replication	0.04			0.08			0.10			0.20			0.80		
		Temperature (°F)														
		-65	70	160	-65	70	160	-65	70	160	-65	70	160	-65	70	160
1"	1	349	236	142	276	146	123	215	146	76	111	79	86	41	83	141
	2	301	237	123	286	148	134	203	153	88	121	67	88	40	94	148
	3	383	247	105	292	142	142	207	120	85	96	68	79	40	90	149
2"	1	321	205	110	211	106	80	189	99	73	89	55	68	31	59	56
	2	346	180	92	228	124	73	184	111	97	94	55	60	31	40	56
	3	370	191	120	205	93	94	138	98	88	98	55	61	35	49	52
3"	1	278	181	86	289	66	47	185	55	45	67	49	29	35	30	28
	2	269	171	78	293	70	52	172	60	40	61	55	31	36	30	25
	3	264	215	86	271	68	50	177	56	53	53	53	29	36	32	23

Thickness	Replication	1.00			1.50			1.50			2.60			3.00			3.40		
		Temperature (°F)																	
		-65	70	160	-65	70	160	-65	70	160	-65	70	160	-65	70	160	-65	70	160
1"	1	57	96	183	80	162	178	96	183	269	122	224	264	178	285	323	170	259	324
	2	53	92	193	73	170	108	91	173	253	131	213	292	176	251	313	158	294	337
	3	38	99	189	85	168	184	106	172	258	132	211	296	170	278	308	167	297	284
2"	1	29	29	41	30	52	73	24	72	75	37	77	94	32	85	95	42	122	97
	2	29	29	38	24	57	45	20	74	75	39	87	93	37	92	91	30	123	118
	3	17	35	47	24	56	76	25	79	77	40	76	90	40	97	91	38	120	119
3"	1	22	24	35	18	32	45	18	20	31	24	31	37	20	33	44	16	36	45
	2	24	34	33	16	21	48	18	22	29	24	29	35	17	30	62	26	34	49
	3	23	20	26	17	30	48	18	22	27	25	26	38	18	31	62	19	31	45

Appendix G

FOUR- AND FIVE-INCH THICK MINICEL DATA

MINICEL - 12 in. Drop Height

STRESS LEVELS (PSI)

Replication	0.04			0.10			0.20			0.40			0.80			
	Temperature (°F)															
	-65	70	160	-65	70	160	-65	70	160	-65	70	160	-65	70	160	Thickness
1	210	75	58	117	47	31	71	29	20	40	21	14	20	13	11	4"
2	176	72	53	118	43	30	57	28	19	38	20	16	22	12	11	
3	151	91	60	121	56	35	69	26	21	40	18	15	21	14	11	
1	155	73	48	102	41	25	53	27	16	36	15	12	19	12	9	5"
2	120	106	60	98	34	30	60	26	15	35	16	13	19	8	10	
3	135	84	41	99	39	23	54	24	18	34	41	12	21	12	10	

Replication	1.00			1.60			2.00			2.40			3.00			
	Temperature (°F)															
	-65	70	160	-65	70	160	-65	70	160	-65	70	160	-65	70	160	Thickness
1	16	13	10	13	10	15	10	11	14	10	10	13	8	10	12	4"
2	20	10	12	13	9	15	11	11	15	9	11	13	8	11	10	
3	19	13	12	12	11	12	11	11	11	11	13	12	8	9	12	
1	27	11	6	12	9	13	10	7	12	10	9	9	8	8	9	5"
2	17	10	10	4	9	11	11	9	10	8	8	10	6	8	10	
3	18	11	7	11	9	9	10	7	9	7	8	8	8	8	8	

MINICEL - 12 in. Drop Height (Continued)

STRESS LEVELS (PSI)

		3.60			4.00			4.60			5.00			
		Temperature (°F)												
		-65	70	160	-65	70	160	-65	70	160	-65	70	160	Thickness
Replication	1	9	10	13	5	12	13	9	12	13	6	13	12	4"
	2	9	10	15	9	11	13	5	12	13	5	13	21	
	3	8	12	12	8	13	9	6	13	14	7	13	21	
	1	12	9	15	8	8	13	6	13	10	7	8	13	5"
	2	8	10	12	6	19	13	6	7	11	5	8	11	
	3	7	9	10	7	9	13	7	8	13	8	10	13	

MINICEL - 18 in. Drop Height

STRESS LEVELS (PSI)

		0.04			0.10			0.20			0.40			0.80			
		Temperature (°F)															
		-65	70	160	-65	70	160	-65	70	160	-65	70	160	-65	70	160	Thickness
Replication	1	235	120	61	148	58	41	80	31	26	39	22	21	24	17	19	4"
	2	253	103	66	126	55	42	74	28	24	41	23	19	23	18	17	
	3	206	91	66	139	55	35	71	30	25	37	23	19	25	20	16	
	1	186	94	55	118	50	35	61	26	21	39	18	18	20	14	9	5"
	2	198	81	62	109	46	38	59	29	21	34	22	17	23	16	15	
	3	178	92	55	115	47	35	72	27	22	37	19	18	23	15	14	

MINICEL - 18 in. Drop Height (Continued)

STRESS LEVELS (PSI)

Replication	1.00			1.60			2.00			2.40			3.00			Thickness
	Temperature (°F)															
	-65	70	160	-65	70	160	-65	70	160	-65	70	160	-65	70	160	
1	21	14	16	19	15	18	12	13	16	13	16	20	11	17	19	4"
2	21	15	16	20	13	17	17	13	17	13	13	21	13	14	19	
3	22	16	18	18	12	19	14	15	18	13	18	20	11	14	19	
1	20	14	13	17	10	11	13	12	12	12	12	13	11	12	14	5"
2	19	13	13	15	12	11	13	12	14	11	12	16	12	11	14	
3	20	13	13	13	10	14	10	12	13	9	14	14	11	11	16	

Replication	3.60			4.00			4.60			5.00			Thickness
	Temperature (°F)												
	-65	70	160	-65	70	160	-65	70	160	-65	70	160	
1	12	18	25	11	16	25	12	24	30	13	22	28	4"
2	11	18	22	11	18	25	11	19	28	13	17	30	
3	11	15	23	10	20	25	10	19	30	11	21	31	
1	10	14	16	8	15	17	9	15	20	9	12	21	5"
2	8	11	18	9	13	19	9	13	21	12	18	20	
3	9	13	18	9	13	17	7	17	24	9	13	22	

MINICEL - 24 in. Drop Height

STRESS LEVELS (PSI)

Replication	0.04			0.10			0.20			0.40			0.80			Thickness
	Temperature (°F)															
	-65	70	160	-65	70	160	-65	70	160	-65	70	160	-65	70	160	
1	190	122	59	151	54	39	71	32	27	44	26	26	27	20	21	4"
2	215	102	61	128	53	40	73	36	29	42	25	23	26	20	22	
3	167	120	57	131	56	42	86	34	30	40	24	23	26	20	25	
1	158	102	65	141	49	39	63	31	24	48	22	22	22	18	18	5"
2	187	93	59	174	55	38	64	30	25	35	21	21	29	18	17	
3	237	90	63	102	53	35	71	32	24	42	24	19	28	16	18	

Replication	1.00			1.60			2.00			2.40			3.00			Thickness
	Temperature (°F)															
	-65	70	160	-65	70	160	-65	70	160	-65	70	160	-65	70	160	
1	18	21	21	18	19	25	17	20	26	19	23	28	22	34	40	4"
2	20	19	24	18	20	23	16	20	26	18	26	26	16	28	30	
3	23	22	23	16	22	24	16	26	25	17	24	32	18	27	34	
1	21	15	18	15	15	19	14	18	21	12	16	21	12	18	24	5"
2	23	15	20	14	15	19	14	14	20	13	19	22	14	18	21	
3	21	17	17	15	14	19	13	18	21	14	16	22	12	20	24	

MINICEL - 24 in. Drop Height (Continued)

STRESS LEVELS (PSI)

		3.60			4.00		
	Temperature (°F)						
Replication	-65	70	160	-65	70	160	Thickness
1	20	32	38	12	31	56	4"
2	20	32	43	21	36	40	
3	17	33	36	22	32	42	
1	14	20	24	12	19	29	5"
2	17	18	30	15	23	25	
3	12	21	28	30	25	30	

MINICEL - 30 in. Drop Height

STRESS LEVELS (PSI)

		0.04			0.10			0.20			0.40			0.80		
	Temperature (°F)															
Replication	-65	70	160	-65	70	160	-65	70	160	-65	70	160	-65	70	160	Thickness
1	274	122	75	132	56	47	69	36	33	42	27	25	26	23	27	4"
2	250	126	74	151	60	48	72	37	35	43	29	27	26	27	28	
3	233	106	83	130	57	45	63	37	32	44	28	26	26	24	29	
1	60	96	69	126	56	39	81	33	27	43	24	20	26	18	22	5"
2	252	123	68	136	54	41	63	35	27	40	25	23	25	18	21	
3	222	100	74	152	62	36	87	40	27	49	23	21	27	19	21	

MINICEL - 30 in. Drop Height (Continued)

STRESS LEVELS (PSI)

	1.00			1.60			2.00			2.40			3.00			
	Temperature (°F)															
Replication	-65	70	160	-65	70	160	-65	70	160	-65	70	160	-65	70	160	Thickness
1	25	27	29	18	27	35	15	29	35	23	36	44	24	39	42	4"
2	23	36	30	21	38	36	15	32	38	24	33	43	24	43	43	
3	23	24	30	21	31	33	18	30	32	19	32	41	25	39	42	
1	30	18	25	18	16	22	22	18	20	12	19	23	15	22	30	5"
2	26	21	21	17	19	26	15	18	24	12	15	25	18	24	28	
3	23	13	23	19	18	20	12	25	22	16	20	26	16	20	32	

	3.60			4.00			
	Temperature (°F)						
Replication	-65	70	160	-65	70	160	Thickness
1	23	40	55	29	51	61	4"
2	24	42	56	24	52	57	
3	41	42	58	32	47	57	
1	16	27	38	15	27	40	5"
2	15	28	39	15	32	46	
3	14	29	40	14	52	36	

Appendix H

PROGRAM LISTING OF CUSHION OPT

```
      PROGRAM SEARCH( INPUT=65, OUTPUT,  TAPE5=INPUT,TAPE6=OUTPUT )
      COMMON  TP, OH, TC, SS, GL, NVR, V(51), INC(51),COEFF(51),CONST,NV
      COMMON  CH, CA, CC, DROPH, GLMAX, SSL(3), SSU(3), TYPEM(2)
      DIMENSION  TT(5), PRV(5), NCK(5), TCC(3)
      EQUIVALENCE (TT(1),CH)
CCC   EQUIVALENCE (TT(1),CH),(TT(2),CA),(TT(3),CC),(TT(4),DROPH)
CCC   EQUIVALENCE (TT(5),GLMAX)

      DATA  PRV/160.,70.,-65., 30.,75./
      CALL  ID4020( 14, 14HWAYNE L. JONES )
      CALL DATE( DAY )
      DSS = 0.05
      SSMIN = 0.05
      SSMAX = 5.00
      DTC = 0.5
      TCMIN = 1.0
      TCMAX = 6.0
C
      READ 901, TYPEM
  901 FORMAT(8A10)
      PRINT 900, DAY,  TYPEM
  900 FORMAT(*1HCOANIEL SEARCH PROGRAM*20X,A10 //1X,2A10.)
C          READ EQUATION COEFFICIENTS
      READ 908, J, CONST, NV
  908 FORMAT( I5, E20.8, 5X, I5 )
      PRINT 910, CONST
  910 FORMAT(9X*INDEX  COEFFICIENT*/10X*0*F15.8)
      DO 20 J=1,NV
      READ 908, IND(J), COEFF(J)
   20 PRINT 912, J, IND(J), COEFF(J)
  912 FORMAT(1X(5,I5,E15.8)
C
   10 READ 902, DROPH,GLMAX, CH,CA,CC,      (NCK(J),J=4,5),(NCK(J),J=1,3)
  902 FORMAT(5F10.0, T1,5A10)
      IF( CCF( 5       )  .NE.  0.0 )   GO TO 800
C
      DO 15 J=1,5
      IF( NCK(J) .EQ. 1H   )  TT(J) = PRV(J)
   15 PRV(J) = TT(J)
      DH = DROPH
      PRINT 900, DAY,  TYPEM
      PRINT 904,  DH, GLMAX
  904 FORMAT(* DROP HEIGHT =*F4.0,5X*GLMAX =*F5.0/)
      PRINT 906, ( TT(J), J=1,3 )
  906 FORMAT(* TEMPERATURES =* 3F5.0/)
C
C          INITIALIZE THICKNESS
      TC = TCMIN - DTC
C          HAS MAXIMUM THICKNESS BEEN REACHED ?
  100 IF( TC .GE. TCMAX )  GO TO 500
C         INCREMENT THICKNESS
      TC = TC + DTC
      DO 120 J=1,3
  120 SSL(J) = SSU(J) = TCC(J) = 0.0
C
C          INITIALIZE TEMPERATURE INDEX
      N = 0
```

```
PROGRAM SEARCH      74/74    OPT=1                        FTN 4.2+74278       11/25/74  15.42.24.

      C          HAS THIRD TEMPERATURE BEEN REACHED ?
        200 IF( N .GE. 3 )  GO TO 500
      C        INCREMENT TEMPERATURE
            N = N + 1
            TP = TT(N)
      C
      C          INITIALIZE STATIC STRESS
            SS = SSMIN - OSS
      C          HAS MAXIMUM STATIC STRESS BEEN REACHED ?
        300 IF( SS .GE. SSMAX )  GO TO 100
      C        INCREMENT STATIC STRESS
            SS = SS + OSS
      C
            CALL MODEL
            IF( GL .GE. GLMAX )  GO TO 300
            SSL(N) = SS
            TCC(N) = TC
      C
      C          HAS MAXIMUM STATIC STRESS BEEN REACHED ?
        400 IF( SS .GE. SSMAX )  GO TO 420
      C        INCREMENT STATIC STRESS
            SS = SS + OSS
      C
            CALL MODEL
            IF( GL .LE. GLMAX )  GO TO 400
        420 SSU(N) = SS
      C          MAKE RANGE TEST
            IF( SSU(N)-SSL(N) .GE. 0.2 )  GO TO 200
            GO TO 100
      C
        500 CONTINUE
            PRINT 920, ( TT(J), SSL(J), SSU(J), TCC(J), J=1,3 )
        920 FORMAT(//* TEMP   SSL   SSU   TC*//(1XF4.0,F6.2,F6.2,F5.1))
            IF( SSL(3)  .EQ. 0.0 )  GO TO 510
            TEST = SSU(1) - SSL(3)
            IF( TEST .GE. 0.2 )  GO TO 520
        510 PRINT 922
        922 FORMAT(//* RANGE TEST FAILED*)
            IF( TC .GE. TCMAX )  PRINT 924
        924 FORMAT(//* USE OTHER MATERIAL*)
            IF( TC .GE. TCMAX )   GO TO 10
            GO TO 100
        520 CONTINUE
            PRINT 926, TC, TYPEM, GLMAX, SSL(3), SSU(1)
        926 FORMAT(/1XF3.1,* INCHES OF *2A10/  * GIVES A*F5.0* G PROTECTION*/
           * * USING A STATIC STRESS*/* RANGE OF*F5.2* TO*F5.2)
      C          PLOT CURVES THAT PASS RANGE TEST
            CALL MPLOT
      C
            GO TO 10
        800 CONTINUE
            CALL SMXYV( 0 ,0 )
            CALL USER ID ( 25HQUICKPL TEKTRONICS TEST   )
            CALL TEKTEST
            CALL END JOB
            END
```

```
      SUBROUTINE MODEL
      COMMON  TP, DH, TC, SS, GL, NVR, V(51), IND(51),COEFF(51),CONST,NV
      COMMON  CH, CA, CC, DROPH, GLMAX, SSL(3), SSU(3), TYPE*(2)
C
C********* DYNAMIC CUSHIONING MODEL ******************************************
      SS100 = SS * 100.
      AL = ALOG( SS100 )
      AL2 = AL * AL
      SRDH = SQRT( DH )
      TCSH = TC ** (-3.5)
       TR = (TP+460)/100.
      TR2 = TR * TR
      TR3 = TR * TR2
      TR4 = TR3 * TR
C
      TCOH = TC ** (-0.5)
      TCTH = TC ** (-1.5)
      TCINV= TC ** (-2.5)
C
      V(01) = TR    * TCOH    * 1.0
      V(02) = TR    * TCOH    * 1.0     * AL
      V(03) = TR    * TCOH    * 1.0     * AL2
      V(04) = TR    * TCTH    * SRDH
      V(05) = TR    * TCTH    * SRDH    * AL
      V(06) = TR    * TCTH    * SRDH    * AL2
      V(07) = TR    * TCOH    * SRDH
      V(08) = TR    * TCOH    * SRDH    * AL
      V(09) = TR    * TCOH    * SRDH    * AL2

      V(10) = TR2   * TCOH    * 1.0
      V(11) = TR2   * TCOH    * 1.0     * AL
      V(12) = TR2   * TCOH    * 1.0     * AL2
      V(13) = TR2   * TCTH    * SRDH
      V(14) = TR2   * TCTH    * SRDH    * AL
      V(15) = TR2   * TCTH    * SRDH    * AL2
      V(16) = TR2   * TCOH    * SRDH
      V(17) = TR2   * TCOH    * SRDH    * AL
      V(18) = TR2   * TCTH    * SRDH    * AL2

      V(19) = TR3   * TCOH    * 1.0
      V(20) = TR3   * TCOH    * 1.0     * AL
      V(21) = TR3   * TCOH    * 1.0     * AL2
      V(22) = TR3   * TCTH    * SRDH
      V(23) = TR3   * TCTH    * SRDH    * AL
      V(24) = TR3   * TCTH    * SRDH    * AL2
      V(25) = TR3   * TCOH    * SRDH
      V(26) = TR3   * TCOH    * SRDH    * AL
      V(27) = TR3   * TCTH    * SRDH    * AL2

      V(28) = TR    * TCTH
      V(29) = TR    * TCTH              * AL
      V(30) = TR    * TCTH              * AL2
      V(31) = TR2   * TCTH
      V(32) = TR2   * TCTH              * AL
      V(33) = TR2   * TCTH              * AL2
      V(34) = TR3   * TCTH
```

```
SUBROUTINE MODEL       74/74    OPT=1                           FTN 4.2+74278      11/25/74  15.42.21.

            V(35) = TR3    * TCTH               * AL
            V(36) = TR3    * TCTH               * AL2
C* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C          COMPUTE DYNAMIC CUSHIONING FUNCTION
            GL = CONST
            DO 100 J=1,NV
            I = INC(J)
        100 GL = GL + COEFF(J) * V(I)
            RETURN
            END
```

```
      SUBROUTINE MPLOT
      COMMCN  TP, CH, TC, SS, GL, NVR, V(51) , IND(51),COEFF(51),CCAST,NV
      COMMON  CH, CA, CC, DRGPH, GLMAX, SSL(3), SSU(3), TYPEM(2)
      DIMENSION        X(101), Y(101,3), A(855), GG(101) , S(101)
      DIMENSION  HEAD(10), LEFT(5), BOTTOM(8), NCH(3),  T(3), LAB(3,8)
      DATA  NCH/1HH,1HA,1HC/
      DATA  LEFT /2*1H , 7HG LEVEL, 2*1H  /
      DATA  BOTTCM /2*1H ,10HSTATIC STR ,3HESS, 4*1H  /
      DATA  HEAD (1)/25H DYNAMIC CUSHIONING CURVE /
C
      T(1) = CH
      T(2) = CA
      T(3) = CC
      HEAD(4) = TYPEM(1)
      HEAD(5) = TYPEM(2)
      XMIN = ALOG10( 0.04 )
      XMAX = ALOG10( 5.00 )
      DX = ( XMAX - XMIN ) / 100.
C
      XX= XMIN
      DO 10 JP = 1, 101
      X(JP)= XX
      S(JP) = 10.  ** XX
      XX = XX + DX
      GG(JP) = GLMAX
   10 CONTINUE
C
      DO 30 K=1,3
      TP = T(K)
      DO 20 JP = 1,101
      SS = S(JP)
      CALL MODEL
      Y(JP,K) = GL
   20 CONTINUE
   30 CONTINUE
C
      CALL SETGRID( A, -76, XMIN, XMAX, 0.0, 350. )
      CALL LABGRID( A, 1, 25, 25H      LOG( STATIC STRESS )  )
      CALL LABGRID( A, 2, 30, LEFT )
      CALL LABGRID( A, 3,  40, HEAD )
C
      CALL PLTGRID( A, 1H., 101, X, GG       )
      CALL PLTGRID( A, 1HH, 101, X, Y(1,1)   )
      CALL PLTGRID( A, 1HA, 101, X, Y(1,2)   )
      CALL PLTGRID( A, 1HC, 101, X, Y(1,3)   )
C
      ENCODE( 120,904,LAB )  CH, CA, CC, DROP
  904 FORMAT(*H =*F5.0* DEGREES*14X, *A =*F5.0* DEGREES*14X, *C =*F5.0,
     1* DEGREES*14X,*DROP HEIGHT =*F4.0* INCHES*6X)
      ENCODE( 120,906,LAB(1,5) ) TC,TYPEM, GLMAX, SSL(3), SSL(1)
  906 FORMAT(F3.1* INCHES OF *A10,A6,*GIVES A*F5.0* G PROTECTION*5X,
     1 *USING A STATIC STRESS*9X,*RANGE OF*F5.2* TO*F5.2,9X)
      NY = 76
      DO 40 J=1,8
      NY = NY - 1
      CALL REMGRID( A, 40, NY, 25, LAB(1,J) )
   40 CONTINUE
```

```
SUBROUTINE MPLOT      74/74    OPT=1                              FTN 4.2+74278      11/25/74  15.42.41.

         C
               CALL PRINTPL( A, 6LOUTPUT )
         C
               CALL SC4020
               CALL LABEL( 1, 35, BOTTOM, 5 )
               CALL LABEL( 1, 30, LEFT  , 6 )
               CALL LABEL( 1,  50, HEAD , 3 )
               CALL PLOTA( S, GG, 0.04,5.0 , 0.,350., 101, 5, 4, 1, 100 )
               JX = 420
               JY = 900.
               DO 45 J=1,8
               CALL RNOTE( JX, JY, 30, LAB(1,J) )
            45 JY = JY - 25
               DO 50 K = 1,3
               CALL PLOT3( S, Y(1,K), 101, -0 )
               CALL NOTE( S(100), Y(098,K), 1, NCH(K) )
            50 CONTINUE
               RETURN
               END
```

SUBROUTINE TEKTEST   74/74   OPT=1   FTN 4.2+74278   11/25/74 15.42.44.

```
      SUBROUTINE TEKTEST
      DIMENSION  X(10),Y(10),Z(10)
      DATA  X/1., 2., 3., 4., 5., 6., 7., 8., 9., 10. /
      DATA  Y/100.,50.,70.,50.,60.,50.,70.,50.,70., 40./
      DATA  Z/100.,100.,100.,80.,90.,80.,90.,80.,70.,100./
      CALL QUICKPL(X,Y,10,-2,6HX-AXIS,6HY-AXIS,8H TEKTEST,0.,0.,X(10),
     * Y(10),Y(1),200,1)
      CALL QUICKPL(X,Z,-10,-1)
      CALL PRINTV( 4,4HXXXX,500,500)
      CALL PRINTV( 4,4HOOOO,600,600)
      CALL LINEV( 0,500,500,500)
      CALL LINEV(500,0 ,500,500)
      CALL LINEV( 0,600,600,600)
      CALL LINEV(600,0 ,600,600)
      RETURN
      END
```