CRC-269

The Effects of Reenlistment Bonuses

CENTER FOR NAVAL ANALYSES
1401 Wilson Boulevard
Arlington, Virginia 22209
Institute of Naval Studies

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September 1974

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Prepared for:
OFFICE OF NAVAL RESEARCH
Department of the Navy
Washington, D.C. 20350
OFFICE OF THE CHIEF OF NAVAL OPERATIONS (Op96)
Department of the Navy
Washington, D.C. 20380

02 026900.00

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

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1. REPORT NUMBER: CRC 269	2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle): The Effects of Reenlistment Bonuses	5. TYPE OF REPORT & PERIOD COVERED
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): Samuel D. Kleinman, William F. Shughart II, USN	8. CONTRACT OR GRANT NUMBER(s): N00014-68-A-0091
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Center for Naval Analyses, 1401 Wilson Boulevard, Arlington, Virginia 22209	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS: Office of Naval Research, Department of the Navy, Washington, D.C. 20350	12. REPORT DATE: September 1974
	13. NUMBER OF PAGES: 62
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office): Office of the Chief of Naval Operations (Op96), Department of the Navy, Washington, D.C. 20380	15. SECURITY CLASS. (of this report): Unclassified
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

This Research Contribution does not necessarily represent the opinion of the Department of the Navy.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

enlisted personnel, military personnel, reenlistment, regression analysis, retention, salaries, SRB - Selective Reenlistment Bonus, VRB - Variable Reenlistment Bonus

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

During the period FY 1966-74, the variable reenlistment bonus (VRB) was the primary policy tool used by the Navy to increase retention in enlisted occupations (ratings). Placing the reenlistment decision in an occupational choice framework, this paper employs regression techniques to analyze the effect of reenlistment bonuses on first-term reenlistments, and moreover on lengths of recommitment and second-term reenlistments, two areas that had not been investigated previously. With ratings as the units of observation, data is analyzed for selected intervals up to FY 1973. The

20. results contained in this paper are applied in the Manpower Compensation Study in determining the cost-effectiveness of using reenlistment bonuses to obtain additional careerists relative to first-termers. The techniques of analysis developed in this paper, as well as the general findings, are equally applicable to the selective reenlistment bonus (SRB).

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INTRODUCTION

The variable reenlistment bonus (VRB) is designed to increase first-term reenlistments in specific occupations (ratings). It is a very powerful tool: when coupled with the regular reenlistment bonus, it can be as high as $10,000 depending upon pay grade, length of service, length of reenlistment, and VRB multiple. In FY 1973, over half of Navy first-term reenlistees, excluding ratings with six-year obligors, received a VRB. The total cost of VRB to the Navy in that year was $58.8 million. Because of its wide use and cost, we investigated the impact of VRB on (1) first-term reenlistment rates (FTRR), (2) length of first-term recommitments (LOR), and (3) second-term reenlistment rates (STRR).

Though we examine VRB, the analysis is readily applicable to the new selective reenlistment bonus (SRB). SRB is similar in concept and computation to VRB. In fact, it can be viewed as an expanded VRB when other bonuses (regular reenlistment bonus and pro pay) are eliminated. The effect of Zone A SRB (the bonus awarded prior to the sixth year) on first-term reenlistments would be exactly the same as VRB. Furthermore, the examination of STRR indicates the potential use of Zone B SRB (the bonus awarded after the sixth year). If we obtain a career commitment from individuals with the first-term reenlistment bonus, we may be able to keep down expenditures on the Zone B bonus.

While VRB was in effect, a regular reenlistment bonus (RRB) was awarded to all ratings. It was equal to an individual's monthly base pay multiplied by the number of years for which he reenlisted. The VRB was the regular reenlistment bonus times a VRB multiple assigned to each rating. The multiples ranged from 0 to 4. A multiple of 2 meant that the individual was awarded the regular reenlistment bonus multiplied by two in addition to the regular bonus. There was a ceiling of $2,000 on the regular reenlistment bonus and $10,000 on the total bonus. At most, the individual could only receive the VRB for one reenlistment.

Prior to presenting the empirical analyses, we discuss the occupational choice decision. Two hypotheses are proposed concerning the effect of VRB: that it increases first-term reenlistment rates and that it decreases second-term reenlistment rates. These hypotheses are tested in the empirical section and conclusions drawn.

We begin with an analysis of the effect of VRB on FTRR. Retaining first-termers beyond their original enlistment reduces turnover costs, the most important of which is training expenditures. It is also believed that a more experienced force is more productive, since individuals gain familiarity with their jobs and require less supervision. But there are added costs associated with greater retention: experienced individuals draw higher earnings and are more likely to accumulate sufficient years to draw retirement compensation. The optimum ratio of experienced to first-term personnel is therefore not clearcut. We confine the analysis to the impact of the VRB policy variable and avoid the issue of the optimal experience mix.

VRB has a positive impact on first-term reenlistment rates. For a given rating, a bonus equivalent to $1000 is associated with an average increase of 1.38 precentage points in the reenlistment rate. Lengths of recommitment are also positively related to VRB. An increase of one in the VRB multiple is associated with an increase of .46 to .69 years in the average length of recommitment. Second-term reenlistment rates were not found to be related to VRB.

The findings on the effect of VRB on FTRR are based on three time intervals: FY 1965-67, FY 1968-69, and FY 1971-72. Simple regressions were run using changes in FTRR and VRB as the dependent and independent variables, respectively. Two sets of regressions, each based on different assumptions concerning how variables interact, yielded similar results, with elasticities increasing over time. For FY 1965-67, the two elasticity measures were 2.27 and 2.20. For FY 1971-72, they were considerably higher, 4.04 and 4.24. The elasticities for FY 1968-69 fall between the earlier and latter values. Although the elasticities appear to increase over time, the slopes decrease. For predicting future effects of VRB, we recommend that a 1.2 percentage point increase in reenlistment rates for a $1000 increase in VRB be used as an upper bound, and that a 1 percentage point increase may be more appropriate.

Two biases may be present in these findings. One results from the assumption that changes in non-VRB factors are constant across ratings. The other is the result of a possible misspecification of the model. The former probably introduces a downward bias and the latter an upward bias in the estimates, but the net effect is uncertain. The results, however, are consistent with previous work.

Our conservative estimate indicates that 2,154 of the first-term reenlistments in FY 1972 were induced by VRB. Additional man-years committed for in FY 1972 were 9,216. Over a 27-year period, the Navy will gain 16,488 additional man-years from these individuals. Though the gains appear sizeable, the final evaluation of VRB rests on its cost-effectiveness relative to alternative wage schemes.

The effect of pecuniary incentives on length of recommitment has not been considered in previous analyses. For a given term of service in the Navy by an individual, however, costs may be reduced by increasing the average LOR. For example, the Navy pays travel expenses between duty station and home of record for all individuals who reenlist. Furthermore, at the time of reenlistment, individuals are entitled to payment for unused leave. Finally, depending on the number of early reenlistments which are executed, individuals may accumulate so-called "constructive time," allowing them to retire as much as a year and a half early. This cost is not insignificant since retirement annuities are based on a twenty-year career, rather than the eighteen and a half years actually served. Although no attempt is made to estimate the level of savings associated with changes in lengths of reenlistment, we provide empirical results in this previously uninvestigated area.

The analysis of lengths of recommitment is based on data for fiscal years 1970 through 1973. The effect of VRB was always positive. Multiple regressions including current VRB multiple changes, previous VRB multiple changes, and changes in recommitment rates as independent variables explained half of the changes in length of recommitment. When the years are merged, an increase in a VRB multiple of one increased lengths of recommitment approximately .5 years.

Second-term reenlistments are generally high and reflect the career orientation of those serving beyond the first term. The question arises as to the career commitment of VRB-induced first-term reenlistees versus non-VRB-induced reenlistees. Some argue that all individuals, once enticed beyond the first term, are likely to make the Navy a career, irrespective of bonus inducement. Others argue that the bonus only buys the individual for the length of the contract, and he is then lost. We cast light on this issue.

STRR data was not of sufficient quality to warrant its use. As a proxy, continuation rates between the 6th and 11th years were examined. No relationship was found between the awarding of VRB and the continuation of people to the eleventh year of service. Since most individuals staying through the eleventh year remain until at least the twentieth year, we conclude that VRB-induced individuals are as likely to be careerists as those not induced by VRB.

THE OCCUPATIONAL CHOICE DECISION

The past work in this area has focused on models of the distribution of individual tastes and civilian income streams.[1] The discussion here is complementary to these models. The unit of analysis is the individual. We examine the civilian and military income streams as perceived by the individual and analyze the factors influencing initial enlistment, subsequent reenlistments, and retirement. It is at the point where we aggregate individuals with different tastes and civilian income streams that the distribution models apply. Their exclusion from this paper, therefore, is designed to direct attention to the individual and not to ignore the necessity of studying the wage and taste distributions.

The analysis starts with the assumption of two careers in the job market -- military and civilian. Within each category there are a number of different jobs. We assume that an individual makes his job choice on entry into the job market. The decision endogenous to the model is whether to select a job within a military or civilian context.

Civilian earnings in year t of an individual who has served τ years in the military is $C(t, \tau)$. The present value, at the beginning of period 1, of the returns to a civilian career for an individual who never serves in the military is:

$$P.V.(C) = \int_0^T C(t,0)e^{-rt}dt \tag{1}$$

where T is the expected remaining lifetime and r the individual's marginal rate of time preference.

The present value of returns in year 1 from being employed in the military sector is:

$$P.V.(M) = \int_0^T M(t)e^{-rt}dt \tag{2}$$

where $M(t)$ is the returns from employment in the military in year t. For those who remain in the military up to the twentieth year, the retirement annuity is sufficiently attractive to prompt most individuals to switch to the civilian sector. For these individuals, the present value of the returns from a military career is:

[1]See references 1, 2, 3, 4 and 5. References 6, 7, and 8 develop the individual decision process but stop short of analyzing career paths.

$$P.V.(M) = \int_0^{20} M(t)e^{-rt}dt + \int_{20}^{T} \{R(t) + C(t, 20)\} e^{-rt}dt \tag{3}$$

where $R(t)$ is the retirement annuity in year t .

The returns to each occupation include cash and in-kind payments. In-kind benefits are bountiful in the military, and include housing and subsistence payments, below market prices at commissaries, and medical benefits. The individual also attaches a value to the job content and environment. To the extent that content and environment differ between the military and civilian sector, individuals will consider these attributes of the jobs in choosing a career. The net value attached to these intangibles can be translated into a dollar value and incorporated into the returns from selecting a civilian occupation. If an individual has a preference for the non-monetary attributes of a civilian occupation, the dollar value attached to this component of $C(t, \tau)$ will be positive; a preference for the non-monetary attributes of a military occupation would make this component negative.

Consider an individual who attaches a positive value to the intangibles associated with civilian employment. If cash and in-kind payments were equal and there were no training differences between military and civilian careers, the dollar value attached to the intangibles would indicate the additional payment the military would have to pay to overcome the individual's taste for the civilian sector.

If the individual has a positive preference for the military, the dollar value of the intangibles would be negative and indicate the magnitude by which the military can offer lower cash and in-kind payments than the civilian sector and still attract the individual.

The value placed on the non-monetary aspects of a career will not only differ between individuals but may also differ over time for the same individual. The adventure and travel associated with a military career may be valued highly in younger years, but lose their attractiveness as the individual ages. If this is the case, the difference between military and civilian earnings would have to be increased as individuals become older in order to keep these individuals in the military.

Individuals who have started a military career can switch to a civilian occupation at the end of any contract period. Switching is assumed irrevocable. The decision to remain military, however, binds an individual for only a limited time period and does not rule out future switching to a civilian occupation.

An individual will enlist in the military for four years and then switch to the civilian sector if three conditions are met. These conditions can be expressed as mathematical inequalities. The first condition is that entering the military for four years and switching to the civilian sector is superior to never entering the military.

$$\int_0^4 M(t)\,e^{-rt}dt + \int_4^T C(t,4)\,e^{-rt}dt > \int_0^T C(t,0)\,e^{-rt}dt \ . \tag{4}$$

Rearranging terms, we have

$$\int_0^4 \{M(t) - C(t,0)\}\,e^{-rt}dt + \int_4^T \{C(t,4) - C(t,0)\}\,e^{-rt}dt > 0. \tag{4'}$$

The first term of 4' is the difference in returns in the two sectors during the four years of service. The second term indicates the gain (or loss) in returns in the civilian sector from having the military experience. If the individual receives training in the military that is transferable to the civilian sector and the training is only available in the civilian sector at a higher cost, this term would be positive.[1] If, on the other hand, individuals in the military do not receive training transferable to the civilian sector and individuals in the civilian sector are receiving such training this term is negative. We thus see that enlistments can be encouraged by either higher military earnings or training that is transferable to the civilian sector.

The second condition necessary for the individual to enlist for four years and then switch to the civilian sector is that the returns are higher than those received from re-enlisting for longer military service. Here we only consider recommitments that do not earn the individual a retirement annuity.

$$\int_4^T C(t,4)e^{-rt}dt > \int_4^{t_o} M(t)e^{-rt}dt + \int_{t_o}^T C(t,t_o)e^{-rt}dt \tag{5}$$

$$\text{for } 4 < t_o < 20 \ .$$

Rearranging terms, we have

$$\int_4^{t_o} \{C(t,4) - M(t)\}\,e^{-rt}dt + \int_{t_o}^T \{C(t,4) - C(t,t_o)\}e^{-rt}dt > 0 \tag{5'}$$

$$\text{for } 4 < t_o < 20 \ .$$

This condition states that no extension of service in the military (short of becoming eligible for retirement) yields higher returns than leaving the military after four years. The effect of the military experience on civilian earnings is reflected in the second term. It is not inconceivable that the second terms in inequalities 4' and 5' are of different signs.

[1]If individuals in the civilian sector receive the same training in the first four years, and the training is paid for by the civilian employee in foregone wages but the military employee does not pay for it, the first term in inequality 4' would reflect the training differences.

The final condition is that a twenty-year military career yields lower returns than switching to the civilian sector.

$$\int_4^T C(t,4)e^{-rt}dt > \int_4^{20} M(t)e^{-rt}dt + \int_{20}^T \{C(t,20) + R(t)\}e^{-rt}dt \quad . \tag{6}$$

Rearranging terms, we have

$$\int_4^{20} \{C(t,4) - M(t)\}e^{-rt}dt + \int_{20}^T \{C(t,4) - C(t,20)\}e^{-rt}dt > \int_{20}^T R(t)e^{-rt}dt \; . \tag{6'}$$

Inequality (6) states that the present value of returns to a civilian career after four years in the military is greater than the present value of returns from a twenty-year military career plus the present value of the retirement annuity. Alternatively, inequality (6') states that the retirement annuity is insufficient to offset the losses in civilian returns from remaining in the military.

We next consider the decision to reenlist. Those reenlisting can be broken down into two groups: (1) those who reenlist and then switch to the civilian sector, and (2) those who reenlist and remain in the military until retirement. Assume the reenlistment to be for four years. An individual will reenlist and then leave the military if returns from the military are greater than the returns from the civilian sector for the additional four years only. That is,

$$\int_4^8 \{M(t) - C(t,4)\}e^{-rt}dt + \int_8^T \{C(t,8) - C(t,4)\}e^{-rt}dt > 0 \quad . \tag{7}$$

This condition is similar to inequality (4'). There are two more conditions. These conditions are analogous to inequalities (5') and (6'). The only difference is the time interval over which we are integrating.

The additional two conditions are:

$$\int_8^{t_o} \{C(t,8) - M(t)\}e^{-rt}dt + \int_{t_o}^T \{C(t,8) - C(t,t_o)\}e^{-rt}dt > 0 \tag{8}$$

for $8 < t_o < 20$

and

$$\int_8^{20} \{C(t,8) - M(t)\}e^{-rt}dt + \int_{20}^T \{C(t,8) - C(t,20)\}e^{-rt}dt > \int_{20}^T R(t)e^{-rt}dt \; . \tag{9}$$

There are also individuals who make more than one reenlistment and leave before retirement. For these individuals, the above conditions hold except that the year of exit from the military is substituted for eight in the inequalities.

Those who fall into the second category, reenlisting and remaining in the military until retirement, must satisfy one condition. This condition is that the returns from a twenty-year military career is greater than any other combination of being in the military sector and then switching. This can be written as:

$$\int_{t_o}^{20} \{M(t) - C(t,t_o)\}e^{-rt}dt + \int_{20}^{T} \{R(t) + C(t,20) - C(t,t_o)\}e^{-rt}dt > 0 \qquad (10)$$

for $4 < t_o < 20$.

In general, the decision-making process can be viewed as one of choosing the optimal point to switch from the military sector to the civilian sector. The earliest switching point would be at the beginning of period 1. That is, the individual maximizes his income stream by going civilian from the beginning. In our analysis, the maximum switching point is at 20 years. The individual goes military until retirement and then switches to a civilian occupation, drawing retirement pay and civilian wages. Interior solutions result in some military service and then switching prior to retirement. The optimal point at which to switch is where the return from an additional unit of time is greater in the civilian sector.

We can now consider the effect of a reenlistment bonus on reenlistments. Individuals not reenlisting will have income streams that do not satisfy inequality (7). The left-hand side will be negative. For some of these individuals, a bonus paid over the first reenlistment will make the left-hand side positive (since the bonus increases $\int_4^8 \{M(t) - C(t,4)\}e^{-rt}dt$) . The bonus should have no effect on those previously planning to reenlist, so that the first-term reenlistment rate should increase.

For an individual to remain in the service twenty years, inequality (10) must hold. One would presuppose that individuals enticed to reenlist by the bonus are less likely to have this condition met.[1] Individuals with higher civilian alternatives (or strong distastes for the military) in the fourth through eighth year will probably maintain these attributes beyond the eighth year. Thus they will be more likely to leave after eight years than those who were not induced to reenlist by the bonus. Since, in fact, most individuals staying beyond the second-term reenlistment remain for twenty years, we focused on

[1]For the individual who reenlisted and is currently at the eight year point, the lower constraint on t_o is eight.

the effects of the reenlistment bonus on this second decision point in the empirical section. Given that the bonus should have no effect on the twenty-year military career choice of pre-bonus reenlistees, we expect the overall second-term reenlistment rate to decline.

EMPIRICAL ANALYSES

Since VRB is designed to increase first-term reenlistment rates, it is this relationship we first analyze. But reenlistments are, by definition, recommitments for two or more years. Forty percent of the recommitments at the four-year point are for under two years, and the bulk of these are for under one year. Many individuals counted as VRB-induced reenlistments are individuals who previously made short-term extensions. Therefore, we also analyze the effect of VRB on all recommitments and the lengths of recommitment. Finally, we investigate the effect of VRB on second-term reenlistment rates, using continuation rates as a proxy.

VRB AND FIRST-TERM REENLISTMENT RATES

When an individual recommits after his first term of service, he can either reenlist or extend. Reenlistments range from two to six years, while extensions range from one to 48 months. The VRB is awarded only to those people in selected ratings recommitting for two or more years. Extensions of two or more years are included as reenlistments in the numerator of the reenlistment rate. The number eligible to make a first-term reenlistment are in the denominator of the reenlistment rate. A policy change in declaring individuals eligible could influence the reenlistment rates and cloud any relationship with the bonus. As a rule, however, the percent of separations declared eligible was stable for those designated to ratings. In FY 1969, 84 percent of these individuals were declared eligible; in FY 1973, 84.8 percent were declared eligible.

Reenlistment rates are collected for a variety of reasons in the Navy. Therefore, we used only data appropriate for our theoretical constructs. For example, the Navy is greatly concerned with the retention of experienced personnel. Since first-term enlistments have traditionally been for four years, the data collected focus on the people staying beyong this point. First-term reenlistment rates, reflecting these considerations, generally include all those going beyond the fourth year. But some individuals make a commitment for six years on entry into the Navy. When they pass the four-year point, they are not recommitting themselves but following through on their first commitment. Given that we want to focus on the effects of VRB on the recommitment decision, we will exclude ratings with these six-year obligors (6YOs).

Nondesignated enlisted personnel and stewards are also excluded from the analysis. Nondesignated enlisted personnel are individuals that have not been assigned a rating. As a rule, individuals remaining in this category at the end of the first term are not permitted to reenlist. For example, in FY 1973 only 11 percent were declared eligible to reenlist. Stewards, on the other hand, are excluded because their reenlistment behavior differs significantly from that of the rest of the Navy. This results from the rating being populated almost exclusively by Filipinos until recently. These individuals have a very low alternative wage and need little incentive to reenlist.

VRB is expected to increase FTRR as demonstrated in table 1. It is apparent that, in general, the greater the change in VRB, the greater the change in FTRR. All VRB changes in table 1 occurred at the beginning of the fiscal year and continued through to its completion. Since no VRB multiples changed in FY 1967-68 and FY 1969-70, these intervals are omitted. Since VRB was introduced in the middle of FY 1966, the first interval looks at changes from FY 1965 to FY 1967. An empty cell in the table indicates that no rating fell in that category.[1]

TABLE 1

CHANGES IN FTRR BY CHANGES IN VRB MULTIPLE

Change in VRB multiple / Interval	-3	-2	-1	0	+1	+2	+3	+4
FY 1965-67				-11.9	-2.3	- 2.5	- 1.3	+.3
FY 1968-69				- 2.2	-1.0	+ 3.6		
FY 1970-71			+0.6	+ 3.2	+4.3	+10.5	+29.7	
FY 1971-72	-2.3	+0.6	-1.8	+ 4.1	+8.2	+11.4	+14.4	
FY 1972-73		-5.6		+ 1.8	+4.2	+11.0	+ 8.7	

Note: Excludes stewards, 6YO ratings, and nondesignated personnel.

To obtain an estimate of the impact of VRB, we assume that the FTRR for the ith rating is a function of VRB, other military wages, W, and a vector of non-military wage factors, A,

$$R_i = f(VRB_i + W_i, A_i) \quad . \tag{1}$$

For changes between two periods we can write:

$$\Delta R_i = f_1 (\Delta VRB_i + \Delta W_i) + f_2 \Delta A_i \quad , \tag{2}$$

where the Δ's indicate changes between periods, f_1 is the impact of changes in VRB and

[1]A more detailed presentation of historical trends in reenlistment rates and VRB multiples, by rating, is contained in: Center for Naval Analyses, CNA Memorandum 743-74, "Selected Navy Reenlistment Statistics, FY 1963-FY 1973," Unclassified 17 May 1974.

other military wages on FTRR, and f_2 is the impact of changes in non-military wage factors on FTRR.

For simplicity, assume that the functional form of (1) is linear:

$$R_i = \alpha_1(VRB_i + W_i) + \alpha_2 A_i + u_i \quad , \tag{3}$$

where an error term, u_i , is introduced. If we assume that the change in non-VRB wages and the vector of other factors is the same for all ratings, we can write (2) as

$$\Delta R_i = \beta_0 + \alpha_1(\Delta VRB_i) + v_i \quad , \tag{4}$$

where

$$\beta_0 = \alpha_1(\Delta W) + \alpha_2(\Delta A) = \text{constant},$$

and

$$v_i = u_{i1} - u_{i0} \quad ,$$

where the second subscripts indicate time periods. That is, given these assumptions, the change in non-VRB factors will shift the intercept in the $\Delta R{:}\Delta VRB$ plane, but not bias the estimate of the slope (α_1) .

The above procedure is adopted because of the lack of data that is specific to ratings. For example, we do not know civilian opportunities by rating. By taking the differences between consecutive years, we need only assume that non-VRB factors change uniformly without actually measuring these factors. This assumption may break down over a longer period, but is reasonable within the time frames used. Since a VRB multiple change of 1 between years could increase wages by as much as \$2000 over the length of the reenlistment, we would expect VRB changes to be the most dominant influence on reenlistment changes (except for a possible trend factor).

An alternative specification to (3) is the logistic function which has the form:

$$R_i = \frac{1}{1 + \exp\{-[\alpha_1' (W_i + VRB_i) + \alpha_2' A_i + u_1']\}} \quad . \tag{5}$$

We can rewrite this as:

$$\log\left(\frac{R_i}{1-R_i}\right) = \alpha'_1(W_i+VRB_i) + \alpha'_2 A_i + u'_i \quad .$$

The difference between two years can be written as:

$$\Delta\log\left(\frac{R_i}{1-R_i}\right) = \alpha'_1\ (\Delta W_i) + \alpha'_2\,(\Delta A_i) + \alpha'_1(\Delta VRB_i) + u'_{i1} - u'_{i0} \quad .$$

Invoking the same assumptions as above, we can estimate the simple regression

$$\Delta\log\left(\frac{R_i}{1-R_i}\right) = \beta'_0 + \alpha'_1\ (\Delta VRB_i) + v'_i \quad ,$$

where

$$\beta'_0 = \alpha'_1\,(\Delta W) + \alpha'_2\,(\Delta A) = \text{constant, and}$$

$$v'_i = u'_{i1} - u'_{i0} \quad .$$

This logit specification is theoretically sounder for regressions with probabilities as the dependent variable. It takes into account the constraint that the reenlistment rate lies between 0 and 1 . Results for both specifications are reported below.

Results for those intervals where 10 or more ratings had a change in VRB are reported in table 2. During the Vietnam conflict, a number of direct procurements (or lateral entries) occurred in the seven construction ratings. Since we did not know what impact this would have on our analysis, we ran two sets of regressions: one including and one excluding the construction ratings. Those reported in the text include construction ratings; for results excluding them, see appendix A. The weighting procedure adopted is discussed in appendix B. The construction of the VRB variable is discussed in appendix C.

All results are significant at the 1 percent level. In the linear model, α_1 measures the effect of a \$1 increase in wages and bonuses. It is the slope of the supply curve of reenlistments. If a \$1000 bonus was awarded to a rating in the first period, the reenlistment rate increased 1.74 percentage points on the average. The coefficient of VRB estimated for the logit model, α'_1 , is not as simple to interpret but it can be used to estimate the slope of the supply curve at the mean. The formula is:

$$\alpha_1 = \alpha'_1\ \bar{R}(1-\bar{R}) \quad .$$

TABLE 2

REGRESSION RESULTS OF THE EFFECT OF VRB ON FTRR

	Linear model				Logit model						
Interval	β_0	α_1	R^2	η	β_0'	α_1'	R^2	α_1	η	Ratings with VRB change	Total no. of ratings
FY 1965-67	-7.786 (-11.72)	.00174 (7.08)	.495	2.27	-.485 (-10.85)	.000105 (6.17)	.427	.00168	2.20	29	53
FY 1968-69	-3.010 (-6.70)	.00163 (3.97)	.233	3.62	-.196 (- 5.57)	.000107 (3.25)	.169	.00138	3.07	17	54
FY 1971-72	3.583 (8.70)	.00121 (6.39)	.417	4.04	.325 (8.60)	.000115 (6.07)	.392	.00127	4.24	16	59
Combined*		.00138 (10.18)	.627			.000110 (9.57)	.630			62	166

Note: t-values are in parentheses. R^2 is coefficient of determination. η is the elasticity computed at the mean.

*β_0 and β_0' are not reported because they each take on three values: one for each interval. α_1 for the logit regression is excluded for the same reason. The elasticity at the mean is excluded because it requires computing averages over all intervals.

The bars indicate that the variables are measured at their means. The α_1 computed by this formula is multiplied by 100 so that it is in the same units as that computed for the linear model. The slopes estimated by the two models are in close agreement.

The smaller effect of VRB in the latter time intervals could reflect inflation in the economy.[1] The attractiveness of a $1000 bonus diminished with a decrease in the value of the dollar. Given that military wages and bonuses keep pace with inflation, however, the VRB remains an effective policy instrument.

The elasticities at the means are constructed as follows:

(Linear model)

$$\eta = \frac{\overline{W} + \overline{VRB}}{\overline{R}} \; \alpha_1$$

(Logit model)

$$\eta = (\overline{W} + \overline{VRB})\,(1-\overline{R})\,\alpha_1'$$

where the elasticity is defined as

$$\frac{\overline{VRB} + \overline{W}}{\overline{R}} \cdot \frac{\Delta R}{\Delta(W+VRB)} \;.$$

The construction of the mean wage is described in appendix C. Though the slopes appear to decrease, the elasticities are increasing; this results from an increase in the mean of the other variables used to construct the elasticity. The factor by which α_1 is multiplied breaks down as follows for the linear model:

[1]Correcting for inflation appeared to bring the coefficients closer together. Expressing VRBs in 1967 dollars, we obtained the following estimates:

	Linear	Logit	
	α_1	α_1'	α_1
FY 1965-67	.00174	.000105	.00168
FY 1968-69	.00179	.000117	.00152
FY 1971-72	.00152	.000144	.00159
Combined	.00157	.000119	

	$\overline{Wage} + \overline{VRB}$	$\overline{R}$	$\frac{\overline{Wage} + \overline{VRB}}{\overline{R}}$
FY 1965-67	26,232.28	20.07	1,307.04
FY 1968-69	33,974.59	15.29	2,222.01
FY 1971-72	42,085.17	12.62	3,334.80

It is apparent that higher wages and lower reenlistment rates have resulted in the higher elasticity estimates.

When the data for all intervals is merged, the coefficients are not statistically different. For the linear model, the coefficient is .00138 as reported in the last row of table 2.[1] Since the coefficient appears to decrease over time, the use of this combined estimate would bias upward any prediction of future effects of VRB. The FY 1971-72 estimates would be best for predicting future effects, and from observing the trend, a figure as low as .001 does not appear out of order.

Consideration was also given to combining the data for the three intervals in a joint generalized weighted least squares procedure (see references 10, 11, and 12 for a description of the technique). This would permit the coefficients to differ for each year, yet take into account the correlation between errors of the same ratings across years. The greater the correlation between errors, the greater the gain in efficiency from using this procedure. The simple correlation coefficients, using the errors from the estimates of the logit functional form, are as follows:

	FY 1965-67	FY 1968-69
FY 1968-69	-.325	-
FY 1971-72	-.121	.188

Because of these low correlation coefficients, the joint estimation procedure was not undertaken.

Simplifying assumptions are often introduced to make the analysis tractable. Given the results, we must go back and review the possible biases that may arise if our assumptions are violated. One questionable assumption is the constancy of non-VRB changes. It can be argued that other factors changed differentially for those ratings with VRB changes. This would explain the necessity for the Navy to change its policy: VRB increases (decreases) probably result from sudden decreases (increases) in reenlistments.

[1]This is obtained by aggregating data for all periods and placing a constraint on the slope term. We could not reject the hypothesis at the 5 percent level that the slopes were equal. For a discussion of this test see reference 9.

If this is true, there is a downward bias in the estimates.[1] But one could argue that Navy reaction to changes is slow, and that in the periods in which it changes VRB, all differential non-VRB changes have already occurred.

It was also implicit in the analysis that military and civilian pay have separate coefficients that are not necessarily equal. An alternative hypothesis is that the ratio of military to civilian pay should enter as an independent variable. It is apparent from past work that the estimates of the coefficient of the ratio is smaller than when the two pays enter separately. The table below reports results from three studies of reenlistment behavior for the Gates Commission (see references 1, 3, and 5). If the correct model includes the ratio as an independent variable, it would appear that we have an upward bias in our estimates.[2]

Year of		Elasticities	
data base	Service	Ratio	Absolute values
1967	Army	2.43	3.81
1968	Air Force	2.36	2.40
1968	Navy	2.15	2.63

A bias may also result from lateral conversions (the movement of individuals between ratings). Individuals are encouraged to move from "closed" ratings to "open" ratings. The former are generally ratings with high reenlistment rates; the latter are generally ratings with low reenlistment rates. If this movement occurs on a large scale, it would distort the relationship of VRB to reenlistment rates. People could switch ratings to reenlist in those areas where VRB is offered. Those remaining in non-VRB ratings are likely to be those who do not expect to reenlist. If this is the case, we may observe higher

[1]This results because the VRB variable "picks up" not only its contribution but also the contribution of omitted variables that are correlated with VRB. Since these other variables have an impact counter to VRB, the VRB coefficient is biased downward.

[2]The elasticities in the first and third rows of the table are coefficients of regressions in logarithmic form. The "ratio elasticity" is of the independent variable log (M/C), the logarithm of the ratio of military pay to civilian pay. The "absolute value" elasticity is for the independent variable log M . The second row used the logit specification outlined in this paper. Recent studies have found pay elasticities comparable to those reported by the Gates Commission. In reference 4, the logit specification is applied for fiscal year 1968 reenlistments. The elasticity estimate of 3.17 is for all services combined. A study reported in reference 13 was specifically directed to the analysis of the impact of VRB in the Navy. The reenlistment period investigated was fiscal years 1964-68; the elasticities are reported "to cluster around 2.0."

reenlistment rates in VRB ratings and reduced reenlistment rates in non-VRB ratings as a result of movement between ratings. Though overall reenlistment rates may increase with increases in VRB, this relationship would be more difficult to measure. Furthermore, the method used here would yield results that are biased upward.

Five months of rating conversion data was available and is reported in table 3. This data is generated by the office that approves rating transfers. Gains are reported when approval is made. Losses are recorded only when an individual makes the physical movement from a rating. There is a significant difference between the two numbers. One reason is that individuals may not pass the tests that are prerequisites for transfer. But it is doubtful that this accounts for any significant portion of the difference. Most likely the loss data is subject to serious underreporting. The gain data is biased upward, but it is probably closer to the true level of lateral conversions.

The period covered by the table is July to November 1973. For the five-month period there were 410 reported gains and 54 reported losses. The vast majority of these conversions were in ratings with at least some 6YOs. In fact, only 66 gains and 14 losses are recorded for ratings with no 6YOs. Since the latter are the only ratings included in this analysis, we can conclude that movements between ratings have little or no effect on our results.

The extent and direction of the overall bias is, therefore, unclear. Lateral conversions can, for all practical purposes, be ignored. The other two biases operate in opposing directions. They could cancel each other, but this is unlikely. If neither is operative, there is likely to be no bias in the estimates.

We also attempted to ascertain the effect of VRB in 6YO ratings with corrected reenlistment rates. The results are reported in appendix D. There appears to be a positive relationship between corrected reenlistment rates and VRB, but the roundabout method of constructing the reenlistment rates makes this conclusion very tenuous.

VRB AND LENGTHS OF RECOMMITMENT

An alternative way to analyze the impact of VRB is to look at its effect on lengths of recommitment (LOR). VRB is awarded only to those who recommit for two or more years. The greater is VRB, the greater is the incentive to recommit for at least two years. We also suspect that the individual originally planning to reenlist for at least two years will increase the length of his recommitment if offered a higher reenlistment bonus. Such circumstance could result from either rising civilian returns with age or rising disutility from serving in the military. In either case, additional monetary incentives are necessary to induce additional years of recommitment. In this section, we focus on the additional man-year commitments generated by VRB. To this end, we analyze the effect of VRB (1) on recommitments and (2) on lengths of recommitment.

TABLE 3

MOVEMENTS BETWEEN RATINGS, JULY-NOVEMBER 1973

Rating	Gains	Losses	Rating	Gains	Losses
BM	1	2	HM	3	0
GM*	2	0	PH	1	0
ST*	15	0	YN	1	0
TM	4	0	DP	1	0
FT*	29	14	SK	4	0
MT*	9	4	GM	4	0
ET*	113	2	MM*	96	16
DS*	8	0	EN	1	1
CT*	3	0	BT	4	2
AT*	5	0	EM*	45	2
AX*	0	1	IC*	18	1
AQ*	1	0	AD	11	4
TD	5	0	AM	12	3
SM	0	1	CS	3	0
RM	7	0	SD	2	0
CT	1	1			
PT	1	0			
			Total	410	54

Note: Adapted from Report E-231A.

*Rating contains 6YOs.

Recommitment rates are total extensions and reenlistments divided by the total eligible to reenlist. Data was available for FY 1970 through FY 1973. The procedure adopted to measure the effect of VRB is identical to that developed above for reenlistment rates. The change in the VRB multiple is the independent variable in this and succeeding analyses. The low variability in base pay across ratings indicates that the multiples serve as good proxies for the actual dollar bonus. This issue is further discussed in appendix C. The regression results are reported in table 4. VRB only had a statistically significant effect on recommitment rates in the FY 1970-71 interval. Given the strong influence of VRB on reenlistment rates, its insignificant effect on recommitment rates for the latter two intervals was surprising. It was unfortunate that recommitment rates could not be constructed for earlier years so that further light could be cast on this issue.

TABLE 4

WEIGHTED REGRESSION RESULTS OF CHANGES IN RECOMMITMENT RATES ON CHANGES IN VRB MULTIPLE

	Linear model			Logit model		
Interval	β_0	β_1	R^2	β_0	β_1	R^2
FY 1970-71	.041 (6.09)	.064 (3.81)	.203	.331 (7.66)	.400 (4.57)	.268
FY 1971-72	.073 (6.65)	-.010 (-0.98)	.016	.432 (6.86)	-.060 (-1.02)	.018
FY 1972-73	.021 (2.31)	.021 (1.24)	.029	.108 (2.20)	.121 (1.41)	.037

Note: β_0: intercept

β_1: slope

t-values are in parentheses.

It appears that VRB increases reenlistments by inducing those already extending for under two years to recommit for over two years. The individuals extending for a short period (30 days to 6 months) are the individuals at the margin. They have not made a decision as to whether or not to make another long term commitment to the Navy. Since it is more difficult to recommit once one leaves the Navy, many of those at the margin probably choose to extend for a short period. It is these individuals who are influenced by the bonus.

We now proceed to the empirical investigation of the relationship of VRB to lengths of recommitment. Changes in length of recommitment by VRB multiple change are presented for FY 1970 through FY 1973. This is followed by simple regressions with changes in the VRB multiple as the independent variable. Next, multiple regressions are reported with lagged changes in the VRB multiple and changes in recommitment rates also included as independent variables. We conclude with an estimate of the additional man-year commitments generated by VRB.

To estimate the impact of VRB on the length of recommitment it is assumed that LOR in the ith rating is a function of VRB, other military wages, W , and a vector of non-military wage variables, A:

$$LOR_i = g(VRB_i + W_i,\ A_i) \quad . \tag{6}$$

If we assume that the functional form of (6) is linear, then

$$LOR_i = \gamma_1 (VRB_i + W_i) + \gamma_2 A_i + z_i \tag{7}$$

where an error term, z_i , is introduced. Assuming that changes in other military and non-military wages are the same across all ratings for the periods examined, we have for changes between two periods

$$\Delta LOR_i = \delta_0 + \gamma_1(\Delta VRB_i) + v_i \quad , \tag{8}$$

where

$$\delta_0 = \gamma_1(\Delta W_i) + \gamma_2(\Delta A_i) = \text{constant},$$

and

$$v_i = z_{i1} - z_{i0} \quad .$$

The second subscript on the z's indicates the time period.

Data on LOR by rating was available for fiscal years 1970 through 1973. Lengths of extension were divided into five categories: 0-11 months, 12-23 months, 24 months, 25-36 months, and 37-48 months. In estimating the regression equations, we used the mid-points of these ranges (in years): .50, 1.458, 2, 2.542, and 3.542. Reenlistments were given in yearly intervals of two through six years. After eliminating nondesignated enlisted personnel, stewards, and ratings containing 6 YOs, we had data remaining on 59 ratings for the period FY 1970-71. A new rating (OT) was formed in FY 1971 and included in

FY 1971-72 regressions. In FY 1973, five ratings (AO, CTI, CTR, CTT, DP) experienced changes in VRB multiple in the middle of the year and had to be eliminated.[1] One rating (SF) was discontinued. Thus, 54 observations were used for FY 1972-73. The results reported in the text include construction ratings; for results excluding them, see appendix E.

The number of ratings with VRB multiple changes in the three intervals is:

FY 1970-71	7
FY 1971-72	16
FY 1972-73	5

Table 5 presents the average lengths of reenlistment and extension by VRB multiple. At first glance, the data does not confirm any strong relationship between VRB and LOR beyond multiple 1. But we would be wrong to rely on this table to test such a relationship. Ratings differ in LOR for a variety of reasons. Danger, dirtiness, and work content of a job are some factors affecting lengths of recommitment (along with determining the magnitude of the recommitment rate). We should look at the effect of a change in VRB on a change in LOR for a given rating. This is shown in table 6. In general, changes in VRB and LOR are in the same direction. Table 7 summarizes the results in table 6. In each row we see that, in general, the greater the increase in VRB, the greater the increase in LOR.

Using the data described above, we ran both unweighted and weighted regressions for the intervals FY 1970-71, FY 1971-72, and FY 1972-73.[2] The regressions are in two forms. The first is equation (8). In the second, the dependent variable is the change in the log of the lengths of recommitment. Table 8 summarizes the results. The numbers in parentheses are t-values.

For a one-tailed test a t-value of greater than 1.68 indicates statistical significance: a criterion satisfied by all but one coefficient on VRB. The effect of a VRB multiple change of 1 on LOR is reported in the last column. The numbers tend to cluster around 0.5 years, or 6 months.

Two additional variables were included as independent variables: the change in VRB and the percent of eligibles making a recommitment. As shown below, the coefficient on the change in VRB in the structural equation is the negative of the coefficient on the lagged change in VRB in the regression equation.

[1]Ratings were eliminated because we could not ascertain the portion of the reenlistees that reenlisted at each VRB level.
[2]See appendix B for weights used.

TABLE 5

AVERAGE LENGTHS OF REENLISTMENT AND EXTENSION BY FISCAL YEAR AND VRB MULTIPLE

	Reenlistments & extensions					Extensions					Reenlistments				
	0	1	2	3	4	0	1	2	3	4	0	1	2	3	4
FY 1970	2.718	3.239	3.121	3.605	3.330	.835	.843	.627	.807	.812	4.681	5.151	5.567	5.407	5.400
FY 1971	2.693	3.998	3.023	3.817	3.648	.819	.917	.630	.706	.820	4.469	5.232	5.313	5.276	5.205
FY 1972*	2.424	--	3.442	3.426	3.682	.709	--	.716	.702	.762	4.376	--	5.193	5.214	5.189
FY 1973*	2.801	--	3.185	3.502	3.378	.769	--	.727	.708	.734	4.270	--	5.102	5.121	5.070

Note: Adapted from Machine Report 1133-2030.

*VRB multiple 1 was not assigned to any rating in FY 1972 and FY 1973.

TABLE 6

CHANGES IN VRB MULTIPLE AND LOR, BY RATING

Interval	Rating	VRB_0	VRB_1	ΔVRB	LOR_0	LOR_1	ΔLOR
FY 1970-71	QM	4	3	-1	3.398	3.414	+ .016
	CTA	4	3	-1	3.013	4.217	+1.204
	AM	1	2	+1	3.410	3.786	+ .376
	AG	1	2	+1	2.590	3.312	+ .722
	AC	3	4	+1	2.973	3.689	+ .716
	PM	1	3	+2	2.209	5.200	+2.991
	TD	0	3	+3	2.499	3.425	+ .926
FY 1971-72	GMG	3	0	-3	4.085	2.906	-1.179
	BU	2	0	-2	2.034	1.349	- .685
	SW	2	0	-2	3.008	1.072	-1.936
	EA	2	0	-2	1.987	0.535	-1.452
	UT	2	0	-2	2.415	1.397	-1.018
	CE	2	0	-2	3.012	1.553	-1.459
	EO	2	0	-2	2.447	1.418	-1.029
	CM	2	0	-2	2.482	1.335	-1.147
	DP	4	3	-1	3.094	2.339	- .755
	CS	1	0	-1	4.088	3.055	-1.033
	SK	1	0	-1	3.821	2.499	-1.322
	GMT	3	4	+1	3.408	4.174	+ .766
	HM	0	2	+2	2.779	3.324	+ .545
	IM	2	4	+2	3.406	4.222	+ .816
	AS	0	3	+3	2.331	4.549	+2.218
	PH	0	3	+3	2.104	3.039	+ .935
FY 1972-73	DK	2	0	-2	4.318	2.773	-1.545
	TM	3	4	+1	3.768	3.897	+ .129
	MU	0	2	+2	2.667	3.097	+ .430
	MN	0	3	+3	2.340	3.242	+ .902
	ABE	0	3	+3	1.552	3.445	+1.893

Note: Adapted from Machine Report 1133-2030.

TABLE 7

YEARLY CHANGES IN VRB MULTIPLE AND LOR

Interval	-3	-2	-1	0	+1	+2	+3
FY 1970-71			+0.23	+0.18	+0.50	+2.99	+0.93
FY 1971-72	-1.18	-1.14	-1.14	-0.11	+0.77	+0.55	+1.46
FY 1972-73		-1.54		-0.01	+0.13	+0.43	+1.58

Note: Adapted from Machine Report 1133-2030.

The change in VRB should identify the expectations of individuals at the reenlistment point. An individual can only receive a VRB once. A recommitment for two or more years entitles an individual to the current bonus, but precludes the awarding of a bonus for a future recommitment. A recommitment of under two years does not entitle the individual to a bonus, but leaves open the possibility of collecting the bonus at a future time. The individual thus has to decide whether to reenlist at the current bonus or extend for a short time and be eligible for a higher (lower) bonus in the future. We assume that expectations of future changes in VRB are predicated on past changes.

Additional individuals are acquired as the recommitment rate increases. These marginal individuals are expected to have lower lengths of recommitment than intra-marginal individuals. This would result in an observed decrease in average length of recommitment. We therefore expect the sign of the coefficient on the recommitment rates to be negative. The expanded model for period 0 is:

$$LOR_0 = h(W_0 + V_0, \Delta V_{0,-1}, RC_0, B_0) \quad , \tag{9}$$

where RC is the recommitment rate and B is a vector of other factors affecting LOR.[1] In linear form (9) becomes:

$$LOR_0 = \theta_0 + \theta_1(W_0 + V_0) + \theta_2(\Delta V_{0,-1}) + \theta_3 RC_0 + \theta_4 B_0 \quad . \tag{10}$$

For period 1 we have:

$$LOR_1 = \theta_0 + \theta_1 W_1 + (\theta_1 + \theta_2)V_1 - \theta_2 V_0 + \theta_3 RC_1 + \theta_4 B_1 \quad . \tag{11}$$

[1]The subscript indicating the i^{th} rating is omitted for notational convenience.

TABLE 8

SIMPLE REGRESSION RESULTS OF CHANGES IN LOR ON CHANGES IN VRB MULTIPLE

	Form	$\hat{\delta}_0$	$\hat{\gamma}_1$	R^2	$\frac{\Delta \text{LOR}}{\Delta \text{VRB}}$
FY 1970-71	1	.098	.480	.149	.480
Unweighted		(1.17)	(3.16)		
	2	.023	.149	.150	.470
		(.88)	(3.17)		
FY 1970-71	1	.108	.296	.049	.296
Weighted		(1.84)	(1.71)		
	2	.024	.093	.045	.293
		(1.26)	(1.65)		
FY 1971-72	1	-.168	.522	.423	.522
Unweighted		(-1.99)	(6.52)		
	2	-.074	.224	.508	.687
		(-2.43)	(7.75)		
FY 1971-72	1	-.151	.460	.564	.460
Weighted		(-2.80)	(8.67)		
	2	-.061	.182	.582	.558
		(-2.96)	(9.00)		
FY 1972-73	1	.108	.450	.143	.450
Unweighted		(1.00)	(2.94)		
	2	.075	.153	.102	.469
		(1.69	(2.44)		
FY 1972-73	1	-.021	.537	.161	.537
Weighted		(-.32)	(3.15)		
	2	.006	.186	.133	.570
		(.235)	(2.83)		

Assuming that changes in W and B between periods is constant across ratings, we have:

$$\Delta LOR_{1,0} = \lambda_0 + \lambda_1 \Delta V_{1,0} + \lambda_2 \Delta V_{0,-1} + \lambda_3 \Delta RC_{1,0}$$

where

$$\lambda_0 = \theta_1 \Delta W_{1,0} + \theta_4 \Delta B_{1,0} = \text{constant},$$

$$\lambda_1 = \theta_1 + \theta_2, \text{ and}$$

$$\lambda_2 = -\theta_2 \quad .$$

Regression results are reported in table 9 for FY 1971-72 and FY 1972-73. The effect of VRB after the first period is measured by $\hat{\theta}_1$. The coefficient is always positive and generally significant at the 5 percent level. It is always significant in the weighted regressions. The expectations hypothesis that individuals increase (decrease) their LOR if VRB increased (decreased) in the previous period cannot be rejected, except in the FY 1971-72 weighted regression.[1] The recommitment coefficient is negative and generally significant. The average length of recommitment apparently declines, as predicted, with an increase in percent recommitting.

We now have to measure the effect of VRB at two stages: the period in which it changes and subsequent periods. These effects are reported in the last two columns of table 9. In the period of a VRB multiple change, the range of the effect on LOR is from .461 to .690 years. These estimates are higher than those measured in the simple regressions. In subsequent periods, however, the effect is less pronounced: the range being from .012 to .496 years, or from 0.14 to 5.95 months.

Estimates from merging the intervals are reported in appendix F. The results confirm that an increase of one in the VRB multiple increases LOR by approximately 0.5 years.

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[1]If the expectations hypothesis is correct the lagged change in VRB should also be included in the reenlistment regression equations. In FY 1971-72 weighted regressions, the coefficient in the lagged variable was insignificant, as it was in the LOR regression. The coefficient was significant, however, in the FY 1972-73 interval. Since the lag was insignificant in the FY 1971-72 period, we did not have to correct the reenlistment rate elasticities reported in the previous section for this interval.

TABLE 9

MULTIPLE REGRESSION RESULTS OF CHANGES IN LOR ON CHANGES IN VRB, CHANGES IN LAGGED VRB, AND CHANGES IN RECOMMITMENT RATES

	Form	Intercept	$\hat{\lambda}_1$	$\hat{\lambda}_2$	$\hat{\theta}_1$	$\hat{\delta}_3$	R^2	$\frac{\Delta LOR_0}{\Delta VRB}$	$\frac{\Delta LOR_1}{\Delta VRB}$
FY 1971-72	1	- .116	.528	- .306	.222		.494	.528	.222
Unweighted		(-1.42)	(6.99)	(-2.83)	(1.71)				
	2	- .062	.225	- .072	.153		.535	.690	.477
		(-2.02)	(7.95)	(-1.79)	(3.15)				
	1	.078	.482	- .298	.184	-2.53	.563	.482	.184
		(.77)	(6.63)	(-2.95)	(1.51)	(-2.98)			
	2	.059	.202	- .070	.132	- .327	.698	.630	.411
		(1.77)	(8.60)	(-2.12)	(3.33)	(-5.51)			
FY 1971-72	1	- .139	.461	- .185	.276		.578	.461	.276
Weighted		(-2.55)	(8.75)	(-1.33)	(1.86)				
	2	- .058	.182	- .042	.140		.587	.558	.436
		(-2.77)	(8.98)	(- .78)	(2.45)				
	1	- .021	.463	- .148	.315	-1.63	.620	.463	.315
		(- .30)	(9.18)	(-1.11)	(2.31)	(-2.51)			
	2	.034	.181	- .022	.159	- .280	.720	.564	.496
		(1.35)	(10.74)	(- .49)	(3.34)	(-5.16)			
FY 1972-73	1	.058	.462	- .325	.137		.327	.462	.137
Unweighted		(.60)	(3.38)	(-3.74)	(.117)				
	2	.051	.159	- .155	.004		.365	.487	.012
		(1.34)	(2.98)	(-4.60)	(.064)				
	1	.082	.504	- .305	.199	-1.01	.337	.504	.199
		(.81)	(3.47)	(-3.40)	(1.14)	(- .87)			
	2	.079	.184	- .138	.046	- .288	.417	.564	.141
		(2.02)	(3.48)	(-4.09)	(.97)	(-2.10)			
FY 1972-73	1	- .020	.536	- .182	.354		.281	.536	.354
Weighted		(- .33)	(3.37)	(-2.92)	(2.07)				
	2	.006	.186	- .077	.109		.280	.570	.334
		(.27)	(3.07)	(-3.23)	(1.68)				
	1	.075	.588	- .125	.463	-3.95	.441	.588	.463
		(1.24)	(4.13)	(-2.17)	(2.99)	(-3.78)			
	2	.047	.197	- .063	.134	- .422	.497	.604	.411
		(2.17)	(3.85)	(-3.10)	(2.43)	(-4.64)			

Finally, we estimated the additional man-years committed for in FY 1972 and FY 1973 as a result of the level and changes in VRB in FY 1972. For this purpose, we use the weighted multiple regressions with three independent variables for FY 1971-72. Using the more conservative linear estimates, 9,216 additional man-years were committed for in FY 1972 as a result of VRB. If the FY 1972 VRB levels had been maintained in FY 1973, commitments for 6,270 additional man-years would have been obtained. The effect of the FY 1973 VRB changes are not considered in computing these numbers.

TABLE 10

ADDITIONAL MAN-YEAR COMMITMENTS IN FY 1972 AND FY 1973 RESULTING FROM VRB LEVEL AND CHANGES IN FY 1972

	FY 1972	FY 1973
Linear	9,216	6,270
Log	11,226	9,873

Note: Excludes 6YOs.

VRB AND SECOND-TERM REENLISTMENT RATES

We postulate that individuals induced to reenlist by VRB at the first-term point are less likely to be careerists than those who reenlist without a bonus. Thus, we expect changes in second-term reenlistment rates (STRR) to be inversely related to changes in VRB.

Three types of data were available for investigating this hypothesis: career reenlistment rates and continuation rates for FY 1965 to FY 1973, and second-term reenlistment rates for FY 1968-71. Since the career reenlistment data was for all careerists, i.e., individuals with more than four years of service, it was unsatisfactory for our purposes. The relatively high reenlistment rates of individuals in the latter stages of their career would have masked any differences in reenlistments caused by VRB at the second-term decision point. The STRR data was deficient in that machine reports for the second half of each fiscal year were unaccountably missing. Therefore, we use a proxy for the STRR in our analysis: the continuation rate from the sixth to the eleventh year of service (YOS). This particular interval was chosen since the length of service distributions were determined by Pay Entry Base Date (PEBD). As a result, continuation rates from the fourth to fifth YOS appeared to be biased upward and the fifth to sixth YOS continuation rates biased downward.[1] Since only 4YO data is considered, by the eleventh YOS all individuals should have made a second-term reenlistment decision.

[1]Active Duty Base Date (ADBD) continuation data would be superior for analysis because VRB eligibility is computed by ADBD, and the bias in the continuation rates into the fifth and sixth year would be removed. Using PEBD continuation data from the sixth to eleventh years results in less reliable estimates of the relationship with VRB.

Nondesignated enlisted personnel, ratings containing 6YOs, and the construction ratings were eliminated from the analysis. The relatively large number of lateral entries into the construction ratings during the Vietnam conflict resulted in continuation rates exceeding 100 percent in a large number of LOS cells.

Continuation rates for forty-three ratings were calculated for two base periods (FY 1966 to FY 1971, and FY 1967 to FY 1972) since, in effect, all ratings in these periods experienced a VRB multiple of zero at the first-term reenlistment point. The 6th to 11th YOS continuation rates were then calculated for the period FY 1968 to FY 1973. The changes in the continuation rates between these periods were observed for each rating and then grouped by the VRB multiple assigned to each occupation in FY 1966. The two base periods provide information on cohorts who reenlisted without VRB in FY 1964 and FY 1965. We can compare the continuation behavior of these two groups with reenlistees in the same ratings who received VRB in FY 1966. Table 11 contains the results, which are summarized below:[1]

1. Between the VRB period FY 1968 to FY 1973 and the base period FY 1966 to FY 1971, the continuation rate for those who had never received VRB (multiple zero) fell by about 9 percentage points. The average continuation rate for those receiving VRB fell by about one and one-half percentage points.

2. Comparing the VRB period with the other base (FY 1967 to FY 1972), the continuation rate for those with no VRB fell by about 2 percentage points. The continuation rates for the VRB ratings fell by an average of about one half of one percent.

In neither of the above cases does there appear to be a trend in the data. This lack of a strong inverse relationship between changes in VRB and changes in continuation rates is confirmed in the regression results reported below.

Our initial assumption is that changes in continuation rates are a linear function of changes in VRB:

$$C_1 - C_0 = \omega_0 + \omega_1 \Delta \mathrm{VRB} \qquad (1)$$

where

C_1 = the continuation rate from the 6th to 11th YOS in the VRB period FY 1968-73, and
C_0 = 6th to 11th YOS continuation in a base period: either FY 1966-71 or FY 1967-72.

[1]Since VRB was introduced in the middle of FY 1966, and some ratings experienced a change in multiple three months later, ΔVRB in table 11 is a weighted average of the multiple assigned for the entire fiscal year.

TABLE 11

CHANGES IN CONTINUATION RATES BY WEIGHTED VRB MULTIPLE

ΔVRB	(a)* $N_{6,66}$	(b) $N_{11,71}$	$C_o=b/a$	(c) $N_{6,67}$	(d) $N_{11,72}$	$C'_o=d/c$	(e) $N_{6,68}$	(f) $N_{11,73}$	$C_1=f/e$	C_1-C_0	$C_1-C'_0$
0	5497	3515	.639	4438	2535	.571	3528	1932	.548	-.091	-.023
.25	980	661	.674	891	552	.620	855	552	.646	-.028	.026
.50	865	574	.664	843	542	.643	652	403	.618	-.048	-.025
.75	451	269	.596	383	239	.624	394	221	.561	-.035	-.063
1.00	755	402	.532	740	356	.481	670	365	.545	.013	.064
1.25	1053	640	.608	1015	683	.673	852	535	.628	.020	-.045
1.50	396	178	.449	315	143	.454	305	128	.420	-.029	-.034
1.75	512	297	.580	434	221	.509	441	249	.565	-.015	.056

*N_{ij} = number of individuals in the ith year of service in the jth fiscal year.

Since continuation rates are contingent probabilities, we also ran regressions on an alternative model using a logit transformation of the dependent variable:[1]

$$\ln \frac{C_1}{1-C_1} - \ln \frac{C_0}{1-C_0} = \omega_0' + \omega_1' \Delta VRB \quad . \tag{2}$$

The regression results are summarized in table 12.

TABLE 12

REGRESSION RESULTS FOR CHANGES IN CONTINUATION RATES ON CHANGES IN VRB

Base period	Linear model ω_0	Linear model ω_1	Linear model R^2	Logit model ω_0'	Logit model ω_1'	Logit model R^2	Number of ratings
FY 1966-71	-.059 (-2.187)	.058 (1.525)	.062	-.275 (-2.228)	.261 (1.510)	.061	37
FY 1967-72	-.014 (-0.675)	.034 (1.131)	.035	-.054 (-0.561	.142 (1.036)	.030	37

Note: t-values are in parentheses
ω_0: intercept
ω_1: slope
R^2: coefficient of determination

The findings do not enable us to draw strong conclusions about the relation between VRB and changes in continuation rates. The coefficients on VRB are positive and, for a two-tailed test, the results in the first row approach significance at the 10 percent level. None of the other coefficients on VRB is statistically significant.

Our confidence in the predictive value of the models is diminished by the relatively small coefficients of determination, R^2. From this evidence we felt that we could not

[1]Six ratings (other than the construction ratings) had continuation rates which exceeded 100 percent for the period covered. These were generally ratings which have relatively small numbers of individuals in each LOS cell. For continuation rates greater than one, the logit transformation is undefined. Hence, the observations for these ratings had to be eliminated from the logit specification. For consistency (and to avoid any possible bias) these ratings were also eliminated from the linear model.

reject the null hypothesis (i.e., that $\omega_1 = 0$). We conclude that individuals who are induced to reenlist by VRB are as likely to recommit at the second-term decision point as those who reenlisted with no VRB.

MAN-YEAR GAINS

The empirical estimates in the previous sections can be used to compute man-year gains from VRB. In contrast to our earlier estimates of man-year commitment gains, we are also concerned here with expected man-year gains that have not as yet been committed for. Also, we are concerned with actual survival rates. Many individuals leave before the end of their contract. For this reason, the man-year gains for the 4th through 10th YOS are smaller than that expected from the man-year commitment gains calculated earlier.

We have an estimate of the increase in FTRR resulting from VRB. We also observed that VRB-induced reenlistments are as likely to be in the military by the 11th YOS as are non-VRB-induced individuals. Once we calculate the additional reenlistments induced by VRB, we can use Navy-wide continuation rates to estimate expected survival rates out to the 30th year.

Separate computations were made for the linear and logit models. For the linear model, the effect of VRB on reenlistments for the interval FY 1971-72 is equal to a 1.21 percentage point increase for a $1000 bonus increase. In the logit model, the change in reenlistment depends on the level of the reenlistment rate. The formula is:

$$\Delta R_i = \alpha_1'(1-R_i)R_i \Delta VRB_i$$

where α_1' is equal to .000115. The mean reenlistment rate is used to compute the initial man-year gain. The top row in the last two columns of table 13 gives the estimated number of additional reenlistments resulting from the VRB program in FY 1972. According to the more conservative linear estimate, 1,501 of these individuals will remain in the Navy one year later. In all, using the linear estimate, 16,488 additional man-years will be gained over a 27-year period. The logit model predicts a larger man-year gain of 19,415.

The numbers in table 13 are subject to biases, and should be viewed only as rough estimates. The gain in the first year is an overestimate since many of the individuals reenlisting would have extended for a short period. These short-term extensions are partially responsible for the low continuation rate from the 5th to 6th year.[1] Since VRB increases LOR, a downward bias is also present. Individuals will be less likely to leave because of the termination of a contract in the earlier years.

[1] As was pointed our earlier, the use of Pay Entry Base Date data also gives a downward bias to the continuation rate in this interval.

TABLE 13

MAN-YEAR GAINS FROM USE OF VRB IN FY 1972

From year	To year	Continuation rate	Linear	Logit
4	5		2153.95	2536.34
5	6	.697	1501.30	1767.83
6	7	.910	1366.19	1608.72
7	8	.878	1199.51	1412.46
8	9	.872	1045.97	1231.67
9	10	.895	936.15	1102.34
10	11	.896	838.79	987.70
11	12	.948	795.17	936.34
12	13	.960	763.36	898.88
13	14	.966	737.41	868.32
14	15	.972	716.76	844.01
15	16	.974	698.13	822.06
16	17	.979	683.47	804.80
17	18	.985	673.21	792.73
18	19	.980	659.75	776.87
19	20	.818	539.67	635.48
20	21	.606	327.04	385.10
21	22	.685	224.02	263.80
22	23	.704	157.71	185.71
23	24	.732	115.45	135.94
24	25	.761	87.85	103.45
25	26	.815	71.60	84.31
26	27	.773	55.35	65.17
27	28	.777	43.01	50.64
28	29	.856	36.81	43.35
29	30	.859	31.62	37.24
30	31	.914	28.90	34.03
			16488.16	19415.30

Though the man-year gains appear formidable, they are not acquired cheaply. Besides the cost of the bonus, these individuals will receive wages and benefits throughout their Navy career and a sizeable number will be eligible for retirement annuities. To properly evaluate the merits of obtaining additional man-years through VRB inducements, one should also consider the returns from an alternative investment: namely, increasing wages of first-termers.

REFERENCES

1. Grubert, Harry and Rodney Weiher, "Navy Re-enlistments: The Role of Pay and Draft Pressure," in Studies Prepared for the President's Commission on an All-Volunteer Armed Force, Washington: U.S. Government Printing Office, 1970.
2. Hause, John C. and Anthony C. Fisher, The Supply of First-Term Enlisted Manpower in the Absence of Draft, Study S-293, Institute of Defense Analyses, Apr 1968.
3. Nelson, Gary R., "Economic Analysis of First-Term Re-enlistments in the Army," in Studies Prepared for the President's Commission on an All-Volunteer Armed Force, Washington: U.S. Government Printing Office, 1970.
4. Nelson, Gary R. and Robert C. Wilburn, Career Choice and The Supply of Military Manpower, Research Paper 896, Institute for Defense Analysis, Sep 1972.
5. Wilburn, Robert, "Impact of Income, the Draft and Other Factors on Retention of Air Force Enlisted Men," in Studies Prepared for the President's Commission on an All-Volunteer Armed Force, Washington: U.S. Government Printing Office, 1970.
6. Cook, Alvin A., Jr., Occupational Choice, the Draft, and the Excess Supply of Air Force Volunteers, Research Paper P-4606-1, RAND, 1971.
7. Cook, Alvin A., Jr., "Supply of Air Force Volunteers," in Studies Prepared for the President's Commission on an All-Volunteer Armed Force, Washington: U.S. Government Printing Office, 1970.
8. Fisher, Anthony C., "The Cost of the Draft and the Cost of Ending the Draft," American Economic Review, pp. 239-54, Jun 1969.
9. Chow, Gregory C., "Tests of Equality Between Sets of Coefficients in Two Linear Regressions," Econometrica, pp. 591-605, Jul 1960.
10. Theil, Henri, "Principles of Econometrics," John Wiley & Sons, New York, 1971.
11. Zellner, Arnold, "An Efficient Method of Estimating Seemingly Unrelated Regressions and Tests for Aggregation Bias," Journal of the American Statistical Association, pp. 348-68, Jun 1962.
12. Zellner, Arnold, "Estimators of Seemingly Unrelated Regressions and Equations: Some Exact Finite Sample Results," Journal of the American Statistical Association, pp. 977-92, Dec 1963.
13. Gray, Burton, Summary of Econometric Anaysis of Factors Influencing First-Term Reenlistment Decisions, Memorandum INS 411-72, Center for Naval Analyses, 1972.

APPENDIX A

REGRESSION RESULTS OF THE EFFECT OF VRB ON FTRR EXCLUDING CONSTRUCTION RATINGS

TABLE A-1

REGRESSION RESULTS OF THE EFFECT OF VRB ON FTRR, EXCLUDING CONSTRUCTION RATINGS

Interval	Linear model β_0	Linear model α_1	Linear model R^2	Linear model η	Logit model β_0'	Logit model α_1'	Logit model R^2	Logit model α_1	Logit model η	Ratings with VRB changes	Total number of ratings
FY 1965-67	--7.710 (-11.19)	.00177 (6.92)	.521	2.31	- -.481 (-10.39)	.000106 (6.04)	.453	.00170	2.22	22	46
FY 1968-69	- 3.690 (- 7.82)	.00197 (4.97)	.354	4.38	- .203 (- 5.51)	.000110 (3.29)	.194	.00142	3.16	17	47
FY 1971-72	3.334 (7.67)	.00153 (6.18)	.433	5.10	.306 (7.79)	.000134 (6.27)	.440	.00148	4.94	9	52
Combined*		.00166 (10.57)	.671			.000117 (9.60)	.652			48	145

Note: t-values are in parentheses. R^2 is coefficient of determination. η is the elasticity computed at the mean.
* β_0 and β_0' are not reported because they each take on three values: one for each interval. α_1 for the logit regression is excluded for the same reason. The elasticity at the mean is excluded because it requires computing averages over all intervals.

APPENDIX B

WEIGHTING PROCEDURES IN LEAST SQUARES ESTIMATIONS

Ordinary least squares gives each error equal weight when minimizing the sum of their squares. But often we know (or suspect) that the errors come from distributions with different variances, and this information can be used to produce more efficient estimators. Assuming, for example, that the individuals coming up for reenlistment each year are a sample from a larger population, we have for the linear model that

$$V(\hat{R}_i) = \frac{R_i(1-R_i)}{N_i-1}$$

$$\doteq \frac{R_i(1-R_i)}{N_i}$$

where R_i is now the true reenlistment rate and $\hat{R}$ is the observed rate. N_i is the number of individuals eligible to reenlist in the <u>ith</u> rating. We ran regressions for the first difference and thus have

$$V(\hat{R}_{1i}-R_{0i}) = \frac{R_{1i}(1-R_{1i})}{N_{1i}} + \frac{R_{0i}(1-R_{0i})}{N_{0i}} + \text{covariance term.} \tag{1}$$

The first subscripts represent time periods.

Since we had no estimate of the covariance term, we assumed that it equalled zero. This results in some loss of efficiency, but introduces no bias in our estimates. To standardize the variance of the errors, we simply multiply through by the inverse of the right hand side of equation (1), excluding the covariance term. We can obtain this effect by multiplying both sides of the regression equation by the square root of the inverse.

$$\text{weight} = \sqrt{\frac{N_{0i}N_{1i}}{N_{1i}\hat{R}_{0i}(1-\hat{R}_{0i}) + N_{0i}\hat{R}_{1i}(1-\hat{R}_{1i})}}$$

where we are now using the observed reenlistment rates to obtain the weight.

For the logit function, the variance of the regression approaches $1/N_iR_i(1-R_i)$ for large samples. Assuming no covariance between errors from different years, the weight for the first differences regression is

$$\text{weight} = \sqrt{\frac{N_{1i}\hat{R}_{1i}(1-\hat{R}_{1i})N_{0i}\hat{R}_{0i}(1-\hat{R}_{0i})}{N_{0i}\hat{R}_{0i}(1-\hat{R}_{0i}) + N_{1i}\hat{R}_{1i}(1-\hat{R}_{1i})}} \quad .$$

The regressions reported in the text used these weights. Estimates using other weights produced similar results.

The LOR regressions use mean values of ratings as observations. This would indicate that the variance of the estimate for rating i is

$$V(\hat{LOR}_i) = \frac{\sigma^2}{N_i}$$

where σ^2 is the unknown error of each individual from the regression. For regressions of first differences, we have

$$V(LOR_{1i} \hat{-} LOR_{0i}) = \sigma^2 \left(\frac{N_{0i}+N_{1i}}{N_{0i}N_{1i}}\right)$$

where again we assume that the covariance is zero.

The term in parentheses on the right hand side indicates that the variance is smaller for those ratings with larger number of eligibles. To correct for this, we weight the observations by:

$$\sqrt{\frac{N_{0i}N_{1i}}{N_{0i}+N_{1i}}} \quad .$$

APPENDIX C

CONSTRUCTION OF VARIABLES

NON-VRB WAGES

In calculating the wage variable, we first assumed that an individual reenlists for four years. The wage variable was constructed as follows:

$$W_i = \sum_{j=5}^{8} E_{ij}(1+r)^{5-j} + RB$$

where

$$E_{ij} = \sum_k P_{ijk} \cdot EXP(BP/k, j) \cdot (1.4) \quad .$$

E_{ij} is expected earnings in year j for an individual in rating i, P_{ijk} is the probability of being in pay grade k in year j for rating i, and $EXP(BP/k, j)$ is the expected yearly base pay given the individual is in pay grade k in year j. We assume that the individual's expected pay increases at the average of the prior two years.[1] The multiplication by 1.4 is designed to bring base pay up to the Regular Military Compensation rate calculated by DoD. Besides base pay, RMC includes the basic allowance for quarters, subsistence, and the tax advantage from the two untaxed allowances. The r is the individual's discount rate, assumed equal to .1. RB is the regular reenlistment bonus received by all reenlistees calculated in the same manner as VRB with a multiple of one. After collecting the data from a variety of reports, we discovered that the average length of reenlistment for those eligible for VRB was 5.15. We, therefore, multiplied W_i by (5.15/4) = 1.2875.

After wage estimates were constructed for those ratings with VRB changes, it became apparent that nonbonus pay did not vary significantly between ratings. The table below indicates the unweighted mean and the range within which all expected wages fell.

Interval	Unweighted mean	Lowest wage	Highest wage	$\frac{\text{Range}}{\text{mean}}$
FY 1965-67	23422.06	22861.66	24316.48	.062
FY 1968-69	29941.12	29330.10	30704.34	.046
FY 1971-72	37205.60	35947.09	38252.58	.062

[1]The pay increase of late 1971, however, was treated as a one-time increase.

This lack of variation is partly the result of excluding 6YO ratings and nondesignated enlisted personnel. The former probably have faster promotions and hence higher wages, and the latter slower promotions and lower wages. But it may indicate an overall low variability across ratings.

Given the low variability of the wages, we used the unweighted mean of those ratings with VRB changes.

VRB

The VRB estimate for the analyses of first-term reenlistments is calculated for each rating with a VRB change between years. We based it on expected base pay in the third year of service and thus have

$$VRB_i = \frac{5.15}{12} \sum_k P_{ik} \cdot EXP(BP/k, 3) \cdot (VM) \quad .$$

Division by 12 gives us monthly base pay; multiplication by 5.15 results from assuming a 5.15 year reenlistment; and VM is the VRB multiple for the rating.

Based on the low variability of expected base pay across ratings, the VRB multiple was used as the independent variable in the length of reenlistment and second-term reenlistment portions of the study.

APPENDIX D

ANALYSIS OF 6YO DATA

Six year obligors are excluded from our study because of the way their FTRR are calculated. Individuals who commit for six years on entry into the Navy are recorded as four-year enlistments with two-year extensions. At the end of four years, their extension becomes effective, and it is recorded as a first-term reenlistment. Since all 6YOs must make this extension, there is, in effect, a 100 percent reenlistment rate for 6YOs at the 4-year point. When the six-year commitment expires these individuals make their first true recommitment decision. If they reenlist, they are again counted as a first-term reenlistment. We thus have double counting and an upwardly biased first term reenlistment rate.

It would be of interest to measure first-term reenlistments of 6YOs at the six-year point, but it is not easy to disentangle the data. The task is complicated by the presence of 4YOs and 6YOs in the same ratings. We do, however, know 6YO eligibles by rating for FY 1972 and FY 1973. We also know the number of 6YO reenlistments by rating and length of service. By assuming that all those recommiting prior to the fifth year of service were not making a true first-term reenlistment decision (even though some of these individuals were not making just two-year extensions), and subtracting this number from total eligibles, we arrived at an estimate of those eligible to reenlist and free to leave the service. All first-term reenlistments beyond the fourth year of service were considered true first-term reenlistments from this group of eligibles. We thus arrived at a reenlistment rate for 6YOs comparable to 4YOs. The data is reported in table D-1.

One should be wary of generalizing from this data. The reenlistment rates are probably subject to wide error resulting from their indirect estimation and small eligible populations. Over half the cells have less than 100 eligibles. Overall averages may be less subject to error. The FTRR of 6YOs at the six-year point for 1972 is computed as 8.49 percent; for 1973 it is 11.94 percent. For the 4YOs used in our work FTRR were 16.07 percent in 1972 and 17.14 percent in 1973. The lower FTRR for 6YOs is consistent with our theory. These individuals are generally the most highly trained personnel with the greatest opportunities in the civilian sector and are hence the least likely to remain in the Navy.

Seven of the twelve 6YO ratings had a drop in VRB from FY 1972 to FY 1973. Six of these reductions came four months into FY 1973, but we still may be able to detect some effect of the VRB change. Table D-2 indicates that the reduction in VRB resulted in smaller increases in reenlistment rates.

TABLE D-1

CORRECTED 6YO FIRST-TERM REENLISTMENT RATES

	FY 1972					FY 1973				
Rating	Total 6YO elig.	1-4 LOS reen.	Elig. with LOS > 4	Reen. with LOS > 4	RR	6YO elig.	1-4 LOS reen	Elig. with LOS > 4	Reen. with LOS > 4	RR
All ST	465	326	139	15	10.79	490	294	196	38	19.39
GMM*	103	79	24	4	16.67	99	37	62	13	20.97
FTG	254	203	51	3	5.88	219	126	93	3	3.23
FTM*	471	293	178	14	7.87	288	119	169	15	8.88
FTB*	179	175	4	1	25.00	181	164	17	4	23.53
MT*	343	316	27	4	14.81	203	181	22	4	18.18
All ET*	2356	1741	715	56	7.83	1943	1181	762	68	8.92
DS*	352	271	81	9	11.11	172	85	87	20	22.99
CTM*	316	230	86	4	4.65	165	75	90	12	13.33
MM	785	585	200	21	10.50	1006	727	279	35	12.54
EM	404	284	120	11	9.17	444	349	95	11	11.58
IC	171	112	59	0	0.00	132	95	37	5	13.51

Note: Adapted from Report 1133-2030.

*Experienced VRB decline in 1973.

TABLE D-2

CHANGES IN REENLISTMENT RATES BY CHANGES IN VRB
FOR 6YOs IN FY 1972-73

	1972	1973	Change
Ratings with no VRB change	8.79	13.14	+4.25
Ratings with reduction in VRB	8.25	11.25	+3.00

APPENDIX E

LENGTH OF RECOMMITMENT DATA EXCLUDING CONSTRUCTION RATINGS

To determine whether lateral entry into construction ratings introduced bias in the analysis of the effect of VRB on length of recommitment, the regression equations were re-estimated with these ratings excluded. Table E-1 reports average LOR by VRB multiple for fiscal 1970-73. Table E-2 summarizes the simple regression results. The multiple regression results are reported in table E-3. The exclusion of construction ratings does not appear to alter the findings reported in the text.

TABLE E-1

AVERAGE LENGTHS OF REENLISTMENT AND EXTENSION BY FISCAL YEAR AND VRB MULTIPLE, EXCLUDING CONSTRUCTION RATINGS

	Reenlistments & extensions					Extensions					Reenlistments				
	0	1	2	3	4	0	1	2	3	4	0	1	2	3	4
1970	2.718	3.239	3.133	3.605	3.330	.835	.843	.624	.807	.812	4.681	5.151	5.256	5.407	5.400
1971	2.693	3.998	3.697	3.817	3.648	.819	.917	.739	.706	.820	4.469	5.232	5.280	5.276	5.205
1972	2.697	-	3.442	3.426	3.682	.783	-	.716	.702	.762	4.341	-	5.193	5.214	5.189
1973	2.799	-	3.185	3.502	3.378	.769	-	.727	.708	.734	4.259	-	5.102	5.121	5.070

Note: Also excludes non-designated enlisted personnel and 6YOs.

TABLE E-2

SIMPLE REGRESSION RESULTS OF CHANGES IN LOR ON CHANGES IN VRB, EXCLUDING CONSTRUCTION RATINGS

	Form	Intercept	$\hat{Y}_1$	R^2	$\frac{\Delta LOR}{\Delta VRB}$
FY 1970-71 Unweighted	1	.165 (1.85)	.457 (3.02)	.154	.457
	2	.045 (1.67)	.142 (3.09)	.160	.453
FY 1970-71 Weighted	1	.189 (3.53)	.257 (1.73)	.057	.257
	2	.051 (3.08)	.080 (1.71)	.055	.255
FY 1971-72 Unweighted	1	- .165 (-1.77)	.516 (4.72)	.304	.516
	2	- .046 (-1.56)	.159 (4.66)	.298	.516
FY 1971-72 Weighted	1	- .147 (-2.53)	.452 (6.88)	.481	.452
	2	- .042 (-2.29)	.141 (6.73)	.470	.457
FY 1972-73 Unweighted	1	- .077 (- .82)	.498 (4.01)	.263	.498
	2	- .021 (- .72)	.178 (4.60)	.320	.561
FY 1972-73 Weighted	1	- .079 (-1.41)	.548 (3.85)	.248	.548
	2	- .023 (-1.36)	.192 (4.52)	.312	.605

TABLE E-3

MULTIPLE REGRESSION RESULTS OF CHANGES IN LOR ON CHANGES IN VRB, CHANGES IN LAGGED VRB, AND CHANGES IN RECOMMITMENT RATES, EXCLUDING CONSTRUCTION RATINGS

	Form	Intercept	$\hat{\lambda}_1$	$\hat{\lambda}_2$	$\hat{\theta}_1$	$\hat{\delta}_3$	R^2	$\frac{\Delta LOR_0}{\Delta VRB}$	$\frac{\Delta LOR_1}{\Delta VRB}$
FY 1971-72	1	- .106	.508	- .308	.200		.395	.508	.200
Unweighted		(-1.17)	(4.94)	(-2.73)	(1.33)				
	2	- .030	.157	- .082	.075		.364	.510	.243
		(-1.04)	(4.78)	(-2.28)	(1.56)				
	1	.100	.582	- .279	.303	-3.54	.503	.582	.303
		(.95)	(6.01)	(-2.69)	(2.19)	(-3.26)			
	2	.046	.175	- .075	.100	- .252	.508	.568	.325
		(1.40)	(5.94)	(-2.33)	(2.34)	(-3.79)			
FY 1971-72	1	- .134	.451	- .187	.264		.498	.451	.264
Weighted		(-2.29)	(6.91)	(-1.30)	(1.67)				
	2	- .039	.140	- .050	.090		.482	.454	.292
		(-2.08)	(6.72)	(-1.08)	(1.78)				
	1	.015	.540	- .112	.428	-2.61	.572	.540	.428
		(.20)	(7.92)	(- .82)	(2.91)	(-2.91)			
	2	.016	.163	- .031	.132	- .20	.565	.529	.428
		(.63)	(7.87)	(- .72)	(4.52)	(-3.05)			
FY 1972-73	1	- .066	.495	- .086	.409		.275	.495	.409
Unweighted		(- .69)	(3.97)	(- .83)	(2.56)				
	2	- .018	.177	- .022	.155		.327	.550	.488
		(- .61)	(4.55)	(- .67)	(3.10)				
	1	- .046	.517	- .085	.432	- .587	.279	.517	.432
		(- .45)	(3.88)	(- .81)	(2.63)	(- .50)			
	2	- .003	.186	- .028	.158	- .112	.341	.586	.498
		(- .10)	(4.64)	(- .70)	(3.33)	(- .96)			
FY 1972-73	1	- .073	.547	- .076	.471		.273	.547	.471
Weighted		(-1.29)	(3.86)	(-1.25)	(3.06)				
	2	- .021	.191	- .021	.170		.333	.593	.535
		(-1.24)	(4.53)	(-1.18)	(3.71)				
	1	.024	.590	- .050	.540	-3.49	.432	.590	.540
		(.41)	(4.64)	(- .91)	(3.93)	(-3.46)			
	2	.014	.199	- .019	.180	- .322	.583	.627	.567
		(.95)	(5.88)	(-1.33)	(4.91)	(-5.08)			

APPENDIX F

ALTERNATIVE LENGTH OF RECOMMITMENT REGRESSION MODELS

We merged the intervals to obtain one set of coefficient estimates. Since there were no VRB changes in FY 1969-70, we could not test the equality of the lagged VRB coefficients for all three intervals. Table F-1 shows that entering lagged VRB changes into the other two regressions did not alter the estimated coefficients of VRB and recommitment rates. We therefore merged the three intervals excluding the lagged VRB variable. The results reported below exclude construction ratings.

We first sought to test the superiority of the model.

$$\text{Model I: } \Delta LOR = \rho + D_1 + D_2 + \epsilon_0 \Delta VRB + \epsilon_1 D_1 \Delta VRB + \epsilon_2 D_2 \Delta VRB + \xi_0 \Delta RR + \xi_1 D_1 \Delta RR + \xi_2 D_2 \Delta RR$$

to the merged model

$$\text{Model II: } \Delta LOR = \rho + D_1 + D_2 + \epsilon_0 \Delta VRB + \xi_0 \Delta RR$$

where

D_1 is 1.0 if observation is from FY 1971-72 interval and 0.0 otherwise, and

D_2 is 1.0 if observation is from FY 1972-73 interval and 0.0 otherwise.

The test examines the significance of the coefficients ϵ_1, ϵ_2, ξ_1, and ξ_2. An F test was used that tested the significance jointly. To this end, we examined the additional explanatory power of Model I over Model II. Let SSE_1 and SSE_2 be the sum of squared errors of Models I and II, respectively. The degrees of freedom attached to their difference is 4, the number of coefficients being tested. The difference over the degrees of freedom is the numerator of the F statistic. The denominator is SSE_1 over its degrees of freedom. The degrees of freedom is 152 (the number of observations) minus 9 (the number of coefficients).

The test statistic is therefore

$$F_{4,143} = \frac{(SSE_2 - SSE_1)/4}{SSE_1/143} \quad .$$

For the unweighted regressions, we could not reject the hypothesis that the four coefficients were jointly equal to zero at the 5 percent level of significance. For the weighted

TABLE F-1

ADDITIONAL MULTIPLE REGRESSION RESULTS FOR CHANGES IN LOR ON CHANGES IN VRB, WITH CHANGES IN LAGGED VRB INCLUDED AND EXCLUDED

	Form	Intercept	$\hat{\lambda}_1$	$\hat{\lambda}_2$	$\hat{\lambda}_3$	R^2	$\frac{\Delta LOR}{\Delta VRB}$
FY 1971-72 Unweighted	1	.031 (.30)	.474 (6.13)		-2.59 (-2.86)	.495	.474
	2	.048 (1.41)	.200 (8.28)		- .329 (-5.38)	.674	.623
	1	.078 (.77)	.482 (6.63)	- .298 (-2.95)	-2.53 (-2.58)	.563	.482
	2	.059 (1.77)	.202 (8.60)	- .070 (-2.12)	- .327 (-5.51)	.698	.630
FY 1971-72 Weighted	1	-.025 (-.36)	.463 (9.15)		-1.71 (-2.65)	.612	.463
	2	.033 (1.33)	.181 (10.81)		- .282 (-5.26)	.719	.564
	1	-.021 (-.30)	.463 (9.18)	- .148 (-1.11)	-1.63 (-2.51)	.620	.463
	2	.034 (1.35)	.181 (10.74)	- .022 (- .49)	- .280 (-5.16)	.720	.564
FY 1972-73 Unweighted	1	.149 (1.36)	.532 (3.35)		-1.99 (-1.62)	.185	.532
	2	.112 (2.57)	.191 (3.15)		- .426 (-2.80)	.222	.586
	1	.082 (.81)	.504 (3.47)	- .305 (-3.40)	-1.01 (- .87)	.337	.504
	2	.079 (2.02)	.184 (3.48)	- .138 (-4.09)	- .288 (-2.10)	.417	.564
FY 1972-72 Weighted	1	.089 (1.43)	.596 (4.05)		-4.55 (-4.35)	.388	.596
	2	.051 (2.17)	.198 (3.58)		- .464 (-4.77)	.400	.607
	1	.075 (1.24)	.588 (4.13)	- .125 (-2.17)	-3.95 (-3.78)	.441	.588
	2	.047 (2.17)	.197 (3.85)	- .063 (-3.10)	- .422 (-4.64)	.497	.604

regression, however, we rejected such an hypothesis. The values of SSE_1, SSE_2, and F are as follows:

		SSE_2	SSE_1	F
Arithmetic	Unweighted	58.2261	56.5426	1.064
	Weighted	21.4546	19.8695	2.852
Log	Unweighted	5.5048	5.3050	1.346
	Weighted	2.0227	1.8381	3.589

We could not, however, distinguish between Model I and the following model for the weighted regressions.

Model III: $\Delta LOR = \rho + D_1 + D_2 + \theta_0 \wedge VRB + \tau_0 \Delta RR + \tau_1 D_1 \Delta RR + \tau_2 D_2 \wedge RR.$

The sum of squared errors and F values is:

		SSE_3	SSE_1	F
Arithmetic	Weighted	20.4983	19.8695	2.263
Log	Weighted	1.9028	1.8381	2.517

This indicates that we cannot reject the hypothesis that $\theta_1 = \theta_2 = 0.0$ for the weighted regressions.

In table F-2 we report the results for Model II unweighted and Model III weighted; t-values are reported in parentheses.

TABLE F-2

REGRESSION RESULTS FOR LOR MODELS II AND III

		ρ	D_1	D_2	ϵ_0	ξ_0	ξ_1	ξ_2	R^2	$\frac{\Delta LOR}{VRB}$
Model II Unweighted	Arithmetic	.242 (2.61)	- .297 (-2.41)	-.258 (-2.03)	.571 (7.66)	-1.868 (-2.76)			.311	.571
	Log	.081 (2.73)	- .082 (-2.16)	- .081 (-2.05)	.179 (8.07)	- .150 (-3.07)			.329	.556
Model III Weighted	Arithmetic	.170 (2.43)	- .167 (-1.64)	- .147 (-1.55)	.505 (8.89)	.052 (.05)	-2.57 (-2.01)	-3.60 (-2.43)	.434	.505
	Log	.050 (2.00)	- .036 (-1.14)	- .036 (-1.14)	.156 (9.33)	- .011 (- .16)	- .186 (-2.06)	- .310 (-2.93)	.455	.484