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CAUSTIC-MATCHING: A NEW TECHNIQUE FOR  
IMPROVING LIMITED-SCAN ANTENNAS

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## Caustic-Matching: a New Technique for Improving Limited-Scan Antennas

### I. INTRODUCTION

Phased line source feeds located along regions of intense focal fields have been successfully used with spherical reflector antennas to obtain high gain and high aperture efficiency. This method of aberration correction is exemplified in the 300-meter Arecibo antenna.

The technique reported here demonstrates that it is not necessary to shape a physical feed to lie along high-intensity ridgelines of a large-aperture antenna. The required correction can be obtained by matching the caustics (focal surfaces) of a subreflector feed to those of the large-aperture antenna. Applied specifically to paraboloids and parabolic cylinder antennas, caustic-matching is a new concept, leading to improved designs that yield high-gain beam-scanning over limited sectors.

The focal surfaces of paraboloids and ridgeline correctors for paraboloidal antennas have been analytically treated;<sup>1</sup> and subreflector designs (discussed on pages 51, 63, and 83 of the work cited) have recently been given considerable attention.<sup>2</sup> In the present paper we derive the caustics for parabolic cylinders. (Received for publication 16 December 1974)

1. Collins, R. E., and Zucker, F. J., Eds. (1969) Antenna Theory, McGraw-Hill, Part II, Chaps. 16 and 17.
2. Fitzgerald, W. D. (1971) Limited Electronic Scanning with an Offset-Feed Near-Field Gregorian System, ESD-TR-71-272, Contract No. F19628-70-C0230, Lincoln Laboratory, 24 Sept. 1971.

## 2. THE PARABOLOIDAL ANTENNA

Plane waves incident on a paraboloidal reflector from angles tilted with respect to the axis of the antenna are reflected in a family of rays that envelop the caustic surfaces. As shown by geometric optics calculations the energy is generally spread over two sheets, with high-energy densities located on distinct ridges on the surfaces. More exact diffraction solutions to focal fields of the paraboloid can be calculated<sup>3, 4</sup> but these ray optic surfaces are sufficient to provide the general structure of the focal fields. Experimentally they have proved adequate guides for antenna feed design.

A set of parametric equations describing best focal lines in regions near the geometric focus can be obtained from suitable limiting curves derived from the caustic surfaces. These curves are functions of the  $x, y, z$  coordinates of the main reflector, the plane-wave arrival angle  $\theta$ , and the focal length  $f$ .

Our main objective in the work reported here was to design a subreflector that, properly located in the focal region of a large reflector and fed by a plane wave, would generate a caustic field distribution similar to that of the large reflector. Complete success required matching not only the limiting ridgelines but the phase amplitude and polarization over the entire caustic surfaces, and this matching had to hold for a large range of tilt angles of the plane waves illuminating the subreflector. The solution would allow a small-aperture phased array to scan a large-aperture reflector and would yield cost savings by decreasing the number of phasing control elements.

For this strategy to be workable the subreflector must have as large a caustic as the main reflector. We see that the expressions for the paraboloid ridgeline coordinates do scale in the sense that the curves are normalized to  $f$ , that is, the longer the focal length the larger the caustic surface for a given angle  $\theta$ . A more interesting property of the curves, however, is that they maintain their shape and expand linearly with increasing angle  $\theta$ . Hence, two reflectors that have different aperture diameters  $D_1$  and  $D_2$  but the same  $f/D$ ,

$$\frac{f_1}{D_1} = \frac{f_2}{D_2} = C,$$

3. James, G. L., and Kerdelmelidis, V. (1973) Reflector antenna radiation pattern analysis by equivalent edge currents, IEEE Tr. AP AP-21(No. 1):19-24.
4. Rusch, W. V. T., and Ludwig, A. C. (1973) Determination of the maximum scan-gain contours of a beam-scanning paraboloid and their relation to the Petzval surface. IEEE Tr. AP AP-21(No. 2):141-147.

will have identical caustics if the tilt angle  $\theta$  on the smaller reflector is increased by  $D_1/D_2$  (Figure 1).

Given these scaling properties, it is interesting to examine the pair of confocal inverted paraboloids suggested by E. A. Dudkowski. Effective caustic-matching requires that the ridgeline for the  $\pm\theta$  tilts on the main reflector be identical with the  $\mp\theta$  tilts on the subreflector. Although the fit is fairly good we can easily show that the  $+\theta$  curves for the large surface are not the same as the  $-\theta$  curves for the subreflector.

To examine this 'end for end' symmetry of the ridgelines for a 'whole' paraboloid rather than for an off-axis section of it, it is instructive to consider the parametric equations for the  $y_0, z_0$  intercepts of the ridgeline rays in the  $yz$  plane:

$$\begin{aligned} y_0 &= - \frac{\sin \theta (x^2 + y^2 + 4f^2)}{4f \cos \theta - 2y \sin \theta} \\ z_0 &= \frac{\cos \theta (x^2 + y^2 + 4f^2)}{4f \cos \theta - 2y \sin \theta} - 2f + z. \end{aligned} \quad (1)$$

Substituting  $-\theta$  for  $+\theta$  in Eqs. (1) reduces the magnitude of both  $y_0$  and  $z_0$  except at  $y = 0$  (vertex zone reflections). Caustic-matching by this confocal inverted geometry of subreflector and off-axis parabolic section can therefore be only an approximation because it depends on caustics being produced on the subreflector by the  $\mp\theta$  inclined waves to match  $\pm\theta$  caustics on the main reflector (Figure 2).

On the other hand, in caustic-matching by plotting the coincidence of subreflector and main-reflector caustic surfaces, the antenna designer has a new tool. He can utilize it to determine how to rotate the subreflector about the focal point to obtain better congruence or to modify both the reflector shapes and sizes to obtain better caustic-matching over the scan sectors of interest.

It is important to remember that caustics and ridgelines for the parabolic cylinder antenna are not the same as for the paraboloid. In Sec. 3 we apply the caustic-matching technique to the parabolic cylinder reflector.

### 3. THE PARABOLIC CYLINDER ANTENNA

The configurations treated in this section are cylindrical in nature and can therefore be analyzed by considering their performance in a single cross-section plane. Using geometric optics analysis, we assume parallel rays incident on a parabola from a direction that makes an angle  $\theta$  with the parabola axis (Figure 3). After reflection from the parabola the rays will envelop a caustic curve whose parametric form is

$$\frac{x}{f} = \frac{[6(h/f)^2 - 8] \sin^2 \theta + [(h/f)^3 - 12(h/f)] \sin \theta \cos \theta}{8},$$

$$\frac{y}{f} = \frac{[12(h/f) - (h/f)^3] \sin^2 \theta + [6(h/f)^2 - 8] \sin \theta \cos \theta}{8}. \quad (2)$$

The parameter  $h$  is the  $y$  coordinate of the intersection of the incident ray with the parabola, and  $f$  is the focal length of the parabola. A set of caustics for an upper-half parabola,  $y \geq 0$ ,  $f = 1$ ,  $f/D = 0.5$  ( $D$  is the height of the aperture), for various positive angles of incidence  $\theta$  is shown by the solid curves in Figure 4.

It is clear from the caustic equations that for fixed incident wave angle  $\theta$  and fixed  $f/D$ , the extent of the caustic and its displacement from the focal point scale directly with  $f$ . It is also clear from Figure 4 (solid curves) that for fixed  $f/D$ , the extent of the caustic and its displacement from the focal point vary almost directly with the plane-wave incidence angle  $\theta$ .

A lower-half parabola,  $y \leq 0$ ,  $f = 0.25$ ,  $f/D = 0.5$ , with vertex  $V$  at  $(f, 0)$  and focus  $F$  at  $(0, 0)$ , is shown in Figure 5. A set of caustic curves for this parabola is shown (broken curves) in Figure 4. The incidence angle  $\phi$  for these curves is measured clockwise from the negative  $x$  axis (see Figure 5).

Inspection of Figure 4 indicates that a rotation of the smaller parabola ( $f = 0.25$ ) approximately  $45^\circ$  clockwise about its focus point would bring the two sets of caustics into near-coincidence. The configuration of the parabolas after such a rotation is shown in Figure 6. Figure 7 shows the two sets of caustics superposed.

Ray traces were performed on the rotated configuration, starting with a plane wave incident on the smaller parabola and finally determining the angles of the rays emerging from the larger parabola. The results, shown in Figure 8, indicate that the system does scan well over an angular sector  $0^\circ$  to  $10^\circ$  with the aperture of the main reflector about  $2f$ , that is, from  $y = 0$  to  $y = 2$ . Reduction in aperture phase error and increase in scan angle could be achieved by phase compensation in the phased array that feeds the smaller reflector.

#### 4. CONCLUSIONS

This approach to the design of a reflector-subreflector system for high-performance limited-sector-scanning is based on relating the focal surfaces, or caustics, of the main reflector and subreflector.

A confocal off-axis paraboloidal reflector-subreflector system can be improved through a simple rotation and slight extension of the subreflector to make the caustics coincide more accurately. Since the designer can see which parts of

the reflector (locations where rays are reflected) are contributing to scanning errors, the technique can be applied to modify the reflector or subreflector contours until the focal surfaces are more closely matched.

The antenna consisting of confocal parabolic cylinder sections is examined in more detail. In this case it is shown that a rather large rotation of the subreflector with reference to the main reflector results in excellent ray collimation and hence in high-quality beam-scanning over a limited sector.

The technique is economical and has important practical applications. In air traffic control radars and satellite communications, especially, the small-aperture subreflector can be illuminated by a plane-wave source (such as a small phased array located in a low-shadowing region of an off-axis set of parabolic sections) to obtain high-gain limited-scan beam-steering on a large-aperture reflector.

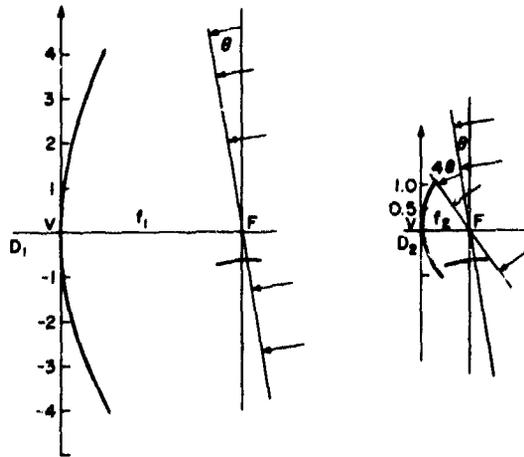


Figure 1. Scaling Properties of Caustic Ridgelines on Paraboloid ( $4\theta$  ridgelines on small paraboloid are the same as  $\theta$  ridgelines on main dish;  $N = D_1/D_2 = f_1/f_2$ )

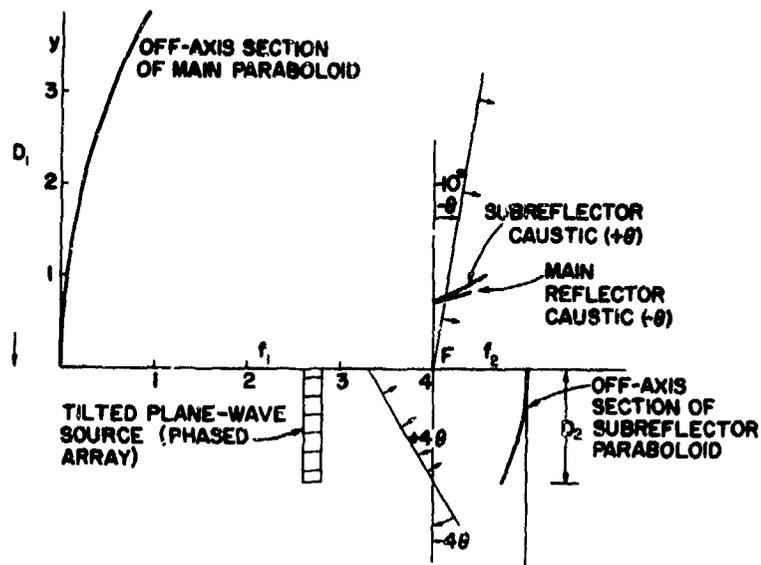


Figure 2. Caustic-Matching With Off-Axis Paraboloid Sections ( $f_1/f_2 = N = 4$ ;  $f_1/D_1 = f_2/D_2$ )

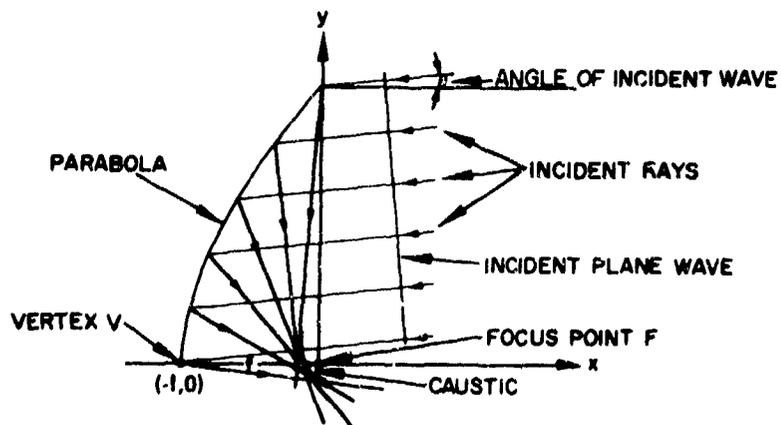


Figure 3. Off-Axis Plane Wave Incident on Upper-Half Parabola ( $f = 1$ )

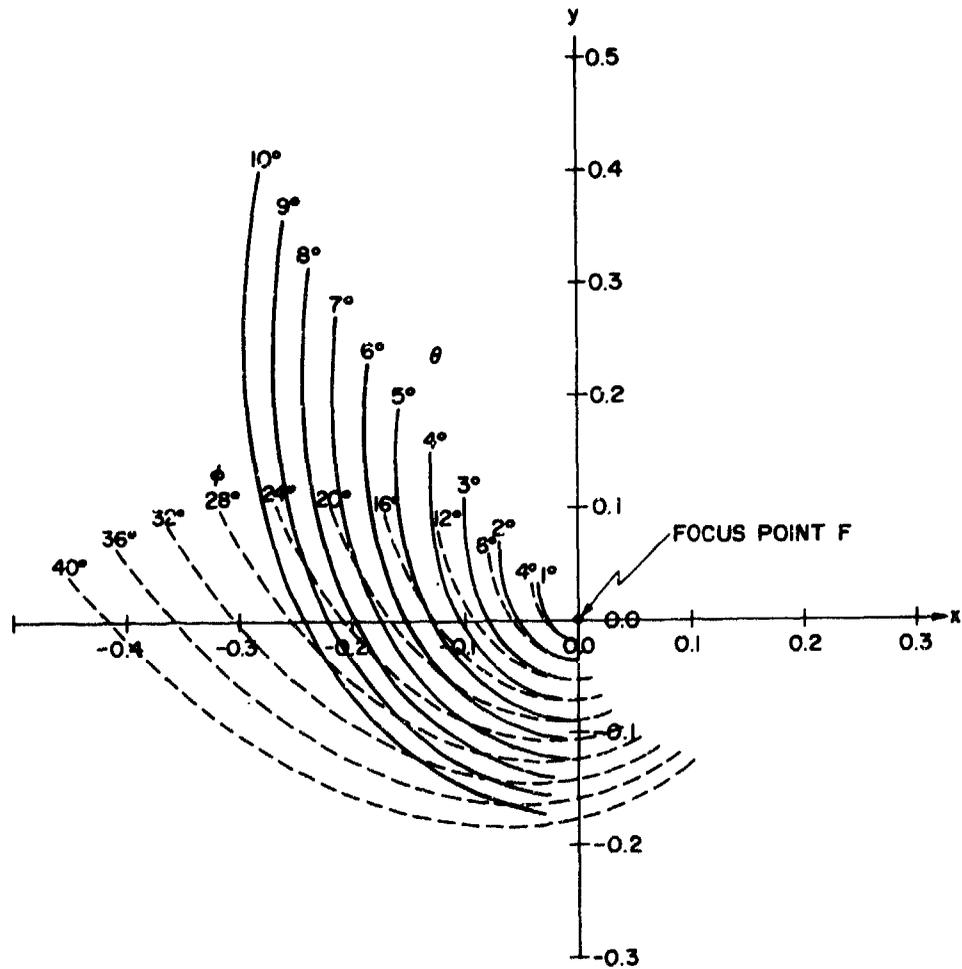


Figure 4. Caustic Curves [ — for upper-half parabola of Figure 3 ( $f = 1$ );  
 --- for lower-half parabola of Figure 5 ( $f = 0.25$ )]

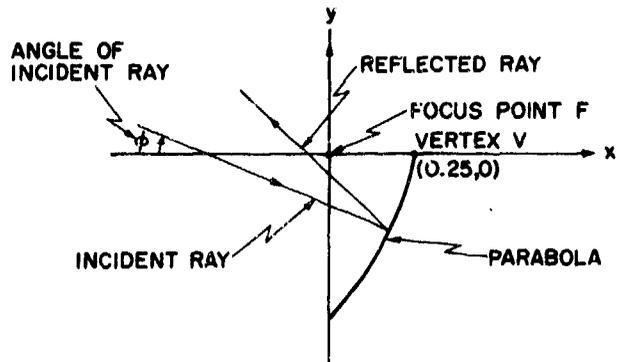


Figure 5. Lower-Half Parabola ( $y \leq 0$ ,  $f = 0.25$ ,  $f/D = 0.5$ )

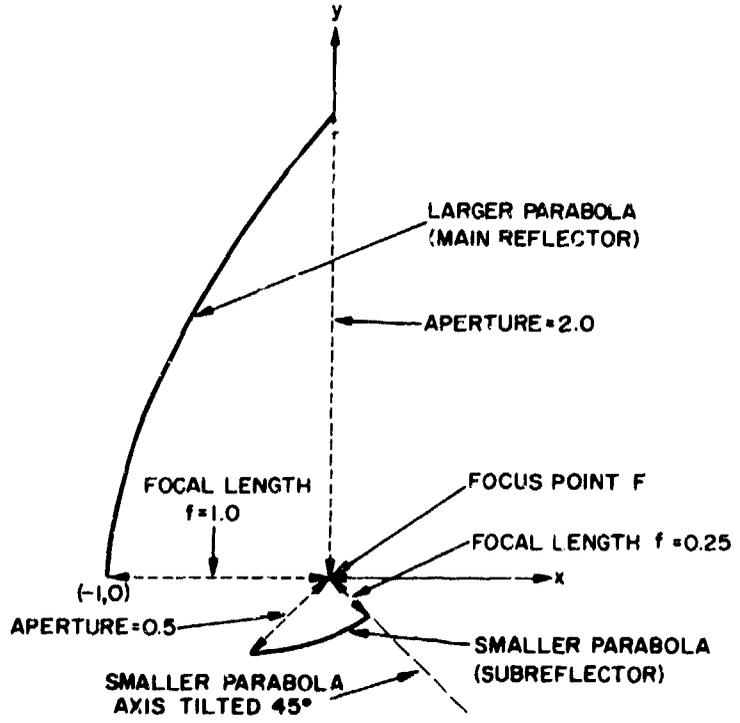


Figure 6. Smaller Parabola Rotated 45° Clockwise About Its Focus Point

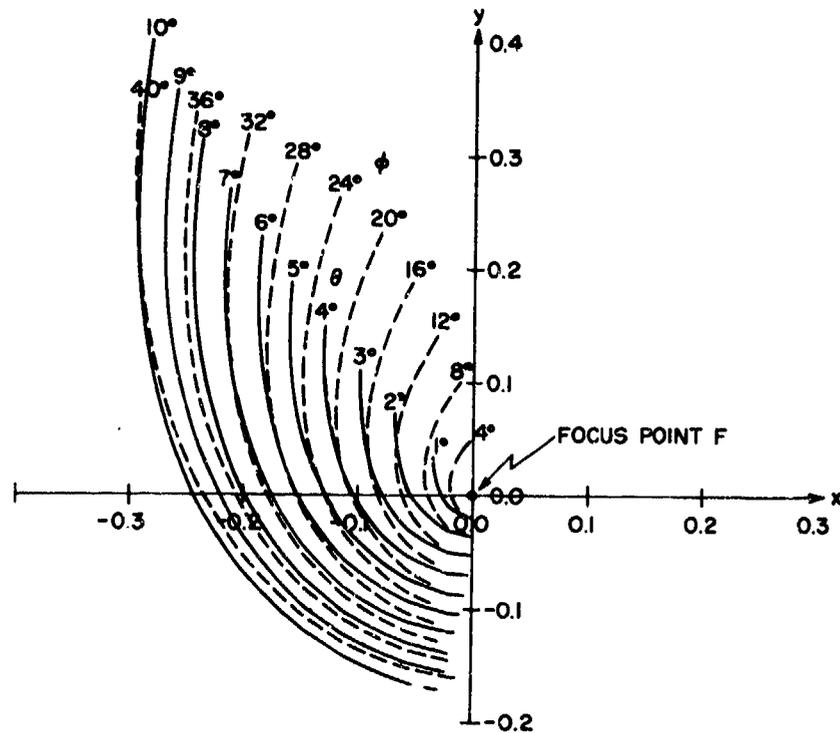


Figure 7. Superposition of Caustics for Reflector Configuration Shown in Figure 6 [—— caustics for larger parabola; ---- caustics for smaller parabola]

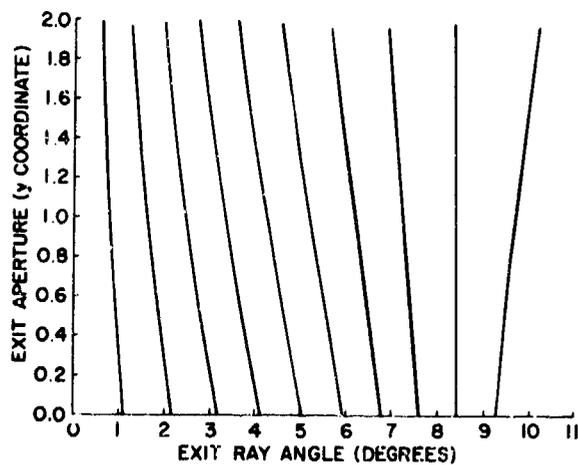


Figure 8. Exit Ray Angle Versus Exit-Ray Aperture Coordinate for Various Tilted Plane Waves Incident on the Smaller Parabola