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A USER'S MANUAL FOR THE REPSIL CODE

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Item 20 (Continued) ABSTRACT

The manual gives instructions for correctly setting up problems and estimating machine time and storage requirements. Two illustrative problems are set up and the resulting solutions given. The numerical algorithm employed by REPSIL is outlined and instructions for programming additional initial geometries and loadings are given. The REPSIL plotting program, which produces isometric and cross-sectional displays and time histories of energies, deflections and strains, is also described. Listings of both the REPSIL program and the REPSIL plotting program are included.

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LIST OF SYMBOLS

	Page
a	Determinant of surface metric $a_{\alpha\beta}$ 34
a_0	Initial value of determinant, a , at time $t = 0$ 28
$a_{\alpha\beta}$	Covariant components of middle surface metric 34
$a^{\alpha\beta}$	Contravariant components of middle surface metric 34
A	Coefficient in quadratic equation for $\Delta\lambda$, see (2.26) 38
$\Lambda_{\alpha\beta}$	Initial values of $a_{\alpha\beta}$ at strain locations on bounding surfaces of shell 52
$b_{\alpha\beta}$	Covariant components of 2nd fundamental tensor of middle surface 34
b^{α}_{β}	Mixed components of 2nd fundamental tensor of middle surface. 34
B	Coefficient in quadratic equation for $\Delta\lambda$, see (2.26) 38
$B_{\alpha\beta}$	Initial values of $b_{\alpha\beta}$ at strain locations on bounding surfaces of shell 52
c	Speed of longitudinal waves in plate 24
C	Coefficient in quadratic equation for $\Delta\lambda$, see (2.26) 38
d, d_j	Empirical constants used to model strain rate dependence 37,63
D	Coefficient of viscous damping 42
E	Young's modulus. 24
E_1, E_2	Exact elongational surface strains along η^1 and η^2 coordinate directions. 52

LIST OF SYMBOLS (Cont'd)

		Page
E_θ	Exact elongational surface strain along direction making angle θ with n^1 coordinate curve	53
g	Determinant of surface metric $g_{\alpha\beta}$	35
$g_{\alpha\beta}$	Covariant components of metric for surface ζ distant from middle surface.	35
$g^{\alpha\beta}$	Contravariant components of metric for surface ζ distant from middle surface.	35
$G_{\alpha\beta}$	Initial values of $g_{\alpha\beta}$ at strain locations on bounding surfaces of shell	52
h	Thickness dimension of shell	22
k	Integer specifying layer station, see (2.2).	22
K	Number layers into which shell thickness is divided.	22
l	Number of time steps from initial to current time	30
L	Number of subincrements into which elastic stress increment is divided for the plasticity calculations.	131
m	Integer specifying mesh point along n^1 coordinate curve, also see (6.1) and (6.2).	22
$M^{\alpha\beta}$	Components of stress moment resultant, called the bending resultant tensor	39
n	Integer specifying mesh point along n^2 coordinate curve, also see (6.1) and (6.2).	22
n^i	Cartesian components of unit normal to middle surface at current time t , also see (2.11)	32
n_-^i	Values of N^i components at time t_-	30

LIST OF SYMBOLS (Cont'd)

		Page
$N^{i\alpha}$	Cartesian components of stress resultant tensor	39
p, p_j	Empirical constants used to model strain rate dependence	37,63
P	Pressure acting on shell	32
P^*	Force due to pressure acting on unit element, $\Delta\eta^1\Delta\eta^2$, of material coordinate area, called the augmented pressure.	34
$Q^{\alpha\beta}$	Membrane or tangential components of the stress resultant tensor.	39
t	Current time; $t = l\Delta t$	21
t_-	Time at one increment before current time; $t = (l-1)\Delta t$	30
T	Kinetic energy of shell at current time t	45
T_-	Kinetic energy of shell at time $t - \frac{1}{2}\Delta t$	48
T_+, T_{++}	Kinetic energy of shell at time $t + \frac{1}{2}\Delta t$ and $t + \frac{3}{2}\Delta t$	48,49
T^*	Kinetic energy removed by previous KEA operation.	49
u_α^i	First partial derivatives of Δu^i with respect to η^α	32
$u_{\alpha\beta}^i$	Second partial derivatives of Δu^i with respect to η^α	32
U^i	Total current displacement at initially specified location	53
v	Magnitude of initial normal velocity imparted to shell	28

LIST OF SYMBOLS (Cont'd)

		Page
v^i	Cartesian components of initial normal velocity imparted to shell	28
V	Elastic strain energy.	44
W	Total work performed by pressure on shell up to current time.	46
W_D	Energy dissipated by damping and KEA procedure, called the damping work	48
W_P	Energy dissipated by plastic flow, called the plastic work	46
y^i	Cartesian coordinates of middle surface at current time t	32
y_-^i	Cartesian coordinates of middle surface at time t_-	30
y_α^i	First partial derivatives of y^i with respect to η^α	32
$y_{\alpha\beta}^i$	Second partial derivatives of y^i with respect to η^α	32
α	Coefficient, function of θ , used in computing surface strains.	52
β	Coefficient, function of θ , used in computing surface strains.	52
γ	Shear component of tangential surface strain	52
Γ_0	Mass per unit initial middle surface area; $\Gamma_c = \rho h$	28
$\Gamma_{\alpha\beta}^\gamma$	Christoffel symbol based on middle surface metric, $a_{\alpha\beta}$	34
δ	Coefficient, function of $G_{\alpha\beta}$, used in computing surface strains.	52

LIST OF SYMBOLS (Cont'd)

		Page
$\dot{\epsilon}$	Second invariant of the strain rate deviator	37
ϵ_1, ϵ_2	Elongational components of tangential surface strain	52
$\epsilon_j, \epsilon_j^0 (j=1,2,\dots)$	Strain coordinates on polygonal approximation to strain hardening stress-strain curve where slopes change	63
$\epsilon_{\alpha\beta}$	Covariant components of tangential strain. . .	50
ζ	Normal distance of particles from middle surface.	21
η^a	Material coordinate of particles on middle surface.	21
θ	Angle specifying direction of elongational surface strain relative to η^1 coordinate curve, see Figure 3.5.	50
ν	Poisson's ratio.	24
ρ	Initial mass density	24
τ_0	Yield stress	37
$\sigma_j, \sigma_j^0 (j=1,2,\dots)$	Stress coordinates on polygonal approximation to strain hardening stress-strain curve where slopes change.	63
$\sigma^{\alpha\beta}$	Contravariant components of tangential stress at time t	32
$\sigma_{-}^{\alpha\beta}$	Contravariant components of tangential stress at time t_{-}	30
$\frac{D\alpha}{D\beta}$	Components of plastic flow corrector stress. .	37
$\frac{T\alpha}{T\beta}$	Components of trial stress (assuming stress increment to be elastic)	37

LIST OF SYMBOLS (Cont'd)

		Page
σ_{β}^{α}	Mixed components of tangential stress at time t	38
$\sigma_{-\beta}^{\alpha}$	Mixed components of tangential stress at time t_{-}	35
ϕ	Function specifying von Mises yield surface. .	37
$\Delta a_{\alpha\beta}$	Increment in $a_{\alpha\beta}$ during interval Δt	35
$\Delta b_{\alpha\beta}$	Increment in $b_{\alpha\beta}$ during interval Δt	35
Δn	Normal component of incremental change in n^i during interval Δt	35
Δn_{α}	Tangential components of incremental change in n^i during interval Δt	35
Δt	Finite difference time increment	23
Δt^*	Diminish time increment to prevent instability during KEA procedure	49
Δt_B	Maximum stable time increment for membrane motion of plate.	24
Δt_M	Maximum stable time increment for bending motion of plate.	24
ΔU_n	Correction applied to normal component of Δu_{+}^i one mesh spacing in from clamped boundary	43
Δu^i	Cartesian components of displacement increment undergone by middle surface during interval $\Delta t = t - t_{-}$	30
Δu_{+}^i	Value of Δu^i for time interval following current time t	32
Δu_{+}^i	Value of Δu_{+}^i one mesh spacing in from clamped boundary before application of normal correction ΔU_n	43

LIST OF SYMBOLS (Cont'd)

		Page
ΔW	Increment in W during the interval Δt	45
$\Delta \epsilon_{\alpha\beta}$	Increment in $\epsilon_{\alpha\beta}$ during the interval Δt	35
$\Delta \epsilon_{\beta}^{\alpha}$	Mixed components form of $\Delta \epsilon_{\alpha\beta}$	35
$\Delta \zeta$	Thickness of individual shell layers; $\Delta \zeta = h/k$	22
$\Delta \eta^{\alpha}$	Finite difference increment in material coordinates.	22
$\Delta \lambda$	Parameter measuring amount of plastic flow . .	38
$\Delta \sigma_{\beta}^{\alpha}$	Mixed components of elastic stress increment.	37

1. INTRODUCTION

REPSIL* is a FORTRAN IV computer program developed at the BRL to treat the large transient deformations of anelastic shells under blast loadings. The program uses the finite difference technique to solve the equations governing the motion of thin Kirchhoff shells of negligible rotational inertia. These equations are derived in a recent BRL report [1]**. That report, which also treats more general Kirchhoff shells, constitutes the theoretical documentation for REPSIL. The present report is a companion user's manual.

The equations on which REPSIL is based impose certain restrictions on the types of shells and deformations that can be treated. As already mentioned, only thin Kirchhoff shells of negligible rotational inertia can be treated. Moreover, as presently formulated, the program can only handle shells of uniform thickness having no cutouts, stiffeners, or bifurcations. On the other hand, the formulation does treat the geometric nonlinearities due to large displacements exactly.

Within the limits of the above geometric restrictions, REPSIL can accept a wide variety of initial shell geometries. This is made possible by the finite difference formulation being coded independent of any particular initial geometry. Hence, accommodating any initial geometry simply entails programming it into the initial geometry subroutine according to the instructions contained in Chapter 6. The current version of REPSIL comes with initial geometry subroutines for the rectangular plate, the cylindrical shell and the conical shell; Section 2.3 contains instructions for their use.

The program also can accept arbitrary distributions over the shell surface of initial impulse velocities and time varying pressures. The minor programming necessary to specify such distributions is described in Chapter 6. Presently, initial velocities at mesh points and over rectangular regions of the shell surface can be specified on input cards without need of programming, as detailed in Section 3.2.

REPSIL can simulate a number of boundary conditions along the four edges bounding the shell. Three edges can have any combination of clamped or hinged boundary conditions. The remaining edge and two of the previous edges can be edges constrained to move in symmetry planes of the problem. The boundary conditions are described in detail in Section 3.2, where the procedure for selecting them is given.

* Response of Elastic-Plastic Shells to Impulsive Loadings

** Numbers in brackets correspond to the list of references on page 126.

REPSIL can model a variety of anelastic material responses, although the present version is limited to isotropic materials. Within that limitation, the response can be either completely elastic or elastoplastic. In the elastic range the behavior is linear. The plastic response can be perfect or strain hardening and in either case it can be made strain rate dependent. Section 3.2 gives the procedure for eliciting these material properties.

A useful feature of the code is the automatic determination of an optimum, stable time increment. REPSIL uses an explicit finite difference approximation to the equations of motion; as is well known, such explicit schemes are subject to numerical instability unless the size of the time increment is limited to some maximum increment. The REPSIL program initially computes such a maximum increment based on criteria given in Section 2.2 and then uses this increment or the increment specified by the initial data (see Section 3.2), whichever is smaller, for the (constant) time increment used to generate the solution. Hence, a stable solution is guaranteed.

The program is provided with a damping option that permits the rapid attainment of final deformed configurations by numerically damping out the motion of the shell. Damping is mainly achieved through the artifice of automatically annihilating the kinetic energy whenever it reaches a local maximum. Details on this procedure are given in Section 2.4, while instructions for executing the option are in Section 3.2.

REPSIL comes equipped with a number of print options for outputting results of the calculations at regular intervals. Printed out are such local measures of the deformation as the displacements and surface strains, and such global measures as the kinetic energy, strain energy and plastic work. There is also a companion plotting program that at regular intervals draws isometric and cross-sectional views of the deforming shell and that graphs time histories of the printed measures of deformations just mentioned. The output options are described in Chapter 4 and illustrated by example problems in Chapter 5. Appendix D contains a description and a listing of the plotting program.

This report is organized as follows: Chapter 2 presents the assumptions and equations of the REPSIL formulation and outlines the computational algorithm used to solve the equations; Chapter 3 describes the input data, shows how to initiate and continue a problem and select the various options, and gives rules-of-thumb for estimating the memory and machine time requirements for a problem; Chapter 4 describes the various output data generated by REPSIL and the formats in which they are printed; Chapter 5 gives two example problems, describing how they are set up and the kind of output data generated; Chapter 6 gives instructions for programming arbitrary loadings and initial geometries. Chapter 3, 4 and 5 contain the necessary information to run the program as listed in Appendix E, with the associated loading and initial geometry subroutines. A user desiring to model other loadings or initial geometries

should read Chapter 6 for programming instructions. Such a user would find a reading of Chapter 2 useful as background. On the other hand, the analyst or programmer who intends to make extensive changes in the formulation or numerics of the program would do well to thoroughly study Chapter 2 before proceeding to the study of Appendix E with the aid of Appendix C, the list of program variables.

2. DESCRIPTION OF PROGRAM

2.1 Synopsis of Program

REPSIL treats the transient deformation of shells by approximating the initial value problem with an explicit nonlinear finite difference scheme. The finite difference equations are solved at each time increment to update the deformation variables. This procedure is repeated cyclically in order to generate the time history of the deformation. This chapter presents the equations of the REPSIL formulation and describes the computational algorithm by which REPSIL solves them.

The formulation is based on equations that use a material or Lagrangian description: the dependent variables of the theory are functions of η^α *, the material coordinates of the middle surface of the shell; moreover variables defined outside the middle surface also depend upon ζ , the normal distance from the middle surface**. All variables are functions of t , the time.

Some comments on the physical significance of the material coordinates of the middle surface are in order. For simple initial geometries, the middle surface material coordinates often have significance as distances, arc lengths, or angles***; however, this need not be the case, not even for simple geometries. It is better to regard the material coordinates abstractly as a pair of parameters defining the position of the middle surface in space. In other words, the material coordinates serve as parameters for the parametric representation of the middle surface in space. That the parameters η^α are also material coordinates simply means that as the image of the middle surface changes in space as a function of time, the pair of values η^α associated with a given material particle on the middle surface does not change, but remains fixed.

* Index notation is used, with Greek indices ranging over the integers 1 and 2; hence, $\eta^\alpha \equiv \eta^1, \eta^2$

** The concept of material coordinates for a shell is defined more precisely in [1].

*** For example, see the cases of the flat plate, cylindrical shell and conical shell in Section 3.2.

The finite difference scheme is obtained by making the dependent variables of the theory discrete functions of η^1 , η^2 , ζ , t . Discretizing the dependence on η^α makes the dependent variables functions of the intersection points of the two dimensional rectangular mesh resulting from the division of the domain of η^α into increments (see Figure 3.1 and 6.1); more precisely, the variables become functions of the mesh number pair (m,n) , the integer pair corresponding to the mesh intersection whose coordinates η^α satisfy the relations

$$\eta^1 = m \Delta\eta^1 + \eta_0^1, \quad \eta^2 = n \Delta\eta^2 + \eta_0^2, \quad (2.1)$$

with $\Delta\eta^\alpha$ constant increments of the coordinates and η_0^α the coordinates of a conveniently chosen origin.

The dependence on ζ is made discrete by conceptually dividing the shell into K layer of equal thickness. Within each layer those variables defined outside the middle surface are assumed constant with respect to ζ . Numbering each layer in sequence, $k = 1, \dots, K$, the variables become functions of the layer number k , as well as functions of (m,n) . When needed, the value of ζ in each layer is taken to be its mid-layer value:

$$\zeta = \left(k - \frac{K+1}{2}\right) \frac{h}{K}; \quad k = 1, \dots, K, \quad (2.2)$$

where h is the thickness of the shell.

The principal reason for making the dependence on ζ discrete is in order to model the variations of the stresses through the thickness. For, while the functional form of the dependence of the strains on ζ is prescribed by the Kirchhoff hypothesis *, no such simplification is possible for the stresses. Plasticity makes the *a priori* determination of the dependence of the stresses on ζ not feasible.

Strain hardening behavior is modelled using the "mechanical sublayer model" **: the stress tensor at a point is assumed to be a weighted sum of J "sublayer" stress tensors or, more simply, J "substress" tensors, with each sublayer obeying the same linear incremental stress-strain relation, but having different yield stresses. Using this model, there will be K stress tensor values at each mesh point (m,n) and $J \times K$ substress tensor values.

* Cf equation (2.15) of this report.

** This method of modelling strain hardening behavior has been developed and extensively used at the Aeroelastic and Structure Research Lab. of MIT. A detailed description of the method is given in [2; Sect. 5.4.2].

The dependence on t , the time, is made discrete by replacing time derivatives by equivalent finite difference operators. As already mentioned, an explicit finite difference scheme results. The scheme characterizes the change undergone by the dependent variables during a time interval Δt . A constant time interval is used, which is automatically determined by the program to assure numerical stability.

The basic function of the finite difference scheme is to advance the values of the following fundamental variables.

- $y^i(m,n)^*$, the Cartesian coordinates of the middle surface at mesh point (m,n) ;
- $n^i(m,n)$, the Cartesian components of the unit normal to the middle surface at mesh point (m,n) ;
- $\Delta u^i(m,n)$, the Cartesian components of the displacement increment undergone by the middle surface at mesh point (m,n) during the time interval Δt ;
- $\sigma^{a\beta}(m,n,k)$, the contravariant components of the stress tensor (or, in the case of strain hardening, the substress tensor) tangential to the middle surface at mesh point (m,n) at layer (or sublayer) station k .

The dependence of the variables on (m,n) or (m,n,k) is indicated explicitly to emphasize that their values are stored as 2- and 3-dimensional arrays in the program. From these fundamental variables all the other dependent variables of the theory are determined.

For the sake of clarity, the description of the computational algorithm used by REPSIL is separated into four groups of calculations:

- Initialization calculations,
- Finite difference calculations,
- Energy calculations,
- Surface strains calculations.

Within each group the description is organized by units of calculations. Units can occur within the main program or as subroutines. In the latter case, the subroutine description, which follows the subroutine name, is indented. When a subroutine occurs within a subroutine, the description

*Latin indices indicate Cartesian components in Euclidean 3-space and range over the integers 1, 2, and 3.

of the embedded subroutine is further indented. All subroutine names are completely capitalized.

2.2 Initialization Calculations

The initialization calculations determine the program constants and the initial values of the dependent variables from the input data. The flow chart in Figure 2.1 summarizes the calculations, whose description follows.

START Figure 2.2 outlines this subroutine. The input data, including material properties, is read into storage and program constants are calculated. The minimum and maximum values of the mesh numbers are determined from the boundary conditions. Subroutine INGEOM is called within this subroutine.

INGEOM This subroutine generates the initial geometry of the shell. First, the dimensions characterizing the particular geometry being treated are read off the input cards. Next, the finite difference increments $\Delta\eta^a$ are calculated. Finally, the subroutine computes the initial Cartesian coordinates $y^1(m,n)$ of the middle surface and stores the resulting arrays. The user not finding a suitable geometry among those described in Section 3.2 (flat plate, cylinder or cone) is referred to Section 6.4 for instructions on programming initial geometries.

Returning to **START**, the program calculates the interpolation coefficients at locations where **STRAIN** * computes the surface strains and the mesh number of the mesh point bracketing these locations. Next, a stable time interval Δt is found from the criteria

$$\Delta t_M = \frac{2}{c} \left[\frac{1}{(\Delta\eta^1)^2} + \frac{1}{(\Delta\eta^2)^2} \right]^{-\frac{1}{2}}, \quad c = \sqrt{\frac{E}{\rho(1-\nu^2)}}; \quad (2.3)$$

$$\Delta t_B = \frac{1}{2hc} \left[\frac{1}{12} \left(1 - \frac{1}{K^2} \right) \left(\frac{1}{(\Delta\eta^1)^4} + \frac{1}{(\Delta\eta^2)^4} + \frac{1+15\nu}{8(\Delta\eta^1\Delta\eta^2)^2} \right) \right]^{-\frac{1}{2}}.$$

These criteria result from a von Neumann stability analysis of an elastic flat plate **; Δt_M is the maximum stable interval for membrane motion and Δt_B for bending motion. The program

* Subroutine **STRAIN** is described in Section 2.5.

** A report describing this analysis is being prepared.

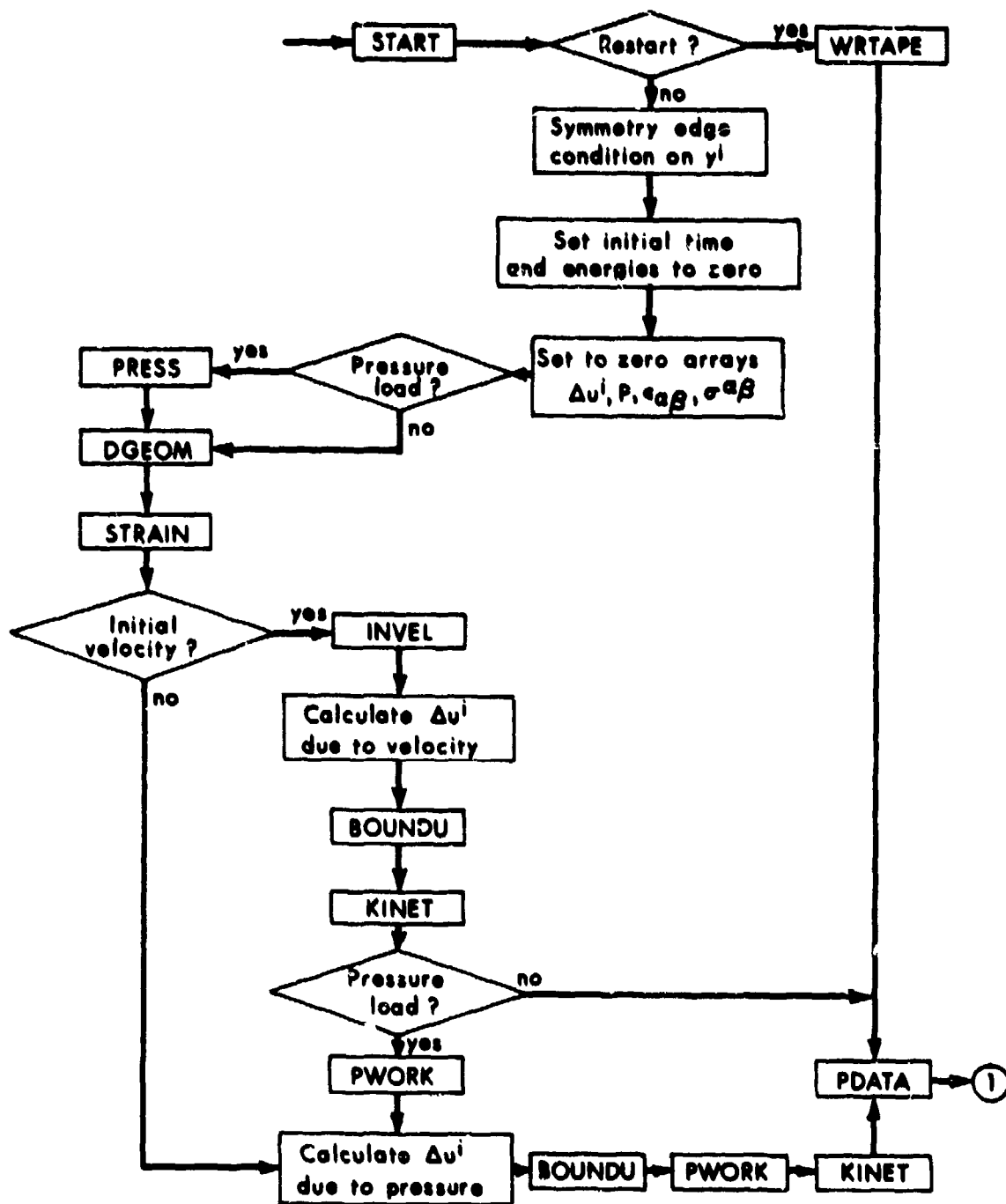


Figure 2.1 Flow Chart for Initialization Calculations

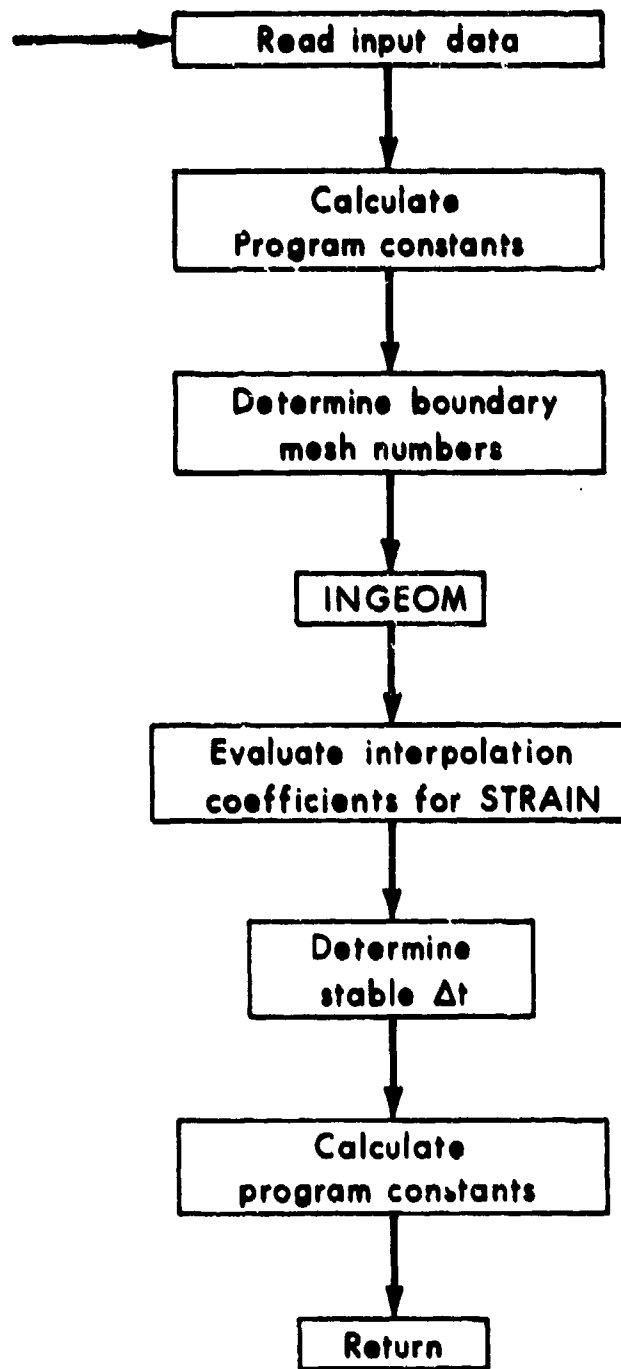


Figure 2.2 Flow Chart for Subroutine START

selects for use in the finite difference calculations the minimum of Δt_M , Δt_B and the Δt prescribed in the input data.

The subroutine ends with the calculation of the remaining program constants, most of which require for their determination the values of Δn^a computed in INGEOM.

The main program next determines whether the problem being solved is a new one, starting from an initial (stress-free) configuration at time $t_0 = 0$ or a continuation of a problem whose solution up to some time $t_1 > 0$ has already been calculated, called a restart problem or, simply, a restart. If it is a restart, subroutine WRTAPE is called in order to read data off the restart tape, after which subroutine PDATA is called and the initialization calculations end (see Figure 2.1).

WRTAPE Depending on the instructions of the main program, this subroutine either reads off or writes on a tape, the restart tape, the values of the fundamental variables y^i , n^i , Δu^i and σ^{ab} , the surface strains and the energy variables at time step prescribed by the initial data. This information is sufficient to permit the program to continue the solution of the problem from any of the prescribed time steps. During the initialization portion of the program this subroutine reads this information off the tape and during the finite difference portion it stores the information.

PDATA This subroutine stores on a tape, the plotting tape, the data required by the REPSIL plotting program. A description of these data are given in Section 4.2. Appendix D describes and lists the plotting program.

If, on the other hand, the program is starting a new problem, WRTAPE is not called at all and PDATA is not called until later (see Figure 2.1). Rather, for a new problem the program proceeds to generate the information otherwise obtained from WRTAPE.

As discussed in Section 3.2, the program treats symmetric deformations about one or two planes. For such deformations, the shell variables, in particular the middle surface coordinates y^i , are symmetrically distributed about symmetry planes. The program imposes this condition numerically by relating the coordinates one mesh spacing outside the symmetry edge to the coordinates one mesh spacing inside the edge. Typically, for the symmetry plane located in the y^2 , y^3 coordinate plane and intersecting the middle surface along the curve with mesh number m (see Figures 3.6 - 3.8 where $m = 2$) these relations are

$$\begin{aligned}
y^1(m-1,n) &= -y^1(m+1,n) \\
y^2(m-1,n) &= y^2(m+1,n) \\
y^3(m-1,n) &= y^3(m+1,n) \\
y^1(m,n) &= 0, \qquad (2.4)
\end{aligned}$$

for all allowable n .

The initial time t_0 , kinetic energy T , strain energy V , damping work W_D and external work W are next set equal to zero. This is followed by the arrays for the displacement increments Δu^i , the pressure P , the tangential strain components $\epsilon_{\alpha\beta}$ and the tangential stress components all being cleared to zero. If pressure loads are acting on the shell, the initial pressure distribution is obtained from subroutine PRESS, whose description is postponed until Section 2.3. If no pressure loads are acting, the program skips PRESS and goes directly to DGEOM. DGEOM is described in Section 2.3, where it is shown that at this point in the program it is called in order to calculate the normal n^i , the augmented pressure P^* , the time constant $\Delta t^2 / (a_0^{1/2} \Gamma_0)$ and the initial values of the tensor $a_{\alpha\beta}$ and $b_{\alpha\beta}$. Next the program goes to STRAIN where the values of $a_{\alpha\beta}$ and $b_{\alpha\beta}$ are interpolated to give the metric tensor $G_{\alpha\beta}$ on the bounding surfaces of the shell; the details of this calculation are given in Section 2.5.

The remainder of the initialization calculations are concerned with obtaining the initial values of the displacement increment arrays $\Delta u^i(m,n)$ and the associated values of the kinetic energy T and external work W . The initial displacement increments can arise in three ways: (1) from an initial impulse velocity distribution; (2) from an initial pressure distribution or (3) from a combination of both. Should they be due to an initial velocity distribution or combination, the program calls INVEL.

INVEL This subroutine determines the magnitude of the initial velocity v at each mesh point and multiplies this by the normal $n^i(m,n)$ to give the initial velocity distribution:

$$v^i(m,n) = v n^i(m,n). \qquad (2.5)$$

The initial velocity magnitudes can either be read off input data cards, as shown in Section 3.2, or the user can program an analytical expression for these magnitudes, as shown in Section 6.2.

The program then calculates the displacement increments due to the initial velocity distribution

$$\Delta u^i(m,n) = v^i(m,n) \Delta t \quad (2.6)$$

and proceeds to subroutine BOUNDU. This subroutine, described in Section 2.3, adjusts the displacement increments one mesh space in from clamped edge boundaries so that clamped edge conditions are satisfied and also generates the displacement increment one mesh spacing outside symmetry boundaries from the values one mesh spacing inside, using equations for Δu^i very much like (2.3) for y^i . Next the kinetic energy T associated with the initial displacement increment distribution is determined in subroutine KINET, described in Section 2.4. If there is no initial pressure distribution, the displacement increments $\Delta u^i(m,n)$ and kinetic energy just determined are associated with the time interval from $t_0 = 0$ to $t_1 = \Delta t$ and the initialization calculations terminate with the calling of subroutine PDATA. On the other hand, if the shell is also subjected to an initial pressure distribution, the displacement increments $\Delta u^i(m,n)$ and kinetic energy T just determined are associated with the time interval from $t_{-1} = -\Delta t$ to $t_0 = 0$ * and the external work due to the pressure distribution acting on the displacement increments just determined is computed in subroutine PWORK, described in Section 2.4. The program then proceeds to calculate the displacement increments for the time interval $[t_0, t_1]$ using an equation which is nothing more than the equation of motion (2.35) with the stresses set equal to zero:

$$\Delta u_+^i = \Delta u^i - \frac{\Delta t^2}{r_0^2 \Gamma_0} P^* n^i, \quad (2.7)$$

with Δu_+^i and Δu^i the displacement increments for the time intervals $[t_0, t_1]$ and $[t_{-1}, t_0]$, respectively. When there is no initial impulse velocity distribution, but only a pressure distribution the program goes from STRAIN to directly computing the displacement increments for the interval $[t_0, t_1]$ using (2.7), with, however, the displacement

* The reason that the displacement increments were previously associated with the time interval $[t_0, t_1]$ rather than the interval $[t_{-1}, t_0]$ is that with no pressure acting and the shell being stress free at the time t_0 , the displacement increments are the same for both intervals. On the other hand, a pressure distribution at time t_0 will cause the displacement increments for $[t_0, t_1]$ to change from those for $[t_{-1}, t_0]$.

increments for $[t_{-1}, t_0]$ set equal to zero.

Again, the new Δu^i are adjusted in BOUNDU to satisfy clamped edge and symmetry edge conditions and the additional external work resulting for the initial pressure distribution acting on the displacement increments for the interval $[t_0, t_1]$ is computed in PWORK. Lastly, the kinetic energy during the time interval $[t_0, t_1]$ is calculated in KINET and the program proceed to PDATA where information is gathered for the plotting program. With this last operation the initialization calculation end and the program proceeds to the finite difference calculational loop.

2.3 Finite Difference Calculations

The finite difference calculations follow the initialization calculations and are repeated each time step. They solve the finite difference equations for the current values of the fundamental variables. In this way, the values of these variables are advanced each time step and the history of the deformation is generated.

Aside from minor simplifications in notation, the finite difference equations presented in this section are identical with those given in [1; Sect. 7.3]. They are written in compact form with only finite time derivatives being shown explicitly; finite differences with respect to material coordinates η^α are symbolically represented by their corresponding partials. At interior points and along symmetry boundaries the program uses central difference operators, while along hinged and clamped boundaries it uses forward or backward difference operator, all operators being of order $|\Delta\eta^\alpha|^2$ accuracy *.

Figure 2.3 outlines the finite difference calculations. They comprise a sequence or, better still, a loop of calculations, which, as already mentioned, are repeated every time step. The description of this loop begins after $l-1$ time steps have elapsed, at the generic time $t_- = (l-1) \Delta t$. The values of the middle surface coordinates, the normal to the middle surface and the tangential stress or substress component are assumed known at this time and are denoted by appending subscripted minus signs: y_-^i , n_-^i and $\sigma_-^{\alpha\beta}$. Also the components of the displacement increment for the next time interval, from the time $t_- = (l-1) \Delta t$ to the time $t = l\Delta t$, are known and are denoted simply as Δu^i . Using these values of the fundamental variables, the finite difference calculations generate succeeding values of the fundamental

* Appendix A summarizes the explicit forms of these operators.

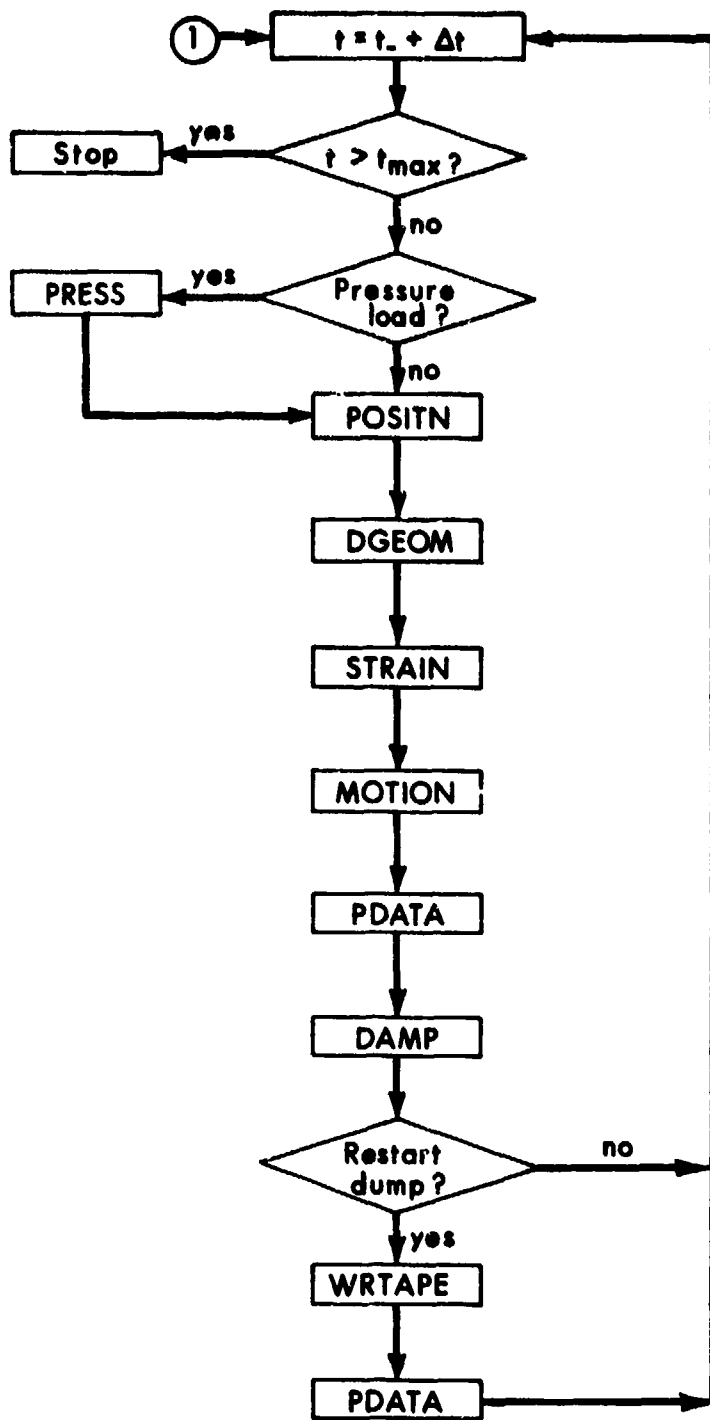


Figure 2.3 Flow Chart for Finite Difference, Energy and Surface Strain Calculations

variable y^i , n^i , $\sigma^{\alpha\beta}$ and Δu^i at the time $t = \ell\Delta t$ as follows.

First the time is updated:

$$t = t_{-} + \Delta t, \quad (2.8)$$

and checked against a prescribed maximum, t_{\max} . If the maximum is exceeded, then the solution has progressed to completion and calculations stop. Otherwise, the program checks as to whether there is a pressure loading acting on the shell at this time; if there is then the program calls subroutines PRESS, POSITN, DGEOM, STRAIN, MOTION, PDATA and DAMP in that order; if not, it skips PRESS and calls the remainder of the sequence.

PRESS This subroutine supplies the values of pressure at mesh intersection points for the given time and stores them in the array $P(m,n)$. The user is expected to supply this information either by programming some analytical expression for the pressure, as outlined in Section 6.3, or by generating a tape with this data in numerical form.

POSITN This subroutine simply calculates the coordinates of the middle surface at the current time t using the formula

$$y^i = y_{-}^i + \Delta u^i, \quad (2.9)$$

storing the values in the arrays $y^i(m,n)$.

DGEOM This subroutine is summarized in Figure 2.4. It begins by calling subroutine GRAD.

GRAD This subroutine, using appropriate finite difference operators, determines the first and second gradients of y^i and Δu^i with respect to the material coordinates η^α :

$$\begin{aligned} y_{\alpha}^i &= \frac{\partial y^i}{\partial \eta^\alpha}, & y_{\alpha\beta}^i &= \frac{\partial^2 y^i}{\partial \eta^\alpha \partial \eta^\beta}, \\ u_{\alpha}^i &= \frac{\partial \Delta u^i}{\partial \eta^\alpha}, & u_{\alpha\beta}^i &= \frac{\partial^2 \Delta u^i}{\partial \eta^\alpha \partial \eta^\beta}. \end{aligned} \quad (2.10)$$

After GRAD the program stores the preceding values of the components of the normal n_{-}^i for later use in calculating strain increments. From the first and second gradient of y^i , the program determines the differential geometric quantities characterizing

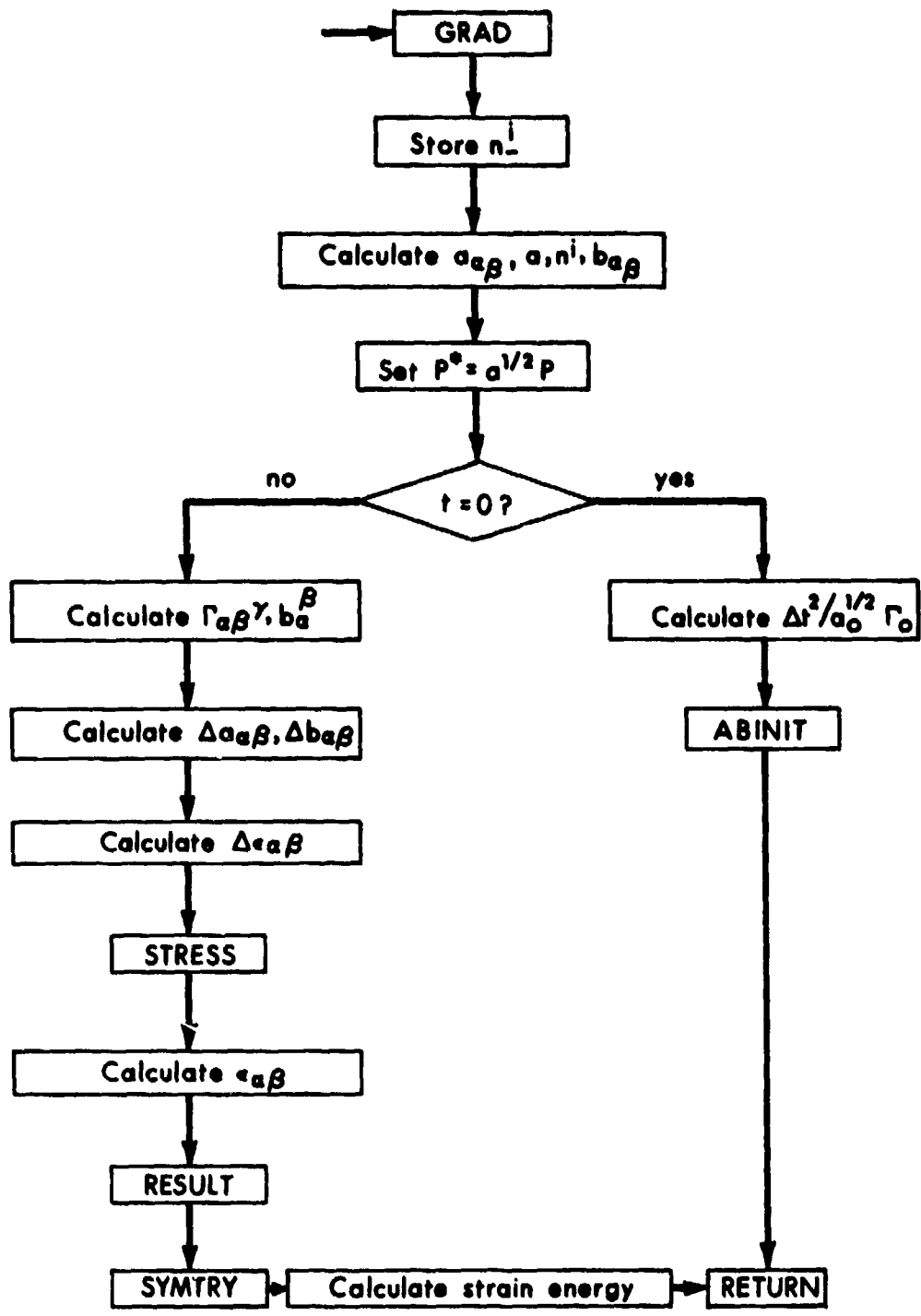


Figure 2.4 Flow Chart for Subroutine DGEOM

the current position of the middle surface: the covariant components of the metric $a_{\alpha\beta}$, the second fundamental tensor $b_{\alpha\beta}$ and the current normal n^i , and also the determinant a of the metric as follows.

$$a_{\alpha\beta} = y_{\alpha}^i y_{\beta}^{i*}, \quad a = a_{11} a_{22} - (a_{12})^2, \quad (2.11)$$

$$n^i = \epsilon^{ijk} y_1^j y_2^k / a^{1/2}, \quad b_{\alpha\beta} = n^i y_{\alpha\beta}^i,$$

where ϵ^{ijk} is the permutation symbol.** Next, the pressure distribution is modified for use in the equation of motion:

$$p^* = a^{1/2} p. \quad (2.12)$$

The remainder of the subroutine's calculations depend on the time. Since initially the shell is assumed to be in a stress-free state, there is no need to calculate the stress field in DGEOM. Rather, the subroutine determines certain time constants and stores these for later use: $\Delta t^2 / (a_0^{1/2} \Gamma_0)$ is calculated for use in the equations of motion and the expression for the kinetic energy and the initial values of $a_{\alpha\beta}$ and $b_{\alpha\beta}$ at prescribed mesh points are selected using subroutine ABINIT for use in STRAIN, as described in Section 2.5. At all subsequent times, the subroutine calculates the stress field and other stress related quantities as follows. First, the contravariant components $a^{\alpha\beta}$ of the metric, the Christoffel symbols $\Gamma_{\alpha\beta}^{\gamma}$ and the mixed components b_{β}^{α} of the second fundamental tensor are calculated:

$$a^{11} = a_{22}/a, \quad a^{12} = -a_{12}/a, \quad a^{22} = a_{11}/a, \quad (2.13)$$

$$\Gamma_{\alpha\beta}^{\gamma} = a^{\gamma\delta} y_{\delta}^i y_{\alpha\beta}^i, \quad b_{\beta}^{\alpha} = a^{\alpha\delta} b_{\delta\beta}$$

* We introduce the summation convention: terms or products of terms having the same index appearing twice are to be summed over the range of the index. In the case of repeated Latin indices both will be superscript since their basis is Cartesian, while repeated Greek indices will always appear as paired superscript and subscript.

** That is, $\epsilon^{ijk} = 1$ for i, j, k an even permutation of 1,2,3; $= -1$ for i, j, k an odd permutation; $= 0$ for i, j, k non-distinct.

Next the incremental changes in $a_{\alpha\beta}$ and $b_{\alpha\beta}$ due to the incremental displacement Δu^i are determined:

$$\Delta n_{\alpha} = -n_{\alpha}^i u_{\alpha}^i, \quad \Delta n = \frac{a^{\alpha\beta} \Delta n_{\alpha} \Delta n_{\beta}}{1 + n_{\alpha}^i n_{\alpha}^i},$$

$$\Delta a_{\alpha\beta} = \gamma_{\alpha}^i u_{\beta}^i + \gamma_{\beta}^i u_{\alpha}^i - u_{\alpha}^i u_{\beta}^i, \quad (2.14)$$

$$\Delta b_{\alpha\beta} = n_{\alpha\beta}^i u_{\alpha\beta}^i + \Gamma_{\alpha\beta}^{\gamma} \Delta n_{\gamma} + b_{\alpha\beta} \Delta n.$$

These expressions are exact with no approximations based on the smallness of Δu^i being used. The corresponding increments undergone by the tangential strain components $\epsilon_{\alpha\beta}$ at the station $\zeta(k)$ distant from the middle surface are then calculated using the equation

$$\Delta \epsilon_{\alpha\beta} = \frac{1}{2} \Delta a_{\alpha\beta} - \zeta \Delta b_{\alpha\beta} \quad (2.15)$$

This equation is based on the thin shell approximation *. Subroutine STRESS follows.

STRESS The principal function of this subroutine, outlined in Figure 2.5, is to calculate the current stress or sub-stress component from their preceding values and the incremental change in the strain component. Calculations begin by evaluating the metric $g_{\alpha\beta}$ and inverse $g^{\alpha\beta}$ for the lamella ζ distance from the middle surface:

$$g_{\alpha\beta} = a_{\alpha\beta} - 2\zeta b_{\alpha\beta}, \quad g = g_{11} g_{22} - (g_{12})^2, \quad (2.16)$$

$$g^{11} = g_{22}/g, \quad g^{12} = -g_{12}/g, \quad g^{22} = g_{11}/g.$$

With these terms evaluated, the mixed components of the incremental strain and preceding stress are immediately obtained:

$$\Delta \epsilon_{\beta}^{\alpha} = g^{\alpha\delta} \Delta \epsilon_{\delta\beta}, \quad \sigma_{-\beta}^{\alpha} = g_{\beta\delta} \sigma^{\delta\alpha} \quad (2.17)$$

Next, assuming that the incremental deformation is elastic, the increments in the stress components are calculated using the linear isotropic law:

* Cf [1; Sect. 7.1].

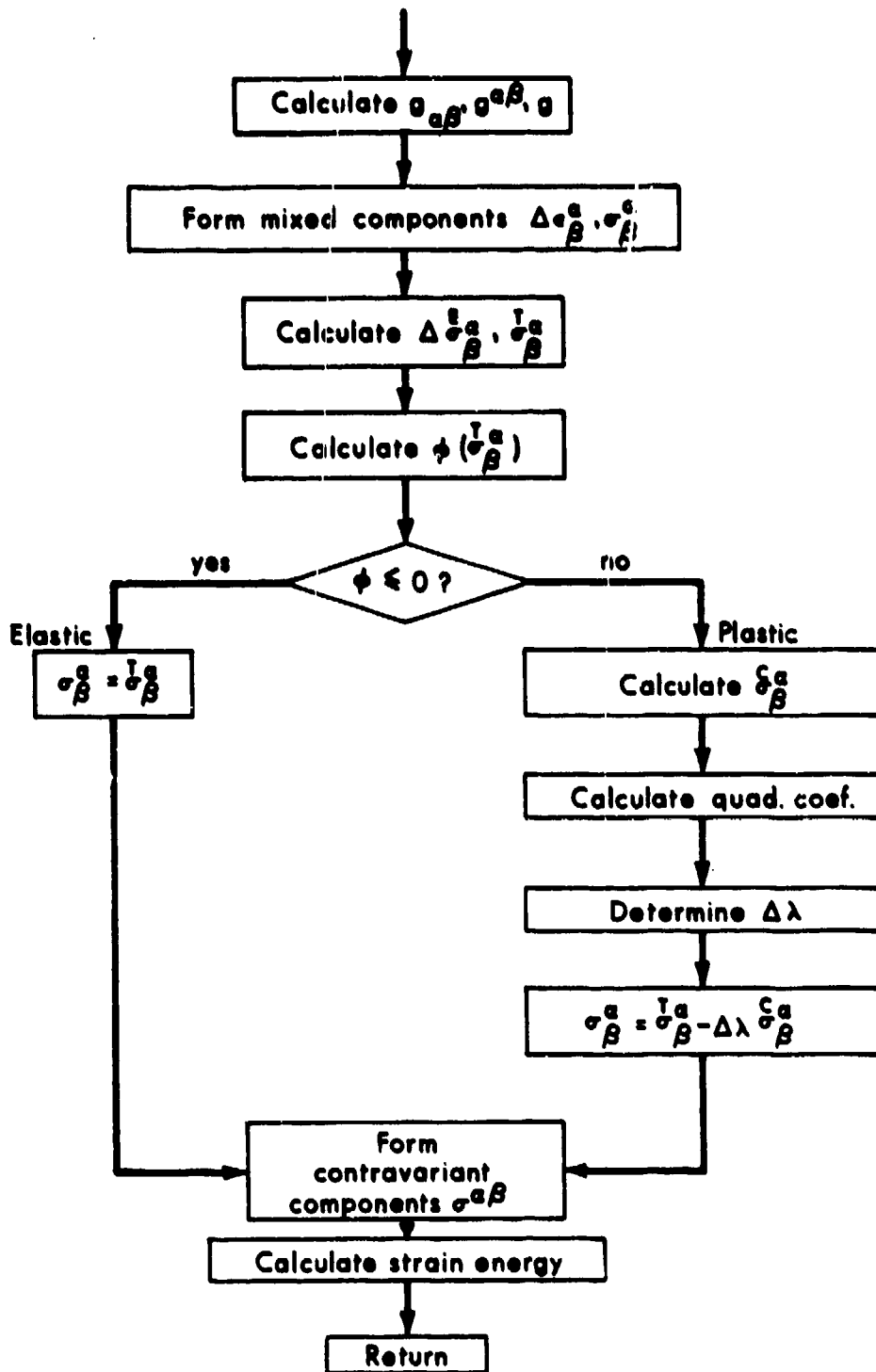


Figure 2.5 Flow Chart for Subroutine STRESS

$$\Delta \sigma_{\beta}^{\alpha} = \frac{E}{1+\nu} (\Delta \epsilon_{\beta}^{\alpha} + \frac{\nu}{1+\nu} \delta_{\beta}^{\alpha} \Delta \epsilon_{\gamma}^{\gamma}) . \quad (2.18)$$

In the case of strain hardening, $\Delta \sigma_{\beta}^{\alpha}$ are elastic increments in the substress components and are the same for each layer. These increments are added to the preceding values to give the components of a trial stress or substress

$$\sigma_{\beta}^{\alpha} = \sigma_{-\beta}^{\alpha} + \Delta \sigma_{\beta}^{\alpha} . \quad (2.19)$$

The trial stress is tested against the yield function

$$\phi(\sigma_{\beta}^{\alpha}) \equiv \frac{3}{2} \frac{\sigma_{\beta}^{\alpha} \sigma_{\alpha}^{\beta}}{\sigma_{\alpha}^{\alpha}} - \frac{1}{2} (\sigma_{\alpha}^{\alpha})^2 - \sigma_0^2 , \quad (2.20)$$

where σ_0 is the uniaxial yield stress or, in the case of strain hardening, the yield substress; for rate sensitive behavior σ_0 is assumed to depend on the second invariant of the strain rate deviator:

$$\sigma_0 = \sigma_0(\text{static}) \left[1 + \left(\frac{\dot{\epsilon}}{D} \right)^{\frac{1}{p}} \right] , \quad (2.21)$$

with

$$\dot{\epsilon} = \frac{1}{\Delta t} \left[\frac{3}{2} \Delta \epsilon_{\beta}^{\alpha} \Delta \epsilon_{\alpha}^{\beta} - \frac{1}{2} (\Delta \epsilon_{\gamma}^{\gamma})^2 \right]^{\frac{1}{2}} . \quad (2.22)$$

If $\phi(\sigma_{\beta}^{\alpha}) \leq 0$, then the trial stress is on or within the yield surface and its components define acceptable values for the current stress components $\sigma_{\beta}^{\alpha} = \frac{\sigma_{\beta}^{\alpha}}{\sigma_{\beta}^{\beta}}$.

Hence, the stress increment is indeed elastic and the calculations for a plastic stress increment are skipped, the program proceeding directly to (2.29) below.

On the other hand, if $\phi(\sigma_{\beta}^{\alpha}) > 0$, then the trial stress is outside the yield surface and, hence, unacceptable. In this case, the subroutine determines a correction to the trial stress due to plastic flow. First, the components of a corrector stress are determined:

$$\Delta \sigma_{\beta}^{\alpha} = 3 (1-\nu) \sigma_{-\beta}^{\alpha} - (1-2\nu) \sigma_{-\gamma}^{\gamma} \delta_{\beta}^{\alpha} \quad (2.23)$$

The corrector stress gives the direction in stress space in which to apply a correction to T_{α}^{β} in order to bring the resultant stress components

$$\sigma_{\beta}^{\alpha} = T_{\alpha}^{\beta} - \Delta\lambda \frac{D_{\alpha}}{\sigma_{\alpha}} \quad (2.24)$$

back to the yield surface, so that

$$\phi(\sigma_{\beta}^{\alpha} - \Delta\lambda \frac{D_{\alpha}}{\sigma_{\alpha}}) = 0. \quad (2.25)$$

This gives a quadratic equation in $\Delta\lambda$

$$A \Delta\lambda^2 - 2 B \Delta\lambda + C = 0, \quad (2.26)$$

where

$$A = \frac{D_{\alpha}}{\sigma_{\beta}} \frac{D_{\beta}}{\sigma_{\alpha}} - \frac{1}{3} \frac{D_{\alpha}}{\sigma_{\alpha}} \frac{D_{\beta}}{\sigma_{\beta}}$$

$$B = \frac{D_{\alpha}}{\sigma_{\beta}} \frac{T_{\beta}}{\sigma_{\alpha}} - \frac{1}{3} \frac{D_{\alpha}}{\sigma_{\alpha}} \frac{T_{\beta}}{\sigma_{\beta}} \quad (2.27)$$

$$C = \frac{T_{\alpha}}{\sigma_{\beta}} \frac{T_{\beta}}{\sigma_{\alpha}} - \frac{1}{3} \frac{T_{\alpha}}{\sigma_{\alpha}} \frac{T_{\beta}}{\sigma_{\beta}} - \frac{2}{3} \sigma_0^2,$$

which the program solves for the smallest positive value of $\Delta\lambda$:

$$\Delta\lambda = B - (B^2 - AC)^{1/2} \quad (2.28)$$

With $\Delta\lambda$ known, current stress or substress components are determined from (2.24) and are then put into contravariant form

$$\sigma^{\alpha\beta} = g^{\alpha\delta} \sigma_{\delta}^{\beta}. \quad (2.29)$$

* If $\Delta\lambda$ turns out to be negative or complex, then the subroutine uses a procedure described in [3; Sect. IV], which divides the elastic increment components $\Delta\sigma_{\beta}^{\alpha}$ into L elastic subincrements and applies a correction to each step such that $\Delta\lambda$ is always positive.

In the case of strain hardening, a weighted sum of the substress components of the layer is taken to obtain the layer stress components before being put in contravariant form. The subroutine finishes by computing the contribution to the strain energy of the current stress using (2.39) of Section 2.4 and returning to DGEOM.

Next, the components of strain at prescribed locations on the bounding surfaces of the shell are evaluated using (2.55) as explained in Section 2.5. Subroutines RESULT and SYMTRY are called next.

RESULT This subroutine numerically integrates the stress components (note: not the substress components) and their moments through the thickness to give components of the membrane and bending resultants:

$$Q^{\alpha\beta} = a^{1/2} \sum_k \sigma^{\alpha\beta} (1-\zeta b_Y^Y) \Delta\zeta ,$$

$$M^{\alpha\beta} = a^{1/2} \sum_k [\sigma^{\alpha\beta} (1-\zeta b_Y^Y) - \frac{\zeta}{2} (\sigma^{\alpha\delta} b_\delta^\beta + \sigma^{\beta\delta} b_\delta^\alpha)] \zeta \Delta\zeta . \quad (2.30)$$

From these components the subroutine determines the components of the stress resultant:

$$N^{i\alpha} = Q^{\alpha\beta} y_B^i + \Gamma_{\beta\gamma}^{\alpha} M^{\beta\gamma} n^i . \quad (2.31)$$

SYMTRY This subroutine imposes the symmetry edge conditions on n^i , $M^{\alpha\beta}$ and $N^{i\alpha}$ relating the values of these variables one mesh spacing outside the symmetry edge to their values one mesh spacing inside. Typically, for the symmetry plane located in the y^2 , y^3 coordinate plane and intersecting the middle surface along the m equals a constant curve these relations are:

$$n^1(m-1,n) = -n^1(m+1,n) ,$$

$$n^2(m-1,n) = n^2(m+1,n) , \quad (2.32)$$

$$n^3(m-1,n) = n^3(m+1,n) ,$$

$$\begin{aligned}
M^{11}(m-1,n) &= M^{11}(m+1,n) , \\
M^{12}(m-1,n) &= -M^{12}(m+1,n) , \\
M^{22}(m-1,n) &= M^{22}(m+1,n) ,
\end{aligned}
\tag{2.33}$$

$$\begin{aligned}
N^{11}(m-1,n) &= N^{11}(m+1,n) , \\
N^{21}(m-1,n) &= -N^{21}(m+1,n) , \\
N^{31}(m-1,n) &= -N^{31}(m+1,n) , \\
N^{12}(m-1,n) &= -N^{12}(m+1,n) , \\
N^{22}(m-1,n) &= N^{22}(m+1,n) , \\
N^{32}(m-1,n) &= N^{32}(m+1,n) ,
\end{aligned}
\tag{2.34}$$

for all admissible n . These relations as well as relations for other symmetry plane locations used in the program are derived in [1; Sect. 6.4 and App. B].

Returning to DGEOM, the program determines the strain energy by summing the contribution of each mesh point and layer as described in Section 2.5. This ends DGEOM and calculations return to the main program.

Next, the strain components at selected points on the bounding surfaces are determined in subroutine STRAIN, as described in Section 2.5, after which subroutine MOTION is called.

MOTION This subroutine is summarized in Figure 2.6. Its principal function is to determine the values Δu_{\pm}^i of the components of the displacement increments undergone by the middle surface in the time interval $[t, t + \Delta t]$. First a check is performed to determine whether a pressure distribution is currently acting on the shell. If a pressure distribution is acting, then subroutine PWORK, which is described in Section 2.4, is called in order to determine the contribution to the external work of the pressure acting through the displacement increments Δu_{\pm}^i of the time interval $[t - \Delta t, t]$. With no pressure loads acting, the subroutine skips PWORK and proceeds directly to determine Δu_{\pm}^i from the finite difference form of the equations of motion

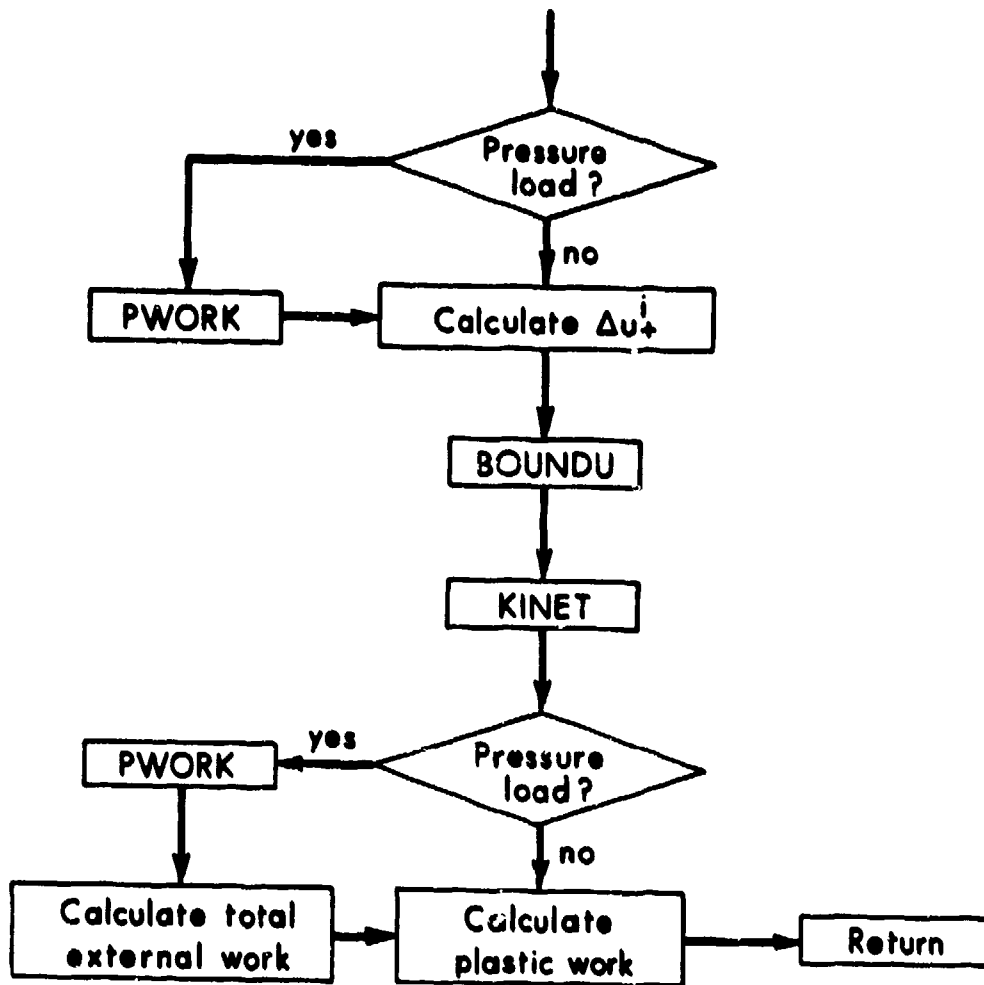


Figure 2.6 Flow Chart for Subroutine MOTION

$$\begin{aligned}
& a_0^{1/2} \left[\Gamma_0 \frac{\Delta u_+^i - \Delta u^i}{\Delta t^2} + D \frac{\Delta u_+^i + \Delta u^i}{2 \Delta t} \right] \\
& = \frac{\rho^2 (M^{\alpha\beta} n^i)}{\partial \eta^\alpha \partial \eta^\beta} + \frac{\rho N^{i\alpha}}{\partial \eta^\alpha} - p^* n^i .
\end{aligned} \tag{2.35}$$

As indicated earlier, the partials appearing in the right hand side of these equations represent finite difference derivatives. These equations do not coincide with those given in [1; Eq. (7.38)]

due to the addition of the term $D \frac{\Delta u_+^i + \Delta u^i}{2 \Delta t}$, representing a linear viscous damping effect. However, this damping term only appears when the damping option is used otherwise, the term is set equal to zero. Next, subroutine BOUNDU is called.

BOUNDU This subroutine generates the additional values of Δu_+^i need along symmetry edges and modifies the values of Δu_+^i along clamped edges so that clamped edge conditions are satisfied. As already described in subroutine SYMTRY, the values of variables one mesh spacing outside a symmetry edge are related to their values one mesh spacing in. The conditions imposed on Δu_+^i are similar to those imposed on y^i in (2.4): for the symmetry plane located in the y^2, y^3 coordinate plane intersecting the middle surface along the m equals a constant curve

$$\begin{aligned}
\Delta u_+^1(m-1, n) &= -\Delta u_+^1(m+1, n) , \\
\Delta u_+^2(m-1, n) &= \Delta u_+^2(m+1, n) , \\
\Delta u_+^3(m-1, n) &= \Delta u_+^3(m+1, n) , \\
\Delta u_+^1(m, n) &= 0
\end{aligned} \tag{2.36}$$

for all admissible n . Along clamped edges the components of the normal n^i must remained fixed at their initial values. This condition is achieved by adjusting Δu_+^i one mesh spacing in from the clamped edge so that its component in the direction of the edge normal is 1/4 the value of the corresponding component of Δu_+^i two mesh spacings in from the edge. This adjustment guarantees

that the tangent to the middle surface at a clamped edge, which is computed using a one-sided finite derivative, will always lie in a fixed plane perpendicular to the original normal. Typically, for a clamped edge fixed in the y^3, y^1 coordinate plane intersecting the middle surface along a curve with fixed mesh number n , the relations used are

$$\Delta u_{+}^i(m, n+1) = \Delta u_{*}^i(m, n+1) - \Delta u_n n^i(m, n) \quad , \quad (2.37)$$

where

$$\Delta u_n = n^j(m, n) [\Delta u_{*}^j(m, n+1) - \frac{1}{4} \Delta u_{+}^j(m, n+2)] \quad (2.38)$$

and Δu_{*}^i are the unadjusted components of the displacement increments one mesh spacing in from the clamped edge as obtained from the equations of motion.

The remainder of MOTION is concerned with energy calculations, the details of which are covered in Section 2.4. Briefly, the kinetic energy due to Δu_{+}^i is calculated and, if there is a pressure loading, the portion of the external work due Δu_{+}^i is also calculated. Then the total external work is computed and finally the energy dissipated by plasticity is determined.

Returning to the main program, see Figure 2.3, the next subroutine called is PDATA, which is described in Section 2.2. Following PDATA, subroutine DAMP is called in order to compute the energy removed by the damping, as described in Section 2.4. A check is next performed to determine if information for a restart should be collected by subroutine WRTAPE at this time step. If no restart information need be gathered, then the finite difference calculational loop is complete; otherwise, WRTAPE and PDATA are called, again completing the finite difference loop. This sequence of calculation has generated new values $y^i, n^i, \sigma^{\alpha\beta}$ and Δu_{+}^i of the fundamental variables from the old values $y_{-}^i, n_{-}^i, \sigma_{-}^{\alpha\beta}$ and Δu_{-}^i ; the solution has been advanced a time step.

2.4 Energy Calculations

The energy calculations and the finite difference calculations are performed concurrently. The energy calculations use the results of the finite difference calculations to determine the current values of the kinetic energy, the strain energy, the external (pressure) work and the plastic work (i.e. the energy dissipated by plasticity). When the damping option is used, they also determine the energy removed by damping, called

the damping work. The energy calculations have no influence on the finite difference calculations and, hence, on the solution, except when the damping option is employed. This section presents the equations and procedure used in the REPSIL energy calculations; a theoretical report justifying the use of these will be forthcoming shortly.

Because the energy calculations are embedded in the finite difference calculations, the flow chart, Figure 2.3, used in describing the latter calculation still pertains and will be referred to in the discussion that follows.

The first calculation performed is for the strain energy. Immediately after the current values of the mixed components of stress σ_{β}^{α} are computed in STRESS, Figure 2.5, the strain energy density ϕ per unit material coordinate volume at mesh point (m,n) and layer k is calculated using:

$$\phi = \frac{1}{2E} \left[(\sigma_1^1 + \sigma_2^2)^2 - 2(1+\nu) (\sigma_1^1 \sigma_2^2 - \sigma_2^1 \sigma_1^2) \right] g^{\frac{1}{2}} \quad (2.39)$$

and simultaneously summed over all mesh points and layers. The strain energy calculation is completed in DGEOM, where the last sum is multiplied by the finite difference volume element giving the strain energy of the shell:

$$V = \sum_{m,n,k} \phi \Delta \eta^1 \Delta \eta^2 \Delta \zeta \quad (2.40)$$

The kinetic energy calculation is accomplished in subroutine KINET, which is, as shown in Figure 2.6, called by subroutine MOTION.

KINET The kinetic energy density ψ per unit material coordinate area of the middle surface at the mesh point (m,n) is determined:

$$\psi = \frac{1}{2} \rho_0 a_0^{\frac{1}{2}} \frac{\Delta u_{\pm}^i}{\Delta t} \frac{\Delta u_{\pm}^i}{\Delta t} \quad (2.41)$$

Summing ψ over all mesh numbers (m,n) and multiplying by the finite difference area element, the kinetic energy of the shell is obtained:

$$T(t + \frac{1}{2} \Delta t) = \sum_{m,n} \psi \Delta \eta^1 \Delta \eta^2 \quad (2.42)$$

Notice, since the displacement increments Δu_{\pm}^i are for the time interval $[t, t+\Delta t]$, the kinetic energy is properly centered at the time $t + 1/2 \Delta t$, as indicated. The kinetic energy at the

time t is determined by averaging the values of the kinetic energy at times $t + \frac{1}{2}\Delta t$ and $t - \frac{1}{2}\Delta t$:

$$T(t) = \frac{1}{2} [T(t + \frac{1}{2}\Delta t) + T(t - \frac{1}{2}\Delta t)] \quad (2.43)$$

The external work calculation is done in two steps, accomplished by MOTION calling subroutine PWORK twice, see Figure 2.6.

PWORK This subroutine calculates the work ω per unit material coordinate area of the middle surface at the mesh point (m,n) due to the pressure P acting during half the displacement increment Δu^i using the equation

$$\omega = -\frac{1}{2} P \Delta u^i n^i, \quad (2.44)$$

where the negative sign is a consequence of the pressure being oppositely directed to n^i . The subroutine then sums ω over the mesh points and returns to MOTION.

The first time PWORK uses the values of the displacement increments for the time interval $[t - \Delta t, t]$, determining the contribution to the external work of the interval $[t - 1/2 \Delta t, t]$; the second time it uses the values for the increment $[t, t + \Delta t]$, determining the contributions of the interval $[t, t + 1/2 \Delta t]$. These contributions are added and the result multiplied by the finite difference area element to give the total external work during the time interval $[t - 1/2 \Delta t, t + 1/2 \Delta t]$:

$$\begin{aligned} \Delta W(t) &= \sum_{m,n} [\omega(t - \frac{1}{4}\Delta t) + \omega(t + \frac{1}{4}\Delta t)] \Delta \eta^1 \Delta \eta^2, \\ &= - \sum_{m,n} \left[\frac{\Delta u^i + \Delta u^i}{2} n^i P^* \right] \Delta \eta^1 \Delta \eta^2. \end{aligned} \quad (2.45)$$

This work increment, which is properly centered at time t , is then averaged with the work increment at time $t - \Delta t$ to give the work increment for the time interval $[t - \Delta t, t]$ centered at time $t - 1/2 \Delta t$:

$$\Delta W(t - 1/2 \Delta t) = 1/2 [\Delta W(t - \Delta t) + \Delta W(t)]. \quad (2.46)$$

This average work increment is added to the total external work up to the time $t - \Delta t$ to give the external work done up to the current time t :

$$W(t) = W(t - \Delta t) + \Delta W(t - 1/2 \Delta t). \quad (2.47)$$

The reasons for doing the external work calculation in this rather elaborate way are made clear in the forthcoming theoretical report on the energy calculations.

If the damping option is not in effect, then the only means by which energy is dissipated is through plastic flow. This unavailable energy is measured by the plastic work W_p , which is simply the difference of the total external work and the sum of the kinetic energy and strain energy:

$$W_p = W - T - V. \quad (2.48)$$

The plastic work is computed in MOTION, just following the external work calculation, see Figure 2.6. With this calculation, the generation of current values of W , T and V is complete and the energy calculations end.

On the other hand if the damping option is in force, then there is an additional means of energy dissipation, measured by the damping work W_D . In this case the plastic work W_p is computed from

$$W_p = W - T - V - W_D. \quad (2.49)$$

The damping work W_D is computed in subroutine DAMP which is called after MOTION, in which the above calculation for W_p is performed. Hence, for a proper sequencing of calculations, DAMP must compute the damping work up to the next time step $t + \Delta t$.

DAMP This subroutine, which is schematically summarized in Figure 2.7, controls the entire damping operations. As already mentioned in the introduction, these operations remove the kinetic energy of the system efficiently so that the shell approaches a static equilibrium configuration quickly. The kinetic energy is removed in two ways: first through viscous damping and second through the use of a kinetic energy annihilation (KEA) procedure. The KEA procedure is the principal means of energy removal, while the viscous damping mainly serves to smooth out disturbances in the solution caused by the abrupt nature of the KEA procedure. Conceptually, the KEA procedure involves "freezing" the position of the shell whenever the kinetic energy achieves a local maximum, so that the velocity and hence the kinetic energy vanish instantaneously, followed by an immediate "release" of the shell.

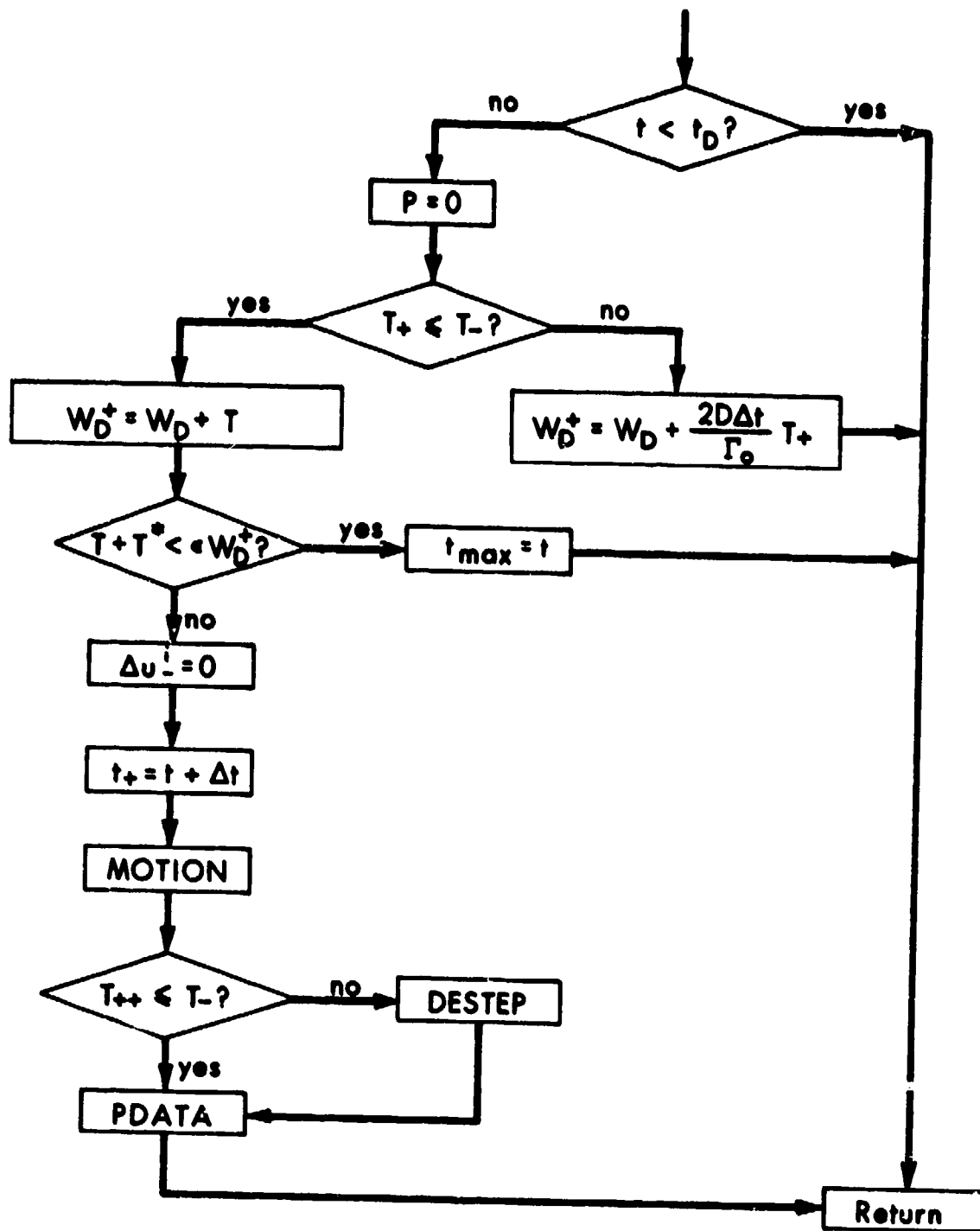


Figure 2.7 Flow Chart for Subroutine DAMP

Since it is highly unlikely that the shell will be "frozen" in a static equilibrium configuration, it will resume its motion on being "released", but with the total energy reduced by the amount of kinetic energy present when the maximum was achieved. This procedure is repeated at each maximum of the kinetic energy until the energy removed at some maximum is a small enough fraction of the total energy removed, at which time the shell is considered sufficiently close to its static equilibrium configuration and computations terminate*. This procedure is motivated by two considerations: first, the removal of energy is accomplished efficiently, since it occurs when the kinetic energy is at a peak and, second, should the kinetic energy be at its absolute maximum, then the corresponding configuration would be a static equilibrium configuration.

The subroutine begins by determining if the damping procedure is in force by checking the time t against the prescribed time t_D at which this procedure is to begin. If t_D is not exceeded the remainder of the subroutine is not used and calculations return to the main program. When t_D is exceeded, first the pressure P is set to zero. Second, the kinetic energy is checked as to whether it has just reached a local maximum by comparing its value T_+ at the time $t + 1/2 \Delta t$ to its value T_- at $t - 1/2 \Delta t$.** If $T_+ > T_-$ then no maximum has been reached and the subroutine adds the increase in the damping work due to viscous damping over this interval, which is simply a linear function of T_+ , to the damping work W_D at the time t to obtain the damping work W_D at the time $t + \Delta t$:

$$W_D^+ = W_D + \frac{2 D \Delta t}{\Gamma_0} T_+ \quad (2.50)$$

On the other hand, if $T_+ \leq T_-$, a local maximum has been reached and the KEA procedure goes into effect. To maintain the energy balance, the kinetic energy removed at this time t is added to the damping work:

$$W_D^+ = W_D + T. \quad (2.51)$$

* This method for reducing kinetic energy appears to be in common use; See, for example, DAHL, BEELER and BOURQUIN [4] who use this method to obtain computer solutions of some solid state physics problems.

** Cf. the description of subroutine KINET, where the notation $T(t+1/2 \Delta t)$ and $T(t-1/2 \Delta t)$ was used for T_+ and T_- .

The sum of kinetic energy T removed at this time and the kinetic energy T^* removed by the previous KEA calculation is compared to the damping work W_D^+ . If this sum is a small enough fraction of W_D^+ , then the maximum time for the problem t_{\max} is set equal to the present time t , after which the main program causes the problem to terminate, see Figure 2.3. If the sum is not sufficiently small, then the displacement increments Δu_{+}^i are set equal to zero, making the velocities at time $t + 1/2 \Delta t$ and, hence, the kinetic energy T_{+} vanish. Since $\Delta u_{+}^i = 0$, the position of the shell and stress field at the time $t + \Delta t$ will remain the same as at the previous time t . Hence, these variables need not be recalculated and the subroutine can proceed to increase the time step and call MOTION in order to calculate the displacement increments Δu_{++}^i for the interval $[t + \Delta t, t + 2 \Delta t]$ and the kinetic energy T_{++} at the time $t + 3/2 \Delta t$. The kinetic energy T_{++} is next compared with T_- . If $T_{++} \leq T_-$ then the subroutine finishes the damping operations by calling PDATA in order to collect some plotting information and returns to the main program. However, if $T_{++} > T_-$ then experience has shown that a numerical instability due to the KEA operations is likely to occur. To remedy this the subroutine calls subroutine DESTEP in order to compute a smaller stable time increment Δt^* .

DESTEP This subroutine calculates a decreased time step Δt^* that prevents the KEA operations from causing an instability:

$$\Delta t^* = \left(\frac{T_-}{T_{++}} \right)^{1/2} \Delta t . \quad (2.52)$$

Then the values of Δu_{++}^i and T_{++} are adjusted for the decreased time step by scaling:

$$\Delta u_{++}^{*i} = \frac{2 \Gamma_0 + D \Delta t}{2 \Gamma_0 + D \Delta t^*} \left(\frac{\Delta t^*}{\Delta t} \right)^2 \Delta u_{++}^i , \quad (2.53)$$

$$T_{++}^* = \left(\frac{2 \Gamma_0 + D \Delta t}{2 \Gamma_0 + D \Delta t^*} \right)^2 \left(\frac{\Delta t^*}{\Delta t} \right)^2 T_{++} . \quad (2.54)$$

Returning from DESTEP back to DAMP, PDATA is called and the calculations return to the main program as before.

2.5 Surface Strain Calculations

The surface strain calculations, like the energy calculations, are embedded in the finite difference sequence of calculations. They do not influence the solution, but their results are a consequence of the solution and provide useful local measures of the validity and reliability of the solution. The elongational strains in prescribed directions at prescribed locations on the bounding surfaces of the shell are calculated. The equations used are derived in [1; App. D] and give the exact elongation per unit initial length, with no approximations based on the smallness of strain being invoked. The strains are intended to simulate the readings of strain gages bonded to the shell at these locations.

The strain calculations take place in subroutine STRAIN. However, before these calculations are performed, the interpolation coefficients and the mesh numbers bracketing the strain locations are calculated in START, see Figure 2.2, and subroutine ABINIT is called during the initial pass through DGEOM, see Figure 2.4.

ABINIT This subroutine uses the mesh numbers of the mesh points bracketing the strain locations, as determined to START, to select the initial values of $a_{\alpha\beta}$ and $b_{\alpha\beta}$ at the bracketing mesh points. These values are stored in arrays for later use in STRAIN.

The above calculations are performed initially and only once. The covariant components of strain $\epsilon_{\alpha\beta}$ on the bounding surfaces of shell are computed in DGEOM each time step for every mesh point using the equation

$$\epsilon_{\alpha\beta} = \epsilon_{\alpha\beta}^- + \frac{1}{2} (\Delta a_{\alpha\beta} \pm h \Delta b_{\alpha\beta}), \quad (2.55)$$

where $\epsilon_{\alpha\beta}^-$ are the covariant strain components at the previous time $(t - \Delta t)$, h is the shell thickness and the $+$ or $-$ sign depends on whether the bounding surface is on the negative or positive side of the normal n^i , respectively. The covariant components of strain are used in STRAIN, immediately following DGEOM.

STRAIN This subroutine calculates the elongational strains in predetermined directions at predetermined locations on the surfaces of the shell. It also computes the components of the total displacement of a predetermined locations on the middle surface. These directions and locations are specified in the input data, see Section 3.2. At each strain location four elongational strains are found: two along the coordinate curves and two in directions specified in the input data by the angles θ made with the n^1 coordinate curve, see Figure 3.5. The first time

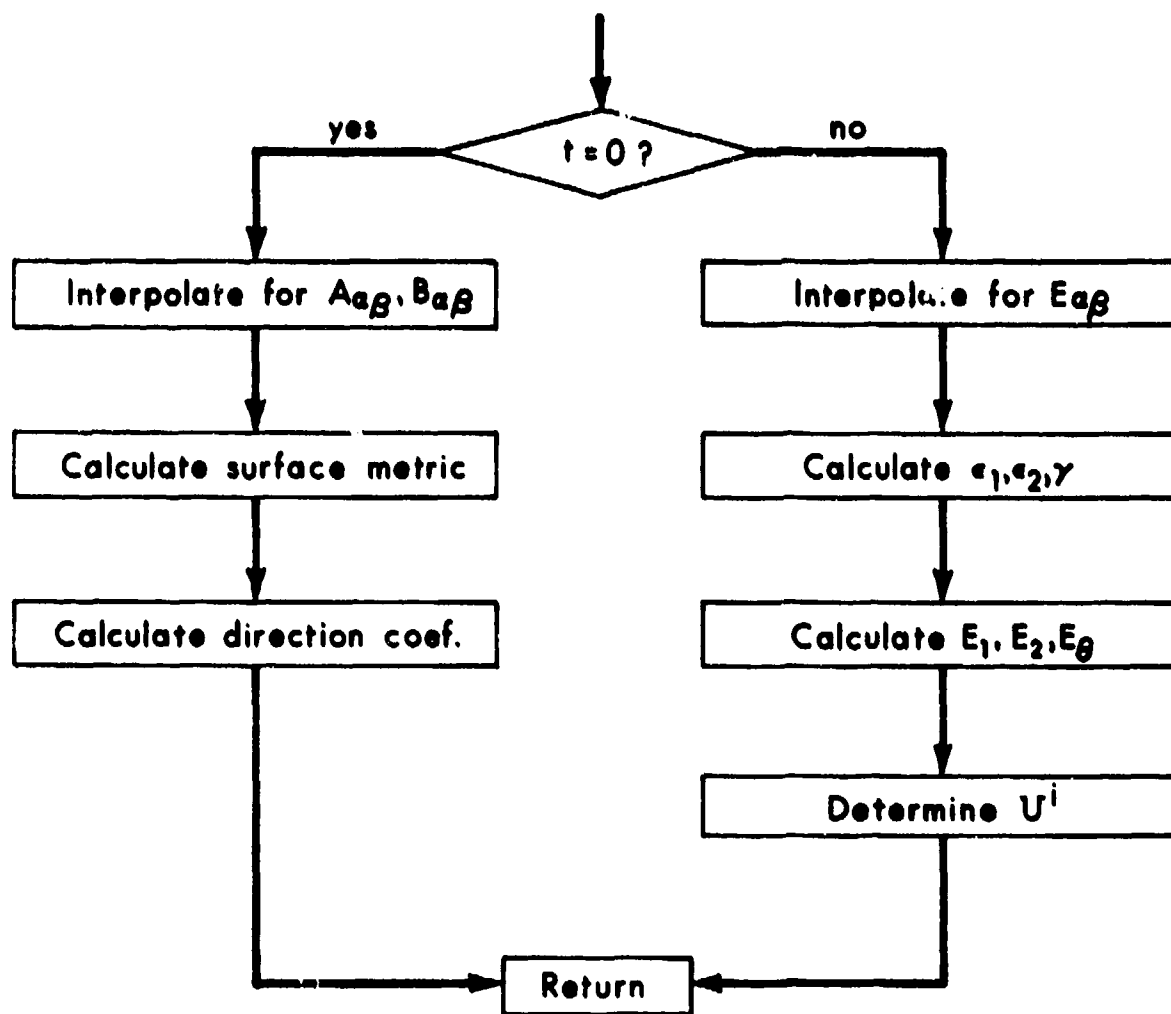


Figure 2.8 Flow Chart for Subroutine STRAIN

this subroutine is executed, see Figure 2.8, the interpolation coefficients calculated in START are used to linearly interpolate the initial values of $a_{\alpha\beta}$ and $b_{\alpha\beta}$ to obtain the corresponding values $A_{\alpha\beta}$ and $B_{\alpha\beta}$ at the strain locations. From these values, the components of the surface metric $G_{\alpha\beta}$ at these locations are calculated:

$$G_{\alpha\beta} = A_{\alpha\beta} \pm h B_{\alpha\beta} \quad (2.56)$$

where, as in (2.55), + or - depends on the surface lying in the negative or positive direction of n^1 . Also, the direction coefficients α and β are calculated from the angle θ specifying the strain directions:

$$\alpha = \sin \theta / \sqrt{1 - \delta^2} \quad \beta = \cos \theta - \alpha\delta \quad (2.57)$$

where δ is a function of $G_{\alpha\beta}$:

$$\delta = G_{12} / \sqrt{G_{11}G_{22}} \quad (2.58)$$

This calculation ends the initial pass through the subroutine. For all subsequent time steps, the subroutine uses a different computational loop to calculate the elongational strains and the components of total displacement, as shown in Figure 2.8. First, the values $\epsilon_{\alpha\beta}$ at the mesh points bracketing the strain locations are linearly interpolated to give the covariant components of strain $E_{\alpha\beta}$ at the strain locations. These components are combined with $G_{\alpha\beta}$ to give the intermediate strain components

$$\epsilon_1 = E_{11}/G_{11} \quad , \quad \epsilon_2 = E_{22}/G_{22} \quad , \quad \gamma = E_{12} / \sqrt{G_{11}G_{22}} \quad (2.59)$$

From these intermediate components, the subroutine determines the elongational strains along the η^1 and η^2 coordinate curves:

$$E_1 = \sqrt{1 + 2\epsilon_1} - 1 \quad , \quad E_2 = \sqrt{1 + 2\epsilon_2} - 1 \quad (2.60)$$

and in the direction specified by θ :

$$E_{\theta} = \sqrt{1 + 2 (\beta^2 \epsilon_1 + 2 \alpha \beta \gamma + \alpha^2 \epsilon_2)} - 1 \quad (2.61)$$

The remainder of the subroutine involves determining the total displacement components U^i at a predetermined location. The components of displacement increments Δu^i at the mesh point surrounding the displacement location are linearly interpolated to give the components of displacement increment ΔU^i at this location. These components are added to the previous values of the components of the total displacement U^i to give their current values.

$$U^i = U_{-}^i + \Delta U^i \quad (2.62)$$

3. DESCRIPTION OF INPUT

3.1 Input Cards

The data needed to run REPSIL are supplied on input cards and, in the case of certain pressure loadings, on a user-generated input tape. Instructions for generating the pressure input tape are given in Section 6.2. The input cards assign values to the FORTRAN variables listed in Table 3.1 in that order using the formats indicated.

Table 3.1 List of Input Cards

CARD	VARIABLES	FORMAT
1	TITLE	10A8
2	MESH, NMESH, LAYER, YLDFAC	3I5,E12.6
3	MAXC, NCONT, NWRITE, DELTAT	3I5,E12.6
4	IBEC1, IBEC2, IBEC3, IBEC4	4I5
5	LOAD, LPRESS, MDAMP, DAMPF, DFACT	3I5,2E12.6
6	E, FNU, SIGZ, RHO, THICKN, NSFL, ISR	5E12.6,2I5
7	(SSIG(J), SEPS(J), DSR(J), PSR(J), J=1, NSFL)	4E15.7
8	NPRINT, (JCHK(J), J=1,3)	4I5
9	NUMCY, (NCYCH(J), J=1, NUMCY)	16I5
10	NLPRIN, (JCYNLP(J), J=1,NLPRIN)	16I5
11	N3D (NC3DP(J), J=1, N3D)	16I5
12	ETADI, ETAD2, NSTRN	2E10.4,I5
13	(ETAG1(I), ETAG2(I), ANGLE(I), ANGLB(I), NETAG(I), I=1, NSTRN)	4E10.4,I5
14	LENGTH, WIDTH [for flat plate]	2E12.6
	LENGTH, RADIUS, THETA [for cylindrical shell]	3E12.6
	LENGTH, RADI, RADF, THETA, MASH [for conical shell]	4E12.6,I5
15	MI, MF, NI, NF, VR, NV	4I5,E12.6,I5
16	M,N,V	2I5,E12.6

The following rules should be obeyed in preparing the input cards.

- Omit card 7 if either NSFL = 0 or NSFL = 1 and ISR = 0; otherwise, the number of card 7's must match NSFL.
- The number of card 13's must match NSTRAN \geq 1.
- Only one card 14 is used, with the data matching the particular subroutine INGEOM used.
- Omit cards 15 and 16 if LOAD = 1 or NCONT > 0.
- Omit card 16 if NV = 0; otherwise, the number of card 16's must equal NV.

The input cards can be grouped according to the type of data they supply:

Cards 2,3	Data controlling the finite difference and numerical analysis.
Card 4	Parameters for selecting boundary conditions.
Card 5	Data controlling the type of loading and the damping option.
Card 6,7	Material properties.
Card 8,9,10,11	Printing and plotting control numbers.
Cards 12,13	Data specifying locations where displacement components and surface strains are to be calculated.
Card 14	Dimensions of shell.
Card 15,16	Data characterizing the initial impulse velocity.

3.2 Description of Input Variables

The input variables are described below in the order in which they appear on the input cards, as listed in Table 3.1. The dimensions of a variable are indicated by capital letters in square brackets following the short underlined description of the variable, with F representing force, L length, and T time. The program is written to accept any consistent set of dimension units. For example, the mass density in the pound-inch-second system of units for a material weighing 1 pound per cubic inch would be $\frac{1}{386} \frac{\text{lb-sec}^2}{\text{in}^4}$.

Card 1 TITLE Title to identify run. Not to exceed 80 alphanumeric characters.

Card 2 MESH Number of mesh intervals in the n^1 direction.

NMESH Number of mesh intervals in the n^2 direction.

Figures 3.6 - 3.8 show the orientation of the mesh relative to the n^1 and n^2 directions for the initial geometries presently programmed in REPSIL. The choice of MESH and NMESH should only be based on the portion of the shell to be actually analyzed; additional intervals due to exterior mesh points along symmetry boundaries should be disregarded. MESH and NMESH are limited by the maximum number of M and N mesh elements permitted by DIMENSION and COMMON statements (see Section 3.3):

$$\text{MESH} \leq M_{\text{max}} - \begin{cases} 2 & ; \text{IBCE3} = 1,3 \\ 3 & ; \text{IBCE3} = 2 \end{cases}$$

$$\text{NMESH} \leq N_{\text{max}} - \begin{cases} 1 & ; \text{IBCE2} = 1,3 \\ 2 & ; \text{IBCE2} = 2 \end{cases}$$

LAYER Number of layers into which the shell thickness is divided. The shell is divided into layers, within which the stress is assumed constant, in order to facilitate the modelling through the thickness of the stress profile resulting from plasticity. Hence, the greater the number of layers used, the more accurately is the stress profile presumably modelled, but at the expense of longer computation times and greater memory requirements. LAYER = 4 has been found to be a good compromise giving reasonably accurate deflections.

YLDFAC Parameter controlling the "thickness" of ellipsoidal annuli surrounding yield surface in stress space. The ellipsoidal annuli divided the excursions of the stress increment outside yield surface into subincrements making the calculation of the stress on the yield surface more accurate, see Appendix B. Accuracy increases with value of YLDFAC, but at the expense of increased computation times, with YLDFAC = 1 a good compromise. In order not to use this option set YLDFAC = 0.

Card 3 MAXC Time step at which it is desired to terminate the problem.

 NCONT Time step at which it is desired to begin the problem.
 For new problems NCONT = 0 and for restart problems
 NCONT = time step from which solution is continued.
 Notice that always NCONT < MAXC.

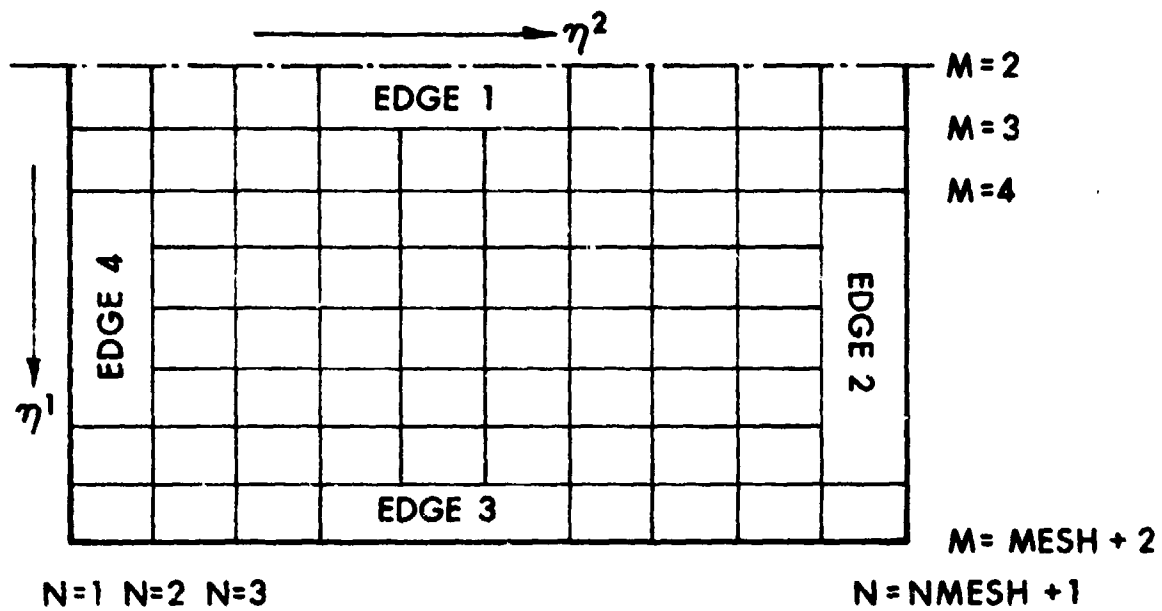
 NRITE Time steps elapsed between the gathering of restart data. Restart information written on the restart tape(tape #1)every NRITE time steps. If it is planned to restart a problem, NRITE ≤ MAXC; if restart information is not desired, make NRITE > MAXC and omit tape.

 DELTAT Finite difference time increment [T]. Using equations (2.3), the program calculates time increments that assure numerical stability in the membrane and bending modes of vibration and then chooses the minimum of these and the input DELTAT to use in the finite difference calculations. If DELTAT = 0.0, the program chooses the minimum stable time increment.

Card 4 IBEC1 } Number prescribing boundary conditions along the edges
 IBEC2 } of the shell. In Figure 3.1 the edges are inumerated
 IBEC3 } relative to the (M,N) grid and the admissible values of
 IBEC4 } the boundary control numbers at each edge are listed.

Clamped edge condition (1). Coordinates of middle surface y^i and components of normal n^i are fixed at their initial values along this edge.

Symmetry edge condition (2). Edge lies in a symmetry plane about which the shell and the loads are symmetrically distributed. Edge 1 is always a symmetry edge located in the $y^1 = 0$ symmetry plane, see Figures 3.6 - 3.8. The symmetry plane for edge 2 is $y^2 = \text{LENGTH}$, see Figure 3.6 and 3.7. The symmetry plane for edge 3 is the same as that for edge 1, namely $y^1 = 0$, and hence is applicable to shell intersecting this plane twice, such as cylinders and cones, see Figure 3.7 and 3.8. Care should be taken that the symmetry edge condition be compatible with the particular shell geometry treated, e.g. although IBCE3 = 2 is admissible, it is certainly not appropriate to the flat plate (Figure 3.6) or the cylindrical panel with $\text{THETA} < \pi$ (Figure 3.7).



- EDGE 1: IBCE1 = 2 (SYMMETRY)
- EDGE 2: IBCE2 = 1 (CLAMPED), 2 (SYMMETRY) OR 3 (HINGED)
- EDGE 3: IBCE3 = 1 (CLAMPED), 2 (SYMMETRY) OR 3 (HINGED)
- EDGE 4: IBCE4 = 1 (CLAMPED) OR 3 (HINGED)

Figure 3.1 Admissible Boundary Conditions

Hinged edge conditions (3). Coordinates of middle surface y^i are fixed at their initial values along this edge, but normal n^i is free to rotate about edge.

Card 5 LOAD

Number controlling mode of loading

- = -1, initial impulse velocity and pressure-time loading,
- = 0, initial impulse velocity,
- = 1, pressure-time loading.

Initial velocity distribution, other than those representable by cards 15 and 16, and pressure-time histories must be supplied by the user in appropriate form through sub-routines INVEL and PRESS; instructions for doing this are in Sections 6.2 and 6.3, respectively.

LPRESS

Distribution of pressures over shell after time steps
LPRESS fixed at the LPRESS distribution. If LOAD = 0, set LPRESS = 0. If user does not desire to fix pressure distribution, make LPRESS > MAXC.

MDAMP

Time step at which damping starts. Numerical damping is used to rapidly slow down the motion of the shell in order to obtain a final equilibrium configuration. MDAMP is selected after most of the plastic dissipation is over. This entails a preliminary run in order to estimate from the energy balance when plastic deformation is virtually finished; the damping run is continued from the time step closest to MDAMP as a restart problem (see Section 3.4). If damping is not desired set MDAMP > MAXC, otherwise values for DAMPF and DFACT must be supplied below.

DAMPF

Viscous damping coefficient used in smoothing solution during damping $[FT/L^3]$. Should not be too large in order to avoid overdamping and consequently prolonging the time to reach a final configuration.

DFACT

Parameter controlling termination of problem during damping. If the ratio of the sum of the energies removed in two consecutive kinetic energy annihilations to the damping work is less than DFACT, the problem terminates (see Figure 2.7). The smaller DFACT is made, the less the residual kinetic energy at termination, but at the expense of longer machine times.

Card 6 E Young's modulus $[F/L^2]$.

FNU Poisson's ratio.

SIGZ Yield stress $[F/L^2]$. For perfectly plastic behavior SIGZ is the maximum stress σ_0 on the uniaxial loading curve, Figure 3.2; for strain hardening behavior SIGZ is the stress σ_1 at the first change in slope in the polygonal approximation to the loading curve, Figure 3.3.

RHO Initial mass density per unit volume $[FT^2/L^4]$.

THICKN Thickness of shell $[L]$.

NSFL Number of changes in slope in the polygonal approximations to the uniaxial loading curve (equal to the number of stress sublayers):

 = 0, no plasticity \equiv elastic behavior,
 = 1, elastoplastic with no strain hardening,
 > 1, elastoplastic with strain hardening.

ISR Strain rate sensitivity control

 = 0, plasticity is strain rate independent,
 = 1, plasticity is strain rate dependent.

Card 7* SSIG(J),SEPS(J) Stress $[F/L^2]$ and strain $[L/L]$ at points of slope change of the polygonal approximation to the uniaxial loading curve, Figure 3.3, where $J = 1, NSFL$. The program automatically makes these data compatible with those on Card 6 by setting $SSIG(1) = SIGZ$ and $SEPS(1) = SIGZ/E$. For the strain rate sensitive case, take these data from the static loading curve.

DSR(J),PSR(J) Emperical constants used to model strain rate sensitive behavior, d and p of equation (2.21). Pair of constants must be specified for each slope change (i.e. each stress sublayer) on the polygonal approximation to the loading curve, $J = 1, NSFL$. On the stress-strain diagrams, Figure 3.4, the straight line portions of the

* Omit card 7 if either $NSFL = 0$ or $NSFL = 1$ and $ISR = 0$.

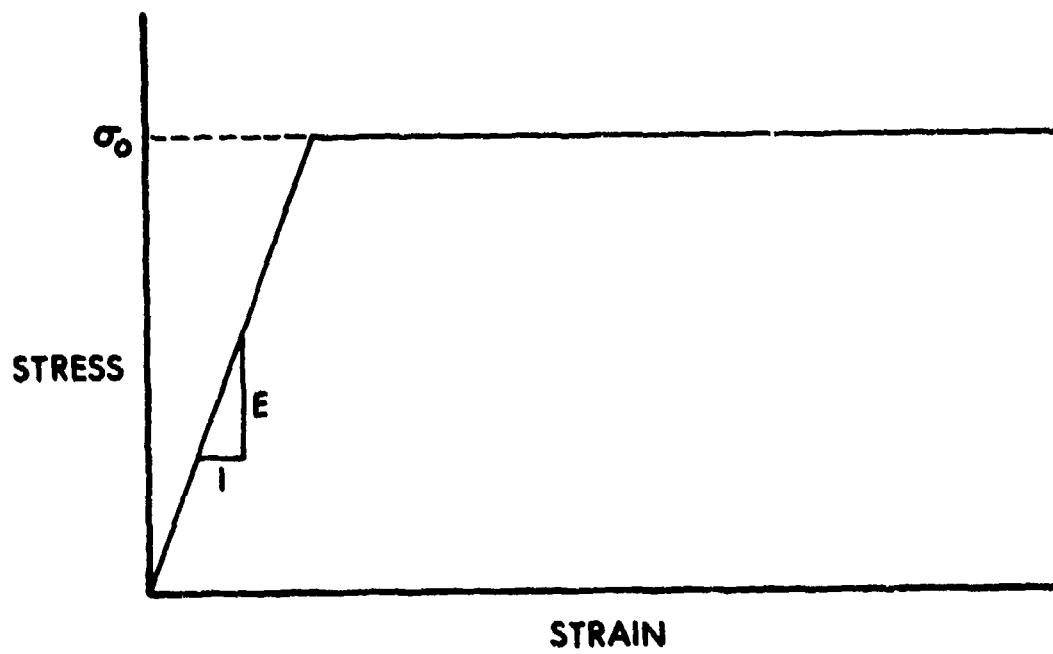


Figure 3.2 Uniaxial Loading Curve for the Elastic/Perfectly-Plastic Constitutive Model

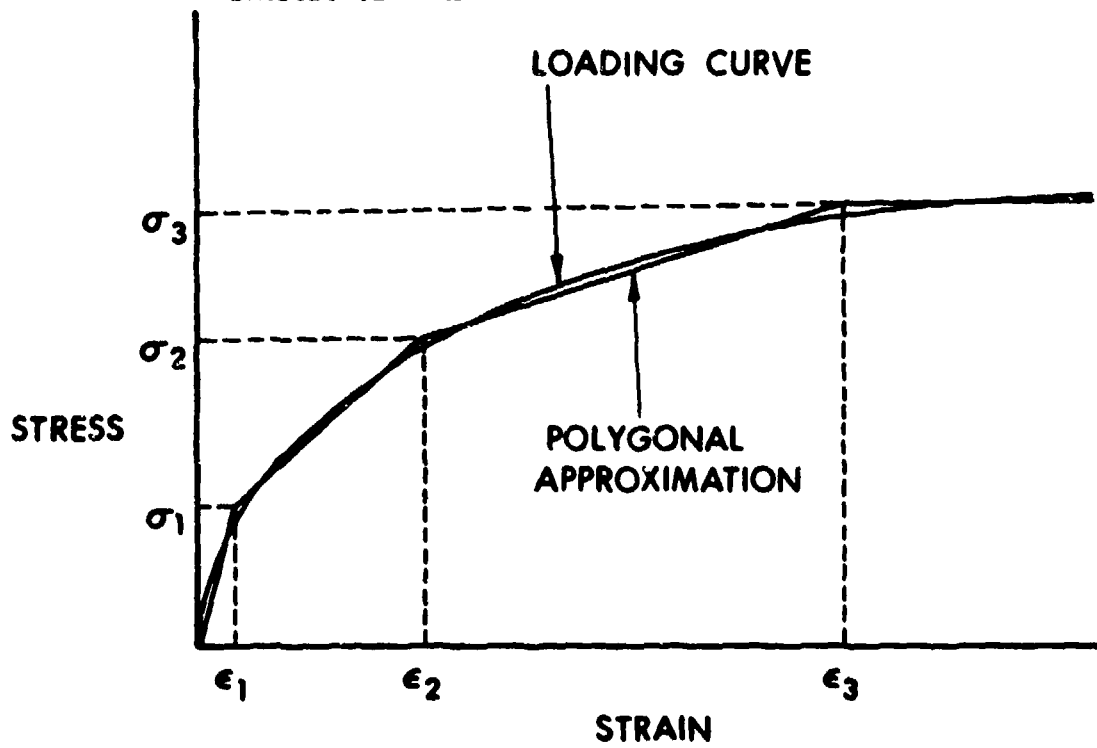
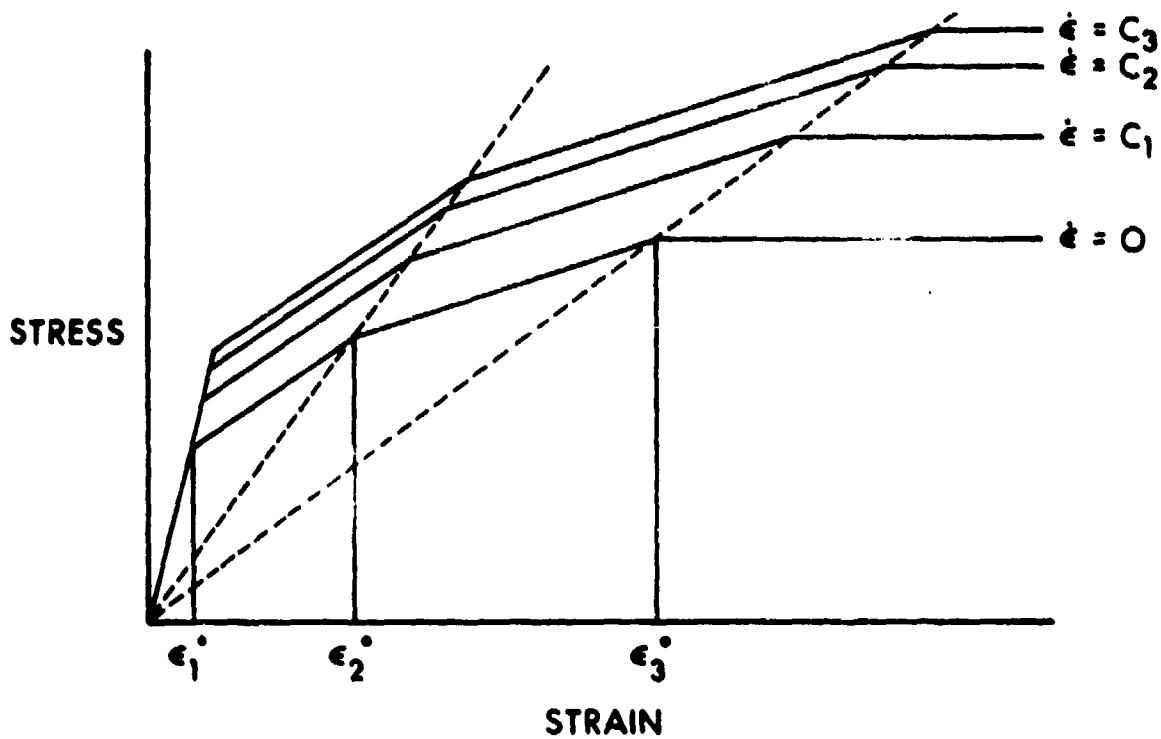
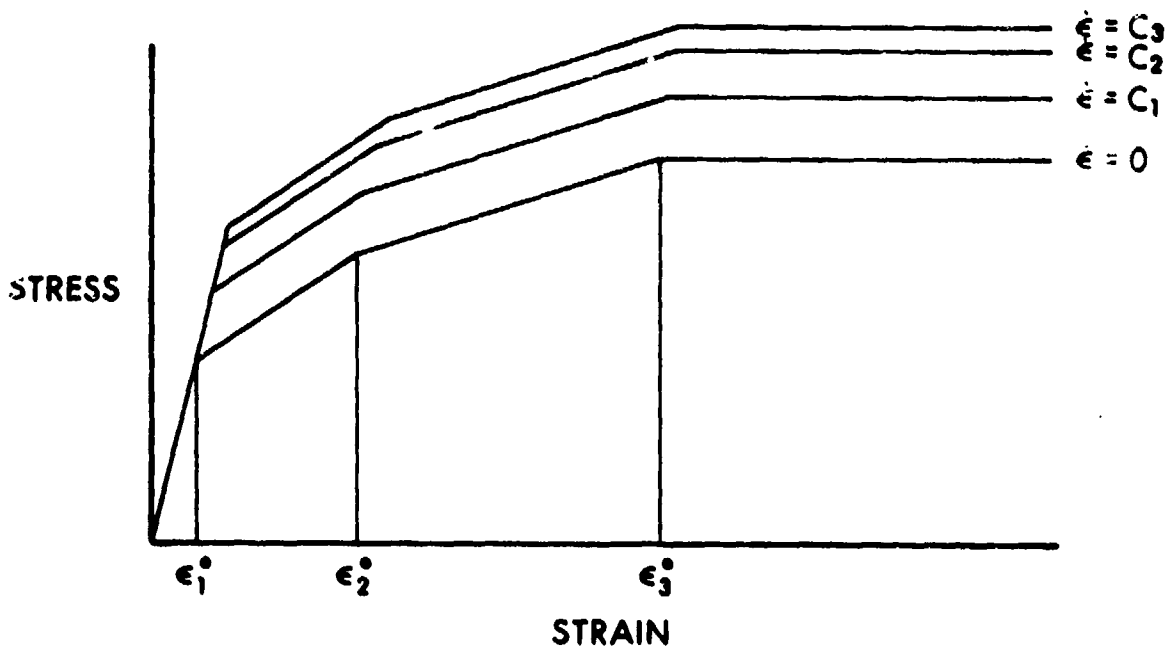


Figure 3.3 Uniaxial Loading Curve for the Strain Hardening Constitutive Model and Polygonal Approximation to Loading Curve



a. Strain rate parameters the same at each point of slope change.



b. Strain rate parameters differ at each point of slope change.

Figure 3.4 Polygonal Approximations to the Uniaxial Loading Curve at Various Constant Strain Rate Levels for Strain Rate Sensitive Materials

constant strain rate curves are parallel to the corresponding portions of the static loading curve. The strain ϵ_j at each point of slope change is magnified from the corresponding static strain ϵ_j° by the rate sensitivity factor:

$$\epsilon_j = \epsilon_j^\circ \left[1 + \left(\frac{\dot{\epsilon}}{d_j} \right)^{\frac{1}{p_j}} \right],$$

where $\dot{\epsilon}$ is the strain rate, see (2.22). If the same values of DSR(J) and PSR(J) are used for all J, the stress σ_j at each point of slope change is also magnified from the corresponding static stress σ_j° :

$$\sigma_j = \sigma_j^\circ \left[1 + \left(\frac{\dot{\epsilon}}{d_j} \right)^{\frac{1}{p_j}} \right],$$

so that

$$\sigma_j = (\sigma_j^\circ / \epsilon_j^\circ) \epsilon_j,$$

as illustrated in Figure 3.4a.

Card 8 NPRINT Number of elapsed time steps between surface strain prints. This print is described in Section 4.1.5. The remainder of the strain print data is specified on Card 13. If this print is not desired, set NPRINT > MAXC.

JCHK(J) Numbers controlling the printing of output data:

JCHK(1), components of displacement increments,

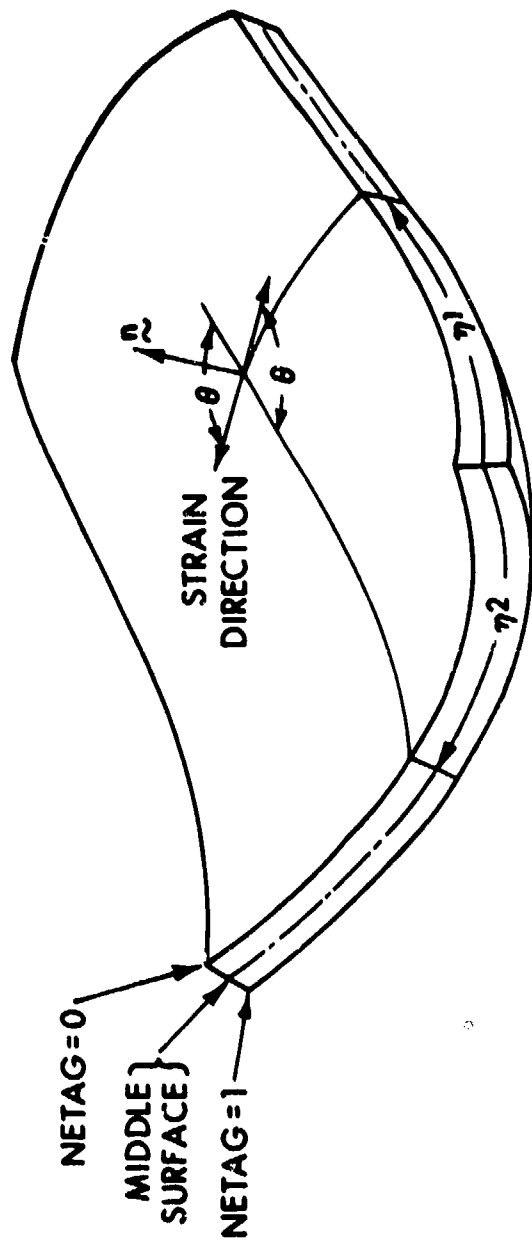
JCHK(2), coordinates of middle surface and the pressure,

JCHK(3), components of surface normal;

JCHK(J) = $\begin{cases} 0, & \text{data not printed,} \\ 1, & \text{data printed.} \end{cases}$

Card 9 NUMCY Number of time steps for which JCHK(J) controlled data and energy balance data are to be printed. Sections 4.1.2 and 4.1.3 describe these prints.

- NCYCH(J) Time steps at which JCHK(J) controlled data and energy balance data are printed. If these prints are not desired, set NUMCY = 1 and NCYCH(1) > MAXC.
- Card 10 NLPRIN Number of time steps for which the LMAT (M,N,K) array is printed. See Section 4.1.4 for a description of this print.
- JCYNLP(J) Time steps at which the LMAT (M,N,K) array is printed. If this print is not desired, set NLPRIN = 1 and JCYNLP(1) > MAXC.
- Card 11 N3D Number of time steps for which isometric and cross-sectional plots are drawn. A description of the plotting capabilities of REPSIL is given in Section 4.2.
- NC3DP(J) Time steps at which isometric and cross-sectional plots are drawn. If plots are not desired, set N3D=1 and NC3DP (1) > MAXC.
- Card 12 ETADI,ETAD2 Material coordinates of location at which the components of displacements are calculated and plotted [dimensions correspond to those for ETAG1(I), ETAG2(I) below].
- NSTRN Number of locations at which surface strains are calculated and plotted.
- Card 13 ETAG1(I), ETAG2(I) Material coordinates of locations at which surface strains are calculated and plotted, Figure 3.5. Dimensions depend on subroutine INGEOM:
- Flat plate, distance along width and length,
Figure 3.6,
- Cylinder, angle in degrees from the symmetry plane
and distance along axis, Figures 3.7.
- Cone, angle in degrees from the symmetry plane and
arclength along the cone generator, Figure 3.8.
- ANGLE(I), ANGLB(I) Angles θ measured in degrees from the η^1 direction counterclockwise about the normal of the directions in which surface strains are calculated, Figure 3.5, where $0 < \theta < 180^\circ$.
- NETAG(I) Number selecting the bounding surface on which surface strain calculations are performed, Figure 3.5:
- = 0, surface on positive side of normal,
= 1, surface on negative side of normal.



6

Figure 3.5 Material Coordinates η^α Locating Surface Strains Position and Orientation of Strain Direction Relative to the η^1 Direction

Card 14

Dimensions of shell specified on this card. Data specifying dimensions must be compatible with the particular subroutine INGEOM used. The next three figures show the orientations relative to the Cartesian coordinate axes y^i and the associated compatible boundary conditions for the three INGEOM subroutines presently programmed in REPSIL.

LENGTH Length of plate along symmetry boundary [L], Figure 3.6. Length of cylinder axis [L], Figure 3.7. Length of cone along axis [L], Figure 3.8.

WIDTH Width of plate up to symmetry boundary [L], Figure 3.6.

RADIUS Radius of cylinder [L], Figure 3.7.

RADI,RADF Small and large radii of cone [L], Figure 3.8.

THETA Angle subtended by cylindrical or conical panel measured from the symmetry plane [Degrees], Figures 3.7 and 3.8.

MASH Number controlling mesh proportions for cone:

= 0, equal mesh intervals along meridian, Figure 6.2a,

= 1, constant mesh proportions, Figure 6.2b.

For the details of this option see Section 6.4.

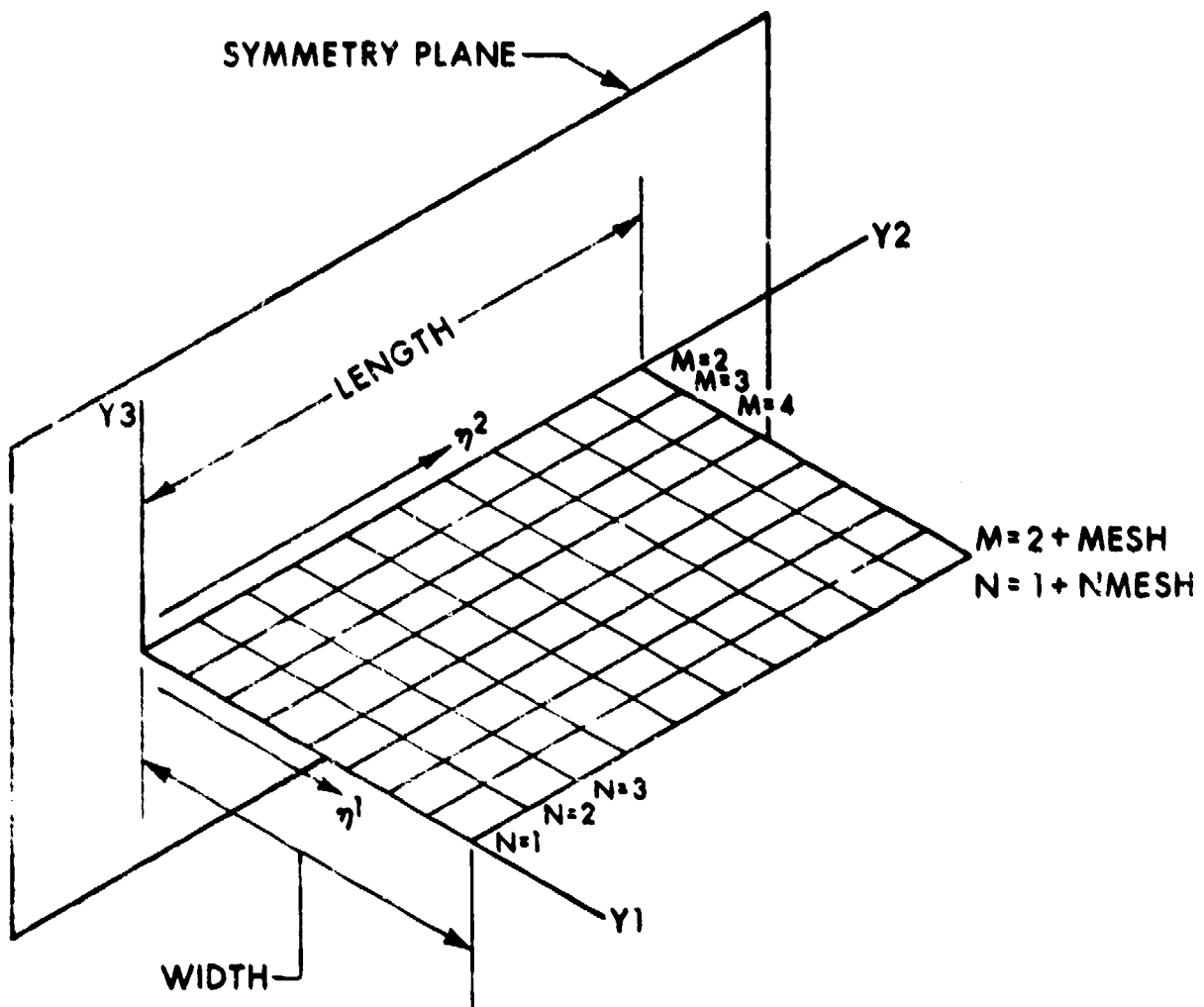
Card 15*

This card gives the data on the uniform initial impulse velocity field and gives the number of points with nonuniform impulse velocity, see Figure 3.9. As indicated in the figure, velocities are directed in the opposite sense to the normal.

MI,MF Minimum and maximum values of mesh number M for point receiving uniform initial impulse velocity VR.
 $2 \leq MI < MF \leq MESH + 2.$

NI,NF Minimum and maximum values of mesh number N for point receiving uniform initial impulse velocity VR.
 $1 \leq NI < NF \leq NMESH + 1.$

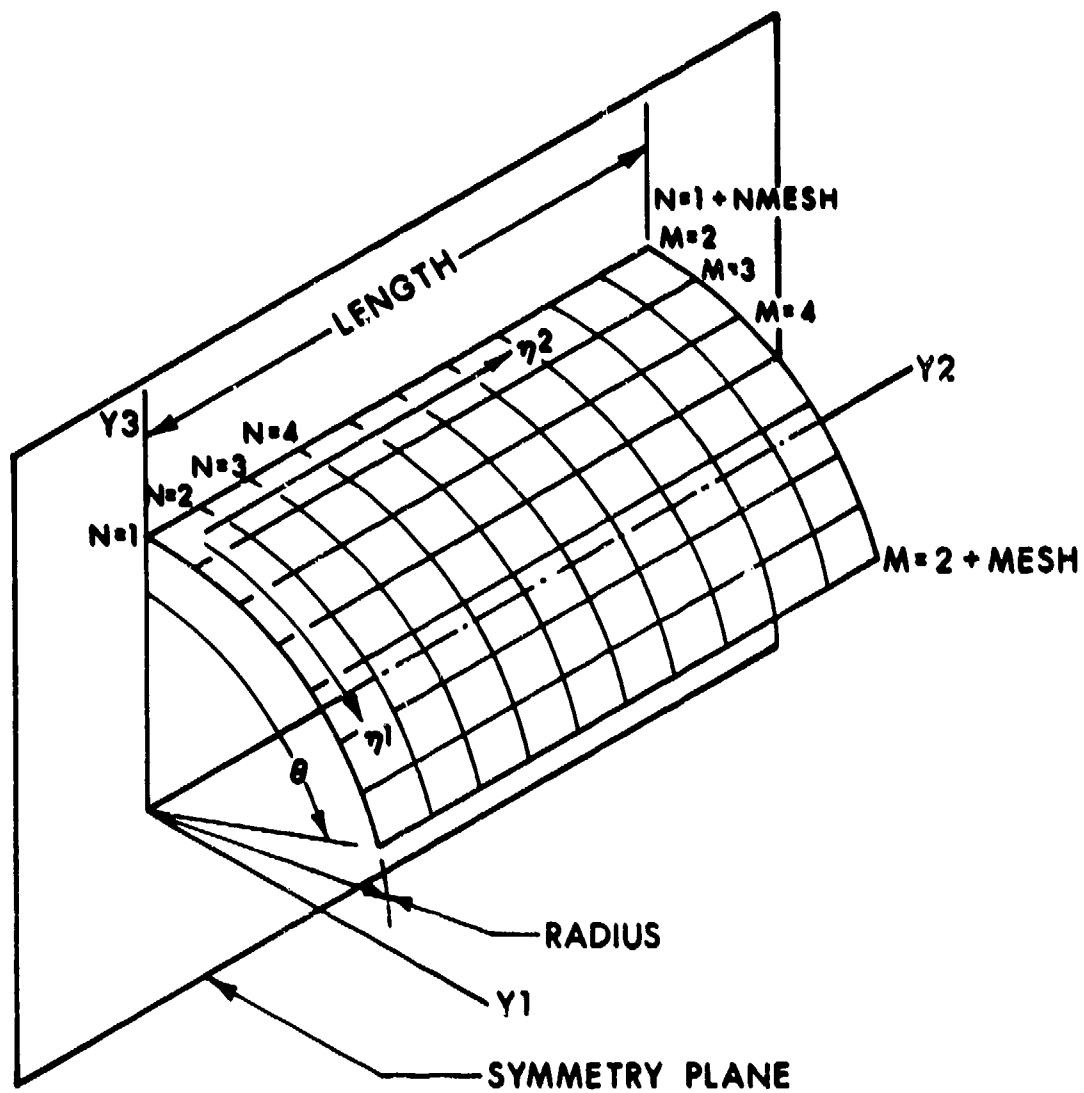
* Cards 15 and 16 omitted if $LOAD = 1$ or $NCONT > 0$.



Boundary conditions compatible
with flat plate geometry (see
Figure 3.1 for key):

$$\left\{ \begin{array}{l} \text{IBCE1} = 2 \\ \text{IBCE2} = 1, 2 \text{ or } 3 \\ \text{IBCE3} = 1 \text{ or } 3 \\ \text{IBCE4} = 1 \text{ or } 3 \end{array} \right.$$

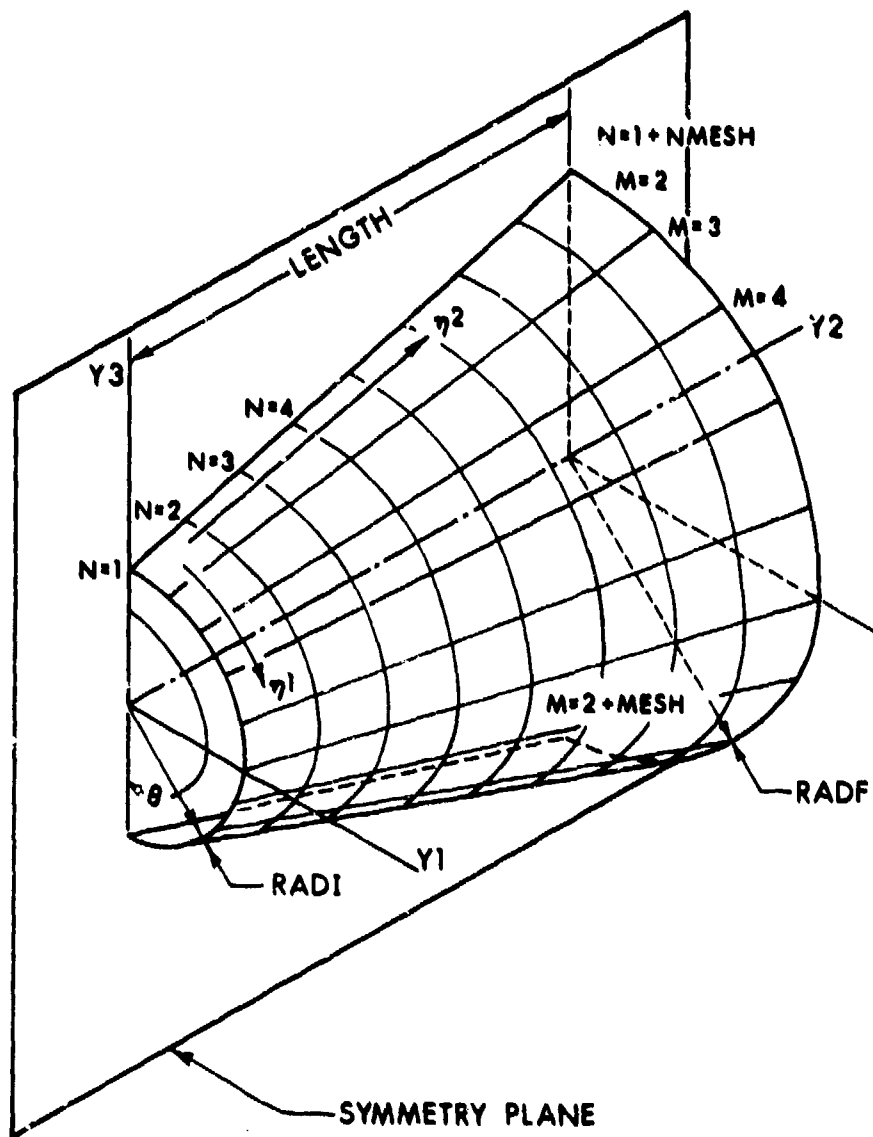
Figure 3.6 Flat Plate Geometry



Boundary conditions compatible with cylindrical geometry (see Figure 3.1 for key):

$$\left\{ \begin{array}{l} IBCE1 = 2 \\ IRCE2 = 1, 2 \text{ or } 3 \\ IBCE3 = 1 \text{ or } 3 \text{ when } 0 < \sigma < 180^\circ \text{ (as shown)} \\ \quad = 1, 2 \text{ or } 3 \text{ when } \sigma = 180^\circ \\ IBCE4 = 1 \text{ or } 3 \end{array} \right.$$

Figure 3.7 Cylindrical Shell Geometry



Boundary conditions compatible with conical geometry (see Figure 3.1 for key):

- IBCE3 = 2
- IBCE2 = 1 or 3*
- IBCE3 = 1 or 3 when $0 < \theta < 180^\circ$
= 1, 2 or 3 when $\theta = 180^\circ$ (as shown)
- IBCE4 = 1 or 3

*IBCE2 = 2 would result in discontinuous slope across boundary .

Figure 3.8 Conical Shell Geometry

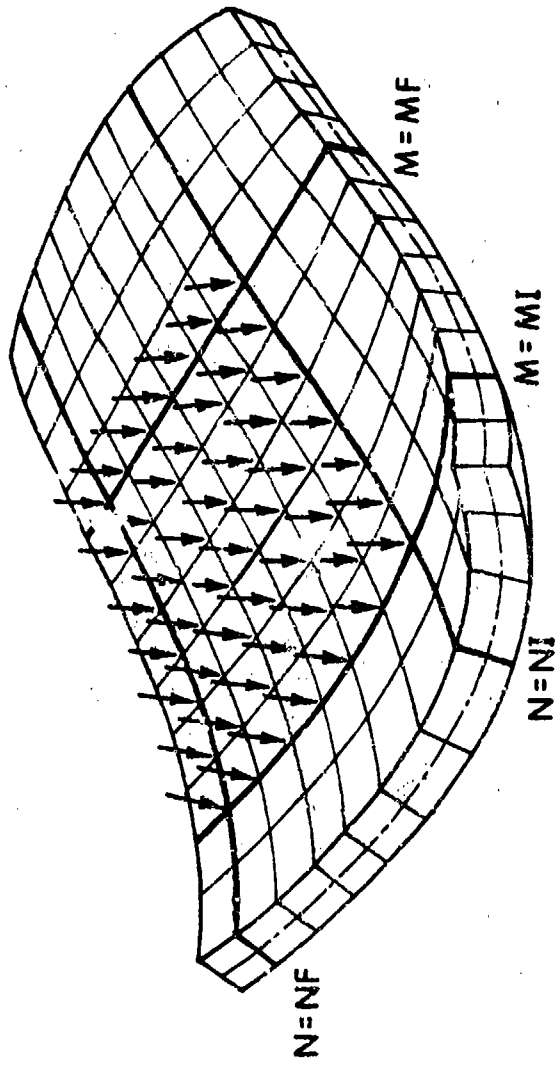


Figure 3.9 Uniform Initial Impulse Velocity Distributed over a Rectangular Region Defined by the Limits $MI \leq M \leq MF$ and $NI \leq N \leq NF$.

VR	<u>Uniform initial impulse velocity received by mesh points (M,N) in the intervals $MI \leq M \leq MF$ & $NI \leq N \leq NF$. [L/T].</u>
NV	<u>Number of mesh point receiving nonzero initial impulse velocity other than uniform velocity VR.</u>
Card 16*	<u>Each card 16 gives data for each mesh point receiving nonuniform initial impulse velocity; hence total number of card 16's must equal NV.</u>
M,N	<u>Mesh numbers of point receiving initial impulse velocity V other than uniform velocity VR.</u>
V	<u>Initial impulse velocity at mesh point (M,N) [L/T].</u>

3.3 Array Size, Memory Requirements and Computation Times

Often, the size of arrays must be adjusted in order for the program to accommodate a new problem. Such adjustments entail changing DIMENSION and COMMON statements to assure that array size equals or exceeds the maximum values of array indices required by the problem.

The maximum values of array indices are easily determined from the input data, as shall now be shown. First, denote the maximum values of the indices M,N,K,J and KJ (see Appendix C.1 for their definitions) by appending the subscript "max" to each. Then M_{\max} and N_{\max} depend on the number of mesh intervals in the η^1 and η^2 directions (specified on input card 2) and the boundary conditions along edges 3 and 2 respectively (card 4):

$$M_{\max} = \begin{cases} \text{MESH} + 2 & ; \text{IBCE3} = 1,3 \\ \text{MESH} + 3 & ; \text{IBCE3} = 2 \end{cases} \quad (3.1)$$

$$N_{\max} = \begin{cases} \text{NMESH} + 1 & ; \text{IBCE2} = 1,3 \\ \text{NMESH} + 2 & ; \text{IBCE2} = 2 \end{cases} \quad (3.2)$$

Moreover, K_{\max} equals the number of layers into which the shell is divided (card 2):

$$K_{\max} = \text{LAYER} \quad , \quad (3.3)$$

* Cards 15 and 16 omitted if LOAD = 1 or NCONT > 0.

J_{\max} equals the number of breaks in the polygonal approximation to the loading curve (Card 6):

$$J_{\max} = \text{NSFL}, \quad (3.4)$$

and KJ_{\max} is the product of K_{\max} and J_{\max} :

$$KJ_{\max} = \text{LAYER} * \text{NSFL} . \quad (3.5)$$

The arrays affected by changes in the maximum values of these indices are listed in Appendices C.2 and C.4. The maximum values of the remaining array indices usually need no adjusting because they are sufficiently large for most problems.

The storage of arrays constitutes the major portion of the memory requirements of REPSIL, with the rest of the memory requirements used to store the remainder of the program. The memory needed to store the remainder is more or less fixed and depends on the compiler that the computer uses; for example, the BRLESC computer at the BRL uses between 11,000 and 12,000 words for this purpose. The memory required to store the arrays depends on their size, specified by prescribing the maximum values of array indices, and hence may change with the problem being solved.

It is useful to have an estimate of the memory required by a problem, to see if the computer can accommodate it. For an estimate it is sufficient to consider only the large arrays — the two and three dimensional arrays. A count of these arrays, listed in Appendices C.2 and C.4, shows that there are 26 two dimensional (M,N) arrays, 3 three dimensional (M,N,KJ) arrays and 1 three dimensional (M,N,K) array. Hence, a problem whose maximum index values are M_{\max} , N_{\max} , K_{\max} and $KJ_{\max} \equiv K_{\max} \times J_{\max}$ will use

$$S_A = M_{\max} \times N_{\max} \times (26 + K_{\max} \times (3 \times J_{\max} + 1)) \quad (3.6)$$

words of memory to store these arrays. An estimate on the amount of memory a given computer uses to store the remainder of the program is most easily obtained as the difference between the total memory used on a given problem and S_A for the given problem. If this difference is S_R , then for any new problem a good estimate of the total number of words of memory needed is

$$S_T = S_R + S_A = S_R + M_{\max} \times N_{\max} \times (26 + K_{\max} \times (3 \times J_{\max} + 1)). \quad (3.7)$$

The maximum values M_{\max} , N_{\max} , K_{\max} and J_{\max} can also be used to estimate the time a given problem will take on a computer. This follows from the fact that the computer takes approximately the same time to solve the finite difference scheme for a given substress at a given layer and mesh point; hence, the total computation time is roughly proportional to the product of the number of substresses J_{\max} , of layers K_{\max} , of mesh points $M_{\max} \times N_{\max}$ and the total number of time steps

$$N_t \cong \text{MAXC} - \text{NCONT} . \quad (3.8)$$

Therefore, once the time T^* taken by the computer to solve a problem with the values J_{\max}^* , K_{\max}^* , M_{\max}^* , N_{\max}^* and N_t^* is established, then an estimate of the time T for any other problem with the values J_{\max} , K_{\max} , M_{\max} , N_{\max} and N_t is given by the ratio

$$\frac{T}{T^*} = \frac{M_{\max} \times N_{\max} \times J_{\max} \times K_{\max} \times N_t}{M_{\max}^* \times N_{\max}^* \times J_{\max}^* \times K_{\max}^* \times N_t^*} . \quad (3.9)$$

This relation only gives approximate times because it neglects such factors as the operations at boundary point differing from those at interior points, the different number of iterations for the plastic stress at different mesh points, the compile times not being proportional. However, the relation serves as a useful rule-of-thumb, giving over-estimates as the number of mesh intervals increases.

3.4 Continuation of Problem (Restart)

Every NRITE number of time steps information is written on the restart tape about the current state of the solution. As discussed in Chapter 2, this information is sufficient for the program to continue the solution from any of these prescribed time steps; such a continuation is called a restart. The restart of a problem entails certain necessary, advisable and permissible changes in the input data, as follow.

Necessary changes are changes without which the restart problem cannot be solved. These changes are confined to Card 3. The initial time step NCONT must be set equal to some multiple of NRITE at which time step there is written information on the restart tape; this is usually chosen as the last time step on the tape. Also, the last time step MAXC must be changed to a value greater than NCONT.

Advisable changes are changes that either reduce the amount of input data or assure meaningful output data. First, Cards 15 and 16 may be omitted. Second, the time steps specified on Cards 9, 10, and 11 at which output data is collected should be changed so as to fall within the new interval between NCONT and MAXC.

Permissible changes are changes that affect the collecting of restart and output data or the functioning of the damping operations. On Card 3 NRITE may be changed so that restart information is collected at a different interval. Also, on Card 8 the interval between the times at which surface strains are printed may be changed by altering NPRINT and a different selection of arrays may be printed by altering JCHK(J). On Card 5, DAMPF and DFACT, which control damping operations, may be freely changed. Also, if LPRESS and MDAMP were greater than the new value of NCONT, they may be changed.

Finally, a few words of caution. If it is planned to continue a problem, provide a tape to collect the restart information; otherwise, omit the restart tape. Any changes in the input data that affect the size of arrays should never be made. Lastly, note that there is no restriction on repeatedly continuing the solution of a problem.

4. DESCRIPTION OF OUTPUT

REPSIL outputs the results of calculations in two forms: printed output and plotted output. This chapter describes the various output options available and how these are controlled through the input data. Reference will be made to tables and figures in Chapter 5 as samples of printed and plotted output.

4.1 Printed Output

4.1.1 Input Data. The printed output begins with a title page reiterating the input data and giving the critical Δt resulting from the REPSIL stability check, see Tables 5.3 and 5.15. If $LOAD = 0$ or -1 , the uniform initial impulse velocity specified on Cards 15 and 16 will be printed out next, Table 5.16. There follows a print of the initial values of the Cartesian coordinates Y_i and pressure P at all mesh intersections (M,N) . Table 5.4 and 5.16 give samples of these arrays at select values of M . The user need not request any of the above prints; they are automatically produced by REPSIL for any initial run. For a restart run only the title page is printed.

4.1.2 Displacement Increment, Cartesian Coordinate, Pressure and Surface Normal Arrays. By setting the input variables $JCHK(I) = 1$ for $I = 1, 2, 3$ (see Section 3.2, Card 8), the values of the displacement increments U_i , Cartesian coordinates Y_i and pressures P , and surface normals SN_i at every intersection (M,N) are printed at each $NCYCH(J)$ time step (Card 9). Samples of these arrays at select values of M appear in Tables 5.5 - 5.7, 5.9 - 5.11 and 5.17 - 5.22. Notice that the displacement increments are for the time increment just preceding time step $NCYCH(J)$.

4.1.3 Energy Balance. The program prints the values of the kinetic energy CINET, strain energy STREN, plastic work PLAST and external work TNRG for the entire shell every NCYCH(J) time steps (Section 3.2, Card 9), as shown in Tables 5.13 and 5.23.

4.1.4 Stress Subincrement Array. The stress subincrement array LMAT (M,N,K) is printed every JCYNLP(J) time steps (Section 3.2, Card 10), see Tables 5.8, 5.12, 5.18, 5.20 and 5.22. The value of LMAT at the location (M,N,K) is an approximate measure of the amount of plasticity occurring there during the given time interval: if LMAT = 0 the stress increment lies within the yield surface and, hence, is elastic; otherwise LMAT equal the number of stress annuli outside the yield surface traversed by the stress increment, see Appendix B for a detailed description.

4.1.5 Surface Strains. The surface strains EPSSI(I), EPSS2(I), EPSANB(I) and EPSANG(I) at locations specified on input Card 13 are printed every NPRINT elapsed time steps (Card 8). These strains, shown in Tables 5.13 and 5.23, simulate the reading of strain gages alined at the angles indicated there relative to the η^1 direction, see also Figure 3.5.

4.1.6 Error Messages. An inability to satisfactorily calculate a plastic stress increment at a location (M,N,K) results in an error print, wherein the values of the quantities involved in computing the stress at this location are printed and the calculations terminate. This print occurs for two reasons: either the lead coefficient AA in the quadratic equation for TAMBDA is negative or the values of TAMBDA continue to remain complex even after the use of 100 stress subdivision. The reasons that both these results are unacceptable and do not permit the continuation of the solution are given in [3; Sect III].

4.2 Plotted Output

The plotted output is generated by a separate plotting program described in Appendix D. This program works from a tape generated by REPSIL on which plotting data is stored. The program employs the Cal Comp Standard Plotting Package SCOOP. The plots shown in this report are generated by the Cal Comp Model 780 Plotter.

4.2.1 Isometric and Cross-sectional Plots. REPSIL stores on tape the Cartesian coordinate array Y_i at the initial time step and at subsequent time steps as specified by the values of NC3DP(J) on Card 11. From these data the plotting program generates two types of plots at each of these time step: an isometric drawing of the distorted image of the finite difference mesh passing through the middle surface; and a pair of cross-sectional drawing through the $Y_1 = 0$ symmetry plane and a plane normal to the Y_2 axis, as specified in the input to the plotting program. The scale of the drawing and a factor to magnify the displacements from the initial position must also be specified as input. These plots are illustrated by Figures 5.3 and 5.8. The zeroth time step plots are

automatically generated without the user requesting them. Notice that these plots print the input data: the mesh number N of the crosssection, the scale and the magnification factor.

4.2.2 Energy, Displacement and Surface Strain Histories. At every time step REPSIL also stores on tape three groups of data: the kinetic energy CINET, strain energy STREN, plastic work PLAST, external work TNRG and, when the damping procedure is used, the damping work TDAMP; the components of the displacement at the location specified on Card 12; and the elongational surface strains in the η^1 and η^2 coordinate direction at the location specified on Card 13.

The plotting program, using the first group of data, plots a time history of the balance of energies during deformation, as illustrated in Figures 5.4 and 5.9; notice that graph b of Figure 5.9 is an enlargement of the last 400 microseconds of graph a to bring out the details of the energy balance during damping operations. In Figures 5.4 and 5.9a the top line represents the external work, with that in Figure 5.4 being due to a pressure loading and in Figure 5.9a due to an initial impulse velocity; in Figure 5.9b the external work line falls off the graph. In all three graphs, the bottom line is the kinetic energy and the line second from the bottom is the total energy of the shell. Hence, the difference between the bottom line and the second from bottom line represents the strain energy and the difference between the top line and the second from bottom line represents the energy dissipated. When damping operations are not used, as in Figure 5.4 and the first 405 microseconds in Figure 5.9, the energy dissipated is solely due to the plastic work. However, at the inception of the damping procedure (405 microseconds in Figure 5.9b) a third line appears dividing the energy dissipated into two parts: the plastic work represented by the difference between the top line and this new line and the damping work represented by the remaining difference.*

The second group of data is used by the plotting program to plot a time history of the components of displacement at a given location, as illustrated in Figures 5.5 and 5.10. The location as specified in the REPSIL input is printed with the plot.

* In principle, the plastic work and the damping work are monotone increasing function of time and when the external work is constant the total energy is monotone decreasing. That this is only approximately true of Figure 5.9b is a consequence of the numerical inaccuracy of the finite difference solution. For the same reason, in purely elastic problems it is found that the total energy oscillates about the external work rather than coinciding. However, an excessive rise of the total energy over the external work is usually an indication that something is going wrong with the solution, e.g. too large a time increment leading to a numerical instability.

The third group is used to plot time histories of the elongational strains on the surface of the shell in the η^1 and η^2 coordinate directions (see Figures 3.6 - 3.8 for the coordinate directions of the various geometries employed, as well as Figure 3.5) at prescribed locations. These plots are illustrated by Figures 5.6 and 5.11, with the solid line being the strain in the η^1 direction and the dashed line the strain in the η^2 direction. The locations as specified in REPSIL, including which bounding surface, are reproduced on the plots. A maximum of 6 locations can be plotted at present.

5. EXAMPLE PROBLEMS

This chapter demonstrates the use of REPSIL to solve two typical shell problems. Correct preparation of input data, including proper implementation of various options available in REPSIL is illustrated. Portions of the printed and plotted output are displayed. These serve as checks on the proper functioning of the code, especially useful to users adapting REPSIL to their computers. For reasons of economy, no attempt is made to discuss the accuracy or significance of results.

5.1 Example 1: Pressure Loaded Flat Plate

The first problem involves finding the deformation history of a simple-supported, rectangular, steel plate subjected to loads resulting from the detonation of an explosive charge. Figure 5.1 shows the dimensions and orientation of the plate and charge. This example illustrates the use of the following REPSIL options.

- Flat plate initial geometry
- Analytically specified pressure loading
- Hinged edge and symmetry edge boundaries
- Strain hardening - strain rate dependent material behavior
- Problem restart or continuation

The material properties of the steel in the elastic range are:

$$\text{Young's modulus } E = 30 \times 10^6 \text{ psi}$$

$$\text{Poisson's ratio } \nu = 0.3$$

$$\text{Mass density } \rho = 7.3235 \times 10^{-4} \frac{\text{lb} \cdot \text{sec}^2}{\text{in}^4}$$

In the plastic range the steel is assumed to strain harden in a strain rate dependent manner. This behavior is approximated by using a 3 substress model with the following values of the parameters.

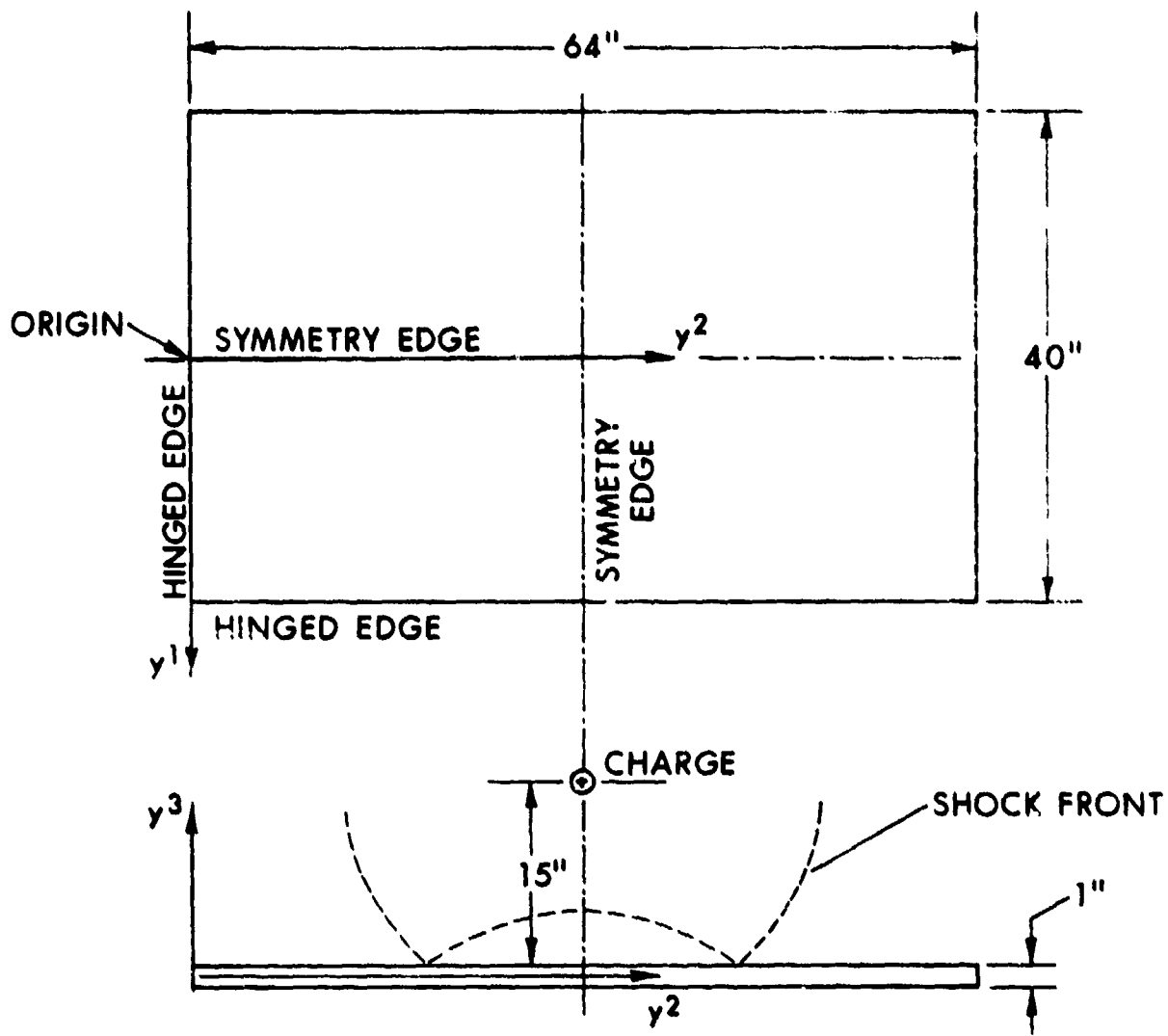


Figure 5.1 Geometry for Example Problem 1

$\sigma_1 = 78,000 \text{ psi}$	$\epsilon_1 = .0026 \text{ in/in}$	$d_1 = 40 \text{ in/in-sec}$	$P_1=5$
$\sigma_2 = 120,000 \text{ psi}$	$\epsilon_2 = .0082 \text{ in/in}$	$d_2 = 400 \text{ in/in-sec}$	$P_2=3$
$\sigma_3 = 180,000 \text{ psi}$	$\epsilon_3 = .0482 \text{ in/in}$	$d_3 = 4000 \text{ in/in-sec}$	$P_3=1$

Figure 5.2 shows the resulting polygonal approximations to the uniaxial loading curves at four strain rate levels.

The pressure loading is simulated by programming into subroutine PRESS the pressure relation

$$P(m,n) = \begin{cases} 0; & \text{for } t < t_a \\ \frac{225 \times 24465.5}{225 + r^2(m,n)} e^{-13000(t-t_a)} & ; \text{ for } t \geq t_a, \end{cases}$$

where $r(m,n)$ is the distance from the center of the plate to the mesh intersection (m,n) and

$$t_a = \frac{\sqrt{225 + r^2(m,n)} - 15}{144000}$$

is the arrival time of the shock front at (m,n) .

Taking advantage of the two fold symmetry of the problem, only the lower right quarter of the plate, as shown in Figure 5.1, is treated. Consequently, edges 1 and 2 are symmetry boundaries and edges 3 and 4 hinged boundaries (compare with Figures 3.1 and 3.6). Also, only the half width and half length of the plate are prescribed as input dimensions. The problem uses a 20 x 32 square mesh and 4 layers through the thickness. A time increment of 4 microseconds is prescribed, a figure well below the critical time increment predicted by the REPSIL stability criteria (see Table 5.3).

The problem is solved in three successive runs. The initial run is set for a maximum of 400 time steps, giving a solution for the first 1600 microseconds. Table 5.1 gives the input for this run in the same order as outlined in Table 3.1. The first restart or continuation run is prescribed for the next 400 time step. Only the input data on Cards 3, 9, 10 and 11 are changed, as shown in Table 5.2. The second restart run continues the solution 389 time steps further and requires changes in Card 3, 9, 10 and 11 as before.

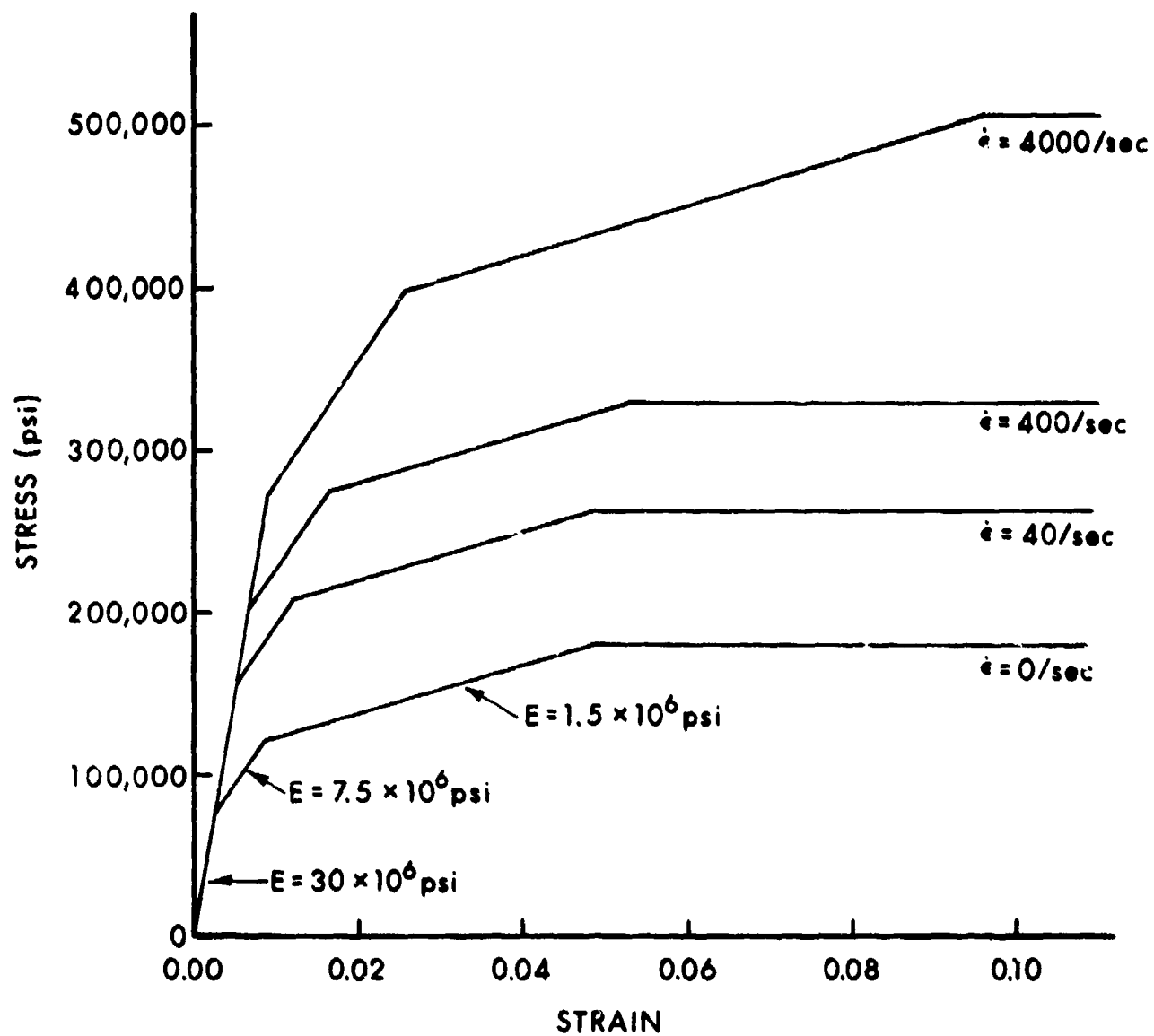


Figure 5.2 Polygonal Approximations to Uniaxial Loading Curves at Four Levels of Strain Rate

Table 5.1 Input Data Cards for Initial Run of Flat Plate Problem

Card 1	EXAMPLE 1 FLAT PLATE WITH PRESSURE LOADING						
Card 2	20	32	4	.100000E 01			
Card 3	400	0	100	.400000E-05			
Card 4	2	2	3	3			
Card 5	1	1500	1500	0.0	0.0		
Card 6	.300000E 08	.300000E 00	.780000E 05	.732350E-03	.100000E 00	3	1
Card 7a	78000.0	.0026		40.0	5.0		
Card 7b	120000.0	.0082		400.0	3.0		
Card 7c	180000.0	.0482		4000.0	1.0		
Card 8	100	1	1	1			
Card 9	4	100	200	300	400		
Card 10	4	100	200	300	400		
Card 11	4	100	200	300	400		
Card 12	0.0	32.0			6		
Card 13a	0.0	32.0	45.0	22.5		0	
Card 13b	0.0	32.0	45.0	22.5		1	
Card 13c	0.0	0.0	45.0	22.5		0	
Card 13d	0.0	0.0	45.0	22.5		1	
Card 13e	20.0	32.0	45.0	22.5		0	
Card 13f	20.0	32.0	45.0	22.5		1	
Card 14	32.0	20.0					

Table 5.2 Input Data Cards Changed for the First Restart Run

Card 3	800	400	100	.400000E-05
Card 9	4	500	600	700 800
Card 10	4	500	600	700 800
Card 11	4	500	600	700 800

Sample listings of the printed output for the initial run are given in Tables 5.3 - 5.13. The program prints the U_i (M,N), Y_i (M,N), S_{Ni} (M,N), arrays over the entire range of (M,N), of which Tables 5.4 - 5.7 and 5.9 - 5.11 give the values for $M = 2, 10$ and $1 \leq N \leq 33$ (i.e. the values along $y^1 = 0$ and $y^1 = 8$, see Figure 5.1). Also the program prints the LMAT (M,N,K) array at every station through the thickness, of which Tables 5.6 and 5.12 give the array values at $K = 3, 4$. Table 5.13 shows the surface strains and energy balance prints.

Figures 5.3 - 5.6 present examples of the plotted output as obtained on the Cal Comp Model 780 Plotter using the plotting program described in Appendix E. Notice that the plots with time as the abscissa, Figures 5.4 - 5.6, are for the initial run plus the two successive restarts, an automatic feature of the plotting program.

Table 5.3 First Page of Printed Output Summarizing the Input Data and Results of Stable Time Increment Check

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ORL REPRIL CODE

EXAMPLE 1 FLAT PLATE WITH PRESSURE LOADING

20 MESHES IN THE X1A1 DIRECTION (DETA1 =0.100000E 01)
32 MESHES IN THE X2A2 DIRECTION (DETA2 =0.100000E 01)

BENDING TIME INCREMENT= 0.514304E-05
MEMBRANE TIME INCREMENT= 0.668953E-05
INPUT TIME INCREMENT= 0.400000E-05

TIME INCREMENT USED BY REPRIL= 0.400000E-05

YOUNG'S MODULUS =0.300000E 08
POISSON'S RATIO =0.300000E 00 YIELD STRESS =0.780000E 05
MASS DENSITY =0.732390E-03 THICKNESS =0.100000E 01

START AT TIME STEP 0
FINAL TIME STEP 400
SURFACE STRAINS EVERY 100 TIME STEP
RESTART WRITE EVERY 100 TIME STEP

LAYER = 4 NSTRN = 6
LOAD = 1 LPRESS = 1500

BOUNDARY CONDITIONS
1/2/3 = CLAMPED/SYMMETRY/HINGED
EDGE1 = 2
EDGE2 = 2
EDGE3 = 3
EDGE4 = 3

PRINT OPTION CONTROL CARD
0/1 = NO PRINT/PRINT
1 DISPLACEMENT INCREMENTS
1 CARTESIAN COORDINATES, PRESSURE
1 SURFACE NORMAL VECTOR COMPONENTS

PRINT INFORMATION AT THE FOLLOWING TIME STEPS
100 200 300 400
PRINT L MATRIX (LMAT) AT THE FOLLOWING TIME STEPS
100 200 300 400
3-D PLOTS FOR THE FOLLOWING TIME STEPS
100 200 300 400
CONSTITUTIVE RELATION ELASTOPLASTIC-WORK HARDENING-STRAIN RATE DEPENDENT
STRESS-STRAIN APPROXIMATION HAS 3 SURF LAYERS

STRESS-STRAIN AND STRAIN RATE PARAMETERS
L SSIG(J) SEPS(J) DSR(J) 1/PSR(J)
1 7.800000E 04 2.6000000E-03 4.0000000E 01 2.0000000E-01
2 1.2000000E 05 8.2000000E-03 4.0000000E 02 3.3333334E-01
3 1.8000000E 05 4.8200000E-02 4.0000000E 03 1.0000000E 00

START DAMPING AFTER TIME STEP 1500 TIME =0.6000E-02
DAMPF =0.0000E 00 DFACT =0.0000E 00

```


Table 5.5 Sample of Displacement Increments Array During Time Increment 199 to 200 for M = 2 and 10

DISPLACEMENT INCREMENTS BETWEEN T.S. 199 AND 200				
M	N	U1(M,N)	U2(M,N)	U3(M,N)
2	1	0.000000000000000E 00	0.000000000000000E 00	0.000000000000000E 00
	2	0.000000000000000E 00	-0.1381206393653739E-04	-0.4168488328986917E-03
	3	0.000000000000000E 00	-0.2375343470663745E-04	-0.8474448913925587E-03
	4	0.000000000000000E 00	-0.4720040488698901E-04	-0.1423860541853202E-02
	5	0.000000000000000E 00	-0.5008229783695276E-04	-0.1779364250648565E-02
	6	0.000000000000000E 00	-0.9719565487315917E-04	-0.2303383105088261E-02
	7	0.000000000000000E 00	-0.8292546569873348E-04	-0.2736982933096866E-02
	8	0.000000000000000E 00	-0.1024470004462928E-03	-0.3295136604832684E-02
	9	0.000000000000000E 00	-0.1083337421897568E-03	-0.3708735256807441E-02
	10	0.000000000000000E 00	-0.1310749969676948E-03	-0.4504531168145721E-02
	11	0.000000000000000E 00	-0.1536371012469078E-03	-0.5020667473699230E-02
	12	0.000000000000000E 00	-0.2164696823739369E-03	-0.5522972854020365E-02
	13	0.000000000000000E 00	-0.2042007798745957E-03	-0.6104938562385423E-02
	14	0.000000000000000E 00	-0.1892760728979844E-03	-0.6431512541719008E-02
	15	0.000000000000000E 00	-0.2163818744548673E-03	-0.6974581950601418E-02
	16	0.000000000000000E 00	-0.1545698046836804E-03	-0.7231816325527253E-02
	17	0.000000000000000E 00	-0.2130450507842500E-03	-0.7705063805455238E-02
	18	0.000000000000000E 00	-0.2293594139886983E-03	-0.7850570817561841E-02
	19	0.000000000000000E 00	-0.1977772677079695E-03	-0.8172739998317206E-02
	20	0.000000000000000E 00	-0.2003865942925029E-03	-0.8185992246671717E-02
	21	0.000000000000000E 00	-0.1976957044141982E-03	-0.9011574519626505E-02
	22	0.000000000000000E 00	-0.1176605779205775E-03	-0.7838282354107286E-02
	23	0.000000000000000E 00	-0.2183472079786409E-03	-0.7625775726690378E-02
	24	0.000000000000000E 00	-0.1198983869082797E-03	-0.7495273269677910E-02
	25	0.000000000000000E 00	-0.8685676002769613E-04	-0.7320251921932451E-02
	26	0.000000000000000E 00	-0.1141018167286585E-03	-0.7177705436477719E-02
	27	0.000000000000000E 00	-0.7751586966518130E-04	-0.7359178246913641E-02
	28	0.000000000000000E 00	-0.5338714258114968E-04	-0.7323212650284600E-02
	29	0.000000000000000E 00	-0.8783717403050155E-04	-0.7419899805486476E-02
	30	0.000000000000000E 00	-0.7163153237329932E-04	-0.755131572122282E-02
	31	0.000000000000000E 00	-0.1932272522140175E-04	-0.7910157657475296E-02
	32	0.000000000000000E 00	0.6771410273254223E-04	-0.7656193690124109E-02
	33	0.000000000000000E 00	0.000000000000000E 00	-0.8156761964262543E-02
10	1	0.000000000000000E 00	0.000000000000000E 00	0.000000000000000E 00
	2	-0.9367931064029214E-06	-0.1934307357308220E-04	-0.2212602323124786E-03
	3	-0.5019807073676166E-05	-0.8790685298558954E-05	-0.4354805858474219E-03
	4	-0.1463650466932884E-05	-0.1839791110262883E-04	-0.5046060999896601E-03
	5	-0.2577635059557088E-04	0.5542688208307563E-05	-0.6259591087167490E-03
	6	-0.7332686709386931E-05	0.2139139744171304E-04	-0.8831368253513934E-03
	7	0.9736339864651590E-05	0.1013419980963647E-05	-0.1197763245342692E-02
	8	0.1444166746512143E-04	0.1895683078942489E-04	-0.1504622949863701E-02
	9	0.4252064687469055E-05	0.4523530138226414E-05	-0.1812568261607337E-02
	10	0.3797795316647549E-04	-0.1243705795220926E-04	-0.2218658747899459E-02
	11	0.2065600270568753E-04	0.1046776067769080E-04	-0.2479624271162489E-02
	12	0.3675218863317805E-04	-0.3867953625348610E-04	-0.2816299042642461E-02
	13	-0.1268702946147533E-05	-0.2245857195802538E-04	-0.3038635553745534E-02
	14	0.4087752628936709E-04	-0.5080187558670575E-04	-0.3462661713695602E-02
	15	0.5603813836442154E-04	-0.5771193611169779E-04	-0.3769302093188341E-02
	16	0.6593726844771896E-04	-0.4061778392411700E-04	-0.4133857448863767E-02
	17	0.1067078279420197E-03	-0.9097365160805708E-04	-0.4594870595072928E-02
	18	0.9490949490945175E-04	-0.9357838581299170E-04	-0.4910451120878080E-02
	19	0.1013614742883946E-03	-0.9812288277081476E-04	-0.5124806084896764E-02
	20	0.9890476997011357E-04	-0.6369638946751248E-04	-0.5326621093544308E-02
	21	0.1418853757259157E-03	-0.5487998986584590E-04	-0.5229782576825541E-02
	22	0.1754142348527070E-03	-0.2434882159727478E-04	-0.5173626422641814E-02
	23	0.1694491410346638E-03	-0.6753179360222226E-04	-0.4968211830931578E-02
	24	0.1404496526327821E-03	-0.8052488354613026E-04	-0.4991519678772019E-02
	25	0.1562774257328265E-03	-0.4054505851603422E-04	-0.4914070665289152E-02
	26	0.1019830274512511E-03	0.1517890049126742E-04	-0.4762491235036106E-02
	27	0.1569721740601215E-03	-0.4144627974546997E-04	-0.4788485323506877E-02
	28	0.1627043896126920E-03	-0.2998060196229850E-04	-0.4702140416624566E-02
	29	0.2069522821120799E-03	-0.1194682230405943E-04	-0.4716081576773645E-02
	30	0.2219073771893363E-03	-0.1061741629519551E-03	-0.4879693121440284E-02
	31	0.2211373807693563E-03	0.5802377953685049E-04	-0.4767498558516651E-02
	32	0.156550647510459E-03	0.3517116302069398E-04	-0.4659847749990656E-02
	33	0.312591618292227E-03	0.000000000000000E 00	-0.4805206926758087E-02

Table 5.6 Sample of Cartesian Coordinates and Pressure Arrays at Time Step 200 for M = 2 and 10

TIME STEP 200		TIME 0.8000000E-03		PRESSURE		
M	N	V1(M,N, 200)	CARTESIAN COORDINATES V2(M,N, 200)	V3(M,N, 200)		P(M,N)
2	1	0.00000000000000000000 00	0.00000000000000000000 00	0.00000000000000000000 00	C.CCCCCCCCCCCCCCCCCC CC	
2	2	0.00000000000000000000 00	0.9999659602400206E 00	-0.5787704347785275E-01	0.0167919806216318E 00	
3	3	0.00000000000000000000 00	0.1999923771346841E 01	-0.1194194180527340E 00	0.7940755914351119E 00	
4	4	0.00000000000000000000 00	0.2999951592891313E 01	-0.1735274702788973E 00	0.7732311151081857E 00	
5	5	0.00000000000000000000 00	0.399994225224123E 01	-0.2321244121180228E 00	0.7541906674416129E 00	
6	6	0.00000000000000000000 00	0.4999819444485916E 01	-0.2908728205803803E 00	0.73689426478162E 00	
7	7	0.00000000000000000000 00	0.5999734150757253E 01	-0.3497134110940949E 00	0.7212894718344779E 00	
8	8	0.00000000000000000000 00	0.6999629057411831E 01	-0.4088972533546703E 00	0.70733105C3354542E 00	
9	9	0.00000000000000000000 00	0.7999518618175310E 01	-0.4680544849847771E 00	0.6949805537267748E 00	
10	10	0.00000000000000000000 00	0.8999332246246000E 01	-0.5274922347870467E 00	0.6842058641830763E 00	
11	11	0.00000000000000000000 00	0.9999129364243030E 01	-0.5872477041867224E 00	0.6749806236460235E 00	
12	12	0.00000000000000000000 00	0.10998733584464E 02	-0.6469788637248207E 00	0.6672835121779712E 00	
13	13	0.00000000000000000000 00	0.1199857482334034E 02	-0.7058701201216622E 00	0.6610973131379158E 00	
14	14	0.00000000000000000000 00	0.1299821908145814E 02	-0.7634156479555029E 00	0.6564076886322452E 00	
15	15	0.00000000000000000000 00	0.1399810552353361E 02	-0.8203182542318846E 00	0.652021572C448323E 00	
16	16	0.00000000000000000000 00	0.1499790077229687E 02	-0.8762680872269327E 00	0.6514650674768556E 00	
17	17	0.00000000000000000000 00	0.1599768831595512E 02	-0.9313939286794805E 00	0.6511807318168086E 00	
18	18	0.00000000000000000000 00	0.1699747641950878E 02	-0.9854320513624732E 00	0.65232491071540835E 00	
19	19	0.00000000000000000000 00	0.1799732157523514E 02	-0.1039145206000769E 01	0.6548593978182595E 00	
20	20	0.00000000000000000000 00	0.1899702545286974E 02	-0.1091934426954038E 01	0.65873417CC359105E 00	
21	21	0.00000000000000000000 00	0.1999693108628917E 02	-0.1143305319578919E 01	0.6638731501693902E 00	
22	22	0.00000000000000000000 00	0.2099675662475275E 02	-0.1193162672642931E 01	0.67017121CC147296E 00	
23	23	0.00000000000000000000 00	0.2199646436769383E 02	-0.1241584269589963E 01	0.6774859788583391E 00	
24	24	0.00000000000000000000 00	0.2299653673669818E 02	-0.1288421227415664E 01	0.6856308002228533E 00	
25	25	0.00000000000000000000 00	0.2399629462953092E 02	-0.1333424987944081E 01	0.694369107673655CE 00	
26	26	0.00000000000000000000 00	0.2499631710599600E 02	-0.1376328957298834E 01	0.7034123458259726E 00	
27	27	0.00000000000000000000 00	0.2599630172617388E 02	-0.1416967904244330E 01	0.712421600C3C5318E 00	
28	28	0.00000000000000000000 00	0.2699655985293373E 02	-0.14545646612176837E 01	0.7210181691608256E 00	
29	29	0.00000000000000000000 00	0.2799682030232250E 02	-0.1487699217488606E 01	0.72879937766728E 00	
30	30	0.00000000000000000000 00	0.2899739719405393E 02	-0.15156154667688002E 01	0.7353683952219711E 00	
31	31	0.00000000000000000000 00	0.2999807554187096E 02	-0.1536646919434049E 01	0.7403602763472827E 00	
32	32	0.00000000000000000000 00	0.3099913507074701E 02	-0.1549453092611953E 01	0.74343829591273441E 00	
33	33	0.00000000000000000000 00	0.32000000000000000000 00	-0.1554093976030353E 01	0.7454545913421148E 00	
10	1	0.89300000000000000000 00	0.00000000000000000000 00	0.00000000000000000000 00	0.00000000000000000000 00	
10	2	0.7939732288727414E 01	0.9999173396170459E 00	-0.5149942277337489E-01	0.841857590869C388E 00	
10	3	0.7939590794483458E 01	0.1999912262528011E 01	-0.1031642973493428E 00	0.817939027534618E 00	
10	4	0.7999536726516393E 01	0.2999829982678408E 01	-0.1552214871938103E 00	0.7958936023236643E 00	
10	5	0.7979790304261775E 01	0.3999814784672533E 01	-0.2072499291925028E 00	0.7756447922254503E 00	
10	6	0.8000386257755872E 01	0.4999748951929792E 01	-0.2592449504297181E 00	0.7571231874321815E 00	
10	7	0.8000492779399781E 01	0.5999702780932307E 01	-0.3110198833167050E 00	0.7402660064382205E 00	
10	8	0.8001315429169993E 01	0.6999647236652598E 01	-0.3628448816667178E 00	0.729016595C967601E 00	
10	9	0.8001667152741545E 01	0.7999536035512374E 01	-0.4130536777016116E 00	0.71132357649C2778E 00	
10	10	0.800241153107C063E 01	0.8999357261968186E 01	-0.4678280098973082E 00	0.6991417376968476E 00	
10	11	0.8003305992049259E 01	0.999924161679841E 01	-0.5205429355C63548E 00	0.6884283237008068E 00	
10	12	0.8004221716739729E 01	0.1099895057387506E 02	-0.5726151304326850E 00	0.679145207C864633E 00	
10	13	0.8005180335642457E 01	0.1199892013007349E 02	-0.6232931835693734E 00	0.671256396233698E 00	
10	14	0.8006250598936349E 01	0.1299864029760508E 02	-0.6729463838276628E 00	0.6647271425937266E 00	
10	15	0.8007273446325064E 01	0.1399861653185099E 02	-0.721623A835841471E 00	0.6595224989188151E 00	
10	16	0.8008465342822045E 01	0.1499840784600662E 02	-0.7696297739780766E 00	0.6556055862873217E 00	
10	17	0.8009655283411928E 01	0.1599835272143853E 02	-0.8166219981386795E 00	0.652935533377289E 00	
10	18	0.80109713204755921E 01	0.1699813101678216E 02	-0.8627041446464915E 00	0.6514650674768526E 00	
10	19	0.8012060269510901E 01	0.1799805803583359E 02	-0.9078296898116915E 00	0.6511377286251281E 00	
10	20	0.8013294971527412E 01	0.1899783435451526E 02	-0.9518733515048169E 00	0.6518848172342464E 00	
10	21	0.8014521506989777E 01	0.1999784269054336E 02	-0.9942925143136804E 00	0.6536221241927485E 00	
10	22	0.8015001097612963E 01	0.2099767461723362E 02	-0.1035153785756821E 01	0.656246691E0C11299E 00	
10	23	0.801734807330956E 01	0.2199764641700348E 01	-0.1074382423047936E 01	0.6596339164C48673E 00	
10	24	0.8019707116027690E 01	0.2299743323580055E 02	-0.1112274072629487E 01	0.663635449472125E 00	
10	25	0.8020208553114021E 01	0.2399751727890038E 02	-0.1148709762267235E 01	0.6680784786228699E 00	
10	26	0.8021590148948941E 01	0.2499745502537334E 02	-0.1183757954084259E 01	0.6727670346418C82E 00	
10	27	0.8023205770228908E 01	0.2599759027155849E 02	-0.1217172482523426E 01	0.677489788583391E 00	
10	28	0.8024477136090069E 01	0.2649770094731253E 02	-0.1248121777988287E 01	0.6820081223591669E 00	
10	29	0.8025939743951624E 01	0.2799788291673956E 02	-0.1275942349123790E 01	0.686104594049458E 00	
10	30	0.8026915356775315E 01	0.2899823638221196E 02	-0.1299234403183508E 01	0.689580202247872E 00	
10	31	0.8027934473152458E 01	0.2999879817848022E 02	-0.1316989443710241E 01	0.6921774026142735E 00	
10	32	0.8029359966594166E 01	0.3099931135186901E 02	-0.1328353277272808E 01	0.6938130129742256E 00	
10	33	0.8028805322487589E 01	0.32000000000000000000 00	-0.1332175654565791E 01	0.694369187487365CE 00	

Table 5.7 Sample of Surface Normal Components Array at Time Step 200 for M = 2 and 10

TIME STEP		200	TIME 0.40000000E-03		
		SURFACE NORMAL VECTOR COMPONENTS			
M	N	SN1(M,N)	SN2(M,N)	SN3(M,N)	
2	1	0.0000000000000000E 00	0.5794857481663093E-01	0.9983195694148850E 00	00
	2	0.0000000000000000E 00	0.5761603959888880E-01	0.9982388162824138E 00	00
	3	0.0000000000000000E 00	0.5772939082038021E-01	0.9983322680528201E 00	00
	4	0.0000000000000000E 00	0.5825372372086324E-01	0.9983018099115385E 00	00
	5	0.0000000000000000E 00	0.5857560281164920E-01	0.9982829752906997E 00	00
	6	0.0000000000000000E 00	0.5869835111796001E-01	0.9982757652953579E 00	00
	7	0.0000000000000000E 00	0.5891532016478697E-01	0.9982629839124961E 00	00
	8	0.0000000000000000E 00	0.5907356954013085E-01	0.9982536317899309E 00	00
	9	0.0000000000000000E 00	0.5920226871409443E-01	0.9982460074446103E 00	00
	10	0.0000000000000000E 00	0.5950259421219625E-01	0.9982281509164218E 00	00
	11	0.0000000000000000E 00	0.5965477281262225E-01	0.9982190681812656E 00	00
	12	0.0000000000000000E 00	0.5922352270819250E-01	0.9982447467219811E 00	00
	13	0.0000000000000000E 00	0.5813488481200881E-01	0.9983067373993550E 00	00
	14	0.0000000000000000E 00	0.5714396880024767E-01	0.9983659483528489E 00	00
	15	0.0000000000000000E 00	0.5634554452449292E-01	0.9984113278665453E 00	00
	16	0.0000000000000000E 00	0.5546391738165598E-01	0.9984606922000890E 00	00
	17	0.0000000000000000E 00	0.5451239004557585E-01	0.9985130942213623E 00	00
	18	0.0000000000000000E 00	0.5380745730216241E-01	0.9985513294461513E 00	00
	19	0.0000000000000000E 00	0.5318780513532761E-01	0.9985845269104094E 00	00
	20	0.0000000000000000E 00	0.5201994904933800E-01	0.9986460458545381E 00	00
	21	0.0000000000000000E 00	0.5055619382739656E-01	0.9987212179311304E 00	00
	22	0.0000000000000000E 00	0.4909143323011414E-01	0.9987942887218635E 00	00
	23	0.0000000000000000E 00	0.4758056388384516E-01	0.9988674035829255E 00	00
	24	0.0000000000000000E 00	0.4587590520799637E-01	0.9989471464103327E 00	00
	25	0.0000000000000000E 00	0.4391628158072367E-01	0.9990352147007244E 00	00
	26	0.0000000000000000E 00	0.417349153779273E-01	0.9991287188538C73E 00	00
	27	0.0000000000000000E 00	0.3908319619295600E-01	0.99923596C0091178E 00	00
	28	0.0000000000000000E 00	0.3633441056067107E-01	0.9993755447429C10E 00	00
	29	0.0000000000000000E 00	0.3049895754374618E-01	0.999534798581957E 00	00
	30	0.0000000000000000E 00	0.2445118785445006E-01	0.999701025013219E 00	00
	31	0.0000000000000000E 00	0.1890121029083301E-01	0.9998571643443403E 00	00
	32	0.0000000000000000E 00	0.8714811380069492E-02	0.9999620253102665E 00	00
	33	0.0000000000000000E 00	0.0000000000000000E 00	0.1000000000000000E 01	00
10	1	0.0000000000000000E 00	0.5135303855117731E-01	0.9986805622577925E 00	00
	2	-0.1669875511733491E-02	0.5151550387932586E-01	0.9986707988000022E 00	00
	3	-0.3433129381139298E-02	0.5179373465996648E-01	0.9986517026530844E 00	00
	4	-0.5315371846915252E-02	0.5197480191562081E-01	0.9986342507584856E 00	00
	5	-0.7068453344170014E-02	0.5194390815117043E-01	0.9986249883582444E 00	00
	6	-0.9883560520375210E-02	0.5181929040065411E-01	0.9986169653550119E 00	00
	7	-0.1084420144573609E-01	0.5173521629106604E-01	0.9986019580845619E 00	00
	8	-0.1272981648847809E-01	0.5195419283062555E-01	0.9985683324493358E 00	00
	9	-0.1482421940176093E-01	0.5243168492688684E-01	0.9985144770782527E 00	00
	10	-0.1678118036782868E-01	0.5271034011176938E-01	0.9984688337903860E 00	00
	11	-0.1892192880714826E-01	0.52335932174507905E-01	0.9984500448447374E 00	00
	12	-0.2092210665899830E-01	0.5129911142981256E-01	0.998464157481061E 00	00
	13	-0.2308955639646668E-01	0.5012038509115521E-01	0.9984762487829796E 00	00
	14	-0.2523982722416899E-01	0.49124082466675829E-01	0.998472230610994E 00	00
	15	-0.2740011884383273E-01	0.4830305198158600E-01	0.9984568336471078E 00	00
	16	-0.2967589225060644E-01	0.4746629210724347E-01	0.9984319168239367E 00	00
	17	-0.3171868369336157E-01	0.4650798143124404E-01	0.9984142090174535E 00	00
	18	-0.3395643679461171E-01	0.4557765871688824E-01	0.9983835121966411E 00	00
	19	-0.3615365871932697E-01	0.4456155915458730E-01	0.9983522326348144E 00	00
	20	-0.3840959818430712E-01	0.4321293880032829E-01	0.9983272733365341E 00	00
	21	-0.4075611307752912E-01	0.4162796652239704E-01	0.9983015835157338E 00	00
	22	-0.4320748193769214E-01	0.4003825031897457E-01	0.9982635183236493E 00	00
	23	-0.4536163035890641E-01	0.3855777476909315E-01	0.9982262371306604E 00	00
	24	-0.4759602745287188E-01	0.3716631855114924E-01	0.9981749761119338E 00	00
	25	-0.4986118137038644E-01	0.3576105269169298E-01	0.9981157296139218E 00	00
	26	-0.5215152728775184E-01	0.3424158140153603E-01	0.9980519687394423E 00	00
	27	-0.5400547021522983E-01	0.3217728766428866E-01	0.9980220595484860E 00	00
	28	-0.5563692932312091E-01	0.2939845058623659E-01	0.9980181677698171E 00	00
	29	-0.5699550374619865E-01	0.2556903728477747E-01	0.9980469612623682E 00	00
	30	-0.5798106291915533E-01	0.20533116650318789E-01	0.9981065010656085E 00	00
	31	-0.5902355804941417E-01	0.1456730969411512E-01	0.9981502958389698E 00	00
	32	-0.5990213295317994E-01	0.7600774562291877E-02	0.9981992408392105E 00	00
	33	-0.5998980305902625E-01	0.0000000000000000E 00	0.9982050547197992E 00	00

Table 5.8 Sample of Stress Subincrement Array at Time Step 200 for Layers K = 3 and 4

LMAT(M,N, 3)

N	M=	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
1		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24		0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
25		0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
26		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27		0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0
28		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29		1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30		1	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
31		0	1	1	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
32		0	1	0	0	1	0	0	1	0	0	0	1	1	0	0	0	0	0	0	0	0
33		0	1	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0

LMAT(M,N, 4)

N	M=	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
1		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21		1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
22		0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
23		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24		1	0	0	0	1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0
25		1	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
26		0	1	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
27		0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28		1	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
29		1	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
30		0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
31		0	1	1	0	0	0	1	0	0	0	0	0	0	0	0	1	1	0	0	0	0
32		0	1	0	0	1	0	0	1	0	0	0	0	1	0	0	1	0	0	0	0	0
33		0	1	0	0	1	0	0	0	0	0	0	1	0	0	1	0	1	0	0	0	0

Table 5.9 Sample of Displacement Increments Array During Time Increment 399 to 400 for M = 2 and 10

DISPLACEMENT INCREMENTS BETWEEN T.S. 399 AND 400				
M	N	U1(M,N)	U2(M,N)	U3(M,N)
2	1	0.0000000000000000E 00	0.0000000000000000E 00	0.0000000000000000E 00
	2	0.0000000000000000E 00	-0.7827086426084424E-04	-0.1887415645534234E-03
	3	0.0000000000000000E 00	-0.5242663369540224E-04	-0.1525946892134154E-03
	4	0.0000000000000000E 00	0.1259292905004027E-04	-0.3351811502311134E-03
	5	0.0000000000000000E 00	0.6876599370272482E-03	-0.1929053428044718E-03
	6	0.0000000000000000E 00	-0.1612526561347885E-03	-0.2397995865387863E-03
	7	0.0000000000000000E 00	0.1583384681163329E-04	-0.2566655180610989E-03
	8	0.0000000000000000E 00	-0.1358299143285851E-03	-0.3740715647691516E-03
	9	0.0000000000000000E 00	-0.7863966704239485E-04	-0.3440130872141165E-03
	10	0.0000000000000000E 00	-0.9831094348567919E-04	-0.4518981913863369E-03
	11	0.0000000000000000E 00	-0.2156389773652301E-03	-0.1262405221485340E-03
	12	0.0000000000000000E 00	-0.2008738904093247E-03	-0.9512025336342927E-04
	13	0.0000000000000000E 00	-0.4747457837557605E-04	0.3013084685941810E-03
	14	0.0000000000000000E 00	0.8123271123307386E-04	0.5622971945173987E-03
	15	0.0000000000000000E 00	-0.1962291869902040E-04	0.9676818325278087E-03
	16	0.0000000000000000E 00	0.2551052649710283E-04	0.1258792139171492E-02
	17	0.0000000000000000E 00	0.5168390723675440E-05	0.1708308245429524E-02
	18	0.0000000000000000E 00	-0.6688755092780272E-04	0.2062872361405049E-02
	19	0.0000000000000000E 00	0.627118555497538E-04	0.2503357487101452E-02
	20	0.0000000000000000E 00	-0.1107670087610243E-03	0.2915668046368819E-02
	21	0.0000000000000000E 00	0.7423969718964788E-04	0.3062659663769643E-02
	22	0.0000000000000000E 00	0.7643984354499332E-04	0.3299257071485427E-02
	23	0.0000000000000000E 00	-0.5225682527510861E-04	0.3319655856228385E-02
	24	0.0000000000000000E 00	-0.271979446518273E-04	0.3269716456263198E-02
	25	0.0000000000000000E 00	0.5203559279456842E-04	0.3152022264604700E-02
	26	0.0000000000000000E 00	0.1377554471292373E-03	0.3147978291306039E-02
	27	0.0000000000000000E 00	0.1667237905957889E-03	0.3001532797809580E-02
	28	0.0000000000000000E 00	-0.3762807274488110E-04	0.2711703056054493E-02
	29	0.0000000000000000E 00	0.1963360302321745E-04	0.2837568213226276E-02
	30	0.0000000000000000E 00	0.1504186692483091E-03	0.2786485665205505E-02
	31	0.0000000000000000E 00	0.146046543131196E-03	0.2720954831598117E-02
	32	0.0000000000000000E 00	0.6460786216664402E-04	0.2917537258768694E-02
	33	0.0000000000000000E 00	0.0000000000000000E 00	0.2877853662753165E-02
10	1	0.0000000000000000E 00	0.0000000000000000E 00	0.0000000000000000E 00
	2	-0.4770754949355578E-04	0.2473546957679517E-04	-0.2474497746349073E-03
	3	-0.6611567314536658E-07	-0.5415396282889874E-04	-0.4311794024616110E-03
	4	-0.3549289901720193E-04	-0.5084043547789977E-04	-0.4625365204107660E-03
	5	-0.5109789316327889E-04	-0.1106942912018887E-03	-0.579093877055993E-03
	6	-0.7643822324891558E-04	-0.1622304223478397E-04	-0.6513186887140760E-03
	7	0.6515251803264600E-04	-0.5096290738040740E-04	-0.7861093078455202E-03
	8	0.5377344060740208E-04	-0.9325751764368133E-04	-0.8671590706444568E-03
	9	0.2190741636500994E-03	-0.8007435645953491E-04	-0.8010588450977380E-03
	10	0.1466801126356095E-04	-0.1647272390227941E-03	-0.9106303605888153E-03
	11	0.6565010450412602E-04	-0.5206558818505338E-04	-0.7197508718649239E-03
	12	0.2886485583920303E-04	-0.1198274021712756E-03	-0.6188107592455552E-03
	13	0.8720845378807517E-04	-0.1223861543029325E-03	-0.3109125740850136E-03
	14	0.5123516951277059E-04	-0.4889679634169021E-04	-0.4588112105118111E-04
	15	-0.4110106531020584E-05	-0.1429679941905883E-03	0.2418446107460484E-03
	16	0.2394082691085453E-04	-0.4932855563784358E-04	0.3943115343312060E-03
	17	-0.8962762593595190E-04	0.4942813523319602E-04	0.5776552695912365E-03
	18	0.7320169563966877E-04	0.9358721557879336E-04	0.8879721353125315E-03
	19	0.1826628027494228E-04	-0.4698440830289077E-04	0.118138338289804E-02
	20	0.3938612508707786E-06	0.1263560251653363E-04	0.1472633158258596E-02
	21	-0.2424859029020859E-04	-0.4465879753781846E-04	0.1702726138893248E-02
	22	-0.1128101383877705E-03	0.2861447988606730E-04	0.1838758678379059E-02
	23	-0.8913274648727732E-04	0.4844514744573254E-04	0.1934487681056446E-02
	24	-0.2636798199886760E-04	-0.1127962430924300E-03	0.2024249062362166E-02
	25	-0.5677616242576935E-04	-0.7070288151711585E-04	0.1938314404194014E-02
	26	-0.9606452080636078E-04	-0.2899596416791084E-05	0.1917558599208607E-02
	27	-0.1079222540335658E-03	-0.8519200370223044E-04	0.1883803019306176E-02
	28	-0.2413125134087902E-04	-0.3057252624073605E-04	0.1899094448887461E-02
	29	-0.1027788702619851E-03	0.5713905678287226E-04	0.1815292378896000E-02
	30	0.1194832407458604E-04	-0.8948041228277551E-04	0.1730785814518515E-02
	31	-0.9991466763069059E-04	0.1015698865135332E-03	0.1969018282427332E-02
	32	-0.4390814225568451E-04	0.3949491148134010E-04	0.1929858169380239E-02
	33	-0.8849211113053738E-04	0.0000000000000000E 00	0.1787040383487114E-02

Table 5.10 Sample of Cartesian Coordinates and Pressure Arrays at Time Step 400 for M = 2 and 10

TIME STEP 400		TIME 0.16000000-02				
M	N	V(I,M,N, 400)		CARTESIAN COORDINATES		PRESSURE
				X(I,M,N, 400)	Y(I,M,N, 400)	P(I,M,N)
2	1	0.3000000000000000	00	0.0000000000000000	00	0.0000000000000000
2	2	0.3000000000000000	00	0.9033716777133409E	00	-0.1262648178627175E
2	3	0.0000000000000000	00	0.1991031583460056E	01	-0.2123616051585924E
2	4	0.0000000000000000	00	0.2986489925456179E	01	-0.3783307401836761E
2	5	0.0000000000000000	00	0.3981943722267893E	01	-0.5036247382130053E
2	6	0.0000000000000000	00	0.4977779215126816E	01	-0.6278885780324873E
2	7	0.0000000000000000	00	0.5973873659621578E	01	-0.7498195417249140E
2	8	0.0000000000000000	00	0.6970204679392300E	01	-0.868727933135340E
2	9	0.0000000000000000	00	0.7967066056050100E	01	-0.9841017077096345E
2	10	0.0000000000000000	00	0.8964784084262459E	01	-0.1093678575097817E
2	11	0.0000000000000000	00	0.9962482990478790E	01	-0.1196440963171498E
2	12	0.0000000000000000	00	0.1096164737018491E	02	-0.1292568567481262E
2	13	0.0000000000000000	00	0.1196069987603772E	02	-0.1380266888208165E
2	14	0.0000000000000000	00	0.1296155315233641E	02	-0.1459436689658249E
2	15	0.0000000000000000	00	0.1396168960406734E	02	-0.1530887197758526E
2	16	0.0000000000000000	00	0.1496314750395679E	02	-0.1595319604052816E
2	17	0.0000000000000000	00	0.1596420592298380E	02	-0.1653251449847307E
2	18	0.0000000000000000	00	0.1696605528139823E	02	-0.1706222462739868E
2	19	0.0000000000000000	00	0.1796778549909062E	02	-0.1753665331937867E
2	20	0.0000000000000000	00	0.1897011181167700E	02	-0.1796563006683662E
2	21	0.0000000000000000	00	0.1997216669402501E	02	-0.1834433154015361E
2	22	0.0000000000000000	00	0.2097471467396310E	02	-0.1868393878357596E
2	23	0.0000000000000000	00	0.2197707201239687E	02	-0.1898383244352068E
2	24	0.0000000000000000	00	0.22979658861916499E	02	-0.1924200070565617E
2	25	0.0000000000000000	00	0.2398177576110006E	02	-0.1946892060707336E
2	26	0.0000000000000000	00	0.2498428952722194E	02	-0.1966539713263535E
2	27	0.0000000000000000	00	0.2598635558126021E	02	-0.1984097669059082E
2	28	0.0000000000000000	00	0.2698858661179954E	02	-0.1999234854365450E
2	29	0.0000000000000000	00	0.2799079851420297E	02	-0.2011879686532295E
2	30	0.0000000000000000	00	0.2899301181364056E	02	-0.2022202472731050E
2	31	0.0000000000000000	00	0.2999541595949241E	02	-0.2030107855960577E
2	32	0.0000000000000000	00	0.30997473394152207E	02	-0.2025092570681502E
2	33	0.0000000000000000	00	0.3200000000000000E	02	-0.2036881489392345E
10	1	0.8000000000000000	01	0.0000000000000000	00	0.0000000000000000
10	2	0.7999712056712909E	01	0.9972687769364465E	00	-0.1019446296688460E
10	3	0.7999712056712909E	01	0.1994560102525932E	01	-0.2033917672831318E
10	4	0.3000279342965516E	01	0.299199070208322E	01	-0.3037297981393019E
10	5	0.8000933741571326E	01	0.3989626578884287E	01	-0.4028601341717271E
10	6	0.8001180868231910E	01	0.4987182640603776E	01	-0.5003549457274481E
10	7	0.8003644651410247E	01	0.5985142456867779E	01	-0.595646029358369E
10	8	0.8005206092566362E	01	0.6982969745122238E	01	-0.6876265795014543E
10	9	0.8007325287163890E	01	0.7981535830587295E	01	-0.7756523272206940E
10	10	0.8009504886269153E	01	0.8980090870271886E	01	-0.8642292741471640E
10	11	0.8011690810294600E	01	0.9979242960477335E	01	-0.9374560981248174E
10	12	0.8013961555925172E	01	0.1097833447980587E	02	-0.1010123607985328E
10	13	0.8016141112411534E	01	0.1197816145756306E	02	-0.1076726606181304E
10	14	0.8018490031285267E	01	0.1297821270313844E	02	-0.1137200379552373E
10	15	0.8020643868701781E	01	0.1397853311481140E	02	-0.1192097204185232E
10	16	0.8022526733123851E	01	0.1497917457007360E	02	-0.1241783766971052E
10	17	0.8024348048116800E	01	0.1597995096213136E	02	-0.1286970468835966E
10	18	0.8025943686447051E	01	0.1698082567746380E	02	-0.132789763304332E
10	19	0.8027541754670854E	01	0.1798191073335599E	02	-0.1364717373946746E
10	20	0.8028617792631183E	01	0.1898296550550313E	02	-0.1397734612435461E
10	21	0.8030181365741550E	01	0.199841430358736E	02	-0.1427248917172914E
10	22	0.8030905396357420E	01	0.2098539559123829E	02	-0.1453394068321960E
10	23	0.8032030358436995E	01	0.2198664234622984E	02	-0.147657800781553E
10	24	0.8032768695292153E	01	0.2298808939246823E	02	-0.149694971011888E
10	25	0.8033256649617593E	01	0.2398956322261444E	02	-0.1515151815096388E
10	26	0.8033864385934990E	01	0.2499090073740315E	02	-0.153097867743081E
10	27	0.8034198177058198E	01	0.2599221203961742E	02	-0.1544449433396688E
10	28	0.8034760980703831E	01	0.2699333131426417E	02	-0.1555817904898587E
10	29	0.8034674052310275E	01	0.2799472303612451E	02	-0.1565422473393227E
10	30	0.8035437569953836E	01	0.2899589898968563E	02	-0.1573032499027969E
10	31	0.8035243403329555E	01	0.2999732226299996E	02	-0.1578527657924640E
10	32	0.80351344705692E	01	0.3099859116475941E	02	-0.1582186082213971E
10	33	0.8035287096507280E	01	0.3200000000000000E	02	-0.1583462396072416E

Table 5.11 Sample of Surface Normal Components Array at Time Step 400 for M = 2 and 10

TIME STEP 400		TIME 0.15000000 -02 SURFACE NORMAL VECTOR COMPONENTS		
M	N	SN1(M,N)	SN2(M,N)	SN3(M,N)
2	1	0.0000000000000000E 00	0.1259438236377335E 00	0.9920373749448670E 00
	2	0.0000000000000000E 00	0.1257431427542407E 00	0.9920628317048709E 00
	3	0.0000000000000000E 00	0.1255925629204617E 00	0.9920819059629451E 00
	4	0.0000000000000000E 00	0.1252118350543883E 00	0.9921300293649581E 00
	5	0.0000000000000000E 00	0.1243460511827116E 00	0.9922389125383393E 00
	6	0.0000000000000000E 00	0.1226627383524021E 00	0.9924484130774204E 00
	7	0.0000000000000000E 00	0.1200098782666495E 00	0.9927726976092886E 00
	8	0.0000000000000000E 00	0.1167375176263626E 00	0.9931628023533890E 00
	9	0.0000000000000000E 00	0.1120704948096137E 00	0.9937002581626024E 00
	10	0.0000000000000000E 00	0.1058160330552835E 00	0.9943857235240474E 00
	11	0.0000000000000000E 00	0.9911081021817243E-01	0.9950764027439789E 00
	12	0.0000000000000000E 00	0.9160816448666335E-01	0.9957951315407395E 00
	13	0.0000000000000000E 00	0.8315004582592052E-01	0.9965370398897480E 00
	14	0.0000000000000000E 00	0.7506055834267283E-01	0.9971789772058405E 00
	15	0.0000000000000000E 00	0.6773044949466867E-01	0.9977036565039104E 00
	16	0.0000000000000000E 00	0.6099148543176072E-01	0.9981382863635817E 00
	17	0.0000000000000000E 00	0.5528623890943591E-01	0.9984705462792825E 00
	18	0.0000000000000000E 00	0.5005440718302291E-01	0.9987664925702773E 00
	19	0.0000000000000000E 00	0.4503310742402104E-01	0.9989854950076784E 00
	20	0.0000000000000000E 00	0.4025297035755706E-01	0.99918941178440573E 00
	21	0.0000000000000000E 00	0.3580995034336639E-01	0.9993586180427953E 00
	22	0.0000000000000000E 00	0.3188060268951205E-01	0.9994916943936940E 00
	23	0.0000000000000000E 00	0.2782353947274755E-01	0.9996128503831914E 00
	24	0.0000000000000000E 00	0.2419041776303528E-01	0.9997373690277820E 00
	25	0.0000000000000000E 00	0.2111620737046566E-01	0.99977702749884290E 00
	26	0.0000000000000000E 00	0.1855710670578592E-01	0.9998278020692916E 00
	27	0.0000000000000000E 00	0.1632049397970176E-01	0.9998668118485900E 00
	28	0.0000000000000000E 00	0.1385888760395115E-01	0.9999039610054462E 00
	29	0.0000000000000000E 00	0.1144761094095897E-01	0.9999344739550409E 00
	30	0.0000000000000000E 00	0.9092715383698147E-02	0.9999586604099945E 00
	31	0.0000000000000000E 00	0.6430697034112497E-02	0.9999793228540536E 00
	32	0.0000000000000000E 00	0.3279282361539989E-02	0.9999946231191413E 00
	33	0.0000000000000000E 00	0.0000000000000000E 00	0.1000000000000000E 01
10	1	0.0000000000000000E 00	0.1019415412234536E 00	0.9947903910739121E 00
	2	-0.5643700761522574E-02	0.1014437297862687E 00	0.9949252702503917E 00
	3	-0.1143812664749513E-01	0.1006425423177331E 00	0.9948568982193464E 00
	4	-0.1736998395783445E-01	0.9948132439688992E-01	0.994887876213138E 00
	5	-0.2335073289916596E-01	0.9806647370523801E-01	0.9949058799746275E 00
	6	-0.2917157596688620E-01	0.9607055304207587E-01	0.9949469674541989E 00
	7	-0.3497242302421939E-01	0.9342370123232040E-01	0.9950120133461687E 00
	8	-0.4102808415402189E-01	0.8989056784954705E-01	0.9951062163025031E 00
	9	-0.4694960672765971E-01	0.8561549222968173E-01	0.9952214436588410E 00
	10	-0.528325374939417E-01	0.8072284801207883E-01	0.9953477827759427E 00
	11	-0.5789988963237005E-01	0.7530118006662580E-01	0.9954785034644958E 00
	12	-0.6298870339570264E-01	0.6950597019779783E-01	0.9955909518227811E 00
	13	-0.6763709120161888E-01	0.6342168184504954E-01	0.9956921784194797E 00
	14	-0.7180309514265984E-01	0.5758492549537625E-01	0.9957551365087836E 00
	15	-0.7559503674215125E-01	0.5219804655082300E-01	0.9957713608882089E 00
	16	-0.7908747727443144E-01	0.4735468694097715E-01	0.9957422883218866E 00
	17	-0.8238978550265740E-01	0.4297591378091432E-01	0.9956731361644351E 00
	18	-0.8530158412225167E-01	0.3880057981875988E-01	0.9955993947125331E 00
	19	-0.8797728368294347E-01	0.3484216052599513E-01	0.9955129438664531E 00
	20	-0.9027237920854672E-01	0.3120622659290592E-01	0.9954280832756631E 00
	21	-0.9215066123071945E-01	0.2777219183202221E-01	0.9953577050465760E 00
	22	-0.937364425234159E-01	0.2460251474355664E-01	0.9952706518601994E 00
	23	-0.9529374439535324E-01	0.2173305518367674E-01	0.9952119254667716E 00
	24	-0.9635656945744200E-01	0.1922648531231980E-01	0.9951602014517759E 00
	25	-0.9744707574357098E-01	0.1696068707256063E-01	0.9950961775240891E 00
	26	-0.9833780519372098E-01	0.1460344726292553E-01	0.9950459363254145E 00
	27	-0.9889205959605420E-01	0.1238696191573141E-01	0.9950210669746390E 00
	28	-0.9949050037364559E-01	0.1044447584608925E-01	0.9949836849220726E 00
	29	-0.9992156550623991E-01	0.8586928754512229E-02	0.9949582631454131E 00
	30	-0.1005123056289959E 00	0.6539137351687736E-02	0.9949153361151389E 00
	31	-0.100993682423629E 00	0.4545797566577934E-02	0.994952371832677E 00
	32	-0.101194037037222E 00	0.2453722013247306E-02	0.9948641534533077E 00
	33	-0.1013195159584104E 00	0.0000000000000000E 00	0.9948539368691819E 00

Table 5.12 Sample of Stress Subincrement Array at Time Step 400 for
Layers K = 3 and 4

LMAT(M,N, 3)

N	M=	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
1		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9		1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10		0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14		0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
15		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

LMAT(M,N, 4)

N	M=	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
1		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
4		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7		0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10		0	0	0	1	0	0	1	0	0	0	0	1	1	0	0	0	0	0	0	0	0
11		0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
12		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15		0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
16		0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0
17		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 5.13 Samples of Energy Balance and of Surface Strains Printed Output at Time Steps 200 and 400

TIME STEP 200 TIME= 0.80000000E-03

SURFACE STRAINS		STRAIN GAGE READING					
ETA1	ETA2	M	N	FACE	ANGLE 0	ANGLE 90	ANGLE
0.000	32.000	2.000	33.000	OUTER	0.51017702E-03	-0.37827347E-02	45.00
0.000	32.000	2.000	33.000	INNER	0.72733858E-02	0.54911106E-02	45.00
0.000	0.000	2.000	1.000	OUTER	0.00000000E 00	0.10399005E-02	45.00
0.000	0.000	2.000	1.000	INNER	0.00000000E 00	0.22662113E-02	45.00
20.000	32.000	22.000	33.000	OUTER	0.34576470E-02	0.00000000E 00	45.00
20.000	32.000	22.000	33.000	INNER	0.37381704E-02	0.00000000E 00	45.00

TIME STEP 200 TIME= 0.80000000E-03 KINETIC= 0.84687997E 06 ELASTIC= 0.34998610E 06 PLASTIC= 0.60309890E 05
 TOTAL ENERGY= 0.12571560E 07

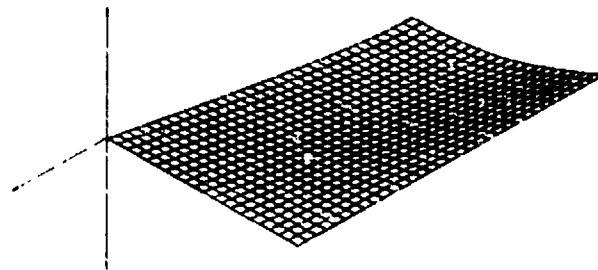
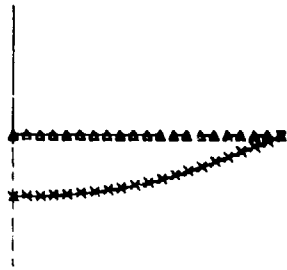
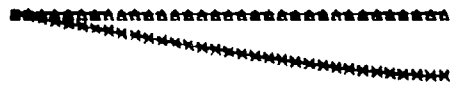
TAPE 1 WRITTEN, NCYCLE= 200 TIME= 0.80000000E-03

TIME STEP 400 TIME= 0.16000000E-02

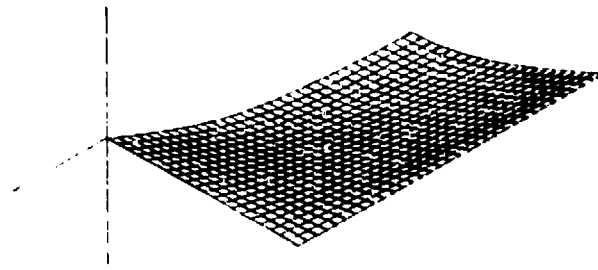
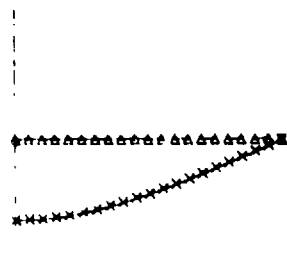
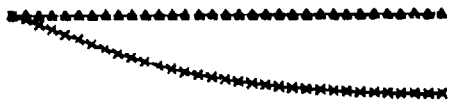
SURFACE STRAINS		STRAIN GAGE READING					
ETA1	ETA2	M	N	FACE	ANGLE 0	ANGLE 90	ANGLE
0.000	32.000	2.000	33.000	OUTER	-0.14012169E-02	0.97995294E-03	45.00
0.000	32.000	2.000	33.000	INNER	0.14749341E-01	0.41496587E-02	45.00
0.000	0.000	2.000	1.000	OUTER	0.00000000E 00	0.30641016E-02	45.00
0.000	0.000	2.000	1.000	INNER	0.00000000E 00	0.33674854E-02	45.00
20.000	32.000	22.000	33.000	OUTER	0.64226842E-02	0.00000000E 00	45.00
20.000	32.000	22.000	33.000	INNER	0.51591539E-02	0.00000000E 00	45.00

TIME STEP 400 TIME= 0.16000000E-02 KINETIC= 0.93843398E 05 ELASTIC= 0.68588570E 06 PLASTIC= 0.47751912E 06
 TOTAL ENERGY= 0.12572482E 07

TAPE 1 WRITTEN, NCYCLE= 400 TIME= 0.16000000E-02



CYCLE 200
800.000 MICROSECONDS



CYCLE 400
1600.000 MICROSECONDS

Figure 5.3 Isometric and Cross-sectional Cal Comp Plots of the Deformed Middle Surface at Time Steps 200 and 400 with Displacements Magnified by Factor of Three

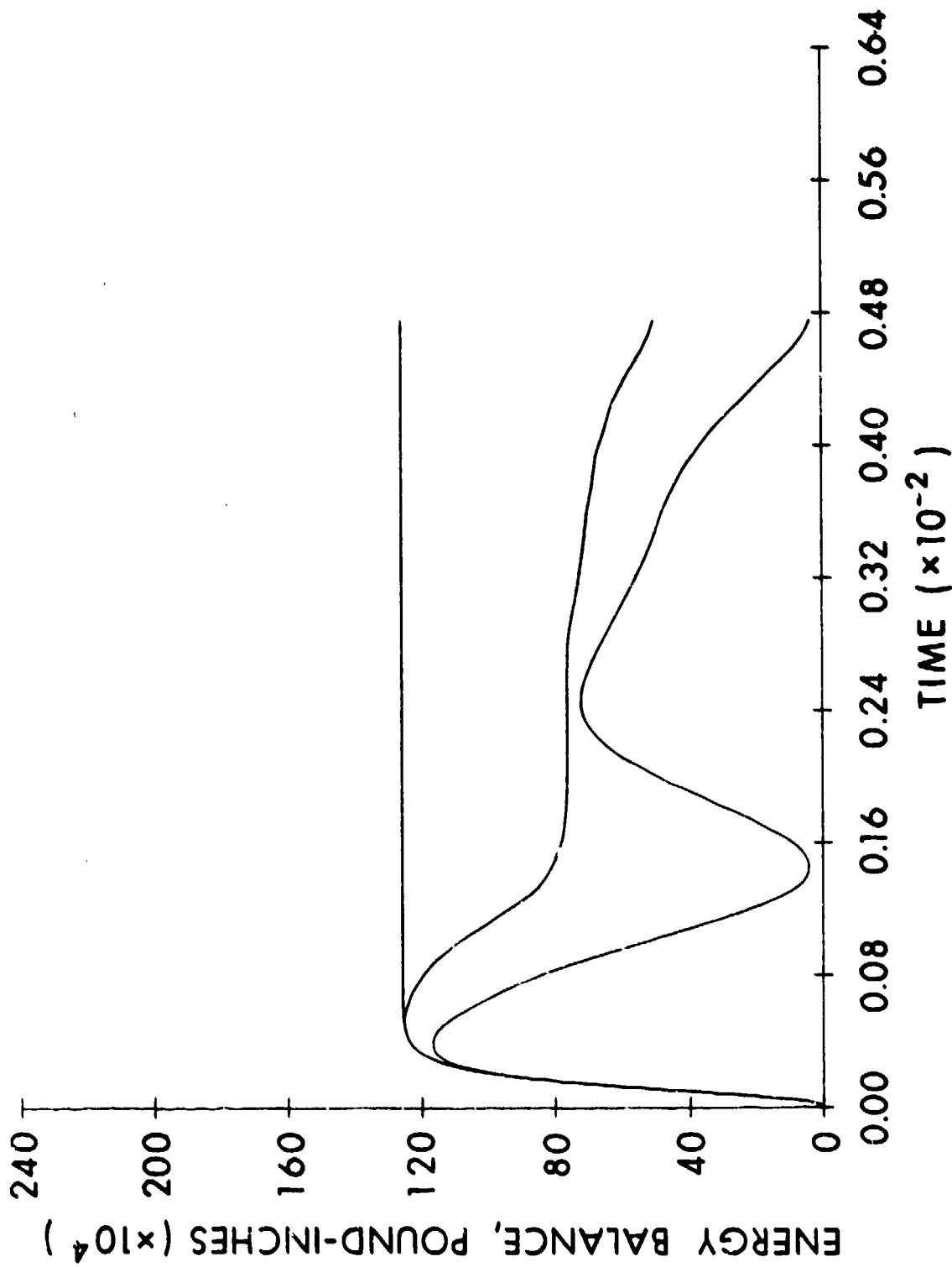


Figure 5.4 Cal Comp Plot of the History of the Energy Balance, Showing in Ascending Order the Kinetic Energy, Total Energy and External Work

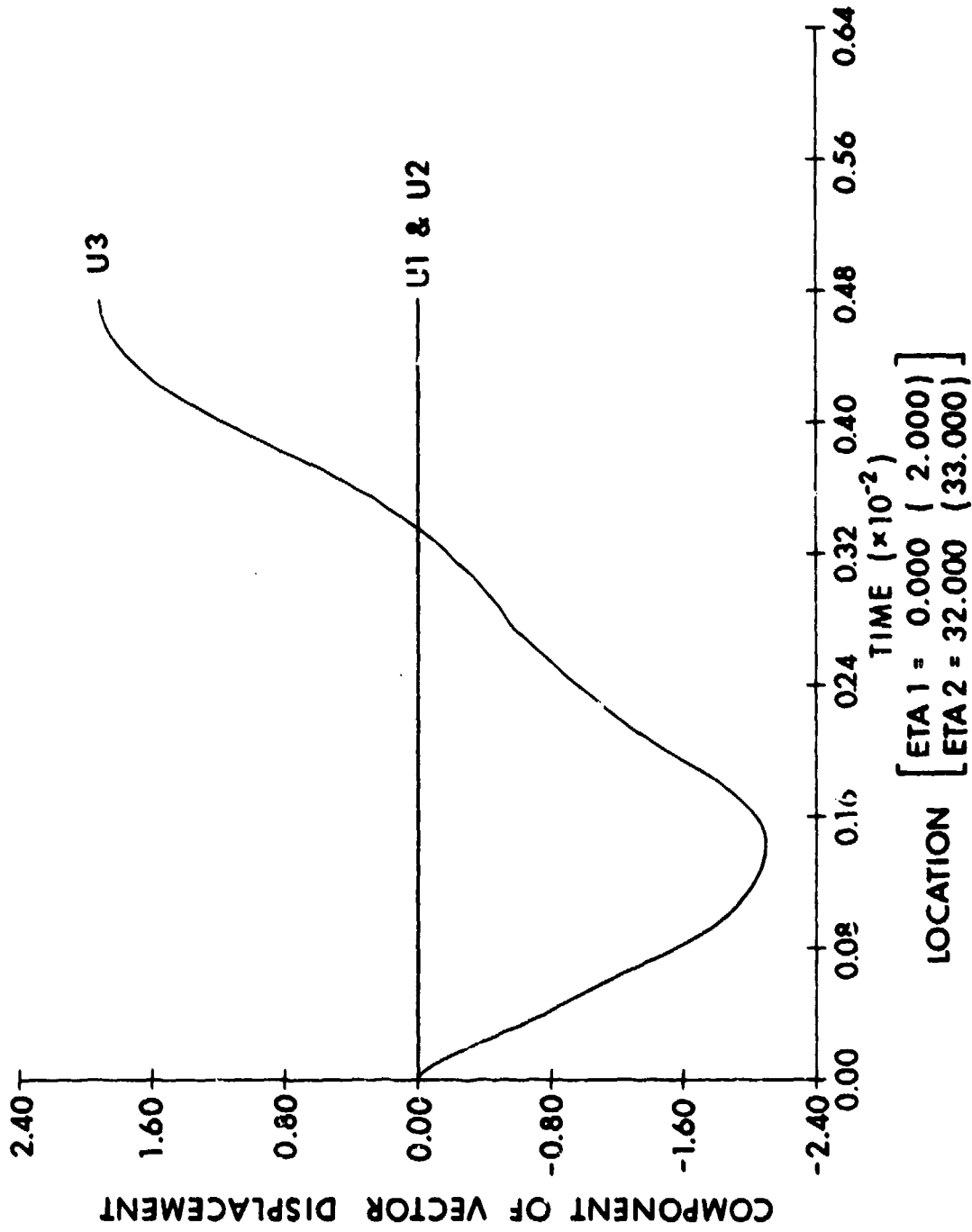


Figure 5.3 Cal Comp Plot of the History of the Deflection at the Center of the Plate

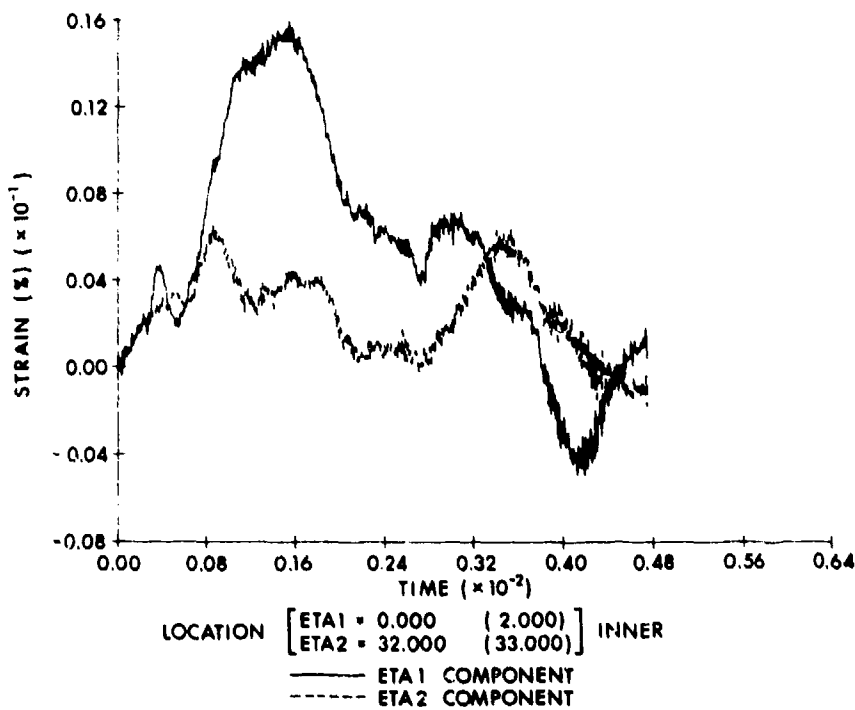
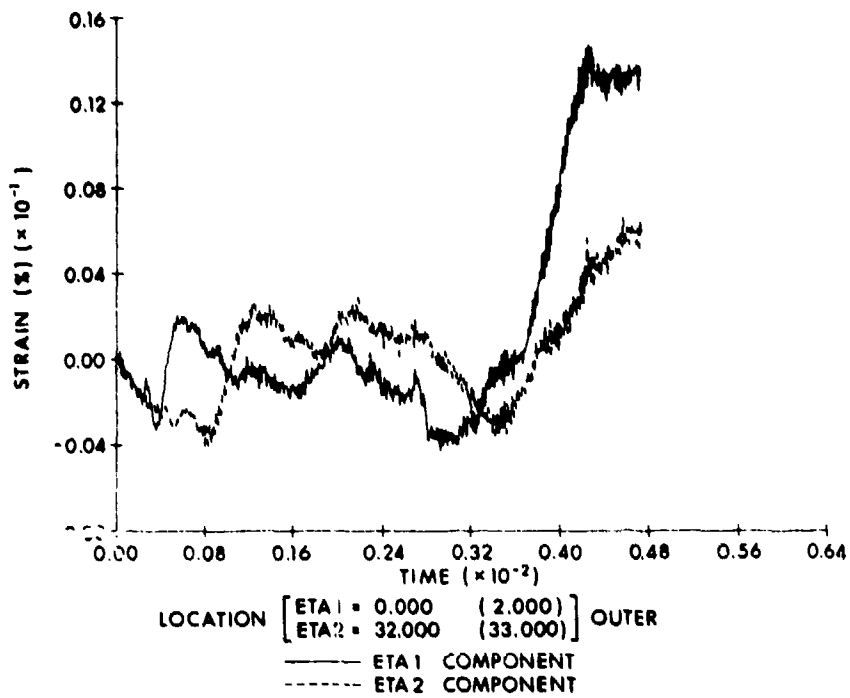


Figure 5.6 Cal Comp Plots of the History of the Surface Elongational Strains Along the Mesh Directions at the Center of Plate

5.2 Example 2: Impulsively Loaded Cylinder

The second example involves a dynamic buckling problem: determine the final equilibrium configuration of a clamped end, aluminum cylinder due to an inwardly directed, instantaneous impulse delivered over a rectangular region of its surface. The impulse imparts a uniform velocity of 7500 in/sec over an 180° arc extending the entire length of the shell. The dimensions and geometry of the problem are shown in Figure 5.7. The example employs the following REPSIL options.

- Cylindrical shell initial geometry
- Input card specified impulsive loading
- Clamped edge and symmetry edge boundaries
- Elastic-perfectly plastic, strain rate insensitive material behavior
- Damping procedure

The material properties of the aluminum are:

$$\text{Young's Modulus } E = 10.7 \times 10^6 \text{ psi}$$

$$\text{Poisson's Ratio } \nu = 1/3$$

$$\text{Yield Stress } \sigma_0 = 42,000 \text{ psi}$$

$$\text{Mass Density } \rho = 2.59066 \times 10^{-4} \frac{\text{lb} \cdot \text{sec}^2}{\text{in}^4}$$

As already indicated, the material is assumed perfectly plastic at the yield stress and independent of the strain rate.

Advantage is taken of the symmetry of the problem to restrict the analysis to the portion of the shell on the positive side of the y^2 , y^3 coordinate plane between the y^1 , y^3 coordinate plane and the cross-sectional symmetry plane, located midway between the ends of the cylinder, Figure 5.7. This makes edges 1, 2 and 3 symmetry boundaries and edge 4 a clamped boundary (compare with Figures 3.1 and 3.7). Also, only 180 degrees of the circumference and half the length are prescribed as the input dimensions and the loaded region is restricted to the first 90 degrees arc. The example uses 20 mesh intervals in the circumferential direction and 12 in the axial direction, giving an almost square mesh. The thickness is divided into 4 layers. The time increment is deliberately set equal to zero in order to assure that the stable time increment determined by the program is used to solve the example.

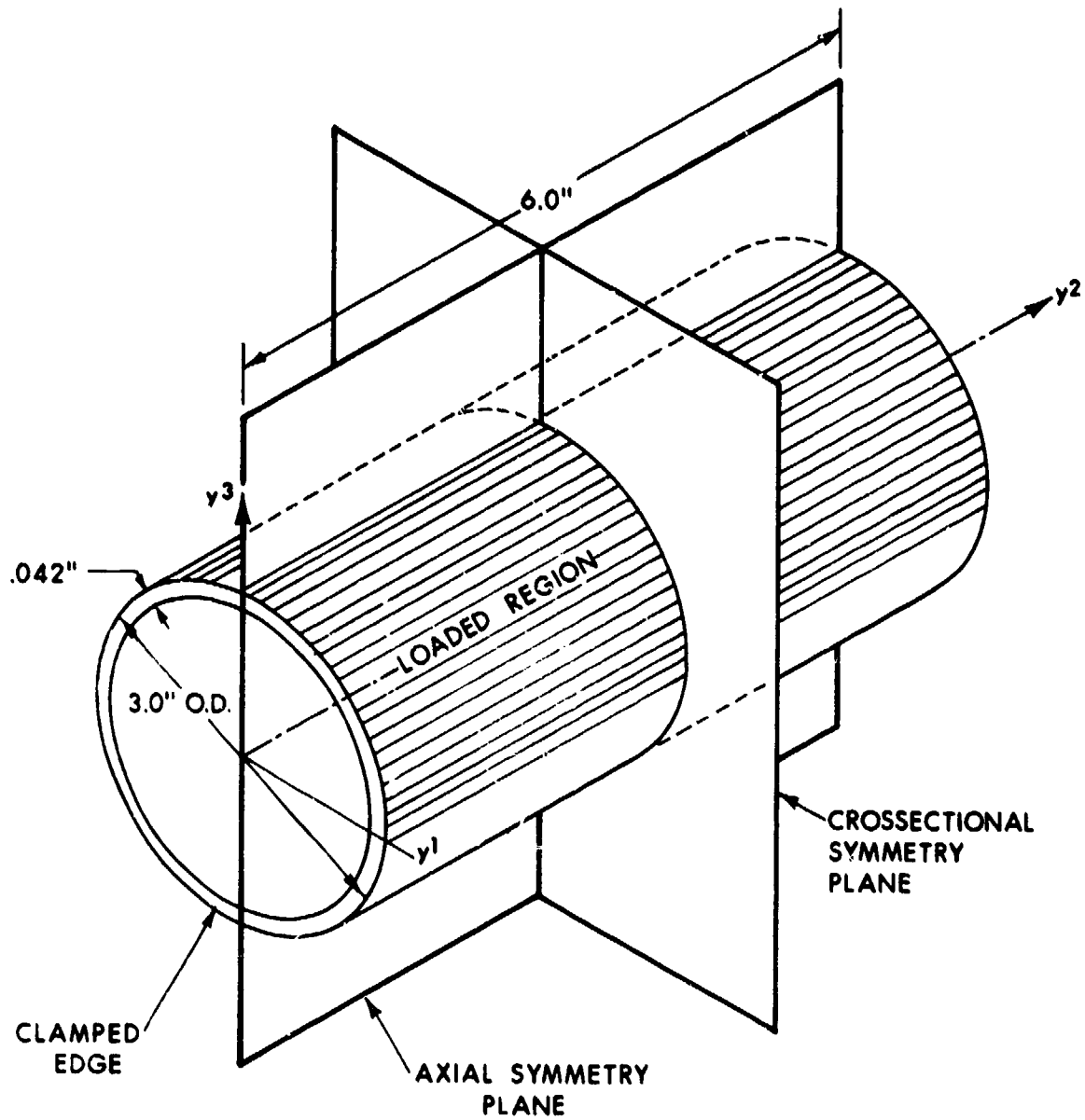


Figure 5.7 Geometry for Example Problem 2

The damping option is used to obtain a final equilibrium configuration. Based on a preliminary solution of the example without the use of damping it is determined that most of the plastic work is accomplished by time step 270. Hence, the damping for this example is picked to begin at time step 270. The example is set to run until time step 600. Since the run may not be terminated by damping before time step 600, provision is made for collecting information for a restart run every 100 time steps. Table 5.14 lists the input data for the run in the order shown in Table 3.1. Notice that while the diameter to the outer surface is specified in Figure 5.9, Card 14 calls for the radius to the middle surface. Also, because half the area associated with each mesh point along the 90° line (i.e. along $M = 12$) receives the full impulse, only half the impulse velocity is assigned to these points in Cards 16a - 16l.

Tables 5.15 - 5.23 give sample listings of the printed output. For the sake of economy, only the values of arrays $U_i(M,N)$, $Y_i(M,N)$, $SN_i(M,N)$ at $M = 2, 12, 22$ and $1 \leq N \leq 13$ (i.e. along the $0^\circ, 90^\circ, 180^\circ$ meridians) are given. Also, the $LMAT(M,N,K)$ array is given at $K = 1, 2$. Figures 5.8 - 5.11 give examples of the plotted output. Notice that the damping operations terminate the run at time step 525, corresponding to approximately 643 microseconds.

Table 5.14 Input Data Cards for Cylindrical Shell Problem

Card 1	EXAMPLE 2	FULL CYLINDER WITH IMPULSIVE LOADING											
Card 2	20	12	4	0.0									
Card 3	600	0	100	0.0									
Card 4	2	2	2	1									
Card 5	0	0	270	.100000E 00 .500000E-02									
Card 6	.107000E 08	.33333333	.420000E 05	.259066E-03	.420000E-01	1	0						
Card 8	50	1	1	1									
Card 9	7	100	125	200	250	300	400	500					
Card 10	4	100	200	300	400								
Card 11a	24	25	50	75	100	125	150	175	200	225	250	275	300
Card 11b	325	350	375	400	425	450	475	500	525	550	575	600	
Card 12	90.0	1.5	6										
Card 13a	0.0	3.0	45.0	135.0	1								
Card 13b	0.0	3.0	45.0	135.0	0								
Card 13c	45.0	3.0	45.0	135.0	1								
Card 13d	45.0	3.0	45.0	135.0	0								
Card 13e	90.0	1.5	45.0	135.0	1								
Card 13f	90.0	1.5	45.0	135.0	0								
Card 14	.300000E 01	.295800E 01	.360000E 03										
Card 15	2	11	2	13	.750000E 04	13							
Card 16a	12	2	.375000E 04										
Card 16b	12	3	.375000E 04										
Card 16c	12	4	.375000E 04										
Card 16d	12	5	.375000E 04										
Card 16e	12	6	.375000E 04										
Card 16f	12	7	.375000E 04										
Card 16g	12	8	.375000E 04										
Card 16h	12	9	.375000E 04										
Card 16i	12	10	.375000E 04										
Card 16j	12	11	.375000E 04										
Card 16k	12	12	.375000E 04										
Card 16l	12	13	.375000E 04										

Table S.15 First Page of Printed Output Summarizing the Input Data and Results of Stable Time Increment Check

OR: REPSIL CODE

*** EXAMPLE 2 FULL CYLINDER WITH IMPULSIVE LOADING ***

20 MESHES IN THE ETA1 DIRECTION (DETA1 = 0.232321E 00)
 12 MESHES IN THE ETA2 DIRECTION (DETA2 = 0.250000E 00)

BENDING TIME INCREMENT = 0.689333E-05
 MEMBRANE TIME INCREMENT = 0.157900E-05
 INPUT TIME INCREMENT = 0.000000E 00

TIME INCREMENT USED BY REPSIL = 0.150000E-05

YOUNG'S MODULUS = 0.107000E 08
 POISSON'S RATIO = 0.333333E 00 YIELD STRESS = 0.420000E 05
 MASS DENSITY = 0.259066E-03 THICKNESS = 0.420000E-01

START AT TIME STEP 0
 FINAL TIME STEP 600
 SURFACE STRAINS EVERY 50 TIME STEP
 RESTART WRITE EVERY 100 TIME STEP

LAYER = 4 NSTRN = 6
 LOAD = 0 LPRESS = 0

BOUNDARY CONDITIONS
 1/2/3 = CLAMPED/SYMMETRY/HINGED
 EDGE1 = 2
 EDGE2 = 2
 EDGE3 = 2
 EDGE4 = 1

PRINT OPTION CONTROL CARD
 0/1 = NO PRINT/PRINT
 1 DISPLACEMENT INCREMENTS
 1 CARTESIAN COORDINATES, PRESSURE
 1 SURFACE NORMAL VECTOR COMPONENTS

PRINT INFORMATION AT THE FOLLOWING TIME STEPS
 100 125 200 250 300 400 500
 PRINT L MATRIX (LMAT) AT THE FOLLOWING TIME STEPS
 100 200 300 400
 3-D PLOTS FOR THE FOLLOWING TIME STEPS
 25 50 75 100 125 150 175 200 225 250 275 300 325 350 375 400
 425 450 475 500 525 550 575 600

CONSTITUTIVE RELATION ELASTOPLASTIC-NO WORK HARDENING-STRAIN RATE INDEPENDENT

STRESS-STRAIN AND STRAIN RATE PARAMETERS
 1 SSIG(J) SEPS(J) DSR(J) 1/PSR(J)
 1 4.2000000E 04 3.9252336E-03 0.0000000E 00 0.0000000E 00

START DAMPING AFTER TIME STEP 270 TIME = 0.4050E-03
 DAMPF = 0.1000E 00 DFACT = 0.9000E-02

Table S.10 Second Page of Printed Output Summarizing the Input Impulse Velocities and Sample of Third Page of Output with Initial Values of Cartesian Coordinates Array for M = 2, 12, and 22 (M = 2, 11) AND (M = 2, 13) RECEIVE FULL VELOCITY (VH) = 0.750000E 04

OTHER VELOCITY DISTRIBUTION

M	N	V
12	2	0.375000E 04
12	3	0.375000E 04
12	4	0.375000E 04
12	5	0.375000E 04
12	6	0.375000E 04
12	7	0.375000E 04
12	8	0.375000E 04
12	9	0.375000E 04
12	10	0.375000E 04
12	11	0.375000E 04
12	12	0.375000E 04
12	13	0.375000E 04

M	N	INITIAL CARTESIAN COORDINATES		
		X1(M,N)	Y2(M,N)	Y3(M,N)
2	1	0.0000000000000000E 00	0.0000000000000000E 00	0.1479000000000000E 01
2	2	0.0000000000000000E 00	0.2500000000000000E 00	0.1479000000000000E 01
2	3	0.0000000000000000E 00	0.5000000000000000E 00	0.1479000000000000E 01
2	4	0.0000000000000000E 00	0.7500000000000000E 00	0.1479000000000000E 01
2	5	0.0000000000000000E 00	0.1000000000000000E 01	0.1479000000000000E 01
2	6	0.0000000000000000E 00	0.1250000000000000E 01	0.1479000000000000E 01
2	7	0.0000000000000000E 00	0.1500000000000000E 01	0.1479000000000000E 01
2	8	0.0000000000000000E 00	0.1750000000000000E 01	0.1479000000000000E 01
2	9	0.0000000000000000E 00	0.2000000000000000E 01	0.1479000000000000E 01
2	10	0.0000000000000000E 00	0.2250000000000000E 01	0.1479000000000000E 01
2	11	0.0000000000000000E 00	0.2500000000000000E 01	0.1479000000000000E 01
2	12	0.0000000000000000E 00	0.2750000000000000E 01	0.1479000000000000E 01
2	13	0.0000000000000000E 00	0.3000000000000000E 01	0.1479000000000000E 01
12	2	0.1479000000000000E 01	0.0000000000000000E 00	0.1284639477134642E -13
12	3	0.1479000000000000E 01	0.2500000000000000E 00	0.1284639477134642E -13
12	4	0.1479000000000000E 01	0.5000000000000000E 00	0.1284639477134642E -13
12	5	0.1479000000000000E 01	0.7500000000000000E 00	0.1284639477134642E -13
12	6	0.1479000000000000E 01	0.1000000000000000E 01	0.1284639477134642E -13
12	7	0.1479000000000000E 01	0.1250000000000000E 01	0.1284639477134642E -13
12	8	0.1479000000000000E 01	0.1500000000000000E 01	0.1284639477134642E -13
12	9	0.1479000000000000E 01	0.1750000000000000E 01	0.1284639477134642E -13
12	10	0.1479000000000000E 01	0.2000000000000000E 01	0.1284639477134642E -13
12	11	0.1479000000000000E 01	0.2250000000000000E 01	0.1284639477134642E -13
12	12	0.1479000000000000E 01	0.2500000000000000E 01	0.1284639477134642E -13
12	13	0.1479000000000000E 01	0.2750000000000000E 01	0.1284639477134642E -13
22	3	0.5154557508554767E -15	0.0000000000000000E 00	-0.1479000000000000E 01
22	4	0.5154557508554767E -15	0.2500000000000000E 00	-0.1479000000000000E 01
22	5	0.5154557508554767E -15	0.5000000000000000E 00	-0.1479000000000000E 01
22	6	0.5154557508554767E -15	0.7500000000000000E 00	-0.1479000000000000E 01
22	7	0.5154557508554767E -15	0.1000000000000000E 01	-0.1479000000000000E 01
22	8	0.5154557508554767E -15	0.1250000000000000E 01	-0.1479000000000000E 01
22	9	0.5154557508554767E -15	0.1500000000000000E 01	-0.1479000000000000E 01
22	10	0.5154557508554767E -15	0.1750000000000000E 01	-0.1479000000000000E 01
22	11	0.5154557508554767E -15	0.2000000000000000E 01	-0.1479000000000000E 01
22	12	0.5154557508554767E -15	0.2250000000000000E 01	-0.1479000000000000E 01
22	13	0.5154557508554767E -15	0.2500000000000000E 01	-0.1479000000000000E 01
22	14	0.5154557508554767E -15	0.2750000000000000E 01	-0.1479000000000000E 01
22	15	0.5154557508554767E -15	0.3000000000000000E 01	-0.1479000000000000E 01

Table 5.17 Samples for M = 2, 12, and 22 of Displacement Increments Array During Time Increment 124 to 125 and of Cartesian Coordinates and Pressure Arrays at Time Step 125

DISPLACEMENT INCREMENTS BETWEEN T.S. 124 AND 125

M	N	U1(M,N)	U2(M,N)	U3(M,N)
2	1	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00
2	2	0.0000000000000000 00	-0.4919296673098917E-04	-0.5991174090041111E-04
2	3	0.0000000000000000 00	-0.8038866734761084E-04	-0.7344449636014444E-04
2	4	0.0000000000000000 00	-0.6392770284339266E-04	-0.1299249115097174E-04
2	5	0.0000000000000000 00	-0.8009929049347683E-04	0.2940714170962202E-04
2	6	0.0000000000000000 00	-0.1989039715441333E-04	0.4830413100045111E-04
2	7	0.0000000000000000 00	0.2163460087768616E-04	0.1197991969426498E-04
2	8	0.0000000000000000 00	-0.1339194769439386E-04	0.1001190917630634E-04
2	9	0.0000000000000000 00	-0.2803674484941836E-04	0.1174967140000792E-04
2	10	0.0000000000000000 00	-0.3501429880611099E-04	0.1449919945746602E-04
2	11	0.0000000000000000 00	-0.5987874938007100E-04	0.1030297103358744E-04
2	12	0.0000000000000000 00	-0.4108822474672329E-04	0.2024951023110244E-04
2	13	0.0000000000000000 00	0.0000000000000000 00	0.2100993177730497E-04
12	1	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00
12	2	0.0000000000000000 00	-0.607403977899897E-04	0.1940000000000000 00
12	3	0.0000000000000000 00	-0.641043199949499E-04	0.1594709211709799E-04
12	4	0.0000000000000000 00	-0.745474199888704E-04	-0.786649848484259E-04
12	5	0.0000000000000000 00	-0.145961540327744E-03	-0.424030548891804E-04
12	6	0.0000000000000000 00	-0.1931311272228607E-03	-0.969119434016177E-04
12	7	0.0000000000000000 00	-0.189888544447866E-03	-0.266016214337639E-04
12	8	0.0000000000000000 00	-0.182439778481477E-03	-0.248038943594349E-04
12	9	0.0000000000000000 00	-0.292418926887370E-03	-0.177495211726264E-04
12	10	0.0000000000000000 00	-0.2079241480213711E-03	-0.274395544530790E-04
12	11	0.0000000000000000 00	-0.2019109091934917E-03	-0.190914933864964E-04
12	12	0.0000000000000000 00	-0.1981700842414389E-03	-0.458201334387056E-04
12	13	0.0000000000000000 00	0.0000000000000000 00	-0.537339873044447E-04
22	1	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00
22	2	0.0000000000000000 00	-0.417662793021811E-05	-0.110944379457020E-04
22	3	0.0000000000000000 00	-0.4697711806740312E-04	-0.474014591448000E-04
22	4	0.0000000000000000 00	-0.1057095711070925E-03	0.1146449307117693E-03
22	5	0.0000000000000000 00	-0.226941450302984E-04	0.543143458441987E-03
22	6	0.0000000000000000 00	-0.387789809781054E-04	0.114844606475149E-04
22	7	0.0000000000000000 00	-0.122411357158393E-03	0.1441461150180744E-04
22	8	0.0000000000000000 00	-0.1027148211048899E-03	0.159746971207440E-04
22	9	0.0000000000000000 00	-0.299529828345030E-04	0.179849791376167E-04
22	10	0.0000000000000000 00	-0.841609430201870E-04	0.163143726767849E-04
22	11	0.0000000000000000 00	-0.7989180999116461E-04	0.193449076645312E-04
22	12	0.0000000000000000 00	-0.1119278127045637E-04	0.214849856421634E-04
22	13	0.0000000000000000 00	0.0000000000000000 00	0.231944024943399E-04

TIME STEP 125 TIME 0.18750000E-01 CARTESIAN COORDINATES

M	N	V1(M,N, 125)	V2(M,N, 125)	V3(M,N, 125)	PRESSURE P(M,N)
2	1	0.0000000000000000 00	0.0000000000000000 00	0.1479000000000000E 01	0.0000000000000000 00
2	2	0.0000000000000000 00	0.2427202771471525E 01	0.1421736773847553E 01	0.0000000000000000 00
2	3	0.0000000000000000 00	0.4616879754695976E 01	0.124994779443019E 01	0.0000000000000000 00
2	4	0.0000000000000000 00	0.7248977819407737E 01	0.120627919802300E 01	0.0000000000000000 00
2	5	0.0000000000000000 00	0.976829239184454E 01	0.1194931170655431E 01	0.0000000000000000 00
2	6	0.0000000000000000 00	0.1238626941777644E 01	0.1181443382214056E 01	0.0000000000000000 00
2	7	0.0000000000000000 00	0.1488924027871278E 01	0.1168647997416319E 01	0.0000000000000000 00
2	8	0.0000000000000000 00	0.1743282033043640E 01	0.1159013494143610E 01	0.0000000000000000 00
2	9	0.0000000000000000 00	0.1997163896697443E 01	0.1152077744748973E 01	0.0000000000000000 00
2	10	0.0000000000000000 00	0.224662988883431E 01	0.1152077744748973E 01	0.0000000000000000 00
2	11	0.0000000000000000 00	0.250120044794028E 01	0.1152077744748973E 01	0.0000000000000000 00
2	12	0.0000000000000000 00	0.2746227176792653E 01	0.1151308477966174E 01	0.0000000000000000 00
2	13	0.0000000000000000 00	0.3000000000000000 00	0.1151308477966174E 01	0.0000000000000000 00
12	1	0.1479000000000000 01	0.0000000000000000 00	0.1209670477148692E-19	0.0000000000000000 00
12	2	0.1471237108239441E 01	0.2959014124814434E 00	-0.1310262749439254E-01	0.0000000000000000 00
12	3	0.1447948432957640E 01	0.5115940280035692E 00	-0.3369149499548211E-01	0.0000000000000000 00
12	4	0.1380130418269254E 01	0.7489731120694224E 00	-0.9240011110294559E-01	0.0000000000000000 00
12	5	0.130939087126191E 01	0.9901210011473038E 00	-0.1445292443349384E 00	0.0000000000000000 00
12	6	0.1257993033349513E 01	0.1231967339900959E 01	-0.1906217459479535E 00	0.0000000000000000 00
12	7	0.120910458147043E 01	0.1478609170805791E 01	-0.2277246499413100E 00	0.0000000000000000 00
12	8	0.1175432985101032E 01	0.1725758274181429E 01	-0.2584149733043766E 00	0.0000000000000000 00
12	9	0.1142433530926913E 01	0.19780520688107E 01	-0.2854946813187930E 00	0.0000000000000000 00
12	10	0.1116944941952716E 01	0.2224966717815739E 01	-0.3179007431005164E 00	0.0000000000000000 00
12	11	0.10903117363516E 01	0.248202739306824E 01	-0.327208534977743E 00	0.0000000000000000 00
12	12	0.10855154617478E 01	0.2739110028630017E 01	-0.339024991146616E 00	0.0000000000000000 00
12	13	0.10806590931183E 01	0.3000000000000000 01	-0.3424805780911059E 00	0.0000000000000000 00
22	1	0.5158557908594767E-15	0.0000000000000000 00	-0.1479000000000000E 01	0.0000000000000000 00
22	2	0.5158557908594767E-15	0.2918608990203314E 00	-0.1473454782011211E 01	0.0000000000000000 00
22	3	0.5158557908594767E-15	0.5023330007732099E 00	-0.145601619044449E 01	0.0000000000000000 00
22	4	0.5158557908594767E-15	0.7522100908569182E 00	-0.1439149839214213E 01	0.0000000000000000 00
22	5	0.5158557908594767E-15	0.1002277807614370E 01	-0.1418993999186472E 01	0.0000000000000000 00
22	6	0.5158557908594767E-15	0.129097659532654E 01	-0.1402280247999200E 01	0.0000000000000000 00
22	7	0.5158557908594767E-15	0.1501991699004186E 01	-0.1384792013674729E 01	0.0000000000000000 00
22	8	0.5158557908594767E-15	0.1741723667956799E 01	-0.1376093268274813E 01	0.0000000000000000 00
22	9	0.5158557908594767E-15	0.2001492230894970E 01	-0.1366814089266527E 01	0.0000000000000000 00
22	10	0.5158557908594767E-15	0.2250813183436625E 01	-0.135549519257903E 01	0.0000000000000000 00
22	11	0.5158557908594767E-15	0.2500488099930661E 01	-0.1342499557707983E 01	0.0000000000000000 00
22	12	0.5158557908594767E-15	0.2750148476098599E 01	-0.1329014708685218E 01	0.0000000000000000 00
22	13	0.5158557908594767E-15	0.3000000000000000 01	-0.1324394094273664E 01	0.0000000000000000 00

Table 5.18 Samples at Time Step 125 of Surface Normal Components Array for M = 2, 12, and 22 and of Stress Subincrement Array for Layers K = 1 and 2

TIME STEP	M	K	TIME	Surface Normal Vector Components	SHR(M,N)	SHR(M,N)
125	2	1	1.0750000E-01	0.0000000000000000E+00	0.0000000000000000E+00	0.1000000000000000E+01
	2	2	1.0750000E-01	0.4444312447793719E+00	0.4444312447793719E+00	0.8988624895587438E+00
	2	3	1.0750000E-01	0.4076729271029899E+00	0.4076729271029899E+00	0.8153458542059798E+00
	2	4	1.0750000E-01	0.1061934680428447E+00	0.1061934680428447E+00	0.2123869160856894E+00
	2	5	1.0750000E-01	0.4824024784497269E-01	0.4824024784497269E-01	0.9648049568994538E-01
	2	6	1.0750000E-01	0.5121084098946893E-01	0.5121084098946893E-01	0.1024216817888678E-01
	2	7	1.0750000E-01	0.4444224479193961E-01	0.4444224479193961E-01	0.8888448958387922E-01
	2	8	1.0750000E-01	0.1242616993039747E-01	0.1242616993039747E-01	0.2485233986079494E-01
	2	9	1.0750000E-01	0.1748649961421888E-01	0.1748649961421888E-01	0.3497299922843776E-01
	2	10	1.0750000E-01	-0.1646224886038859E-02	-0.1646224886038859E-02	-0.3292449772071718E-02
	2	11	1.0750000E-01	-0.2114901256378383E-02	-0.2114901256378383E-02	-0.4229802512736766E-02
	2	12	1.0750000E-01	0.259419746726503E-02	0.259419746726503E-02	0.518839493453006E-02
	2	13	1.0750000E-01	0.3000000100000000E+00	0.3000000100000000E+00	0.6000000200000000E+00
	12	1	1.0750000E-01	0.0000000000000000E+00	0.0000000000000000E+00	0.0000000000000000E+00
	12	2	1.0750000E-01	0.7014094897682427E-01	0.7014094897682427E-01	0.1402819795377454E-01
	12	3	1.0750000E-01	0.239426109968181E+00	0.239426109968181E+00	0.478852219936362E+00
	12	4	1.0750000E-01	0.3314021389067471E+00	0.3314021389067471E+00	0.662804277813494E+00
	12	5	1.0750000E-01	0.2807944209882338E+00	0.2807944209882338E+00	0.561588841976467E+00
	12	6	1.0750000E-01	0.2249100953430371E+00	0.2249100953430371E+00	0.449820190686074E+00
	12	7	1.0750000E-01	0.1798644716880590E+00	0.1798644716880590E+00	0.359728943376118E+00
	12	8	1.0750000E-01	0.1445618338765072E+00	0.1445618338765072E+00	0.289123767753014E+00
	12	9	1.0750000E-01	0.1193436510729441E+00	0.1193436510729441E+00	0.238687302145888E+00
	12	10	1.0750000E-01	0.974182448871918E-01	0.974182448871918E-01	0.194836497743736E-01
	12	11	1.0750000E-01	0.7146797751080610E-01	0.7146797751080610E-01	0.142935950216012E-01
	12	12	1.0750000E-01	0.5542301874504544E-01	0.5542301874504544E-01	0.110846034848889E-01
	12	13	1.0750000E-01	0.0000000000000000E+00	0.0000000000000000E+00	0.0000000000000000E+00
	22	1	1.0750000E-01	0.0000000000000000E+00	0.0000000000000000E+00	-0.1000000000000000E+01
	22	2	1.0750000E-01	0.4411946021811244E-01	0.4411946021811244E-01	-0.8823892043622488E-01
	22	3	1.0750000E-01	0.4874667968190674E-01	0.4874667968190674E-01	-0.974933593637414E-01
	22	4	1.0750000E-01	0.754238254402329E-01	0.754238254402329E-01	-0.150847608880458E-01
	22	5	1.0750000E-01	0.7347183234392299E-01	0.7347183234392299E-01	-0.14693764747959E-01
	22	6	1.0750000E-01	0.604247973722121E-01	0.604247973722121E-01	-0.120849594744242E-01
	22	7	1.0750000E-01	0.4064970099790474E-01	0.4064970099790474E-01	-0.812994019958094E-01
	22	8	1.0750000E-01	0.4377940799719221E-01	0.4377940799719221E-01	-0.875588159942444E-01
	22	9	1.0750000E-01	0.4372600947241624E-01	0.4372600947241624E-01	-0.874520189448244E-01
	22	10	1.0750000E-01	0.481188652736347E-01	0.481188652736347E-01	-0.962376905472694E-01
	22	11	1.0750000E-01	0.428223421255977E-01	0.428223421255977E-01	-0.856446842511955E-01
	22	12	1.0750000E-01	0.3678863723863258E-01	0.3678863723863258E-01	-0.735772744772711E-01
	22	13	1.0750000E-01	0.0000000000000000E+00	0.0000000000000000E+00	-0.1000000000000000E+01

Time Step 125 TIME 1.0750000E-01 SUBDIVISIONS OF TIME INCREMENT IN STRESS

UMAT(M,N: 1)

M	N	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	2	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	4	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	5	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	6	1	1	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	7	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	8	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
1	9	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
1	10	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
1	11	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
1	12	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
1	13	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0

UMAT(M,N: 2)

M	N	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	4	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	5	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	6	0	0	1	0	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
1	7	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	8	0	0	1	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
1	9	0	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
1	10	1	0	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
1	11	0	0	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
1	12	1	0	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
1	13	1	0	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0

Table 5.19 Samples for M = 7, 12, and 22 of Displacement Increments Array During Time Increment 249 to 250 and of Cartesian Coordinates and Pressure Arrays at Time Step 250

DISPLACEMENT INCREMENTS BETWEEN T,2: 249 AND 250

M	N	U1(M,N)	U2(M,N)	U3(M,N)
7	1	0.000000000000000000 00	0.000000000000000000 00	0.000000000000000000 00
	2	0.000000000000000000 00	-0.40650921549622772E-05	0.79009791100953970E-04
	3	0.000000000000000000 00	0.10141706412067060E-03	0.30303116479981440E-03
	4	0.000000000000000000 00	0.11739736961676706E-03	0.29922739032444700E-03
	5	0.000000000000000000 00	0.10611773180064200E-03	0.34400943223190000E-04
	6	0.000000000000000000 00	0.1707000406020961E-03	-0.20913069740943990E-04
	7	0.000000000000000000 00	0.23692796207896000E-03	-0.40404768346209350E-04
	8	0.000000000000000000 00	0.10967956083440010E-03	-0.00673793219227920E-03
	9	0.000000000000000000 00	0.10719090002405500E-03	-0.30944956431007900E-02
	10	0.000000000000000000 00	0.167024-013007197E-03	-0.131070000470070E-02
	11	0.000000000000000000 00	0.79970000000000000E-04	-0.14030690024049400E-02
	12	0.000000000000000000 00	0.2909477970243400E-04	-0.1500173039933000E-02
	13	0.000000000000000000 00	0.000000000000000000 00	-0.19070011022900270E-02
12	1	0.000000000000000000 00	0.000000000000000000 00	0.000000000000000000 00
	2	0.77077911410622002E-04	0.90127030361029031E-05	-0.67247619239726000E-04
	3	0.31039000047449.49E-03	0.13142792139960790E-04	0.34026300073391070E-04
	4	0.413263199240577E-03	0.99492730000230490E-04	0.497000313713070E-04
	5	0.449790300740420E-03	0.00291177302313000E-04	0.3643640953994970E-04
	6	0.3127000070264001E-03	0.77291077102311000E-04	-0.721274000761010E-04
	7	0.210300000000000000 00	-0.04100771019010200E-05	-0.2610963117073230E-04
	8	0.1463900001231471E-03	-0.49210970413193000E-04	-0.4707771662900470E-03
	9	0.7-0.17430007100400E-04	-0.4061342064790091E-04	-0.90970000000104490E-03
	10	0.545599273191130E-04	0.10294100009769900E-04	-0.92497044769721770E-03
	11	0.312215 1070449144E-04	-0.31170047129794470E-04	-0.71309004110011000E-03
	12	0.640000000000000000 00	-0.44300000000101000E-04	-0.0093004793174190E-03
	13	0.274103240470542E-04	0.000000000000000000 00	-0.47036131041004400E-03
22	1	0.000000000000000000 00	0.000000000000000000 00	0.000000000000000000 00
	2	0.000000000000000000 00	0.612294000704209070E-04	-0.10574411367203510E-03
	3	0.000000000000000000 00	-0.14002714147472390E-04	-0.42290404000174000E-03
	4	0.000000000000000000 00	0.109000000000000000 00	-0.999000000000000000 00
	5	0.000000000000000000 00	-0.41240034007620900E-04	-0.92047000000470000E-03
	6	0.000000000000000000 00	-0.006700000000000000 00	-0.39107994330000000E-03
	7	0.000000000000000000 00	-0.662700000000000000 00	-0.91671100010103300E-03
	8	0.000000000000000000 00	0.70010070001700040E-04	-0.36007470200643300E-04
	9	0.000000000000000000 00	0.130973.47260400000E-03	-0.33741097041671470E-03
	10	0.000000000000000000 00	-0.0204033931700000E-04	-0.000000000000000000 00
	11	0.000000000000000000 00	-0.114934020000000000 00	-0.140670130000000000 00
	12	0.000000000000000000 00	-0.1160046344790100E-03	-0.240090000000000000 00
	13	0.000000000000000000 00	0.000000000000000000 00	-0.30341.47232000000E-02

TIME STEP	250	TIME	0.17500000E-03	CARTESIAN COORDINATES	PRESSURE
M	N	U1(M,N, 250)	U2(M,N, 250)	U3(M,N, 250)	P(M,N)
7	1	0.000000000000000000 00	0.000000000000000000 00	0.470000000000000000 03	0.000000000000000000 00
	2	0.000000000000000000 00	0.241596400 0200992E 00	0.14700471903012741E 03	0.000000000000000000 00
	3	0.000000000000000000 00	0.4577034106110670E 00	0.174400000 0600071E 01	0.000000000000000000 00
	4	0.000000000000000000 00	0.721400000000000000 00	0.170394700040000000 01	0.000000000000000000 00
	5	0.000000000000000000 00	0.972600000000000000 00	0.17039006700130310E 01	0.000000000000000000 00
	6	0.000000000000000000 00	0.123504000000000000 00	0.170300000000000000 01	0.000000000000000000 00
	7	0.000000000000000000 00	0.140466000000000000 00	0.17006746740040217E 01	0.000000000000000000 00
	8	0.000000000000000000 00	0.173000000000000000 00	0.12124912400276400E 01	0.000000000000000000 00
	9	0.000000000000000000 00	0.100120000000000000 00	0.121701137.7043250E 01	0.000000000000000000 00
	10	0.000000000000000000 00	0.22479199940776400E 01	0.17299160002002240E 01	0.000000000000000000 00
	11	0.000000000000000000 00	0.2490500770906173E 01	0.172727000000000000 01	0.000000000000000000 00
	12	0.000000000000000000 00	0.27451997141001600E 01	0.123404070000000000 01	0.000000000000000000 00
	13	0.000000000000000000 00	0.300000000000000000 00	0.1719400104661530E 01	0.000000000000000000 00
12	1	0.147900000000000000 01	0.000000000000000000 00	0.120000000000000000 00	0.000000000000000000 00
	2	0.1466170192784.4E 01	0.2444020111957033E 00	-0.17421909103933010E 01	0.000000000000000000 00
	3	0.1428-0.1242111067E 01	0.5006340910470000E 00	-0.3730311170991240E 01	0.000000000000000000 00
	4	0.139942747073632E 01	0.7499327991001163E 00	-0.096030000000000000 00	0.000000000000000000 00
	5	0.1290-0.1910966420E 01	0.006767464.449007E 00	-0.139770709000000000 00	0.000000000000000000 00
	6	0.1239-0.194763786007E 01	0.122600000000000000 00	-0.1023694001000446E 00	0.000000000000000000 00
	7	0.1192-0.19600000000000 01	0.147994000000000000 00	-0.2157363350574921E 00	0.000000000000000000 00
	8	0.1159-0.20403910269E 01	0.172311030000000000 01	-0.241031350000000000 00	0.000000000000000000 00
	9	0.1120-0.10200000000000 01	0.197304040000000000 01	-0.20335907470440173E 00	0.000000000000000000 00
	10	0.11054279460110.04 01	0.272915953.170753E 01	-0.2704001300234000E 00	0.000000000000000000 00
	11	0.108610200000000000 01	0.24702027900001444E 01	-0.292409000000000000 00	0.000000000000000000 00
	12	0.107410010000000000 01	0.273007193500010E 01	-0.3013120210669400E 00	0.000000000000000000 00
	13	0.1070001244499173E 01	0.300000000000000000 01	-0.304067000004242E 00	0.000000000000000000 00
22	1	0.9190597900994767E-19	0.000000000000000000 00	-0.147900000000000000 03	0.000000000000000000 00
	2	0.9190597900994767E-19	0.250000000000000000 00	-0.14769-3960067590E 03	0.000000000000000000 00
	3	0.9190597900994767E-19	0.300000000000000000 00	-0.1470654243070330E 01	0.000000000000000000 00
	4	0.9190597900994767E-19	0.750000000000000000 00	-0.1463037306457940E 01	0.000000000000000000 00
	5	0.9190597900994767E-19	0.100000000000000000 00	-0.145163931197430E 01	0.000000000000000000 00
	6	0.9190597900994767E-19	0.1200432379040097E 01	-0.1430992778600791E 01	0.000000000000000000 00
	7	0.9190597900994767E-19	0.190023900000000000 01	-0.142477304000000000 01	0.000000000000000000 00
	8	0.9190597900994767E-19	0.179000200000000000 01	-0.1411744261350639E 01	0.000000000000000000 00
	9	0.9190597900994767E-19	0.200000000000000000 01	-0.140001700000000000 01	0.000000000000000000 00
	10	0.9190597900994767E-19	0.224000000000000000 01	-0.139000000000000000 01	0.000000000000000000 00
	11	0.9190597900994767E-19	0.240000000000000000 01	-0.138000000000000000 01	0.000000000000000000 00
	12	0.9190597900994767E-19	0.270000000000000000 01	-0.140000000000000000 01	0.000000000000000000 00
	13	0.9190597900994767E-19	0.300000000000000000 01	-0.140000000000000000 01	0.000000000000000000 00

Table 5.21 Samples for $N = 2, 12, \text{ and } 22$ of Displacement Increments Array During Time Increment 499 to 500 and of Cartesian Coordinates and Pressure Arrays at Time Step 500

DISPLACEMENT INCREMENTS ARRAY FOR $N = 2, 12, \text{ AND } 22$

	$u(i,m)$	$v(i,m)$	$w(i,m)$
2	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00
1	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00
2	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00
3	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00
4	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00
5	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00
6	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00
7	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00
8	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00
9	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00
10	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00
11	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00
12	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00
13	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00
14	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00
15	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00
16	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00
17	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00
18	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00
19	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00
20	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00
21	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00
22	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00

	$u(i,m)$	$v(i,m)$	$w(i,m)$	Pressure
2	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00
1	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00
2	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00
3	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00
4	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00
5	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00
6	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00
7	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00
8	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00
9	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00
10	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00
11	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00
12	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00
13	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00
14	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00
15	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00
16	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00
17	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00
18	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00
19	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00
20	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00
21	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00
22	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00	0.0000000000000000 00

Table 5.22 Samples at Time Step 500 of Surface Normal Components Array for M = 2, 12, and 22 and of Stress Subincrement Array for Layers K = 1 and 2

TIME STEP	M	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
500	2	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000
500	12	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000
500	22	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000

TIME STEP 500 TIME 6.1997766E-04 SUBDIVISIONS OF TIME INCREMENT IN STRESS

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 5.23 Samples of Energy Balance and of Surface Strains Printed Output at Time Steps 250 and 500

TIME STEP 250 TIME= 0.37500000E-03

SURFACE STRAINS

ETA1	ETA2	M	N	FACE	ANGLE 0	ANGLE 90	ANGLE
1.000	3.000	2.000	13.000	INNER	-0.21843286E 00	0.18997300E-01	45.00
1.000	3.000	2.000	13.000	OUTER	-0.13090572E 00	0.19724857E-01	45.00
1.162	3.000	7.000	13.000	INNER	-0.10754889E 00	0.39073041E-01	45.00
1.162	3.000	7.000	13.000	OUTER	-0.11384974E 00	0.28376254E-01	45.00
2.323	1.500	12.000	7.000	INNER	-0.15332724E-01	0.11780287E-01	45.00
2.323	1.500	12.000	7.000	OUTER	0.72246290E-02	0.56118482E-02	45.00

STRAIN GAGE READING

ANGLE	ANGLE
135.00	-0.91924415E-01
135.00	-0.52592050E-01
135.00	-0.31459405E-01
135.00	-0.40098962E-01
135.00	-0.57773029E-02
135.00	-0.82882890E-02
135.00	0.91157369E-02

TIME STEP 250 TIME= 0.37500000E-03 KINETIC= 0.48234038E 02 ELASTIC= 0.51063366E 02 PLASTIC= 0.71923544E 04
 TOTAL ENERGY= 0.72916228E 04

110

TIME STEP 500 TIME= 0.61997784E-03

SURFACE STRAINS

ETA1	ETA2	M	N	FACE	ANGLE 0	ANGLE 90	ANGLE
1.000	3.000	2.000	13.000	INNER	-0.21902752E 00	0.19023327E-01	45.00
1.000	3.000	2.000	13.000	OUTER	-0.13087125E 00	0.19016056E-01	45.00
1.162	3.000	7.000	13.000	INNER	-0.11230353E 00	0.37450252E-01	45.00
1.162	3.000	7.000	13.000	OUTER	-0.10330033E 00	0.29364833E-01	45.00
2.323	1.500	12.000	7.000	INNER	-0.13369569E-01	0.12458008E-01	45.00
2.323	1.500	12.000	7.000	OUTER	0.70857540E-02	0.64973669E-02	45.00

STRAIN GAGE READING

ANGLE	ANGLE
135.00	-0.92161030E-01
135.00	-0.52660856E-01
135.00	-0.34534025E-01
135.00	-0.37480294E-01
135.00	0.66152859E-02
135.00	0.74820235E-02
135.00	0.96698031E-02

TIME STEP 500 TIME= 0.61997784E-03 KINETIC= 0.11639291E 00 ELASTIC= 0.31063351E 02 PLASTIC= 0.72053024E 04
 TOTAL ENERGY= 0.72916228E 04

TAPE 1 WRITTEN, NOCYCLE= 500 TIME= 0.61997784E-03

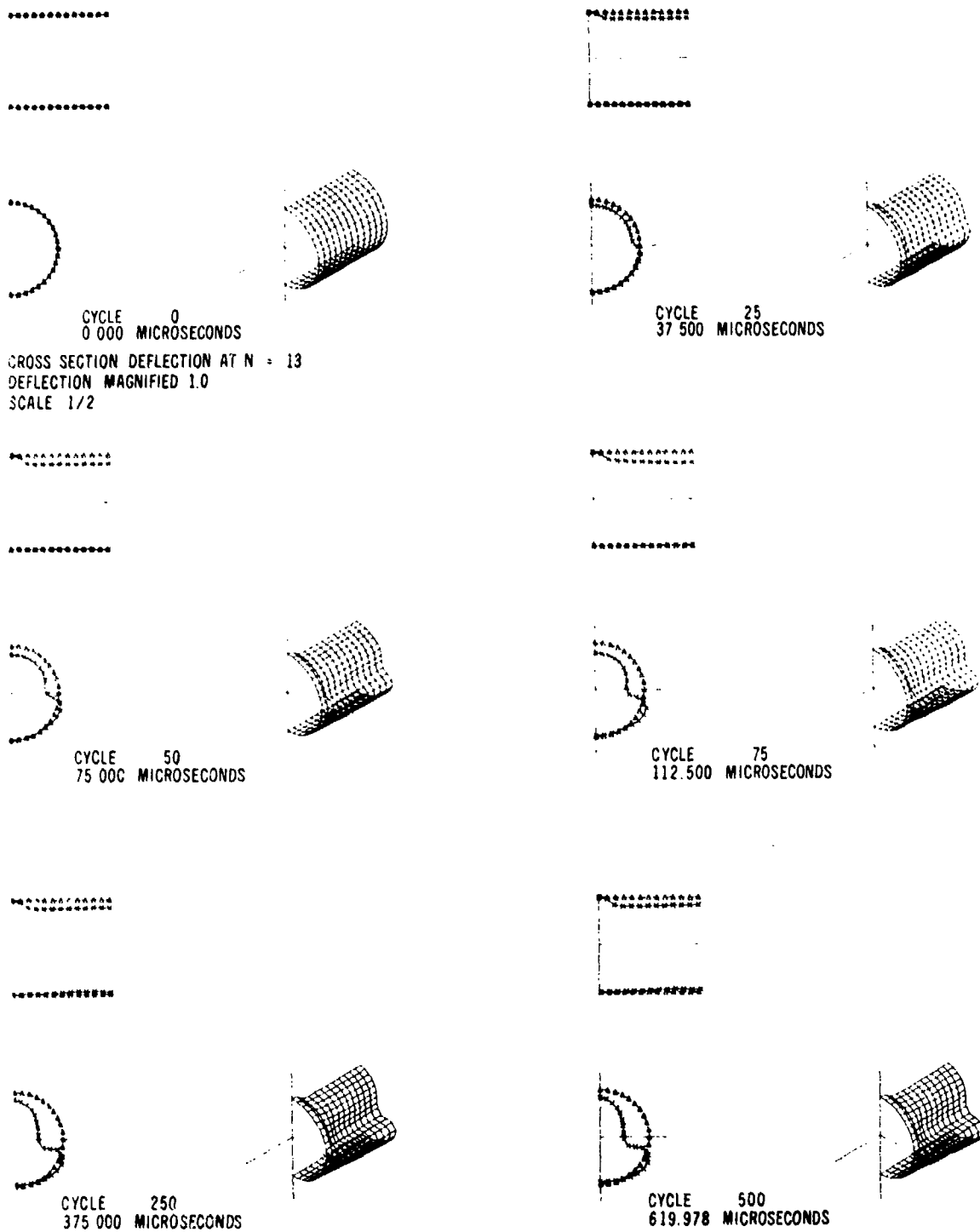
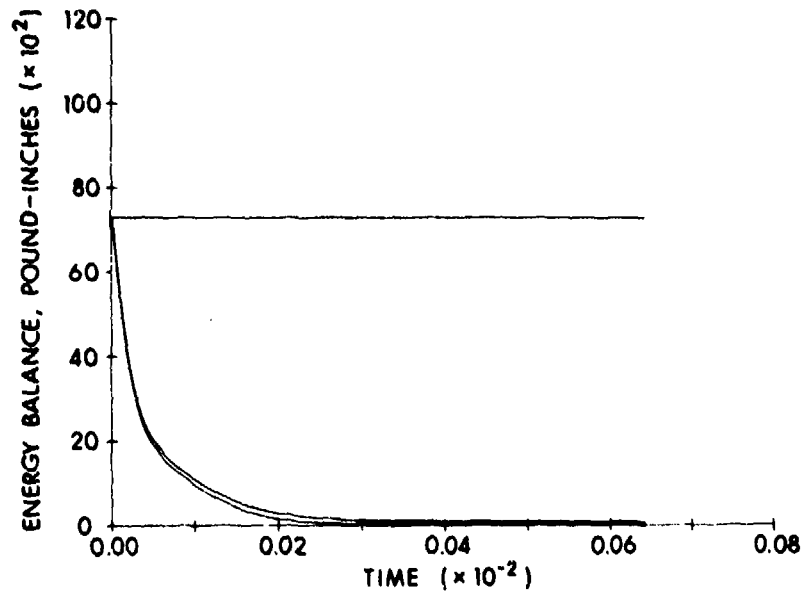
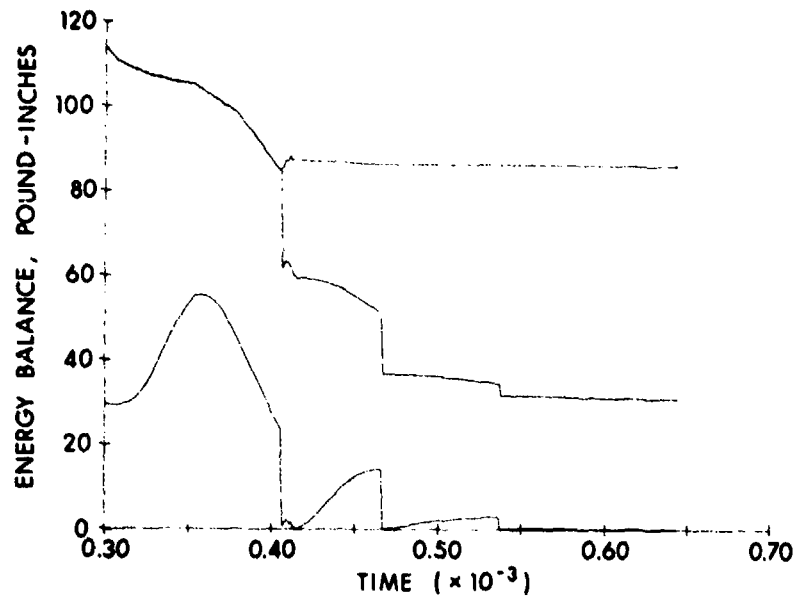


Figure 5.8 Isometric and Cross-sectional Cal Comp Plots of the Deforming Middle Surface at Selected Time Steps Showing True (Unmagnified) Displacements

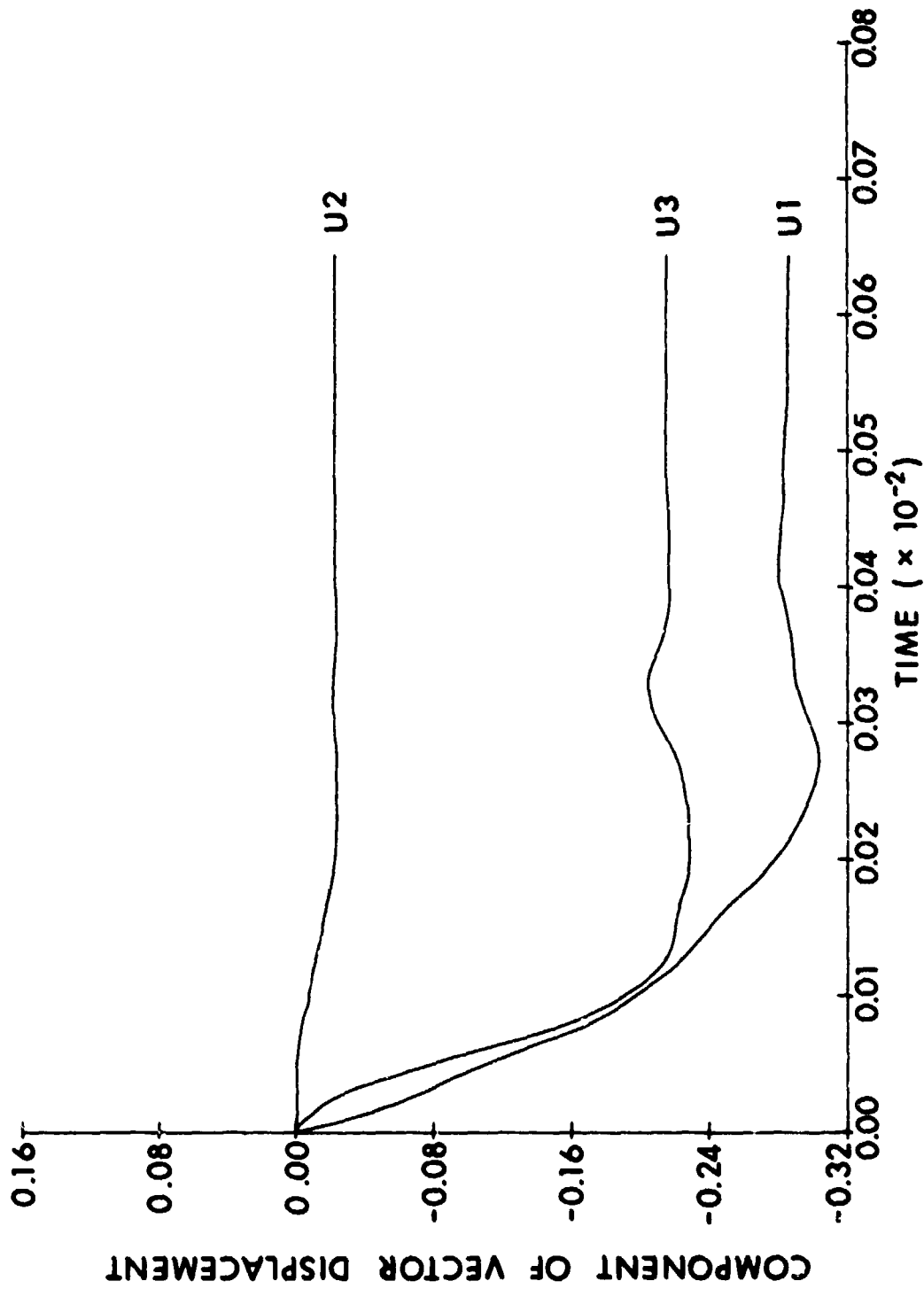


a. Entire history of the energy balance from 0 to 643 microseconds, showing in ascending order the kinetic energy, total energy, total energy plus damping work, and external work.



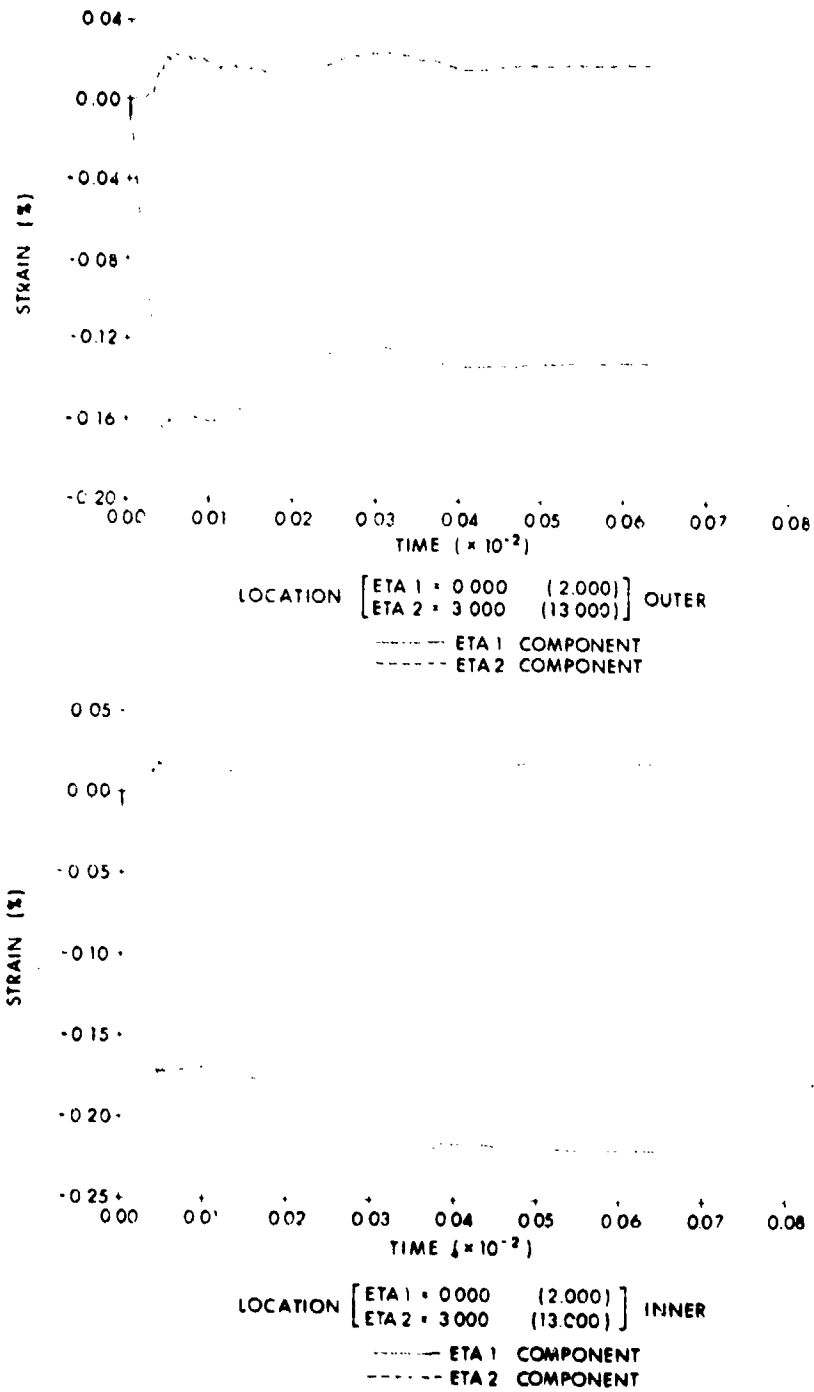
b. Blow up of the history of the energy balance from 300 to 643 microsecond, showing in ascending order the kinetic energy, total energy, and total energy plus damping work.

Figure 5.9 Cal Comp Plots of the Energy Balance for the Impulsively Loaded Cylinder



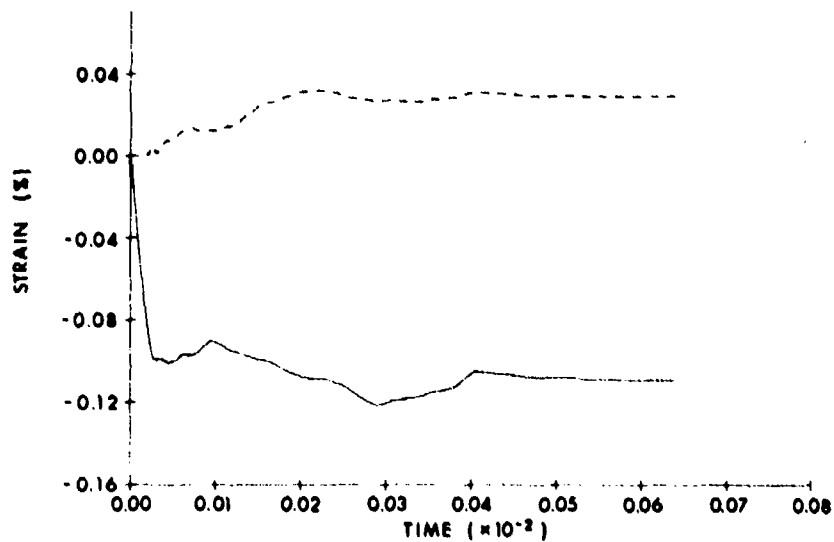
LOCATION [ETA 1 = 2.323 (12.000)]
 [ETA 2 = 1.500 (7.000)]

Figure 5.10 Cal Comp Plot of the History of the Deflection at Point 90° from Crown Line and Midway Between Clamped and Symmetry Edges

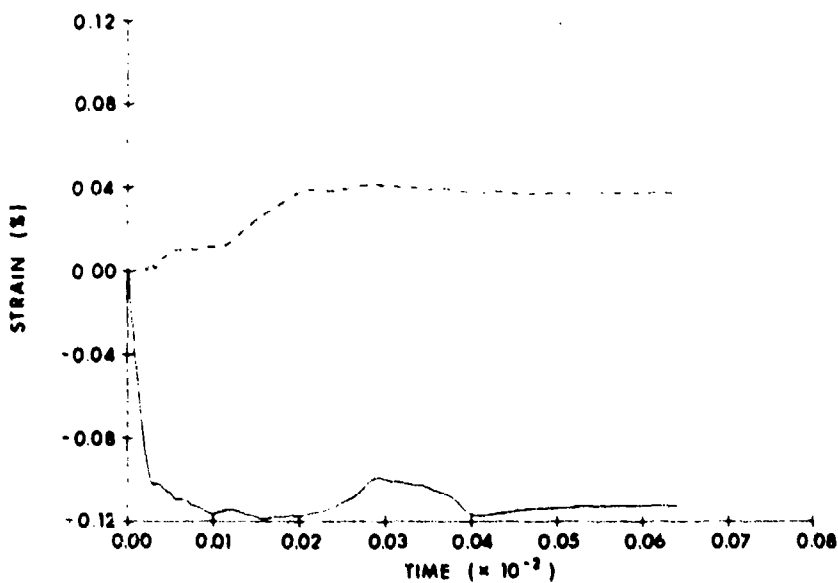


a. Strains on the crown line at half the axial length.

Figure 5.11 Cal Comp Plots of the History of the Surface Elongational Strains Along the Mesh Directions



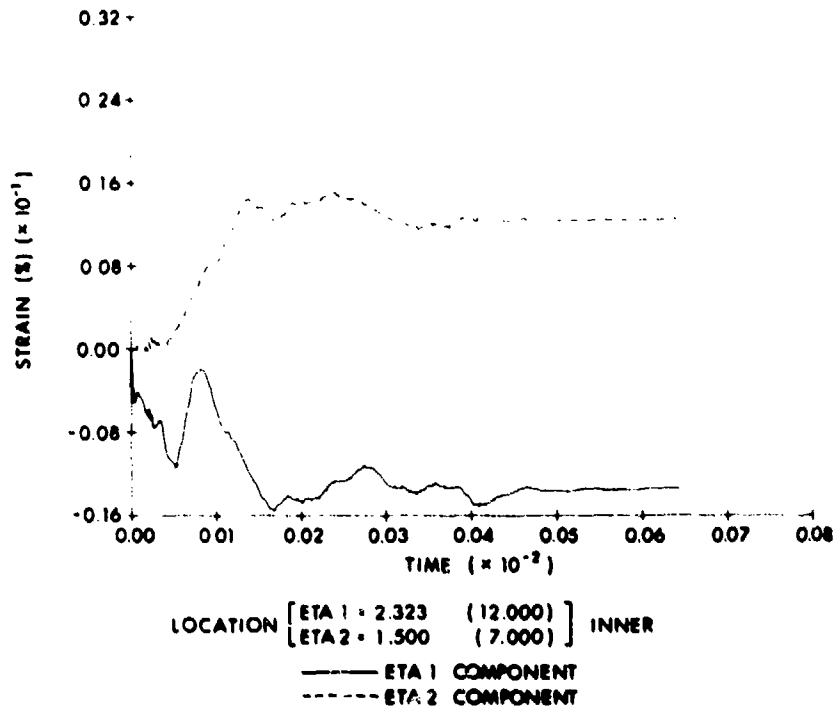
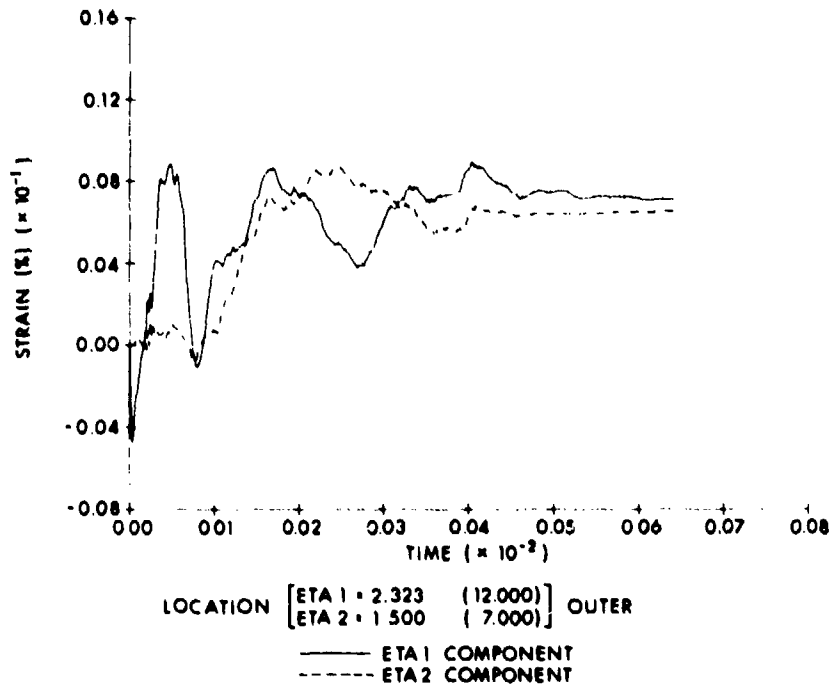
LOCATION [ETA 1 = 1.162 (7.000)] OUTER
 [ETA 2 = 3.000 (13.000)]
 ——— ETA 1 COMPONENT
 - - - - - ETA 2 COMPONENT



LOCATION [ETA 1 = 1.162 (7.000)] INNER
 [ETA 2 = 3.000 (13.000)]
 ——— ETA 1 COMPONENT
 - - - - - ETA 2 COMPONENT

b. Strains at 45° from the crown line and half the axial length.

Figure 5.11 (Continued)



c. Strains at 90° from the crown line and quarter the axial length.

Figure 5.11 (Continued)

6. IMPLEMENTATION OF LOADINGS AND INITIAL GEOMETRIES

REPSIL can be made to accept arbitrary impulse loadings, arbitrary time and space varying pressure loadings, and, within certain limits, arbitrary initial shell geometries. This involves a small amount of programming confined to certain REPSIL subroutine: INVEL, PRESS and INGEOM. This chapter describes the acceptable ways of implementing loadings and geometries into REPSIL so as to be compatible with the code's formulation.

6.1 Two Approaches to Implementing Loads and Geometries

Implementing a loading or an initial geometry in REPSIL basically involves assigning to each mesh point (m,n) a value of initial impulse velocity or a value of pressure for each time step, or values of the initial coordinates of the middle surface. These values can be assigned in two ways: first, they can be read off a tape or cards and assigned to the mesh points by the appropriate subroutine; second, an analytical expression programmed into the appropriate subroutine can generate these values, which the subroutine then assigns to the mesh points. The first approach requires that data be generated outside the program and be read onto a tape or punched onto cards. The data itself can be generated analytically, numerically or experimentally -- this approach will accept all; therein lies its advantage, especially for ad hoc problems. Its drawbacks are the tediousness of reading onto a tape or punching onto cards large numbers of values especially when many mesh points are employed, and necessity of regenerating new data for a given physical problem whenever the number of mesh intervals is changed. The second approach has none of these drawbacks, but does require that the data be expressible analytically. These analytical expressions are most conveniently written as functions of the underlying material coordinates (η^1, η^2) rather than the mesh numbers (m,n), for then the forms of the expressions are not affected by a change in the number of mesh intervals. The value of such an expression at a mesh point (m,n) is obtained as the value of the function at the material coordinates (η^1, η^2) corresponding to the point (m,n) :

$$\eta^1(m) = \eta_0^1 + (m-1) \Delta\eta^1, \quad \eta^2(n) = \eta_0^2 + (n-1) \Delta\eta^2 \quad * \quad (6.1)$$

* These equations are closer to the indexing of m and n used in the program than (2.1), reflecting the fact that FORTRAN does not permit zero values of array indices.

where, for $M \times N$ mesh intervals, $1 \leq m \leq M + 1$, $1 \leq n \leq N + 1$ and

$$\Delta \eta^1 = \frac{\eta_f^1 - \eta_o^1}{M}, \quad \Delta \eta^2 = \frac{\eta_f^2 - \eta_o^2}{N}. \quad (6.2)$$

In the subsequent sections, in which the implementation of the loading and geometry subroutines are individually described, the two approaches are again covered, as they apply to each subroutine.

6.2 Subroutine INVEL

Initial impulse loadings are specified in subroutine INVEL. At the zeroth time step INVEL assigns a value of impulse velocity to each mesh point. The velocity is free to vary spatially over the shell surface, but at present is restricted to be directed along the normal - a minor restriction that is easily remedied by allowing tangential components to be defined through the middle surface basis vectors, see Section 2.3.

As stated in the previous section, two approaches can be used in programming INVEL. With the first approach, INVEL simply reads off cards or a tape a sequence of values corresponding to the normal velocity v at each mesh point. Each value is immediately multiplied by the surface normal at the mesh point to give components of the velocity

$$v^i = v n^i, \quad (6.3)$$

which are then stored in the as yet unused three $M \times N$ arrays $\Delta u^i(m,n)$. A form of this approach is used for the subroutine INVEL presently in REPSIL, which is specified on input Card 15 and 16, see Section 3.2. (A listing of this subroutine is given in Appendix E.)

The second approach requires some analytic expressions of the form

$$v = v(\eta^1, \eta^2) \quad (6.4)$$

for the normal velocity as a function of the material coordinates. The material coordinates are limited to the rectangular domain specified in INGEOM, see (6.10) of Section 6.4. With this approach, (6.1) and (6.4) are both programmed into INVEL; (6.2) need not be included since the increments $\Delta \eta^a$ are calculated in INGEOM. INVEL determines the normal velocity at mesh points by calculating the values of the material

coordinates at the mesh point (m,n) using (6.1) and substituting these values in (6.4) to obtain

$$v(m,n) = v(n^1(m), n^2(n)). \quad (6.5)$$

As with the first approach, these values are immediately multiplied by the normal to obtain the components of the velocity at (m,n), which are then stored in the three M x N arrays $\Delta u^i(m,n)$.

6.3 Subroutine PRESS

Time/space varying pressure loads are specified in subroutine PRESS. Each time step this subroutine assigns a value of pressure to each mesh point.

With the first of the two approaches to programming already mentioned, at each time step PRESS reads off cards or (more usually) a tape a sequence of values of the pressure P at each mesh point and stores these values in the M x N array P(m,n) for later use in subroutine MOTION, see Section 2.3. This approach has been used in REPSIL for a number of cases, in which the pressure data has been experimentally/numerically generated; reports on these cases are in preparation.

For the second approach, analytic expressions for the pressure as a function of the material coordinates and the time:

$$P = F (n^1, n^2, t) \quad (6.6)$$

are programmed into PRESS, along with (6.1); the material coordinates are limited to the rectangular domain (6.10) specified in INGEOM. At each time step ℓ , PRESS calculates the pressure P at each mesh point (m,n) by determining the material coordinates of the mesh point from (6.1), the value of time at time step ℓ from

$$t(\ell) = \ell \Delta t \quad (6.7)$$

and substituting these values in (6.6):

$$P(m,n) = P(n^1(m), n^2(n), t(\ell)). \quad (6.8)$$

As with the first approach, these values are stored in the M x N array

$P(m,n)$. This approach is used to generate the pressure data for example problem 1 (Section 5.1); a listing of this PRESS subroutine is in Appendix E.

6.4 Subroutine INGEOM

The initial geometry of the shell is specified in subroutine INGEOM. A new subroutine INGEOM must be written for each initial geometry or at least each family of initial geometries.* The only restrictions on the admissible geometries are that the middle surface be simply connected and bounded by four smooth edges, such that none of the corners formed by intersecting edges are reentrant or straight.

Particularizing the remarks of Section 6.1 to initial geometries, the basic function of INGEOM is to set up a correspondence between mesh numbers (m,n) and coordinates y^i in 3-space through which the middle surface of the shell initially passes; this correspondence should be one-to-one. As already mentioned, two approaches can be used in programming INGEOM. First, INGEOM can just comprise instructions for reading off cards or a tape a sequence of ordered triplexes (y^1, y^2, y^3) and storing these in three $M \times N$ arrays $y^1(m,n)$, $y^2(m,n)$ and $y^3(m,n)$. Of course, the sequence of triplexes y^i must be chosen carefully, not only for sake of obtaining a one-to-one correspondence, but also in order that the correspondence be topologically continuous in the following sense: neighboring points in space are assigned adjacent mesh numbers in the proper order, as indicated in Figure 6.1. This approach requires that the increments in material coordinates, Δn^1 and Δn^2 , be assigned convenient values that are subsequently used to form finite difference quotients. Also, it is important that the number of mesh intervals M and N assigned elsewhere in the program jibe with the spacing implied by subroutine INGEOM.

The second approach uses one or more sets of analytic expressions giving the Cartesian coordinates y^i of the middle surface parametrically as functions of the material coordinates n^a :

$$y^1 = y^1(n^1, n^2), \quad y^2 = y^2(n^1, n^2), \quad y^3 = y^3(n^1, n^2). \quad (6.9)$$

These functions are required to be continuous and one-to-one in the domain over which the material coordinates vary; the domain itself is

* REPSIL has INGEOM subroutines for a flat plate, cylindrical shell and conical shell programmed, see Section 3.2 and Appendix E.

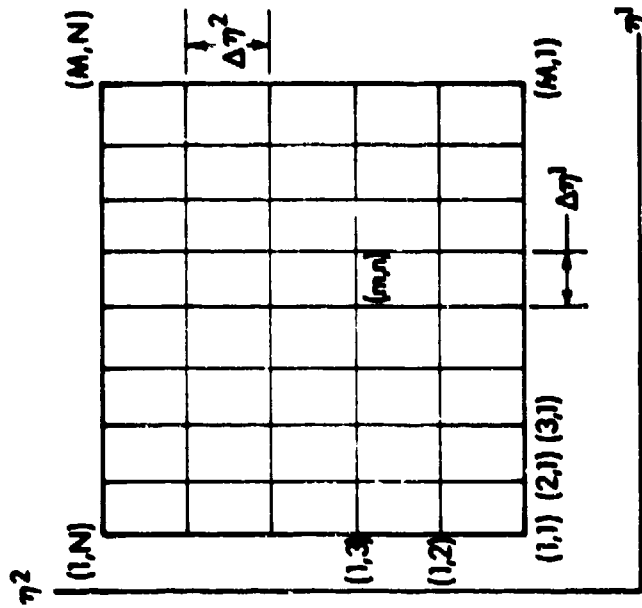
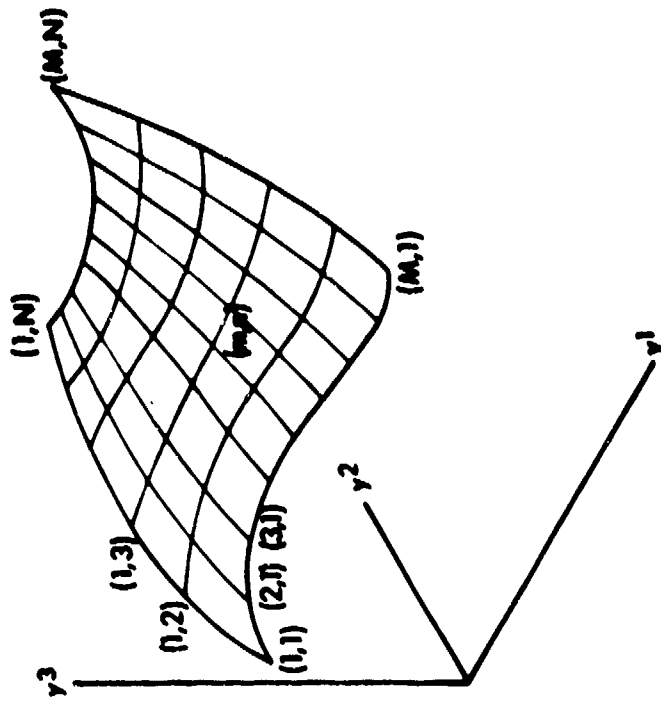


Figure 6.1 Mapping of Material Coordinate Mesh onto the Middle Surface of Shell

limited to some rectangle in the material coordinates plane:

$$\eta_0^1 \leq \eta^1 \leq \eta_f^1, \quad \eta_0^2 \leq \eta^2 \leq \eta_f^2. \quad (6.10)$$

Consequently, the parametric representation of the middle surface is simply a one-to-one continuous map of a rectangle in the η^1, η^2 plane into 3-space, from which automatically follow the aforementioned topological continuity and restrictions on the boundary. It should be noted that the parametric representation is not unique--many exist for a given surface. Moreover, the material coordinates need not have physical significance, such as arc length, angle, etc., although often a simple transformation can give them such meaning.

With the second approach, equations (6.9) are programmed into INGEOM, as well as (6.1) and (6.2). For a given physical problem, the extent of the shell is fixed by the limits on the material coordinates (6.10). Once the number of mesh intervals $M \times N$ are specified, the subroutine computes the constant intervals $\Delta\eta^a$ using (6.2) and the values of the material coordinates $\eta^1(m)$ and $\eta^2(n)$ at the mesh points (m,n) using (6.1). Substituting these values of the material coordinates into the analytical expression (6.9), the subroutine calculates the Cartesian coordinates of the mesh points:

$$y^1 = y^1(m,n), \quad y^2 = y^2(m,n), \quad y^3 = y^3(m,n), \quad (6.11)$$

thus mapping the rectangular $M \times N$ mesh in the η^1, η^2 plane into a curvilinear mesh in 3-space, as pictured in Figure 6.1. As with the first approach, the subroutine stores these values in three $M \times N$ arrays.

The second approach to programming INGEOM will be illustrated with an example of a shell having for its initial middle surface a frustum of a circular cone with an axial length L , a small radius R_0 and large radius R_f . Let the frustum be located relative to the y^1, y^2, y^3 axis as shown in Figure 3.8. With this orientation the coordinates of the frustum are parametrically given by

$$y^1 = \eta^1 \sin \eta^2, \quad y^2 = \frac{\eta^1 - R_0}{R_f - R_0} L, \quad y^3 = \eta^1 \cos \eta^2, \quad (6.12)$$

with the material coordinates limited to the domain

$$R_0 \leq \eta^1 \leq R_f \text{ \& } 0 \leq \eta^2 < 2\pi . \quad (6.13)$$

In this representation both parameters have physical significance: η^1 being the radius from the cone axis and η^2 the angle about the axis.

This representation is not unique; for example, η^1 can be replaced by a parameter $\hat{\eta}^1$ measuring arclength along generators through the transformation

$$\hat{\eta}^1 = \eta^1 \csc \alpha ,$$

with α the cone angle:

$$\alpha = \arctan \frac{R_f - R_0}{L} , \quad (6.14)$$

resulting in the representation

$$y^1 = \hat{\eta}^1 \sin \alpha \sin \eta^2 , \quad y^2 = (\hat{\eta}^1 - S_0) \cos \alpha , \quad y^3 = \hat{\eta}^1 \sin \alpha \cos \eta^2 , \quad (6.15)$$

with domain

$$S_0 (= R_0 \csc \alpha) \leq \hat{\eta}^1 \leq S_f (= R_f \csc \alpha) , \quad 0 \leq \eta^2 < 2\pi \quad (6)$$

Clearly, a transformation replacing η^2 by circumferential arclength cannot exist, for it is impossible for the same angle η^2 at different location along the cone axis to subtend equal arclength on the surface of a cone. For the same reason, the image of an $M \times N$ mesh under the representation (6.15) or for that matter (6.12) will yield a curvilinear rectangular mesh with nonequal rectangles: as shown in Figures 3.8 and 6.2a, while the meridional lengths of rectangles are equal, the circumferential lengths increase with the radius. However, it is possible through a judicious transformation of η^1 or $\hat{\eta}^1$ into say $\bar{\eta}^1$ to obtain nonuniform meridional increments that give similar rectangles (i.e. rectangles of constant side ratio). Using differentials, the condition that constant increments $\Delta \bar{\eta}^1$ and $\Delta \eta^2$ give rectangles with a constant meridional to circumferential side ratio of ν can be written as

$$\left| \frac{\partial y^1}{\partial \bar{\eta}^1} \Delta \bar{\eta}^1 \right| / \left| \frac{\partial y^1}{\partial \eta^2} \Delta \eta^2 \right| = \kappa$$

Assuming that $\hat{\eta}^1$ is a function of $\bar{\eta}^1$ and using (6.15), this condition is shown to be equivalent to the differential equation

$$\frac{d\hat{\eta}^1}{d\bar{\eta}^1} = K \sin \alpha \hat{\eta}^1, \quad K = \kappa \frac{\Delta \eta^2}{\Delta \bar{\eta}^1}.$$

Choosing the lower limit on $\bar{\eta}^1$ to be zero for the sake of convenience, the solution of this differential equation yields the transformation

$$\hat{\eta}^1 = S_0 e^{K \sin \alpha \bar{\eta}^1},$$

in the range $0 \leq K \sin \alpha \bar{\eta}^1 \leq \ln \frac{S_f}{S_0}$. This transformation is substituted in (6.15) to give the parametric representation

$$\begin{aligned} y^1 &= S_0 \sin \alpha \times \sin \eta^2 e^{K \sin \alpha \bar{\eta}^1} \\ y^2 &= S_0 \cos \alpha \times (e^{K \sin \alpha \bar{\eta}^1} - 1) \\ y^3 &= S_0 \sin \alpha \times \cos \eta^2 e^{K \sin \alpha \bar{\eta}^1} \end{aligned} \quad (6.17)$$

over the domain

$$0 \leq K \sin \alpha \bar{\eta}^1 \leq \ln \frac{S_f}{S_0}, \quad 0 \leq \eta^2 < 2\pi. \quad (6.18)$$

Since the program specified the number of mesh intervals M and N rather than the increments $\Delta \eta^\alpha$, the constant K can be conveniently set equal to unity. Also (6.15) and (6.17) can be summarized in a single general representation by a change in the scale of $\hat{\eta}^1$, giving the representation

$$\begin{aligned} y^1 &= S_0 \sin \alpha \sin \eta^2 f(\hat{\eta}^1 \sin \alpha) \\ y^2 &= S_0 \cos \alpha [f(\hat{\eta}^1 \sin \alpha) - 1] \\ y^3 &= S_0 \sin \alpha \cos \eta^2 f(\hat{\eta}^1 \sin \alpha), \end{aligned} \quad (6.19)$$

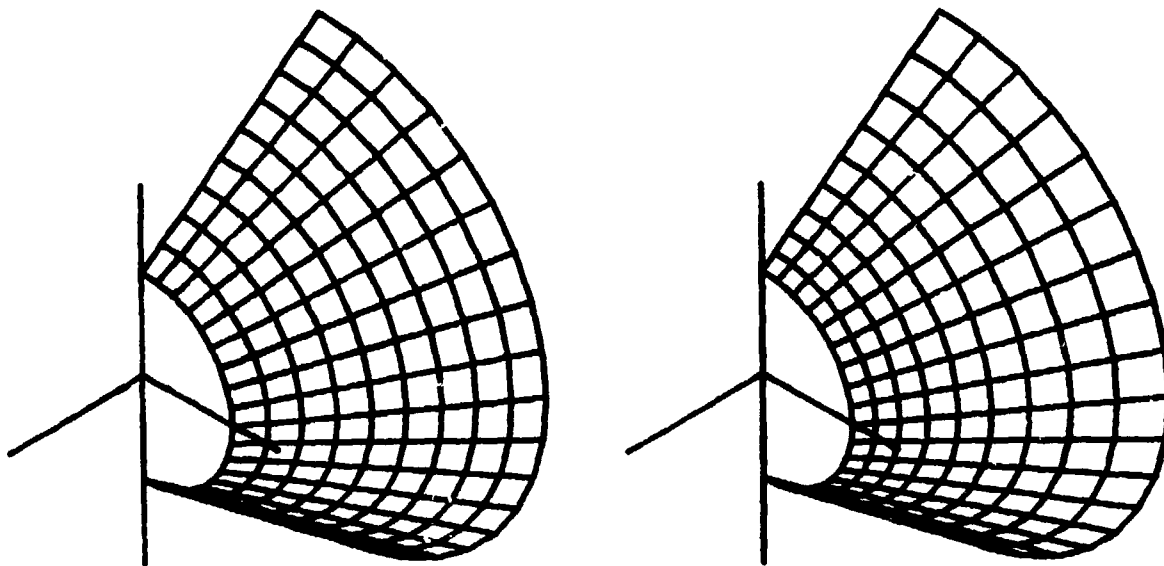
where for a uniform meridional spacing

$$f(\eta^1 \sin \alpha) = \eta^1 \sin \alpha \quad ; \quad 1 \leq \eta^1 \sin \alpha \leq \frac{S_f}{S_o} \quad (6.20)$$

and for a constant side ratio

$$f(\eta^1 \sin \alpha) = \exp(\eta^1 \sin \alpha) \quad ; \quad 0 \leq \eta^1 \sin \alpha \leq \ln \frac{S_f}{S_o} \quad (6.21)$$

The programming of this representation is straightforward and is included in the listing in Appendix E. Figure 6.2 shows the difference between the two types of spacing for given M x N mesh intervals, reproduced from Cal Comp plots.



a. Constant meridional increment b. Constant (almost square) side ratio

Figure 6.2 Comparison of the 18 x 9 Mesh Generated by Subroutine INGEOM for the Frustum of a Cone Using the Constant Meridional Increment Option and the Constant Mesh Proportions Option

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APPENDIX A.
FINITE DIFFERENCE OPERATORS

The partial differential equations solved by the REPSIL code employ the following partial derivatives with respect to the material coordinates:

$$\frac{\partial}{\partial n^1}, \quad \frac{\partial}{\partial n^2}, \quad \frac{\partial^2}{(\partial n^1)^2}, \quad \frac{\partial^2}{\partial n^1 \partial n^2}, \quad \frac{\partial^2}{(\partial n^2)^2}.$$

These partials are approximated in the code by finite difference operators of order $|\Delta n|^2$ in accuracy. These operators, which were symbolized by their corresponding partials in Section 2.3, will now be given explicitly.

Let $F(m,n)$ represent a typical mesh function (the position coordinates y^i or the bending resultants $M^{\alpha\beta}$, for example) defined over the domain of mesh points $m_1 \leq m \leq m_f$ and $n_1 \leq n \leq n_f$. At interior mesh points where the mesh numbers (m,n) satisfy $m_1 < m < m_f$ and $n_1 < n < n_f$, central difference operators, symbolized by the superscript c , are used:

$$\frac{\Delta F^c}{\Delta n^1}(m,n) \equiv \frac{F(m+1,n) - F(m-1,n)}{2\Delta n^1},$$

$$\frac{\Delta F^c}{\Delta n^2}(m,n) \equiv \frac{F(m,n+1) - F(m,n-1)}{2\Delta n^2},$$

$$\frac{\Delta^2 F^c}{(\Delta n^1)^2}(m,n) \equiv \frac{F(m+1,n) - 2F(m,n) + F(m-1,n)}{(\Delta n^1)^2},$$

$$\frac{\Delta^2 F^c}{(\Delta n^2)^2}(m,n) \equiv \frac{F(m,n+1) - 2F(m,n) + F(m,n-1)}{(\Delta n^2)^2},$$

$$\frac{\Delta^2 F^c}{\Delta n^1 \Delta n^2}(m,n) \equiv \frac{F(m+1,n+1) - F(m-1,n+1) - F(m+1,n-1) + F(m-1,n-1)}{\Delta n^1 \Delta n^2},$$

with $\Delta n^1, \Delta n^2$ (constant) increments in the material coordinates. Notice that the mixed partials operator involves the successive application of the first partial operators:

$$\frac{\Delta^2 F^c}{\Delta\eta^1 \Delta\eta^2}(m,n) = \frac{\frac{\Delta F^c}{\Delta\eta^2}(m+1,n) - \frac{\Delta F^c}{\Delta\eta^2}(m-1,n)}{2\Delta\eta^1} = \frac{\frac{\Delta F^c}{\Delta\eta^1}(m,n+1) - \frac{\Delta F^c}{\Delta\eta^1}(m,n-1)}{2\Delta\eta^2}$$

or more compactly:

$$\frac{\Delta^2 F^c}{\Delta\eta^1 \Delta\eta^2} = \frac{\Delta}{\Delta\eta^1} \left(\frac{\Delta F^c}{\Delta\eta^2} \right)^c = \frac{\Delta}{\Delta\eta^2} \left(\frac{\Delta F^c}{\Delta\eta^1} \right)^c$$

Along the boundaries where $m = m_i$ or m_f or $n = n_i$ or n_f central difference operators cannot be used to approximate all partials due to $F(m,n)$ not being defined outside the domain of mesh points. Rather, some of the above central difference operators are replaced by either forward or backward difference operators. Specifically, along $m = m_i$ or m_f only the central difference operators $\frac{\Delta F^c}{\Delta\eta^2}$ and $\frac{\Delta^2 F^c}{(\Delta\eta^2)^2}$ are employed. The remaining operators are replaced by forward difference operators (denoted by the superscript f) along $m = m_i$:

$$\frac{\Delta F^f}{\Delta\eta^1}(m,n) = - \frac{3F(m,n) - 4F(m+1,n) + F(m+2,n)}{2\Delta\eta^1},$$

$$\frac{\Delta^2 F^f}{(\Delta\eta^1)^2}(m,n) = \frac{2F(m,n) - 5F(m+1,n) + 4F(m+2,n) - F(m+3,n)}{(\Delta\eta^1)^2},$$

$$\frac{\Delta^2 F^f}{\Delta\eta^1 \Delta\eta^2}(m,n) = - \frac{3 \frac{\Delta F^c}{\Delta\eta^2}(m,n) - 4 \frac{\Delta F^c}{\Delta\eta^2}(m+1,n) + \frac{\Delta F^c}{\Delta\eta^2}(m+2,n)}{2\Delta\eta^1}$$

and by backward difference operators (denoted by the superscript b) along $m = m_f$:

$$\frac{\Delta F^b}{\Delta\eta^1}(m,n) = \frac{3F(m,n) - 4F(m-1,n) + F(m-2,n)}{2\Delta\eta^1},$$

$$\frac{\Delta^2 F^b}{(\Delta\eta^1)^2}(m,n) = \frac{2F(m,n) - 5F(m-1,n) + 4F(m-2,n) - F(m-3,n)}{(\Delta\eta^1)^2}$$

$$\frac{\Delta^2 F^b}{\Delta \eta^1 \Delta \eta^2} (m,n) = \frac{3 \frac{\Delta F^c}{\Delta \eta^2} (m,n) - 4 \frac{\Delta F^c}{\Delta \eta^2} (m-1,n) + \frac{\Delta F^c}{\Delta \eta^2} (m-2,n)}{2\Delta \eta^1} .$$

Notice again that the mixed partials operator involves the successive application of first partial operators (forward or backward with respect to η^1 and central with respect to η^2):

$$\frac{\Delta^2 F^f}{\Delta \eta^1 \Delta \eta^2} (m,n) = \frac{\Delta}{\Delta \eta^1} \left(\frac{\Delta F^c}{\Delta \eta^2} \right)^f , \quad \frac{\Delta^2 F^b}{\Delta \eta^1 \Delta \eta^2} = \frac{\Delta}{\Delta \eta^1} \left(\frac{\Delta F^c}{\Delta \eta^2} \right)^b .$$

Where possible the code takes advantage of this fact to reduce calculations.

Along the boundary $n = n_i$ or n_f the central operators $\frac{\Delta F^c}{\Delta \eta^1}$ and $\frac{\Delta^2 F^c}{(\Delta \eta^1)^2}$ are retained, while the remaining operators are replaced by appropriate forward or backward operators obtained from the above operators by interchanging the roles of m and n , and η^1 and η^2 simultaneously.

At the corners of the domain where $m = m_i$ or m_f and $n = n_i$ or n_f the remaining central operators are replaced by the forward or backward operators appropriate to the common boundaries; in particular, the mixed partials operators involve the successive application of the appropriate forward or backward first partial operators -- for example, at $m = m_i$ and $n = n_f$

$$\frac{\Delta^2 F}{\Delta \eta^1 \Delta \eta^2} = \frac{\Delta}{\Delta \eta^1} \left(\frac{\Delta F^b}{\Delta \eta^2} \right)^f .$$

APPENDIX B
COMMENTS ON ELASTOPLASTIC STRESS EVALUATIONS

In the REPSIL code stresses at time t are calculated in the manner of (finite) incremental plasticity using the incremental strains $\Delta \epsilon_{\beta}^{\alpha}$ which occur between time $t - \Delta t$ and time t as well as stored values of the stresses at time $t - \Delta t$. In the finite difference analysis these calculations are made at every (m,n) mesh node for each k layer. Additionally, where the strain-hardening constitutive option is used, stresses or, more precisely, substresses are calculated for each j sublayer of the mechanical sublayer model (see [2; Section 5.4.2]) and the stresses in each k layer are determined as weighted averages of the stresses in the j sublayers.

At locations where the incremental strains $\Delta \epsilon_{\beta}^{\alpha}$ entail plastic flow, the flow parameter $\Delta \lambda$ of (2.24) must be evaluated as the root of a quadratic equation, see (2.26). An algorithm for dealing with the various types of roots which may occur has been developed by Huffington [3]. If complex roots occur this algorithm subdivides the elastic stress increment, defined by (2.18), into L equal subincrements (for purposes of stress calculation only) in each of which elastic stress increments $E_{\alpha} \Delta \sigma_{\beta} / L$ take place. Stress calculations are performed for each of the L subincrements, consecutively. If at any stage of this process a complex root is obtained, the value of L is increased and the calculation is re-initiated. This procedure is continued until real stresses are determined for time t .

Experience with this procedure has shown that under certain circumstances it may be desirable to use the subdivided increment algorithm even when real roots are obtained for the full increment. An illustration of this is depicted in Figure B.1, where for simplicity it is assumed that σ_1^1 and σ_2^2 are principal stresses ($\sigma_2^1 = 0$). The values shown on this figure were taken from an actual computer solution. Starting from the stress state labeled "1", the components of the trial elastic stress vector $\Delta \sigma^E$ were calculated using (2.18) and the correction stress vector $-\Delta \lambda \sigma^D$ was determined by use of (2.23) and (2.28). These calculations which resulted in a real root for $\Delta \lambda$, indicate that the actual stress increment $\Delta \sigma$ for the full time step is represented by the vector joining points "1" and "2". Similarly calculations for the next time step result in a stress increment from point "2" to point "3" and subsequent calculations indicate continued oscillations between the third and fourth quadrants.

Suspecting that these calculations were inaccurate, the calculations for the first time step were performed using the subdivided stress increment algorithm with $L=2$. At the end of the first half increment, the stress state labeled "1.5" was predicted and, for end of the second

half step, the point labeled "2*". Thus, the revised stress increment for the full cycle is presented by the vector $\Delta\sigma^*$. Repeat applications of this algorithm using larger values of the subincrement counter L did not produce any appreciable revision of the stress increment vector $\Delta\sigma^*$ so that this procedure appears to be rapidly convergent.

It is apparent that the inaccuracy associated with the use of $L=1$ in this example results from the large excursion outside the yield surface performed by the $\Delta\sigma$ vector. As a rational approach to arriving at an appropriate value of L to use in a given case, consider that the yield surface is surrounded by concentric ellipsoidal annuli which are labeled $L=1,2,3,\dots$ as one progresses radially outward from the yield surface (see Figure B.1). The elastic region within the yield surface is arbitrarily designated $L=0$. Then the appropriate value of L for the subincrement calculation would be determined by the designation of the ellipsoidal annulus in which the tip of the $\Delta\sigma^E$ vector is located i.e., the trial stress state $\bar{\sigma}$ given by (2.19). The analytical formulation of the criterion for selecting L may be expressed as:

$$L = \text{INT} \left\{ \text{YLDFAC} \left(\sqrt{\frac{\phi^T + \sigma_0^2}{\sigma_y^2}} - 1 \right) \right\} + 1$$

where INT signifies the "integer part of" the quantity within the curly brackets. Note that ϕ^T , the function defined by (2.20) with the components of σ substituted, is already calculated within the STRESS subroutine to test for yielding. The coefficient YLDFAC is a parameter (not necessarily integral) which permits variation of the "thickness" of the concentric ellipsoidal annuli in stress space. The user should choose YLDFAC to suit his compromise between minimum computer time (YLDFAC=0, for which no subdivision of the elastic stress increment occurs as long as a real $\Delta\lambda$ root is obtained) and maximum accuracy (YLDFAC $\rightarrow \infty$, corresponding to differential subintervals). Although no systematic study of the effect of varying YLDFAC has been conducted, it appears that reasonably accurate stress determinations have been obtained using YLDFAC = 1 with only nominal increases in machine time. For this value of YLDFAC the ellipsoidal annulus interfaces intersect the σ_y^Y axes at integral multiples of the uniaxial yield stress σ_0 .

The REPSIL code now provides an optional printout of L (m,n) arrays for each k layer of the structural shell at specified time cycles. These arrays provide a definition of the regions of the structural shell within which plastic flow is occurring as well as a

qualitative indication of the magnitude of the flow activity. Where a strain-hardening constitutive model is being employed, the largest value of L for any of the sublayers within the k th layer will be printed. Samples of the L array print format are included with the example problems, see Tables 5.8, 5.12, 5.18, 5.20 and 5.22.

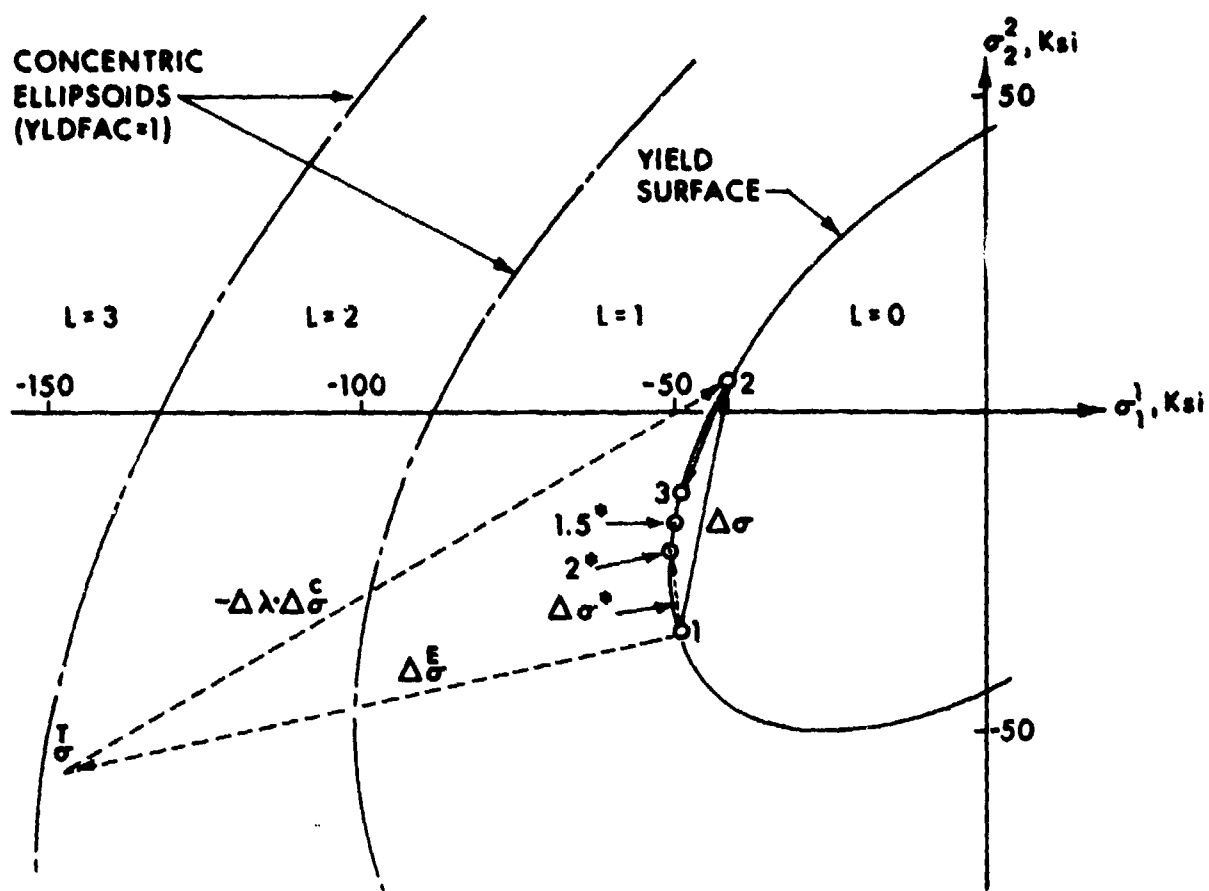


Figure B.1 Graphical Representation of Stress Increments

APPENDIX C.
DEFINITION OF PROGRAM VARIABLES

In this appendix we list the FORTRAN variables used in the program and give a brief description of each, identifying where possible the variable with the symbol used in the body of the report. The variables are grouped according to whether they are integer or real and whether they represent an array or not. Within each group the variables are listed alphabetically. Index notation is applied to certain sets of FORTRAN variables, with Greek indices ranging over the integers 1, 2 and Latin indices over 1, 2, 3, as before. These indices are not subscripted or superscripted. Latin indices are distinguished from letters in the FORTRAN names by not being capitalized. Input variables already described in Section 3.2 are identified by a superscript \S .

C.1 Integer Variables

Name	Symbol	Description
I		Index corresponding to surface strain locations, $1 \leq I \leq NSTRN$.
I1,I2		Dummy indices for elements of the arrays MI1(I) and MI2(I).
IBCE1 \S IBCE2 \S IBCE3 \S IBCE4 \S		Numbers controlling boundary conditions along edges (input data, card 4).
IFLAG		Control number used in subroutine PDATA.
II		Dimension of DAT(J) array, $II=2 \cdot NSTRN + 8$.
ISRS \S		Strain rate sensitivity control (input data, card 6).
J		Index corresponding to stress sublayers, $1 \leq J \leq NSFL$. Also a general dummy index.
J1,J2		Dummy indices for elements of the arrays NI1(I) and NI2(I).
K	k	Index corresponding to layer stations, $1 \leq K \leq LAYER$.
KD		Dummy argument replacing K.

KEY Control number governing mode of operation of subroutine WRTAPE: 1, write data on tape
2, read data off tape.

KJ Index corresponding to the Jth stress sublayer of the Kth layer, $1 \leq KJ \leq KJMAX$.

KJMAX Total number of stress sublayers in the entire thickness of shell, $KJMAX = NSFL * LAYER$.

KN Total number of stress sublayers in the first (K-1) layers, $KN = NSFL * (K-1)$.

L L Number of subincrements into which the elastic stress increment $\Delta\sigma_{\beta}^{\alpha}$ is divided for plastic flow calculations described in Appendix B, $L \leq 100$.

LAYERS K or K_{max} Total number of layers into which the thickness is divided (input data, card 2).

LC Counter for the elastic stress subincrement used in plastic flow calculations described in Appendix B, $1 \leq LC \leq L + 1$.

LINK Control number used in subroutines STRAIN and PDATA.

LMNK Counter for the maximum number of subincrements L occurring in a layer, supplying entrees for the array LMAT (M,N,K).

LOADS Control number governing mode of loading (input data, card 5).

LPRESSS Last time step at which pressure is nonconstant (input data, card 5).

M m Mesh number in the η^1 direction, $1 \leq M \leq MM$.

M1 Maximum value of M at which strains and stresses are calculated, a function of the edge conditions along boundary 3:

$$M1 = \begin{cases} MS, & \text{symmetry edge} \\ MM, & \text{clamped or hinged edge.} \end{cases}$$

MASHS Number controlling mesh proportion of conical shell (input data, card 14).

MAXC Final time step (input data, card 3).

MB1 Maximum value of M at which the displacement increments are not modified by clamped edge conditions along boundary 3:

$$\text{MB1} = \begin{cases} \text{MR, clamped edge} \\ \text{MS, symmetry or hinged edge.} \end{cases}$$

MD Dummy argument replacing M.

MDAMP\$ Time step at which damping operations begin (input data, card 5).

MESH\$ Number of mesh intervals in the η^1 direction (input data, card 2).

MF\$ Maximum value of M for points receiving uniform initial velocity VR (input data, card 15).

MI\$ Minimum value of M for points receiving uniform initial velocity VR (input data, Card 15).

MM Total number of mesh points in the η^1 direction.

MQ1, MQ2 Values of M at mesh points bracketing the location at which the displacement U^i is determined, see subroutine STRAIN.

MR $\text{MR} = \text{MM} - 2$.

MRITE Time steps at which restart data is collected by subroutine WRTAPE.

MS $\text{MS} = \text{MM} - 1$.

N n Mesh number in the η^2 direction, $1 \leq N \leq \text{NN}$.

N1 Maximum value of N at which strains and stresses are calculated, a function of the edge conditions along boundary 2:

$$\text{N1} = \begin{cases} \text{NS, symmetry edge} \\ \text{NN, clamped or hinged edge} \end{cases}$$

N3D\$ Number of time steps at which 3D plots are drawn (input data, card 1!).

NB1 Minimum value of N at which the displacement increments are not modified by clamped edge conditions along boundary 4:

$$NB1 = \begin{cases} 2, & \text{hinged edge} \\ 3, & \text{clamped edge.} \end{cases}$$

NB2 Maximum value of N at which the displacement increments are not modified by clamped edge conditions along boundary 2:

$$NB2 = \begin{cases} NR, & \text{clamped edge} \\ NS, & \text{symmetry or hinged edge} \end{cases}$$

NCNTS Initial time step (input data, card 3).

NCYC $NCYC = NCYCLE - 1.$

NCYCLE & Time step.

ND Dummy argument replacing N.

NDELP Number of time steps between surface strain prints, replaces NPRINT in program.

NFS Maximum value of N for points receiving uniform initial velocity VR (input data, card 15).

NIS Minimum value of N for points receiving uniform initial velocity VR (input data, card 15).

NLP Counter indexing the array JCYNLP(J).

NLPRINS Number of times the array LMAT(M,N,K) is printed (input data, card 10).

NMESHs Number of mesh intervals in the η^2 direction (input data, card 2).

NN Total number of mesh points in the η^2 direction.

NNSD Counter indexing the array NC3DP(J).

NNN Counter indexing the array NCYCH(J).

NFLOT Plotting tape unit number.

NPRINTS Input on card 8 as the number of time steps between surface strain prints, replaced by NDELP in program. In program NPRINT gives the time step, at which surface strains are printed.

NQ1,NQ2 Values of N at mesh points bracketing the location at which the displacement U^1 is determined, see MQ1, MQ2.

NR NR = NN-2

NRITES Number of time steps between collection of restart data (input data, card 3).

NS NS = NN-1.

NSPLS J Total number of stress sublayers in each layer - a plasticity modelling control (input data, card 6).

NSTRNS Total number of locations at which surface strain are computed (input data, card 12).

NUMCY\$ Number of time steps at which JCHK(J) controlled data and energy-work data are printed (input data, card 9).

NV. Number of mesh points receiving initial velocities V different from uniform initial velocity VR (input data, card 15).

C. Integer Arrays

JCHK(J)\$ Print control (input data, card 8).

JCYNLP(J)\$ Time steps at which array LMAT(M,N,K) is printed (input data, card 10).

LMAT(M,NK) L(r,n) Matrix of maximum stress subincrements for the Kth layer, see description of L and LMNK, and Appendix B.

MI1(I) MI2(I) Values of M at mesh points bracketing the Ith surface strain location, $MI1(I) \leq PM(I) \leq MI2(I) \equiv MI1(I) + 1$.

NC3DP(J)\$ Time steps at which 3D plots are obtained (input data, card 11).

NCYCH(J)\$ Time steps at which JCHK(J) controlled data and energy-work data are printed (input data, card 9).

NETAG(I)\$ Control selecting surface on which Ith surface strain location is situated (input data, card 13).

NI1(I), NI2(I) Values of N at mesh points bracketing the Ith surface strain location, $NI1(I) \leq PN(I) \leq NI2(I) \equiv NI1(I) + 1$.

C.3 Real Variables

A11	} $A_{\alpha\beta}$	$a_{\alpha\beta}$	Covariant components of middle surface metric.
A12			
A22			
AA		A	Coefficient of quadratic equation for $\Delta\lambda$.
ALFN			Alphanumeric print indicating surface on which strain are determined.
ANGEL		θ	ANGLB(I) or ANGLE(I) in radians.
AR11	} $AR_{\alpha\beta}$	$a^{\alpha\beta}$	Contravariant components of middle surface metric.
AR12			
AR22			
AVGRAD			Cylinder radius or average cone radius, zero for flat plate.
B		-2B	Coefficient of quadratic equation for $\Delta\lambda$.
B11	} $B_{\alpha\beta}$	$b_{\alpha\beta}$	Covariant components of middle surface 2nd fundamental tensor.
B12			
B22			
BM11	} $BM_{\alpha\beta}$	b_{β}^{α}	Mixed components of 2nd fundamental tensor.
BM12			
BM21			
BM22			
BT		b_{γ}^{γ}	Trace of 2nd fundamental tensor.
C		c	Sound speed, $c = \sqrt{\frac{E}{\rho(1-\nu^2)}}$
C1			Program constant used in equations of motion, $C1 = C2/(4 + C2)$.
C1OLD			Old value of C1 in subroutine DESTEP.
C2		$2D\Delta t/\Gamma_0$	Program constant used in damping work calculations.
CA		$\Delta\eta^1\Delta\eta^2$	Program constant used in kinetic energy and work calculations.
CB		$\Delta\eta^1\Delta\eta^2\Delta z/E$	Program constant used in elastic energy calculations.
CINEP		T*	Kinetic energy removed by previous KEA operation.

CINER	T_-	Kinetic energy at time $t - 1/2 \Delta t$.
CINES	T_+, T_{++}	Kinetic energy at time $t + 1/2 \Delta t$ or at time $c + 3/2 \Delta t$ in subroutines DAMP and DESTEP.
CINES1	T_-	Previous value of CINER in subroutines DAMP and DESTEP.
CINET	T	Kinetic energy at current time t .
CM,CN		Weighing factor accounting for reduced areas at boundaries in summing for the kinetic energy and work.
CS111 CS112 CS122 CS211 CS212 CS222	$CS_{\alpha\beta\gamma} \Gamma_{\beta\gamma}^\alpha$	Christoffel symbols for the middle surface.
CSM1 CSM2	CSM_α	Normal components of $FNT_{\alpha i}$.
D	$B^2 - AC$	Discriminant of quadratic equation for $\Delta\lambda$.
D1 D2 D3	$D_i U^i$	Components of the displacement of the middle surface particle with material coordinate ETAD1, ETAD2.
DA	a	Determinant of the surface metric $a_{\alpha\beta}$.
DA11 DA12 DA22	$1/2 \Delta a_{\alpha\beta}$	One half the incremental change in $a_{\alpha\beta}$ during the time interval from $t - \Delta t$ to t .
DAMPF δ	D	Viscous damping coefficient (input data, card 5).
DB11 DB12 DB22	$\Delta b_{\alpha\beta}$	Incremental change in $b_{\alpha\beta}$ during the time interval from $t - \Delta t$ to t .
DELB	Δt_B	Critical bending time increment for stability.
DELGAM	$\Delta t^2 / \Gamma_0$	Program constant used to calculate TEMP(M,N).
DELIN	Δt	Temporary storage of input DELTAT.
DELM	Δt_M	Critical membrane time increment for stability.

DELMIN		Minimum of DELB and DELM rounded off.
DELR	$\left(\frac{\Delta t^*}{\Delta t}\right)^2$	Factor used in computing Δu^i and T in subroutine DESTEP, see (2.53 and 2.54).
DELS	$\left(\frac{\Delta t^*}{\Delta t}\right)^2 \frac{2\Gamma_o + D\Delta t}{2\Gamma_o + D\Delta t^*}$	Factor used in computing Δu^i and T in subroutine DESTEP, see (2.53 and 2.54).
DELSQ	Δt^2	DELSQ = DELTAT**2.
DELTA	δ	Time constant used in subroutine STRAIN.
DELTAT\$	Δt	Time increment (calculated by program or input on card 3).
DEPS11 DEPS12 DEPS21 DEPS22	DEPS $_{\alpha\beta}$ $\Delta \epsilon_{\beta}^{\alpha}$	Mixed components of the strain increment $\Delta \epsilon_{\alpha\beta}$
DETA1} DETA2}	$\Delta \eta^{\alpha}$	Increments in the η^1 and η^2 material coordinates, respectively.
DETAN	$\frac{1}{12} \left(1 - \frac{1}{K2}\right)$	Factor used in calculating DELB, see (2.3).
DEFACT\$		Factor for terminating damping operations (input data, card 5).
DG	g	Determinant of $g_{\alpha\beta}$.
DN	Δn	Normal component of increment in n^i .
DNL1} DNL2}	Δn_{α}	Tangential components of increment in n^i .
DNR1} DNR2}	Δn^{α}	Contravariant form of Δn_{α} , above; $\Delta n^{\alpha} = a^{\alpha\beta} \Delta n_{\beta}$
DSG11L DSG12L DSG21L DSG22L	$\frac{E_{\alpha}}{\Delta \sigma_{\beta} / L}$	Subincrements into which $\Delta \sigma_{\beta}^{\alpha}$ below is divided.
DSIG11 DSIG12 DSIG21 DSIG22	DSIG $_{\alpha\beta}$ $\Delta \sigma_{\beta}^{\alpha}$	Mixed components of elastic stress increment.
DSQOLD	Δt^2	Old values of DELSQ in subroutine DESTEP.

DUSN } DUSN1 } DUSN2 }	Δu_n	Corrections applied to the normal components of Δu^i along clamped edges and at clamped corners, respectively.
DW	$n^i \Delta u^i$	Normal component of Δu^i .
DX4	$1/(\Delta n^1)^4$ } $1/(\Delta n^2)^4$ }	Factors used in calculating DELB, see (2.3)
DY4		
ES	E	Young's modulus (input data, card 6).
EN	$\Delta W(t - \frac{1}{2} \Delta t)$	Work increment in the time interval $t - \Delta t$ to t .
ENR	$\frac{\Delta W(t - \Delta t)}{\Delta n^1 \Delta n^2}$	Value of ENS at the time $t - \Delta t$.
ENS	$\frac{\Delta W(t)}{\Delta n^1 \Delta n^2}$	Proportional to work increment in the time interval $t - 1/2 \Delta t$ to $t + 1/2 \Delta t$.
EPSDOT	$\dot{\epsilon}$	Deviator strain rate used in strain rate dependence law.
EPSR1 } EPSR2 } GRAMMAR }	ϵ_1 } ϵ_2 } 2γ }	Intermediate components of strain used in calculating surface strains.
ETA1 } ETA2 }	η^1 } η^2 }	Material coordinates η^1 and η^2 of middle surface particles.
ETAD1s } ETAD2s }	η^1 } η^2 }	Material coordinates of the point at which the displacements U^i are calculated (input data, card 12).
F11 } F12 } F22 }	$\frac{M^{\alpha\beta}}{a^{1/2} \Delta \zeta}$	Proportional to bending resultant $FM_{\alpha\beta}$ (M,N).
FLAYER	$\left(\frac{K}{2}\right)^2$	Factor used in calculating DELB.
FLOATL	1/L	Fraction of stress subincrement.
FNT11 } FNT12 } FNT13 } FNT21 } FNT22 } FNT23 }	$\frac{N^{i\alpha}}{a^{1/2} \Delta \zeta}$	Proportional to stress resultant $FN_{\alpha i}$ (M,N).
FNU\$	ν	Poisson's ratio (input data, card 6).
G	$\frac{E}{2(1+\nu)}$	Shear Modulus.

G11 } G12 } G22 }	$g_{\alpha\beta}, G_{\alpha\beta}$	Used in subroutine STRESS for the metric $g_{\alpha\beta}$ of the lamella ζ distance from the middle surface and in subroutint STRAIN for the initial metric $G_{\alpha\beta}$ on the bounding surface.
GAMMAR		See EPSR1 above.
GAMZ	$\Gamma_0 = \rho h$	Mass per initial middle surface area.
GR	$1 / \sqrt{G_{11} G_{22}}$	Time constant computed from $G_{\alpha\beta}$.
GR11 } GR12 } GR22 }	$g^{\alpha\beta}$	Inverse of metric $g_{\alpha\beta}$.
GTWO	$\frac{E}{1+\nu}$	Twice the shear modulus G .
H	$1/2 h$	Half the shell thickness, $H = \frac{\text{THICKN}}{2}$.
LENGTHS		Length of plate, cylinder, or cone (input data, card 14).
PHIT	ϕ	Yield function.
PI		The mathematical constant π .
PLAST	W_p	Plastic work.
PM1		PM1 = MI1(I)
PM2		PM2 = MI2(I) Conversion from integer to
PN1		PN1 = NI1(I) floating point form.
PN2		PN2 = NI2(I)
PRAT	$\frac{E}{1-\nu^2}$	Material constant.
Q11 } Q12 } Q22 }	$\frac{Q^{\alpha\beta}}{a^2 \Delta \zeta}$	Proportional to membrane components of stress resultant.
QM } QN }	m, n	Mesh numbers corresponding to material coordinates ETAD1 and ETAD2.
QM1, QM2 } QN1, QN2 }		Interpolation coefficient used to compute the displacement components D_i .
RA	$1/a$	$RA = \frac{1}{DA}$.
RADIS		Small radius of conical shell (input data, card 14).
RADIUS		Initial radius of cylindrical shell (input data, card 14).

RADFS		Large radius of conical shell (input data, card 14).
RD11	$\left. \begin{array}{l} 1/(\Delta\eta^1)^2 \\ 1/4\Delta\eta^1\Delta\eta^2 \\ 1/(\Delta\eta^2)^2 \end{array} \right\}$	Program constants used in computing second order finite difference derivatives, see Appendix A.
RD12		
RD22		
RG	$1/g$	$RG = \frac{1}{DG}$
RHC\$	ρ	Initial mass density (input data, card 6).
RN1	$\left. \begin{array}{l} \\ \\ \\ \end{array} \right\} n_i^1$	Cartesian components of unit normal SNi (M,N) at the previous time t - Δt.
RN2		
RN3		
RR'	$1/a^{1/2}$	$RRA = \frac{1}{SRA}$
RSUM		Factor used in rounding off DELMIN.
RTD1	$\left. \begin{array}{l} 1/2\Delta\eta^1 \\ 1/2\Delta\eta^2 \end{array} \right\}$	Program constants used in computing first order finite difference derivatives, see Appendix A.
RTD2		
SA,SB	α, β	Constants for computing surface strain in θ direction.
SIG11	$\left. \begin{array}{l} \\ \\ \\ \end{array} \right\} SIG_{\alpha\beta} \quad \sigma_{\beta}^{\alpha}$	Mixed components of stress at time t.
SIG12		
SIG21		
SIG22		
SIG11D	$\left. \begin{array}{l} \\ \\ \\ \end{array} \right\} SIG_{\alpha\beta D} \quad D_{\beta}^{\alpha}$	Mixed components of plastic flow corrector stress.
SIG12D		
SIG21D		
SIG22D		
SIG11I	$\left. \begin{array}{l} \\ \\ \\ \end{array} \right\} SIG_{\alpha\beta I} \quad \sigma_{\beta}^{\alpha}$	Mixed components of stress at time t - Δt.
SIG12I		
SIG21I		
SIG22I		
SIG11L	$\left. \begin{array}{l} \\ \\ \\ \end{array} \right\} SIG_{\alpha\beta L} \quad T_{\beta}^{\alpha}$	Mixed components of trial stress.
SIG12L		
SIG21L		
SIG22L		
SIGYSQ	σ_o^2	Square of yield stress or square of yield stress magnified by rate sensitivity factor.
SIGZ\$	σ_o	Uniaxial yield stress (input data, card 6).

SNN	$n^i n^i$	Scalar product of normals at times t and t - Δt.
SRA	$a^{1/2}$	SRA = SQRT (DA).
SRDEL	$1/\sqrt{1-\delta^2}$	Time constant used in subroutine STRAIN.
SRG	$g^{1/2}$	SRG = SQRT (DG).
SS11 SS12 SS21 SS22	$SS_{\alpha\beta} \sigma_{\beta}^{\alpha}$	Weighted sum of the mixed components of sublayer stress for given layer, giving the layer stress.
STREN	V	Elastic strain energy.
SUM		Factor used in rounding off DELMIN.
SUMA11 SUMA12 SUMA22	$\sum_k \sigma^{\alpha\beta}$	Sum of components of layer stress SS1MN(k), etc. over all layers.
SUMB11 SUMB12 SUMB22	$\sum_k \sigma^{\alpha\beta} \zeta$	Sum of first moment of components of layer stress SS1MN(k), etc. over all layers.
SUMC11 SUMC12 SUMC22	$\sum_k \sigma^{\alpha\beta} \zeta^2$	Sum of second moments of components of layer stress SS1MN(k), etc. over all layers.
T2	$\frac{1 + 15 \nu}{8(\Delta\eta^1 \Delta\eta^2)}$	Factor used in calculating DELB, see (2.3).
TA	$\Delta\zeta$	Layer thickness, $TA = \frac{THICKN}{ZAYER}$.
TAMBDA	$\Delta\lambda$	Factor measuring amount of plastic flow.
TB	$a^{1/2} \Delta\zeta$	Program constant used in calculating resultants.
TDAMP	W_D	Damping work.
THETA:		Angle subtended by cylindrical and conical panels (input data, card 14).
THICKNS	h	Thickness of shell (input data, card 6).
TIME	t	Current time, TIME = NCYCLE*DELTAT.
TNRG	W	Total external work (due to pressure).
TRD	$1/(2\Delta\eta^1 \Delta\eta^2)$	TRD = 2*RD12.

U11	} U _{ai}	u _α ⁱ	First finite difference derivatives of the displacement increment U _i (M,N) with respect to material coordinates ET _{Aα} .
U12			
U13			
U21			
U22			
U23			
U111	} U _{αβi}	u _{αβ} ⁱ	Second finite difference derivatives of the displacement increment U _i (M,N) with respect to material coordinates ET _{Aα} .
U112			
U113			
U121			
U122			
U123			
U221			
U222			
U223			
U1R	}	Δu ⁱ	Components of displacement increment during the time interval [t - Δt, t].
U2R			
U3R			
U1S	}	Δu ₊ ⁱ	Components of displacement increment during the time interval [t, t + Δt].
U2S			
U3S			
V _§		v	Initial velocity at mesh points not receiving uniform initial velocity V _R (input data, card 16).
VF1	}	-P*n ⁱ	Force due to the pressure P (M,N) in the equations of motion.
VF2			
VF3			
VM1	}	$\frac{\partial^2 (M^{\alpha\beta} n^i)}{\partial \eta^\alpha \partial \eta^\beta}$	Force due to bending resultants F _{Mαβ} (M,N) in the equations of motion.
VM2			
VM3			
VN1	}	$\frac{\partial N^{i\alpha}}{\partial \eta^\alpha}$	Force due to stress resultants F _{Nαi} (M,N) in the equations of motion.
VN2			
VN3			
VR _§		v	Uniform initial velocity (input data, card 15).
WIDTH _§			Half width of plate (input data, card 12).
Y11	} Y _{ai}	y _α ⁱ	First finite difference derivatives of the position vector Y _i (M,N) with respect to material coordinate ET _{Aα} , forming local basis for middle surface.
Y12			
Y13			
Y21			
Y22			
Y23			

Y111
Y112
Y113
Y121
Y122
Y123
Y221
Y222
Y223

$Y_{\alpha\beta i}$ $y_{\alpha\beta}^i$

Second finite difference derivatives of the position vector Y_i (M,N) with respect to material coordinates ETA_{α} .

YLDFAC\$

Parameter governing the "thickness" of stress ellipsoidal annuli, see Appendix B (input data, card 2).

YR11
YR12
YR13
YR21
YR22
YR23

$Y_{R\alpha i}$ $a^{\alpha\beta} y_{\beta}^i$

Dual or reciprocal basis to the basis $Y_{\alpha i}$.

ZAYER

K

Floating point representation of LAYER.

ZETAK

2ζ

ZETAK = 2 * ZETA(K).

C.4 Real Arrays

A111(I) } A121(I) } A221(I) }	A $\alpha\beta$ 1(I)	Initial $a_{\alpha\beta}$ at mesh point (MI1(I), NI1(I)).
A112(I) } A122(I) } A222(I) }	A $\alpha\beta$ 2(I)	Initial $a_{\alpha\beta}$ at mesh point (MI1(K), NI2(I)).
A113(I) } A123(I) } A223(I) }	A $\alpha\beta$ 3(I)	Initial $a_{\alpha\beta}$ at mesh point (MI2(I), NI1(I)).
A114(I) } A124(I) } A224(I) }	A $\alpha\beta$ 4(I)	Initial $a_{\alpha\beta}$ at mesh point (MI2(I), NI2(I)).
ANGLB(I)§ } ANGLE(I)§ }	θ	Angle specifying direction of surface strains E_{θ} at location I (input data, card 13).
ASA(I), ASB(I)	$2\alpha^2$	Constant for computing surface strain in θ direction.
B111(I) } B121(I) } B221(I) }	B $\alpha\beta$ 1(I)	Initial $b_{\alpha\beta}$ at mesh point (MI1(I), NI1(I)).
B112(I) } B122(I) } B222(I) }	B $\alpha\beta$ 2(I)	Initial $b_{\alpha\beta}$ at mesh point (MI1(I), NI2(I)).
B113(I) } B123(I) } B223(I) }	B $\alpha\beta$ 3(I)	Initial $b_{\alpha\beta}$ at mesh point (MI2(I), NI1(I)).
B114(I) } B124(I) } B224(I) }	B $\alpha\beta$ 4(I)	Initial $b_{\alpha\beta}$ at mesh point (MI2(I), NI2(I)).
BSA(I), BSB(I)	$2\beta^2$	Constant for computing surface strain in θ direction.
CSA(I), CSB(I)	$2\alpha\beta$	Constant for computing surface strain in θ direction.
DAT(J)		Array storing energy, displacement and strain plotting data each time step.
DEPS1(K) } DEPS2(K) } DGAMMA(K) }	$\Delta\epsilon_{11}$ } $\Delta\epsilon_{22}$ } $\Delta\epsilon_{12}$ }	Elongational components in the η^1 and η^2 directions and shear component of strain increment in the Kth layer.

DM1(I), DM2(I) } DN1(I), DN2(I) }		Interpolation coefficient used to compute components of surface strain at location I.
DSR(J) §	d_j	Constant in strain rate dependence law (input data, card 7).
EPSANB(I) } EPSANG(I) }	E_θ	Elongational surface strains in the θ directions at location I.
EPSL1(M,N) } EPSL2(M,N) } GAMMAL(M,N) }	ϵ_{11} } ϵ_{22} } ϵ_{12} }	Elongational components in the η^1 and η^2 directions and shear component of strain on the negative bounding surface at mesh point (M,N).
EPSS1(I) } EPSS2(I) }	E_1 } E_2 }	Elongational surface strains in the η^1 and η^2 directions at location I.
EPSU1(M,N) } EPSU2(M,N) } GAMMAU(M,N) }	ϵ_{11} } ϵ_{22} } ϵ_{12} }	Elongational components in the η^1 and η^2 directions and shear component of strain on the positive bounding surface at mesh point (M,N).
ETAG1(I) § } ETAG2(I) § }	η^α	Material coordinates of the Ith surface strain location (input data, card 13).
FM11(M,N) } FM12(M,N) } FM22(M,N) }	$FM_{\alpha\beta}(M,N)$	$M^{\alpha\beta}$ Components of the bending resultant tensor at the mesh point (M,N).
FN11(M,N) } FN12(M,N) } FN13(M,N) } FN21(M,N) } FN22(M,N) } FN23(M,N) }	$FN_{\alpha i}(M,N)$	$N^{i\alpha}$ Components of the stress resultant tensor at the mesh point (M,N).
GAMMAL(M,N)		See EPSL1(M,N) above.
GAMMAU(M,N)		See EPSU1(M,N) above.
GI11(I) } GI12(I) } GI22(I) }	$1/G_{11}$ } $2/\sqrt{G_{11}G_{22}}$ } $1/G_{22}$ }	Time constants computed from the initial surface metric $G_{\alpha\beta}$ at strain location I.
P(M,N)	P, P*	The pressure or augmented pressure at mesh point (M,N).
PM(I) } PN(I) }	m, n	Mesh numbers corresponding to material coordinates ETAG1(I) and ETAG2(I).
PSR(J) §	p_j	Constant in strain rate dependent law (input data, card 7).

SE(J)		Slopes of Jth segments of polygonal approximation to strain hardening stress-strain curve.
SEPS(J) §	ϵ_j	Strains at corners of polygonal approximation to strain hardening stress-strain curve (input data, card 7).
SIG1(M,N,KJ)	$\left. \begin{matrix} \sigma^{11} \\ \sigma^{22} \\ \sigma^{12} \end{matrix} \right\}$	Normal and tangential components of stress in the η^1 and η^2 directions at mesh point (M,N), layer K, sublayer J.
SIG2(M,N,KJ)		
TAU(M,N,KJ)		
SIGZSQ(J)	σ_0^2	Square of yield stress at Jth sublayer.
SN1(M,N)	$\left. \begin{matrix} \\ \\ \end{matrix} \right\} n^i$	Cartesian components of unit normal to middle surface at mesh point (M,N).
SN2(M,N)		
SN3(M,N)		
SS1MN(K)	$\left. \begin{matrix} \sigma^{11} \\ \sigma^{22} \\ \sigma^{12} \end{matrix} \right\}$	Normal and tangential components of stress in layer K, obtained from the mixed component form $SS\alpha\beta$ by raising index.
SS2MN(K)		
STMN(K)		
SSIG(J) §	σ_j	Stresses at corners of polygonal approximation to strain hardening stress-strain curve (input data, card 7).
STMN(K)		See SS1MN(K) above.
TAU(M,N,KJ)		See SIG1(M,N,KJ) above.
TEMP(M,N)	$\Delta t^2 / (a_0^{1/2} \Gamma_0)$	Value of time constant at mesh point (M,N).
U1(M,N)	$\left. \begin{matrix} \\ \\ \end{matrix} \right\} \Delta u^i$	Cartesian components of displacement increment undergone by the mesh point (M,N) in the time interval Δt .
U2(M,N)		
U3(M,N)		
WT(J)		Weighing factors used in summing the sublayer stress to obtain layer stress, see $SS\alpha\beta$.
Y1(M,N)	$\left. \begin{matrix} \\ \\ \end{matrix} \right\} y^i$	Cartesian coordinate of the mesh point (M,N) on the middle surface.
Y2(M,N)		
Y3(M,N)		
ZETA(K)	ζ	Distance along the normal from middle surface to midpoint of Kth layer.
ZETASQ(K)	ζ^2	Square of ZETA(K).

APPENDIX D REPSIL PLOTTING PROGRAM

This is an independent program, separate from the REPSIL program. It was written to satisfy the requirement for a visual display of the output from REPSIL and PETROS structural response programs. This plotting program is useful in quickly interpreting results that in tabular form would be extremely difficult if not impossible, to understand.

The program makes use of the Cal Comp Standard Plotting Package SCOOP which is standard software for a large number of computing systems. If SCOOP is not available it can easily be adapted to any particular plotting system.

D.1 Description of Main Program and Subroutines

The main program reads the plotting data tape and controls the flow of information. If the variable IFLAG equals one, data is read and stored in the plotting data arrays. If it equals two, data is read and the program calls subroutine PLOT3D. When IFLAG equals 99999 the program calls subroutine GRAPH.

If the number of cycles are greater than MAXAR-2 an ERROR PRINT will occur indicating the need for enlargement of the following data arrays: TIM(MAXAR), U1(MAXAR), U2(MAXAR), U3(MAXAR), CIN(MAXAR), STC(MAXAR) TNR(MAXAR), DAMPLT(MAXAR), and EPSS1(N), EPSS2(N), where subscript N is equal to NSTRN*MAXAR. If memory is available MAXAR and the plotting data arrays can easily be enlarged.

Subroutine PLOT3D reads one control card and plots a cross-section, profile and isometric view of the middle surface of the shell. The coordinate data points Y1 (M,N), Y2 (M,N) and Y3 (M,N) are mapped into XP, YP for each M and N to form the isometric view of the surface. Every line lying on the surface is plotted, not just those which are visible (cf. Figure 5.8).

The surface is plotted by connecting successive (M,N) mesh points with straight-line segments for each M over all N on the mesh and, in turn, connecting successive (M,N) mesh points for each N over all M.

Input parameters on the control card allow considerable flexibility in plotting the three views. The deflection can be magnified in order to bring out the details of the deformation pattern. The scale to which the plots are drawn can be adjusted in two ways. First, the scaling factor SF is found automatically, all three dimensions are scaled independently in order to fit the surface into a cube. Then the maximum scale of the three dimensions is selected as the desired scaling. If the maximum is less than one, SF is set to one. Second, the user specifies the scaling factor SF on the input control card.

It is recommended that the first way be used with a cube size, SOFC, of 3.5 inches for every new surface to automatically find SF. See Figures 5.3 and 5.8 for examples of the plotted output.

Subroutine GRAPH produces two graphs. First, time vs displacements U1, U2, U3 at the coordinate point (ETAD1, ETAD2), see Figure 5.5 and 5.10. The second graph is energy balance information (time vs kinetic energy, strain energy, total energy and total damping work) which is useful for detection of numerical instabilities and as an indication of when the solution may be terminated, see Figures 5.4 and 5.9.

Subroutine STRGRA produces NSTRN graphs of time vs elongational strain in the η^1 and η^2 directions at the coordinate point (ETAG1, ETAG2) on the inner or outer surface of the shell. The results can be compared with experimental measurements recorded by strain gauge mounted on the surface, see Figures 5.6 and 5.11. See REPSIL input, Section 3.2 for the description of NSTRN.

Subroutine SEDSHL will plot a dashline instead of a solid line.

Subroutine MAXMIN finds the maximum and minimum point for two or more arrays of data on the Y-axis for a given plot.

D.2 Input Plotting Control Card

Variables	Format
DEFLM, SOFC, SF, NPT	3E10.4, IS
DEFLM	Deflection magnification factor.
SOFC	Cube size, automatically finds scale factor SF.
SF	=0, Scaling factor SF is found automatically. >0, Desired scale factor. Example: Scale 1/2, punch a real number 2 in columns 21 to 30. Scale 1/10, punch a real number 10 in columns 21 to 30.
NPT	Mesh point location in the η^2 direction at which a cross-section is desired.

Following are some examples of the plotting control card.

Example 1

DEFLM = 1.0, SOFC = 3.5, SF = 0.0, NPT = 13

Example 2

DEFLM = 3.0, SOFC = 0.0, SF = 2.0, NPT = 33

Example 3

DFFLM = 100.0, SOFC = 0.0, SF = 10.0, NPT = 10

D.3 Description of Variables

Name	Definition
A11, A12, A21 A22, A23	Constants used in the mapping functions XM, YM to rotate and transform [Y1, Y2, Y3] into [XP, YP]
CIN(1500)	The array of numbers, corresponding to kinetic energy used in the energy balance plot.
DAMPLT(1500)	The array of numbers, corresponding to total damping work used in the energy balance plot.
DAT(20)	Array used in the plotting data tape input list.
EPSS1(9000)	The array of numbers, corresponding to the strains in η^1 direction.
EPSS2(9000)	The array of numbers, corresponding to the strains in the η^2 direction.
ETAD1, ETAD2	See Section 3.2 and Appendix C.
ETAG1(6), ETAG2(6)	See Section 3.2 and Appendix C.
HEAD1(4)	First line of the title that appears on the first isometric plot.
HEAD2(3)	Second line of the title that appears on the first isometric plot.
HEAD3(2)	Title that appears on every isometric plot.
I1	Number of mesh points in the η^1 direction.
I2	Number of mesh points in the η^2 direction.
IBCE3	Boundary condition value.
IBUF(1000)	Array used by the Cal Comp basic software package
IPEN	Control for pen during movement.

IFLAG	A flag. When it equals 1, the program will read data for the time wise plots. When it equals 2, the program will read data for the cross-section, profile and isometric views. When it equals 99999, the program will plot the time wise plots.
M1	Same as I1.
MAXAR	Size of plotting data arrays.
MAXA	MAXA = MAXAR-2
NCYCL	The number of time steps for each curve on the time wise plots.
NCYCLE	Time cycle counter.
NETAG(6)	Zero or one, corresponding to inner or outer surface used in the title of the strain plots.
NPLOT	Magnetic tape input unit number.
NSTRN	Number of strain plots.
PA(4)	Array that stores the dash line pattern.
PM(6), PN(6)	The arrays which stores the mesh location (M,N) that appear in the title on the strain plots.
QM, QN	Mesh location (M,N) that appear in the title on the vector displacement plot.
SF	Scale factor.
SOFC	Size of cube.
STC(1500)	The array of numbers corresponding to strain energy used in the energy balance plot.
TE ₁ , TE ₂ , TE ₃ XYZ	Values used in the mapping functions.
TI	TI = TIME x 1.0 x 10 ⁶
TIM (1500)	The array of numbers corresponding to time used in the time wise plots.
TIME	Current values of time.
TNR (1500)	The array of numbers corresponding to total energy used in the energy plot.

U ₁ (1500) U ₂ (1500) U ₃ (1500)	The arrays of numbers corresponding to the vector displacement U1, U2, U3.
X(25, 35) Y(25, 35) Z(25, 35)	The arrays of numbers corresponding to the initial shape, Y1(M,N), Y2(M,N), Y3(M,N).
X1(90)	The array of XP values for plotting the cross-section and profile views.
X2(90)	The array of YP values for plotting the cross-section and profile views.
XBAR, YBAR	Board coordinate point on the plotting paper in inches, measured from the lower left corner of the page.
XL	Length of X-axis in inches.
XM (A1, A2)	X-axis mapping function used in isometric view.
XMAX	Maximum X-coordinate.
XMIN	Minimum X-coordinate.
XP, YP	X, Y coordinates on the plotting surface.
XPAGE	Plotting page length.
Y1(25,35), Y2(25,35), Y3(25,35)	See Appendix C.
YM(A1, A2, A3)	Y-axis mapping function used in isometric view.
YMAX, YMIN	Maximum and Minimum Y-coordinate.
YP	See XP.
ZMAX, ZMIN	Maximum and Minimum Z-coordinate.

D.4 FORTRAN Listing of Plotting Program

A complete FORTRAN listing is given in the following order.

1. MAIN Program
2. PLOT 3D
3. GRAPH
4. STRGRA
5. MAXMIN
6. SCDSHL

C	REPSIL PLOTTING PROGRAM (CALCOMP SOFTWARE PACKAGE SCOUP)	MAIN	1
	COMMON ETAG1(6),ETAG2(6),PM(6),PN(6),NETAG(6),NSTRN,MAXAR	MAIN	2
	COMMON Y1(25,35),Y2(25,35),Y3(25,35),X(25,35),Y(25,35),Z(25,35)	MAIN	3
	COMMON DAT(20),NCYCLE,TIME,IBCE3,ETAD1,ETAD2,QM,QN,NCYCL	MAIN	4
	COMMON TIM(1500),U1(1500),U2(1500),U3(1500),CIN(1500),STC(1500),	MAIN	5
	ITNR(1500),DAMPLT(1500),EPSS1(9000),EPSS2(9000)	MAIN	6
C	NCYCL=0	MAIN	7
	NPLOT=3	MAIN	8
	REWIND NPLOT	MAIN	9
	MAXAR=1500	MAIN	10
	MAXA=MAXAR-2	MAIN	11
C	READ(NPLOT) IBCE3,ETAD1,ETAD2,QM,QN,NSTRN	MAIN	12
	READ(NPLOT) (ETAG1(I),ETAG2(I),PM(I),PN(I),NETAG(I),I=1,NSTRN)	MAIN	13
	READ(NPLOT) NCYCLE,TIME,M1,N1,(Y1(M,N),Y2(M,N),Y3(M,N),M=1,M1),	MAIN	14
	IN=1,N1)	MAIN	15
	II=2*NSTRN+8	MAIN	16
C	SAVE INITIAL SHAPE NEEDED FOR DEFLECTION MAGNIFICATION IN PLOT3D	MAIN	17
	DO 5 M=1,M1	MAIN	18
	DO 5 N=1,N1	MAIN	19
	X(M,N)=Y1(M,N)	MAIN	20
	Y(M,N)=Y2(M,N)	MAIN	21
	Z(M,N)=Y3(M,N)	MAIN	22
5	CONTINUE	MAIN	23
	CALL PLOT3D(M1,N1)	MAIN	24
C	10 READ(NPLOT) IFLAG	MAIN	25
	IF(IFLAG .EQ. 99999)GOTO 30	MAIN	26
	GOTO(20,25),IFLAG	MAIN	27
C	20 READ (NPLOT) NCYCLE,(DAT(I),I=1,II)	MAIN	28
	NCYCL=NCYCL+1	MAIN	29
	TIM(NCYCL)=DAT(1)	MAIN	30
	U1(NCYCL)=DAT(2)	MAIN	31
	U2(NCYCL)=DAT(3)	MAIN	32
	U3(NCYCL)=DAT(4)	MAIN	33
	CIN (NCYCL)=DAT(5)	MAIN	34
	STC (NCYCL)=DAT(6)	MAIN	35
	TNR (NCYCL)=DAT(7)	MAIN	36
	DAMPLT(NCYCL)=DAT(8)	MAIN	37
	DO 22 I=9,II,2	MAIN	38
	J=NCYCL+MAXAR*(I-9)/2	MAIN	39
	EPSS1(J)=DAT(I)	MAIN	40
	EPSS2(J)=DAT(I+1)	MAIN	41
22	CONTINUE	MAIN	42
	IF(NCYCL .GE. MAXA)GOTO 28	MAIN	43
	GOTO 10	MAIN	44
C	25 READ(NPLOT) NCYCLE,TIME,M1,N1,(Y1(M,N),Y2(M,N),Y3(M,N),M=1,M1),	MAIN	45
	IN=1,N1)	MAIN	46
	CALL PLOT3D(M1,N1)	MAIN	47
	GOTO 10	MAIN	48
C	28 WRITE(6,100) NCYCL	MAIN	49
	30 CALL GRAPH	MAIN	50
	CALL EXIT	MAIN	51
C	100 FORMAT(15HIERROR NCYCL = ,I5,29H ENLARGE PLOTTING DATA ARRAYS)	MAIN	52
	END	MAIN	53
		MAIN	54
		MAIN	55
		MAIN	56
		MAIN	57
		MAIN	58
		MAIN	59
		MAIN	60

	SUBROUTINE PLOT3D (I1,I2)	****	1
	DIMENSION IBUF(1000),X1(90),X2(90),HEAD1(4),HEAD2(3),HEAD3(2)	PLOT	2
	COMMON(USE MAIN)	PLOT	3
	COMMON /PLOTD/ YNP,XDIST,PA(4)	PLOT	4
	DATA 1/0/,A11/.707107/,A12/.707107/,A21/-.408248/,A22/.408248/,	PLOT	5
	A23/.816497/	PLOT	6
	DATA (HEAD1(IK),IK=1,4)/10HCROSS SECT,10HION DEFLEC,10HTION AT N,	PLOT	7
	12H: /	PLOT	8
	DATA (HEAD2(IK),IK=1,3)/10HDEFLECTION,10H MAGNIFIED,1H /	PLOT	9
	DATA (HEAD3(IK),IK=1,2)/10H MICROSECO,3HMDS/	PLOT	10
	DATA (PA(J),J=1,4)/.1,.1,.1,.1/	PLOT	11
C		PLOT	12
	XM(A1,A2)=SF*(A11*A1+A12*A2)	PLOT	13
C		PLOT	14
	YM(A1,A2,A3)=SF*(A21*A1+A22*A2+A23*A3)	PLOT	15
	IF(1.EQ.0)GOTO 10	PLOT	16
	I=I+1	PLOT	17
	XBARN=-4.25	PLOT	18
	YBAR=10.0	PLOT	19
	YNP=YNP+10.0	PLOT	20
	IF(1.LE.3)GOTO 30	PLOT	21
	CALL PLOT (XDIST,-YNP,-3)	PLOT	22
	GOTO 20	PLOT	23
10	XPAGE=200.0	PLOT	24
	READ (5,11) DEFLM,SOFC,SF,N12	PLOT	25
	XDIST=6.75	PLOT	26
11	FORMAT(3E10.4,15)	PLOT	27
C	----- SCALE FACTOR FOR 3D PLOT -----	PLOT	28
	XMAX=X(2,1)	PLOT	29
	XMIN=X(2,1)	PLOT	30
	YMAX=Y(2,1)	PLOT	31
	YMIN=Y(2,1)	PLOT	32
	ZMAX=Z(2,1)	PLOT	33
	ZMIN=Z(2,1)	PLOT	34
	DO 12 M=2,11	PLOT	35
	DO 12 N=1,12	PLOT	36
	XMAX=AMAX1(X(M,N),XMAX)	PLOT	37
	XMIN=AMIN1(X(M,N),XMIN)	PLOT	38
	YMAX=AMAX1(Y(M,N),YMAX)	PLOT	39
	YMIN=AMIN1(Y(M,N),YMIN)	PLOT	40
	ZMAX=AMAX1(Z(M,N),ZMAX)	PLOT	41
	ZMIN=AMIN1(Z(M,N),ZMIN)	PLOT	42
12	CONTINUE	PLOT	43
	IF(SF .NE. 0.0)GOTO 13	PLOT	44
	XS=(XMAX-XMIN)/SOFC	PLOT	45
	YS=(YMAX-YMIN)/SOFC	PLOT	46
	ZS=(ZMAX-ZMIN)/SOFC	PLOT	47
	SF=AMAX1(XS,YS,ZS)	PLOT	48
	SF=AMIN1(SF)	PLOT	49
	IF(SF .LT. 1.0) SF=1.0	PLOT	50
13	CALL PLOTS (IBUF,1000,XPAGE)	PLOT	51
	CALL SYMBOL (1.0,1.0,.1,HEAD1,0.0,32)	PLOT	52
	CALL NUMBER (999.,999.,.1,N12,0.0,2H15)	PLOT	53
	CALL SYMBOL (1.0,0.8,.1,HEAD2,0.0,21)	PLOT	54
	CALL NUMBER (999.,999.,.1,DEFLM,0.0,1)	PLOT	55
	CALL SYMBOL (1.0,0.6,.1,8HSCALE 1/,0.0,8)	PLOT	56
	CALL NUMBER (999.,999.,.1,SF,0.0,-1)	PLOT	57
	SF=1.0/SF	PLOT	58
C	-----	PLOT	59
20	YBAR=4.3	PLOT	60

	XBAR=1.0	PLOT 61
	YNP=YBAR	PLOT 62
	I=1	PLOT 63
30	XBAR=XHARN	PLOT 64
C	----- MIDSECTION DEFLECTION -----	PLOT 65
	CALL PLOT (XBAR,YBAR,-3)	PLOT 66
	CALL PLOT (0.0,-1.0,3)	PLOT 67
	CALL PLOT (0.0,1.0,2)	PLOT 68
	CALL PLOT (0.0,0.0,3)	PLOT 69
	CALL PLOT (1.0,0.0,2)	PLOT 70
	K=0	PLOT 71
	DO 90 M=2,11	PLOT 72
	K=K+1	PLOT 73
	X1(K)=SF*X(M,NI2)	PLOT 74
	X2(K)=SF*Z(M,NI2)	PLOT 75
90	CONTINUE	PLOT 76
	X1(K+1)=0.0	PLOT 77
	X1(K+2)=1.0	PLOT 78
	X2(K+1)=0.0	PLOT 79
	X2(K+2)=1.0	PLOT 80
	CALL SCDS=M (X1,X2,K,1,PA,4)	PLOT 81
	CALL LINE (X1,X2,K,1,-1,2)	PLOT 82
	K=0	PLOT 83
	DO 95 M=2,11	PLOT 84
	K=K+1	PLOT 85
	X1(K)=SF*(X(M,NI2)+DEFLM*(Y1(M,NI2)-X(M,NI2)))	PLOT 86
	X2(K)=SF*(Z(M,NI2)+DEFLM*(Y3(M,NI2)-Z(M,NI2)))	PLOT 87
95	CONTINUE	PLOT 88
	CALL LINE (X1,X2,K,1,1,4)	PLOT 89
C	-----	PLOT 90
	XB=2.25	PLOT 91
	YB=-2.0	PLOT 92
	CALL SYMBOL (XB,YB,-1,6HCYCLE,0.0,6)	PLOT 93
	CALL NUMBER (999.,999.,.1,NCYCLE,0.0,2H15)	PLOT 94
	YB=YB-0.2	PLOT 95
	TI=TIME*1.00E 06	PLOT 96
	CALL NUMBER (XB,YB,-1,TI,0.0,3)	PLOT 97
	CALL SYMBOL (999.,999.,.1,HEAD3,0.0,13)	PLOT 98
C	----- CENTER LINE DEFLECTION PROFILE -----	PLOT 99
	CALL PLOT (0.0,3.0,-3)	PLOT 100
	XB=SF*Y(2,12)	PLOT 101
	YB=0.0	PLOT 102
	CALL PLOT (XB,YB,2)	PLOT 103
	XB=0.0	PLOT 104
	YB=-SF*Z(2,1)	PLOT 105
	CALL PLOT (XB,YB,3)	PLOT 106
	YB=SF*Z(2,1)	PLOT 107
	CALL PLOT (XB,YB,2)	PLOT 108
C		PLOT 109
	L=2	PLOT 110
	DO 85 J=1,2	PLOT 111
	K=0	PLOT 112
	DO 82 N=1,12	PLOT 113
	K=K+1	PLOT 114
	X1(K)=SF*Y(L,N)	PLOT 115
	X2(K)=SF*Z(L,N)	PLOT 116
82	CONTINUE	PLOT 117
	X1(K+1)=0.0	PLOT 118
	X1(K+2)=1.0	PLOT 119
	X2(K+1)=0.0	PLOT 120

X2(K+2)=1.0	PLOT 121
CALL SCDSHL (X1,X2,K,1,PA,4)	PLOT 122
CALL LINE (X1,X2,K,1,-1,2)	PLOT 123
K=0	PLOT 124
DO 83 N=1,12	PLOT 125
K=K+1	PLOT 126
X1(K)=SF*(Y(L,N)+DEFLM*(Y2(L,N)-Y(L,N)))	PLOT 127
X2(K)=SF*(Z(L,N)+DEFLM*(Y3(L,N)-Z(L,N)))	PLOT 128
83 CONTINUE	PLOT 129
CALL LINE (X1,X2,K,1,1,4)	PLOT 130
IF(1BCE3 .NE. 2)GOTO 86	PLOT 131
L=11	PLOT 132
85 CONTINUE	PLOT 133
C ----- DRAW AXIS -----	PLOT 134
86 CALL PLOT (4.25,-3.0,-3)	PLOT 135
CALL PLOT (0.0,-1.0,3)	PLOT 136
CALL PLOT (0.0,1.0,2)	PLOT 137
CALL PLOT (0.0,0.0,3)	PLOT 138
CALL PLOT (-A12,-A22,2)	PLOT 139
CALL PLOT (0.0,0.0,3)	PLOT 140
CALL PLOT (A11,A21,2)	PLOT 141
C -----	PLOT 142
DO 50 M=2,11	PLOT 143
IPEN=3	PLOT 144
DO 40 N=1,12	PLOT 145
TE1=X(M,N)+DEFLM*(Y1(M,N)-X(M,N))	PLOT 146
TE2=Y(M,N)+DEFLM*(Y2(M,N)-Y(M,N))	PLOT 147
TE3=Z(M,N)+DEFLM*(Y3(M,N)-Z(M,N))	PLOT 148
XP=XM(TE1,TE2)	PLOT 149
YP=YM(TE1,TE2,TE3)	PLOT 150
35 CALL PLOT (XP,YP,IPEN)	PLOT 151
IF(IPEN .LT. 3)GOTO 40	PLOT 152
IPEN=2	PLOT 153
GOTO 35	PLOT 154
40 CONTINUE	PLOT 155
50 CONTINUE	PLOT 156
DO 70 N=1,12	PLOT 157
IPEN=3	PLOT 158
DO 60 M=2,11	PLOT 159
TE1=X(M,N)+DEFLM*(Y1(M,N)-X(M,N))	PLOT 160
TE2=Y(M,N)+DEFLM*(Y2(M,N)-Y(M,N))	PLOT 161
TE3=Z(M,N)+DEFLM*(Y3(M,N)-Z(M,N))	PLOT 162
XP=XM(TE1,TE2)	PLOT 163
YP=YM(TE1,TE2,TE3)	PLOT 164
45 CALL PLOT (XP,YP,IPEN)	PLOT 165
IF(IPEN .LT. 3)GOTO 60	PLOT 166
IPEN=2	PLOT 167
GOTO 45	PLOT 168
60 CONTINUE	PLOT 169
70 CONTINUE	PLOT 170
RETURN	PLOT 171
END	PLOT 172

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SUBROUTINE GRAPH                                     **** 1
DIMENSION SYM1(4),SYM2(3),ETA1(3),ETA2(3),X1(4),X2(4) GRAP 2
COMMON (USE MAIN) GRAP 3
COMMON /PLOT/ YNP,XDIST,PA(4) GRAP 4
DATA(SYM1(I),I=1,4)/10HCOMPONENT ,10HOF VECTOR ,10HDISPLACEME,2HNTGRAP 5
:/ GRAP 6
DATA(ETA1(I),I=1,3)/10HETA1 = ,10H ( ,4H )/, GRAP 7
1 (ETA2(I),I=1,3)/10HETA2 = ,10H ( ,4H )/, GRAP 8
DATA(SYM2(I),I=1,3)/10HENERGY BAL,10HANCE, POUN,8HD-INCHES/ GRAP 9
C GRAP 10
XDIST=8.0 GRAP 11
CALL PLOT (XDIST,-YNP,-3) GRAP 12
XBAR=0.0 GRAP 13
YNP=2.0 GRAP 14
XL=8.0 GRAP 15
YL=6.0 GRAP 16
N=NCYCL GRAP 17
CALL SCALE (TIM,XL,N,1) GRAP 18
XMIN=TIM(N+1) GRAP 19
XS=TIM(N+2) GRAP 20
C ----- GRAPH ONE (VECTOR DISPLACEMENT) ----- GRAP 21
CALL MAXMIN (U1,YMIN,YMAX,1,N,1) GRAP 22
CALL MAXMIN (U2,YMIN,YMAX,1,N,2) GRAP 23
CALL MAXMIN (U3,YMIN,YMAX,1,N,2) GRAP 24
X1(1)=YMIN GRAP 25
X1(2)=YMAX GRAP 26
CALL SCALE (X1,YL,2,1) GRAP 27
YMIN=X1(3) GRAP 28
YS=X1(4) GRAP 29
U1(N+1)=YMIN GRAP 30
U1(N+2)=YS GRAP 31
U2(N+1)=YMIN GRAP 32
U2(N+2)=YS GRAP 33
U3(N+1)=YMIN GRAP 34
U3(N+2)=YS GRAP 35
CALL PLOT (XBAR,YNP,-3) GRAP 36
CALL AXIS (0.0,0.0,4HTIME,-4,XL,0.0,XMIN,XS) GRAP 37
CALL AXIS (0.0,0.0,SYM1,32,YL,90.0,YMIN,YS) GRAP 38
CALL LINE (TIM,U1,N,1,0,0) GRAP 39
TX=TIM(N)/XS+.2 GRAP 40
TY=(U1(N)-YMIN)/YS GRAP 41
CALL SYMBOL (TX,TY,.1,2HU1,0.0,2) GRAP 42
CALL LINE (TIM,U2,N,1,0,0) GRAP 43
TY=(U2(N)-YMIN)/YS GRAP 44
CALL SYMBOL (TX,TY,.1,2HU2,0.0,2) GRAP 45
CALL LINE (TIM,U3,N,1,0,0) GRAP 46
TY=(U3(N)-YMIN)/YS GRAP 47
CALL SYMBOL (TX,TY,.1,2HU3,0.0,2) GRAP 48
CALL SYMBOL (2.1,-1.0,.1,8HLOCATION,0.0,8) GRAP 49
CALL SYMBOL (3.0,-1.15,.4,1H ,0.0,1) GRAP 50
CALL SYMBOL (5.3,-1.15,.4,1H ,0.0,1) GRAP 51
CALL SYMBOL (3.1,-0.9,.1,ETA1,0.0,24) GRAP 52
CALL NUMBER (3.8,999.,.1,ETAD1,0.0,3) GRAP 53
CALL NUMBER (4.8,999.,.1,ON,0.0,3) GRAP 54
CALL SYMBOL (3.1,-1.1,.1,ETA2,0.0,24) GRAP 55
CALL NUMBER (3.8,999.,.1,ETAD2,0.0,3) GRAP 56
CALL NUMBER (4.8,999.,.1,ON,0.0,3) GRAP 57
C ----- GRAPH TWO (ENERGY BALANCE) ----- GRAP 58
XBAR=0.0 GRAP 59
YNP=10.0 GRAP 60

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CALL MAXMIN (CIN,YMIN,YMAX,.,N,1)	GRAP 61
CALL MAXMIN (STC,YMIN,YMAX,.,N,2)	GRAP 62
CALL MAXMIN (TNR,YMIN,YMAX,1,N,2)	GRAP 63
CALL MAXMIN (DAMPLT,YMIN,YMAX,1,N,2)	GRAP 64
X1(1)=YMIN	GRAP 65
X1(2)=YMAX	GRAP 66
CALL SCALE (X1,YL,2,1)	GRAP 67
YMIN=X1(3)	GRAP 68
YS=X1(4)	GRAP 69
CIN(N+1)=YMIN	GRAP 70
CIN(N+2)=YS	GRAP 71
STC(N+1)=YMIN	GRAP 72
STC(N+2)=YS	GRAP 73
TNR(N+1)=YMIN	GRAP 74
TNR(N+2)=YS	GRAP 75
DAMPLT(N+1)=YMIN	GRAP 76
DAMPLT(N+2)=YS	GRAP 77
CALL PLOT (XBAR,YNP,-3)	GRAP 78
CALL AXIS (0.0,0.0,4HTIME,-4,XL,0.0,XMIN,XS)	GRAP 79
CALL AXIS (0.0,0.0,SYM2,28,YL,90.0,YMIN,YS)	GRAP 80
CALL LINE (TIM,CIN,N,1,0,0)	GRAP 81
CALL LINE (TIM,STC,N,1,0,0)	GRAP 82
CALL LINE (TIM,TNR,N,1,0,0)	GRAP 83
CALL LINE (TIM,DAMPLT,N,1,0,0)	GRAP 84
C ----- SURFACE STRAIN GRAPHS -----	GRAP 85
DO 100 I=1,NSTRN	GRAP 86
J=1+MAXAR*(I-1)	GRAP 87
CALL STRGRA (TIM,EPSS1,EPSS2,ETAG1(I),ETAG2(I),PM(I),PN(I),	GRAP 88
INETAG(I),J,N)	GRAP 89
100 CONTINUE	GRAP 90
RETURN	GRAP 91
END	GRAP 92

```

SUBROUTINE STRGRA (X,Y,Z,ETA1,ETA2,PM1,PM1,NETA,J,N)      0000 1
C X IS THE NAME OF THE ARRAY CONTAINING THE X COORDINATE DATA.  STRG 2
C Y,Z IS THE NAMES OF THE ARRAYS CONTAINING THE Y COORDINATE DATA.  STRG 3
C ETA1,ETA2 LOCATION POINT USED IN THE TITLE.  STRG 4
C PM1,PM1 MESH LOCATION USED IN THE TITLE.  STRG 5
C NETA IS THE CONTROL FOR INNER OR OUTER USED IN THE TITLE.  STRG 6
C J IS THE LOCATION OF THE FIRST DATA POINT IN ARRAYS Y AND Z FOR  STRG 7
C EACH STRAIN PLOT.  STRG 8
C N IS THE NUMBER OF DATA POINTS.  STRG 9
C DIMENSION X(1),Y(1),Z(1),SYM1(3),SYM2(3),SYM3(2),SYM4(2),X1(4),  STRG 10
  X2(4)  STRG 11
COMMON /PLOTD/ YNP,XDIST,PA(4)  STRG 12
DATA(SYM1(K),K=1,3)/10HETA1 = ,10H ( ,4H )/  STRG 13
1 (SYM2(K),K=1,3)/10HETA2 = ,10H ( ,4H )/  STRG 14
2 (SYM3(K),K=1,2)/10HETA2 COMPO,4HNENT/,  STRG 15
3 (SYM4(K),K=1,2)/10HETA1 COMPO,4HNENT/  STRG 16
C  STRG 17
  IF(1.EQ.0)GOTO 10  STRG 18
  I=I+1  STRG 19
  YNP=YNP+10.0  STRG 20
  YBAR=10.0  STRG 21
  IF(1.LE.3)GOTO 25  STRG 22
  YNP=YNP-10.0  STRG 23
  CALL PLOT (XDIST,-YNP,-3)  STRG 24
  GOTO 20  STRG 25
10 XDIST=13.0  STRG 26
  CALL PLOT (XDIST,-YNP,-3)  STRG 27
  XL=8.0  STRG 28
  YL=6.0  STRG 29
  CALL SCALE (X,XL,N,1)  STRG 30
  XMIN=X(N+1)  STRG 31
  XS=X(N+2)  STRG 32
20 XBAR=0.0  STRG 33
  YNP=0.0  STRG 34
  YBAR=0.0  STRG 35
  I=1  STRG 36
25 CALL MAXMIN (Y,YMIN,YMAX,J,N,1)  STRG 37
  CALL MAXMIN (Z,YMIN,YMAX,J,N,2)  STRG 38
  X1(1)=YMIN  STRG 39
  X1(2)=YMAX  STRG 40
  CALL SCALE (X1,YL,2,1)  STRG 41
  YMIN=X1(3)  STRG 42
  YS=X1(4)  STRG 43
  JM=J+N  STRG 44
  JS=J+N+1  STRG 45
  Y(JM)=YMIN  STRG 46
  Y(JS)=YS  STRG 47
  Z(JM)=YMIN  STRG 48
  Z(JS)=YS  STRG 49
  CALL PLOT (XBAR,YBAR,-3)  STRG 50
  CALL AXIS (0.0,0.0,4HTIME,-4,XL,0.0,XMIN,XS)  STRG 51
  CALL AXIS (0.0,0.0,10HSTRAIN ( ),10,YL,90.0,YMIN,YS)  STRG 52
  CALL SYMBOL (1.8,-1.0,.1,8HLOCATION,0.0,8)  STRG 53
  CALL SYMBOL (2.7,-1.15,.4,1H ,0.0,1)  STRG 54
  CALL SYMBOL (5.0,-1.15,.4,1H ,0.0,1)  STRG 55
  IF(NETA .NE. 0)GOTO 30  STRG 56
  CALL SYMBOL (5.3,-1.0,.1,5HOUTER,0.0,5)  STRG 57
  GOTO 35  STRG 58
30 CALL SYMBOL (5.3,-1.0,.1,5HINNER,0.0,5)  STRG 59
35 CALL SYMBOL (2.8,-0.9,.1,SYM1,0.0,24)  STRG 60

```


CALL NUMBER (3.5,999.,.1,ETA1,0.0,3)	STRG 61
CALL NUMBER (4.5,999.,.1,PM1,0.0,3)	STRG 62
CALL SYMBOL (2.8,-1.1,.1,SYM2,0.0,24)	STRG 63
CALL NUMBER (3.5,999.,.1,ETA2,0.0,3)	STRG 64
CALL NUMBER (4.5,999.,.1,PM1,0.0,3)	STRG 65
X1(1)=2.9	STRG 66
X2(1)=-1.3	STRG 67
X1(2)=3.4	STRG 68
X2(2)=-1.3	STRG 69
X1(3)=0.0	STRG 70
X2(3)=0.0	STRG 71
X1(4)=1.0	STRG 72
X2(4)=1.0	STRG 73
CALL LINE (X1,X2,2,1,0,0)	STRG 74
CALL SYMBOL (3.6,-1.3,.1,SYM4,0.0,14)	STRG 75
X2(1)=-1.5	STRG 76
X2(2)=-1.5	STRG 77
CALL SCOSML (X1,X2,2,1,PA,4)	STRG 78
CALL SYMBOL (3.6,-1.5,.1,SYM3,0.0,14)	STRG 79
CALL LINE (X,Y(J),N,1,0,0)	STRG 80
CALL SCOSML (X,Z(J),N,1,PA,4)	STRG 81
RETURN	STRG 82
END	STRG 83

```

SUBROUTINE MAXMIN (A,AMIN,AMAX,J,N,KEY)
DIMENSION A(1)
GOTO (10,20),KEY
10 J1=J
   J2=J+N-1
   AMIN=A(J1)
   AMAX=A(J2)
20 DO 100 I=J1,J2
   AMAX=AMAX1(AMAX,A(I))
   AMIN=AMIN1(AMIN,A(I))
100 CONTINUE
RETURN
END

```

```

**** 1
MAXM 2
MAXM 3
MAXM 4
MAXM 5
MAXM 6
MAXM 7
MAXM 8
MAXM 9
MAXM 10
MAXM 11
MAXM 12
MAXM 13

```

	SUBROUTINE SCDSHL(X,Y,M,INC,PA,N)	****	1
C(1)	X,Y ARE THE NAMES OF THE ARRAYS CONTAINING THE X AND Y	SCDS	2
C	COORDINATES, AND THE SCALING PARAMETERS. (SEE 'SCALE'.)	SCDS	3
C(1)	M IS THE NUMBER OF POINTS IN THE X AND Y ARRAYS. THIS DOES	SCDS	4
C	NOT INCLUDE THE TWO EXTRA LOCATIONS FOR THE SCALING PARAMETERS.	SCDS	5
C(1)	INC IS THE INCREMENT THAT THE SCDSHL SUBROUTINE IS TO USE	SCDS	6
C	IN GETTING DATA FROM THE X AND Y ARRAYS. (SEE 'SCALE'.)	SCDS	7
C(1)	PA IS THE NAME OF THE LINEAR ARRAY WHICH CONTAINS THE ELEMENTS	SCDS	8
C	OF THE PATTERN WHICH IS TO BE REPEATED UNTIL THE CURVE	SCDS	9
C	IS DRAWN.	SCDS	10
C	PA(1), FOR I=1,3,5,....., CORRESPOND TO THE DASHES IN	SCDS	11
C	THE PATTERN, WHILE	SCDS	12
C	PA(2), FOR I=2,4,6,....., CORRESPOND TO THE SPACES	SCDS	13
C	BETWEEN THE DASHES.	SCDS	14
C(1)	N IS THE NUMBER OF ELEMENTS IN THE PATTERN DESCRIPTION.	SCDS	15
	DIMENSION PA(1),X(1),Y(1)	SCDS	16
	NP=M*INC+1	SCDS	17
	NQ=NP+INC	SCDS	18
	DX=1.0/X(NQ)	SCDS	19
	FX=X(NP)	SCDS	20
	DY=1.0/Y(NQ)	SCDS	21
	FY=Y(NP)	SCDS	22
	K=0	SCDS	23
	DLN=0.0	SCDS	24
	DX2=(X(1)-FX)*DX	SCDS	25
	DY2=(Y(1)-FY)*DY	SCDS	26
	I=1	SCDS	27
	J=1	SCDS	28
	DTD=PA(1)	SCDS	29
	CALL PLOT(DX2,DY2,3)	SCDS	30
	GOTO 30	SCDS	31
20	DSI=DY2-DY1	SCDS	32
	DCO=DX2-DX1	SCDS	33
	DLN=SQRT(DSI*DSI+DCO*DCO)	SCDS	34
	K=MOD(J,2)	SCDS	35
	IF(DLN.LE.DTD)GOTO 30	SCDS	36
	DX1=DX1+DTD*DCO/DLN	SCDS	37
	DY1=DY1+DTD*DSI/DLN	SCDS	38
	CALL PLOT(DX1,DY1,3-K)	SCDS	39
	J=J+1	SCDS	40
	IF(J.GT.N)J=1	SCDS	41
	DTD=PA(J)	SCDS	42
	GOTO 20	SCDS	43
30	IF(K.EQ.0)CALL PLOT(DX2,DY2,2)	SCDS	44
	DX1=DX2	SCDS	45
	DY1=DY2	SCDS	46
	I=I+INC	SCDS	47
	IF(I.GT.M) RETURN	SCDS	48
	DX2=(X(1)-FX)*DX	SCDS	49
	DY2=(Y(1)-FY)*DY	SCDS	50
	DTD=DTD-DLN	SCDS	51
	GO TO 20	SCDS	52
	END	SCDS	53

APPENDIX E.
FORTRAN LISTING OF THE REPSIL PROGRAM

The listing is given in the following order, consisting of Main program and 22 subroutines.

- | | |
|-----------------|-----------------------------|
| 1. MAIN Program | 12. BOUNDU |
| 2. START | 13. ABINIT |
| 3. INVEL | 14. SYMTRY |
| 4. POSITN | 15. KINET |
| 5. DGEOM | 16. PWORK |
| 6. GRAD | 17. DAMP |
| 7. STRESS | 18. DESTEP |
| 8. RESULT | 19. PDATA |
| 9. MOTION | 20. PRESS* |
| 10. WRTAPE | 21a. INGEOM (Flat Plate) |
| 11. STRAIN | 21b. INGEOM (Full Cylinder) |
| | 21c. INGEOM (Cone) |

There are three nonstandard FORTRAN statements used:

1. COMMON (USE MAIN) is used instead of repeating long COMMON statements in the subroutines that appear in the MAIN program.
2. CALL SKIPFILE (t,n) is used in subroutine PDATA to move tape t forward n file marks. The next READ or WRITE statement will begin with the information after the file mark.
3. CALL BACKFILE (t,n) is used in subroutine PDATA to move tape t backward n file marks. The next READ statement will begin with the information after the file mark, the WRITE will erase the file mark.

*Subroutine PRESS gives the pressure loading applied in the flat plate example, Section 5.1.

```

C      MAIN PROGRAM                                MAIN  1
C
C      TAPE 1 RESTART INFORMATION                 MAIN  2
C      TAPE (NPLOT) PLOTTING DATA               MAIN  3
C      COMMON Y1(23,34),Y2(23,34),Y3(23,34),U1(23,34),U2(23,34),U3(23,34) MAIN  4
1,FM11(23,34),FM12(23,34),FM22(23,34),FN11(23,34),FN12(23,34),    MAIN  5
2FN13(23,34),FN21(23,34),FN22(23,34),FN23(23,34),SN1(23,34),    MAIN  6
3SN2(23,34),SN3(23,34),TEMP(23,34),P(23,34),EPSL1(23,34),      MAIN  7
4EPSL2(23,34),GAMMAL(23,34),EPSU1(23,34),EPSU2(23,34),GAMMAU(23,34) MAIN  8
5,SIG1(23,34,12),SIG2(23,34,12),TAU(23,34,12),LMAT(23,34,4),   MAIN  9
6DEPS1(4),DEPS2(4),DGAMMA(4),ZETA(4),SS1MN(4),SS2MN(4),STMN(4), MAIN 10
7ZETASQ(4),NCYCH(50),NC3DP(50),JCYNLP(50)                        MAIN 11
COMMON MM,MR,MS,NN,NR,NS,N1,M1,RO11,RO12,RO22,RTD1,RTD2,MB1,    MAIN 12
10ETA1,ETA2,TRD,NN3D,DAMPF,DFACT,MDAMP,TDAMP,LOAD,DELGAH       MAIN 13
COMMON E,FNU,G,PRAT,SIGZ,GAMZ,H,LAYER,DELTAT,TIME,LPRESS,GTHO,  MAIN 14
11NN,NCYCLE,NRITE,NCONT,NSTRN,CINER,CINES,CINEP,C1,C2,NPLOT,   MAIN 15
20ELSQ,TA,MAXC,WRITE,CA,CB,CINET,STREN,PLAST,TNRG,NB1,NB2,ENS,ENR MAIN 16
COMMON NPRINT,NDELP,MESH,NMESH,IBCE1,IBCE2,IBCE3,IBCE4,ISR,NSFL, MAIN 17
1KJMAX,YLDFAC,NLP                                              MAIN 18
COMMON MI1(6),MI2(6),NI1(6),NI2(6),DM1(6),DM2(6),DN1(6),DN2(6), MAIN 19
1PM(6),PN(6),ETAG1(6),ETAG2(6),ANGLE(6),ANGLB(6),NETAG(6),EPSS1(6), MAIN 20
2EPSS2(6),JCHK(3)                                             MAIN 21
COMMON QM,QN,MQ1,MQ2,NQ1,NQ2,QM1,QM2,QN1,QN2,D1,D2,D3,ETAD1,ETAD2 MAIN 22
COMMON GI11(6),GI22(6),GI12(6),ASA(6),BSA(6),CSA(6),ASB(6),BSB(6), MAIN 23
1CSB(6),B111(6),B121(6),B221(6),A111(6),A121(6),A221(6),B112(6), MAIN 24
2B122(6),B222(6),A112(6),A122(6),A222(6),B113(6),B123(6),B223(6), MAIN 25
3A113(6),A123(6),A223(6),B114(6),B124(6),B224(6),A114(6),A124(6), MAIN 26
4A224(6),DSR(3),PSR(3),SSIG(3),SEPS(3),SE(3),SIGZSQ(3),WT(3)  MAIN 27
COMMON Y11,Y12,Y13,Y21,Y22,Y23,U11,U12,U13,U21,U22,U23,      MAIN 28
1 Y111,Y112,Y113,Y121,Y122,Y123,Y221,Y222,Y223,              MAIN 29
2 U111,U112,U113,U121,U122,U123,U221,U222,U223              MAIN 30
COMMON A11,A12,A22,SRA,CS111,CS112,CS122,CS211,CS212,CS222,   MAIN 31
1 B11,B12,B22,BT,BM11,BM12,BM21,BM22,CINES1                 MAIN 32
C
C      NPLOT=3                                         MAIN 33
C      NLP=1                                           MAIN 34
C      NNN=1                                           MAIN 35
C      CALL START                                       MAIN 36
C
C      CHECK IF THIS IS A RESTART                     MAIN 37
C      IF(NCONT .EQ. 0)GOTO 15                         MAIN 38
C      CALL WRTAPE (2)                                  MAIN 39
C      NPRINT=(NCYCLE-MOD(INCYCLE,NDELP))+NDELP       MAIN 40
C      GOTO 49                                           MAIN 41
C
C      SET CYCLE NUMBER = 0                             MAIN 42
15 NCYCLE=0                                             MAIN 43
TIME=0.0                                               MAIN 44
CINES=0.0                                             MAIN 45
TDAMP=0.0                                             MAIN 46
CINEP=0.0                                             MAIN 47
ENS=0.0                                               MAIN 48
C1=0.0                                                MAIN 49
C2=0.0                                                MAIN 50
C3=0.0                                                MAIN 51
C
C      SET SYMMETRY BOUNDARY CONDITIONS FOR EDGE1,EDGE2,EDGE3 MAIN 52
C      IF(IBCE2 .NE. 2)GOTO 17                         MAIN 53
C      DO 16 M=2,M1                                     MAIN 54
C      Y1(M,NN)=Y1(M,NR)                               MAIN 55
C      Y2(M,NN)=-Y2(M,NR)+2.0*Y2(M,NS)                MAIN 56
C      Y3(M,NN)=Y3(M,NR)                               MAIN 57

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16	CONTINUE	MAIN	61
17	DO 20 N=1,NN	MAIN	62
	Y1(1,N)=-Y1(3,N)	MAIN	63
	Y2(1,N)= Y2(3,N)	MAIN	64
	Y3(1,N)= Y3(3,N)	MAIN	65
	IF(1BCE3 .NE. 2)GOTO 20	MAIN	66
	Y1(MM,N)=-Y1(MR,N)	MAIN	67
	Y2(MM,N)= Y2(MR,N)	MAIN	68
	Y3(MM,N)= Y3(MR,N)	MAIN	69
20	CONTINUE	MAIN	70
C		MAIN	71
C	SET INITIAL DISPLACEMENT,PRESSURE,STRAIN AND STRESS = 0	MAIN	72
	DO 28 M=1,MM	MAIN	73
	DO 28 N=1,NN	MAIN	74
	U1(M,N)=0.0	MAIN	75
	U2(M,N)=0.0	MAIN	76
	U3(M,N)=0.0	MAIN	77
	P(M,N)=0.0	MAIN	78
	EPSL1(M,N)=0.0	MAIN	79
	EPSL2(M,N)=0.0	MAIN	80
	GAMMAL(M,N)=0.0	MAIN	81
	EPSU1(M,N)=0.0	MAIN	82
	EPSU2(M,N)=0.0	MAIN	83
	GAMMAU(M,N)=0.0	MAIN	84
	DO 28 K=1,KJMAX	MAIN	85
	SIG1(M,N,K)=0.	MAIN	86
	SIG2(M,N,K)=0.	MAIN	87
	TAU(M,N,K)=0.	MAIN	88
28	CONTINUE	MAIN	89
C		MAIN	90
	IF(LOAD) 29,30,29	MAIN	91
29	CALL PRESS	MAIN	92
30	DELGAM=DELSQ/GAMZ	MAIN	93
C	WRITE INITIAL CARTESIAN COORDINATES, PRESSURE	MAIN	94
	WRITE(6,300)	MAIN	95
	DO 46 M=2,M1	MAIN	96
46	WRITE(6,400)M,(N,Y1(M,N),Y2(M,N),Y3(M,N),P(M,N),N=1,N1)	MAIN	97
C		MAIN	98
	CALL DGECM	MAIN	99
	CALL STRAIN	MAIN	100
	IF(LOAD) 31,31,43	MAIN	101
C		MAIN	102
31	CALL INVEL	MAIN	103
C		MAIN	104
	DO 35 M=2,MS	MAIN	105
	DO 35 N=2,NS	MAIN	106
	U1(M,N)=DELTAT*U1(M,N)	MAIN	107
	U2(M,N)=DELTAT*U2(M,N)	MAIN	108
	U3(M,N)=DELTAT*U3(M,N)	MAIN	109
35	CONTINUE	MAIN	110
	CALL BOUNDU	MAIN	111
	CALL KINET	MAIN	112
	CINES=2.0*CINET	MAIN	113
	TNRG=CINES	MAIN	114
C		MAIN	115
	IF(LOAD) 42,45,43	MAIN	116
C		MAIN	117
42	CALL PWORK	MAIN	118
C		MAIN	119
43	DO 44 M=2,MS	MAIN	120

DO 44 N=2,NS	MAIN 121
U1(M,N)=U1(M,N)-P(M,N)*SN1(M,N)*TEMP(M,N)	MAIN 122
U2(M,N)=U2(M,N)-P(M,N)*SN2(M,N)*TEMP(M,N)	MAIN 123
U3(M,N)=U3(M,N)-P(M,N)*SN3(M,N)*TEMP(M,N)	MAIN 124
44 CONTINUE	MAIN 125
CALL BOUNDU	MAIN 126
CALL PWORK	MAIN 127
ENR=ENS	MAIN 128
CALL KINET	MAIN 129
TNRG=CINET	MAIN 130
45 IF(NCYC(1) .EQ. 0)NNM=2	MAIN 131
49 CALL PDATA (1)	MAIN 132
C	MAIN 133
C END INITIALIZATION	MAIN 134
50 NCYCLE=NCYCLE+1	MAIN 135
TIME=TIME+DELTAT	MAIN 136
C CHECK FOR FINAL STEP	MAIN 137
IF(CYCLE-MAXC) 60,60,70	MAIN 138
C CHECK IF CALL PRESS IS NEEDED	MAIN 139
50 IF((LPRESS-NCYCLE) 64,62,62	MAIN 140
62 CALL PRESS	MAIN 141
64 CALL POSITN	MAIN 142
CALL DGEOM	MAIN 143
CALL STRAIN	MAIN 144
CALL MOTION	MAIN 145
CALL PDATA (2)	MAIN 146
CALL DAMP	MAIN 147
C CHECK FOR RESTART DUMP	MAIN 148
IF(NCYCLE-NRITE) 50,66,50	MAIN 149
66 CALL MRTAPE (1)	MAIN 150
NRITE=NRITE+NRITE	MAIN 151
CALL PDATA (3)	MAIN 152
GOTO 50	MAIN 153
70 IF(NCYCLE .LT. 25)CALL EXIT	MAIN 154
CALL PDATA (3)	MAIN 155
CALL PDATA (4)	MAIN 156
CALL EXIT	MAIN 157
C	MAIN 158
300 FORMAT(1H1,32X,29H:INITIAL CARTESIAN COORDINATES,32X,8HPRESSURE/3X,	MAIN 159
11H,3X,1H,9X,7HY1(M,N),18X,7HY2(M,N),18X,7HY3(M,N),20X,6HP(M,N))	MAIN 160
400 FORMAT(2I4,4(2X,E23.16)/(I8,4(2X,E23.16)))	MAIN 161
END	MAIN 162

	SUBROUTINE START	****	1
	DIMENSION TITLE(8)	STAR	2
	COMMON (USE MAIN)	STAR	3
	READ(5,100) TITLE	STAR	4
	READ(5,105) MESH,NMESH,LAYER,YLOFAC	STAR	5
	READ(5,105) MAXC,NCONT,NRITE,DEL TAT	STAR	6
	READ(5,110) IBCE1,IBCE2,IBCE3,IBCE4	STAR	7
	READ(5,115) LOAD,LPRESS,MDAMP,DAMPF,DFACT	STAR	8
	READ(5,120) E,FNU,SIGZ,RMO,THICKN,NSFL,ISR	STAR	9
	IF(NSFL .EQ. 1 .AND. ISR .EQ. 0)GOTO 700	STAR	10
	IF(NSFL .EQ. 0)GOTO 700	STAR	11
	READ (5,125) (SSIG(J),SEPS(J),DSR(J),PSR(J),J=1,NSFL)	STAR	12
700	IF(NSFL .LT. 1) ISR=-1	STAR	13
	IF(NSFL .LT. 1) NSFL=1	STAR	14
	READ(5,110) NPRINT,(JCHK(J),J=1,3)	STAR	15
	READ(5,110) NUMCY,(NCYCH(J),J=1,NUMCY)	STAR	16
	READ(5,110) NLPRIN,(JCYNLP(J),J=1,NLPRIN)	STAR	17
	READ(5,110) N3D,(NC3DP(J),J=1,N3D)	STAR	18
	SSIG(1)=SIGZ	STAR	19
	SEPS(1)=SIGZ/E	STAR	20
	KJMAX=LAYER*NSFL	STAR	21
	SE(1)=E	STAR	22
	SIGZSQ(1)=SSIG(1)**2	STAR	23
	DO 795 J=1,NSFL	STAR	24
	IF(ISR .LT. 1)GOTO 794	STAR	25
	IF(DSR(J) .GT. 0.0 .AND. PSR(J) .GT. 0.0)GOTO 793	STAR	26
791	WRITE(6,792)	STAR	27
792	FORMAT(/46H ERROR IN STRAIN HARDENING OR STRAIN RATE DATA)	STAR	28
	CALL EXIT	STAR	29
793	PSR(J) = 1.0/PSR(J)	STAR	30
794	IF(J .EQ. 1) GOTO 795	STAR	31
	IF(SEPS(J).LE.SEPS(J-1)) GOTO 791	STAR	32
	SE(J)=(SSIG(J)-SSIG(J-1))/(SEPS(J)-SEPS(J-1))	STAR	33
	WT(J-1)=(SE(J-1)-SE(J))/E	STAR	34
	SIGZSQ(J)=(E*SEPS(J))**2	STAR	35
795	CONTINUE	STAR	36
	WT(NSFL)=SE(NSFL)/E	STAR	37
C	CHECK NUMBER OF MESH POINTS NEEDED FOR ETA1,ETA2 DIRECTIONS	STAR	38
	MM=MESH+2	STAR	39
	NN=NMESH+1	STAR	40
	N1=NN	STAR	41
C	CHECK BOUNDARY CONDITIONS FOR EDGE2	STAR	42
	IF((IBCE2 .NE. 2))GOTO 4	STAR	43
	NN=NMESH+2	STAR	44
	N1=NN-1	STAR	45
C		STAR	46
4	M1=MM	STAR	47
	MB1=MM-2	STAR	48
	NB1=3	STAR	49
	NB2=NN-2	STAR	50
C	CHECK BOUNDARY CONDITIONS FOR EDGE3	STAR	51
	IF((IBCE3 .NE. 2))GOTO 5	STAR	52
	NN=MESH+3	STAR	53
	M1=MM-1	STAR	54
	MB1=MM-1	STAR	55
5	MS=MM-1	STAR	56
	NS=NN-1	STAR	57
	MR=MM-2	STAR	58
	NR=NN-2	STAR	59
	IF((IBCE3 .EQ. 3)) MB1=MS	STAR	60

	IF(I BCE4 .EQ. 3) NB1=2	STAR 61
	IF(I BCE2 .NE. 1) NB2=NS	STAR 62
C		STAR 63
	READ(5,130) ETAD1,ETAD2,NSTRN	STAR 64
	READ(5,15)(ETAG1(I),ETAG2(I),ANGLE(I),ANGLB(I),NETAG1(I),I=1,NSTRN)	STAR 65
C		STAR 66
	CALL INGEOM	STAR 67
	QM=2.0+ETAD1/DETA1	STAR 68
	QN=1.0+ETAD2/DETA2	STAR 69
	MQ1=QM	STAR 70
	MQ2=MQ1+1	STAR 71
	NQ1=QN	STAR 72
	NQ2=NQ1+1	STAR 73
	QM1=QM-FLOAT(MQ1)	STAR 74
	QM2=FLOAT(MQ2)-QM	STAR 75
	QN1=QN-FLOAT(NQ1)	STAR 76
	QN2=FLOAT(NQ2)-QN	STAR 77
	DO 20 I=1,NSTRN	STAR 78
	PM(I)=2.0+ETAG1(I)/DETA1	STAR 79
	PN(I)=1.0+ETAG2(I)/DETA2	STAR 80
	M1(I)=PM(I)	STAR 81
	M12(I)=M1(I)+1	STAR 82
	N1(I)=PN(I)	STAR 83
	N12(I)=N1(I)+1	STAR 84
	PM1=M1(I)	STAR 85
	PM2=M12(I)	STAR 86
	PN1=N1(I)	STAR 87
	PN2=N12(I)	STAR 88
	DM1(I)=PM(I)-PM1	STAR 89
	DM2(I)=PM2-PM(I)	STAR 90
	DN1(I)=PN(I)-PN1	STAR 91
	DN2(I)=PN2-PN(I)	STAR 92
20	CONTINUE	STAR 93
C	TIME INCREMENT BY VON NEUMAN	STAR 94
	C=SQR(E/(RHO*(1.0-FNU**2)))	STAR 95
	DELM=2.0/(C*SQR(1.0/DETA1**2+1.0/DETA2**2))	STAR 96
	ZAYER=LAYER	STAR 97
	FLAYER=(ZAYER/2.0)**2	STAR 98
	DETAN=.08333333333333333-(1.0/(48.0*FLAYER))	STAR 99
	DX4=1.0/DETA1**4	STAR 100
	DY4=1.0/DETA2**4	STAR 101
	T2=(1.0+15.0*FNU)/(8.0*DETA1**2*DETA2**2)	STAR 102
	DELB=1.0/(2.0*THICKN*C*SQR(DETAN*(DX4+DY4+T2)))	STAR 103
	DELMIN=AMIN1(DELB,DELM)	STAR 104
	SUM=1.0	STAR 105
25	RSUM=1.0/SUM	STAR 106
	SUM=SUM*10.0	STAR 107
	IF(RSUM .LT. DELMIN)GOTO 30	STAR 108
	GOTO 25	STAR 109
30	DELMIN=AINT(DELMIN*SUM)/SUM	STAR 110
	DELIN=DELTAT	STAR 111
	IF(DELIN .GT. 0.0)DELMIN=AMIN1(DELIN,DELMIN)	STAR 112
	DELTAT=DELMIN	STAR 113
C	PROGRAM CONSTANTS	STAR 114
	H=0.5*THICKN	STAR 115
	GAMZ=THICKN*RHO	STAR 116
	NDELP=NPRINT	STAR 117
	NRITE=NCONT+NWRITE	STAR 118
	DO 3 K=1,LAYER	STAR 119
	ZETA(K)=H*(1.0-(2.0*FLOAT(K)-1.0)/ZAYER)	STAR 120

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ZETASQ(K) = ZETA(K)**2
3 CONTINUE
TA=M*(2.0/ZAYER)
DELSQ=DEL TAT**2
Q=.5*E/(1.0+FNU)
GTWO=G**2.0
PRAT=E/(1.0-FNU**2)
RTD1=1.0/(2.0*DETA1)
RTD2=1.0/(2.0*DETA2)
RD11=1.0/DETA1**2
RD22=1.0/DETA2**2
RD12=0.25/(DETA1*DETA2)
CA=DETA1*DETA2
CB=TA*DETA1*DETA2/E
TRD=2.0*RD12
C DAMPING CONSTANTS
C2=2.0*DEL TAT*DAMPF/GANZ
C1=C2/(4.0+C2)
C
WRITE(6,130)
WRITE(6,140) TITLE
WRITE(6,150) MESH,DETA1,NMESH,DETA2
WRITE(6,300) DELB,DELM,DELIN,DEL TAT
WRITE(6,160) E,FNU,SIGZ,RHO,THICKN
WRITE(6,170) NCONT,MAXC,NPRINT,WRITE
WRITE(6,175) LAYER,NSTRN,LOAD,LPRESS
WRITE(6,180) IBCE1,IBCE2,IBCE3,IBCE4
WRITE(6,185) (JCHK(I),I=1,3)
WRITE(6,190) (NCYCH(I),I=1,NUMCY)
WRITE(6,500) (JCYNLP(J),J=1,NLPRIN)
WRITE(6,195) (NC3DP(I),I=1,N3D)
IF(ISR.EQ.-1) WRITE(6,400)
IF(NSFL.EQ.1.AND.ISR.EQ.1) WRITE(6,405)
IF(NSFL.GT.1.AND.ISR.EQ.0) WRITE(6,410)
IF(NSFL.EQ.1.AND.ISR.EQ.0) WRITE(6,415)
IF(NSFL.GT.1.AND.ISR.EQ.1) WRITE(6,420)
IF(NSFL.GT.1)WRITE(6,820)NSFL
WRITE(6,821)(J,SSIG(J),SEPS(J),USR(J),PSR(J),J=1,NSFL)
WRITE(6,110)
TIME=DEL TAT*FLOAT(MDAMP)
WRITE(6,200) MDAMP,TIME,DAMPF,DFACT
RETURN
15 FORMAT(4E10.4,I5)
100 FORMAT(8A10)
105 FORMAT(3I5,E12.6)
110 FORMAT(16I5)
115 FORMAT(3I5,2E12.6)
120 FORMAT(5E12.6,2I5)
125 FORMAT(4E15.7)
130 FORMAT(2E10.4,I5)
C
140 FORMAT(1H1,53X,15HBRL REPSIL CODE//24X,8A10/)
150 FORMAT(38X,14,29H MESHES IN THE ETA1 DIRECTION,3X,8H(DETA1 =E12.6,STAR 173
11H)/38X,14,29H MESHES IN THE ETA2 DIRECTION,3X,8H(DETA2 =E12.6,1H)STAR 174
2/)
160 FORMAT(57X,17HYOUNG'S MODULUS =E12.6 STAR 176
1 /32X,17HPOISSON'S RATIO =E12.6,10X,17HYIELD STRESS *E12.6STAR 177
2 /32X,17HMASS DENSITY =E12.6,10X,17HTHICKNESS *E12.6STAR 178
3/)
170 FORMAT(55X,18HSTART AT TIME STEPI5/55X,18HFINAL TIME STEP ,15/ STAR 180

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1	55X,21HSURFACE STRAINS EVERYIS,10H TIME STEP/	STAR 181
2	55X,21HRESTART WRITE EVERYIS,10H TIME STEP/)	STAR 182
175	FORMAT(43X,7HLAYER =15,18X,8HNSTRN =15/	STAR 183
1	43X,7HLOAD =15,18X,8HLPRESS =15/)	STAR 184
180	FORMAT(54X,19HBOUNDARY CONDITIONS/47X,31H1/2/3 = CLAMPED/SYMMETRY/STAR 185	
1	HINGED/57X,7HEDGE1 =14/57X,7HEDGE2 =14/57X,7HEDGE3 =14/	STAR 186
2	57X,7HEDGE4 =14/)	STAR 187
185	FORMAT(50X,25HPRINT OPTION CONTROL CARD/52X,20H0/1 = NO PRINT/PRINSTAR 188	
1	7/50X,14,24H DISPLACEMENT INCREMENTS/	STAR 189
2	50X,14,32H CARTESIAN COORDINATES, PRESSURE/	STAR 190
3	50X,14,33H SURFACE NORMAL VECTOR COMPONENTS/)	STAR 191
190	FORMAT(24X,45HPRINT INFORMATION AT THE FOLLOWING TIME STEPS/(24X, STAR 192	
1	(1615)))	STAR 193
195	FORMAT(24X,38H3-D PLOTS FOR THE FOLLOWING TIME STEPS/(24X,(1615)))STAR 194	
200	FORMAT(/46X,29HSTART DAMPING AFTER TIME STEPI5,5X,6HTIME =E10.4/ STAR 195	
1	40X,7HDAMPF =E10.4,16X,7HDFACT =E10.4)	STAR 196
300	FORMAT(47X,25HBENDING TIME INCREMENT= E12.6/47X,25HMEMBRANE TIME STAR 197	
1	INCREMENT= E12.6/47X,25HINPUT TIME INCREMENT= E12.6//44X,31HTIMSTAR 198	
2	E INCREMENT USED BY REPSIL= E12.6/)	STAR 199
400	FORMAT(/37X,32HCONSTITUTIVE RELATION ELASTIC) STAR 200	
405	FORMAT(/24X,78HCONSTITUTIVE RELATION ELASTOPLASTIC-NO WORK HARDSTAR 201	
1	ENING-STRAIN RATE DEPENDENT)	STAR 202
410	FORMAT(/24X,77HCONSTITUTIVE RELATION ELASTOPLASTIC-WORK HARDENISTAR 203	
1	ING-STRAIN RATE INDEPENDENT)	STAR 204
415	FORMAT(/24X,80HCONSTITUTIVE RELATION ELASTOPLASTIC-NO WORK HARDSTAR 205	
1	ENING-STRAIN RATE INDEPENDENT)	STAR 206
420	FORMAT(/37X,75HCONSTITUTIVE RELATION ELASTOPLASTIC-WORK HARDENISTAR 207	
1	ING-STRAIN RATE DEPENDENT)	STAR 208
500	FORMAT(24X,49HPRINT L MATRIX (LMAT) AT THE FOLLOWING TIME STEPS/ STAR 209	
1	(24X,1615))	STAR 210
820	FORMAT(35X,31HSTRESS-STRAIN APPROXIMATION HAS,13,10H SUBLAYERS) STAR 211	
821	FORMAT(/46X,40HSTRESS-STRAIN AND STRAIN RATE PARAMETERS/ STAR 212	
1	130X,1HJ,9X,7HSSIG(J),8X,7HSEPS(J),14X,6HDSR(J),8X,8H1/PSR(J)/	STAR 213
2	(26X,15,5X,1P2E15.7,5X,2E15.7))	STAR 214
	END	STAR 215

	SUBROUTINE INVEL	****	1
	COMMON (USE MAIN)	INVE	2
C	EVALUATE THE INITIAL VELOCITY AT TIME=0 FOR ALL MESH POINTS	INVE	3
	READ(5,100) MI,MF,NI,NF,VR,NV	INVE	4
	WRITE(6,200) MI,MF,NI,NF,VR	INVE	5
C		INVE	6
	DO 30 M=MI,MF	INVE	7
	DO 30 N=NI,NF	INVE	8
	U1(M,N)=-VR*SN1(M,N)	INVE	9
	U2(M,N)=-VR*SN2(M,N)	INVE	10
	U3(M,N)=-VR*SN3(M,N)	INVE	11
30	CONTINUE	INVE	12
	IF(NV)50,50,40	INVE	13
40	WRITE(6,300)	INVE	14
	DO 45 K=1,NV	INVE	15
	READ(5,500) M,N,V	INVE	16
	WRITE(6,400) M,N,V	INVE	17
	U1(M,N)=-V*SN1(M,N)	INVE	18
	U2(M,N)=-V*SN2(M,N)	INVE	19
	U3(M,N)=-V*SN3(M,N)	INVE	20
45	CONTINUE	INVE	21
50	RETURN	INVE	22
C		INVE	23
100	FORMAT(4I5,E12.6,I5)	INVE	24
200	FORMAT(1H1,3IX,3H(M=,I3,1H,,I3,9H) AND (N=,I3,1H,,I3,30H) RECEIVE	INVE	25
	IFULL VELOCITY,(VR)= ,E12.6/)	INVE	26
300	FORMAT(52X,27HOTHER VELOCITY DISTRIBUTION/57X,1HM,4X,1HN,8X,1HV/)	INVE	27
400	FORMAT(54X,2I5,2X,E12.6)	INVE	28
500	FORMAT(2I5,E12.6)	INVE	29
	END	INVE	30

SUBROUTINE POSITM	****	1
COMMON(USE MAIN)	POSI	2
DO 50 M=1,MM	POSI	3
DO 50 N=1,NN	POSI	4
Y1(M,N)=Y1(M,N)+U1(M,N)	POSI	5
Y2(M,N)=Y2(M,N)+U2(M,N)	POSI	6
Y3(M,N)=Y3(M,N)+U3(M,N)	POSI	7
50 CONTINUE	POSI	8
IF(NCYCLE .NE. NCYCH(MN))GOTO 75	POSI	9
WRITE DISPLACEMENT INCREMENTS	POSI	10
IF(JCHK(1)) 63,63,55	POSI	11
55 NCYC=NCYCLE-1	POSI	12
WRITE (6,509) NCYC,NCYCLE	POSI	13
WRITE (6,9915)	POSI	14
DO 60 M=2,M1	POSI	15
60 WRITE (6,9917) M, (N,U1(M,N),U2(M,N),U3(M,N),N=1,N1)	POSI	16
WRITE CARTESIAN COORDINATES, PRESSURE	POSI	17
63 IF(JCHK(2)) 75,75,65	POSI	18
65 WRITE (6,99991) NCYCLE,TIME	POSI	19
WRITE (6,903) NCYCLE,NCYCLE,NCYCLE	POSI	20
DO 70 M=2,M1	POSI	21
70 WRITE (6,902) M, (N,Y1(M,N),Y2(M,N),Y3(M,N),P(M,N),N=1,N1)	POSI	22
75 RETURN	POSI	23
C	POSI	24
509 FORMAT(1H1,21X,36HDISPLACEMENT INCREMENTS BETWEEN T.S.,I4.4H AND,	POSI	25
1I4,7)	POSI	26
9915 FORMAT(1H ,5X,1HM,4X,1HN,10X,7HU1(M,N),18X,7HU2(M,N),18X,7HU3(M,N)	POSI	27
1)	POSI	28
9917 FORMAT(16,15,3(2X,E23.16)/(111,3(2X,E23.16)))	POSI	29
99991 FORMAT(10HITIME STEP,15,6X,5H TIME,E16.8)	POSI	30
903 FORMAT(36X,21HCARTESIAN COORDINATES,36X,8HPRESSURE/3X,1HM,3X,1HN,	POSI	31
19X,7HY1(M,N,I4,1H),13X,7HY2(M,N,I4,1H),13X,7HY3(M,N,I4,1H)	POSI	32
2,15X,6HP(M,N))	POSI	33
902 FORMAT(2I4,4(2X,E23.16)/(18,4(2X,E23.16)))	POSI	34
END	POSI	35

SUBROUTINE DGEOM	****	1
COMMON (USE MAIN)	DGEO	2
STREN=0.0	CGEO	3
CO 90 M=2,M1	CGEO	4
CO 90 N=1,N1	DGEO	5
CALL GRAC (M,N)	DGEO	6
RN1=SN1(M,N)	CGEO	7
RN2=SN2(M,N)	DGEO	8
RN3=SN3(M,N)	CGEO	9
A11=Y11**2+Y12**2+Y13**2	CGEO	10
A22=Y21**2+Y22**2+Y23**2	DGEO	11
A12=Y11*Y21+Y12*Y22+Y13*Y23	CGEO	12
CA=A11*A22-A12**2	CGEO	13
RA=1.0/DA	DGEO	14
SRA=SQRT(DA)	CGEO	15
RRR=1.0/SRA	DGEO	16
IF (LPRESS .GE. NCYCLE) P(M,N)*SRA*P(M,N)	DGEO	17
IF (NCYCLE .EQ. 0) GOTO 86	DGEO	18
IF (M.EQ.MM .AND. IBCE3.EQ.1) GOTO 87	CGEO	19
IF (N.EQ.1 .AND. IBCE4.EQ.1) GOTO 87	DGEO	20
IF (M.EQ.MM .AND. IBCE2.EQ.1) GOTO 87	DGEO	21
86 SN1(M,N)=RRR*(Y12*Y23-Y13*Y22)	CGEO	22
SN2(M,N)=RRR*(Y13*Y21-Y11*Y23)	DGEO	23
SN3(M,N)=RRR*(Y11*Y22-Y12*Y21)	DGEO	24
87 B11=SN1(M,N)*Y111+SN2(M,N)*Y112+SN3(M,N)*Y113	DGEO	25
B12=SN1(M,N)*Y121+SN2(M,N)*Y122+SN3(M,N)*Y123	DGEO	26
B22=SN1(M,N)*Y221+SN2(M,N)*Y222+SN3(M,N)*Y223	DGEO	27
IF (NCYCLE.GT.0) GOTO 88	CGEO	28
TEMP(M,N)=DELGAM*RRR	DGEO	29
CALL ABINIT(M,N)	DGEO	30
GOTO 90	CGEO	31
88 AR11= RA*A22	DGEO	32
AR22= RA*A11	DGEO	33
AR12=-RA*A12	DGEO	34
BM11=AR11*BM11+AR12*BM12	DGEO	35
BM12=AR11*BM12+AR12*BM22	DGEO	36
BM21=AR12*BM11+AR22*BM12	DGEO	37
BM22=AR12*BM12+AR22*BM22	DGEO	38
BT=BM11+BM22	DGEO	39
YR11=AR11*Y11+AR12*Y21	CGEO	40
YR12=AR11*Y12+AR12*Y22	DGEO	41
YR13=AR11*Y13+AR12*Y23	DGEO	42
YR21=AR12*Y11+AR22*Y21	CGEO	43
YR22=AR12*Y12+AR22*Y22	CGEO	44
YR23=AR12*Y13+AR22*Y23	CGEO	45
CS111=YR11*Y111+YR12*Y112+YR13*Y113	CGEO	46
CS112=YR11*Y121+YR12*Y122+YR13*Y123	CGEO	47
CS122=YR11*Y221+YR12*Y222+YR13*Y223	DGEO	48
CS211=YR21*Y111+YR22*Y112+YR23*Y113	DGEO	49
CS212=YR21*Y121+YR22*Y122+YR23*Y123	CGEO	50
CS222=YR21*Y221+YR22*Y222+YR23*Y223	DGEO	51
CNL1=-RN1*U11-RN2*U12-RN3*U13	CGEO	52
CNL2=-RN1*U21-RN2*U22-RN3*U23	CGEO	53
CNR1=AR11*DNL1+AR12*DNL2	DGEO	54
CNR2=AR12*DNL1+AR22*DNL2	CGEO	55
SNN=RN1*SN1(M,N)+RN2*SN2(M,N)+RN3*SN3(M,N)	CGEO	56
CN=(DNL1*DNR1+DNL2*DNR2)/(1.0+SNN)	CGEO	57
CA11=Y11*U11+Y12*U12+Y13*U13-C.5*(U11**2+U12**2+U13**2)	CGEO	58
CA22=Y21*U21+Y22*U22+Y23*U23-C.5*(U21**2+U22**2+U23**2)	CGEO	59
CA12=0.5*(Y11*U21+Y21*U11+Y12*U22+Y22*U12+Y13*U23+Y23*U13)	CGEO	60

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1          -U11*U21-U12*U22-U13*U23)
CB11=RN1*U11+RN2*U112+RN3*U113+CS111*DNL1+CS211*DNL2+D11*DN
DB22=RN1*U221+RN2*U222+RN3*U223+CS122*DNL1+CS222*DNL2+D12*DN
CB12=RN1*U121+RN2*U122+RN3*U123+CS112*DNL1+CS212*DNL2+D12*DN
CO 89 K=1,LAYER
CEPS1(K)=DA11-ZETA(K)*DB11
CEPS2(K)=DA22-ZETA(K)*DB22
CGAMMA(K)=DA12-ZETA(K)*DB12
CALL STRESS (M,N,K)
89 CONTINUE
EPSL1(M,N)=EPSL1(M,N)+DA11+M*DB11
EPSL2(M,N)=EPSL2(M,N)+DA22+M*DB22
GAMMAL(M,N)=GAMMAL(M,N)+DA12+M*DB12
EPSU1(M,N)=EPSU1(M,N)+DA11-M*DB11
EPSU2(M,N)=EPSU2(M,N)+DA22-M*DB22
GAMMAU(M,N)=GAMMAU(M,N)+DA12-M*DB12
CALL RESULT (M,N)
90 CONTINUE
IF(NCYCLE .EQ. 0)GOTO 820
CALL SYNTY
100 STREN=CB*STREN
IF(1/STREN .EQ. 2) STREN=2.0*STREN
C      WRITE LMATRIX
      IF(NCYCLE.NE.JCYNLP(NLP))GOTO 820
      NLP=NLP+1
      WRITE(6,811)NCYCLE,TIME
      DO 802 K=1,LAYER
      WRITE(6,812)K,(M,M=2,M1)
      CO 802 N=1,N1
802 WRITE(6,813)N,(LMAT(M,N,K),M=2,M1)
C      WRITE SURFACE NORMAL VECTOR
820 IF(NCYCLE .NE. NCYCH(NNM))GOTO 700
IF(JCHK(1)) 700,700,800
800 WRITE(6,900) NCYCLE,TIME
WRITE(6,910)
CO 810 M=2,M1
810 WRITE(6,920) M,(N,SN1(M,N),SN2(M,N),SN3(M,N),N=1,N1)
700 RETURN
C
811 FORMAT(10H1TIME STEP,15,6X,4HTIME,1PE16.7,1H.,10X,
1 40HSUBDIVISIONS OF TIME INCREMENT IN STRESS/)
812 FORMAT(//20X,9HLMAT(M,N,12,1K)//9H N M=,40I3/)
900 FORMAT(13,5X,40I3)
813 FORMAT(10H1TIME STEP,15,6X,5H TIME,E16.8)
910 FORMAY(22X,32HSURFACE NORMAL VECTOR COMPONENTS/3X,1
1HM,3X,1HM,9X,8HSN1(M,N),17X,8HSN2(M,N),17X,8HSN3(M,N))
920 FORMAT(214,9(2X,E29.16)/(18,3(2X,E29.16)))
ENC
DGEO 61
DGEO 62
DGEO 63
CGEO 64
CGEO 65
DGEO 66
CGEO 67
CGEO 68
DGEO 69
CGEO 70
CGEO 71
DGEO 72
DGEO 73
DGEO 74
DGEO 75
DGEO 76
CGEO 77
CGEO 78
DGEO 79
CGEO 80
DGEO 81
DGEO 82
DGEO 83
DGEO 84
DGEO 85
CGEO 86
DGEO 87
DGEO 88
CGEO 89
DGEO 90
DGEO 91
DGEO 92
DGEO 93
DGEO 94
CGEO 95
CGEO 96
DGEO 97
CGEO 98
CGEO 99
DGEO 100
DGEO 101
CGEO 102
CGEO 103
DGEO 104
DGEO 105
DGEO 106
DGEO 107
CGEO 108

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SUBROUTINE GRAD (ND,NO)	****	1
COMMON (USE MAIN)	GRAD	2
M=ND	GRAD	3
N=ND	GRAD	4
IF (M.EQ.MN) GOTO 3	GRAD	5
Y11= RTD1*(Y1(M+1,N)-Y1(M-1,N))	GRAD	6
Y12= RTD1*(Y2(M+1,N)-Y2(M-1,N))	GRAD	7
Y13= RTD1*(Y3(M+1,N)-Y3(M-1,N))	GRAD	8
U11= RTD1*(U1(M+1,N)-U1(M-1,N))	GRAD	9
U12= RTD1*(U2(M+1,N)-U2(M-1,N))	GRAD	10
U13= RTD1*(U3(M+1,N)-U3(M-1,N))	GRAD	11
Y111=RD11*(Y1(M+1,N)-2.0*Y1(M,N)+Y1(M-1,N))	GRAD	12
Y112=RD11*(Y2(M+1,N)-2.0*Y2(M,N)+Y2(M-1,N))	GRAD	13
Y113=RD11*(Y3(M+1,N)-2.0*Y3(M,N)+Y3(M-1,N))	GRAD	14
U111=RD11*(U1(M+1,N)-2.0*U1(M,N)+U1(M-1,N))	GRAD	15
U112=RD11*(U2(M+1,N)-2.0*U2(M,N)+U2(M-1,N))	GRAD	16
U113=RD11*(U3(M+1,N)-2.0*U3(M,N)+U3(M-1,N))	GRAD	17
IF (M.EQ.1) GOTO 1	GRAD	18
IF (M.EQ.MN) GOTO 2	GRAD	19
Y121=RD12*(Y1(M+1,N+1)-Y1(M-1,N+1)-Y1(M+1,N-1)+Y1(M-1,N-1))	GRAD	20
Y122=RD12*(Y2(M+1,N+1)-Y2(M-1,N+1)-Y2(M+1,N-1)+Y2(M-1,N-1))	GRAD	21
Y123=RD12*(Y3(M+1,N+1)-Y3(M-1,N+1)-Y3(M+1,N-1)+Y3(M-1,N-1))	GRAD	22
U121=RD12*(U1(M+1,N+1)-U1(M-1,N+1)-U1(M+1,N-1)+U1(M-1,N-1))	GRAD	23
U122=RD12*(U2(M+1,N+1)-U2(M-1,N+1)-U2(M+1,N-1)+U2(M-1,N-1))	GRAD	24
U123=RD12*(U3(M+1,N+1)-U3(M-1,N+1)-U3(M+1,N-1)+U3(M-1,N-1))	GRAD	25
GOTO 4	GRAD	26
1 Y121=RD12*(Y1(M-1,N+2)-Y1(M+1,N+2)	GRAD	27
1 -4.0*(Y1(M-1,N+1)-Y1(M+1,N+1)))-3.0*RTD2*Y11	GRAD	28
Y122=RD12*(Y2(M-1,N+2)-Y2(M+1,N+2)	GRAD	29
1 -4.0*(Y2(M-1,N+1)-Y2(M+1,N+1)))-3.0*RTD2*Y12	GRAD	30
Y123=RD12*(Y3(M-1,N+2)-Y3(M+1,N+2)	GRAD	31
1 -4.0*(Y3(M-1,N+1)-Y3(M+1,N+1)))-3.0*RTD2*Y13	GRAD	32
U121=RD12*(U1(M-1,N+2)-U1(M+1,N+2)	GRAD	33
1 -4.0*(U1(M-1,N+1)-U1(M+1,N+1)))-3.0*RTD2*U11	GRAD	34
U122=RD12*(U2(M-1,N+2)-U2(M+1,N+2)	GRAD	35
1 -4.0*(U2(M-1,N+1)-U2(M+1,N+1)))-3.0*RTD2*U12	GRAD	36
U123=RD12*(U3(M-1,N+2)-U3(M+1,N+2)	GRAD	37
1 -4.0*(U3(M-1,N+1)-U3(M+1,N+1)))-3.0*RTD2*U13	GRAD	38
GOTO 4	GRAD	39
2 Y121=RD12*(Y1(M+1,N-2)-Y1(M-1,N-2)	GRAD	40
1 -4.0*(Y1(M+1,N-1)-Y1(M-1,N-1)))+3.0*RTD2*Y11	GRAD	41
Y122=RD12*(Y2(M+1,N-2)-Y2(M-1,N-2)	GRAD	42
1 -4.0*(Y2(M+1,N-1)-Y2(M-1,N-1)))+3.0*RTD2*Y12	GRAD	43
Y123=RD12*(Y3(M+1,N-2)-Y3(M-1,N-2)	GRAD	44
1 -4.0*(Y3(M+1,N-1)-Y3(M-1,N-1)))+3.0*RTD2*Y13	GRAD	45
U121=RD12*(U1(M+1,N-2)-U1(M-1,N-2)	GRAD	46
1 -4.0*(U1(M+1,N-1)-U1(M-1,N-1)))+3.0*RTD2*U11	GRAD	47
U122=RD12*(U2(M+1,N-2)-U2(M-1,N-2)	GRAD	48
1 -4.0*(U2(M+1,N-1)-U2(M-1,N-1)))+3.0*RTD2*U12	GRAD	49
U123=RD12*(U3(M+1,N-2)-U3(M-1,N-2)	GRAD	50
1 -4.0*(U3(M+1,N-1)-U3(M-1,N-1)))+3.0*RTD2*U13	GRAD	51
GOTO 4	GRAD	52
3 Y11= RTD1*(Y1(M-2,N)-4.0*Y1(M-1,N)+3.0*Y1(M,N))	GRAD	53
Y12= RTD1*(Y2(M-2,N)-4.0*Y2(M-1,N)+3.0*Y2(M,N))	GRAD	54
Y13= RTD1*(Y3(M-2,N)-4.0*Y3(M-1,N)+3.0*Y3(M,N))	GRAD	55
U11= RTD1*(U1(M-2,N)-4.0*U1(M-1,N)+3.0*U1(M,N))	GRAD	56
U12= RTD1*(U2(M-2,N)-4.0*U2(M-1,N)+3.0*U2(M,N))	GRAD	57
U13= RTD1*(U3(M-2,N)-4.0*U3(M-1,N)+3.0*U3(M,N))	GRAD	58
Y111=RD11*(2.0*Y1(M,N)-5.0*Y1(M-1,N)+4.0*Y1(M-2,N)-Y1(M-3,N))	GRAD	59
Y112=RD11*(2.0*Y2(M,N)-5.0*Y2(M-1,N)+4.0*Y2(M-2,N)-Y2(M-3,N))	GRAD	60

Y113=RD11*(2.0*Y3(M,N))-5.0*Y3(M-1,N)+4.0*Y3(M-2,N)-Y3(M-3,N)	GRAD 61
U111=RD11*(2.0*U1(M,N))-5.0*U1(M-1,N)+4.0*U1(M-2,N)-U1(M-3,N)	GRAD 62
U112=RD11*(2.0*U2(M,N))-5.0*U2(M-1,N)+4.0*U2(M-2,N)-U2(M-3,N)	GRAD 63
U113=RD11*(2.0*U3(M,N))-5.0*U3(M-1,N)+4.0*U3(M-2,N)-U3(M-3,N)	GRAD 64
4 IF (M.EQ.1) GOTO 5	GRAD 65
IF (M.EQ.MM) GOTO 6	GRAD 66
Y21= RTD2*(Y1(M,N+1))-Y1(M,N-1)	GRAD 67
Y22= RTD2*(Y2(M,N+1))-Y2(M,N-1)	GRAD 68
Y23= RTD2*(Y3(M,N+1))-Y3(M,N-1)	GRAD 69
U21= RTD2*(U1(M,N+1))-U1(M,N-1)	GRAD 70
U22= RTD2*(U2(M,N+1))-U2(M,N-1)	GRAD 71
U23= RTD2*(U3(M,N+1))-U3(M,N-1)	GRAD 72
Y221=RD22*(Y1(M,N+1))-2.0*Y1(M,N)+Y1(M,N-1)	GRAD 73
Y222=RD22*(Y2(M,N+1))-2.0*Y2(M,N)+Y2(M,N-1)	GRAD 74
Y223=RD22*(Y3(M,N+1))-2.0*Y3(M,N)+Y3(M,N-1)	GRAD 75
U221=RD22*(U1(M,N+1))-2.0*U1(M,N)+U1(M,N-1)	GRAD 76
U222=RD22*(U2(M,N+1))-2.0*U2(M,N)+U2(M,N-1)	GRAD 77
U223=RD22*(U3(M,N+1))-2.0*U3(M,N)+U3(M,N-1)	GRAD 78
IF (M.NE.MM) GOTO 7	GRAD 79
Y121=RD12*(Y1(M-2,N+1))-Y1(M-2,N-1)	GRAD 80
1 -4.0*(Y1(M-1,N+1))-Y1(M-1,N-1)))+3.0*RTD1*Y21	GRAD 81
Y122=RD12*(Y2(M-2,N+1))-Y2(M-2,N-1)	GRAD 82
1 -4.0*(Y2(M-1,N+1))-Y2(M-1,N-1)))+3.0*RTD1*Y22	GRAD 83
Y123=RD12*(Y3(M-2,N+1))-Y3(M-2,N-1)	GRAD 84
1 -4.0*(Y3(M-1,N+1))-Y3(M-1,N-1)))+3.0*RTD1*Y23	GRAD 85
U121=RD12*(U1(M-2,N+1))-U1(M-2,N-1)	GRAD 86
1 -4.0*(U1(M-1,N+1))-U1(M-1,N-1)))+3.0*RTD1*U21	GRAD 87
U122=RD12*(U2(M-2,N+1))-U2(M-2,N-1)	GRAD 88
1 -4.0*(U2(M-1,N+1))-U2(M-1,N-1)))+3.0*RTD1*U22	GRAD 89
U123=RD12*(U3(M-2,N+1))-U3(M-2,N-1)	GRAD 90
1 -4.0*(U3(M-1,N+1))-U3(M-1,N-1)))+3.0*RTD1*U23	GRAD 91
GOTO 7	GRAD 92
5 Y21=-RTD2*(Y1(M,N+2))-4.0*Y1(M,N+1)+3.0*Y1(M,N)	GRAD 93
Y22=-RTD2*(Y2(M,N+2))-4.0*Y2(M,N+1)+3.0*Y2(M,N)	GRAD 94
Y23=-RTD2*(Y3(M,N+2))-4.0*Y3(M,N+1)+3.0*Y3(M,N)	GRAD 95
U21=-RTD2*(U1(M,N+2))-4.0*U1(M,N+1)+3.0*U1(M,N)	GRAD 96
U22=-RTD2*(U2(M,N+2))-4.0*U2(M,N+1)+3.0*U2(M,N)	GRAD 97
U23=-RTD2*(U3(M,N+2))-4.0*U3(M,N+1)+3.0*U3(M,N)	GRAD 98
Y221=RD22*(2.0*Y1(M,N))-5.0*Y1(M,N+1)+4.0*Y1(M,N+2)-Y1(M,N+3)	GRAD 99
Y222=RD22*(2.0*Y2(M,N))-5.0*Y2(M,N+1)+4.0*Y2(M,N+2)-Y2(M,N+3)	GRAD 100
Y223=RD22*(2.0*Y3(M,N))-5.0*Y3(M,N+1)+4.0*Y3(M,N+2)-Y3(M,N+3)	GRAD 101
U221=RD22*(2.0*U1(M,N))-5.0*U1(M,N+1)+4.0*U1(M,N+2)-U1(M,N+3)	GRAD 102
U222=RD22*(2.0*U2(M,N))-5.0*U2(M,N+1)+4.0*U2(M,N+2)-U2(M,N+3)	GRAD 103
U223=RD22*(2.0*U3(M,N))-5.0*U3(M,N+1)+4.0*U3(M,N+2)-U3(M,N+3)	GRAD 104
IF (M.NE.MM) GOTO 7	GRAD 105
Y121=RD12*(Y1(M-2,N+2))-4.0*(Y1(M-1,N+2)+Y1(M-2,N+1)	GRAD 106
1 -4.0*Y1(M-1,N+1))-9.0*Y1(M,N))+3.0*(RTD1*Y21-RTD2*Y11)	GRAD 107
Y122=RD12*(Y2(M-2,N+2))-4.0*(Y2(M-1,N+2)+Y2(M-2,N+1)	GRAD 108
1 -4.0*Y2(M-1,N+1))-9.0*Y2(M,N))+3.0*(RTD1*Y22-RTD2*Y12)	GRAD 109
Y123=RD12*(Y3(M-2,N+2))-4.0*(Y3(M-1,N+2)+Y3(M-2,N+1)	GRAD 110
1 -4.0*Y3(M-1,N+1))-9.0*Y3(M,N))+3.0*(RTD1*Y23-RTD2*Y13)	GRAD 111
U121=RD12*(U1(M-2,N+2))-4.0*(U1(M-1,N+2)+U1(M-2,N+1)	GRAD 112
1 -4.0*U1(M-1,N+1))-9.0*U1(M,N))+3.0*(RTD1*U21-RTD2*U11)	GRAD 113
U122=RD12*(U2(M-2,N+2))-4.0*(U2(M-1,N+2)+U2(M-2,N+1)	GRAD 114
1 -4.0*U2(M-1,N+1))-9.0*U2(M,N))+3.0*(RTD1*U22-RTD2*U12)	GRAD 115
U123=RD12*(U3(M-2,N+2))-4.0*(U3(M-1,N+2)+U3(M-2,N+1)	GRAD 116
1 -4.0*U3(M-1,N+1))-9.0*U3(M,N))+3.0*(RTD1*U23-RTD2*U13)	GRAD 117
GOTO 7	GRAD 118
6 Y21= RTD2*(Y1(M,N-2))-4.0*Y1(M,N-1)+3.0*Y1(M,N)	GRAD 119
Y22= RTD2*(Y2(M,N-2))-4.0*Y2(M,N-1)+3.0*Y2(M,N)	GRAD 120

Y23=	RTD2*(Y3(M,N-2)-4.0*Y3(M,N-1)+3.0*Y3(M,N))	GRAD 121
U21=	RTD2*(U1(M,N-2)-4.0*U1(M,N-1)+3.0*U1(M,N))	GRAD 122
U22=	RTD2*(U2(M,N-2)-4.0*U2(M,N-1)+3.0*U2(M,N))	GRAD 123
U23=	RTD2*(U3(M,N-2)-4.0*U3(M,N-1)+3.0*U3(M,N))	GRAD 124
Y221=	RD22*(2.0*Y1(M,N)-5.0*Y1(M,N-1)+4.0*Y1(M,N-2)-Y1(M,N-3))	GRAD 125
Y222=	RD22*(2.0*Y2(M,N)-5.0*Y2(M,N-1)+4.0*Y2(M,N-2)-Y2(M,N-3))	GRAD 126
Y223=	RD22*(2.0*Y3(M,N)-5.0*Y3(M,N-1)+4.0*Y3(M,N-2)-Y3(M,N-3))	GRAD 127
U221=	RD22*(2.0*U1(M,N)-5.0*U1(M,N-1)+4.0*U1(M,N-2)-U1(M,N-3))	GRAD 128
U222=	RD22*(2.0*U2(M,N)-5.0*U2(M,N-1)+4.0*U2(M,N-2)-U2(M,N-3))	GRAD 129
U223=	RD22*(2.0*U3(M,N)-5.0*U3(M,N-1)+4.0*U3(M,N-2)-U3(M,N-3))	GRAD 130
IF	(M.NE.MM) GOTO 7	GRAD 131
Y121=	RD12*(Y1(M-2,N-2)-4.0*(Y1(M-1,N-2)+Y1(M-2,N-1)-4.0*Y1(M-1,N-1))-9.0*Y1(M,N))+3.0*(RTD1*Y21+RTD2*Y11)	GRAD 132
1		GRAD 133
Y122=	RD12*(Y2(M-2,N-2)-4.0*(Y2(M-1,N-2)+Y2(M-2,N-1)-4.0*Y2(M-1,N-1))-9.0*Y2(M,N))+3.0*(RTD1*Y22+RTD2*Y12)	GRAD 134
1		GRAD 135
Y123=	RD12*(Y3(M-2,N-2)-4.0*(Y3(M-1,N-2)+Y3(M-2,N-1)-4.0*Y3(M-1,N-1))-9.0*Y3(M,N))+3.0*(RTD1*Y23+RTD2*Y13)	GRAD 136
1		GRAD 137
U121=	RD12*(U1(M-2,N-2)-4.0*(U1(M-1,N-2)+U1(M-2,N-1)-4.0*U1(M-1,N-1))-9.0*U1(M,N))+3.0*(RTD1*U21+RTD2*U11)	GRAD 138
1		GRAD 139
U122=	RD12*(U2(M-2,N-2)-4.0*(U2(M-1,N-2)+U2(M-2,N-1)-4.0*U2(M-1,N-1))-9.0*U2(M,N))+3.0*(RTD1*U22+RTD2*U12)	GRAD 140
1		GRAD 141
U123=	RD12*(U3(M-2,N-2)-4.0*(U3(M-1,N-2)+U3(M-2,N-1)-4.0*U3(M-1,N-1))-9.0*U3(M,N))+3.0*(RTD1*U23+RTD2*U13)	GRAD 142
1		GRAD 143
7	RETURN	GRAD 144
	END	GRAD 145

	SUBROUTINE STRESS(MD,ND,KD)	****	1
C	CONSTITUTIVE RELATION--LINEARLY ELASTIC, OR ELASTIC-(PERFECTLY	STRE	2
C	PLASTIC OR -STRAIN HARDENING). OPTIONAL STRAIN RATE DEPENDENCE	STRE	3
	COMMON(USE MAIN)	STRE	4
	K=MD	STRE	5
	N=ND	STRE	6
	K=KD	STRE	7
	SS11=0.0	STRE	8
	SS12=0.0	STRE	9
	SS21=0.0	STRE	10
	SS22=0.0	STRE	11
	LMNK=0	STRE	12
	KN=(K-1)*NSFL	STRE	13
	ZETAK=2.0*ZETA(K)	STRE	14
	G11=A11-ZETAK*B11	STRE	15
	G12=A12-ZETAK*B12	STRE	16
	G22=A22-ZETAK*B22	STRE	17
	DG=G11*G22-G12**2	STRE	18
	SRG=SQRT(DG)	STRE	19
	RG=1.0/DG	STRE	20
	GR11=RG*G22	STRE	21
	GR12=-RG*G12	STRE	22
	GR22=RG*G11	STRE	23
	DEPS11=GR11*DEPS1(K)+GR12*DGAMMA(K)	STRE	24
	DEPS12=GR12*DEPS2(K)+GR11*DGAMMA(K)	STRE	25
	DEPS21=GR12*DEPS1(K)+GR22*DGAMMA(K)	STRE	26
	DEPS22=GR22*DEPS2(K)+GR12*DGAMMA(K)	STRE	27
	DSIG11=PRAT*(DEPS11+FNU*DEPS22)	STRE	28
	DSIG12=GTWO*DEPS12	STRE	29
	DSIG21=GTWO*DEPS21	STRE	30
	DSIG22=PRAT*(DEPS22+FNU*DEPS11)	STRE	31
	IF(ISR.GT.0)EPSDOT=SQRT(DEPS11*(DEPS11-DEPS22)+DEPS22**2	STRE	32
	1+3.0*DEPS12*DEPS21)/DELTAT	STRE	33
	DO J=1,NSFL	STRE	34
	KJ=N+J	STRE	35
	SIG11I=G11*SIG1(M,N,KJ)+G12*TAU(M,N,KJ)	STRE	36
	SIG12I=G12*SIG1(M,N,KJ)+G22*TAU(M,N,KJ)	STRE	37
	SIG21I=G12*SIG2(M,N,KJ)+G11*TAU(M,N,KJ)	STRE	38
	SIG22I=G22*SIG2(M,N,KJ)+G12*TAU(M,N,KJ)	STRE	39
	SIGYSQ=SIGZSQ(J)	STRE	40
	IF(ISR.GT.0)SIGYSQ=SIGYSQ*(1.0+(EPSDOT/DSR(J))**PSR(J))**2	STRE	41
	I=1	STRE	42
	LC=1	STRE	43
	SIG11L=SIG11I+DSIG11	STRE	44
	SIG12L=SIG12I+DSIG12	STRE	45
	SIG21L=SIG21I+DSIG21	STRE	46
	SIG22L=SIG22I+DSIG22	STRE	47
	IF(ISR.LT.0)GOTO 2	STRE	48
	PHIT*SIG11L*(SIG11L-SIG22L)+SIG22L**2+3.0*SIG12L*SIG21L-SIGYSQ	STRE	49
	IF(PHIT.LE.0.0)GOTO 2	STRE	50
	L=INT(YLDFAC*(SORT((PHIT+SIGYSQ)/SIGYSQ)-1.0))+1	STRE	51
100	SIG11=SIG11I	STRE	52
	SIG12=SIG12I	STRE	53
	SIG21=SIG21I	STRE	54
	SIG22=SIG22I	STRE	55
	IF(L.EQ.1)GOTO 3	STRE	56
	LC=1	STRE	57
	FLOATL=1.0/FLOAT(L)	STRE	58
	DSG11L=DSIG11*FLOATL	STRE	59
	DSG12L=DSIG12*FLOATL	STRE	60

	DSG21L = DSIG21*FLOATL	STRE 61
	DSG22L = DSIG22*FLOATL	STRE 62
101	SIG11L= SIG11+DSG11L	STRE 63
	SIG12L= SIG12+DSG12L	STRE 64
	SIG21L= SIG21+DSG21L	STRE 65
	SIG22L= SIG22+DSG22L	STRE 66
	PHIT=SIG11L*(SIG11L-SIG22L)+SIG22L**2+3.0*SIG12L*SIG21L-SIGY5Q	STRE 67
	IF(PHIT .GT. 0.0)GOTO 3	STRE 68
C	ELASTIC	STRE 69
2	SIG11 = SIG11L	STRE 70
	SIG12 = SIG12L	STRE 71
	SIG21 = SIG21L	STRE 72
	SIG22 = SIG22L	STRE 73
	GO TO 9	STRE 74
C	PLASTIC	STRE 75
3	SIG11D = (2.0-FNU)*SIG11-(1.0-2.0*FNU)*SIG22	STRE 76
	SIG12D = 3.0*(1.0-FNU)*SIG12	STRE 77
	SIG21D = 3.0*(1.0-FNU)*SIG21	STRE 78
	SIG22D = (2.0-FNU)*SIG22-(1.0-2.0*FNU)*SIG11	STRE 79
	AA = SIG11D**2-SIG11D*SIG22D+SIG22D**2+3.0*SIG12D*SIG21D	STRE 80
	B = -(SIG11L*(2.0*SIG11D-SIG22D)+SIG22L*(2.0*SIG22D-SIG11D)+3.0*(S	STRE 81
	IG12L*SIG21D+SIG21L*SIG12D))	STRE 82
	D=B**2-4.*AA*PHIT	STRE 83
	IF(L .GT. LMNK) LMNK=L	STRE 84
	IF(AA)8,16,4	STRE 85
8	WRITE(6,10)	STRE 86
10	FORMAT(1H ,4X,14HAA NEGATIVE AT)	STRE 87
	GOTO 12	STRE 88
4	IF(D .LT. 0.0 .OR. B .GT. 0.0)GOTO 16	STRE 89
	TAMBDA=(-B-SQRT(D))/(2.*AA)	STRE 90
	SIG11 = SIG11L-TAMBDA*SIG11D	STRE 91
	SIG12 = SIG12L-TAMBDA*SIG12D	STRE 92
	SIG21 = SIG21L-TAMBDA*SIG21D	STRE 93
	SIG22 = SIG22L-TAMBDA*SIG22D	STRE 94
9	LC=LC+1	STRE 95
	IF(LC-L)101,101,102	STRE 96
16	L=L+1	STRE 97
	IF(L-100)100,100,103	STRE 98
103	WRITE(6,104)	STRE 99
104	FORMAT(1H ,4X,36HSTRESS CALCULATION UNSATISFACTORY AT)	STRE 100
12	WRITE(6,105)NCYCLE,M,N,K,L,LC	STRE 101
105	FORMAT(1H ,9X,9HTIME STEP,14,5X,2HM=,12,5X,2HN=,12,5X,2HK=,12,>X,2STRE .02	
	1HL=,13,5X,3HLC=,13)	STRE 103
	WRITE(6,106) EPSL1(M,N),EPSL2(M,N),GAMMAL(M,N),EPSU1(M,N),	STRE 104
	1EPSU2(M,N),GAMMAU(M,N),DEPS1(K),DEPS2(K),DGAMMA(K),	STRE 105
2	SIG1(M,N,KJ),SIG2(M,N,KJ),TAU(M,N,KJ),DSIG11,	STRE 106
2DS	IG12,DSIG21,DSIG22,SIG11,SIG12,SIG21,SIG22,SIG11L,SIG12L,SIG21L,STRE 107	
3S	IG22L,SIG11D,SIG12D,SIG21D,SIG22D,AA,B,PHIT	STRE 108
106	FORMAT(1H ,9X,7HEPSL1 =E15.8,3X,7HEPSL2 =E15.8,3X,8HGAMMAL =E15.8,STRE 109	
	1/1H ,9X,7HEPSU1 =E15.8,3X,7HEPSU2 =E15.8,3X,8HGAMMAU =E15.8,	STRE 110
1	/1H ,9X,7HDEPS1 =,E15.8,3X,7HDEPS2 =,E15.8,3X,7HDGAMMA=,E15.8,/1STRE 111	
2H	,9X,7HSIG1 =,E15.8,3X,7HSIG2 =,E15.8,3X,7HTAU =,E15.8,/1H ,9STRE 112	
3X,	7HDSIG11=,E15.8,3X,7HDSIG12=,E15.8,3X,7HDSIG21=,E15.8,3X,7HDSIG2STRE 113	
42=,	E15.8,/1H ,9X,7HSIG11 =,E15.8,3X,7HSIG12 =,E15.8,3X,7HSIG21 =,ESTRE 114	
515.	8,3X,7HSIG22 =,E15.8,/1H ,9X,7HSIG11L=,E15.8,3X,7HSIG12L=,E15.8STRE 115	
6,	3X,7HSIG21L=,E15.8,3X,7HSIG22L=,E15.8,/1H ,9X,7HSIG11D=,E15.8,3X,STRE 116	
77H	SIG12D=,E15.8,3X,7HSIG21D=,E15.8,3X,7HSIG22D=,E15.8,/1H ,9X,7HAASTRE 117	
8	=,E15.8,3X,7H8 =,E15.8,3X,7HPHIT =,E15.8)	STRE 118
	NC3UP(NN3D)=NCYCLE	STRE 119
	CALL PDATA (2)	STRE 120

CALL PDATA (3)	STRE 121
CALL PDATA (4)	STRE 122
CALL EXIT	STRE 123
102 SS11=SS11+SIG11*WT(J)	STRE 124
SS12=SS12+SIG12*WT(J)	STRE 125
SS21=SS21+SIG21*WT(J)	STRE 126
SS22=SS22+SIG22*WT(J)	STRE 127
SIG1(M,N,KJ)=GR11*SIG11+GR12*SIG12	STRE 128
SIG2(M,N,KJ)=GR22*SIG22+GR12*SIG21	STRE 129
TAU (M,N,KJ)=GR12*SIG22+GR11*SIG21	STRE 130
003 CONTINUE	STRE 131
IF(M .EQ. 2 .OR. M .EQ. M1)SRG=0.5*CRG	STRE 132
IF(N .EQ. 1 .OR. N .EQ. N1)SRG=0.5*SRG	STRE 133
STREN=STREN+((SS11+SS22)*2-(1.+FNU)*2.*(SS11*SS22-SS12*SS21))*SRG	STRE 134
SS1M(K)=GR11*SS11+GR12*SS12	STRE 135
SS2M(K)=GR22*SS22+GR12*SS21	STRE 136
STM(K)=GR12*SS22+GR11*SS21	STRE 137
LMAT(M,N,K) = LMNK	STRE 138
RETURN	STRE 139
END	STRE 140

	SUBROUTINE RESULT(MD,ND)	****	1
	COMMON (USE MAIN)	RESU	2
C	THE CALCULATION OF THE STRESS AND MOMENT RESULTANTS FOLLOW	RESU	3
	M=MD	RESU	4
	N=ND	RESU	5
	TB=TA*SRA	RESU	6
	SUMA11 = SS1MN(1)	RESU	7
	SUMA22 = SS2MN(1)	RESU	8
	SUMA12 = STMN(1)	RESU	9
	SUMB11 = SS1MN(1)*ZETA(1)	RESU	10
	SUMB22 = SS2MN(1)*ZETA(1)	RESU	11
	SUMB12 = STMN(1)*ZETA(1)	RESU	12
	SUMC11 = SS1MN(1)*ZETASQ(1)	RESU	13
	SUMC22 = SS2MN(1)*ZETASQ(1)	RESU	14
	SUMC12 = STMN(1)*ZETASQ(1)	RESU	15
	IF(LAYER .EQ. 1)GOTO 5	RESU	16
	DO 4 K=2,LAYER	RESU	17
	SUMA11 = SUMA11 + SS1MN(K)	RESU	18
	SUMA22 = SUMA22 + SS2MN(K)	RESU	19
	SUMA12 = SUMA12 + STMN(K)	RESU	20
	SUMB11 = SUMB11 + SS1MN(K)*ZETA(K)	RESU	21
	SUMB22 = SUMB22 + SS2MN(K)*ZETA(K)	RESU	22
	SUMB12 = SUMB12 + STMN(K)*ZETA(K)	RESU	23
	SUMC11 = SUMC11 + SS1MN(K)*ZETASQ(K)	RESU	24
	SUMC22 = SUMC22 + SS2MN(K)*ZETASQ(K)	RESU	25
4	SUMC12 = SUMC12 + STMN(K)*ZETASQ(K)	RESU	26
5	CONTINUE	RESU	27
	Q11=SUMA11-BT*SUMB11	RESU	28
	Q22=SUMA22-BT*SUMB22	RESU	29
	Q12=SUMA12-BT*SUMB12	RESU	30
	F11=SUMB11-(BT+BM11)*SUMC11-BM12*SUMC12	RESU	31
	F22=SUMB22-(BT+BM22)*SUMC22-BM21*SUMC12	RESU	32
	F12=SUMB12-1.5*BT*SUMC12-0.5*(BM21*SUMC11+BM12*SUMC22)	RESU	33
	IF(1BCE4 .NE. 3)GOTO 1	RESU	34
	IF(N .NE. 1)GOTO 1	RESU	35
	F22=0.0	RESU	36
1	IF(1BCE2 .NE. 3)GOTO 2	RESU	37
	IF(N .NE. NN)GOTO 2	RESU	38
	F22=0.0	RESU	39
2	IF(1BCE3 .NE. 3)GOTO 3	RESU	40
	IF(M .NE. MM)GOTO 3	RESU	41
	F11=0.0	RESU	42
3	CSM1=CS111*F11+CS122*F22+2.0*CS112*F12	RESU	43
	CSM2=CS211*F11+CS222*F22+2.0*CS212*F12	RESU	44
	FNT11=Q11*Y11+Q12*Y21+CSM1*SN1(M,N)	RESU	45
	FNT12=Q11*Y12+Q12*Y22+CSM1*SN2(M,N)	RESU	46
	FNT13=Q11*Y13+Q12*Y23+CSM1*SN3(M,N)	RESU	47
	FNT21=Q12*Y11+Q22*Y21+CSM2*SN1(M,N)	RESU	48
	FNT22=Q12*Y12+Q22*Y22+CSM2*SN2(M,N)	RESU	49
	FNT23=Q12*Y13+Q22*Y23+CSM2*SN3(M,N)	RESU	50
	FM11(M,N) = TB*F11	RESU	51
	FM22(M,N) = TB*F22	RESU	52
	FM12(M,N) = TB*F12	RESU	53
	FN11(M,N) = TB*FNT11	RESU	54
	FN21(M,N) = TB*FNT21	RESU	55
	FN13(M,N) = TB*FNT13	RESU	56
	FN12(M,N) = TB*FNT12	RESU	57
	FN22(M,N) = TB*FNT22	RESU	58
	FN23(M,N) = TB*FNT23	RESU	59
	RETURN	RESU	60

END

RESU 61

	SUBROUTINE MOTION	****	1
	COMMON(USE MAIN)	MOTI	2
C	VF1=0.0	MOTI	3
	VF2=0.0	MOTI	4
	VF3=0.0	MOTI	5
	IF(LOAD) 10,30,10	MOTI	6
10	ENS=0.0	MOTI	7
	CALL PWORK	MOTI	8
C		MOTI	9
		MOTI	10
30	DO 130 M=2,MS	MOTI	11
	DO 130 N=2,NS	MOTI	12
	VM1=RD11*(FM11(M+1,N)*SN1(M+1,N)-2.0*FM11(M,N)*SN1(M,N)	MOTI	13
1	+FM11(M-1,N)*SN1(M-1,N))+ TRD *(FM12(M+1,N+1)*SN1	MOTI	14
2	(M+1,N+1)-FM12(M+1,N-1)*SN1(M+1,N-1)-FM12(M-1,N+1)*SN1(M-1,N+1)	MOTI	15
3	+FM12(M-1,N-1)*SN1(M-1,N-1))+RD22*(FM22(M,N+1)*SN1(M,N+1)	MOTI	16
4	-2.0*FM22(M,N)*SN1(M,N)+FM22(M,N-1)*SN1(M,N-1))	MOTI	17
	VM2=RD11*(FM11(M+1,N)*SN2(M+1,N)-2.0*FM11(M,N)*SN2(M,N)+FM11(M-1,N)	MOTI	18
1	*SN2(M-1,N))+ TRD *(FM12(M+1,N+1)*SN2(M+1,N+1)-FM12(M+1,N-1)	MOTI	19
2	*SN2(M+1,N-1)-FM12(M-1,N+1)*SN2(M-1,N+1)+FM12(M-1,N-1)*SN2(M	MOTI	20
3	-1,N-1))+RD22*(FM22(M,N+1)*SN2(M,N+1)-2.0*FM22(M,N)*SN2(M,N)+FM	MOTI	21
4	M22(M,N-1)*SN2(M,N-1))	MOTI	22
	VM3=RD11*(FM11(M+1,N)*SN3(M+1,N)-2.0*FM11(M,N)*SN3(M,N)+FM11(M-1,N)	MOTI	23
1	*SN3(M-1,N))+ TRD *(FM12(M+1,N+1)*SN3(M+1,N+1)-FM12(M+1,N-1)	MOTI	24
2	*SN3(M+1,N-1)-FM12(M-1,N+1)*SN3(M-1,N+1)+FM12(M-1,N-1)*SN3	MOTI	25
3	(M-1,N-1))+RD22*(FM22(M,N+1)*SN3(M,N+1)-2.0*FM22(M,N)*SN3(M,N)	MOTI	26
4	+FM22(M,N-1)*SN3(M,N-1))	MOTI	27
	VN1=RTD1*(FN11(M+1,N)-FN11(M-1,N))+RTD2*(FN21(M,N+1)-FN21(M,N-1))	MOTI	28
	VN2=RTD1*(FN12(M+1,N)-FN12(M-1,N))+RTD2*(FN22(M,N+1)-FN22(M,N-1))	MOTI	29
	VN3=RTD1*(FN13(M+1,N)-FN13(M-1,N))+RTD2*(FN23(M,N+1)-FN23(M,N-1))	MOTI	30
	IF(LOAD) 40,50,40	MOTI	31
40	VF1=-SN1(M,N)*P(M,N)	MOTI	32
	VF2=-SN2(M,N)*P(M,N)	MOTI	33
	VF3=-SN3(M,N)*P(M,N)	MOTI	34
C		MOTI	35
		MOTI	36
50	U1R=U1(M,N)	MOTI	37
	U2R=U2(M,N)	MOTI	38
	U3R=U3(M,N)	MOTI	39
	U1S=U1R+(VM1+VN1+VF1)*TEMP(M,N)	MOTI	40
	U2S=U2R+(VM2+VN2+VF2)*TEMP(M,N)	MOTI	41
	U3S=U3R+(VM3+VN3+VF3)*TEMP(M,N)	MOTI	42
	IF(TDAMP .EQ. 0.0)GOTO 115	MOTI	43
C		MOTI	44
		MOTI	45
		MOTI	46
115	U1(M,N)=U1S	MOTI	47
	U2(M,N)=U2S	MOTI	48
	U3(M,N)=U3S	MOTI	49
130	CONTINUE	MOTI	50
	CALL BOUNDU	MOTI	51
	CALL KINET	MOTI	52
	IF(LOAD) 65,75,65	MOTI	53
65	CALL PWORK	MOTI	54
	EN=0.5*CA*(ENS+ENR)	MOTI	55
	ENR=ENS	MOTI	56
	IF(1BCE2 .NE. 2)GOTO 74	MOTI	57
	EN=2.0*EN	MOTI	58
74	TNRG=TNRG+EN	MOTI	59
75	PLAST=TNRG-CINET-STREN-TDAMP	MOTI	60

C	IF(NCYCLE .NE. NCYCH(NNN)) GOTO 140	MOTI 61
	WRITE(6,99991) NCYCLE,TIME,CINET,STREN,PLAST,TNRG	MOTI 62
	NNN=MNN+1	MOTI 63
140	RETURN	MOTI 64
		MOTI 65
C		MOTI 66
99991	FORMAT(/10H TIME STEP,15,3X,5HTIME=,E16.8,3X,8HKINETIC=,E15.8,3X,	MOTI 67
	10HELASTIC=,E15.8,3X,8HPLASTIC=,E15.8/14H TOTAL ENERGY=,E15.8)	MOTI 68
	END	MOTI 69

	SUBROUTINE WRTAPE (KEY)	****	1
C	WRITE + READ CONTINUATION RUN TAPE (KEY=1, WRITE KEY=2, READ)	WRTA	2
	COMMON(USE MAIN)	WRTA	3
	GOTO(10,20),KEY	WRTA	4
10	WRITE(1) NCYCLE, TIME, CINES, CINEP, TNRG, TDAMP, D1, D2, D3, ENR	WRTA	5
	WRITE(1) ((U1(M,N), U2(M,N), U3(M,N), Y1(M,N), Y2(M,N), Y3(M,N),	WRTA	6
	1EPSL1(M,N), EPSL2(M,N), GAMMAL(M,N), EPSU1(M,N), EPSU2(M,N),	WRTA	7
	2GAMMAU(M,N), P(M,N), N=1, NN), M=1, MM)	WRTA	8
	WRITE(1) ((SIG1(M,N,K), SIG2(M,N,K), TAU(M,N,K),	WRTA	9
	1K=1, KJMAX), N=1, NN), M=1, MM)	WRTA	10
	WRITE(1) ((SN1(M,N), SN2(M,N), SN3(M,N), TEMP(M,N), N=1, NN), M=1, MM)	WRTA	11
	WRITE(1) (G111(N), G122(N), G112(N), ASA(N), BSA(N), CSA(N), ASB(N),	WRTA	12
	1BSB(N), CSB(N), N=1, NSTRN)	WRTA	13
	WRITE(6,100) NCYCLE, TIME	WRTA	14
	RETURN	WRTA	15
C		WRTA	16
20	REWIND 1	WRTA	17
25	READ (1) NCYCLE, TIME, CINES, CINEP, TNRG, TDAMP, D1, D2, D3, ENR	WRTA	18
	READ (1) ((U1(M,N), U2(M,N), U3(M,N), Y1(M,N), Y2(M,N), Y3(M,N),	WRTA	19
	1EPSL1(M,N), EPSL2(M,N), GAMMAL(M,N), EPSU1(M,N), EPSU2(M,N),	WRTA	20
	2GAMMAU(M,N), P(M,N), N=1, NN), M=1, MM)	WRTA	21
	READ (1) ((SIG1(M,N,K), SIG2(M,N,K), TAU(M,N,K),	WRTA	22
	1K=1, KJMAX), N=1, NN), M=1, MM)	WRTA	23
	READ (1) ((SN1(M,N), SN2(M,N), SN3(M,N), TEMP(M,N), N=1, NN), M=1, MM)	WRTA	24
	READ (1) (G111(N), G122(N), G112(N), ASA(N), BSA(N), CSA(N), ASB(N),	WRTA	25
	1BSB(N), CSB(N), N=1, NSTRN)	WRTA	26
	IF(NCYCLE.NE.NCONT)GOTO 25	WRTA	27
	WRITE(6,200) NCYCLE, TIME	WRTA	28
	RETURN	WRTA	29
C		WRTA	30
100	FORMAT(//24H TAPE 1 WRITTEN, NCYCLE=I4,7H TIME=E15.8)	WRTA	31
200	FORMAT(//39H INFORMATION READ FROM TAPE 1 FOR CYCLE I4,7H TIME=E15.8)	WRTA	32
	1.8)	WRTA	33
	END	WRTA	34

	SUBROUTINE STRAIN	****	1
C		STRA	2
C	PRINT STRAINS ON INNER OR OUTER SURFACE	STRA	3
C		STRA	4
	DIMENSION EPSANG(6),EPSANB(6)	STRA	5
	COMMON (USE MAIN)	STRA	6
	DATA P1/3.141592653589793/	STRA	7
	IF(NCYCLE ² .GT. 0)GOTO 25	STRA	8
C	----- INITIAL ENTRY -----	STRA	9
	DO 20 I=1,NSTRN	STRA	10
	A11=DM2(I)*(DN2(I)*A111(I)+DN1(I)*A112(I))	STRA	11
	1 +DN1(I)*(DN2(I)*A113(I)+DN1(I)*A114(I))	STRA	12
	A12=DM2(I)*(DN2(I)*A121(I)+DN1(I)*A122(I))	STRA	13
	1 +DN1(I)*(DN2(I)*A123(I)+DN1(I)*A124(I))	STRA	14
	A22=DM2(I)*(DN2(I)*A221(I)+DN1(I)*A222(I))	STRA	15
	1 +DN1(I)*(DN2(I)*A223(I)+DN1(I)*A224(I))	STRA	16
	B11=DM2(I)*(DN2(I)*B111(I)+DN1(I)*B112(I))	STRA	17
	1 +DN1(I)*(DN2(I)*B113(I)+DN1(I)*B114(I))	STRA	18
	B12=DM2(I)*(DN2(I)*B121(I)+DN1(I)*B122(I))	STRA	19
	1 +DN1(I)*(DN2(I)*B123(I)+DN1(I)*B124(I))	STRA	20
	B22=DM2(I)*(DN2(I)*B221(I)+DN1(I)*B222(I))	STRA	21
	1 +DN1(I)*(DN2(I)*B223(I)+DN1(I)*B224(I))	STRA	22
	IF(NETAG(I).EQ.1)GOTO 10	STRA	23
	G11 = A11-2.0*H*B11	STRA	24
	G12 = A12-2.0*H*B12	STRA	25
	G22 = A22-2.0*H*B22	STRA	26
	GOTO 15	STRA	27
10	G11 = A11+2.0*H*B11	STRA	28
	G12 = A12+2.0*H*B12	STRA	29
	G22 = A22+2.0*H*B22	STRA	30
15	G11(I)=1.0/G11	STRA	31
	G12(I)=1.0/G12	STRA	32
	GR=SQRT(G11(I)*G12(I))	STRA	33
	DELTA=G12*GR	STRA	34
	G12(I)=2.0*GR	STRA	35
	SRDEL=1.0/SQRT(1.0-DELTA**2)	STRA	36
	ANGEL=ANGLE(I)*PI/180.0	STRA	37
	SA=SRDEL*SIN(ANGEL)	STRA	38
	SB=COS(ANGEL)-DELTA*SA	STRA	39
	ASA(I)=2.0*SA**2	STRA	40
	BSA(I)=2.0*SB**2	STRA	41
	CSA(I)=2.0*SA*SB	STRA	42
	ANGEL=ANGLB(I)*PI/180.0	STRA	43
	SA=SRDEL*SIN(ANGEL)	STRA	44
	SB=COS(ANGEL)-DELTA*SA	STRA	45
	ASB(I)=2.0*SA**2	STRA	46
	BSB(I)=2.0*SB**2	STRA	47
	CSB(I)=2.0*SA*SB	STRA	48
20	CONTINUE	STRA	49
	GOTO 71	STRA	50
C	-----	STRA	51
25	LINK=1	STRA	52
C	CHECK FOR SURFACE STRAIN PRINT	STRA	53
	IF(NCYCLE-NPRINT) 40,30,30	STRA	54
30	NPRINT=NPRINT+NDEL*	STRA	55
	LINK=2	STRA	56
40	DO 46 I=1,NSTRN	STRA	57
	I1=N11(I)	STRA	58
	J1=N11(I)	STRA	59
	I2=N12(I)	STRA	60

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J2=N12(I)
IF(NETAG(I).EQ.1)GOTO 44
EPSR1=(DN2(I)*DN2(I)*EPSU1(I1,J1)+DN1(I)*EPSU1(I1,J2))
1 +DM1(I)*(DN2(I)*EPSU1(I2,J1)+DN1(I)*EPSU1(I2,J2))*G11(I)
EPSR2=(DN2(I)*DN2(I)*EPSU2(I1,J1)+DN1(I)*EPSU2(I1,J2))
1 +DM1(I)*(DN2(I)*EPSU2(I2,J1)+DN1(I)*EPSU2(I2,J2))*G12(I)
GAMMAR=(DN2(I)*DN2(I)*GAMMAU(I1,J1)+DN1(I)*GAMMAU(I1,J2))
1 +DM1(I)*(DN2(I)*GAMMAU(I2,J1)+DN1(I)*GAMMAU(I2,J2))*G12(I)
GOTO 45
44 EPSR1=(DN2(I)*DN2(I)*EPSL1(I1,J1)+DN1(I)*EPSL1(I1,J2))
1 +DM1(I)*(DN2(I)*EPSL1(I2,J1)+DN1(I)*EPSL1(I2,J2))*G11(I)
EPSR2=(DN2(I)*DN2(I)*EPSL2(I1,J1)+DN1(I)*EPSL2(I1,J2))
1 +DM1(I)*(DN2(I)*EPSL2(I2,J1)+DN1(I)*EPSL2(I2,J2))*G12(I)
GAMMAR=(DN2(I)*DN2(I)*GAMMAL(I1,J1)+DN1(I)*GAMMAL(I1,J2))
1 +DM1(I)*(DN2(I)*GAMMAL(I2,J1)+DN1(I)*GAMMAL(I2,J2))*G12(I)
45 EPSS1(I)=SQRT(1.0+2.0*EPSR1)-1.0
EPSS2(I)=SQRT(1.0+2.0*EPSR2)-1.0
EPSANG(I)=SQRT(1.0+BSA(I)*EPSR1+ASA(I)*EPSR2+CSA(I)*GAMMAR)-1.0
EPSANB(I)=SQRT(1.0+BSB(I)*EPSR1+ASB(I)*EPSR2+CSB(I)*GAMMAR)-1.0
46 CONTINUE
C COMPONENTS OF VECTOR DISPLACEMENT
D1=D1+(QN2*(QN2*U1(MQ1,NQ1)+QN1*U1(MQ1,NQ2))
1 +QM1*(QN2*U1(MQ2,NQ1)+QN1*U1(MQ2,NQ2)))
D2=D2+(QN2*(QN2*U2(MQ1,NQ1)+QN1*U2(MQ1,NQ2))
1 +QM1*(QN2*U2(MQ2,NQ1)+QN1*U2(MQ2,NQ2)))
D3=D3+(QN2*(QN2*U3(MQ1,NQ1)+QN1*U3(MQ1,NQ2))
1 +QM1*(QN2*U3(MQ2,NQ1)+QN1*U3(MQ2,NQ2)))
GOTO (71,50),LINK
50 WRITE(6,60) NCYCLE,TIME
DO 70 I=1,NSTRN
ALFN=5HINNER
IF(NETAG(I).EQ.1)GOTO 67
ALFN=5HOUTER
67 WRITE(6,65) ETAG1(I),ETAG2(I),PM(I),PN(I),ALFN,EPSS1(I),EPSS2(I),
1 ANGLE(I),EPSANG(I),ANGLB(I),EPSANB(I)
70 CONTINUE
71 RETURN
C
60 FORMAT(//10H TIME STEP,15,3X,5HTIME=,E16.8//14X,15HSURFACE STRAINS,STRA
1,37X,19HSTRAIN GAGE READING//2X,4HETA1,4X,4HETA2,6X,1HM,7X,1HN,5X,STRA
24HFACE,8X,7HANGLE 0,10X,8HANGLE 90,6X,5HANGLE,18X,5HANGLE/)
65 FORMAT(1H ,F7.3,1X,F7.3,2X,F7.3,1X,F7.3,2X,A5,1X,2(2X,E15.8),
12(2X,F6.2,2X,E15.8))
END
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STRA 104

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	SUBROUTINE BOUNDU	****	1
	COMPUTATION OF U1,U2,U3 AT THE BOUNDARY	BOUN	2
C	COMMON (USE MAIN)	BOUN	3
	IF (IBCE4 .EQ. 3) GOTO 121	BOUN	4
	DO 120 M=2,MB1	BOUN	5
	DUSN=(U1(M,2)-0.25*U1(M,3))*SN1(M,1)	BOUN	6
	1 +(U2(M,2)-0.25*U2(M,3))*SN2(M,1)	BOUN	7
	2 +(U3(M,2)-0.25*U3(M,3))*SN3(M,1)	BOUN	8
	U1(M,2)=U1(M,2)-SN1(M,1)*DUSN	BOUN	9
	U2(M,2)=U2(M,2)-SN2(M,1)*DUSN	BOUN	10
120	U3(M,2)=U3(M,2)-SN3(M,1)*DUSN	BOUN	11
121	IF (IBCE2 .NE. 1) GOTO 123	BOUN	12
	DO 122 M=2,MB1	BOUN	13
	DUSN=(U1(M,NS)-0.25*U1(M,NR))*SN1(M,NN)	BOUN	14
	1 +(U2(M,NS)-0.25*U2(M,NR))*SN2(M,NN)	BOUN	15
	2 +(U3(M,NS)-0.25*U3(M,NR))*SN3(M,NN)	BOUN	16
	U1(M,NS)=U1(M,NS)-SN1(M,NN)*DUSN	BOUN	17
	U2(M,NS)=U2(M,NS)-SN2(M,NN)*DUSN	BOUN	18
122	U3(M,NS)=U3(M,NS)-SN3(M,NN)*DUSN	BOUN	19
123	IF (IBCE3 .NE. 1) GOTO 50	BOUN	20
	DO 124 N=NB1,NB2	BOUN	21
	DUSN=(U1(MS,N)-0.25*U1(MR,N))*SN1(MM,N)	BOUN	22
	1 +(U2(MS,N)-0.25*U2(MR,N))*SN2(MM,N)	BOUN	23
	2 +(U3(MS,N)-0.25*U3(MR,N))*SN3(MM,N)	BOUN	24
	U1(MS,N)=U1(MS,N)-SN1(MM,N)*DUSN	BOUN	25
	U2(MS,N)=U2(MS,N)-SN2(MM,N)*DUSN	BOUN	26
124	U3(MS,N)=U3(MS,N)-SN3(MM,N)*DUSN	BOUN	27
	IF (IBCE4 .NE. 1) GOTO 125	BOUN	28
	DUSN1=(U1(MS,2)-0.25*U1(MS,3))*SN1(MS,1)	BOUN	29
	1 +(U2(MS,2)-0.25*U2(MS,3))*SN2(MS,1)	BOUN	30
	2 +(U3(MS,2)-0.25*U3(MS,3))*SN3(MS,1)	BOUN	31
	DUSN2=(U1(MS,2)-0.25*U1(MR,2))*SN1(MM,2)	BOUN	32
	1 +(U2(MS,2)-0.25*U2(MR,2))*SN2(MM,2)	BOUN	33
	2 +(U3(MS,2)-0.25*U3(MR,2))*SN3(MM,2)	BOUN	34
	U1(MS,2)=U1(MS,2)-0.5*(SN1(MS,1)*DUSN1+SN1(MM,2)*DUSN2)	BOUN	35
	U2(MS,2)=U2(MS,2)-0.5*(SN2(MS,1)*DUSN1+SN2(MM,2)*DUSN2)	BOUN	36
	U3(MS,2)=U3(MS,2)-0.5*(SN3(MS,1)*DUSN1+SN3(MM,2)*DUSN2)	BOUN	37
125	IF (IBCE2 .NE. 1) GOTO 50	BOUN	38
	DUSN2=(U1(MS,NS)-0.25*U1(MR,NS))*SN1(MM,NS)	BOUN	39
	1 +(U2(MS,NS)-0.25*U2(MR,NS))*SN2(MM,NS)	BOUN	40
	2 +(U3(MS,NS)-0.25*U3(MR,NS))*SN3(MM,NS)	BOUN	41
	DUSN1=(U1(MS,NS)-0.25*U1(MS,NR))*SN1(MS,NN)	BOUN	42
	1 +(U2(MS,NS)-0.25*U2(MS,NR))*SN2(MS,NN)	BOUN	43
	2 +(U3(MS,NS)-0.25*U3(MS,NR))*SN3(MS,NN)	BOUN	44
	U1(MS,NS)=U1(MS,NS)-0.5*(SN1(MS,NN)*DUSN1+SN1(MM,NS)*DUSN2)	BOUN	45
	U2(MS,NS)=U2(MS,NS)-0.5*(SN2(MS,NN)*DUSN1+SN2(MM,NS)*DUSN2)	BOUN	46
	U3(MS,NS)=U3(MS,NS)-0.5*(SN3(MS,NN)*DUSN1+SN3(MM,NS)*DUSN2)	BOUN	47
C	SET SYMMETRY BOUNDARY CONDITIONS FOR EDGE1,EDGE2,EDGE3	BOUN	48
50	IF (IBCE2 .NE. 2) GOTO 30	BOUN	49
	DO 25 M=2,M1	BOUN	50
	U1(M,NN)=U1(M,NR)	BOUN	51
	U2(M,NN)=-U2(M,NR)	BOUN	52
	U3(M,NN)=U3(M,NR)	BOUN	53
	U2(M,NS)=0.0	BOUN	54
25	CONTINUE	BOUN	55
30	DO 5 N=1,NN	BOUN	56
	U1(2,N)=0.	BOUN	57
	U1(1,N)=-U1(3,N)	BOUN	58
	U2(1,N)=U2(3,N)	BOUN	59
	U3(1,N)=U3(3,N)	BOUN	60

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IF (IBCE3 .NE. 2) GOTO 5
U1(MS,N) = 0.0
U1(MN,N) = -U1(MR,N)
U2(MN,N) = U2(MR,N)
U3(MN,N) = U3(MR,N)
5 CONTINUE
RETURN
END
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BOUN 61
BOUN 62
BOUN 63
BOUN 64
BOUN 65
BOUN 66
BOUN 67
BOUN 68
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SUBROUTINE ABINIT(M,N)	****	1
COMMON (USE MAIN)	ABIN	2
DO 200 I=1,NSTRM	ABIN	3
IF(M.EQ.MI1(I) .AND. (N.EQ.NI1(I) .OR. N.EQ.NI2(I)))GOTO 205	ABIN	4
IF(M.EQ.MI2(I) .AND. (N.EQ.NI1(I) .OR. N.EQ.NI2(I)))GOTO 220	ABIN	5
GOTO 200	ABIN	6
205 IF(N .EQ. NI2(I))GOTO 210	ABIN	7
A11(I)=A11	ABIN	8
A12(I)=A12	ABIN	9
A22(I)=A22	ABIN	10
B11(I)=B11	ABIN	11
B12(I)=B12	ABIN	12
B22(I)=B22	ABIN	13
GOTO 200	ABIN	14
210 A12(I)=A11	ABIN	15
A122(I)=A12	ABIN	16
A222(I)=A22	ABIN	17
B12(I)=B11	ABIN	18
B122(I)=B12	ABIN	19
B222(I)=B22	ABIN	20
GOTO 200	ABIN	21
220 IF(N .EQ. NI2(I))GOTO 225	ABIN	22
A113(I)=A11	ABIN	23
A123(I)=A12	ABIN	24
A223(I)=A22	ABIN	25
B113(I)=B11	ABIN	26
B123(I)=B12	ABIN	27
B223(I)=B22	ABIN	28
GOTO 200	ABIN	29
225 A114(I)=A11	ABIN	30
A124(I)=A12	ABIN	31
A224(I)=A22	ABIN	32
B114(I)=B11	ABIN	33
B124(I)=B12	ABIN	34
B224(I)=B22	ABIN	35
200 CONTINUE	ABIN	36
RETURN	ABIN	37
END	ABIN	38

SUBROUTINE SYMTRY	****	1
COMMON (USE MAIN)	SYMT	2
DO 10 N=1,N1	SYMT	3
SN1(1,N)=-SN1(3,N)	SYMT	4
SN2(1,N)=SN2(3,N)	SYMT	5
SN3(1,N)=SN3(3,N)	SYMT	6
FM1(1,N)=FM1(3,N)	SYMT	7
FM2(1,N)=FM2(3,N)	SYMT	8
FM12(1,N)=-FM12(3,N)	SYMT	9
FN1(1,N)=FN1(3,N)	SYMT	10
FN2(1,N)=-FN2(3,N)	SYMT	11
FN13(1,N)=-FN13(3,N)	SYMT	12
FN12(1,N)=-FN12(3,N)	SYMT	13
FN22(1,N)=FN22(3,N)	SYMT	14
FN23(1,N)=FN23(3,N)	SYMT	15
10 CONTINUE	SYMT	16
IF(10CE3 .NE. 2)GOTO 30	SYMT	17
DO 20 N=1,N1	SYMT	18
SN1(M,N)=-SN1(M,N)	SYMT	19
SN2(M,N)=SN2(M,N)	SYMT	20
SN3(M,N)=SN3(M,N)	SYMT	21
FM1(M,N)=FM1(M,N)	SYMT	22
FM2(M,N)=FM2(M,N)	SYMT	23
FM12(M,N)=-FM12(M,N)	SYMT	24
FN1(M,N)=FN1(M,N)	SYMT	25
FN2(M,N)=-FN2(M,N)	SYMT	26
FN13(M,N)=-FN13(M,N)	SYMT	27
FN12(M,N)=-FN12(M,N)	SYMT	28
FN22(M,N)=FN22(M,N)	SYMT	29
FN23(M,N)=FN23(M,N)	SYMT	30
20 CONTINUE	SYMT	31
30 IF(10CE2 .NE. 2)GOTO 50	SYMT	32
DO 40 M=1,MM	SYMT	33
SN1(M,NN)=SN1(M,NR)	SYMT	34
SN2(M,NN)=-SN2(M,NR)	SYMT	35
SN3(M,NN)=SN3(M,NR)	SYMT	36
FM1(M,NN)=FM1(M,NR)	SYMT	37
FM12(M,NN)=-FM12(M,NR)	SYMT	38
FM2(M,NN)=FM2(M,NR)	SYMT	39
FN1(M,NN)=FN1(M,NR)	SYMT	40
FN2(M,NN)=-FN2(M,NR)	SYMT	41
FN13(M,NN)=FN13(M,NR)	SYMT	42
FN12(M,NN)=-FN12(M,NR)	SYMT	43
FN22(M,NN)=FN22(M,NR)	SYMT	44
FN23(M,NN)=-FN23(M,NR)	SYMT	45
40 CONTINUE	SYMT	46
50 RETURN	SYMT	47
END	SYMT	48

SUBROUTINE KINET	****	1
COMMON (USE MAIN)	KINE	2
CINET=0.	KINE	3
DO 20 M=2,MS	KINE	4
CM=1.0	KINE	5
IF(M.EQ.2 .OR. (IBCE3.EQ.2 .AND. M.EQ.MS))CM=0.5	KINE	6
DO 20 N=2,NS	KINE	7
CN=1.0	KINE	8
IF((IBCE2 .EQ. 2 .AND. N .EQ. NS))CN=0.5	KINE	9
20 CINET=CINET+(U1(M,N)**2+U2(M,N)**2+U3(M,N)**2)/TEMP(M,N)*CM*CN	KINE	10
IF((IBCE2 .NE. 2))GOTO 25	KINE	11
CINET=2.0*CINET	KINE	12
25 CINER=CINES	KINE	13
CINES=CA*CINET	KINE	14
CINET=0.5*(CINES+CINER)	KINE	15
RETURN	KINE	16
END	KINE	17

SUBROUTINE PWORK	****	1
COMMON (USE MAIN)	PWOR	2
DO 20 M=2,MS	PWOR	3
CM=1.0	PWOR	4
IF(M.EQ.2 .OR. (IBCE3.EQ.2 .AND. M.EQ.MS))CM=0.5	PWOR	5
DO 20 N=2,NS	PWOR	6
CN=1.0	PWOR	7
IF(1BCE2 .EQ. 2 .AND. N .EQ. NS)CN=0.5	PWOR	8
DW=U1(M,N)*SN1(M,N)+U2(M,N)*SN2(M,N)+U3(M,N)*SN3(M,N)	PWOR	9
ENS=ENS-CM*CN*DW*P(M,N)	PWOR	10
20 CONTINUE	PWOR	11
RETURN	PWOR	12
END	PWOR	13

<pre> SUBROUTINE DAMP COMMON (USE MAIN) CHECK FOR START OF DAMPING IF(NCYCLE .LT. MDAMP)RETURN DO 15 M=1,MM DO 15 N=1,NN P(M,N)=0.0 15 CONTINUE IF(CINES-CINER)20,20,40 20 TDAMP=TDAMP+CINET IF(CINET+CINEP .LE. DFACT*TDAMP)GOTO 50 DO 30 M=1,MM DO 30 N=1,NN U1(M,N)=0.0 U2(M,N)=0.0 U3(M,N)=0.0 30 CONTINUE CINEP=CINET CINES1=CINER CINES=0.0 NCYCLE=NCYCLE+1 TIME=TIME+DELTAT CALL MOTION IF(CINES .LE. CINES1)GOTO 39 CALL DESTEP 39 CALL PDATA(2) 40 TDAMP=TDAMP+C2*CINES 45 RETURN 50 WRITE(6,100) NCYCLE MAXC=NCYCLE NC3DP(NN3D)=NCYCLE CALL PDATA (2) GOTO 45 100 FORMAT(1H1,10X,30HRUN SEL(-TERMINATED TIME STEP,15) END </pre>	<pre> *** 1 DAMP 2 DAMP 3 DAMP 4 DAMP 5 DAMP 6 DAMP 7 DAMP 8 DAMP 9 DAMP 10 DAMP 11 DAMP 12 DAMP 13 DAMP 14 DAMP 15 DAMP 16 DAMP 17 DAMP 18 DAMP 19 DAMP 20 DAMP 21 DAMP 22 DAMP 23 DAMP 24 DAMP 25 DAMP 26 DAMP 27 DAMP 28 DAMP 29 DAMP 30 DAMP 31 DAMP 32 DAMP 33 DAMP 34 DAMP 35 DAMP 36 </pre>
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C	SUBROUTINE DESTEP	****	1
	CHANGE DELTAT	DEST	2
	COMMON (USE MAIN)	DEST	3
	DELTAT=SQRT(CINES1/CINES)*DELTAT	DEST	4
	DSQOLD=DELSQ	DEST	5
	C1OLD=C1	DEST	6
	DELSQ=DELTAT**2	DEST	7
	C2=2.0*DELTAT*DAMPF/GAMZ	DEST	8
	C1=C2/(4.0+C2)	DEST	9
	DELR=DELSQ/DSQOLD	DEST	10
	DELS=DELR*(1.0-C1)/(1.0-C1OLD)	DEST	11
	CINES=CINES*DELS**2/DELR	DEST	12
	CINET=0.5*(CINES+CINER)	DEST	13
	PLAST=TNRG-CINET-STREN-TDAMP	DEST	14
	DO 10 N=2,M1	DEST	15
	DO 10 N=1,M1	DEST	16
	TEMP(M,N)=DELR*TEMP(M,N)	DEST	17
	U1(M,N)=DELS*U1(M,N)	DEST	18
	U2(M,N)=DELS*U2(M,N)	DEST	19
	U3(M,N)=DELS*U3(M,N)	DEST	20
10	CONTINUE	DEST	21
	RETURN	DEST	22
	END	DEST	23

	SUBROUTINE PDATA(LINK)	****	1
	DIMENSION DAT(20)	PDAT	2
	COMMON (USE MAIN)	PDAT	3
C	PDATA SELECTS AND WRITES DATA ON TAPE(NPLOT, FOR THE REPSI	PDAT	4
C	PLOTTING PROGRAM	PDAT	5
C	GOTO (10,40,50,60),LINK	PDAT	6
10	NN3D=1	PDAT	7
	II=2*NSTRN+8	PDAT	8
	IF(INCYCLE .EQ. 0)GOTO 15	PDAT	9
12	CALL SKIPFILE (NPLOT,1)	PDAT	10
	READINPLOT) IFLAG	PDAT	11
	IF(IFLAG .EQ. 99999)GOTO 14	PDAT	12
	READINPLOT) IFLAG	PDAT	13
	IF(IFLAG .NE. NCONT+1)GOTO 12	PDAT	14
14	CALL BACKFILE (NPLOT,1)	PDAT	15
	GOTO 50	PDAT	16
15	WRITE(NPLOT) IBCE3,ETAD1,ETAD2,QM,QN,NSTRN	PDAT	17
	WRITE(NPLOT) (ETAG1(I),ETAG2(I),PM(I),PN(I),NETAG(I),I=1,NSTRN)	PDAT	18
	WRITE(NPLOT) NCYCLE,TIME,M1,N1,((Y1(M,N),Y2(M,N),Y3(M,N),M=1(M1),	PDAT	19
	IN=1,N1)	PDAT	20
C	DO 25 I=1,II	PDAT	21
	DAT(I)=0.0	PDAT	22
25	CONTINUE	PDAT	23
	IF(LOAD) 30,30,35	PDAT	24
30	DAT(5)=TNRG	PDAT	25
	DAT(6)=TNRG	PDAT	26
	DAT(7)=TNRG	PDAT	27
	DAT(8)=TNRG	PDAT	28
35	IFLAG=1	PDAT	29
	WRITE(NPLOT) IFLAG	PDAT	30
	WRITE(NPLOT) NCYCLE,(DAT(I),I=1,II)	PDAT	31
	GOTO 100	PDAT	32
C	GOTO 100	PDAT	33
C	GOTO 100	PDAT	34
40	DAT(1)=TIME	PDAT	35
	DAT(2)=D1	PDAT	36
	DAT(3)=D2	PDAT	37
	DAT(4)=D3	PDAT	38
	DAT(5)=CINET	PDAT	39
	DAT(6)=STREN+CINET	PDAT	40
	DAT(7)=TNRG	PDAT	41
	DAT(8)=DAT(6)+TDAMP	PDAT	42
	J=9	PDAT	43
	DO 45 I=1,NSTRN	PDAT	44
	DAT(J)=EPSS1(I)	PDAT	45
	DAT(J+1)=EPSS2(I)	PDAT	46
	J=J+2	PDAT	47
45	CONTINUE	PDAT	48
	IFLAG=1	PDAT	49
	WRITE(NPLOT) IFLAG	PDAT	50
	WRITE(NPLOT) NCYCLE,(DAT(I),I=1,II)	PDAT	51
C	CHECK FOR 3D PLOT	PDAT	52
	IF(INCYCLE .NE. NC3DP(NN3D))GOTO 100	PDAT	53
	NN3D=NN3D+1	PDAT	54
	IFLAG=2	PDAT	55
	WRITE(NPLOT) IFLAG	PDAT	56
	WRITE(NPLOT) NCYCLE,TIME,M1,N1,((Y1(M,N),Y2(M,N),Y3(M,N),M=1(M1),	PDAT	57
	IN=1,N1)	PDAT	58
	GOTO 100	PDAT	59
		PDAT	60

```
C 50 END FILE NPLOT  
    GOTO 100  
C 60 IFLAG=99999  
    WRITE(NPLOT) IFLAG  
100 RETURN  
    END
```

```
PDAT 61  
PDAT 62  
PDAT 63  
PDAT 64  
PDAT 65  
PDAT 66  
PDAT 67  
PDAT 68
```

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SUBROUTINE PRESS
COMMON (USE MAIN)
COMMON /IPR/ RSQ(23,34)
DO 10 M=2,MS
DO 10 N=2,NS
TI=(SQRT(RSQ(M,N)+225.0)-15.0)/144000.0
P(M,N)=0.0
IF(TIME .LT. TI)GOTO 10
P(M,N)=5504737.5*EXP(-13000.0*(TIME-TI))/(RSQ(M,N)+225.0)
10 CONTINUE
RETURN
END

```

```

**** 1
PRES 2
PRES 3
PRES 4
PRES 5
PRES 6
PRES 7
PRES 8
PRES 9
PRES 10
PRES 11
PRES 12

```

	SUBROUTINE INGEOM	****	1
C	FLAT PLATE	INGE	2
	COMMON (USE MAIN)	INGE	3
	COMMON /IPR/ RSQ(23,34)	INGE	4
	REAL LENGTH	INGE	5
C	READ (5,100) LENGTH,WIDTH	INGE	6
	DETA1=WIDTH/FLOAT(MESH)	INGE	7
	DETA2=LENGTH/FLOAT(NMESH)	INGE	8
C	DO 10 M=2,M1	INGE	9
	DO 10 N=1,N1	INGE	10
	Y1(M,N)=FLOAT(M-2)*DETA1	INGE	11
	Y2(M,N)=FLOAT(N-1)*DETA2	INGE	12
	Y3(M,N)=0.0	INGE	13
	RSQ(M,N)=Y1(M,N)**2+(Y2(M,N)-LENGTH)**2	INGE	14
10	CONTINUE	INGE	15
	RETURN	INGE	16
C	100 FORMAT(2E10.4)	INGE	17
	END	INGE	18
		INGE	19
		INGE	20
		INGE	21

	SUBROUTINE INGEOM	****	1
C	CYLINDER	INGE	2
	COMMON (USE MAIN)	INGE	3
	REAL LENGTH	INGE	4
C	DATA DTOR/.1745329251994329E-01/	INGE	5
	READ (5,100) LENGTH,RADIUS,THETA	INGE	6
	DETA1=THETA*RADIUS*DTOR/FLOAT(MESH)	INGE	7
	DETA2=LENGTH/FLOAT(MESH)	INGE	8
	DO 10 M=2,M1	INGE	9
	ETA1=FLOAT(M-2)*DETA1	INGE	10
	DO 10 N=1,N1	INGE	11
	ETA2=FLOAT(N-1)*DETA2	INGE	12
	Y1(M,N)=RADIUS*SIN(ETA1/RADIUS)	INGE	13
	Y2(M,N)=ETA2	INGE	14
	Y3(M,N)=RADIUS*COS(ETA1/RADIUS)	INGE	15
10	CONTINUE	INGE	16
C	ETAD1=ETAD1*RADIUS*DTOR	INGE	17
	DO 20 I=1,NSTRN	INGE	18
	ETAG1(I)=ETAG1(I)*RADIUS*DTOR	INGE	19
20	CONTINUE	INGE	20
	RETURN	INGE	21
C	100 FORMAT(3E12.6)	INGE	22
	END	INGE	23
		INGE	24
		INGE	25

	SUBROUTINE INGEOM	0000	1
	CONICAL SHELL	INGE	2
C	COMMON (US9 MAIN)	INGE	3
	REAL LENGTH	INGE	4
	DATA DTOR/.1748329251994329E-01/	INGE	5
C		INGE	6
	RADIUS,1001 LENGTH,RADI,RADF,THETA,MASH	INGE	7
	ALPHA=ATAN(RADF-RADI)/LENGTH	INGE	8
	SINALF=SIN(ALPHA)	INGE	9
	CSCALF=1.0/SINALF	INGE	10
	COTALF=LENGTH/(RADF-RADI)	INGE	11
	ETA1F=RADI*THETA*DTOR	INGE	12
	IRINASH .EQ. 01GOTO 1	INGE	13
	ETA2I=0.0	INGE	14
	ETA2F=RADI*CSCALF*ALOG(RADF/RADI)	INGE	15
	GOTO 2	INGE	16
1	ETA2I=RADI*CSCALF	INGE	17
	ETA2F=RADF*CSCALF	INGE	18
2	DETA1=ETA1F/FLOAT(MESH)	INGE	19
	DETA2=(ETA2F-ETA2I)/FLOAT(INMESH)	INGE	20
	DCHI1=DETA1/RADI	INGE	21
	DCHI2=DETA2*SINALF/RADI	INGE	22
	CHI2I=ETA2I*SINALF/RADI	INGE	23
	MU=1	INGE	24
	IR(THETA .LT. 180.0)GOTO 5	INGE	25
	MN=1+MS/2	INGE	26
	MN1=MN+1	INGE	27
	MU=MN	INGE	28
	IR(2*MN .EQ. MS+2) MU=1+MN/2	INGE	29
5	DO 10 N=1,MN1	INGE	30
	CHI2=FLOAT(MN-1)*DCHI2+CHI2I	INGE	31
	FINK=CHI2	INGE	32
	IR(MASH .GT. 0) FINK=EXP(CHI2)	INGE	33
	DO 10 N=2,MU	INGE	34
	CHI1=FLOAT(N-2)*DCHI1	INGE	35
	Y1(N,N)=RADI*SIN(CHI1)*FINK	INGE	36
	Y2(N,N)=RADI*COTALF*(FINK-1.0)	INGE	37
	Y3(N,N)=RADI*COS(CHI1)*FINK	INGE	38
10	CONTINUE	INGE	39
	IR(THETA .LT. 180.0)GOTO 50	INGE	40
	IR(2*MN .LT. MS+2)GOTO 30	INGE	41
	MN1=MU+1	INGE	42
	IR(2*MU .EQ. MN+2) MU1=KU	INGE	43
	DO 20 N=1,MN1	INGE	44
	DO 30 N=MN1,MN	INGE	45
	MN1=MN+2-N	INGE	46
	Y1(N,N)=Y3(MN,N)	INGE	47
	Y2(N,N)=Y2(MN,N)	INGE	48
20	Y3(N,N)=Y1(MN,N)	INGE	49
30	DO 40 N=1,MN1	INGE	50
	DO 40 N=MN1,MS	INGE	51
	MN1=MS+2-N	INGE	52
	Y1(N,N)=Y1(MN,N)	INGE	53
	Y2(N,N)=Y2(MN,N)	INGE	54
40	Y3(N,N)=-Y3(MN,N)	INGE	55
C		INGE	56
	CSCRAD=CSCALF*RADI	INGE	57
	ETAD1=ETAD1+RADI*DTOR	INGE	58
	IR(MASH .EQ. 1)ETAD2=CSCRAD*ALOG(1.+ETAD2/CSCRAD)	INGE	59
	DO 45 I=1,MSYRN	INGE	60

ETAG1(1)-ETAG1(1)*RADI*DTOR	INGE 61
IRINASH .FB. 1(ETAG2(1)-CSCRAD*ALOG(1.+ETAG2(1)/CSCRAD)	INGE 62
45 CONTINUE	INGE 63
50 RETURN	INGE 64
C	INGE 65
100 FORMAT(4E10.4,15)	INGE 66
END	INGE 67