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THRUST DEDUCTION IN CONTRAROTATING PROPELLERS

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Naval Ship Research and Development Center
Bethesda, Maryland

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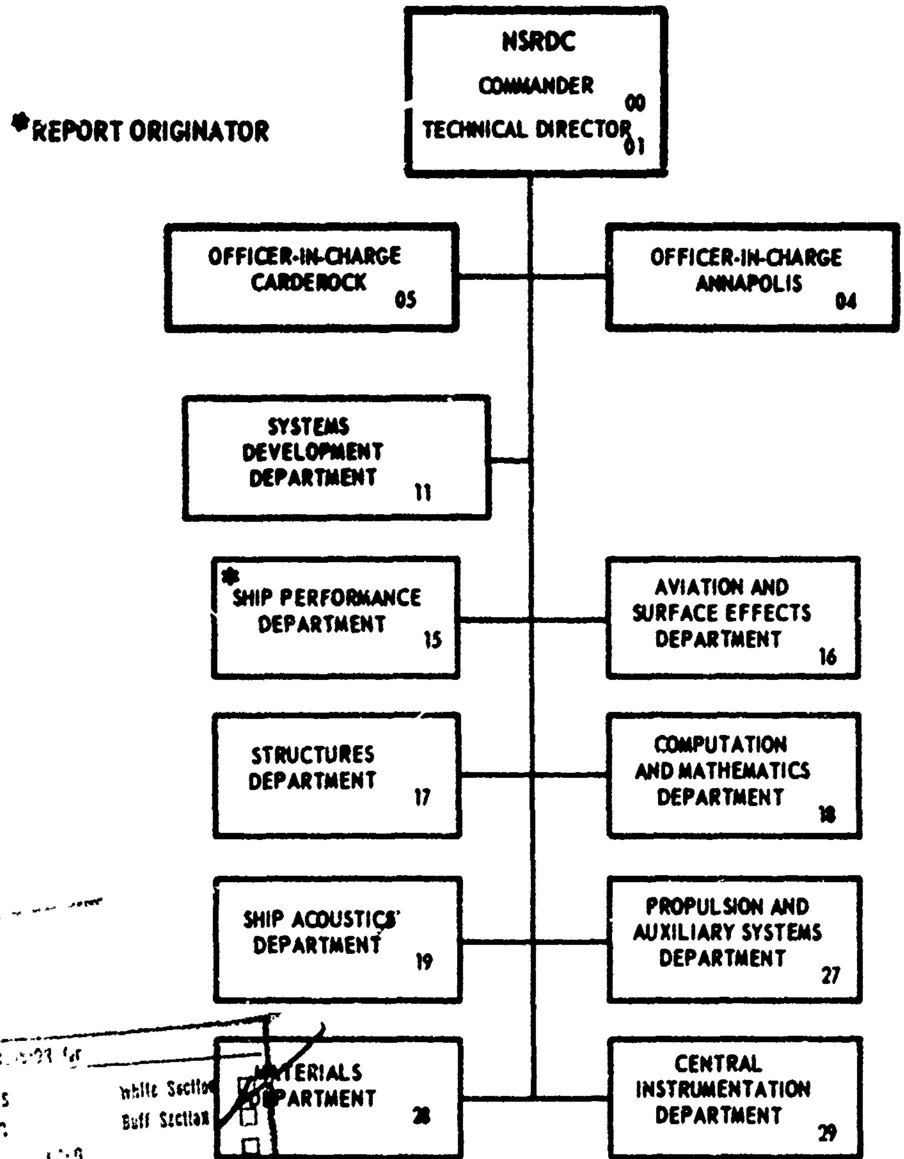

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A theoretical method is presented for calculating the steady propulsive interaction (thrust deduction) force in contrarotating propellers. Contrarotating propellers operating at off-design loading and spacing as well as the contribution of a rudder were investigated. The importance of the separate thrust deduction of the forward and aft propellers in analyzing the behavior of a CR propeller set was shown. Numerical results are given for a MARAD high-speed containership. Some principal findings for the subject ship are:

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(1) good agreement between theory and experiment with regard to the thrust deduction of a centerline rudder, (2) at equal thrust the forward and aft propellers produced 73 percent and 27 percent of the total thrust deduction, respectively, and (3) the total thrust deduction is reduced by unbalancing the propelling thrust with smaller thrust carried on the forward propeller.

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NOTATION

A_0	Propeller disk area
C_D	Drag augmentation coefficient $\dfrac{D}{2\pi\rho V^2 R_1^2}$
C_{Th}	Thrust loading coefficient $\dfrac{T}{1/2\,\rho(A_0)_1\, V_a^2}$
C_{Ths}	Thrust loading coefficient $\dfrac{T}{1/2\,\rho(A_0)_1\, V^2}$
D	Augmented drag or propeller diameter
D_B	Maximum diameter of a body of revolution
d	Axial distance between CR propeller planes
F_N	Froude number $V/\sqrt{gL}$
F_Q	Force (source)
f_a	Circumferential average factor for interference velocities
G^*	Total nondimensional circulation at each radius of one propeller, $Z\Gamma/\pi DV$
g_a	Lerbs' distance factor
L	Body length
L_0	Reference length (distance)
Q	Output of a point source
q_m	Output or strength density of a surface source
R	Propeller radius
r	Directed distance in general
S	Surface area
T	Total thrust CR set
T′	Propeller thrust ratio T_1/T and T_2/T
t	Thrust deduction coefficient
U_a	Axial induced velocity
U_t	Tangential induced velocity
V	Free-stream velocity
V_a	Propeller speed of advance $(1 - w)V$
V_x	Local axial velocity component

V_∇	Volume mean velocity
v	Coaxial velocity induced by point sources
w	Wake fraction
x	Propeller radius fraction
$\bar{\delta}$	Average contraction ratio at aft propeller
ϵ	Influence coefficient
$\bar{\zeta}$	Average circulation factor at aft propeller
ρ	Mass density

SUBSCRIPTS

0	Particular
1	Forward propeller
2	Aft propeller
a	Axial
f	Frictional
h	Hub
i	Interference induced
p	Potential
s	Self induced
t	Tangential
x	Local

ABSTRACT

A theoretical method is presented for calculating the steady propulsive interaction (thrust deduction) force in contrarotating propellers. Contrarotating propellers operating at off-design loading and spacing as well as the contribution of a rudder were investigated. The importance of the separate thrust deduction of the forward and aft propellers in analyzing the behavior of a CR propeller set was shown. Numerical results are given for a MARAD high-speed containership. Some principal findings for the subject ship are: (1) good agreement between theory and experiment with regard to the thrust deduction of a centerline rudder, (2) at equal thrust the forward and aft propellers produced 73 percent and 27 percent of the total thrust deduction, respectively, and (3) the total thrust deduction is reduced by unbalancing the propelling thrust with smaller thrust carried on the forward propeller.

ADMINISTRATIVE INFORMATION

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INTRODUCTION

To the author's knowledge the important and intriguing problem of the drag augmentation (thrust deduction) of a ship with contrarotating propellers has not been solved previously in sufficient detail for practical design purposes.[1] The problem is inherently more complex than single-screw propulsion which has received considerable attention,[2-10] mainly because

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[1] Hickling, R., "Propellers in the Wake of an Axisymmetric Body," Institution of Naval Architects, Quarterly Transactions, Vol. 99, No. 4 (Oct 1957). A complete listing of references is given on page 29.

[2] Tsakonas, S. and W.R. Jacobs, "Analytical Study of the Thrust Deduction of a Single-Screw Thin Ship," Stevens Institute of Technology, Davidson Laboratory Report 816 (Mar 1962).

[3] Beveridge, J.L., "Thrust Deduction Due to a Propeller Behind a Hydrofoil," David Taylor Model Basin Report 1603 (Oct 1962).

[4] Beveridge, J.L., "Effect of Axial Position of Propeller on the Propulsion Characteristics of a Submerged Body of Revolution," David Taylor Model Basin Report 1456 (Mar 1963).

[5] Nowacki, H., "Potential Wake and Thrust Deduction Calculations for Ship-Like Bodies," Transactions STG (1963) (in German).

[6] Wald, Q., "Performance of a Propeller in a Wake and the Interaction of Propeller and Hull," Journal of Ship Research, Vol. 9, No. 1 (Jun 1965).

[7] Amtsberg, H., "Investigations on the Interaction between Hull and Propeller of Bodies of Revolution," David Taylor Model Basin Translation 309 (Dec 1965).

[8] Dreger, W., "A Method of Calculation of Potential Thrust Deduction," David Taylor Model Basin Translation 328 (Mar 1966).

[9] Pohl, K.H., "The Interaction between Hull and Propeller," David Taylor Model Basin Translation 334 (Feb 1967).

[10] Beveridge J.L., "Analytical Prediction of Thrust Deduction for Submersibles and Surface Ships," Journal of Ship Research, Vol. 13, No. 4 (Dec 1969).

of the additional singularities involved and the attendant multiple integrations required for the potential part of the thrust deduction force. The frictional component of the thrust deduction is known to be very small and can be approximated.[11–13] This elucidation of the problem, i.e., as a Lagally type of force, has been adequately investigated and applied successfully.[3,10] It has its basis in a well known principle of reciprocity (Newton) for finding the forces between bodies in a flow field.

In apportioning the thrust load between a pair of contrarotating propellers, current design methods do not consider the actual division of the thrust deduction between the forward and after propellers. Thus, the design condition of either thrust or torque balance between propellers at equal rates of rotation (rpm) may not produce the minimum shaft horsepower. The investigation reported herein develops a theoretical method for calculating the thrust deduction due to contrarotating stern propellers. The scope of the work includes the development of analytical methods for determining the effect of rudder, variations in the ratio of thrust between forward and after propellers, and the spacing between propellers. Computational results are presented for a MARAD high-speed containership.[14]

The plan and principal points of this report include: (1) the derivation of analytical expressions for the thrust deduction for contrarotating propellers, and (2) the introduction of an influence coefficient to account for the effect of wave-making and certain geometric particulars on the potential wake of a high-speed merchant-ship type of hull.

STATEMENT OF PROBLEM

In potential flow, mathematical singularities which represent solid bodies or boundaries are widely used. The Lagally steady-motion equation[15,16] relates the force on a source (or sink) of given strength in an arbitrary flow to the velocity which the flow would possess at the location of the source if the source did not exist. The thrust-deduction problem is analogous to the familiar problem of finding the forces between bodies in a flow field. A force arises from the mutual influence of the hull on the contrarotating propellers and these

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[11] Dickmann, H.E., "The Interaction between Propeller and Ship with Special Consideration to the Influence of Waves," Jahrbuch der Schiffbautechnischen Gesselschaft, 40 (1939).

[12] Van Lammeren, W.P.A., "Analysis of Propulsion Components in Relation to Scale Effect by Model Tests," David Taylor Model Basin Translation 68 (Sep 1950).

[13] Beveridge, J.L., "Pressure Distribution on Towed and Propelled Streamline Bodies of Revolution at Deep Submergence," David Taylor Model Basin Report 1665 (Jun 1966).

[14] Strom-Tejsen, J., "A Comparison of Contrarotating Propellers with other Propulsion Systems," Marine Technology, Vol. 9, No. 1 (Jan 1972).

[15] Durand, W.F., "Aerodynamic Theory," Div. C, Vol. 1, p. 260, Dover Publications (1963).

[16] Betz, A., "The Method of Singularities for the Determination of Forces and Moments Acting on a Body in Potential Flow," David Taylor Model Basin Translation 241, Revised Edition (Jan 1951).

propellers on the hull. Simply stated, a hull affects contrarotating propeller performance through its wake, and a hull (from the point of view of resistance) is affected by the self-induced velocity field of the contrarotating propellers.

The Lagally theorem provides a means of circumventing the detail of integrating pressures over the hull surface to obtain the thrust-deduction force. Since the interaction force is to be obtained directly from relations between singularities, the main problem is to find appropriate generating singularities for hull and propellers. Essential parts of the problem are determination of the perturbation velocities due to a hull in a uniform potential flow and determination of the radial distribution of propeller thrust. Specifically, these essential parts enter into the solution of the interaction or thrust-deduction force, using the Lagally theorem, by providing the required hull-disturbance velocity and propeller-sink strengths.

The principal assumptions and limitations that are involved in the analysis and treatment of the problem are summarized as follows:

1. Interference effects which involve changes in boundary conditions and lead to iterations are assumed to be small[4,13] and are not considered.

2. The effects of various singularities can be combined. Exact linear superposition of flows is, of course, limited to those which satisfy the Laplace equation. It is assumed that the effect of the propellers on the hull is mainly potential in origin, whereas the effect of the hull on the propellers is essentially viscous in origin. The effect on the propellers is treated, however, as a potential problem through the use of the circumferential average of the total wake at each propeller radius.

3. It is assumed that the actual propeller blades can be replaced by lifting lines where the induced velocity is based on the actual radial thrust distribution. For more than three blades, calculations show the circumferential variation (fluctuation) of induced velocity from lifting-line theory has a negligible effect on the thrust deduction. Lifting-line theory is used strictly as a matter of convenience to obtain the required steady sink disk strength which is radially variable. This mathematical propeller model gives the propeller induced velocity as a function of propeller thrust in a viscous fluid and radius, but does not consider propeller blade thickness. A check of the effect of propeller blade thickness was made and was found to be small for the propellers considered.

4. It is assumed for this investigation that the separation point on the hull is not changed by the propellers. In principle an iteration procedure and boundary-layer theory can be used to consider changes in the separation point.

5. The theory presented is applicable, strictly speaking, only to ships without significant wavemaking unless the potential part considers the free surface. To make the theory more general, an influence coefficient is used to correct the potential wake for the ship wave system. In the present report, the influence coefficient, also, contained some geometrical effects.

6. At off-design propeller loadings the propeller singularity strengths are assumed to be proportional to $(1 + 1/2\, C_{Th})$. This assumption restricts the analysis to cases for which the relative loading difference between the two propellers is not great.

SOLUTION

THRUST DEDUCTION AS A LAGALLY FORCE

The Lagally steady-motion equation[16]

$$F_Q = -\rho \frac{QQ_0}{4\pi r^2} \tag{1}$$

connects the force F_Q on a source at some point in the flow field with the output Q and the flow velocity $Q_0/4\pi r^2$ at the location of the source Q, where r is the distance from a source Q_0 to a source Q. For contrarotating propellers Equation (1) can be rewritten in the more general form

$$\begin{aligned} F_{Q_1} &= \rho Q_1\, [V(w_p)_1 + (U_{ai})_1] \text{ for forward propeller} \\ F_{Q_2} &= -\rho Q_2\, [V(w_p)_2 + (U_{ai})_2] \text{ for aft propeller} \end{aligned} \tag{2}$$

where w_p is the hull nondimensional-disturbing velocity and U_{ai} is the propeller induced interference velocity and at each conjugate propeller can vary spatially but not temporally. The hull-disturbing velocity w_p is the potential flow perturbation velocity caused by the hull moving at velocity V. The hull can be considered as generated by a surface distribution of sources and the interaction force of a contrarotating propeller-hull system is obtained, by reciprocity, from the hull-disturbing velocity

$$w_p = \frac{1}{V} \int_s \frac{q_m\ (S)\ ds}{4\pi r^2}$$

at the contrarotating propellers. Each propeller is represented by a sink disk of strength

$$\int_{A_0} q_m\ (A_0)\ dA_0$$

where q_m = strength of a surface source
S = surface area of the hull and
A_0 = prepeller disk area

Expressions for the propeller interference-axial velocity U_{ai} will be given later.

Equation (2) can be written in terms of w_p and U_{ai} at each propeller disk (of the contrarotating pair) and distributed surface sinks of strength $q_m = U_{as}$. The total drag augmentation coefficient C_D is then given by

$$
\begin{aligned}
C_D = \frac{D}{2\pi\rho V^2 R_1^2} &= \int_{x_h}^{1} \left[(w_p)_1 + \frac{(U_{ai})_1}{V}\right] \frac{(U_{as})_1}{V} \, xdx \\
&+ \left(\frac{R_2}{R_1}\right)^2 \int_{x_h}^{1} \left[(w_p)_2 + \frac{(U_{ai})_2}{V}\right] \frac{(U_{as})_2}{V} \, xdx \\
&= \int_{x_h}^{1} (w_p)_1 \frac{(U_{as})_1}{V} \, xdx + \int_{x_h}^{1} \left[\frac{(U_{ai})_1 \, (U_{as})_1}{V^2}\right] xdx \\
&+ \left(\frac{R_2}{R_1}\right)^2 \int_{x_h}^{1} (w_p)_2 \frac{(U_{as})_2}{V} \, xdx \\
&+ \left(\frac{R_2}{R_1}\right)^2 \int_{x_h}^{1} \left[\frac{(U_{ai})_2 \, (U_{as})_2}{V^2}\right] xdx \\
&= I_1 + I_{12} + I_2 + I_{21}
\end{aligned}
\tag{3}
$$

where the variables within the integrals are understood to be average values in the circumferential direction.

Interaction force has been presented as a drag-augmentation coefficient C_D. The thrust-deduction coefficient is related to the drag-augmentation coefficient by

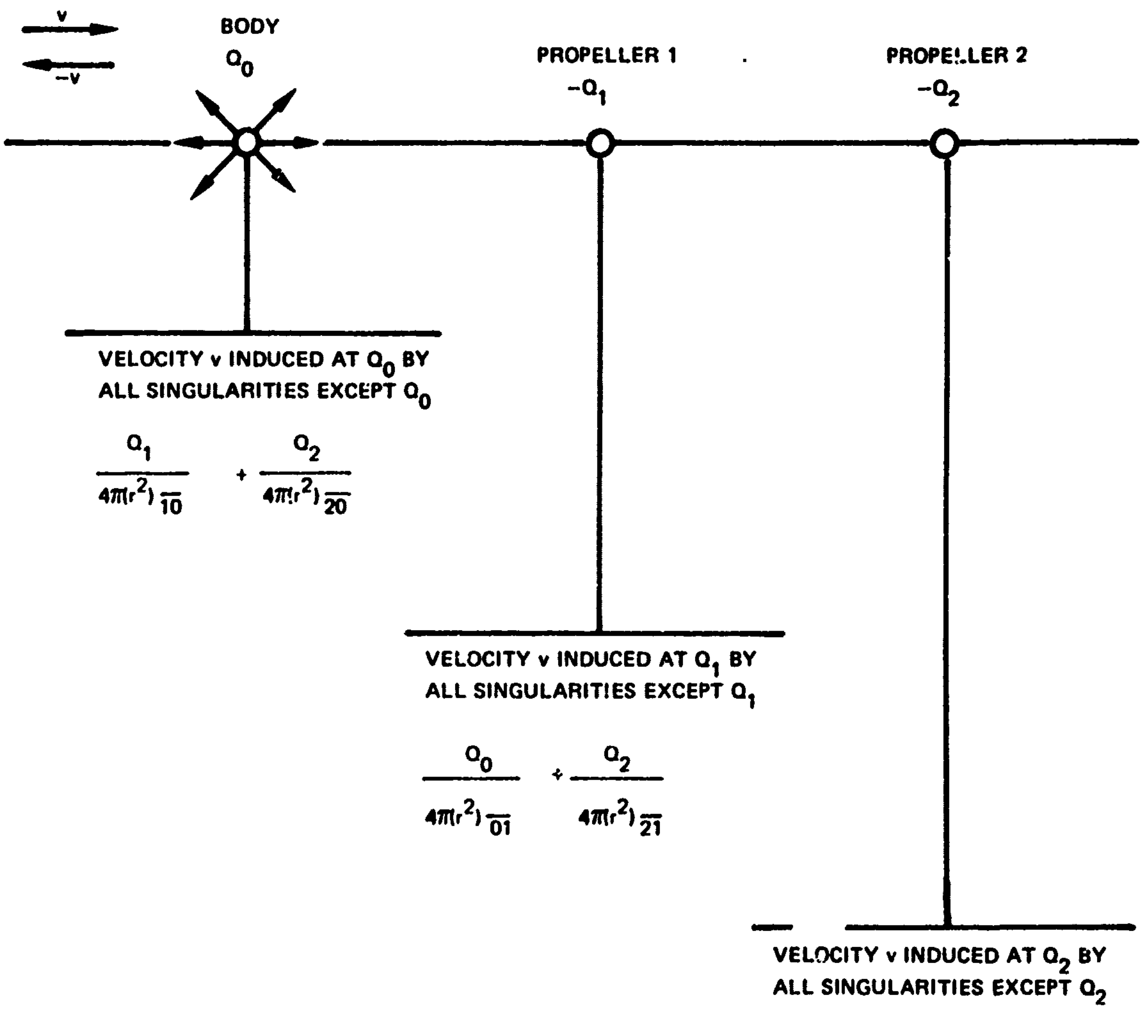
$$
t = t_f + t_1 + t_2 = t_f + \frac{4(C_{D_1} + C_{D_2})}{C_{Ths}} = t_f + \frac{4C_D}{C_{Ths}} \tag{4}
$$

where t_f is the frictional thrust deduction. For $t_f = 0$

$$
\frac{t_1}{t} = \frac{C_{D_1}}{C_D} \quad \text{and} \quad \frac{t_2}{t} = \frac{C_{D_2}}{C_D}
$$

Some comments will be made later concerning the frictional thrust deduction coefficient t_f.

The interference force integrals I_{12} and I_{21} in Equation (3) cancel since only the force on the body is wanted. The simple case sketched below is illustrative. Assume all perturbations are caused by three point sources (representing: body, Propeller 1, and Propeller 2) enclosed by spheres of infinitesimal radius and located coaxially. On the axis, distance r between singularities is indicated by bar (–). From the system as sketched we can write the following force equations:



$$\frac{Q_1}{4\pi(r^2)_{\overline{10}}} + \frac{Q_2}{4\pi(r^2)_{\overline{20}}}$$

$$\frac{Q_0}{4\pi(r^2)_{\overline{01}}} + \frac{Q_2}{4\pi(r^2)_{\overline{21}}}$$

$$\frac{Q_0}{4\pi(r^2)_{\overline{02}}} - \frac{Q_1}{4\pi(r^2)_{\overline{12}}}$$

$$-\frac{4\pi F_{Q_0}}{\rho} = Q_0 \left[\frac{Q_1}{(r^2)_{\overline{10}}} + \frac{Q_2}{(r^2)_{\overline{20}}}\right] \tag{5}$$

$$-\frac{4\pi F_{Q_1}}{\rho} = -Q_1 \left[\frac{Q_0}{(r^2)_{\overline{01}}} + \frac{Q_2}{(r^2)_{\overline{21}}}\right] \tag{6}$$

$$-\frac{4\pi F_{Q_2}}{\rho} = -Q_2 \left[\frac{Q_0}{(r^2)_{\overline{02}}} - \frac{Q_1}{(r^2)_{\overline{12}}}\right] \tag{7}$$

Equating the force on the body to the force on the propellers, we obtain

$$\frac{Q_0 Q_1}{(r^2)_{\overline{10}}} + \frac{Q_0 Q_2}{(r^2)_{\overline{20}}} = -\frac{Q_1 Q_0}{(r^2)_{\overline{01}}} - \frac{Q_1 Q_2}{(r^2)_{\overline{21}}} - \frac{Q_2 Q_0}{(r^2)_{\overline{02}}} + \frac{Q_2 Q_1}{(r^2)_{\overline{12}}} \tag{8}$$

or

$$-F_{Q_0} = F_{Q_1} + F_{Q_2}$$

It can be seen from Equation (8) that the interference terms $Q_1 Q_2$ on the right-hand side cancel since the distance $r_{\overline{21}} = r_{\overline{12}}$.

INDUCED VELOCITY IN CONTRAROTATING PROPELLERS

A design procedure for contrarotating propellers which is based on Lerbs' theory has been presented by Morgan.[17] Since the theory and calculation of wake-adapted contrarotating propellers are not within the scope of this report,[18] the procedure used here for obtaining the propeller source strength $q_m/V = U_{as}/V$ will be discussed only briefly. To derive the thrust deduction from the Lagally theorem, it is important to emphasize that the required singularity strength (far downstream) q_m/V is equal to twice* the circumferential average of the axial component of the propeller self-induced velocity at the propeller disk.[19] It was shown in the previous section that the propeller-induced interference terms $(U_{ai})_1/V$ and $(U_{ai})_2/V$ in Equation (3) do not contribute to the thrust deduction.

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[17] Morgan, W.B., "The Design of Counterrotating Propellers Using Lerbs' Theory," Transactions SNAME, Vol. 68 (1960).

[18] Lerbs, H.W., "Contra-Rotating Optimum Propellers Operating in a Radially Non-Uniform Wake," David Taylor Model Basin Report 941 (May 1955).

[19] Weinblum, G., "The Thrust Deduction," American Society of Naval Engineers, Vol. 63 (1951).

*The actual numerical values of the self-induced axial velocity at the lifting line of each propeller were multiplied by two to obtain the final (far downstream) velocity U_{as}/V.

Lerbs' theory for either lightly or moderately loaded contrarotating propellers considers propellers having finite hub, finite number of blades, and an arbitrary radial load distribution. His theory is based on potential flow in which viscous effects are neglected; however, the drag forces on the blades which exist in a real fluid are introduced later as a correction to the potential theory. The inflow velocities required for the calculation of a wake-adapted propeller are based on the total wake. The nominal inflow V_x/V is corrected by V_a/V_∇, which is the ratio of the effective velocity to the volume mean velocity. The volume mean velocity ratio is defined as

$$\frac{V_\nabla}{V} = \frac{2}{1 - x_h^2} \int_{x_h}^{1} \frac{V_x}{V} \, x dx$$

where V_x/V is the circumferential average of the local velocity at each radius.

Calculations for moderately loaded contrarotating (CR) propellers using Lerbs' induction-factor method are programmed for a high-speed digital computer at this Center.* A numerical solution which includes the total propeller-induced velocity at the lifting lines (see Figure 1 for velocity component diagram) for the propeller operating in a real flow is obtained from the program. Because the propeller self-induced velocity U_{as} is needed in the calculation of the thrust deduction and only the propeller total induced velocity U_{aT} is included in the computer output (U_{as} not convenient to retrieve) U_{as} must be calculated from the computer output by:

$$(U_{as})_1 = (U_{aT})_1 - (U_{ai})_1 \quad \text{for forward propeller}$$

$$(U_{as})_2 = (U_{aT})_2 - (U_{ai})_2 \quad \text{for aft propeller}$$

where from Morgan[17]

$$\begin{aligned} (U_{ai})_1 &= (U_{as})_2 \, (f_a)_2 \, [1 - (g_a)_2] \\ (U_{ai})_2 &= (U_{as})_1 \, (f_a)_1 \, [1 + (g_a)_1] \end{aligned} \qquad (9)$$

$f_a \approx \dfrac{G^*}{2x\,(U_t/V)}$ = factor for obtaining the circumferential average of interference velocities

$(g_a)_1 \approx (g_a)_2$ = distance factor for obtaining the effect of axial distance on interference velocities

---

*Reprogramming of the present CR propeller computer program is currently underway. The so-called "equivalent propeller concept" will no longer be needed and the self-induced velocity U_{as} will be provided in the output. Consequently, the method used to obtain U_{as} in the present investigation will not be necessary in the future.

G^* = total nondimensional circulation at each radius of one propeller ($Z\Gamma/\pi DV$)

x = nondimensional radius

U_t = tangential velocity

D = propeller diameter

V = ship speed

Z = number of blades, and

Γ = circulation of each section

The computer program uses an iterative-type method of solution to determine the induced velocities. Before proceeding with the design of two actual propellers with a specified axial spacing an "equivalent" propeller[17] which produces one half the total thrust is introduced. The expressions for the self-induced axial velocities, appearing in Equation (9), of each actual CR propeller in terms of the induced velocity U_a from the equivalent propeller are:

$$\begin{aligned}(U_{as})_1 &= (U_a)_1 \\ (U_{as})_2 &= (1 + \bar{\delta})(1 + \bar{\zeta})(U_a)_2\end{aligned} \qquad (10)$$

where $\bar{\delta}$ is the average contraction ratio of the slipstream at the aft propeller and $\bar{\zeta}$ is the average circulation factor at the aft propeller. According to Morgan[17] both $\bar{\delta}$ and $\bar{\zeta}$ are small in absolute value. In the present investigation they are assumed to be zero.

Lerbs' distance factor[18] g_a has been calculated, for the present investigation, to cover a higher range of propeller spacing d/R and curves of g_a versus d/R are shown by Figure 2. Recent design experience at the Center has indicated a preference for these factors over those presented by Morgan.[17]

THRUST DEDUCTION AT OFF-DESIGN LOADING

The minimum horsepower for a ship propelled by CR propellers is a function of the propeller efficiency and the hull efficiency. Equal thrust or equal torque on each propeller does not necessarily give the minimum shaft horsepower since the minimum thrust deduction does not usually occur when either the thrusts or torques are equal. Therefore, to obtain minimum horsepower, it is necessary to investigate the effect of unequal thrust loadings of the propellers on the thrust deduction. To calculate the thrust deductions at off-design loadings, it is assumed that the propeller self-induced axial velocity U_{as} is proportional to the propeller thrust coefficient C_{Th}.

For a particular propeller, the average factor f_a in Equation (9) is essentially constant with a change in thrust only. Thus, for a specified CR propeller spacing, i.e., distance factor g_a, we observe that $(U_{ai})_1/(U_{as})_2 = k_1 \approx$ constant and $(U_{ai})_2/(U_{as})_1 = k_2 \approx$ constant and the *off-design* relations are:

$$2T_1' (U_{as})_1 + k_1 (U_{as})_2 \; 2T_2' = (\text{Total Axial})_1$$

$$2T_2' (U_{as})_2 + k_2 (U_{as})_1 \; 2T_1' = (\text{Total Axial})_2$$

At design condition (assumed)

$$T_1' = T_2' = 0.5$$

The necessity for utilizing an off-design approximation lies in the fact that the present CR propeller design computer program (lifting-line computation) considers only equal torque balance, i.e., $T_1' \simeq T_2'$.

With propeller off-design singularity strength taken as proportional to T' in Equation (3), Equation (4) would become

$$t = t_f + \frac{8(T_1' C_{D_1} + T_2' C_{D_2})}{C_{Ths}} \tag{4a}$$

and for $t_f = 0$

$$\frac{t_1}{t} = \frac{1}{1 + \dfrac{T_2'}{T_1'} \left(\dfrac{C_{D_2}}{C_{D_1}}\right)} \quad \text{and} \quad \frac{t_2}{t} = \frac{1}{1 + \dfrac{T_1'}{T_2'} \left(\dfrac{C_{D_1}}{C_{D_2}}\right)}$$

where the drag augmentation coefficients C_{D_1} and C_{D_2} are *at design condition.*

THRUST DEDUCTION AT OFF-DESIGN SPACING

The spacing between propellers and between propeller and hull can also affect the minimum shaft horsepower. As soon as the position of either CR propeller is changed, all perturbation velocities must be recalculated at both propellers. For off-design spacing, in this investigation, the forward propeller location remained fixed and the aft propeller was moved downstream. Large propeller spacing (defined by axial distance between propeller planes as a fraction of forward propeller radius, d/R_1) requires the physical removal of the rudder for the typical single-screw ship. Therefore, thrust deduction was determined mathematically with the rudder removed when the parameter d/R_1 was varied.

To approximate the change in the hull-potential wake fraction w_p with distance downstream, a method based on the variation of thrust deduction with propeller axial position behind a body of revolution, as derived by the author,[4] was used together with the exact functional relation between the potential wake fraction and thrust-deduction coefficient for uniform flow. The relation can be expressed in the following convenient form:

$$w_p = \frac{t_p}{2} \left(1 + \sqrt{1 + C_{Th}}\right) \tag{11}$$

Consider the fact that for the same propeller average total thrust, we have the same average total induced velocity. By a judicious choice of d/R_1 some reasonable assumptions concerning off-design performance can be made as follows: (1) If we do not make too large a change in spacing, the assumption is reasonable that the radial distribution of thrust for each CR propeller remains approximately the same. With this assumption we calculate new self-induced velocities U_{as} at the new location. (2) If the aft propeller is positioned downstream where $w_p \approx 0$ (about $d/R_1 = 10.8$ for the MARAD Containership) then $t_2 = 0$ and $t \approx t_1$. It is known that T_1 will change at the same rpm but the ratio D/T remains almost constant for nonconfigurational changes in propeller loading.

THRUST DEDUCTION DUE TO RUDDER

A simple relation between w_p and t_p was given (Equation (11)) with the assumption of uniform flow. Several simple and reasonable approximations to the disturbing velocity w_p induced by a rudder may be made. For example, a point source in a two-dimensional uniform flow (represents a rudder of infinite span and chord) may be an adequate model for the present investigation inasmuch as only the flow field ahead of the rudder is needed and points beyond the propeller slipstream lack a point of application. Isowake lines can be constructed at the propeller plane and these integrated circumferentially at several radii to provide an average radial distribution of w_p.

Harvald[20] has constructed a two-dimensional flow model* by a conformal transformation technique that produces a more typical rudder section shape than a two-dimensional half body. His Diagram 89, Chapter V, was used to evaluate w_p for the MARAD Containership rudder. Equation (11) was then used to calculate the thrust deduction due to the rudder.

COMPUTATIONAL EXAMPLE

MARAD CONTAINERSHIP

Figures 3 and 4 show, respectively, the lines and stern view of the Center Model 5218, representing a 25.5-knot containership equipped with a CR propeller stern arrangement

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[20] Harvald, S.A., "Wake of Merchant Ships," The Danish Technical Press, Copenhagen (1950).

*Harvald also examined the effect of a convergent flow field and it appears that in general an assumption of a free rudder in a parallel flow would be adequate for the purpose of the present investigation.

and a single rudder. This containership has been investigated extensively by Strom-Tejsen[14] with model tests of several design variations including twin rudders, single screw, and overlapping propellers. Ship and model data for this model are given in Table 1. A drawing and pertinent geometrical data appear in Figure 5 for the design CR propellers, Numbers 4458 and 4459. Surface-sink strengths of these propellers as a function of propeller radius for the design condition (equal thrust) and at an off-design spacing $d/R_1 = 1.5$ are given in Figures 6 and 7, respectively. Table 2 gives predicted propulsion results at 25.5 knots for model experiments for the ship equipped with contrarotating propellers. The experiments conducted with Model 5218 included propulsion data for the conditions: with, and without rudder, off-design thrust loading between CR propellers, and comparative single-screw results. Thus, it seemed to be an excellent vehicle for performing a theoretical analysis of the thrust deduction for a high-speed single screw ship.

INFLUENCE COEFFICIENT FOR POTENTIAL WAKE

To effect a major saving in time and money the potential-wake fraction w_p was obtained for the subject containership by application of an influence coefficient. The influence coefficient would not generally be needed. In essence this coefficient was determined numerically from propulsion data[14] (Table 2 of this report) and the experimental radial distribution of w_p reported for the single-screw surface ship SIMON BOLIVAR.[10]

Introducing the influence coefficient ϵ into Equation (4), we can write

$$t_{experimental} = t_f + \frac{4\epsilon C_D}{C_{Ths}} \tag{4b}$$

where $\epsilon C_D = \epsilon(I_1 + I_2)$ by Equation (3) and the integrals I_1 and I_2 have been computed for the MARAD Containership using a w_p distribution from SIMON BOLIVAR already adjusted Equation (11) for differences in propeller diameter and spacing relative to the hull. Let

$t_f = 0.015$ from Beveridge[10]

$(t)_{exp} = 0.188$ from Table 2

$C_{Ths} = 0.6924$ from Strom-Tejsen[14] (actual test value for CR set) and

$C_D = 0.0240$ from Equation (3)

Equation (4b) solved for ϵ is

$$\epsilon = \frac{C_{Ths}\,[(t)_{exp} - t_f]}{4C_D} = 1.25 \tag{12}$$

The influence coefficient ϵ mainly corrects the assumed w_p distribution for differences in the free-wave system and geometry between SIMON BOLIVAR and the MARAD Containership. The effect of geometry appears to be dominant. Although other unaccountable effects may be lumped into ϵ it is believed to operate chiefly on w_p (as intended) by a comparison with wake data of Harvald,[21] and Nowacki and Sharma.[22] Final curves of w_{p1} and w_{p2} versus x for the containership are presented in Figures 8 and 9 at the design CR propeller spacing $d/R_1 = 0.52$ and at off-design spacing $d/R_1 = 1.50$.

A check of the Harvald data reveals $w = 0.22 \approx w_p$ for a small wave wake while the present containership potential wake (Figure 8) shows $w_p = 0.21$ at 0.7 propeller radius. Another check on w_p for the containership can be obtained from the relation of t_p and w_p for uniform flow (Equation (11)). With $C_{Th} = 1.214$ (computed for $1 - w_0 = 0.76$) and $t_p = 0.188 - 0.015 = 0.173$, we have

$$w_p = \frac{t_p}{2}\left(1 + \sqrt{1 + C_{Th}}\right) = 0.22$$

A small wave-wake component w_w is indicated at design Froude number, $F_N = 0.272$, for the containership and some substance is given to this by the results of Nowacki and Sharma, which are given in Figure 10, for a somewhat idealized mathematical hull having $C_B = 0.64$ and $C_X = C_P = C_W = 0.8$.

NUMERICAL RESULTS

Although the frictional component of the thrust-deduction coefficient t_f was introduced earlier, it was assumed equal to zero in all numerical computations involving ratios of thrust deduction (e.g., t_1/t and t_2/t). This assumption avoids any superficial division such as $(t_f)_1$ and $(t_f)_2$ for an already small quantity.[11,13] However, all absolute values of t are based on $t_f = 0.015$.[13] A most important observation is made at this point concerning the influence coefficient ϵ that was used in determining potential wake; to wit, ϵ cancels if we deal with thrust deduction ratios in our analysis. In a very real sense percentage or fractional changes in thrust deduction with parametric variations are what we want anyway.

RUDDER CONTRIBUTION AND SPACING BETWEEN PROPELLERS

As described previously the Harvald[20] diagram showing the dependence of w_p on the dimensions and position of the rudder (in parallel flow) may be used in conjunction with

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[21] Harvald, S.A., "Wake and Thrust Deduction at Extreme Propeller Loadings," Swedish State Shipbuilding Experimental Tank Publication No. 61, Goteborg (1967).

[22] Nowacki, H. and S.D. Sharma, "Free-Surface Effects in Hull Propeller Interaction," The University of Michigan College of Engineering Report 112 (Sep 1971).

Equation (11) to estimate the thrust deduction due only to the rudder For the MARAD Containership the following rudder input has been used:

$$t^*/\ell = 0.43$$

$$D/\ell = 2.6$$

$$a/\ell = 0.45$$

where t^* = maximum section thickness of the rudder (symbol t^* not to be confused with thrust deduction coefficient),

D = propeller diameter

a = axial clearance between the rudder leading edge and a representative propeller plane. Note: Propeller plane defined at 0.7D when rake is present and

ℓ = rudder chord length

For the subject rudder and CR propeller set the midplane was chosen in computing (a), and (ℓ) was taken as the rudder chord at $0.7R_1$. A w_p of 0.05 results and (t_p) rudder = 0.04 was computed from Equation (11). The experimental result for the containership from tests *with and without rudder* gave

$\Delta t = 0.056$ Design CR propellers (Table 2)

$\Delta t = 0.029$ Single Screw, Strom-Tejsen[14]

Very good agreement between the experimental and theoretical results for the thrust deduction *due solely* to the rudder is revealed when it is realized that choosing a midplane location for the CR propellers is tantamount to placing the propeller disk loading closer to the rudder compared to the single screw location and moving the aft CR propeller farther ahead of the rudder. Thus, the computed incremental value, $\Delta t = (t_p)_{rudder} = 0.04$, lies between the two experimental values.

A simple monotonic variation of the thrust deduction coefficient with axial distance between CR propellers was found as expected (see Figure 11). The rudder has been removed (physically necessary for large spacing) by subtracting its known contribution from the total thrust deduction since it had been determined experimentally. Selection of particular discrete d/R_1 ratios for performing thrust deduction computations was discussed previously. There is nothing striking or unusual about the results obtained and the variation for the MARAD CR propellers is similar to the curve (shown in Figure 11 for comparison) derived for a bare-hull, single-screw submarine form.[4]

Although the present work was limited to an investigation of thrust deduction, some more far-reaching comments seem desirable at this point. It is generally accepted that axial spacing has little effect on the open-water efficiency of CR propellers, provided they are operated at their design spacing. Limited space available usually dictates that propeller spacing be kept to a minimum. Hecker[23] has investigated CR propeller performance in uniform flow for distance ratios from $d/R_1 \approx 0.2$ to 0.8. Purely from a hydrodynamic standpoint, the concept that behind a hull the potential flow field decays at a greater rate than does the frictional flow field, is a basis for speculating that an optimum propeller spacing might exist in the wake-adapted case. As an example it has been shown for a submerged body of revolution[4] that an optimum (based on maximum propulsive coefficient) downstream location does occur for a single screw. Based on these statements a spacing for optimizing the propulsive coefficient probably exists for a CR propeller pair installed on a submersible and certain surface ships where free surface and wave effects would not be significant.

Practical application of these ideas may be feasible because submersibles are presently configured with the stern control surfaces forward of the propeller, and a twin-rudder arrangement for a surface ship would permit the aft propeller of a CR set to be positioned farther downstream. In retrospect, investigative effort along these lines seems warranted.

THRUST BALANCE

Table 3 gives calculated thrust deduction results at several discrete values of relative thrust loading between the forward and aft propellers designed for the MARAD Containership. For equal thrust loading $T_1/T_2 = 1.0$ the division of the thrust deduction is seen to be 73 percent of the total produced by the forward propeller and 27 percent produced by the aft propeller. This result may be important in the propeller design problem since an assumed 50–50 distribution of the thrust deduction would thus produce approximately a 5 percent difference in the division of useful propeller thrust between propellers.

As mentioned earlier a design condition $T_1/T_2 = 1.0$ was assumed in the theoretical analysis. However, the experimental results for the MARAD Containership reported by Strom-Tejsen[14] revealed that at equal rpm a value $T_1/T_2 = 0.95$ was obtained in contrast to equal thrusts for which they were designed. In Figure 12 the computed $(1 - t)$ values of Table 3 have been plotted as fractions of the design value with an incremental shift of 0.05 on T_1/T_2 to give a unity thrust deduction ratio at $T_1/T_2 = 0.95$. In addition two other experimental points are shown corresponding to the rpm ratios of 0.95 and 1.05 in Table 2. The $(1 - t)$ ratio curve of Figure 12 shows about an 8 percent decrease when the propeller thrust ratio T_1/T_2 is increased from about 0.4 to 2.0. The two off-design experimental points do not lie on the theoretical curve. This is believed to be due to either experimental test accuracy, or the rudder effect, or to both. In percent the differences are small, but it

[23] Hecker, R. and N.A. McDonald, "The Axial Spacing and Optimum Diameter of Counterrotating Propellers," David Taylor Model Basin Report 1342 (Feb 1960).

should be pointed out that the rudder effect on thrust deduction was treated theoretically as a constant disturbance factor in the singularity system, while in physical reality one might expect a thrust-loading condition that would produce a minimum thrust deduction with a rudder. This problem requires further investigation. For the present case experimental data at more extreme propeller thrust loadings would have been informative. The earlier discussion concerning the rudder contribution to the thrust deduction and its overall effect on CR propulsion (Table 2) seem to show that the single centerline rudder offers no advantage in this propulsion system.

Also, depicted in Figure 12 is the theoretical variation of the separate thrust-deduction coefficients versus T_1/T_2 for the forward and aft propellers. Of special interest is the intersection of the curves for $t_1 = t_2$ which shows that a thrust-loading ratio of about 0.4 is needed to accomplish equal division of the thrust deduction in this CR propeller set. The curves have been extended to the limiting values as $T_1 \to 0$. Reverse limiting values are obtained at the upper limit $T_1 \to \infty$. A calculation shows that $t_1/t \approx 0.98$ and $t_2/t \approx 0.02$ at $T_1/T_2 = 20$. These statements concerning extreme values of T_1/T_2 have only academic meaning, since they are not practical. However, the required linearity (Assumption 6, Statement of Problem) is preserved because the total-thrust coefficient

$$C_{Ths} = \frac{T_1 + T_2}{\frac{1}{2}\rho(A_0)_1 V^2} = 0.6924$$

is constant.

Determination of the separate thrust deduction coefficients t_1 and t_2 for the CR propellers was essential to finding the total thrust deduction of the CR propeller system and this point is emphasized. The variation of t_1 and t_2 with the thrust ratio T_1/T_2 may be important to the total propulsion problem. As discussed in the section on THRUST DEDUCTION AT OFF-DESIGN LOADING, equal thrust or equal torque may not necessarily give minimum shaft horsepower. For the containership, Figure 12 shows that the total thrust deduction is reduced by increasing the thrust on the aft propeller relative to the forward propeller. It may be possible to design a CR propeller pair (without centerline rudder) for unequal thrust producing a lower thrust deduction and no loss in propeller efficiency compared to an equal thrust condition. That this is possible is not reflected by the horsepower given in Table 2 for the containership when the rpm ratio n_1/n_2 was varied. However, it is noted in this case that the thrust unbalance is relatively small, the rudder effect is unclear and the off-design propeller efficiency may not be the best efficiency obtainable.

CONCLUDING REMARKS

A theoretical method was developed to determine the steady interaction force, thrust deduction, of contrarotating propellers. Numerical values of this force are needed to determine the propeller thrust required to propel a ship at design conditions, and to make parametric investigations of propulsive performance. In the method the force was obtained as a Lagally-type force which has its basis in a principle of reciprocity for finding forces between bodies in a flow field. Certain interference forces were shown to cancel. The methodology involved a combination of potential and viscous flow effects.

Application of the method was made to secure numerical results for a set of CR propellers designed for a MARAD high-speed containership. Some salient features and implications concerning the thrust deduction of the containership were:

1. Determination of the separate thrust deduction for the forward and aft propeller of the CR pair at design and off-design loading. At design loading 73 percent of the total thrust deduction was produced by the forward propeller and 27 percent by the aft. The total thrust deduction was in exact agreement with the experimental result because of the use of an influence coefficient in determining the final potential wake. Indications are that the distribution of the thrust deduction between propellers might be important with regard to minimum shaft horsepower. However, additional investigation will be required to clarify this point.

2. Computation of the rudder contribution shows agreement with experiment within better than two percent for one minus the thrust deduction.

3. As expected a simple monotonic reduction in thrust deduction was observed when the forward propeller location remained fixed and the aft propeller was moved downstream.

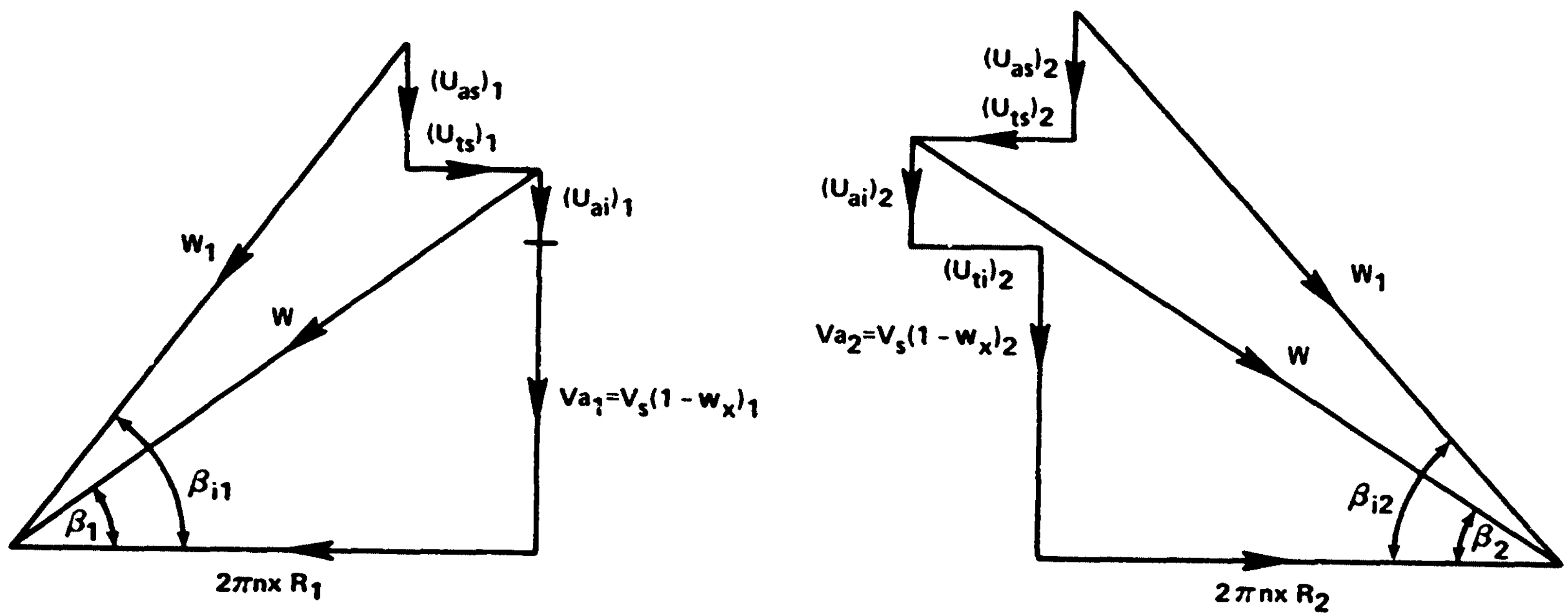


Figure 1 – Flow Velocity at Blade Section for CR Propellers

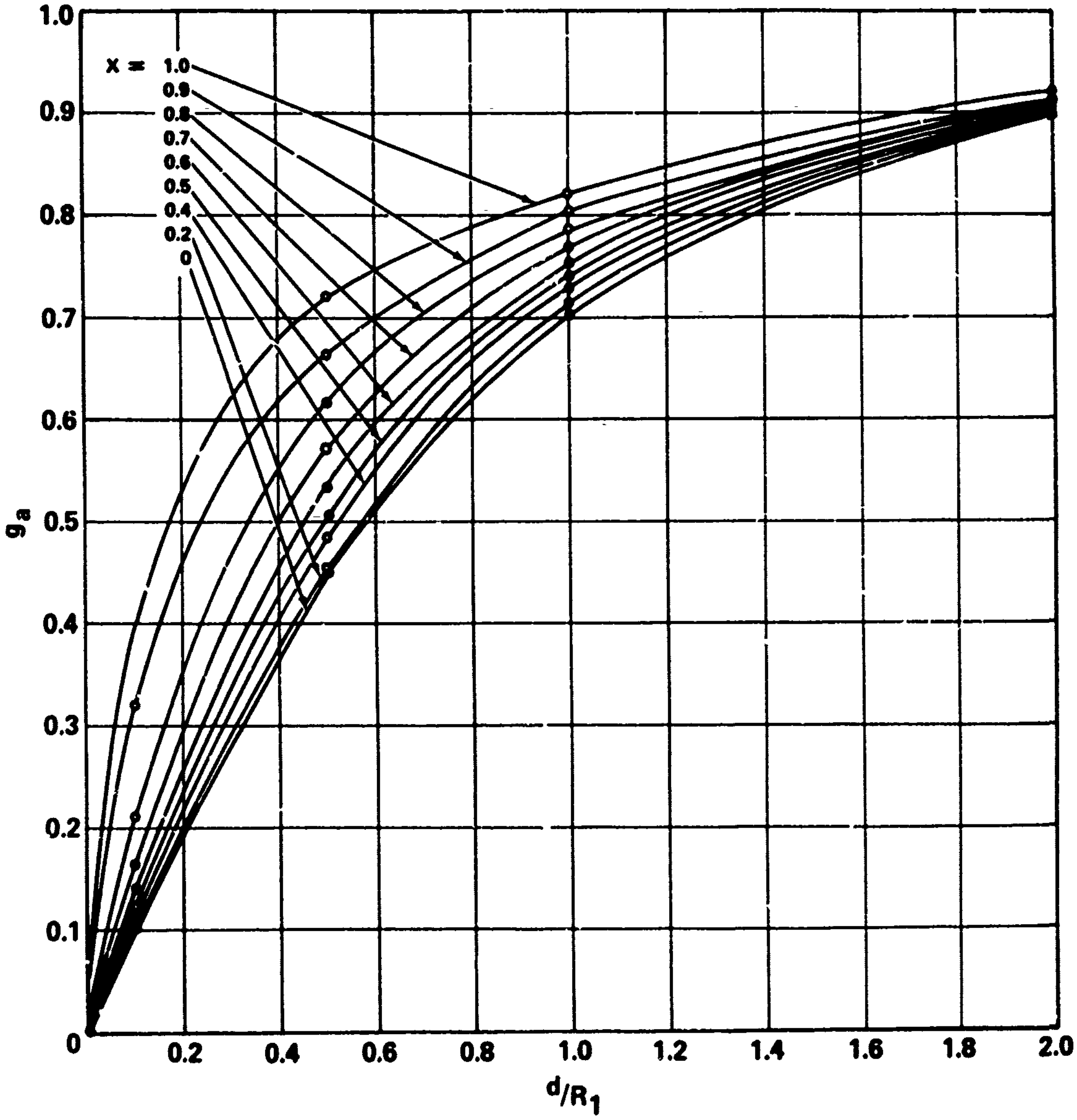


Figure 2 – Lerbs' Distance Factor

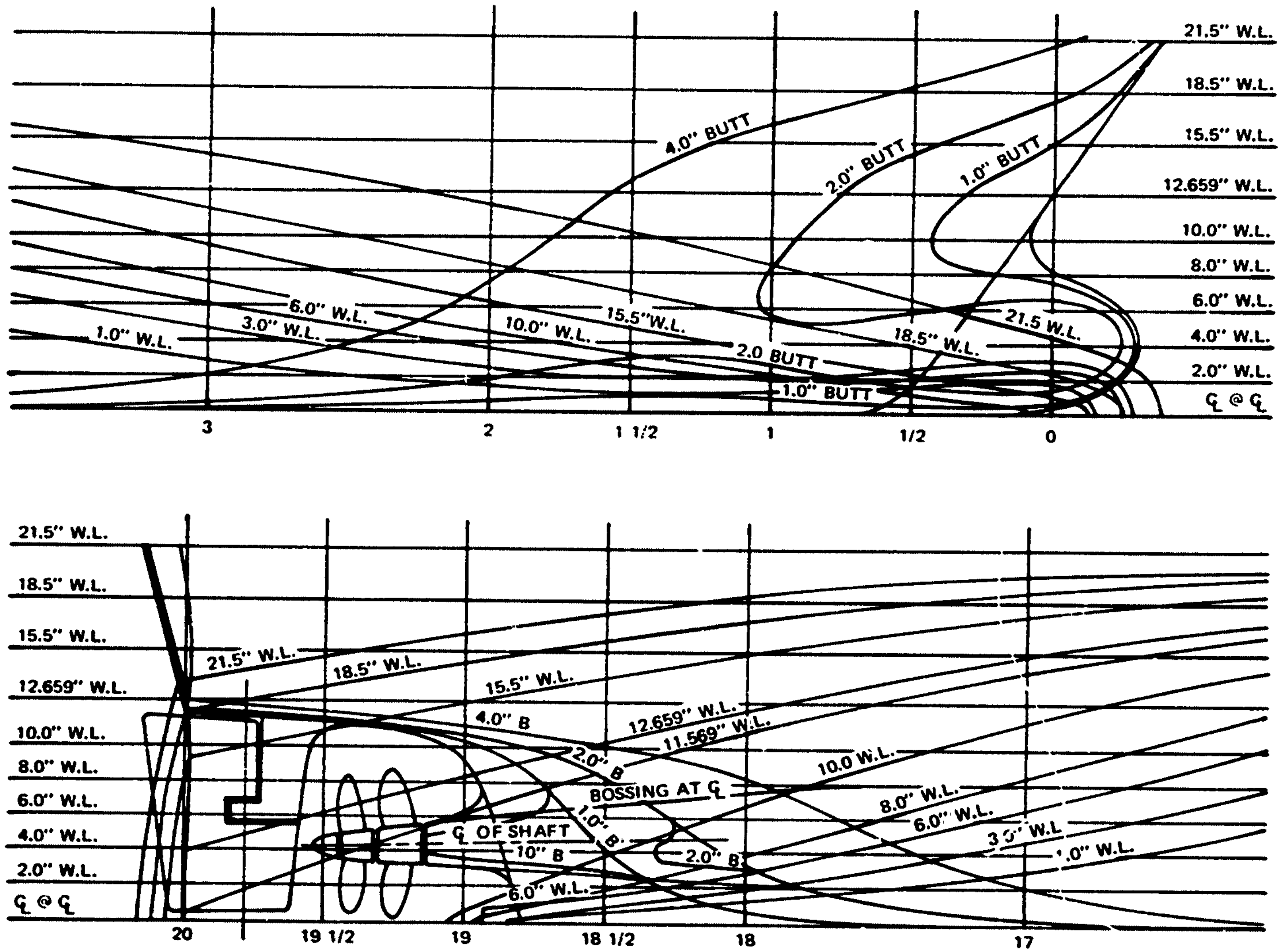


Figure 3 – Model 5218 Representing Single Rudder Contrarotating Containership Design

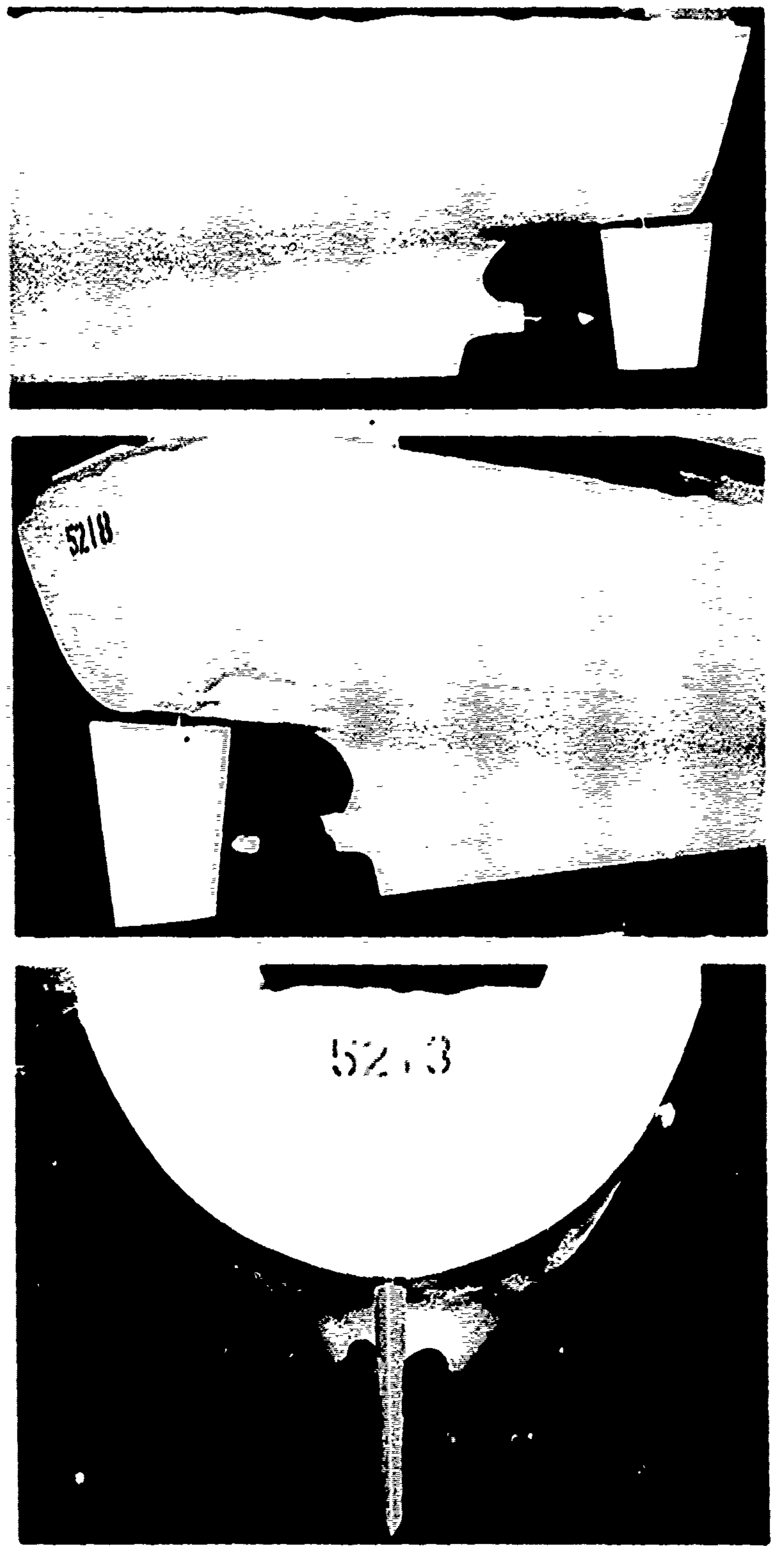
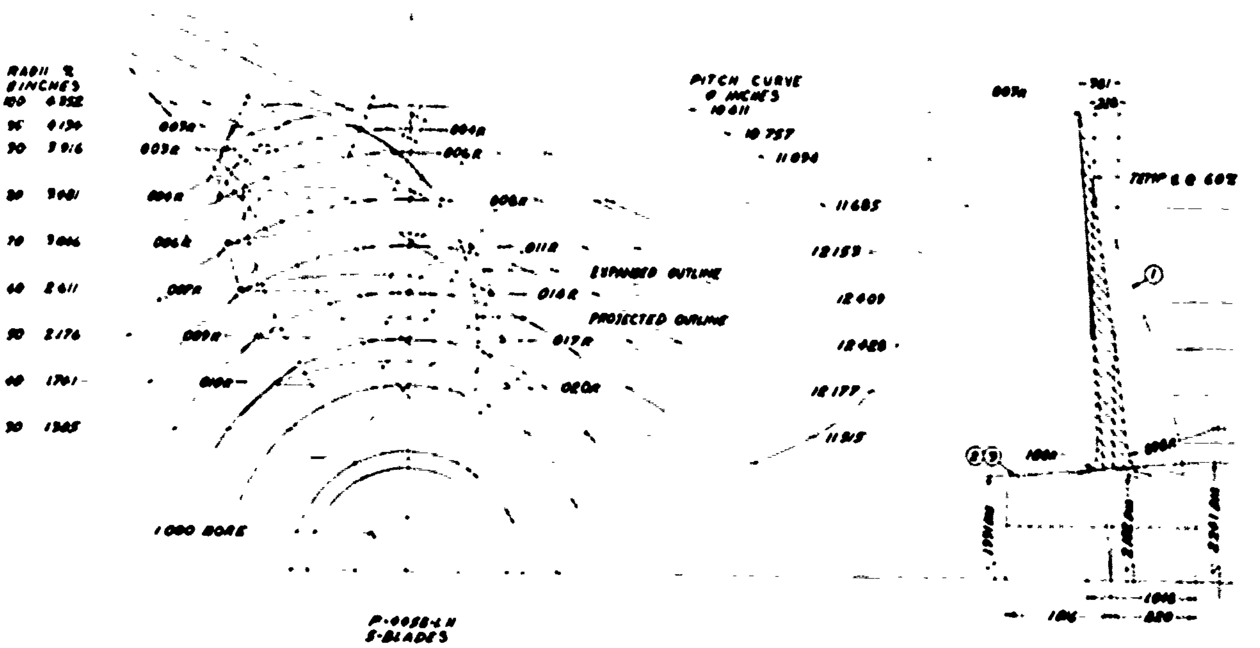
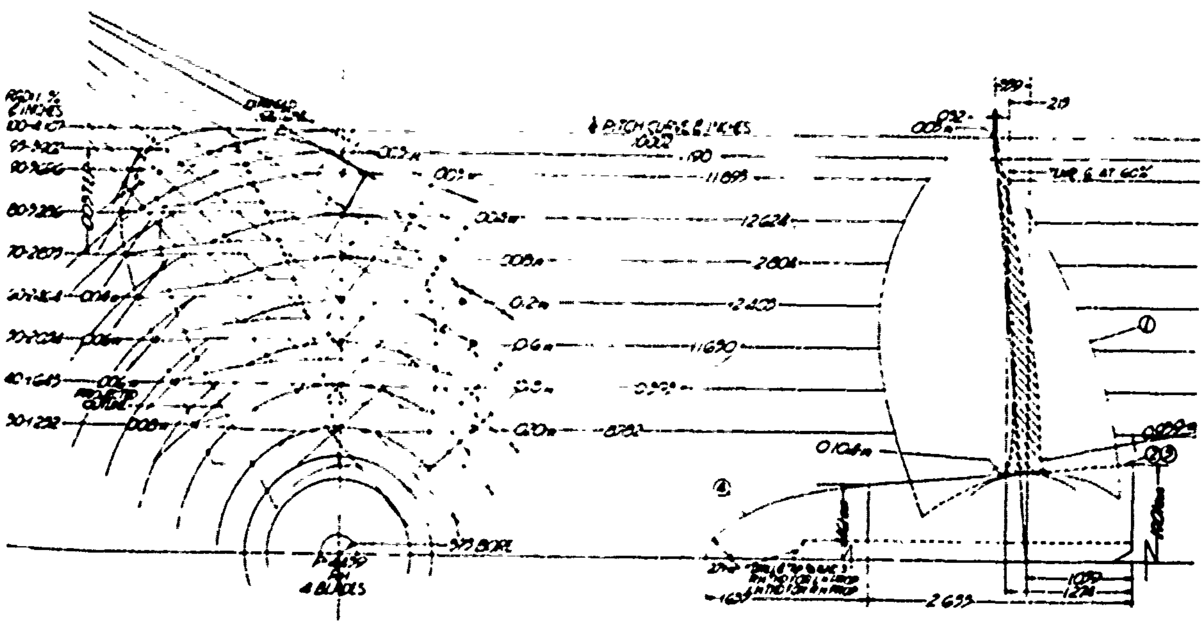


Figure 4 – Model 5218, Stern View



Number of blades	5	Propeller 4458	Diameter, in.	8.703
Exp. area ratio	0.576		Pitch at 0.7R, in.	12.153
MWR	0.238		Rotation	L.H.
BTF	0.045		Designed by	NSRDC
P/D at 0.7R	1.396			

Figure 5a – Characteristics of Contrarotating, Forward Propeller 4458 Designed for 25.5-Knot Containership



Number of blades	4	Propeller 4459	Diameter, in.	8.214
Exp. area ratio	0.701		Pitch at 0.7R, in.	12.804
MWR	0.349		Rotation	R.H.
BTF	0.048		Designed by	NSRDC
P/D at 0.7R	1.559			

Figure 5b – Characteristics of Contrarotating Aft Propeller 4459 Designed for 25.5-Knot Containership

Figure 5 – Characteristics of Contrarotating Propeller 4458 (Forward) and 4459 (Aft)

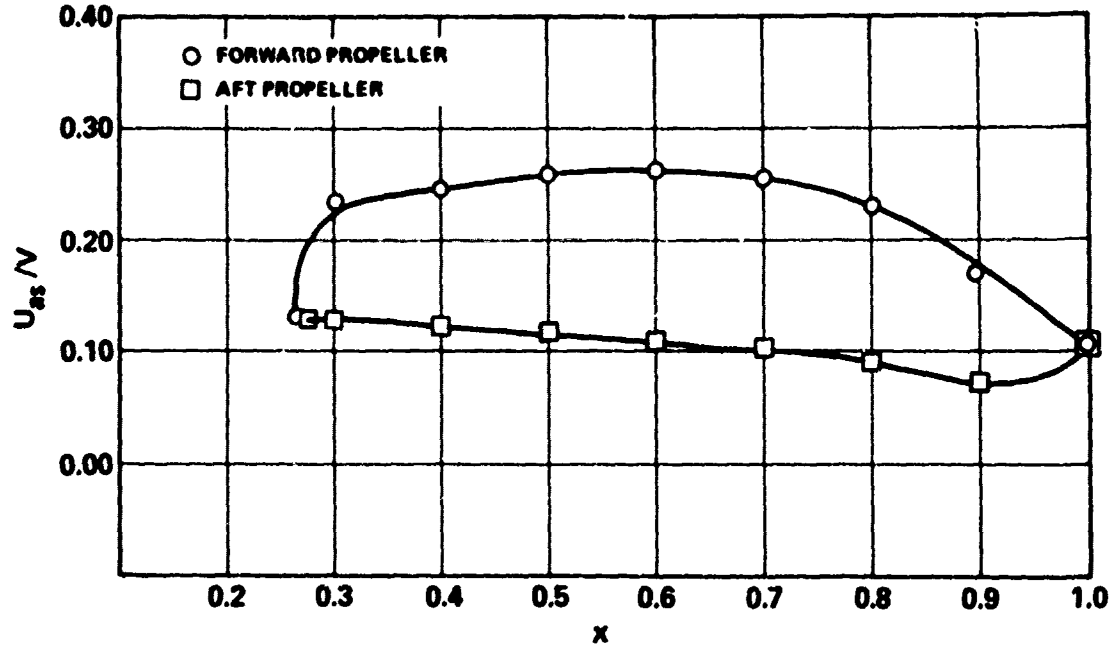


Figure 6 - Surface Sink Strength at Design Spacing. $d/R_1 = 0.52$

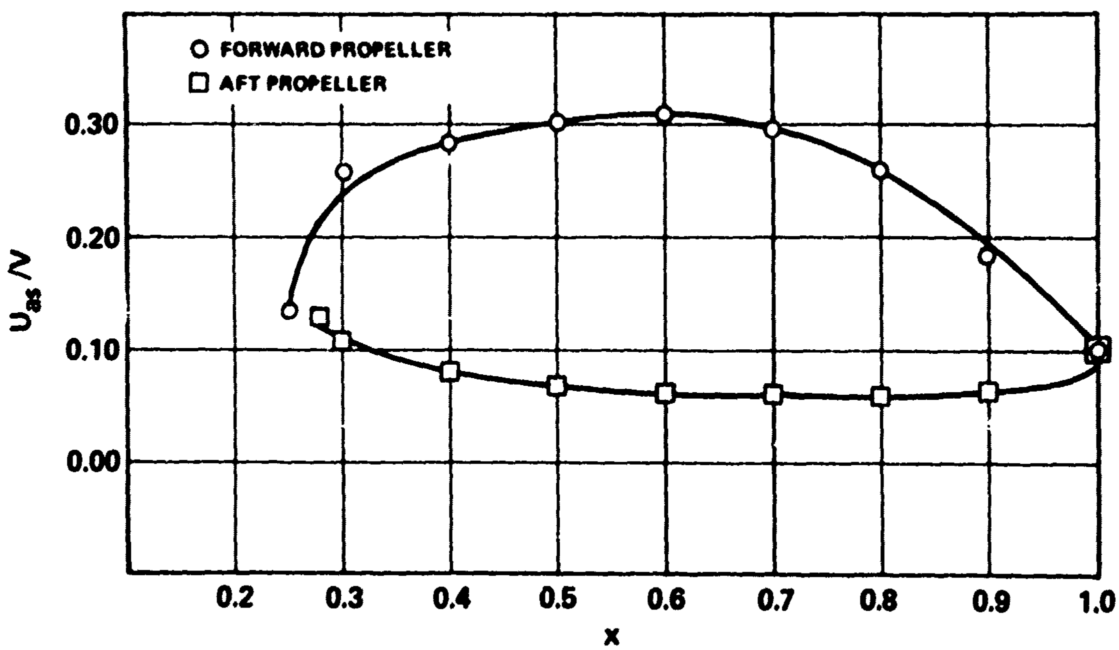


Figure 7 – Surface Sink Strength at Off-Design Spacing, $d/R_1 = 1.5$

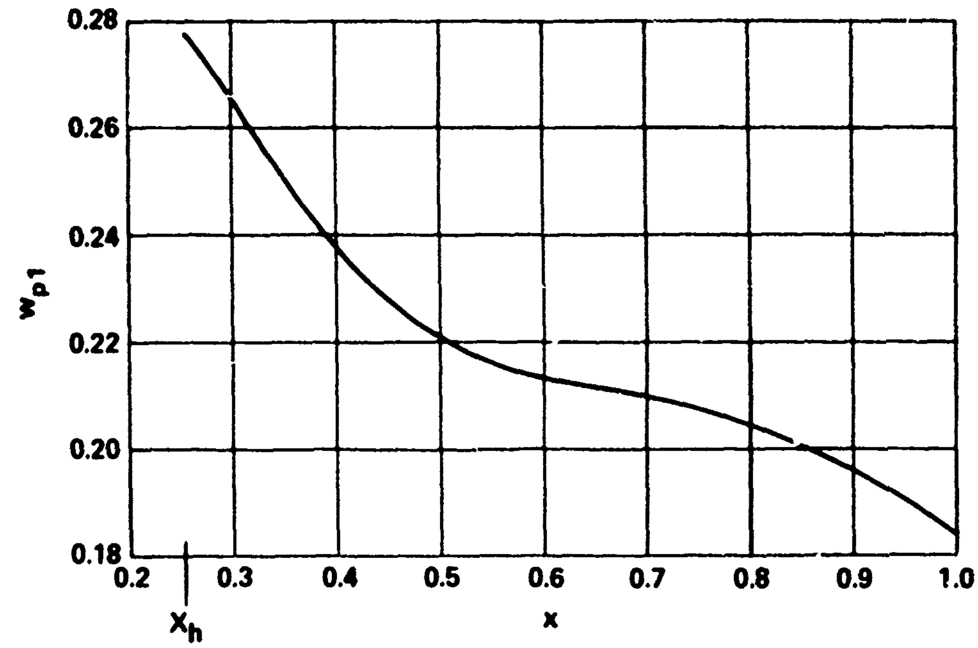


Figure 8 – Potential Wake Fraction for MARAD Containership at Forward Propeller

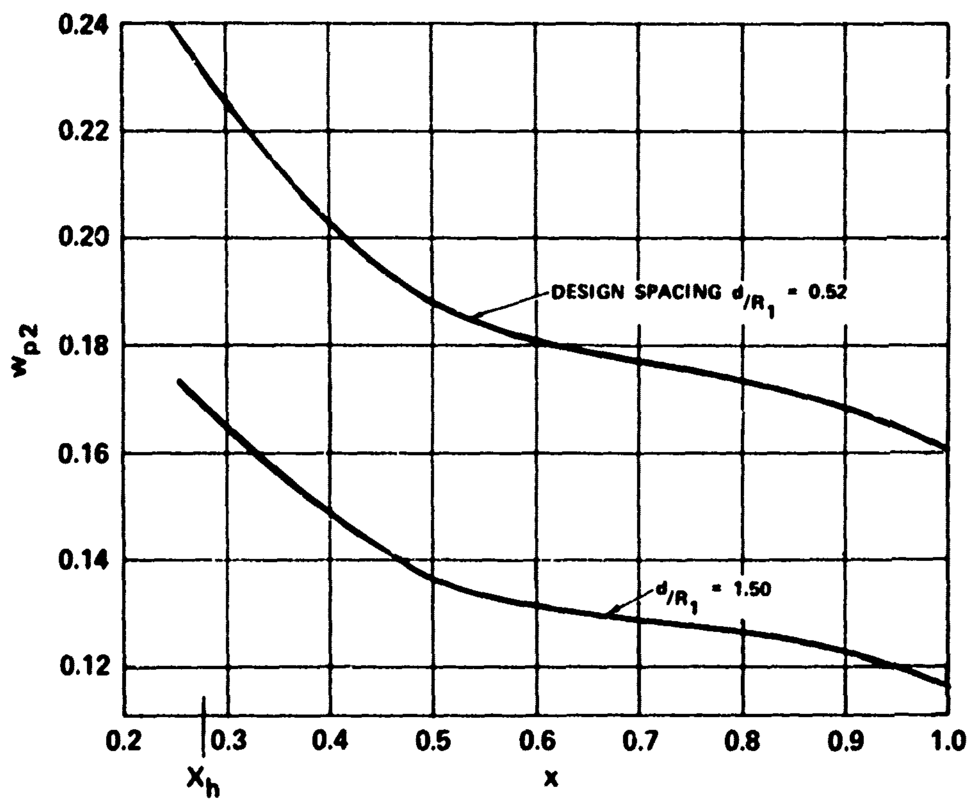


Figure 9 Potential Wake Fraction for MARAD Containership at Aft Propeller

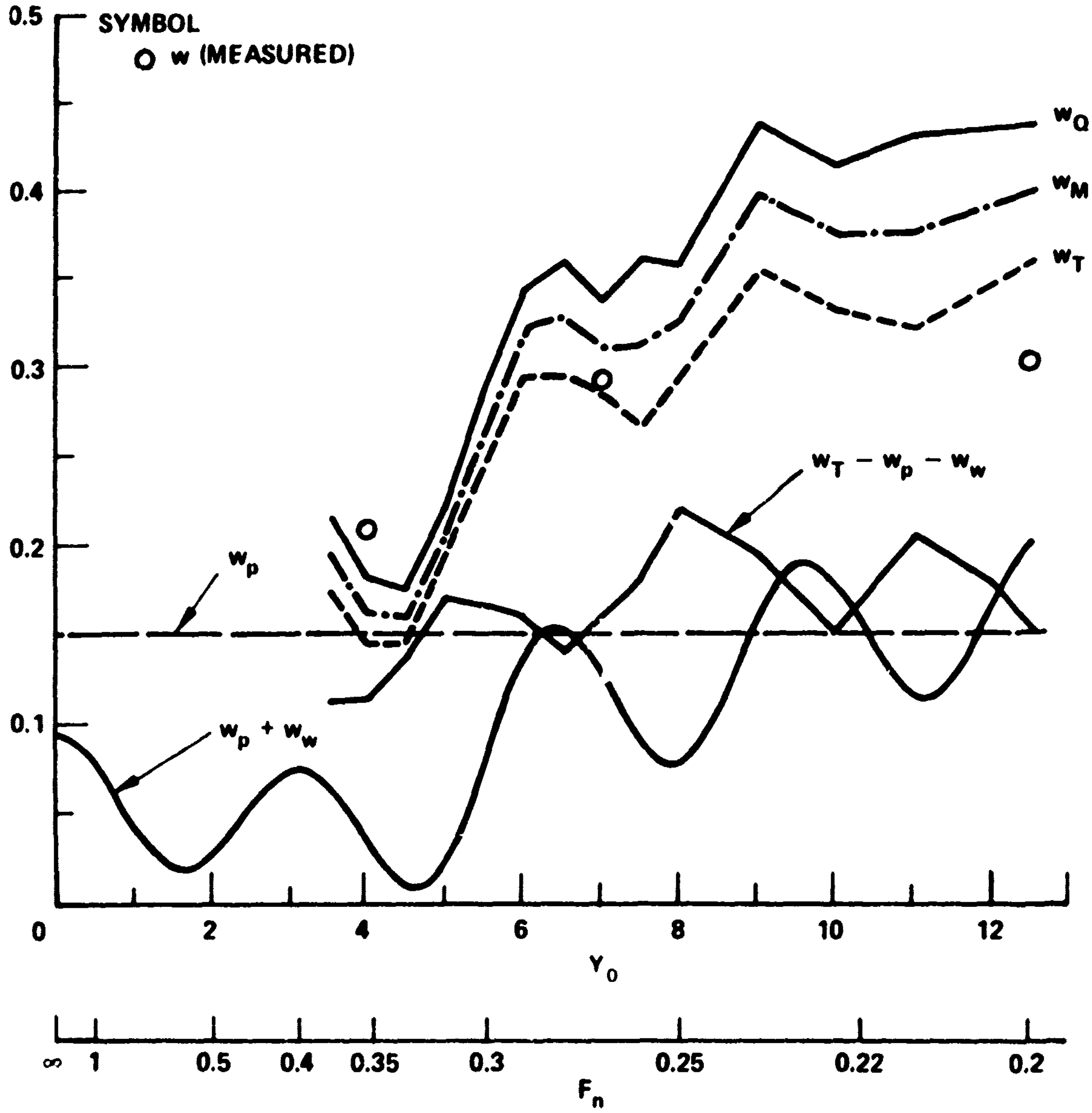


Figure 10 – Wake Components for an Idealized Hull Form (From Reference 22)

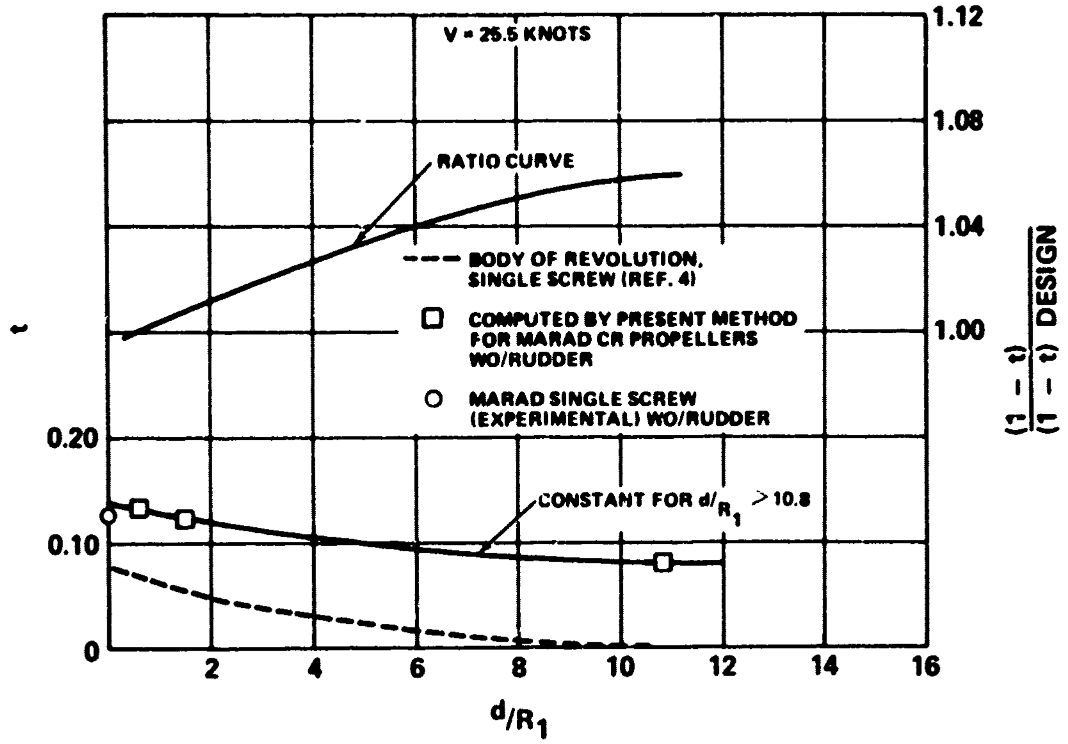


Figure 11 – Thrust Deduction as a Function of CR Propeller Spacing for MARAD Containership

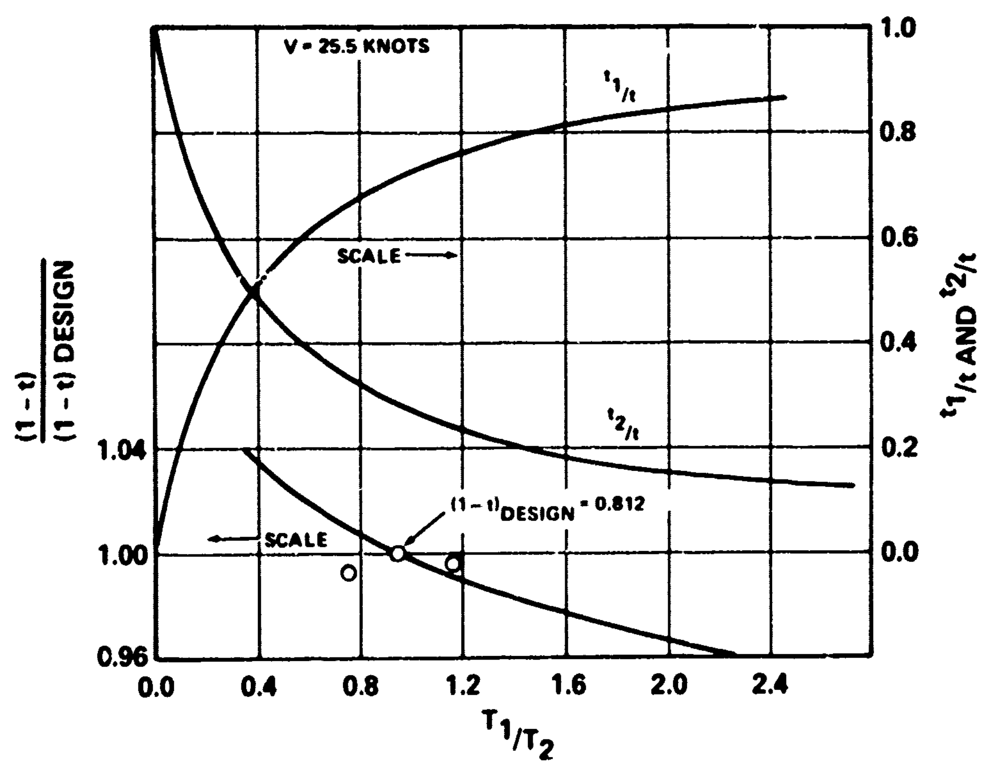


Figure 12 – Thrust Deduction as a Function of Relative Thrust Loading between CR Propellers for MARAD Containership

TABLE 1 - SHIP AND MODEL DATA FOR MODEL 5218
CONTRAROTATING CONTAINERSHIP DESIGN

APPENDAGES: Propeller Bossing (No Rudder or Bilge Keels)

DIMENSIONS			LWL COEFFICIENTS			
	Ship	Model	C_B	0.555	C_{WF}	0.62
Length (LWL), ft	778.8	25.673	C_P	0.590	C_{WA}	0.78
Length (LWP), ft	780.0	25.714	C_X	0.940	L_E/L	0.52
Beam (B_x), ft	103.4	3.408	C_W	0.697	L_P/L	0.00
Draft (H), ft	30.0	0.989	C_{PF}	0.57	L_R/L	0.48
Displ, tons	38520SW	1.342FW	C_{PA}	0.62	L/Bx	7.53
Wetted Surf, sqft	83920	91.7	C_{PE}	0.58	Bx/H	3.45
Design v, knots	25.5	4.63	C_{PR}	0.60	$\Delta/(0.01L)^3$	81.1
LCB_{LWL} = 394.8 Aft of FP			C_{PV}	0.80	$S/\sqrt{\Delta L}$	15.45
LCB_{LBP} = 395.5 Aft of FP			C_{PVA}	0.74	f	0.09
WL Entrance Half Angle = 7.0°			C_{PVF}	0.86	t	0.05
λ = 30.334 $V/\sqrt{L_{LWL}}$ = 0.914			LBP COEFFICIENTS			
K = 2.561 P = 0.887			C_B	0.555	L/Bx	7.53
Lines NSRDC Afterbody lines, Model 5218, 10 Nov 69			C_P	0.590	$\Delta/(0.01L)^3$	81.1

TABLE 2 – RESULTS OF SHIP TESTS AT 25.5 KNOTS FOR SHIP MODEL 5218, WITH CONTRAROTATING PROPELLER MODELS 4458 AND 4459

Rudder Arrangement	RPM-Ratio n_1/n_2	P_E hp	P_D hp	η_D	$1 - t$	$1 - w_T$	$1 - w_Q$	η_H	η_R	η_O	J_T	n_T RPM
Single	1.0	30830	40670	0.758	0.812	0.757	0.772	1.073	1.025	0.689	0.992	89.5
Single	1.05	30830	40730	0.757	0.810	0.770	0.788	1.053	1.031	0.697	0.981	92.1
Single	0.95	30830	40670	0.758	0.806	0.745	0.774	1.082	1.047	0.669	0.998	87.6
No Rudder	1.0	31120	39040	0.797	0.868	0.792	0.811	1.095	1.035	0.703	1.032	90.1

TABLE 3 THRUST DEDUCTION FOR VARIATIONS IN THRUST RATIO T_1/T_2

	T_1/T_2	T_1'	T_2'	t_1/t	t_2/t	t	$1 - t$
	0.4286	0.3	0.7	0.53	0.47	0.157	0.843
	0.6666	0.4	0.6	0.64	0.36	0.172	0.828
(Design)	1.0000	0.5	0.5	0.73	0.27	0.188	0.812
	1.500	0.6	0.4	0.80	0.20	0.203	0.797
	2.333	0.7	0.3	0.86	0.14	0.219	0.781

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