AD/A-002 942

ANALYSIS OF A PROPOSED TWO-FREQUENCY RADAR WAVEFORM

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Washington, D. C.

November 1974

AD/A 002 942

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS
BEFORE COMPLETING FORM

1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
NRL Memorandum Report 2926		

4. TITLE (and Subtitle)	5. TYPE OF REPORT & PERIOD COVERED
ANALYSIS OF A PROPOSED TWO-FREQUENCY RADAR WAVEFORM	Final Report
	6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(s)	8. CONTRACT OR GRANT NUMBER(s)
Dr. Merrill I. Skolnik	

9. PERFORMING ORGANIZATION NAME AND ADDRESS	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
Naval Research Laboratory Washington, D.C. 20375	65378 NC

11. CONTROLLING OFFICE NAME AND ADDRESS	12. REPORT DATE
Office of Naval Research Navy Department Washington, D.C. 20360	November 1974
	13. NUMBER OF PAGES
	~~68~~ 64

14. MONITORING AGENCY NAME & ADDRESS(If different from Controlling Office)	15. SECURITY CLASS. (of this report)
	UNCLASSIFIED
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, If different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Radar
Moving Target Indication (MTI) Radar
Frequency Agility

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The conventional MTI radar is not compatible with pulse-to-pulse frequency agility since the phase from a stationary clutter target will vary from pulse to pulse as the frequency changes. This can result in an uncancelled residue at the output of a delay line canceller. This report examines a proposed (unsuccessful) method for achieving frequency agility by radiating two frequencies which may take on any value from pulse to pulse, provided the mean frequency remains unchanged. On reception,
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20. (Continued Abstract)

the signals are combined so as to process the mean-frequency component. That is, the two frequencies on each pulse transmission are $f_o + \Delta f_1$ and $f_o - \Delta f_1$, where Δf_1 can be selected arbitrarily, and the processing is accomplished at f_o. The receiver utilizes an envelope detector as well as a coherent reference at the mean frequency. It is shown that the signal received from a single echo depends only on the mean frequency and not on the difference. This gave encouragement to the idea that MTI processing could be undertaken even though the difference between the two frequencies was changed pulse-to-pulse. Unfortunately, when clutter was considered, the phase of the processed signal was found to depend on the difference between the two frequencies. This gives rise to an uncancelled residue. Hence, this form of frequency agility is incompatible with MTI. It is possible, however, to utilize two or more widely spaced frequencies, either simultaneously or in time sequence, in a more conventional MTI system if the same frequencies are repeated.

CONTENTS

ANALYSIS OF A PROPOSED TWO-FREQUENCY RADAR WAVEFORM

1. INTRODUCTION

The use of the doppler frequency shift of the echo signal from a moving target is a well known method for separating desired moving targets from undesired stationary clutter in CW radar, MTI (Moving Target Indication) radar and pulse doppler radar.[1] Another radar technique, not as widely used, is frequency agility in which the transmitted frequency is changed from pulse-to-pulse. Generally, the frequency band in which agility is to occur is relatively wide compared to the spectral width of an individual pulse. Frequency agility is of interest for increasing the detectability of targets with fluctuating cross section, reducing the glint (angular wander) in tracking, improving low-altitude tracking, mitigating the effects of distributed clutter, and as an electronic counter-countermeasure to avoid spot jamming.[2-4] But whatever the reason for wishing to utilize frequency agility, a problem arises if doppler frequency extraction is also desired simultaneously. Conventional doppler processing and frequency agility are not compatible. Conventional MTI radar, which will be taken as the model of a doppler radar, requires the transmitted frequency to remain fixed for the duration of the doppler processing time (usually the time on target). This is, of course, inconsistent with pulse-to-pulse frequency agility. The advantages of both frequency agility and MTI in some applications are important enough to consider how these two competing processes might be accomodated.

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This report considers the problem of trying to achieve simultaneous frequency agility and MTI. When proper matched-filter processing is used there is no basic reason why the doppler frequency shift of a moving target cannot be extracted from the clutter when the transmitted signal spectrum extends over a wide band as in radar with the familiar FM-CW, FM (chirp) pulse compression, or noise-like waveforms that produce the thumb-tack ambiguity diagram. Therefore, it does not seem unreasonable to search for some type of frequency-agile waveform that would be compatible with MTI. The chief technique examined in this report is a radar waveform consisting of symmetrical pairs of frequencies which may be transmitted simultaneously or in sequence. The doppler signal extracted is that corresponding to the <u>mean</u>, or average, frequency of the pair of frequencies. (This is not to be confused with the so-called two-frequency MTI system which extracts the doppler frequency associated with the difference of the two frequencies.[5,6]) Preliminary analysis of this technique was encouraging — but deceptive. A more thorough examination revealed a fundamental weakness that negates its use as a frequency agile MTI waveform. It is possible to use this technique to transmit two or more widely spaced frequencies either simultaneously or in time sequence and to perform MTI provided the same frequencies are retransmitted. However, it does not seem possible to transmit an entirely different set of frequencies and still perform MTI.

The basis for interest in this particular waveform and processing, and the reason why it fails, is described in this report. Although the result is negative, it seems worthwhile to document this fact for the benefit of those who want to try again. Too often, ideas which turn out to be negative are not documented and are periodically reinvented.

Most of the details of the analysis are given in the appendices with only the highlights sketched in the text.

Before discussing the proposed technique and its limitations, the

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reason for the failure of frequency agility with conventional MTI will be described in the next section. The reasons for the failure of frequency agility in the basic MTI and in the two-frequency MTI are similar.

2. LIMITATION OF CONVENTIONAL MTI AND FREQUENCY AGILITY

In a conventional MTI radar of constant frequency, the phase of the echo signal from stationary (clutter) targets remains fixed from pulse to pulse. But from a moving target, the phase varies. This difference in behavior allows the desired moving target to be separated from the undesired stationary targets by filtering. When the frequency is changed, the phase from stationary clutter also changes and results in an uncancelled residue at the output of a delay line canceller. This uncancelled residue can be mistaken for a moving target.

Consider the transmitted pulse to have a form

$$e_t(t) = A(t) \cos (\omega_o t + \varphi_o) \tag{1}$$

where $A(t)$ represents the pulse modulation, ω_o is the frequency and φ_o is the phase. The signal received from a target at a range R_o is

$$e_r(t) = a_r \cos[(\omega_o + \omega_d)t - 2\omega_o R_o/c + \varphi_o] \tag{2}$$

where $\omega_d = 2\omega_o v_r/c$ is the doppler (angular) frequency shift from a target moving toward the radar with a relative velocity v_r. In the receiver, the signal is mixed (as in a phase detector) with a reference signal that is coherent with the transmitter to obtain

$$e_m(t) = a_m \cos(\omega_d t - 2\omega_o R_o/c) \tag{3}$$

This is from a moving target.

The signal from a stationary (clutter) target is found by setting $\omega_d = 0$, and is

$$e_c(t) = a_c \cos(2\omega_o R_o/c) \tag{4}$$

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Equation (3) shows that the signal amplitude received from a moving target varies with time in accordance with the doppler frequency, but that from a clutter target as in Eq. (4), is constant from pulse to pulse. A delay-line canceller that subtracts successive received pulses therefore removes the clutter, but energy from doppler-shifted signals remains.

When the carrier frequency ω_o changes, as in a frequency agile radar, Eq. (4) indicates that the phase of the clutter signal also changes. The result is that the delay-line canceller does not cancel successive clutter echoes and the residue at the output of the canceller can be mistaken for a moving target echo. This is the basic incompatibility between MTI radar and frequency agility. The change in frequency results in uncancelled residue from stationary targets that would not exist if the frequency were fixed. Thus it does not appear, in general, that conventional MTI processing can be used when succeeding pulses are not at the same frequency. Appendix I elaborates on this problem.

3. TWO-FREQUENCY WAVEFORM WITH MEAN-FREQUENCY PROCESSING

Consider a waveform consisting of two simultaneous pulses whose frequencies are symmetrically spaced about the mean frequency ω_o. One pulse is at a frequency $\omega_o + \Delta\omega_i$ and the other is at $\omega_o - \Delta\omega_i$, where $2\Delta\omega_i$ is the frequency separation. The subscript i is an index that corresponds to a specific pair of pulses in a periodic train of pulse pairs. The choice of $\Delta\omega_i$ can cover a wide range of frequencies. It is assumed that $\Delta\omega_i$ is larger than the spectral width of a single pulse. (If it is less than the pulse spectral width, the technique will work, but this case is uninteresting for practical applications of agility.) By transmitting a two-frequency signal, but processing the signal at the mean frequency, ω_o, it is possible for the phase shift associated with the higher frequency component to "compensate" that of the lower component, independent of the choice of $\Delta\omega_i$, when a single target echo is involved. This fact is what gave encouragement to the examination

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of the proposed technique. However, when clutter is properly con-
sidered, a limitation similar to that described in Sec. 2 appears.
The analysis and its consequences are outlined in the present section,
with the details given in Appendix II.

The signal transmitted by this radar is represented as

$$u_t(t) = a_t \cos \left[(\omega_o + \Delta\omega_1)t + \varphi_o + \varphi_1 \right]$$
$$+ a_t \cos \left[(\omega_o - \Delta\omega_1)t + \varphi_o - \varphi_1 \right] \tag{5}$$

The phase φ_o is associated with the oscillator generating the frequency
ω_o, and φ_1 is the phase associated with the oscillator of frequency
$\Delta\omega_1$. (These phases will be set equal to zero, since they have no
fundamental affect on the result. They are retained, however, in the
analysis of Appendix II.) The two radiated signals are obtained by
mixing the oscillator outputs of frequency ω_o and $\Delta\omega_1$ and taking their
sum and difference frequencies. A sketch showing the basic features of
the time waveform of the transmitted signal $u_t(t)$ is shown in Fig. 1a.
It consists of a carrier of frequency ω_o modulated by a frequency $\Delta\omega_1$.
The envelope of this time waveform is similar to the radiation pattern
of an interferometer antenna of two widely spaced elements. The
spectrum of the transmitted signal is sketched in Fig. 1b.

Single Target - The received signal is assumed to be from a
single target located at range R_o at a time $t=0$. It moves with a
velocity v_r relative to the radar. The received signal can be written:

$$u_r(t) = a_1 \cos \left[(\omega_o + \omega_d + \Delta\omega_1 + \Delta\omega_{d1})t - 2(\omega_o + \Delta\omega_1)R_o/c \right]$$
$$+ a_2 \cos \left[(\omega_o + \omega_d - \Delta\omega_1 - \Delta\omega_{d1})t - 2(\omega_o - \Delta\omega_1)R_o/c \right] \tag{6}$$

where $\omega_d = 2\omega_o v_r/c$ is the doppler shift of the target echo, and $\Delta\omega_{d1} = 2\Delta\omega_1 v_r/c$ is one-half the difference of the target doppler shifts at the
two radiated frequencies. The frequency $\Delta\omega_{d1}$ arises since the doppler
shifts at the two frequencies are not the same. In most conventional
radar applications, the spectral width of the transmitted signal is
narrow enough to assume the doppler shift is constant for each spectral
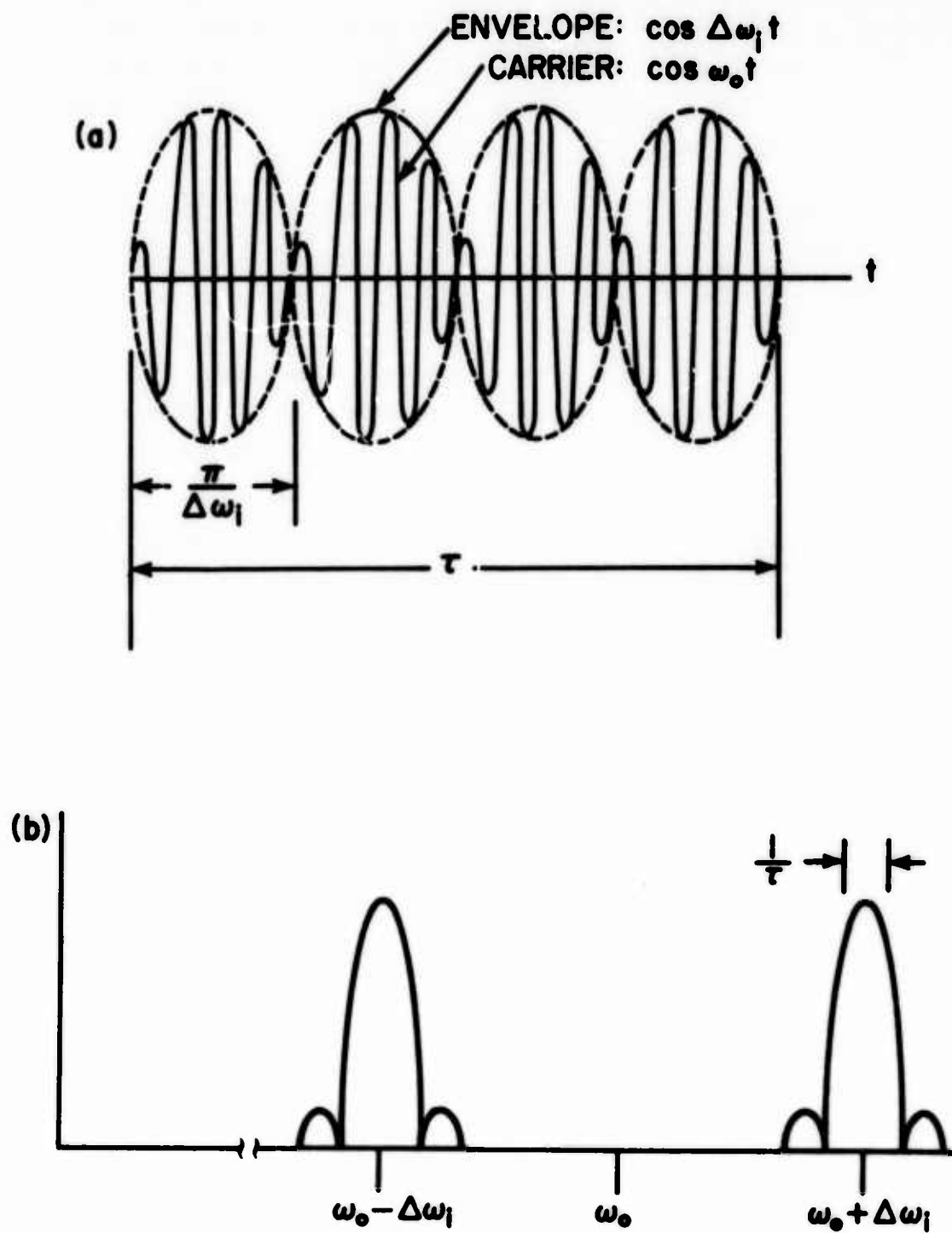component. When this is true, $\Delta\omega_{d1}$ may be neglected. In this study,

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FIG.1 (a) TIME WAVEFORM OF THE SUM OF TWO PULSES OF WIDTH τ AND OF FREQUENCY $\omega_o + \Delta\omega_i$ AND $\omega_o - \Delta\omega_i$. (b) SPECTRUM OF (a).

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however, wide separation between the two frequency components is desired so that this term cannot be neglected in general.

The type of processing to be employed with the received signal of Eq. (6) is outlined in Fig. 2. On reception, the echo is heterodyned with the reference signal $\cos(\omega_o t + \varphi_o)$ and the filter selects the component centered on $\Delta\omega_1$. Then the output of the filter is

$$u(t) = A \cos\left[(\Delta\omega_1 + \omega_d + \Delta\omega_{di})t - 2(\omega_o + \Delta\omega_1)R_o/c\right]$$

$$+ A \cos\left[(\Delta\omega_1 - \omega_d + \Delta\omega_{di})t + 2(\omega_o - \Delta\omega_1)R_o/c\right] \qquad (7)$$

In the above the amplitudes of the received signals at the two frequencies were assumed equal, hence $a_1 = a_2 = A$. (This is a reasonable assumption if the frequency separation is small. However, in the frequency-agile MTI, wide frequency separation is desired and the echoes might not be equal. For purposes of obtaining preliminary answers and a general insight into the problem, the assumption of equal-amplitude signals will be made. However, the effect of unequal amplitudes cannot be ignored in any complete analysis of this problem. It is discussed further in Appendix VI.) In Eq. (7), advantage was taken of the fact that $\Delta\omega_1$ is greater than ω_d so that the argument of the second term could be written as a positive quantity; i.e., $\cos(-\theta) = \cos\theta$.

Using trigonometry, Eq. (7) can be written as

$$u(t) = 2A \cos(\omega_d t - 2\omega_o R_o/c) \cos\left[(\Delta\omega_1 + \Delta\omega_{di})t - 2\Delta\omega_1 R_o/c\right] \qquad (8)$$

This is of the form $u_m(t) \cos(\omega t + \varphi)$, where $u_m(t)$ is a modulation of the carrier frequency $\Delta\omega_1 + \Delta\omega_{di}$. An envelope detector will extract the magnitude of $u_m(t)$. The output of the envelope detector is then

$$R(t) = 2A\left|\cos(\omega_d t - 2\omega_o R_o/c)\right| \qquad (9)$$

Equation (9) shows that the envelope of the output of the receiver when the input is the sum of two frequencies, is dependent only on the average frequency ω_o. It is independent of the spread $2\Delta\omega_1$ or of
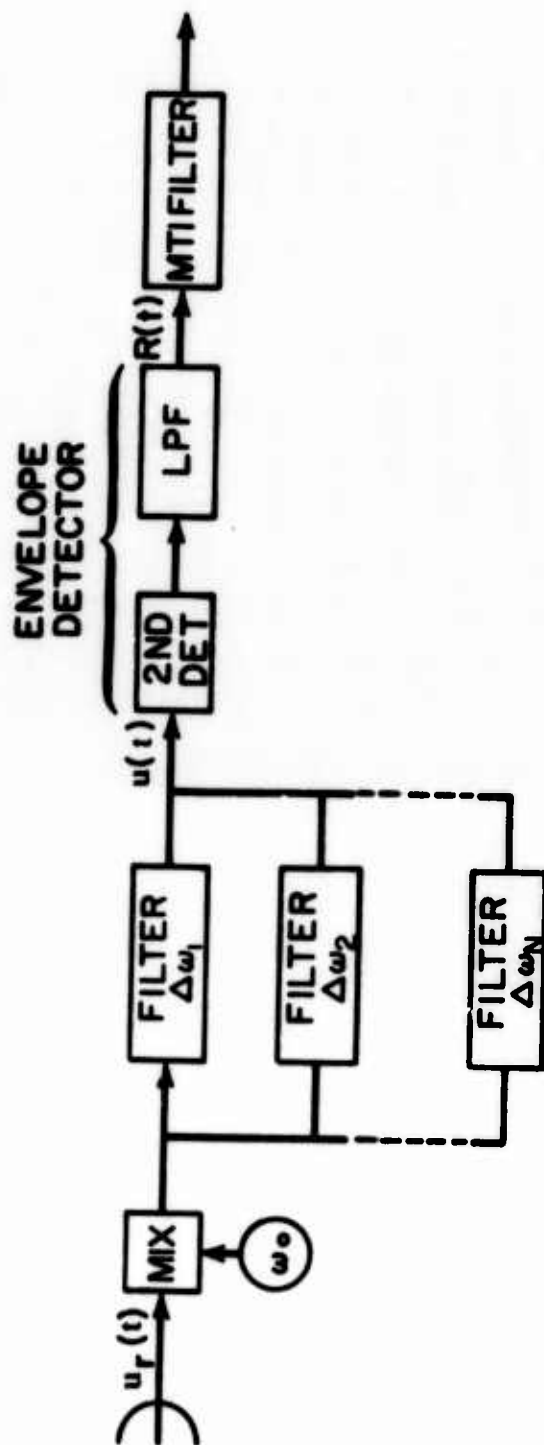
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FIG. 2 BLOCK DIAGRAM OF SUM SIGNAL (MEAN FREQUENCY) PROCESSING USING ENVELOPE DETECTION WHEN TRANSMITTING $\omega_0 + \Delta\omega_i$ AND $\omega_0 - \Delta\omega_i$ SIMULTANEOUSLY.

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either frequency of the transmitted symmetrical-frequency pair. There-fore, if on the next transmission a different value of $\Delta\omega_i$ were used, the receiver output will still be at the same doppler frequency ω_d and of the same phase $2\omega_o R_o/c$ as before (assuming frequency independence of the cross section). From this it might seem that the two pulses could be processed by a conventional MTI delay line canceller or any other MTI doppler processor. If Eq. (9) represented the envelope from a single clutter echo, two successive signals with different values of $\Delta\omega_i$ would cancel in a delay-line canceller. (It should be understood that superimposed on all these signals is the modulation of a pulse of width τ. The width of the "IF" filter centered at $\Delta f_i = \Delta\omega_i/2\pi$ must approximate the matched filter bandwidth.)

The expression for the envelope given by Eq. (9) is similar to the output of the phase detector in a conventional MTI radar operating at the single frequency ω_o. It is like a full-wave rectified sine wave. The consequences of this are discussed in Appendix III. If a square-law envelope detector were used, a sine wave at twice the doppler frequency would be obtained, which could be processed conventionally.

The block diagram of Fig. 2 shows an envelope detector rather than a conventional phase detector. The presence of an envelope detector does not mean that this is a noncoherent radar. The radar of Fig. 2 is coherent, with the "local oscillator" at frequency ω_o acting as the reference signal. It will be recalled that this reference signal is of the same frequency and phase as that of the transmitted signal.

So far, so good. It appears that we have a method for obtaining a doppler signal independent of the frequency difference, provided the mean of the pair of frequencies remains the same. The case of target and clutter is considered next, which will deflate the opti-mistic appraisal obtained from consideration of only a single target.

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Target and clutter - The transmitted signal is as before (Eq.(5)). The received signal is assumed to be from a single clutter echo at range R_o moving with a relative velocity v_c. Again, the target is at a range R_o with a relative velocity v_r. The signal returned from the target and clutter, after heterodyning each frequency with $\cos \omega_o t$ and passing through a filter center at $\Delta\omega_i$, can be written

$$
\begin{aligned}
u(t) = &\ A \cos[(\Delta\omega_i + \omega_d + \Delta\omega_{di})t - 2(\omega_o + \Delta\omega_i)R_o/c] \\
&+ A \cos[(\Delta\omega_i - \omega_d + \Delta\omega_{di})t + 2(\omega_o - \Delta\omega_i)R_o/c] \\
&+ C \cos[(\Delta\omega_i + \omega_c + \Delta\omega_{ci})t - 2(\omega_o + \Delta\omega_i)R'_o/c] \\
&+ C \cos[(\Delta\omega_i - \omega_c + \Delta\omega_{ci})t + 2(\omega_o - \Delta\omega_i)R'_o/c]
\end{aligned}
\tag{10}
$$

In the above it has been assumed that the amplitudes of the target echo at each frequency are both equal to A, and the amplitudes of the clutter echo are both equal to C.

The envelope of this signal cannot be readily obtained by simple trigonometric manipulation as was the case for the single target echo. The classical definition of the envelope as formulated by Rice[10] has to be used. This definition is given in Appendix III and its application to Eq. (10) is found in Appendix II. The square of the envelope $R(t)$ is

$$
\begin{aligned}
R^2(t) = 4 \Big\{ &A^2 \cos^2(\omega_d t - 2\omega_o R_o/c) \\
&+ 2AC \cos(\omega_d t - 2\omega_o R_o/c) \cos(\omega_c t - 2\omega_o R'_o/c) \\
&\quad \times \underline{\cos[(\Delta\omega_{di} - \Delta\omega_{ci})t - 2\Delta\omega_i(R_o - R'_o)/c]} \\
&+ C^2 \cos(\omega_c t - 2\omega_o R'_o/c) \Big\}
\end{aligned}
\tag{11}
$$

This is the output of a square law envelope detector. For a linear envelope detector, the square root of Eq. (11) has to be taken, but this is not easily obtained in closed form. It is noted that if in the second term, the third cosine factor (indicated by the dashed underline) can be set equal to unity, $R^2(t)$ is then a perfect square,

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and

$$R(t) \approx \left| 2A \cos(\omega_d t - 2\omega_o R_o/c) \right.$$
$$\left. + 2C \cos(\omega_c t - 2\omega_o R_o'/c) \right| \tag{12}$$

Thus if the underlined cosine factor has no effect, the operation of the envelope detector produces the sum of the two signals detected separately. Up to this point, it is still possible to be encouraged since Eq. (12) is independent on the choice of $\Delta\omega_i$. The question, however, is whether the third cosine factor in the second term can be ignored. (It can not!)

Both the frequency and the phase terms of the cosine factor must be examined. Consider the frequency term $\Delta\omega_{di} - \Delta\omega_{ci}$. We know that if the two frequencies $\omega_o + \Delta\omega_i$ and $\omega_o - \Delta\omega_i$ are close enough, the envelope detector will operate as desired. In this case $\Delta\omega_{di} \approx 0$ and $\Delta\omega_{ci} \approx 0$. This is the usual assumption made in an MTI radar; i.e., the doppler shift is taken to be the same for all components of the pulse spectrum. This is not the case, however, in a frequency agile radar. Appendix II gives the maximum change in $\Delta\omega_i$; i.e., $\delta\Delta\omega_i = (\Delta\omega_i - \Delta\omega_{i+1})$ that can be used before the differences in dopplers must be taken into account. This frequency change is less than is desired for the frequency-agile MTI discussed here. In fact it is less than the spectral width of the pulse. Therefore the frequency term of the cosine factor cannot be ignored so that Eq. (12) is not a complete description of the output from the envelope detector.

It might be argued that although this factor cannot be ignored it does not necessarily mean the frequency agile MTI will be degraded. In the presence of clutter only ($\omega_d = 0$), no new frequency components are generated. If Eq. (11) were to represent the echo from two clutter targets, new frequencies are generated but the spread is small since $\Delta\omega_i \ll \omega_o$. Furthermore, when target and clutter are present and the clutter doppler is spread due to $\Delta\omega_{ci} \neq 0$, the target doppler will likewise spread since $\Delta\omega_{dc} \neq 0$. Filtering is then possible when symmetrical frequency pairs are used if filtering is possible in the

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single frequency MTI. Thus, on the basis of the frequency term, there seems to be little problem.

However, when the _phase term_ of this factor is examined, $[\Delta\omega_i(R_o - R_o')/c]$, it is seen to depend on the choise of $\Delta\omega_i$, just as the phase of the conventional MTI of Eqs. (3) or (4) depends on the frequency ω_o. _Consequently, if the frequency difference $\Delta\omega_i$ is changed in a random manner, an uncancelled residue will result._ This fact, which was not evident from the analysis of a single target signal, is what prohibits the use of frequency agility even when two-frequencies are transmitted and the processing is performed at the mean frequency. The envelope detector cannot be considered as a linear device to which the principle of superposition can be applied.

The use of a phase detector as in a conventional MTI in place of the envelope detector does not change the conclusion. If the signal at the output of the "IF" filter at a frequency $\Delta\omega_i$ (as represented by Eq. (10)) is fed to a phase detector whose reference signal is $\cos \Delta\omega_i t$ the result, after removing the higher frequency terms, is

$$u_v(t) = 2A \cos(\omega_d t - 2\omega_o R_o/c) \cos(\Delta\omega_{di} t - 2\Delta\omega_i R_o/c)$$
$$+ 2C \cos(\omega_i t - 2\omega_o R_o'/c) \cos(\Delta\omega_{ci} t - 2\Delta\omega_i R_o'/c) \qquad (13)$$

This is not independent of $\Delta\omega_i$. In fact, when there is no target signal and only a single clutter echo of zero doppler there will be a factor $\cos 2\Delta\omega_i R_o'/c$ which varies as $\Delta\omega_i$ is varied. Consequently, the same problem exists in this form of processing. (Note that in the envelope detector, the echo from a single clutter target was independent of $\Delta\omega_i$; which is what made the consideration of that scheme attractive, until two clutter targets were considered.)

The two-frequency waveform with envelope detection was examined because the output signal from a single target was independent of $\Delta\omega_i$, but the output from a phase detector for a single target is not. When more than one signal is present, both techniques prove unsatisfactory. When the two (or more) frequency MTI is used with the same frequencies

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on each transmission, the type of detector that should be employed is determined by practical considerations.

4. DISCUSSION

The proposed frequency agility technique that radiates a pair of frequencies and combines the received signals so as to process the mean frequency component through an envelope detector, appeared attractive at first. A simple analysis of the signals from a single echo found them to be dependent only on the mean of the two frequencies. Thus the two frequencies could be changed on successive transmissions and doppler processing employed so long as the mean of the two frequencies remained constant. This attractive property evaporated when multiple echoes were present simultaneously within the radar resolution cell. When stationary, distributed clutter is considered, the phase of the processed signal depends upon the difference of the two frequencies and stationary clutter echoes will result in uncancelled residue from a delay line canceller if the frequencies are changed.

It is possible to use the two-frequency waveform for MTI provided the two frequencies remain fixed, if there were a desire to do so. The spacing between the frequencies can be wide and more than two can be used. They may be radiated simultaneously or they can be transmitted in time sequence provided the proper time delays are inserted on reception. The frequencies may be changed but the same frequencies need to be repeated if MTI processing is to be performed.

If it is possible to use a continuous waveform of sufficiently long duration, the frequency can be changed at random and still extract the doppler frequency shift. This type of waveform yields the classical "thumbtack" ambiguity diagram.[9] It may not be well suited for the air surveillance radar application since the required time duration of the continuous signal for doppler resolution is greater than the repetition period of the usual radar. The problem arises when a sampled rather than continuous waveform must be used. The aliasing caused by the

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sampling results in the clutter spectrum being spread over the entire available frequency domain, and thus causes energy to spread into the doppler region when frequency agility is attempted. This could conceivably be employed for MTI detection with a bank of contiguous filters covering the doppler band. Assuming a sufficiently large number of random frequencies so that the clutter energy is distributed uniformly, the improvement will be proportional to the number of parallel filters used.

If the material in the appendices to this report seem overly detailed for a study that has a negative result it is because the author performed much of this work thinking the result would be positive. A large part of the analysis was discarded after the negative result was realized, since it was no longer germane. The appendices that are included are, for the most part, in support of the main conclusion of this report.

APPENDIX I

ANALYSIS OF A CONVENTIONAL MTI PROCESSOR WHEN THE
CARRIER FREQUENCY IS VARIED PULSE TO PULSE

In Sec 2, the problem encountered is a conventional MTI was
briefly discussed. It was shown that when the carrier frequency of an
MTI radar is changed there is an uncancelled residue from a stationary
clutter target that appears as if it were from a moving target. This
appendix illustrates the problem in more detail. A possible correction
for the phase change with frequency is described, along with a dis-
cussion of the practical difficulties involved in its implementation.

To illustrate the nature of the problem, a simple model of the
MTI radar can be taken. Consider a transmitted pulse whose form is

$$e_t(t) = A(t) \cos(\omega_o t + \varphi_o) \tag{1}$$

The amplitude $A(t)$ is assumed to represent the pulse modulation, ω_o is
the frequency and φ_o is the phase of the transmitted signal. An echo
from a target at a range R will be received $T = 2R/c$ seconds after
transmission, where c is the velocity of propagation. The received
signal is then

$$e_r(t) = a_t A(t-T) \cos[\omega_o(t-T) + \varphi_o] \tag{2}$$

where a_t represents the reflectivity of the target. If the target is
moving with a relative velocity v_r, then
$R = R_o - v_r t$, where R_o is the range at $t = 0$. The received signal is

$$e_r(t) = a_t A(t-T) \cos[(\omega_o + \omega_d)t - \varphi_r + \varphi_o] \tag{3}$$

where $a_t A(t-T)$ is the received amplitude, $\omega_d = 2\omega_o v_r/c$ is the doppler
(angular) frequency shift and $\varphi_r = 2\omega_o R_o/c$. On reception the signal is

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mixed to baseband in a phase detector whose reference signal is $\cos(\omega_o t + \varphi_o)$. Therefore we have:

for a moving target—

$$e_m(t) = a_m \cos(\omega_d t - 2\omega_o R_o/c) \tag{4a}$$

for a stationary clutter target ($\omega_d = 0$) —

$$e_c(t) = a_c \cos(2\omega_o R_o/c) \tag{4b}$$

Equation 4b represents a single clutter echo whereas in reality clutter consists of the summation of the individual contributions from many scatterers, so that

$$E_c(t) = \sum_{i=1}^{N} a_i \cos(2\omega_o R_i/c) \tag{4c}$$

The above applies to a single pulse. If we compare this pulse with one transmitted T_p seconds earlier, the phase (argument of the sinusoidal factor) will change for the moving target because of the factor $\omega_d t$, but the phase will be unchanged for the stationary clutter. Thus it is possible to subtract successive pulses in a delay-line canceller and eliminate the fixed clutter targets, but not the moving targets.

Assume that the earlier pulse was transmitted with a frequency ω_2 and the later pulse at a frequency ω_1. The two received signals can be written:

for a moving target —

pulse 1 $$e_{m1}(t) = a_m \cos(\omega_{d1} t - 2\omega_1 R_o/c) \tag{5a}$$

pulse 2 $$e_{m2}(t) = a_m \cos(\omega_{d2} t - 2\omega_2 R_o/c) \tag{5b}$$

for a single stationary clutter target —

pulse 1 $$e_{c1}(t) = a_c \cos(2\omega_1 R_o/c) \tag{5c}$$

pulse 2 $$e_{c2}(t) = a_c \cos(2\omega_2 R_o/c) \tag{5d}$$

for a group of clutter scatterers —

pulse 1
$$E_{c1}(t) = \sum_1 a_1 \cos(2\omega_1 R_1/c) \qquad (5e)$$

pulse 2
$$E_{c2}(t) = \sum_1 a_1 \cos(2\omega_2 R_1/c) \qquad (5f)$$

For the case of a moving target the phase of the signal (Eqs. 5a and 5b) is different on each pulse, just as in the case of a single frequency, and there would be an output from the delay line canceller. (If a conventional, parallel-bank of narrow-band filters were used insread of a delay line canceller, the different values of doppler frequency resulting from the different carrier (RF) frequencies might cause the received signal energy to be spread among several filters and so reduce the energy in any one of them.)

The phase of the signal from a stationary clutter target (Eqs. 5c and 5d) is seen to vary as the frequency is changed. Hence a residue would appear at the output of a delay line canceller and a stationary target would result in a false alarm.

Note that if the range to the target R_o is <u>precisely</u> known it is possible to compensate for the difference in phase, which is $\Delta\varphi = 2(\omega_2-\omega_1)R_o/c$. This type of phase compensation is similar to that of a phased array. In the array, the proper phase at each element must be selected for a particular angle of arrival. In the frequency-agile radar the proper phase shift at each frequency must be selected for a particular range. Figure I.1 illustrates this for two frequencies.

In practice, however, it might not be too convenient to make such a compensation because of the large number of range elements. If the phase shift, $\Delta\varphi$, must be known to 0.1 radian and if the two frequencies are separated by 10 MHz, then the range must be known to an accuracy of about 25 cm. Thus there should be a range gate every 25 cm, but this is shorter than most radar pulses or radar targets. In a 100 km range, there are 4×10^5 range intervals 25 cm in extent. However, because of the modulo 2π nature of the phase there are needed only 63 sets of phase corrections. Although this approach seems to have its
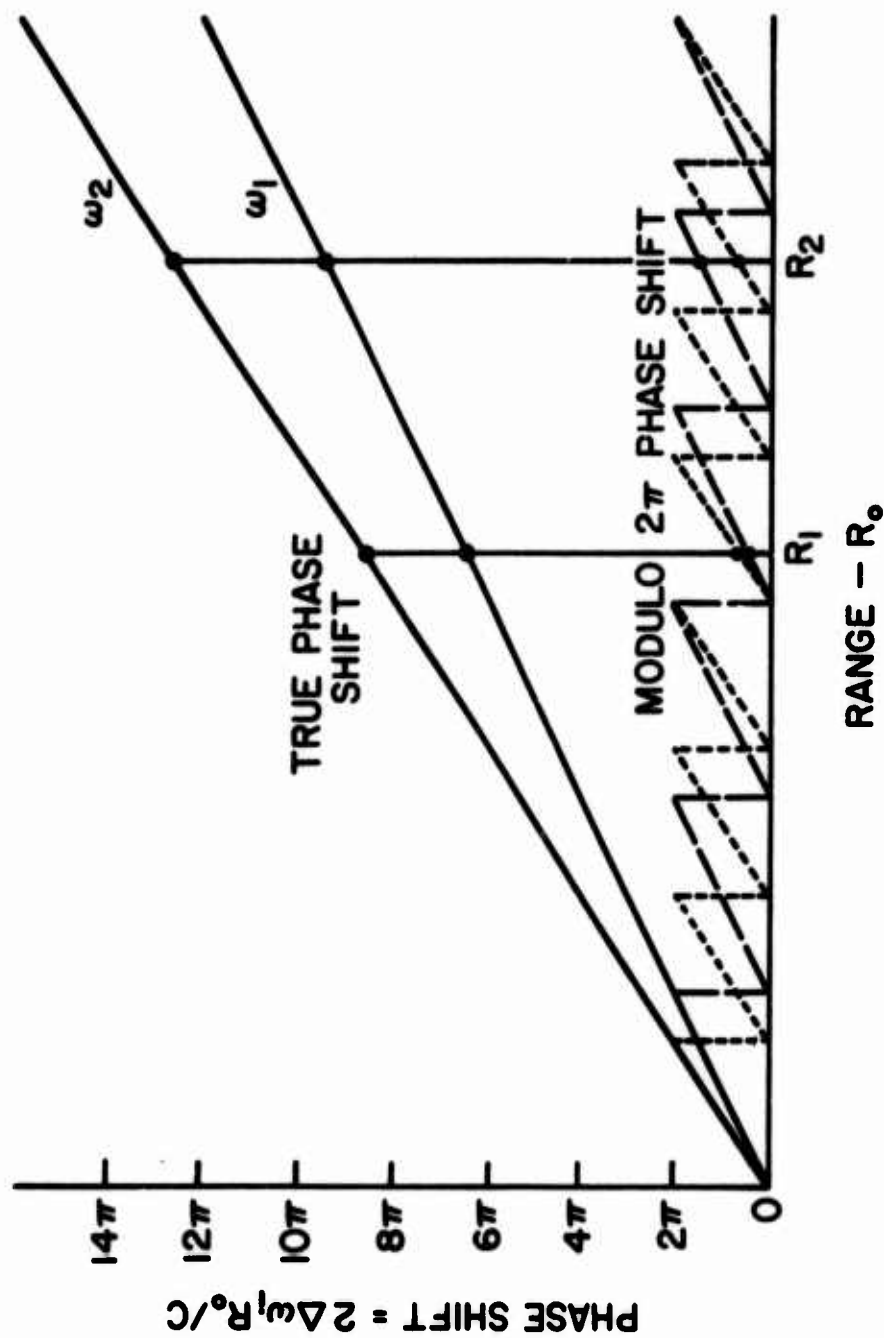
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FIG I.I VARIATION OF THE REQUIRED PHASE CORRECTION AS A FUNCTION OF THE RANGE WHEN TWO FREQUENCIES, ω_1 AND ω_2, ARE TRANSMITTED.

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difficulties, it would seem desirable to study it further (this is not done here) to determine if it is practical to implement.

Even if the range to a single clutter target were known and compensations were possible, the presence of multiple clutter scatterers within a pulse width might further complicate the problem, especially since it is unlikely that the separation of clutter scatterers within the pulse is known. Furthermore to make a proper compensation, each of the contributions from the individual clutter scatterers might have to be separated somehow and the proper phase shift separately applied to each. It does not seem that this is easy to do.

OUTPUT OF THE ENVELOPE DETECTOR WHEN THE OUTPUT
IS TARGET SIGNAL AND CLUTTER

In this appendix the output of the envelope detector is calcu-
lated when the input consists of a two-frequency waveform reflected
from a moving target and from clutter. The processing is assumed to
be like that of Fig 2. of the text, in which the mean of the two
frequencies is utilized. The details here are in support of the
discussion presented in Sec 3. Some of the preliminaries of Sec 3
are repeated since it is desired that this appendix be complete in
itself, except for its use of the definition of the envelope detector
given in Appendix III.

The radar is assumed to radiate two pulses simultaneously on each
transmission. One is at a frequency $\omega_o + \Delta\omega_i$ and the other at $\omega_o - \Delta\omega_i$.
the subscript i is an index designating the pulse-pair transmission.
The two frequencies radiated are taken symmetrically around the frequency
ω_o, but $\Delta\omega_i$ is assumed to be a random value within a wide band of fre-
quencies. The frequency ω_o is the mean of the pair of frequencies and
remains the same for each pulse-pair transmission. With frequency
agility, it is $\Delta\omega_i$ that is varied. It is assumed that $\Delta\omega_i$ is large
compared to the spectral width of a single pulse. The transmitted
signal can be written

$$u_t(t) = a_t \cos[(\omega_o + \Delta\omega_i)t + \varphi_o + \varphi_i]$$
$$+ a_t \cos[(\omega_o - \Delta\omega_i)t + \varphi_o - \varphi_i] \tag{1}$$

The phases φ_o and φ_i appear (with their respective signs) because it is
assumed that the two transmitted signals are generated by mixing the

outputs of two oscillators with frequency and phase ω_o, φ_o and $\Delta\omega_1$, φ_1, respectively, and taking the sum and difference frequencies. (Superimposed on this waveform is the pulse of duration τ. It is omitted in this formulation so as not to overly complicate the analysis. Its effect must be kept in mind, however, when specifying the bandwidth of the filters within the receiver.)

The received signal from a single target is

$$u_r(t) = a_1 \cos\left[(\omega_o + \Delta\omega_1)(t-T) + \varphi_o + \varphi_1\right]$$

$$+ a_2 \cos\left[(\omega_o - \Delta\omega_1)(t-T) + \varphi_o - \varphi_1\right] \quad (2)$$

where the time delay $T = (2R_o/c) - 2v_r t/c$, R_o = range to the target at time $t = 0$, c = velocity of propagation, v_r = relative velocity of the target. The assumption is made that the amplitudes (a positive quantity) of the two frequency components are equal so that $a_1 = a_2$ = A. Strictly speaking, this assumption is correct only for a fictitious scatterer, but it is taken here for convenience. This assumption is examined further in Appendix VI. Substituting for T, Eq. 2 becomes

$$u_r(t) = A \cos\left[(\omega_o + \omega_d + \Delta\omega_1 + \Delta\omega_{d1})t - 2(\omega_o + \Delta\omega_1)R_o/c + \varphi_o + \varphi_1\right]$$

$$+ A \cos\left[(\omega_o + \omega_d - \Delta\omega_1 - \Delta\omega_{d1})t - 2(\omega_o - \Delta\omega_1)R_o/c + \varphi_o - \varphi_1\right] \quad (3)$$

where $\omega_d = 2\omega_o v_r/c$ is the doppler shift that would be experienced if a frequency ω_o were transmitted and $\Delta\omega_{d1} = 2\Delta\omega_1 v_r/c$ is one-half the difference in the doppler shifts associated with the two transmitted frequencies.

The object is to process this signal so that MTI (doppler) discrimination of a moving target from stationary clutter can be accomplished. One approach to MTI operation is to extract the difference frequency $2\Delta\omega_1$. This is an old idea, and has been examined in the literature.[5-7] Its purpose is to obtain the benefit of the higher first blind speed of the difference frequency, as compared to

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the blind speed of either $\omega_o - \Delta\omega_i$. The difference frequency is extracted by mixing the two frequency components in a nonlinear device. Although this has some interesting consequences, especially for MTI on a moving platform (AMTI), the nonlinear processing results in some undesirable properties, such as a spreading of the clutter spectrum as compared to that of a single frequency system.[7] A comparison of this form of two-frequency MTI with that proposed here is given in Appendix VII.

The transmitted waveform used here (Eq 1) is the same as that in the dual-frequency MTI which extracts the difference signal after mixing, but the processing is significantly different. Instead of extracting the difference of the two frequencies, processing is performed at the average, or mean, frequency (ω_o). To accomplish such processing it is necessary to convert the two frequency components of Eq 3 to an "IF" frequency equal to the offset frequency $\Delta\omega_i$. Therefore, the received signal is heterodyned with the reference signal $2\cos(\omega_o t + \varphi_o)$ and the output is passed through a filter centered at $\Delta\omega_i$, as illustrated in Fig 2 of the text. The filter must be wide enough to pass the frequency components due to the pulse of width τ, as well as the frequency shifts due to ω_d and $\Delta\omega_{di}$, which are usually small compared to the spectral width of the pulse when the target is an aircraft. The output of the filter is then

$$u(t) = A\cos\left[(\Delta\omega_i + \omega_d + \Delta\omega_{di})t - 2(\omega_o + \Delta\omega_i)R_o/c + \varphi_1\right]$$
$$+ A\cos\left[(\Delta\omega_i - \omega_d + \Delta\omega_{di})t + 2(\omega_o - \Delta\omega_i)R_o/c + \varphi_1\right] \qquad (4)$$

Note in the above that $\Delta\omega_i$ is larger than ω_d or $\Delta\omega_{di}$, so the sign of the argument of the second term has been changed to recognize that the frequency is positive. This is permitted since $\cos(-\phi) = \cos\phi$. Using the trigonometric identity $\cos\alpha + \cos\beta = 2\cos[(\alpha - \beta)/2]\cos[(\alpha + \beta/2]$, Eq 4 can be written

$$u(t) = 2A\cos(\omega_d t - 2\omega_o R_o/c)\cos\left[(\Delta\omega_i + \Delta\omega_{di})t - 2\Delta\omega_i R_o/c + \varphi_1\right] \qquad (5)$$

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After passing through an envelope detector, the following is obtained

$$R(t) = 2A|\cos(\omega_d t - 2\omega_o R_o/c)| \tag{6}$$

Note that this is independent of the choice of $\Delta\omega_i$, a result that is desired. Therefore, if on succeeding pulses different values of $\Delta\omega_i$ are selected so as to achieve the effects of frequency agility, the output of the envelope detector will still be of the form given by Eq 6. This is what is required for combining frequency agility with MTI. Keep in mind, however, that the above applies to a single target. (When clutter is considered it will be found that linear superposition does not apply, and the consequences will be different.)

The output of the envelope detector as given by Eq 6 is a full-wave rectified cosine wave. Its major frequency component is at $2\omega_d$, as described in Appendix III, plus higher harmonics. If the distortion of the higher harmonics were to prove harmful, a square-law envelope detector can be used to give an envelope

$$R^2(t) = 4A^2\cos^2(\omega_d t - 2\omega_o R_o/c)$$

$$= 2A^2[1 + \cos(2\omega_d t - 4\omega_o R_o/c)] \tag{7}$$

After filtering the dc component, a pure cosine wave remains at a frequency $2\omega_d$. Higher harmonics are absent. The doppler filters would have to be designed on the basis of double the doppler frequency that would be expected from a conventional MTI radar which operated at ω_o.

In the above, each frequency component of the received signal was heterodyned by the mean frequency ω_o to obtain two signals, each of the same approximate carrier frequency. This has the effect of removing ω_o and φ_o from Eq 3. It can be considered a linear operation just as the first mixer of a superheterodyne is a "linear" device so long as the reference signal is large compared to the input. If ω_o is not removed from the input signals, a different answer will result. For example, Eq 3 can be written, after trigonometric manipulation, as

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$$u_r(t) = 2A \cos[(\omega_o + \omega_d)t - 2\omega_o R_o/c + \varphi_o]$$

$$x \cos[(\Delta\omega_i + \Delta\omega_{di})t - 2\Delta\omega_i R_o/c + \varphi_i] \qquad (8)$$

Since $\omega_o \gg \Delta\omega_i$, this is a signal centered at ω_o and modulated by $\Delta\omega_i$. The envelope is $|\cos[(\Delta\omega_i + \Delta\omega_{di})t - 2\Delta\omega_i R_o/c + \varphi_i]|$ which is at the frequency $\Delta\omega_i$. The doppler shift of a moving target can be extracted if the signal is compared with $\Delta\omega_i$ as the reference in a phase detector. However, frequency agility by changing $\Delta\omega_i$ is not possible because of the presence of $\Delta\omega_i$ in the phase term of the envelope.

<u>Target and Clutter</u> - Analyses of processing systems with a single signal sometimes can be misleading, especially if nonlinear processing is involved. (This is one of the limitations of the multiplicative two-frequency MTI discussed in Appendix VII.) Therefore we next consider the case of a moving target and clutter. The transmitted signal is a symmetrical frequency pair as was described by Eq 1. The received signal consists of four components: two from the target and two from the clutter. The target components are represented as in Eq 3 and the clutter components are represented similarly. The clutter is assumed to be from a single scatterer at a distance R_o' (at time $t = 0$) which is moving with a relative velocity v_c. The target is at a range R_o and a relative velocity v_r. As before, we heterodyne with the reference signal at frequency ω_o to produce signals with a carrier in the vicinity of $\Delta\omega_i$. Then the received signal at the output of the "IF" filter can be represented as:

$$u_r(t) = A \cos[(\Delta\omega_i + \omega_d + \Delta\omega_{di})t - 2(\omega_o + \Delta\omega_i)R_o/c + \varphi_i]$$

$$+ A \cos[(\Delta\omega_i - \omega_d + \Delta\omega_{di})t + 2(\omega_o - \Delta\omega_i)R_o/c + \varphi_i]$$

$$+ C \cos[(\Delta\omega_i + \omega_c + \Delta\omega_{ci})t - 2(\omega_o + \Delta\omega_i)R_o'/c + \varphi_i]$$

$$+ C \cos[(\Delta\omega_i - \omega_c + \Delta\omega_{ci})t + 2(\omega_o - \Delta\omega_i)R_o'/c + \varphi_i] \qquad (9)$$

In the above, we have assumed the amplitudes of the target echoes at both frequencies to be equal, and that the clutter echoes are also

equal.

When the definition of the envelope detector as represented by Eq 8 of Appendix III is applied to the above, the square of the envelope is

$$R^2(t) = 4A^2 \cos^2(\omega_d t - 2\omega_o R_o/c) + 4C^2 \cos^2(\omega_c t - 2\omega_o R'_o/c)$$

$$+ 8AC \cos(\omega_d t - 2\omega_o R_o/c) \cos(\omega_c t - 2\omega_o R'_o/c)$$

$$\times \cos\left[(\Delta\omega_{di} - \Delta\omega_{ci})t - \boxed{2\Delta\omega_i(R_o - R'_o)/c}\right] \qquad (10)$$

It is not obvious how to extract the square root of this expression. However, if it were possible to set equal to unity the third cosine factor of the third term of Eq 10 (the one with frequency $\Delta\omega_{di} - \Delta\omega_{ci}$ which contains the phase term outlined by a dashed box), what remains is a perfect square. The output of the envelope detector under such a condition is the linear superposition of the two signals operated on individually. Thus, when this is true

$$R(t) = \left| 2A \cos(\omega_d t - 2\omega_o R_o/c) + 2C \cos(\omega_c t - 2\omega_o R'_o/c) \right| \qquad (11)$$

Setting this particular cosine factor equal to unity is akin to saying that the two spectral components, $\omega_o + \Delta\omega_i$ and $\omega_o - \Delta\omega_i$, are close enough in frequency that the doppler shifts associated with each may be considered the same. A similar assumption is generally made in a conventional MTI radar when the doppler shift associated with each spectral componenet of the signal waveform is taken to be the same as that of the carrier. It is seldom, if ever, suggested that there is a limit to the spectral width of a conventional MTI radar pulse waveform before this assumption is invalid. Yet, there must be some limit beyond which it cannot apply. (This problem has been recognized in sonar doppler systems since the percentage bandwidths used in sonar are generally much wider than those of radar.[8])

Thus, if the two spectral components are not too far apart, the effect of the doppler-difference cosine term can be ignored. In a

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frequency agile system, however, it is desired that wide frequency separations be used so that this factor cannot be ignored, unfortunately.

The factor in question is

$$\cos[(\Delta\omega_{di} - \Delta\omega_{ci})t - 2\Delta\omega_i(R_o - R_o')/c] \tag{12}$$

We next see what the limits are to $\Delta\omega_i$ in order to make the argument of the cosine negligible. Consider the phase term of Eq 12 (which is shown outlined by the dashed box of Eq 10.) Since the clutter echo and the target echo will always be within the range resolution cell of the radar, we have that $2(R_o - R_o')/c \leq \tau$, where τ = pulse width. If the phase term is to be less than 0.1 rad, we have the condition that $\delta(\Delta\omega_i) \leq 0.1/\tau$. If the pulse width were 10 μsec, the value of $\delta(\Delta f_i) = \delta(\Delta\omega_i)/2\pi$ must be less than 1.6 kHz. This is too small to be of practical value in an agile system.

The frequency term $\Delta\omega_{di}$ or $\Delta\omega_{ci}$ will be small if the frequency offset $\Delta\omega_i$ is small or the time of observation, ΔT, is small. Assume it is desired that $\delta(\Delta\omega_{di})\Delta T \leq 0.1$ rad. If we select $\Delta T = 1/60$ sec, $v_r = 300$ m/s (600 knots), then $\delta(\Delta f_i) \leq 480$ kHz. By either criterion (maintaining small phase or small frequency terms in the argument of the cosine factor), the allowable $\Delta\omega_i$ is much less than is desired for the agile MTI radar waveform. Thus, it is not likely that the simple expression for the output of an envelope detector, as given by Eq 11, can be assumed here. The criteria expressed above also apply to the change in $\Delta\omega_i$ that can be tolerated with each pulse-pair transmission.

If in the presence of target and clutter, energy is put into the doppler pass-band by the cosine factor expressed in (12), then there should be no adverse effect. It should be helpful. When clutter only is present, this term is absent. Thus it might seem that the cosine factor of (12) is not harmful. However, when multiple clutter sources are present it seems that this factor could cause an energy spread, but the extent is not clear from the simple model of the present analysis. A limitation arises when $\Delta\omega_i$ is changed, as it would in a

frequency-agile system. To see this effect clearly, the envelope detector is assumed to be square law.

When a square-law envelope detector is employed instead of a linear detector, the output can be expressed as follows:

$$R^2(t) = 2A^2 + 2A^2 \cos(2\omega_d t - 4\omega_o R_o/c)$$
$$+ 2C^2 + 2C^2 \cos(2\omega_c t - 4\omega_o R_o'/c)$$
$$+ 2AC \left\{ \cos[(\omega_d - \omega_c + \Delta\omega_{di} - \Delta\omega_{ci})t - 2(\omega_o + \Delta\omega_i)(R_o - R_o')/c] \right.$$
$$+ \cos[(\omega_d - \omega_c - \Delta\omega_{di} + \Delta\omega_{ci})t - 2(\omega_o - \Delta\omega_i)(R_o - R_o')/c]$$
$$+ \cos[(\omega_d + \omega_t + \Delta\omega_{di} - \Delta\omega_{ci})t - 2\omega_o(R_o + R_o')/c - 2\Delta\omega_i(R_o - R_o')/c]$$
$$\left. + \cos[(\omega_d + \omega_c - \Delta\omega_{di} + \Delta\omega_{ci})t - 2\omega_o(R_o + R_o')/c + 2\Delta\omega_i(R_o - R_o')/c] \right\} \quad (13)$$

This is obtained from Eq 10, or by application of Eq 7 of Appendix III. With trigonometric manipulation,

$$R^2(t) = 2A^2 + 2A^2 \cos(2\omega_d t - 4\omega_o R_o/c)$$
$$+ 2C^2 + 2C^2 \cos(2\omega_i t - 4\omega_o R_o'/c)$$
$$+ 4AC \left\{ \cos[(\Delta\omega_{di} - \Delta\omega_{ci})t - 2\Delta\omega_i(R_o - R_o')/c] \right.$$
$$\times \left[\cos[(\omega_d - \omega_c)t - 2\omega_o(R_o - R_o')/c] \right.$$
$$\left. \left. + \cos[(\omega_d + \omega_c)t - 2\omega_o(R_o + R_o')/c] \right] \right\} \quad (14)$$
$$= u_v(t)$$

This represents the output of a square-law envelope detector. The following components are present:

   a) A dc component of value $2A^2 + 2C^2$.
   b) A component at twice the doppler frequency, $2\omega_d$.
   c) A component at twice the clutter doppler frequency, $2\omega_c$.
   d) Two cross-product terms at $\omega_d - \omega_c \pm (\Delta\omega_{di} - \Delta\omega_{ci})$.
   e) Two cross product terms at $\omega_d + \omega_c \pm (\Delta\omega_{di} - \Delta\omega_{ci})$.

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The doubling of the target and clutter frequencies (b and c) presents no fundamental problem. The cross product terms do, however.

The cross product terms are multiplied by a factor

$$\cos[(\Delta\omega_{di} - \Delta\omega_{ci})t - 2\Delta\omega_i(R_o - R_o')/c] \qquad (15)$$

The phase term of the argument contains the frequency offset $\Delta\omega_i$ which is to be varied pulse to pulse if agility is to be employed. In the presence of clutter, therefore, the output of the envelope detector will vary as $\Delta\omega_i$ is varied. There will be an uncancelled residue from the delay-line canceller which can be mistaken for a moving target. This is similar to the limitation discussed in Appendix I for the conventional MTI when frequency agility is attempted. Thus the presence of the phase term in Eqs 14 or 15, which depend on $\Delta\omega_i$, prohibit the use of frequency agility with MTI using a two-frequency waveform. The single target analysis of this system gave encouragement that it could be used, but the analysis of the single target with a single clutter echo has shown it necessary to revise the conclusion. Note that the fact that Eq 14 reduces to a satisfactory answer when no target is present (A=0) is of little help. Clutter is almost always a distributed target rather than a single echo as assumed in Eq 14. Consider, for example that Eq 14 represents the signal from a clutter model consisting of two scatterers and that there is no target. (A and C now represent the clutter amplitudes, and ω_d and ω_c represent the doppler shifts of each clutter scatterer.) The offending phase term with $\Delta\omega_i$ is present and when it is changed pulse to pulse, the clutter spectrum will change and cause energy to appear in the doppler bands. Since the phase changes at a rate determined by the pulse repetition period, the value of $\Delta\phi/\Delta T_p$ = angular frequency, will be comparable to the prf and will thus be in the range of the doppler frequencies from a moving target. A more general analysis of a multiple-clutter model will give the same conclusion. The two-frequency MTI system is not compatible with frequency agility. The two-frequency waveform can be used with MTI, however, provided the same frequencies are utilized on each trans-

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mission.

Phase Detection - The use of a phase detector instead of an envelope detector at the output of the IF filter centered at $\Delta\omega_i$ offers no change in the conclusion regarding frequency agility. If the signal as represented by Eq 9 is heterodyned with a reference signal $\cos(\Delta\omega_i t + \varphi_i)$ the output, after removing the higher frequency terms, is

$$
\begin{aligned}
u_r(t) = & \; A \cos[(\omega_d + \Delta\omega_{di})t - 2(\omega_o + \Delta\omega_i)R_o/c] \\
& + A \cos[(\omega_d - \Delta\omega_{di})t - 2(\omega_o - \Delta\omega_i)R_o/c] \\
& + C \cos[(\omega_c + \Delta\omega_{ci})t - 2(\omega_o + \Delta\omega_i)R_o'/c] \\
& + C \cos[(\omega_c - \Delta\omega_{ci})t - 2(\omega_o - \Delta\omega_i)R_o'/c]
\end{aligned}
\tag{16}
$$

with trigonometric manipulation this becomes

$$
\begin{aligned}
u_r(t) = & \; 2A \cos(\Delta\omega_{di} t - 2\Delta\omega_i R_o/c) \cos(\omega_d t - 2\omega_o R_o/c) \\
& + 2C \cos(\Delta\omega_{ci} t - 2\Delta\omega_i R_o'/c) \cos(\omega_c t - 2\omega_o R_o'/c)
\end{aligned}
\tag{17}
$$

Again, there is a phase term that depends on $\Delta\omega_i$ and an uncancelled residue will result from clutter even in the absence of target. Note that in the case of a phase detector, a single clutter target, even with $\omega_c = 0$, will yield a signal dependent on $\Delta\omega_i$. This did not occur with the envelope detector. More than one clutter echo was needed there to yield a signal that depends on $\Delta\omega_i$.

Waveforms with Thumbtack Ambiguity Functions - The class of waveforms that give rise to a thumbtack ambiguity diagram[9] is of interest here because they generally involve random frequency shifting. This type of ambiguity diagram is characterized by having a narrow spike at the origin whose width is $1/B$ along the time axis and $1/T$ along the doppler-frequency axis, where B is the spectral width and T is the time duration of the waveform. In addition to the spike there is a plateau about the origin of width T along the time axis and width B along the doppler axis. The average height of the plateau is $(BT)^{-1}$ when the

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height of the central spike is unity. Such an ambiguity diagram can be achieved with a noiselike waveform or a series of pulses with frequencies shifted at random so as to contiguously cover the band B.

This, then, is a form of frequency agility that can be used with doppler processing. The narrow spike of width $1/T$ in the frequency domain allows the use of a bank of contiguous matched filters, each of width $1/T$, to cover the band of expected doppler shifts. Pulse-to-pulse random frequency hopping within the band B provides a form of frequency agility.

Although this type of waveform can be used to obtain pulse-to-pulse frequency agility and doppler processing, it has two important drawbacks. First, the frequencies used with each pulse must be selected from the band B. Even though the frequencies may be selected at random, the thumbtack ambiguity diagram requires that they be used uniformly. That is, if the width of each pulse is τ, and N pulses are used the total band occupied is N/τ. The usual concept of frequency agility is not so restricted. Generally, if N pulses of width τ are used, the available band from which their frequencies are chosen is large compared to N/τ, rather than equal to N/τ.

Second, the average value of the plateau may be less than the center spike by a factor of B_T, but the fluctuations about the average can be large. Hence, there can be undesirably large sidelobe levels in the ambiguity diagram, especially if BT is not very large.

The thumbtack ambiguity diagram gives evidence that there are waveforms that allow some degree of frequency agility and doppler processing, if their limitations can be tolerated.

It would appear that if a sufficiently large number of random frequencies are used pulse-to-pulse, the clutter energy can be spread more or less uniformly over the doppler frequency domain and that banks of narrow band filters might be used to extract the moving target echo.

APPENDIX III

THE ENVELOPE DETECTOR

The definition of the envelope of a multichromatic waveform $u(t)$ is given here according to the formulation of Rice[10] and Dugundji.[11] The waveform at the input of the detector is

$$u(t) = \sum_n c_n \cos(\omega_n t + \varphi_n) \tag{1}$$

A frequency q called the "midband frequency" is selected and Eq 1 is rewritten

$$u(t) = \sum_n c_n \cos[(\omega_n - q)t + \varphi_n + qt] \tag{2}$$

$$= \cos qt \sum_n c_n \cos[(\omega_n - q)t + \varphi_n]$$

$$- \sin qt \sum_n c_n \sin[(\omega_n - q)t + \varphi_n]$$

or $\qquad u(t) = I_c \cos qt - I_s \sin qt$

where $I_c = \sum_n c_n \cos[(\omega_n - q)t + \varphi_n]$ and $I_s = \sum_n c_n \sin[(\omega_n - q)t + \varphi_n]$

Then the envelope of $u(t)$ is defined as

$$R(t) = (I_c^2 + I_s^2)^{\frac{1}{2}} \tag{3}$$

Although a particular "midband frequency" was chosen to implement this definition, it has been shown[11] that $R(t)$ depends only on the given input waveform and not on the selected value of the "midband frequency."

<u>Envelope of Two Frequencies</u> - Consider the input signal to be

$$u(t) = \cos(\omega_1 t + \varphi_1) + \cos(\omega_2 t + \varphi_2) \tag{4}$$

This is of the form of Eq 7 in Sec 3 of the text. Then to find its envelope we obtain

$$I_c = \cos[(\omega_1 - q)t + \varphi_1] + \cos[(\omega_2 - q)t + \varphi_2]$$

$$I_s = \sin[(\omega_1 - q)t + \varphi_1] + \sin[(\omega_2 - q)t + \varphi_2]$$

$$I_c^2 = \cos^2[(\omega_1 - q)t + \varphi_1] + 2 \cos[(\omega_1 - q)t + \varphi_1]\cos[(\omega_2 - q)t + \varphi_2]$$

$$+ \cos^2[(\omega_2 - q)t + \varphi_2]$$

$$I_s^2 = \sin^2[(\omega_1 - q)t + \varphi_1] + 2 \sin[(\omega_1 - q)t + \varphi_1]\sin[(\omega_2 - q)t + \varphi_2]$$

$$+ \sin^2[(\omega_2 - q)t + \varphi_2]$$

$$I_c^2 + I_s^2 = 2 + 2 \cos[A]\cos[B] + 2 \sin[A] \sin[B]$$

$$= 2 \{1 + \cos[A - B]\}$$

Then $I_c^2 + I_s^2 = 2 \left\{1 + \cos[(\omega_1 - \omega_2)t + \varphi_1 - \varphi_2]\right\}$

$$= 4 \cos^2\left[\left(\frac{\omega_1 - \omega_2}{2}\right) t + \frac{\varphi_1 - \varphi_2}{2}\right]$$

The envelope is then

$$R(t) = 2 \cos\left[\left(\frac{\omega_1 - \omega_2}{2}\right) t + \frac{\varphi_1 - \varphi_2}{2}\right] \tag{5}$$

Applying this to Eq 7 of the text gives the same answer as Eq 9, found by other means.

Envelope of Four Frequency Waveform - When the echo is from target and clutter, such as was given by Eq 10 of the text, the input is of the form

$$u(t) = A \cos(\omega_1 t + \varphi_1) + A \cos(\omega_2 t + \varphi_2)$$

$$+ C \cos(\omega_3 t + \varphi_3) + C \cos(\omega_4 t + \varphi_4) \tag{6}$$

Then we take

$$I_c = A \left\{\cos[(\omega_1 - q)t + \varphi_1] + \cos[(\omega_2 - q)t + \varphi_2]\right\}$$

$$+ C \left\{\cos[(\omega_3 - q)t + \varphi_3] + \cos[(\omega_4 - q)t + \varphi_4]\right\}$$

$$I_s = A \left\{ \sin[(\omega_1 - q)t + \varphi_1] + \sin[(\omega_2 - q)t + \varphi_2] \right\}$$
$$+ C \left\{ \sin[(\omega_3 - q)t + \varphi_3] + \sin[(\omega_4 - q)t + \varphi_4] \right\}$$

$$R^2(t) = I_c^2 + I_s^2$$

$$= 2A^2 + 2C^2$$

$$+ 2A^2 \left\{ \cos[(\omega_1 - q)t + \varphi_1] \cos[(\omega_2 - q)t + \varphi_2] \right.$$
$$\left. + \sin[(\omega_1 - q)t + \varphi_1] \sin[(\omega_2 - q)t + \varphi_2] \right\}$$

$$+ 2C^2 \left\{ \cos[(\omega_3 - q)t + \varphi_3] \cos[(\omega_4 - q)t + \varphi_4] \right.$$
$$\left. + \sin[(\omega_3 - q)t + \varphi_3] \sin[(\omega_4 - q)t + \varphi_4] \right\}$$

$$+ 2AC \left\{ \cos[(\omega_1 - q)t + \varphi_1] \cos[(\omega_3 - q)t + \varphi_3] \right.$$
$$+ \sin[(\omega_1 - q)t + \varphi_1] \sin[(\omega_3 - q)t + \varphi_3]$$
$$+ \cos[(\omega_1 - q)t + \varphi_1) \cos[(\omega_4 - q)t + \varphi_4]$$
$$+ \sin[(\omega_1 - q)t + \varphi_1) \sin[(\omega_4 - q)t + \varphi_4]$$
$$+ \cos[(\omega_2 - q)t + \varphi_2] \cos[(\omega_3 - q)t + \varphi_3]$$
$$+ \sin[(\omega_2 - q)t + \varphi_2] \sin[(\omega_3 - q)t + \varphi_3]$$
$$+ \cos[(\omega_2 - q)t + \varphi_2] \cos[(\omega_4 - q)t + \varphi_4]$$
$$\left. + \sin[(\omega_2 - q)t + \varphi_2] \sin[(\omega_4 - q)t + \varphi_4] \right\}$$

Since $\cos \alpha \cos \beta + \sin \alpha \sin \beta = \cos (\alpha - \beta)$, we can write the above as

$$R^2(t) = 2A^2 + 2C^2$$
$$+ 2A^2 \cos[(\omega_1 - \omega_2)t + \varphi_1 - \varphi_2] + 2C^2 \cos[(\omega_3 - \omega_4)t + \varphi_3 - \varphi_4]$$
$$+ 2AC \left\{ \cos[(\omega_1 - \omega_3)t + \varphi_1 - \varphi_3] + \cos[(\omega_1 - \omega_4)t + \varphi_1 - \varphi_4] \right.$$
$$\left. + \cos[(\omega_2 - \omega_3)t + \varphi_2 - \varphi_3] + \cos[(\omega_2 - \omega_4)t + \varphi_2 - \varphi_4] \right\}$$

$$(7)$$

This can be rewritten in the form

$$R^2(t) = 4 A^2 \cos^2\left[\frac{(\omega_1 - \omega_2)}{2} t + \frac{\varphi_1 - \varphi_2}{2}\right]$$

$$+ 4 C^2 \cos^2\left[\frac{(\omega_3 - \omega_4)}{2} t + \frac{\varphi_3 - \varphi_4}{2}\right]$$

$$+ 8 AC \left\{\cos\left[\frac{(\omega_1 - \omega_2)}{2} t + \frac{\varphi_1 - \varphi_2}{2}\right] \cos\left[\frac{(\omega_3 - \omega_4)}{2} t + \frac{\varphi_3 - \varphi_4}{2}\right]\right.$$

$$\left. \times \cos\left[\left(\frac{\omega_1 + \omega_2}{2} - \frac{\omega_3 + \omega_4}{2}\right)t + \frac{\varphi_1 + \varphi_2}{2} - \frac{\varphi_3 + \varphi_4}{2}\right]\right\} \quad (8)$$

This is the form used to obtain Eq 11 in Sec 3 of the text.

Distortion - The signal given by Eq 8 in Sec 3 of the text that is operated on by the envelope detector is reproduced below:

$$u(t) = 2A\cos(\omega_d t - 2\omega_o R_o/c) \cos[(\Delta\omega_i + \Delta\omega_{di})t - 2\Delta\omega_i R_o/c + \varphi_i] \quad (9)$$

This was depicted in Fig 1a. The envelope of this signal is a rectified cosine wave. Such a waveform results in harmonics of the modulation frequency ω_d. The fourier series expansion of a full-wave rectified cosine wave of amplitude a and frequency ω is

$$\frac{2a}{\pi}\left(1 - \frac{2}{3}\cos 2\omega t - \frac{2}{15}\cos 4\omega t - \frac{2}{35}\cos 6\omega t - \ldots - \frac{2}{(n-1)(n+1)}\cos n\omega t - \ldots\right)$$

The fundamental frequency is twice ω. The relative magnitudes of the higher harmonics compared to the fundamental are -14 db, -21.3 db, -26.4 db, -30.4 db, etc. The magnitudes of the higher harmonics drop off fairly respectably. For a moving target, the higher harmonics are of little concern. Their energy, if it were to lie within the doppler pass band, would be recognized as that of a moving target. The question to be addressed is what is the effect of the harmonics of a clutter signal.

If the clutter were perfectly stationary there would be no problem since the doppler is zero. Clutter, however, will have a finite velocity spectrum and we will have to examine the effect of the

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harmonics of the clutter that appear in the doppler pass band. For
example, wooded hills in a 40 kt wind result in a clutter spectrum with
a standard deviation of 0.32 kts. This is small compared to the 10-12
kt cutoff that one might employ in an L band air-surveillance doppler
processor like that of the ARSR-2 FAA air traffic control radar. Hence,
it is not likely that the higher harmonics of ground clutter will be
serious. Clutter from birds, however, could spread into the pass band.

The higher harmonics of the envelope could be reduced by making
unequal the amplitudes of the two components in Eq 9. The shape of the
envelope when the signals are equal is like a full-wave rectified sine
wave, as already mentioned. The more unequal the two amplitudes, the
closer the envelope approaches a sine wave variation. The closer the
envelope is to a sine wave, the less will be the harmonics.

Another approach to making the envelope like that of a sine wave
is to add a carrier at frequency $\Delta\omega_1$ to the signal of Eq 9. It then
appears as a double-sideband modulated carrier and the envelope will be
a sine wave. This is equivalent to reinserting the carrier in a
suppressed-carrier, double-sideband modulated signal. The phase of the
reinserted carrier must be maintained coherent with those of the two
"sideband" components. Since the radar is coherent, it should not be
difficult to establish a signal of the proper phase, except for the
presence of the phase term $2\Delta\omega_1 R_o/c$ in Eq 9. With a moving target,
imperfect phasing might not be too critical. With completely stationary
clutter $2\Delta\omega_1 R_o/c$ is zero and there is no problem.

Still another way to eliminate the problem of higher harmonics of
the envelope of the signal given by Eq. 9 is to use a square law
detector rather than a linear one. The output of such a detector is
proportional to the square of the envelope. Therefore the signal will
be of the form $\cos^2(\omega_d t - 2\omega_o R_o/c)$ rather than $|\cos(\omega_d t - 2\omega_o R_o/c)|$. The
square produces a dc term and a sinusoidal term of the form $\cos 2(\omega_d t - 2\omega_o R_o/c)$. Practical detectors are néither linear nor square law so that
the harmonics that might be achieved in practice are likely to be less

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than that of a perfect linear detector, but not completely absent as in a perfect square-law detector.

The width of the clutter notch in a doppler processor need not be very wide if only ground echoes are present. If no other clutter were present, there would be no problem as to the higher harmonics since the filter cutoff probably could be made high enough to exclude them. With sea clutter, the notch has to be wider because of the motion of the sea itself. If the radar is located on a moving platform such as a ship, the notch has to be wider to account for the velocity of the platform. Weather, chaff, or birds can also widen the clutter spectrum that must be rejected in an MTI doppler processor. Higher harmonics from such clutter sources could spill over into the doppler passband and reduce the subclutter visibility. If such is the case, other methods might have to be employed to reduce this type of clutter, rather than widen the doppler notch. A tunable notch such as in TACCAR might be employed. Also STC or dual elevation beams might be used to reduce the effect of birds.

APPENDIX IV

OPERATION WITH MORE THAN TWO FREQUENCIES

In this appendix we consider the case where many frequencies are transmitted rather than a single pair. As before, we will assume for ease of analysis that the amplitudes of the target echoes and the clutter echoes are independent of frequency. Each of the N frequencies are assumed to be radiated simultaneously on each transmission. There are two reasons for wanting to consider the use of more than two frequencies in the type of systems under discussion. First, there is the hope that by some means the use of more frequencies might make agility and MTI possible, whereas two frequencies do not. No success was achieved in this regard. Second, if a two frequency waveform can be used with MTI (although the same frequencies are radiated each time), then it ought to be possible to use more than two frequencies. This is, of course, possible in the limit where the frequencies as spaced contiguously within a band B so as to produce a pulse of width 1/B. In this sense, the question as to whether a signal with more than two frequency components can be used for MTI is a trivial one, since the answer should be yes.

Consider the transmitted waveform to consist of two pairs of frequencies, each with the same frequency separation $\Delta\omega_i$, (four frequency components, in total.) One pair is centered at a frequency ω_o, the other pair at ω_o'. The pair of frequencies centered about ω_o are heterodyned with the reference signal $\cos(\omega_o t + \varphi_o)$ and passed through a filter centered at $\Delta\omega_i$. The output of this filter is given by Eq 7 of the main text. This equation is reproduced below for the case of a single target:

$$u_1(t) = A \cos[(\Delta\omega_i + \omega_d + \Delta\omega_{di})t - 2(\omega_o + \Delta\omega_i)R_o/c + \varphi_i] \quad (1a)$$

$$+ A \cos[(\Delta\omega_i - \omega_d + \Delta\omega_{di})t + 2(\omega_o - \Delta\omega_i)R_o/c + \varphi_i]$$

$$= 2A \cos(\omega_d t - 2\omega_o R_o/c) \cos[(\Delta\omega_i + \Delta\omega_{di})t - 2\Delta\omega_i R_o/c + \varphi_i]$$

The signal received from transmission of the pair of frequencies centered about ω_o', i.e. $\omega_o' - \Delta\omega_i$ and $\omega_o' + \Delta\omega_i$ may be expressed

$$u_2(t) = 2A \cos(\omega_o' t - 2\omega_o' R_o/c) \cos[\Delta\omega_i + \Delta\omega_{di})t - 2\Delta\omega_i R_o/c + \varphi_i] \quad (1b)$$

The sum of these two is

$$u(t) = u_1(t) + u_2(t)$$

$$= 4A \cos[(\omega_d + \omega_d')t/2 - (\omega_o + \omega_o')R_o/c]$$

$$\times \cos[(\omega_d - \omega_d')t/2 - (\omega_o - \omega_o')R_o/c]$$

$$\times \cos[(\Delta\omega_i + \Delta\omega_{di})t - 2\Delta\omega_i R_o/c + \varphi_i] \quad (2)$$

where $\omega_d' = 2\omega_o' v_r/c$. From Eq 2 we can describe three properties.

First, equation 2 states directly that the envelope of the carrier will consist of two frequencies, one of which is at the doppler ω_d' corresponding to ω_o'. These two spectral components are similar to the two components at $\omega_d \pm \Delta\omega_{di}$, obtained when only a single pair of frequencies are transmitted.

Second, when the two frequencies ω_o and ω_o' are sufficiently close so that $\omega_d \approx \omega_d'$, and $\omega_d + \omega_d' \approx 2\omega_d$, Eq 2 becomes

$$u(t) = 4A \cos[(\omega_d t - (\omega_o + \omega_o')R_o/c]$$

$$\times \cos[(\Delta\omega_i + \Delta\omega_{di})t - 2\Delta\omega_i R_o/c + \varphi_i] \quad (3)$$

In this case the envelope of the signal is at a single doppler frequency.

Third, let the signal given by Eq 2 be heterodyned with the reference signal $\cos[\Delta\omega_i t + \varphi_i]$ so as to give

$$u_v(t) = 4A \cos[(\omega_d + \omega_d')t/2 - (\omega_o + \omega_o')R_o/c]$$
$$\cos[(\omega_d - \omega_d')t/2 - (\omega_o - \dot\omega_o')R_o/c]$$
$$\cos[(\Delta\omega_{di}t - 2\Delta\omega_i R_o/c] \tag{4}$$

Again we see that there is a phase term that depends on $\Delta\omega_i$ as well as on $\omega_o - \omega_o'$. As long as they remain fixed from pulse to pulse, they should not interfere with MTI.

When clutter is present, the signal of Eq 2 becomes

$$u(t) = 4A \cos[(\omega_d + \omega_d')t/2 - (\omega_o + \omega_o')R_o/c]$$
$$\times \cos[(\omega_d - \omega_d')t/2 - (\omega_o - \omega_o')R_o/c]$$
$$\times \cos[(\Delta\omega_i + \Delta\omega_{di})t - 2\Delta\omega_i R_o/c + \varphi_i]$$
$$+ 4C \cos[(\omega_c + \omega_c')t/2 - (\omega_o + \omega_o')R_o'/c]$$
$$\times \cos[(\omega_c - \omega_c')t/2 - (\omega_o - \omega_o')R_o'/c]$$
$$\times \cos[(\Delta\omega_i + \Delta\omega_{ci})t - 2\Delta\omega_i R_o'/c + \varphi_i] \tag{5}$$

No attempt will be made to find the envelope of the signal represented by Eq 5 with the use of the definition of the envelope presented in Appendix III. Instead we will assume that this signal is heterodyned with the reference signal $\cos(\Delta\omega_i t + \varphi_i)$. The result, which we will not write down, is the same as Eq 5, but with $\varphi_i = 0$ and $\Delta\omega_i$ set equal to zero in the frequency terms (but not in the phase terms). We then have a complicated looking signal that can be described as a collection of doppler-shifted frequencies due to the target and a collection due to the clutter. With this form of processing there are no cross terms between target and clutter, although there would have been if envelope detection were used. Again there are phase terms that depend on $\Delta\omega_i$ and $\omega_o - \omega_o'$, so that agility is not possible.

If the frequencies of the transmitted signals are not too far apart so that the doppler frequency shifts can be assumed the same for each component, then in Eq 5 we can set $\Delta\omega_{di} = 0$, $\Delta\omega_{ci} = 0$,

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$\omega_d = \omega_d'$ and $\omega_c = \omega_c'$. The resultant signal is

$$u(t) = 4A \cos(\omega_d t - (\omega_o + \omega_o')R_o/c)$$
$$\times \cos(\Delta\omega_1 t - 2\Delta\omega_1 R_o/c + \varphi_1)$$
$$+ 4C \cos(\omega_c t - (\omega_o + \omega_o')R_o'/c)$$
$$\times \cos(\Delta\omega_1 t - 2\Delta\omega_1 R_o'/c + \varphi_1) \tag{6}$$

When this signal is heterodyned with the reference signal
$\cos(\Delta\omega_1 t + \varphi_1)$ we get

$$u_v(t) = k_1 \cos(\omega_d t - \Phi_1) + k_2 \cos(\omega_c t - \Phi_2) \tag{7}$$

where k_1 and k_2 are constants if $\Delta\omega_c$ is fixed and Φ_1 and Φ_2 are phases whose specific values are constant if $(\omega_o + \omega_o')$ is fixed. From Eq 7 it is seen that the output is the linear superposition of the signals that would have been received separately from target only and from clutter only.

Next, consider the case where signals are equally spaced in frequency with a separation $\Delta\omega_1$. Assuming that the number N of components is odd with the center component of frequency ω_o, the transmitted signal can be represented as

$$e_t(t) = \cos\left\{\left[\omega_o - \frac{(N-1)}{2}\Delta\omega_1\right]t + \varphi_o - \frac{(N-1)}{2}\varphi_1\right\}$$
$$+ \cos\left\{\left[\omega_o - \frac{(N-2)}{2}\Delta\omega_1\right]t + \varphi_o - \frac{(N-2)}{2}\varphi_1\right\} + \ldots$$
$$+ \cos[\omega_o t + \varphi_o] + \cos\left\{(\omega_o + \Delta\omega_1)t + \varphi_o + \varphi_1\right\}$$
$$+ \ldots + \cos\left\{\left[\omega_o + \frac{(N-1)}{2}\Delta\omega_1\right]t + \varphi_o + \frac{(N-1)}{2}\varphi_1\right\} \tag{8}$$

This is similar to the form of the equation for an equally spaced phased array. Using the trigonometric relation

$$\cos\left[\alpha - \frac{(N-1)}{2}\delta\right] + \ldots + \cos(\alpha - \delta) + \cos\alpha + \ldots + \cos(\alpha + \frac{(N-1)}{2}\delta)$$
$$= \frac{\sin N\delta/2}{\sin \delta/2}\cos\alpha \tag{9}$$

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we can write the transmitted signal in the form:

$$u_t(t) = \frac{\sin N(\Delta\omega_1 t + \varphi_1)/2}{\sin(\Delta\omega_1 t + \varphi_1)/2} \cos(\omega_o t + \varphi_o) \tag{10}$$

The received signal from a single target is

$$u_r(t) = \frac{\sin N[(\Delta\omega_1 + \Delta\omega_{d1})t - 2\Delta\omega_1 R_o/c + \varphi_1]/2}{\sin[(\Delta\omega_1 + \Delta\omega_{d1})t - 2\Delta\omega_1 R_o/c + \varphi_1]/2}$$

$$\times \cos[(\omega_o + \omega_d)t - 2\omega_o R_o/c + \varphi_o] \tag{11}$$

The carrier frequency of this waveform is at ω_o shifted by the doppler frequency ω_d. There is a modulation of the form $\sin NX/X$. Superimposed on this is the modulation envelope due to the pulse transmission of width τ.

If the received signal of Eq 11 is heterodyned with the reference signal $\cos(\omega_o t + \varphi_o)$ we get

$$u(t) = \cos(\omega_d t - 2\omega_o R_o/c) \frac{\sin N[(\Delta\omega_1 + \Delta\omega_{d1})t - 2\Delta\omega_1 R_o/c + \varphi_1]/2}{\sin[(\Delta\omega_1 + \Delta\omega_{d1})t - 2\Delta\omega_1 R_o/c + \varphi_1]/2} \tag{12}$$

If the $\sin NX/X$ factor is considered as a carrier, proper processing can extract the modulation term $\cos(\omega_d t - 2\omega_o R_o/c)$ for MTI processing. (In a conventional pulse radar with a pulse repetition frequency equal to f_p and pulse width $\tau \ll 1/f_p$, the effect of the $\sin NX/X$ factor is small and can be replaced by unity. In the type of waveforms considered here, this is not the case.)

There are several possible methods for extracting the modulation of Eq 12. These may be listed as

1. Envelope detection of the entire signal occupying a band $N\Delta\omega_1$.

2. Envelope detection after passing the signal through a comb filter with components centered at $\Delta\omega_1$, $2\Delta\omega_1$,...$N\Delta\omega_1$. This is superior to No. 1 since it is closer to a matched filter and eliminates much of the noise.

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3. Heterodyning with a comb of frequencies of $n\Delta\omega_1$, $n=0,1,2,\ldots,N$.

4. Filtering the component at $N\Delta\omega_1$ and then taking the envelope or heterodyning with $N\Delta\omega_1$.

5. Filtering the component at $\Delta\omega_1$ and taking the envelope or heterodyning with $\Delta\omega_1$. This is the approach that might be tried first.

If the spacings between the frequencies are not equal but are arranged unequally the modulation envelope of the transmitted or the received frequency will not be a regular pattern, but will have a central peak with random-like time sidelobes. This is by analogy to the spatial pattern of the unequally spaced array antenna. Even though it might be possible to operate with unequal, randomly chosen spacings, it is not desirable to do so since energy is diverted to the time sidelobes which is difficult to extract efficiently. (This might not apply to a frequency spectrum where the unequal frequencies are symmetrically spaced and each pair processed independently as described for the single symmetrical frequency pair. The signals are combined at video.)

APPENDIX V

TIME-SEQUENCE TRANSMISSION

It has been assumed in most of this discussion that the basic transmitted waveform consists of a pair of frequencies $\omega_o + \Delta\omega_1$ and $\omega_o - \Delta\omega_1$ that are radiated simultaneously. On reception, the two frequencies are heterodyned with a coherent reference of frequency ω_o, combined, passed through a filter centered at $\Delta\omega_1$ and envelope detected. We have seen that it is not possible to use a different pair of frequencies on succeeding transmissions because of the dependence of $\Delta\omega_1$ (as described in Appendix II.) Although this waveform cannot be used for frequency agility and MTI, it can be employed as an MTI waveform if the frequencies remain the same pulse to pulse.

In this appendix we examine the possibility of transmitting the two (or more) frequencies at different times, rather than simultaneously. To perform MTI, each frequency must be repeated at a later time. At time $t = 0$ the frequency component $\omega_o + \Delta\omega_1$ is transmitted, followed T_1 seconds later by $\omega_o - \Delta\omega_1$. The transmitted signal is

$$u_t(t) = a_t(t) \cos[(\omega_o + \Delta\omega_1)t + \varphi_o + \varphi_1]$$
$$+ a_t(t-T_1) \cos[(\omega_o - \Delta\omega_1)(t-T_1) + \varphi_o - \varphi_1] \tag{1}$$

where $a_t(t)$ represents the modulation due to a pulse of width τ. signal received from a moving target at range R_o is

$$u_r(t) = a_r(t) \cos[(\omega_o + \omega_d + \Delta\omega_1 + \Delta\omega_{d1})t - 2(\omega_o + \Delta\omega_1)R_o/c + \varphi_o + \varphi_1]$$
$$+ a_r(t-T_1)\cos[(\omega_o + \omega_d - \Delta\omega_1 - \Delta\omega_{d1})t - 2(\omega_o - \Delta\omega_1)R_o/c - (\omega_o - \Delta\omega_1)T_1 + \varphi_o - \varphi_1]$$

$$\tag{2}$$

where the symbols are the same as used for the discussion of Eq 3 in Appendix II. There are two operations that must be performed on this received signal for them to be the same as the simultaneously transmitted pairs discussed in Appendix II. Since it is assumed that $T_1 > \tau$, the first pulse at frequency $\omega_o + \Delta\omega_1$ must be delayed at time T_1 on reception in order to bring it in time coincidence with the second pulse of frequency $\omega_o - \Delta\omega_1$ transmitted T_1 seconds later. Then the phase of the one signal must be compensated to make the two components equivalent to the symmetrical frequency pair described by Eq 3, Appendix II.

If the component centered at a frequency $\omega_o + \Delta\omega_1$ is passed through a time delay of T_1, we obtain

$$
\begin{aligned}
u_r(t) = &\; a_r(t-T_1)\cos[(\omega_o + \omega_d + \Delta\omega_1 + \Delta\omega_{d1})t - 2(\omega_o + \Delta\omega_1)R_o/c \\
&\; -(\omega_o + \omega_d + \Delta\omega_1 + \Delta\omega_{d1})T_1 + \varphi_o + \varphi_1] \\
&+ a_r(t-T_1)\cos[(\omega_o + \omega_d - \Delta\omega_1 - \Delta\omega_{d1})t - 2(\omega_o - \Delta\omega_1)R_o/c \\
&\; -(\omega_o - \Delta\omega_1)T_1 + \varphi_o - \varphi_1]
\end{aligned}
\tag{3}
$$

This operation brings the two signals in time coincidence, but the phase terms of the two components are not as they should be if they are to be similar to Eq 3 of Appendix II. If a compensating phase shift equal to $(\omega_d + \Delta\omega_{d1})T_1$ is added to the first signal then we get

$$
\begin{aligned}
u_r(t) = &\; a_r\cos[(\omega_o + \omega_d + \Delta\omega_1 + \Delta\omega_{d1})t - 2(\omega_o + \Delta\omega_1)R_o/c \\
&\; -(\omega_o + \Delta\omega_1)T_1 + \varphi_o + \varphi_1] \\
&+ a_r\cos[(\omega_o + \omega_d - \Delta\omega_1 - \Delta\omega_{d1})t - 2(\omega_o - \Delta\omega_1)R_o/c \\
&\; -(\omega_o - \Delta\omega_1)T_1 + \varphi_o - \varphi_1]
\end{aligned}
\tag{4}
$$

This equation is identical to Eq 3 of Appendix II except for the addition of a phase term in each cosine component. In the lower frequency component this added phase term is $-(\omega_o - \Delta\omega_1)T_1$ and in the upper

component it is $-(\omega_o + \Delta\omega_i)T_1$. To see the effect of these phases we follow through the processing as outlined by Eqs 3 to 6 of Appendix II.

The received signal represented by Eq 4 is heterodyned with the reference $\cos(\omega_o t + \varphi_o)$. The signal that passes through an appropriate filter centered at $\Delta\omega_i$ is

$$u(t) = A \cos[(\Delta\omega_i + \omega_d + \Delta\omega_{di})t - 2(\omega_o + \Delta\omega_i)R_o/c - (\omega_o + \Delta\omega_i)T_1 + \varphi_i]$$
$$+ A \cos[(\Delta\omega_i - \omega_d + \Delta\omega_{di})t + 2(\omega_o - \Delta\omega_i)R_o/c + (\omega_o - \Delta\omega_i)T_1 + \varphi_i] \quad (5)$$

where the amplitudes have been set equal to A. This can be written as

$$u(t) = 2A \cos[\omega_d t - 2\omega_o R_o/c - \omega_o T_1] \cos[(\Delta\omega_i + \Delta\omega_{di})t$$
$$- 2\Delta\omega_i R_o/c - \Delta\omega_i T_1 + \varphi_i] \quad (6)$$

The envelope of this signal is then

$$R(t) = 2A \cos[\omega_d t - 2\omega_o R_o/c - \omega_o T_1] \quad (7)$$

This is the same as Eq 6 of Appendix II except for the phase $-\omega_o T_1$. Since this term is fixed from pulse to pulse, it is of no consequence in the MTI processing. Therefore it can be concluded that it should be possible in principle to transmit the two frequencies of the symmetrical frequency pair at different times rather than simultaneously.

There are two ranges of values for the time separation T_1 that may be of interest to examine. One situation is when T_1 is comparable to the pulse width τ and the other when it is comparable to the pulse repetition period T_p.

In the former situation, the pulse transmission might consist of a pulse divided into two halves. The first half is $\omega_o + \Delta\omega$. (Alternatively, there may be a separation between these two pulses that might be a few pulse widths.) Let us take, for purposes of example, the time separation between the two frequencies to be 10 μsec. The generation and transmission of two pulses of different frequencies separated by 10 μsec can be readily accomplished in practice. On reception, the first half of the pulse must be delayed. An RF delay of about 10 μsec

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is difficult to achieve, but not impossible. We assume it can be accomplished. The compensating phase shift to be added is $(\omega_d + \Delta\omega_{d1})T_1$. If $\omega_o + \Delta\omega_1 = 2\pi \times 10^9$ sec^{-1} (L band) and if the target velocity is 600 knots (300m/s), the required phase shift is equal to 0.06 radian $\approx 3.6^\circ$. This is small enough to be neglected and no phase compensation is required. If the phase compensation were more than an order of magnitude greater it could not be neglected and would have to be accounted for. There ought to be no major problem in doing so in principle, although in practice it might be complex since the doppler frequency ω_d would have to be estimated. However, it is not necessary to compensate for this phase shift for a moving target. It is only necessary to compensate for moving clutter which will be one to two orders of magnitude less in doppler frequency than the 600 kts assumed here. Thus, this compensation might not be necessary in any event.

It might be noted that the transmission of two closely spaced pulses of different frequencies can have uses other than for MTI. They could provide a means for simple coding of one's own transmission so as to minimize RFI or they might prove effective in mitigating the effects of certain types of repeater jammers.

Next we examine the situation where the time delay T_1 is the same as the pulse repetition period T_p, which we take to be one millisecond by way of example. It is not practical to delay an RF microwave signal for as long as a millisecond. It is possible to achieve such delays at lower frequencies, at what would normally be considered IF frequencies. Therefore we convert the signals at $\omega_o \pm \Delta\omega_1$ to a frequency $\Delta\omega_1$ before introducing the time delay. (This could also have been done with the shorter delays described above.) The received signal as expressed by Eq 2, after heterodyning by the reference $\cos(\omega_o t + \varphi_o)$ becomes

$$u(t) = a_r(t) \cos[(\Delta\omega_1 + \omega_d + \Delta\omega_{d1})t - 2(\omega_o + \Delta\omega_1)R_o/c + \varphi_1]$$
$$+ a_r(t - T_1) \cos[(\Delta\omega_1 - \omega_d + \Delta\omega_{d1})t + 2(\omega_o - \Delta\omega_1)R_o/c + (\omega_o - \Delta\omega_1)T_1 + \varphi_1]$$

$$(8)$$

The component represented by the first term is then delayed a time T_1 to give

$$u(t) = a_r(t-T_1) \cos[(\Delta\omega_i + \omega_d + \Delta\omega_{di})t - 2(\omega_o + \Delta\omega_i)R_o/c$$
$$- (\Delta\omega_i + \omega_d + \Delta\omega_{di})T_1 + \varphi_i]$$
$$+ a_r(t-T_1) \cos[(\Delta\omega_i - \omega_d + \Delta\omega_{di})t + 2(\omega_o - \Delta\omega_i)R_o/c$$
$$+ (\omega_o - \Delta\omega_i)T_1 + \varphi_i] \qquad (9)$$

To make this signal more like that of Eq 3 of Appendix III (when both frequencies are transmitted simultaneous) a phase shift of $+ \omega_o T_1$ must be inserted into the first term. With this insertion, Eq 9 can be written (with $a_r(t-T_1) = A$) as

$$u(t) = A \cos[(\Delta\omega_i + \omega_d + \Delta\omega_{di})t - 2(\omega_o + \Delta\omega_i)R_o/c + (\omega_o - \Delta\omega_i - \omega_d - \Delta\omega_{di})T_1 + \varphi_i]$$
$$+ A \cos[(\Delta\omega_i - \omega_d + \Delta\omega_{di})t + 2(\omega_o - \Delta\omega_i)R_o/c + (\omega_o - \Delta\omega_i)T_1 + \varphi_i]$$

By trigonometric manipulation, we get

$$u(t) = 2A \cos[\omega_d t - 2\omega_o R_o/c - (\omega_d + \Delta\omega_{di})T_1/2]$$
$$\times \cos[(\Delta\omega_i + \Delta\omega_{di})t - 2\Delta\omega_i R_o/c + (\omega_o - \Delta\omega_i)T_1 - (\omega_d + \Delta\omega_{di})T_1/2 + \varphi_i] \qquad (10)$$

The envelope of which is

$$R(t) = 2A \cos[\omega_d t - 2\omega_o R_o/c - (\omega_d + \Delta\omega_{di})T_1/2] \qquad (11)$$

The only difference between this and the case where both frequencies are transmitted simultaneously is the presence of the phase shift $(\omega_d + \Delta\omega_{di})T_1/2$. This phase can be neglected for the normal values of clutter doppler and pulse repetition periods encountered in air-search radar.

Next we consider the need to compensate for the phase term $\omega_o T_1$, as was assumed in proceeding from Eq 9 to Eq 10. Assuming for sake of illustration that $\omega_o = 2\pi \times 10^9$ sec^{-1} and $T_1 = 10^{-3}$ sec we have $\omega_o T_1 = 2\pi \times 10^6$ radians. Since this is not changed pulse to pulse, no correction is necessary, but both the time delay T_1 and the frequency ω_o must be maintained to an accuracy of almost one part in 10^8. It is possible

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to obtain a frequency stability of this accuracy, but it is not clear whether delay lines are this accurate or whether it might be possible to measure the delay to the desired accuracy. In any event, it certainly does not look easy to achieve. Therefore, the shorter values of T_1, that are perhaps no more than a few tens of microseconds rather than milliseconds, are likely to be easier to implement in this manner.

To summarize, each successive pair of frequencies transmitted are of the same average frequency ω_o. Assume, for sake of illustration, that there are three pairs of frequencies. After the six pulses are transmitted, they are repeated. There is no fundamental need to always transmit the higher frequency first or always transmit the lower one first, provided proper compensation is made. The choice could be at random. Each of the three pairs of frequencies are translated to its particular $\Delta\omega_i$, where they are summed to obtain a single signal whose envelope is of the same doppler as if the mean frequency ω_o were transmitted. The envelopes from successive sets of transmissions can then be processed as in a conventional MTI radar.

If six different frequencies are available for use in an MTI radar they might be transmitted on successive pulses and processed in six separate receivers in the conventional manner. The only advantage of the mean-frequency processing described here is that a single MTI processor is used rather than the six processors required of conventional parallel MTI systems. It is not clear, however, whether this advantage compensates for the complexity of the mean-frequency processing.

APPENDIX VI

EFFECT OF THE VARIATION OF TARGET CROSS SECTION

WITH FREQUENCY

In all of the analyses thus far it has been assumed that the radar cross sections of both the target and clutter were independent of frequency. Such an assumption is only true for a fictitious target. (That is, it is not true in general.) We examine in this appendix the validity of such an assumption and its effect on processing. The discussion in this appendix is only "academic" since there are other, simpler reasons for discarding the system which is the subject on this report. It is included here since it has some applicability to other frequency agile systems.

Consider the simple case of the signal reflected from a single target when the transmitted waveform consists of two frequencies as described in Sec. 3 of the report. Equation 9 of Sec. 3, reproduced below as equation 1, represents the envelope of the processed signal:

$$R(t) = 2A \left| \cos(\omega_d t - 2\omega_o R_o/c) \right| \tag{1}$$

In this equation the amplitudes of the echo signals at the two frequencies $\omega_o + \Delta\omega_1$ and $\omega_o - \Delta\omega_1$ were assumed the same and equal to A. When this assumption is not true, the received signal is

$$u(t) = A_1 \cos\left[(\Delta\omega_1 + \omega_d + \Delta\omega_{d1})t - 2(\omega_o + \Delta\omega_1)R_o/c + \varphi_1 \right] \tag{2}$$

$$+ A_2 \cos\left[(\Delta\omega_1 - \omega_d + \Delta\omega_{d1})t + 2(\omega_o - \Delta\omega_1)R_o/c + \varphi_1 \right]$$

This is similar to Eq. 7 of the text except that the amplitudes A_1 and A_2 of the two terms are not equal. Using the definition of the envelope given in Appendix III we have

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$$R^2(t) = A_1{}^2 + A_2{}^2 + 2A_1A_2 \cos (2\omega_d t - 4\omega_o R_o/c) \tag{3}$$

When $A_1 = A_2 = A$, the square root can be taken and Eq. 3 yields the same form as Eq. 9 of Sec. 3 (or Eq. 1 of this appendix). When $A_1 \neq A_2$, there is no simple expression for $R(t)$.

The envelope $R(t)$ can be diagramed as a vector summation as sketched in Fig. VI.1. The time rotation of the vector A_2 at a rate $2\omega_d$ results in the angle Y being a function of time. The angle Y represents the phase of the envelope. The relationship between $Y(t)$ and $\omega_d t$ for the case shown in Fig. VI.1 is

$$Y(t) = \text{arc tan} \left[\frac{A_2 \ \sin(2\omega_d t - 4\omega_o R_o/c)}{A_1 + A_2 \ \cos(2\omega_d t - 4\omega_o R_o/c)} \right] \tag{4}$$

Except in special cases, there is no simple way to express the relation between ω_d and $Y(t)$. One can use either the envelope $R(t)$ or the phase $Y(t)$ in an MTI process since the same target factors are included in both.

In the case of a moving target the variation of σ, and hence the amplitude A, with frequency is of little consequence since an uncancelled residue is produced in the MTI delay line canceller no matter whether A is constant or not. With a clutter signal, the doppler frequency is zero and the envelope of Eq. 3 reduces to

$$R(t) = [A_1^2 + A_2^2 + 2A_1A_2 \ \cos 4\omega_o R_o/c]^{\frac{1}{2}} \tag{5}$$

which is a constant for a clutter echo at a fixed range. If, on the succeeding pulse transmission, the two transmitted frequencies remain the same, Eq. 5 will not change and cancellation of the clutter is possible. However, if a different pair of frequencies are transmitted on the succeeding pulse, the echo amplitudes A_1 and A_2 might not be the same and each clutter target will produce a different value for the envelope. Consequently, when succeeding pulses are processed in a delay line canceller, the frequency dependence will result in fixed clutter producing an uncancelled residue. This can be mistaken for a
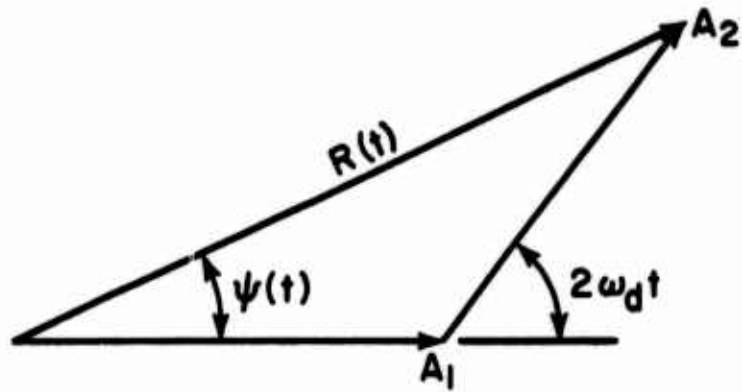
FIG. VI.I RELATIONSHIP OF ENVELOPE AND ITS
TWO COMPONENTS A_1 AND A_2.

moving target and the MTI action is negated. Hence, it does not seem possible to utilize the envelope detector as described in an agile dual-frequency radar when the clutter cross section is frequency dependent. Even if the clutter echo were completely independent of frequency the amplitudes of the received signals will vary since the gain of the usual radar antenna will change with frequency. Also, it is not likely that the transmitter power will be constant over its frequency band. Thus the echo signal strength should always be expected to vary with frequency whether due to the nature of the clutter, the antenna or the transmitter. Any processing technique employed with the agile dual-frequency radar therefore must ignore the amplitude and work with the phase of the individual signals.

The variation of the echo signal amplitude with frequency can be eliminated by hard limiting in the radar receiver. Limiting must be accomplished before the envelope is taken (before the frequency components are summed) or else we will obtain cos $\Psi(t)$, as given by Eq. 4, which depends on the values of A_1 and A_2. Thus to achieve frequency independence of the echo in a dual-frequency agile system requires that hard limiting be accomplished for each frequency component before summation and extraction of the envelope.

In the operation of a conventional MTI radar, limiting is employed to make the residue of the cancelled clutter comparable to that of receiver noise. If the clutter level is greater than receiver noise, the clutter residue will so obscure the scope that it will be impossible to detect desired targets in the clutter area. Generally, the limit level is set above noise by an amount equal to the clutter attenuation provided by the MTI delay line canceller. Hard limiting has seldom been used in MTI radar.

Although the use of limiting in conventional MTI radar is essential, it causes some degradation in the improvement factor. Shrader[12] has pointed out the effect limiting has on lowering the Improvement Factor. With a two-pulse canceller, the reduction in the Improvement Factor due

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to hard limiting as compared with no limiting might be from 5 to 8 db depending on the number of hits, but for a three-pulse canceller the loss varies from about 15 to 40 db over the same conditions. Therefore when using limiting, the theoretical improvement anticipated with the use of higher-order-pulse cancellers should not be expected.

Another consequence of the limiter is that strong signals will suppress weak ones. This problem has not been treated in the MTI literature. It is seldom, if ever, mentioned. Its neglect might mean that it is too difficult a problem to either analyze or experimentally measure, or that it is not important, or that it is accepted as a way of life.

APPENDIX VII

THE MULTIPLICATIVE TWO-FREQUENCY MTI SYSTEM

In this appendix we consider the analysis of the two-frequency MTI system discussed by Kroszcynski[5,6] and by Hsiao[7]. The two frequencies are beat together in a nonlinear device and the component at the difference frequency is taken. The motivation for this technique is to achieve the higher blind speeds associated with the difference frequency. It also seems to have some advantage to offer when the radar platform is in motion, as in an AMTI radar. One of its disadvantages is that the clutter spectrum is caused to spread. Although two frequencies are transmitted simultaneously in this system as it is in the agile MTI, the two frequencies remain unchanged from pulse-to-pulse and thus it is not pulse-to-pulse frequency agile as is the system discussed in this report which linearly adds the two frequencies and operates on their mean. The purpose of this appendix is to examine the processing of the multiplicative two-frequency MTI so as to compare it to the agile MTI using symmetrical frequency pairs.

In the multiplicative two-frequency MTI the two frequencies $\omega_o + \Delta\omega_1$ and $\omega_o - \Delta\omega_1$ are mixed together and the difference frequency is extracted. For a single target the echo signal can be represented as in Eq (6) of Sec. 3 of the text, which is

$$u_r(t) = a_1 \cos\left[(\omega_o + \omega_d + \Delta\omega_1 + \Delta\omega_{d1})t - 2(\omega_o + \Delta\omega_1)R_o + \varphi_o + \varphi_1\right]$$

$$+ a_2 \cos\left[(\omega_o + \omega_d - \Delta\omega_1 - \Delta\omega_{d1})t - 2(\omega_o - \Delta\omega_1)R_o + \varphi_o - \varphi_1\right] \tag{1}$$

When these two components are multiplied together, the difference signal is

$$u_d(t) = a_1 a_2 \cos\left[2(\Delta\omega_1 + \Delta\omega_{d1})t - 4\Delta\omega_1 R_o/c + \varphi_1\right] \tag{2}$$

The result is as if the difference frequency ($2\Delta\omega_i$) had been transmitted. Note that if the difference frequency is maintained constant, but if the mean is changed pulse-to-pulse, agility and MTI appear compatible since Eq. 2 is independent of ω. However, when considered in isolation it can be misleading for the same reasons the mean-frequency processing was misleading. Also, the nonlinear process causes undesirable by-products in the presence of more than one signal.

To get a better idea of the effect of more than one target we assume the echo consists of components from a single target and a single clutter echo, as was discussed in Sec 3. In addition to a target signal like Eq. (1) there will be a clutter echo at range R_o' with a doppler shift ω_c. One frequency channel will be

$$u_1(t) = a_1 \cos[(\omega_o + \omega_d + \Delta\omega_i + \Delta\omega_{di})t - 2(\omega_o + \Delta\omega_i)R_o/c + \varphi_o + \varphi_i]$$
$$+ C_1 \cos[(\omega_o + \omega_c + \Delta\omega_i + \Delta\omega_{ci})t - 2(\omega_o + \Delta\omega_i)R_o'/c + \varphi_o + \varphi_i] \qquad (3)$$

and the other will be

$$u_2(t) = a_2 \cos[(\omega_o + \omega_d - \Delta\omega_i - \Delta\omega_{di})t - 2(\omega_o - \Delta\omega_i)R_o/c + \varphi_o - \varphi_i]$$
$$+ C_2 \cos[(\omega_o + \omega_c - \Delta\omega_i - \Delta\omega_{ci})t - 2(\omega_o - \Delta\omega_i)R_o'/c + \varphi_o - \varphi_i] \qquad (4)$$

When Eqs. 3 and 4 are multiplied together and the difference terms extracted we get a target signal component of the form:

$$\text{Target Comp.} = a_1 a_2 \cos[2(\Delta\omega_i + \Delta\omega_{di})t - 4\Delta\omega_i R_o/c + 2\varphi_i] \qquad (5)$$

and a clutter term of the form:

$$\text{Clutter Comp.} = C_1 C_2 \cos[(\Delta\omega_i + \Delta\omega_{ci})t - 4\Delta\omega_i R_o'/c + 2\varphi_i] \qquad (6)$$

as well as cross terms between clutter and target of the form:

$$\text{CxT Comp.} = a_1 C_2 \cos[(\omega_d - \omega_c + 2\Delta\omega_i + \Delta\omega_{di} + \Delta\omega_{ci})t - 2\Delta\omega_i(R_o - R_o')/c + 2\varphi_i]$$
$$+ a_2 C_1 \cos[(\omega_c - \omega_d + 2\Delta\omega_i + \Delta\omega_{ci} + \Delta\omega_{di})t - 2\Delta\omega_i(R_o' - R_o)/c + 2\varphi_i]$$
$$\qquad (7)$$

These components are beat with a reference signal at the difference frequency $\cos[2\Delta\omega_i t + 2\varphi_i]$ to give

$$u(t) = a_1 a_2 \cos[2\Delta\omega_{di} t - 4\Delta\omega_i R_o/c]$$
$$+ C_1 C_2 \cos[2\Delta\omega_{ci} t - 4\Delta\omega_i R_o'/c]$$
$$+ a_1 C_2 \cos[(\omega_d - \omega_c + \Delta\omega_{di} + \Delta\omega_{ci})t - 2\Delta\omega_i (R_o - R_o')/c]$$
$$+ a_2 C_1 \cos[(\omega_d - \omega_c - \Delta\omega_{di} - \Delta\omega_{ci})t - 2\Delta\omega_i (R_o - R_o')/c] \tag{8}$$

In the above the target signal and the clutter signal (the first and second terms) appear as they would if they were alone. The two cross term signals, however, are due to the simultaneous appearance of both target and clutter. Since these cross product terms are absent when the target is absent, they are credited to the target. Thus the $C_1 C_2$ term represents clutter and the other three terms represent target. The frequencies associated with each of these components are sketched in Fig. VII.1. The single dashed line represents clutter and the three solid lines represents target signal.

Although the two spectral components centered around $\omega_d - \omega_c$ represent target energy it is not certain that it would be best to retain them. For example, a low pass filter might be employed which rejects those frequencies greater than the maximum expected value of $2\Delta\omega_{di}$. Then the signals that are passed can be treated as if they were transmitted by the difference frequency $2\Delta\omega_i$ (this is not quite true because of the spread in the clutter spectrum as mentioned later). The blind speeds will be those of the difference frequency. However, with all components utilized, Hsiao[7] shows the blind speed is increased just as in a staggered prf MTI radar. Thus no matter whether the components around $\omega_d - \omega_c$ are rejected or remain, the blind speeds can be greater than would be obtained with either frequency separately. It also appears that in this simple model when no target is present, the clutter signal is dependent only on the choice of the difference frequency so that a form of frequency agility can be employed. This, however, has to be modified in a more realistic analysis.
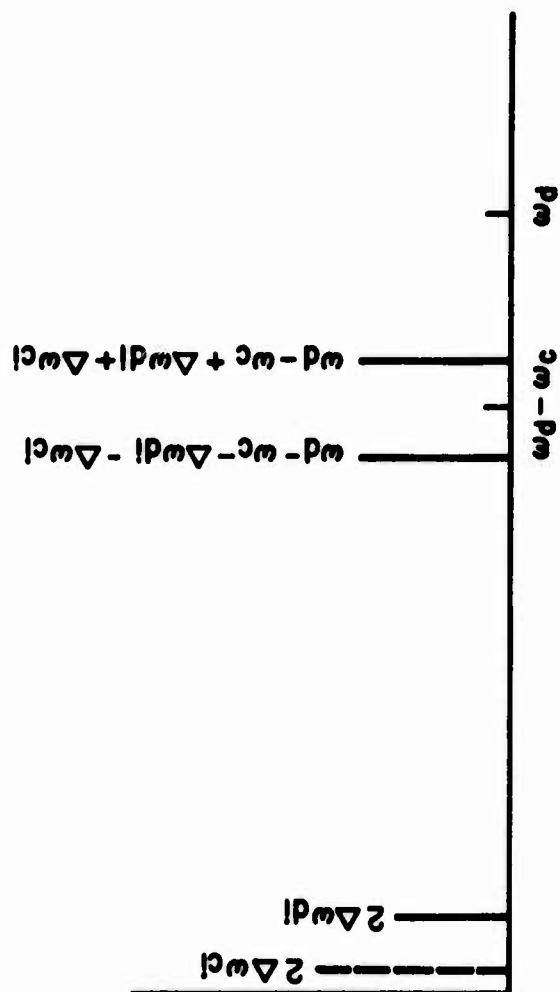
FIG. VII.1 SPECTRUM OF THE SIGNALS IN THE MULTIPLICATIVE DUAL-FREQUENCY MTI FOR A SINGLE TARGET AND A SINGLE CLUTTER ECHO.

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Hsiao[7] has pointed out that modeling the clutter signal as a single spectral component can be misleading, especially when the clutter spectrum has a finite width. He shows that the clutter doppler variance of a dual-frequency MTI is increased. If σ_1^2 and σ_2^2 represent the clutter variances at the two RF frequencies, then the variance of the clutter in this system is $\sigma_1^2 + \sigma_2^2$. Note that not only is the clutter spectrum increased, but the width of the clutter at the difference frequency is not that associated with the difference frequency. Instead it is greater than the width of the clutter at either of the two RF frequencies. This can be seen from an examination of the cross product terms of Eq 8 if ω_d and ω_c are considered two components of clutter.

The transfer and widening of the RF spectrum to the difference frequency tends to negate the potential of this approach. It is concluded that because of this factor, agility is not practical and there is little the system has to offer.

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