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A NOTE ON ADMISSIBLE EXCHANGES IN
SPANNING TREES

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ABSTRACT

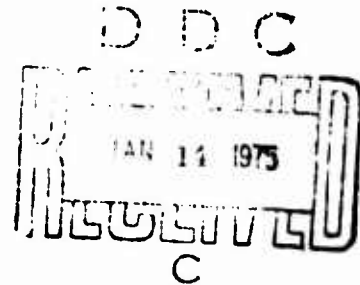
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A NOTE ON
ADMISSIBLE EXCHANGES
IN SPANNING TREES

by

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April, 1974



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1.

Spanning trees appear in numerous guises and applications throughout graph theory, ranging from the minimum spanning tree problem [1, 8, 9, 10, 13] to the network "basis structures" of linear programming whose exploitation has resulted in efficient algorithms for minimum cost flow problems [2, 3, 7, 12]. Spanning trees have also recently been shown to have relevance for solving traveling salesman problems [5,6].

Uses of spanning trees are frequently based on "swaps" involving the deletion of an "in-tree" edge and the addition of an "out-of-tree" edge, such that the result is also a spanning tree. This type of swap, which we will call an admissible exchange, corresponds precisely to a linear programming basis exchange step (i.e., "pivot operation") in a network. Thus, it is well known (and immediately apparent), for example, that given any two distinct spanning trees, T and T' , one can always select any edge of T that is not in T' , and thereupon find some edge of T' not in T , so that swapping these edges (deleting the first and adding the second) will give an admissible exchange relative to T .

The purpose of this note is to establish a considerably stronger result, concerning the ability to "pair" all edges of $T - T'$ with all edges of $T' - T$ (treating these trees as edge sets) so that each pair gives an admissible exchange relative to T . Furthermore, we will provide a constructive procedure in the proof of this result for identifying precisely such a pairing.

Letting $T'(e)$ denote the unique edge-simple path in T' joining the endpoints of the edge e (or alternatively, the collection of edges on this path) we make the following preliminary observation.

Lemma: Let $e_1, \dots, e_k$ be any k edges of $T - T'$. Then the union of $T'(e_1), \dots, T'(e_k)$ contains at least k distinct edges of $T' - T$.

Proof: The union of $T'(e_i)$, $i = 1, \dots, k$ is a forest in T' . The endpoints of each edge e_i lie in exactly one tree of this forest. Now add the edges $e_1, \dots, e_k$ to this forest and delete all edges of $T' - T$. The result is a subgraph of T . If the forest contains fewer than k edges of $T' - T$ to be deleted, then a cycle is introduced in at least one tree of this forest by adding $e_1, \dots, e_k$. Since T has no cycles, this is impossible, completing the proof.

Note that the sets $T'(e_i)$ in the foregoing lemma can be also replaced with the sets $T^*(e_i) = T'(e_i) - T$. Utilizing this lemma, we now establish the following result.

Theorem: There is a one-one pairing (matching) of the edges of $T - T'$ with the edges of $T' - T$ so that each pair gives rise to an admissible exchange in T .

Proof: We establish the theorem by a constructive process that generates the desired pairing.* Assume that the edges in a subset E of $T - T'$ have been paired with the edges in a subset E' of $T' - T$. Select any edge $e_0 \in (T - T') - E$. Let $U_0 = \emptyset$, and for $i \geq 1$ define $V_i =$ the union of $T^*(e_0)$ with all sets $T^*(e)$ such that $e \in U_{i-1}$, and define $U_i = \{e \in E: e \text{ pairs with some } e' \in V_i\}$. As long as $V_i \not\subseteq E'$ we have $|U_i| = |V_i| \geq |U_{i-1}| + 1$, where the inequality follows from the definition of V_i in terms of U_{i-1} and the lemma. Thus there must be a finite least k such that $V_k \subseteq E'$. Identify any $e'_k \in V_k - E'$, and for each $i < k$ identify e_i such that $e_i \in U_i \cup \{e_0\}$ and $e'_{i+1} \in T^*(e_i)$, and identify e'_i such that e'_i currently pairs with e_i . It follows that the pairing $(e_0, e'_1), (e_1, e'_2), \dots, (e_{k-1}, e'_k)$, with all other pairs in E and E' unchanged, creates a 1-1 correspondence of the edges in $E \cup \{e_0\}$ and $E' \cup \{e'_k\}$.

*We are indebted to Hal Gabow for pointing out that a nonconstructive proof of the theorem can be based on applying Hall's theorem for distinct set representatives to the foregoing lemma (see, e.g., [4a]p.53).

The ability to identify and construct the one-one matching of the theorem in the manner indicated is a useful tool for characterizing various types of optimality criteria involving spanning trees. If, for example, one seeks to minimize $F(T)$, where T is a spanning tree, then the theorem establishes the validity of an algorithm that utilized sequential admissible exchanges, all of which are "improving," provided the difference $F(T) - F(T')$ can be expressed in terms of an appropriate function of the edges of $T - T'$ and of $T' - T$. We shall not attempt to fully specify the conditions of an "appropriate function," which can be of a highly complex character, but note that the objective function evaluation for feasible linear programming bases falls in the proper category. As demonstrated in [4], the theorem is also useful for establishing optimality conditions in certain problems where not all improving moves can be restricted to single admissible exchanges.

4

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