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RECEIVER CANONIC MODELS

L. Vears, et al

Signatron, Incorporated

Prepared for:

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Factors such as complexity and realism of the interference environment are used to establish the necessary cases and boundary values to be approximated by the

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RECEIVER CANONIC MODELS

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FOREWORD

This Final Technical Report was prepared by Signatron, Incorporated, 27 Hartwell Ave., Lexington Industrial Park, Lexington, MA 02173, under Contract F30602-73-C-0206, Job Order 45400256, for Rome Air Development Center, Griffiss Air Force Base, New York. Mr. John F. Spina (RBCT) was the RADC Project Engineer.

The effort described was accomplished during 8 December 1973 to 7 March 1974.

This report has been reviewed by the RADC Information Office (OI), and is releasable to the National Technical Information Service (NTIS).

This technical report has been reviewed and is approved.

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ABSTRACT

This report is concerned first with the computer determination of optimum parameters of a tapped-delay line canonic model of third order interference generation, and second with the use of this model in simulating receiver response to waveforms.

Factors such as complexity and realism of the interference environment are used to establish the necessary cases and boundary values to be approximated by the model. The convergence of the tapped-delay-line model to a frequency-polynomial model, and the rate of this convergence, are used to establish limits on group delay and tap spacing. An algorithm is developed embodying these limits, that recursively searches for the best group delay and tap spacing, and explicitly determines the best tap coefficients. Sampled data techniques are developed for generation of AM/FM or noiselike signals and interferers, for generation of interference products using the model, for simulation of the equivalent linear filter of the cascade model, and for calculation of distortion in phase demodulator or amplitude-demodulator outputs. The functioning of the computer programs is demonstrated with printouts from the tests.

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INTRODUCTION AND SUMMARY

This technical report for the period 8 December 1973 to 7 March 1974 discusses the computer determination of optimum parameters of a tapped-delay line canonic model of third order interference generation, and second, with the use of this model in simulating receiver response to waveforms.

1.1 Introduction

In earlier studies by SIGNATRON extensive effort has been devoted to the determination of the response of nonlinear circuits to multiple sinusoidal inputs as characterized by the nonlinear transfer function of the circuit. The present effort is concerned with the development of canonic models that will permit easier determination of the network response to modulated input signals.

1.1.1 Specific Program Objectives

The development of canonic models falls naturally into a sequence of steps which form the specific objectives of this program:

a. The determination of analytically tractable approximations to the nonlinear circuit response that are particularly useful for the small-percentage-bandwidth signals of interest in communications.

b. Determining the minimum number of parameters necessary to characterize these approximations.

c. Determine the minimum number and most effective set of measurements that will permit the model parameters to be extracted, both for entire receivers, and for component amplifiers and mixers.

d. Verification of the feasibility of the measurement procedure.

e. Determine necessary computer programs to calculate the model parameters either from measured data or from analytic predictions of circuit response.

f. Determine necessary computer programs for prediction of response to specific modulated input waveforms using the measured/calculated model parameters.

1.2 Summary of this Report

This report is concerned entirely with items (e) and (f) in the list of objectives: the determination and generation of computer programs to calculate optimum model coefficients and simulate the effect on waveforms of the receiver being modeled.

Section 2 defines the objectives of the computer programs and establishes appropriate limits on model complexity, interference cases, and model parameters.

Section 3 deals with the calculation of optimum model parameters including tap spacing, group delay and optimum tap coefficients. The rate of approach to a frequency-polynomial approximation is used to define search limits for tap spacing while the limiting behavior provides a guideline to setting search limits for group delay. An algorithm is then developed that uses a recursive search for optimum group delay and tap spacing and an explicit optimization of the tap coefficients.

Section 4 derives properties of second-order sampled data Butterworth filters used in signal filtering and noise generation.

It is useful to have available an analytically well-understood nonlinear circuit for testing program routines. Section 5 describes such a circuit and the associated formulas for calculation of H_3 and H_1 transfer functions.

In Section 6 we discuss the generation of signal and interference waveforms which may be sinusoidally amplitude modulated, phase modulated, or both, or may be noiselike in character. These are available as an alternative to sampled data tapes derived from actual signal sources.

Computer programs have been successfully written that embody all of the features described in Sections 2 to 6. In Section 7 we discuss the results of an end-to-end test of these programs.

1.3 Contributors

The work reported on here was performed by L.H.Vears, J.N. Pierce, N.Johnson, H.Gish and S.H.Richman. This report was prepared by Ms. Vears and Mr. Pierce.

1.4 Acknowledgments

We are indebted for program guidance and technical suggestions to Mr. John F. Spina of RADC and Prof.D. Weiner of Syracuse University.

SECTION 2

COMPUTATIONAL OBJECTIVES

The bulk of the work discussed in this report relates to the generation of computer programs related to the canonic modelling effort. In this section we discuss the objectives of this software effort.

2.1 General Objectives

Our Technical Report #2 demonstrated that, at the present time, the scope of canonic modelling should be limited to the modelling of third-order nonlinear transfer functions. We further established that for computational purposes only a very few models were practical. For the purposes of this effort we have chosen the most useful of these, the generalized tapped-delay line model, which involves complex-exponential approximations to the third-order nonlinear transfer functions.

The software necessary to use canonic modelling must provide the following capabilities:

a) A program to calculate the parameters of the tapped-delay line model either from measured values of the transfer function or from transfer function values calculated by programs which analyze the nonlinear circuits.

b) A program that accepts arbitrary signal and interference inputs consistent with the model bandwidth and sampling rates, and generates the complex envelope of the corresponding third-order interference.

c) Routines to generate realistic signal and interference inputs to be used with the program in (b) above.

d) Routines to display the effect of the interference on the baseband output of a receiver's demodulator.

2.2 Spectral Structure of Signals and Interference

To put some structure on the computer modelling, we can visualize the RF spectrum as consisting of a large number of equally spaced channels with a separation of W Hz between the center frequencies of adjacent channels. This structure is, in fact, quite typical of military spectral allocations. The same number W will also be roughly equal to the typical signal bandwidth, and typical receiver IF bandwidth, if we take these bandwidths to be defined by the (-20 dB) or (-60 dB) points on the spectra, for example. Since the objective here is the modelling of nonlinear effects, it is an adequate approximation to equate the -3 dB bandwidths to W as long as the software routines avoid any linear adjacent channel interference effects. The basic framework will then be taken as a desired signal at the tuning frequency ν, and potential interferers at $\nu \pm W$, $\nu \pm 2W$, etc., all with equal bandwidth W, which is also to be taken as the IF bandwidth.

Now let ν_1, ν_2, ν_3 be the carrier frequencies of the three interfering signals. Then, as was pointed out in TR #2, the bandwidth of the third-order interaction is 3W so that interference to the desired signal can occur if

$$\nu_1 + \nu_2 - \nu_3 = \nu. \tag{2-1a}$$

or

$$\nu_1 + \nu_2 - \nu_3 = \nu \pm W. \tag{2-1b}$$

The most general type of computer modelling would then admit

a) an arbitrary signal modulation at the carrier frequency ν,

b) three independent interference modulations at carrier frequencies ν_1, ν_2, ν_3,

c) the interaction carrier frequency located at either ν or one of the adjacent carrier frequencies $\nu \pm W$.

If we take into account, however, the relative importance or relative probability of the various types of interference, the scope of the modelling can be reduced with no loss of utility. We will now develop these specific restrictions. Before proceeding to this it is helpful to review some results from TR #2 and TR #3.

2.3 Use of Equivalent Receiver

In Section 2.1 of TR#3 we introduced the concept of an equivalent receiver. We repeat the relevant part of Fig. 2.5 of that report as Fig. 2.1 here. The essence of the equivalent receiver is to replace the distributed ($H_1 - H_3$) structure of the actual receiver with a single linear filter (with transfer function $H(f)$) following a parallel combination of a unit-gain amplifier and a third-order transfer function $K(f_1,f_2,f_2)$.

This equivalent receiver structure also forms an excellent signal flow chart for computer simulation in that the possibly complicated tapped-delay line structure for the equivalent IF filter $H(f)$ can be applied to the one-dimensional output of the third-order filter $K(f_1,f_2,f_3)$ rather than having its effects incorporated in the three-dimensional tapped-delay line structure which synthesizes the nonlinear response.

2.4 Formulas for Third-Order Zonal Output

In Section 2 of TR#2 we developed formulas for the zonal outputs of the third-order transfer function. We repeat here the necessary formulas; we are substituting the equivalent transfer

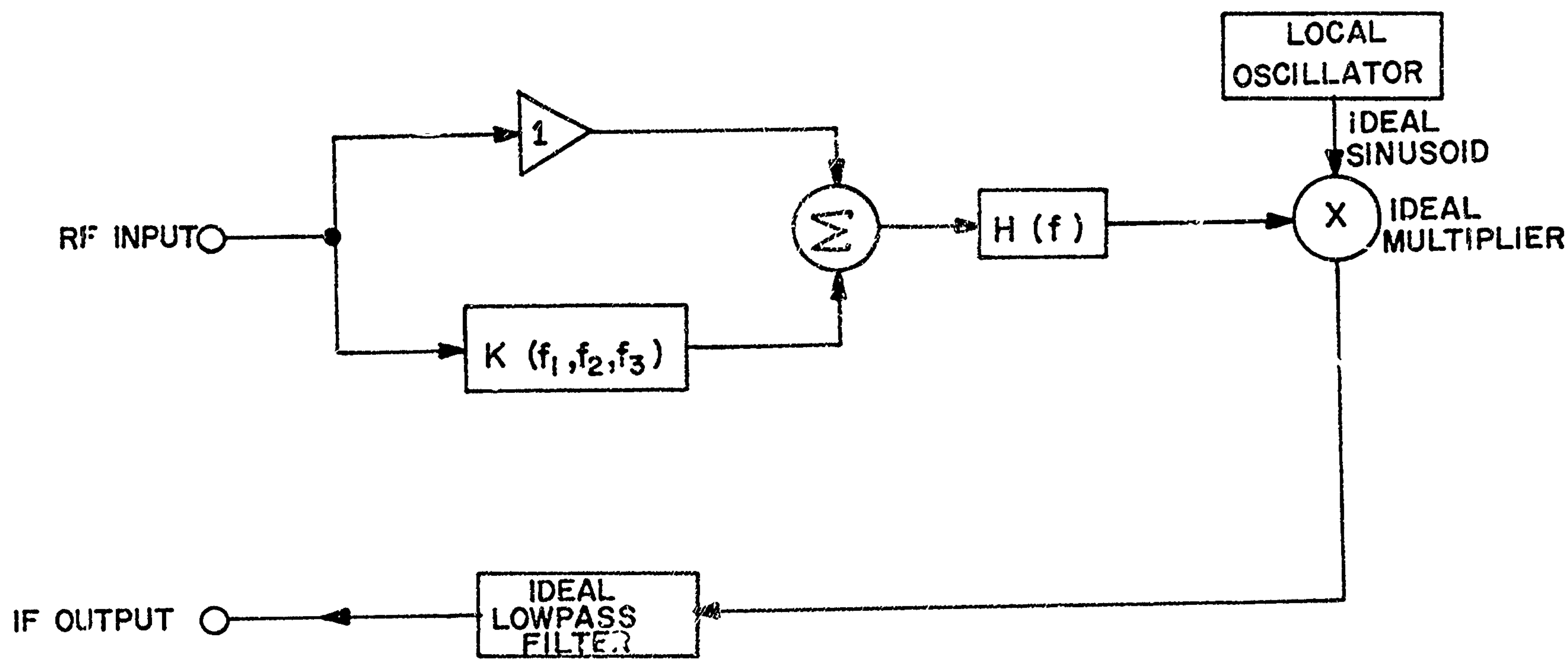

Fig. 2.1 Equivalent Receiver

function K for H_3 wherever appropriate, and making very minor notational changes where useful.

$$X(t) = \text{total input signal} = \sum_{p=1}^{P} x_p(t); \tag{2-2}$$

$$x_p(t) = \mathrm{Re}[z_p(t) \exp(j2\pi \nu_p t)]; \tag{2-3}$$

$$z_{-p}(t) = z_p^*(t); \tag{2-4}$$

$$\nu_{-p} = -\nu_p; \tag{2-5}$$

$$y_K(t) = \text{third-order output}; \tag{2-6}$$

$$y_K(t) = \frac{1}{8} \sum_{p_1,p_2,p_3} a_{p_1,p_2,p_3}(t) \exp[j2\pi(\nu_{p_1} + \nu_{p_2} + \nu_{p_3})t]; \tag{2-7}$$

$$\begin{aligned} a_{p_1,p_2,p_3}(t) &= \iiint_{-W/2}^{W/2} df_1 df_2 df_3 \, G_{p_1,p_2,p_3}(\underline{f}) \exp[j2\pi t(f_1 + f_2 + f_3)] \\ &\qquad z_{p_1}(f_1)\, z_{p_2}(f_2)\, z_{p_3}(f_3); \end{aligned} \tag{2-8}$$

$$G_{p_1,p_2,p_3}(\underline{f}) = K(f_1 + \nu_{p_1}, f_2 + \nu_{p_2}, f_3 + \nu_{p_3}). \tag{2-9}$$

The sum in Eq. (2-7) contains $8P^3$ terms altogether, which consist of $4P^3$ terms and their conjugates. Many of these terms are identical because they represent subscript permutations. Furthermore most of them will not fall at carrier frequencies that can create interference; this is the case for any term all of whose subscripts are positive, for example.

We will now specialize Eq.(2-7) to those cases that might be of interest for computer modelling; we will subsequently narrow this list down even more.

In the tabulation that follows we will write the outputs in the form

$$y_K(t) = \text{Constant} \cdot \mathrm{Re}\,\{a_{p_1,p_2,p_3}(t)\,\exp[j2\pi\nu_T t]\}, \tag{2-10a}$$

with

$$\begin{aligned} p_1 &> 0 \\ p_2 &> 0 \\ p_3 &< 0 \end{aligned} \tag{2-10b}$$

and

$$\nu_T = \nu_{p_1} + \nu_{p_2} + \nu_{p_3}. \tag{2-10c}$$

The "Real part of" consolidates terms in Eq. (2-7) with their conjugates and the constant takes into account the number of permutations that lead to identical terms.

It will be recalled from Section 2.4.3 of TR#2 that for certain combinations of interfering signals, many different interactions will lead to inband interference. We have decided that to keep the computer programs manageable it is reasonable to require the operator either to ascertain by inspection of the relative power levels which component is the most significant, or to run all cases separately and combine the outputs afterwards.

In all of the cases of interest we allow the possibility that none of the interacting frequencies is the desired frequency. The computer program must thus allow for the possibility that the linearly amplified component is distinct from any of the interfering complex envelopes.

We now tabulate the cases.

$$\left.\begin{aligned}&\nu_{p_1} = \nu_1, \nu_{p_2} = \nu_1, \nu_{p_3} = -\nu_1\\ &y_K(t) = (3/4)\ \mathrm{Re}[a_{1,1,-1}(t)\ \exp(j2\pi\nu_T(t)]\end{aligned}\right\}. \tag{2-11}$$

$$\left.\begin{aligned}&\nu_{p_1} = \nu_1, \nu_{p_2} = \nu_1, \nu_{p_3} = -\nu_2\\ &y_K(t) = (3/4)\ \mathrm{Re}[a_{1,1,-2}(t)\ \exp(j2\pi\nu_T t)]\end{aligned}\right\}. \tag{2-12}$$

$$\left.\begin{aligned}&\nu_{p_1} = \nu_1, \nu_{p_2} = \nu_2, \nu_{p_3} = -\nu_2\\ &y_K(t) = (3/2)\ \mathrm{Re}[a_{1,2,-2}(t)\ \exp(j2\pi\nu_T t)]\end{aligned}\right\}. \tag{2-13}$$

$$\left.\begin{aligned}&\nu_{p_1} = \nu_1, \nu_{p_2} = \nu_2, \nu_{p_3} = -\nu_3\\ &y_K(t) = (3/2)\ \mathrm{Re}[a_{1,2,-3}(t)\ \exp(j2\pi\nu_T t)]\end{aligned}\right\}. \tag{2-14}$$

2.5 Interference Spectrum

It is useful to have some idea of the shape of the spectrum of the interference envelopes $a_{\underline{p}}(t)$. To this end, let us write the time domain analog of Eq. (2-8); we will drop some of the subscripting where it will cause no confusion. We have

$$a(t) = \iiint dt_1 dt_2 dt_3\ g(t_1, t_2, t_3)$$

$$z_{p_1}(t-t_1)\ z_{p_2}(t-t_2)\ z_{p_3}(t-t_3), \tag{2-15a}$$

where $g(t_1, t_2, t_3)$ is any function whose transform equals G on the cube of integration in Eq. (2-8):

$$\iiint dt_1 dt_2 dt_3 \; g(t_1,t_2,t_3)$$

$$\exp[-j2\pi(t_1f_1 + t_2f_2 + t_3f_3)]$$

$$= G(f_1,f_2,f_3) \text{ when } |f_i| \le W/2, \; i = 1,2,3. \qquad (2\text{-}15b)$$

For future reference we should keep in mind that since $p_3 < 0$,

$$z_{p_3}(t) = z^*_{|p_3|}(t). \qquad (2\text{-}16)$$

Let $R_a(\tau)$ be the autocorrelation function of $a(t)$:

$$R_a(\tau) = E[a(t)a^*(t+\tau)]. \qquad (2\text{-}17)$$

Substitution of Eq. (2-15a) in Eq. (2-17), with the introduction of new dummy variables, yields

$$R_a(\tau) = \int\cdots\int dt_1 \ldots dt_6 \; g(t_1,t_2,t_3) \; g^*(t_4,t_5,t_6)$$

$$U(t_1,\ldots,t_6;\tau) \qquad (2\text{-}18a)$$

where

$$U(t_1,\ldots,t_6;\tau) = E\begin{bmatrix} z_{p_1}(t-t_1) \; z^*_{p_1}(t+\tau-t_4) \\ \cdot z_{p_2}(t-t_2) \; z^*_{p_2}(t+\tau-t_5) \\ \cdot z_{p_3}(t-t_3) \; z^*_{p_3}(t+\tau-t_6) \end{bmatrix}. \qquad (2\text{-}18b)$$

As usual, by E() we mean the "expected value of".

Before going any further, we should observe that the form of Eq.(2-15a) indicates that in the case defined by Eqs.(2-11), (2-13) and (2-14), the interference is highly correlated with the component envelope $z_{p_1}(t)$. Consequently, in any of these three cases, if $z_{p_1}(t)$ is the envelope of the desired signal, the power spectrum of the interference may be of little interest because the interference actually bears useful signal information. We will therefore exclude those cases from consideration in evaluating Eq. (2-18b).

That equation is hopeless to evaluate as it stands because the determination of the expectation of the sixfold products requires information on the joint statistics of envelopes at six time instants. However, some progress can be made if we assume that the envelopes $\{z_{p_1}(t)\}$ are complex Gaussian processes with identical covariance functions. (They will be identical processes when the subscripts coincide.) We will make this assumption, and write the common covariance as

$$R_z(\tau) = E\left\{z_{p_i}(t)\ z^*_{p_i}(t+\tau)\right\},\ i = 1,2,3. \tag{2-19a}$$

We note that this covariance satisfies

$$R_z(-\tau) = R_z{}^*(\tau). \tag{2-19b}$$

It will be convenient to approximate these autocorrelations as being associated with a rectangular power spectrum of bandwidth W:

$$R_z(\tau) = \int_{-W/2}^{W/2} \exp\ (j2\pi f\tau)df. \tag{2-20a}$$

(It perhaps should be pointed out that we are ignoring the scale factors on these autocorrelation functions and power spectra, which are immaterial to the shape of the interference spectrum.) It will be observed that where convenient we may use equally well

$$R_z(\tau) = \int_{-W/2}^{W/2} \exp\ (-j2\pi f\tau)df \tag{2-20b}$$

because the $R_z(\tau)$ defined by Eq. (2-20a) is pure real.

In the appendix to this section we derive the general form of expectations of the type in Eq. (2-18b). The results these may be used in conjunction with Eq. (2-18) to derive the autocorrelation function of the interference envelope, and hence the power spectrum. We wish to restrict attention here to the special cases described by Eqs. (2-12) and (2-14).

In Eq. (2-12) there are two distinct carrier frequencies ν_1 and ν_2, and consequently two distinct interferer envelopes so that

$$\left.\begin{aligned} z_{p_1}(t) &= z_{p_2}(t) = z_1(t) \\ z_{p_3}(t) &= z^*_2(t) \end{aligned}\right\}. \qquad (2\text{-}21)$$

Equation (2-18b) then becomes

$$U(t_1,\ldots,t_6;\tau) = E\left\{\begin{array}{l} z_1(t-t_1)\ z_1(t-t_2)\ z_2(t+\tau-t_6) \\ z_1^*(t+\tau-t_4)\ z_1^*(t+\tau-t_5)\ z_2^*(t-t_3) \end{array}\right\} \qquad (2\text{-}22)$$

which, from Eq. (2-A7) in the appendix is

$$U(t_1,\ldots,t_6;\tau) = R_z(t_6-t_3-\tau)\left[\begin{array}{l} R_z(\tau-t_4+t_1)\ R_z(\tau-t_5+t_2) \\ +R_z(\tau-t_4+t_2)\ R_z(\tau-t_5+t_1) \end{array}\right]. \qquad (2\text{-}23)$$

If we now assume the autocorrelation function given by Eq. (2-20), this may be substituted in Eq. (2-18a) to yield

$$R_a(\tau) = \iiint_{-W/2}^{W/2} df_1 df_2 df_3 \int \cdots \int dt_1 \cdots dt_6 \; g(t_1,t_2,t_3) \; g^*(t_4,t_5,t_6)$$

$$\cdot \left\{ \begin{array}{l} \exp[-j2\pi(f_1t_1 - f_1t_4 + f_1\tau + f_2t_2 - f_2t_5 + f_2\tau + f_3t_3 - f_3t_6 + f_3\tau)] \\ + \exp[-j2\pi(f_1t_1 - f_1t_5 + f_1\tau + f_2t_2 - f_2t_4 + f_2\tau + f_3t_3 - f_3t_6 + f_3\tau)] \end{array} \right\} .$$

(2-24)

Evaluation of the integrals in $t_1,\ldots,t_6$ yields

$$R_a(\tau) = \iiint_{-W/2}^{W/2} df_1 df_2 df_3 \; \exp[-j2\pi\tau(f_1 + f_2 + f_3)]$$

$$\cdot \left[\begin{array}{l} G(f_1,f_2,f_3) \;\; G^*\;(f_1,f_2,f_3) \\ + G(f_1,f_2,f_2) \;\; G^*\;(f_2,f_1,f_3) \end{array} \right] . \qquad (2\text{-}25)$$

Since

$$\begin{aligned} G(f_2,f_1,f_3) &= K(f_2+\nu_1, f_1+\nu_1, f_3-\nu_2) \\ &= K(f_1+\nu_1, f_2+\nu_1, f_3-\nu_2) \\ &= G(f_1,f_2,f_3) \;, \end{aligned}$$

the large bracket in Eq.(2-25) is actually equal to twice the first summand in it. We can write down by inspection the power spectrum of the interference as

$$\begin{aligned} G_a(f) &= \int_{-\infty}^{\infty} d\tau \; \exp(-j2\pi f\tau) \; R_a(\tau) \\ &= 2 \iiint_{-W/2}^{W/2} df_1 \; df_2 \; df_3 \; \delta(f-f_1-f_2-f_3) \\ &\qquad\qquad |G(f_1,f_2,f_3)|^2 \end{aligned}$$

or

$$G_a(f) = 2 \iint_{(f_1, f_2)\in\Omega_f} df_1 df_2 \left| G(f_1, f_2, f-f_1-f_2) \right|^2$$

where

$$\Omega_f = \left\{ \left(f_1, f_2\right) : |f_i| \le W/2, |f-f_1-f_2| \le W/2 \right\}, \tag{2-26b}$$

when

$$\nu_{p_1} = \nu_{p_2} = \nu_1, \; \nu_{p_3} = -\nu_2. \tag{2-26c}$$

We are particularly interested in the tail of the spectrum where

$$W/2 \le |f| \le 3W/2,$$

in comparison with the peak value. Suppose we let

$$G_m = \min_{|f_i| \le W/2} |G(f_1, f_2, f_3)| \tag{2-27a}$$

and

$$G_M = \max_{|f_i| \le W/2} |G(f_1, f_2 f_3)|. \tag{2-27b}$$

Then for any f

$$G_a(f) \le 2 \, G_M^2 \iint_{\Omega_f} df_1 df_2, \tag{2-28a}$$

$$G_a(f) \ge 2 \, G_m^2 \iint_{\Omega_f} df_1 \, df_2. \tag{2-28b}$$

The integral over Ω_f can be evaluated fairly readily; we find

$$\int_{\Omega_f} df_1 df_2 = \begin{cases} 3W^2/4 - f^2 & , \; |f| \le W/2 \\ (3W/2 - |f|)^2/2 & , \; W/2 \le |f| \le 3W/2 \end{cases} . \tag{2-29}$$

From Eqs(2-28) and (2-29) we can bound the ratio of interference power in the channels centered on $\nu_T \pm W$ to the interference power in the channel centered on ν_T. We have

$$\frac{\text{adjacent channel interference power}}{\text{direct channel interference power}} \le \frac{G_M}{4G_m}. \tag{2-30}$$

We can repeat the whole procedure now for the case in Eq. (2-14) where all three carriers are distinct:

$$\nu_{p_1} = \nu_1, \nu_{p_2} = \nu_2, \nu_{p_3} = \nu_3. \tag{2-31}$$

The expectation U is given by

$$U(t_1,\ldots,t_6;\tau) = E\left\{\begin{matrix} z_1(t-t_1) \; z_2(t-t_2) \; z_3(t+\tau-t_6) \\ z_1^*(t+\tau-t_4) \; z_2^*(t+\tau-t_5) \; z_3^*(t-t_3) \end{matrix}\right\} \tag{2-32}$$

from which

$$U(t_1,\ldots,t_6;\tau) = R(\tau-t_4+t_1)\, R(\tau-t_5+t_2)\, R(t_6-t_3-\tau). \tag{2-33}$$

A comparison with Eq. (2-23) shows that the interference spectrum is exactly one-half that found in the previous case so that Eq. (2-30) applies in this case also.

2.6 Final Selection of Interference Combinations

We are now in a position to make a selection of the interference cases to be modelled. We recall that Eqs. (2-11) to (2-14) defined four basic combinations of interfering frequencies, and for each of these cases it would in general be possible to examine situations where the interference carrier ν_T fell on either the desired carrier ν or the adjacent channel carriers $\nu \pm W$.

The case given by Eq. (2-11) involves interaction of a signal with itself so that the interference band is centered on the same carrier frequency. The only two situations of interest are those where $\nu_1 = \nu$ and $\nu_1 = \nu + W$. (The lower adjacent channel case is essentially identical to the upper adjacent channel and need not be treated separately.) We find it reasonable to assume that a well-designed AGC circuit will preclude significant self-interference of the desired signal. We therefore restrict this case to

Case 1: $\nu_{p_1} = \nu_{p_2} = -\nu_{p_3} = \nu_T = \nu + W$. (2-34)

The situation in Eq. (2-12) is that of carriers at ν_1 and ν_2 producing an intermodulation carrier at $2\nu_1 - \nu_2$. We again assume that an adequate AGC makes the case $\nu_1 = \nu$ uninteresting. This leaves only the question of whether to allow modelling of the cases

$$\nu_2 = 2\nu_1 - \nu \pm W,$$

as well as

$$\nu_2 = 2\nu_1 - \nu.$$

The conclusion we draw from Eq. (2-30) is that the adjacent channel interference effect is unlikely to be as strong as the direct channel interference effect so that for any reasonably well behaved K we can restrict attention to the situation where the intermodulation carrier falls on the desired carrier. We are thus led to take as the second case:

Case 2: $\nu_{p_1} = \nu_{p_2} = \nu_1; \nu_{p_3} = \nu - 2\nu_1; \nu_T = \nu$. (2-35)

The situation in Eq. (2-13) is a cross-modulation interference where the interference carrier is at the frequency of one of the two interfering carriers. The classic case involves crossmodulation of the desired signal, which certainly must be evaluated. However, it would appear equally important to consider the situation where a moderately strong adjacent channel signal is splattered into the desired band as a result of crossmodulation by a strong out-of-band signal. We are thus led to two more cases:

$$\text{Case 3:} \quad \nu_{p_1} = \nu_T = \nu, \; \nu_{p_2} = -\nu_{p_3} = \nu_2; \tag{2-36}$$

$$\text{Case 4:} \quad \nu_{p_1} = \nu_T = \nu + W, \; \nu_{p_2} = -\nu_{p_3} = \nu_2. \tag{2-37}$$

The final situation is that of Eq.(2-14) involving three-frequency intermodulation. We note that Case 2 and Case 4 above are special cases of Eq.(2-14). Furthermore, from the results of Section 6 of TR#3 we know that the probability that three carriers have sufficient power to produce this interference is small compared to the probability of the two-carrier interactions described by Eqs.(2-35) and (2-37). We therefore exclude this option from the modelling capability.

In summary, the cases described by Eqs. (2-34) through (2-37) will form the basis of our modelling.

2.7 Scaling

The complex envelopes appearing in the several expressions include implicitly a scale factor proportional to the square root of the nominal carrier power. These implicit scale factors are multiplied together (also implicitly) in determining the peak voltage of the complex envelope of the interference.

Considerable efficiency can be achieved in the operation of the computer programs if these scale factors are made explicit and their product used to scale the interference output <u>after</u> it has been calculated. In this way the effect of varying power level can be determined by scaling a single output sequence rather than by repeating the entire triple summation involved in the tapped-delay line model.

It is also appropriate to include the peak magnitude of $K(f_1, f_2, f_3)$ in this final scaling so that the tapped-line coefficients have a relatively restricted set of magnitudes.

2.8 <u>Tap Spacing and Sample Spacing</u>

We would like to discuss here the interrelation between the tap spacing and the sample spacing. Let

δ = time interval between adjacent samples of the complex envelopes; (2-38)

t_o = time interval between taps in tapped delay line model. (2-39)

The range of values of t_o is determined by the requirement of getting a good fit to the transfer function. Values of t_o near zero will be used, for example, in approximating polynomial fits to the transfer function. At the other extreme we can assume that $t_o < 1/W$ which is the largest value that permits a Fourier representation of $K(f_1, f_2, f_3)$. We thus have

$$0 < t_o < 1/W. \tag{2-40}$$

A second constraint is imposed by compatibility with the sampled-data representation of the complex envelopes. We clearly must constrain t_o to be an integer multiple of δ:

$$t_o = \text{integer}\cdot\delta. \tag{2-41}$$

The sampling interval itself is constrained by the need for adequate representation of the complex envelope of the interference. Since, by the earlier assumptions, this envelope has a spectrum occupying the interval $(-3W/2, 3W/2)$ a sampling rate of 3W samples/second is the minimum allowable to permit Nyquist sampling. To avoid the need for (sin x/x) sampled data filters it is wise to allow at least some margin and require that

$$\delta < 1/4W. \tag{2-42}$$

For this initial effort, which must be looked on as a validation of the possibility of computer simulation of the models, we have chosen to satisfy all of the constraints simultaneously by requiring that

$$\delta = t_o \tag{2-43}$$

and

$$0 < t_o < 1/4W. \tag{2-44}$$

The penalty imposed by this lack of flexibility in setting $\delta=t_o$ is a restriction of the "wildness" of the transfer functions that can be accommodated by the model. We should point out, however, that this same restriction greatly reduces the measurement or computational burden in determining the values of the transfer function on a cubic lattice.

APPENDIX TO SECTION 2:

CALCULATION OF TRIPLE MOMENT OF COMPLEX GAUSSIAN RANDOM VARIABLES

Let $x_1, \ldots x_6$ be complex Gaussian variables and let μ be the moment:

$$R = E(x_1 x_2 x_3 x_4^* x_5^* x_6^*). \tag{2-A1}$$

By a Gram-Schmidt procedure we can represent $\{x_A\}$ as a transformation on uncorrelated unit variance variables in the form

$$\left.\begin{aligned} x_1 &= a_{11} y_1 \\ x_2 &= a_{21} y_1 + a_{22} y_2 \\ &\cdots \\ x_6 &= a_{61} y_1 + \ldots + a_{66} y_6 \end{aligned}\right\} \tag{2-A2}$$

where

$$E\left\{y_m\, y_n^*\right\} = \delta_{mn}, \text{ the Kronecker delta.} \tag{2-A3}$$

If we substitute Eq. (2-A2) in Eq. (2-A1), the result is a sum containing 6! products of the $\left\{y_n\right\}$ and their conjugates. It would be bad judgment to write this sum out because most of the terms vanish when we take the expectation. In fact the only non-zero expectations are those of the form

$$E\left\{|y_i|^2\ |y_j|^2\ |y_k|^2\right\}$$

in which i,j,k may or may not be distinct subscripts. We see immediately that y_4, y_5 and y_6 will never enter into the calculation of R because they never appear unconjugated in the product. We will need the three moments of the unit-mean exponential distribution in writing down the expected products:

$$E\left\{|y_i|^{2p}\right\} = p!. \tag{2-A4}$$

We then have

$$\begin{aligned} R = {} & 6(a_{11}a_{21}a_{31})(a_{41}a_{51}a_{61})^* \\ & +2(a_{11}a_{22}a_{31}+a_{11}a_{21}a_{32})(a_{51}a_{61}a_{42}+a_{41}a_{61}a_{52}+a_{41}a_{51}a_{62})^* \\ & +2(a_{11}a_{21}a_{33})(a_{51}a_{61}a_{43}+a_{41}a_{61}a_{53}+a_{41}a_{51}a_{63})^* \\ & +2(a_{11}a_{22}a_{32})(a_{51}a_{62}a_{41}+a_{42}a_{62}a_{51}+a_{42}a_{52}a_{61})^* \\ & +(a_{11}a_{22}a_{33})(a_{41}a_{52}a_{63}+a_{51}a_{62}a_{43}+a_{61}a_{42}a_{53}+a_{41}a_{53}a_{62} \\ & \qquad +a_{51}a_{63}a_{42}+a_{61}a_{43}a_{52})^* \end{aligned} \tag{2-A5}$$

This latter expression needs to be rephrased in terms of the covariances of pairs of $\left\{x_n\right\}$:

$$R_{mn} = E\left\{X_m X_n^*\right\} = \sum_i \alpha_{mi}\, \alpha_{ni}^*. \tag{2-A6}$$

Now if X_1, X_2, and X_3 were independent and if X_4, X_5, X_6 were permutation of them, then R would contain a product of the form

$$R_{1,p_1}\, R_{2,p_2} R_{3,p_3}$$

where (p_1,p_2,p_3) was a permutation of (4,5,6). We are thus led to conjecture that

$$\begin{aligned} R = {} & R_{14}\, R_{25}\, R_{36} + R_{15}\, R_{26}\, R_{34} + R_{15}\, R_{24}\, R_{35} \\ & + R_{14}\, R_{26}\, R_{35} + R_{15}\, R_{24}\, R_{36} + R_{16}\, R_{25}\, R_{34}. \end{aligned} \tag{2-A7}$$

If we write out this sum using Eq. (2-A6) we can verify its coincidence with Eq. (2-A5) and thus correctness of our conjecture.

SECTION 3

DETERMINATION OF MODEL PARAMETERS

We start here with Eq. (2-10) of the last section which we repeat here:

$$y_K(t) = \text{constant} \cdot \mathrm{Re}\{a_{p_1,p_2,p_3}(t) \exp[j2\pi\nu_T t]\} \tag{3-1a}$$

where

$$\left.\begin{array}{l} p_1 > 0 \\ p_2 > 0 \\ p_3 < 0 \end{array}\right\} \tag{3-1b}$$

and

$$\nu_T = \nu_{p_1} + \nu_{p_2} + \nu_{p_3}. \tag{3-1c}$$

The complex envelope $a_{\underline{p}}(t)$ in Eq. (3-1a) is given by Eq. (2-8) as

$$a_{\underline{p}}(t) = \iiint_{-W/2}^{W/2} d\underline{f}\ G_{\underline{p}}(\underline{f}) \exp[j2\pi t(f_1 + f_2 + f_3)]$$

$$\cdot Z_{p_1}(f_1)\ Z_{p_2}(f_2)\ Z_{p_3}(f_3), \tag{3-2}$$

in which

$$G_{\underline{p}}(\underline{f}) = K(f_1 + \nu_{p_1},\ f_2 + \nu_{p_2},\ f_3 + \nu_{p_3}). \tag{3-3}$$

Our modelling procedure relies on the property that if

$$G_{\underline{p}}(\underline{f}) \approx \hat{G}(f) \text{ for } |f_i| < W/2,\ i = 1,2,3, \tag{3-4}$$

then

$$a_{\underline{p}}(t) \approx \iiint_{-\infty}^{\infty} d\underline{f}\ \hat{G}(\underline{f}) \exp[j2\pi t(f_1 + f_2 + f_2)]$$

$$Z_{p_1}(f_1)\ Z_{p_2}(f_2)\ Z_{p_3}(f_3), \tag{3-5}$$

by virtue of the bounded support of the input spectra $\{Z_{p_i}\}$. We have furthermore chosen to restrict attention to approximations $\hat{G}(\underline{f})$ of the form

$$\hat{G}(\underline{f}) = \sum_{n_1} \sum_{n_2} \sum_{n_3} B(n_1,n_2,n_3) \exp[-j2\pi\delta(n_1 f_1 + n_2 f_2 + n_3 f_3)]. \tag{3-6}$$

Substitution of Eq. (3-6) in Eq. (3-5) then leads to

$$a_{\underline{p}}(t) = \sum_{n_1} \sum_{n_2} \sum_{n_3} B(n_1,n_2,n_3) \cdot \prod_{i=1}^{3} \int_{-\infty}^{\infty} df_i \exp[j2\pi f_i(t-n_i\delta)] Z_{p_i}(f_i) \tag{3-7}$$

or

$$a_{\underline{p}}(t) = \sum_{n_1} \sum_{n_2} \sum_{n_3} B(n_1,n_2,n_3) \prod_{i=1}^{3} z_{p_i}(t-n_i\delta). \tag{3-8}$$

3.1 Choice of Model Parameters: General Considerations

Referring to Eq.(3-6) the parameters that must be specified are

a) the tap spacing δ

b) the range of indices $\{n_1,n_2,n_3\}$ in the sum

c) the coefficient set $\{B(n_1,n_2,n_3)\}$.

For computational purposes it is reasonable to require that the set of indices be identical for each coordinate so that Eq.(3-6) can be specialized to read

$$\hat{G}(f) = \sum_{N_I+1 \le \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix} \le N_I+1} \sum \sum B(n_1,n_2,n_3) \prod_{i=1}^{3} e^{-j2\pi\delta n_i f_i}. \tag{3-9}$$

The analogous time response of Eq. (3-8) then becomes

$$a_{\underline{p}}(t) = \sum_{N_I+1 \le \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix} \le N_I + N} \sum \sum B(n_1,n_2,n_3) \prod_{i=1}^{3} z_{p_1}(t-n_i\delta). \tag{3-10}$$

In these expressions N is then the number of delays used in the model for each input sequence, while the number N_I can be thought of as an overall delay of the output sample sequence relative to the linear components of the model. The number N can be assumed to be fixed ahead of time by complexity limitations on the computer programming. It is therefore necessary to determine $N^3 + 2$ model parameters:

a) the delay spacing δ

b) the overall delay N_I

c) the N^3 values of the $\{B(n_1,n_2,n_3)\}$.

If the function $G(\underline{f})$ were specified at all values of $\underline{f}$ by an analytic description, it would be reasonbly straightforward conceptually to find a choice of the $N^3 + 2$ parameters that minimized the quadratic approximation error

$$\iiint d\underline{f}\,|\hat{G}(\underline{f}) - G(\underline{f})|^2.$$

What we will actually be working with, however, is a finite set of calculated values as measurements of $G(\underline{f})$ in the form

$$G(m_1 f_o,\ m_2 f_o,\ m_3 f_o)$$

as m_1, m_2, and m_3 range over some small set of integers. If M is the number of measurements per frequency coordinate so that the total number of measurements is M^3, then clearly we require that

$$M^3 > N^3 + 2,$$

so that the number of measurements will exceed the number of parameters to be determined. Thus we must take

$$M \geq N + 1.$$

On the other hand, the determination of each of the M^3 data points will require either a significant measurement effect or significant computation time. Therefore we believe that it is reasonable to use only the minimum number and hence require that

$$M = \underset{\sim}{N} + 1, \tag{3-11a}$$

where $\underset{\sim}{N}$ is the largest value of N for which modelling is to be done.

At this point we should observe that the smallest "interesting value of N is N = 2 corresponding to a two-tap model, or, in the limit of small tap-spacing δ, a frequency power series with linear terms in each frequency. We have chosen to allow values of N as large as 4 which provides considerable flexibility beyond the minimally interesting model; the corresponding value of M = 5 which requires 125 measurements or calculations is probably as large as can be conveniently accomplished with any reasonable economy of either computer or measurement time. We thus will restrict attention to

$$M \leq 5 \tag{3-12a}$$

and, from Eq. (3-11),

$$N \leq 4. \tag{3-12b}$$

The most favorable location of the frequency lattice points is not immediately apparent. To be more specific, it is not apparent how close to the band edges of $\pm$ W/2 the extreme data points should be. Although an argument could be made that choosing the frequencies of $\pm$ W/2 as 2 of the data coordinates on each dimension gives undue weight to possibly anomalous band-edge phenomena, this choice is a conservative one in that it will make any model weaknesses most apparent. We therefore will henceforth assume that the measured or calculated data consist of the values

$$\Upsilon(m_1,m_2,m_3) = G\begin{bmatrix} -W/2 + (m_1-1)W/\underset{\sim}{N}, \\ -W/2 + (m_2-1)W/\underset{\sim}{N}, \\ -W/2 + (m_3-1)W/\underset{\sim}{N} \end{bmatrix}, \tag{3-13a}$$

for

$$1 \le m_1,m_2,m_3 \le \underset{\sim}{N}+1. \tag{3-13b}$$

We now define $\hat{\Upsilon}$ in the obvious way as the value of the approximating function $\hat{G}$ at the same lattice frequencies, and define an error criterion

$$V = \sum_{m_1=1}^{\underset{\sim}{N}+1} \sum_{m_2=1}^{\underset{\sim}{N}+1} \sum_{m_3=1}^{\underset{\sim}{N}+1} \left|\hat{\Upsilon}(m_1,m_2,m_3) - \Upsilon(m_1,m_2,m_3)\right|^2 \tag{3-14}$$

which is a discrete version of the quadratic error criterion. It should be observed that V is implicitly a function of the parameters N_I, δ, and $\{B(n_1,n_2,n_3)\}$. The objective of the parameter-extraction program is then to minimize V by the choice of these implicit arguments.

Before proceeding further it is helpful notationally to let

$$\beta(n_1,n_2,n_3) = B(n_1+N_I,n_2+N_I,n_3+N_I) \tag{3-15}$$

so that Eqs. (3-9) and (3-10) can be rewritten as

$$\hat{G}(\underline{f}) = \exp\left[-j2\pi N_I\delta(f_1+f_2+f_3)\right]$$

$$\cdot \sum_{n_1=1}^{N} \sum_{n_2=1}^{N} \sum_{n_3=1}^{N} \beta(n_1,n_2,n_3) \prod_{i=1}^{3} e^{-j2\pi\delta n_i f_i} . \tag{3-16}$$

$$a_p(t) = \sum_{n_1=1}^{N} \sum_{n_2=1}^{N} \sum_{n_3=1}^{N} \beta(n_1,n_2,n_3) \prod_{i=1}^{3} z_{p_i}(t-n_i\delta-N_I\delta). \tag{3-17}$$

It can be verified that for any fixed δ and N_I, the determination of the best values of $\beta(n_1,n_2,n_3)$ is a routine quadratic minimization. (We will present the derivation of these coefficients subsequently.) The problem thus reduces to finding an efficient algorithm for determining the best choice of δ and N_I.

3.2 Dependence of V on δ

Although it is theoretically possible to find the optimum value of δ by differentiating the quadratic error V with respect to δ and equating this derivative to zero, it appears to be more realistic computationally to search for the minimum V by evaluating Eq.(3-14) for several discrete values of δ. This approach also guarantees that the value of δ we select will be an approximation to the value yielding a global minimum rather than one which yields only a local minimum. We now need to investigate the limits of this computer search, or, more exactly, the lower limit, since we have already determined that $\delta < 1/4W$. We thus need to investigate the behavior of the optimizing solutions as $\delta \to 0$ and then determine the largest possible positive δ which permits approximating this limiting behavior.

From Eqs. (3-13) and (3-16) we have

$$\hat{Y}(m_1,m_2,m_3) = \exp\{-j2\pi N_I \delta W[-3/2-3/\underset{\sim}{N}+(m_1+m_2+m_3)/\underset{\sim}{N}]\}$$

$$\cdot \sum_{n_1=1}^{N} \sum_{n_2=1}^{N} \sum_{n_3=1}^{N} \beta(n_1,n_2,n_3)$$

$$\prod_{i=1}^{3} \exp[-j2\pi\delta n_i W(-1/2\ -1/\underset{\sim}{N}+m_i/\underset{\sim}{N})]. \tag{3-18}$$

We clearly must have each of the $\hat{\gamma}$ approaching a limiting value as $\delta \to 0$; we will denote this limit by the subscript zero:

$$\hat{\gamma}_0(m_1,m_2,m_3) = \lim_{\delta \to 0} \hat{\gamma}(m_1,m_2,m_3). \tag{3-19}$$

In Eq. (3-18), let us denote the triple sum by $\sigma(m_1,m_2,m_3)$. By writing the product of exponentials as the exponential of the sum of arguments, σ can be written as

$$\sigma(m_1,m_2,m_3) = \sum_{n_1=1}^{N} \sum_{n_2=1}^{N} \sum_{n_3=1}^{N} \beta(n_1,n_2,n_3) \cdot$$

$$\cdot \exp\left[-j2\pi\delta W \sum_{i=1}^{3} n_i(-1/2-1/\underset{\sim}{N}+m_i/\underset{\sim}{N})\right]. \tag{3-20}$$

If we now expand the exponential in a power series in δ, the summation in the power series can be commuted with the triple finite sum on (n_1,n_2,n_3) to yield

$$\sigma(m_1,m_2,m_3) = \sum_{k=0}^{\infty} (-j2\pi W\delta)^k \cdot$$

$$\cdot \sum_{n_1=1}^{N} \sum_{n_2=1}^{N} \sum_{n_3=1}^{N} \beta(n_1,n_2,n_3) \cdot$$

$$\cdot \left[\sum_{i=1}^{3} n_i \left(-\frac{1}{2} - \frac{1}{\underset{\sim}{N}} + \frac{m_i}{\underset{\sim}{N}}\right)\right]^k. \tag{3-21}$$

Now, if the coefficients $\{\beta(n_1,n_2,n_3)\}$ were constants independent of δ, then, for sufficiently small δ, the leading term in the infinite sum would be the dominant term, and $\sigma(m_1,m_2,m_3)$ would be independent of m_1,m_2, and m_3. However, we can expect that the dependence of the $\{\beta(n_1,n_2,n_3)\}$ on δ will be reflected in relations of the form

$$\sum_{(n_1,n_2,n_3)} \beta(n_1,n_2,n_3) \to \text{constant as } \delta\to 0,$$

$$\sum_{(n_1,n_2,n_3)} n_i\ \beta(n_1,n_2,n_3) \to \frac{\text{constant}}{\delta} \quad \text{as } \delta\to 0,$$

$$\sum_{(n_1,n_2,n_3)} n_i n_j\ \beta(n_1,n_2,n_3) \to \frac{\text{constant}}{\delta^2} \quad \text{as } \delta\to 0,$$

and so forth. It will thus be possible to obtain polynomials in $m_1, m_2,$ and m_3 as $\delta\to 0$.

Suppose, then, that we postulate that each β has a finite Laurent development of the form

$$\begin{aligned}\beta(n_1,n_2,n_3) &= \beta_0(n_1 \cdot n_2,n_3) + \beta_1(n_1,n_2,n_3)\delta^{-1} \\ &\quad +\ldots+ \beta_L(n_1,n_2,n_3)\delta^{-L} \qquad (3\text{-}22) \\ &\quad + \text{terms of the order of } \delta \text{ or smaller.}\end{aligned}$$

If we let

$$P_D(n_1,n_2,n_3) = \text{homogeneous polynomial in } (n_1,n_2,n_3) \text{ of degree } D, \qquad (3\text{-}23)$$

then clearly we must have

$$\sum_{(n_1,n_2,n_3)} \beta_\ell(n_1,n_2,n_3)\ P_D(n_1,n_2,n_3) = 0 \text{ if } D < \ell \qquad (3\text{-}24)$$

for otherwise there would be infinite values of σ as $\delta\to 0$. Equation (3-24) imposes

$$\binom{\ell+2}{3} = \frac{\ell(\ell+1)(\ell+2)}{6}$$

linear constraints on the $\{\beta_\ell(n_1,n_2,n_3)\}$ corresponding to the number of types of homogeneous polynomials of degree $< \ell$. Since there are only N^3 of these coefficients, it follows that

$$L(L+1)(L+2) \le 6N^3 \tag{3-25}$$

or

$$\left.\begin{array}{l} L \le 1 \text{ if } N = 1 \\ L \le 2 \text{ if } N = 2 \\ L \le 4 \text{ if } N = 3 \\ L \le 6 \text{ if } N = 4 \end{array}\right\} . \tag{3-26}$$

The limiting value of $\sigma(\ )$, which we will denote by $\sigma_o(\ )$, is then

$$\sigma_o(m_1, m_2, m_3) = \sum_{\ell=0}^{L} (-j2\pi W)^{\ell} \cdot \sum_{n_1=1}^{N} \sum_{n_2=1}^{N} \sum_{n_3=1}^{N} \beta_{\ell}(n_1, n_2, n_3) \left[\sum_{i=1}^{3} n_i\left(-\frac{1}{2} - \frac{1}{\underset{\sim}{N}} + \frac{m_i}{\underset{\sim}{N}}\right)\right]^{\ell} . \tag{3-27}$$

The terms in this expression can be rearranged to yield

$$\sigma_o(m_1, m_2, m_3) = \sum_{\underline{\alpha} \in A} c_{\underline{\alpha}}\, m_1^{\alpha_1}\, m_2^{\alpha_2}\, m_3^{\alpha_3} \tag{3-28a}$$

where

$$\underline{\alpha} = (\alpha_1, \alpha_2, \alpha_3) \tag{3-28b}$$

and

$$A = \{\underline{\alpha}: \Sigma\alpha_i \le L\}. \tag{3-28c}$$

The number of coefficients in Eq. (3-28a), or, equivalently, the cardinality of A, is

$$\text{Card } (A) = \binom{L+3}{3}. \tag{3-29}$$

This might suggest the inference that there are more degrees of freedom in the limiting case than in any of the expressions for nonzero δ. This inference would be false, however. In addition to the linear constraints of Eq. (3-24) there are linear constraints between the different $\{\beta_\ell\}$ and the measurement values. In fact, if δ_1 is some value of δ sufficiently small so that

$$\sigma_{\delta_1}(m_1,m_2,m_3) \approx \sigma_o(m_1,m_2,m_3)$$

then the fact that there are only N^3 independent coefficients in Eq. (3-20) implies that of the $M^3 = (\underset{\sim}{N}+1)^3$ values of σ_{δ_1}, only N^3 are linearly independent.

We now need to investigate the rate of approach to the limiting form of Eq. (3-28a). To this end, let us define

$$\Gamma_\delta(\underline{f}) = \sum_{\underline{n}} \sum_{\ell=0}^{L} \beta_\ell(\underline{n})\, \delta^{-\ell} \exp\left[-j2\pi\delta\Sigma n_i f_i\right]. \tag{3-30}$$

The rate of approach of $\Gamma_\delta(f)$ to its limiting value is at least as good as would be obtained if the summation on ℓ included positive powers in δ. We find

$$\lim_{\delta\to 0} \delta^{-\ell} \sum_{\underline{n}} \beta_\ell(\underline{n}) \exp[-j2\pi\delta\Sigma n_i f_i] = P_\ell(\underline{f}) \tag{3-31}$$

where P_ℓ is some homogeneous polynomial of degree ℓ in 3 variables. It is actually one part of the frequency polynomial approximation:

$$P_\ell(\underline{f}) = \sum_{\underline{\ell}} \frac{\ell!}{\ell_1!\ell_2!\ell_3!}(-j2\pi)^\ell f_1^{\ell_1} f_2^{\ell_2} f_2^{\ell_3} \sum_{\underline{n}} \beta_\ell(\underline{n})\, n_1^{\ell_1} n_2^{\ell_2} n_3^{\ell_3}; \tag{3-32a}$$

in which

$$\underline{\ell} = (\ell_1, \ell_2, \ell-\ell_1-\ell_2). \tag{3-32b}$$

It is not difficult to verify that the slowest convergence is obtained when $\ell = L$. We will therefore restrict attention to that case.

We now observe that

$$n_1^{m_1} n_2^{m_2} n_3^{m_3} = \frac{1}{(2\pi j)^3} \iiint \frac{(n_1z_1+n_2z_2+n_3z_3)^D}{z_1^{1+m_1}\, z_2^{1+m_2}\, z_3^{1+m_3}}\, dz_1dz_2dz_3 \tag{3-33}$$

where all three integrals are on the unit circle. Hence any homogenous polynomial of degree D in $\underline{n}$ can be approximated by a linear combination of polynomials of the form $(n_1z_1+n_2z_2+n_3z_3)^D$. The linear constraints of Eq. (3-24) can therefore be phrased as

$$\sum_{\underline{n}} \beta_L(\underline{n})\ (n_1z_1+n_2z_2+n_3z_3)^D = 0$$

for every nonzero $\underline{z}$ and for every $D < L$. (3-34)

Suppose that we define

$$g(w,\underline{z}) = \sum_{D=0}^{\infty} \frac{w^D}{D!} \sum_{\underline{n}} \beta_L(\underline{n})\,(n_1z_1+n_2z_2+n_3z_3)^D. \tag{3-35}$$

Transposing the order of the two sums,

$$g(w,\underline{z}) = \sum_{\underline{n}} \beta_L(\underline{n}) \exp\,[w(n_1z_1+n_2z_2+n_3z_3)]. \tag{3-36}$$

The constraint equation now becomes, from Eqs. (3-34) and (3-35)

$$\lim_{w\to 0} w^{-L}\, g(w,\underline{z}) < \infty \text{ for every nonzero } \underline{z}. \tag{3-37}$$

We observe now that the right hand side of Eq. (3-36) is a polynomial in

$$e^{wz_1},\ e^{wz_2},\ \text{and } e^{wz_3}.$$

This polynomial is of degree N in each variable with the zero degree terms in each variable missing. It is readily seen that the only polynomial of this form that satisfies Eq. (3-37) is

$$g(w,\underline{z}) = \exp(w\,\Sigma\, z_i)\prod_{m=1}^{L}\left\{1-\exp[w(\mu_{m1}z_1+\mu_{m2}z_2+\mu_{m3}z_3)]\right\}, \qquad (3\text{-}38)$$

up to a constant multiplier. In this expression the $\{\mu_{mi}\}$ are non-negative integers with at least one μ_{mi} nonzero for each m. Furthermore since the product includes a term of the form

$$\exp\left[w\sum_m\sum_i \mu_{mi}\, z_i\right],$$

the exponent in this product must be no greater than

$$(N-1)(z_1+z_2+z_3).$$

We therefore have

$$\sum_i \mu_{mi} > 0 \text{ for every m.} \qquad (3\text{-}39a)$$

$$\sum_m \mu_{mi} \leq N-1 \text{ for every i.} \qquad (3\text{-}39b)$$

The coefficients $\{\beta_L(\underline{n})\}$ could be found by writing out the L-fold product in Eq.(3-38) and matching coefficients with Eq.(3-36). However, it is much more to the point to compare Eq.(3-30) with Eq.(3-36) for the special case where $\beta_\ell(\underline{n})$ is zero for $\ell < L$, and note that in this case

$$\Gamma_\delta(\underline{f}) = \delta^{-L} g(-j2\pi\delta,\underline{f}) \qquad (3\text{-}40)$$

so that from Eq.(3-38)

$$\Gamma_\delta(\underline{f}) = \delta^{-L}\exp(-j2\pi\delta\Sigma f_i)\prod_{m=1}^{L}\{1-\exp[-j2\pi\delta\sum_i\mu_{mi}f_i]\}. \qquad (3\text{-}41)$$

The limiting form of this can be written by inspection as

$$\Gamma_0(\underline{f}) = (j2\pi)^L\prod_{m=1}^{L}\sum_{i=1}^{3}\mu_{mi}f_i. \qquad (3\text{-}42)$$

Let us write

$$\varphi(\delta) = \Gamma_\delta(f)/\Gamma_o(f) \tag{3-43}$$

in order to evaluate the approach to the limit. Then

$$\varphi(\delta) = \exp(-j2\pi\delta\Sigma f_i) \cdot \prod_{m=1}^{L} \frac{1-\exp[-j2\pi\delta\sum_i \mu_{mi} f_i]}{j2\pi\delta\sum_i \mu_{mi} f_i} . \tag{3-44}$$

The multiplier

$$\exp(-j2\pi\delta\Sigma f_i)$$

can actually be subsumed in the multiplier

$$\exp\{-j2\pi N_I \delta \ \Sigma f_i\}$$

which appears in its discrete version as the first multiplier in (Eq.3-18). We can therefore take the approach to the limit to be that of

$$\tilde{\varphi}(\delta) = \prod_{m=1}^{L} \frac{1-\exp[-j2\pi\delta\sum_i \mu_{mi} f_i]}{j2\pi\delta\sum_i \mu_{mi} f_i} , \tag{3-45}$$

which, for sufficiently small δ is approximately

$$\tilde{\varphi}(\delta) \approx 1 - j\pi\delta \sum_m \sum_i \mu_{mi} f_i . \tag{3-46}$$

The absolute fractional difference between $\Gamma_\delta(\underline{f})$ and $\Gamma_o(\underline{f})$ can then be taken to be

$$\epsilon(\delta,\underline{f}) = \pi\delta \sum_m \sum_i \mu_{mi} f_i . \tag{3-47}$$

This is clearly largest at the band edges so that

$$|\epsilon| \leq (\pi\delta W/2) \sum_m \sum_i \mu_{mi} . \tag{3-48}$$

From Eq.(3-39b) this can in turn be bounded above by

$$|\epsilon| \le 3(N-1)\pi\delta W/2, \tag{3-49}$$

or, using

$$N \le \underset{\sim}{N} = 4, \tag{3-50}$$

$$|\epsilon| \le 9\pi\delta W/2. \tag{3-51}$$

It is a numerical convenience to have the smallest value of δW be an integer power of 1/2. At $\delta W = 1/128$ we have

$$|\epsilon| \le 9\pi/256 \text{ if } \delta W = 1/128. \tag{3-52}$$

This represents about a 6° discrepancy from the limiting value when this discrepancy is 90° out of phase as suggested by the form of Eq. (3-46).

We can also check the relative amplitudes using Eq.(3-44). We have

$$|\varphi(\delta)| = \prod_{m=1}^{L} \frac{\left|\sin \pi\delta \sum_i \mu_{mi} f_i\right|}{\left|\pi\delta \sum_i \mu_{mi} f_i\right|} \quad , \tag{3-53}$$

which for small δ, is approximately

$$|\varphi(\delta)| \approx 1- (\pi^2\delta^2 W^2/24) \sum_m \left(\sum_i \mu_{mi}\right)^2,$$

$$\text{if } |f_i| = W/2. \tag{3-54}$$

The maximum of this under the constraints of Eq.(3-39) occurs, for $N = 4$ and $L = 6$, when

$\mu_{m1} = \mu_{m2} = 1, \quad 1 \le m \le 3$

$\mu_{m3} = 1 \qquad , \quad 4 \le m \le 6$

$\mu_{mi} = 0 \qquad$, otherwise.

We then have

$$|\varphi(\delta) \approx 1-5(\pi\delta W)^2/8, \quad N = 4. \tag{3-55}$$

For $\delta W = 1/128$, this represents an error of about 2.3×10^{-4} which is negligible.

3.3 Choice of Values of δ in Parameter Fitting

Bas d on the results of the previous subsection we will confine δ to the range

$$1/128 \leq \delta W \leq 1/4. \tag{3-56}$$

It is also necessary to select the grid of points for δ within this range. We have somewhat arbitrarily chosen to start at $\delta W = 1/4$ and successively halve the value of δ to get the next trial value. In this manner, each tapped line can be looked on as a refinement and truncation of the previous one. We therefore have the final selection of values

$$\delta W = 1/4,\ 1/8,\ 1/16,\ 1/32,\ 1/64,\ \text{and } 1/128. \tag{3-57}$$

3.4 Range of N_I

Referring back to Eqs. (3-18) and (3-20), let us write

$$\hat{v}(m_1,m_2,m_3) = \sigma(m_1,m_2,m_3)$$

$$\cdot \exp\{-j2\pi N_I \delta W[-3/2-3/\underset{\sim}{N}+(m_1+m_2+m_3)/\underset{\sim}{N}]\}. \tag{3-58}$$

For sufficiently small δ we can replace $\sigma(m_1,m_2,m_3)$ by its limiting value $\sigma_o(m_1,m_2,m_3)$. Unless $N_I\delta$ is very small throughout the range where this approximation holds true, the values of $\hat{Y}$ will change as $\delta\to 0$ unless $N_I\delta$ remains fixed. We therefore can assume that

$$N_I \sim \text{constant}/\delta \text{ as } \delta \to 0. \qquad (3\text{-}59)$$

We will use this property to pick the range of N_I after the first trial.

At the other extreme, we observe that a change of m_i by 1 causes a change of

$$j2\pi(N_I + n_i)\delta W/\underset{\sim}{N}$$

in the argument of some exponential term. More accurately, if we use an arbitrary lattice of M^3 measurement points, the argument of the exponential changes by

$$j2\pi(N_I + n_i)\delta W/(M-1),$$

which for $\delta W = 1/4$ is

$$j\pi(N_I + n_i)/2(M-1).$$

This implies that if $|N_I + n_i| > M-1$, then one of the exponentials is incurring more than 90° phase change between adjoining lattice points. We believe that such a situation corresponds to an under-sampled transfer function (in an engineering sense) and that therefore the restriction should be assumed that

$$|N_I + n_i| \le M-1,$$

or, since

$$1 \le n_i \le N,$$

we will require that

$$N_I + 1 \ge 1-M$$

$$N_I + N \le M-1$$

or

$$-M \le N_I \le M-1-N. \qquad (3\text{-}60)$$

Let us now denote by $\hat{N}_I(\delta)$ the apparent best choice of N_I for a particular value of δ. From Eq.(3-58) we would expect that

$$\hat{N}_I(\delta/2) \approx 2\hat{N}_I(\delta), \tag{3-61}$$

so that the new interval of delays covered by the tap locations falls within the old interval. We note that with the tap spacing δ the actual delays are $[\hat{N}_I(\delta)+1]\delta,\ldots,[\hat{N}_I(\delta)+4]\delta$. If we then choose the minimum and maximum values of $N_I(\delta/2)$ to cover all possible overlaps of this range we must have

$$[\mathrm{Min}\ N_I(\delta/2) + 1](\delta/2) = [\hat{N}_I(\delta) + 1]\delta$$

$$[\mathrm{Max}\ N_I(\delta/2) + 4](\delta/2) = [\hat{N}_I(\delta) + 4]\delta$$

so that

$$2\hat{N}_I(\delta) + 1 \le N_I(\delta/2) \le 2\hat{N}_I(\delta) + 4 \tag{3-62}$$

3.5 Best Choice of $\{\beta(\underline{n})\}$

We can now assume that δ and N_I are temporarily fixed at some trial value; the values of $\{\beta(\underline{n})\}$ must be calculated. We have available the lattice of measurements $\gamma(m_1,m_2,m_3)$ for $1 \le m_i \le M$, $1 \le i \le 3$. It will be a labelling convenience both computationally and to derive the best choice of the $\{\beta(\underline{n})\}$ if we define two indexing variables

$$\mu = M^2(m_1-1) + M(m_2-1) + m_3 \tag{3-63}$$

$$\nu = N^2(n_1-1) + N(n_2-1) + n_3. \tag{3-64}$$

The ranges of these variables are

$$1 \le \mu \le M^3; \tag{3-65}$$

$$1 \le \nu \le N^3. \tag{3-66}$$

We modify the measurement values by the first complex exponential factor in Eq.(3-18) by setting

$$S(\mu) = \gamma(m_1,m_2,m_3)\exp\left\{j2\pi N_I\delta W\left[-\frac{3}{2}+\frac{m_1+m_2+m_3-3}{M-1}\right]\right\}. \tag{3-67}$$

We further define a coefficient array equal to the triple product of Eq.(3-18):

$$C(\mu,\nu) = \exp\left[-j2\pi\delta W\sum_i n_i\left(-\frac{1}{2} + \frac{m_i - 1}{M-1}\right)\right]. \tag{3-68}$$

We now want to find that se . of $\beta(\nu)$ for which

$$V = \sum_{\mu} \left|S(\mu) - \sum_{\nu} \beta(\nu)\; C(\mu,\nu)\right|^2 \tag{3-69}$$

is minimum.

We can interpret the array

$$C = \{C(\mu,\nu\;)\}$$

as an M^3-row by N^3-column matrix; the array

$$\beta = \{\beta(\nu)\}$$

as an N^3-entry column matrix; and the array

$$S = \{S(\mu)\}$$

as an M^3-entry column matrix. We can then rewrite Eq.(3-69) as the matrix equation

$$V = (S - C\beta)^{*T}(S - C\beta) \tag{3-70}$$

where

$* =$ conjugate

$T =$ transpose.

In expanded form this is

$$V = S^{*T} S - \beta^{*T} C^{*T} S - S^{*T} C\beta + \beta^{*T} C^{*T} C\beta. \tag{3-71}$$

Let us now conjecture that we can rewrite V as

$$V = (\beta-\alpha)^{*T} C^{*T} C\;(\beta-\alpha) + U \tag{3-72}$$

where U is a scalar constant and α is a column matrix with N^3 entries. Expanding the product we have

$$V = \beta^{*T} C^{*T} C\beta - \alpha^{*T} C^{*T} \beta - \beta^{*T} C^{*T} C\alpha$$

$$+ \alpha^{*T} C^{*T} C\alpha + U. \tag{3-73}$$

If we equate corresponding terms in Eqs.(3- 1) and (3-73) we find that

$$C^{*T} C\alpha = C^{*T} S$$

so that

$$\alpha = (C^{*T} C)^{-1} C^{*T} S; \tag{3-74}$$

and that

$$\alpha^{*T} C^{*T} C\alpha + U = S^{*T} S$$

so that

$$U = S^{*T} S - \alpha^{*T} C^{*T} C\alpha \tag{3-75}$$

Substitution of the value for α from Eq.(3-74) in this last expression gives

$$U = S^{*T} S - S^{*T} C(C^{*T} C)^{-1} C^{*T} C (C^{*T} C)^{-1} C^{*T} S$$

or

$$U = S^{*T} S - S^{*T} C(C^{*T} C)^{-1} C^{*T} S. \tag{3-76}$$

This last result can also be phrased as

$$U = S^{*T} S - S^{*T} C\alpha.$$

Clearly, the minimum value of V is attained when

$$\beta = \alpha \tag{3-77a}$$

where

$$V = U. \tag{3-77b}$$

3.6 Program Outline

We are now in a position to summarize the programs for determining the model parameters.

3.6.1 Input Data from Measurements

If the input data are to be acquired by measurements it is necessary to specify

a) the size of the measurement lattice M,

b) the three carrier frequencies ν_1, ν_2, and ν_3 (where ν_3 is negative),

c) the bandwidth W.

The output of the measurements should be a tape record or card deck containing the following data.

a) lattice size M

b) bandwidth W

c) a three dimensional array $\gamma(m_1, m_2, m_3)$ where

$$\gamma(m_1, m_2, m_2) = \frac{1}{G_o} G \begin{Bmatrix} -W/2 + (m_1-1) \ W/(M-1), \\ -W/2 + (m_2-1) \ W/(M-1), \\ -W/2 + (m_3-1) \ W/(M-1) \end{Bmatrix} \tag{3-78}$$

where

$$G(f_1, f_2, f_3) = \frac{H_3(\nu_1+f_1, \ \nu_2+f_2, \ \nu_3+f_3)}{H_1 \ (\nu_1+\nu_2+\nu_3+f_1+f_2+f_3)} \tag{3-79a}$$

and

$$G_o = \max_{f_1, f_2, f_3} G(f_1, f_2, f_3)$$

d) the normalizing constant G_o.

3.6.2 Input Data by Computations

If the input data are to be determined by computer analyses of the nonlinear circuit it is necessary to duplicate the same type of output as in Section 3.6.1.

Using SIGNCAP for example, we must specify

a) the lattice dimension M

where

$$2 \le M \le 5. \tag{3-80}$$

b) the bandwidth W

c) the carrier frequencies

ν_1, ν_2, ν_3 where

$$\nu_3 < 0. \tag{3-81}$$

One then calculates the three dimensional array variable $\gamma(m_1, m_2, m_3)$ from

$$G(m_1, m_2, m_3) = \frac{H_3[\nu_1 + f(m_1),\ \nu_2 + f(m_2),\ \nu_3 + f(m_3)]}{H_1[\nu_1 + \nu_2 + \nu_3 + f(m_1) + f(m_2) + f(m_3)]} \tag{3-82a}$$

where

$$f(m_i) = -W/2 + (m_i - 1)W/(M-1)$$
$$\text{for } i = 1,2,3;\ m_i = 1,2,\ldots,M, \tag{3-82b}$$

by setting

$$\gamma(m_1, m_2, m_3) = \frac{1}{G_o} G(m_1, m_2, m_3) \tag{3-83a}$$

with

$$G_o = \underset{\text{all}\{m_i\}}{\text{Max}}\ G(m_1, m_2, m_3). \tag{3-83b}$$

Note that the maximum data generated by this program are 125 complex numbers. The program output should then be

a) the lattice dimension M

b) the bandwidth W

c) the array variables $\gamma(m_1, m_2, m_3)$

d) the normalizer G_o.

3.6.3 Computation of Model Parameters

The input to the program consists of the output of the lattice value computation routine or the measurement results. Specifically the input data should contain

a) the lattice dimension M

b) the bandwidth W

c) the array variables $\gamma(m_1, m_2, m_3)$ for $1 \le m_i \le M$; i = 1,2, and 3.

d) the normalizer G_o.

In addition

e) the number N of taps/coordinate must be specified. This number must satisfy

$$1 \le N \le M-1. \tag{3-84}$$

We define a normalized tap spacing

$$D = W\delta, \tag{3-85}$$

and take the "group delay" N_I as defined before. For each D we define the minimum value of N_I by $N_m(D)$, and the maximum value of N_I by $N_M(D)$. We will reserve the symbol V for the final minimum value of the approximation error and use U for the approximation error for a specific N_I and D:

$$U(N_I, D) = (S-C\beta)^{*T}(S-C\beta). \tag{3-86}$$

It will be helpful to define partial minimizations of U

$$V_T(N_I, D) = U(K_T, D) = \min_{n_I \le N_I} U(n_I, D), \tag{3-87}$$

$$V_T(D) = U(K_o, D) = \min_{n_I \le N_M} U(n_I, D),$$

and

$$V_R(D) = U(K_R, D_R) = \min_{\substack{d \ge D \\ N_I}} U(N_I, d). \tag{3-88}$$

Note that K_T is implicitly a function of N_I and D, K_o is implicitly a function of D, and that K_R, D_R are implicitly functions of D.

We can then write the recursion relations

$$N_M(D) = 2K_o(2D) + 4, \tag{3-89a}$$

$$N_m(D) = 2K_o(2D) + 1; \tag{3-39b}$$

$$V_T(N_I,D) = \begin{cases} V_T(N_I-1,D) & \text{if } U(N_I,D) > V_T(N_I-1,D) \\ U(N_I\,.\,,D) & \text{if } U(N_I,D) \le V_T(N_I-1,D) \end{cases} \tag{3-90}$$

$$K_T(N_I,D) = \begin{cases} K_T(N_I-1,D) & \text{if } U(N_I,D) > V_T(N_I-1,D) \\ N_I \text{ if } U(N_I,D) \le V_T(N_I-1,D) \end{cases}; \tag{3-91}$$

$$V_T(D) = V_T(N_M,D), \tag{3-92}$$

$$K_o = K_T(N_M,D); \tag{3-93}$$

$$V_R(D) = \begin{cases} V_R(2D) & \text{if } V_T(D) > V_R(2D) \\ V_T(2D) & \text{if } V_T(D) \le V_R(2D) \end{cases}, \tag{3-94}$$

$$K_R(D) = \begin{cases} K_R(2D) & \text{if } V_T(D) > V_R(2D) \\ K_o(d) & \text{if } V_T(D) \le V_R(2D) \end{cases}, \tag{3-95}$$

$$D_R(D) = \begin{cases} D_R(2D) & \text{if } V_T(D) > V_R(2D) \\ D & \text{if } V_T(D) \le V_R(2D) \end{cases}, \tag{3-96}$$

For each of these recursions we also need initial conditions. For N_m and N_M we have

$$\left\{ \begin{array}{l} N_m = -M \\ N_M = M-1-N \end{array} \right\} \text{ when } D = 1/2. \tag{3-97}$$

Since we have constrained the values of γ by

$$|\gamma(m_1,m_2,m_3)|^2 \leq 1,$$

it follows that

$$S^{*T}S \leq 125$$

so that V_R and V_T can be initialized by setting them equal to any number larger than 125, say 200. We have then

$$V_T(N_m-1,D) = 200 \tag{3-98}$$

$$V_R(1/2) = 200.$$

The final values of the parameters then yield the optimum parameter values:

$$\text{Optimum value of } \delta W = D_R(1/128) \tag{3-99a}$$

$$\text{Optimum value of } N_I = K_R(1/128)$$

$$\text{Mimimum of } V = V_R(1/128).$$

We then have the following scheme for calculations:

a) Initialize record keeping parameters:

$$V_T = V_R = 200$$

b) D is initialized at

$$D = 1/4$$

c) N_I is initialized at

$$N_I = -M$$

N_M is initialized at $M - 1 - N$

d) For each m_i

$$1 \leq m_i \leq M;\ i = 1,2,3$$

define an integer

$$\mu = M^2(m_1-1) + M(m_2-1) + m_3,$$

and an array variable

$$S(\mu) = \gamma(m_1,m_2,m_3)\exp\left\{j2\pi N_I D\left[-\frac{3}{2} + \frac{m_1+m_2+m_3-3}{M-1}\right]\right\}.$$

Note that under the program restrictions,

$$1 \le \mu \le M^3 \le 125.$$

e) For each m_i,

$$1 \le m_i \le M;\ i = 1,2,3$$

and for each n_i

$$1 \le n_i \le N;\ i = 1,2,3,$$

define μ as before and define

$$\nu = N^2(n_1 - 1) + N(n_2 - 1) + n_3,$$

and a two dimensional array variable

$$C(\mu,\nu) = \exp\left[-j2\pi D \sum_{i=1}^{3} n_i\left(-\frac{1}{2} + \frac{m_i - 1}{M-1}\right)\right].$$

Note that

$$1 \le \mu \le M^3 \le 125,$$

$$1 \le \nu \le N^3 \le (M-1)^3 \le 64.$$

f) Interpreting $S(\mu)$ as a column matrix and $C(\mu,\nu)$ as a rectangular matrix with M^3 rows and N^3 columns, calculate the number

$$U = [C(C^{*T}C)^{-1}C^{*T}S-S]^{*T}[C(C^{*T}C)^{-1}C^{*T}S-S]$$

g) If $U < V_T$,

$$\text{set} \left\{ \begin{matrix} V_T = U \\ K_T = N_I \end{matrix} \right\}.$$

Make available for print out N_I, D, and U.

h) Replace N_I by $N_I + 1$

i) If $N_I \le N_M$ go back to step (d)

j) Set $K_o = K_T$

k) If $V_T < V_R$ set $\{V_R = V_T, K_R = K_T, D_R = D\}$.

l) Set
$$N_M = 2K_O + 5$$
$$N_I = 2K_O + 1$$
$$D = D/2$$
$$V_T = 200.$$

m) If 129 D ≥ 1 go back to step (d)

n) Set $D = D_R$, $N_I = K_R$, $V = V_R$, $K = K_R$

o) Calculate the array variables S and C as in steps (d) and (e)

p) Calculate the array variables $\beta(\nu)$ for $\nu = 1$ to N^3 by interpreting β as a column matrix given by

$$\beta = (C^{*T}C)^{-1}\, C^{*T}S.$$

q) for $1 \le n_i \le N$, for $i = 1$ to 3 set

$$B(n_1,n_2,n_3) = \beta\left[(n_1-1)N^2 + (n_2-1)\,N + n_3\right].$$

r) Calculate the normalized error

$$\rho = \frac{V}{S^{*T}S}\,.$$

s) Make available for print out
G_o, Γ D, ρ and the array variables $B(n_1,n_2,n_3)$

t) Make av able on cards
G_o, K, δ = D/W, N, and the array variables
$B(n_1,n_2,n_3)$.

SECTION 4

SAMPLED DATA BUTTERWORTH FILTER

4.1 Use of Approximate Linear Filter Response

After adding the third order distortion terms to the linearly amplified components, it is necessary to filter the resultant signal so that its bandwidth is equal to the nominal IF bandwidth of the receiver. Simulation of the exact IF response on the computer would, in general, require a convolution involving a very large number of delayed replicas of the input process, and would be very time consuming.

It will be adequate for most purposes to replace the impulse response of the actual linear circuit by <u>any</u> filter having the correct 3 dB bandwidth and adequately fast roll off. It should be emphasized that such a change does not have the drastic effect on nonlinear spectra that would result from changing the actual IF transfer function. The effect of this actual receiver filter will have been incorporated in the equivalent transfer function $K(f_1, f_2, f_3)$; the substitution of an approximate filter for H(f) causes only linear distortion of the output.

The filter that is chosen to approximate H(f) should obviously be selected for ease of simulation on the computer as well as for reasonable match to IF filter characteristics. The second-order Butterworth filter meets all these requirements satisfactorily: it has maximally flat inband response, it yields 12 dB rejection at one bandwidth separation from its center frequency, and it can be simulated by a two stage recursive filter on the computer.

4.2 Use of Butterworth Filter for Spectral Shaping of Noiselike Signals

In order to simulate the effects of random interference it is desirable to include a noiselike waveform as one of the possible inputs. This waveform can be generated conveniently using a sequenc of random complex numbers at the sampling rate of the simulation. However, it is necessary to provide spectral shaping in order to approximate the bandwidth characteristics of the interference. We have again chosen a second-order Butterworth sampled data filter for this function.

4.3 Properties of Second-Order Filter

Let $\{Z_n: -\infty \le n \le \infty\}$ be a sequence of complex numbers derived from sampling an input process using a sampling interval δ, and let

$$Y_n = (2\rho\cos\varphi)\, Y_{n-1} - \rho^2\, Y_{n-2} + AZ_n, \tag{4-1}$$

where ρ and φ and A are real constants. We assume the nonrecursive definition of Y_n to be

$$Y_n = \sum_{m=0}^{\infty} (a_1 b_1^{\ m} + a_2 b_2^{\ m})\, Z_{n-m}. \tag{4-2}$$

Then since Eq. (4-1) can b written as

$$Y_n - (2\rho\cos\varphi)\, Y_{n-1} + \rho^2\, Y_{n-2} = AZ_n \tag{4-3}$$

we can determine a_1, b_1, a_2, b_2 by substitution of Eq. (4-2) in Eq. (4-3) to yield

$$(a_1 + a_2) Z_n + (a_1 b_1 + a_2 b_2) Z_{n-1} + \sum_{k=0}^{\infty} (a_1 b_1^{2+k} + a_2 b_2^{2+k}) Z_{n-2-k}$$

$$- (2\rho\cos\varphi)\left[(a_1 + a_2) Z_{n-1} + \sum_{k=0}^{\infty}\left(a_1 b_1^{\ 1+k} + a_2 b_2^{\ 1+k}\right) Z_{n-2-k}\right]$$

$$+ \rho^2\left[\sum_{k=0}^{\infty}\left(a_1 b_1^{\ k} + a_2 b_2^{\ k}\right) Z_{n-2-k}\right]$$

$$= AZ_n.$$

From this equation we deduce that

$$b_i^2 - (2\rho\cos\varphi)b_i + \rho^2 = 0, \ i = 1,2; \tag{4-4}$$

$$a_1 + a_2 = A; \tag{4-5}$$

$$a_1 b_1 + a_2 b_2 - (2\rho\cos\varphi)(a_1 + a_2) = 0. \tag{4-6}$$

Equation (4-4) has the solution

$$b_i = \frac{2\rho\cos\varphi \pm \sqrt{4\rho^2\cos^2\varphi - 4\rho^2}}{2}$$

or

$$b_1 = \rho \exp(j\varphi) \tag{4-7a}$$

$$b_2 = \rho \exp(-j\varphi). \tag{4-7b}$$

We observe that $a_2 = a_1^*$ since $b_2 = b_1^*$. We can thus write

$$a_1 = |a| \exp(j\alpha), \tag{4-8a}$$

$$a_2 = |a| \exp(-j\alpha), \tag{4-8b}$$

so that Eqs. (4-5) and (4-6) become

$$2|a|\cos\alpha = A \tag{4-9}$$

$$-2|a|\rho\cos(\rho-\alpha) = 0. \tag{4-10}$$

We deduce immediately that

$$\alpha = \varphi - \pi/2$$

and

$$|a| = A/(2\sin\varphi).$$

Hence

$$a_1 = (A/2)(1-j\cot\varphi), \tag{4-11a}$$

$$a_2 = (A/2)(1+j\cot\varphi). \tag{4-11b}$$

We thus have

$$Y_n = (A/2)\sum_{m=0}^{\infty} \rho^m Z_{n-m} \begin{bmatrix} (1-j\cot\varphi)\exp(jm\varphi) \\ +(1+j\cot\varphi)\exp(-jm\varphi) \end{bmatrix}. \tag{4-12}$$

4.3.1 Response to Sampled Cissoid

Now let $\{Z_m\}$ be the samples of a complex cissoid with frequency f:

$$Z_m = \exp\ (j2\pi f\delta m). \tag{4-13}$$

We can shorten subsequent expressions by setting

$$\beta = 2\pi f\delta \tag{4-14}$$

so that

$$Z_m = \exp\ (jm\beta). \tag{4-15}$$

Substitution of this in Eq. (4-12) yields

$$\begin{aligned}
&2Y_n \exp(-jn\beta)/A \\
&\quad = (1-j\cot\varphi)\sum_{m=0}^{\infty} [\rho \exp\ (j\varphi-j\beta)]^m \\
&\quad\quad +(1-j\cot\varphi)\sum_{m=0}^{\infty}[\rho \exp\ (-j\varphi-j\beta)]^m \\
&\quad = \frac{1-j\cot\varphi}{1-\rho\exp(j\varphi-j\beta)} + \frac{1+j\cot\varphi}{1-\rho\exp(-j\varphi-j\beta)},
\end{aligned}$$

or

$$\begin{aligned}
&2jY_n(\sin\varphi)\exp(-jn\beta)/A \\
&\quad =[\exp(-j\varphi) - \rho \exp\ (-j\beta)]^{-1} \\
&\quad -[\exp(j\varphi) - \rho \exp(-j\beta)]^{-1}. \\
&\quad =2j\sin\varphi\ [1-(2\rho\cos\varphi) \exp(-j\beta) + \rho^2 \exp(-j2\beta)]^{-1}
\end{aligned}$$

so that

$$Y_n = A\exp(jn\beta)[1-(2\rho\cos\varphi) \exp(-j\beta) + \rho^2\exp(-j2\beta)]^{-1}. \tag{4-16}$$

We then have the magnitude of Y_n given by the expression

$$|Y_n|^2 = A^2|1-(2\rho\cos\varphi)\ \exp(-j\beta) + \rho^2 \exp(-j2\beta)|^{-2}$$

$$= A^2|[\exp(j\varphi) - \rho\ \exp(-j\beta)][\exp(-j\varphi) - \rho\ \exp(-j\beta)]|^{-2}$$

$$= A^2[1-2\rho\cos(\beta+\varphi) + \rho^2]^{-1}[1-2\rho\cos(\beta-\varphi) + \rho^2]^{-1}$$

or

$$|Y_n|^2/A^2 = \left[(1+\rho^2)^2 - 4\rho(1+\rho^2)\cos\beta\ \cos\varphi + 2\rho^2(\cos 2\beta + \cos 2\varphi)\right]^{-1}. \tag{4-17}$$

If we now write

$$\cos\beta = 1-2\sin^2\gamma \tag{4-18a}$$

where

$$\gamma = \dot{\beta}/2, \tag{4-18b}$$

and then use the relation

$$\cos 2\beta = 2\cos^2\beta - 1 = 1-8\sin^2\gamma + 8\sin^4\gamma, \tag{4-18c}$$

Eq. (4-17) can be rewritten as

$$A^2/|Y_n|^2 = (1+\rho^2)^2 - 4\rho(1+\rho^2)\cos\varphi + 2\rho^2(1+\cos 2\varphi)$$
$$+8\rho(1+\rho^2)\cos\varphi\ \sin^2\gamma - 16\rho^2\sin^2\gamma + 16\rho^2\sin^4\gamma$$

or as

$$A^2/|Y_n|^2 = (1-2\rho\cos\varphi+\rho^2)^2$$
$$+ 8\rho[(1+\rho^2)\ \cos\varphi - 2\rho]\ \sin^2\gamma$$
$$+ 16\rho^2\sin^4\gamma. \tag{4-19}$$

4.3.2 Special Choice of ρ

We now choose ρ so that the coefficient of $\sin^2\gamma$ vanishes in order to have a maximally flat response:

$$(1+\rho^2)\cos\varphi - 2\rho = 0$$

or

$$\cos\varphi = 2\rho/(1+\rho^2). \tag{4-20}$$

We then have

$$\sin\varphi = (1-\rho^2)/(1+\rho^2) \tag{4-21}$$

so that

$$\rho^2 = \frac{1-\sin\varphi}{1+\sin\varphi} = \frac{1-\cos(\varphi-\pi/2)}{1+\cos(\varphi-\pi/2)} \tag{4-22a}$$

$$= \tan^2(\varphi/2 - \pi/4)$$

or

$$\rho = \tan(\varphi/2 - \pi/4). \tag{4-22b}$$

With this choice, the leading term of Eq.(4-19) becomes

$$(1-2\rho\cos\varphi + \rho^2)^2$$

$$= \left[1 - \frac{4\rho^2}{1+\rho^2} + \rho^2\right]^2$$

$$= \left[\frac{(1-\rho^2)^2}{1+\rho^2}\right]^2$$

$$= \left[\frac{2\sin^2\varphi}{1+\sin\varphi}\right]^2.$$

Hence

$$|A^2|/|Y_n|^2 = \left[\frac{2\sin^2\varphi}{1+\sin\varphi}\right]^2 + 16\left[\frac{1-\sin\varphi}{1+\sin\varphi}\right]\sin\gamma$$

or

$$|Y_n|^2 = A^2\left[\frac{1+\sin\varphi}{2\sin^2\varphi}\right]\left[1 + \frac{4\cos^2\varphi\sin^4\gamma}{\sin^4\varphi}\right]^{-1} \tag{4-23}$$

In order to have unity gain at zero frequency we take

$$A = \frac{2\sin^2\varphi}{1+\sin\varphi}. \tag{4-24}$$

Furthermore, the 3 dB attenuation point is achieved when

$$\frac{4\cos^2\varphi \sin^4\gamma}{\sin^4\varphi} = 1 \tag{4-25}$$

If we wish this 3 dB point to be at $f = W/2$, then when $\beta = \pi W\delta$ (see Eq.(4-14)) or when $\gamma = \pi W\delta/2$ (see Eq. (4-18)), Eq.(4-25) must be satisfied. We thus require

$$\frac{\sin^4\varphi}{\cos^2\varphi} = 4\sin^4\gamma_o \tag{4-26a}$$

where

$$\gamma_o = \pi W\delta/2. \tag{4-26b}$$

We can solve Eq.(4-26a) immediately to arrive at

$$\varphi = \text{arc sin } S \tag{4-27a}$$

where

$$S = \left[\frac{2\sin^2\gamma_o}{[1+\sin^4\gamma_o]^{1/2} + \sin^2\gamma_o}\right]^{1/2}. \tag{4-27b}$$

We thus have, upon substitution in Eq. (4-24),

$$A = 2S^2/(1+S), \tag{4-28}$$

and, upon substitution in Eq. (4-22a),

$$\rho = [(1-S)/(1+S)]^{1/2}. \tag{4-29}$$

Finally, since

$$\cos\varphi = (1-S^2)^{1/2} = [(1+S)(1-S)]^{1/2}, \tag{4-30}$$

the recursion relation Eq.(4-1) can be written as

$$Y_n = 2(1-S)\,Y_{n-1} - [(1-S)/(1+S)]\,Y_{n-2} + [2S^2/(1+S)]Z_n. \tag{4-31}$$

4.3.3 Response to Independent Samples

We now investigate the response of this sampled data filter to a sequence of independent complex samples. Repeating Eq. (4-12) here,

$$Y_n = (A/2) \sum_{m=0}^{\infty} \rho^m Z_{n-m} \begin{bmatrix} (1-j\cot\varphi) \exp(jm\varphi) \\ +(1+j\cot\varphi) \exp(-jm\varphi) \end{bmatrix}. \tag{4-32}$$

Letting

$$V_Z = E\left\{|Z_n|^2\right\} \tag{4-33}$$

be the common variance of the input sequence, we have

$$V_Y = E\left\{|Y_n|^2\right\} = (A^2 V_Z/4)$$

$$\cdot \sum_{m=0}^{\infty} \rho^{2m} \left| \begin{matrix} (1-j\cot\varphi) \exp(jm\varphi) \\ +(1+j\cot\varphi) \exp(-jm\varphi) \end{matrix} \right|^2. \tag{4-34}$$

We can immediately rewrite this as

$$4(\sin^2\varphi) V_Y / A^2 V_Z$$

$$= \sum_{m=0}^{\infty} \rho^{2m} \left| \exp(j\varphi + jm\varphi) - \exp(-j\varphi - jm\varphi) \right|^2$$

$$= -\exp(j2\varphi) \sum_{m=0}^{\infty} [\rho^2 \exp(j2\varphi)]^m$$

$$- \exp(-j2\varphi) \sum_{m=0}^{\infty} [\rho^2 \exp(-j2\varphi)]^m$$

$$+ 2 \sum_{m=0}^{\infty} \rho^{2m}$$

$$= -\frac{\exp(j2\varphi)}{1-\rho^2 \exp(j2\varphi)} - \frac{\exp(-j2\varphi)}{1-\rho^2 \exp(-j2\varphi)} + \frac{2}{1-\rho^2}$$

$$= 2 \frac{(1+\rho)[1-\cos(2\varphi)]}{(1-\rho^2)[1+\rho^4 - 2\rho^2 \cos(2\varphi)]}.$$

If we now substitute Eqs. (4-27) through (4-30) in this we obtain

$$V_Y = \frac{S}{2-S^2} V_Z \qquad (4\text{-}35)$$

In order to provide a unit variance output, we therefore set

$$V_Z = \frac{2-S^2}{S} . \qquad (4\text{-}36)$$

SECTION 5

NONLINEAR TRANSFER FUNCTION FOR TESTING COMPUTER ROUTINES

It is helpful to have available a routine for generating the first and third order transfer functions $H_1(f)$ and $H_3(f_1, f_2, f_3)$ that does not require the use of SIGNCAP. This section presents an outline of a routine for providing such transfer functions.

Consider the circuit diagram shown in Fig. 5.1, consisting of two single tuned circuits separated by an amplifier that exhibits a cubic distortion term. If $H_A(f)$ is the transfer function of the first filter then an input of the form

$$v_o(t) = \sum_n a_n e^{j2\pi\nu_n t}, \tag{5-1}$$

where the $\{\nu_n\}$ occur in pairs of positive and negative frequencies, yields an input to the nonlinearity of the form

$$v_1(t) = \sum_n a_n H_A(\nu_n)\, e^{j2\pi\nu_n t}. \tag{5-2}$$

The output of this nonlinearity is

$$v_2(t) = \sum_n a_n H_A(\nu_n)\, e^{j2\pi\nu_n t} + B \sum_{n_1} \sum_{n_2} \sum_{n_3} a_{n_1} a_{n_2} a_{n_3} H_A(\nu_{n_1})\, H_A(\nu_{n_2})\, H_A(\nu_{n_3}) \exp\left[j2\pi t(\nu_{n_1} + \nu_{n_2} + \nu_{n_3})\right]. \tag{5-3}$$

The second linear filter has the same transfer function as the first so that the final output is

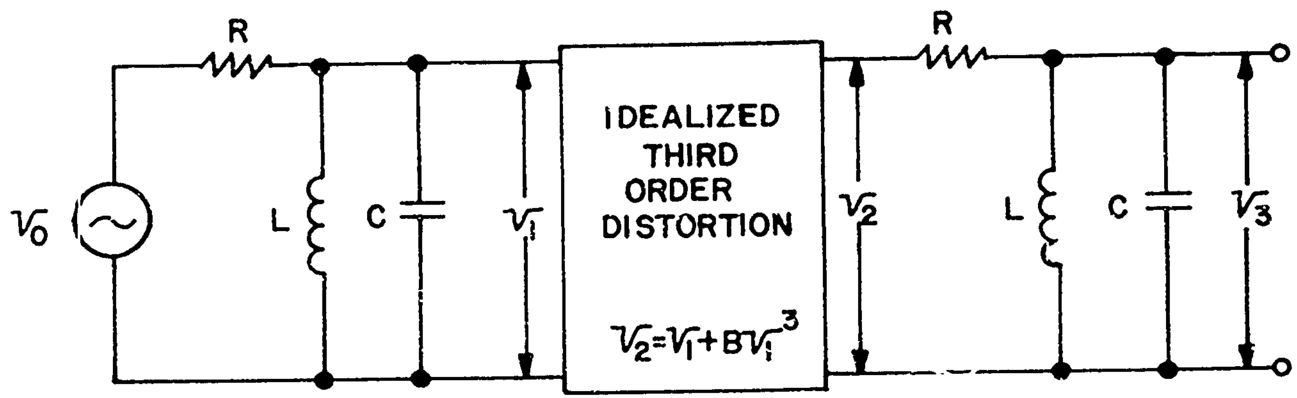

Fig. 5.1 Simplified Circuit for Program Tests

$$v_3(t) = \sum_n a_n H_A^2(\nu_n)\, e^{j2\pi\nu_n t}$$

$$+ B \sum_{n_1} \sum_{n_2} \sum_{n_3} a_{n_1} a_{n_2} a_{n_3} H_A(\nu_{n_1}) H_A(\nu_{n_2}) H_A(\nu_{n_3})$$

$$H_A(\nu_{n_1} + \nu_{n_2} + \nu_{n_3})$$

$$\exp\left[j2\pi t(\nu_{n_1} + \nu_{n_2} + \nu_{n_3}) \right]. \qquad (5\text{-}4)$$

Since we can also write

$$v_3(t) = \int df\, e^{j2\pi ft}\, H_1(f) \sum_n a_n\, \delta(f-\nu_n)$$

$$+ \iiint df_1 df_2 df_3\, e^{j2\pi t(f_1+f_2+f_3)}$$

$$H_3(f_1, f_2, f_3)$$

$$\sum_{n_1} \sum_{n_2} \sum_{n_3} \prod_{i=1}^{3} a_{n_i}\, \delta(f_i - \nu_i), \qquad (5\text{-}5)$$

we have the immediate correspondence

$$H_1(f) = H_A^2(f) \qquad (5\text{-}6)$$

$$H_3(f_1, f_2, f_3) = BH_A(f_1 + f_2 + f_3) \prod_{i=1}^{3} H_A(f_i). \qquad (5\text{-}7)$$

The transfer function of the single-tuned filter is just

$$H_A(f) = \{1 + jR[2\pi fC - 1/(2\pi fL)]\}^{-1}. \qquad (5\text{-}8)$$

If we use the usual notation

$$f_o = 1/2\pi(LC)^{1/2}, \qquad (5\text{-}9)$$

and

$$Q = R/(2\pi f_o L), \qquad (5\text{-}10)$$

we can write the transfer function as

$$H_A(f) = [1 + jQ(f/f_o - f_o/f)]^{-1} . \tag{5-11}$$

For numerical calculations to test various program routines it is necessary to assign numerical values to Q and f_o, and also to select the bandwidths and center frequencies of the signals to be accommodated by the canonic model. To this end we take

$$f_o = 50 \text{ MHz}$$

or

$$f_o = 5 \times 10^7 \qquad \text{(5-12a}$$

and

$$Q = 10^2 \qquad \text{(5-12b}$$

to characterize the filter. We will then model a crossmodulation situation where the desired carrier frequency ν_1 is at the center frequency of the filter:

$$\nu_1 = f_o = 5 \times 10^7, \qquad \text{(5-13a}$$

and where the interfering carrier is 1 MHz removed from the desired carrier:

$$\nu_2 = 5.1 \times 10^7 \qquad \text{(5-13b}$$

$$\nu_3 = - 5.1 \times 10^7. \qquad \text{(5-13c}$$

We will take the nominal bandwidth of the signals to be 0.5 MHz:

$$W = 0.5 \times 10^6. \qquad \text{(5-13d}$$

These choices of reasonably realistic transfer functions and numerical values will permit testing of the parameter-fitting routines without the necessity of time consuming calculations with nonlinear circuit analysis programs.

It is an additional convenience in testing program routines to include the effect of computational or experimental error. This can be accomplished by introducing deliberate round-off error in the H_3 or H_1 outputs.

SECTION 6

COMPUTER GENERATED WAVEFORMS

After having computed the model coefficients by the procedure outlined in Section 3, the necessary parameters are available for calculating the complex envelope of a third order interaction term. The general form of this complex envelope can be found by substitution of the tapped delay line model into Eq.(2-7) to yield

$$a_{\underline{p}}(t) \; \sum_{\underline{n}} \beta(\underline{n}) \prod_{i=1}^{3} z_{p_i}(t - n_i\delta - K\delta) \tag{6-1}$$

where K is the optimum value of N_I found by the procedures outlined in Section 3. For purposes of computation we will actually evaluate $a_{\underline{p}}(t)$ only at integer multiples of δ, however. Let us therefore introduce the notation

$$Y_3(J) = a_{\underline{p}}(J\delta + K\delta) \tag{6-2}$$

and

$$Z_i(J) = z_{p_i}(J\delta). \tag{6-3}$$

(It will be noted that these represent a duplication of earlier use of upper case letters for spectra; no confusion should result in the balance of this material where only sampled data sequences are to be considered.) We then have

$$Y_3(J) = \sum_{\underline{N}} \beta(\underline{N}) \prod_{i=1}^{3} Z_i(J - N_i). \tag{6-4}$$

This sampled output $Y_3(J)$ is <u>advanced</u> by K samples with respect to the linear output term and this discrepancy of alignment must be compensated in the program by delaying Y_3 by K samples before combining if K is positive, or delaying the samples of the linear output by $|K|$ samples if K is negative.

It will also be recalled that in Interference Cases #1 and #4 of Section 2.6, (Eqs.(2-34) and (2-37)), the interference envelope is actually modulating the adjacent carrier frequency separated by W Hz from the linear output. Hence a transformation of the samples $\{Y_3(J)\}$ equivalent to this frequency translation must must be affected.

Finally, as discussed in Section 2.7, the amplitude of the third order product must be scaled relative to the linear component to take into account both the relative amplitudes of interferers and desired signal, and to incorporate the normalizing constant G_o of Section 3.

Let $\{Y(J)\}$ be the samples $\{Y_3(J)\}$ corrected for the frequency offset:

$$Y(J) = e^{2\pi AW\delta J} \sum_{\underline{N}} B(\underline{N}) \prod_i Z_i(J-N_i). \tag{6-5a}$$

where

$$A = \begin{Bmatrix} 0 \text{ if interference product is at nominal carrier} \\ 1 \text{ if interference product is at adjacent carrier} \end{Bmatrix} \tag{6-5b}$$

and let

$$Y_L(J) = Z_o(J+K) \tag{6-6}$$

with $\{Z_o(J)\}$ the samples from the desired signal. We can then write the samples of the total output from the unit gain amplifier and idealized third order transfer function as

$$Y_T(J) = C_L Y_L(J) + C_T Y(J) \tag{6-7}$$

where C_L and C_T are normalizing constants that include the constant G_o and the relative powers of the signal and interferers as well as the overall gain of the receiver. For most computations where only relative distortion is important, only the ratio of C_T to C_L need be specified, and the absolute scaling can be accomplished for computational convenience.

6.1 The Four Signal/Interference Combinations

In Section 2 we discussed the four cases appropriate for analysis. We now list these cases in the notation of this section, using the additional notation

$$\left.\begin{array}{ll} S(t) & = \text{signal waveform} \\ U_1(t) & = \text{interfering waveform} \\ U_2(t) & = \text{different interfering waveform} \end{array}\right\} . \qquad (6\text{-}8)$$

Table 6.1 Signal/Interference Combinations

	Case 1	Case 2	Case 3	Case 4
$z_o(t)$	$S(t)$	$S(t)$	$S(t)$	$S(t)$
$z_1(t)$	$U_1(t)$	$U_1(t)$	$S(t)$	$U_1(t)$
$z_2(t)$	$U_1(t)$	$U_1(t)$	$U_1(t)$	$U_2(t)$
$z_3(t)$	$U_1{}^*(t)$	$U_2(t)$	$U_1{}^*(t)$	$U_2{}^*(t)$
A	1	0	0	1

6.2 Equivalent IF Filter

It is necessary to include the effect of the linear filter $H(f)$ of the equivalent receiver to determine the overall impact of the interference on inband interference. As we discussed earlier, it is adequate to approximate this filter by any filter which restricts the bandwidth to W Hz; and a sampled data second-order Butterworth filter is adequate for this purpose.

Using the results of Section 4, we define

$$\gamma = \pi W\delta/2 \tag{6-9}$$

$$S = \left[\frac{2\sin^2\gamma}{(1+\sin^4\gamma)^{1/2} + \sin^2\gamma}\right]^{1/2}. \tag{6-10}$$

The filtered output sequence is then given by

$$Y_{TF}(J) = 2(1-S)Y_{TF}(J-1) - \frac{1-S}{1+S} Y_{TF}(J-2) + \frac{2S^2}{1+S} Y_T(J). \tag{6-11}$$

We can also examine the filtered version of the third order produc without the linear term by defining

$$Y_F(J) = Y_{TF}(J) \text{ when } C_L = 0. \qquad \text{(6-12a}$$

Correspondingly, we can define a linearly filtered signal by

$$Y_{LF}(J) = Y_{TF}(J) \text{ when } C_T = 0. \qquad \text{(6-12b}$$

6.3 Signal and Interference Waveforms

The model, as it stands, will accept any choice of waveforms for the signal and interferers of Table 6.1. It is a program requirement that it be possible to generate typical waveforms internally during the computations. We believe that suitable waveforms can be provided by the following repertoire:

a) Signal waveform:
   - i) CW
   - ii) Sinusoidal amplitude modulation
   - iii) Sinusoidal frequency modulation
   - iv) Combined FM/AM with different modulating frequencies

b) First Inteferer:

   Same possible characteristics as for signal

c) Second Interferer:

Noiselike waveform with second-order Butterworth frequency characteristics.

We therefore define the following sampled data sequences

$$S_{IG}(J) = [1 + \mu_s \cos(2\pi f_{s_{AM}} J\delta + \theta_s)]$$

$$\cdot \exp[jD_s \cos(2\pi f_{s_{FM}} J\delta)]; \tag{6-13}$$

$$U(J) = [1 + \mu_I \cos(2\pi f_{I_{AM}} J\delta + \theta_I)]$$

$$\cdot \exp[j D_I \cos(2\pi f_{I_{FM}} J\delta)]; \tag{6-14}$$

$$\eta(J) = 2(1-S)\eta(J-1) - \frac{1-S}{1+S}\eta(J-2) + \frac{2C_5 S^2}{1+S} G(J), \tag{6-15a}$$

where

$$C_5 = \left[\frac{[1-(1-2S^2)(1-S^2)^{1/2}]}{2S^3(1+S)[1+(1-S^2)^{1/2}]}\right]^{1/2}, \tag{6-15b}$$

$\{G(J)\}$ are independent, zero mean complex Gaussian random variables, with unit variance (6-15c)

and the initial conditions are

$$\eta(1) = \eta(2) = 0. \tag{6-15d}$$

(The normalizing constant C_5 was derived in Section 3.)

In choosing the modulating frequencies and deviation ratios

$$f_{S_{AM}},\ f_{S_{FM}},\ f_{I_{AM}},\ f_{I_{FM}},\ D_S,\ D_I,$$

it is necessary to insure that the resultant bandwidth of the complex envelopes does not exceed W. We note that a deviation ratio of approximately 2.405 permits generating an FM waveform having complete carrier suppression; we therefore have chosen to restrict the maximum deviation ratio to 2.5:

$$\begin{Bmatrix} 0 \le D_S \le 2.5 \\ 0 \le D_I \le 2.5 \end{Bmatrix}. \qquad (6\text{-}16)$$

At this peak deviation ratio, the sideband power distribution is as given in Table 6.2.

Table 6.2

Sideband Power for Deviation Ratio of 2.5

Component	Fraction of Total Power
Carrier	≈ 0
Sideband # $\pm$ 1	0.25
Sideband # $\pm$ 2	0.20
Sideband # $\pm$ 3	0.03
Sideband # $\pm$ 3	0.005

Restriction of the FM modulating frequency so that the fourth sidebands are included in (-W/2,W/2) is adequate to meet the bandwidth restriction. In the AM and combined AM/FM cases, the amplitude modulation introduces an additional spreading of every component equal to the amplitude modulating frequency. We therefore need to restrict the pairs of modulating frequencies by some relation of the form

$$f_{S_{AM}} + 4f_{S_{FM}} \le W/2 , \qquad (6\text{-}16a$$

$$f_{I_{AM}} + 4f_{I_{FM}} \le W/2. \qquad (6\text{-}16b$$

6.3.1 Drift Frequency

Because of the choice of sample spacing to be an integer submultiple of the reciprocal bandwidth, and because of the placement of an interfering carrier exactly one bandwidth away from the desired signal in Cases 1 and 4 of Table 6.1, it is possible for a CW or AM interferer to yield an interference product in a fixed phase relation relative to the desired signal. To avoid this program artifact it is useful to introduce a "drift" frequency in Eq.(6-14). This drift frequency should be chosen to be less than a few percent of the bandwidth, and irrationally related to it. These requirements can be met by taking the drift frequency to be $W/20\pi$ so that Eq.(6-14) can be replaced by

$$U(J) = [1 + \mu_I \cos(2\pi f_{I_{AM}} J\delta + \theta_I)]$$

$$\exp\left\{j\left[0.1J\delta W + D_I \cos(2\pi f_{I_{FM}} J\delta)\right]\right\}. \qquad (6\text{-}17)$$

6.3.2 Interference Cases with Internal Routines

In Table 6.1 we listed the possible signal/interference combinations. With the internal routines available for generation of "modulated" interference (the sequence $U(J)$) and noiselike interference (the sequence $\eta(J)$), it is possible to create two distinct interference products for each case, depending on how we associate U_1 and U_2 with U and η. We can therefore expand the table to yield the eight cases shown in Table 6.3.

6.4 Demodulated Outputs

It is of some help in evaluating the impact of nonlinear distortion to determine the distortion voltage after detection. It is possible to provide this option in the computer routines for both envelope detection and phase detection. It should be observed that for both of these types of detection the "filtered" output version $\{Y_{TF}(J)\}$ or $\{Y_{LF}(J)\}$ in Eqs.(6-11a) and (6-12b) should be used, since the unfiltered distortion products have a bandwidth of 3W.

Table 6.3

Internally Generated Signal/Interference Combinations

Case	Offset Parameter A	$Z_o(J)$	$Z_1(J)$	$Z_2(J)$	$Z_3(J)$
1a	1	$S_{IG}(J)$	$U(J)$	$U(J)$	$U^*(J)$
1b	1	$S_{IG}(J)$	$\eta(J)$	$\eta(J)$	$\eta^*(J)$
2a	0	$S_{IG}(J)$	$U(J)$	$U(J)$	$\eta(J)$
2b	0	$S_{IG}(J)$	$\eta(J)$	$\eta(J)$	$U(J)$
3a	0	$S_{IG}(J)$	$S_{IG}(J)$	$U(J)$	$U^*(J)$
3b	0	$S_{IG}(J)$	$S_{IG}(J)$	$\eta(J)$	$\eta^*(J)$
4a	1	$S_{IG}(J)$	$U(J)$	$\eta(J)$	$\eta^*(J)$
4b	1	$S_{IG}(J)$	$\eta(J)$	$U(J)$	$U^*(J)$

A simplified presentation of the distortion in the detector outputs is possible if only the distortion is made available at the output. In the case of the phase detector, this also simplifie the computation in that it eliminates computational errors of 2π in computing the arc tangent. We therefore define an envelope distortion

$$env(J) = |Y_{TF}(J)| - |Y_{LF}(J)| \tag{6-18}$$

and a phase distortion

$$ph(J) = arg[Y_{TF}(J)] - arg|Y_{LF}(J)| .$$

It will be noted that this latter expression can be written as

$$ph = \mathrm{Im}\ \{\ \log(Y_{TF}/Y_{LF})\}$$

$$= \mathrm{Im}\left\{\log\left[1 + \frac{Y_{TF} - Y_{LF}}{Y_{LF}}\right]\right\}$$

which, if the distortion is small enough to be tolerated at all, can be approximated by

$$ph \approx \mathrm{Im}\left\{\frac{Y_{TF} - Y_{LF}}{Y_{LF}}\right\}$$

$$= \mathrm{Im}\ \{Y_{TF}/Y_{LF}\}.$$

We will use this approximate formula as sufficiently precise for the purpose of estimating phase distortion. (The phase distortion can be equated to π radians when Y_{LF} is small.)

6.5 Necessary Computer Routines

6.5.1 Generation of Signal, Interference and Noise

(a) The necessary input parameters are given in Table 6.4.

Table 6.4

Input Parameters for Waveform Generation

Description	Textual Notation
Bandwidth	W
Tap spacing	δ
AM modulation index, signal	μ_S
AM modulation index, interferer	μ_I
FM deviation ratio, signal	D_S
FM deviation Ratio, interferer	D_I
AM modulating frequency, signal	$f_{S_{AM}}$
AM modulating frequency, interferer	$f_{I_{AM}}$
AM phase, signal	θ_S
AM phase, interferer	θ_I
FM modulating frequency, signal	$f_{S_{FM}}$
FM modulating frequency, interferer	$f_{I_{FM}}$

(b) Restrictions on parameters

$W\delta \le 1/4;$

$0 \le \mu_S \le 1,$

$0 \le \mu_I \le 1;$

$0 \le D_S \le 2.5,$

$0 \le D_I \le 2.5$

$$\text{If } \begin{Bmatrix} \mu_S = 0 \\ D_S > 0 \end{Bmatrix} \text{ then } \begin{Bmatrix} f_{S_{AM}} = 0 \\ 0 \le f_{S_{FM}} \le W/8 \\ \theta_S = 0 \end{Bmatrix} .$$

$$\text{If } \begin{Bmatrix} D_S = 0 \\ \mu_S > 0 \end{Bmatrix} \text{ then } \begin{Bmatrix} f_{S_{FM}} = 0 \\ 0 \le f_{S_{AM}} \le W/2 \\ \theta_S = 0 \end{Bmatrix} .$$

$$\text{If } \begin{Bmatrix} D_S > 0 \\ \mu_S > 0 \end{Bmatrix} \text{ then } \begin{Bmatrix} 0 \le f_{S_{AM}} \le W/10 \\ 0 \le f_{S_{FM}} \le W/10 \\ 0 \le \theta_S \le 2\pi \end{Bmatrix} .$$

$$\text{If } \begin{Bmatrix} \mu_I = 0 \\ D_I > 0 \end{Bmatrix} \text{ then } \begin{Bmatrix} f_{I_{AM}} = 0 \\ 0 \le f_{I_{FM}} \le W/8 \\ \theta_I = 0 \end{Bmatrix} .$$

$$\text{If } \begin{Bmatrix} D_I = 0 \\ \mu_I > 0 \end{Bmatrix} \quad \text{then} \quad \begin{Bmatrix} f_{I_{FM}} = 0 \\ 0 \le f_{I_{AM}} \le W/2 \\ \theta_I = 0 \end{Bmatrix}.$$

$$\text{If } \begin{Bmatrix} D_I > 0 \\ \mu_I > 0 \end{Bmatrix} \quad \text{then} \quad \begin{Bmatrix} 0 \le f_{I_{AM}} \le W/10 \\ 0 \le f_{S_{AM}} \le W/10 \\ 0 \le \theta_I \le 2\pi \end{Bmatrix}.$$

(c) Recommended Values of Parameters for Tests

(c-1) AM Signal and AM Interferer:

$\mu_S = 0.3$

$\mu_I = 1$

$f_{S_{AM}} = 0.45\ W,$

$f_{I_{AM}} = 0.5\ W,$

$\theta_S = \theta_I = D_S = D_I = f_{S_{FM}} = f_{I_{FM}} = 0.$

(c-2) FM Signal and FM Interferer

$\mu_S = \mu_I = \theta_S = \theta_I = f_{S_{AM}} = f_{I_{AM}} = 0$

$D_S = D_I = 2.405$

$f_{S_{FM}} = 0.11$

$f_{I_{FM}} = 0.12$

(d) Signal generation: for some large number of positive values of J, set

$$S_{IG}(J) = \left[1 + \mu_S \cos(2\pi f_{S_{AM}} J\delta + \theta_S)\right] \cdot \exp\left[j D_S \cos(2\pi f_{S_{FM}} J\delta)\right]$$

(e) Interferer generation: for some large number of positive values of J, set

$$U(J) = \left[1 + \mu_I \cos(2\pi f_{I_{AM}} J\delta + \theta_I)\right] \cdot \exp\left\{j\left[0.1J\delta W + D_I \cos(2\pi f_{I_{FM}} J\delta)\right]\right\}.$$

(f) Noise generation

Calculate

$$\gamma = \pi W\delta/2$$

$$S = \left[\frac{2\sin^2\gamma}{(1+\sin^4\gamma)^{1/2} + \sin^2\gamma}\right]^{1/2}$$

$$C_5 = \left[\frac{\left[1-\left(1-2S^2\right)\left(1-S^2\right)^{1/2}\right]}{2S^3(1+S)\left[1+(1-S^2)^{1/2}\right]}\right]^{1/2}.$$

Generate a sequence of independent, identically distributed, zero-mean, unit variance, complex Gaussian variables $\{G(J)\}$.

Set

$$\eta(1) = \eta(2) = 0.$$

For a large number of positive integers J, set

$$\eta(J) = 2(1-S)\eta(J-1) - \frac{1-S}{1+S}\eta(J-2) + \frac{2C_5S^2}{1+S}G(J).$$

6.5.2 Generation of Linear and Nonlinear Interference Outputs

a) Case selection

The input data sequences must be matched to the appropriate sequences for computation. There are four main cases which can be described by the titles:

Case a) Splatter of adjacent channel

Case b) Two-frequency intermodulation

Case c) Cross-modulation

Case d) Cross-modulation splatter of adjacent channel.

In addition, each case is subdivided into two cases according to the assignment of interferer and noise waveforms to the internal data sequences. The internal data sequences are labelled Z_0, Z_1, Z_2 and Z_3 and there is an additional labelling variable A which describes whether the interference spectrum is centered on the desired channel or the adjacent channel. Table 6.3 lists the possible cases.

b) Calculate the third-order interference product,

$$Y(J) = \exp(j2\pi AW\delta J) \cdot \sum_{N_1=1}^{N} \sum_{N_2=1}^{N} \sum_{N_3=1}^{N} B(N_1, N_2, N_3) \cdot Z_1(J-N_1)\, Z_2(J-N_2)\, Z_3(J-N_3),$$

and the linear term

$$Y_L(J) = Z_o(J+K).$$

The required input information consists of

(1) the four data sequences $Z_o, \ldots, Z_4$ from (a) preceding,

(2) the number of taps/coordinate N from the routine described in Section 3

(3) the coefficients $B(N_1, N_2, N_3)$ from Section 3

(4) the group delay K of the nonlinear product, from Section 3.

Note that the extreme range of K values that can occur is

$$-129 \le K \le 251$$

and consequently the allowable range of arguments of Y and Y_L must be adjusted accordingly.

c) Calculate γ and S as in Step (f) of 6.5.1

d) Calculate, as needed for output requirements,

(d-1) The total output

$$Y_T(J) = Y_L(J) + C_T Y(J).$$

Required input parameter is C_T.

(d-2) The filtered linear output

$$Y_{LF}(J) = 2(1-S)Y_{LF}(J-1) - \frac{1-S}{1+S} Y_{LF}(J-2) + \frac{2S^2}{1+S} Y_L(J) \text{ for } J \ge J_o$$

with initial values

$$Y_{LF}(J_o-2) = Y_{LF}(J_o-1) = 0,$$

with J_o chosen sufficiently large so that $Y_L(J_o)$ is defined.

(d-3) The filtered total output

$$Y_{TF}(J) = 2(1-S)Y_{TF}(J-1) - \frac{1-S}{1+S} Y_{TF}(J-2) + \frac{2S^2}{1+S} Y_T(J) \text{ for}$$

$$J \ge J_o$$

with initial values

$$Y_T(J_o-2) = Y_T(J_o-1) = 0,$$

with J_o chosen sufficiently large so that $Y_T(J_o)$ is defined.

(e) Calculate, as needed for output requirements the distortion terms:

$$ENV(J) = |Y_{TF}(J)| - |Y_{LF}(J)|$$

$$PH(J) = \begin{cases} Im\{Y_{TF}/Y_{LF}\} & \text{if } |Y_{TF}| \le 2|Y_{LF}| \\ \pi & \text{if } |Y_{TF}| > 2|Y_{LF}| \end{cases}.$$

SECTION 7

TYPICAL OUTPUTS

We have written computer programs that incorporate all the features described in Sections 3, 5 and 6. Figures 7.1 to 7.11 show the results of computer runs using these programs.

The particular nonlinear calculation involved cross-modulation of a desired signal by a nearby modulated carrier. We used the model circuit of Section 5 to create transfer functions H_1 and H_3. Figures 7.1-a and 7.1-b show the form of the computer output for these functions. The normalized nonlinear transfer function is then computed from these values and is shown in Figures 7.2-a and 7.2-b. The normalizer G_o is also included in the output data for later use.

From the values of the normalized transfer function the optimum model coefficients are determined. We show in Figs. 7.3-a to 7.4-c part of the print out associated with the intermediate steps in this, where the distortion vs group delay and normalized tap spacing are made available after each computation of delay line coefficients. The optimum normalized tap spacing for this case proves to be the initial value of D = 1/4, and the associated group delay N_J or K is -1. The computer automatically selects this case for data transfer to the next program segment, along with the correct tap coefficients. This output in printed form is shown in Fig. 7.4.

The generation of sampled data sequences corresponding to two amplitude modulated signals and one noise waveform is shown in Fig. 7.5.

The model parameters, case selection and library of three input waveforms are then made available to the program segment that calculates nonlinear outputs. Figure 7.6 shows the verification of the input selection and model coefficients, while Fig. 7.7 shows the selection of the model inputs according to

$$Z_0(J) = S_{IG}(J)$$

$$Z_1(J) = S_{IG}(J)$$

$$Z_2(J) = U(J)$$

$$Z_3(J) = U^*(J).$$

The next program segment calculates the third order interference terms, and replicates $Z_0(J)$ as the linear output; these are shown in Fig. 7.8. The combined output, with the weighting of the interference set at G_O, is shown in Fig. 7.9. This includes the proper group delay N_I.

The linear and combined outputs are then filtered in a sampled data second order Butterworth filter. These filtered outputs are shown in Fig. 7.10. As a note of caution, the first few outputs include the transient response of the filter, which has a response time roughly equal to 1/D samples. These first few samples should not be used in subsequent data reduction.

The envelope and phase distortion are then calculated as indications of the distortion to be expected in amplitude or phase demodulators. These are shown in Fig. 7.11.

M = 4
N = 3
W = 0.5000000E 06

H3		H1	
-0.7812121E-04	0.1163288E-03	-0.7918239E-01	0.5882762E-01
-0.9220759E-04	0.1776747E-03	-0.1064314E 00	0.1108112E 00
-0.9144409E-04	0.3520127E-03	-0.1246572E 00	0.2316873E 00
-0.1259978E-05	0.5054665E-03	-0.1243234E-02	0.4987524E 00
-0.6375134E-04	0.1310775E-03	-0.1064261E 00	0.1107971E 00
-0.6190402E-04	0.2093698E-03	-0.1246575E 00	0.2316790E 00
-0.8753268E-06	0.3511654E-03	-0.1243234E-02	0.4987524E 00
0.2800112E-03	0.4917828E-03	0.7196492E 00	0.5402430E 00
-0.4472164E-04	0.1570375E-03	-0.1246575E 00	0.2316790E 00
-0.6414339E-06	0.2573342E-03	-0.1243234E-02	0.4987524E 00
0.1941610E-03	0.3540451E-03	0.7196913E 00	0.5402142E 00
0.4516756E-03	0.1753779E-03	0.7204866E 00	-0.5396639E 00
-0.4896283E-04	0.1964063E-03	-0.1243234E-02	0.4987524E 00
0.1425408E-03	0.2662227E-03	0.7196913E 00	0.5402142E 00
0.3254658E-03	0.1367151E-03	0.7204363E 00	-0.5396987E 00
0.3135479E-03	-0.3834347E-04	0.1285904E-02	-0.5012850E 00
-0.1019918E-04	0.2247135E-03	-0.1064261E 00	0.1107971E 00
0.4273668E-04	0.3492371E-03	-0.1246575E 00	0.2316790E 00
0.2238910E-03	0.5197307E-03	-0.1243234E-02	0.4987524E 00
0.7293371E-03	0.5471159E-03	0.7196492E 00	0.5402430E 00
0.3772220E-04	0.2533957E-03	-0.1246590E 00	0.2316438E 00
0.1655667E-03	0.3692340E-03	-0.1258433E-02	0.4987371E 00
0.5066965E-03	0.3803787E-03	0.7196492E 00	0.5402430E 00
0.7591411E-03	-0.4169920E-04	0.7205286E 00	-0.5396351E 00
0.1266476E-03	0.2741320E-03	-0.1258433E-02	0.4987371E 00
0.3713074E-03	0.2797416E-03	0.7196492E 00	0.5402430E 00
0.5420595E-03	-0.2108615E-04	0.7204866E 00	-0.5396639E 00
0.4122749E-03	-0.2567737E-03	0.1304924E-02	-0.5012997E 00
0.2823972E-03	0.2127473E-03	0.7196492E 00	0.5402430E 00
0.4055258E-03	-0.1165584E-04	0.7204866E 00	-0.5396639E 00
0.3049716E-03	-0.1790660E-03	0.1289904E-02	-0.5012850E 00
0.2085540E-03	-0.2262870E-03	-0.1244653E 00	-0.2354797E 00
0.2063473E-03	0.2775007E-03	-0.1246575E 00	0.2316790E 00
0.4105074E-03	0.2572640E-03	-0.1243234E-02	0.4987524E 00
0.7500961E-03	0.4256550E-04	0.7196913E 00	0.5402142E 00
0.7301658E-03	-0.5446287E-03	0.7204866E 00	-0.5396639E 00
0.3026973E-03	0.1705833E-03	-0.1258433E-02	0.4987371E 00
0.5420263E-03	0.2166753E-04	0.7196492E 00	0.5402430E 00

Fig. 7.1a Computer-Generated H_3 and H_1 Values; Part a of Data Output

H3		H1	
0.5072861E-02	-0.3799710E-03	0.7204866E 00	-0.5396639E 00
0.2261068E-02	-0.5204678E-03	0.1304924E-02	-0.5012997E 00
0.4055128E-03	0.1213367E-04	0.7196492E 00	0.5402430E 00
0.3717393E-02	-0.2794429E-03	0.7204866E 00	-0.5396639E 00
0.1672474E-02	-0.3687750E-03	0.1289904E-02	-0.5012850E 00
0.4449677E-04	-0.3512742E-03	-0.1244653E 00	-0.2354797E 00
0.2837270E-02	-0.2125193E-03	0.7204866E 00	-0.5396639E 00
0.1278409E-02	-0.2744875E-03	0.1289904E-02	-0.5012850E 00
0.3904224E-04	-0.2548485E-03	-0.1244673E 00	-0.2354456E 00
-0.9170661E-05	-0.2270922E-03	-0.1075900E 00	-0.1139259E 00
0.3132495E-02	0.3993143E-04	-0.1243234E-02	0.4987524E 00
0.4532770E-02	-0.1744145E-02	0.7196912E 00	0.5402142E 00
0.2872648E-02	-0.4922578E-03	0.7204363E 00	-0.5396987E 00
0.1307257E-05	-0.5080330E-03	0.1289904E-02	-0.5012850E 00
0.3264495E-03	-0.1356491E-02	0.7196492E 00	0.5402430E 00
0.1957657E-02	-0.3544092E-02	0.7204866E 00	-0.5396639E 00
0.9081850E-06	-0.3529494E-02	0.1289904E-02	-0.5012850E 00
-0.9040684E-04	-0.2951477E-03	-0.1244653E 00	-0.2354797E 00
0.1437382E-03	-0.2665673E-03	0.7204866E 00	-0.5396639E 00
0.6655027E-06	-0.2586406E-03	0.1289904E-02	-0.5012850E 00
-0.6113111E-04	-0.2115498E-02	-0.1244673E 00	-0.2354456E 00
-0.9213922E-04	-0.1505123E-02	-0.1075900E 00	-0.1139259E 00
0.5079783E-06	-0.1974056E-03	0.1289904E-02	-0.5012850E 00
-0.4412707E-04	-0.1586749E-03	-0.1244673E 00	-0.2354456E 00
-0.6364801E-04	-0.1231479E-02	-0.1075887E 00	-0.1139224E 00
-0.7855117E-04	-0.1187664E-03	-0.8080417E-01	-0.6116995E-01

Fig. 7.1b Computer-Generated H_3 and H_1 Values; Part b of Data Output.

M = 4
N = 3
W = 0.500000E 04
GO = 0.114761E-02 -0.101028E-02

GAMMA(M1,M2,M3)

(1,1,1)	0.1[illegible]E 00	0.3458212E 00
(1,1,2)	0.7736547E 00	0.3[illegible]345E 00
(1,1,3)	0.6556864E 00	0.285670[illegible] 00
(1,1,4)	0.497[illegible]E 00	0.437[illegible]30E 00
(1,2,1)	0.[illegible]E 00	0.2469[illegible]E 00
(1,2,2)	0.472191[illegible]E 00	0.2676636E 00
(1,2,3)	0.345651[illegible]E 00	0.20428[illegible]E 00
(1,2,4)	0.17[illegible]E 00	0.2721946E 00
(1,3,1)	0.2551279E 00	0.1966215E 00
(1,3,2)	0.[illegible]E 00	0.2229821E 00
(1,3,3)	0.1206532E 00	0.2675325E 00
(1,3,4)	-0.5757760E-01	0.2473026E 00
(1,4,1)	0.1933741E 00	0.1701902E 00
(1,4,2)	0.9971911E-01	0.2009741E 00
(1,4,3)	-0.4850865E-01	0.2517223E 00
(1,4,4)	-0.2218021E 00	0.3407191E 00
(2,1,1)	0.1000000E 01	0.0
(2,1,2)	0.[illegible]974735E 00	0.9293276E-01
(2,1,3)	0.7061[illegible]13E 00	0.2282256E 00
(2,1,4)	0.4975[illegible]E 00	0.4379921E 00
(2,2,1)	0.6349[illegible]19[illegible] 00	0.5113567E-01
(2,2,2)	0.5064[illegible]11E 00	0.1547[illegible]E 00
(2,2,3)	0.2456[illegible]13E 00	0.3042894[illegible] 00
(2,2,4)	0.1425362E 00	0.5[illegible]18E 00
(2,3,1)	0.379[illegible]E 00	0.1119245E 00
(2,3,2)	0.2[illegible]E 00	0.2270821E 00
(2,3,3)	0.[illegible]E-01	0.3822751E 00
(2,3,4)	-0.1022795[illegible] 00	0.6252272E 00
(2,4,1)	0.19[illegible]E 00	0.1701901E 00
(2,4,2)	0.68[illegible]E-01	0.2966654E 00
(2,4,3)	-0.9[illegible]E-01	0.4537701E 00
(2,4,4)	-0.2798142E 00	0.7060497E 00
(3,1,1)	0.6679402E 00	-0.3715096E 00
(3,1,2)	0.60[illegible]E 00	-0.1325433E 00
(3,1,3)	0.5476413E 00	0.7375988E-01
(3,1,4)	0.4975309E 00	0.4379920E 00
(3,2,1)	0.4395657E 00	-0.1444700E 00
(3,2,2)	0.3015472E 00	0.4625720E-01
(3,2,3)	0.2456[illegible]13E 00	0.3042891E 00
(3,2,4)	0.2164097E 00	0.6[illegible]E 00
(3,3,1)	0.2921580E 00	0.2172733E-01

Fig. 7.2a Generation of Normalized Third-Order Transfer Function; Part (a) of data

GAMMA (M1,M2,M3)

(3,3,2)	0.25[illegible] 00	0.2229[illegible]1E 00
(3,3,3)	0.2181[illegible] 00	0.48111187E 00
(3,3,4)	0.2039[illegible]E 00	0.845[illegible]26E 00
(3,4,1)	0.19332240E 00	0.1701[illegible]E 00
(3,4,2)	0.1[illegible] 00	0.2614[illegible]E 00
(3,4,3)	0.132[illegible]E 00	0.6191806E 00
(3,4,4)	0.1291760[illegible] 00	0.98187790E 00
(4,1,1)	0.[illegible]1014[illegible] 00	-0.2746011E 00
(4,1,2)	0.[illegible] 00	-0.10072[illegible]E 00
(4,1,3)	0.[illegible] 00	0.1275[illegible] 00
(4,1,4)	0.[illegible] 00	0.47[illegible]E 00
(4,2,1)	0.244[illegible] 00	-0.7974[illegible]E-01
(4,2,2)	0.2811[illegible]E 00	0.6[illegible]3779E-01
(4,2,3)	0.245[illegible]11E 00	0.304[illegible]1E 00
(4,2,4)	0.4646[illegible]E 00	0.5[illegible]E 00
(4,3,1)	0.2109[illegible] 00	0.6[illegible]427E-01
(4,3,2)	0.25[illegible] 00	0.222[illegible]30[illegible] 00
(4,3,3)	0.3246[illegible]10E 00	0.43[illegible] 00
(4,3,4)	0.45[illegible]E 00	0.714[illegible]1E 00
(4,4,1)	0.15[illegible]241E 00	0.1701[illegible]2E 00
(4,4,2)	0.23[illegible]E 00	0.22[illegible]4[illegible] 00
(4,4,3)	0.31[illegible] 00	0.5[illegible] 00
(4,4,4)	0.44[illegible] 00	0.5017[illegible]1E 00

Fig. 7.2b Generation of Normalized Third-Order Transfer Function; Part (b) of data

```
B-ARRAY GENERATION M = 4 N = 2

NI =    -4
D = 0.25000000E 00
U = 0.0

DR = 0.0              E = 0.25000000E 00

VR =  0.20000000E 03
VT =  0.20000000E 03

KR =     0   KI =     0   KC =     0

NM =     0   NI =    -4

NI =    -4
D = 0.25000000E 00
U = 0.19934127E 01

NI =    -3
D = 0.25000000E 00
U = 0.4358411E 00

NI =    -2
D = 0.25000000E 00
U = 0.97606193E-01

NI =    -1
D = 0.25000000E 00
U = 0.40267772E-01

NI =     0
D = 0.25000000E 00
U = 0.1294269E 01
```

Fig. 7.3a Generation of Model Coefficients; Part (a) of Data; $D = 2^{-2}$

```
DR = 0.2500000E 00   D = 0.1250000E 00

VR =  0.4026773E-01
VT =  0.2000000E 03

KR =    -1   KT =    -1   KD =    -1

NM :     3   NI =    -1

NI =    -1
D = 0.1250000E 00
U = 0.1306700E 02

NI =     0
D = 0.1250000E 00
U = 0.1757500E 00

NI =     1
D = 0.1250000E 00
U = 0.1127250E 01

NI =     2
D = 0.1250000E 00
U = 0.4817698E 01

NI =     3
D = 0.1250000E 00
U = 0.1071460E 02
```

Fig. 7.3b Generation of Model Coefficients; Part (b) of Data; $D = 2^{-3}$

```
DR =  0.25000000E 00   D = 0.62500000E-01

VR =  0.40267727E-01
VT =  0.20000000E 02

KR =      -1  KT =       0  KO =       0

NM =       5  NI =       1

NI =       1
D = 0.62500000E-01
U = 0.27966135E 01

NI =       2
D = 0.62500000E-01
U = 0.10799528E 03

NI =       3
D = 0.62500000E-01
U = 0.52871515E 03

NI =       4
D = 0.62500000E-01
U = 0.93474079E 03

NI =       5
D = 0.62500000E-01
U = 0.12622555E 04
```

Fig. 7.3c Generation of Model Coefficients; Part (c) of Data; $D = 2^{-4}$

B-ARRAY

(1,1,1)	0.6303[illegible] 00	0.5[illegible]7214E 00
(1,1,2)	-0.3790[illegible] 00	0.[illegible]01[illegible]41E-01
(1,1,3)	0.[illegible]E-01	-0.7704[illegible]E-01
(1,2,1)	0.[illegible]E-01	-0.[illegible] 00
(1,2,2)	-0.[illegible]E-01	-0.[illegible]77E-01
(1,2,3)	0.44[illegible]E-01	0.[illegible]E-01
(1,3,1)	-0.[illegible]E-01	0.1[illegible] 00
(1,3,2)	0.4[illegible]E-01	0.2[illegible]0146[illegible]E-01
(1,3,3)	-0.[illegible]E-01	-0.20[illegible]E-01
(2,1,1)	0.1[illegible]441E 00	0.[illegible]224E-01
(2,1,2)	-0.131[illegible]07E 00	-0.[illegible]E-01
(2,1,3)	0.[illegible]E-01	0.411[illegible]E-01
(2,2,1)	-0.1[illegible] 00	-0.1[illegible]27E 00
(2,2,2)	0.1277[illegible]E 00	0.112[illegible]E 00
(2,2,3)	-0.[illegible]E-01	-0.577[illegible]E-01
(2,3,1)	0.4[illegible]E-01	0.[illegible]44E-01
(2,3,2)	-0.[illegible]E-01	-0.[illegible]171[illegible]E-01
(2,3,3)	0.[illegible]417[illegible]E-01	0.31[illegible]741E-01
(3,1,1)	0.4[illegible] 00	0.[illegible] 00
(3,1,2)	-0.[illegible]77[illegible] 00	-0.1[illegible]74[illegible] 00
(3,1,3)	0.7[illegible]14E-01	-0.1[illegible]E-01
(3,2,1)	-0.1[illegible] 00	-0.[illegible]7E 00
(3,2,2)	-0.1[illegible] 00	-0.1[illegible]274[illegible] 00
(3,2,3)	0.[illegible]E-01	0.4721[illegible]E-01
(3,3,1)	-0.[illegible]E-02	0.[illegible]E-01
(3,3,2)	0.[illegible]1[illegible]E-01	0.714111[illegible]E-01
(3,3,3)	-0.[illegible]E-01	-0.2[illegible]04E-01

K = -1

[illegible] = 0.[illegible]000[illegible] 00

R[illegible] = 0.1973[illegible]E-0[illegible]

DELTA = 0.[illegible]E-06

G0 = 0.1147[illegible]E-02 -0.1[illegible]102[illegible]E-02

Fig. 7.4 Optimum Model Coefficients

SWINC TAPE GENERATION

20 DATA SEQUENCES TO BE GENERATED

```
MUS  = 0.300000E 00   MUI  = 0.100000E 01
THS  = 0.0            THI  = 0.0
DS   = 0.0            DI   = 0.0
FSAM = 0.250000E 06   FIAM = 0.250000E 06
FSFM = 0.0            FIFM = 0.0

W = 0.500000E 06   DELTA = 0.500000E-06

S  = 0.5031143E 00
C  = 0.3367995E 00
C1 = 0.9937713E 00
C2 =-0.3305709E 00
C5 = 0.1863363E 01
```

SIC		U		ETA	
0.1228122E 01	0.0	0.1706573E 01	0.4267319E-01	0.5171514E 00	-0.1788676E 00
0.1046930E 01	0.0	0.9987512E 00	0.4997918E-01	0.4244204E 00	0.2012417E 00
0.8432508E 00	0.0	0.2920712E 00	0.2194647E-01	0.8587005E 00	0.6234251E 00
0.7146832E 00	0.0	0.0	0.0	0.1119857E 01	0.6551824E 00
0.7228356E 00	0.0	0.2906051E 00	0.3651597E-01	0.9816405E 00	-0.1049306E 00
0.8638024E 00	0.0	0.9887683E 00	0.1494375E 00	0.8647919E-01	0.9799641E-01
0.1070032E 01	0.0	0.1681029E 01	0.2972202E 00	-0.1020768E 01	-0.2807005E 00
0.1242704E 01	0.0	0.1960133E 01	0.3973383E 00	-0.1260220E 01	-0.7134588E 00
0.1299075E 01	0.0	0.1664081E 01	0.3809669E 00	-0.4310688E 00	0.6826949E-01
0.1212132E 01	0.0	0.9689162E 00	0.2474046E 00	-0.3151357E 00	0.9337074E 00
0.1023538E 01	0.0	0.2818900E 00	0.7953483E-01	-0.2443892E 00	0.6812584E-01
0.8236655E 00	0.0	0.0	0.0	-0.1387761E 00	-0.9946856E 00
0.7082896E 00	0.0	0.2775556E 00	0.9352165E-01	0.7132500E-01	-0.1073957E 01
0.7326974E 00	0.0	0.9393672E 00	0.3428954E 00	0.4284915E 00	-0.1288701E 01
0.8851938E 00	0.0	0.1586472E 01	0.6252644E 00	0.3357148E-01	-0.9288895E 00
0.1092703E 01	0.0	0.1842122E 01	0.7788361E 00	-0.5711730E 00	-0.7382005E 00
0.1255791E 01	0.0	0.1555243E 01	0.7038763E 00	-0.5450040E 00	-0.9263064E 00
0.1296307E 01	0.0	0.9004532E 00	0.4349682E 00	-0.4920520E 00	-0.3687907E 00
0.1194835E 01	0.0	0.2604712E 00	0.1339530E 00	0.3581973E 00	-0.6084734E-01
0.1000001E 01	0.0	0.0	0.0	0.1052943E 01	0.2446743E 00

Fig. 7.5 Signal, Interference and Noise

20 SWING TAPE

CASE 3A N = 3
GROUP DELAY K = -1

CT = 0.1147600E-02 -0.1010300E-02

Z0 = SIG
Z1 = SIG
Z2 = U
Z3 = U*

W = 0.5000000E 05 DELTA = 0.5000000E-06

S = 0.5031143E 00
C = 0.3367995E 00
C1 = 0.9937713E 00
C2 =-0.3305709E 00

B(N,N,N)

(1,1,1)	0.6303000E 00	0.5507000E 00
(1,1,2)	-0.3789999E 00	0.7999998E-01
(1,1,3)	0.8996000E-01	-0.7699996E-01
(1,2,1)	0.3837000E-01	-0.3947000E 00
(1,2,2)	-0.8449995E-01	-0.6860000E-01
(1,2,3)	0.4455000E-01	0.3640000E-01
(1,3,1)	-0.6814998E-01	0.1010000E 00
(1,3,2)	0.4320000E-01	0.3900000E-01
(1,3,3)	-0.2330000[illegible]1	-0.2070000E 00
(2,1,1)	0.1070000E 00	0.9819996E-01
(2,1,2)	-0.1320000E 00	-0.9809995E-01
(2,1,3)	0.6388998E-01	0.4120000E-01
(2,2,1)	-0.1094000E 00	-0.1214000E 00
(2,2,2)	0.1277000E 00	0.1129000E 00
(2,2,3)	-0.6889999E-01	-0.5780000E-01
(2,3,1)	0.4690000E-01	0.5900000E-01
(2,3,2)	-0.6619995E-01	-0.6320000E-01
(2,3,3)	0.3420000E-01	0.3110000E-01
(3,1,1)	0.4610000E-01	0.3579000E 00
(3,1,2)	-0.2774000E 00	-0.1075000E 00
(3,1,3)	0.7966995E-01	-0.1390000E-01
(3,2,1)	-0.1373000E 00	-0.2608999E 00
(3,2,2)	-0.1840000E 00	-0.1526999E 00
(3,2,3)	0.8039999E-01	0.4730000E-01
(3,3,1)	-0.2400000E-02	0.8299994E-01
(3,3,2)	0.6350000E-01	0.7139999E-01
(3,3,3)	-0.2940000E-01	-0.2520000E-01

NUMBER OF INPUT SEQUENCES ON TAPE =

JMIN = 1 JMAX = 23

Fig. 7.6 Input Data for Nonlinear Response Calculation

Z0		Z1		Z2		Z3	
0.12281E 01	0.0	0.12281E 01	0.0	0.17066E 01	0.42673E-01	0.17066E 01	-0.42673E-01
0.10469E 01	0.0	0.10469E 01	0.0	0.99875E 00	0.49979E-01	0.99875E 00	-0.49979E-01
0.34325E 00	0.0	0.84325E 00	0.0	0.29207E 00	0.21946E-01	0.29207E 00	-0.21946E-01
0.71468E 00	0.0	0.71468E 00	0.0	0.0	0.0	0.0	0.0
0.72234E 00	0.0	0.72234E 00	0.0	0.29061E 00	0.36516E-01	0.29061E 00	-0.36516E-01
0.86380E 00	0.0	0.86380E 00	0.0	0.98877E 00	0.14944E 00	0.98877E 00	-0.14944E 00
0.10700E 01	0.0	0.10700E 01	0.0	0.16810E 01	0.29722E 00	0.16810E 01	-0.29722E 00
0.12427E 01	0.0	0.12427E 01	0.0	0.19601E 01	0.39734E 00	0.19601E 01	-0.39734E 00
0.12991E 01	0.0	0.12991E 01	0.0	0.16641E 01	0.38087E 00	0.16641E 01	-0.38087E 00
0.12121E 01	0.0	0.12121E 01	0.0	0.96892E 00	0.24740E 00	0.96892E 00	-0.24740E 00
0.10235E 01	0.0	0.10235E 01	0.0	0.28189E 00	0.79535E-01	0.28189E 00	-0.79535E-01
0.82367E 00	0.0	0.82367E 00	0.0	0.0	0.0	0.0	0.0
0.70829E 00	0.0	0.70829E 00	0.0	0.27756E 00	0.93522E-01	0.27756E 00	-0.93522E-01
0.73270E 00	0.0	0.73270E 00	0.0	0.93937E 00	0.34290E 00	0.93937E 00	-0.34290E 00
0.88519E 00	0.0	0.88519E 00	0.0	0.15885E 01	0.62526E 00	0.15885E 01	-0.62526E 00
0.10927E 01	0.0	0.10927E 01	0.0	0.18421E 01	0.77884E 00	0.18421E 01	-0.77884E 00
0.12558E 01	0.0	0.12558E 01	0.0	0.15552E 01	0.70388E 00	0.15552E 01	-0.70388E 00
0.12963E 01	0.0	0.12963E 01	0.0	0.90045E 00	0.43497E 00	0.90045E 00	-0.43497E 00
0.11948E 01	0.0	0.11948E 01	0.0	0.26047E 00	0.13395E 00	0.26047E 00	-0.13395E 00
0.10000E 01	0.0	0.10000E 01	0.0	0.0	0.0	0.0	0.0

Fig. 7.7 Model Inputs Selected from Tape

THIRD ORDER INTERFERENCE PRODUCT - Y OUTPUT

SEQ NO.	I	Y(I)	
1	4	-0.275682E 00	-0.662829E 00
2	5	-0.334783E-01	-0.161800E 00
3	6	0.600404E-01	0.657727E-01
4	7	0.417135E 00	0.595764E 00
5	8	0.816096E 00	0.136553E 01
6	9	0.327190E 00	0.114013E 01
7	10	-0.832532E 00	-0.458089E 00
8	11	-0.970597E 00	-0.136089E 01
9	12	-0.297369E 00	-0.777586E 00
10	13	-0.384805E-01	-0.186357E 00
11	14	0.626764E-01	0.733756E-01
12	15	0.338240E 00	0.560390E 00
13	16	0.600180E 00	0.116504E 01
14	17	0.326157E 00	0.100263E 01
15	18	-0.579662E 00	-0.382213E 00
16	19	-0.845045E 00	-0.136217E 01
17	20	-0.296088E 00	-0.87[illegible]709E 00

LINEAR OUTPUT - YL

SEQ NO.	I	I+K	YL(I)	
1	2	1	0.1228122E 01	0.0
2	3	2	0.1046930E 01	0.0
3	4	3	0.8432508E 00	0.0
4	5	4	0.7148832E 00	0.0
5	6	5	0.7228358E 00	0.0
6	7	6	0.8633024E 00	0.0
7	8	7	0.1370032E 01	0.0
8	9	8	0.1242704E 01	0.0
9	10	9	0.1299075E 01	0.0
10	11	10	0.1212132E 01	0.0
11	12	11	0.1023538E 01	0.0
12	13	12	0.8230655E 00	0.0
13	14	13	0.7082896E 00	0.0
14	15	14	0.7325974E 00	0.0
15	16	15	0.8351938E 00	0.0
16	17	16	0.1092703E 01	0.0
17	13	17	0.1255791E 01	0.0
18	19	18	0.1296307E 01	0.0
19	20	19	0.1194835E 01	0.0

Fig. 7.8 Interference and Linear Outputs

LINEAR TERM = YL, TOTAL OUTPUT = YT

SEQ NO.	JJ	YL(JJ)		YT(JJ)	
1	4	0.84325E 00	0.0	0.84226E 00	-0.45214E-03
2	5	0.71468E 00	0.0	0.71448E 00	-0.15186E-03
3	6	0.72264E 00	0.0	0.72297E 00	0.14322E-04
4	7	0.86380E 00	0.0	0.86408E 00	0.20227E-03
5	8	0.10700E 01	0.0	0.10723E 01	0.74256E-03
6	9	0.12427E 01	0.0	0.12442E 01	0.97766E-03
7	10	0.12991E 01	0.0	0.12977E 01	0.31551E-03
8	11	0.12121E 01	0.0	0.12096E 01	-0.58106E-03
9	12	0.10235E 01	0.0	0.10224E 01	-0.59196E-03
10	13	0.82367E 00	0.0	0.82343E 00	-0.17499E-03
11	14	0.70629E 00	0.0	0.70644E 00	0.20384E-04
12	15	0.73270E 00	0.0	0.73365E 00	0.30138E-03
13	16	0.88519E 00	0.0	0.88706E 00	0.73064E-03
14	17	0.10927E 01	0.0	0.10941E 01	0.62110E-03
15	18	0.12553E 01	0.0	0.12547E 01	0.14700E-03
16	19	0.12963E 01	0.0	0.12940E 01	-0.70856E-03
17	20	0.11943E 01	0.0	0.11936E 01	-0.70009E-03

Fig. 7.9 Combined Output

FILTERED LINEAR OUTPUT = YLF

SEQ NO.	JO	YLF(JO)		
1	4	0.2840064E	00	0.0
2	5	0.5229423E	00	0.0
3	6	0.6692515E	00	0.0
4	7	0.7831417E	00	0.0
5	8	0.9174149E	00	0.0
6	9	0.1071359E	01	0.0
7	10	0.1198941E	01	0.0
8	11	0.1245558E	01	0.0
9	12	0.1186191E	01	0.0
10	13	0.1044466E	01	0.0
11	14	0.8843915E	00	0.0
12	15	0.7803850E	00	0.0
13	16	0.7813029E	00	0.0
14	17	0.8864856E	00	0.0
15	18	0.1045637E	01	0.0
16	19	0.1182673E	01	0.0
17	20	0.1232067E	01	0.0

FILTERED TOTAL OUTPUT -- YFT

SEQ NO.	JO	YFT(JO)		
1	4	0.2836743E	00	-0.1623845E-03
2	5	0.5225443E	00	-0.2125190E-03
3	6	0.6690114E	00	-0.1525237E-03
4	7	0.7833985E	00	0.7010240E-05
5	8	0.9185294E	00	0.3074876E-03
6	9	0.1072895E	01	0.6325969E-03
7	10	0.1199622E	01	0.6332716E-03
8	11	0.1244888E	01	0.2245086E-03
9	12	0.1184920E	01	-0.1855914E-03
10	13	0.1043346E	01	-0.3175866E-03
11	14	0.8837478E	00	-0.2472233E-03
12	15	0.7834368E	00	-0.3919388E-04
13	16	0.7821956E	00	0.2838537E-03
14	17	0.8878226E	00	0.5765562E-03
15	18	0.1046316E	01	0.5269891E-03
16	19	0.1182115E	01	0.9447016E-04
17	20	0.1230878E	01	-0.3161149E-03

Fig. 7.10 Filtered Outputs

DISTORTION TERMS

SEQ NO.	JO	EMV(JO)	PH(JO)
1	4	-0.3320575E-03	-0.5717636E-03
2	5	-0.3980598E-03	-0.4063908E-03
3	6	-0.2401471E-03	-0.2279019E-03
4	7	0.2568364E-03	0.8551431E-05
5	8	0.1114488E-02	0.3351672E-03
6	9	0.1536369E-02	0.5904622E-03
7	10	0.6809235E-03	0.5281924E-03
8	11	-0.[illegible]694794E-03	0.1802474E-03
9	12	-0.[illegible]270294E-02	-0.1564600E-03
10	13	-0.[illegible]119614E-02	-0.3040659E-03
11	14	-0.6437302E-03	-0.2795404E-03
12	15	0.5179644E-04	-0.5022377E-04
13	16	0.8926392E-03	0.3697076E-03
14	17	0.1337171E-02	0.6503840E-03
15	18	0.6790161E-03	0.5039885E-03
16	19	-0.557899[illegible]-03	0.7987853E-04
17	20	-0.1189232E-02	-0.2565726E-03

Fig. 7.11 Distortion Terms

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