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THE THERMAL STRESS AND DEFORMATION IN CARTRIDGE CASES

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) A refined theoretical model for predicting the states of stress, strain and displacement in a cartridge case is presented. The results are valid during loading of the case, but prior to the time at which the case is extracted from the weapon. The combined effects of thermal and mechanical loading due to propellant gas products are included. The applied temperature may be steady state of transient. The case is modeled as a thermo-elasto-plastic metal with linear workhardening using the Tresca yield criterion. The weapon

20. ABSTRACT (Cont'd)

chamber is elastic. The simple expressions for the stresses, strains and displacements are algebraic, requiring only the solution of several 4 X 4 matrices. No numerical results are presented.

TABLE OF CONTENTS

NOMENCLATURE

C,	constant, Equation 13
D,	constant, Equation 15
E,	modulus of elasticity, Equation 5
$[E]$,	matrix, Equation 27
$[F]$,	matrix, Equation 27
$[G]$,	matrix, Equation 27
K_1,	constant, Equation (A-1)
K_2,	constant, Equation (A-1)
M,	constant, Equation 14
N,	constant, Equation 14
P,	applied longitudinal load, Equation 11
T,	temperature rise, Equation 5
a,	inner case radius, Equation 11
b,	outer case radius, Equation 11
c,	inner chamber radius, Equation 24
d,	outer chamber radius, Equation 24
ϵ_r	total radial strain, Equation 1
ϵ_z	total axial strain, Equation 4
ϵ_θ	total tangential strain, Equation 2
$\bar{\epsilon}^p$	equivalent plastic strain, Equation 8
k_0	yield stress in shear, Equation 7
p,	propellant pressure, Equation 20
r,	radial coordinate, Equation 1
u,	radial displacement, Equation 3
z,	axial coordinate, Equation 4

NOMENCLATURE (Cont'd)

α coefficient of thermal expansion, Equation 5

γ_1 constant, Equation 16

γ_2 constant, Equation 16

σ_z axial stress, Equation 5

σ_r radial stress, Equation 1

σ_θ tangential stress, Equation 1

θ tangential coordinate, Equation 1

τ constant, Equation 8

ν Poisson's ratio, Equation 5

Superscripts

e, elastic, Equation 4

p, plastic, Equation 4

', chamber, Equation A-1

INTRODUCTION

The objectives of the Small Arms Project 1J562604A607 are to provide exploratory development of new or improved munitions components, and ammunition simulation. Task 25 of this project deals with the mathematical modeling of ammunition, and part of the work performed under that task is reported here.

This study was an extension of the modeling of case extraction described in an earlier report[1] by the author. In that report an analytical model for determining the force required to extract a spent cartridge case from a conventional weapon was presented. It was recommended in that report that additional refinement of the mechanics in the model be performed. Part of this additional refinement is presented herein.

In Reference 1 the cartridge case was modeled as a thin membrane shell being deformed and stressed, initially, by the mechanical pressure from burning propellant gases, and subsequently by interaction with the inner chamber wall. The chief purpose of the model was to calculate the state of residual radial stress in the case at extraction. This calculated residual radial stress at the outside diameter of the case wall, when multiplied by the area on which it acts times a coefficient of friction, and less the blowback thrust, was essentially the force of extraction. (See Equation 6 of Reference 1). Those factors which were neglected or over-simplified in that model and are corrected here are:

1. The ability to include thermal (steady state and transient) strains.

2. The compatibility of the radial and hoop strains.

3. The use of simplified equilibrium and constitutive equations in which the state of stress was independent of the elastic and plastic material properties of the case.

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1 P. Gordon, "Analytic Study of Extraction in the M16 Weapon", Frankford Arsenal Report M73-30-1, Oct 1973, pp 1-32.

These factors are corrected by extending an analysis due initially to Bland[2] to the present cartridge case problem. In Bland's analysis[2] the state of stress and strain in an elastoplastic, workhardening cylinder subject to internal and external pressure and temperature was given. Because the piece-wise linear Tresca's yield criterion was adopted, the resulting equations were linear, and solvable in closed form for linear work hardening. Bland assumed, however, that the cylinder wall was so thick that, at all levels of loading, part of the cylinder was elastic, and part plastic. His formulation can not readily handle the more difficult case in which the entire cylinder goes plastic before loading is complete.

In the present analysis, it is assumed that Bland's analysis is applicable until the entire case has yielded. Then Bland's equations are reformulated under the assumption that the entire case experiences some plastic deformation and, thus, that no region remains purely elastic. Expressions for the states of stress, strain and displacement as functions of internal pressure are presented. These expressions are valid during the loading of the case. Unloading and the associated evaluation of the radial stress at extraction have not been treated.

THEORY

Cartridge Case Model

Consider the cartridge case to be a cylinder undergoing axisymmetric loading. Then the radial equation of equilibrium is

$$\sigma_\theta - \sigma_r - r\frac{d\sigma_r}{dr} = 0 \qquad (1)$$

[2] D. R. Bland, "Elastoplastic Thick-Walled Tubes of Workhardening Material Subject to Internal and External Pressures and Temperature Gradients", J. Mech. Phys. of Solids, Vol 4, 1956, pp 209-229.

The small strain - small displacement equations are

$$\varepsilon_r = \frac{du}{dr} \tag{2}$$

$$\varepsilon_\theta = u/r \tag{3}$$

where σ_r σ_θ u ε_r ε_θ are, respectively, the radial and tangential stresses, the radial displacement, the radial and tangential strains.

The total strains are assumed to be made up of elastic and plastic parts, i.e.:

$$\varepsilon_r = \varepsilon_r^e + \varepsilon_r^p \tag{4.a}$$

$$\varepsilon_\theta = \varepsilon_\theta^e + \varepsilon_\theta^p \tag{4.b}$$

$$\varepsilon_z = \varepsilon_z^e + \varepsilon_z^p \tag{4.c}$$

where the superscripts "e" and "p" denote elastic and plastic components, and is the total axial strain.

The elastic strains are related to the stresses through three Hooke's Law[3] equations of the form

$$E\,\varepsilon_r^e = \sigma_r - \nu(\sigma_\theta + \sigma_z) + E\alpha T \tag{5.a}$$

$$E\,\varepsilon_\theta^e = \sigma_\theta - \nu(\sigma_z + \sigma_r) + E\alpha T \tag{5.b}$$

$$E\,\varepsilon_z^e = \sigma_z - \nu(\sigma_r + \sigma_\theta) + E\alpha T \tag{5.c}$$

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[3] A. E. H. Love, Mathematical Theory of Elasticity, Fourth Edition, Cambridge University Press, 1927, pg 108.

where E is the Young's modulus of the material and ν is the Poisson's ratio, both of which are temperature insensitive. T is the temperature rise and α the coefficient of thermal expansion.

In order to relate the plastic strain components of Equations 4 to the stresses, the following assumptions are made:

1. The material is linear work-hardening after yielding.

2. The Tresca yield criterion is used.

3. The stresses are such that:

$$\sigma_\theta \geq \sigma_z \geq \sigma_r \tag{6}$$

4. The difference between any two stresses never drops below the yield strength of the material.

As a result of these assumptions Bland[2] shows that the associated (Tresca) flow rule provides the following expressions for the plastic constitutive equations:

$$\sigma_r - \sigma_\theta = K_0(1 + \eta \bar{\varepsilon}^p) \tag{7}$$

$$\bar{\varepsilon}^p = (2/\sqrt{3})\varepsilon_\theta^p \tag{8}$$

$$\varepsilon_z^p = 0 \tag{9}$$

$$\delta \varepsilon_\theta^p = -\delta \varepsilon_r^p \tag{10}$$

[2] D. R. Bland, "Elastoplastic Thick-Walled Tubes of Workhardening Material Subject to Internal and External Pressures and Temperature Gradients", J. Mech. Phys. of Solids, Vol 4, 1956, pp 209-229.

where τ is the yield stress in shear, $(k_t \tau)$ is the slope of the plastic portion of the stress-strain curve in shear, $\bar{\varepsilon}^p$ is the equivalent plastic strain, and the δ denotes an increment of strain.

An additional equation is required in order to maintain equilibrium in the longitudinal direction. If P is the net applied longitudinal load, including possible friction at r = b due to chamber interference, then

$$P = 2\pi \int_a^b r \sigma_z \, dr \tag{11}$$

is the required expression of longitudinal equilibrium. Here b and a are, respectively, the outer and inner case radii.

It is important to note several facts at this point:

1. The constitutive Equations 7 to 10 are <u>linear</u> while still satisfying an exact theory of plasticity. This is a considerable simplification over the classical treatment[4] which used the non-linear von-Mises criterion, and which may be solved only by a numerical (finite-difference) method.

2. The equilibrium equations above, 1 and 2, cannot be solved explicitly to obtain the stresses as a function only of the applied loads without regard to the constitutive equations and material properties as well as was done in the previous case extraction study.[4]

3. Lastly, because Equations 2 and 3 are to be solved simultaneously, the resulting total strains ε_θ and ε_r will be compatible. Physically this means that the cartridge case will now "fit together" after deformation.

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[4] R. Hill, Mathematical Theory of Plasticity, Clarendon Press, 1950, pp 109-120.

At this point, there are 14 Equations (1 to 5, 7 to 11) in as many unknowns: (3 stresses, 3 total strains, 3 plastic strains, 3 elastic strains, 1 equivalent strain, and 1 displacement). Thus the problem is determinate.

By combining Equations 1, 7 and 8 we know that

$$r\frac{d\sigma_r}{dr} = K_o\left[1 + (2\eta/\sqrt{3})\epsilon_\theta^p\right] \tag{12}$$

A value for ϵ_θ^p is given by Bland's[2] Equation (19). This when combined with Equation 12 leads to the following differential equation for σ_r :

$$r\frac{d\sigma_r}{dr}\left(1 + K_o\eta 2/\sqrt{3}\cdot\frac{1-\nu^2}{E}\right) = K_o + \left[K_o\eta\cdot 2/\sqrt{3}\right]\left[C/r^2 + \left\langle\frac{2}{r^2}\int^r T(r)r\,dr - T(r)\right\rangle\alpha(1+\nu)\right] \tag{13}$$

C is, as yet, an undetermined constant, and T (r) is the radial temperature distribution.

It is assumed that the case is heated non-uniformally to a temperature T_a (above ambient) due to the propellant at $r = a$, and is heated to T_b at $r = b$ at the barrel case interface. The steady state temperature through the wall is then a solution of Laplace's equation, which is

$$T(r) = M + N\ln r \tag{14.a}$$

[2] D. R. Bland, "Elastoplastic Thick-Walled Tubes of Workhardening Material Subject to Internal and External Pressures and Temperature Gradients", J. Mech. Phys. of Solids, Vol 4, 1956, pp 209-229.

where

$$M = \left[T_a \ln b - T_b \ln a\right] \ln(a/b) \tag{14.b}$$

$$N = \left[T_b - T_a\right] \ln(a/b) \tag{14.c}$$

Introduction of Equation 14 into 13 and integration leads to the following expression for σ_r :

$$\sigma_r = \gamma_3 \ln r - \gamma_1 C/2r^2 + D \tag{15}$$

where

$$\gamma_1 = \left(R_c \eta\, 2/\sqrt{3}\right)\left[1 + \frac{R_c \eta^2}{\sqrt{3}}(1-\nu^2)\right]^{-1} \tag{16.a}$$

$$\gamma_3 = \gamma_1 \left[\frac{\sqrt{3}}{2\eta} - \frac{\alpha(1+\nu)}{2} N\right] \tag{16.b}$$

are material constants introduced for convenience. C and D are constants to be determined by the boundary conditions.

The hoop stress, σ_θ , is obtained from Equation 7 as

$$\sigma_\theta = \sigma_r + R_c\left[1 + \eta^2/\sqrt{3}\ \varepsilon_\theta^p\right] \tag{17}$$

The value for σ_r in Equation 17 is obtained from Bland's[1] Equation 19 with the aid of 15. Substituting that result in 17 the final expression for σ_θ is

$$\sigma_\theta = k_0 + \frac{2k_0}{\sqrt{3}}\eta\Big[c/\rho^2 - \left(\frac{1-\nu^2}{E}\right)\left(\gamma_3 + \frac{c_1\gamma_1}{\rho^2}\right) + - \frac{\alpha(1+\nu)}{2}N \Big] + \sigma_r \tag{18}$$

where σ_r is given by 15.

The expression for the axial stress, σ_z, may be obtained from Equation 15 and Bland's[2] Equation 13 as

$$\sigma_z = E_z + \nu\Big[2\gamma_3 \ln r - \gamma_1 c/r^2 + 2D + \gamma_3 + \gamma_1 c/r^2\Big] - E\alpha T \tag{19}$$

where the longitudinal strain, ε_z, is given by

$$\varepsilon_z = \Big[P/\pi - 2\nu(a^2p - b^2q) + 2E\alpha\int_a^b rT\,dr\Big]\Big[E(b^2-a^2)\Big]^{-1} \tag{20}$$

where p and q are, respectively, the internal (propellant) and external (interference between case and chamber) pressures, or equivalently, $p = -\sigma_r(a)$, $q = -\sigma_r(b)$

[2] D. R. Bland, "Elastoplastic Thick-Walled Tubes of Workhardening Material Subject to Internal and External Pressures and Temperature Gradients", J. Mech. Phys. of Solids, Vol 4, 1956, pp 209-229.

As mentioned previously, P is the net axial force to which the cartridge case is subjected at extraction. If experimental values for the thrust from the gas expulsion at the case mouth were known, a value of P could be determined. This would in turn allow the average longitudinal strain and stress to be determined. Such experimental data is not as yet available.[5]

A unique feature of this analysis is that the formula for σ_r, Equation 15, does not depend on ϵ_z, and, hence, does not depend on the axial thrust, P. This is a great simplification over the classic treatment[4], in which the value of σ_r was nonlinearly coupled to both the axial strain and axial stress. Thus the lack of data for P does not, in the present analysis, explicitly influence the determination of the force of extraction, since the force is a function only of σ_r.[4] As will be shown below, however, the radial displacement, u, does explicitly depend upon ν and P. During extraction u is used to determine the point of unloading and thus affects σ_r.

In order to calculate u, the radial displacement, use is made of Equations 14 and 15 by substitution into Bland's Equation 16.[2] That result is:

$$u(r) = \frac{(1-2\nu)(1+\nu)}{E}\left[r\gamma_3 \ln r - \frac{\gamma_1 C r}{2} + Dr\right] - \nu\epsilon_z r + (1+\nu)\alpha\left[Mr + N\left(r\ln r - \frac{r}{2}\right)\right] + C/r \qquad (21)$$

The results may be summarized as follows: The stresses are given by Equations 15, 18, and 19, respectively. The axial strain, ϵ_z, is given by 20, and the radial displacement, u, by Equation 21. With u known the total strains, ϵ_r, ϵ_θ, follow from Equations 2 and 3; the elastic strains ϵ_θ^e, ϵ_r^e, follow from 5.a and 5.b and the

[2] D. R. Bland, "Elastoplastic Thick-Walled Tubes of Workhardening Material Subject to Internal and External Pressures and Temperature Gradients", J. Mech. Phys. of Solids, Vol 4, 1956, pp 209-229.

[4] R. Hill, Mathematical Theory of Plasticity, Clarendon Press, 1950, pp 109-120.

[5] S. Goldstein, Private Communication.

plastic strains, $\epsilon_r^p, \epsilon_\theta^p$, from Equations 4.a-4.c.

The above equations for displacements, strains and stresses contain two arbitrary constants, C and D. As previously mentioned these are determined by the boundary conditions. The boundary conditions are:

a. During expansion but prior to chamber contact:

$$\sigma_r(a) = -P \qquad \sigma_r(b) = 0 \tag{22}$$

b. During expansion and chamber contact but prior to unloading:

$$\sigma_r(a) = -P \tag{23}$$

$$\sigma_r(c) = \sigma_r'(c) \tag{24}$$

$$u(c) = u'(c) \tag{25}$$

$$\sigma_r(d) = 0 \tag{26}$$

where c and d are the inner and outer chamber radii, respectively, and the primed ($'$) quantities pertain to the revelant constants within the chamber.

For case (a), the conditions given in Equation 22 and Equation 15 to evaluate C and D. Those equations are in matrix form, [E][F] = [G], where the elements of the column matrix [F] are the unknowns C and D:

$$\begin{bmatrix} -\frac{1}{2}\gamma_1 a^{-2} & 1 \\ -\frac{1}{2}\gamma_1 b^{-2} & 1 \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} -P & -\gamma_3 \ln a \\ 0 & -\gamma_3 \ln b \end{bmatrix} \tag{27}$$

Hence the formal solution for F is

$$[F] = [E]^{-1}[G] \tag{28}$$

For case (b), the radial stress and displacement given in Equations 15 and 22 must be used to match the radial stress and displacement in the elastic chamber. Expressions for the state of stress and displacement in the chamber are summarized in the Appendix. Using Equations A-1, A-4, 15 and 22 yields in the same matrix notation [E][F] = [G]:

$$\begin{bmatrix} -\frac{1}{2}\gamma_1 a^{-2} & 1 & 0 & 0 \\ 0 & 0 & 1 & -a^{-2} \\ \frac{1}{2}\gamma_1 b^{-2} & -1 & 1 & -b^{-2} \\ E_{41} & E_{42} & E_{43} & E_{44} \end{bmatrix} \begin{bmatrix} C \\ D \\ E \\ F \end{bmatrix} = \begin{bmatrix} -(P+\gamma_3 \ln a) \\ 0 \\ \gamma_3 \ln b \\ G_4 \end{bmatrix} \tag{29}$$

where

$$\begin{aligned} E_{41} &= \left[1-\gamma_1(1-2\nu)(1+2\nu)/2E\right]b^{-1} \\ E_{42} &= \left[(1-2\nu)(1+\nu)b\right]E^{-1} \\ E_{43} &= -\left[1-\nu'-2(\nu')^2\right]/E' \\ E_{44} &= -(1+\nu')/E'b \\ G_4 &= \{(1-2\nu)(1+\nu)b\gamma_3 \ln b/E - \nu b\epsilon_z + \\ &\quad + \alpha(1+\nu)\left[Mb + N(b\ln b - b/2)\right]\} \end{aligned} \tag{30}$$

The solution matrix, $[F]$, is again given by Equation 28.

The actual calculating procedure is summarized as follows: If chamber contact has not been made, Equation 27 is used to calculate C and D. If contact has been made, Equation 29 is used to calculate C, D, K_1, K_2. The remaining variables are calculated as

	Variable	From Equation
Cartridge Case	σ_r	15
	σ_θ	18
	ϵ_z	20
	σ_z	19
	u	21
	ϵ_r	2
	ϵ_θ	3
Chamber	σ_r'	A-1
	σ_θ'	A-2
	σ_z'	A-3
	u'	A-4

The values of σ_θ, σ_z, and σ_r calculated above are used to check the inequality given by Equation 6 and also that no difference between any two of the stresses drops below the yield stress K_0. A discussion of these requirements is given in Bland[2]. If either of these requirements is not fulfilled the analysis is not valid.

As mentioned in the Introduction the present analysis is valid only during loading of the case, unloading has not as yet been treated for this formulation. Unloading for crude theories of plasticity has

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[2] D. R. Bland, "Elastoplastic Thick-Walled Tubes of Workhardening Material Subject to Internal and External Pressures and Temperature Gradients", J. Mech. Phys. of Solids, Vol 4, 1956, pp 209-229.

been treated approximately in other studies.[1, 6]

Transient Heating

The steady state heat transfer problem is presented above. A more accurate solution can be obtained, however, by considering the propellant heating to be transient.

The transient temperature distribution, $T(r,t)$, is assumed to be known by separate analysis. References 8 and 9 give the details for such calculations. Equations 13 may be integrated in general form to yield.

$$\sigma_r(r,t) = (\gamma_1 B/2\eta)\ln r - \gamma_1 C/2r^2 + D + \gamma_1 \alpha(1+\nu)\int^r \frac{2}{r^3}\left(\int^r r\,T(r,t) - T(r,t)\right) dr\,dr \tag{31}$$

for the transient radial stress. Expressions for $\sigma_\theta, u, \epsilon_z, \epsilon_\theta$ follow closely the analysis given in the preceding section.

RECOMMENDATIONS

A theoretical analysis of the states of stress, strain and displacement in a cartridge case and chamber has been performed. The cartridge case has been modeled as a right circular cylinder of thermo elastic-plastic, linear strain hardening metal. The chamber is elastic.

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[1] P. Gordon, "Analytic Study of Extraction in the M16 Weapon", Frankford Arsenal Report M73-30-1, Oct 1973, pp 1-32.

[6] L. M. Gold, "Cartridge Case Chamber Interaction During Firing", Frankford Arsenal Report M73-35-1, December 1973.

[8] A. Carslaw, J. Jaegar, <u>Conduction of Heat in Solids</u>, Second Edition Oxford University Press, 1959.

[9] B. Boley & J. Weiner, <u>Theory of Thermal Stresses</u>, New York, John Wiley, 1960.

It was found that by extending an analysis of Bland[2] the shortcomings in a previous simplified model[1] could be corrected. These corrections included:

1. Thermal strains due to propellant heating.

2. Compatibility of the radial and hoop strains.

3. Improved representation of plastic flow (consitutive equations).

It was also noted that the present analysis is still far simpler than the classical[4] analysis, which requires extensive numerical integration of nonlinear equations. The present analysis reduces to algebraic expressions for the stresses, strains and displacements, requiring only the inversion of either a 2 x 2 or a 4 x 4 matrix. While the formulation has been specifically used for a steady state temperature effect (viz, Equation 14), the extension to transient heating is demonstrated.

It is recommended that:

1. This analysis be programmed and the results compared with experiment when available.

2. The analysis be extended to include unloading and the actual calculation of extraction forces.

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[1] P. Gordon, "Analytic Study of Extraction in the M16 Weapon", Frankford Arsenal Report M73-30-1, Oct 1973, pp 1-32.

[2] D. R. Bland, "Elastoplastic Thick-Walled Tubes of Workhardening Material Subject to Internal and External Pressures and Temperature Gradients", J. Mech. Phys. of Solids, Vol 4, 1956, pp 209-229.

[4] R. Hill, Mathematical Theory of Plasticity, Clarendon Press, 1950, pp 109-120.

APPENDIX

State of Stress and Displacement in the Chamber

Consider the chamber to be a long right circular annulus, $c \leq r \leq d$, in a state of plane strain. Then the most general solutions to the elastic isothermal governing equations[7] are

$$\sigma_r' = K_1' - K_2'/r^2 \qquad \text{(A-1)}$$

$$\sigma_\theta' = K_1' + K_2'/r^2 \qquad \text{(A-2)}$$

$$\sigma_z' = 2\nu' K_1' \qquad \text{(A-3)}$$

$$u' = K_1' r\left(1-\nu'-2(\nu')^2\right)/E + K_2'(1+\nu')/E'r \qquad \text{(A-4)}$$

where K_1 and K_2 are constants to be determined.

If the chamber is too hot for the thermal strains in the chamber to be neglected than A-1 through A-4 can be easily modified for thermal strain as shown by Nadai.[7]

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[7] A. Nadai, Theory of Flow and Fracture in Solids, Vol 2, McGraw-Hill, 1963, pg 389.

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1. P. Gordon, "Analytic Study of Extraction in the M16 Weapon", Frankford Arsenal Report M73-30-1, Oct 1973, pp 1-32.

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