AD/A-001 682

FORCED VIBRATION CALCULATION USING GENERAL BENDING RESPONSE PROGRAM (GBRP) AND THE FAST FOURIER TRANSFORM

Francis M. Henderson

Naval Ship Research and Development Center
Bethesda, Maryland

August 1974

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER 4481	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER AD/A-001682
4. TITLE (and Subtitle) FORCED VIBRATION CALCULATION USING GENERAL BENDING RESPONSE PROGRAM (GBRP) AND THE FAST FOURIER TRANSFORM		5. TYPE OF REPORT & PERIOD COVERED Final
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Francis M. Henderson		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Ship Research & Development Center Bethesda, Maryland 20084		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS SF 53 532 106, Task 15326 Work Unit 1-1844-916
11. CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE August 1974
		13. NUMBER OF PAGES 49
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

APPROVED FOR PUBLIC RELEASE: DISTRIBUTION UNLIMITED.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Vibration	Beam-Spring Systems
Ship Hull	Bending Response
Transient Response	Coupled Bending-Torsion Response
Forced Vibration	Whirling Response (Propellers) (CONT).

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A new capability for calculating the transient response of non-uniform beam-spring models has been incorporated into the General Bending Response Program (GBRP), a digital computer program package used for ship vibration analysis. This method, which replaces the previously existing one based upon a Fourier transform approach, uses a Fourier series representation of the excitation force and response, and considers transient forces as periodic with

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-014-6601

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

19. continued:

Finite Differences
Fourier Series
Harmonic Analysis
Fast Fourier Transform
Matrix Inversion

20. continued:

sufficiently long periods.

The method is implemented through a program interface developed between GBRP and a fast Fourier transform subroutine in use at the Computation and Mathematics Department. Test calculations were made for two mass-spring models, each subjected to a transient rectangular forcing function. The computed response in each case exhibits generally good agreement with theoretical results.

MAJOR NSRDC ORGANIZATIONAL COMPONENTS

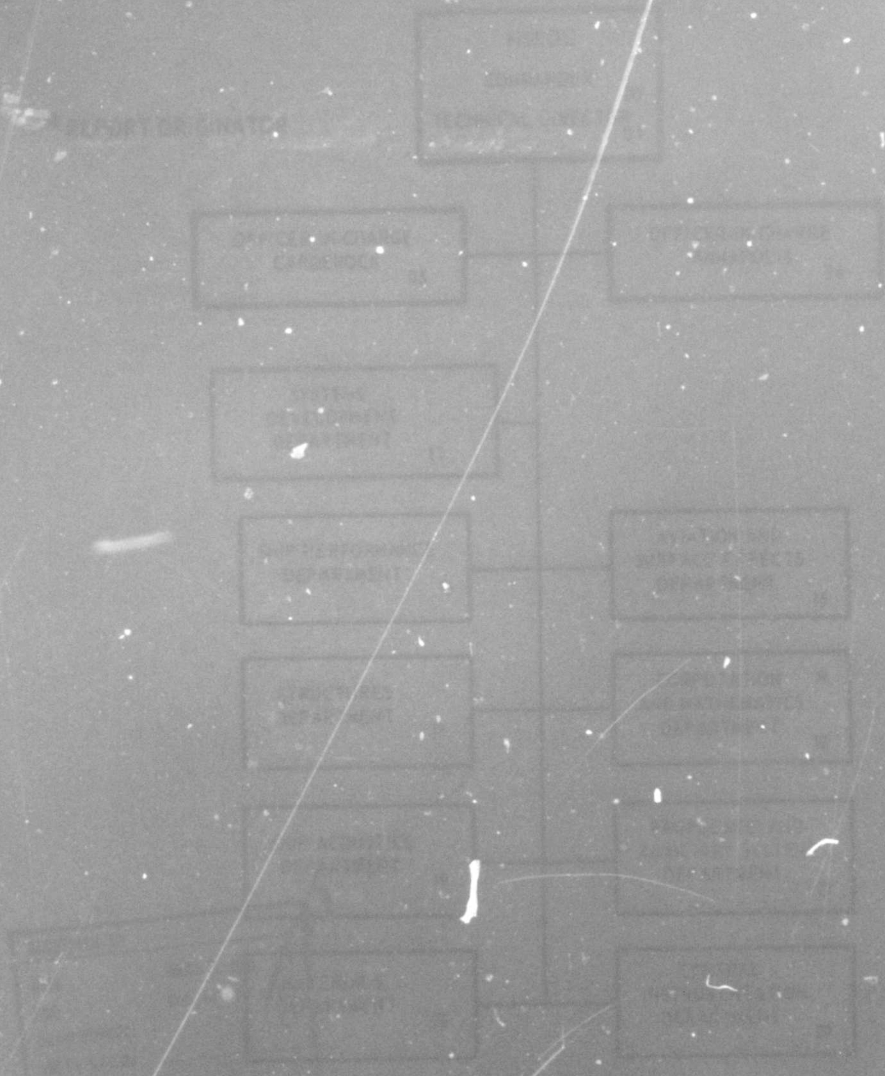

TABLE OF CONTENTS

LIST OF FIGURES

I. INTRODUCTION

A method for computing the response of elastically coupled non-uniform beam-spring systems to a transient forcing function, based upon the use of the Fourier integral transform, has already been described.[1] The original version of this method incorporated into the General Bending Response Program (GBRP) treated only the case in which the forcing function had a Fourier transform which could be expressed in closed form.

Recently, the GBRP transient response capability has been augmented by replacing the Fourier transform approach with one which is based upon the Fourier series and the use of the fast Fourier transform (FFT) to obtain the series coefficients. The advantage of this approach is that it enables the program to handle not only general periodic forcing functions but transient impulses as well by considering these impulses as periodic with suitably long periods.

In the following pages we will discuss the application of FFT to forced vibration analysis, focusing on the GBRP-FFT interface and demonstrating the results via several practical calculations.

---

[1] Henderson, F., "Transient Response Calculation In the Frequency Domain with General Bending Response Program (GBRP)," Naval Ship Research and Development Center Report 3613 (Feb 1971).

II. MATHEMATICAL FORMULATION

The Fourier series transformation of the equations of motion for a non-uniform beam follows the classical procedure for this type of analysis described in many textbooks. The steps in the procedure parallel those for the integral transform approach.[1]

The equations for the bending motion of a non-uniform beam used in that approach were

$$\frac{\partial V(x,t)}{\partial x} = -\mu(x)\,\frac{\partial^2 y(x,t)}{\partial t^2} - c(x)\,\frac{\partial y(x,t)}{\partial t} + P(x,t) \tag{1}$$

$$\frac{\partial M(x,t)}{\partial x} = V(x,t) + I_{\mu z}(x)\,\frac{\partial^2 \gamma(x,t)}{\partial t^2} + Q(x,t) \tag{2}$$

$$\frac{\partial y(x,t)}{\partial x} = \gamma(x,t) - \frac{V(x,t)}{KAG(x)} \tag{3}$$

$$\frac{\partial \gamma(x,t)}{\partial x} = \frac{M(x,t)}{EI(x)} \tag{4}$$

with the following notation:

x	Distance in the direction of the beam axis measured from the origin of coordinates
t	Time variable
y	Displacement normal to x in the xy-plane of bending
γ	Angular displacement relative to the z-axis
V	Shearing force in the direction of flexural vibration (y-direction)
M	Bending moment
μ	Effective mass per unit length
$I_{\mu z}$	Effective rotary inertia per unit length

KAG	Shear rigidity
EI	Bending rigidity
P	External forcing function, force in the y-direction
c	Viscous damping coefficient
Q	External forcing moment per unit length

If the forcing functions acting on the structure, assumed initially at rest, are periodic in time with period T, they can be expressed as

$$P(x,t) = \sum_{m=-\infty}^{\infty} P_m(x)e^{\frac{2\pi imt}{T}} \tag{5}$$

$$Q(x,t) = \sum_{m=-\infty}^{\infty} Q_m(x)e^{\frac{2\pi imt}{T}} \tag{6}$$

The variables for the structural response to these forces then have the form

$$V(x,t) = \sum_{m=-\infty}^{\infty} V_m(x)e^{\frac{2\pi imt}{T}} \tag{7}$$

$$y(x,t) = \sum_{m=-\infty}^{\infty} y_m(x)e^{\frac{2\pi imt}{T}} \tag{8}$$

$$\gamma(x,t) = \sum_{m=-\infty}^{\infty} \gamma_m(x)e^{\frac{2\pi imt}{T}} \tag{9}$$

$$M(x,t) = \sum_{m=-\infty}^{\infty} M_m(x)e^{\frac{2\pi imt}{T}} \tag{10}$$

After substituting the expressions (5) through (10) into Equations (1) through (4) and performing the indicated differentiations, we can then, for each m, set the sum of the coefficients of

$e^{\frac{2\pi imt}{T}}$ in each equation equal to zero, divide out the common factor $e^{\frac{2\pi imt}{T}}$, and obtain the system

$$\frac{dV_m(x)}{dx} = \mu(x)\omega^2 y_m - ic(x)\omega y_m + P_m \tag{11}$$

$$\frac{dM_m(x)}{dx} = V_m(x) - I_{\mu z}(x)\omega^2 \gamma_m(x) + Q_m \tag{12}$$

$$\frac{dy_m(x)}{dx} = \gamma_m(x) - \frac{V_m(x)}{KAG(x)} \tag{13}$$

$$\frac{d\gamma_m(x)}{dx} = \frac{M_m(x)}{EI(x)} \tag{14}$$

where $\omega = 2\pi m(\frac{1}{T})$.

This gives precisely the system of differential equations given by Cuthill and Henderson,[2] from which may be obtained the set of approximating finite-difference equations whose matrix form is

$$A \vec{Z}_m = \vec{P}_m \tag{15}$$

with

$$\vec{Z}_m = \begin{vmatrix} y_{m_1} \\ M_{m_1} \\ y_{m_2} \\ M_{m_2} \\ \vdots \\ y_{m_N} \\ M_{m_N} \end{vmatrix}, \qquad \vec{P}_m = \begin{vmatrix} \tilde{P}_{m_1} \\ \tilde{P}'_{m_1} \\ \tilde{P}_{m_2} \\ \tilde{P}'_{m_2} \\ \vdots \\ \tilde{P}_{m_N} \\ \tilde{P}'_{m_N} \end{vmatrix} \tag{16}$$

---

[2] Cuthill, E.H. and F.M. Henderson, "Description and Usage of General Bending Response Code 1 (GBRC1)," David Taylor Model Basin Report 1925 (July 1965).

and the matrix A and $\tilde{P}$ as defined there.[2] In Equation (16), the subscript m with the components of $\vec{P}_m$ denotes the m^{th} Fourier coefficient in the series representation of P; for the components of $\vec{Z}_m$ the subscript denotes the m^{th} coefficient in the series representing the deflection y or bending moment M. The subscripts 1,2,...,N are the usual GBRP labeling for midpoints of beam subdivisions (sections) at which the response is calculated.

For the purpose of performing a computation, this approach suggests the following three steps:

Step 1 Compute the Fourier coefficients for the series given by Equations (5) or (6) truncated to a finite number of terms.

Step 2 For each term of the truncated series compute the complex frequency response of the structure, thereby obtaining the Fourier coefficients for the truncated series approximating Equations (8) and (10).

Step 3 With the coefficients generated in Step 2, form the summations [Equations (8) and (10)] to obtain response as a function of time.

This procedure is implemented by using FFT to accomplish Steps 1 and 3 and GBRP to accomplish Step 2. Since, from the standpoint of computing, each step is independent of the other two, the total sequence of calculations is handled by suitably interfacing FFT with GBRP. The term "interface" here implies a linking of the two programs such that data generated by FFT in Step 1 becomes the input to GBRP (Step 2) which in turn supplies data to FFT (Step 3) for the final calculation. In other words, the interface involves only a manipulation of the conventional data supplied to and generated by the two programs. The interface will be explored more fully in Section IV.

III. NOTES ON FFT

In Equation (5) from the previous section,

$$P(x,t) = \sum_{m=-\infty}^{\infty} P_m(x) e^{\frac{2\pi imt}{T}}$$

P(x,t) designates a vertical forcing function acting at a distance x along the beam. If we assume this function is defined only for the interval, $0<t<T$, the Fourier coefficients are given by

$$P_m(x) = \frac{1}{T} \int_0^T P(x,t) e^{-\frac{2\pi imt}{T}} dt \tag{17}$$

It was the purpose of the present project to determine how FFT might be used to compute the coefficients of the series (truncated) in Equation (5) and to compute the series (truncated) in Equations (8) and (10).

To apply FFT[3] it is necessary to have a periodic function, g(t), which is assumed to be known at N equally spaced time points g(0), g(1), g(2), ..., g(N-2), g(N-1), (that is, at $t = t_0, t_1, t_2, \ldots, t_{N-2}, t_{N-1}$). N quantities G(0), G(1), G(2), ..., G(N-1) are then sought such that

$$g(k) = \sum_{q=0}^{N-1} G(q) e^{\frac{2\pi i}{N} kq} \quad , k=0,1,2,\ldots,N-1 \tag{18}$$

Equation (18) specifies a system of N equations in N unknowns which can be solved to obtain

$$G(q) = \frac{1}{N} \sum_{k=0}^{N-1} g(k) e^{-\frac{2\pi i}{N} kq} \quad , q=0,1,2,\ldots,N-1 \tag{19}$$

---

[3] Theilheimer, Feodor, "Some Applications of the Fast Fourier Transform," Naval Ship Research and Development Center Report 4231 (July 1973).

The set of G(q) is called the discrete Fourier transform for the given data, g(k). The alternate process of computing the g(k) from the G(q) by Equation (18) is called the inverse Fourier transform for that set of data.

The quantities G(q) may be referred to as the discrete Fourier transform coefficients for the data g(k). It is worthwhile to consider their relationship to the conventional Fourier coefficients for it is the latter which are to be approximated in the present application. One way of doing this has been demonstrated by Ralph B. Johnson, Jr. of the Computation and Mathematics Department in some unpublished notes.

To summarize Johnson's approach we can begin with the Fourier series representation of a periodic function, g(t), with period T

$$g(t) = \sum_{m=-\infty}^{\infty} a_m e^{i \frac{2\pi mt}{T}} \tag{20}$$

Assuming that the interval in which the function is defined, $0<t<T$, is divided into N equal subintervals Δt and substituting in Equation (20) for integer multiples of this subinterval of time, $k\Delta t$, then

$$g(k\Delta t) = \sum_{m=-\infty}^{\infty} a_m e^{i \frac{2\pi m}{T}(k\Delta t)} = \sum_{m=-\infty}^{\infty} a_m e^{i \frac{2\pi mk}{N}} \tag{21}$$

since $\Delta t = \frac{T}{N}$. Noting that

$$e^{i \frac{2\pi k}{N}(pN+q)} = e^{i \frac{2\pi kq}{N}} \tag{22}$$

where q are the integers modulo N, and setting $m=pN+q$, $q=0,1,2,\ldots,N-1$, $p=\ldots,-1,0,1,\ldots$, we obtain the following relationship:

$$g(k\Delta t) = \sum_{q=0}^{N-1} \left\{ \sum_{p=-\infty}^{\infty} a_{(pN+q)} \right\} e^{i \frac{2\pi kq}{N}} \tag{23}$$

Defining

$$G(q) = \sum_{p=-\infty}^{\infty} a_{(pN+q)} \tag{24}$$

and substituting into Equation (23), provides the relationship

$$g(k) = \sum_{q=0}^{N-1} G(q) e^{i\frac{2\pi kq}{N}} \tag{18)(25}$$

with the right hand side yielding at most N distinct values as $k=0,1,2,...,N-1$. As stated by Johnson, this definition of G indicates a grouping of the amplitudes of certain frequencies as a single frequency, an effect called "aliasing". For this discussion it is sufficient to observe that in Equation (24) as N becomes large, the discrete Fourier coefficient for a particular q approaches the q^{th} conventional coefficient, because the only terms $a_{(pN+q)}$ being added to a_q will be the ever smaller relative amplitude terms associated with the higher frequencies.

Another way of demonstrating this relationship is to show that the discrete Fourier coefficients correspond to a trapezoidal rule approximation to the formula for conventional coefficients. Consider that the quantities a_m in Equation (20) are computed from

$$a_m = \frac{1}{T} \int_0^T g(t) e^{-\frac{2\pi imt}{T}} dt \tag{26}$$

With the interval $0 \leq t \leq T$ subdivided into N equal subintervals, Δt, the trapezoidal rule for the integral is

$$I \cong \frac{T}{N} [\frac{1}{2} g(0) + g(\Delta t) e^{-\frac{2\pi im(\Delta t)}{T}} + g(2\Delta t) e^{-\frac{2\pi im(2\Delta t)}{T}} + \ldots$$

$$+ \frac{1}{2} g(N\Delta t)] \tag{27}$$

Since $N\Delta t = T$ and, by periodicity, $g(0) = g(T)$, the first and last terms may be combined, giving

$$I \doteq \frac{T}{N} [g(0)+g(\Delta t)e^{-\frac{2\pi im(\Delta t)}{T}} + g(2\Delta t)e^{-\frac{2\pi im(2\Delta t)}{T}} + \cdots \tag{28}$$

$$+ g\{(N-1)\Delta t\}e^{-\frac{2\pi im\{(N-1)\Delta t\}}{T}}] = \frac{T}{N}\sum_{k=0}^{N-1} g(k)e^{-\frac{2\pi imk\Delta t}{T}}$$

Substituting $T = N\Delta t$ yields the relationship

$$I \doteq \frac{T}{N}\sum_{k=0}^{N-1} g(k)e^{-\frac{2\pi imk}{N}} \tag{29}$$

Substituting this approximation to the integral into Equation (26), we obtain the equation

$$a_m \doteq \frac{1}{T} I = \frac{1}{N}\sum_{k=0}^{N-1} g(k)e^{-\frac{2\pi imk}{N}} \tag{30}$$

Again setting $m = pN+q$, as was done on page 7, and noting that the right hand side then produces only q distinct values, $q=0,1,2,\ldots N-1$, as $p=\ldots,-1,0,1,\ldots$, we obtain

$$a_q \doteq \frac{1}{N}\sum_{k=0}^{N-1} g(k)e^{-\frac{2\pi ikq}{N}} = G(q) \tag{31}$$

The relationships given by Equations (24) and (31) indicate that for a given value of N, the discrepancy obtained in approximating a_q by $G(q)$ will be greater as the value of q increases. From Johnson's approach this discrepancy is seen as the effect of the magnitude of a_q approaching, as q increases, the magnitude of the nearest terms $a_{(pN+q)}$ being added to it (the magnitude of Fourier coefficients $\rightarrow 0$ with increasing q). In the latter approach it is the problem of numerically integrating Equation (26) when the integrand becomes increasingly oscillatory with large values of q (i.e., $m = pN+q$). In other words, N must be increased in order to gain accuracy in the discrete Fourier coefficients associated with higher frequencies.

Having considered the relationship between the discrete Fourier coefficients and the corresponding conventional Fourier coefficients, we will now describe the use of the FFT algorithm to compute the discrete Fourier coefficients, hereafter designated DFC.

Assume that we have sampled data for a function and wish to compute the DFC. The sampled data consist of N points of data, $g(k) \equiv g(t_k)$ on the interval of definition for the function as indicated in Figure 1. For the particular FFT subroutine used here, N must be a power of 2. However, this condition on N stems from the method used to implement FFT for machine computation and not from the basic theory. The quantities g(k) may be real- or complex-valued, but for the present applications they will be considered real since they represent points of a forcing function that is assumed to be real-valued in time. With the g(k) as input data, we can then designate an option of the subroutine which requests computation of the discrete Fourier transform, G(q) [Equation (19)]. If, however, we wish to obtain the quantities, g(k), from the G(q) set considered as input data, the option for the inverse discrete transform must be selected.

Remember that the FFT algorithm calculates <u>only</u> coefficients approximating the a_m where $m = 0,1,2,\ldots$, whereas the formal series we are concerned with here [Equation (20)] requires a summation over the a_{-m} as well. Recall that this equation is simply the complex representation of the trigonometric series

$$g(t) = \frac{1}{2} c_0 + \sum_{m=1}^{\infty} \left(c_m \cos\frac{2\pi mt}{T} + b_m \sin\frac{2\pi mt}{T}\right) \tag{32}$$

with
$$a_0 = \frac{1}{2} c_0$$
$$a_m = \frac{1}{2}(c_m - ib_m)$$
$$a_{-m} = \frac{1}{2}(c_m + ib_m)$$

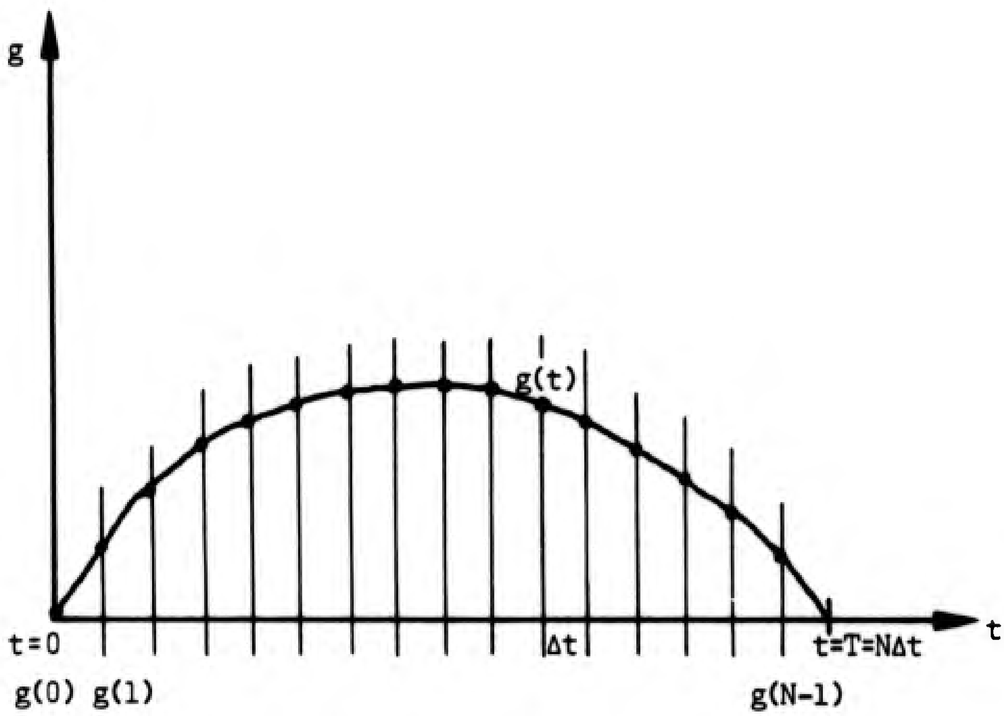

Figure 1 - Sampled Data Points for a Function

A term in the summation on the right-hand side of Equation (20) for $m>0$ is then

$$a_m e^{i\frac{2\pi mt}{T}} = \frac{1}{2}(c_m - i\, b_m)\,(\cos\frac{2\pi mt}{T} + i \sin\frac{2\pi mt}{T})$$

$$= \frac{1}{2}(c_m \cos\frac{2\pi mt}{T} + b_m \sin\frac{2\pi mt}{T}) + \frac{1}{2}i(c_m \sin\frac{2\pi mt}{T} - b_m \cos\frac{2\pi mt}{T}) \tag{33}$$

Similarly, the $-m$ term gives

$$a_{-m} e^{-i\frac{2\pi mt}{T}} = \frac{1}{2}(c_m \cos\frac{2\pi mt}{T} + b_m \sin\frac{2\pi mt}{T}) + \frac{1}{2}i(-c_m \sin\frac{2\pi mt}{T} + b_m \cos\frac{2\pi mt}{T}) \tag{34}$$

The sum of these two terms is

$$a_m e^{i\frac{2\pi mt}{T}} + a_{-m} e^{-i\frac{2\pi mt}{T}} = c_m \cos\frac{2\pi mt}{T} + b_m \sin\frac{2\pi mt}{T}$$

$$= 2\mathcal{R}\; a_m e^{i\frac{2\pi mt}{T}} \tag{35}$$

where $\mathcal{R}$ denotes "real part of". Equation (35) indicates that Equation (20) can then be expressed in terms of positive m,

$$g(t) = a_0 + 2\sum_{m=1}^{\infty} \mathcal{R}\; a_m e^{i\frac{2\pi mt}{T}} \tag{36}$$

Since it is the intent here to use Equation (18) to obtain an approximation to Equation (21) the expression above suggests we compute

$$g(k\Delta t) \doteq 2\sum_{q=0}^{N-1} G(q) e^{\frac{2\pi ikq}{N}} - G(0),\; k=0,1,2,\ldots,N-1 \tag{37}$$

in the present application.

The particular version of the FFT algorithm used for combination with GBRP is one which was developed in the Computation and Mathematics Department and designated FFT5. For this application, only minor changes were introduced so that the subroutine could handle up to 16384 data points for the sets $g(k)$ or $G(k)$.

These few notes summarize those aspects of FFT which pertain directly to the present work and are essential to an understanding of the FFT-GBRP interface and its use as explored in later sections.

For a discussion of the significance of the term "fast" associated with the algorithm the reader may refer to a paper by Dr. F. Theilheimer of the Computation and Mathematics Department.[4]

IV. DESCRIPTION OF GBRP-FFT INTERFACE

As previously stated at the close of the discussion of the mathematical formulation, the interface between GBRP and FFT is essentially a data interface with each program performing its particular part of the total calculation independently.

The nature of the interface is perhaps best demonstrated by considering the form of the data (input and output) associated with the steps (page 5) of a typical calculation, and the procedure for processing FFT data for use with GBRP and vice versa.

Assume a beam-spring system acted upon by a set of periodic forcing functions. The structure is divided into sections

[4] Theilheimer, Feodor, "A Matrix Version of the Fast Fourier Transform," IEEE Transactions on Audio and Electroacoustics, Vol. AU-17, No. 2 (June 1969).

numbered 1, 2, 3, ..., L, and the forcing functions are specified as acting at the section midpoints. The objective is to compute the bending response as a function of time.

The forcing functions are defined by their respective sets of sampled data points: $g_1(0)$, $g_1(1)$, $g_1(2)$, ..., $g_1(N-1)$ for the force at section 1; $g_2(0)$, $g_2(1)$, $g_2(2)$, ..., $g_2(N-1)$ for the force at section 2; etc. These sets are arranged in this order on an input file for FFT which calculates the discrete coefficients. From Equation (19) it is seen that N coefficients will be generated for each set. These data provide the steady-state force amplitudes to be used in the complex frequency response calculation (Step 2 with GBRP) indicated in Equation (15). However, since the solution for each $\vec{Z}_m$ involves a matrix inversion, it becomes highly advantageous from the standpoint of computing time to make this calculation for only those MN coefficients which are expected to produce significant amplitudes of response in the structure. "Significant" here refers to the relative effect these amplitudes (which are in turn the discrete coefficients of the time response) are likely to exert in the summation appearing in Equation (18). Accordingly, MN discrete coefficients for the force at each section are retrieved from the N computed by FFT and written on an output file in the following order: $G_1(0)$, $G_1(1)$, $G_1(2)$, ..., $G_1(MN)$ for section 1; $G_2(0)$, $G_2(1)$, $G_2(2)$, ..., $G_2(MN)$ for section 2; etc.

In preparation for Step 2, these G sets are sent to the interface which forms the vectors $\vec{P}_m$ of Equation (16). Assuming only vertical forces of excitation (i.e., no moments), the G's in GBRP notation become the $\tilde{P}_m$'s, and the required vectors, written sequentially on tape, are

$$\vec{G}_m = \begin{vmatrix} G_1(0) \\ G_2(0) \\ G_3(0) \\ \cdot \\ \cdot \\ \cdot \\ G_L(0) \end{vmatrix}, \begin{vmatrix} G_1(1) \\ G_2(1) \\ G_3(1) \\ \cdot \\ \cdot \\ \cdot \\ G_L(1) \end{vmatrix}, \ldots \begin{vmatrix} G_1(MN) \\ G_2(MN) \\ G_3(MN) \\ \cdot \\ \cdot \\ \cdot \\ G_L(MN) \end{vmatrix}, \quad m=0,1,2,\ldots,MN \tag{38}$$

Comparing the notation in these vectors with that in Equation (16) it is clear that the integers in parentheses (Equation (38)) correspond to m, which is the multiplying factor of the fundamental frequency. GBRP then solves Equation (15), using the successive right hand sides $\vec{G}_m$, and produces the vectors (Equation (16))

$$\vec{Z}_m = \begin{vmatrix} y_1(0) \\ M_1(0) \\ y_2(0) \\ M_2(0) \\ \cdot \\ \cdot \\ \cdot \\ y_L(0) \\ M_L(0) \end{vmatrix}, \begin{vmatrix} y_1(1) \\ M_1(1) \\ y_2(1) \\ M_2(1) \\ \cdot \\ \cdot \\ \cdot \\ y_L(1) \\ M_L(1) \end{vmatrix}, \ldots \begin{vmatrix} y_1(MN) \\ M_1(MN) \\ y_2(MN) \\ M_2(MN) \\ \cdot \\ \cdot \\ \cdot \\ y_L(MN) \\ M_L(MN) \end{vmatrix}, \quad m=0,1,2,\ldots,MN \tag{39}$$

which are written out on a tape. These data, the output of Step 2, give the discrete Fourier coefficients of the time response in deflection and moment at each section.

From Equation (39) we see that the coefficients relating to a particular section are distributed among all the vectors and thus require reordering before FFT can perform the inverse discrete transform. Consequently, these data are sent to the interface which forms the sets: $y_1(0)$, $y_1(1)$, $y_1(2)$, ..., $y_1(MN)$ and $M_1(0)$, $M_1(1)$, $M_1(2)$, ..., $M_1(MN)$ for section 1; $y_2(0)$, $y_2(1)$, $y_2(2)$, ..., $y_2(MN)$ and $M_2(0)$, $M_2(1)$, $M_2(2)$, ..., $M_2(MN)$ for section 2; etc; and writes them on tape in the order shown.

In Step 3, FFT computes the discrete inverse transform (Equation (18)) for the augmented data sets: $y_1(0)$, $y_1(1)$, $y_1(2)$, ..., $y_1(MN)$, $y_1(MN+1)$, ..., $y_1(N-1)$; $M_1(0)$, $M_1(1)$, $M_1(2)$, ..., $M_1(MN)$, $M_1(MN+1)$, ..., $M_1(N-1)$; $y_2(0)$, $y_2(1)$, $y_2(2)$, ..., $y_2(MN)$, $y_2(MN+1)$, ..., $y_2(N-1)$; $M_2(0)$, $M_2(1)$, $M_2(2)$, ..., $M_2(MN)$, $M_2(MN+1)$, ..., $M_2(N-1)$; etc.; the coefficients with subscripts greater than MN being assigned the value zero. Making notational substitutions of y or M for G and $\tilde{y}$ or $\bar{M}$ for g in Equation (18) for example, results at section j in

$$\tilde{y}_j(k) = \sum_{q=0}^{N-1} y_j(q) e^{\frac{2\pi i}{N} kq} \quad , \; k=0,1,2,\ldots,N-1$$

and

$$\bar{M}_j(k) = \sum_{q=0}^{N-1} M_j(q) e^{\frac{2\pi i}{N} kq} \quad , \; k=0,1,2,\ldots,N-1$$

The sets of data $\tilde{y}_j(k)$, $\bar{M}_j(k)$, for $j = 1,2,3,\ldots,L$ are written sequentially on tape by FFT as they are produced.

With the third step of the calculation completed, the final data sets obtained from FFT are ready for editing. It is important to recall that the response computed at each section is given at the intervals $k\cdot\Delta t = \frac{k\cdot T}{N}$ determined by the FFT process. As will be discussed in the next section, the program (GBRP-FFT) allows the user to specify that only every n^{th} point of the final data sets received be edited for printing. The transformation of FFT results indicated in Equation (37) is performed on each n^{th} point just prior to editing.

If the user wishes to see the complete time history of response for each section separately the final sets of data, $\tilde{y}_j(k)$, $\tilde{M}_j(k)$, can be edited directly from the FFT output tape because the results are ordered sequentially first with respect to k, then with respect to j. If alternately the user desires to see the response of the entire beam system for each time step (i.e., through the usual GBRP editing subroutines), the final data sets may be rechanneled to the interface which will form the vectors

$$\begin{vmatrix} \tilde{y}_1(0) \\ \tilde{M}_1(0) \\ \tilde{y}_2(0) \\ \tilde{M}_2(0) \\ \vdots \\ \tilde{y}_L(0) \\ \tilde{M}_L(0) \end{vmatrix}, \quad \begin{vmatrix} \tilde{y}_1(1) \\ \tilde{M}_1(1) \\ \tilde{y}_2(1) \\ \tilde{M}_2(1) \\ \vdots \\ \tilde{y}_L(1) \\ \tilde{M}_L(1) \end{vmatrix}, \quad \begin{vmatrix} \tilde{y}_1(2) \\ \tilde{M}_1(2) \\ \tilde{y}_2(2) \\ \tilde{M}_2(2) \\ \vdots \\ \tilde{y}_L(2) \\ \tilde{M}_L(2) \end{vmatrix}, \quad \cdots \quad \begin{vmatrix} \tilde{y}_1(N-1) \\ \tilde{M}_1(N-1) \\ \tilde{y}_2(N-1) \\ \tilde{M}_2(N-1) \\ \vdots \\ \tilde{y}_L(N-1) \\ \tilde{M}_L(N-1) \end{vmatrix}$$

which have the same form of the right-hand sides of Equation (16) for $\vec{Z}_m$.

In concluding this section, a few notes on the software aspects of this interface are in order.

The prospect of incorporating such an interface in GBRP suggested that a new version of the program be created using a system of overlays. The overlay facility would remove the constraint of a fixed amount of computer core for instructions pursuant to the addition of new subroutines and would enable the larger program to execute on the machine with a significantly reduced memory requirement.

The basic overlay version of GBRP was developed using the NASTRAN Linkage Editor.[5] Four subroutines were then added to this basic version in making the present modification for general forced vibration calculations. Two of these subroutines, FFT5 and IRVING, combine to perform the discrete transform and inverse transform. These two subroutines were obtained as already operational program decks; FFT5 was modified as noted on page 13. The other two subroutines, TRNSFM and SORT, were developed specifically for the present application.

Subroutine SORT provides the data interface between GBRP and FFT referred to on pages 14, 15, 16, and 17. The subroutine establishes two arrays in computer storage. One, a singly dimensioned array, acts as a buffer to receive data coming from GBRP or FFT calculations. The other, a doubly dimensioned array, serves as working space for making data from either program compatible as input to the other. As an illustration, assume an output file from FFT with the sets of discrete coefficients shown on page 14: $G_1(0)$, $G_1(1)$, $G_1(2),\ldots,G_1(MN)$; $G_2(0)$, $G_2(1)$, $G_2(2),\ldots,G_2(MN)$; etc.; in this order. SORT first reads all elements of the first set into the buffer array, and then regroups all elements into m-word subsets; if $m<MN$, the subsets are written onto a random access file. This process continues for each succeeding original set. The first m-word subset of each original set is then read into the two-dimensional array as follows (assume, for example, m=3)

$$\begin{array}{lll} G_1(0) & G_2(0)\ \ldots & G_L(0) \\ G_1(1) & G_2(1)\ \ldots & G_L(1) \\ G_1(2) & G_2(2)\ \ldots & G_L(2) \end{array}$$

[5] Martin, R.J., "A General Purpose Overlay Loader For CDC-6000-Series Computers; Modification of the NASTRAN Linkage Editor," Naval Ship Research and Development Center Rept. 4062 (Apr 1973).

and these elements are written out by rows on a file as the vectors $\vec{G}_m$, m=1,2,3 (page 15). This procedure continues until all subsets of the original set have been treated and the vectors $\vec{G}_m$, m=4,5,6,...,MN are completed. The $\vec{G}$ file is then in a form suitable for use with GBRP. As shown, the two-dimensional storage array must have at least L columns, although only m rows are necessary, depending upon how much core is to be used in a particular run. In applications where the number of beam sections or exciting forces, L, is small, m may be set equal to MN, thereby bypassing the regrouping of the original sets and writing of the random access file.

In the overlay structure FFT5, IRVING, TRNSFM, and SORT are grouped into a single program segment which in Linkage-Editor terminology is called a link, and which we designate by the name LINK2. TRNSFM serves as a master controlling routine governing the order of calling other members of LINK2 when this link is in memory. All of the original subroutines of GBRP are grouped in another segment designated LINK1. Because FFT5 requires a large amount of core storage for its arrays (page 13), this use of links proves highly beneficial. Not only need just one of the links (LINK2 for calculation steps 1 and 3 or LINK1 for step 2) be present in memory at a time during transient calculations, but for steady-state applications only LINK1 need ever be present. In other words, the core requirement for a job can always be restricted to the minimum amount required for the particular application.

V. INPUT DATA DESCRIPTION

This section outlines the data input to a GBRP-FFT job on the computer. The cards are discussed in order of their appearance in the data deck. The specific contents of standard GBRP data cards are discussed in detail only insofar as items of data receive a particular interpretation for this type of application. The material of this section supersedes that given in Section IV of Henderson's work.[1]

The first two data cards are not to be read by GBRP's data reading subroutine and therefore do not have the usual "type number" (columns 3 and 4) assigned. They are referred to as Data Cards 1 and 2.

- Data Card 1 (Option to select GBRP alone or in conjunction with FFT)

Columns	Contents	
1	2	Indicates a forced vibration run involving GBRP and FFT
	1	All other GBRP applications

- Data Card 2 (Data input to LINK2 - refer to page 19)

Columns	Contents
1-5	Number of sampled time points describing one period of the forcing function(s), i.e., each forcing function acting on the structure is assumed represented by this number of points. This number must be an integer power of 2 $\leq$ 16384.
6-10	Number of sampled time points to be printed out for each forcing function; used in verifying the input data.

Columns	Contents
11-16	Number of blocks of sampled time data input. Each block originates from a forcing function acting at a particular beam section.
17-22	Number of discrete Fourier coefficients (refer to page 14) to be used for the Step 2 complex frequency response calculations. A particular value for this number, say N_{DFC}, is equivalent to using $2N_{DFC}-1$ terms (symmetric about a_0) of Equation (21).
23-26	Total number of beam sections.
27-30	An integer, L, designating that every L^{th} input time point or time response coefficient (from complex frequency response calculation) is to be printed for verification purposes.
31-34	An integer, K, designating that every K^{th} discrete Fourier coefficient of the time data transform is to be printed for verification purposes.
35-42	An approximate time interval, δt (in seconds), for editing time response of the structure. Since FFT produces this response only at $\Delta t = k \cdot \frac{T}{N}$, the program selects that multiple of T/N <u>nearest</u> the specified interval.
43-50	Maximum time $\leq N \cdot \frac{T}{N}$ for which the time response is to be edited and printed
51-58	Period (in seconds) of the forcing function(s).
59-60	01 Sampled data points for forcing function(s) are to be read in binary form from tape.

Columns	Contents	
	02	Sampled data points are to be generated internally by the program (for example, see Problem 1, page 28, Data Card 2).
61-62	01	Sampled data points for forcing function(s) and time response coefficients (from complex frequency response calculation) are not to be printed.
	02	The points are to be printed.
63-64	01	The discrete Fourier coefficients of the forcing function transform are not to be printed.
	02	The coefficients are to be printed.
65-66	Maximum number of response variables (for example, deflection, moment, twist, etc.) that may be present in the GBRP solution vector at any station for the particular application.	
67-68	The number of these variables that are not identically zero at every section. For example, in longitudinal calculations with GBRP, variables for both deflection and moment occur in the solution vector although moments are defined in this calculation to be zero. This information with that in the previous two card columns is utilized in the program to insure that the inverse transform is performed only on arrays not identically zero.	

The standard GBRP data deck, headed by the title card, is placed behind these two data cards. The contents of three of these GBRP data cards merit special attention for the present use.

- Forcing Function Location Card - "32" Card

Columns	Contents
3-4	32
7-9	Total number of beam sections
10-11, 12-13, 14-15, ..., 71-72	Section numbers of those sections where forcing functions act

- Frequency Range and Increment for Generating $\vec{Z}_m$ Card

We recall from the discussion on mathematical formulation, pages 3 and 4, that the angular frequency, ω, associated with $\vec{Z}_m$ is equal to $m \cdot \frac{2\pi}{T}$. Step 2 calculations are therefore made for frequencies corresponding to m=0,1,2,...,MN. This range of frequencies is specified to the program as follows.

- General Data Card - "30" card

Columns	Contents
3-4	30
9-16	Starting frequency in CPS This is the zero frequency with m=0. Since GBRP cannot make its calculation, however, using an identically zero frequency, a very small number, for example $\omega=10^{-6}$ may be selected.
17-24	Final frequency in CPS This quantity will be $MN \cdot \frac{1}{T}$.
25-32	Frequency increment in CPS This quantity will be $\frac{1}{T}$.

The factor 2π in the latter two quantities is supplied automatically by the program which always computes in terms of angular frequency.

- Option Control Card - "20" Card

Column	Contents	
21-24	0003	This number signifies to GBRP matrix subroutines that the elements of $\vec{P}_m$ are to be read from a tape rather than generated as usual. The tape contains the discrete Fourier coefficients of the forcing function at each beam section.
37-40	0003	This option designates a bypass of GBRP's detailed editing (EDITA) of computed output for each frequency. These are only intermediate results which may be accessed for review purposes in a more economical fashion (see seventh option on Data Card 2).

VI. SAMPLE CALCULATIONS

This section outlines and presents the results of two calculations performed as a test of the GBRP-FFT interface.

The first problem considered was the one F. Henderson[1] used to test the previous GBRP capability for transient response. For convenience we have reproduced the physical details. A diagram of the mechanical system follows.

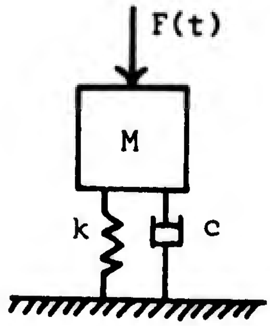

Figure 2 - Simple Harmonic Oscillator

The transient force F(t) represents a constant 0.5 lb force acting on the system for two seconds. The mass $M = 1.0 \frac{\text{lb-in}}{\text{sec}^2}$, the spring constant k = 8.4 lbs/in, the damping constant $c = 0.6 \frac{\text{lb-sec}}{\text{in}}$. The natural frequency of this oscillator is 0.4613 CPS.

In the present approach the transient force F(t) is replaced by a force of period T as shown in the following figure.

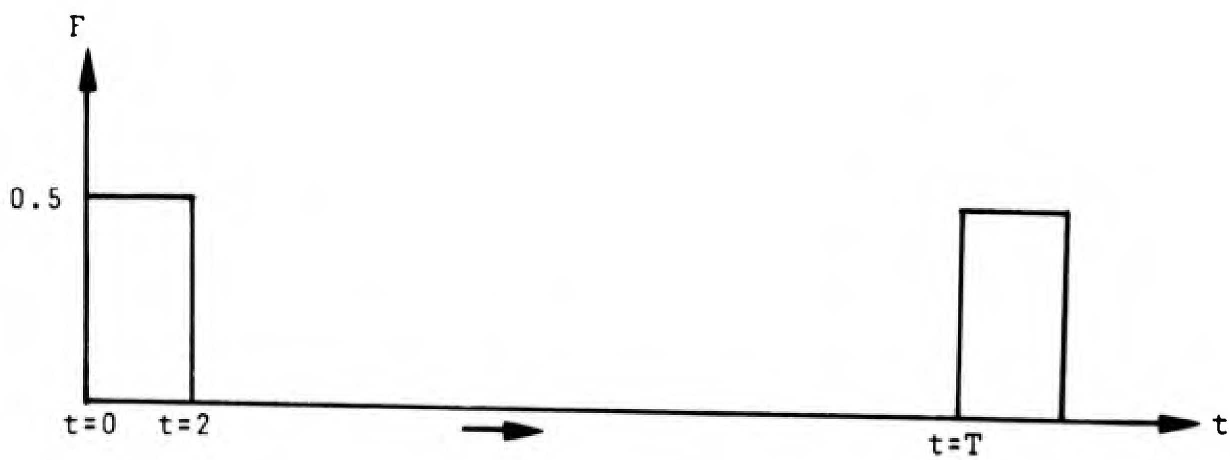

Figure 3 - Transient Rectangular Force as Limiting Case of a Periodic Force With Period $T \to \infty$

T must be of a duration sufficient to allow the effect of the periodic force on the oscillator to approach that of the original F(t).

Some preliminary calculations were made as an aid to selecting a suitable value of T and to estimating the number of Fourier coefficients to include in the analysis. In these calculations we take advantage of the fact that for this application simple formulas for the Fourier series coefficients of the forcing function and hence the oscillator response can be obtained. We can then readily determine the oscillator response by automatic calculation, without, however, any recourse at this point to the larger program. These calculations using various values of T and various numbers of coefficients indicate that with T = 20 (secs) and 601 coefficients in the Fourier series for the response, a good agreement can be obtained with the response solution calculated for the original transient force. The latter solution is obtained utilizing the formula

$$Z(t) = \frac{F_0}{M\omega_0^2} [\ \psi(t)-\psi(t-\tau)U(t-\tau)\ ] \tag{40}$$

where F_0 is the magnitude of the force

M is the mass

$\omega_0^2 = k/M$

$\psi(t) = 1-e^{-bt}[\cos \omega t + \frac{b}{\omega} \sin \omega t]$

with $b = c/2M$

$$U(t-\tau) = \begin{cases} 0, & t<\tau \\ 1, & t>\tau \end{cases}$$

with τ the force duration.

A comparison of the two solutions for oscillator response computed for time 0 to 4 seconds at a 0.04 second interval is shown in Figure 4.

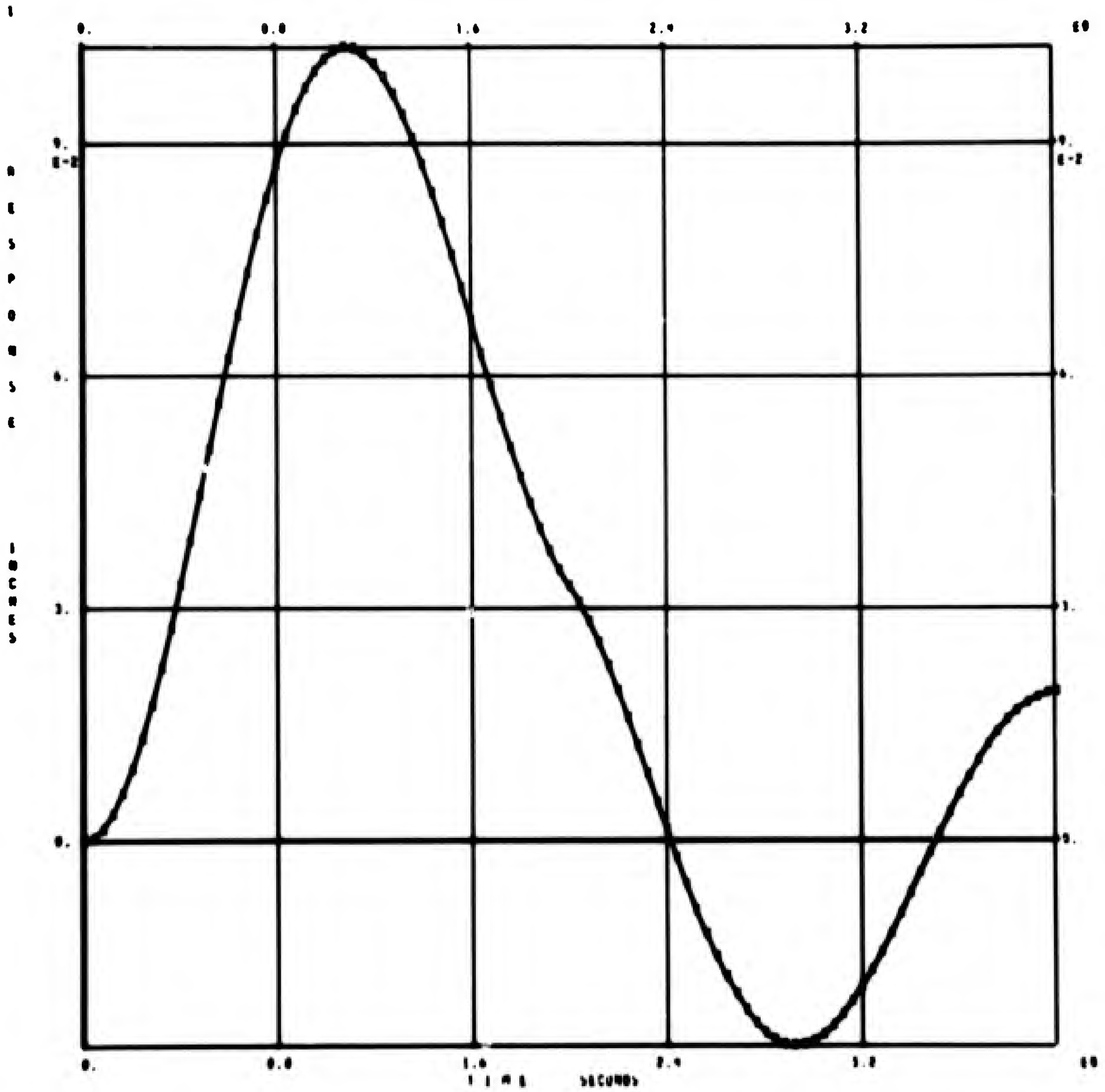

Figure 4 - Simple Harmonic Oscillator
Response to a Transient Rectangular Force
Comparison of Laplace Transform and Fourier Series Solutions

Using the previous estimation for period and number of coefficients, these data for the harmonic oscillator problem were then prepared for input to GBRP-FFT. This process can be facilitated through the use of standard GBRP data forms as shown on pages 29 through 32. Computer runs were made using N = 2048, 4096, 8192, and 16384 in sampling the period T of the periodic forcing function (Figure 3). The sampled points of the time function were generated by the program internally. The response computed with the discrete Fourier coefficients is compared with the Laplace transform solution and found to be in good agreement (Figure 5, page 33).

The input data for the run using 2048 sampled points of the forcing function are included in the following four pages.

Data Card 1

1
2

Data Card 2

2	8	16	20	26	30	34	39	48	53	60	62	64	66	68
2048	100	1	300	1	1	2	0.04	4.0	20.0	2	2	2	2	1

TITLE ______ PROGRAMMER ______ DATE ______

PROBLEM NO ______ PHASE ______ LABEL ______ SHEET 1 OF 4

RUN TITLE CARD

000000(62HAA SIMPLE HARMONIC OSCILLATOR

DATA CONTROL CARD

	NO	TYPE	NO	TYPE	etc											
0090	01	10	01	20	01	30	01	32	01	41	01	43	01	57	01	53

CASE TITLE CARD

001000(62HAA RESPONSE TO RECTANGULAR FORCING FUNCTION

OPTION CONTROL CARD

	OP1	OP2	OP3	OP4	OP5	OP6	OP7	OP8	OP9	OP10
00200000				0003				0003		

EDIT CONTROL CARD

00210000

GENERAL DATA CARD

	NO SECTIONS	ω_1 (CPS)	ω_2 (CPS)	$\Delta\omega$ (CPS)
0030	0001	.000001	14.95	0.05

SYSTEMS DATA CARDS

	SYSTEMS	RADIUS	INITIAL J
0031			
0031			

FORCING FUNCTION LOCATION CARD ___ (Used only if forces are nonharmonic)

0032 | 00 | 1 | 01

TITLE ______________________ PROGRAMMER ______________ DATE __________

PROBLEM NO. ______________________ PHASE ______ LABEL. ______ SHEET 2 OF 4

SECTION DATA CARDS

	5 SECTION NO	9 END CONDN	13 SYSTEM	17 MASS	25 WATER INERTIA	33 $(\Delta x/EI)_n$	41 $(\Delta x)_{n,n+1}$	49 $(\Delta x/\kappa AG)_{n,n+1}$	57 $(I_{HZ}\Delta x)_{n,n+1}$	65 P_n	73
REAL PART OF SCALING FACTORS											
00410000			0001	1.0							
00410000											
00410000											
IMAGINARY PART OF SCALING FACTORS											
00420000											
00420000											
00420000											
PARAMETER VALUES - UNSCALED											
0043	0001		0001	1.0							
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											
0043											

TITLE ____________ PROGRAMMER ____________ DATE ______

PROBLEM NO. ____________ PHASE ______ LABEL ______ SHEET 3 OF 4

5 9 13 17 25 33 41 49 57 65 73

SPECIAL CONNECTION CARDS

	n	m	SYSTEM	$K_{n,m}$	$C_{n,m}$	$C_{n,m}/\omega$	$\Delta X_{n,m}$	$(\Delta X/KAG)_{n,m}$	$(I_{\mu z}\Delta x)_{n,m}$	$Q_{n,m}$
REAL PART OF SCALING FACTORS										
0051			0001	1.0	1.0					
0051										
0051										
IMAGINARY PART OF SCALING FACTORS										
0052										
0052										
0052										
PARAMETER VALUES - UNSCALED										
0053	0001	0000	0001	8.4	0.6					
0053										
0053										
0053										
0053										
0053										
0053										
0053										
0053										
0053										

TITLE ______ PROGRAMMER ______ DATE ______

PROBLEM NO. ______ PHASE ______ LABEL ______ SHEET 4 OF 4

CHARACTRON PLOTTING CHARACTERS

006000	ΔΔΔΔΔΔ						
34	7	13	19	25	31	37	43

NATURAL FREQUENCY SELECTION CARD

0070000		
34	9	11

START NEW DATA SET CARD

0098
34

END OF DATA CARD

✓ 0099
34

Δ indicates blank space

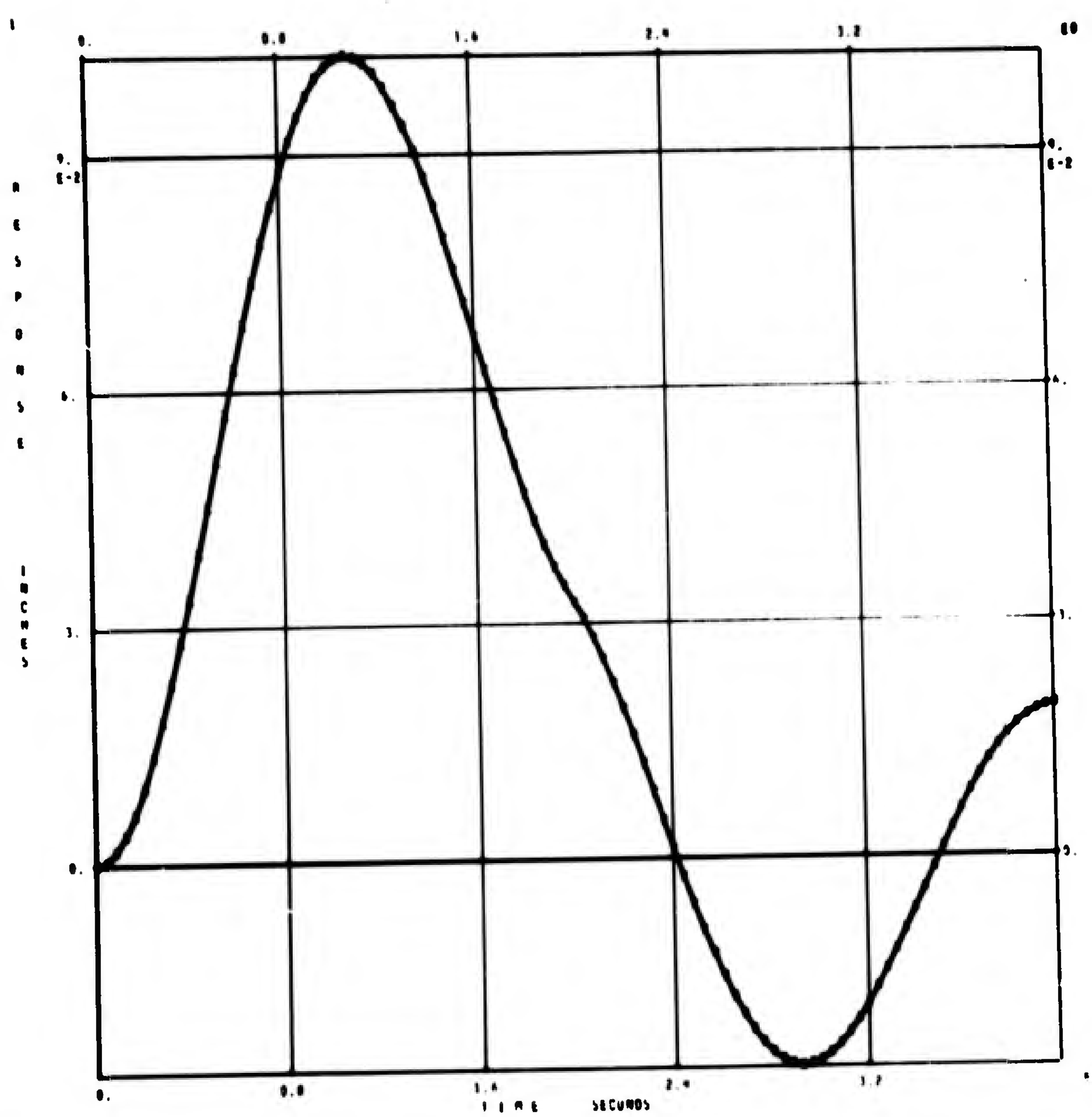

Figure 5 - Simple Harmonic Oscillator, Response to a Transient Rectangular Force, Comparison of Laplace Transform and Discrete Fourier Transform Solutions

To check the functioning of the interface for a case involving several masses, we next made test computations of the response of the spring-mass system given in Figure 6, using the following data:

$$M_1 = 1 \frac{\text{lb-sec}^2}{\text{in}} \quad k_{1,2} = 2000 \frac{\text{lb}}{\text{in}} \quad c_{1,2} = 100 \frac{\text{lb-sec}}{\text{in}}$$

$$M_2 = 1 \frac{\text{lb-sec}^2}{\text{in}} \quad k_{2,3} = 4000 \frac{\text{lb}}{\text{in}} \quad c_{2,3} = 200 \frac{\text{lb-sec}}{\text{in}}$$

$$M_3 = 2 \frac{\text{lb-sec}^2}{\text{in}} \quad k_{3,0} = 6000 \frac{\text{lb}}{\text{in}} \quad c_{3,0} = 300 \frac{\text{lb-sec}}{\text{in}}$$

The system configuration and values of M and k used here are from Biggs.[6] To these data were added the damping constants whose values are assumed to be 5 percent of the respective spring constants with which they are associated. F(t) is the same transient rectangular force (page 25) used with the first problem. The undamped natural frequencies of the system as reported in Biggs[6] are 4.445 CPS, 9.375 CPS, and 14.862 CPS.

The initial computer runs for this problem were made with the period T (see Figure 3) equal to 20 seconds and utilizing 300 coefficients of the discrete transform of the forcing function, as was done in the first problem. In successive trials the period was increased to 30 seconds and the number of coefficients increased to 500. The latter increase was made since in this calculation we attempt to keep all of the undamped natural frequencies of the structure within the range of frequencies included in the analysis. For example, from page 4 we can see that for T = 30, the frequencies (in CPS) associated with the

[6] Biggs, John M., "Introduction to Structural Dynamics," McGraw-Hill, Inc., New York, 1964.

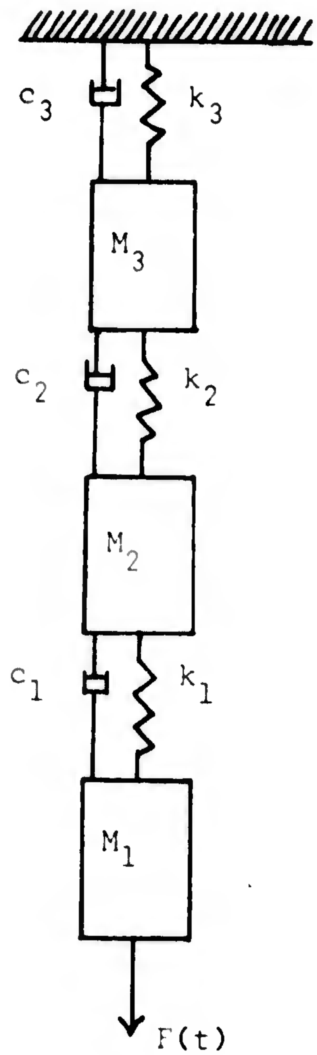

Figure 6 - System of Springs and Masses

Fourier series components are $\frac{\omega}{2\pi} = m(\frac{1}{T}) = m \times 0.05$. If only 300 discrete transform coefficients, corresponding to m = 299, were used, the highest frequency obtained would be 9.966 CPS and therefore the structure's highest frequency would be excluded from the range.

The results of the calculation with T = 30 seconds and N = 2048, 4096, and 8192 are compared in Figures 7, 8, and 9 with an analytic solution obtained using Laplace transforms. The time range for which the response is shown is t = 0 to 3 seconds at intervals of 0.04 seconds. Program data for the case N = 2048 precede the comparison of the results.

Data Card 1

1
2

Data Card 2

2	8	16	20	26	30	34	39	48	53	60	62	64	66	68
2048	100	1	500	3	1	2	0.04	4.0	30.0	2	2	2	2	1

TITLE ______ PROGRAMMER ______ DATE ______

PROBLEM NO ______ PHASE ______ LABEL ______ SHEET 1 OF 4

RUN TITLE CARD

000000(62HΔΔ SYSTEM OF SPRINGS AND MASSES

DATA CONTROL CARD

NO TYPE NO TYPE etc

0090	01	10	01	20	01	30	01	32	01	41	03	43	01	57	03	53

CASE TITLE CARD

001000(62HΔΔ RESPONSE TO RECTANGULAR FORCING FUNCTION

OPTION CONTROL CARD

	OP1	OP2	OP3	OP4	OP5	OP6	OP7	OP8	OP9	OP10
00200800				0003				0003		
	9	13	17	21	25	29	33	37	41	45

EDIT CONTROL CARD

00210000						
	9	13	17	21	25	29

GENERAL DATA CARD

	NO SECTIONS	ω_1 (CPS)	ω_2 (CPS)	$\Delta\omega$ (CPS)
0030	0003	.000001	16.63317	.0333333
	5	9	17	25

SYSTEMS DATA CARDS

	SYSTEMS	RADIUS	INITIAL J
0031			
0031			

FORCING FUNCTION LOCATION CARD (Used only if forces are nonharmonic)

0032			1	1
	6	7	9	11

Reproduced from best available copy.

TITLE ______ PROGRAMMER ______ DATE ______

PROBLEM NO. ______ PHASE ______ LABEL. ______ SHEET 2 OF 4

SECTION DATA CARDS

5 9 13 17 25 33 41 49 57 65 73

SECTION NO.	END COND.	SYSTEM	MASS	WATER INERTIA	$(\Delta x/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta x/\kappa AG)_{n,n+1}$	$(I_{\mu z}\Delta x)_{n,n+1}$	D_n	
REAL PART OF SCALING FACTORS										
00410000		0001	1.0							
00410000										
00410000										
IMAGINARY PART OF SCALING FACTORS										
00420000										
00420000										
00420000										
PARAMETER VALUES - UNSCALED										
0043 0001		0001	1.0							
0043 0002		0001	1.0							
0043 0003		0001	2.0							
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										
0043										

TITLE ______ PROGRAMMER ______ DATE ______

PROBLEM NO. ______ PHASE ______ LABEL ______ SHEET 3 OF 4

5 9 13 17 25 33 41 49 57 65 73

SPECIAL CONNECTION CARDS

	n	m	SYSTEM	$K_{n,m}$	$C_{n,m}$	$C_{n,m}/\omega$	$\Delta X_{n,m}$	$(\Delta x/KAG)_{n,m}$	$(I_{yz}\Delta x)_{n,m}$	$Q_{n,m}$
REAL PART OF SCALING FACTORS										
0051			0001	1.0	1.0					
0051										
0051										
IMAGINARY PART OF SCALING FACTORS										
0052										
0052										
0052										
PARAMETER VALUES - UNSCALED										
0053	0001	0002	0001	2000.	100.					
0053	0002	0003	0001	4000.	200.					
0053	0003	0000	0001	6000	300.					
0053										
0053										
0053										
0053										
0053										
0053										
0053										

TITLE ________ PROGRAMMER ________ DATE ________

PROBLEM NO. ________ PHASE ______ LABEL ☐☐☐☐ ______ SHEET 4 OF 4

CHARACTRON PLOTTING CHARACTERS

00600ΔΔΔΔΔΔ

3 4 7 13 19 25 31 37 43

NATURAL FREQUENCY SELECTION CARD

00700000

3 4 9 11

START NEW DATA SET CARD

0098

3 4

END OF DATA CARD

0099

34

Δ indicates blank space

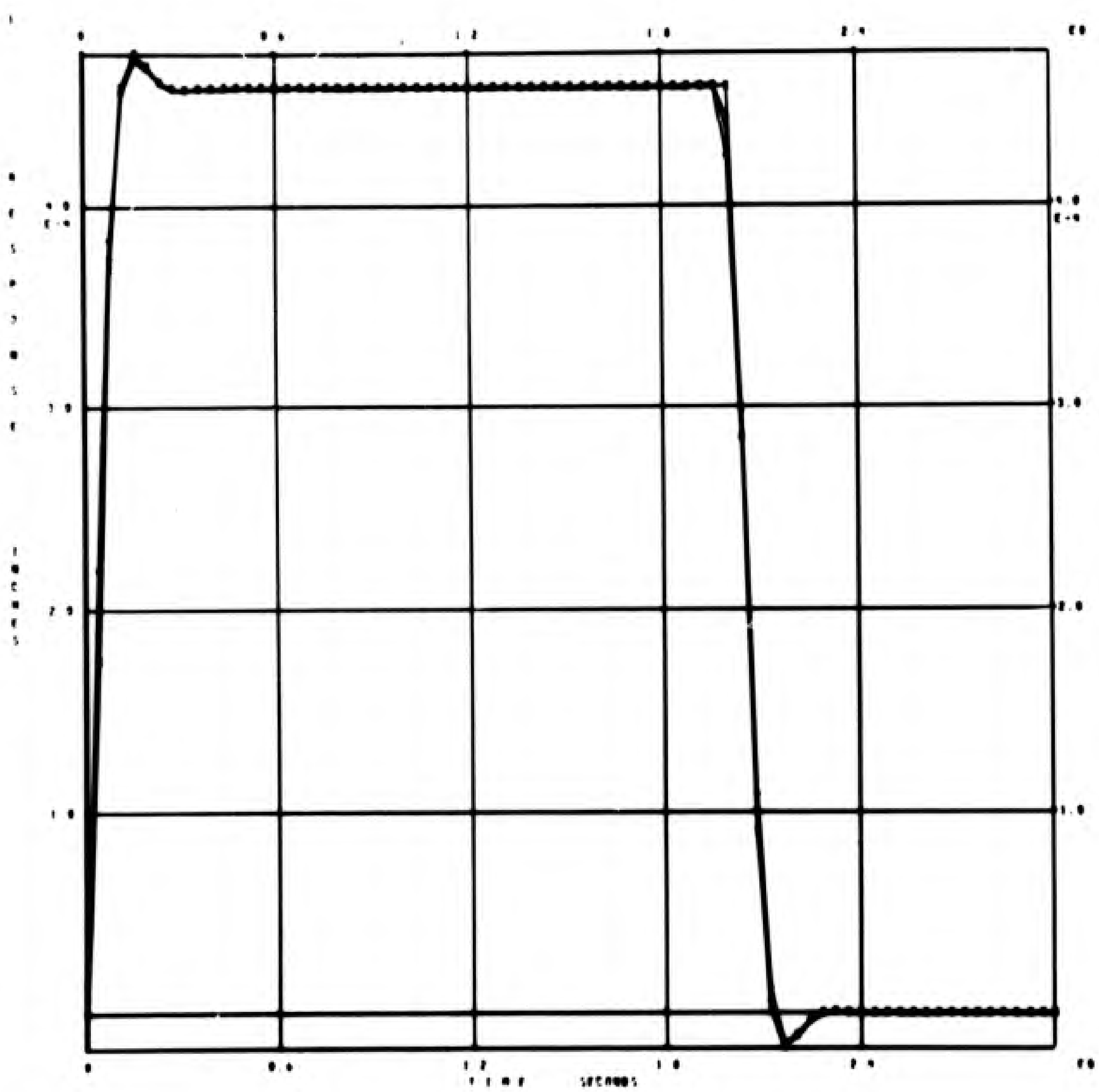

Figure 7 - System of Springs and Masses, Response to a Transient Rectangular Force, Comparison of Laplace Transform and Discrete Fourier Transform Solutions at Mass 1.

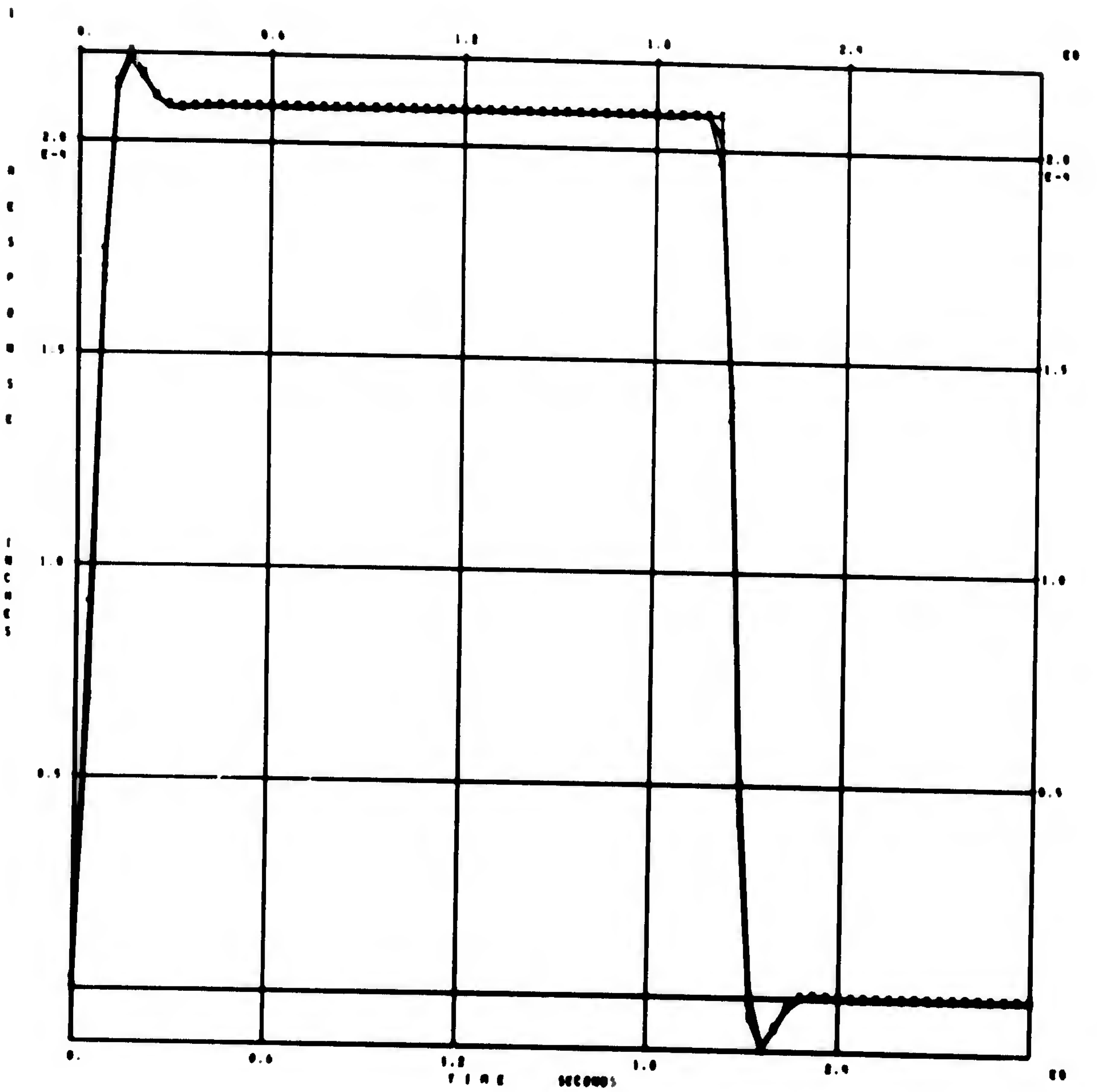

Figure 8 - System of Springs and Masses, Response to a Transient Rectangular Force, Comparison of Laplace Transform and Discrete Fourier Transform Solutions at Mass 2.

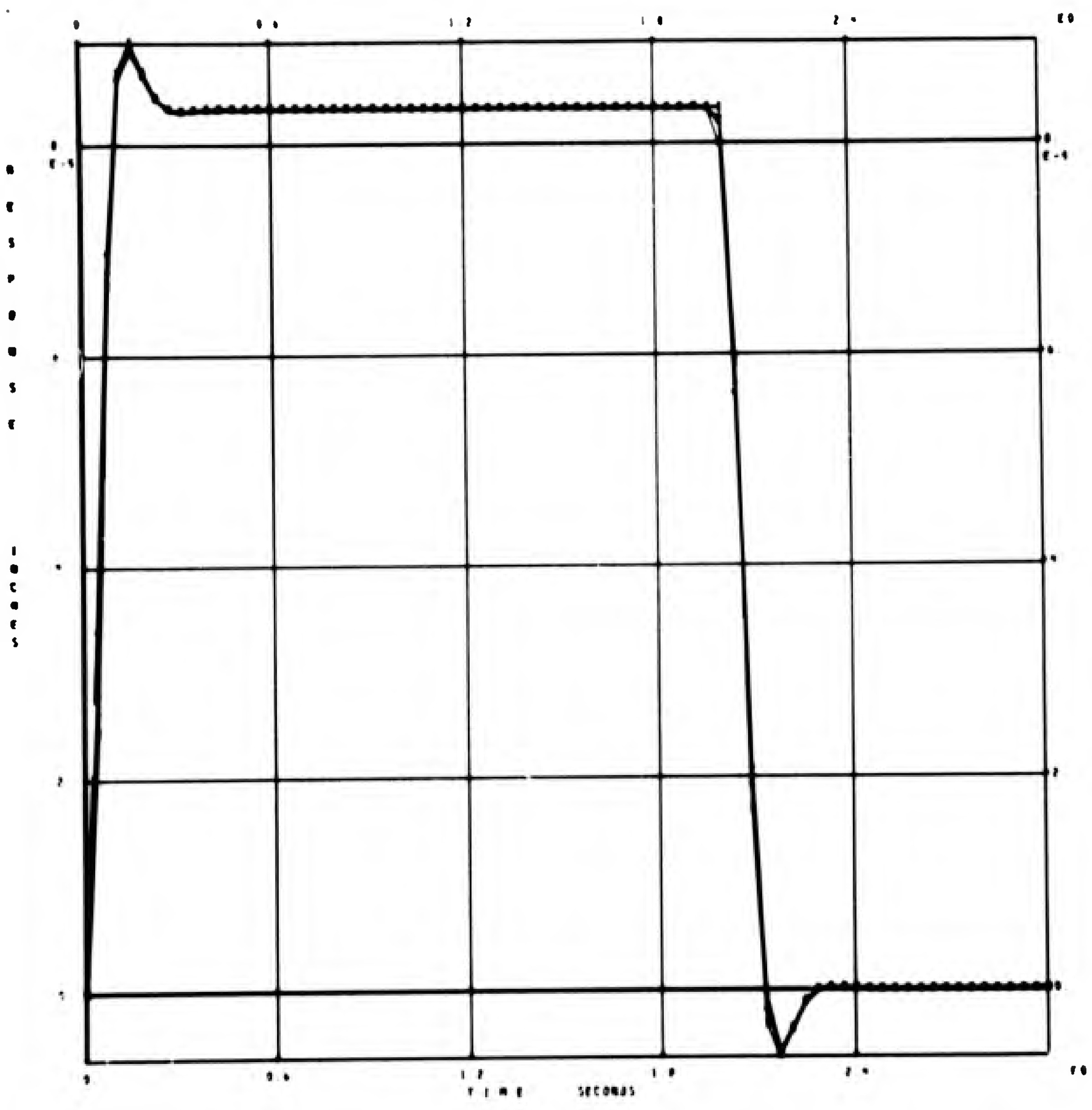

Figure 9 - System of Springs and Masses, Response to a Transient Rectangular Force, Comparison of Laplace Transform and Discrete Fourier Transform Solutions at Mass 3

The graphs in Figures 4, 5, 7, 8, and 9 were obtained through use of the plotting facilities of the NASTRAN[7] program. This operation was performed separately from the computer runs which produced the data sets.

Table 1 records some computer running times for the second problem, run on the CDC-6400.

TABLE 1 - COMPUTING TIMES ASSOCIATED WITH THE SECOND SAMPLE CALCULATION

N	Discrete Transform of Forcing Function by FFT5 (seconds)	Complex Frequency Response by GBRP (seconds)	Discrete Inverse Transform by FFT5 (seconds)
8192	8.89	27.6	8.8
4096	4.10	28.0	4.1
2048	1.91	27.7	1.9

ACKNOWLEDGMENTS

The author wishes to express his indebtedness to the following persons: Dr. Feodor Theilheimer for his consultations and interest in this project; Mr. Ralph B. Johnson, Jr. for allowing the use of his unpublished manuscript; Mr. George Gluck for his modifications to the FFT5 subroutine; Mr. Michael E. Golden for establishing the GBRP overlay via the NASTRAN Linkage Editor; and Dr. Gordon C. Everstine for devising the special use of NASTRAN's plotting facilities to obtain the graphs for this report.

[7] McCormick, C.W., ed., "The NASTRAN User's Manual, NASA SP-222 (01), Washington, D.C. (1973).